UCSB, Physics 129L, Computational Physics: Final Exam

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Final Exam Instructions

Location: Web 1100
Time: Tuesday, March 18, 2025 4:00 PM - 7:00 PM

Before you start, **write your name and perm number clearly on the blue book cover**. This is a closed-book, pen-and-paper exam. Please label which question you are answering and write your answer clearly in the blue book.

There are 20 questions and 1 extra credit question. You have a maximum of **three hours to complete the exam**. You are allowed to use **a single (front and back) handwritten note sheet**, and you must complete the final exam without consulting any other information sources. Academic dishonesty will not be tolerated and carries significant consequences, including disciplinary action and loss of credibility at the university level.

If you finish the exam early, you can turn in your blue book and leave early. You may keep the final exam page.

Total point: 260+5

Good luck.

1

List 5 topics that you find *interesting* in this course. (5 points)

2

List 5 topics that you find *not interesting* in this course. (5 points)

3

What is the difference between tensors defined in computation and tensors defined in physics?(5 points)

4

When performing a 2 dimensional Delaunay triangulation, how many triangles share one vertex? (5 points)

5

a) In parallel transport, we see the transported vector "changes" its direction in the global frame. When it gets back to the same point, it appears to have a different direction comparing to the initial vector. What is the reason? (5 points)

6

What is the difference between a NP problem and a NP-hard problem? (5 points)

7

Given the following recurrence expression, use the Master Theorem to calculate the time complexity to the leading order,

$$T(n) = 3T\left(\frac{n}{2}\right).$$

(10 points)

8

Consider the following overdetermined system A:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

Solve the following normal equation,

$$A^T A\mathbf{x} = A^T \mathbf{b}, \tag{1}$$

and find a $\mathbf{x}$ that minimize the following functional,

$$E = \min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|^2. \tag{2}$$

What is the minimized value $\min E$ and the vector $\mathbf{x}$? (15 points)

9

Let's consider a Markov chain on the following state space $\{S_1, S_2, S_3\}$ with the following transition matrix,

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}.$$

a) If we start from the state S_1, what is the probability remaining at state S_1 after three Markov steps? b) What is the stationary distribution of this Markov chain? (20 points)

10

Let W_t be a standard Brownian motion and consider the following quadratic function,

$$g(t, W_t) = \mu t + \sigma^2 (W_t)^2. \tag{3}$$

Calculate the differential dg using the Ito's Lemma,

$$dg = \frac{\partial g}{\partial t}\, dt + \frac{\partial g}{\partial W_t}\, dW_t + \frac{1}{2}\frac{\partial^2 g}{\partial W_t^2}(dW_t)^2.$$

(10 points)

11

In the Langevin equation, the stochastic differential of the velocity dv is given by,

$$dv = -\frac{\gamma}{m} v \, dt + e^{-\frac{\gamma}{m}t} \frac{\sqrt{D}}{m} \, dW_t, \quad (4)$$

where γ is the friction coefficient, and D is the diffusion constant. Calculate the following ensemble average, $\langle v_t \rangle$. (10 points)

12

A pure birth process describes a system where transitions occur sequentially from state j to $j+1$ at a constant rate λ. Given the initial state at $j = 0$, we look at the probability $P_{0j}(t)$ of being in state j at time t. For $P_{01}(t)$, write down the transition equation and find the solution with initial condition $P_{01}(0) = 0$. (20 points)

13

Let's consider the following linear regression with observation (x_i, y_i). To the first order, write down the fixed point equation associated with the following model,

$$y(x|a, b, c) = a \cdot x^b e^{-c \cdot x}, \quad (5)$$

under the Gaussian noise assumption: Recall that,

$$\chi^2(a, b, c) = \sum_i \frac{(y_i - y(x_i \mid a, b, c))^2}{\sigma_i^2} \quad (6)$$

(20 points). *Hint*: you need to minimize the χ^2 with respect to each hyperparameter.

14

Let's consider the famous Prisoner's Dilemma. Two criminals, A and B, are arrested for a serious crime. The police do not have enough evidence to convict them of the main crime, but they do have enough to convict them of a lesser offense. The two prisoners are interrogated separately and cannot communicate with each other.

If both stay silent, they each get 1 year in prison for the lesser offense. If one betrays and the other stays silent, the betrayer goes free, while the silent one gets 3 years in prison. If both betray, they each get 2 years in prison.

a) List all possible outcomes in a table (5 points).

b) Let's first consider a uniform prior where all possible outcomes have an equal probability of occurring. What is the expected prison sentence for each prisoner (10 points).

c) If prisoner B betrays prisoner A with probability p, what is the expected prison sentence for prisoner A?(5 points)

d) A *Nash equilibrium* occurs when neither prisoner can improve their outcome by changing their decision, assuming the other prisoner's decision remains unchanged. Following the above information, what is the expected prison sentence for prisoner A under Nash equilibrium?(10 points)

15

The probability mass function (PMF) of the Poisson distribution is given by,

$$P(x = x_i|\lambda) = \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}.$$

Let's consider n independent observations $x_1, x_2, \ldots, x_n$, the joint likelihood is:

$$L(\lambda) = \prod_{i=1}^{n} \frac{e^{-\lambda}\lambda^{x_i}}{x_i!}.$$

Find the maximum likelihood estimator for λ_{MLE}, and calculate the variance associated with the λ_{MLE}. (20 points)

16

Write a pseudocode that describes the Monte Carlo integration procedure via importance sampling. It should contain both necessary mathematical expressions and important operations, such as "initialize an array", "start a loop", ..., etc. (20 points)

17

(Same question from the practice exam) Show that the first Order Runge-Kutta Method (RK1) is the Euler method at x_0,

$$y_{n+1} = y_n + hF(x_n, y_n),$$

with the following differential equation,

$$y'(x_0) = F(x_0, y_0), \quad y(x_0) = y_0.$$

The Euler method is given by,

$$y(x_0 + h) = hy'(x_0) + \mathcal{O}(h^2).$$

(10 points)

18

Let's consider the following Hamiltonian

$$H(p, q) = \frac{1}{2}(p^2 + q^2).$$

The **symplectic Euler method** is given by,

$$q' = q + \frac{\partial H}{\partial p}\Delta t, \quad p' = p, \qquad (7)$$

$$q' = q, \quad p' = p - \frac{\partial H}{\partial q}\Delta t. \qquad (8)$$

Show that to the first order, the above integrator is symplectic, i.e. to the first order, we have,

$$\tilde{H}(t+\Delta t) = \frac{1}{2}\left(p'^2 + q'^2 + \Delta t p' q'\right) = \tilde{H}(t).$$

(20 points)

19

We are given two discrete distributions,

$$\mu = [0.2, 0.8], \quad \nu = [0.4, 0.6],$$

and we want to find the optimal transport plan $\pi = [\pi_{ij}]$ that minimizes the transportation cost,

$$C(\pi) = \sum_{i,j} \pi_{ij} \cdot c(i, j),$$

where the cost function is given by,

$$c = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

What is the optimal transport plan in this case? *Hint:* 1) You will get a degenerate equation. 2) You have $\pi_{ij} \geq 0$. 3) You should keep tracking the total transport cost and make sure it is minimized. (20 points)

20

What is the difference between a Multilayer perceptron and a recurrent neural network? (5 points)

Extra Credit

What is the main difference between Monte Carlo and quantum Monte Carlo? (5 points)