Out of 25 pts

D100 Test 1

5pts

Week 1

1 $P(Y=0) = P(Y=0, X=1) + \cdots + P(Y=0, X=6)$ ①

$= \frac{1}{12} + \cdots + \frac{1}{12} = \boxed{\frac{1}{2}}$

$\frac{1}{12} / \frac{2}{15} = \frac{1}{12} \cdot \frac{15}{2}$

$\frac{15}{24} = \boxed{\frac{5}{8}}$

$P(X=3) = P(Y=0, X=3) + P(Y=1, X=3) = \boxed{\frac{1}{12} + \frac{1}{20} =} = \boxed{\frac{2}{15}} \approx .1\bar{3}$

2. $P(X=6 \mid Y=1) = P(X=6, Y=1) / P(Y=1) = 1/4 / 1/2 = \boxed{1/2}$ ①

$\frac{1}{12} + \frac{1}{20} =$

3 $P(Y=0 \mid X=4) = P(Y=0, X=4) / P(X=4) = \boxed{1/12 / \left(\frac{1}{12} + \frac{1}{20}\right)}$ ①

$\boxed{5/8} =$

$\frac{20}{240} + \frac{12}{240} = \frac{32}{240}$

$\frac{32}{240} = \frac{4}{30} = \frac{2}{15}$

$\frac{2}{15} \approx .13\bar{3}$

4 $E(X \mid Y=0) = 1 \cdot P(X=1 \mid Y=1) + \cdots + 6 \cdot P(X=6 \mid Y=1)$ ①

$= 3.5$

$P(X \mid Y=1) = \begin{cases} 1/20 / (1/2) & \text{if } X=1 & 1/10 \\ \vdots & & \\ 1/20 / (1/2) & \text{if } X=5 & 1/10 \\ 1/4 / (1/2) & \text{if } X=6 & 1/2 \end{cases}$

so $E(X \mid Y=1) = \frac{1}{2} \cdot 6 + \frac{1}{10}(1 + \cdots + 5) = \boxed{4.5}$ $\boxed{\frac{45}{10}}$

$E(X \mid Y=0) = \boxed{3.5}$ $\boxed{\frac{21}{6}}$

5 $E(X) = E[E[X \mid Y]] = \frac{1}{2} \underbrace{3.5}_{E[X \mid Y=0]} + \frac{1}{2} \cdot \underbrace{4.5}_{E[X \mid Y=1]} = \boxed{4}$ ①

$E[X] = E[E[X \mid Y]] = Pr(Y=0) \cdot E[X \mid Y=0] + Pr(Y=1) \cdot E[X \mid Y=1] = \underline{4}$

Ans.

4pts

Week 2

1. $\boxed{Cov(Y_i, Y_j)} = Corr(Y_i, Y_j) \times \sigma_{Y_i} \times \sigma_{Y_j}$ ①

$= \boxed{\rho \sigma_Y^2}$

2. $E(\bar{Y}) = E\left[\frac{1}{2}Y_1 + \frac{1}{2}Y_2\right] = \frac{1}{2}E(Y_1) + \frac{1}{2}E(Y_2) = 2 \cdot \frac{1}{2}\mu_Y = \mu_Y$ ①

3. $Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2ab\, Cov(X, Y)$ ①

$Var(\frac{1}{2}Y_1 + \frac{1}{2}Y_2) = \frac{1}{4}\sigma_Y^2 + \frac{1}{4}\sigma_Y^2 + 2 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \rho \sigma_Y^2$

$= \frac{1}{2}\sigma_Y^2 + \frac{1}{2}\rho\sigma_Y^2$

4. Yes. (Check that they know what unbiased mean. ①

Ans.

4pts

Week 3

1. 95%-CI is given by $[\bar{Y}_{male} - 1.96 \cdot SE(\bar{Y}_{male}),$
$\bar{Y}_{male} + 1.96\, SE(\bar{Y}_{male}).]$ (1)

$\bar{Y}_{male} = 2700$

$SE(\bar{Y}_{male}) = S_Y/\sqrt{n} = 100/10 = 10.$

So the CI is $[2700 - 19.6, 2700 + 19.6]$

2. Since $\mu_{male,0} = 2000$ is not in the 95%-CI above, it will be rejected. Reject $H_0: \mu_{male} = 2000$

(Connection between hypothesis tests and confidence intervals.) (1)

3. $\bar{Y}_{male} - \bar{Y}_{female} = 700$

$SE(\bar{Y}_{male} - \bar{Y}_{female}) = \sqrt{\frac{100^2}{100} + \frac{100^2}{100}} = \sqrt{200} = \sqrt{2} \cdot 10$ (1)

95%-CI is $[700 - 1.96 \cdot 10 \cdot \sqrt{2},$
$700 + 1.96 \cdot 10 \cdot \sqrt{2}]$

4. "It is either in there or not, so $\in \{0,1\}$" (1)

Wrong answer: 95%

Ans.

$\boxed{6\text{pts}}$ $\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y}) = -1 \cdot -2 = 2$
$+ 0 \cdot 2 = 2$ $\Big\} = 4$
$+ 0 \cdot 0 = 0$

Week 4 ~~$(x_i-\bar{x})$~~

1. $\hat{\beta}_1 = \frac{2+0+2}{1^2+0^2+1^2} = \frac{4}{2} = 2$ ✓ $(\bar{X}=1, \bar{Y}=6, \text{etc.})$ 2 $\hat{\beta}_0 = 6 - 2(1)$, $\hat{\beta}_0 = \boxed{4}$

2. 0! (They must provide a computation.) 2

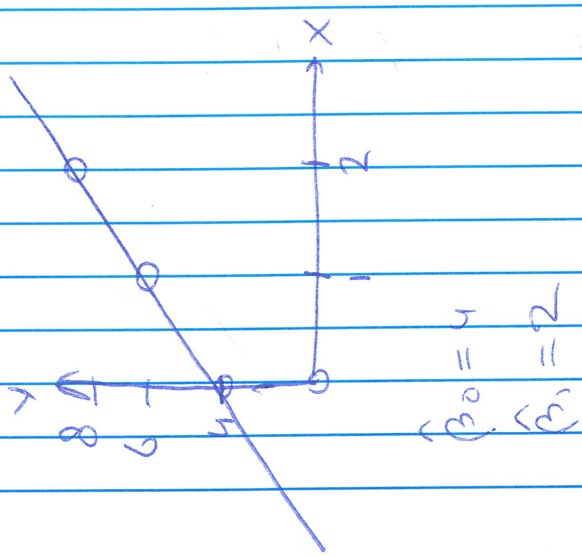
3. Decreases. Under 2, $R^2 = 1 = 1 - \frac{0}{SST}$. 2

With 1 point off the line, it will no longer be a perfect fit, so $R^2 < 1$.

$\hat{\beta}_0 = 6 - 4(1) = \hat{\beta}_0 = 2$

$Y_i = \hat{\beta}_0 + \hat{\beta}_1(X_i) + \varepsilon_i$

2. $4 = 4 + 2(0) + \varepsilon_1 \Rightarrow \varepsilon_1 = 0$ $\varepsilon_1^2 = 4$
$8 = 4 + 2(2) + \varepsilon_2 \Rightarrow \varepsilon_2 = 0$ $\varepsilon_2^2 = 4$ $\Big\}$ SSR = 0!
$6 = 4 + 2(1) + \varepsilon_3 \Rightarrow \varepsilon_3 = 0$



ANS.

5pts

Week 5

ANS.

1. Teacher quality
Parents income
⋮

(be generous)

(1)

2. $E(u_i|X_i)=0$ or "the factors mentioned in 1 are not related to whether a student was assigned to small or large class." (1)

3. Students were randomly assigned! randomization (1)
(~2 sentences?)

4. 95%-CI from $[\hat{\beta}_1 - 1.96\,SE(\hat{\beta}_1), \hat{\beta}_1 + 1.96\,SE(\hat{\beta}_1)]$ (1)

so $[13.9 - 1.96 \cdot 2.5, \ 13.9 + 1.96 \cdot 2.5]$

5. Given that assumption 1 holds, it is the causal effect of being in a small v large class on test scores. (1)

Required answer ?: "β_1 is the change in the expected test score, (conditional on class size) when we move somebody from a large to a small class, ceteris paribus."

1pt

6. Self-Study

a $E[\hat{\beta}_0] = \beta_0$, b yes.

.5pt .5pt

ANS.