BUEC 333, Test 1

June 10, 2015, 14:30-17:20

Important!

- On the front page of **your answer sheet**, write (i) your name; (ii) your student ID.
- On the front page of **this document** (the questions), write: (i) your name; and (ii) your student ID.
- Once you **finish** this exam, **signal** it to us, and we will come to pick up your exam. Stay seated until somebody comes to cllect your exam.
- Unless otherwise noted, provide: (i) the answer; (ii) an explanation. No explanation = no points. A correct answer with correct explanation earns 1 point for each subquestion.

1 Probability

1. Consider a random variable X with sample space $\{-a, a\}$ and probability distribution given by $P(X=-a)$=P$(X=a)=0.5$.
   (a) Compute $E(X)$.
   (b) Compute $\text{Var}(X)$.
   (c) Compute $\text{Var}\left(-\frac{1}{a}X\right)$.

2. Let X and Y be two RVs, both with sample space $\{0,1\}$. The joint probabilities are all equal:
$$P(X=0,Y=0)=P(X=0,Y=1)=P(X=1,Y=0)=P(X=1,Y=1)=0.25.$$
   (a) Compute $E(X|\,Y=0)$.
   (b) What is $\text{Cov}(X,Y)$?

3. **PS.8** Let X and Y be two RVs, both with sample space $\{0,10\}$. You know that $P(X=10)=0.3$ and $P(Y=10)=0.4$.
   (a) Assume that X and Y are independent. Compute $P(X=10,Y=0)$.
   (b) Assume that X and Y are independent. Compute $P(X=10|Y=0)$.
   (c) If X and Y are not independent, can you compute $P(X=10,Y=0)$? Explain.
   (d) Assume that $P(X=10,Y=10)=0.1$. Compute $E[X|\,Y=10]$.

4. Let $(X_1, \cdots, X_n)$ be a random sample from a distribution with mean $E(X) = \mu_x$ and $\text{Var}(X) = \sigma^2$.

   (a) Can you express $E(X_1)$ in terms of μ_x?

   (b) Express $\text{Var}(X_1 + X_2)$ in terms of σ^2.

2 Statistics

5. Assume that you have a random sample $(7, 8, 9)$ for a RV X with mean μ_x and variance σ_x^2.

   (a) For this sample, compute $\bar{X}$.

   (b) For this sample, compute the sample variance.

   (c) Construct a 95%-CI for μ_x based on the sample mean and sample variance. Assume that the Central Limit Theorem applies, so that

$$0.95 = P\left(-1.96 < \frac{\bar{X} - \mu_x}{SE\left(\bar{X}\right)} < 1.96\right)$$

   (d) What is the coverage probability of your confidence interval?

   (e) Interpret the numbers in your answer to question (c).

6. We have a random sample of size $n = 10$, $(X_1, X_2, \cdots, X_{10})$ from a distribution X with mean μ_x and variance σ_x^2. Consider two estimators for μ_x. First, the sample average $\bar{X} = \frac{1}{10}(X_1 + \cdots + X_{10})$. We know that $E\left(\bar{X}\right) = \mu_x$ and that $\text{Var}\left(\bar{X}\right) = \sigma^2/10$. Second, the average for the odd half of the sample: $\tilde{X} = \frac{1}{5}(X_1 + X_3 + X_5 + X_7 + X_9)$.

   (a) Is $\tilde{X}$ a random variable?

   (b) Is $\bar{X}$ unbiased?

   (c) Is $\tilde{X}$ unbiased?

   (d) Which of the two estimators is more efficient?