MIE1622H
Computational Finance and Risk Management

Assignment 3

Credit Risk Modeling and Simulation

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1. Introduction

The purpose of this assignment is to model a credit-risk portfolio of corporate bonds. We need to generate the following 3 sets of scenarios:

- Monte Carlo approximation 1 : 5000 in-sample scenarios (N = 1000 5 = 5000 (1000 systemic scenarios and 5 idiosyncratic scenarios for each systemic), non-Normal distribution of losses);
- Monte Carlo approximation 2: 5000 in-sample scenarios (N = 5000 (5000 systemic scenarios and 1 idiosyncratic scenario for each systemic), non-Normal distribution of losses);
- True distribution: 100000 out-of-sample scenarios (N = 100000 (100000 systemic scenarios and 1 idiosyncratic scenario for each systemic), non-Normal distribution of losses).

The VaR and CVaR at quantile levels 99% and 99.9% are evaluated for the following two portfolios:

1) One unit invested in each of 100 bonds
2) Equals value (dollar amount) is invested in each of 100 bonds;

2. Simulation and Evaluation

2.1 Simulation Results

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Portfolio 1:

Out-of-sample: VaR 99.0% = $37668009.00, CVaR 99.0% = $45434628.99
In-sample MC1: VaR 99.0% = $37254411.71, CVaR 99.0% = $44714065.19
In-sample MC2: VaR 99.0% = $36997505.76, CVaR 99.0% = $44555137.57
In-sample No: VaR 99.0% = $26341979.95, CVaR 99.0% = $29250839.69
In-sample N1: VaR 99.0% = $26182840.59, CVaR 99.0% = $29070565.97
In-sample N2: VaR 99.0% = $26153088.26, CVaR 99.0% = $29037641.23

Out-of-sample: VaR 99.9% = $55197450.92, CVaR 99.9% = $63751102.95
In-sample MC1: VaR 99.9% = $53885863.74, CVaR 99.9% = $61069345.64
In-sample MC2: VaR 99.9% = $53736938.80, CVaR 99.9% = $61650490.34
In-sample No: VaR 99.9% = $32899233.86, CVaR 99.9% = $35275806.32
In-sample N1: VaR 99.9% = $32692452.66, CVaR 99.9% = $35051758.09
In-sample N2: VaR 99.9% = $32655549.00, CVaR 99.9% = $35012262.54
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Portfolio 2:

Out-of-sample: VaR 99.0% = $27507296.24, CVaR 99.0% = $33947339.63
In-sample MC1: VaR 99.0% = $27417068.27, CVaR 99.0% = $33372015.56
In-sample MC2: VaR 99.0% = $27337376.90, CVaR 99.0% = $33406545.90
In-sample No: VaR 99.0% = $21274509.21, CVaR 99.0% = $23461445.55
In-sample N1: VaR 99.0% = $21077013.23, CVaR 99.0% = $23238278.14
In-sample N2: VaR 99.0% = $21093718.74, CVaR 99.0% = $23258214.28

Out-of-sample: VaR 99.9% = $42508340.49, CVaR 99.9% = $49772063.26
In-sample MC1: VaR 99.9% = $40995165.00, CVaR 99.9% = $46368226.17
In-sample MC2: VaR 99.9% = $40925519.96, CVaR 99.9% = $47968668.80
In-sample No: VaR 99.9% = $26204377.99, CVaR 99.9% = $27991130.57
In-sample N1: VaR 99.9% = $25949012.57, CVaR 99.9% = $27714791.29
In-sample N2: VaR 99.9% = $25973000.69, CVaR 99.9% = $27741418.87
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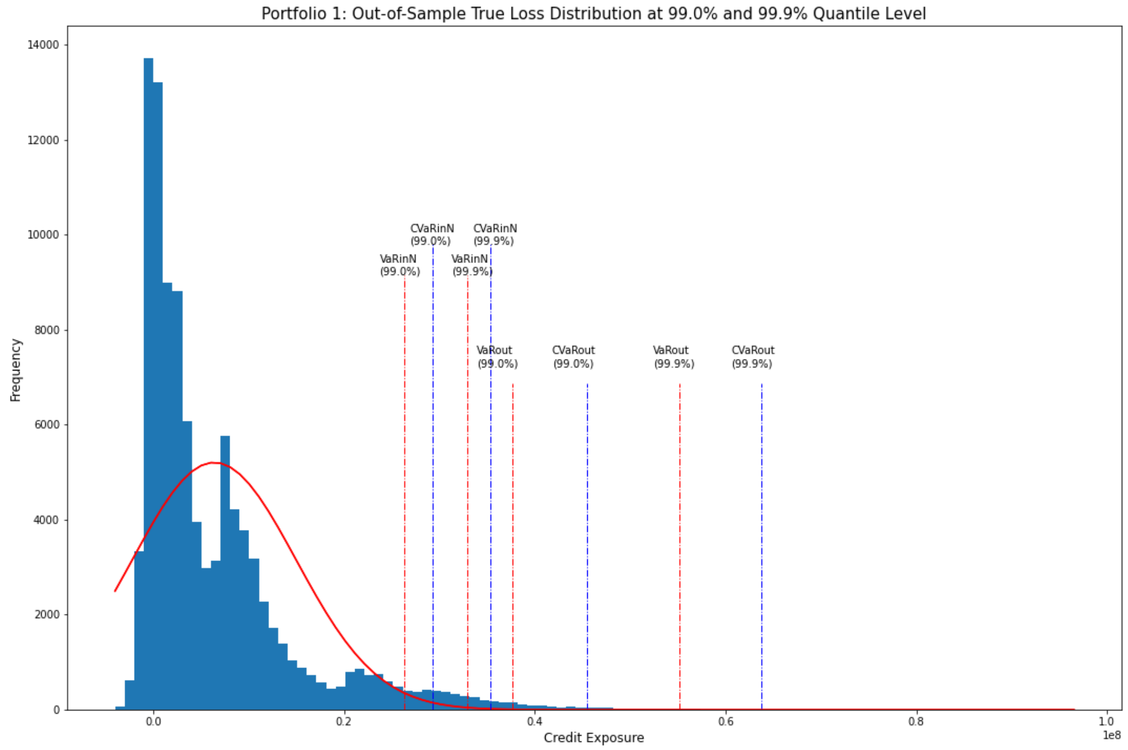
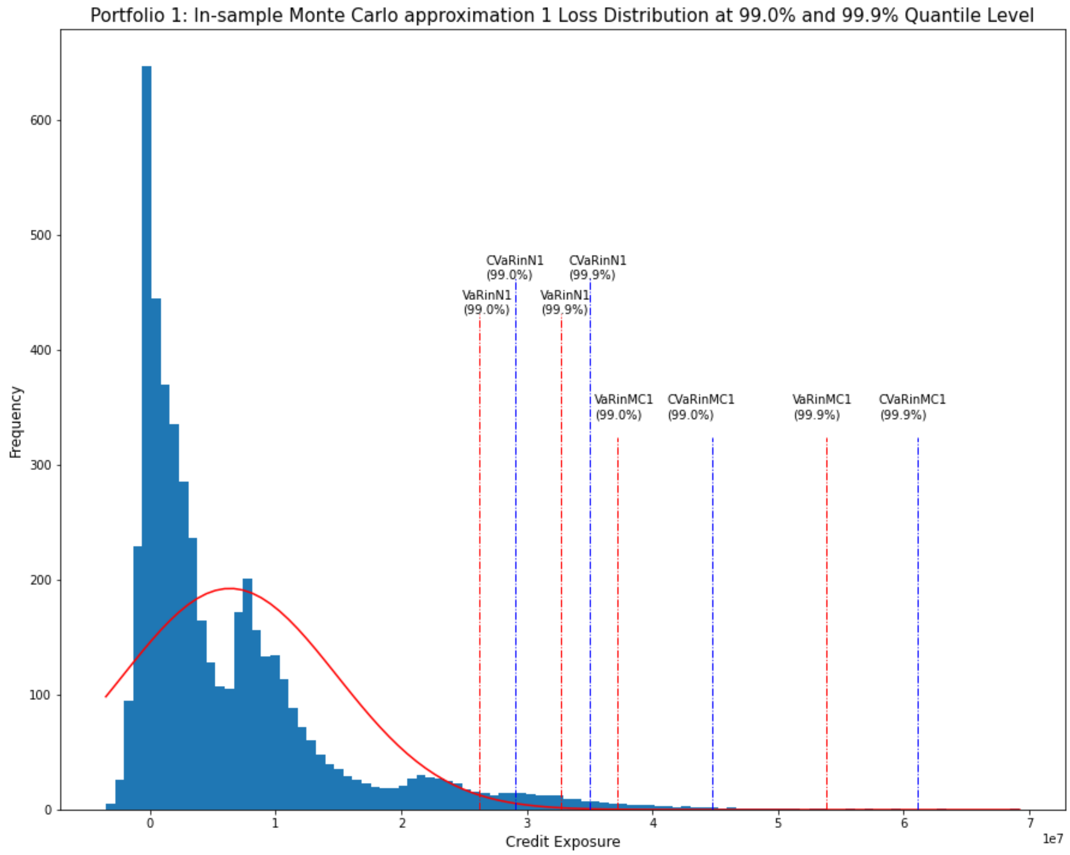
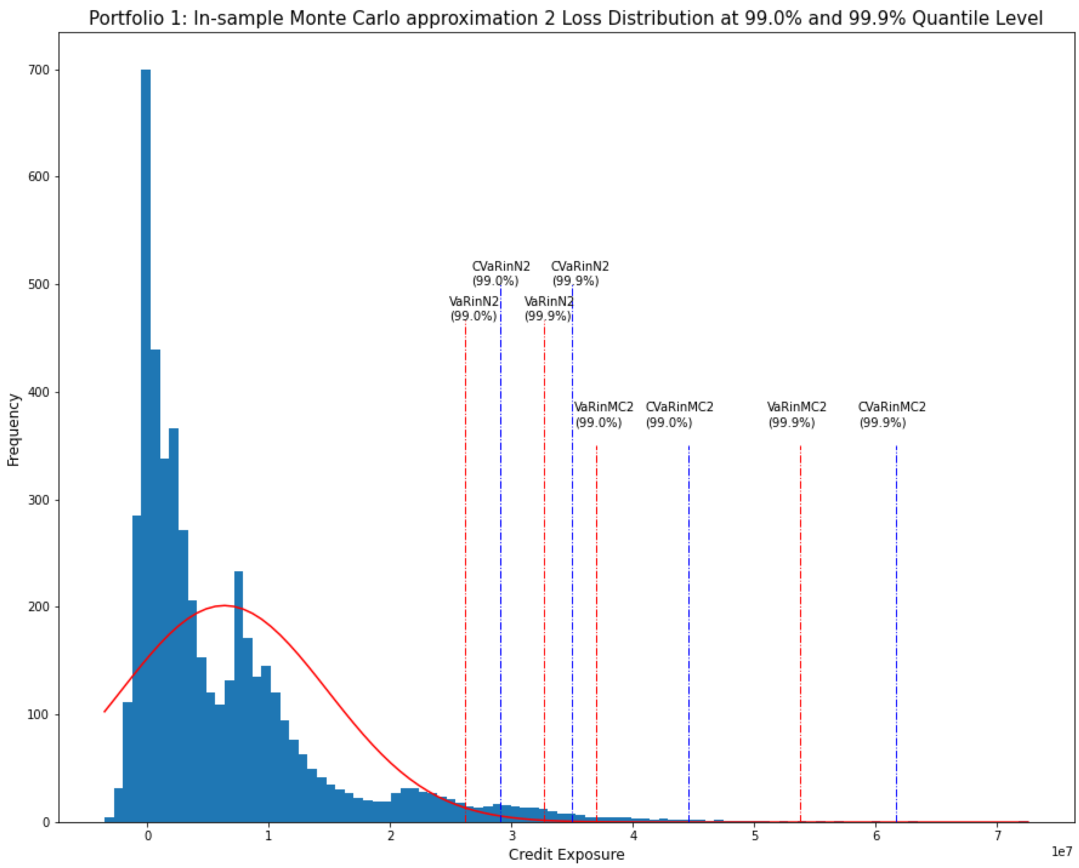
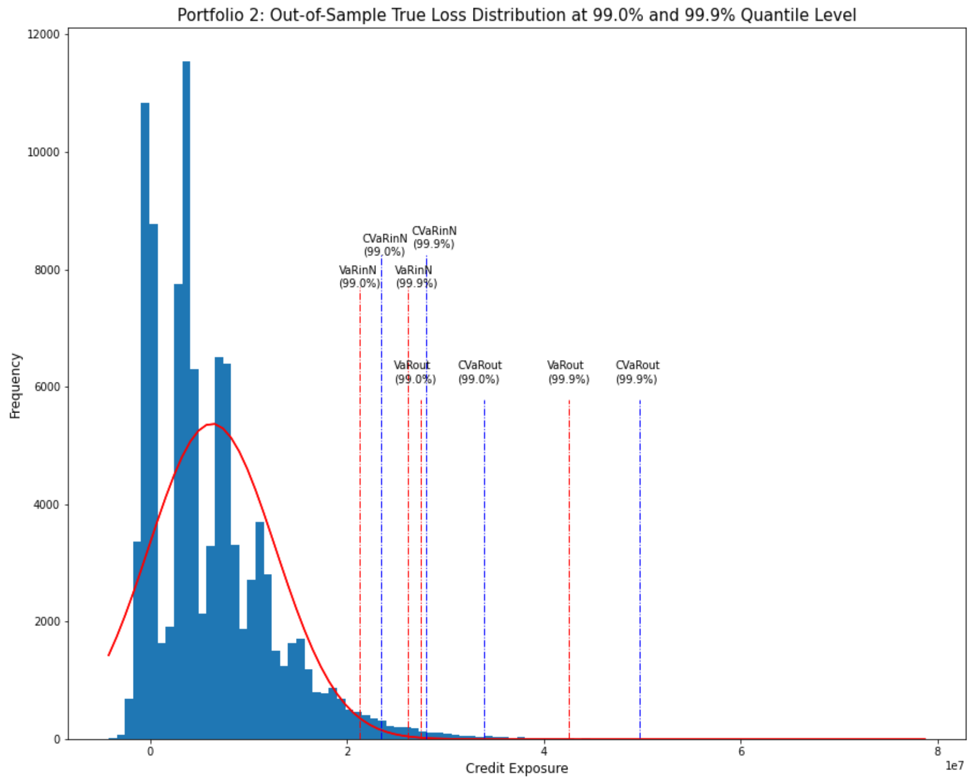
2.2 Loss Distribution Plot

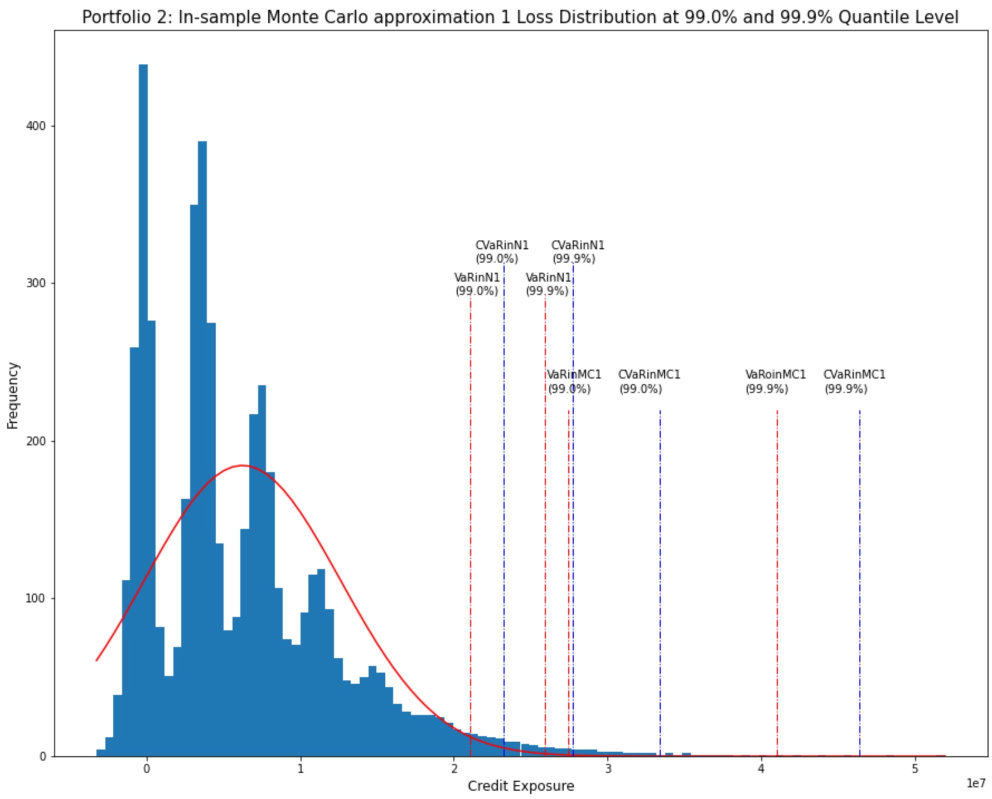
There are 6 plots in total for 3 sets of scenarios for each portfolio. In each plot, the VaR and CVaR of both non-normal and Normal distribution have been plotted for better comparison.

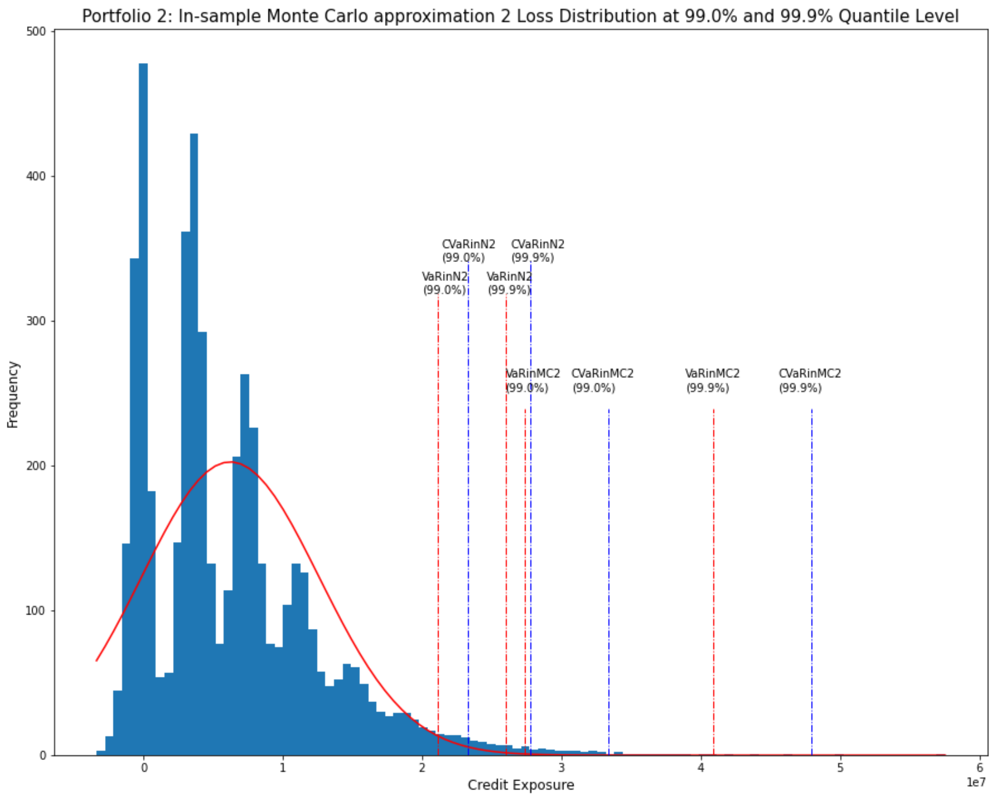



Portfolio 1: In-sample Monte Carlo approximation 1 Loss Distribution at 99.0% and 99.9% Quantile Level
Frequency
Credit Exposure
1e7
CVaRinN1 (99.0%)
CVaRinN1 (99.9%)
VaRinN1 (99.0%)
VaRinN1 (99.9%)
VaRinMC1 (99.0%)
CVaRinMC1 (99.0%)
VaRinMC1 (99.9%)
CVaRinMC1 (99.9%)


Portfolio 1: In-sample Monte Carlo approximation 2 Loss Distribution at 99.0% and 99.9% Quantile Level
Frequency
Credit Exposure
1e7
CVaRinN2 (99.0%)
CVaRinN2 (99.9%)
VaRinN2 (99.0%)
VaRinN2 (99.9%)
VaRinMC2 (99.0%)
CVaRinMC2 (99.0%)
VaRinMC2 (99.9%)
CVaRinMC2 (99.9%)


Portfolio 2: Out-of-Sample True Loss Distribution at 99.0% and 99.9% Quantile Level
Frequency
Credit Exposure
1e7
CVaRinN
(99.0%)
CVaRinN
(99.9%)
VaRinN
(99.0%)
VaRinN
(99.9%)
VaRout
(99.0%)
CVaRout
(99.0%)
VaRout
(99.9%)
CVaRout
(99.9%)


Portfolio 2: In-sample Monte Carlo approximation 1 Loss Distribution at 99.0% and 99.9% Quantile Level
Frequency
Credit Exposure
1e7
CVaRinN1
(99.0%)
CVaRinN1
(99.9%)
VaRinN1
(99.0%)
VaRinN1
(99.9%)
VaRinMC1
(99.0%)
CVaRinMC1
(99.0%)
VaRoinMC1
(99.9%)
CVaRinMC1
(99.9%)


Portfolio 2: In-sample Monte Carlo approximation 2 Loss Distribution at 99.0% and 99.9% Quantile Level
Frequency
Credit Exposure
1e7
0
100
200
300
400
500
0
1
2
3
4
5
6
CVaRinN2
(99.0%)
CVaRinN2
(99.9%)
VaRinN2
(99.0%)
VaRinN2
(99.9%)
VaRinMC2
(99.0%)
CVaRinMC2
(99.0%)
VaRinMC2
(99.9%)
CVaRinMC2
(99.9%)

2.3 Error Analysis

1) **Sampling error**

   Sampling error is the deviation between random sampling (Monte Carlo approximation) and the true population distribution(out-of-sample). In this assignment, the sampling error represents the difference between the loss distributions of the non-normal Monte Carlo approximation and the loss distribution of the out-of-sample(true) data.

Table 1: Portfolio 1 - VaR and CVaR Sampling Error between In-sample and Out-of-sample Scenarios

	VaR (99.0%)	Error (%)	VaR (99.9%)	Error (%)	CVaR (99.0%)	Error (%)	CVaR (99.9%)	Error (%)
True distribution	37668009.00	-	55197450.92	-	45434628.99	-	63751102.95	-
Monte Carlo 1	37254411.71	-1.10%	53885863.74	-2.38%	44714065.19	-1.59%	61069345.64	-4.21%
Monte Carlo 2	36997505.76	-1.78%	53736938.80	-2.65%	44555137.57	-1.94%	61650490.34	-3.30%

Table 2: Portfolio 2 - VaR and CVaR Sampling Error between In-sample and Out-of-sample Scenarios

	VaR (99.0%)	Error (%)	VaR (99.9%)	Error (%)	CVaR (99.0%)	Error (%)	CVaR (99.9%)	Error (%)
True distribution	27507296.24	-	42508340.49	-	33947339.63	-	49772063.26	-
Monte Carlo 1	27417068.27	-0.33%	40995165.00	-3.56%	33372015.56	-1.69%	46368226.17	-6.84 %
Monte Carlo 2	27337376.90	-0.62%	40925519.96	-3.72%	33406545.90	-1.59%	47968668.80	-3.62 %

Observation:

- Both Monte Carlo approximations have a satisfying performance in predicting the true population distribution for portfolios 1 and 2. The sampling errors in prediction VaR and CVaR are minor ranging from 0.33% - 6.84%. The sampling error has an insignificant impact on the accuracy of the prediction.

2) **Model error**

Model error is the error that arises when the approximation wrongly assumed to be normally distributed for the portfolio loss. In this case, it is the difference between the in-sample normal approximations (Normal with mean/standard deviation computed from N=10005, 5000, 100000) and the true (out-of-sample) loss distribution.

Table 3: Portfolio 1 - VaR and CVaR Model Error between In-sample and Out-of-sample Scenarios

	VaR (99.0%)	Error (%)	VaR (99.9%)	Error (%)	CVaR (99.0%)	Error (%)	CVaR (99.9%)	Error (%)
True distribution	37668009.00	-	55197450.92	-	45434628.99	-	63751102.95	-
Normal distribution	26341979.95	-30.07%	32899233.86	-40.40%	29250839.69	-35.62%	35275806.32	-44.67%
Monte Carlo N1	26182840.59	-30.49%	32692452.66	-40.77%	29070565.97	-36.02%	35051758.09	-45.02%
Monte Carlo N2	26153088.26	-30.57%	32655549.00	-40.84%	29037641.23	-36.09%	35012262.54	-45.08%

Table 4: Portfolio 2 - VaR and CVaR Model Error between In-sample and Out-of-sample Scenarios

	VaR (99.0%)	Error (%)	VaR (99.9%)	Error (%)	CVaR (99.0%)	Error (%)	CVaR (99.9%)	Error (%)
True distribution	27507296.24	-	42508340.49	-	33947339.63	-	49772063.26	-
Normal distribution	21274509.21	-22.66%	26204377.99	-38.35%	23461445.55	-30.89%	27991130.57	-43.76%
Monte Carlo N1	21077013.23	-23.38%	25949012.57	-38.96%	23238278.14	-31.55%	27714791.29	-44.32%
Monte Carlo N2	21093718.74	-23.32%	25973000.69	-38.90%	23258214.28	-31.49%	27741418.87	-44.26%

Observation:

- The model errors between normal approximations and the true distribution are significant ranging between -22.66% - 45.08%. As the model errors are all negative, this suggests the Normal approximation significantly underestimates the credit risk. This is lethal in portfolio management because underestimating risk may lead to great losses.

3. Discussion

3.1 If you report the in-sample VaR and CVaR to decision-makers in your bank, what consequences for the bank capital requirements it may have?

Assuming that the portfolios are the same, based on the above analysis, if we report the non-Normal Monte Carlo in-sample approximations to the decision-makers, they are able to get an accurate prediction on future credit risks and also be able to prepare the bank capital better for possible extreme credit risk. With the information provided by the simulation, it is unlikely that the bank capital requirements suffer from the loss of assets due to credit risk.

If we report the normal approximations to the decision-makers, the bank capital requirements will face a serious risk of losses, since the normal approximations significantly underestimate the credit risks. The decision-makers are likely to be misled by the estimates and the bank capital requirements may suffer a higher credit exposure than expected.

3.2 Techniques for Minimizing Impacts of Sampling and Model Errors

1) Sampling Errors
    - Increase the sampling sizes, methods of approximation (scenarios), and run more trials

2) Model Errors
    - Do not assume any underlying patterns such as normal distribution on the data since real-world data may not follow a certain type of distribution.
3) Other
    - Decision-makers should not fully rely on the predictions of models since the models are built using historical data and these historical patterns cannot exactly reflect the future. Decision-makers should combine the results of credit risk models and the current circumstances when making significant decisions. If the bank cannot suffer any loss, they should be more conservative and not overestimating the credit risks.