Change-point detection in Phase I for autocorrelated Poisson profiles with random or unbalanced designs

Supplementary Material

December 4, 2019

A. Random design scenario

In this section, other simulation results in the random design scenario are given corresponding to three underlying correlation structures: AR(1), exchangeable, and independent.

A.1. *AR(1) correlation*

When the true correlation structure is AR(1), performance comparisons of different methods in the random design scenario are presented in Tables 1 – 2 and Figs. 1 – 2.

Table 1. *Performance comparison with AR(1) correlation coefficient* $\rho = 0.9$ *when* $\tau = 10$ *and* 50 *in random design scenario.*

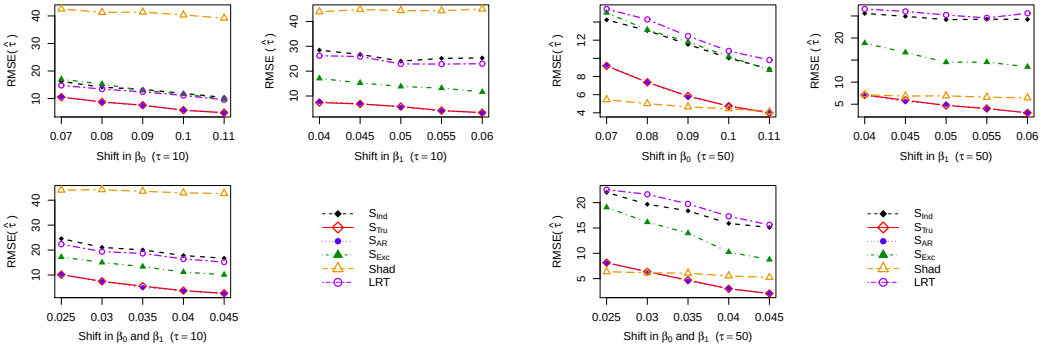
Shift	Signal probabilities							$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$					
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
						$\tau = 10$							
(0, 0.04)	0.058	0.614	0.595	0.159	0.057	0.060	0.052	0.110	0.534	0.530	0.347	0.000	0.114
(0, 0.05)	0.067	0.855	0.838	0.238	0.048	0.068	0.048	0.105	0.632	0.634	0.395	0.000	0.114
(0, 0.06)	0.076	0.965	0.959	0.367	0.056	0.077	0.050	0.119	0.745	0.745	0.501	0.000	0.125
(0.07, 0)	0.191	0.443	0.440	0.197	0.063	0.203	0.065	0.293	0.449	0.454	0.352	0.000	0.295
(0.09, 0)	0.306	0.725	0.721	0.329	0.082	0.322	0.080	0.378	0.591	0.592	0.448	0.000	0.380
(0.11, 0)	0.475	0.906	0.904	0.520	0.097	0.493	0.099	0.470	0.703	0.700	0.559	0.000	0.475
(0.025, 0.025)	0.085	0.512	0.501	0.144	0.059	0.087	0.056	0.118	0.474	0.473	0.341	0.000	0.129
(0.035, 0.035)	0.132	0.869	0.858	0.297	0.060	0.139	0.061	0.221	0.639	0.643	0.457	0.000	0.224
(0.045, 0.045)	0.199	0.989	0.986	0.500	0.066	0.211	0.063	0.317	0.792	0.791	0.549	0.000	0.316
						$\tau = 50$							
(0, 0.04)	0.055	0.664	0.652	0.142	0.069	0.052	0.051	0.091	0.584	0.579	0.244	0.142	0.092
(0, 0.05)	0.069	0.885	0.875	0.225	0.084	0.068	0.049	0.098	0.697	0.693	0.343	0.131	0.094
(0, 0.06)	0.080	0.978	0.972	0.321	0.093	0.076	0.047	0.115	0.790	0.789	0.422	0.131	0.106
(0.07, 0)	0.205	0.507	0.506	0.207	0.256	0.197	0.054	0.290	0.490	0.488	0.310	0.272	0.292
(0.09, 0)	0.330	0.784	0.781	0.351	0.410	0.311	0.062	0.368	0.622	0.624	0.421	0.337	0.370
(0.11, 0)	0.480	0.936	0.932	0.544	0.575	0.463	0.075	0.455	0.727	0.726	0.531	0.396	0.458
(0.025, 0.025)	0.088	0.559	0.550	0.143	0.104	0.081	0.057	0.141	0.513	0.517	0.214	0.154	0.131
(0.035, 0.035)	0.130	0.907	0.896	0.292	0.160	0.120	0.054	0.203	0.708	0.709	0.398	0.211	0.193
(0.045, 0.045)	0.212	0.995	0.994	0.511	0.274	0.199	0.058	0.280	0.839	0.839	0.545	0.285	0.283

Table 2. *Performance comparison with AR(1) correlation coefficient* $\rho = 0.3$ *when* $\tau = 10$ *and* 50 *in random design scenario.*

Shift	Signal probabilities							$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$					
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
						$\tau = 10$							
(0, 0.06)	0.384	0.418	0.423	0.390	0.073	0.391	0.101	0.433	0.437	0.433	0.428	0.000	0.436
(0, 0.08)	0.654	0.710	0.709	0.668	0.112	0.660	0.159	0.523	0.546	0.547	0.543	0.002	0.525
(0.05, 0)	0.595	0.513	0.514	0.459	0.102	0.602	0.139	0.495	0.475	0.473	0.469	0.000	0.496
(0.07, 0)	0.908	0.866	0.864	0.810	0.184	0.912	0.277	0.674	0.665	0.661	0.659	0.009	0.672
(0.035, 0.035)	0.734	0.698	0.700	0.641	0.129	0.741	0.191	0.564	0.557	0.558	0.554	0.002	0.563
(0.045, 0.045)	0.934	0.926	0.927	0.890	0.229	0.937	0.354	0.690	0.695	0.687	0.688	0.021	0.692
						$\tau = 50$							
(0, 0.06)	0.310	0.374	0.379	0.328	0.359	0.303	0.090	0.364	0.408	0.408	0.397	0.332	0.359
(0, 0.08)	0.559	0.655	0.659	0.598	0.636	0.544	0.125	0.494	0.548	0.550	0.542	0.440	0.495
(0.05, 0)	0.588	0.511	0.508	0.451	0.646	0.578	0.131	0.476	0.465	0.465	0.459	0.421	0.478
(0.07, 0)	0.890	0.847	0.850	0.788	0.922	0.885	0.261	0.633	0.634	0.631	0.617	0.553	0.634
(0.035, 0.035)	0.669	0.650	0.653	0.594	0.730	0.656	0.162	0.533	0.545	0.546	0.521	0.459	0.529
(0.045, 0.045)	0.900	0.897	0.897	0.848	0.927	0.894	0.289	0.648	0.661	0.663	0.652	0.548	0.645

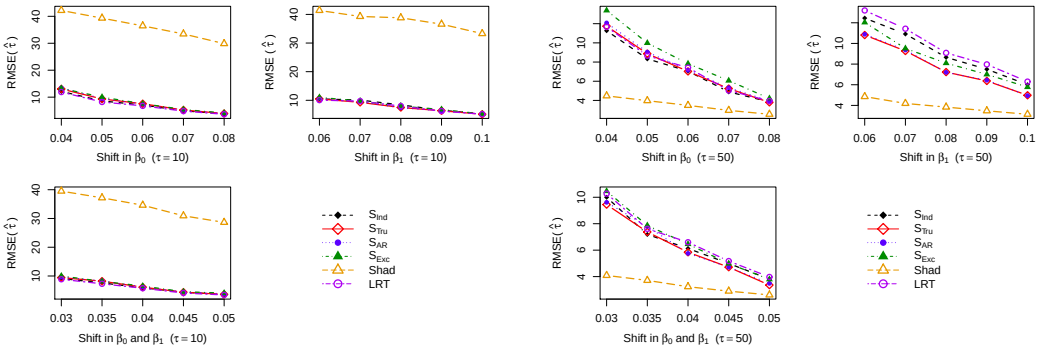
A.2. *Exchangeable correlation*

When the true correlation structure is exchangeable, performance comparisons of different methods in the random design scenario are shown in Tables 3 – 4 and Figs. 3 –



(a) $\tau = 10$ (b) $\tau = 50$

Figure 1. *Values of* RMSE($\hat{\tau}$) *with AR(1) correlation coefficient* $\rho = 0.9$ *when* $\tau = 10$ *and* 50 *in random design scenario.*



(a) $\tau = 10$ (b) $\tau = 50$

Figure 2. *Values of* RMSE($\hat{\tau}$) *with AR(1) correlation coefficient* $\rho = 0.3$ *when* $\tau = 10$ *and* 50 *in random design scenario.*

4.

Table 3. *Performance comparison with exchangeable correlation coefficient* $\rho = 0.9$ *when* $\tau = 10$ *and* 50 *in random design scenario.*

Shift	Signal probabilities							$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$					
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
						$\tau = 10$							
(0, 0.04)	0.059	0.685	0.329	0.637	0.053	0.065	0.047	0.061	0.545	0.416	0.553	0.000	0.071
(0, 0.05)	0.063	0.894	0.557	0.862	0.055	0.067	0.048	0.070	0.672	0.542	0.675	0.000	0.063
(0, 0.06)	0.062	0.982	0.767	0.969	0.051	0.067	0.044	0.081	0.783	0.660	0.787	0.000	0.093
(0.07, 0)	0.132	0.416	0.242	0.395	0.064	0.143	0.059	0.190	0.440	0.326	0.444	0.000	0.193
(0.09, 0)	0.204	0.684	0.438	0.649	0.070	0.225	0.064	0.279	0.562	0.444	0.569	0.000	0.284
(0.11, 0)	0.303	0.886	0.657	0.869	0.086	0.322	0.067	0.347	0.679	0.589	0.678	0.000	0.356
(0.025, 0.025)	0.077	0.556	0.276	0.519	0.057	0.080	0.055	0.109	0.505	0.365	0.504	0.000	0.125
(0.035, 0.035)	0.102	0.898	0.579	0.865	0.062	0.110	0.059	0.182	0.667	0.554	0.674	0.000	0.180
(0.045, 0.045)	0.143	0.993	0.852	0.989	0.066	0.154	0.055	0.207	0.814	0.688	0.813	0.000	0.217
						$\tau = 50$							
(0, 0.04)	0.059	0.688	0.382	0.637	0.064	0.056	0.046	0.061	0.587	0.512	0.593	0.100	0.068
(0, 0.05)	0.066	0.895	0.603	0.858	0.072	0.063	0.052	0.082	0.702	0.610	0.706	0.114	0.076
(0, 0.06)	0.073	0.983	0.817	0.970	0.084	0.069	0.058	0.115	0.793	0.697	0.794	0.152	0.125
(0.07, 0)	0.146	0.443	0.287	0.416	0.176	0.138	0.054	0.224	0.469	0.394	0.466	0.208	0.218
(0.09, 0)	0.219	0.718	0.512	0.690	0.272	0.206	0.056	0.288	0.590	0.512	0.593	0.253	0.276
(0.11, 0)	0.320	0.892	0.723	0.873	0.383	0.302	0.059	0.341	0.690	0.609	0.692	0.314	0.348
(0.025, 0.025)	0.078	0.580	0.325	0.540	0.085	0.073	0.055	0.105	0.532	0.436	0.536	0.147	0.091
(0.035, 0.035)	0.110	0.902	0.649	0.872	0.125	0.106	0.053	0.160	0.699	0.600	0.707	0.194	0.163
(0.045, 0.045)	0.146	0.993	0.903	0.988	0.175	0.135	0.051	0.221	0.838	0.748	0.837	0.237	0.223

Table 4. *Performance comparison with exchangeable correlation coefficient* $\rho = 0.3$ *when* $\tau = 10$ *and* 50 *in random design scenario.*

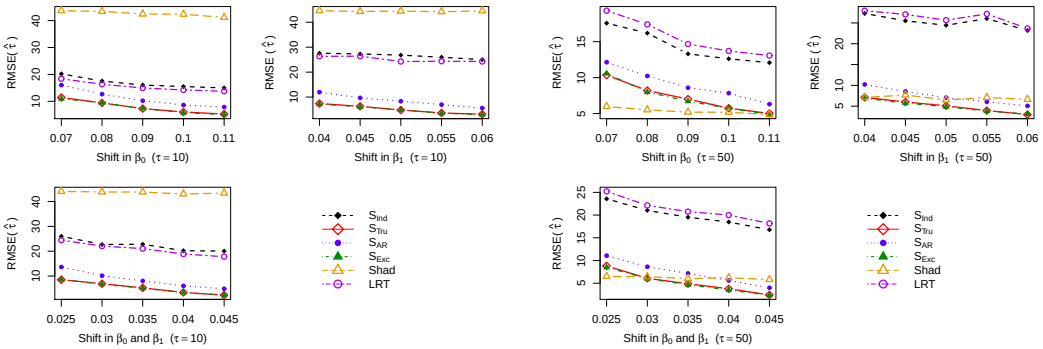
Shift	Signal probabilities							$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$					
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
						$\tau = 10$							
(0, 0.06)	0.117	0.289	0.126	0.290	0.060	0.122	0.058	0.196	0.351	0.215	0.348	0.000	0.208
(0, 0.08)	0.175	0.515	0.210	0.515	0.065	0.179	0.066	0.270	0.465	0.327	0.470	0.000	0.279
(0.05, 0)	0.166	0.143	0.161	0.138	0.063	0.176	0.062	0.273	0.268	0.269	0.269	0.000	0.274
(0.07, 0)	0.338	0.284	0.325	0.266	0.077	0.351	0.081	0.372	0.343	0.365	0.352	0.000	0.375
(0.035, 0.035)	0.218	0.288	0.230	0.286	0.066	0.230	0.067	0.297	0.369	0.315	0.370	0.000	0.302
(0.045, 0.045)	0.357	0.505	0.383	0.502	0.084	0.372	0.084	0.397	0.498	0.426	0.498	0.000	0.397
						$\tau = 50$							
(0, 0.06)	0.104	0.261	0.117	0.266	0.123	0.098	0.053	0.185	0.379	0.200	0.377	0.174	0.182
(0, 0.08)	0.150	0.490	0.194	0.495	0.188	0.141	0.061	0.213	0.484	0.282	0.483	0.237	0.211
(0.05, 0)	0.178	0.163	0.183	0.161	0.219	0.169	0.064	0.238	0.248	0.249	0.248	0.243	0.233
(0.07, 0)	0.326	0.302	0.333	0.288	0.389	0.316	0.073	0.363	0.373	0.360	0.360	0.358	0.361
(0.035, 0.035)	0.210	0.306	0.227	0.305	0.253	0.200	0.061	0.278	0.370	0.309	0.377	0.267	0.279
(0.045, 0.045)	0.337	0.506	0.359	0.502	0.403	0.323	0.074	0.371	0.487	0.395	0.491	0.336	0.379

A.3. *Independent*

When the Poisson profile samples are independent, performance comparisons of different methods in the random design scenario are provided in Table 5 and Figure 5.

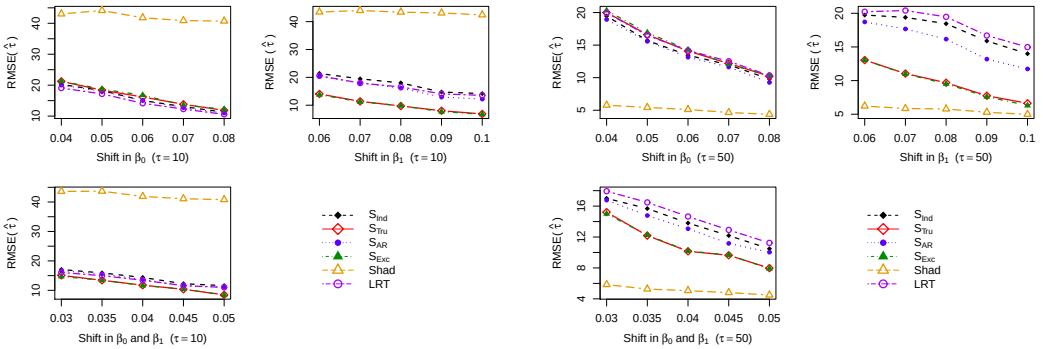
B. Unbalanced design scenario

In this section, the critical values L of different detection methods for three correlation settings in the unbalanced design scenario are listed in Table 6. Then, other simulation results in the unbalanced design scenario are given corresponding to three true correlation structures: AR(1), exchangeable, and independent.



(a) $\tau = 10$ (b) $\tau = 50$

Figure 3. *Values of* RMSE($\hat{\tau}$) *with exchangeable correlation coefficient* $\rho = 0.9$ *when* $\tau = 10$ *and* 50 *in random design scenario.*

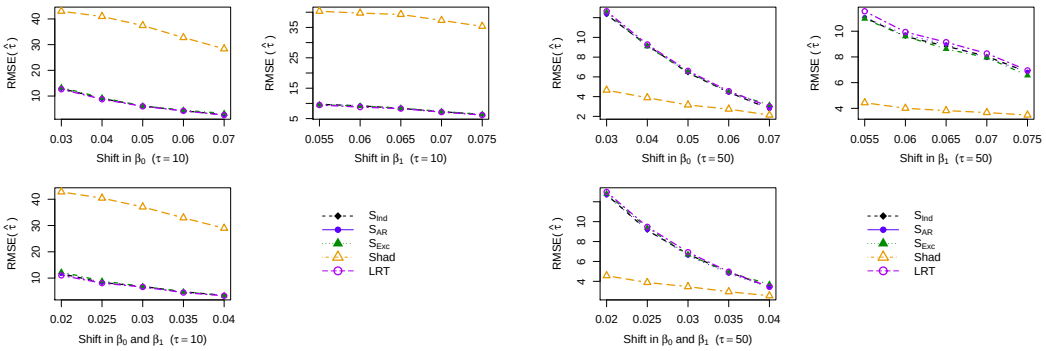


(a) $\tau = 10$ (b) $\tau = 50$

Figure 4. *Values of* RMSE($\hat{\tau}$) *with exchangeable correlation coefficient* $\rho = 0.3$ *when* $\tau = 10$ *and* 50 *in random design scenario.*

Table 5. *Performance comparison for independent setting when* $\tau = 10$ *and* 50 *in random design scenario.*

Shift	Signal probabilities						$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$				
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
$\tau = 10$											
(0, 0.06)	0.582	0.580	0.546	0.103	0.588	0.162	0.476	0.472	0.471	0.000	0.477
(0, 0.07)	0.753	0.747	0.722	0.131	0.761	0.230	0.571	0.568	0.566	0.002	0.570
(0.05, 0)	0.815	0.810	0.759	0.134	0.819	0.237	0.611	0.609	0.604	0.000	0.613
(0.07, 0)	0.992	0.991	0.982	0.335	0.992	0.561	0.801	0.801	0.793	0.051	0.802
(0.025, 0.025)	0.614	0.608	0.550	0.108	0.620	0.163	0.513	0.516	0.507	0.000	0.515
(0.035, 0.035)	0.924	0.921	0.888	0.198	0.925	0.351	0.680	0.681	0.673	0.016	0.684
$\tau = 50$											
(0, 0.06)	0.468	0.469	0.452	0.552	0.456	0.121	0.481	0.477	0.475	0.407	0.477
(0, 0.07)	0.645	0.643	0.621	0.717	0.635	0.150	0.535	0.539	0.540	0.472	0.538
(0.05, 0)	0.795	0.789	0.732	0.848	0.788	0.223	0.617	0.615	0.610	0.531	0.616
(0.07, 0)	0.988	0.987	0.975	0.995	0.987	0.513	0.797	0.798	0.788	0.677	0.797
(0.025, 0.025)	0.527	0.524	0.479	0.601	0.516	0.124	0.493	0.494	0.488	0.437	0.497
(0.035, 0.035)	0.881	0.877	0.845	0.917	0.876	0.265	0.647	0.649	0.648	0.551	0.645



(a) $\tau = 10$ (b) $\tau = 50$

Figure 5. *Values of* RMSE($\hat{\tau}$) *for independent setting when* $\tau = 10$ *and* 50 *in random design scenario.*

Table 6. *Critical values* L *of different chart statistics for different correlation settings in unbalanced design scenario.*

Chart	Independent	AR(1)			Exchangeable		
Statistics	$\rho = 0$	$\rho = 0.3$	$\rho = 0.6$	$\rho = 0.9$	$\rho = 0.3$	$\rho = 0.6$	$\rho = 0.9$
$\boldsymbol{S_{Ind}}$	11.8385	17.9364	34.3145	100.1897	60.7770	112.1658	165.0564
$\boldsymbol{S_{Tru}}$	11.8385	12.0693	11.9865	12.1532	11.7783	12.1198	12.2513
$\boldsymbol{S_{AR}}$	11.7923	12.0948	11.9344	12.0951	34.0610	33.0242	19.6546
$\boldsymbol{S_{Exc}}$	11.7031	12.2972	13.0888	15.5624	11.7677	12.1309	12.4207
Shad	8.8277	13.0197	24.0843	71.9877	42.9252	80.6522	116.5058
LRT	11.8303	17.9810	34.4138	99.5253	60.8572	112.3274	164.9352
GLMM	0.7486	3.8003	15.5897	100.7394	51.7593	117.6187	192.2147

B.1. *AR(1) correlation*

When the true correlation structure is AR(1), performance comparisons of different methods in the unbalanced design scenario are summarised in Tables 7 – 9 and Figs. 6 – 8.

Table 7. *Performance comparison with AR(1) correlation coefficient* $\rho = 0.9$ *in unbalanced design scenario.*

Shift	Signal probabilities							$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$					
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
						$\tau = 10$							
(0, 0.04)	0.056	0.562	0.551	0.102	0.046	0.062	0.052	0.071	0.519	0.517	0.177	0.000	0.068
(0, 0.06)	0.070	0.947	0.941	0.219	0.052	0.076	0.053	0.100	0.739	0.737	0.362	0.000	0.123
(0.07, 0)	0.170	0.421	0.425	0.120	0.060	0.187	0.065	0.252	0.471	0.471	0.254	0.000	0.247
(0.10, 0)	0.359	0.805	0.806	0.273	0.085	0.382	0.093	0.382	0.642	0.643	0.441	0.000	0.385
(0.03, 0.03)	0.096	0.672	0.661	0.129	0.054	0.105	0.063	0.163	0.561	0.563	0.258	0.000	0.168
(0.04, 0.04)	0.143	0.935	0.928	0.239	0.063	0.156	0.067	0.214	0.720	0.720	0.371	0.000	0.222
						$\tau = 30$							
(0, 0.04)	0.079	0.923	0.916	0.155	0.065	0.083	0.060	0.099	0.613	0.612	0.200	0.015	0.094
(0, 0.06)	0.114	1.000	1.000	0.446	0.080	0.116	0.058	0.136	0.840	0.842	0.421	0.018	0.131
(0.07, 0)	0.389	0.809	0.805	0.253	0.251	0.392	0.079	0.311	0.495	0.496	0.282	0.112	0.311
(0.10, 0)	0.718	0.991	0.990	0.609	0.526	0.723	0.101	0.435	0.714	0.715	0.472	0.203	0.434
(0.03, 0.03)	0.185	0.969	0.965	0.259	0.128	0.189	0.062	0.207	0.663	0.662	0.315	0.048	0.205
(0.04, 0.04)	0.319	1.000	1.000	0.528	0.204	0.322	0.076	0.291	0.818	0.820	0.455	0.099	0.293
						$\tau = 50$							
(0, 0.04)	0.067	0.642	0.625	0.066	0.071	0.066	0.053	0.110	0.602	0.604	0.096	0.143	0.109
(0, 0.06)	0.090	0.977	0.973	0.164	0.107	0.087	0.049	0.160	0.810	0.811	0.349	0.205	0.145
(0.07, 0)	0.232	0.500	0.500	0.107	0.270	0.225	0.065	0.303	0.500	0.502	0.218	0.299	0.303
(0.10, 0)	0.424	0.862	0.860	0.269	0.506	0.412	0.071	0.436	0.691	0.688	0.447	0.380	0.431
(0.03, 0.03)	0.120	0.755	0.748	0.111	0.140	0.116	0.061	0.202	0.640	0.641	0.193	0.212	0.200
(0.04, 0.04)	0.179	0.968	0.963	0.204	0.225	0.172	0.054	0.309	0.792	0.794	0.399	0.302	0.299

Table 8. *Performance comparison with AR(1) correlation coefficient* $\rho = 0.6$ *when* $\tau = 10$ *and* 50 *in unbalanced design scenario.*

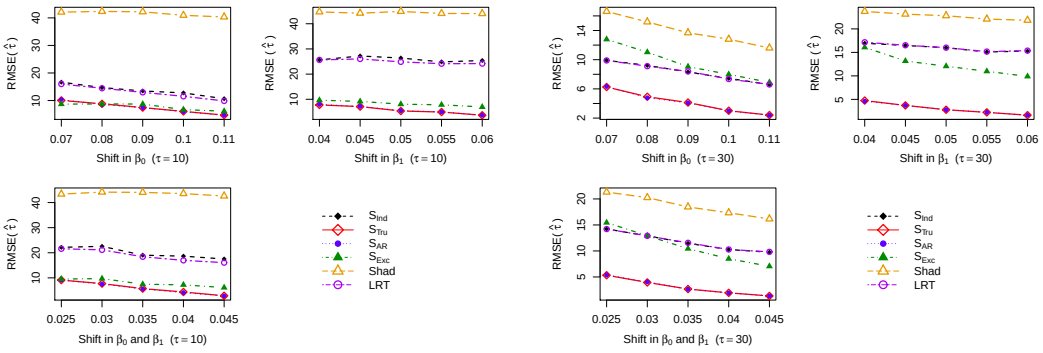
Shift	Signal probabilities							$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$					
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
						$\tau = 10$							
(0, 0.06)	0.143	0.386	0.391	0.235	0.059	0.148	0.068	0.242	0.443	0.442	0.352	0.000	0.248
(0, 0.08)	0.269	0.692	0.694	0.436	0.069	0.277	0.083	0.352	0.562	0.564	0.466	0.000	0.355
(0.07, 0)	0.526	0.508	0.502	0.397	0.090	0.536	0.127	0.465	0.474	0.478	0.473	0.000	0.463
(0.10, 0)	0.884	0.891	0.888	0.794	0.173	0.889	0.264	0.655	0.679	0.677	0.658	0.002	0.654
(0.040, 0.040)	0.453	0.615	0.614	0.446	0.088	0.462	0.106	0.436	0.522	0.520	0.476	0.000	0.437
(0.045, 0.045)	0.566	0.754	0.754	0.556	0.093	0.579	0.136	0.512	0.591	0.590	0.538	0.000	0.515
						$\tau = 50$							
(0, 0.06)	0.162	0.438	0.440	0.207	0.209	0.154	0.064	0.317	0.498	0.500	0.349	0.295	0.311
(0, 0.08)	0.299	0.759	0.755	0.416	0.382	0.284	0.081	0.428	0.653	0.658	0.539	0.389	0.425
(0.07, 0)	0.599	0.587	0.580	0.417	0.692	0.587	0.120	0.528	0.528	0.523	0.501	0.442	0.528
(0.10, 0)	0.914	0.914	0.911	0.802	0.946	0.910	0.264	0.692	0.719	0.714	0.687	0.594	0.692
(0.040, 0.040)	0.501	0.673	0.672	0.423	0.602	0.487	0.110	0.509	0.583	0.585	0.514	0.432	0.509
(0.045, 0.045)	0.617	0.802	0.799	0.568	0.711	0.604	0.131	0.580	0.671	0.671	0.591	0.501	0.580

B.2. *Exchangeable correlation*

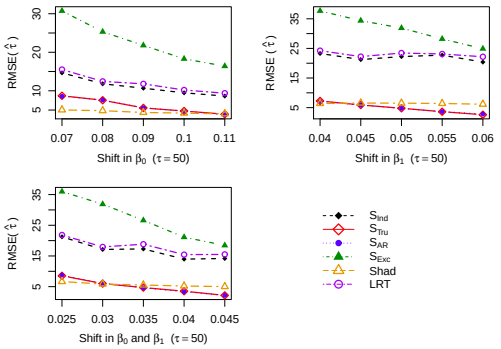
When the true correlation structure is exchangeable, performance comparisons of different methods in the unbalanced design scenario are illustrated in Tables 10 – 12 and Figs. 9 – 11.

Table 9. *Performance comparison with AR(1) correlation coefficient $\rho = 0.3$ in unbalanced design scenario.*

Shift	Signal probabilities							$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$					
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
						$\tau = 10$							
(0, 0.08)	0.571	0.651	0.646	0.583	0.083	0.579	0.143	0.517	0.554	0.554	0.535	0.000	0.516
(0, 0.10)	0.829	0.880	0.875	0.838	0.120	0.834	0.247	0.607	0.656	0.658	0.628	0.000	0.605
(0.05, 0)	0.519	0.442	0.432	0.398	0.081	0.527	0.120	0.473	0.473	0.468	0.478	0.000	0.470
(0.07, 0)	0.869	0.812	0.804	0.757	0.148	0.873	0.237	0.621	0.620	0.622	0.622	0.000	0.617
(0.030, 0.030)	0.500	0.464	0.455	0.419	0.079	0.509	0.117	0.444	0.452	0.453	0.445	0.000	0.446
(0.045, 0.045)	0.894	0.885	0.881	0.839	0.148	0.897	0.282	0.662	0.672	0.672	0.667	0.001	0.662
						$\tau = 30$							
(0, 0.08)	0.919	0.951	0.951	0.920	0.777	0.919	0.275	0.625	0.653	0.650	0.619	0.327	0.624
(0, 0.10)	0.993	0.997	0.997	0.993	0.957	0.993	0.492	0.751	0.781	0.783	0.744	0.425	0.751
(0.05, 0)	0.871	0.824	0.817	0.746	0.688	0.870	0.216	0.515	0.514	0.514	0.497	0.282	0.517
(0.07, 0)	0.997	0.994	0.994	0.981	0.978	0.997	0.480	0.702	0.702	0.702	0.680	0.407	0.701
(0.030, 0.030)	0.864	0.844	0.837	0.771	0.676	0.863	0.218	0.527	0.533	0.536	0.508	0.282	0.526
(0.045, 0.045)	0.998	0.998	0.997	0.992	0.986	0.998	0.581	0.757	0.764	0.762	0.742	0.452	0.755
						$\tau = 50$							
(0, 0.08)	0.615	0.689	0.687	0.608	0.697	0.603	0.156	0.590	0.614	0.611	0.592	0.504	0.589
(0, 0.10)	0.855	0.909	0.907	0.853	0.909	0.849	0.274	0.714	0.749	0.750	0.727	0.598	0.714
(0.05, 0)	0.578	0.507	0.497	0.427	0.657	0.570	0.137	0.530	0.526	0.525	0.510	0.446	0.527
(0.07, 0)	0.895	0.857	0.850	0.781	0.928	0.891	0.288	0.677	0.667	0.668	0.664	0.569	0.678
(0.030, 0.030)	0.534	0.504	0.496	0.426	0.622	0.524	0.134	0.525	0.515	0.517	0.503	0.438	0.528
(0.045, 0.045)	0.928	0.920	0.915	0.866	0.950	0.924	0.334	0.730	0.733	0.732	0.720	0.601	0.730

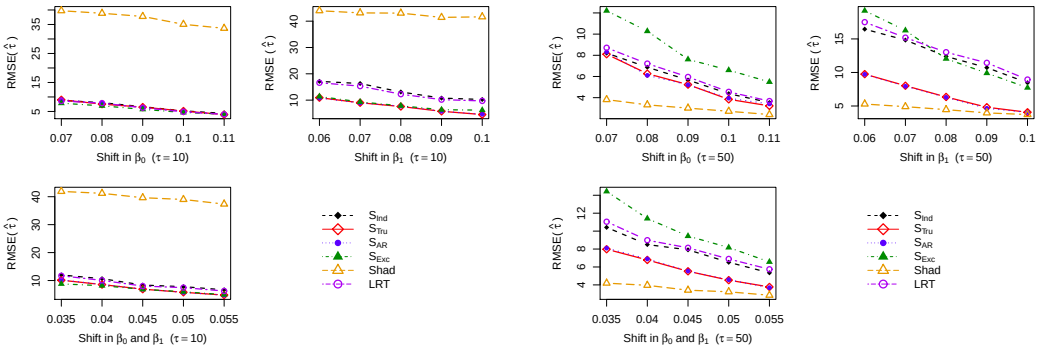


(a) $\tau = 10$ (b) $\tau = 30$



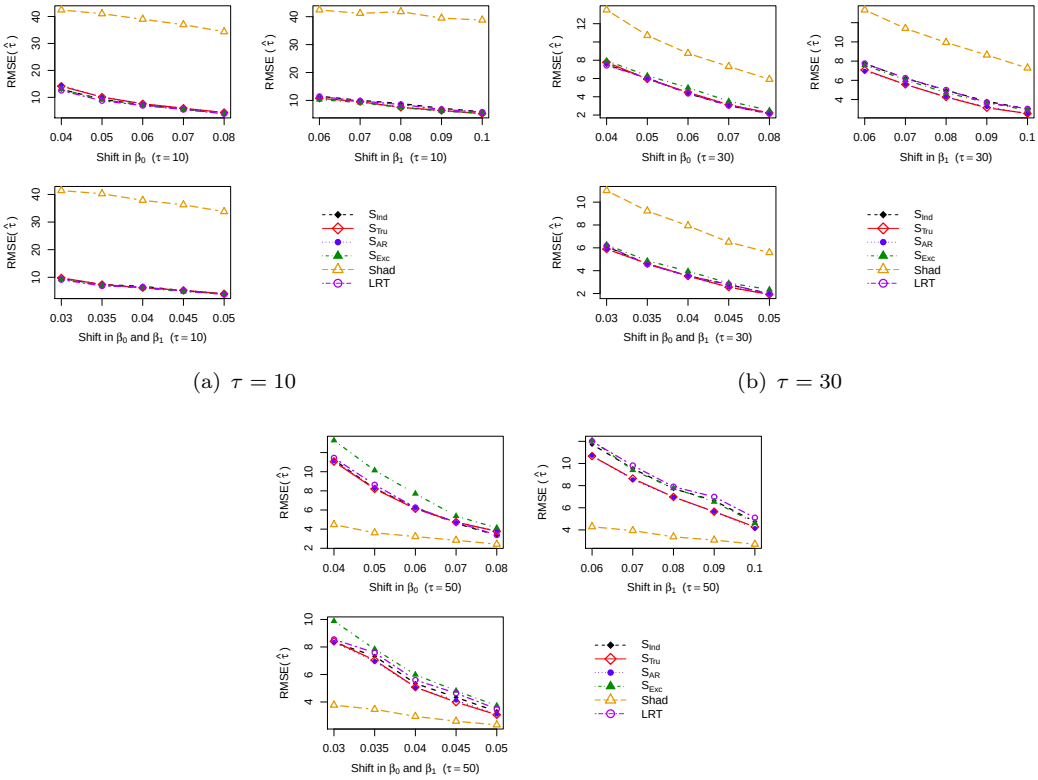
(c) $\tau = 50$

Figure 6. *Values of* RMSE($\hat{\tau}$) *with AR(1) correlation coefficient $\rho = 0.9$ in unbalanced design scenario.*



(a) $\tau = 10$ (b) $\tau = 50$

Figure 7. *Values of* RMSE($\hat{\tau}$) *with AR(1) correlation coefficient* $\rho = 0.6$ *when* $\tau = 10$ *and* 50 *in unbalanced design scenario.*



(a) $\tau = 10$ (b) $\tau = 30$

(c) $\tau = 50$

Figure 8. *Values of* RMSE($\hat{\tau}$) *with AR(1) correlation coefficient* $\rho = 0.3$ *in unbalanced design scenario.*

Table 10. *Performance comparison with exchangeable correlation coefficient* $\rho = 0.9$ *in unbalanced design scenario.*

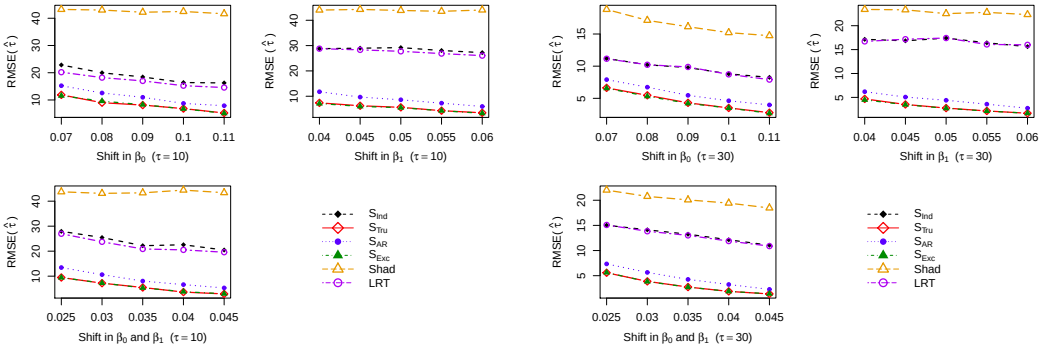
Shift	Signal probabilities							$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$					
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
						$\tau = 10$							
(0, 0.04)	0.047	0.635	0.300	0.587	0.047	0.053	0.051	0.034	0.534	0.404	0.537	0.000	0.038
(0, 0.06)	0.059	0.968	0.724	0.954	0.055	0.061	0.056	0.061	0.751	0.637	0.752	0.000	0.062
(0.08, 0)	0.130	0.503	0.297	0.480	0.064	0.140	0.050	0.212	0.511	0.405	0.506	0.000	0.231
(0.10, 0)	0.197	0.741	0.514	0.717	0.073	0.214	0.069	0.296	0.619	0.543	0.621	0.000	0.303
(0.03, 0.03)	0.070	0.708	0.383	0.662	0.056	0.077	0.050	0.097	0.582	0.475	0.589	0.000	0.099
(0.04, 0.04)	0.102	0.953	0.697	0.938	0.057	0.109	0.059	0.138	0.737	0.632	0.737	0.000	0.152
						$\tau = 30$							
(0, 0.04)	0.068	0.942	0.688	0.915	0.064	0.065	0.049	0.077	0.593	0.511	0.596	0.003	0.077
(0, 0.06)	0.085	1.000	0.985	1.000	0.072	0.087	0.052	0.122	0.830	0.744	0.830	0.014	0.114
(0.08, 0)	0.303	0.875	0.689	0.845	0.193	0.304	0.059	0.280	0.528	0.469	0.538	0.114	0.285
(0.10, 0)	0.476	0.983	0.902	0.975	0.317	0.477	0.071	0.333	0.655	0.586	0.656	0.131	0.331
(0.03, 0.03)	0.133	0.972	0.789	0.957	0.101	0.132	0.060	0.148	0.648	0.557	0.654	0.028	0.147
(0.04, 0.04)	0.202	1.000	0.980	1.000	0.144	0.201	0.056	0.205	0.810	0.730	0.811	0.044	0.211
						$\tau = 50$							
(0, 0.04)	0.054	0.624	0.366	0.580	0.068	0.052	0.056	0.093	0.602	0.495	0.601	0.106	0.088
(0, 0.06)	0.073	0.971	0.813	0.951	0.086	0.068	0.051	0.126	0.806	0.709	0.809	0.192	0.124
(0.08, 0)	0.183	0.524	0.381	0.501	0.229	0.169	0.059	0.263	0.548	0.475	0.556	0.248	0.259
(0.10, 0)	0.292	0.778	0.614	0.757	0.347	0.278	0.065	0.356	0.664	0.576	0.663	0.327	0.346
(0.03, 0.03)	0.096	0.717	0.468	0.675	0.123	0.086	0.051	0.166	0.640	0.544	0.637	0.170	0.146
(0.04, 0.04)	0.132	0.956	0.790	0.940	0.164	0.121	0.056	0.212	0.788	0.700	0.791	0.243	0.206

Table 11. *Performance comparison with exchangeable correlation coefficient* $\rho = 0.6$ *when* $\tau = 10$ *and* 50 *in unbalanced design scenario.*

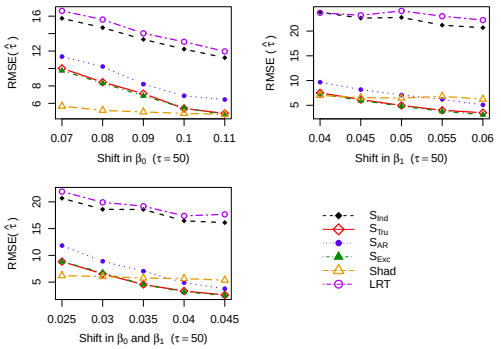
Shift	Signal probabilities							$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$					
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
						$\tau = 10$							
(0, 0.06)	0.073	0.385	0.104	0.375	0.052	0.077	0.053	0.112	0.437	0.198	0.443	0.000	0.125
(0, 0.08)	0.089	0.689	0.172	0.670	0.055	0.098	0.055	0.139	0.563	0.323	0.569	0.000	0.143
(0.08, 0)	0.195	0.247	0.199	0.243	0.063	0.209	0.059	0.268	0.346	0.293	0.345	0.000	0.278
(0.10, 0)	0.313	0.417	0.334	0.415	0.077	0.331	0.078	0.378	0.471	0.405	0.468	0.000	0.382
(0.04, 0.04)	0.133	0.402	0.169	0.398	0.058	0.144	0.054	0.172	0.454	0.282	0.462	0.000	0.189
(0.05, 0.05)	0.193	0.646	0.288	0.638	0.065	0.204	0.061	0.265	0.543	0.382	0.544	0.000	0.270
						$\tau = 50$							
(0, 0.06)	0.081	0.384	0.122	0.380	0.094	0.079	0.050	0.160	0.489	0.248	0.492	0.202	0.148
(0, 0.08)	0.111	0.702	0.219	0.688	0.136	0.103	0.051	0.224	0.629	0.419	0.633	0.239	0.231
(0.08, 0)	0.270	0.289	0.274	0.289	0.320	0.252	0.064	0.323	0.385	0.348	0.392	0.304	0.318
(0.10, 0)	0.380	0.443	0.400	0.445	0.446	0.362	0.075	0.422	0.504	0.450	0.504	0.369	0.418
(0.04, 0.04)	0.172	0.425	0.224	0.417	0.209	0.161	0.057	0.271	0.479	0.336	0.490	0.275	0.266
(0.05, 0.05)	0.253	0.679	0.368	0.669	0.307	0.237	0.062	0.329	0.595	0.455	0.605	0.309	0.329

Table 12. *Performance comparison with exchangeable correlation coefficient* $\rho = 0.3$ *in unbalanced design scenario.*

Shift	Signal probabilities							$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$					
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{Tru}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
						$\tau = 10$							
(0, 0.08)	0.148	0.476	0.174	0.472	0.072	0.154	0.060	0.218	0.458	0.271	0.456	0.000	0.222
(0, 0.10)	0.223	0.723	0.297	0.722	0.077	0.233	0.070	0.308	0.578	0.390	0.582	0.000	0.310
(0.07, 0)	0.291	0.294	0.291	0.278	0.076	0.304	0.073	0.343	0.361	0.347	0.348	0.000	0.343
(0.08, 0)	0.376	0.375	0.375	0.357	0.085	0.395	0.091	0.394	0.428	0.400	0.433	0.000	0.397
(0.04, 0.04)	0.239	0.384	0.258	0.375	0.076	0.251	0.078	0.301	0.413	0.330	0.414	0.000	0.304
(0.05, 0.05)	0.378	0.592	0.409	0.585	0.087	0.393	0.086	0.405	0.519	0.435	0.518	0.000	0.405
						$\tau = 30$							
(0, 0.08)	0.320	0.840	0.434	0.839	0.203	0.322	0.061	0.310	0.526	0.387	0.527	0.108	0.312
(0, 0.10)	0.512	0.972	0.684	0.971	0.331	0.510	0.078	0.433	0.649	0.526	0.649	0.151	0.435
(0.07, 0)	0.620	0.582	0.612	0.552	0.433	0.618	0.098	0.391	0.387	0.396	0.382	0.174	0.393
(0.08, 0)	0.752	0.722	0.754	0.701	0.575	0.751	0.112	0.427	0.433	0.439	0.428	0.213	0.428
(0.04, 0.04)	0.525	0.727	0.571	0.718	0.355	0.522	0.085	0.375	0.450	0.403	0.447	0.136	0.376
(0.05, 0.05)	0.759	0.914	0.811	0.910	0.565	0.760	0.117	0.475	0.580	0.512	0.578	0.242	0.476
						$\tau = 50$							
(0, 0.08)	0.168	0.504	0.210	0.505	0.218	0.158	0.058	0.329	0.52	0.377	0.521	0.326	0.322
(0, 0.10)	0.263	0.742	0.348	0.738	0.337	0.252	0.064	0.386	0.629	0.477	0.633	0.376	0.382
(0.07, 0)	0.351	0.327	0.349	0.304	0.429	0.336	0.077	0.394	0.398	0.398	0.390	0.357	0.396
(0.08, 0)	0.451	0.424	0.452	0.402	0.541	0.437	0.081	0.442	0.446	0.445	0.441	0.384	0.443
(0.04, 0.04)	0.292	0.410	0.310	0.399	0.357	0.281	0.062	0.359	0.462	0.383	0.462	0.337	0.357
(0.05, 0.05)	0.451	0.627	0.485	0.622	0.533	0.438	0.078	0.456	0.567	0.495	0.562	0.400	0.454

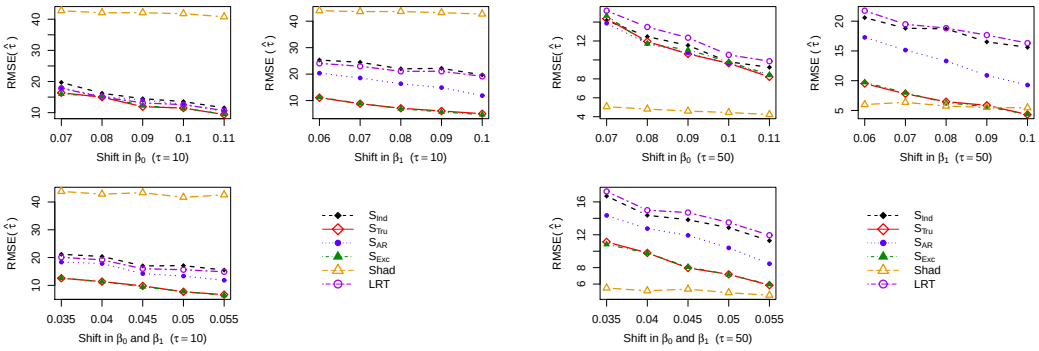


(a) $\tau = 10$ (b) $\tau = 30$



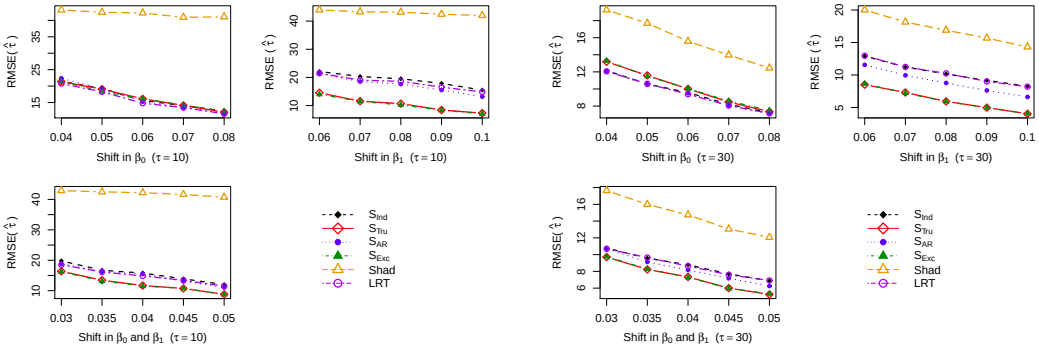
(c) $\tau = 50$

Figure 9. *Values of* RMSE($\hat{\tau}$) *with exchangeable correlation coefficient* $\rho =$ 0.9 *in unbalanced design scenario.*

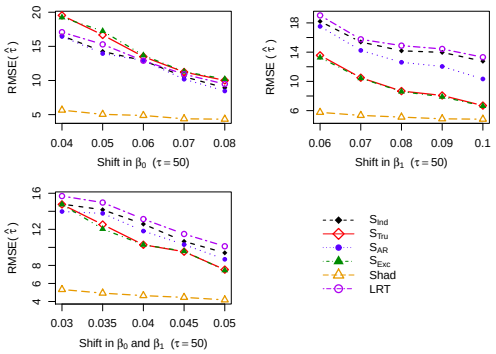


(a) $\tau = 10$ (b) $\tau = 50$

Figure 10. *Values of* RMSE($\hat{\tau}$) *with exchangeable correlation coefficient* $\rho = 0.6$ *when* $\tau = 10$ *and* 50 *in unbalanced design scenario.*



(a) $\tau = 10$ (b) $\tau = 30$



(c) $\tau = 50$

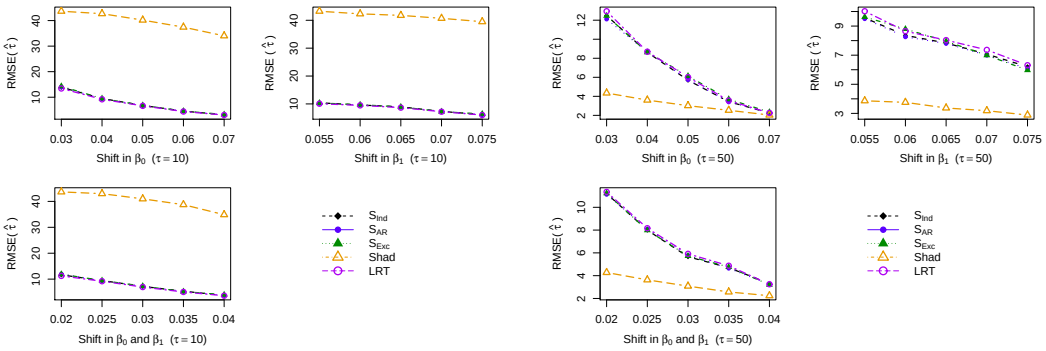
Figure 11. *Values of* RMSE($\hat{\tau}$) *with exchangeable correlation coefficient* $\rho = 0.3$ *in unbalanced design scenario.*

B.3. *Independent*

When the Poisson profile samples are independent, performance comparisons of different methods in the unbalanced design scenario are reported in Table 13 and Fig. 12.

Table 13. *Performance comparison for independent setting when* $\tau = 10$ *and* 50 *in unbalanced design scenario.*

Shift	Signal probabilities						$P(\lvert\hat{\tau} - \tau\rvert) \leq 1$				
$(\delta_0,\ \delta_1)$	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*	*GLMM*	$\boldsymbol{S_{Ind}}$	$\boldsymbol{S_{AR}}$	$\boldsymbol{S_{Exc}}$	*Shad*	*LRT*
$\tau = 10$											
(0, 0.06)	0.518	0.520	0.495	0.081	0.528	0.132	0.461	0.462	0.457	0.000	0.460
(0, 0.07)	0.689	0.689	0.668	0.091	0.698	0.183	0.545	0.544	0.546	0.000	0.547
(0.04, 0)	0.523	0.517	0.469	0.074	0.532	0.135	0.483	0.484	0.489	0.000	0.486
(0.06, 0)	0.922	0.920	0.888	0.135	0.927	0.323	0.695	0.694	0.689	0.000	0.695
(0.025, 0.025)	0.540	0.537	0.492	0.086	0.547	0.146	0.486	0.488	0.488	0.000	0.486
(0.035, 0.035)	0.885	0.880	0.847	0.127	0.888	0.291	0.656	0.657	0.656	0.000	0.654
$\tau = 50$											
(0, 0.06)	0.539	0.543	0.511	0.615	0.531	0.149	0.545	0.545	0.541	0.465	0.545
(0, 0.07)	0.714	0.710	0.682	0.770	0.708	0.217	0.638	0.64	0.636	0.541	0.638
(0.04, 0)	0.568	0.563	0.502	0.636	0.559	0.169	0.533	0.535	0.523	0.456	0.536
(0.06, 0)	0.943	0.942	0.912	0.965	0.94	0.419	0.739	0.738	0.732	0.601	0.739
(0.025, 0.025)	0.579	0.573	0.526	0.646	0.569	0.162	0.549	0.549	0.548	0.471	0.551
(0.035, 0.035)	0.915	0.910	0.880	0.939	0.913	0.357	0.727	0.725	0.721	0.606	0.726



(a) $\tau = 10$ (b) $\tau = 50$

Figure 12. *Values of* RMSE($\hat{\tau}$) *for independent setting when* $\tau = 10$ *and* 50 *in unbalanced design scenario.*