

Review of Tarjan's algorithm

① using DFS to label each vertex in G

in such format $\frac{n}{n} \rightarrow \frac{\text{1st visit time}}{\text{2nd visit time}}$

with the following rule:

for each unvisit node:

mark the time as 1st visit time, and visited the node as

for each neighbor n

if n is unvisit;

recursive-call

mark the time as 2nd visit time

② sort the vertices based on 2nd visit time (decreasing)

③ reverse every edge in the Graph G

and start with the 1st element in the sorted vertices to find SCC and store each SCC in scc-list

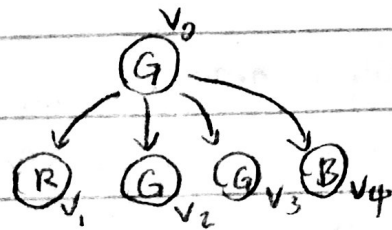
Before applying tarjan's algorithm, let's make the following changes:

① pre-process the graph: for each pair of adjacent green vertices (or adjacent red vertices) add necessary edge to make them compose a SCC:

For example: $\dots \rightarrow G \rightarrow G \rightarrow \dots \Rightarrow \dots \rightarrow G \rightleftarrows G \rightarrow \dots$

② modify the get-neighbors method, now only the neighbors with the same color as the parent are neighbors.

For example :



neighbors(v_0) returns
 $[v_2, v_3]$

After the above 2 changes we made run tarjan algorithm on the modified graph twice:

1st run : find all the green SCC

2nd run : find all the red SCC

if there exist $\text{Size}(\text{SCC}_i^{\text{Red}}) \geq \text{Size}(\text{SCC}_j^{\text{Green}})$

return true

else return false

2. The problem is to find the max network flow subject to the constraint $C_1 + C_2 \leq K$, in other words, to make sure finding the max flow, $C_1 + C_2 = K-1$

My idea is to apply greedy hill climbing algorithm.

based of C_1 , the range of C_1 is $[0, K-1]$

① randomly select value for C_1 from $[0, K-1]$

then $C_2 = K-1 - C_1$

② plug current C_1, C_2 into the max-flow algorithm to get current max M_{mid} .

③ repeat step 1 and 2 with (C_1-1) and (C_1+1) to get M_{left} and M_{right} accordingly

④ if $M_{\text{mid}} \geq \max(M_{\text{left}}, M_{\text{right}})$

M_{mid} is the local maximum

else $M_{mid} = \max(M_{left}, M_{right})$, $C_1 = \max(C_1^{M_{left}}, C_1^{M_{right}})$
and repeat from step 2

Restart the whole process for a few time with different values
start C_1 value,

- 3
- ① using BFS to find all the yellow nodes
 - ② detach every yellow nodes from the original graph
 - ③ run tarjan's algorithm on updated graph
for each SCC that has size > 1 :
if scc contains both red nodes and green
return true;
endfor
return false

Dear Prof. Dang,

On the homework set, you state that α is a infinite
long path. Shouldn't α be a walk instead? I did
the question with the assumption that α is a walk.

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4 (1) path-count = 0

DFS (v, v')

for each neighbor V_n of v :

if $V_n == v'$, path-count ++;

else DFS (V_n , v');

End for

end DFS

(2) good-path-counter = 0

DFS (v, v', green-count = 0, yellow-counter = 0)

if $v.color == green$, green-count ++

elif $v.color == yellow$, yellow-counter ++

for each neighbor V_n of v :

if ($V_n == v'$)

if $V_n.color == green$, green-counter ++

elif $V_n.color == yellow$, yellow-counter ++

if green-counter > yellow-counter

good-path-counter ++

End for

End DFS