Problem Set 7

Due: April 14th, 11.59pm PST on Gradescope EE483 Spring 2025

Total 107 pts

1. **(25 pts, 5 pts each) Region of Convergence** Consider the discrete-time signals $x_1[n] = (-5)^{-n}u[-n-1]$ and $x_2[n] = -(-5)^{-n}u[-n-6]$.

   (a) Compute the z-transforms of $x_1[n]$ and $x_2[n]$.

   (b) Compute and sketch the ROC, zeros and poles for $X_1(z)$ and $X_2(z)$.

   (c) Define $x[n] = x_1[n] + x_2[n]$. Compute the z-transform $X(z)$ and express it in rational form.

   (d) Compute and sketch the ROC, zeros and poles of $X(z)$.

   (e) You should see a big difference between your answers to (b) and (c). Explain what happened.

2. **(32 pts, 4,4,4,4,16) ROC and DTFT** Let $x[n] = e^{-\alpha n}u[n]$, where $\alpha \in \mathbb{R}$.

   (a) Compute the z-transforms of $x_[n]$.

   (b) Compute and sketch the ROC, zeros and poles for $X(z)$.

   (c) Find the values of α for which DTFT of $x[n]$ exist.

   (d) Assume α satisfies the condition in (c), give the DTFT of $x[n]$.

   (e) Let $y[n] = e^{(3-2\alpha)n}u[-n-1]$. Repeat part (a)-(d) for $y[n]$.

3. **(40 pts, 10 pts each)**Let $x_1[n]$ and $x_2[n]$ be defined as follows:

$$x_1[n] = \alpha^n u[n]$$

$$x_2[n] = \beta^n u[-n-1]$$

   Let $x[n] = x_1[n] + x_2[n]$. For each value of α and β below, (i) find the z-transforms of $x[n]$ and express it in rational form, (ii) Compute and sketch the ROC, zeros and poles, (iii) give the DTFT if it exist.

   (a) $\alpha = -\frac{1}{3}, \quad \beta = \frac{1}{2}$

   (b) $\alpha = \frac{1}{2}, \quad \beta = -2$

   (c) $\alpha = -\frac{1}{2}, \quad \beta = \frac{1}{3}$

   (d) $\alpha = -\frac{4}{3}, \quad \beta = \frac{5}{2}$

4. **(10 pts) LTI systems and Z-transform** Let $h[n]$ be the impulse response of an LTI system and $H(z)$ be its $z-$transform. Prove that if the ROC of $H(z)$ contains the unit circle, then the LTI system is BIBO stable.