Problem Set 2

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1. **Properties (24 pts, 1pt for each answer of causality, stability, time invariance)** For the following systems, determine whether they are i) causal or non causal, ii) stable or unstable, iii) time invariant or time variant. Justify your answer with a proof or a counter example.

(a) $y[n] = x[n^2]$

i) when $n=2$, $y[2] = x[4]$

$\therefore$ non causal

ii) if $|x[n]| \le M$, then $|y[n]| = |x[n^2]| \le M$

$\therefore$ stable

iii) $y[n-n_0] = x[(n-n_0)^2] = x[n^2 - 2n_0 n + n_0^2]$

$\neq x[n^2 - n_0]$

$\therefore$ time variant

(b) $y[n] = x[n](u[n] - u[n-6])$

i) $\because u[n] - u[n-6] = \begin{cases} 1 & 0 \le n \le 5 \\ 0 & \text{else} \end{cases}$

$\therefore y[n] = x[n](u[n] - u[n-6]) = \begin{cases} x[n], & 0 \le n \le 5 \\ 0, & \text{else} \end{cases}$

$\therefore$ causal

ii) if $|x[n]| \le M$, then $|y[n]| \le M \times 1 = M$

$\therefore$ stable

iii) $y[n-n_0] = x[n-n_0](u[n-n_0] - u[n-n_0-6])$

$\neq x[n-n_0](u[n] - u[n-6])$

$\therefore$ time variant

(c) $y[n] = \log_{10}(1 + |x[n]|)$

i) $y[n]$ only depends on $x[n]$

$\therefore$ causal

ii) if $|x[n]| \le M$, then $1 + |x[n]| \le M+1$,

$\because \log_{10} x$ is a strictly increasing function

$\therefore |y[n]| \le |\log_{10}(1+M)|$

$\therefore$ stable

iii) $y[n-n_0] = \log_{10}(1+|x[n-n_0]|)$

$\therefore$ time invariant

(d) $y[n] = (x[n])^2$

i) $\because$ $y[n]$ only depends on $x[n]$

$\therefore$ causal

ii) if $|x[n]| \le M$, then $|y[n]| \le M^2$

$\therefore$ stable

iii) $y[n-n_0] = (x[n-n_0])^2$

$\therefore$ time invariant

(e) $y[n] = nx[-3n+2]$

i) when $n=-1$, $y[-1] = -x[5]$

$\therefore$ non causal

ii) if $|x[n]| \le M$, then $|y[n]| \le |n| \cdot M$

when $n \to \infty$, $|y[n]| \to +\infty$ not bounded

$\therefore$ unstable

iii) $y[n-n_0] = (n-n_0)x[-3(n-n_0)+2] = (n-n_0)x[-3n+3n_0+2]$

$\neq nx[-3n+2-n_0]$

$\therefore$ time variant

(f) $y[n] = -x[n] + 2x[n+1] + 3$

i) when $n=1$, $y[1] = -x[1] + 2x[2] + 3$

$\therefore$ non causal

ii) if $|x[n]| \le M$, then $|y[n]| \le M + 2M + 3 = 3M+3$

$\therefore$ stable

iii) $y[n-n_0] = -x[n-n_0] + 2x[n-n_0+1] + 3$

$\therefore$ time invariant

(g) $y[n]=2^n x[n]+(-1)^n x[2n-1]$

i) when $n=2$, $y[2]=2^2 x[2]+(-1)^2 x[3]$

$\therefore$ non causal

ii) if $|x[n]|\le M$, $|y[n]|\le |2^n|\cdot M+M$

when $n\to+\infty$, $|y[n]|\to+\infty$ not bounded

$\therefore$ unstable

iii) $y[n-n_0]=2^{n-n_0}x[n-n_0]+(-1)^{n-n_0}x[2n-2n_0-1]$

$\neq 2^n x[n-n_0]+(-1)^n x[2n-1-n_0]$

$\therefore$ time variant

(h) $y[n]=(\frac{1}{2})^n x[n-2]+(\frac{1}{3})^n x[n+3]$

i) when $n=1$, $y[1]=(\frac{1}{2})^1 x[-1]+(\frac{1}{3})^n x[4]$

$\therefore$ non causal

ii) if $|x[n]|\le M$, then $|y[n]|\le |(\frac{1}{2})^n|\cdot M+|(\frac{1}{3})^n|\cdot M$

when $n\to-\infty$, $|y[n]|\to+\infty$ not bounded

$\therefore$ unstable

iii) $y[n-n_0]=(\frac{1}{2})^{n-n_0}x[n-n_0-2]+(\frac{1}{3})^{n-n_0}x[n-n_0+3]$

$\neq(\frac{1}{2})^n x[n-2-n_0]+(\frac{1}{3})^n x[n+3-n_0]$

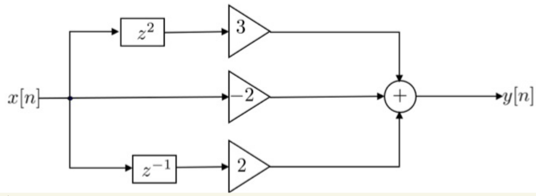
$\therefore$ time variant

2. **Impulse response and properties 1 (12 pts, 2pt each impulse response, 1 pt for stability, 1pt causality)** For the following LTI systems, find the inpulse response $h[n]$ of the system and determine whether they are i) causal or non causal, and ii) stable or unstable. Give a brief explanation for your answer.

(a) $y[n] = 2x[n-2] - x[n-1] + 4x[n] + \frac{1}{2}x[n+1]$

(b) $y[n] = \sum_{k=-M}^{M} a_k x[n-k]$ for $a_k \in \mathbb{R}$.

(c)



(a) Let $x[n] = \delta[n]$

$h[n] = 2\delta[n-2] - \delta[n-1] + 4\delta[n] + \frac{1}{2}\delta[n+1]$

i) when $n=-1$, $h[-1] = 2\delta[-3] - \delta[-2] + 4\delta[-1] + \frac{1}{2}\delta[0] = \frac{1}{2} \neq 0$

$\therefore$ non causal

ii) $\because |\delta[n]| \leq 1$

$\therefore \sum_{n=-\infty}^{+\infty} |h[n]| = 2+1+4+\frac{1}{2} = 7.5 < \infty$

$\therefore$ stable

(b) Let $x[n] = \delta[n]$

$h[n] = \sum_{k=-M}^{M} a_k \delta[n-k]$

$= a_{-M}\delta[n+M] + a_{(-M+1)}\delta[n+M-1] + \cdots + a_M\delta[n-M]$

i) when $n<0$, $h[n] = 0 + \cdots + 0 + a_n\delta[n-n] + 0 + \cdots + 0 = a_n\delta[0] = a_n$

$\therefore$ if when $k<0$, $a_k = 0$, then causal

eles non causal

ii) $\because \sum_{n=-\infty}^{+\infty} |h[n]| = \sum_{k=-M}^{+M} |a_k|$ and $a_k \in \mathbb{R}$

$\therefore \sum_{n=-\infty}^{+\infty} |h[n]| < \infty$

$\therefore$ stable

(c) $y[n] = 3x[n+2] - 2x[n] + 2x[n-1]$

Let $x[n] = \delta[n]$

$h[n] = 3\delta[n+2] - 2\delta[n] + 2\delta[n-1]$

i) when $n = -2$, $h[-2] = 3\delta[0] - 2\delta[-2] + 2\delta[-3] = 3 \neq 0$

$\therefore$ non causal

ii) $\sum_{n=-\infty}^{+\infty} |h[n]| = 3+2+2 = 7 < \infty$

$\therefore$ stable

3. **Impulse response and Properties 2 (20 pts, 2pts for each answer)** For the following impulse responses of the LTI systems, determine whether they are i) causal or non causal, and ii) stable or unstable. Give a brief explanation for your answer.

(a) $h[n] = 2^n u[n]$

(b) $h[n] = \left(\frac{1}{2}\right)^n u[n] + 3^n u[-n-1]$

(c) $h[n] == \left\{\ldots, 0, 0, 8, \underset{\uparrow}{4}, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, 0, 0, \ldots\right\}$

(d) $h[n] = \left(\frac{2}{3}\right)^n u[-n+3]$

(e) $h[n] = \delta[n] + 3\delta[n-1] + 4\delta[n-3] + 2\delta[n-3]$

(a) $h[n] = 2^n u[n]$

i) $\because u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

$\therefore h[n] = \begin{cases} 2^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$

$\therefore$ causal

ii) $\sum_{n=-\infty}^{+\infty} |h[n]| = \sum_{n=0}^{+\infty} |2^n| = \lim_{k \to +\infty} 2^k - 1 = \infty$

$\therefore$ unstable

(b) $h[n] = \left(\frac{1}{2}\right)^n u[n] + 3^n u[-n-1]$

i) $\because u[-n-1] = \begin{cases} 1, & n \leq -1 \\ 0, & n > -1 \end{cases}$

$\therefore$ when $n \leq -1$, $h[n] = 0 + 3^n = 3^n \neq 0$

$\therefore$ non causal

ii) $\sum_{n=-\infty}^{+\infty} |h[n]| = \sum_{n=0}^{+\infty} \left(\frac{1}{2}\right)^n + \sum_{n=-\infty}^{-1} 3^n = \lim_{k \to \infty} \left(2 - \left(\frac{1}{2}\right)^{k-1}\right) + \lim_{k \to \infty} \frac{1}{2}\left(1 - \left(\frac{1}{3}\right)^k\right) = 2 + \frac{1}{2} = 2.5 < \infty$

$\therefore$ stable

(c) $h[n] = \{\cdots, 0, 0, 8, \underset{\uparrow}{4}, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, 0, 0, \cdots\}$

i) $\because h[-1] = 8 \neq 0$

$\therefore$ non causal

ii) $\sum_{n=-\infty}^{+\infty} |h[n]| = 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} < \infty$

$\therefore$ stable

(d) $h[n] = (\frac{2}{3})^n u[-n+3]$

i) $\because u[-n+3] = \begin{cases} 1, & n \le 3 \\ 0, & n > 3 \end{cases}$

$\therefore$ when $n < 0$, $h[n] = (\frac{2}{3})^n \neq 0$

$\therefore$ non causal

ii) $\sum_{n=-\infty}^{+\infty} |h[n]| = \sum_{n=-\infty}^{3} |(\frac{2}{3})^n| = \sum_{n=-3}^{+\infty} (\frac{3}{2})^n = \infty$

$\therefore$ unstable

(e) $h[n] = \delta[n] + 3\delta[n-1] + 4\delta[n-3] + 2\delta[n-3]$

i) when $n < 0$, $h[n] = 0$

$\therefore$ causal

ii) $\sum_{n=-\infty}^{+\infty} |h[n]| = 1 + 3 + 4 + 2 = 10 < \infty$

$\therefore$ stable

4. **Composition of LTI systems. (10 pts, 5 pts each part)** Consider LTI systems with impulse responses $h_1[n]$ and $h_2[n]$. In this question you will have to prove or disprove BIBO stability. Justify your answers with a proof or a counter example.

(a) If $h_1[n]$ is BIBO stable and $h_2[n]$ is bounded, is $h[n] = h_1[n] \circledast h_2[n]$ also BIBO stable?

(b) If $h_1[n]$ and $h_2[n]$ are not BIBO stable, can $h[n] = h_1[n] + h_2[n]$ be BIBO stable?

(a) $h_1[n]$ is BIBO $\longrightarrow \sum_{n=-\infty}^{\infty} |h_1[n]| < \infty$

$h_2[n]$ is bounded $\longrightarrow \exists M > 0, \forall n, |h_2[n]| \leq M$

$\because h[n] = h_1[n] \circledast h_2[n]$

$\therefore \sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{+\infty} \left| \sum_{k=-\infty}^{\infty} h_1[k] h_2[n-k] \right|$

$\leq \sum_{n=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} |h_1[k]| |h_2[n-k]|$

$= \sum_{k=-\infty}^{+\infty} |h_1[k]| \cdot \sum_{n=-\infty}^{+\infty} |h_2[n-k]|$

$\because \sum_{n=-\infty}^{+\infty} |h_2[n-k]| \leq M \cdot \infty = \infty$

$\therefore$ It is possible that $\sum_{n=-\infty}^{\infty} |h[n]| = \infty$

$\therefore h[n]$ is not a BIBO system

For example: $h_1[n] = (\frac{1}{2})^n u[n]$ is BIBO,

$h_2[n] = u[n]$ is bounded, but is not BIBO

$\therefore h[n] = \sum_{k=0}^{+\infty} (\frac{1}{2})^k u[n-k]$

$\therefore \sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{+\infty} \left| \sum_{k=0}^{+\infty} (\frac{1}{2})^k u[n-k] \right|$

$= \sum_{n=-\infty}^{+\infty} \sum_{k=0}^{+\infty} (\frac{1}{2})^k u[n-k]$

$= \sum_{n=k}^{+\infty} \sum_{k=0}^{+\infty} (\frac{1}{2})^k$

$= \sum_{k=0}^{+\infty} (\frac{1}{2})^k \sum_{n=k}^{+\infty} 1$

$= \sum_{k=0}^{+\infty} (\frac{1}{2})^k \cdot \infty$

$= \infty$

$\therefore h[n]$ is not BIBO stable

(b) Suppose $h_1[n] = u[n]$, $h_2[n] = -u[n]$ are not BIBO

$\therefore h[n] = h_1[n] + h_2[n] = 0$

$\sum_{n=-\infty}^{+\infty} |h[n]| = 0 + 0 + \cdots = 0 < \infty$

$\therefore$ can be BIBO stable

5. **Discrete convolution. (10 pts, 2 pts each part)** For each of the following pair of $x_1[n]$ and $x_2[n]$, compute the discrete-time convolution of the signals $y[n] = x_1[n] \circledast x_2[n]$. Give your answer in the most simplified form.

(a) $x_1[n] = \left\{\ldots, 0, 0, 2, \underset{\uparrow}{1}, 3, 0, 0, \ldots\right\}$, $x_2[n] = \left(\frac{1}{2}\right)^n u[n]$

(b) $x_1[n] = u[n]$, $x_2[n] = u[n-5]$

(c) $x_1[n] = \left(\frac{1}{2}\right)^n u[n-3]$, $x_2[n] = u[n]$

(d) $x_1[n] = \left\{\ldots, 0, 0, 1, 1, 2, \underset{\uparrow}{2}, 3, 3, 0, 0, \ldots\right\}$, $x_2[n] = \left\{\ldots, 0, 0, \underset{\uparrow}{1}, 1, 2, 0, 0, \ldots\right\}$

(e) $x_1[n] = \cos\left(\frac{\pi n}{2}\right)$, $x_2[n] = \left(\frac{1}{3}\right)^n u[n-3]$ (Hint: use Euler's formula for the cosine)

(a) $y[n] = \sum_{k=-\infty}^{+\infty} x_1[k]x_2[n-k]$

$= x_1[-1]x_2[n+1] + x_2[0]x_2[n] + x_1[1]x_2[n-1]$

$= 2\cdot(\frac{1}{2})^{n+1}u[n+1] + 1\cdot(\frac{1}{2})^{n}u[n] + 3\cdot(\frac{1}{2})^{n-1}u[n-1]$

$= (\frac{1}{2})^{n}u[n+1] + (\frac{1}{2})^{n}u[n] + 6\cdot(\frac{1}{2})^{n}u[n-1]$

(b) $y[n] = \sum_{k=-\infty}^{+\infty} x_1[k]x_2[n-k]$

$= \sum_{k=-\infty}^{+\infty} u[k]u[n-k-5]$

$= \sum_{k=0}^{n-5} u[k]u[n-k-5]$

$= (n-5+1)u[n-5]$

$= (n-4)u[n-5]$

(c) $y[n] = \sum_{k=-\infty}^{+\infty} x_1[k]x_2[n-k]$

$= \sum_{k=-\infty}^{+\infty} x_2[k]x_1[n-k]$

$= \sum_{k=-\infty}^{+\infty} u[k]\cdot(\frac{1}{2})^{n-k}u[n-k-3]$

$= \sum_{k=0}^{n-3} (\frac{1}{2})^{n-k}u[k]u[n-k-3]$

$= \dfrac{(\frac{1}{2})^3(1-(\frac{1}{2})^{n-2})}{1-\frac{1}{2}} \quad u[n-3]$

$= (\frac{1}{4}-(\frac{1}{2})^{n})u[n-3]$

(d) $y[n] = \sum_{k=-\infty}^{+\infty} x_2[k]x_1[n-k]$

$= x_2[0]x_1[n] + x_2[1]x_1[n-1] + x_2[2]x_1[n-2]$

$= x_1[n] + x_1[n-1] + 2x_1[n-2]$

$\therefore y[-3] = 1+0+0 = 1 \quad y[-2] = 1+1+0 = 2 \quad y[-1] = 2+1+1\times2 = 5 \quad y[0] = 2+2+1\times2 = 6$

$y[1] = 3+2+2\times2 = 9 \quad y[2] = 3+3+2\times2 = 10 \quad y[3] = 0+3+3\times2 = 9 \quad y[4] = 0+0+3\times2 = 6$

$\therefore y[n] = \{\cdots, 0, 0, 1, 2, 5, \underset{\uparrow}{6}, 9, 10, 9, 6, 0, 0, \cdots\}$

(e) $x_1[n] = \dfrac{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}{2}$

$$\begin{aligned}\therefore y[n] &= \sum_{k=-\infty}^{+\infty} x_1[k]\, x_2[n-k] \\ &= \sum_{k=-\infty}^{+\infty} \cos(\tfrac{\pi}{2}k) \cdot (\tfrac{1}{3})^{n-k} u[n-k-3] \\ &= \sum_{k=-\infty}^{n-3} \cos(\tfrac{\pi}{2}k) \cdot (\tfrac{1}{3})^{n-k} u[n-k-3]\end{aligned}$$

$\because x_1[n] = \cos(\frac{\pi}{2}n) = \{\cdots, 1, 0, -1, 0, \underset{\uparrow}{1}, 0, -1, 0, 1, \cdots\}$

$T = 4$

$\therefore$ when $n = 4m+1$ or $n = 4m+3$, $x_1[n] = 0$. $\forall m \in \mathbb{R}$

$$\begin{aligned}\therefore y[n] &= \sum_{m=-\infty}^{\lfloor\frac{n-3}{2}\rfloor} (-1)^m (\tfrac{1}{3})^{n-2m} \\ &= \frac{(-1)^{\lfloor\frac{n-3}{2}\rfloor+1} \cdot (\frac{1}{3})^{n-2\lfloor\frac{n-3}{2}\rfloor}}{1 + (\frac{1}{3})^2} \\ &= 0.9\, (-1)^{\lfloor\frac{n-3}{2}\rfloor+1} (\tfrac{1}{3})^{n-2\lfloor\frac{n-3}{2}\rfloor}\end{aligned}$$

(I don't know how to use Euler's formula to calculate this question)

6. **Systems. (15 pts, 5 points each part)** Let $\mathcal{H}$ denote a system and let $y_1[n] = \mathcal{H}\{x_1[n]\}$. $y_2[n] = \mathcal{H}\{x_2[n]\}$ and $y_3[n] = \mathcal{H}\{x_3[n]\}$ be the output of the system for the input signals $x_1[n]$, $x_2[n]$, and $x_3[n]$ respectively. The signals $x_1[n]$, $y_1[n]$, $x_2[n]$, and $x_3[n]$ are shown in Fig.1.

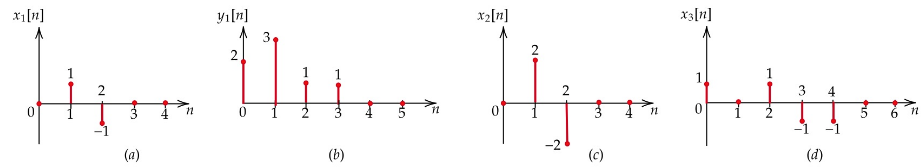


Figure 1: Plots of $x_1[n]$, $y_1[n]$, , $x_2[n]$, and $x_3[n]$.

(a) Express $x_2[n]$ and $x_3[n]$ shown in Fig. 1 (c) and (d) respectively as a linear combination of the time-shifted versions of $x_1[n]$.

(b) If it is given only that $\mathcal{H}$ is linear, can you find $y_2[n]$ and $y_3[n]$? If yes, find and plot the signals. If not, give a brief explanation.

(c) If it is given that $\mathcal{H}$ is linear and time invariant, can you find $y_2[n]$ and $y_3[n]$? If yes, find and plot the signals. If not, give a brief explanation.

(a) $x_2[n] = 2x_1[n]$

$\because \{\underline{1}, 0, 1, -1, -1, 0, 0\}$

$\underline{0}\ \ 1\ \ -1\ \ 0\ \ 0$

$+\ \ \underline{0}\ \ 1\ \ -1\ \ 0\ \ 0$

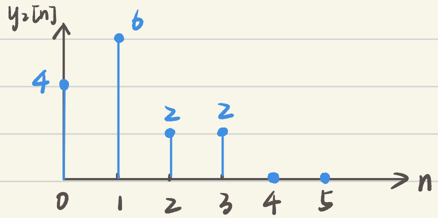
$+\ \ \underline{0}\ \ 2\ \ -2\ \ 0\ \ 0$

$+\ \ \underline{0}\ \ 1\ \ -1\ \ 0\ \ 0$

$\therefore x_3[n] = x_1[n-1] + x_1[n] + 2x_1[n+1] + x_1[n+2]$

(b) ① $\because y_1[n] = H\{x_1[n]\}$ and H is linear

$\therefore y_2[n] = H\{2x_1[n]\} = 2H\{x_1[n]\} = 2y_1[n]$

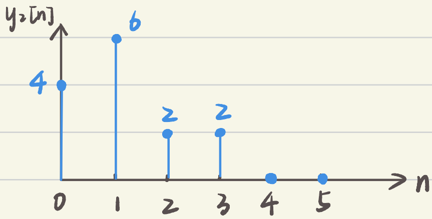


② we don't know whether H is time invariant or not

$\therefore H\{x_1[n-1]\} \neq y_1[n-1]$

$\therefore$ we cannot find $y_3[n]$

(c) ① $y_2[n]$ is same as (b)①



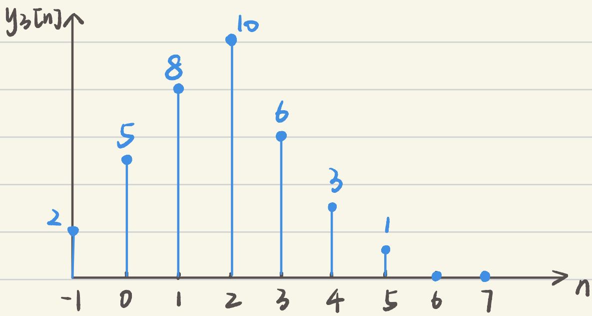
② $\because$ H is time invariant

$\therefore y_3[n] = H\{x_1[n-1] + x_1[n] + 2x_1[n+1] + x_1[n+2]\}$

$= y_1[n-1] + y_1[n] + 2y_1[n+1] + y_1[n+2]$

$$
\begin{array}{cccccccccc}
= & 2 & \underline{3} & 1 & 1 & 0 & 0 & & & \\
+ & & \underline{2} & 3 & 1 & 1 & 0 & 0 & & \\
+ & & & 4 & 6 & 2 & 2 & 0 & 0 & \\
+ & & & & 2 & 3 & 1 & 1 & 0 & 0
\end{array}
$$

$= \{2, \underset{\uparrow}{5}, 8, 10, 6, 3, 1, 0, 0\}$



7. **LTI systems.(19 pts, 7,4,4,4 pts for each part)** Given that $\mathcal{H}$ is an LTI system. For a signal $x_1[n]$, the output of the system is $y_1[n] = \mathcal{H}\{x_1[n]\}$. The signals $x_1[n]$ and $y_1[n]$ are shown in Fig.2 (a) and (b) respectively.

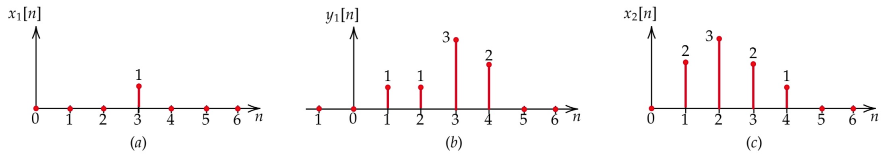


Figure 2: Plots of $x_1[n], y_1[n]$ and $x_2[n]$.

(a) Find the impulse response of the system, $h[n]$, and plot $h[n]$ by hand.

(b) Is the system $\mathcal{H}$ causal? Give a brief explanation for your answer.

(c) Is the system $\mathcal{H}$ BIBO stable? Give a brief explanation for your answer.

(d) For the input signal $x_2[n]$ shown in Fig. 2 (c), compute $y_2[n] = \mathcal{H}\{x_2[n]\}$.

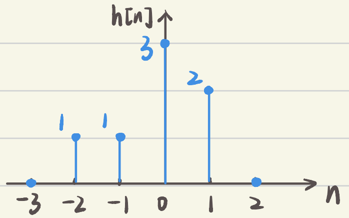
(a) $\because x_1[n] = \delta[n-3]$

$y_1[n] = \delta[n-1] + \delta[n-2] + 3\delta[n-3] + 2\delta[n-4]$

$= x_1[n+2] + x_1[n+1] + 3x_1[n] + 2x_1[n-1]$

$y_1[n] = H\{x_1[n]\} = x_1[n] \circledast h[n]$

$\therefore h[n] = \delta[n+2] + \delta[n+1] + 3\delta[n] + 2\delta[n-1] = \{1, 1, \underset{\uparrow}{3}, 2, 0, \cdots\}$



(b) $\because h[n] = \delta[n+2] + \delta[n+1] + 3\delta[n] + 2\delta[n-1]$

$\therefore$ when $n=-2$, $h[-2] = \delta[0] + 0 + 0 + 0 = 1 \neq 0$

$\therefore$ LTL system H is non causal

(c) $\sum_{n=-\infty}^{+\infty} |h[n]| = 1+1+3+2 = 7 < \infty$

$\therefore$ LTL system H is BIBO stable

(d) $y_2[n] = H\{x_2[n]\} = x_2[n] \circledast h[n]$

$\therefore y_2[n] = \sum_{k=-\infty}^{+\infty} x_2[k]h[n-k]$

$= x_2[1]h[n-1] + x_2[2]h[n-2] + x_2[3]h[n-3] + x_2[4]h[n-4]$

$= 2h[n-1] + 3h[n-2] + 2h[n-3] + h[n-4]$

$\therefore y_2[-1] = 2\times1+0+0+0 = 2$ $\quad y_2[0] = 2\times1+3\times1+0+0 = 5$

$y_2[1] = 2\times3+3\times1+2\times1+0 = 11$ $\quad y_2[2] = 2\times2+3\times3+2\times1+1 = 16$

$y_2[3] = 0+3\times2+2\times3+1 = 13$ $\quad y_2[4] = 0+0+2\times2+3 = 7$

$y_2[5] = 0+0+0+2 = 2$

$\therefore y_2[n] = \{\cdots, 0, 0, 2, \underset{\uparrow}{5}, 11, 16, 13, 7, 2, 0, 0, \cdots\}$