

MIE1622H
Computational Finance and Risk Management

Assignment 2
Risk-Based and Robust Portfolio Selection Strategies

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1. Introduction

The objective of this assignment is to compare the performance of the following four computational investment strategies:

1. Buy and Hold
2. Equally Weighted
3. Minimum Variance
4. Maximum Sharpe's Ratio
5. Equal Risk Contribution
6. Leveraged Equal Risk Contribution
7. Robust Mean-Variance Optimization (robust MVO)

The seven investment strategies are implemented to rebalance a portfolio bi-monthly between the trading period of 2019-2020 in a total of 24 months. The portfolio value for each strategy is recorded at the end of each day. The general approach of constructing the portfolio with each strategy is shown below. Detailed implementation of each strategy can be found in the IPython file '*Assignment2_ZhiXinZhu.ipynb*'.

1. Compute the portfolio value based on the current price and current position held for each stock.
2. Compute the re-balance optimal positions based on the strategy chosen
3. Compute the transaction cost required to re-balance the portfolio
4. Calculate the balance of the cash account after rebalancing the portfolio
5. If the balance of the cash account is negative, adjust the optimal position while still maintaining the same weight of the portfolio.

To ensure that we only purchase integer numbers of shares for each stock, the *np.floor()* function is used when we calculate the *x_optimal* given the weight in dollar values. Rounding down the number of shares reduces the possibility of resulting in a negative cash account after each re-balance.

In the situation where a negative cash account occurs after re-balance, we will modify the optimal position for each stock while still maintaining the same portfolio weight distribution. Negative cash suggests that we do not have sufficient funds to purchase/sell stocks and pay transaction fees. To solve this problem we will reduce the number of shares for each stock based on its weight in the portfolio to accommodate the negative fund. The *np.ceil()* function is used to round up the number of shares we need to deduct from the optimal position.

2. Result

Figure 1. Start and End Portfolio Value between 2019 -2020 for all Strategies

Period 1: start date 01/02/2019, end date 02/28/2019
Strategy "Buy and Hold", value begin = \$ 1000070.06, value end = \$ 1121179.83
Strategy "Equally Weighted Portfolio", value begin = \$ 991124.38, value end = \$ 1097031.81
Strategy "Minimum Variance Portfolio", value begin = \$ 991702.16, value end = \$ 1057442.13
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 990119.39, value end = \$ 1016524.41
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 991376.60, value end = \$ 1086492.14
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 987645.11, value end = \$ 1178612.18
Strategy "Robust Optimization Portfolio", value begin = \$ 992347.84, value end = \$ 1074448.71

Period 2: start date 03/01/2019, end date 04/30/2019
Strategy "Buy and Hold", value begin = \$ 1126131.27, value end = \$ 1075001.89
Strategy "Equally Weighted Portfolio", value begin = \$ 1103260.47, value end = \$ 1188731.33
Strategy "Minimum Variance Portfolio", value begin = \$ 1055386.60, value end = \$ 1107919.46
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1007118.16, value end = \$ 1076636.79
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1087320.63, value end = \$ 1154023.42
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1182214.44, value end = \$ 1327325.54
Strategy "Robust Optimization Portfolio", value begin = \$ 1074243.63, value end = \$ 1122702.97

Period 3: start date 05/01/2019, end date 06/28/2019
Strategy "Buy and Hold", value begin = \$ 1070867.54, value end = \$ 969057.81
Strategy "Equally Weighted Portfolio", value begin = \$ 1181234.03, value end = \$ 1169139.09
Strategy "Minimum Variance Portfolio", value begin = \$ 1091866.19, value end = \$ 1099453.77
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1060340.11, value end = \$ 1073413.37
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1144515.21, value end = \$ 1133780.67
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1301571.61, value end = \$ 1276943.20
Strategy "Robust Optimization Portfolio", value begin = \$ 1107722.78, value end = \$ 1108859.95

Period 4: start date 07/01/2019, end date 08/30/2019
Strategy "Buy and Hold", value begin = \$ 976973.31, value end = \$ 933721.61
Strategy "Equally Weighted Portfolio", value begin = \$ 1179634.22, value end = \$ 1149869.96
Strategy "Minimum Variance Portfolio", value begin = \$ 1097286.63, value end = \$ 1129362.38
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1071292.98, value end = \$ 1140352.70
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1134847.84, value end = \$ 1118390.93
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1284597.91, value end = \$ 1247463.84
Strategy "Robust Optimization Portfolio", value begin = \$ 1109245.63, value end = \$ 1110415.42

Period 5: start date 09/03/2019, end date 10/31/2019
Strategy "Buy and Hold", value begin = \$ 922211.42, value end = \$ 1028337.74
Strategy "Equally Weighted Portfolio", value begin = \$ 1138167.02, value end = \$ 1252745.95
Strategy "Minimum Variance Portfolio", value begin = \$ 1115625.47, value end = \$ 1182543.87
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1139167.02, value end = \$ 1246565.34
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1108203.71, value end = \$ 1208437.02
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1219216.90, value end = \$ 1440059.41
Strategy "Robust Optimization Portfolio", value begin = \$ 1100595.14, value end = \$ 1183689.37

Period 6: start date 11/01/2019, end date 12/31/2019
Strategy "Buy and Hold", value begin = \$ 1037933.42, value end = \$ 1099403.03
Strategy "Equally Weighted Portfolio", value begin = \$ 1270461.87, value end = \$ 1373479.86
Strategy "Minimum Variance Portfolio", value begin = \$ 1184550.14, value end = \$ 1255966.80
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1248640.31, value end = \$ 1370079.26
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1222772.82, value end = \$ 1313787.16
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1466441.36, value end = \$ 1685744.18
Strategy "Robust Optimization Portfolio", value begin = \$ 1191507.10, value end = \$ 1260629.28

Period 7: start date 01/02/2020, end date 02/28/2020
Strategy "Buy and Hold", value begin = \$ 1112112.69, value end = \$ 900207.54
Strategy "Equally Weighted Portfolio", value begin = \$ 1396296.22, value end = \$ 1258330.19
Strategy "Minimum Variance Portfolio", value begin = \$ 1256246.44, value end = \$ 1159357.93
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1379472.80, value end = \$ 1284770.75
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1325992.42, value end = \$ 1201583.41
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1722510.84, value end = \$ 1398801.56
Strategy "Robust Optimization Portfolio", value begin = \$ 1269153.63, value end = \$ 1173590.96

Period 8: start date 03/02/2020, end date 04/30/2020
Strategy "Buy and Hold", value begin = \$ 924774.25, value end = \$ 856285.51
Strategy "Equally Weighted Portfolio", value begin = \$ 1312225.31, value end = \$ 1215208.23
Strategy "Minimum Variance Portfolio", value begin = \$ 1209662.55, value end = \$ 1077487.18
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1340490.22, value end = \$ 1417345.41
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1253012.14, value end = \$ 1139513.23
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1528337.56, value end = \$ 1252175.80
Strategy "Robust Optimization Portfolio", value begin = \$ 1226955.48, value end = \$ 1106673.92

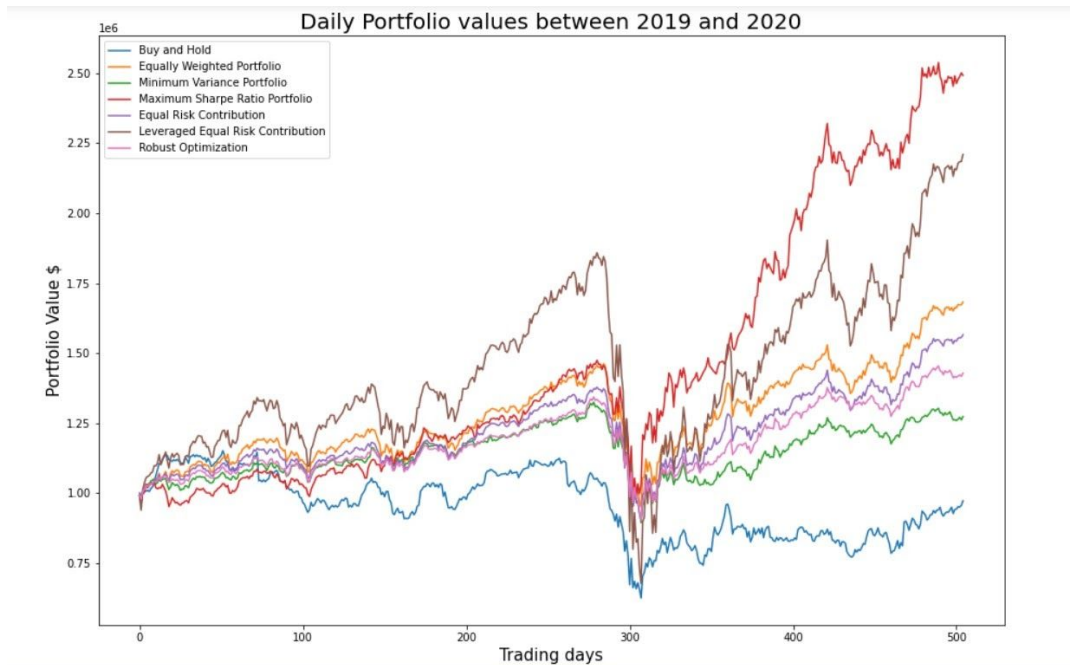
Period 9: start date 05/01/2020, end date 06/30/2020
Strategy "Buy and Hold", value begin = \$ 822532.65, value end = \$ 875128.45
Strategy "Equally Weighted Portfolio", value begin = \$ 1171040.04, value end = \$ 1316082.51
Strategy "Minimum Variance Portfolio", value begin = \$ 1046751.56, value end = \$ 1081069.86
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1357646.73, value end = \$ 1638005.29
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1098684.66, value end = \$ 1223884.68
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1146737.42, value end = \$ 1412621.72
Strategy "Robust Optimization Portfolio", value begin = \$ 1070111.61, value end = \$ 1172806.66

Period 10: start date 07/01/2020, end date 08/31/2020
Strategy "Buy and Hold", value begin = \$ 852159.31, value end = \$ 852474.32
Strategy "Equally Weighted Portfolio", value begin = \$ 1307022.80, value end = \$ 1493983.95
Strategy "Minimum Variance Portfolio", value begin = \$ 1084089.11, value end = \$ 1242113.55
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 1699102.28, value end = \$ 2238611.66
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1218855.60, value end = \$ 1405452.69
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1397132.48, value end = \$ 1825665.01
Strategy "Robust Optimization Portfolio", value begin = \$ 1181312.14, value end = \$ 1344390.82

Period 11: start date 09/01/2020, end date 10/30/2020
Strategy "Buy and Hold", value begin = \$ 857122.42, value end = \$ 795062.75
Strategy "Equally Weighted Portfolio", value begin = \$ 1504676.72, value end = \$ 1407362.52
Strategy "Minimum Variance Portfolio", value begin = \$ 1244312.01, value end = \$ 1193038.04
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 2285965.91, value end = \$ 2163817.00
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1415963.57, value end = \$ 1329487.86
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1843351.89, value end = \$ 1616298.04
Strategy "Robust Optimization Portfolio", value begin = \$ 1350597.39, value end = \$ 1293588.08

Period 12: start date 11/02/2020, end date 12/31/2020
Strategy "Buy and Hold", value begin = \$ 811070.20, value end = \$ 972162.37
Strategy "Equally Weighted Portfolio", value begin = \$ 1419803.51, value end = \$ 1682239.09
Strategy "Minimum Variance Portfolio", value begin = \$ 1203694.61, value end = \$ 1273010.91
Strategy "Maximum Sharpe Ratio Portfolio", value begin = \$ 2161858.95, value end = \$ 2492620.00
Strategy "Equal Risk Contributions Portfolio", value begin = \$ 1332899.84, value end = \$ 1565697.49
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = \$ 1637050.24, value end = \$ 2209842.08
Strategy "Robust Optimization Portfolio", value begin = \$ 1303557.99, value end = \$ 1428424.21

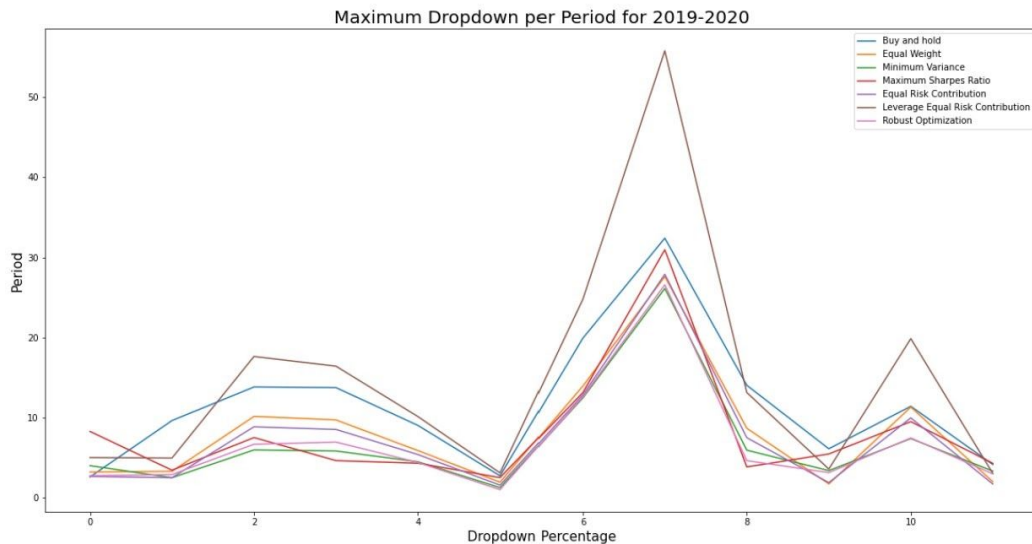
Figure 2. Daily Portfolio Value between 2019-2020



The following discussion is made based on the observation of the result above:

- All seven curves have a similar shape except the fluctuation of the portfolio is different
- In terms of rate of return, the 'Leveraged Equal Risk Contribution' strategy performs best during the period between day 0 to day 280. However, the 'Maximum Sharpe's Ratio' strategy has the better performance during the period of trading days 310 - 500 (late march 2020 till the end of 2020), right after the 2020 Coronavirus Crash. The global market re-entered a bull market shortly after the crash, therefore a more volatile portfolio like the Maximum Sharpe Ratio portfolio will perform extremely well.
- The best strategy following 'Leverage Equal Risk Portfolio' and 'Maximum Sharpe's Ratio' is the 'Equally Weighted' strategy. As the strategy equally distributes the funds to all stocks, its performance demonstrates the general trend of the stock market during the trading period of 2019-2020. Compared to the other strategies, the 'Equally Weighted' strategy has some degree of hedging capability, but its growth heavily relies on the general movement of the market and the industries of the chosen stock.
- The 'Equal Risk Contribution' strategy lies in the middle. This strategy actively seeks risk diversification of the portfolio, it performs well in terms of managing the trade-off between risk and volatility.
- The 'Minimum Variance' strategy is the best choice for risk-averse investors. Although it does not provide the highest rate of return, the 'Minimum Variance' portfolio has the best ability in managing risk. The fluctuation of the minimum variance portfolio is the smallest out of all four strategies. During the Coronavirus Crash, the value of this portfolio dropped the least. In times of Bear Market, the 'Minimum Variance' strategy will have the best performance among all other strategies.
- The 'Robust MVO' strategy performs slightly better than the "Minimum Variance" strategy in terms of return. It is a model that provides an optimal trade-off between expected return and volatility. The expected return of 'Robust MVO' is usually higher than 'Minimum Variance' (if the target return is the optimal return of a Minimum Variance portfolio) and it is less risky than a 'Max Sharpe Ratio' portfolio.
- The 'Buy and Hold' Strategy performs the worst. It has zero capability in managing risk, zero flexibility and its performance solely depends on the initial judgement of the investor. In times of Bull market, the strategy works fine. However, during times of market crisis, this strategy will not give the investor a chance to shift to a less risky position.

Figure 3. Maximum Dropdown per Period between 2019 and 2020



The maximum drawdown(MDD) is a specific measure that looks for the greatest movement from a high point to a low point before a new peak is achieved. It is a useful measure to assess the relative riskiness between strategies. Figure 3, illustrates the MDD for every strategy over 12 periods of time. From figure 3, we can see that even though the ‘Leveraged Equal Risk Contribution’ portfolio yields a high expected return, its MDD is also the highest among all strategies. This suggests that the losses from this portfolio are higher than all other strategies. The ‘Equal Weighted’ strategy has the second-highest MDD. This strategy, not only yields the lowest expected return but also suffered great losses over 2019-2020. All other strategies have similar MDD curves because all these strategies focus on mitigating risk in some way.

2.1 Dynamic Change in Portfolio Allocation

The dynamic change in portfolio allocation graphs illustrates the position or weight of all selected stocks over the trading period between 2019 to 2020. Dynamic change plots are developed for the following strategy: minimum variance, maximum Sharpe ratio, robust MVO. The graphs in figure 4, show the portfolio allocation in terms of portfolio value weight. The x-axis represents the trading period and the y-axis represents the value weight of the stock with respect to the total portfolio value. The graphs in figure 5 show the portfolio allocation in terms of unit share. The x-axis represents the trading period and the y-axis represents the percentage of shares of each stock in terms of the total number of shares. The ‘Minimum Variance’ portfolio holds stocks with low volatility for a long period of time. This strategy performs well in mitigating risk in the early stages of the crash to avoid significant portfolio loss.

Contrarily, the ‘Maximum Sharpe’s Ratio’ portfolio tends to maximize portfolio return by doing short-term buying and selling. From the top right graph of figure 4, we can see the allocation of each stock changes dramatically from period to period. As soon as a stock appears in a declining trend or other stocks perform better, the algorithm immediately sells the stock and purchases other stocks that provide a better rate of return.

The ‘Robust MVO’ trades a lot less than the two strategies above. The portfolio is allocated more evenly over all assets.

Figure 10 consists of four line charts arranged in a 2x2 grid, showing the dynamic change of value weight for different trading rules over 12 trading periods. The top row displays results for the Minimum Variance Strategy, and the bottom row displays results for the Maximum Sharpe Ratio. The left column plots 'Value Weight' (0.0 to 1.0) against 'Trading Periods' (0 to 12). The right column plots 'Population Weight Proportion' (0.0 to 1.0) against 'Trading Periods' (0 to 12). Each chart contains 18 lines representing different trading rules: F, PM, GDOGS, HFO, C, HOS, VZ, AMFL, BM, T, CSO, BAC, BNC, AMO, SMI, INDA, ANCA, HS, and BK. The charts show how the value weight of each rule changes over time for each strategy.

3. Implementation of Strategies for years of 2008 - 2009

The 'Maximum Sharpe Ratio' is slightly modified to accommodate a negative risk premium. If the risk premium is negative, the strategy will not rebalance and remain the same portfolio allocation for the next period.

Figure 6. Start and End Portfolio Value between 2008 -2009 for all Strategies

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Period 1: start date 01/02/2008, end date 02/29/2008
Strategy "Buy and Hold", value begin = $ 789230.94, value end = $ 749509.71
Strategy "Equally Weighted Portfolio", value begin = $ 782163.77, value end = $ 669696.11
Strategy "Minimum Variance Portfolio", value begin = $ 781380.07, value end = $ 666665.15
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 789230.94, value end = $ 749509.71
Strategy "Equal Risk Contributions Portfolio", value begin = $ 782136.81, value end = $ 674054.51
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 778862.15, value end = $ 562065.95
Strategy "Robust Optimization Portfolio", value begin = $ 782111.89, value end = $ 679092.09

Period 2: start date 03/03/2008, end date 04/30/2008
Strategy "Buy and Hold", value begin = $ 754361.26, value end = $ 752687.89
Strategy "Equally Weighted Portfolio", value begin = $ 659796.35, value end = $ 726864.96
Strategy "Minimum Variance Portfolio", value begin = $ 657682.64, value end = $ 745012.05
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 754361.26, value end = $ 752687.89
Strategy "Equal Risk Contributions Portfolio", value begin = $ 664232.77, value end = $ 739641.44
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 538812.49, value end = $ 661579.35
Strategy "Robust Optimization Portfolio", value begin = $ 670930.14, value end = $ 740531.39

Period 3: start date 05/01/2008, end date 06/30/2008
Strategy "Buy and Hold", value begin = $ 779329.50, value end = $ 663602.44
Strategy "Equally Weighted Portfolio", value begin = $ 750646.94, value end = $ 632893.40
Strategy "Minimum Variance Portfolio", value begin = $ 760108.57, value end = $ 716195.31
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 779329.50, value end = $ 663602.44
Strategy "Equal Risk Contributions Portfolio", value begin = $ 761706.27, value end = $ 660885.29
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 695129.04, value end = $ 510202.43
Strategy "Robust Optimization Portfolio", value begin = $ 757883.41, value end = $ 695712.49

Period 4: start date 07/01/2008, end date 08/29/2008
Strategy "Buy and Hold", value begin = $ 674748.24, value end = $ 619979.82
Strategy "Equally Weighted Portfolio", value begin = $ 633824.42, value end = $ 647087.17
Strategy "Minimum Variance Portfolio", value begin = $ 716912.15, value end = $ 725969.60
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 674748.24, value end = $ 619979.82
Strategy "Equal Risk Contributions Portfolio", value begin = $ 660148.64, value end = $ 668556.65
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 505798.42, value end = $ 518791.49
Strategy "Robust Optimization Portfolio", value begin = $ 692490.29, value end = $ 695280.54

Period 5: start date 09/02/2008, end date 10/31/2008
Strategy "Buy and Hold", value begin = $ 621151.79, value end = $ 579282.75
Strategy "Equally Weighted Portfolio", value begin = $ 648678.95, value end = $ 474129.85
Strategy "Minimum Variance Portfolio", value begin = $ 709849.52, value end = $ 548171.04
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 621151.79, value end = $ 579282.75
Strategy "Equal Risk Contributions Portfolio", value begin = $ 668066.27, value end = $ 496683.96
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 515904.03, value end = $ 249378.61
Strategy "Robust Optimization Portfolio", value begin = $ 685633.90, value end = $ 529463.06

Period 6: start date 11/03/2008, end date 12/31/2008
Strategy "Buy and Hold", value begin = $ 576738.59, value end = $ 500698.25
Strategy "Equally Weighted Portfolio", value begin = $ 471245.36, value end = $ 407527.29
Strategy "Minimum Variance Portfolio", value begin = $ 548930.00, value end = $ 505662.59
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 576738.59, value end = $ 500698.25
Strategy "Equal Risk Contributions Portfolio", value begin = $ 494962.17, value end = $ 435919.10
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 244096.04, value end = $ 185166.33
Strategy "Robust Optimization Portfolio", value begin = $ 531144.23, value end = $ 493075.76

Period 7: start date 01/02/2009, end date 02/27/2009
Strategy "Buy and Hold", value begin = $ 505855.81, value end = $ 414408.26
Strategy "Equally Weighted Portfolio", value begin = $ 424896.34, value end = $ 348178.16
Strategy "Minimum Variance Portfolio", value begin = $ 522070.15, value end = $ 497626.02
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 505855.81, value end = $ 414408.26
Strategy "Equal Risk Contributions Portfolio", value begin = $ 453983.97, value end = $ 388305.46
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 202134.35, value end = $ 143000.80
Strategy "Robust Optimization Portfolio", value begin = $ 509595.14, value end = $ 463425.20

Period 8: start date 03/02/2009, end date 04/30/2009
Strategy "Buy and Hold", value begin = $ 400004.61, value end = $ 475987.18
Strategy "Equally Weighted Portfolio", value begin = $ 331313.10, value end = $ 532702.15
Strategy "Minimum Variance Portfolio", value begin = $ 478003.54, value end = $ 650588.98
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 400004.61, value end = $ 475987.18
Strategy "Equal Risk Contributions Portfolio", value begin = $ 372266.69, value end = $ 555674.27
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 127596.01, value end = $ 255335.80
Strategy "Robust Optimization Portfolio", value begin = $ 446877.89, value end = $ 608504.11

Period 9: start date 05/01/2009, end date 06/30/2009
Strategy "Buy and Hold", value begin = $ 483627.06, value end = $ 538125.39
Strategy "Equally Weighted Portfolio", value begin = $ 531813.99, value end = $ 559778.43
Strategy "Minimum Variance Portfolio", value begin = $ 645542.43, value end = $ 652503.18
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 478815.13, value end = $ 494173.15
Strategy "Equal Risk Contributions Portfolio", value begin = $ 554696.70, value end = $ 576025.76
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 253692.49, value end = $ 273436.01
Strategy "Robust Optimization Portfolio", value begin = $ 603932.00, value end = $ 612413.10

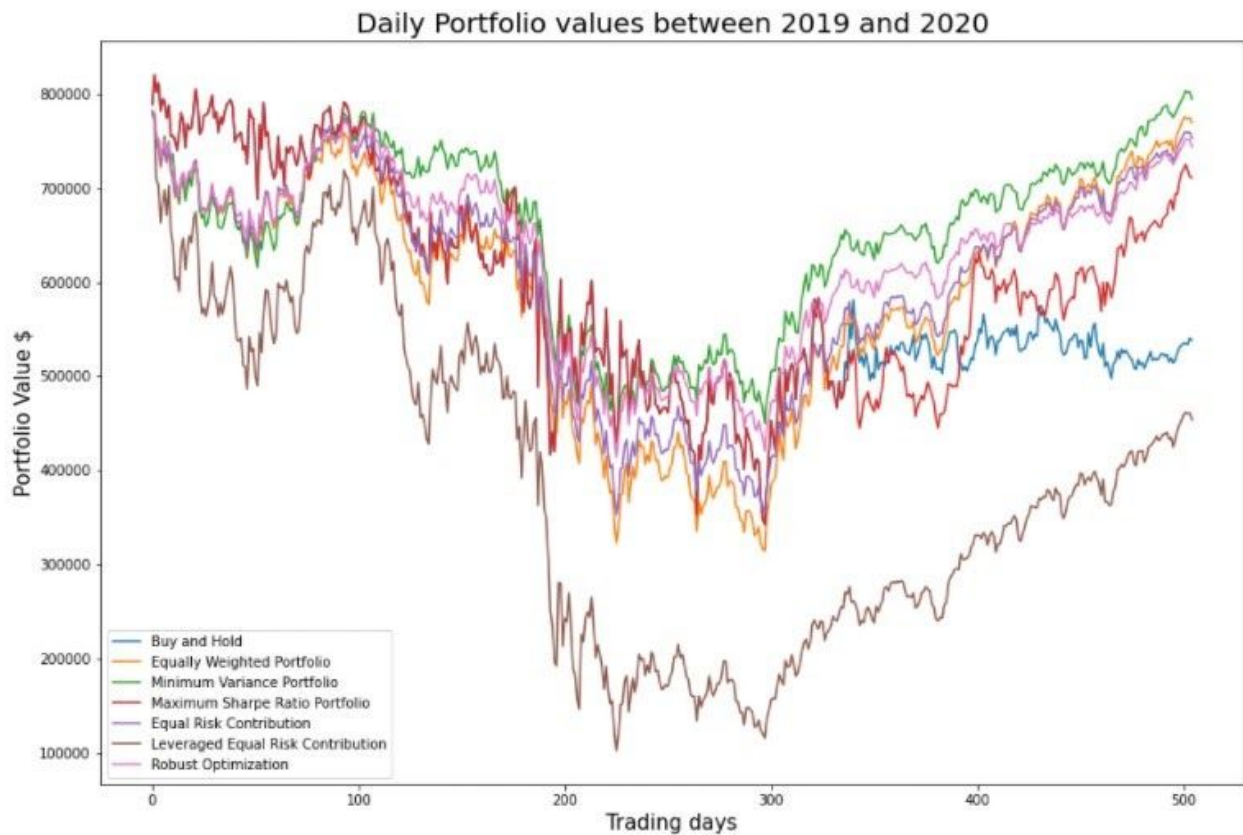
Period 10: start date 07/01/2009, end date 08/31/2009
Strategy "Buy and Hold", value begin = $ 528549.59, value end = $ 554215.70
Strategy "Equally Weighted Portfolio", value begin = $ 559139.95, value end = $ 659133.18
Strategy "Minimum Variance Portfolio", value begin = $ 651882.92, value end = $ 695331.93
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 487028.26, value end = $ 600898.24
Strategy "Equal Risk Contributions Portfolio", value begin = $ 576022.85, value end = $ 654441.72
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 272371.20, value end = $ 347344.62
Strategy "Robust Optimization Portfolio", value begin = $ 612210.93, value end = $ 654901.89

Period 11: start date 09/01/2009, end date 10/30/2009
Strategy "Buy and Hold", value begin = $ 529171.54, value end = $ 510275.57
Strategy "Equally Weighted Portfolio", value begin = $ 635416.56, value end = $ 672535.95
Strategy "Minimum Variance Portfolio", value begin = $ 678713.81, value end = $ 712963.86
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 575143.97, value end = $ 575792.16
Strategy "Equal Risk Contributions Portfolio", value begin = $ 634724.20, value end = $ 672786.71
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 327283.14, value end = $ 367011.24
Strategy "Robust Optimization Portfolio", value begin = $ 639730.34, value end = $ 669055.94

Period 12: start date 11/02/2009, end date 12/31/2009
Strategy "Buy and Hold", value begin = $ 515205.91, value end = $ 538768.59
Strategy "Equally Weighted Portfolio", value begin = $ 675305.57, value end = $ 769734.96
Strategy "Minimum Variance Portfolio", value begin = $ 707686.74, value end = $ 794707.90
Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 600073.67, value end = $ 711049.33

Strategy "Equal Risk Contributions Portfolio", value begin = $ 674094.30, value end = $ 753224.16
Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 366984.57, value end = $ 454275.27
Strategy "Robust Optimization Portfolio", value begin = $ 665308.81, value end = $ 743891.51
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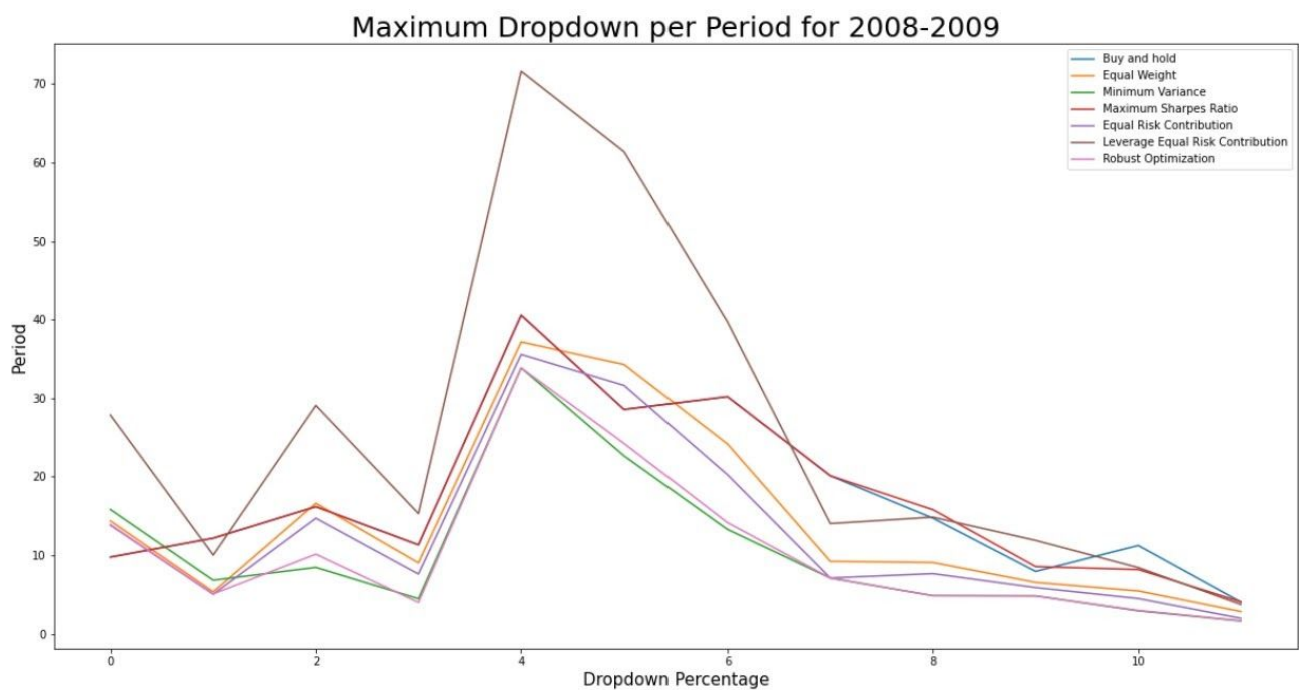
Figure 7. Daily Portfolio Value between 2008-2009



The following discussion is made based on the observation of the result above:

- ‘Leveraged Equal Risk Contribution’ portfolio performed the worst. During the financial crisis, all stocks are dropping drastically. A strategy like LERC where we borrow money to buy stock will yield great losses.
- Out of all performance, minimum variance suffers the least losses because it allocates all money to stocks that are least volatile.

Figure 8. Maximum Dropdown per Period between 2008 and 2009



3.1 Dynamic Change of Portfolio

The Maximum Sharpes Ratio portfolio trades drastically in period 9. Comparing to the other two strategies, the ‘Robust MVO’ strategy reduces trading. Its holdings do not change a lot every period.

Figure 9 Dynamic Change of Portfolio Value Weight for 2008-2009

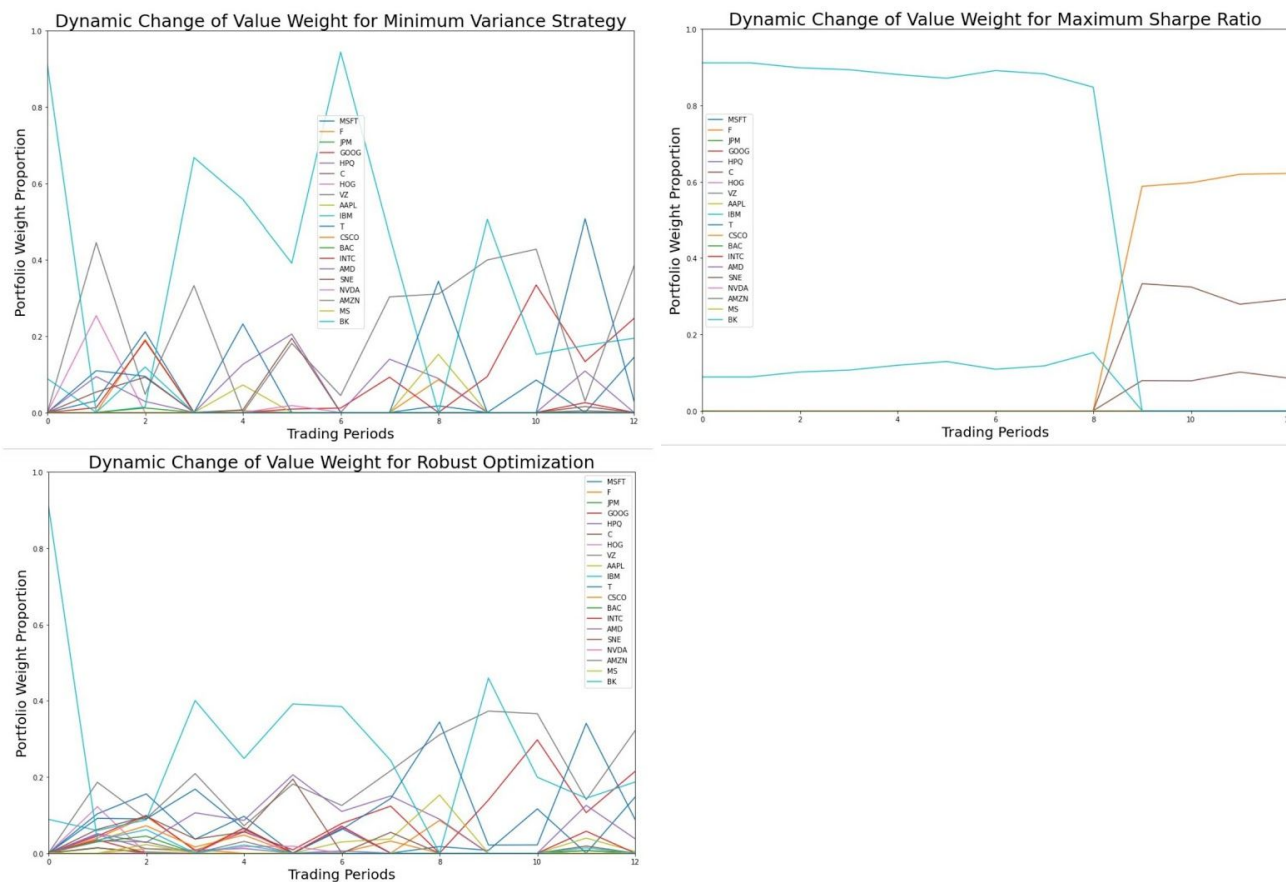
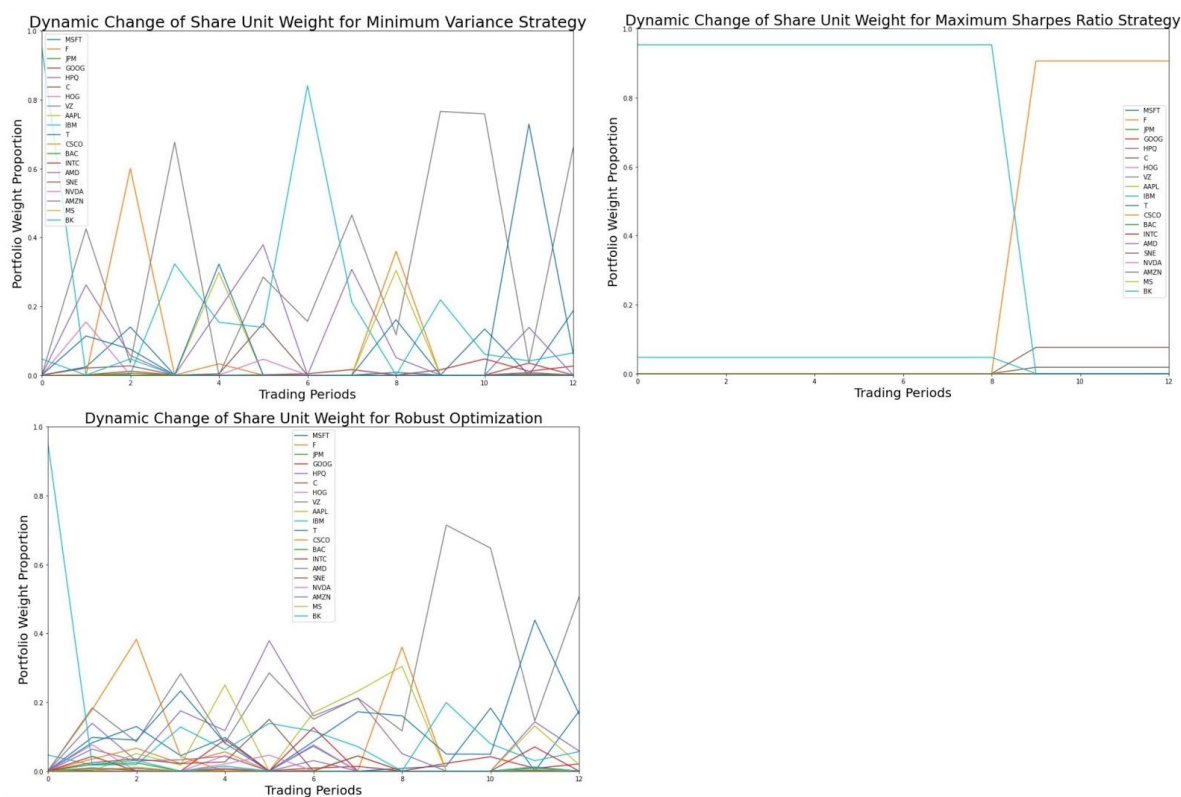


Figure 10. Dynamic Change of Share Unit for 2019-2020



In general, all strategies perform poorly during 2008-2009. The strategy that suffers the least is the 'Minimum Variance' strategy. The best strategies in 2019- 2020, 'Max Sharpes' and 'LERC', are the worst strategy in 2008 - 2009. The reason for that is, in the period between 2019-2020, the crisis did not last a long time, the market enters Bull market shortly after the crisis, therefore volatile strategies work well. The crisis in 2008-2009 last a long time. Conservative strategies like 'Minimum Variance' suffer the least in crisis. Volatile strategies like the LERC and Maximum Sharpes Ratio suffer the most in turbulent markets.

In 2008 - 2009, I would definitely choose the minimum variance strategy. In turbulent times, strategies that try to mitigate risk are the safest strategy.

