

# Midterm BUEC 333

June 19, 2014, 12:30-14:20

## Handing in your midterm

- On the **front page of your answer sheet**, write (i) your name; (ii) your student ID. On the **front page of this document** (the questions), write: (i) your name; and (ii) your student ID.
- If you finish this exam **before 14:00**, come forward and hand in the documents: (i) the answer sheet, (ii) your answer sheet. If you finish it **after 14:00**, please stay seated until we come by to pick things up at 14:20.

*Unless otherwise noted, provide: (i) the answer; (ii) an explanation. A correct answer with correct explanation earns 1 point per question.*

## 1 Probability

You have a random sample  $(X_1, X_2, \dots, X_n)$  of measurements of  $X$ . The random variable  $X$  has mean  $\mu_x = E(X)$  and variance  $\sigma_x^2 = Var(X)$ . The sample average is  $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ .

1. Which one of the following three statements about  $\bar{X}$  and  $\mu_x$  is true?
  - (a)  $\bar{X} = \mu_x$ .
  - (b)  $Var(X_1) = Var(\mu_x)$ .
  - (c)  $E(\bar{X}) = E(X_1) = \mu_x$ .
2. Show that  $Var(\bar{X}) = \sigma_x^2/n$ . Carefully state what you are doing at each step of the derivation.

Consider the joint probability distribution presented in Table 1.

3. For each of the following statements (a)-(h), state whether the statement is true. No explanation necessary. Correct answer: 0.25. No answer: 0. Incorrect: -0.25 points.

- (a)  $P(X_2 = 1 | X_1 = 3) = 1$

		$X_1$		
		1	2	3
$X_2$	1	0.1	0.2	0.25
	2	0.1	0.2	0
	3	0.05	0.1	0

Table 1: Joint probability distribution.

- (b)  $P(X_2 = 1|X_1 = 2) = 0.4$
- (c)  $P(X_2 = 3|X_1 = 2) = 0.1$
- (d)  $X_1$  and  $X_2$  are uncorrelated
- (e)  $X_1$  and  $X_2$  are independent
- (f)  $E(X_1) > 1$
- (g)  $P(X_2 = 2) = 0.1$
- (h)  $P(X_1 \leq 2) = 0.75$

## 2 Statistics

Assume that you have a random sample  $(2, 3, 4)$  for a RV  $X$  with mean  $\mu_x$  and variance  $\sigma_x^2$ .

4. For this sample, compute  $\bar{X}$ .
5. For this sample, compute the sample variance.
6. Construct a 90%-CI for  $\mu_x$  based on the sample mean and sample variance.
7. Interpret the numbers in your answer to question 6.

## 3 Regression analysis

8. This question is about the error term,  $u_i$ , in regression analysis.
  - (a) Why do we include an error term in our regression equation?
  - (b) What is the difference between an error term and a residual?
9. Using the population regression equation *or* the estimated regression equation, write down *precisely* what the OLS estimator minimizes. Your answer should include “error terms” or “residuals”, as well as a precise mathematical expression for what is being minimized.

Consider the linear regression model, with equation  $Y_i = \beta_0 + \beta_1 X_i + u_i$ . You are given the following random sample of size  $n = 3$ :

$X_i$	$Y_i$	$i$
2	6	1
1	3	2
3	3	3

10. For this sample, Compute  $\hat{\beta}_1$ , the OLS estimator for the intercept  $\beta_1$ .
11. For this sample, Compute  $\hat{\beta}_0$ , the OLS estimator for the intercept  $\beta_0$ . If you did not manage to the previous question, assume that  $\hat{\beta}_1 = 0.5$ .
12. Compute the  $R^2$ . Interpret the number you obtain.