Problem Set 8

Due: Wednesday April 23rd, 11.59pm PST on Gradescope EE483 Spring 2025

130 points

1. **Inverse z-transform**
   (a) (14 pts, 2 each)For $X(z)$ in (i) below, determine all the possible ROCs. Show your derivation.
   (b) (14 pts, 2 each)For $X(z)$ in (i) below, sketch the zero-pole diagram by hand.
   (c) (70 pts, 10 each)For each ROC in (a), find $x[n]$.
   (d) For the remaining z transforms below, repeat (a), (b), and (c).

   i) $X(z) = \frac{2z^{-1}}{(1-\frac{2}{3}z^{-1})(1-5z^{-1})}$
   ii) $X(z) = \frac{z}{z^2+9}$
   iii) $X(z) = \frac{z}{z^2+8z+25}$
   iv) $X(z) = \frac{1}{z^2+6z+9}$
   v) $X(z) = \frac{z^2-z}{z^2-2z+2}$
   vi) $X(z) = \frac{z^3}{(z-1)^2(z-2)}$
   vii) $X(z) = \frac{z^{-1}(\frac{1}{2}-z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{4}{5}z^{-1})^2}$

2. **LTI system input-output (10 pts)** Using the z-transform and its properties, find the output $y[n]$ of an LTI system with impulse response $h[n] = u[n+3] - u[n-4]$, with input

$$x[n] = \begin{cases} 3^n & \text{if } n \geq 3 \\ 2^n & \text{if } n < 3 \end{cases} \tag{1}$$

   Is it possible to compute $y[n]$ using a Fourier based approach?

3. **LTI systems (10 pts)** Consider two LTI systems with impulses response $h_1[n]$ and $h_2[n]$, and define $h[n]$ as the impulse response of the cascade of $h_1[n]$ and $h_2[n]$. Suppose

$$h_1[n] = 4^n u[n-1]. \tag{2}$$

   (a) Using z transform properties, find $h_2[n]$ knowing that $h[n] = 2\delta[n-2]$.
   (b) Is it possible to solve (a) using a Fourier approach?. Explain.

4. **Transfer function (12 pts)**. Consider an LTI system with impulse response $h[n]$ whose DTFT $H(e^{j\omega})$ converges uniformly. The transfer function is

$$H(z) = \frac{z-2}{z-1/2} \tag{3}$$

   (a) Is this system causal?
   (b) Is this system BIBO stable?
   (c) Compute the impulse response $h[n]$ if it is possible, if not explain why.
   (d) Find a linear difference equation that implements the LTI system described by $h[n]$.