Problem Set 1

Due: Friday January 31st, 6pm PST on Gradescope EE483 Spring 2025

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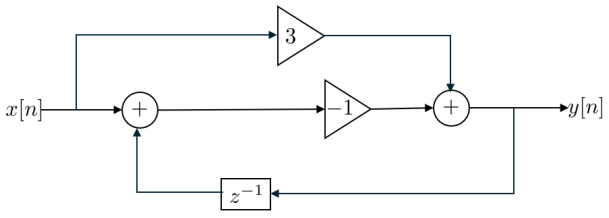
1. **Signal sketching. (15pts, 3pt each)** Sketch by hand the following signals. Here n is the discrete time variable, $u[n]$ is the unit step function and $\delta[n]$ is the delta function. The signals $a[n]$ and $c[n]$ defined in part (a) and (c), respectively, are also used in parts (d) and (e).
   (a) $a[n] = 2^n(u[n+3] - u[n-4])$
   (b) $b[n] = u[-3n+12]u[n+1]$
   (c) $c[n] = 2u[n+2] + 2u[n-2] - u[n-4]$
   (d) $d[n] = a[-n+2] + \delta[n-3]$
   (e) $e[n] = c[1-n^2]$

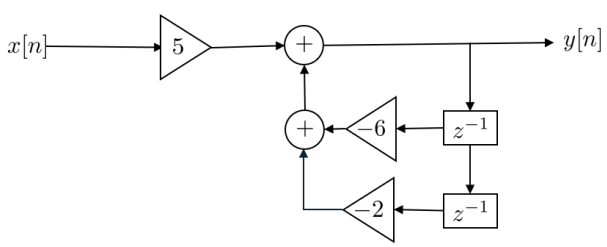
2. **Alternative signal representation.(10 pts, 2pt each)** Represent each signal from part 1 as an ordered list of numbers. For example, the unit impulse $\delta[n]$ is represented by
$$\{\cdots, 0, 0, \underset{\uparrow}{1}, 0, , 0 \cdots\} \tag{1}$$

3. **Sinusoidal sequence (20 pts, 4pt each)**. Consider the sequences
$$x[n] = \sin\left(\frac{\pi}{6}n\right), \quad y[n] = x[n](u[n] - u[n-N]) \tag{2}$$
for some fixed $N > 0$. $u[n]$ is the step function.
   (a) Sketch by hand $x[n]$.
   (b) $x[n]$ is T-periodic, what is is the smallest possible value of T?.
   A discrete time signal $a[n]$ is T-periodic with period $T \in \mathbb{N}$ if for all $n \in \mathbb{Z}$
$$a[n] = a[n+T]. \tag{3}$$
   (c) For $N = 4, 8, 12$ sketch by hand $y[n]$
   (d) For which of the above three values of N the identity
$$\sum_{k\in\mathbb{Z}} y[n - kN] = x[n] \tag{4}$$
   is true?. Give a brief explanation.
   (e) Consider the signal $z[n] = 3^{|x[n]|}$. Briefly explain whether z[n] is a periodic signal? If yes, find the period of this signal z[n].

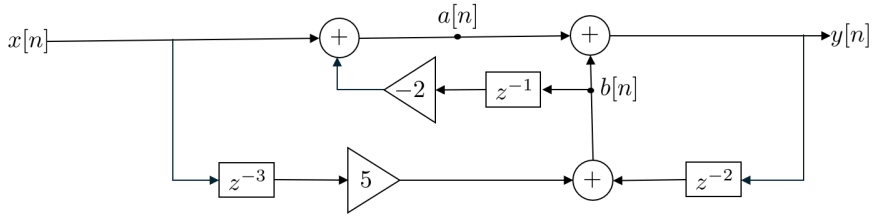
4. **System diagram (20 pts, 5pt each)** For each of the systems below, write a linear difference equation that describes $y[n]$ as a function of its past values, and the present and past values of $x[n]$
   (a)



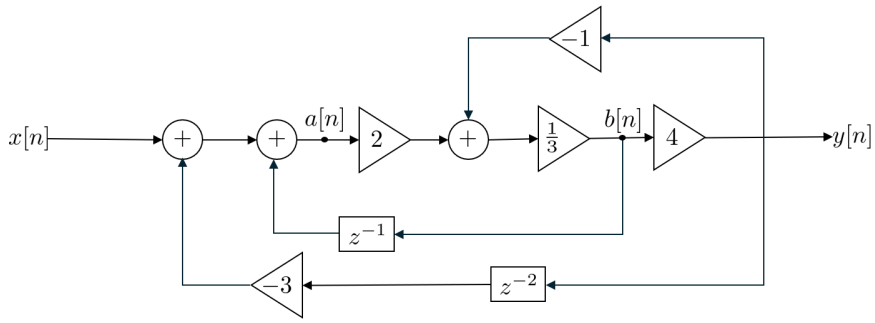
(b)



(c)



(d)



5. **System diagram (5 pts)** Draw the system diagram for the following linear difference equation.

$$y[n] = 2x[n-2] - x[n-1] + 4x[n] + \frac{1}{2}x[n+1]$$

6. **Linear systems (30 pts, 5pt each)** For the following systems, determine whether they are linear or non-linear. Justify your answer with a proof or a counter example.

   (a) $y[n] = x[n^2]$
   (b) $y[n] = x[n](u[n] - u[n-6])$
   (c) $y[n] = \log_{10}(1 + |x[n]|)$
   (d) $y[n] = (x[n])^2$
   (e) $y[n] = nx[-3n+2]$
   (f) $y[n] = -x[n] + 2x[n+1] + 3$