EE530 Homework #12

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1a #3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & 4-\lambda & 6 \\ 3 & 6 & 9-\lambda \end{vmatrix} \Rightarrow -\lambda^3 + 14\lambda^2 + 14\lambda = 0$$

$$\det(B - \lambda I) = \begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & 4-\lambda & 6 \\ 3 & 6 & 9-\lambda \end{vmatrix} \Rightarrow -\lambda^3 + 14\lambda^2 + 14\lambda = 0$$

$$\det(B - \lambda I) = -\lambda^3 + 14\lambda^2 + 14\lambda = 0$$

#4

$$\begin{aligned} &\Phi(t) = e^{At} = \alpha_0 I + \alpha_1 A + \alpha_2 A^2 \\ &\Rightarrow \alpha_0 + \alpha_1 \lambda_1 + \alpha_2 \lambda_1^2 = e^{\lambda_1 t} = e^{-t} \\ &\Rightarrow \alpha_0 + \alpha_1 \lambda_2 + \alpha_2 \lambda_2^2 = e^{\lambda_2 t} = e^{-2t} \end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Row reduction

$$\begin{bmatrix} 0 & 1 & 0 \\ -2 & -3 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \therefore \text{Rank}(A) = 1$$

$$\det \begin{bmatrix} 0 & 1 & 0 \\ -2 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 2$$

$$\begin{aligned} &\Phi(t) = (\alpha_0 I + \alpha_1 A + \alpha_2 A^2) = \begin{bmatrix} \alpha_0 & 0 \\ 0 & \alpha_0 \end{bmatrix} + \begin{bmatrix} \alpha_1 \lambda_1 & \alpha_1 \\ -2\alpha_1 & -3\alpha_1 \end{bmatrix} + \begin{bmatrix} \alpha_2 \lambda_1^2 & \alpha_2 \lambda_1 \\ -2\alpha_2 \lambda_1 & -3\alpha_2 \lambda_1 \end{bmatrix} \\ &= \begin{bmatrix} \alpha_0 & 0 \\ 0 & \alpha_0 \end{bmatrix} + \begin{bmatrix} \alpha_1 \lambda_1 & \alpha_1 \\ -2\alpha_1 & -3\alpha_1 \end{bmatrix} + \begin{bmatrix} \alpha_2 \lambda_1^2 & \alpha_2 \lambda_1 \\ -2\alpha_2 \lambda_1 & -3\alpha_2 \lambda_1 \end{bmatrix} \\ &= \begin{bmatrix} \alpha_0 + \alpha_1 \lambda_1 + \alpha_2 \lambda_1^2 & \alpha_1 + \alpha_2 \lambda_1 \\ -2\alpha_1 - 2\alpha_2 \lambda_1 & \alpha_0 - 3\alpha_1 - 3\alpha_2 \lambda_1 \end{bmatrix} \\ &\Rightarrow \begin{cases} e^{-t} = \alpha_0 + \alpha_1 \lambda_1 + \alpha_2 \lambda_1^2 \\ e^{-2t} = \alpha_0 + \alpha_1 \lambda_2 + \alpha_2 \lambda_2^2 \end{cases} \\ &\Rightarrow \begin{cases} \alpha_1 = e^{-t} \\ \alpha_2 = 0 \end{cases} \end{aligned}$$

$$\Phi(t) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-t} \end{bmatrix} + \begin{bmatrix} -3e^{-t} & e^{-t} \\ -2e^{-t} & -3e^{-t} \end{bmatrix} = \begin{bmatrix} -2e^{-t} & e^{-t} \\ -2e^{-t} & -2e^{-t} \end{bmatrix}$$

$$\Phi(t) = \begin{bmatrix} -2e^{-t} & e^{-t} \\ -2e^{-t} & -2e^{-t} \end{bmatrix}$$

Equivalent $\lambda_1 = -1, \ \lambda_2 = -2, \ \lambda_1 = -1$

Determinant $\lambda_1 = -1, \ \lambda_2 = -2, \ \lambda_1 = -1$

#10

$$E = G_1 G_2 G^{-1} \qquad \text{and} \qquad E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{aligned} &1 = \alpha_1 - 2\beta_1 + \alpha_2 - 2\beta_2 \\ &0 = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{\alpha_1^2 - 2\alpha_1\alpha_2 + \alpha_2^2 + 8\beta_1^2 - 8\beta_1\beta_2 + 8\beta_2^2}}{2} \\ &\quad = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &1 = \alpha_1 - 2\beta_1 \quad \text{and} \quad \alpha_2 = 2\beta_2 \end{aligned}$$

$$\text{So} \quad \alpha_1 = 1 + 2\beta_1 \qquad \alpha_2 = 2\beta_2$$

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$$\therefore \ 0 = \frac{1 - 2\beta_1 - 2\beta_2 - \sqrt{(1 - 2\beta_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \quad \text{where} \quad \alpha = \frac{\alpha_1}{\alpha_2} = \frac{1 + 2\beta_1}{2\beta_2}$$

$$\therefore \ \alpha = 1 \quad \text{and} \quad \alpha = 0$$

$$\text{So} \quad \alpha_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \alpha_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\alpha_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \alpha_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\therefore \ 0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \text{where} \quad \alpha = \frac{\alpha_1}{\alpha_2} = \frac{\alpha_1}{\alpha_2} = \frac{\alpha_1}{\alpha_2}$$

$$\beta_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \beta_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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$$\begin{aligned} &1 = \alpha_1 - 2\beta_1 + \alpha_2 - 2\beta_2 \\ &0 = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &\quad = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &1 = \alpha_1 - 2\beta_1 \quad \text{and} \quad \alpha_2 = 2\beta_2 \end{aligned}$$

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#14

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{and} \qquad E = G_1 G_2 G^{-1}$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{aligned} &1 = \alpha_1 - 2\beta_1 + \alpha_2 - 2\beta_2 \\ &0 = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &\quad = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &1 = \alpha_1 - 2\beta_1 \quad \text{and} \quad \alpha_2 = 2\beta_2 \end{aligned}$$

$$\text{So} \quad \alpha_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \alpha_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\alpha_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \alpha_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\therefore \ 0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \text{where} \quad \alpha = \frac{\alpha_1}{\alpha_2} = \frac{\alpha_1}{\alpha_2} = \frac{\alpha_1}{\alpha_2}$$

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$$\begin{aligned} &1 = \alpha_1 - 2\beta_1 + \alpha_2 - 2\beta_2 \\ &0 = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &\quad = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &1 = \alpha_1 - 2\beta_1 \quad \text{and} \quad \alpha_2 = 2\beta_2 \end{aligned}$$

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$$\beta_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \beta_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Similarly for the second matrix

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \alpha_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\alpha_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \alpha_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$E = \alpha_1 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{aligned} &1 = \alpha_1 - 2\beta_1 + \alpha_2 - 2\beta_2 \\ &0 = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &\quad = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &1 = \alpha_1 - 2\beta_1 \quad \text{and} \quad \alpha_2 = 2\beta_2 \end{aligned}$$

$$\text{So} \quad \alpha_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \alpha_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\alpha_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \alpha_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\therefore \ 0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \text{where} \quad \alpha = \frac{\alpha_1}{\alpha_2} = \frac{\alpha_1}{\alpha_2} = \frac{\alpha_1}{\alpha_2}$$

$$\beta_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \beta_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\beta_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad \beta_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

C #1

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{aligned} &\lambda_1 = \alpha_1 - 2\beta_1 + \alpha_2 - 2\beta_2 \qquad \alpha_2 = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &\lambda_2 = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &\quad = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &\quad = \frac{\alpha_1 - 2\beta_1 - \alpha_2 + 2\beta_2 - \sqrt{(\alpha_1 - \alpha_2)^2 + 8(\beta_1 - \beta_2)^2}}{2} \\ &\lambda_1 = 1 \quad \text{and} \quad \lambda_2 = -1 \end{aligned}$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \begin{aligned} &\lambda_1 = 1 \qquad \lambda_2 = -1 \\ &\lambda_1 = 1 \qquad \lambda_2 = -1 \end{aligned}$$

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)(1-\lambda) - 1 = (1-\lambda)^2 - 1$$

$$= (1-\lambda)(1-\lambda) - 1 = (1-\lambda)(1-\lambda) + (1-\lambda)(1-\lambda)