

1.



— tree edge  
--- back edge

Vertex	DFS#	LOW
A	4	2
B	3	2
C	1	1
D	7	6
E	8	6
F	5	2
G	2	1
H	6	1
I	9	2
J	10	10

edge orders are  
listed leftside of edges.

Articulation points: G, H

2.

Component 1:  $\{ \{A, B\}, \{A, F\}, \{B, G\}, \{F, G\} \}$

Component 2:  $\{ \{C, G\}, \{C, H\}, \{G, I\}, \{G, H\}, \{H, I\} \}$

Component 3:  $\{ \{D, E\}, \{D, H\}, \{H, E\} \}$

Component 4:  $\{ \{H, J\} \}$

3.

Since articulation points are the edge of biconnected components, in other words, a graph are divided into biconnected components by articulation points.

Modification to the art. pt. algorithm: At initialization stage, initilize a stack

At DFS stage, whenever encounters a new edge <sup>or back edge</sup>, push it on the stack until DFS found a articulation point, then pop the stack.

sudo code:

initialization :  $dfscount = 0$ ;  $v.dfs = -1$  for all  $v$ ; initialize stack  $S$

$DFS\_bi\_comp(v)$ :

$v.dfs = dfscount++$

$v.low = v.dfs$

for each edge  $(v, x)$

if  $(x.dfs = -1)$  // new edge

$S.push((v, x))$

$DFS\_bi\_comp(x)$

$v.low = \min(v.low, x.low)$

if  $(x.low \geq v.dfs)$

$output\_bi\_comp((v, x))$

else if  $(x$  is not  $v$ 's parent) // back edge

$S.push((v, x))$

$v.low = \min(v.low, x.dfs)$

$output\_bi\_comp((v, x))$ :

do

$e = S.pop()$

print  $e$

while  $(e \neq (v, x))$

Simulation for finding the first bi-comp :

Start from  $v_1$

$DFS(v_1)$

$DFS(v_1)$

$DFS(v_1)$

$DFS(v_1)$

$DFS(v_1)$

$DFS(v_1)$

$DFS(v_1)$

$DFS(v_1)$

$DFS(v_1)$

$DFS(v_1)$

start from C

dfs(c):

C.dfs = 1, C.low = 1

edge(C, G) new edge, stack { (C, G) }

dfs(G):

G.dfs = 2, G.low = 2

edge(G, B) new edge, stack { (C, G), (G, B) }

dfs(B):

B.dfs = 3, B.low = 3

edge(B, A) new, stack { (C, G), (G, B), (B, A) }

dfs(A):

A.dfs = 4, A.low = 4

edge(A, F) new, stack { (C, G), (G, B), (B, A), (A, F) }

dfs(F):

F.dfs = 5, F.low = 5

edge(F, G) back edge

stack { (C, G), (G, B), (B, A), (A, F), (F, G) }

F.low = G.dfs = 2 < A.dfs

A.low = F.low = 2 < B.dfs

B.low = A.low = 2 = G.dfs

G is articulation point

output edge up to (G, B) from stack

output: (F, G), (A, F), (B, A), (G, B)

which is exactly the 2nd bi-comp