# Three-Dimensional Numerical Simulation of Lightning Discharge Based on DBM Model

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*Abstract*—**The modeling and simulation of lightning discharge are very important for the lightning protection engineering and lightning research. In this paper, with the thundercloud electrostatic model and potential equation in the space, a three-dimensional(3-D) numerical simulation of lightning discharge at 50-m resolution was presented. The dielectric breakdown model(DBM) based on fractal theory and bidirectional leaders independent develop mechanism were employed to simulate the lightning leader progression. The effects of probability exponent $\eta$ and internal E-field $E_{in}$ along the channel on the on-ground E-field distribution were analyzed respectively. The results show that the distribution of E-field at the ground level below the stepped leader is similar to Gauss distribution. The peak value of the E-field at the ground level increases with the probability exponent and internal E-field along the channel decreasing.**

***Keywords-DBM; thundercloud model; lightning simulation; distribution of E-field; parameters of model***

## I. INTRODUCTION

Lightning is a typical damaging source in the nature. Especially with the quick increase of microelectronic devices used in the systems, the probability of lightning damage increases accordingly [1], [2]. However, researches on lightning discharge utilizing natural lightning are quite difficult to carry out because of the randomicity and destructive effect of lightning discharges. Therefore, the modeling and simulation of lightning discharges are valuable for the lightning protection engineering. In the late 1980s, the DBM model was introduced by Niemeyer et al. [3], which was then developed and used in the simulation of lightning discharges. Nevertheless, the DBM model was mostly employed within the confines of 2-D simulation, or 3-D simulation with a low resolution, which can't present truly the 3-D characteristics of lightning discharges [4]-[7]. In the present work, through the combination of a thundercloud electrostatic model with 3-D DBM model, a fine resolution 3-D numerical simulation of lighting discharge was presented according to fractal theory and bidirectional leaders independent develop mechanism. Meanwhile, considering that the distribution of E-field at the ground level below the stepped leader will determine the points of strike to a great extent, the effects of probability exponent $\eta$ and internal E-field $E_{in}$ on the on-ground E-field distribution were also analyzed respectively.

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## II. THE BASIC OF 3-D FRACTAL SIMULATION OF LIGHTNING DISCHARGE

The 3-D fractal simulation of lightning discharge in this paper is derived from the macroscopic phenomenon of lightning discharges and do not deal with the microscopic processes of breakdown. Based on Laplace's equation (or Poisson's equation) and the boundary conditions, the potential distribution in the interested region is fixed. Then the 3-D simulation of lightning discharge is realized according to fractal theory and bidirectional leaders independent develop mechanism. The progression of the lightning leaders is determined by these factors such as thundercloud model, DBM model, parameter settings and so on. The concrete application of each factor is discussed as follows.

### A. Thundercloud Model

The initial boundary conditions of the interested region are determined by the structure of the thundercloud model in the simulation. In the traditional dipole and multiple-dipole models, as a result of oversimplified point concentrations of large distributions of charges, the geometrical variables in question can significantly influence the calculation of the E-field [8]. Thus, a three-level set of charged circular discs representing a generic cell recommended by Amoruso and Lattarulo [8] is used in this paper, as shown in Fig. 1. In this model, the local non-uniformity in the charge distribution of the thundercloud is neglected. Taking into account of the mirroring action of the ground which is assumed to be a perfect conductor, the on-ground E-field on the observation point $P$ can be represented as follows:

$$\boldsymbol{E}(P)=\sum_i \frac{\rho_i}{2\pi\varepsilon}\int_{S_i}\boldsymbol{u}_i\frac{dS_i}{r_i} \tag{1}$$

in which $S_i$ is the surface where the charge of each layer located, $\boldsymbol{u}_i$ is the unit vector of each surface, $\rho_i$ is the volume charge density deposited on the surface $S_i$, $r_i$ is the distance between the elementary surface $dS_i$ and $P$. To each disc surface, the surface integral is given by

$$\int_s \frac{dS}{r} = 2[-(1-\varepsilon')\frac{\pi}{2}h + r_d\,\mathrm{E}(p) + \frac{a^2 - R^2}{r_d}\mathrm{K}(p) + \frac{h^2}{r_d}\frac{a-R}{a+R}\Pi(p,m)] \tag{2}$$

where $a$ is the radius of the charged circular disc, $R$ is the horizontal distance from the observation point to the centre of the disc, $h$ is the height of the disc, $\varepsilon'$=-1, 0, 1 when $R$ is less than, equal to, greater than $a$, respectively, $r_d = \sqrt{(a+R)^2 + h^2}$ , $\mathrm{K}(p)$ , $\mathrm{E}(p)$ and $\Pi(p,m)$ are complete elliptic integrals of the first, second and third kind, respectively, where $p = 2\sqrt{aR}/r_d$ and $m = 2\sqrt{aR}/(a+R)$ . Thus, thunderclouds of different characteristics can be obtained through superposition of different cell systems.

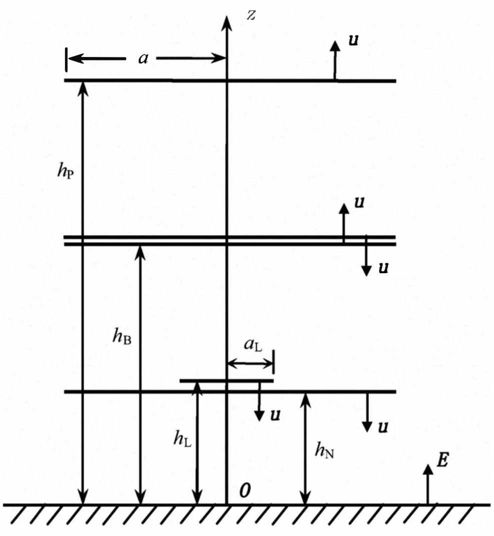


Figure 1. Three-level charged discs model

Considering that the on-axis field of thundercloud increases parabolically with ascending of the height approximately, the field distribution in the space can be obtained by the method of parabolic curve fitting based on the on-ground E-field data. Then, the potential distribution in the interested region can be determined as $V = -\int \boldsymbol{E}\, d\boldsymbol{l}$. And the potential distributions on the boundary of the interested region serve as the initial boundary conditions of the numerical simulation.

### B. DBM Model

In the fractal simulation, the progression of the leader depends on DBM model principally. There are two types of commonly used DBM models: NPW model and WZ model [9], [10]. NPW model, presented by Niemeyer et al. in 1980s, assumes that the selection of the discharge point and the discharge direction during the leader progression is random, and considers the probability of the leader progression is the biggest along the direction where the electric field intensity is the greatest. However, NPW model supposes the leader channel is equipotential, and neglects the critical electric field threshold for propagation, which are unrealistic. Therefore, WZ model developed by Wiesmann and Zeller adds a critical electric field threshold $E_{crit}$ for propagation and an internal electric field $E_{in}$ in the leader channel based on the NPW model. The propagation probability $P_i$ of WZ model can be expressed as

$$P_i = \begin{cases} \dfrac{|E_{ij} - E_{crit}|^{\eta}}{\sum\limits_{j(E_{ij} > E_{crit})} |E_{ij} - E_{crit}|^{\eta}} & \text{for } E_{ij} \ge E_{crit} \\ 0 & \text{for } E_{ij} < E_{crit} \end{cases} \tag{3}$$

where $E_{ij} = (\varphi_i - \varphi_j)/\Delta l$ is the magnitude of the electric field between the $i$th point of the channel and the adjacent non-channel points, $\Delta l$ is the distance between the two points, probability exponent $\eta$ denotes the intensity of Faraday effect, which would influence the fractal dimension. Results of research have shown that the channels visualize as "bush" type for $\eta \le 1$, but "branched" type for $\eta \ge 3$.

In the WZ model, if the reference potential of the initial breakdown point is $\varphi_0$, the potential of the $i$th channel point can be obtained as

$$\varphi_i = \varphi_0 - E_{in}\sum \Delta l \tag{4}$$

where $\sum \Delta l$ is the distance along the channel between the initial breakdown point and the $i$th channel point.

In the 3-D DBM model, the leaders propagate step-by-step from an initial breakdown point. Fig. 2 shows the 18 adjacent non-channel points around the breakdown point. The extension probabilities $P_{ei}$ of the possible channel points in the next step and the propagation probabilities $P_{pi}$ of the 18 adjacent non-channel points around the selected channel points can be calculated from Eq. (3). Then, the breakdown point on the channel and the discharge direction are selected by Monte Carl Methods.

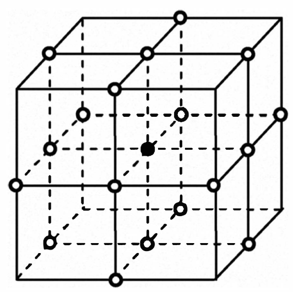

Figure 2. 18 adjacent non-channel points around the breakdown point. Solid sphere indicates the breakdown point. Hollow spheres indicate the possible new breakdown point.

### C. Parameter Settings

Consider a medium-sized thundercloud in this paper. The corresponding parameters are set as follows: the disc radius of the thundercloud is $a$=2km, the charge densities of the positive discs and the negative discs are $\rho_P$=0.5nC·m$^{-3}$, $\rho_N$=-0.5nC·m$^{-3}$, respectively, the height of the upper positive layer, the intermediate double layer and the lower negative layer are set to $h_P$=10km, $h_B$=5.5km, $h_N$=1.5km, respectively. Furthermore, an additional 0.5-km radius positive layer with $\rho_L$=0.1nC·m$^{-3}$ is embedded in the centre of the lower negative layer. Considering that the stepped leaders typically advance in segments on the order of 50m, hence 50-m spatial resolution is adopted in the simulation. Based on the previous works, the critical field threshold $E_{init}$ for the leader initiation is set to be 200kV·m$^{-1}$, and $E_{crit}$ is chosen to be 150kV·m$^{-1}$.

## III. Implementation of 3-D Fractal Simulation

Based on the parameters set above, the flow chart of 3-D fractal simulation of lightning discharge is shown in Fig. 3. Though the breakdown physics mechanisms are different for positive and negative leader propagation, both types of leaders are treated in the same way in this simulation. The leaders initiate at a point, which is chosen at (0, 0, 3.0)km in the paper, while the electric field magnitude in a certain region exceeds the threshold $E_{init}$. Then the bidirectional leaders develop independently.

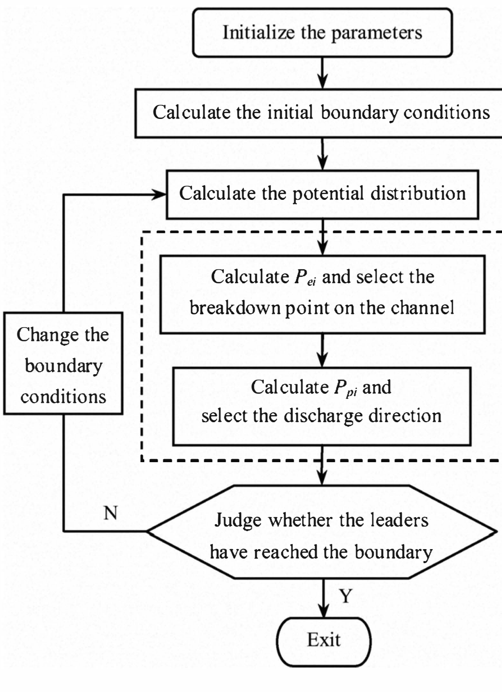


Figure 3. Flow chart of simulation

Fig. 4 shows a negative cloud-to-ground (CG) lightning discharge diagram with $\eta$=3 and $E_{in}$=-17kV·m$^{-1}$, in which the main channel of the bidirectional leaders is visible and branches of the leaders are prolific.

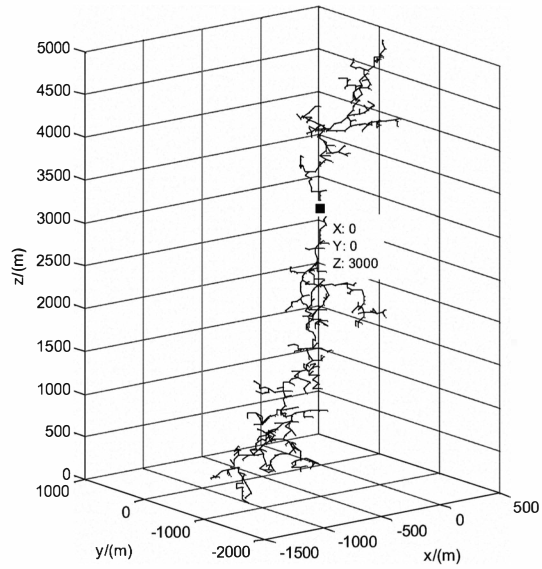


Figure 4. CG lightning discharge diagram

## IV. E-Field Distributions At The Ground Level Below The Leaders

The E-field distribution at the ground level is determined by the thundercloud and the leaders together. Moreover, the downward stepped leader plays an important role in the selection of the point of strike when the leader has penetrated up to a height of hundreds of meters. Here, $h$ represents the tip height of the downward leader above the ground. The effects of the parameter settings on the on-ground E-field distribution were analyzed by taking example for $h$=500m as follows.

### A. Effect of Probability Exponent

The probability exponent $\eta$ can influence the fractal dimension. The greater the value of $\eta$ is, the less the fractal dimension is. It means that while the value of $\eta$ is enlarged, the branches of the leaders will decrease, which will decrease the E-field at the ground level contributed by the leader simultaneously. By remaining the parameters as specified in Section II and setting $E_{in}$=-17kV·m$^{-1}$, numerical simulations of lightning discharge for $\eta$=1, 2, 3, 4 are compared, and the corresponding E-field distributions at the ground level are obtained. The results show that the on-ground E-field distribution below the stepped leader is similar to Gauss distribution, as shown in Fig. 5. And the peak values of the E-field calculated from our simulations vary with the probability exponent, as listed in Table I. When $\eta$ is larger, the peak value of the E-field at the ground level is smaller. Although there are some statistic errors because of the randomicity of the leader development, the calculated results are consistent with the above-mentioned results of theory analysis.

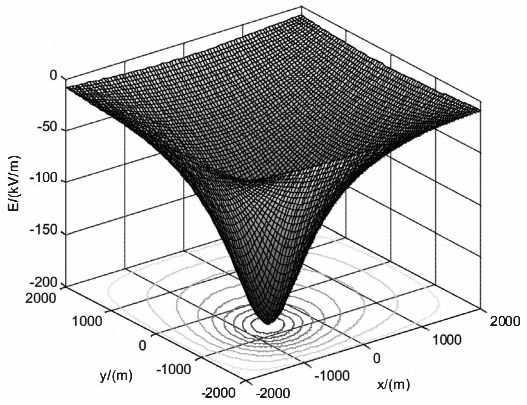


Figure 5. On-ground electric field distribution when $\eta$=3

TABLE I. PEAK VALUES OF THE ON-GROUND E-FIELD FOR DIFFERENT VALUES OF $\eta$

| $\eta$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| **Peak value of the on-ground E-field(kV·m$^{-1}$)** | -240.8 | -213.9 | -198.2 | -153.6 |

### B. Effect of the Internal E-Field

In order to ensure the extensibility of the main lightning channel and the prolific branched structure in the CG lightning simulation simultaneously, the probability exponents for the probabilities $P_{ei}$ and $P_{pi}$ are considered independently. Set $\eta$=3 for the extension probability $P_{ei}$, and $\eta$=1 for the propagation probability $P_{pi}$. By some discrete values of $E_{in}$, the relation between the peak values of the on-ground E-field and the internal E-field is obtained, as shown in Table II. The peak value of the E-field increases with the internal E-field $E_{in}$ decreasing. This is because that the magnitude of internal E-field along the channel is in relation to the channel current. The less internal E-field is, the higher channel current is. Thus, the contribution of the channel to the on-ground E-filed is greater in the situation of the same fractal dimension.

TABLE II. PEAK VALUES OF THE ON-GROUND E-FIELD FOR DIFFERENT VALUES OF $E_{in}$

| $E_{in}$(kV·m$^{-1}$) | -17 | -10 | -5 | 0 |
|---|---|---|---|---|
| **Peak value of the on-ground E-field(kV·m$^{-1}$)** | -191.9 | -220.9 | -252.6 | -294.3 |

## V. CONCLUSIONS

In this paper, a 3-D numerical simulation of lightning discharge at 50-m resolution was presented based on the three-level electrostatic model of a thunderstorm cell recommended by Amoruso and Lattarulo. Then, taking the downward leader tip height $h$=500m for example, the effects of probability exponent $\eta$ and internal E-field $E_{in}$ on the on-ground E-field distribution were discussed according to the simulation, respectively. Results show that the distribution of on-ground E-field is similar to Gauss distribution, and the peak value of the E-field at the ground level increases with the probability exponent and internal E-field decreasing.


## ACKNOWLEDGMENT

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