

MIE1622H
Computational Finance and Risk Management

Final Project

Zhi Xin Zhu
1002117112
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Q1. Part 1 (Detail results and plots are found in the appendix):

- 1) $\text{VaR}(10 \text{ day}) = 10 * \text{VaR}(\text{one day})$
- 2) $\text{CVaR}(10 \text{ day}) = 10 * \text{CVaR}(\text{one day})$

Both statements above are not true. The price movement of 10 days is not the same as 10 times the price movement of 1 day. The reason is stock prices fluctuate (goes up and down). On day 10, you may see that the stock price is the same as day 1, but in that 10 days, the stock price may drop (i.e. on day 2) but go back up again (i.e. on day 9).

Q1. Part 2 (Detail results and plots are found in the appendix):

- 1) $\text{VaR}(\text{Portfolio}) = \text{VaR}(\text{MSFT}) + \text{VaR}(\text{AAPL}) + \text{VaR}(\text{IBM})$
- 2) $\text{VaR}_N(\text{Portfolio}) = \text{VaR}_N(\text{MSFT}) + \text{VaR}_N(\text{AAPL}) + \text{VaR}_N(\text{IBM})$

Both statements above are not true. When calculating the loss, the profit/loss data are sorted. When we separate into each stock by itself, the sorted sequence may not be the same for each stock (i.e. on day x , MSFT loss = 0.03, AAPL = 0.04, IBM = 0.05; on day $x+1$, MSFT = 0.06, AAPL = 0.05, IBM = 0.04), thus the total portfolio VaR for both historical data and normal distribution data does not equal to the sum of VaR for each stock.

Q2. (Detail results and plots are found in the appendix):

Uniform distribution is used to generate random portfolio weights since uniform distribution is 'fair' that every number between 0 and 1 has the same probability to be chosen as a weight. To ensure that we have a more separated cluster of random portfolios (some outliers), around 15% of the 1000 random portfolios are reassigned with new weights ($\frac{1}{3}$ of the weights are set to zero).

Q3. Part 1 (Detail results and plots are found in the appendix):

R-squared (R^2) is a statistical measure of how close the data are to the fitted regression line. R^2 equals to 1 means perfect fit. From the result, we can see that the R^2 in 2020 is higher than the R^2 in 2019. A possible reason for that is the stock market in 2020 is more consistent (predictable) than in 2019 since the market keeps dropping due to the COVID19 pandemic.

If investors used the FF three-factor model to model asset return during the COVID19 pandemic, it is very likely that they were able to shift their investment to less volatile assets and avoid great losses in 2020, since the average R^2 of 2020 is around 0.70, which is quite high (decent fit).

Q3. Part 2 (Detail results and plots are found in the appendix):

The number of steps is chosen to be 504, the same as the number of historical data used to compute 1-day VaR in question 1. The number of scenarios is 10000. The number is large enough (to represent the population) to compute sample mean that is close to true mean.

The VaR in Q3 is much larger than the VaR computed in Q1. The reason is we simulated a large number of scenarios to generate the factors. In Q1, we only have one scenario. As the number of

scenarios increases, it is more likely that we get extreme cases (i.e extreme high volatility), therefore the VaR is much different from the one computed with historical data.

In Q3, the portfolio VaR for both simulated data and normally distributed data does not equal the sum of VaR for all three assets. But the differences are smaller than the ones in Q1. As the number of scenarios increases, the portfolio VaR and VaRn will be closer and closer to the sum of VaR (VaRn) for all three assets. The reason for that is the factors in Q3 are randomly generated with normal distribution. The normally generated factors are used to calculate the stock prices and the VaR. Unlike in, Q1 where the data are real-world stock prices, the stock prices in Q3 are normally distributed, the impact of sorted losses is less significant for normally distributed stock prices.

Appendix:

Q1. Part 1:

Historical 1-day VaR 95.0% = \$2477.25, Historical 1-day CVaR 95.0% = \$4326.98

Normal 1-day VaR 95.0% = \$2646.49, Normal 1-day CVaR 95.0% = \$3339.94

Historical 10-day VaR 95.0% = \$9023.03, Historical 10-day CVaR 95.0% = \$14099.34

Normal 10-day VaR 95.0% = \$7331.15, Normal 10-day CVaR 95.0% = \$9396.60

Does VaR (10 day) = 10 * VaR (one day) : False

Does CVaR (10 day) = 10 * CVaR (one day) : False

The Difference for VaR is: \$ -15749.49260999996

The Difference for CVaR is: \$ -29170.473228734496

Q1. Part 2:

Historical 1-day VaR for portfolio 95.0% = \$2477.25

Historical 1-day VaR for MSFT 95.0% = \$531.61

Historical 1-day VaR for AAPL 95.0% = \$564.25

Historical 1-day VaR for IBM 95.0% = \$1837.73

Does VaR(Portfolio) = VaR(MSFT) + VaR(AAPL) + VaR(IBM) : False

The difference is: \$ -456.3316459999978

Normal 1-day VaR for portfolio 95.0% = \$2646.49

Normal 1-day VaR for MSFT 95.0% = \$556.21

Normal 1-day VaR for AAPL 95.0% = \$592.79

Normal 1-day VaR for IBM 95.0% = \$1936.14

Does VaRn(Portfolio) = VaRn(MSFT) + VaRn(AAPL) + VaRn(IBM) : False

The difference is: \$ -438.6515478974943

Q2. Part 1:

Minimum Variance Portfolio:

Expected Daily Return = 0.0007054004609645362
Variance = 0.00014724575648841358
Standard Deviation = 0.012134486247403042

Maximum Return Portfolio:

Expected Daily Return = 0.0037921461229095984
Variance = 0.0013160293025746026
Standard Deviation = 0.03627711816799403

Efficient Frontier Portfolio:

Expected Daily Return = 0.0022172758872233416
Variance = 0.0003409219514040153
Standard Deviation = 0.01846407190746438

1/N Portfolio:

Expected Daily Return = 0.0012723110526728
Variance = 0.0003624929083287007
Standard Deviation = 0.01903924652733665

Initial portfolio from assignment 1:

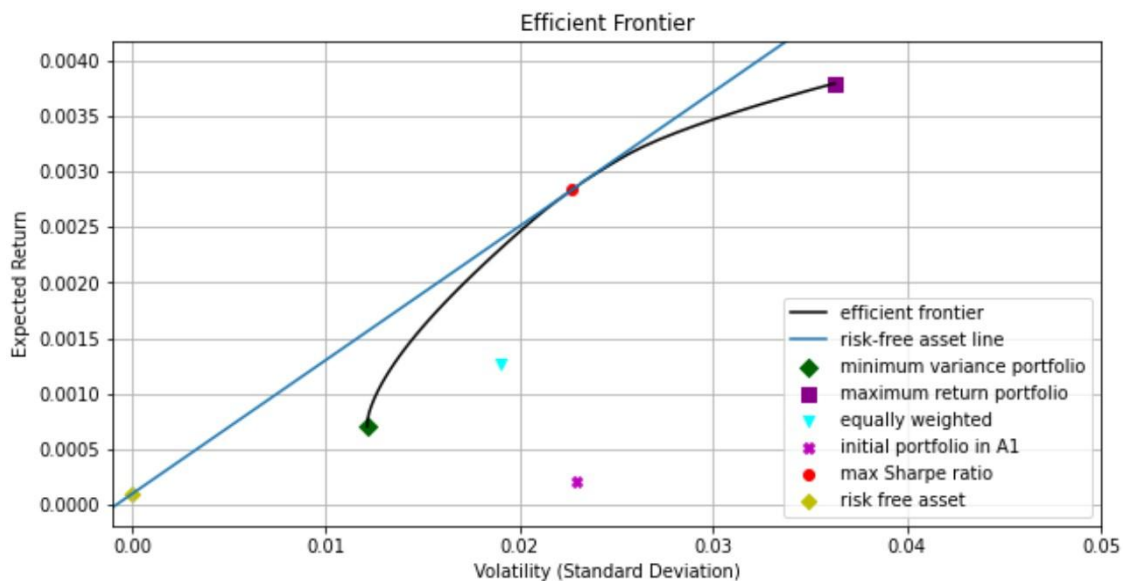
Expected Daily Return = 0.0002168389384752773
Variance = 0.0005249499603611936
Standard Deviation = 0.022911786494317583

Maximum Sharpe Ratio:

Expected Daily Return = 0.002836044560613901
Variance = 0.0005160195911414756
Standard Deviation = 0.02271606460506475

Risk-free Portfolio:

Expected Daily Return = 0.002836044560613901
Variance = 0
Standard Deviation = 0



Q2. Part 2

Efficient Frontier Portfolio:

Expected Daily Return = 0.0037921461229096

Variance = 0.0013160293025746039

Standard Deviation = 0.036277118167994046

MSFT Only Portfolio:
Expected Daily Return = 0.0018457979844508077
Variance = 0.00046064422033872363
Standard Deviation = 0.021462623799030807

C Only Portfolio:
Expected Daily Return = 0.0009273909357525166
Variance = 0.0010188914462825303
Standard Deviation = 0.03192007904568111

T Only Portfolio:
Expected Daily Return = 0.0003572198537467035
Variance = 0.0003155739710634546
Standard Deviation = 0.01776440179300881

F Only Portfolio:
Expected Daily Return = 0.0007558533373962707
Variance = 0.0007788398488431351
Standard Deviation = 0.027907702321100087

HOG Only Portfolio:
Expected Daily Return = 0.0009373691065705685
Variance = 0.0013899155318365356
Standard Deviation = 0.037281570941103535

CSCO Only Portfolio:
Expected Daily Return = 0.00043782161542075805
Variance = 0.00046679542155816326
Standard Deviation = 0.021605448885828854

JPM Only Portfolio:
Expected Daily Return = 0.0009474343224944809
Variance = 0.0006564667072847781
Standard Deviation = 0.025621606258874132

VZ Only Portfolio:
Expected Daily Return = 0.000345952874756518
Variance = 0.00016953901829273667
Standard Deviation = 0.013020714968569762

BAC Only Portfolio:
Expected Daily Return = 0.0008748153152853764
Variance = 0.000789155367664684
Standard Deviation = 0.028091909291906168

GOOG Only Portfolio:
Expected Daily Return = 0.0012275458023695042
Variance = 0.0004072153241672979
Standard Deviation = 0.020179576907539413

AAPL Only Portfolio:
Expected Daily Return = 0.0027390626907027173
Variance = 0.0005679314711885752
Standard Deviation = 0.023831312829732547

INTC Only Portfolio:
Expected Daily Return = 0.0005690878321540904
Variance = 0.0007115931560126024
Standard Deviation = 0.02667570347737061

HPQ Only Portfolio:
Expected Daily Return = 0.0008903419094523814
Variance = 0.0007894100981731359
Standard Deviation = 0.0280964428028378

IBM Only Portfolio:
Expected Daily Return = 0.0005785422869890868
Variance = 0.00041556059427639343
Standard Deviation = 0.020385303389363462

AMD Only Portfolio:
Expected Daily Return = 0.0037921461229095984
Variance = 0.0013160293025746026
Standard Deviation = 0.03627711816799403

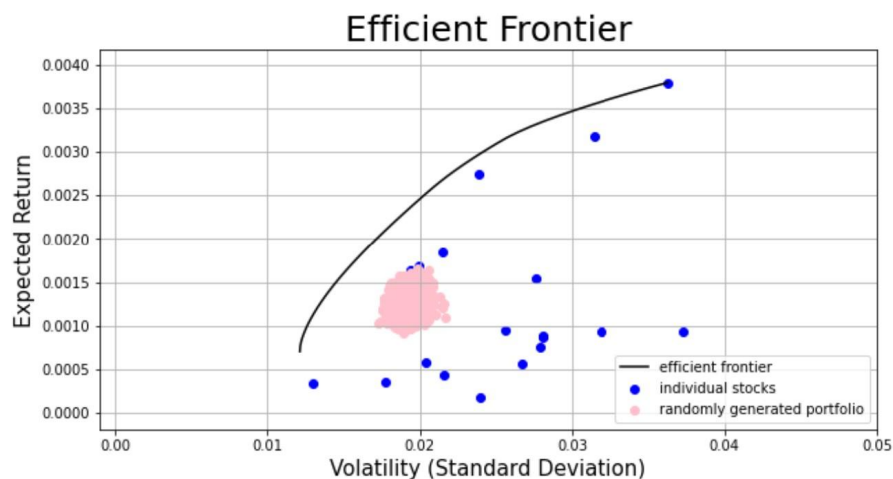
SNE Only Portfolio:
Expected Daily Return = 0.0016365897151436506
Variance = 0.000374333459885317
Standard Deviation = 0.01934769614162192

NVDA Only Portfolio:
Expected Daily Return = 0.0031785178670652526
Variance = 0.0009908749449027403
Standard Deviation = 0.03147816616168642

AMZN Only Portfolio:
Expected Daily Return = 0.001686663587797254
Variance = 0.0003984157754941652
Standard Deviation = 0.01996035509439061

MS Only Portfolio:
Expected Daily Return = 0.0015424759765057191
Variance = 0.0007629921936572324
Standard Deviation = 0.027622313329213256

BK Only Portfolio:
Expected Daily Return = 0.00017559191649274553
Variance = 0.0005722828658448154
Standard Deviation = 0.02392243436284893



Q3. Part 1:

2019 Beta & R-squared

Year 2019	
For Stock: MSFT	For Stock: GOOG
R-squared = 0.7358442793583	R-squared = 0.47028981872629305
alpha = -0.008104728348644858	alpha = -0.00871807428180938
Fitted loadings = [0.01250768 -0.00493042 -0.0052477]	Fitted loadings = [0.01208192 -0.00051884 -0.00444812]
For Stock: F	For Stock: HPQ
R-squared = 0.3226248335219295	R-squared = 0.2752633763959418
alpha = -0.008084396178881527	alpha = -0.009260756284890294
Fitted loadings = [0.01027909 0.00526981 0.00535963]	Fitted loadings = [0.01250746 0.00122724 0.00236997]
For Stock: JPM	For Stock: C
R-squared = 0.7383173340789397	R-squared = 0.7625687353513629
alpha = -0.00773554468730025	alpha = -0.007787085960120605
Fitted loadings = [0.01140387 -0.00317496 0.00994707]	Fitted loadings = [0.01545998 -0.00164176 0.01071079]
For Stock: HOG	For Stock: IBM
R-squared = 0.48909320666951595	R-squared = 0.44754063702735203
alpha = -0.008722638125819002	alpha = -0.008662510706135655
Fitted loadings = [0.01439313 0.0047184 0.01064717]	Fitted loadings = [0.01083393 -0.00283152 0.00189292]
For Stock: VZ	For Stock: T
R-squared = 0.07896417995110372	R-squared = 0.18484514108443628
alpha = -0.008341374348229388	alpha = -0.007533926900745158
Fitted loadings = [0.00349375 -0.00258789 -0.00026857]	Fitted loadings = [5.68445590e-03 -3.23898440e-06 3.08140285e-03]
For Stock: AAPL	For Stock: CSCO
R-squared = 0.5470844185247401	R-squared = 0.44049998643844923
alpha = -0.007391019313861006	alpha = -0.00906622234609741
Fitted loadings = [0.01481455 -0.00189112 -0.00266901]	Fitted loadings = [0.01249855 -0.0015687 -0.00029821]
For Stock: BAC	For Stock: SNE
R-squared = 0.7200359382145569	R-squared = 0.29844477054958707
alpha = -0.007696090224787405	alpha = -0.008067764628639264
Fitted loadings = [0.01306421 0.00016255 0.01262563]	Fitted loadings = [0.01104448 0.00188412 -0.00112806]
For Stock: INTC	For Stock: NVDA
R-squared = 0.3773375762699497	R-squared = 0.506705010787091
alpha = -0.008620218173581464	alpha = -0.008009057143141046
Fitted loadings = [0.01327868 -0.00411736 0.00124403]	Fitted loadings = [0.02134775 0.00414222 -0.00093229]
For Stock: AMD	For Stock: AMZN
R-squared = 0.3970624621828125	R-squared = 0.57515821331006
alpha = -0.006899894935878898	alpha = -0.009013915598442841
Fitted loadings = [0.02458144 0.00210635 -0.00828593]	Fitted loadings = [1.23210409e-02 3.50762260e-05 -5.47951180e-03]
For Stock: MS	
R-squared = 0.6928259701833761	
alpha = -0.008306232501963329	
Fitted loadings = [0.01404408 -0.00157848 0.01049484]	
For Stock: BK	
R-squared = 0.41519513127058205	
alpha = -0.008657267597391799	
Fitted loadings = [0.00990454 0.00013648 0.00882682]	

2020 Beta & R-squared:

Year 2020

For Stock: MSFT

R-squared = 0.8808265273129627

alpha = -0.0015823046007986065

Fitted loadings = [0.01251838 -0.00261475 -0.0051449]

For Stock: F

R-squared = 0.6468421235838635

alpha = -0.0007227843509892131

Fitted loadings = [0.00894802 0.00127282 0.01001591]

For Stock: JPM

R-squared = 0.8870157494122171

alpha = -0.0006158102197929026

Fitted loadings = [0.01052282 -0.00292862 0.01120189]

For Stock: HOG

R-squared = 0.5627757389215908

alpha = -0.0006558612269933419

Fitted loadings = [0.01352433 0.00312532 0.00900089]

For Stock: VZ

R-squared = 0.5508168572049206

alpha = -0.001541874881134121

Fitted loadings = [0.0051123 -0.00292998 0.00118393]

For Stock: AAPL

R-squared = 0.7631500383299246

alpha = -0.00037533170275881426

Fitted loadings = [0.01239125 -0.00410898 -0.00433962]

For Stock: BAC

R-squared = 0.8985467001922405

alpha = -0.0007457056868489538

Fitted loadings = [0.01150781 -0.00354925 0.01203926]

For Stock: INTC

R-squared = 0.5761098900185

alpha = -0.0028911889428446936

Fitted loadings = [0.01209421 -0.00306226 -0.00161541]

For Stock: AMD

R-squared = 0.5324288645831607

alpha = -0.0006052436506695887

Fitted loadings = [0.01340765 -0.00042843 -0.00749781]

For Stock: MS

R-squared = 0.8420877249334742

alpha = 0.00027792207835536305

Fitted loadings = [0.01276553 -0.00280294 0.00804912]

For Stock: BK

R-squared = 0.7547250957878002

alpha = -0.001316131376999587

Fitted loadings = [0.00932977 -0.00274539 0.00829047]

For Stock: GOOG

R-squared = 0.7520354323594565

alpha = -0.0015120901126767664

Fitted loadings = [0.01015526 -0.00122051 -0.00257756]

For Stock: HPQ

R-squared = 0.6067849065499784

alpha = -0.0010579049670522104

Fitted loadings = [0.01058887 0.00733659 0.00355326]

For Stock: C

R-squared = 0.8704396113597668

alpha = -0.0010643081019740385

Fitted loadings = [0.01289465 -0.00028922 0.0132933]

For Stock: IBM

R-squared = 0.7230275656732141

alpha = -0.0017180949147249567

Fitted loadings = [0.00924837 -0.0005354 0.00323231]

For Stock: T

R-squared = 0.7367437851990369

alpha = -0.0020305146234351657

Fitted loadings = [0.00733889 -0.00395129 0.00503508]

For Stock: CSCO

R-squared = 0.6587121768213194

alpha = -0.002144892260988292

Fitted loadings = [0.0099376 -0.00332084 0.00024015]

For Stock: SNE

R-squared = 0.5605899847043344

alpha = -0.000689560985899442

Fitted loadings = [0.00771867 -0.00051055 -0.00150411]

For Stock: NVDA

R-squared = 0.7881974738515525

alpha = -0.0006218838660531235

Fitted loadings = [0.01545765 0.00239574 -0.00910861]

For Stock: AMZN

R-squared = 0.5909114807526449

alpha = -0.0007048239467842209

Fitted loadings = [0.00854037 -0.00187726 -0.0068138]

2019 - 2020 Beta & R-squared:

Year 2019 & 2020

For Stock: MSFT
R-squared = 0.8366451575784701
alpha = -0.004817755055087905
Fitted loadings = [0.01250293 -0.00283259 -0.00524018]

For Stock: F
R-squared = 0.5712923982996787
alpha = -0.004436057469955746
Fitted loadings = [0.00934102 0.00275168 0.00903396]

For Stock: JPM
R-squared = 0.8546270570639595
alpha = -0.004158211898600456
Fitted loadings = [0.01069166 -0.00253881 0.01081837]

For Stock: HOG
R-squared = 0.5452905212953477
alpha = -0.004725287095573795
Fitted loadings = [0.01362473 0.003922 0.00891275]

For Stock: VZ
R-squared = 0.38016508877944455
alpha = -0.005008177911256883
Fitted loadings = [0.00495911 -0.00265409 0.00095122]

For Stock: AAPL
R-squared = 0.6932806280839288
alpha = -0.0038817436037875195
Fitted loadings = [0.01270546 -0.00307278 -0.00453434]

For Stock: BAC
R-squared = 0.8589231922876673
alpha = -0.004267893916294511
Fitted loadings = [0.01175948 -0.00223615 0.01172808]

For Stock: INTC
R-squared = 0.5272484782239311
alpha = -0.005731677799763874
Fitted loadings = [0.01215947 -0.00297473 -0.00143998]

For Stock: AMD
R-squared = 0.44276259252922334
alpha = -0.00348027217819789
Fitted loadings = [0.01497681 0.00152321 -0.00865019]

For Stock: MS
R-squared = 0.7994006899193258
alpha = -0.004036637714362842
Fitted loadings = [0.01289166 -0.00204262 0.00803147]

For Stock: BK
R-squared = 0.6770463754494198
alpha = -0.005048106186105918
Fitted loadings = [0.00944137 -0.0016952 0.00806131]

For Stock: GOOG
R-squared = 0.650872240647149
alpha = -0.005081357832678424
Fitted loadings = [0.01049643 -0.00049912 -0.00314741]

For Stock: HPQ
R-squared = 0.5107104759709691
alpha = -0.005002660955711636
Fitted loadings = [0.01082083 0.0065367 0.00324918]

For Stock: C
R-squared = 0.8467973897027469
alpha = -0.00432859680470642
Fitted loadings = [1.33166599e-02 1.48653457e-05 1.26566628e-02]

For Stock: IBM
R-squared = 0.6481049594176451
alpha = -0.0051117870547992715
Fitted loadings = [0.00948755 -0.00055329 0.00284998]

For Stock: T
R-squared = 0.6064025377475644
alpha = -0.004912151161016898
Fitted loadings = [0.00724101 -0.00290759 0.00466491]

For Stock: CSCO
R-squared = 0.5856309838136764
alpha = -0.005579129761310813
Fitted loadings = [0.01033446 -0.00233917 -0.00023106]

For Stock: SNE
R-squared = 0.4347833154933539
alpha = -0.004347747539257686
Fitted loadings = [0.0081982 0.00069406 -0.00194492]

For Stock: NVDA
R-squared = 0.6681455468689124
alpha = -0.004236991904916467
Fitted loadings = [0.01602425 0.00354335 -0.00872504]

For Stock: AMZN
R-squared = 0.5552349359679734
alpha = -0.004813405323280419
Fitted loadings = [0.00904595 -0.00071416 -0.00717343]

R-squared comparison (2019 and 2020):

2019 R-Squared:

[0.7358442793583, 0.3226248335219295, 0.7383173340789397, 0.47028981872629305, 0.2752633763959418, 0.7625687353513629, 0.48909320666951595, 0.07896417995110372, 0.5470844185247401, 0.44754063702735203, 0.18484514108443628, 0.44049998643844923, 0.7200359382145569, 0.3773375762699497, 0.3970624621828125, 0.29844477054958707, 0.506705010787091, 0.57515821331006, 0.6928259701833761, 0.41519513127058205]

2020 R-Squared:

[0.8808265273129627, 0.6468421235838635, 0.8870157494122171, 0.7520354323594565, 0.6067849065499784, 0.8704396113597668, 0.5627757389215908, 0.5508168572049206, 0.7631500383299246, 0.7230275656732141, 0.7367437851990369, 0.6587121768213194, 0.8985467001922405, 0.5761098900185, 0.5324288645831607, 0.5605899847043344, 0.7881974738515525, 0.5909114807526449, 0.8420877249334742, 0.7547250957878002]

The average R-squared for all stocks in 2020 is: 0.709138386377598

Q3. Part 2

Result from Q1 Part 2

Historical 1-day VaR for portfolio 95.0% = \$2477.25

Historical 1-day VaR for MSFT 95.0% = \$531.61

Historical 1-day VaR for AAPL 95.0% = \$564.25

Historical 1-day VaR for IBM 95.0% = \$1837.73

The difference of VaR(portfolio) and sum of VaR for each stock is: \$ -456.3316459999978

Result from Q3

Monte Carlo Simulated 1-day VaR for portfolio 95.0% = \$5893.73

Monte Carlo Simulated 1-day CVaR for portfolio 95.0% = \$7974.34

Monte Carlo Simulated 1-day VaR for MSFT 95.0% = \$255.16

Monte Carlo Simulated 1-day VaR for AAPL 95.0% = \$253.25

Monte Carlo Simulated 1-day VaR for IBM 95.0% = \$5572.29

Does VaR(Portfolio) = VaR(MSFT) + VaR(AAPL) + VaR(IBM) : False

The difference is: \$ -186.96629804437453

Result from Q1 Part 2

Normal 1-day VaR for portfolio 95.0% = \$2646.49

Normal 1-day VaR for MSFT 95.0% = \$556.21

Normal 1-day VaR for AAPL 95.0% = \$592.79

Normal 1-day VaR for IBM 95.0% = \$1936.14

The difference is: \$ -438.6515478974943

Result from Q3

Normal 1-day VaR for portfolio 95.0% = \$5641.43

Normal 1-day CVaR for portfolio 95.0% = \$7142.15

Normal 1-day VaR for MSFT 95.0% = \$248.99

Normal 1-day VaR for AAPL 95.0% = \$250.87

Normal 1-day VaR for IBM 95.0% = \$5257.45

Does VaRn(Portfolio) = VaRn(MSFT) + VaRn(AAPL) + VaRn(IBM) : False

The difference is: \$ -115.87105364458967