HOMEWORK SET #2

EE 510: Linear Algebra for Engineering

Assigned: 7 September 2024
Due: 14 September 2024

Directions: Please show all work and box answers when appropriate.

1. Introduction to Linear Algebra by Gilbert Strang (5th Edition):

   a) Problem Set 2.3: #3, #25

   b) Problem Set 2.5: #25, #30

   c) Problem Set 2.6: #5, #13, #22.

2. Determine the condition on a, b, c, and d for the following linear system to be consistent:

$$Ax = \begin{bmatrix} 2 & 4 & 1 & 3 \\ -3 & 1 & 2 & -2 \\ 13 & 5 & -4 & 12 \\ 12 & 10 & -1 & 13 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}.$$

3. Show that the inverse of a lower triangular matrix A with nonzero diagonal elements is itself lower triangular. (*Hint*: Use the definition of matrix multiplication)

4. Determine whether $[6, 10, 2]$ is a linear combination of $[1, 3, 2]$, $[2, 8, -1]$, and $[-1, 9, 2]$.

5. Let the system $Ax = b$ be such that $A \in \mathbb{R}^{m \times n}$ and $x \in \mathbb{R}^{n \times 1}$. Is the solution set a subspace of $\mathbb{R}^{n \times 1}$?

6. Show that the intersection of any number of subspaces of a vector space V is a subspace of V.

7. If $S = \{\alpha_1, \alpha_2, ..., \alpha_n\}$ is a finite subset of the vectors in vector space V over field F, the set $\mathcal{L}(S)$ of all linear combinations of S over F forms a subspace of V.