HW3-Q1

K-means

In [1]:

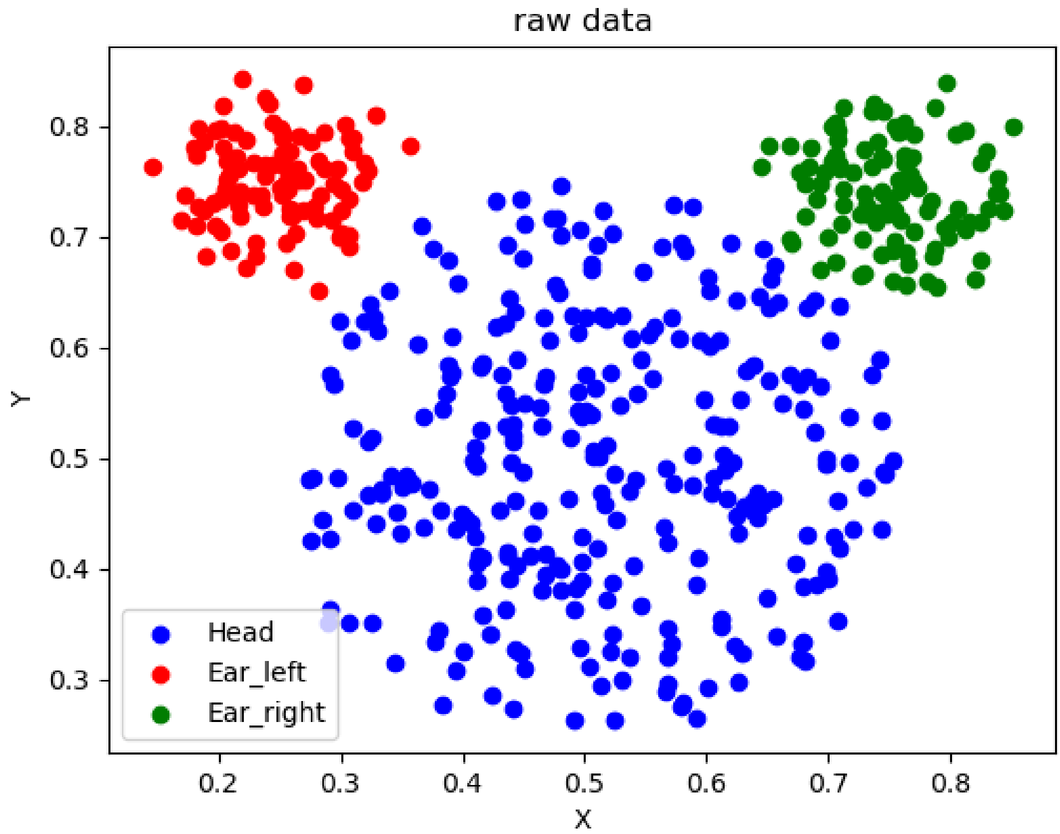
```
import re
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.cluster.vq import kmeans2
```

In [2]:

```
#read data
with open('cluster.txt','r') as f:
    lines=f.readlines()
pattern=re.compile(r'^-?\d+(\.\d+)?\s+-?\d+(\.\d+)?\s+[a-zA-Z]+(_[a-zA-Z]+)*$'
new_lines=[i for i in lines if pattern.match(i.strip())]
with open('new_lines.txt','w') as f:
    f.writelines(new_lines)
pairs=pd.read_table('new_lines.txt',sep=' ',header=None,names=['x','y','label
pairs_3=pairs.to_numpy()
```

In [3]:

```
#draw raw data
plt.figure()
idx=np.where(pairs_3[:,2]=='Head')[0]
plt.scatter(pairs_3[idx,0],pairs_3[idx,1],color='blue')
idx=np.where(pairs_3[:,2]=='Ear_left')[0]
plt.scatter(pairs_3[idx,0],pairs_3[idx,1],color='red')
idx=np.where(pairs_3[:,2]=='Ear_right')[0]
plt.scatter(pairs_3[idx,0],pairs_3[idx,1],color='green')
plt.legend(['Head','Ear_left','Ear_right'])
plt.xlabel('X')
plt.ylabel('Y')
plt.title('raw data')
plt.show()
```

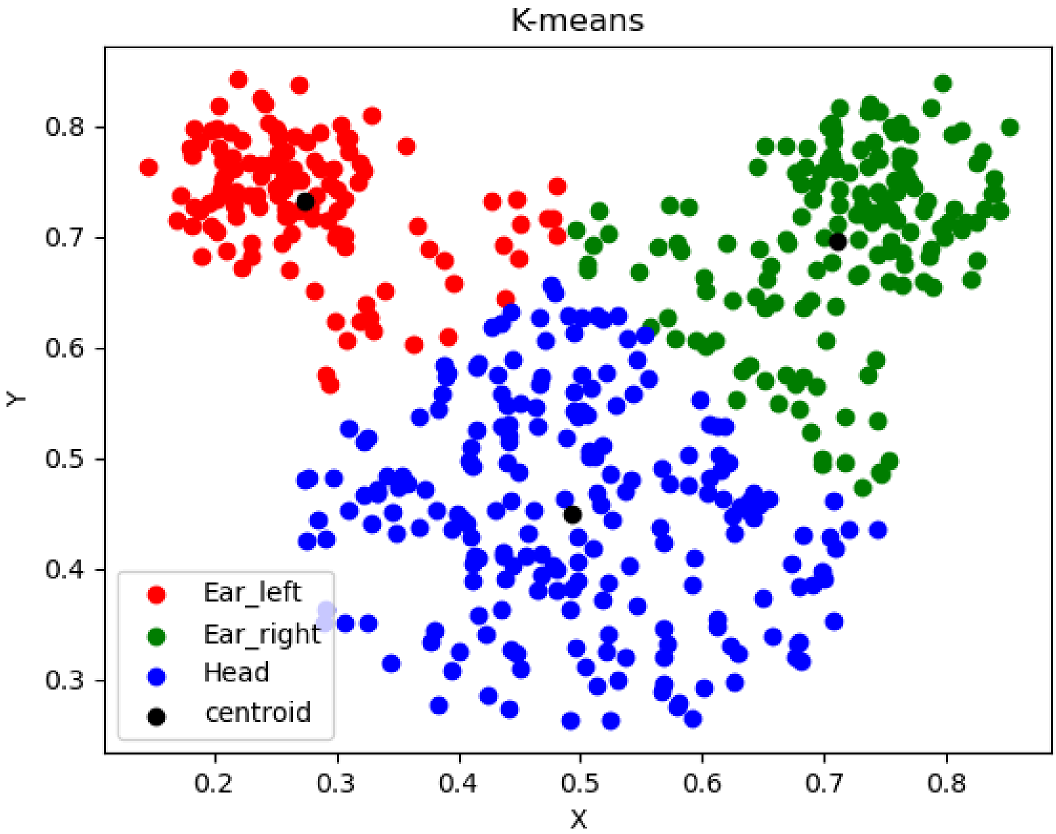


In [16]:

```
#kmean
K=3
pairs_2=pairs_3[:,:2]
pairs_2=pairs_2.astype(float)
centroid,label=kmeans2(pairs_2,K,minit='points')
sorted_centroid=centroid[centroid[:,0].argsort()]
sorted_centroid[[1,2]]=sorted_centroid[[2,1]]
sorted_label=np.zeros_like(label)
for i in range(len(label)):
    for j in range(3):
        if centroid[label[i],0]==sorted_centroid[j,0]:
            sorted_label[i]=j
#print(sorted_label)
```

In [17]:

```
color_dic={0:'red',1:'green',2:'blue'}
plt.figure()
for i in range(3):
    idx=np.where(sorted_label==i)[0]
    plt.scatter(pairs_2[idx,0],pairs_2[idx,1],color=color_dic[i])
plt.scatter(centroid[:,0],centroid[:,1],color='black')
plt.legend(['Ear_left','Ear_right','Head','centroid'])
plt.xlabel('X')
plt.ylabel('Y')
plt.title('K-means')
plt.show()
```



In [18]:

```
#confusion matrix
confusion_matrix = pd.crosstab(pairs_3[:,2],sorted_label,rownames=['Actual'],c
print("Confusion Matrix")
print("(0:Ear_left, 1:Ear_right, 2:Head)")
print(confusion_matrix)
```

```
Confusion Matrix
(0:Ear_left, 1:Ear_right, 2:Head)
Predicted    0    1    2
Actual
Ear_left   100    0    0
Ear_right    0  100    0
Head        25   54  211
```

GMM

In [7]:

```
def draw_label_scater(X,label,title):
    color_dic={0:'red',1:'green',2:'blue'}
    plt.figure()
    for i in range(3):
        idx=np.where(label==i)[0]
        plt.scatter(X[idx,0],X[idx,1],color=color_dic[i])
    plt.legend(['Ear_left','Ear_right','Head'])
    plt.xlabel('X')
    plt.ylabel('Y')
    plt.title(title)
    plt.show()
```

In [8]:

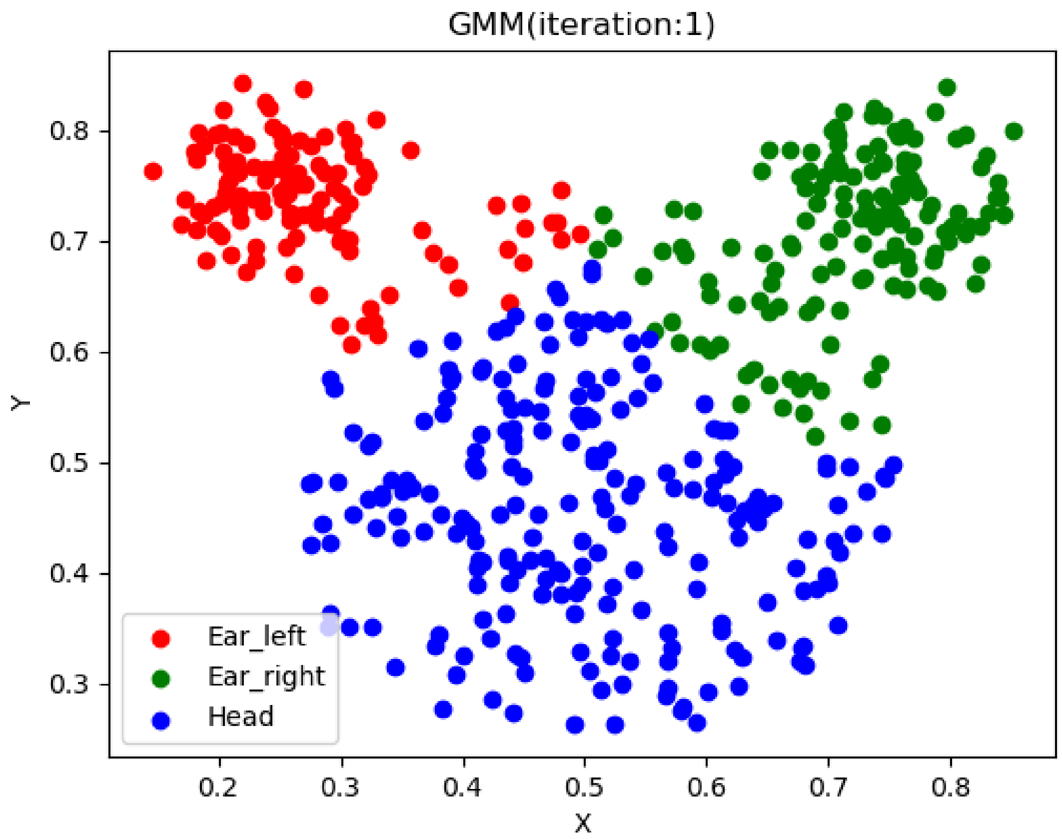
```
#PDF
def jointed_gaussian_pdf(X,mu,sigma):
    N=len(X)
    D=len(X[0])
    sigma_inv=np.linalg.inv(sigma)
    sigma_det=np.linalg.det(sigma)
    lowwer=1/(np.sqrt(((2*np.pi)**D)*sigma_det))
    pdf=np.zeros(N)
    for i in range(N):
        diff=X[i]-mu
        exponent=-0.5*np.dot(diff.T,np.dot(sigma_inv,diff))
        pdf[i]=lowwer*np.exp(exponent)
    return pdf
```

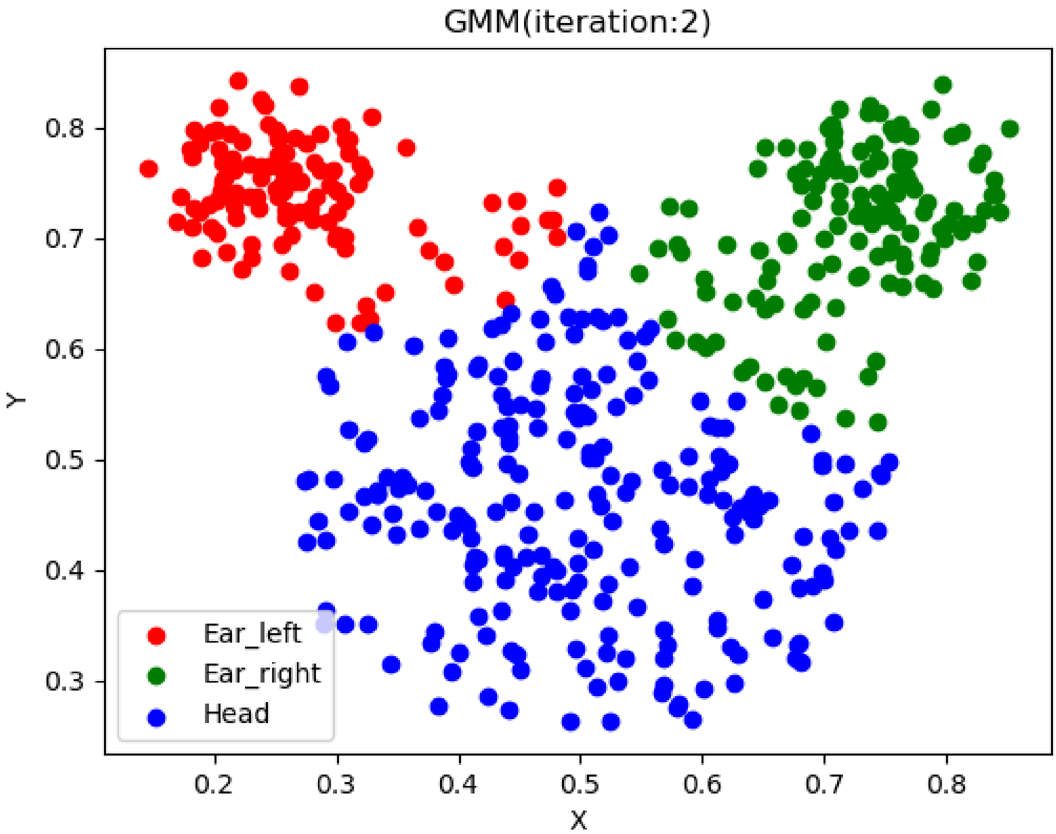
In [9]:

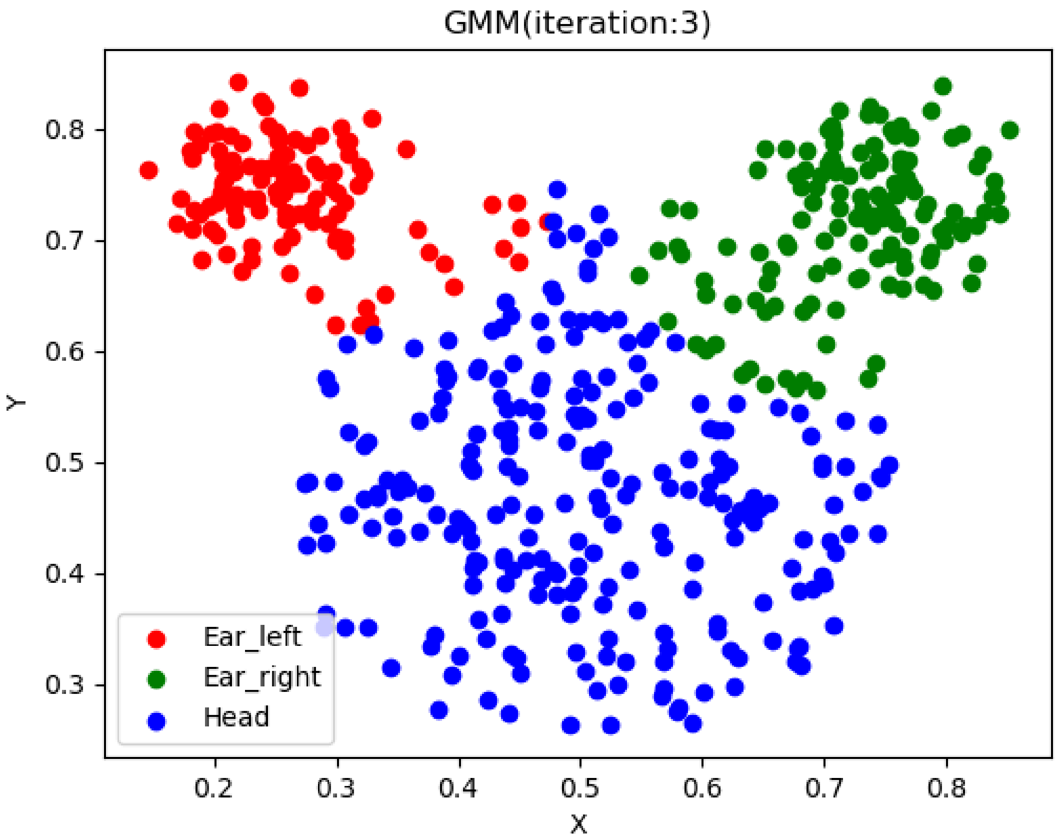
```
#GMM
def gmm(X,K,gamma,pis,mus,sigmas,max_iter=100,tol=1e-4):
    N=len(X)
    D=len(X[0])
    ll_old=0#-np.inf    #negative log-likelihood
    new_label=np.zeros_like(label)
    for iter in range(max_iter):
        #E-step
        for k in range(K):
            gamma[:,k]=pis[k]*jointed_gaussian_pdf(X,mus[k],sigmas[k])
        gamma/=np.sum(gamma,axis=1,keepdims=True)
        #for k in range(K):
        #    gamma[:,k]/=sum(gamma[:,k])

        # M-step
        for k in range(K):
            pis[k]=sum(gamma[:,k])/N
            for d in range(D):
                mus[k,d]=sum(gamma[:,k]*X[:,d])/sum(gamma[:,k])
            diff=X-mus[k]
            sigmas[k]=np.dot(gamma[:,k]*diff.T,diff)/sum(gamma[:,k])

        #update new_label and draw first four iterations
        for n in range(N):
            new_label[n]=np.argmax(gamma[n])
        if iter<4:
            draw_label_scater(pairs_2,new_label,f"GMM(iteration:{iter+1})")

        #negative log-likelihood
        ll_new=0
        for k in range(K):
            ll_new+=pis[k]*jointed_gaussian_pdf(X,mus[k],sigmas[k])
        ll_new=sum(np.log(ll_new))
        if abs(ll_new-ll_old)<tol:
            draw_label_scater(pairs_2,new_label,f"Final result of GMM(iteratio
            break
        ll_old=ll_new
    return new_label
```

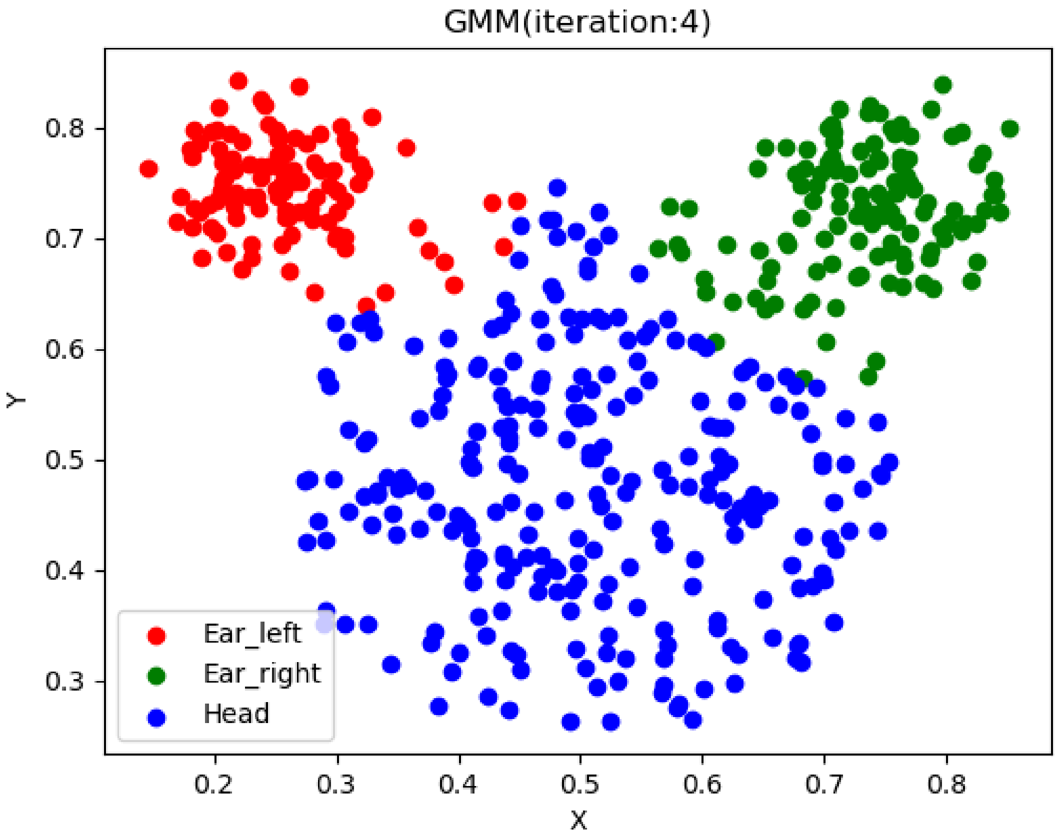
In [10]:

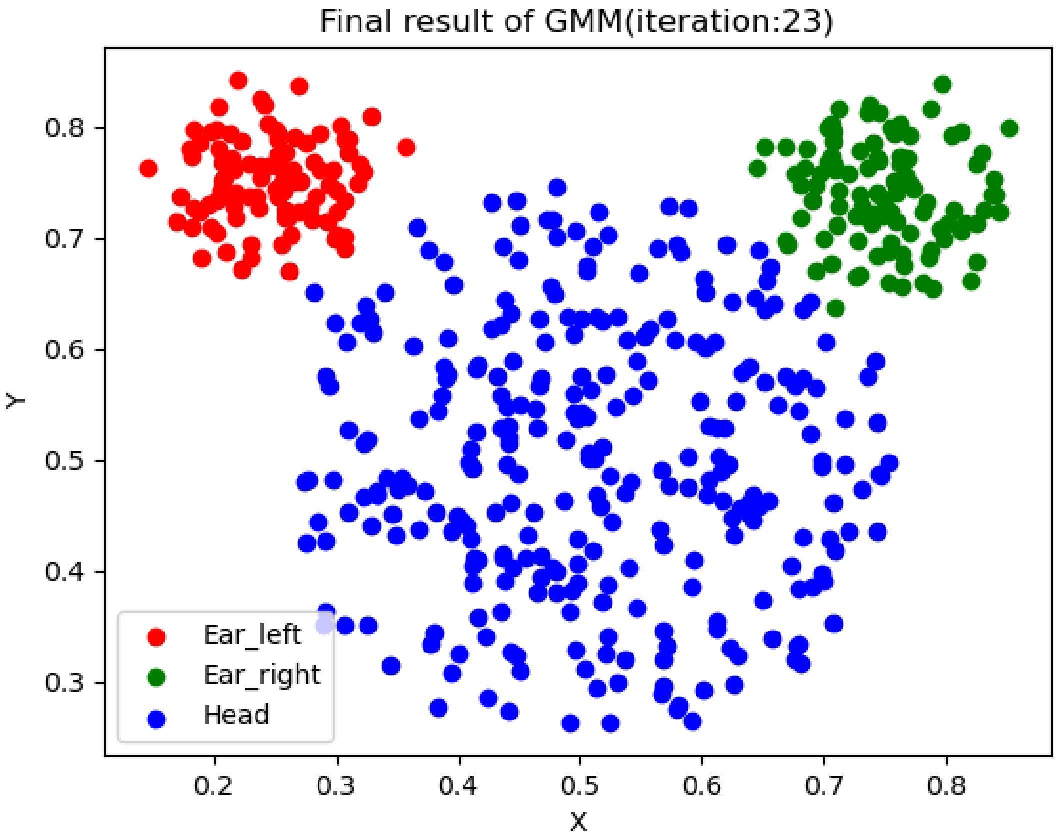
```
#one-hot
one_hot=np.zeros((len(pairs_3),K))
for i in range(len(sorted_label)):
    one_hot[i,sorted_label[i]]=1
#print(one_hot)

cluster_means=np.zeros((K,2))   #mu
cluster_covs=np.zeros((K,2,2))  #sigma
cluster_weights=np.zeros((K))   #pi
for k in range(K):
    cluster_weights[k]=sum(one_hot[:,k])/len(pairs_3)
    cluster_means[k,0]=sum(one_hot[:,k]*pairs_2[:,0])/sum(one_hot[:,k]) #x_mea
    cluster_means[k,1]=sum(one_hot[:,k]*pairs_2[:,1])/sum(one_hot[:,k]) #y_mea
    diff=pairs_2-cluster_means[k]
    cluster_covs[k,0,0]=sum(one_hot[:,k]*(diff[:,0]**2))/sum(one_hot[:,k])  #v
    cluster_covs[k,1,1]=sum(one_hot[:,k]*(diff[:,1]**2))/sum(one_hot[:,k])  #v
    cluster_covs[k,0,1]=cluster_covs[k,1,0]=sum(one_hot[:,k]*diff[:,0]*diff[:,

new_label=gmm(pairs_2,K,one_hot,cluster_weights,cluster_means,cluster_covs)
```




GMM(iteration:2)
Y
X
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Ear_left
Ear_right
Head


GMM(iteration:3)
Y
X
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Ear_left
Ear_right
Head


GMM(iteration:4)
Y
X
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Ear_left
Ear_right
Head


Final result of GMM(iteration:23)
Y
X
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Ear_left
Ear_right
Head

In [11]:

```
#confusion matrix
confusion_matrix = pd.crosstab(pairs_3[:,2],new_label,rownames=['Actual'],coln
print("Confusion Matrix")
print("(0:Ear_left, 1:Ear_right, 2:Head)")
print(confusion_matrix)
```

```
Confusion Matrix
(0:Ear_left, 1:Ear_right, 2:Head)
Predicted   0    1    2
Actual
Ear_left   99    0    1
Ear_right   0  100    0
Head        0    1  289
```

Comment

different

- K-means based purely on distance, so some "Head" points may be misclassified as "Ear_left" or "Ear_right" if thay are closer to those clusters.
- GMM can handle more complex situation, becase it consider both the mean and the covarance matrix of each cluster, allowing for clusters with different shapes and sizes.

perform

- In this problem, I think GMM performs better. GMM takes into account the variations in size and shape of different cluster.
- From the confusion matrices of two methods, we can see that the result of GMM are closer to the true labels compared to K-means.
- When clusters have the same shape and size, K-means may perform well and it computationally faster than GMM.