

## CPT\_S 580 HW2

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### 2.3.21

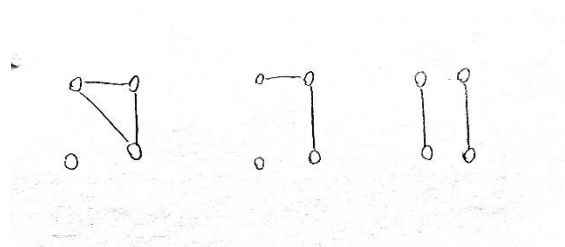
- (a) 1 clique  $(u, v, y)$
- (b)  $w = 3$
- (c)  $uxz, xyz, xyw, xvz, xuw$
- (d)  $a = 3$
- (e) the center is  $v, y$

### 2.3.22

- (a) 3 cliques  $(u, x, y), (u, v, y), (y, v, w)$
- (b)  $w = 3$
- (c)  $xvz, xw, uw, uz, yz$
- (d)  $a = 3$
- (e) the center is  $v, y$

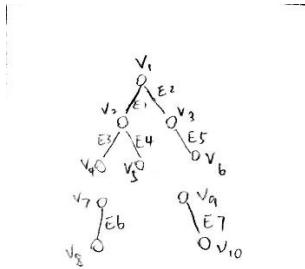
### 2.3.30

All possible isomorphism types of a simple 4-vertex graph with exactly 2 components



### 2.3.36

A forest with 10 vertices, 7 edges, and 3 components



#### 2.4.8

Cut-vertices:  $\{u, v, z\}$

Cut-edges:  $\{uv\}$

#### 2.4.9

Cut-vertices:  $\{u, v, x, z, s\}$

Cut-edges:  $\{uv, xz, zs, st\}$

#### 2.4.30

Since the  $e$  is the cutting-edge, removing it will separate parts that connected by  $e$ , and since any vertex is not removed, which means the two separate parts are connected. Therefore, connected graph  $G - \text{cutting edge } e$  has exactly two components.

#### 2.5.3

The left-side graph has 1 4-edge circle, while the right-side graph has none.

#### 2.5.5

Yes, they are in the pair of isomorphic

Cause we can map one graph to the other one.

Map:  $\{a \rightarrow 1, b \rightarrow 4, c \rightarrow 2, d \rightarrow 3, e \rightarrow 5\}$

#### 3.1.15

Suppose graph  $G$  has  $n$  vertices  $v_1$  to  $v_n$ . And suppose the average degree of its vertices is 2, then,

$$\deg(v_1) + \dots + \deg(v_n) = 2n$$

which implies that the number of edges in  $G$  is  $n$ . The graph  $G$  has  $n$  vertices; thus it cannot be a tree. Thus  $G$  has at least one cycle, say  $C$ . Let  $e$  be an edge in  $C$ . Then the graph  $G - e$  is connected, and has  $n - 1$  edges, hence it is a tree. By a theorem in the notes, adding one edge  $e$  to the tree  $G - e$  creates exactly one cycle. Thus  $G$  has exactly one cycle.

### 3.1.17

Let  $G$  have  $n$  vertices and  $n$  edges. Since  $G$  is a connected graph, it has a spanning tree  $T$  with  $n$  vertices and  $n - 1$  edges. Let  $e$  be the edge not in  $T$ , with its endpoints  $u$  and  $v$ . There is a unique path  $\gamma$  between  $u$  and  $v$  in  $T$  (since  $T$  is a tree). The union of  $e$  and  $\gamma$  is a cycle. Suppose that there is some other cycle  $\delta$ . If  $\delta$  does not contain  $e$ , then it is contained in  $T$ , contradicting that  $T$  has no cycles.

### 3.1.18

We know that  $n$ -vertex simple graph with  $n$  edge has exactly 1 cycles. Then add 1 edge to the graph would connect 2 non-adjacent vertices (since simple graph), which make the 1 cycle into 2 cycles.

### 2.2.10

For every vertex in hypercube graph  $Q_n$ , the vertex-degree is identical, which means any vertex has the same local environment. Thus, for any two vertices of  $Q_n$ , the vertex-bijection that swaps them and fixes all the others is adjacency-preserving. It is its own inverse, so the inverse bijection is also adjacency-preserving.

### 2.6.24

When  $n = 1$ , the entry  $a_{ij}$  is 1 if  $\{i, j\} \in E$ . By definition,  $i \{i, j\} j$  is then an  $i - j$  walk of length 1 and this is the only one. So the statement is true for  $n = 1$ . Now, we assume the statement is true for  $n$  and then prove the statement is also true for  $n + 1$ . Since  $A_{ij}^{n+1} = A_{ij}^n \cdot A_{ij}$ , therefore,  $a_{ij}^{n+1} = \sum_{k=1}^m a_{ik}^n \cdot a_{kj}$ . Because  $a_{ki} = 0$  whenever  $\{k, i\} \notin E$  and  $a_{ki} = 1$  if  $\{k, i\} \in E$ , it follows that  $a_{ik}^n \cdot a_{kj}$  represents the number of those  $i - j$  walks that are  $i - k$  walks of length  $n$  joined by the edge  $\{k, j\}$ . In particular, all walks from  $i$  to  $j$  of length  $n + 1$  are of this form for some vertex  $k$ .

Thus  $a_{ij}^{n+1} = \sum_{k=1}^m a_{ik}^n \cdot a_{kj}$  indeed represents the total number of  $i - j$  walks of length  $n + 1$ . This proves the statement for  $n + 1$ . Then by the principle of induction, we prove the statement for all natural numbers  $n$ .