HOMEWORK SET #1

EE 510: Linear Algebra for Engineering

Assigned: 30 August 2024
Due: 7 September 2024

Directions: Please show all work and box answers when appropriate.

1. Introduction to Linear Algebra by Gilbert Strang (5th Edition):

   a) Problem Set 1.2: #5, #6, #13, and #16

2. Prove: For any $u, v, w \in \mathbb{C}^n$ and $k \in \mathbb{C}$:

   a) $(u + v) \cdot w = u \cdot w + v \cdot w$

   b) $||u|| \geq 0$; and $||u|| = 0$ if and only if $u = \mathbf{0}$

   c) $k(u + v) = ku + kv$

NOTE: $u \cdot v = \sum_{k=1}^{n} u_k v_k^*$.

3. Prove: Let $A \in \mathbb{R}^{m \times n}$ and $B \in \mathbb{R}^{n \times m}$:

   a) If A has a zero row, then AB has a zero row

   b) If B has a zero column, then AB has a zero column

4. Prove: Let $A_1, A_2, ..., A_n \in \mathbb{R}^{m \times m}$:

   a) $(A_1 A_2 ....A_{n-1} A_n)^T = A_n^T A_{n-1}^T .....A_2^T A_1^T$

   b) $(A_1 A_2 ....A_{n-1} A_n)^{-1} = A_n^{-1} A_{n-1}^{-1} .....A_2^{-1} A_1^{-1}$ if the A_k's are invertible.

5. Suppose A and B are unitary. Show that A^H, A^{-1}, $A^H B^{-1}$ are unitary.

6. Let A be a square matrix. Show that

   a) $A + A^H$ is Hermitian

   b) $A - A^H$ is skew-Hermitian

   c) $A = B + C$, where B is Hermitian and C is skew-Hermitian.

7. Suppose A is a orthogonal matrix. Show that the rows of A are mutually orthogonal.