Assigned: 10 September

Homework #2

EE 503: Fall 2024

Instructions: Write your solutions to these homework problems. Submit your work to Brightspace by the due date. Show all work and box answers where appropriate. Do not guess.

Due: Tuesday, 17 September at 12:00.

1. Use truth tables to prove whether these propositional assertions are valid or invalid:

   (a) $P \Leftrightarrow [(P \ \& \ Q) \vee (P \ \& \ \sim Q)]$.

   (b) $\sim (P \rightarrow Q) \Leftrightarrow (P \ \& \ Q) \vee (\sim P \ \& \ \sim Q)$.

   (c) $P\&(Q \vee R) \Leftrightarrow [(P \ \& \ Q) \vee (P \ \& \ R)]$.

   (d) $\{(P \Rightarrow Q) \ \& \ [P \vee (\sim P \ \& \ R)]\} \Leftrightarrow [P \Leftrightarrow (P \ \& \ Q)]$.

2. Use mathematical induction to prove these theorems for all positive integers $n \geq 1$:

   (a) $\dfrac{1}{1 \cdot 2} + \dfrac{1}{2 \cdot 3} + \cdots + \dfrac{1}{n \cdot (n+1)} = 1 - \dfrac{1}{n+1}$.

   (b) $10^n - 1$ is divisible by 11 for every *even* $n \geq 1$.

   (c) Let $x \in \mathbb{R}$ be non-zero. If $x + \frac{1}{x}$ is an integer then $x^n + \frac{1}{x^n}$ is an integer.

3. Prove or disprove:

   (a) $2^{A \cup B} = 2^A \cup 2^B$.

   (b) $2^A \subset 2^B$ if and only if $A \subset B$.

   (c) $2^{f^{-1}(A)} \cap 2^{f^{-1}(B)} = 2^{f^{-1}(A \cap B)}$.

4. Let $\Omega = \{v, w, x, y, z\}$. Find the sigma-algebra $\sigma(\emptyset)$ that the empty set $\emptyset$ generates. Find the sigma-algebra $\sigma(\{w, x\})$. Find the sigma-algebra $\sigma(\{w, x, z\})$. Find $\sigma(\sigma(\{w\}) \cup \sigma(\{z\}))$.

5. We do not know how the Ancient Green engineer Archimedies proved his famous result $\frac{265}{153} < \sqrt{3} < \frac{1351}{780}$ described in "Measurement of a Circle". Use a proof by contradiction to show that $\sqrt{3}$ is an irrational number.

6. Prove by induction that the n eigenvectors $e_1, \ldots, e_n$ of the n-by-n matrix A are linearly independent if the n corresponding eigenvalues $\lambda_1, \ldots, \lambda_n$ are distinct. The column vector e_k is an *eigenvector* of A with scalar *eigenvalue* λ_k if and only if $Ae_k = \lambda_k e_k$ and e_k is not the null vector. The p column vectors $v_1, \ldots, v_p$ are *linearly independent* if and only if the following implication holds for all complex scalars $c_1, \ldots, c_p$: $\sum_{k=1}^{p} c_k v_k = 0$ imples that all the scalars c_k are zero: $c_1 = \cdots = c_p = 0$.