

# CPTS 515 HW 3

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1. a. Build a bipartite graph from the sample from  $C$  by finding the mapping from high bit to low bit  
b. Run max flow algorithm over the bipartite graph to calculate the max matching number  $M$ .  
c. apply the Theorem 2 from the paper Security of Numerical Sensors in Automata, the average Leaking bits is  $\log(M)$

2. Write another program that take  $X$  as input and  $\text{int}[] \{x_1, x_2, \dots, x_7\}$  as output.  
the logic of the program is the same as myFunction but in reversed order. Input a negative number  $X \neq 0$  to the program. if there is a valid output  $\text{int}[] \{x_1, x_2, \dots, x_7\}$  returned, then there are values for  $x_1, x_2, \dots, x_7$  passed to the myFunction the can return a negative integer.

3. For a set  $K = \{1, \dots, k\}$  any subset  $p$  of  $k$  can be expressed as  $\{b_1, \dots, b_k\}$  where  $b_i$  is a boolean variable to indicate whether element  $k_i$  in the subset. (Using  $b_i = 0$  for false,  $b_i = 1$  for true)  
In this way each subset can be encoded into a binary form.  
e.g.  $\emptyset = \{0\}$   
 $K = \{\underbrace{1, 1, 1, 1, \dots, 1}_k\}$

Then convert the binary representation into decimal integer where the bound is  $[0, 2^k]$

Since each binary representation is unique, each converted decimal integer is also unique. (C is 1-1)

The number of subset of  $K$  is  $2^k$ , so  $C_p \in \{1, \dots, B_k\}$

4. The number of boolean variable for nodes  $= \log(2048) = 11$   
11 variables needed to encode every node, so  $11 \times 2 = 22$  needed to encode the graph (each edge has two nodes)

5.  $\log(40) \approx 6$ , There are at least 6 bits needed to hash 40 students that each student has a unique hashcode assigned