UCSB, Physics 129AL, Computational Physics: Section Worksheet, Week 3

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Section Participation and Submission Guidelines

Section attendance is required, but you do not need to complete all the work during the section. At each section, the TA will answer any questions that you might have, and you are encouraged to work with others and look for online resources during the section and outside of sections. Unless otherwise stated, the work will be due one week from the time of assignment. The TA will give you 1 point for each task completed. You can see your grades on Canvas.

We will use GitHub for section worksheet submissions. By the due date, you should have a single public repository on GitHub containing all the work you have done for the section work. Finally, upload a screenshot or a .txt file to Canvas with your GitHub username and repository name so the TA knows who you are and which repository you are using for the section.

Remember: talk to your fellow students, work together, and use GPTs. You will find it much easier than working alone. Good luck! All work should be done in the Docker container, and don't forget to commit it to Git!

Task 1: Turing Machine for binary multiplication

Let's look at a Turing machine that preforms binary multiplication,

$$M = (Q = \{q_0, \cdots, q_{20}, q_{halt}\}, \Gamma = \{0, 1\}, \Sigma = \{B, \#, \$, X, Y, 0, 1\}, \delta, q_0). \quad (1)$$

The tape configurations and Turing machine states for the following binary multiplication,

$$101 \times 110 = 11110 \to 5 \times 6 = 30. \quad (2)$$

are given below.

a). Using a similar idea demonstrated on the tape below, you are asked to create a python program that simulates a Turing machine that preforms arbitrary-length ($\mathcal{L}$) binary multiplications, for example,

$$101001010111\cdots\times 101000101,\ \ L_{12,9} = [\mathcal{L}(101001010111), \mathcal{L}(101000101)] = [12, 9], \tag{3}$$

where $L_{a,b}$ is the net length. For a particular The program should produces a ".dat" file, and each row contains an instance of the tape. You should make sure that each filename is identifiable based on the binary string multiplications provided.

b). Use your program to general the tape files for the following two binary multiplication, $101001010111\cdots\times 101000101$ and $101111\cdots\times 101001$. Those files should be accessible on Github.

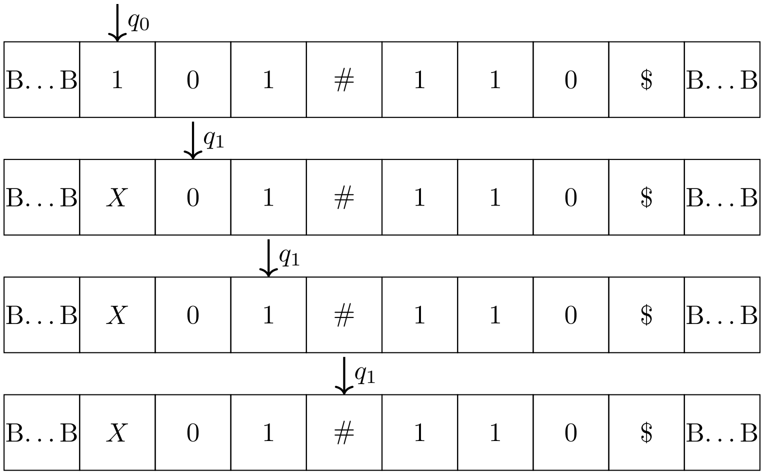
c). The Turing machine that produces the tape below is not optimized, Reduce the size of Q, and draw the corresponding state transition diagram. How many states remains? Report this number in a separated ".txt" file.

d). Let's investigate the computation complexity of the above program, i.e. how many steps n until halting for a total length $L_{a,b}$? Test different binary inputs with the same $L_{a,b}$, and find the statistics associated with $n(L_{a,b})$. In other words, you should be finding the histogram for each n. What are the worst $\max(n)$, best $\max(n)$, and average $\langle n\rangle$ computation complexity for each $L_{a,b}$ for $L_{a,b} = [2, 3], [3, 2], [3, 5], [5, 3], [3, 12], [12, 3]$?

e). Generate a 2D heatmap for the average complexity $\langle n\rangle$ on grid points $L_{a,b} = [a, b], a, b \in Z^+$, and $2 \le a, b \le 30$. Ideally, your program should have $[a, b] = [b, a]$, and why there could be a difference? Hint: think about how Turing machine moves.

f). Design a nondeterministic (probabilistic) Turing machine that reduces the computational complexity and removes the above difference.

Tapes at various steps

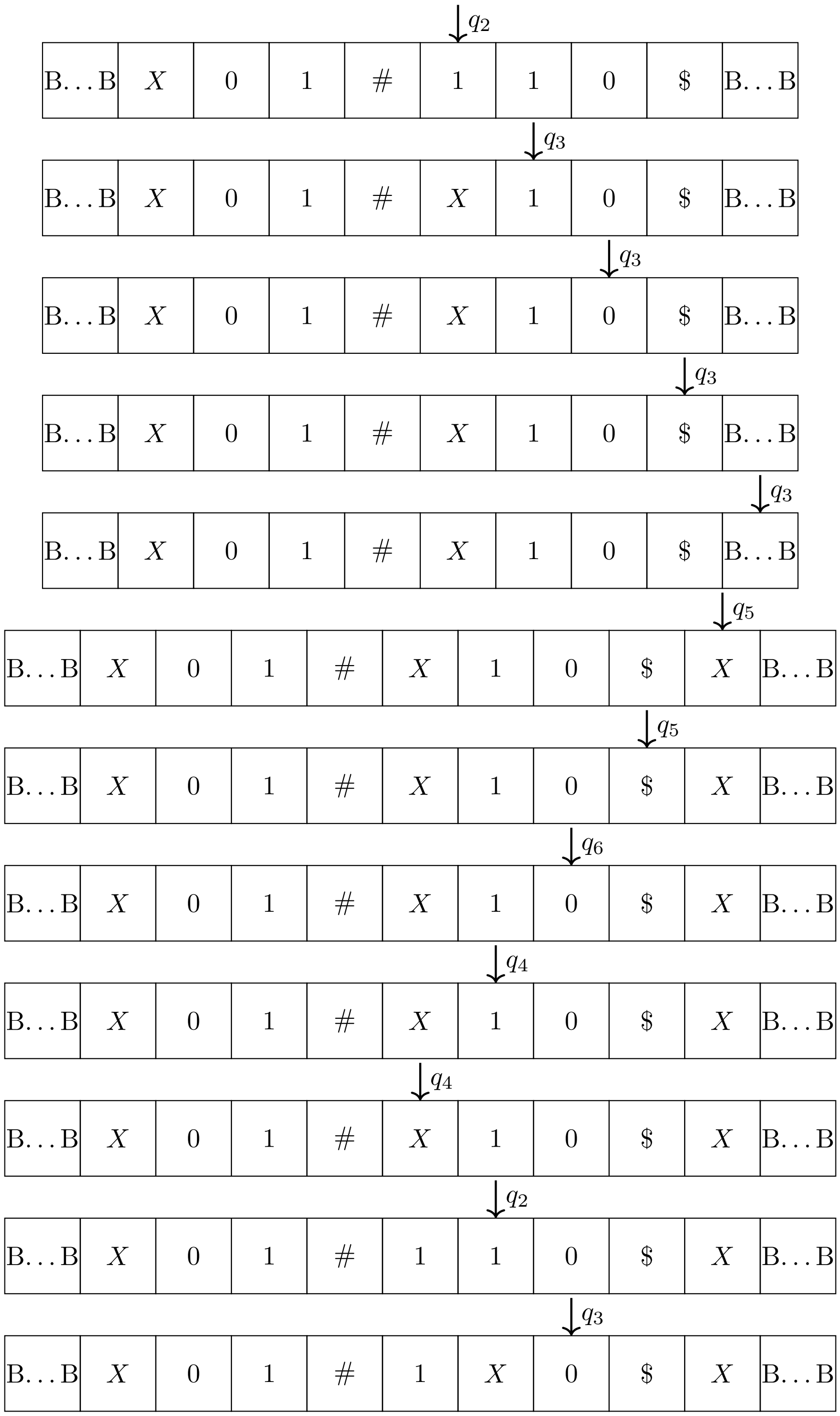


	↓q_0								
B...B	1	0	1	#	1	1	0	$	B...B

		↓q_1							
B...B	X	0	1	#	1	1	0	$	B...B

			↓q_1						
B...B	X	0	1	#	1	1	0	$	B...B

				↓q_1					
B...B	X	0	1	#	1	1	0	$	B...B


q_2
B...B X 0 1 # 1 1 0 \$ B...B
q_3
B...B X 0 1 # X 1 0 \$ B...B
q_3
B...B X 0 1 # X 1 0 \$ B...B
q_3
B...B X 0 1 # X 1 0 \$ B...B
q_3
B...B X 0 1 # X 1 0 \$ B...B
q_5
B...B X 0 1 # X 1 0 \$ X B...B
q_5
B...B X 0 1 # X 1 0 \$ X B...B
q_6
B...B X 0 1 # X 1 0 \$ X B...B
q_4
B...B X 0 1 # X 1 0 \$ X B...B
q_4
B...B X 0 1 # X 1 0 \$ X B...B
q_2
B...B X 0 1 # 1 1 0 \$ X B...B
q_3
B...B X 0 1 # 1 X 0 \$ X B...B

								↓ q_3		
B. . . B	X	0	1	#	1	X	0	$	X	B. . . B

									↓ q_3	
B. . . B	X	0	1	#	1	X	0	$	X	B. . . B

										↓ q_3
B. . . B	X	0	1	#	1	X	0	$	X	B. . . B

										↓ q_5	
B. . . B	X	0	1	#	1	X	0	$	X	X	B. . . B

									↓ q_5		
B. . . B	X	0	1	#	1	X	0	$	X	X	B. . . B

								↓ q_5			
B. . . B	X	0	1	#	1	X	0	$	X	X	B. . . B

							↓ q_6				
B. . . B	X	0	1	#	1	X	0	$	X	X	B. . . B

						↓ q_4					
B. . . B	X	0	1	#	1	X	0	$	X	X	B. . . B

							↓ q_2				
B. . . B	X	0	1	#	1	1	0	$	X	X	B. . . B

								↓ $q_{3'}$			
B. . . B	X	0	1	#	1	1	Y	$	X	X	B. . . B

									↓ $q_{3'}$		
B. . . B	X	0	1	#	1	1	Y	$	X	X	B. . . B

										↓ $q_{3'}$	
B. . . B	X	0	1	#	1	1	Y	$	X	X	B. . . B

											$q_{3'}$
B. . . B	X	0	1	#	1	1	Y	$	X	X	B. . . B

											q_5	
B. . . B	X	0	1	#	1	1	Y	$	X	X	Y	B. . . B

										q_5		
B. . . B	X	0	1	#	1	1	Y	$	X	X	Y	B. . . B

									q_5			
B. . . B	X	0	1	#	1	1	Y	$	X	X	Y	B. . . B

								q_5				
B. . . B	X	0	1	#	1	1	Y	$	X	X	Y	B. . . B

							q_6					
B. . . B	X	0	1	#	1	1	Y	$	X	X	Y	B. . . B

								q_8				
B. . . B	X	0	1	#	1	1	Y	$	X	X	Y	B. . . B

									q_8			
B. . . B	X	0	1	#	1	1	Y	$	X	X	Y	B. . . B

								q_7				
B. . . B	X	0	1	#	1	1	Y	$	1	X	Y	B. . . B

							q_7					
B. . . B	X	0	1	#	1	1	Y	$	1	X	Y	B. . . B

						q_7						
B. . . B	X	0	1	#	1	1	0	$	1	X	Y	B. . . B

					q_7							
B. . . B	X	0	1	#	1	1	0	$	1	1	Y	B. . . B

				↓q_7								
B…B	*X*	0	1	#	1	1	0	$	1	*X*	*Y*	B…B
			↓q_9									
B…B	*X*	0	1	#	1	1	0	$	1	*X*	*Y*	B…B
		↓q_9										
B…B	*X*	0	1	#	1	1	0	$	1	*X*	*Y*	B…B
	↓q_9											
B…B	*X*	0	1	#	1	1	0	$	1	*X*	*Y*	B…B
		↓q_0										
B…B	*X*	0	1	#	1	1	0	$	1	*X*	*Y*	B…B
			↓q_{10}									
B…B	*X*	*Y*	1	#	1	1	0	$	1	*X*	*Y*	B…B
				↓q_{10}								
B…B	*X*	*Y*	1	#	1	1	0	$	1	*X*	*Y*	B…B
						…						
				↓q_{11}								
B…B	*X*	*Y*	1	#	1	1	0	$	1	1	*Y*	B…B
			↓q_{11}									
B…B	*X*	*Y*	1	#	1	1	0	$	1	1	*Y*	B…B
				↓q_{12}								
B…B	*X*	*Y*	*X*	#	1	1	0	$	1	1	*Y*	B…B
					↓q_{13}							
B…B	*X*	*Y*	*X*	#	1	1	0	$	1	1	*Y*	B…B
						↓q_{14}						
B…B	*X*	*Y*	*X*	#	*X*	1	0	$	1	1	*Y*	B…B

							↓ q_{14}					
B. . . B	X	Y	X	#	X	1	0	$	1	1	Y	B. . . B

								↓ q_{14}				
B. . . B	X	Y	X	#	X	1	0	$	1	1	Y	B. . . B

									↓ q_{15}			
B. . . B	X	Y	X	#	X	1	0	$	1	1	Y	B. . . B

										↓ q_{15}		
B. . . B	X	Y	X	#	X	1	0	$	1	1	Y	B. . . B

											↓ q_{15}	
B. . . B	X	Y	X	#	X	1	0	$	1	1	Y	B. . . B

										↓ q_{16}		
B. . . B	X	Y	X	#	X	1	0	$	1	1	1	B. . . B

									↓ q_{16}			
B. . . B	X	Y	X	#	X	1	0	$	1	1	1	B. . . B

								↓ q_{16}				
B. . . B	X	Y	X	#	X	1	0	$	1	1	1	B. . . B

							↓ q_{16}					
B. . . B	X	Y	X	#	X	1	0	$	1	1	1	B. . . B

						↓ q_{17}						
B. . . B	X	Y	X	#	X	1	0	$	1	1	1	B. . . B

					↓ q_{17}							
B. . . B	X	Y	X	#	X	1	0	$	1	1	1	B. . . B

						↓ q_{13}						
B. . . B	X	Y	X	#	1	1	0	$	1	1	1	B. . . B

							q_{14}					
B...B	X	Y	X	#	1	X	0	$	1	1	1	B...B

								q_{14}				
B...B	X	Y	X	#	1	X	0	$	1	1	1	B...B

									q_{15}			
B...B	X	Y	X	#	1	X	0	$	1	1	1	B...B

										q_{15}		
B...B	X	Y	X	#	1	X	0	$	1	1	1	B...B

											q_{15}	
B...B	X	Y	X	#	1	X	0	$	1	1	1	B...B

												q_{15}
B...B	X	Y	X	#	1	X	0	$	1	1	1	B...B

												q_{16}	
B...B	X	Y	X	#	1	X	0	$	1	1	1	X	B...B

											q_{16}		
B...B	X	Y	X	#	1	X	0	$	1	1	1	X	B...B

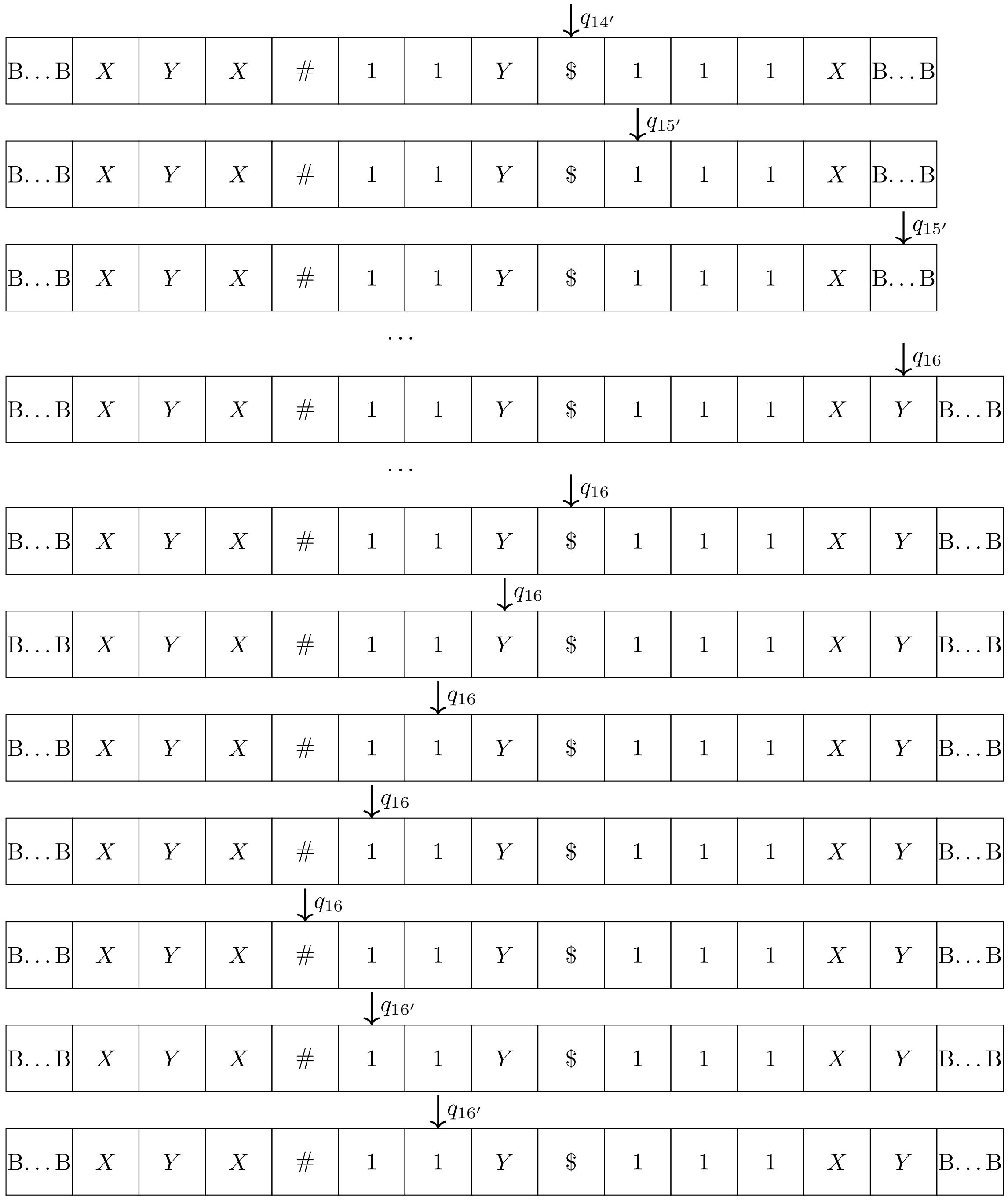
...

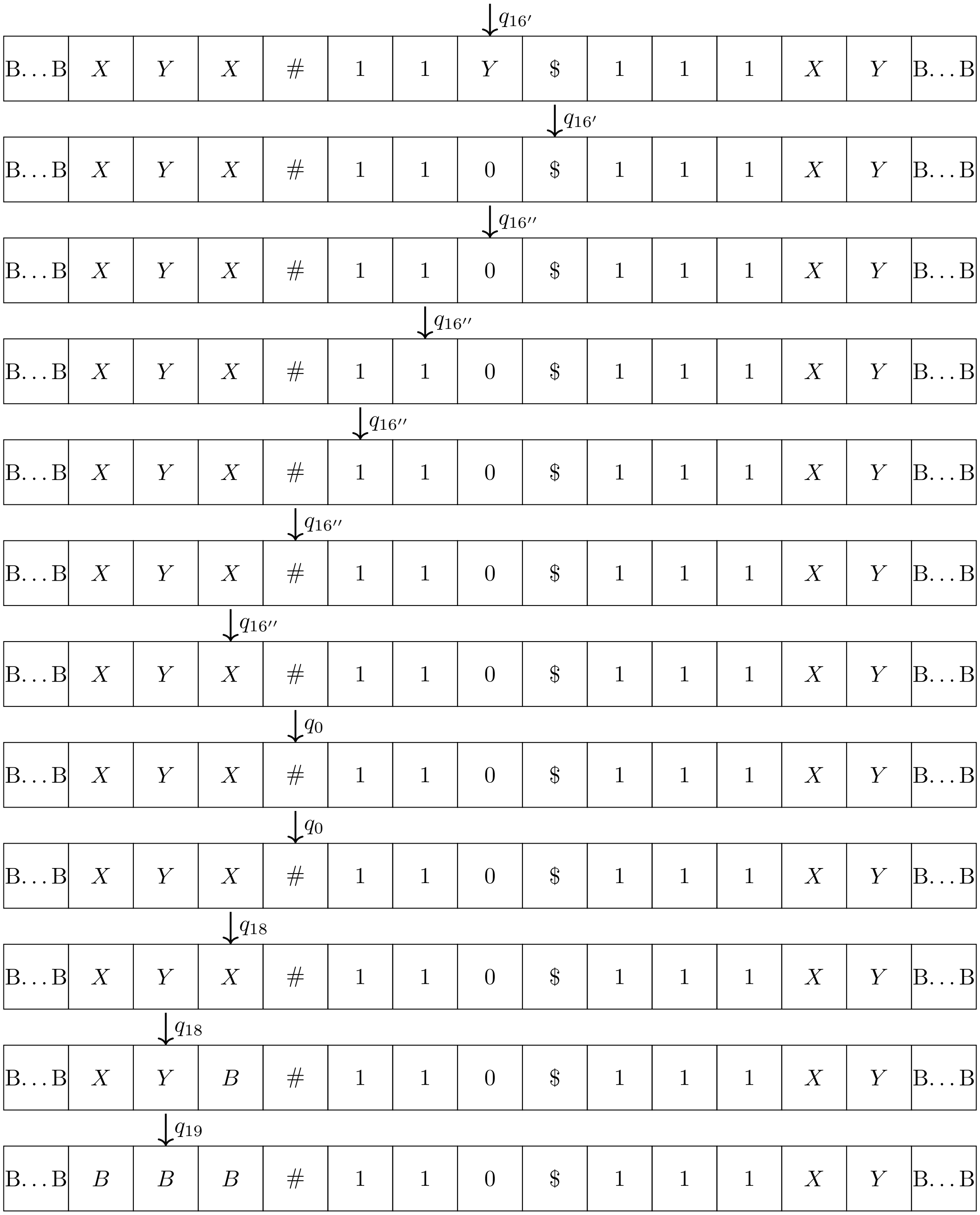
								q_{16}					
B...B	X	Y	X	#	1	X	0	$	1	1	1	X	B...B

							q_{16}						
B...B	X	Y	X	#	1	X	0	$	1	1	1	X	B...B

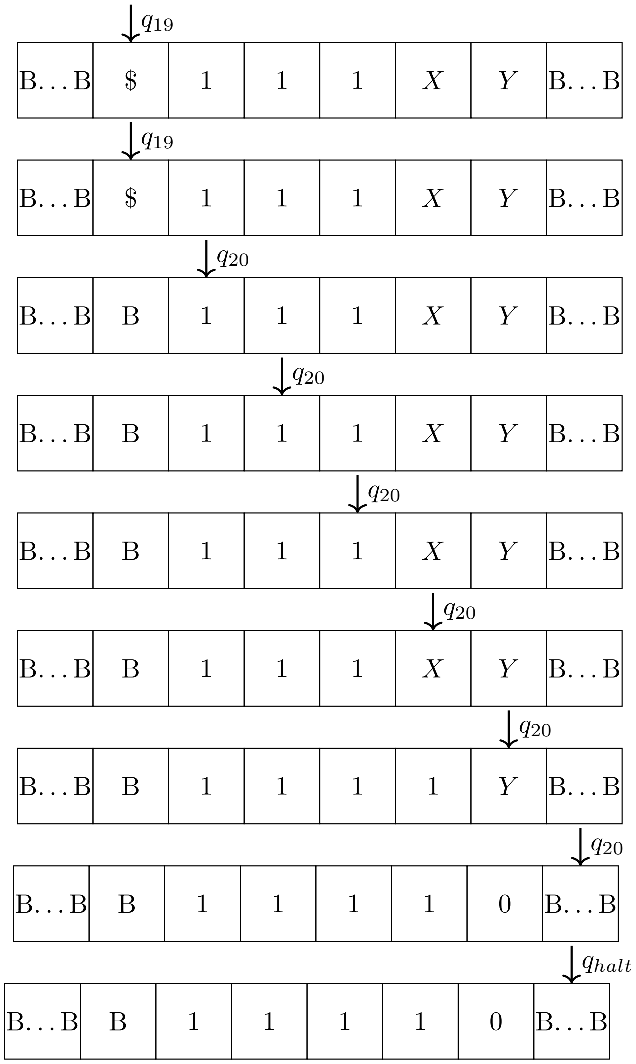
						q_{17}							
B...B	X	Y	X	#	1	X	0	$	1	1	1	X	B...B

							q_{13}						
B...B	X	Y	X	#	1	1	0	$	1	1	1	X	B...B


$q_{14'}$
B. . . B X Y X # 1 1 Y \$ 1 1 1 X B. . . B
$q_{15'}$
B. . . B X Y X # 1 1 Y \$ 1 1 1 X B. . . B
$q_{15'}$
B. . . B X Y X # 1 1 Y \$ 1 1 1 X B. . . B
. . .
q_{16}
B. . . B X Y X # 1 1 Y \$ 1 1 1 X Y B. . . B
. . .
q_{16}
B. . . B X Y X # 1 1 Y \$ 1 1 1 X Y B. . . B
q_{16}
B. . . B X Y X # 1 1 Y \$ 1 1 1 X Y B. . . B
q_{16}
B. . . B X Y X # 1 1 Y \$ 1 1 1 X Y B. . . B
q_{16}
B. . . B X Y X # 1 1 Y \$ 1 1 1 X Y B. . . B
q_{16}
B. . . B X Y X # 1 1 Y \$ 1 1 1 X Y B. . . B
$q_{16'}$
B. . . B X Y X # 1 1 Y \$ 1 1 1 X Y B. . . B
$q_{16'}$
B. . . B X Y X # 1 1 Y \$ 1 1 1 X Y B. . . B


$q_{16'}$
B...B X Y X # 1 1 Y $ 1 1 1 X Y B...B
$q_{16'}$
B...B X Y X # 1 1 0 $ 1 1 1 X Y B...B
$q_{16''}$
B...B X Y X # 1 1 0 $ 1 1 1 X Y B...B
$q_{16''}$
B...B X Y X # 1 1 0 $ 1 1 1 X Y B...B
$q_{16''}$
B...B X Y X # 1 1 0 $ 1 1 1 X Y B...B
$q_{16''}$
B...B X Y X # 1 1 0 $ 1 1 1 X Y B...B
$q_{16''}$
B...B X Y X # 1 1 0 $ 1 1 1 X Y B...B
q_0
B...B X Y X # 1 1 0 $ 1 1 1 X Y B...B
q_0
B...B X Y X # 1 1 0 $ 1 1 1 X Y B...B
q_{18}
B...B X Y X # 1 1 0 $ 1 1 1 X Y B...B
q_{18}
B...B X Y B # 1 1 0 $ 1 1 1 X Y B...B
q_{19}
B...B B B B # 1 1 0 $ 1 1 1 X Y B...B

. . .


q_{19}
B. . . B $ 1 1 1 X Y B. . . B
q_{19}
B. . . B $ 1 1 1 X Y B. . . B
q_{20}
B. . . B B 1 1 1 X Y B. . . B
q_{20}
B. . . B B 1 1 1 X Y B. . . B
q_{20}
B. . . B B 1 1 1 X Y B. . . B
q_{20}
B. . . B B 1 1 1 X Y B. . . B
q_{20}
B. . . B B 1 1 1 1 Y B. . . B
q_{20}
B. . . B B 1 1 1 1 0 B. . . B
q_{halt}
B. . . B B 1 1 1 1 0 B. . . B