Two-dimensional packing problem

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Abstract

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1 Two-dimensional packing problem

Notations

Sets and indices

- I: Set of components, $I = \{0, 1, \ldots, n\}$.
- i: Index of components, $i \in I$.

Parameters

- W_{UB}: Upper bound of the width.
- L_{UB}: Upper bound of the length.
- w_i: Width of component i.
- l_i: Length of component i.

Decision variables

- W: width of the packing.
- L: length of the packing.
- x_i: X-coordinate of component i.
- y_i: Y-coordinate of component i.
- $u_{i,j}$: Binary variable indicating whether component i is placed at the left of component j.
- $v_{i,j}$: Binary variable indicating whether component i is placed at the above of component j.
- μ_i: Binary variable indicating whether component i is in rotation mode.

Optimization model

$$\begin{aligned}
\min \quad & WL && (1)\\
\text{s.t.} \quad & x_i + w_i(1-\mu_i) + l_i\mu_i \le x_j + W_{UB}(1-u_{i,j}) \quad \forall i,j \in I, i \ne j && (2)\\
& y_i + l_i(1-\mu_i) + w_i\mu_i \le y_j + L_{UB}(1-v_{i,j}) \quad \forall i,j \in I, i \ne j && (3)\\
& u_{i,j} + u_{j,i} + v_{i,j} + v_{j,i} \ge 1 \quad \forall i,j \in I, i \ne j && (4)\\
& u_{i,j}, v_{i,j} \in \{0,1\} \quad \forall i,j \in I, i \ne j && (5)\\
& \mu_i \in \{0,1\} \quad \forall i \in I && (6)\\
& 0 \le x_i \le W - w_i(1-\mu_i) - l_i\mu_i \quad \forall i \in I && (7)\\
& 0 \le y_i \le L - l_i(1-\mu_i) - w_i\mu_i \quad \forall i \in I && (8)
\end{aligned}$$