Robust Tail Table

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Constraint setting	Relative ratio/Estimated UpperBound	Converage probability
$(0, \chi^2)$	$16.08/8.04 \times 10^{-2}$	1
$(1, \chi^2)$	$5.46/2.73 \times 10^{-2}$	1
$(2, \chi^2)$	$2.99/1.50 \times 10^{-2}$	1
$(0, \text{KS})$	$14.23/7.11 \times 10^{-2}$	1
$(1, \text{KS})$	$6.64/3.32 \times 10^{-2}$	1
$(2, \text{KS})$	$3.95/1.98 \times 10^{-2}$	1

(a) Data sample size = 500.

Constraint setting	Relative ratio/Estimated UpperBound	Converage probability
$(0, \chi^2)$	$14.19/7.10 \times 10^{-2}$	1
$(1, \chi^2)$	$5.00/2.50 \times 10^{-2}$	1
$(2, \chi^2)$	$2.76/1.38 \times 10^{-2}$	1
$(0, \text{KS})$	$11.65/5.83 \times 10^{-2}$	1
$(1, \text{KS})$	$5.90/2.95 \times 10^{-2}$	1
$(2, \text{KS})$	$3.54/1.77 \times 10^{-2}$	1

(b) Data sample size = 800.

Table 1: Tail probablity estimation with Gamma data source. True value is 0.005.

Constraint setting	Relative ratio/Estimated UpperBound	Converage probability
$(0, \chi^2)$	$15.75/7.88 \times 10^{-2}$	1
$(1, \chi^2)$	$6.35/3.18 \times 10^{-2}$	1
$(2, \chi^2)$	$3.93/1.96 \times 10^{-2}$	1
$(0, \text{KS})$	$14.11/7.05 \times 10^{-2}$	1
$(1, \text{KS})$	$7.94/3.97 \times 10^{-2}$	1
$(2, \text{KS})$	$5.06/2.53 \times 10^{-2}$	1

(a) Data sample size = 500.

Constraint setting	Relative ratio/Estimated UpperBound	Converage probability
$(0, \chi^2)$	$16.98/8.49 \times 10^{-2}$	1
$(1, \chi^2)$	$6.58/3.29 \times 10^{-2}$	1
$(2, \chi^2)$	$3.99/2.00 \times 10^{-2}$	1
$(0, \text{KS})$	$11.55/5.78 \times 10^{-2}$	1
$(1, \text{KS})$	$6.98/3.49 \times 10^{-2}$	1
$(2, \text{KS})$	$4.46/2.23 \times 10^{-2}$	1

(b) Data sample size = 800.

Table 2: Tail probablity estimation with Lognormal data source. True value is 0.005.

Constraint setting	Relative ratio/Estimated UpperBound	Converage probability
$(0, \chi^2)$	$14.78/7.39 \times 10^{-2}$	1
$(1, \chi^2)$	$6.39/3.20 \times 10^{-2}$	1
$(2, \chi^2)$	$4.29/2.15 \times 10^{-2}$	1
$(0, \text{KS})$	$13.63/6.81 \times 10^{-2}$	1
$(1, \text{KS})$	$8.85/4.43 \times 10^{-2}$	1
$(2, \text{KS})$	$5.95/2.98 \times 10^{-2}$	1

(a) Data sample size = 500.

Constraint setting	Relative ratio/Estimated UpperBound	Converage probability
$(0, \chi^2)$	$14.92/7.46 \times 10^{-2}$	1
$(1, \chi^2)$	$6.44/3.22 \times 10^{-2}$	1
$(2, \chi^2)$	$4.23/2.11 \times 10^{-2}$	1
$(0, \text{KS})$	$11.15/5.58 \times 10^{-2}$	1
$(1, \text{KS})$	$7.59/3.79 \times 10^{-2}$	1
$(2, \text{KS})$	$5.14/2.57 \times 10^{-2}$	1

(b) Data sample size = 800.

Table 3: Tail probablity estimation with Pareto data source. True value is 0.005.

Constraint setting	Relative ratio/Estimated UpperBound	Converage probability
$(0, \chi^2)$	$1.83/2.07 \times 10^1$	1
$(1, \chi^2)$	$1.35/1.53 \times 10^1$	1
$(2, \chi^2)$	$1.27/1.44 \times 10^1$	1

(a) Data sample size = 500.

Constraint setting	Relative ratio/Estimated UpperBound	Converage probability
$(0, \chi^2)$	$1.75/1.98 \times 10^1$	1
$(1, \chi^2)$	$1.30/1.47 \times 10^1$	1
$(2, \chi^2)$	$1.22/1.38 \times 10^1$	1

(b) Data sample size = 800.

Table 4: Quantitle estimation with Gamma data source. True value is 11.3147.

Constraint setting	Relative ratio/Estimated UpperBound	Converage probability
$(0, \chi^2)$	$2.64/2.71 \times 10^1$	1
$(1, \chi^2)$	$1.87/1.92 \times 10^1$	1
$(2, \chi^2)$	$1.75/1.80 \times 10^1$	1

(a) Data sample size = 500.

Constraint setting	Relative ratio/Estimated UpperBound	Converage probability
$(0, \chi^2)$	$2.92/2.99 \times 10^1$	1
$(1, \chi^2)$	$2.06/2.11 \times 10^1$	1
$(2, \chi^2)$	$1.93/1.98 \times 10^1$	1

(b) Data sample size = 800.

Table 5: Quantitle estimation with Lognormal data source. True value is 10.2405.

Thresholds	$(2, \chi^2)$	$(2, \mathrm{KS})$
0.60	$1.54 \times 10^{-2}/3.075/1$	$1.98 \times 10^{-2}/3.952/1$
0.62	$1.53 \times 10^{-2}/3.058/1$	$1.98 \times 10^{-2}/3.952/1$
0.65	$1.53 \times 10^{-2}/3.051/1$	$1.98 \times 10^{-2}/3.952/1$
0.68	$1.51 \times 10^{-2}/3.021/1$	$1.98 \times 10^{-2}/3.952/1$
0.70	$1.50 \times 10^{-2}/2.994/1$	$1.98 \times 10^{-2}/3.952/1$
$[0.6, 0.62, 0.65, 0.68, 0.7]$	$1.50 \times 10^{-2}/2.994/1$	$1.98 \times 10^{-2}/3.952/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \mathrm{KS})$
0.60	$1.41 \times 10^{-2}/2.820/1$	$1.77 \times 10^{-2}/3.536/1$
0.62	$1.41 \times 10^{-2}/2.812/1$	$1.77 \times 10^{-2}/3.536/1$
0.65	$1.40 \times 10^{-2}/2.801/1$	$1.77 \times 10^{-2}/3.536/1$
0.68	$1.39 \times 10^{-2}/2.785/1$	$1.77 \times 10^{-2}/3.536/1$
0.70	$1.38 \times 10^{-2}/2.764/1$	$1.77 \times 10^{-2}/3.536/1$
$[0.6, 0.62, 0.65, 0.68, 0.7]$	$1.38 \times 10^{-2}/2.764/1$	$1.77 \times 10^{-2}/3.536/1$

(b) Data sample size = 800.

Table 6: Tail probablity estimation under cutoff threshold(s) from Gamma data source. Case: [0.6, 0.61, 0.62, 0.63, 0.64]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \mathrm{KS})$
0.60	$2.02 \times 10^{-2}/4.040/1$	$2.53 \times 10^{-2}/5.058/1$
0.62	$2.00 \times 10^{-2}/3.999/1$	$2.53 \times 10^{-2}/5.058/1$
0.65	$1.99 \times 10^{-2}/3.980/1$	$2.53 \times 10^{-2}/5.058/1$
0.68	$1.97 \times 10^{-2}/3.949/1$	$2.53 \times 10^{-2}/5.058/1$
0.70	$1.96 \times 10^{-2}/3.927/1$	$2.53 \times 10^{-2}/5.058/1$
$[0.6, 0.62, 0.65, 0.68, 0.7]$	$1.96 \times 10^{-2}/3.927/1$	$2.53 \times 10^{-2}/5.058/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \mathrm{KS})$
0.60	$2.07 \times 10^{-2}/4.145/1$	$2.23 \times 10^{-2}/4.461/1$
0.62	$2.04 \times 10^{-2}/4.089/1$	$2.23 \times 10^{-2}/4.461/1$
0.65	$2.03 \times 10^{-2}/4.060/1$	$2.23 \times 10^{-2}/4.461/1$
0.68	$2.01 \times 10^{-2}/4.029/1$	$2.23 \times 10^{-2}/4.461/1$
0.70	$2.00 \times 10^{-2}/3.991/1$	$2.23 \times 10^{-2}/4.461/1$
$[0.6, 0.62, 0.65, 0.68, 0.7]$	$2.00 \times 10^{-2}/3.991/1$	$2.23 \times 10^{-2}/4.461/1$

(b) Data sample size = 800.

Table 7: Tail probablity estimation under cutoff threshold(s) from Lognormal data source. Case: [0.6, 0.61, 0.62, 0.63, 0.64]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \mathrm{KS})$
0.60	$2.22 \times 10^{-2}/4.431/1$	$2.98 \times 10^{-2}/5.951/1$
0.62	$2.20 \times 10^{-2}/4.396/1$	$2.98 \times 10^{-2}/5.951/1$
0.65	$2.18 \times 10^{-2}/4.354/1$	$2.98 \times 10^{-2}/5.951/1$
0.68	$2.15 \times 10^{-2}/4.309/1$	$2.98 \times 10^{-2}/5.951/1$
0.70	$2.15 \times 10^{-2}/4.293/1$	$2.98 \times 10^{-2}/5.951/1$
$[0.6, 0.62, 0.65, 0.68, 0.7]$	$2.15 \times 10^{-2}/4.293/1$	$2.98 \times 10^{-2}/5.951/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \mathrm{KS})$
0.60	$2.21 \times 10^{-2}/4.413/1$	$2.57 \times 10^{-2}/5.143/1$
0.62	$2.18 \times 10^{-2}/4.355/1$	$2.57 \times 10^{-2}/5.143/1$
0.65	$2.16 \times 10^{-2}/4.320/1$	$2.57 \times 10^{-2}/5.143/1$
0.68	$2.14 \times 10^{-2}/4.279/1$	$2.57 \times 10^{-2}/5.143/1$
0.70	$2.11 \times 10^{-2}/4.230/1$	$2.57 \times 10^{-2}/5.143/1$
$[0.6, 0.62, 0.65, 0.68, 0.7]$	$2.11 \times 10^{-2}/4.230/1$	$2.57 \times 10^{-2}/5.143/1$

(b) Data sample size = 800.

Table 8: Tail probablity estimation under cutoff threshold(s) from Pareto data source. Case: [0.6, 0.61, 0.62, 0.63, 0.64]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.65	$1.53 \times 10^{-2}/3.051/1$	$1.98 \times 10^{-2}/3.952/1$
0.68	$1.51 \times 10^{-2}/3.021/1$	$1.98 \times 10^{-2}/3.952/1$
0.70	$1.50 \times 10^{-2}/2.994/1$	$1.98 \times 10^{-2}/3.952/1$
0.73	$1.49 \times 10^{-2}/2.983/1$	$1.98 \times 10^{-2}/3.952/1$
0.75	$1.49 \times 10^{-2}/2.970/1$	$1.98 \times 10^{-2}/3.952/1$
[0.65, 0.68, 0.7, 0.73, 0.75]	$1.49 \times 10^{-2}/2.970/1$	$1.98 \times 10^{-2}/3.952/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.65	$1.40 \times 10^{-2}/2.801/1$	$1.77 \times 10^{-2}/3.536/1$
0.68	$1.39 \times 10^{-2}/2.785/1$	$1.77 \times 10^{-2}/3.536/1$
0.70	$1.38 \times 10^{-2}/2.764/1$	$1.77 \times 10^{-2}/3.536/1$
0.73	$1.37 \times 10^{-2}/2.739/1$	$1.77 \times 10^{-2}/3.536/1$
0.75	$1.36 \times 10^{-2}/2.727/1$	$1.77 \times 10^{-2}/3.536/1$
[0.65, 0.68, 0.7, 0.73, 0.75]	$1.36 \times 10^{-2}/2.727/1$	$1.77 \times 10^{-2}/3.536/1$

(b) Data sample size = 800.

Table 9: Tail probablity estimation under cutoff threshold(s) from Gamma data source. Case: [0.65, 0.66, 0.67, 0.68, 0.69]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.65	$1.99 \times 10^{-2}/3.980/1$	$2.53 \times 10^{-2}/5.058/1$
0.68	$1.97 \times 10^{-2}/3.949/1$	$2.53 \times 10^{-2}/5.058/1$
0.70	$1.96 \times 10^{-2}/3.927/1$	$2.53 \times 10^{-2}/5.058/1$
0.73	$1.95 \times 10^{-2}/3.906/1$	$2.53 \times 10^{-2}/5.058/1$
0.75	$1.95 \times 10^{-2}/3.896/1$	$2.53 \times 10^{-2}/5.058/1$
[0.65, 0.68, 0.7, 0.73, 0.75]	$1.95 \times 10^{-2}/3.896/1$	$2.53 \times 10^{-2}/5.058/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.65	$2.03 \times 10^{-2}/4.060/1$	$2.23 \times 10^{-2}/4.461/1$
0.68	$2.01 \times 10^{-2}/4.029/1$	$2.23 \times 10^{-2}/4.461/1$
0.70	$2.00 \times 10^{-2}/3.991/1$	$2.23 \times 10^{-2}/4.461/1$
0.73	$1.97 \times 10^{-2}/3.943/1$	$2.23 \times 10^{-2}/4.461/1$
0.75	$1.95 \times 10^{-2}/3.908/1$	$2.23 \times 10^{-2}/4.461/1$
[0.65, 0.68, 0.7, 0.73, 0.75]	$1.95 \times 10^{-2}/3.908/1$	$2.23 \times 10^{-2}/4.461/1$

(b) Data sample size = 800.

Table 10: Tail probablity estimation under cutoff threshold(s) from Lognormal data source. Case: [0.65, 0.66, 0.67, 0.68, 0.69]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.65	$2.18 \times 10^{-2}/4.354/1$	$2.98 \times 10^{-2}/5.951/1$
0.68	$2.15 \times 10^{-2}/4.309/1$	$2.98 \times 10^{-2}/5.951/1$
0.70	$2.15 \times 10^{-2}/4.293/1$	$2.98 \times 10^{-2}/5.951/1$
0.73	$2.13 \times 10^{-2}/4.252/1$	$2.98 \times 10^{-2}/5.951/1$
0.75	$2.11 \times 10^{-2}/4.214/1$	$2.98 \times 10^{-2}/5.951/1$
[0.65, 0.68, 0.7, 0.73, 0.75]	$2.11 \times 10^{-2}/4.214/1$	$2.98 \times 10^{-2}/5.951/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.65	$2.16 \times 10^{-2}/4.320/1$	$2.57 \times 10^{-2}/5.143/1$
0.68	$2.14 \times 10^{-2}/4.279/1$	$2.57 \times 10^{-2}/5.143/1$
0.70	$2.11 \times 10^{-2}/4.230/1$	$2.57 \times 10^{-2}/5.143/1$
0.73	$2.09 \times 10^{-2}/4.171/1$	$2.57 \times 10^{-2}/5.143/1$
0.75	$2.06 \times 10^{-2}/4.127/1$	$2.57 \times 10^{-2}/5.143/1$
[0.65, 0.68, 0.7, 0.73, 0.75]	$2.06 \times 10^{-2}/4.127/1$	$2.57 \times 10^{-2}/5.143/1$

(b) Data sample size = 800.

Table 11: Tail probablity estimation under cutoff threshold(s) from Pareto data source. Case: [0.65, 0.66, 0.67, 0.68, 0.69]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.70	$1.50 \times 10^{-2}/2.994/1$	$1.98 \times 10^{-2}/3.952/1$
0.72	$1.49 \times 10^{-2}/2.983/1$	$1.98 \times 10^{-2}/3.952/1$
0.75	$1.49 \times 10^{-2}/2.970/1$	$1.98 \times 10^{-2}/3.952/1$
0.77	$1.48 \times 10^{-2}/2.953/1$	$1.98 \times 10^{-2}/3.952/1$
0.80	$1.47 \times 10^{-2}/2.934/1$	$1.98 \times 10^{-2}/3.952/1$
[0.7, 0.72, 0.75, 0.77, 0.8]	$1.47 \times 10^{-2}/2.934/1$	$1.98 \times 10^{-2}/3.952/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.70	$1.38 \times 10^{-2}/2.764/1$	$1.77 \times 10^{-2}/3.536/1$
0.72	$1.37 \times 10^{-2}/2.739/1$	$1.77 \times 10^{-2}/3.536/1$
0.75	$1.36 \times 10^{-2}/2.727/1$	$1.77 \times 10^{-2}/3.536/1$
0.77	$1.36 \times 10^{-2}/2.710/1$	$1.77 \times 10^{-2}/3.536/1$
0.80	$1.35 \times 10^{-2}/2.695/1$	$1.77 \times 10^{-2}/3.536/1$
[0.7, 0.72, 0.75, 0.77, 0.8]	$1.35 \times 10^{-2}/2.695/1$	$1.77 \times 10^{-2}/3.536/1$

(b) Data sample size = 800.

Table 12: Tail probablity estimation under cutoff threshold(s) from Gamma data source. Case: [0.7, 0.71, 0.72, 0.73, 0.74]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.70	$1.96 \times 10^{-2}/3.927/1$	$2.53 \times 10^{-2}/5.058/1$
0.72	$1.95 \times 10^{-2}/3.906/1$	$2.53 \times 10^{-2}/5.058/1$
0.75	$1.95 \times 10^{-2}/3.896/1$	$2.53 \times 10^{-2}/5.058/1$
0.77	$1.92 \times 10^{-2}/3.839/1$	$2.53 \times 10^{-2}/5.058/1$
0.80	$1.90 \times 10^{-2}/3.790/1$	$2.53 \times 10^{-2}/5.058/1$
[0.7, 0.72, 0.75, 0.77, 0.8]	$1.90 \times 10^{-2}/3.790/1$	$2.53 \times 10^{-2}/5.058/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.70	$2.00 \times 10^{-2}/3.991/1$	$2.23 \times 10^{-2}/4.461/1$
0.72	$1.97 \times 10^{-2}/3.943/1$	$2.23 \times 10^{-2}/4.461/1$
0.75	$1.95 \times 10^{-2}/3.908/1$	$2.23 \times 10^{-2}/4.461/1$
0.77	$1.92 \times 10^{-2}/3.850/1$	$2.23 \times 10^{-2}/4.461/1$
0.80	$1.91 \times 10^{-2}/3.826/1$	$2.23 \times 10^{-2}/4.461/1$
[0.7, 0.72, 0.75, 0.77, 0.8]	$1.91 \times 10^{-2}/3.826/1$	$2.23 \times 10^{-2}/4.461/1$

(b) Data sample size = 800.

Table 13: Tail probablity estimation under cutoff threshold(s) from Lognormal data source. Case: [0.7, 0.71, 0.72, 0.73, 0.74]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.70	$2.15 \times 10^{-2}/4.293/1$	$2.98 \times 10^{-2}/5.951/1$
0.72	$2.13 \times 10^{-2}/4.252/1$	$2.98 \times 10^{-2}/5.951/1$
0.75	$2.11 \times 10^{-2}/4.214/1$	$2.98 \times 10^{-2}/5.951/1$
0.77	$2.09 \times 10^{-2}/4.176/1$	$2.98 \times 10^{-2}/5.951/1$
0.80	$2.08 \times 10^{-2}/4.153/1$	$2.98 \times 10^{-2}/5.951/1$
[0.7, 0.72, 0.75, 0.77, 0.8]	$2.08 \times 10^{-2}/4.153/1$	$2.98 \times 10^{-2}/5.951/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.70	$2.11 \times 10^{-2}/4.230/1$	$2.57 \times 10^{-2}/5.143/1$
0.72	$2.09 \times 10^{-2}/4.171/1$	$2.57 \times 10^{-2}/5.143/1$
0.75	$2.06 \times 10^{-2}/4.127/1$	$2.57 \times 10^{-2}/5.143/1$
0.77	$2.04 \times 10^{-2}/4.079/1$	$2.57 \times 10^{-2}/5.143/1$
0.80	$2.02 \times 10^{-2}/4.039/1$	$2.57 \times 10^{-2}/5.143/1$
[0.7, 0.72, 0.75, 0.77, 0.8]	$2.02 \times 10^{-2}/4.039/1$	$2.57 \times 10^{-2}/5.143/1$

(b) Data sample size = 800.

Table 14: Tail probablity estimation under cutoff threshold(s) from Pareto data source. Case: [0.7, 0.71, 0.72, 0.73, 0.74]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.75	$1.49 \times 10^{-2}/2.970/1$	$1.98 \times 10^{-2}/3.952/1$
0.78	$1.48 \times 10^{-2}/2.953/1$	$1.98 \times 10^{-2}/3.952/1$
0.80	$1.47 \times 10^{-2}/2.934/1$	$1.98 \times 10^{-2}/3.952/1$
0.82	$1.45 \times 10^{-2}/2.895/1$	$1.98 \times 10^{-2}/3.952/1$
0.85	$1.44 \times 10^{-2}/2.890/1$	$1.98 \times 10^{-2}/3.952/1$
[0.75, 0.78, 0.8, 0.82, 0.85]	$1.44 \times 10^{-2}/2.890/1$	$1.98 \times 10^{-2}/3.952/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.75	$1.36 \times 10^{-2}/2.727/1$	$1.77 \times 10^{-2}/3.536/1$
0.78	$1.36 \times 10^{-2}/2.710/1$	$1.77 \times 10^{-2}/3.536/1$
0.80	$1.35 \times 10^{-2}/2.695/1$	$1.77 \times 10^{-2}/3.536/1$
0.82	$1.34 \times 10^{-2}/2.672/1$	$1.77 \times 10^{-2}/3.536/1$
0.85	$1.33 \times 10^{-2}/2.657/1$	$1.77 \times 10^{-2}/3.536/1$
[0.75, 0.78, 0.8, 0.82, 0.85]	$1.33 \times 10^{-2}/2.657/1$	$1.77 \times 10^{-2}/3.536/1$

(b) Data sample size = 800.

Table 15: Tail probablity estimation under cutoff threshold(s) from Gamma data source. Case: [0.75, 0.76, 0.77, 0.78, 0.79]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.75	$1.95 \times 10^{-2}/3.896/1$	$2.53 \times 10^{-2}/5.058/1$
0.78	$1.92 \times 10^{-2}/3.839/1$	$2.53 \times 10^{-2}/5.058/1$
0.80	$1.90 \times 10^{-2}/3.790/1$	$2.53 \times 10^{-2}/5.058/1$
0.82	$1.87 \times 10^{-2}/3.732/1$	$2.53 \times 10^{-2}/5.058/1$
0.85	$1.83 \times 10^{-2}/3.663/1$	$2.53 \times 10^{-2}/5.058/1$
[0.75, 0.78, 0.8, 0.82, 0.85]	$1.83 \times 10^{-2}/3.663/1$	$2.53 \times 10^{-2}/5.058/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \text{KS})$
0.75	$1.95 \times 10^{-2}/3.908/1$	$2.23 \times 10^{-2}/4.461/1$
0.78	$1.92 \times 10^{-2}/3.850/1$	$2.23 \times 10^{-2}/4.461/1$
0.80	$1.91 \times 10^{-2}/3.826/1$	$2.23 \times 10^{-2}/4.461/1$
0.82	$1.89 \times 10^{-2}/3.779/1$	$2.23 \times 10^{-2}/4.461/1$
0.85	$1.87 \times 10^{-2}/3.749/1$	$2.23 \times 10^{-2}/4.461/1$
[0.75, 0.78, 0.8, 0.82, 0.85]	$1.87 \times 10^{-2}/3.749/1$	$2.23 \times 10^{-2}/4.461/1$

(b) Data sample size = 800.

Table 16: Tail probablity estimation under cutoff threshold(s) from Lognormal data source. Case: [0.75, 0.76, 0.77, 0.78, 0.79]. True value is 0.005.

Thresholds	$(2, \chi^2)$	$(2, \mathrm{KS})$
0.75	$2.11 \times 10^{-2}/4.214/1$	$2.98 \times 10^{-2}/5.951/1$
0.78	$2.09 \times 10^{-2}/4.176/1$	$2.98 \times 10^{-2}/5.951/1$
0.80	$2.08 \times 10^{-2}/4.153/1$	$2.98 \times 10^{-2}/5.951/1$
0.82	$2.05 \times 10^{-2}/4.099/1$	$2.98 \times 10^{-2}/5.951/1$
0.85	$2.02 \times 10^{-2}/4.046/1$	$2.98 \times 10^{-2}/5.951/1$
$[0.75, 0.78, 0.8, 0.82, 0.85]$	$2.02 \times 10^{-2}/4.046/1$	$2.98 \times 10^{-2}/5.951/1$

(a) Data sample size = 500.

Thresholds	$(2, \chi^2)$	$(2, \mathrm{KS})$
0.75	$2.06 \times 10^{-2}/4.127/1$	$2.57 \times 10^{-2}/5.143/1$
0.78	$2.04 \times 10^{-2}/4.079/1$	$2.57 \times 10^{-2}/5.143/1$
0.80	$2.02 \times 10^{-2}/4.039/1$	$2.57 \times 10^{-2}/5.143/1$
0.82	$1.99 \times 10^{-2}/3.979/1$	$2.57 \times 10^{-2}/5.143/1$
0.85	$1.97 \times 10^{-2}/3.937/1$	$2.57 \times 10^{-2}/5.143/1$
$[0.75, 0.78, 0.8, 0.82, 0.85]$	$1.97 \times 10^{-2}/3.937/1$	$2.57 \times 10^{-2}/5.143/1$

(b) Data sample size = 800.

Table 17: Tail probablity estimation under cutoff threshold(s) from Pareto data source. Case: [0.75, 0.76, 0.77, 0.78, 0.79]. True value is 0.005.