MIE1622H
Computational Finance and Risk Management

Assignment 4
Asset Pricing

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1. Introduction

The purpose of this assignment is to implement three pricing functions:

1) Black-Scholes pricing formula for European option in the function BS_european_price.
2) Monte Carlo pricing procedure for European option in the function MC_european_price.
3) Monte Carlo pricing procedure for Barrier knock-in option in the function MC_barrier_knockin_price.

For the Monte Carlo pricing procedure, implement a 1-step and a multi-step simulation. The number of steps chosen is to be 12 (1 year). The number of scenarios is 1,000,000. Other parameters are: S_0 = 100, K = 105, T =1.0, risk-free rate = 0.05, μ = 0.05, σ = 0.2.

2. Result and Evaluation

2.1 Result

The result is shown below:

```
Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455

One-step MC price of an European call option is 8.011202446389978
One-step MC price of an European put option is 7.90852037203274

Multi-step MC price of an European call option is 8.022019838500855
Multi-step MC price of an European put option is 7.903487060061184

One-step MC price of an Barrier call option is 7.807368030859526
One-step MC price of an Barrier put option is 0.0

Multi-step MC price of an Barrier call option is 7.950171460146708
Multi-step MC price of an Barrier put option is 1.2740136174115408
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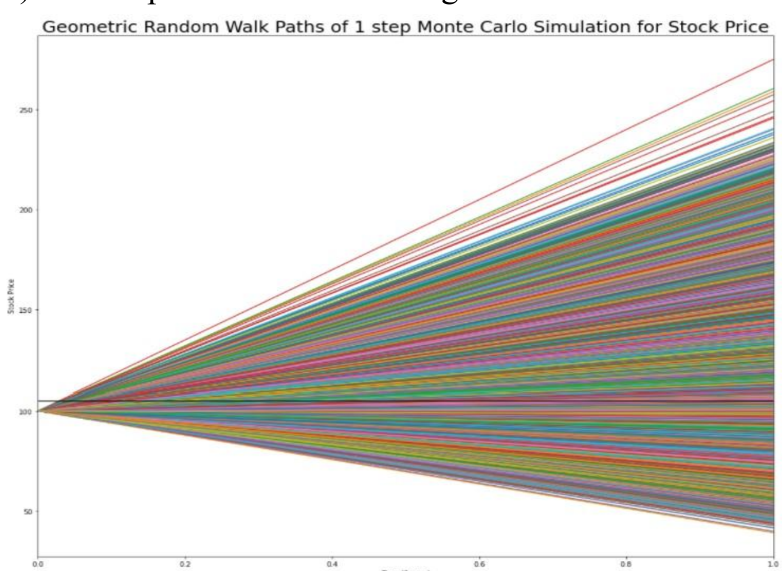
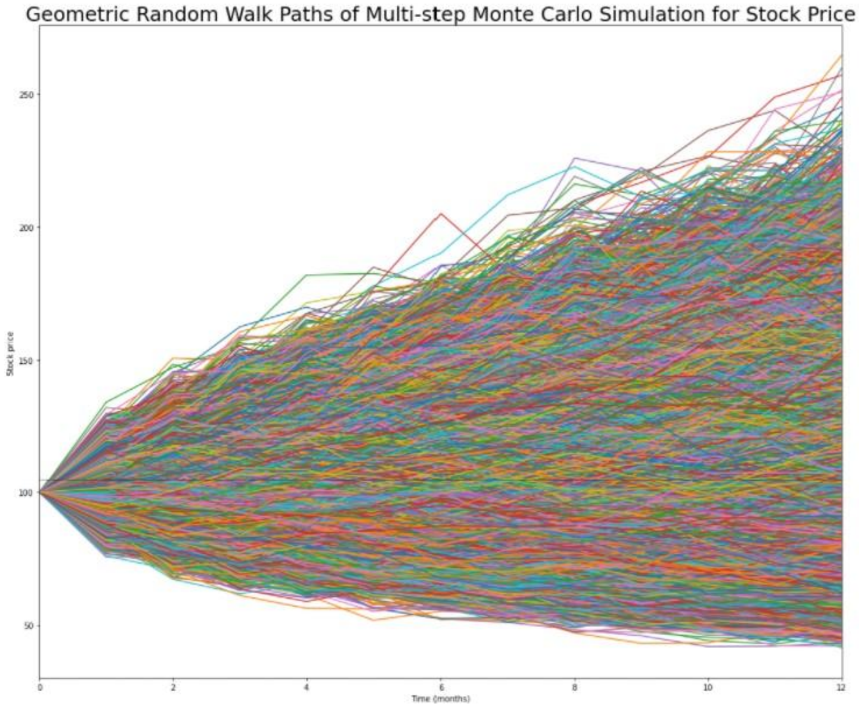
2.2 Monte Carlo Pricing Procedure Plot

1) One-step Monte Carlo Pricing



2) Multi-step Monte Carlo Pricing



2.3 European Option Pricing Strategy Comparison

The Black-Scholes model is based on fixed inputs and performs well on the calculation of standard European options. However, it lacks the flexibility to add in non-standard features like barrier options. On the other hand, the Geometric Random Walk Model uses constant drift and volatility of the pricing model. This model incorporates time-steps thus allowing more flexibility.

In our models, the results of one-step and multi-step Monte Carlo simulation are very close to the result of the closed-form of the Black-Scholes equation. The difference between one-step and multi-step simulation is so minor (0.005- 0.01) that there is no observable impact on the accuracy of the Monte Carlo simulation. However, the number of scenarios/paths plays a significant role. As the number of scenarios increases, the one-step Monte Carlo estimation will approach the result of the closed-form Black-Scholes.

2.4 Difference between European and Barrier Option

The call price and put price of a European option are very similar. However, for a barrier option, the put price is much lower than the call price. This is an up-and-in barrier option, an option that only activates when the stock price reaches the barrier price of $110. In the case of a put option, the option will only profit when the price at expiry is below the strike price of $105. It can be seen in Figure 1 and Figure 2, options that crossed the barrier mostly end up with a price higher or close to the strike price, therefore a put option with barrier results in yields close to or equal to zero. For a call option, the higher the final price, the higher the yield of the option. A call option will yield profit as long as the stock price is higher than the strike price. But in order for a barrier call option to profit, the stock price must reach the barrier at some point and end up with a stock price higher than the strike price at expiry. If the stock price is fluctuating between strike price ($105) and barrier ($110), a standard call option will profit but a barrier call option will be worthless. Therefore the price of a barrier call option is slightly lower than a standard European option.

2.5 Volatility Changes by 10%

1) 10% Increase

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One-step MC price of an Barrier call option with volatility increased by 10% is 8.59863150710071
One-step MC price of an Barrier put option with volatility increased by 10% is 0.0

Multi-step MC price of an Barrier call option with volatility increased by 10% is 8.776179825614784
Multi-step MC price of an Barrier put option with volatility increased by 10% is 1.592178412900364
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2) 10% Decrease

```
One-step MC price of an Barrier call option with volatility decreased by 10% is 7.000429792297398
One-step MC price of an Barrier put option with volatility decreased by 10% is 0.0

Multi-step MC price of an Barrier call option with volatility decreased by 10% 7.1472092297355205
Multi-step MC price of an Barrier put option with volatility decreased by 10% 0.9725423354790903
```

Pricing volatility is the degree of movement the option price exhibits before it is exercised. The results above show that the price of barrier options is directly related to the change of volatility. Higher volatility represents the increased potential of higher yields for the option in the future, thus the price premium is higher.

3. Optimal Parameter to Achieve the Same Price as Black-Scholes Formula

As discussed in earlier sections, the accuracy of the Monte Carlo simulation is dependent on the number of paths/scenario performs. The number of steps does not have a significant impact on our estimations. To find the optimal number of paths in Monte Carlo pricing for the European option to get the same price as given by the Black-Scholes formula, I will loop over different numbers of paths ranging from 10^2 to 10^7, each time increase by a power of 1. The results are shown below.

```
Black-Scholes price of an European call option is 8.021352235143176
Black-Scholes price of an European put option is 7.9004418077181455

The optimal number of paths for call option is 1000000 and the optimal estimation of call option price is 8.0201
The absolute residual between Black-Scholes and the estimation is: 0.0012
The optimal number of paths for put option is 10000000 and the optimal estimation of call option price is 7.9055
The absolute residual between Black-Scholes and the estimation is: 0.0051
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In order for a Monte Carlo estimation to get the same price (error up a 0.01) as given by the Black-Scholes formula, the number of paths must be greater than or equal to 10,000,000.