Midterm BUEC 333

June 19, 2014, 12:30-14:20

Handing in your midterm

- On the **front page of your answer sheet**, write (i) your name; (ii) your student ID. On the **front page of this document** (the questions), write: (i) your name; and (ii) your student ID.

- If you finish this exam **before 14:00**, come forward and hand in the documents: (i) the answer sheet, (ii) your answer sheet. If you finish it **after 14:00**, please stay seated until we come by to pick things up at 14:20.

Unless otherwise noted, provide: (i) the answer; (ii) an explanation. A correct answer with correct explanation earns 1 point per question.

1 Probability

You have a random sample $(X_1, X_2, \cdots, X_n)$ of measurements of X. The random variable X has mean $\mu_x = E(X)$ and variance $\sigma_x^2 = Var(X)$. The sample average is $\bar{X} = \frac{1}{n}\sum_{i=1}^n X_i$.

1. Which one of the following three statements about $\bar{X}$ and μ_x is true?

   (a) $\bar{X} = \mu_x$.

   (b) $Var(X_1) = Var(\mu_x)$.

   (c) $E(\bar{X}) = E(X_1) = \mu_x$.

2. Show that $Var(\bar{X}) = \sigma_x^2/n$. Carefully state what you are doing at each step of the derivation.

Consider the joint probability distribution presented in Table 1.

3. For each of the following statements (a)-(h), state whether the statement is true. No explanation necessary. Correct answer: 0.25. No answer: 0. Incorrect: -0.25 points.

   (a) $P(X_2 = 1 | X_1 = 3) = 1$

		X_1		
		1	2	3
	1	0.1	0.2	0.25
X_2	2	0.1	0.2	0
	3	0.05	0.1	0

Table 1: Joint probability distribution.

(b) $P(X_2 = 1|X_1 = 2) = 0.4$

(c) $P(X_2 = 3|X_1 = 2) = 0.1$

(d) X_1 and X_2 are uncorrelated

(e) X_1 and X_2 are independent

(f) $E(X_1) > 1$

(g) $P(X_2 = 2) = 0.1$

(h) $P(X_1 \leq 2) = 0.75$

2 Statistics

Assume that you have a random sample $(2, 3, 4)$ for a RV X with mean μ_x and variance σ_x^2.

4. For this sample, compute $\bar{X}$.

5. For this sample, compute the sample variance.

6. Construct a 90%-CI for μ_x based on the sample mean and sample variance.

7. Interpret the numbers in your answer to question 6.

3 Regression analysis

8. This question is about the error term, u_i, in regression analysis.

   (a) Why do we include an error term in our regression equation?

   (b) What is the difference between an error term and a residual?

9. Using the population regression equation *or* the estimated regression equation, write down *precisely* what the OLS estimator minimizes. Your answer should include "error terms" or "residuals".

Consider the linear regression model, with equation $Y_i = \beta_0 + \beta_1 X_i + u_i$. You are given the following random sample of size $n = 3$:

X_i	Y_i	i
2	6	1
1	3	2
3	3	3

10. For this sample, Compute $\hat{\beta}_1$, the OLS estimator for the intercept β_1.

11. For this sample, Compute $\hat{\beta}_0$, the OLS estimator for the intercept β_0. If you did not manage to the previous question, assume that $\hat{\beta}_1 = 0.5$.

12. Compute the R^2. Interpret the number you obtain.