Operations Research: Models, Algorithms, and Implementations

2025-08-31

Preface 3

I 4

1 5

1.1 5

1.2 6

2 9

2.1 10

2.2 11

2.3 14

2.4 EOQ 15

References 16

Preface

This is a Quarto book.

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Part I

1

Inventory Management

Just In Time, JIT
JIT JIT
Lean Manufacturing

-

1.
2.

Scientific Inventory Management

1.
2.

1.1

demand (Deterministic) Stochastic
review (Continuous Review) (Periodic Review)
1
lead time 0 0
backorder
planning horizon (Single Period) (Multi Period) (Infinite)

EOQ

Wagner-Whitin

1.2

ordering cost	1	fixed cost	1	K
purchase cost		c		
stockout cost		p		
holding cost		1	h	

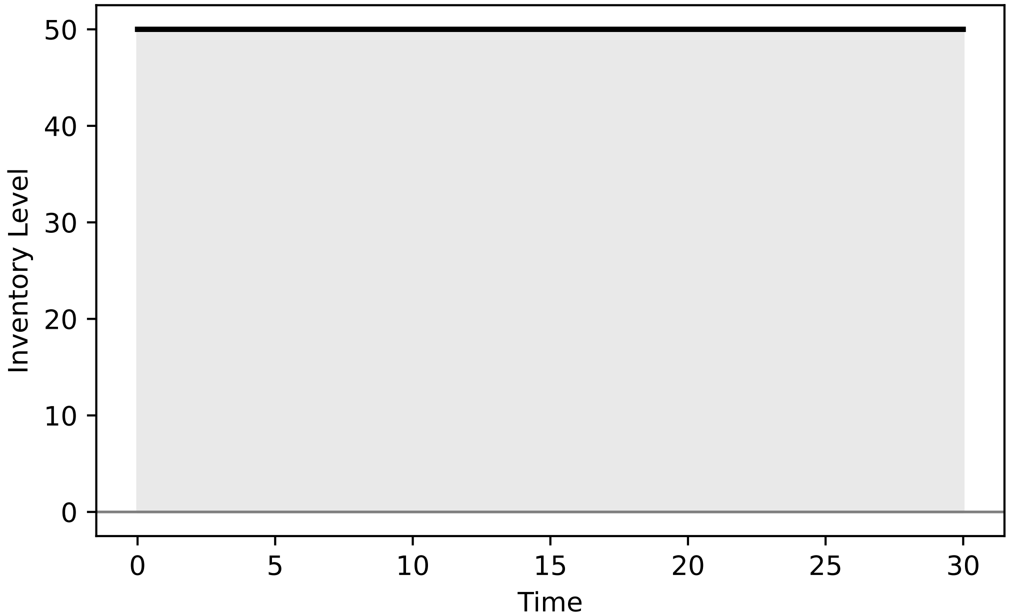
1 1 h 30 50 $30 \times 50 \times h = 1500h$

$\times h = 1500h$

```python
import matplotlib.pyplot as plt
import numpy as np

t = np.linspace(0, 30, 1000)
inventory = np.full_like(t, 50)

# Plotting the inventory level
plt.fill_between(t, inventory, color="lightgray", alpha=0.5, label="Inventory Level")
plt.plot(t, inventory, label="Inventory Level", color="black", linewidth=2)
plt.xlabel("Time")
plt.ylabel("Inventory Level")
plt.axhline(0, color="gray", linewidth=1)
plt.tight_layout()
plt.show()
```

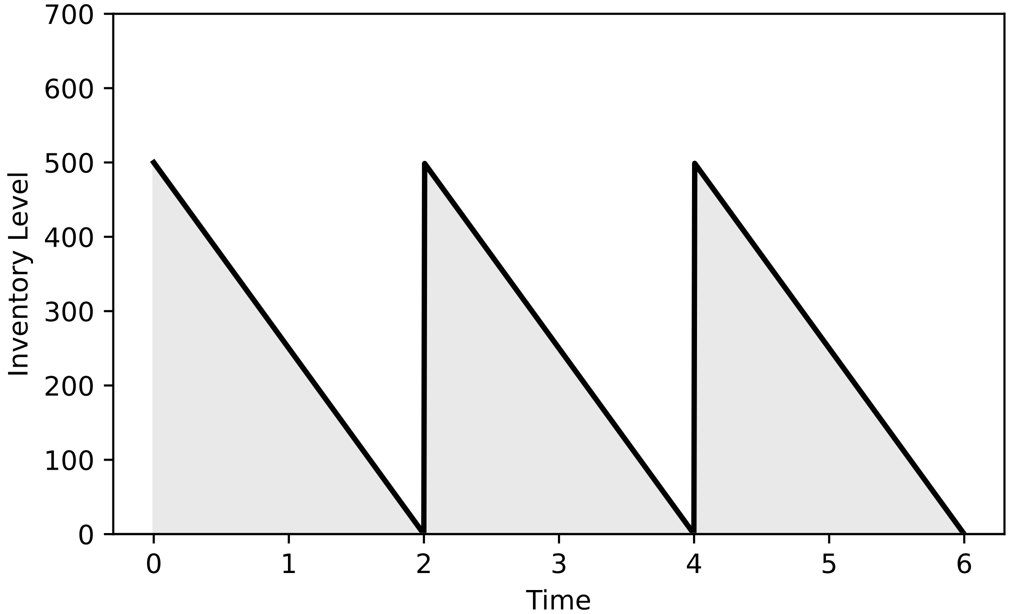


500 6

```
# Parameters
d = 250  # Demand rate
Q = 500  # Order quantity
T = Q / d  # Cycle length
t = np.linspace(0, 2.999 * T, 1000)

# Inventory level over time
inventory = np.maximum(0, Q - (d * t) % Q)

# Plotting the inventory level
plt.fill_between(t, inventory, color="lightgray", alpha=0.5, label="Inventory Level")
plt.plot(t, inventory, label="Inventory Level", color="black", linewidth=2)
plt.xlabel("Time")
plt.ylabel("Inventory Level")
plt.axhline(0, color="gray", linewidth=1)
plt.ylim(bottom=0, top=Q + 200)
plt.tight_layout()
plt.show()
```



$$\frac{2 \times 500}{2} \times 3 \times h$$

2

EOQ: Economic Order Quantity Harris (1990)

EOQ demand rate d 0 Q EOQ

K h c

EOQ (Snyder Shen 2019)

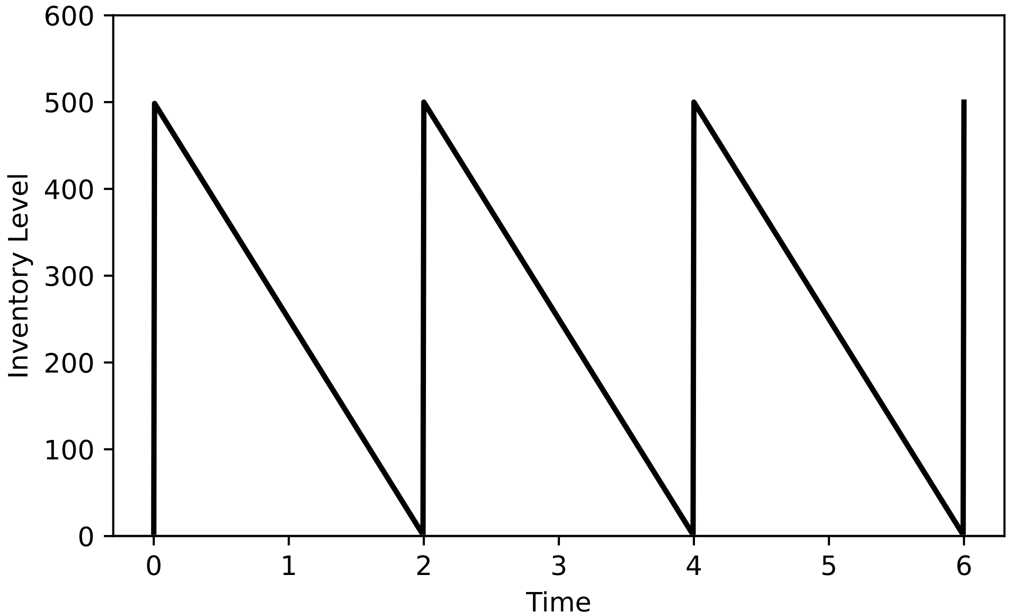
1. Zero-inventory ordering (ZIO). 0 0 0
2. Constant order sizes. d 0

```python
import matplotlib.pyplot as plt
import numpy as np

# Parameters
d = 250  # Demand rate
Q = 500  # Order quantity
T = Q / d  # Cycle length
t = np.linspace(0, 3 * T, 1000)  # Time from 0 to 3 cycles

# Inventory level over time
inventory = np.maximum(0, Q - (d * t) % Q)
inventory[0] = 0

# Plotting the inventory level
plt.plot(t, inventory, label="Inventory Level", color="black", linewidth=2)
plt.xlabel("Time")
plt.ylabel("Inventory Level")
plt.axhline(0, color="gray", linewidth=1)
plt.ylim(bottom=0, top=Q + 100)
plt.tight_layout()
plt.show()
```



(cycle)

$$T = \frac{Q}{d}$$

2.1. A 250 500

$$T = \frac{500}{250} = 2$$

2.1

1

1 K

Q c cQ

T Q 0 $\frac{Q}{2}$ $\frac{hQ}{2}$ T $\frac{Q}{d}$ 1

$$\frac{hQ}{2} \cdot T = \frac{hQ^2}{2d}$$

1

$$K + cQ + \frac{hQ^2}{2d}$$

T $g(Q)$

$$\begin{aligned} g(Q) &= \frac{1}{T}\left(K + cQ + \frac{hQ^2}{2d}\right) \\ &= \frac{d}{Q}\left(K + cQ + \frac{hQ^2}{2d}\right) \\ &= \frac{Kd}{Q} + cd + \frac{hQ}{2} \end{aligned}$$

Q

$$g(Q) = \frac{Kd}{Q} + cd + \frac{hQ}{2}$$

2.2

EOQ $g(Q)$ Q

$g'(Q)$ 0 Q^*

$$g'(Q) = -\frac{Kd}{Q^2} + \frac{h}{2} = 0$$

$$Q^* = \sqrt{\frac{2Kd}{h}}$$

EOQ EOQ formula Q^*

$g''(Q)$

$$g''(Q) = \frac{2Kd}{Q^3} > 0$$

$g''(Q) > 0$ Q^*

Q^*

2.1. *EOQ* Q^*

$$Q^* = \sqrt{\frac{2Kd}{h}}$$

Q^* T^*

$$T^* = \frac{Q^*}{d} = \sqrt{\frac{2K}{hd}}$$

2.1.

1. Q^* c
2. h Q^*
3. K Q^*

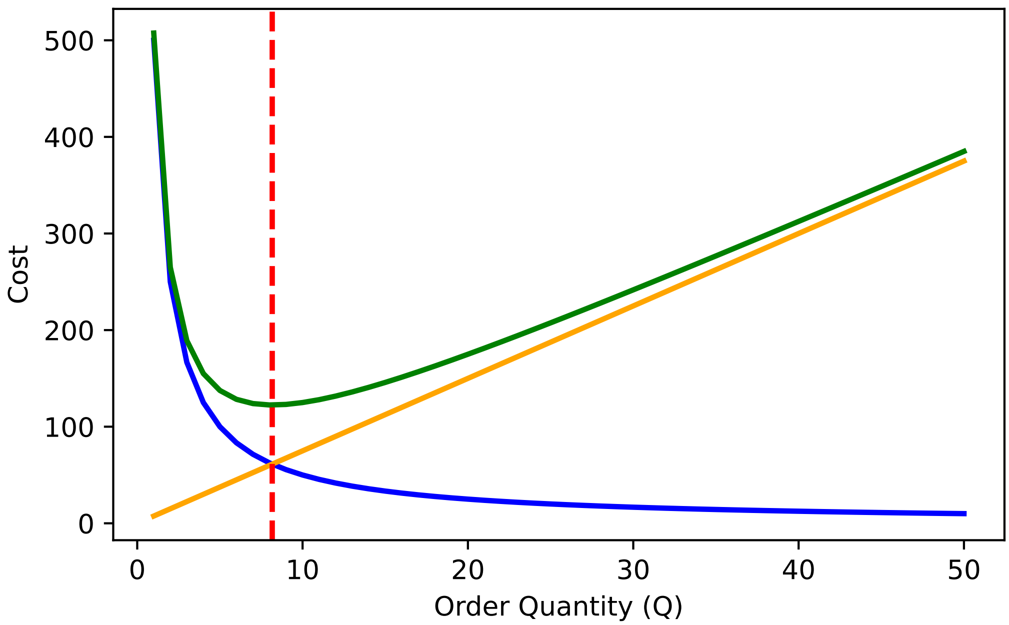
$c = 0$

```
# Parameters
K = 500  # Order cost
h = 15  # Holding cost
c = 0  # Purchase cost
Q = np.linspace(1, 50, 50)

# Average cost function
g = (K / Q) + c + (h * Q / 2)

# Plotting the costs vs order quantity
plt.plot(Q, K / Q, label="Order Cost", color="blue", linewidth=2)
plt.plot(Q, c + (h * Q / 2), label="Holding Cost", color="orange", linewidth=2)
plt.plot(Q, g, label="Average Cost", color="green", linewidth=2)
plt.axvline(
    x=np.sqrt(2 * K / h),
    color="red",
    linestyle="--",
```

```
    label="Optimal Order Quantity",
    linewidth=2,
)
plt.xlabel("Order Quantity (Q)")
plt.ylabel("Cost")
plt.tight_layout()
plt.show()
```



Q^* Q^*

$$\frac{Kd}{Q^*} = \frac{hQ^*}{2} \Longrightarrow Q^* = \sqrt{\frac{2Kd}{h}}$$

Q

2.2. 250 PC 5000 1 150 10 Q^*

$$Q^* = \sqrt{\frac{2 \cdot 5000 \cdot 250}{150}}$$

Q^*

```python
def eoq(K, d, h):
    """
    Calculate the Economic Order Quantity (EOQ).

    Parameters:
    K (float): Order cost
    d (float): Demand rate
    h (float): Holding cost

    Returns:
    float: Optimal order quantity Q*
    """
    return np.sqrt(2 * K * d / h)


if __name__ == "__main__":
    K = 5000  # Order cost
    d = 250  # Demand rate (units per month)
    h = 150  # Holding cost (per unit per month)

    Q_star = eoq(K, d, h)
    print(f"Optimal Order Quantity (Q*): {Q_star:.2f}")
```

```
Optimal Order Quantity (Q*): 129.10
```

PC $g(129)$ $g(130)$

2.3

EOQ 0 $L > 0$ Q^* L Q^*

r reorder point r L dL

$$r = dL$$

2.3. 4 $L = 1/4$

$$r = dL = 250 \times \frac{1}{4} = 62.5$$

PC 63

2.4 EOQ

- EOQ
- quantity discount EOQ
  - all-units discount
  - incremental discount

References

Harris, Ford W. 1990. How many parts to make at once . *Oper. Res.* 38 (6): 947–50.

Snyder, Lawrence V, Zuo-Jun Max Shen. 2019. *Fundamentals of supply chain theory*. 2nd . Nashville, TN: John Wiley & Sons.