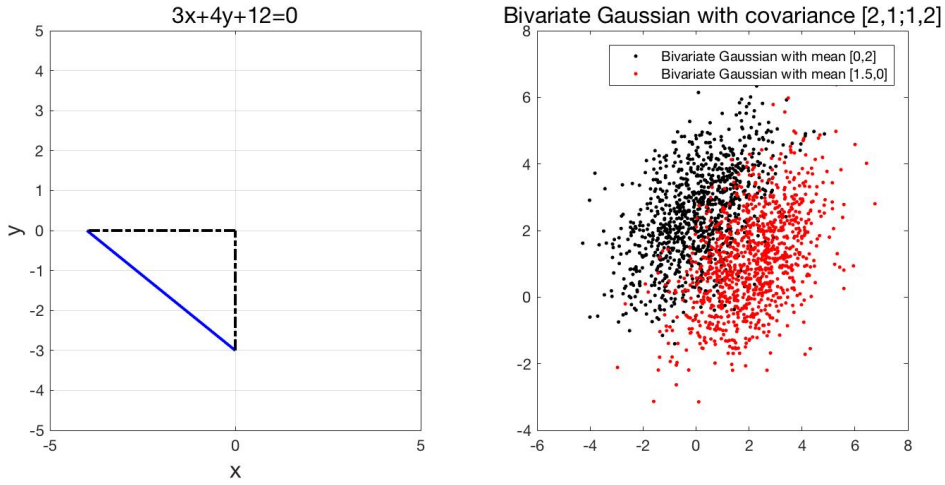
Machine Learning Lab 2

Name: ZHIKUN ZHU; ID:29356822

Q1. The figure of function $3x + 4y + 12 = 0$ is shown in Fig.1 (a), which is the blue line. The black lines are the x-axis and y-axis, respectively. We can found from the plot that three lines are enclosed a triangle area. Then we can easily conscious that the distance between origin to straight line is $\frac{3\times4}{\sqrt{3^2+4^2}}$ by utilizing area formula of triangle.



(a) *(b)*

Fig.1 Plots for Q1 and Q2

Q2. Two classes of bivariate Gaussian random variable were plotted in the Fig.1 (b). The black group have mean = $[0, 2]^T$ and the red group have mean = $[1.5, 0]^T$. Both distribution have covariance matrix = [2, 1; 1, 2].

Q3. From the Bayesian Decision Theory, we can get the function for liner classifier:

$$w^T x + b = 0$$

Where:

$$w = 2C^{-1}(m_2 - m_1) = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$b = (m_1^T C^{-1} m_1 - m_2^T C^{-1} m_2) - \log\frac{p(w_1)}{p(w_2)}$$

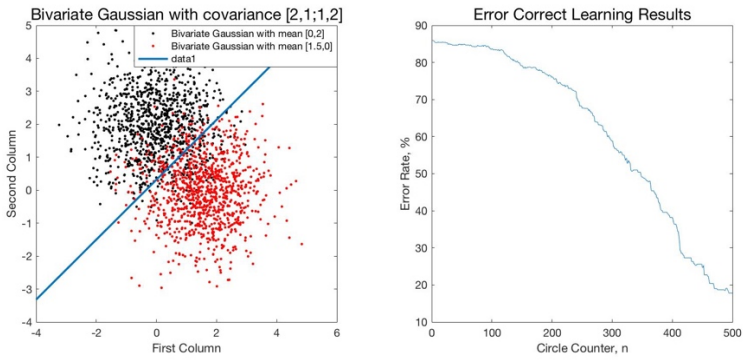
Then, we can get the boundary for Bayes' optimal class boundary:

$$(w_1 \quad w_2)\begin{pmatrix} x \\ y \end{pmatrix} + b = 0$$

Which equals:

$$w_1 x + w_2 y + b = 0$$

Then, the Bayes' optimal class boundary is shown in Fig.2(a).



(a) *(b)*

Fig.2 Bayes' optimal class boundary and learning algorithm performance

Q4. a) The Bayes' optimal boundary is successfully implemented with MATLAB, and its error rate is shown in Fig.2 (b). The error rate decreased as the circle goes on. Moreover, Fig.3 shows the relationship between learning rate and algorithm convergence rate, which indicates that algorithm convergence rate decreases as learning rate decrease.

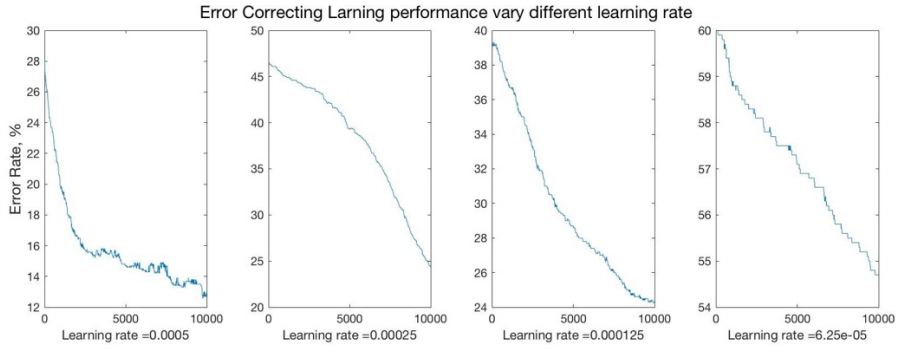


Fig.3 Algorithm convergence rate between different learning rate

b) Fig.4 shows the algorithm error rate varying from different data separations (more separated mean vectors).

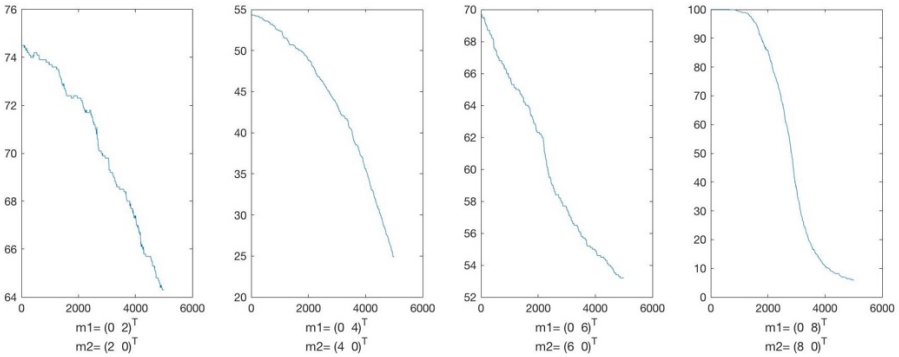


Fig.4 Algorithm convergence rate varying from different means

It is clear from the chart that learning algorithm converge more quickly under more separated data. Besides, the limit of learning algorithm's error rate is getting lower as data getting separated.