

Operations Research: Models, Algorithms, and Implementations

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Preface	3
I	4
1	5
1.1	5
1.2	6
2	9
2.1	10
2.2	11
2.3	14
2.4 EOQ	15
References	16

Preface

This is a Quarto book.

To learn more about Quarto books visit <https://quarto.org/docs/books>.

Part I

1

Inventory Management

i

JIT Just In Time, JIT
 JIT
 Lean Manufacturing

•

- 1.
- 2.

Scientific Inventory Management

- 1.
- 2.

1.1

demand	(Deterministic)	Stochastic	
review	(Continuous Review)	(Periodic Review)	
1			
lead time		0	0
backorder			
planning horizon	(Single Period)	(Multi Period)	(Infinite)

EOQ

1.2

ordering cost	1	fixed cost	1	K
purchase cost		c		
stockout cost		p		
holding cost		1	h	

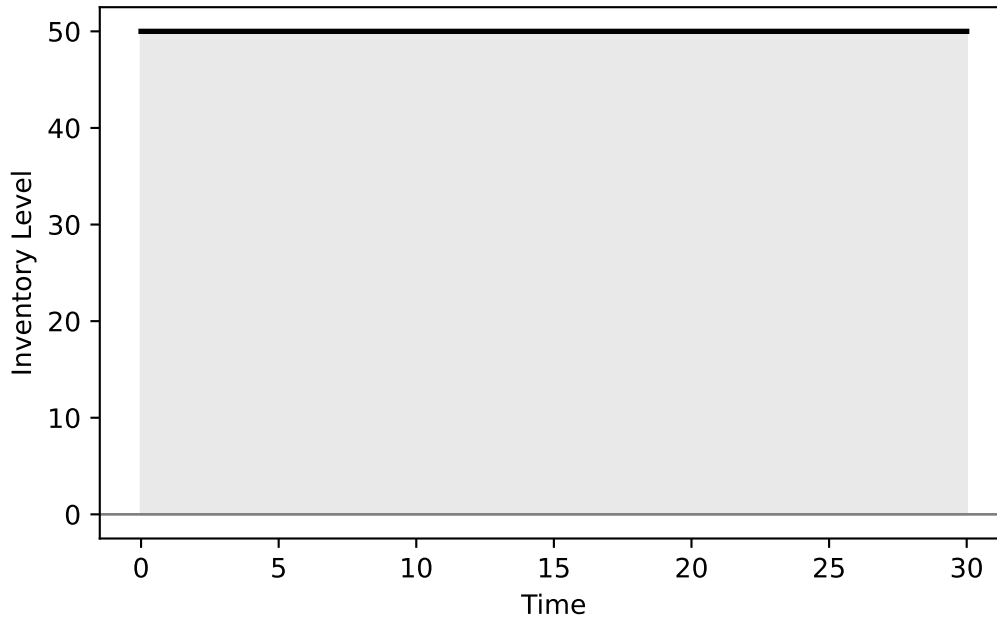
1	1	h	30	50	$30 \times 50 \times h = 1500h$
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 $\times h = 1500h$

```
import matplotlib.pyplot as plt
import numpy as np

t = np.linspace(0, 30, 1000)
inventory = np.full_like(t, 50)

# Plotting the inventory level
plt.fill_between(t, inventory, color="lightgray", alpha=0.5, label="Inventory Level")
plt.plot(t, inventory, label="Inventory Level", color="black", linewidth=2)
plt.xlabel("Time")
plt.ylabel("Inventory Level")
plt.axhline(0, color="gray", linewidth=1)
plt.tight_layout()
plt.show()
```

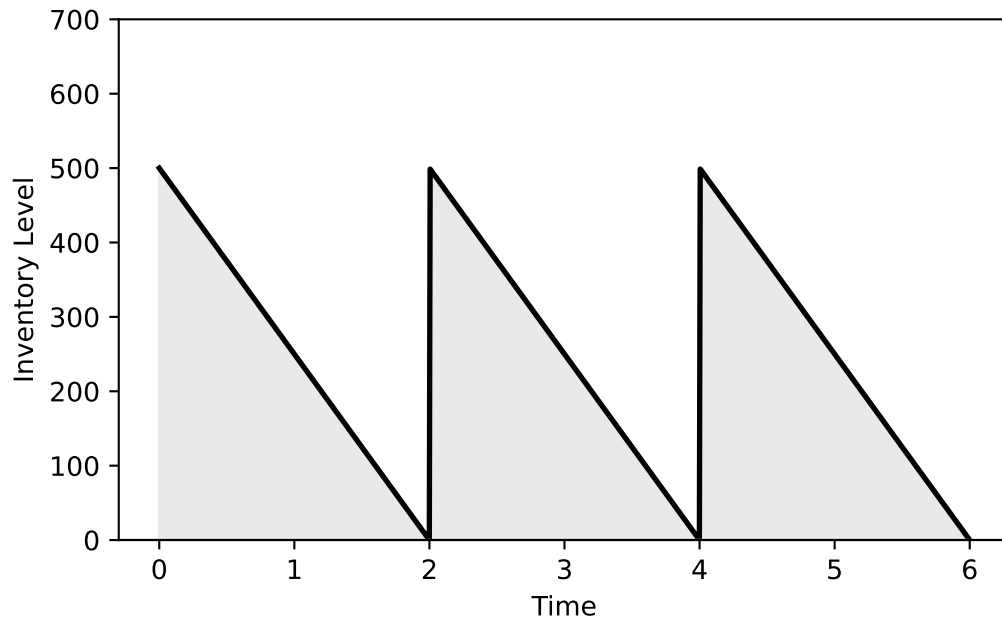


500 6

```
# Parameters
d = 250 # Demand rate
Q = 500 # Order quantity
T = Q / d # Cycle length
t = np.linspace(0, 2.999 * T, 1000)

# Inventory level over time
inventory = np.maximum(0, Q - (d * t) % Q)

# Plotting the inventory level
plt.fill_between(t, inventory, color="lightgray", alpha=0.5, label="Inventory Level")
plt.plot(t, inventory, label="Inventory Level", color="black", linewidth=2)
plt.xlabel("Time")
plt.ylabel("Inventory Level")
plt.axhline(0, color="gray", linewidth=1)
plt.ylim(bottom=0, top=Q + 200)
plt.tight_layout()
plt.show()
```



$$\frac{2 \times 500}{2} \times 3 \times h$$

2

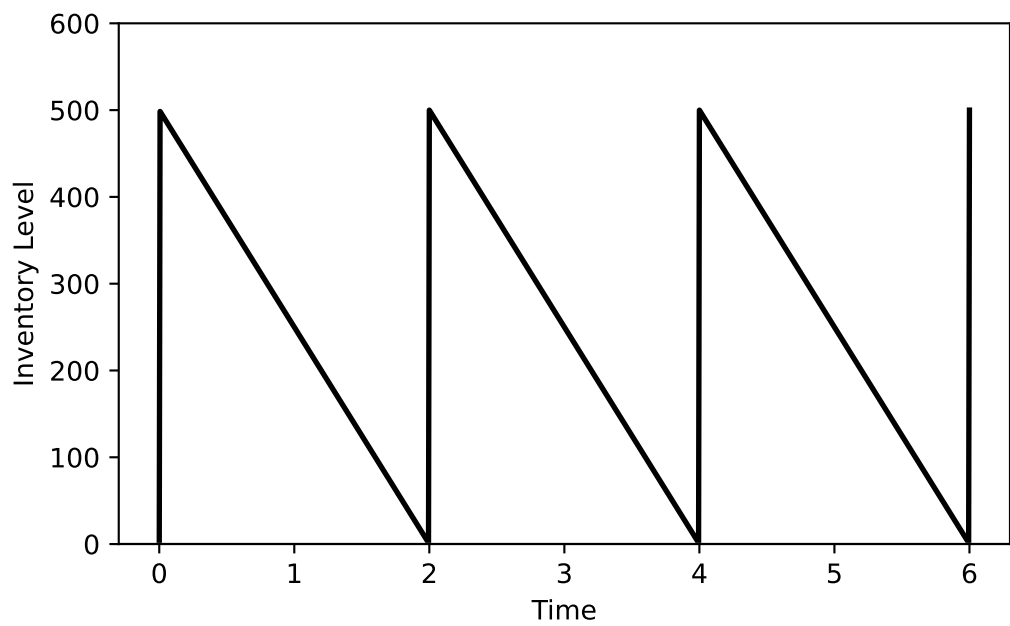
EOQ: Economic Order Quantity			Harris (1990)		
EOQ	Q		EOQ	demand rate	d
	K	h			0
		c			
EOQ	(Snyder Shen 2019)				
1. Zero-inventory ordering (ZIO).	0		0		0
2. Constant order sizes.	d		0		

```
import matplotlib.pyplot as plt
import numpy as np

# Parameters
d = 250 # Demand rate
Q = 500 # Order quantity
T = Q / d # Cycle length
t = np.linspace(0, 3 * T, 1000) # Time from 0 to 3 cycles

# Inventory level over time
inventory = np.maximum(0, Q - (d * t) % Q)
inventory[0] = 0

# Plotting the inventory level
plt.plot(t, inventory, label="Inventory Level", color="black", linewidth=2)
plt.xlabel("Time")
plt.ylabel("Inventory Level")
plt.axhline(0, color="gray", linewidth=1)
plt.ylim(bottom=0, top=Q + 100)
plt.tight_layout()
plt.show()
```



(cycle)

$$T = \frac{Q}{d}$$

2.1. A 250 500

$$T = \frac{500}{250} = 2$$

2.1

$$\begin{array}{ccccccc}
 1 & & & & & & \\
 1 & & K & & & & \\
 Q & c & & cQ & & & \\
 T & Q & 0 & \frac{Q}{2} & \frac{hQ}{2} & T & \frac{Q}{d} & 1
 \end{array}$$

$$\frac{hQ}{2} \cdot T = \frac{hQ^2}{2d}$$

1

$$K + cQ + \frac{hQ^2}{2d}$$

$$T \qquad g(Q)$$

$$\begin{aligned} g(Q) &= \frac{1}{T} \left(K + cQ + \frac{hQ^2}{2d} \right) \\ &= \frac{d}{Q} \left(K + cQ + \frac{hQ^2}{2d} \right) \\ &= \frac{Kd}{Q} + cd + \frac{hQ}{2} \end{aligned}$$

Q

$$g(Q) = \frac{Kd}{Q} + cd + \frac{hQ}{2}$$

2.2

$$\begin{array}{lll} \text{EOQ} & g(Q) & Q \\ & g'(Q) = 0 & Q^* \end{array}$$

$$g'(Q) = -\frac{Kd}{Q^2} + \frac{h}{2} = 0$$

$$Q^* = \sqrt{\frac{2Kd}{h}}$$

$$\begin{array}{lll} \text{EOQ} & \text{EOQ formula} & Q^* \\ g''(Q) & & \end{array}$$

$$g''(Q) = \frac{2Kd}{Q^3} > 0$$

$$g''(Q) > 0 \quad Q^*$$

2.1. EOQ Q^*

$$Q^* = \sqrt{\frac{2Kd}{h}}$$

$$Q^* \quad T^*$$

$$T^* = \frac{Q^*}{d} = \sqrt{\frac{2K}{hd}}$$

2.1.

1. Q^* c
2. h Q^*
3. K Q^*

$$c = 0$$

```
# Parameters
K = 500 # Order cost
h = 15 # Holding cost
c = 0 # Purchase cost
Q = np.linspace(1, 50, 50)

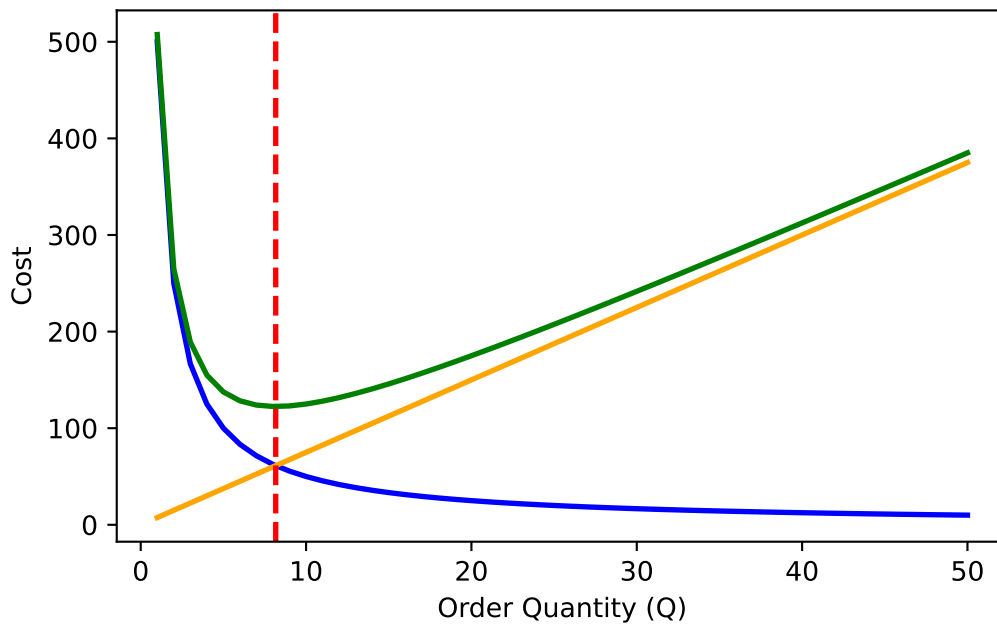
# Average cost function
g = (K / Q) + c + (h * Q / 2)

# Plotting the costs vs order quantity
plt.plot(Q, K / Q, label="Order Cost", color="blue", linewidth=2)
plt.plot(Q, c + (h * Q / 2), label="Holding Cost", color="orange", linewidth=2)
plt.plot(Q, g, label="Average Cost", color="green", linewidth=2)
plt.axvline(
    x=np.sqrt(2 * K / h),
    color="red",
    linestyle="--",
```

```

    label="Optimal Order Quantity",
    linewidth=2,
)
plt.xlabel("Order Quantity (Q)")
plt.ylabel("Cost")
plt.tight_layout()
plt.show()

```



Q^*

Q^*

$$\frac{Kd}{Q^*} = \frac{hQ^*}{2} \Rightarrow Q^* = \sqrt{\frac{2Kd}{h}}$$

Q

2.2.

250 PC

5000

1

150

10

Q^*

$$Q^* = \sqrt{\frac{2 \cdot 5000 \cdot 250}{150}}$$

Q^*

```

def eoq(K, d, h):
    """
    Calculate the Economic Order Quantity (EOQ).

    Parameters:
    K (float): Order cost
    d (float): Demand rate
    h (float): Holding cost

    Returns:
    float: Optimal order quantity Q*
    """
    return np.sqrt(2 * K * d / h)

if __name__ == "__main__":
    K = 5000 # Order cost
    d = 250 # Demand rate (units per month)
    h = 150 # Holding cost (per unit per month)

    Q_star = eoq(K, d, h)
    print(f"Optimal Order Quantity (Q*): {Q_star:.2f}")

```

Optimal Order Quantity (Q*): 129.10

PC $g(129)$ $g(130)$

2.3

EOQ 0 $L > 0$ Q^* L Q^*
 r reorder point r L dL

$$r = dL$$

2.3. 4 $L = 1/4$

$$r = dL = 250 \times \frac{1}{4} = 62.5$$

PC 63

2.4 EOQ

- EOQ
- quantity discount EOQ
 - all-units discount
 - incremental discount

References

- Harris, Ford W. 1990. How many parts to make at once . *Oper. Res.* 38 (6): 947–50.
- Snyder, Lawrence V, Zuo-Jun Max Shen. 2019. *Fundamentals of supply chain theory*. 2nd .
Nashville, TN: John Wiley & Sons.