UCSB, Physics 129L, Computational Physics: Practice Final Exam

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Final Exam Instructions

Location: Web 1100
Time: Tuesday, March 18, 2025 4:00 PM - 7:00 PM

Before you start, **write your name and perm number clearly on the blue book cover**. This is a closed-book, pen-and-paper exam. Please label which question you are answering and write your answer clearly in the blue book.

There are 20 questions and 1 extra credit question. You have a maximum of **three hours to complete the exam**. You are allowed to use **a single (front and back) handwritten note sheet**, and you must complete the final exam without consulting any other information sources. Academic dishonesty will not be tolerated and carries significant consequences, including disciplinary action and loss of credibility at the university level.

If you finish the exam early, you can turn in your blue book and leave early. You may keep the final exam page.

Good luck.

1

What is the major difference between a scripting language and compile language? Write an example for each type. (5 points)

2

What is a Fetch operation in Git? Explain the concept and draw a schematic that demonstrates this operation between a remote branch and a local branch. (10 points)

3

Consider a harmonic potential surface derivative,

$$x^2 + y^2 + \frac{z^2}{3} = V, \quad z \geq 0, \tag{1}$$

where V is a function of x, y, z.

Using the projector method ($\mathbf{N}$ is the surface normal),

$$\mathbf{P} = \mathbf{I} - \mathbf{N}\mathbf{N}^T, \quad \nabla_s \phi = P_{ij} \partial_j \phi, \tag{2}$$

find the surface derivative ∇_s on the following two functions, $\phi = V(x, y, z)$ and $\phi = x^4$. (10 points)

4

What is the difference between surface (facet) norm and vertex norm? (5 points)

5

a) What is the meaning of "parallel" in parallel transport? b) What are geodesics curves, and what happen if you parallel transport their tangent vectors? (10 points)

6

What is the difference between a deterministic and non-deterministic Turing machine? (5 points)

7

Given the following recurrence expression, use the Master Theorem to calculate the time complexity to the leading order,

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n).$$

(10 points)

8

Consider the following matrix A:

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

Use the Gram-Schmidt process to find the QR decomposition of matrix A, where Q is an orthogonal matrix and R is an upper triangular matrix. Construct the upper triangular matrix R using the formula,

$$R = Q^T A.$$

(15 points)

9

Let's consider a Markov chain on the following state space $\{S_1, S_2, S_3\}$ with the following transition matrix,

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}.$$

a) If we start from the state S_1, what is the probability remaining at state S_1

after three Markov steps? b) What is the stationary distribution of this Markov chain? (20 points)

10

Let W_t be a standard Brownian motion and consider the following quadratic function,

$$g(t, W_t) = \mu_1 t + \mu_2 t^2 + \sigma^2 t (W_t)^2. \quad (3)$$

Calculate the differential dg using the Ito's Lemma (10 points)

11

In the Langevin equation, the stochastic differential of the velocity dv is given by,

$$dv = -\frac{\gamma}{m} v\, dt + e^{-\gamma t} \frac{\sqrt{D}}{m} W_t\, dW_t, \quad (4)$$

where γ is the friction coefficient, and D is the diffusion constant. Calculate the following ensemble average, a) $\langle v_t \rangle$. b) $\langle v_t^2 \rangle - \langle v_t \rangle^2$. (20 points)

12

Let's consider a system with a fixed energy, and the multiplicity is given by $\Omega(E) = \frac{(E-E_0)^2}{2}$. Why can you still define the system temperature? Write the temperature as a function of energy $T(E)$. Discuss the implications for the following case for $E < E_0$, $E > E_0$, $E = E_0$. (10 points)

13

What is the Central Limit Theorem? Give an intuitive explanation on why Central Limit Theorem is valid? (5 points)

14

What are the differences between Bayesian inference and Frequentist interpretation? (10 points)

15

The Gamma distribution is a continuous probability distribution with two parameters: the shape parameter k (also denoted as α) and the rate parameter λ (also denoted as $\beta = \frac{1}{\lambda}$, the scale parameter). The probability density function (PDF) of the Gamma distribution is:

$$\gamma(x, k, \lambda) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{\Gamma(k)} \quad \text{for } x > 0,$$

where $\Gamma(k)$ is the Gamma function, and $k > 0$ and $\lambda > 0$. Let's consider a given a set of n independent observations $\{x_1, x_2, \ldots, x_n\}$ drawn from a Gamma distribution, and you want to estimate what are the maximum likelihood of the hyperparameter k, λ. a) Write down the log-likelihood function. b) find the maximum likelihood estimator for $\hat{k}, \hat{\lambda}$. You do not need to solve the differential equation associated with the maximum likelihood estimator. (20 points)

16

The Gaussian quadrature has the following form,

$$\int_{-1}^{1} w(x) g(x) dx \approx \sum_{i=1}^{n} W_i g(x_i),$$

where the weighting function is $w(x) = 1$. Calculate the expression such that it is exact to the third polynomial order x^3. Find the node locations via solving the system of equations. *Hint*: you can write the above,

$$g(x) = g_0 + g_1 x + g_2 x^2 + g_3 x^3, \tag{5}$$

and,

$$\sum_i W_i g(x_i) = \sum_i (W_i g_0 + W_i g_1 x_i + W_i g_2 x_i^2 + W_i g_3 x_i^3). \tag{6}$$

Two sides must be the same for any g_0, g_1, g_2, g_3, and we must have,

$$\int_{-1}^{1} g(x)dx = \sum_i W_i g(x_i), \tag{7}$$

which gives the system of equations, e.g.

$$\int g_0 dx = \sum_i W_i g_0, \cdots . \tag{8}$$

(15 points)

17

Show that the first Order Runge-Kutta Method (RK1) is the Euler method at x_0,

$$y_{n+1} = y_n + hF(x_n, y_n),$$

with the following differential equation,

$$y'(x_0) = F(x_0, y_0), \quad y(x_0) = y_0.$$

The Euler method is given by,

$$y(x_0 + h) = hy'(x_0) + \mathcal{O}(h^2).$$

(10 points)

18

How to categorize different second order partial differential equation using discriminant? Write down the conditions, and discuss the implications. (10 points)

19

a) What are the most important characteristics of a chaos system? b) A classical system is fully deterministic, and why there still exists deterministic chaos? (10 points)

20

What is optimal transport, and how does it relate to the concept of general neural network? (5 points)

Extra Credit

Consider a 3-layer single neuron Multilayer Perceptron with ReLU activation function $f = \mathrm{ReLU}(z) = \max(0, z)$ for a regression problem.

$$z^{(l)} = \theta_1^{(l)} a^{(l-1)} + \theta_0^{(l)}, \tag{9}$$

$$a^{(l)} = f\big(z^{(l)}\big). \tag{10}$$

a) Write down the forward propagation represented by the following transport plan, $\hat{y} = T(x)$, and draw the corresponding directed graph. b) Using the chain rule, write down the backward propagate the error analytically, and draw the corresponding directed graph. (15 points)