MIE1622H
Computational Finance and Risk Management

Assignment 1
Mean-Variance Portfolio Selection Strategies

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1.0 Introduction

The objective of this assignment is to compare the performance of the following four computational investment strategies:
1. Buy and Hold
2. Equally Weighted
3. Minimum Variance
4. Maximum Sharpe's Ratio

The four investment strategies are implemented to rebalance a portfolio bi-monthly between the trading period of 2019-2020 in total 24 months. The portfolio value for each strategy is recorded at the end of each day. The general approach of constructing the portfolio with each strategy is shown below. Detailed implementation of each strategy can be found in the IPython file 'Assignment1_ZhiXinZhu.ipynb'.

1. Compute the portfolio value based on the current price and current position held for each stock.
2. Compute the re-balance optimal positions based on the strategy chosen
3. Compute the transaction cost required to re-balance the portfolio
4. Calculate the balance of the cash account after rebalancing the portfolio
5. If the balance of the cash account is negative, adjust the optimal position while still maintaining the same weight of the portfolio.

To ensure that we only purchase integer numbers of shares for each stock, the np.floor() function is used when we calculate the x_optimal given the weight in dollar values. Rounding down the number of shares reduces the possibility of resulting in a negative cash account after each re-balance.

In the situation where a negative cash account occurs after re-balance, we will modify the optimal position for each stock while still maintaining the same portfolio weight distribution. Negative cash suggests that we do not have sufficient funds to purchase/sell stocks and pay transaction fees. To solve this problem we will reduce the number of shares for each stock based on its weight in the portfolio to accommodate the negative fund. The np.ceil() function is used to round up the number of shares we need to deduct from the optimal position.

2.0 Result

The starting and ending values of the portfolio for each strategy throughout the trading period between 2019-2020 are presented in figure.1 and the daily portfolio values over 2019-2020 for each strategy are presented in figure.2.

Figure 1. Start and End Portfolio Value between 2019 -2020 for all Strategies

Period 1: start date 01/02/2019, end date 02/28/2019
  Strategy "Buy and Hold", value begin = $ 1000070.06, value end = $ 1121179.83
  Strategy "Equally Weighted Portfolio", value begin = $ 991124.38, value end = $ 1097031.81
  Strategy "Mininum Variance Portfolio", value begin = $ 991702.16, value end = $ 1057442.13
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990119.39, value end = $ 1016524.41

Period 2: start date 03/01/2019, end date 04/30/2019
  Strategy "Buy and Hold", value begin = $ 1126131.27, value end = $ 1075001.89
  Strategy "Equally Weighted Portfolio", value begin = $ 1103260.47, value end = $ 1188731.33
  Strategy "Mininum Variance Portfolio", value begin = $ 1055386.60, value end = $ 1107919.46
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1007118.16, value end = $ 1076636.79

Period 3: start date 05/01/2019, end date 06/28/2019
  Strategy "Buy and Hold", value begin = $ 1070867.54, value end = $ 969057.81
  Strategy "Equally Weighted Portfolio", value begin = $ 1181234.03, value end = $ 1169139.09
  Strategy "Mininum Variance Portfolio", value begin = $ 1091866.19, value end = $ 1099453.77
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1060340.11, value end = $ 1073413.37

Period 4: start date 07/01/2019, end date 08/30/2019
  Strategy "Buy and Hold", value begin = $ 976973.31, value end = $ 933721.61
  Strategy "Equally Weighted Portfolio", value begin = $ 1179634.22, value end = $ 1149869.96
  Strategy "Mininum Variance Portfolio", value begin = $ 1097286.63, value end = $ 1129362.38
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1071292.98, value end = $ 1140352.70

Period 5: start date 09/03/2019, end date 10/31/2019
  Strategy "Buy and Hold", value begin = $ 922211.42, value end = $ 1028337.74
  Strategy "Equally Weighted Portfolio", value begin = $ 1138167.02, value end = $ 1252745.95
  Strategy "Mininum Variance Portfolio", value begin = $ 1115625.47, value end = $ 1182543.87
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1139167.02, value end = $ 1246565.34

Period 6: start date 11/01/2019, end date 12/31/2019
  Strategy "Buy and Hold", value begin = $ 1037933.42, value end = $ 1099403.03
  Strategy "Equally Weighted Portfolio", value begin = $ 1270461.87, value end = $ 1373479.86
  Strategy "Mininum Variance Portfolio", value begin = $ 1184550.14, value end = $ 1255966.80
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1248640.31, value end = $ 1370079.26

Period 7: start date 01/02/2020, end date 02/28/2020
  Strategy "Buy and Hold", value begin = $ 1112112.69, value end = $ 900207.54
  Strategy "Equally Weighted Portfolio", value begin = $ 1396296.22, value end = $ 1258330.19
  Strategy "Mininum Variance Portfolio", value begin = $ 1256246.44, value end = $ 1159357.93
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1379472.80, value end = $ 1284770.75

Period 8: start date 03/02/2020, end date 04/30/2020
  Strategy "Buy and Hold", value begin = $ 924774.25, value end = $ 856285.51
  Strategy "Equally Weighted Portfolio", value begin = $ 1312225.31, value end = $ 1215208.23
  Strategy "Mininum Variance Portfolio", value begin = $ 1209662.55, value end = $ 1077487.18
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1340490.22, value end = $ 1417345.41

Period 9: start date 05/01/2020, end date 06/30/2020
  Strategy "Buy and Hold", value begin = $ 822532.65, value end = $ 875128.45
  Strategy "Equally Weighted Portfolio", value begin = $ 1171040.04, value end = $ 1316082.51
  Strategy "Mininum Variance Portfolio", value begin = $ 1046751.56, value end = $ 1081069.86
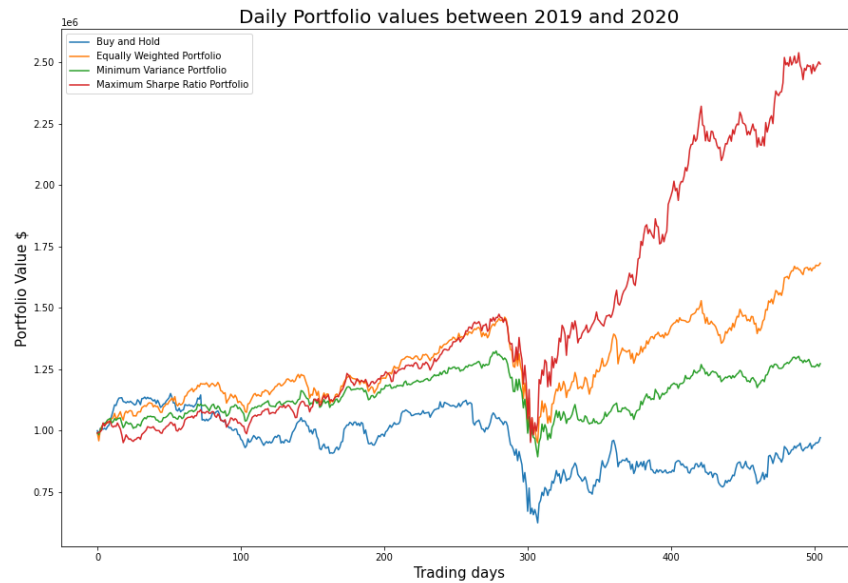 Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1357646.73, value end = $ 1638005.29

Period 10: start date 07/01/2020, end date 08/31/2020
  Strategy "Buy and Hold", value begin = $ 852159.31, value end = $ 852474.32
  Strategy "Equally Weighted Portfolio", value begin = $ 1307022.80, value end = $ 1493983.95
  Strategy "Mininum Variance Portfolio", value begin = $ 1084089.11, value end = $ 1242113.55
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1699102.28, value end = $ 2238611.66

Period 11: start date 09/01/2020, end date 10/30/2020
  Strategy "Buy and Hold", value begin = $ 857122.42, value end = $ 795062.75
  Strategy "Equally Weighted Portfolio", value begin = $ 1504676.72, value end = $ 1407362.52
  Strategy "Mininum Variance Portfolio", value begin = $ 1244312.01, value end = $ 1193038.04
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2285965.91, value end = $ 2163817.00

Period 12: start date 11/02/2020, end date 12/31/2020
  Strategy "Buy and Hold", value begin = $ 811070.20, value end = $ 972162.37
  Strategy "Equally Weighted Portfolio", value begin = $ 1419803.51, value end = $ 1682239.09
  Strategy "Mininum Variance Portfolio", value begin = $ 1203694.61, value end = $ 1273010.91
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2161858.95, value end = $ 2492620.00

Figure 2. Daily Portfolio Value between 2019-2020

The following discussion are made based on the observation of the result above:

- All four curves have a similar shape except the fluctuation of the portfolio is different
- In terms of rate of return, the 'Maximum Sharpe's Ratio' strategy has the best performance, especially during the period of trading days 310 - 500 (late march 2020 till the end of 2020), right after the 2020 Coronavirus Crash. The global market re-entered a bull market shortly after the crash, therefore a more volatile portfolio like the Maximum Sharpe Ratio portfolio will perform extremely well.
- The second best strategy in this case is the 'Equally Weighted' strategy. As the strategy equally distributes the funds to all stocks, its performance demonstrates the general trend of the stock market during the trading period of 2019-2020. Compared to the other strategies, the 'Equally Weighted' strategy has some degree of hedging capability, but its growth heavily relies on the general movement of the market and the industries of the chosen stock.
- The 'Minimum Variance' strategy is the best choice for risk averse investors. Although it does not provide the highest rate of return, the 'Minimum Variance' portfolio has the best ability in managing risk. The fluctuation of the minimum variance portfolio is the smallest out of all four strategies. During the Coronavirus Crash, the value of this portfolio dropped the least. In times of Bear Market, the 'Minimum Variance' strategy will have the best performance among all other strategies.
- The 'Buy and Hold' Strategy performs the worst. It has zero capability in managing risk, zero flexibility and its performance solely depends on the initial judgement of the investor. In times of Bull market, the strategy works fine. However, during times of market crisis, this strategy will not give the investor a chance to shift to a less risky position.

If I want to construct my portfolio, I will choose the 'Maximum Sharpe's Ratio' or 'Minimum Variance Portfolio depending on the trend of the market. In the time of market downturn, a Minimum Variance portfolio will be least affected by the crash. As the market starts to revive, the Maximum Sharpe's Ratio portfolio will provide the highest return.

2.1 Dynamic Change in Portfolio Allocation

The dynamic change in portfolio allocation graphs illustrates the position of all selected stocks over the trading period between 2019 to 2020. The x-axis represents the trading period and the y-axis represents the value weight of the stock with respect to the total portfolio value. For this assignment we have constructed the dynamic change in portfolio allocation graph for the 'Minimum Variance'(figure 3)and 'Maximum Sharpe's Ratio' strategy(figure 4). The 'Minimum Variance' portfolio holds stocks with low volatility for a long period of time. In the graph, we can see that starting from period 1, the portfolio allocates more than 30% of the funds to stock 'VZ'. We can see in figure 5, that the price of stock 'VZ' has a fairly stable price throughout the two years. It only dropped around 20% during the Coronavirus Crash in 2020. We can also see that its weight decreases when its price dropped drastically during the crash and increases when the stock is reviving. This strategy performs well in mitigating risk in early stages of crash to avoid significant portfolio loss. Contrarily, the 'Maximum Sharpe's Ratio' portfolio tends to maximize portfolio return by doing short-term buying and selling. From figure 4, we can see the allocation of each stock changes dramatically from period to period. As soon as a stock appears in a declining trend or other stocks perform better, the algorithm immediately sells the stock and purchases other stocks that provide a better rate of return.

Figure 3. Dynamic Change of Portfolio Allocation for Minimum Variance Strategy
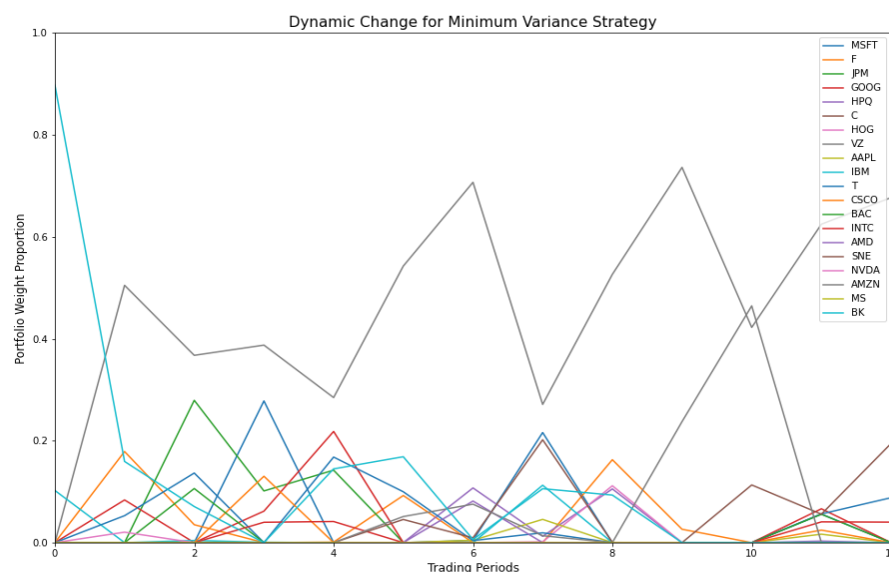
Figure 4. Dynamic Change of Portfolio Allocation for Maximum Sharpe's Ratio Strategy
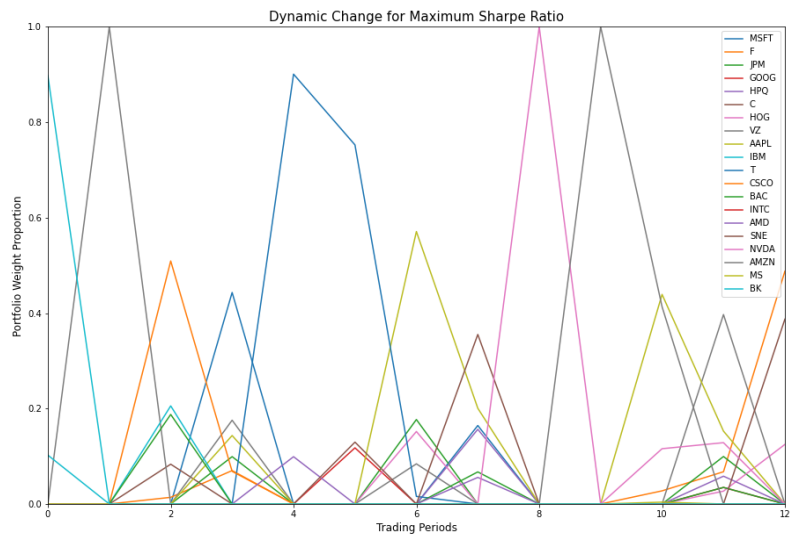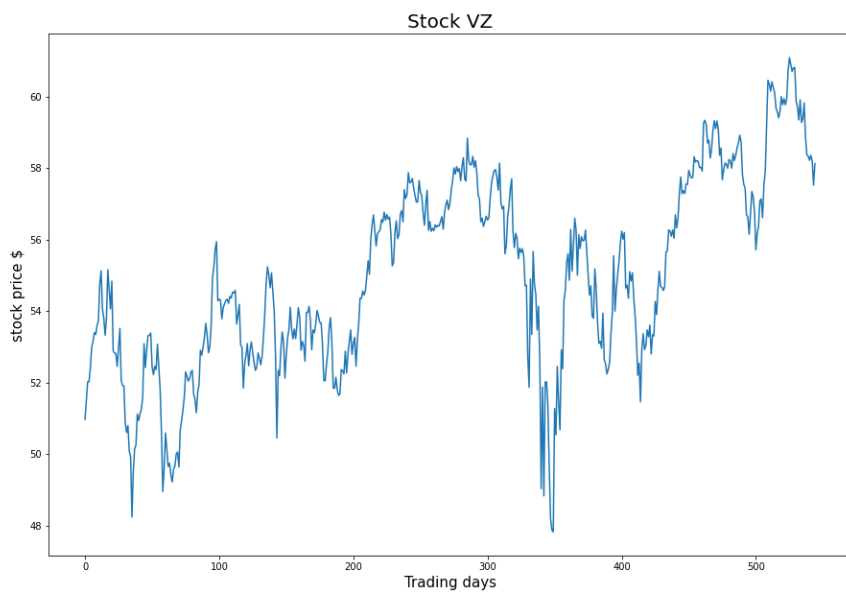

Figure 5. Prices of Stock VZ between 2019-2020

3.0 Variation of Strategies

We have implemented a variation of strategies where we hold the portfolio position after rebalancing on period 1 till the end of the trading period 12. The implementation code can be found in the IPython 'Assignment1_ZhiXinZhu_Variation.ipynb'.

Figure 6. Final Output of Variation of Strategies

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Period 11: start date 09/01/2020, end date 10/30/2020
   Strategy "Buy and Hold", value begin = $ 857122.42, value end = $ 795062.75
   Strategy "Equally Weighted Portfolio", value begin = $ 1676356.15, value end = $ 1520312.59
   Strategy "Mininum Variance Portfolio", value begin = $ 1137258.10, value end = $ 1119964.76
   Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1125495.65, value end = $ 1095756.21

Period 12: start date 11/02/2020, end date 12/31/2020
   Strategy "Buy and Hold", value begin = $ 811070.20, value end = $ 972162.37
   Strategy "Equally Weighted Portfolio", value begin = $ 1528907.70, value end = $ 1788414.66
   Strategy "Mininum Variance Portfolio", value begin = $ 1130354.26, value end = $ 1207975.83
   Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1109407.51, value end = $ 1129595.98
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Figure 7.Daily Portfolio Value between 2019-2020 for Variation of Strategies
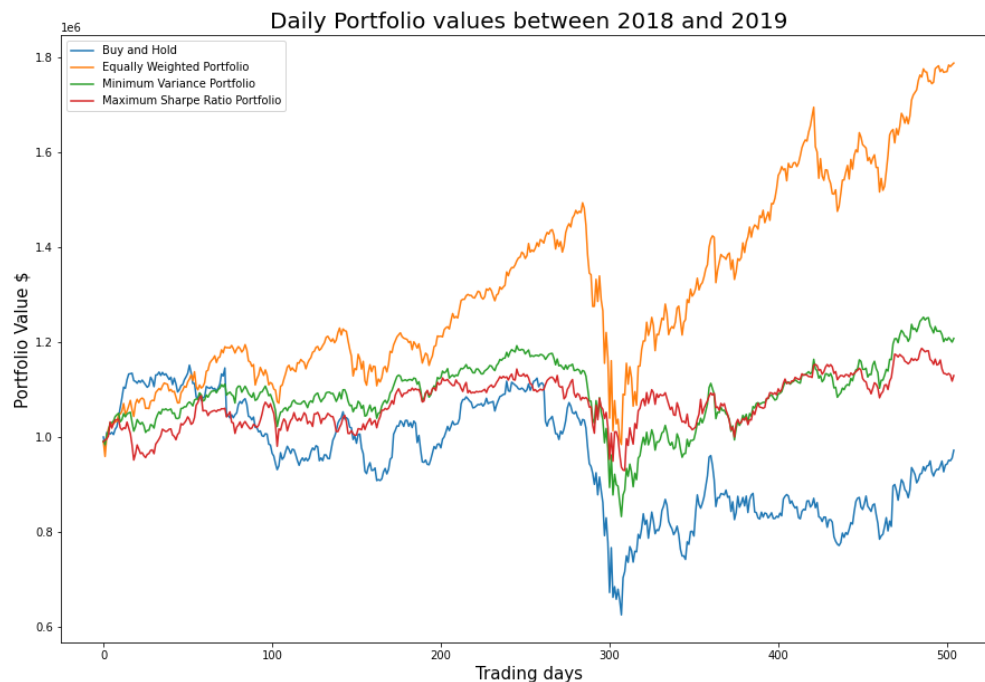
The following discussion are made based on the observation of the result above:
- 'Equally Weighted' strategy results in the highest portfolio value at the end of the trading period.
- 'Buy and Hold' strategy remains the worst performance out of all strategy
- 'Maximum Sharpe's Ratio' strategy performs a lot worse than the original strategy. This is reasonable because the strategy tends to do short buying and selling, holding the same portfolio weight after period contradicts with its concept.
- The performance of the 'twisted Minimum Variance' strategy is slightly lower than the original algorithm.

3.1 Improvement on the Variation of Strategies

The variation implemented above is just a simple twist on the original algorithm. It is definitely not the best change that we can make. Possible improvements that can be done on the algorithm are listed below:

- The length of re-balance period can be shorter for example 2 weeks. The stock market changes everyday. Prices of stock can have a high variance over a period of two months. It is essential for the strategy to react to the change as soon as possible.

- Combining the 'Maximum Sharpe's Ratio' and 'Minimum Variance' strategy may further maximize the rate of return of the portfolio. In the time of market downturn, a Minimum Variance portfolio will be least affected by the crash. As the market starts to revive, the Maximum Sharpe's Ratio portfolio will provide the highest return.