Homework #3
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1. Let $n = 4$.

a) Find $\det(A)$, $\det(B)$, and $\det(A+B)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \\ 1 & 16 & 81 & 256 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 6 & 12 \\ 0 & 6 & 24 & 60 \\ 0 & 14 & 78 & 252 \end{vmatrix} = \begin{vmatrix} 2 & 6 & 12 \\ 6 & 24 & 60 \\ 14 & 78 & 252 \end{vmatrix} = \begin{vmatrix} 2 & 6 & 12 \\ 0 & 6 & 24 \\ 0 & 42 & 84 \end{vmatrix} = \begin{vmatrix} 6 & 24 \\ 42 & 84 \end{vmatrix} = 504 - 1008 = -504$$

$$\det(B) = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & 9 & 16 \\ 1 & 8 & 27 & 64 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 3 & 8 & 15 \\ 0 & 7 & 26 & 63 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 8 & 15 \\ 7 & 26 & 63 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 12 & 42 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 12 & 42 \end{vmatrix} = 84 - 72 = 12$$

$$\det(A+B) = \begin{vmatrix} 2 & 3 & 4 & 5 \\ 2 & 6 & 12 & 20 \\ 2 & 10 & 36 & 80 \\ 2 & 18 & 108 & 320 \end{vmatrix} = \begin{vmatrix} 2 & 3 & 4 & 5 \\ 0 & 3 & 8 & 15 \\ 0 & 7 & 32 & 75 \\ 0 & 15 & 104 & 315 \end{vmatrix} = \begin{vmatrix} 3 & 8 & 15 \\ 7 & 32 & 75 \\ 15 & 104 & 315 \end{vmatrix} = \begin{vmatrix} 3 & 8 & 15 \\ 0 & 4 & 15 \\ 0 & 56 & 180 \end{vmatrix} = \begin{vmatrix} 4 & 15 \\ 56 & 180 \end{vmatrix} = 720 - 840 = -120$$

$$\boxed{\det(A) = -504, \det(B) = 12, \det(A+B) = -120}$$

b) Find $\det(A^{-1})$, $\det(B^{-1})$, and $\det((A+B)^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{-504} = -\frac{1}{504}$$

$$\det(B^{-1}) = \frac{1}{\det(B)} = \frac{1}{12}$$

$$\det((A+B)^{-1}) = \frac{1}{\det(A+B)} = \frac{1}{-120} = -\frac{1}{120}$$

$$\boxed{\det(A^{-1}) = -\frac{1}{504}, \det(B^{-1}) = \frac{1}{12}, \det((A+B)^{-1}) = -\frac{1}{120}}$$

c) Find $\det(A^{-1}B)$, $\det(B^{-1}A)$, and $\det(A^{-1}(A+B))$.

Proof.

$$\det(A^{-1}B) = \det(A^{-1})\det(B) = -\frac{1}{504} \cdot 12 = -\frac{1}{42}$$

$$\det(B^{-1}A) = \det(B^{-1})\det(A) = \frac{1}{12} \cdot -504 = -42$$

$$\det(A^{-1}(A+B)) = \det(A^{-1})\det(A+B) = -\frac{1}{504} \cdot -120 = \frac{5}{21}$$

$$\boxed{\det(A^{-1}B) = -\frac{1}{42}, \det(B^{-1}A) = -42, \det(A^{-1}(A+B)) = \frac{5}{21}}$$

d) Find $\det(A^{-1}(A+B)B^{-1})$.

Proof.

$$\det(A^{-1}(A+B)B^{-1}) = \det(A^{-1})\det(A+B)\det(B^{-1}) = -\frac{1}{504} \cdot -120 \cdot \frac{1}{12} = \frac{5}{21}$$

$$\boxed{\det(A^{-1}(A+B)B^{-1}) = \frac{5}{21}}$$

2. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

3. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

4. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

5. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

6. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

7. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

8. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

9. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

10. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

11. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

12. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

13. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

14. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$

15. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{bmatrix}$.

a) Find $\det(A)$.

Proof.

$$\det(A) = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 6 & 24 \end{vmatrix} = \begin{vmatrix} 2 & 6 \\ 6 & 24 \end{vmatrix} = 48 - 36 = 12$$

$$\boxed{\det(A) = 12}$$

b) Find $\det(A^{-1})$.

Proof.

$$\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{12}$$

$$\boxed{\det(A^{-1}) = \frac{1}{12}}$$

c) Find $\det(A^{-1}A)$.

Proof.

$$\det(A^{-1}A) = \det(A^{-1})\det(A) = \frac{1}{12} \cdot 12 = 1$$

$$\boxed{\det(A^{-1}A) = 1}$$

d) Find $\det(A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{144}$$

$$\boxed{\det(A^{-1}A^{-1}) = \frac{1}{144}}$$

e) Find $\det(A^{-1}A^{-1}A)$.

Proof.

$$\det(A^{-1}A^{-1}A) = \det(A^{-1})\det(A^{-1})\det(A) = \frac{1}{12} \cdot \frac{1}{12} \cdot 12 = \frac{1}{12}$$

$$\boxed{\det(A^{-1}A^{-1}A) = \frac{1}{12}}$$

f) Find $\det(A^{-1}A^{-1}A^{-1})$.

Proof.

$$\det(A^{-1}A^{-1}A^{-1}) = \det(A^{-1})\det(A^{-1})\det(A^{-1}) = \frac{1}{12} \cdot \frac{1}{12} \cdot \frac{1}{12} = \frac{1}{1728}$$

$$\boxed{\det(A^{-1}A^{-1}A^{-1}) = \frac{1}{1728}}$$