Problem sets for BUEC 333
Part 2: Linear regression, one covariate

I will indicate the relevant exercises for each week at the end of the Wednesday lecture. Numbered exercises are back-of-chapter exercises from Stock and Watson. Try to complete the exercises before going to the tutorials. In the tutorials, the TAs will help you if you have any difficulties.

Mechanics and fit

1. (Wooldridge, Exercise 2.3) Table 1 presents a random sample of 8 observations on students' ACT (American College Test) and GPA (grade point average) scores. Consider the model

$$GPA_i = \beta_0 + \beta_1 ACT + u_i$$

   a) Compute the OLS estimators $\hat{\beta}_1$ and $\hat{\beta}_0$.

   b) Does $\hat{\beta}_0$ have a useful interpretation?

   c) What is the interpretation of $\hat{\beta}_1$?

   d) For each of the 8 observations, compute $\hat{u}_i$. Then, compute $\sum_{i=1}^{8} \hat{u}_i$

   e) What is the predicted value of GPA when $ACT = 20$?

   f) How much of the variation in GPA for these eight students is explained by ACT?

2. Consider the data in Table . Note that this data has some observations that depend on k.

   a) Compute R^2 if $k = 2$.

   b) Can you compute $\hat{\beta}_0$ and $\hat{\beta}_1$ for arbitrary values of k?

   c) Compute R^2 if $k = 1$.

   d) Compute R^2 if $k = 3$.

   e) Compute R^2 if $k = 10$.

   f) Can you explain what happens when k becomes very large?

3. Stock and Watson, 4.1 (a)-(c).

Student	GPA	ACT
1	2.8	21
2	3.4	24
3	3.0	26
4	3.5	27
5	3.6	29
6	3.0	25
7	2.7	25
8	3.7	30

Table 1: Students's ACT and GPA scores

i	X_i	Y_i
1	-3	-3
2	-2	$-k$
3	-1	-1
4	1	1
5	2	k
6	3	3

Table 2: Artificial data.

4. Stock and Watson, 4.2.
5. Stock and Watson, 4.3.
6. Stock and Watson, Exercise 4.9.

Estimation and inference

1. Stock and Watson, Exercise 4.4.
2. Stock and Watson, Exercise 4.6.
3. Stock and Watson, Exercise 4.7.
4. Stock and Watson, Exercise 5.2 (a)-(d).
5. Stock and Watson, Exercise 5.3
6. (Final, Summer 2014) Consider the following estimated regression equation that describes the relationship between a student's weight and height:

$$\widehat{WEIGHT} = 100 + 6\,HEIGHT$$

   a) A student has height 5. What is the regression's prediction for that student's weight?

   b) In the sample, the sample average of HEIGHT is 4. What can you say about the sample average for WEIGHT?

   Now, an additional variable is included, is ID, a student's SFU ID. Obviously, this is a nonsensical variable that is not in any way related to a student's weight. The new estimated regression equation is

$$\widehat{WEIGHT} = 100 + 6\,HEIGHT + 0.02\,ID$$

   c) Someone's weight has nothing to do with their SFU ID. Still, the R^2 went up from 0.74 to 0.75. How is this possible?

   d) On the other hand, the adjusted R-squared $\bar{R}^2$ went down from 0.73 to 0.72. Explain how it is possible that the R^2 can go up while the $\bar{R}^2$ goes down.

   e) If the student's SFU ID number is not related to a student's weight, should the estimated coefficient not be equal to 0? How could it be that it is 0.02?

7. (Final, Summer 2012) This question tests your understanding of the concepts involved in regression analysis. Consider the model with one regressor,

$$Y_i = \beta_0 + \beta_1 X_i + u_i.$$

   a) Write down the population regression function. Write down the sample regression function.

b) What does Least Squares Assumption 1, $\mathrm{E}(u_i|X_i) = 0$, mean? In your answer, use the words "other factors".

c) Describe the other two "Least Squares Assumptions"? You only need to use one sentence for each assumption.

d) Is β_1 a random variable? Is $\hat{\beta}_1$ an estimator? Is u_i a random variable? Is $\hat{u}_i$ a random variable? (First: 4x yes/no. Then: short explanations if you are not sure about your yes/no answers.)

Prediction and forecasting

1. Exercise 14.1, page 571.
2. 14.7 (a), (b), and (d), on page 574.
3. Consider the time series $(-1, 1, -1)$.
   a) Compute the first autocovariance.
   b) Compute $\hat{\rho}_1$.
   c) Can you construct a time series that has its first autocovariance equal to zero?
4. What is the difference between a prediction and a forecast?