BUEC 333, Spring 2012
Midterm, Feb 24, 10:30-12:30

Instructor: Chris Muris

This midterm consists of 10 questions. Each question is worth 1 point.

Multiple-choice

Each question has zero, one, or multiple correct answers. If none of the statements are correct, write "None of the statements are true." If only answer (b) is correct, write "(b)". If (a), (b) and (d) are correct, write "(a), (b) and (d)". If you name all the correct statements, you earn 1 point. If you do not name all the correct statements, you earn 0 points.

A random variable X has mean μ and variance σ^2. You have a random sample $(X_1, X_2, \cdots, X_N)$ of measurements of X. The sample average is $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$, and the sample variance is $s^2 = \frac{\sum_{i=1}^{N}(X_i - \bar{X})^2}{N-1}$.

1. Which of the following statements about $\bar{X}$ and μ is/are true?

   (a) $\bar{X} = \mu$.

   (b) $\text{Var}(\bar{X}) = \text{Var}(\mu)$.

   (c) $\text{E}(\bar{X}) = \text{E}(X) = \mu$.

2. Which of the following statements is/are true?

   (a) $\text{Var}(\bar{X}) = \sigma^2$.

   (b) $\text{Var}(\bar{X}) = \sigma^2/N$.

   (c) $\sigma^2 = s^2$.

3. Consider the quantity $\frac{\bar{X}-\mu}{s/\sqrt{N}}$. Which of the following statements is/are true?

   (a) This quantity is called a $Z-$statistic.

   (b) It has a normal distribution with $N-1$ degrees of freedom.

   (c) It is a random variable.

4. This question is about the coefficient of determination, R^2. Which of the following statements is/are true?

   (a) $R^2 = \frac{\text{explained sum of squares}}{\text{total sum of squares}}$

   (b) $R^2 = 1 - \frac{\text{residual sum of squares}}{\text{total sum of squares}}$

   (c) The R^2 never decreases when you increase the number of explanatory variables.

5. Let $\hat{\beta}$ be the OLS estimator of a population regression coefficient β. You know that, given the assumptions that make up the "Classical model", the OLS estimator is *BLUE*. The "B" in BLUE stands for "best". What do we mean by "best"?

   (a) The estimated regression coefficient $\hat{\beta}$ is equal to the population regression coefficient β.

   (b) The expected value of the $\hat{\beta}$ is equal to β.

   (c) The variance of the population regression coefficient, β, is as low as can be.

   (d) Among all unbiased estimators for β, the OLS estimator $\hat{\beta}$ has the lowest variance.

The midterm continues on the next page.

Non-multiple-choice

For each question, provide: (1) the answer (a number); (2) a step-by-step explanation of how you derived the answer. An answer without a full explanation earns 0 points. A wrong answer with a right explanation may earn you 0.5 points. A correct answer with correct explanation earns 1 point.

Assume that you have a random sample $(2, 4, 7, 9, 5, 7, 6, 3, 8)$.

6. For this sample, compute $\bar{X}$.
7. Compute the $t-$statistic, assuming that $\mu = 3$.

Consider the linear regression model, with equation $Y = \beta_0 + \beta_1 X + \epsilon$. Now assume that you have the following data:

X	Y
3	2.2
4	2.4
6	3.3
7	4.1
5	3.1
8	4.3

8. For this sample, Compute $\hat{\beta}_1$, the OLS estimator for the intercept β_1. Remember that
$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^{N}(X_i - \bar{X})(X_i - \bar{X})}.$$
9. For this sample, Compute $\hat{\beta}_0$, the OLS estimator for the intercept β_0. If you did not manage to answer question 8, assume that $\hat{\beta}_1 = 0.5$.
10. One of your data points is $(Y_6 = 4.3,\ X_6 = 8)$. Compute the residual e_6. If you did not manage to answer question 8 and/or 9, then assume that $\hat{\beta}_1 = \hat{\beta}_0 = 0.5$.