HW12 — Handout — Q1

① Proof: $\sum_{k=1}^{n} \frac{c}{(1+r)^k} = c \sum_{k=1}^{n} \left(\frac{1}{1+r}\right)^k$

$$= c \, \frac{\frac{1}{1+r}\left(1-\left(\frac{1}{1+r}\right)^n\right)}{1-\frac{1}{1+r}}$$

$$= c \, \frac{1-\left(\frac{1}{1+r}\right)^n}{r}$$

② $\begin{cases} r = \frac{1}{2} \cdot 11\% = 0.055 \\ C = \$50 \\ M = \$1000 \\ n = 2 \times 20 = 40 \end{cases}$

$$\therefore P = c\left(\frac{1-\frac{1}{(1+r)^n}}{r}\right) + \frac{M}{(1+r)^n}$$

$$= \$50 \cdot \frac{1-\frac{1}{(1+0.055)^{40}}}{0.055} + \frac{\$1000}{(1+0.055)^{40}}$$

$$\approx 802.31 + 117.46$$

$$= 919.77$$

③ $\begin{cases} r' = \frac{1}{2} \cdot 6.8\% = 0.034 \\ C = \$50 \\ M = \$1000 \\ n = 2 \times 20 = 40 \end{cases}$

$$\therefore P = c\left(\frac{1-\frac{1}{(1+r)^n}}{r}\right) + \frac{M}{(1+r)^n}$$

$$= \$50 \cdot \frac{1-\frac{1}{(1+0.034)^{40}}}{0.034} + \frac{\$1000}{(1+0.034)^{40}}$$

$$\approx 1084.52 + 262.53$$

$$= 1347.05$$

④ $\begin{cases} r' = \frac{1}{2} \cdot 10\% = 0.05 \\ C = \$50 \\ M = \$1000 \\ n = 2 \times 20 = 40 \end{cases}$

$$\therefore P = C\left(\frac{1 - \frac{1}{(1+r)^n}}{r}\right) + \frac{M}{(1+r)^n}$$

$$= \$50 \cdot \frac{1 - \frac{1}{(1+0.05)^{40}}}{0.05} + \frac{\$1000}{(1+0.05)^{40}}$$

$$\approx 857.95 + 142.05$$

$$= 1000$$

HW12 - Handout - Q2

①

$$D_{k+1} = \begin{cases} D_k(1+g), & p \\ D_k(1-g), & p_D \\ D_k, & 1-p-p_D \end{cases}$$

$$\therefore E[D_{k+1}] = E[D_k(1+g)p + D_k(1-g)p_D + D_k(1-p-p_D)]$$

$$= [(1+g)p + (1-g)p_D + (1-p-p_D)]\,E[D_k]$$

$$= (1+pg-p_Dg)\,E[D_k]$$

$$= (1+pg-p_Dg)^{k+1} D_0$$

$$\therefore E[P] = \sum_{k=1}^{\infty} \frac{E[D_k]}{(1+r)^k} = \sum_{k=1}^{\infty} \frac{D_0 \cdot (1+pg-p_Dg)^k}{(1+r)^k}$$

$$= D_0 \sum_{k=1}^{\infty} \left(\frac{1+pg-p_Dg}{1+r}\right)^k$$

when $\frac{1+(p-p_D)g}{1+r} < 1 \Rightarrow g < \frac{r}{p-p_D} \quad (p > p_D)$

$$\lim_{k\to\infty} \left(\frac{1+(p-p_D)g}{1+r}\right)^k = 0$$

$$\therefore E[P] = D_0 \frac{\frac{1+(p-p_D)g}{1+r}\left(1 - \left(\frac{1+(p-p_D)g}{1+r}\right)^k\right)}{1 - \frac{1+(p-p_D)g}{1+r}}$$

$$= D_0 \frac{\frac{1+(p-p_D)g}{1+r}}{1 - \frac{1+(p-p_D)g}{1+r}}$$

$$= D_0 \frac{1+(p-p_D)g}{1+r-(1+(p-p_D)g)}$$

$$= D_0 \frac{1+(p-p_D)g}{r-(p-p_D)g} \quad \left(g < \frac{r}{p-p_D}\right)$$

②

$$D_{k+1} = \begin{cases} D_k(1+g) & , \quad p \\ D_k(1-g) & , \quad p_D \\ D_k & , \quad 1-p-p_D-p_B \\ 0 & , \quad p_B \end{cases}$$

$$\therefore E[D_{k+1}] = E[D_k(1+g)p + D_k(1-g)p_D + D_k(1-p-p_D-p_B) + 0\cdot p_B]$$
$$= [(1+g)p + (1-g)p_D + (1-p-p_D-p_B)]\, E[D_k]$$
$$= (1+pg-p_Dg-p_B)\, E[D_k]$$
$$= (1+pg-p_Dg-p_B)^{k+1} D_0$$

$$\therefore E[P] = \sum_{k=1}^{\infty} \frac{E[D_k]}{(1+r)^k} = \sum_{k=1}^{\infty} \frac{D_0(1+pg-p_Dg-p_B)^k}{(1+r)^k}$$

$$= D_0 \sum_{k=1}^{\infty} \left(\frac{1+pg-p_Dg-p_B}{1+r}\right)^k$$

when $\frac{1+(p-p_D)g-p_B}{1+r} < 1 \Rightarrow g < \frac{r+p_B}{p-p_D} \quad (p > p_D)$

$$\lim_{k\to\infty} \left(\frac{1+(p-p_D)g-p_B}{1+r}\right)^k = 0$$

$$\therefore E[P] = D_0 \frac{\frac{1+(p-p_D)g-p_B}{1+r}\left(1-\left(\frac{1+(p-p_D)g-p_B}{1+r}\right)^k\right)}{1-\frac{1+(p-p_D)g-p_B}{1+r}}$$

$$= D_0 \frac{\frac{1+(p-p_D)g-p_B}{1+r}}{1-\frac{1+(p-p_D)g-p_B}{1+r}}$$

$$= D_0 \frac{1+(p-p_D)g-p_B}{1+r-(1+(p-p_D)g-p_B)}$$

$$= D_0 \frac{1+(p-p_D)g-p_B}{r-(p-p_D)g+p_B} \quad \left(g < \frac{r+p_B}{p-p_D}\right)$$

HW12 – Handout – Q3

$$P = D_0 \sum_{k=1}^{\infty} \left(\frac{1+g}{1+r}\right)^k = D_0 \frac{1+g}{r-g}$$

I choose <u>The Coca Cole Company</u> (from 2018 – 2023)

CocaCola Annual Gross Profit (Millions of US $)	
2023	$27,234
2022	$25,004
2021	$23,298
2020	$19,581
2019	$22,647
2018	$21,233

$\Rightarrow \$27.234 = \$21.233\,(1+g)^6$

$\therefore g \approx 0.0424$

$g < r = 0.08$

Dividends

The Company paid dividends of $7,952 million and $7,616 million during the years ended December 31, 2023 and 2022, respectively.

At its February 2024 meeting, our Board of Directors increased our regular quarterly dividend to $0.485 per share, equivalent to a full year dividend of $1.94 per share in 2024. This is our 62nd consecutive annual increase. Our annualized common stock dividend was $1.84 per share and $1.76 per share in 2023 and 2022, respectively.

$\Rightarrow D_0 = \$1.84$

$$\therefore P = D_0 \frac{1+g}{r-g} = 1.84 \times \frac{1+0.0424}{0.08-0.0424} \approx 51.01$$

$\Rightarrow$ In 12/29/2023

it's market price is 58.93

it is larger than RAP 51.01

HW12 - Handout - Q4

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{(x-x_0)^2}{2t}\right)\right)$$

$$= -\frac{1}{2}\frac{1}{\sqrt{2\pi t^3}} \exp\left(-\frac{(x-x_0)^2}{2t}\right) + \frac{1}{\sqrt{2\pi t}} \frac{(x-x_0)^2}{2t^2} \exp\left(-\frac{(x-x_0)^2}{2t}\right)$$

$$= \frac{1}{\sqrt{2\pi t}} \exp\left(\frac{-(x-x_0)^2}{2t}\right)\left(\frac{(x-x_0)^2}{2t^2} - \frac{1}{2t}\right)$$

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{(x-x_0)^2}{2t}\right)\right)$$

$$= \frac{1}{\sqrt{2\pi t}} \frac{\partial}{\partial x}\left(\exp\left(-\frac{(x-x_0)^2}{2t}\right)\right)$$

$$= -\frac{x-x_0}{t} \cdot \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{(x-x_0)^2}{2t}\right)$$

$$\frac{\partial^2 P}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial P}{\partial x}\right) = -\frac{1}{t}\frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{(x-x_0)^2}{2t}\right) - \frac{x-x_0}{t}\frac{1}{\sqrt{2\pi t}}\left(-\frac{x-x_0}{t}\right) \exp\left(\frac{-(x-x_0)^2}{2t}\right)$$

$$= -\frac{1}{t}\frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{(x-x_0)^2}{2t}\right) + \frac{(x-x_0)^2}{t^2}\frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{(x-x_0)^2}{2t}\right)$$

$$= \frac{1}{\sqrt{2\pi t}} \exp\left(\frac{-(x-x_0)^2}{2t}\right)\left(\frac{(x-x_0)^2}{t^2} - \frac{1}{t}\right)$$

$$\therefore \frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial x} = \frac{1}{2}\frac{\partial^2 P}{\partial x} \Rightarrow D = \frac{1}{2}$$ Einstein was right

HW12 – Handout – Q5

① $S=47$, $F=45$, $T=0.5$, $r=0.1$, $\sigma=0.25$

$$\therefore d_1 = \frac{1}{\sigma\sqrt{T}}\left(\ln\frac{S}{F} + \left(r+\frac{\sigma^2}{2}\right)T\right)$$

$$= \frac{1}{0.25\sqrt{0.5}}\left(\ln\frac{47}{45} + \left(0.1+\frac{(0.25)^2}{2}\right)0.5\right)$$

$$\approx 0.6172$$

$$d_2 = d_1 - \sigma\sqrt{T} = 0.6172 - 0.25\sqrt{0.5} \approx 0.4404$$

$$\therefore C(S,t) = SN(d_1) - Fe^{-rT}N(d_2) \approx 5.6906$$

② $S=47$, $F=45$, $T=0.5$, $r=0.1$, $\sigma=0.4$

$$\therefore d_1 = \frac{1}{0.4\sqrt{0.5}}\left(\ln\frac{47}{45} + \left(0.1+\frac{(0.4)^2}{2}\right)0.5\right) \approx 0.4719$$

$$d_2 = 0.4719 - 0.4\sqrt{0.5} \approx 0.1891$$

$$\therefore C(S,t) \approx 7.4185$$

③ Let $r=0.05$, $S=47$, $F=45$, $T=0.5$, $\sigma=0.25$

$\therefore d_1 \approx 0.4758$ $\quad d_2 \approx 0.2990$

$\therefore C(S,t) \approx 4.9928$

④ Let $r=0.15$, $S=47$, $F=45$, $T=0.5$, $\sigma=0.25$

$\therefore d_1 \approx 0.7586$ $\quad d_2 \approx 0.5819$

$\therefore C(S,t) \approx 6.4253$