Problem Set 8

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1. **Inverse z-transform**

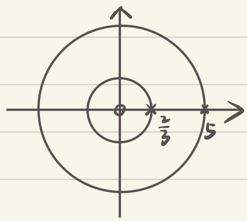
   (a) (14 pts, 2 each)For $X(z)$ in (i) below, determine all the possible ROCs. Show your derivation.

   (b) (14 pts, 2 each)For $X(z)$ in (i) below, sketch the zero-pole diagram by hand.

   (c) (70 pts, 10 each)For each ROC in (a), find $x[n]$.

   (d) For the remaining z transforms below, repeat (a), (b), and (c).

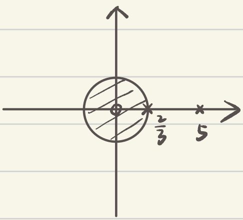
i) $X(z) = \dfrac{2z^{-1}}{(1-\frac{2}{3}z^{-1})(1-5z^{-1})}$ $\quad M<N$

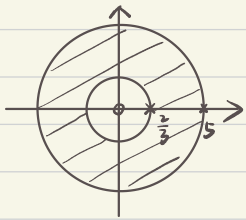


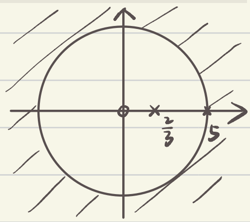
$\therefore \begin{cases} \text{poles: } \lambda_1 = \frac{2}{3},\ \lambda_2 = 5 \\ \text{zeros: } z = 0 \text{ with multiplicity } 2 \end{cases}$

$\therefore$ ROC could be

① $\{z : |z| < \frac{2}{3}\}$ ② $\{z : \frac{2}{3} < |z| < 5\}$ ③ $\{z : 5 < |z|\}$







Let $X(z) = \dfrac{2z^{-1}}{(1-\frac{2}{3}z^{-1})(1-5z^{-1})} = \dfrac{C_1}{1-\frac{2}{3}z^{-1}} + \dfrac{C_2}{1-5z^{-1}}$

$\therefore (1-5z^{-1})C_1 + (1-\frac{2}{3}z^{-1})C_2 = (C_1+C_2) + (-5C_1 - \frac{2}{3}C_2)z^{-1} = 2z^{-1}$

$\therefore \Rightarrow \begin{cases} C_1 + C_2 = 0 \\ -5C_1 - \frac{2}{3}C_2 = 2 \end{cases} \Rightarrow \begin{cases} C_1 = -6/13 \\ C_2 = 6/13 \end{cases}$

$\therefore X(z) = \dfrac{-\frac{6}{13}}{1-\frac{2}{3}z^{-1}} + \dfrac{\frac{6}{13}}{1-5z^{-1}}$

$\therefore$ Let $X_1(z) = \dfrac{-\frac{6}{13}}{1-\frac{2}{3}z^{-1}}$, $|z| > \frac{2}{3} \Rightarrow x_1[n] = -\frac{6}{13}(\frac{2}{3})^n u[n]$

$\tilde{X}_1(z) = \dfrac{-\frac{6}{13}}{1-\frac{2}{3}z^{-1}}$, $|z| < \frac{2}{3} \Rightarrow \tilde{x}_1[n] = \frac{6}{13}(\frac{2}{3})^n u[-n-1]$

$$X_2(z)=\frac{\frac{6}{13}}{1-5z^{-1}},\ |z|>5 \Rightarrow x_2[n]=\frac{6}{13}\cdot 5^n u[n]$$

$$X_2(z)=\frac{\frac{6}{13}}{1-5z^{-1}},\ |z|<5 \Rightarrow \tilde{x}_2[n]=-\frac{6}{13}\cdot 5^n u[-n-1]$$

$\therefore$ for ① ROC $\{z: |z|<\frac{2}{3}\}$

$$x[n]=\tilde{x}_1[n]+\tilde{x}_2[n]=\frac{6}{13}\left(\frac{2}{3}\right)^n u[-n-1]-\frac{6}{13}\cdot 5^n u[-n-1]$$

for ② ROC $\{z: \frac{2}{3}<|z|<5\}$

$$x[n]=x_1[n]+\tilde{x}_2[n]=-\frac{6}{13}\left(\frac{2}{3}\right)^n u[n]-\frac{6}{13}\cdot 5^n u[-n-1]$$

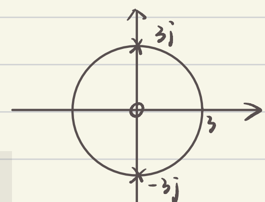
for ③ ROC $\{z: 5<|z|\}$

$$x[n]=x_1[n]+x_2[n]=-\frac{6}{13}\left(\frac{2}{3}\right)^n u[n]+\frac{6}{13}\cdot 5^n u[n]$$

ii) $$X(z)=\frac{z}{z^2+9}=\frac{z}{(z+3j)(z-3j)}=\frac{z^{-1}}{(1+3jz^{-1})(1-3jz^{-1})} \quad M<N$$

$\therefore$ poles: $\lambda_1=-3j$, $\lambda_2=3j$

zeros: $z=0$, $z=\infty$



$\therefore$ ROC could be

① $\{z: |z|>3\}$ ② $\{z: |z|<3\}$

$\because$ $$X(z)=\frac{z}{z^2+9}=\frac{z^{-1}}{1+9z^{-2}}=\frac{1}{3}\cdot\frac{(3\sin\frac{\pi}{2})z^{-1}}{1-(2\times 3\cos\frac{\pi}{2})z^{-1}+3^2z^{-2}}$$

$\therefore$ use z-transform table

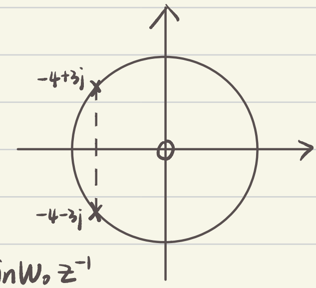
for ① ROC: $\{z: |z|>3\}$

$$x[n]=\frac{1}{3}\left(3^n\sin\frac{\pi n}{2}\right)u[n]=\left(3^{n-1}\sin\frac{\pi n}{2}\right)u[n]$$

for ② ROC: $\{z: |z|<3\}$

$$x[n]=-\frac{1}{3}\left(3^n\sin\frac{\pi n}{2}\right)u[-n-1]=-\left(3^{n-1}\sin\frac{\pi n}{2}\right)u[-n-1]$$

iii) $X(z) = \dfrac{z}{z^2+8z+25}$ $M<N$



$\therefore \begin{cases} \text{poles: let } z^2+8z+25=0 \Rightarrow \lambda_1 = -4+3j,\ \lambda_2 = -4-3j \\ \text{zeros: } z_1 = 0,\ z_2 = \infty \end{cases}$

$\therefore |\lambda_1| = |\lambda_2| = 5$

$\therefore$ ROC could be

① $\{z : |z| > 5\}$ ② $\{z : |z| < 5\}$

$\because X(z) = \dfrac{z}{z^2+8z+25} = \dfrac{z^{-1}}{1+8z^{-1}+25z^{-2}} = \dfrac{\frac{1}{3} \times 5 \sin w_0 z^{-1}}{1-(2\times 5\cos w_0)z^{-1}+5^2 z^{-2}}$

where $\cos w_0 = -\frac{4}{5}$, $\sin w_0 = \frac{3}{5}$

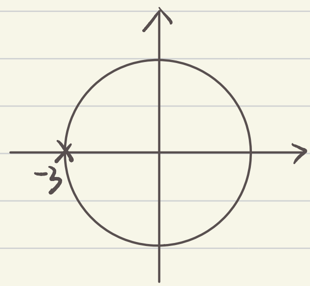
$\therefore$ for ① ROC: $\{z : |z| > 5\}$

$x[n] = \frac{1}{3}(5^n \sin w_0 n)u[n]$

for ② ROC: $\{z : |z| < 5\}$ where $w_0 = \pi - \arcsin(\frac{3}{5})$

$x[n] = -\frac{1}{3}(5^n \sin w_0 n)u[-n-1]$

iv) $X(z) = \dfrac{1}{z^2+6z+9} = \dfrac{1}{(z+3)^2}$ $M<N$



$\therefore \begin{cases} \text{poles: } \lambda = -3 \text{ with multiplicity } 2 \\ \text{zeros: } z = \infty \text{ with multiplicity } 2 \end{cases}$

$\therefore$ ROC could be

① $\{z : |z| > 3\}$ ② $\{z : |z| < 3\}$

$\because X(z) = \dfrac{1}{z^2+6z+9} = \dfrac{z^{-2}}{1+6z^{-1}+9z^{-2}} = \dfrac{z^{-2}}{(1+3z^{-1})^2} = z^{-2}\dfrac{1}{(1-(-3)z^{-1})^2}$

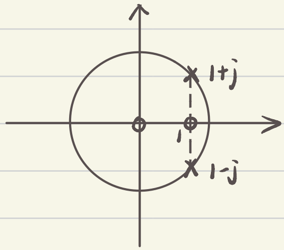
$\therefore$ for ① ROC: $\{z : |z| > 3\}$

$x[n] = ((n-2)+1)(-3)^{n-2}u[n-2] = (n-1)(-3)^{n-2}u[n-2]$

for ② ROC: $\{z : |z| < 3\}$

$x[n] = -((n-2)+1)(-3)^{n-2}u[-(n-2)-1] = -(n-1)(-3)^{n-2}u[-n+1]$

v) $X(z)=\dfrac{z^2-z}{z^2-2z+2}$ $\quad M=N$



$\therefore \begin{cases} \text{poles: let } z^2-2z+2=0 \Rightarrow \lambda_1=1+j,\ \lambda_2=1-j \\ \text{zeros: let } z^2-z=0 \Rightarrow z_1=0,\ z_2=1 \end{cases}$

$\therefore |\lambda_1|=|\lambda_2|=\sqrt{2}$

$\therefore$ ROC could be:

① $\{z:|z|>\sqrt{2}\}$ ② $\{z:|z|<\sqrt{2}\}$

$\because X(z)=\dfrac{z^2-z}{z^2-2z+2}=\dfrac{1-z^{-1}}{1-2z^{-1}+2z^{-2}}=\dfrac{1-(\sqrt{2}\cos\frac{\pi}{4})z^{-1}}{1-(2\times\sqrt{2}\cos\frac{\pi}{4})z^{-1}+(\sqrt{2})^2z^{-2}}$

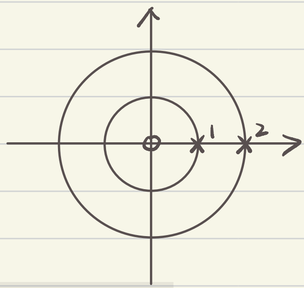
$\therefore$ for ① ROC: $\{z:|z|>\sqrt{2}\}$

$x[n]=(\sqrt{2})^n(\cos\frac{\pi n}{4})u[n]$

for ② ROC: $\{z:|z|<\sqrt{2}\}$

$x[n]=-(\sqrt{2})^n(\cos\frac{\pi n}{4})u[-n-1]$

vi) $X(z)=\dfrac{z^3}{(z-1)^2(z-2)}$ $\quad M=N$



$\therefore \begin{cases} \text{poles: } \lambda_1=\lambda_2=1,\ \lambda_3=2 \\ \text{zeros: } z=0 \text{ with multiplicity } 3 \end{cases}$

$\therefore$ ROC could be

① $\{z:|z|<1\}$ ② $\{z:1<|z|<2\}$ ③ $\{z:2<|z|\}$

$\because X(z)=\dfrac{z^3}{(z-1)^2(z-2)}=\dfrac{z^3}{z^3-4z^2+5z-2}=1+\dfrac{4z^2-5z+2}{(z-1)^2(z-2)}$

$\therefore$ let $\dfrac{4z^2-5z+2}{(z-1)^2(z-2)}=\dfrac{A}{z-1}+\dfrac{B}{(z-1)^2}+\dfrac{C}{z-2}$

$\therefore (z-1)(z-2)A+(z-2)B+(z-1)^2C=4z^2-5z+2$

$(z^2-3z+2)A+(z-2)B+(z^2-2z+1)C=4z^2-5z+2$

$(A+C)z^2+(-3A+B-2C)z+(2A-2B+C)=4z^2-5z+2$

$$\therefore \begin{cases} A+C=4 \\ -3A+B-2C=-5 \\ 2A-2B+C=2 \end{cases} \Rightarrow \begin{cases} A=-4 \\ B=-1 \\ C=8 \end{cases}$$

$$\therefore X(z)=1+\frac{-4}{z-1}+\frac{-1}{(z-1)^2}+\frac{8}{z-2}$$

$\therefore$ Let $X_1(z)=1,\ \forall z \Rightarrow x_1[n]=\delta[n]$

Let $X_2(z)=\dfrac{-4}{z-1}=(-4)z^{-1}\dfrac{1}{1-z^{-1}}$

for $|z|>1 \Rightarrow x_2[n]=(-4)u[n-1]$

$|z|<1 \Rightarrow \tilde{x}_2[n]=-(-4)u[-(n-1)-1]=4u[-n]$

Let $X_3(z)=\dfrac{-1}{(z-1)^2}=-z^{-2}\dfrac{1}{(1-z^{-1})^2}$

for $|z|>1 \Rightarrow x_3[n]=-((n-2)+1)u[n-2]=-(n-1)u[n-2]$

$|z|<1 \Rightarrow \tilde{x}_3[n]=((n-2)+1)u[-(n-2)-1]=(n-1)u[-n+1]$

Let $X_4(z)=\dfrac{8}{z-2}=8z^{-1}\dfrac{1}{1-2z^{-1}}$

for $|z|>2 \Rightarrow x_4[n]=8((n-1)+1)2^{n-1}u[n-1]=n\cdot 2^{n+2}u[n-1]$

$|z|<2 \Rightarrow \tilde{x}_4[n]=-8((n-1)+1)2^{n-1}u[-(n-1)-1]=-n2^{n+2}u[-n]$

$\therefore$ for ① ROC: $\{z:|z|<1\}$

$x[n]=x_1[n]+\tilde{x}_2[n]+\tilde{x}_3[n]+\tilde{x}_4[n]$

$=\delta[n]+4u[-n]+(n-1)u[-n+1]-n\cdot 2^{n+2}u[-n]$

② ROC: $\{z:1<|z|<2\}$

$x[n]=x_1[n]+x_2[n]+x_3[n]+\tilde{x}_4[n]$

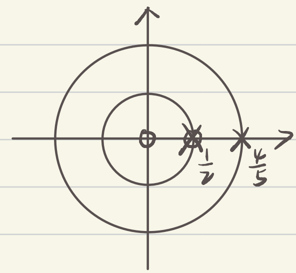
$=\delta[n]-4u[n-1]-(n-1)u[n-2]-n2^{n+2}u[-n]$

③ ROC: $\{z:2<|z|\}$

$x[n]=x_1[n]+x_2[n]+x_3[n]+x_4[n]$

$=\delta[n]-4u[n-1]-(n-1)u[n-2]+n\cdot 2^{n+2}u[n-1]$

vii) $X(z) = \dfrac{z^{-1}(\frac{1}{2} - z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{4}{5}z^{-1})^2}$ $\quad M<N$



$\therefore \begin{cases} \text{poles}: \lambda_1 = \frac{1}{2}, \ \lambda_2 = \lambda_3 = \frac{4}{5} \\ \text{zeros}: z_1 = z_2 = 0, \ z_3 = \frac{1}{2} \end{cases}$

$\therefore$ ROC could be

① $\{z : |z| < \frac{1}{2}\}$ ② $\{z : \frac{1}{2} < |z| < \frac{4}{5}\}$ ③ $\{z : \frac{4}{5} < |z|\}$

$\because X(z) = \dfrac{z^{-1}(\frac{1}{2} - z^{-1})}{(1-\frac{1}{2}z^{-1})(1-\frac{4}{5}z^{-1})^2} = \dfrac{\frac{1}{2}z^{-1} - z^{-2}}{(1-\frac{1}{2}z^{-1})(1-\frac{4}{5}z^{-1})^2}$

$\therefore$ Let $X(z) = \dfrac{A}{1-\frac{1}{2}z^{-1}} + \dfrac{B}{1-\frac{4}{5}z^{-1}} + \dfrac{C}{(1-\frac{4}{5}z^{-1})^2}$

$\therefore (1-\frac{4}{5}z^{-1})^2 A + (1-\frac{1}{2}z^{-1})(1-\frac{4}{5}z^{-1})B + (1-\frac{1}{2}z^{-1})C = -z^{-2} + \frac{1}{2}z^{-1}$

$\therefore (\frac{16}{25}z^{-2} - \frac{8}{5}z^{-1} + 1)A + (\frac{2}{5}z^{-2} - \frac{13}{10}z^{-1} + 1)B + (-\frac{1}{2}z^{-1} + 1)C = -z^{-2} + \frac{1}{2}z^{-1}$

$\therefore (\frac{16}{25}A + \frac{2}{5}B)z^{-2} + (-\frac{8}{5}A - \frac{13}{10}B - \frac{1}{2}C)z^{-1} + (A+B+C) = -z^{-2} + \frac{1}{2}z^{-1}$

$\therefore \begin{cases} \frac{16}{25}A + \frac{2}{5}B = -1 \\ -\frac{8}{5}A - \frac{13}{10}B - \frac{1}{2}C = \frac{1}{2} \\ A+B+C = 0 \end{cases} \Rightarrow \begin{cases} A = -\frac{25}{3} \\ B = \frac{65}{6} \\ C = -\frac{5}{2} \end{cases}$

$\therefore X(z) = \dfrac{-\frac{25}{3}}{1-\frac{1}{2}z^{-1}} + \dfrac{\frac{65}{6}}{1-\frac{4}{5}z^{-1}} + \dfrac{-\frac{5}{2}}{(1-\frac{4}{5}z^{-1})^2}$

$\therefore$ Let $X_1(z) = -\frac{25}{3} \dfrac{1}{1-\frac{1}{2}z^{-1}}$

for $|z| > \frac{1}{2} \Rightarrow x_1[n] = -\frac{25}{3}(\frac{1}{2})^n u[n]$

$|z| < \frac{1}{2} \Rightarrow \tilde{x}_1[n] = \frac{25}{3}(\frac{1}{2})^n u[-n-1]$

Let $X_2(z) = \frac{65}{6} \dfrac{1}{1-\frac{4}{5}z^{-1}}$

for $|z| > \frac{4}{5} \Rightarrow x_2[n] = \frac{65}{6}(\frac{4}{5})^n u[n]$

$|z| < \frac{4}{5} \Rightarrow \tilde{x}_2[n] = -\frac{65}{6}(\frac{4}{5})^n u[-n-1]$

Let $X_3(z) = -\frac{5}{2}\frac{1}{(1-\frac{4}{5}z^{-1})^2}$

for $|z| > \frac{4}{5} \Rightarrow x_3[n] = -\frac{5}{2}(n+1)(\frac{4}{5})^n u[n]$

$|z| < \frac{4}{5} \Rightarrow \tilde{x}_3[n] = \frac{5}{2}(n+1)(\frac{4}{5})^n u[-n-1]$

$\therefore$ for ① ROC: $\{z: |z| < \frac{1}{2}\}$

$x[n] = \tilde{x}_1[n] + \tilde{x}_2[n] + \tilde{x}_3[n]$

$= \frac{25}{3}(\frac{1}{2})^n u[-n-1] - \frac{65}{6}(\frac{4}{5})^n u[-n-1] + \frac{5}{2}(n+1)(\frac{4}{5})^n u[-n-1]$

② ROC: $\{z: \frac{1}{2} < |z| < \frac{4}{5}\}$

$x[n] = x_1[n] + \tilde{x}_2[n] + \tilde{x}_3[n]$

$= -\frac{25}{3}(\frac{1}{2})^n u[n] - \frac{65}{6}(\frac{4}{5})^n u[-n-1] + \frac{5}{2}(n+1)(\frac{4}{5})^n u[-n-1]$

③ ROC: $\{z: \frac{4}{5} < |z|\}$

$x[n] = x_1[n] + x_2[n] + x_3[n]$

$= -\frac{25}{3}(\frac{1}{2})^n u[n] + \frac{65}{6}(\frac{4}{5})^n u[n] - \frac{5}{2}(n+1)(\frac{4}{5})^n u[n]$

2. **LTI system input-output (10 pts)** Using the z-transform and its properties, find the output $y[n]$ of an LTI system with impulse response $h[n] = u[n+3] - u[n-4]$, with input

$$x[n] = \begin{cases} 3^n & \text{if } n \geq 3 \\ 2^n & \text{if } n < 3 \end{cases} \tag{1}$$

Is it possible to compute $y[n]$ using a Fourier based approach?

① $\because h[n] = u[n+3] - u[n-4]$

$\therefore H(z) = \sum_{n=-\infty}^{+\infty} h[n] z^{-n} = \sum_{n=-3}^{+3} z^{-n} = z^3 + z^2 + z + 1 + z^{-1} + z^{-2} + z^{-3}$

$\because x[n] = \begin{cases} 3^n, & n \geq 3 \\ 2^n, & n < 3 \end{cases} = 3^n u[n-3] + 2^n u[-n+2] = x_1[n] + x_2[n]$

$\therefore X_1(z) = \sum_{n=3}^{+\infty} 3^n z^{-n} = \sum_{n=3}^{+\infty} (3z^{-1})^n = (3z^{-1})^3 \sum_{n=0}^{+\infty} (3z^{-1})^n = \frac{(3z^{-1})^3}{1-3z^{-1}}$ ROC: $|z|>3$

$X_2(z) = \sum_{n=-\infty}^{2} 2^n z^{-n} = \sum_{n=-2}^{+\infty} (\frac{1}{2}z)^n = (\frac{1}{2}z)^{-2} \sum_{n=0}^{+\infty} (\frac{1}{2}z)^n = \frac{(\frac{1}{2}z)^{-2}}{1-\frac{1}{2}z}$ ROC: $|z|<2$

$\therefore$ Let $Y_1(z) = H(z)X_1(z) = (z^3+z^2+z+1+z^{-1}+z^{-2}+z^{-3})X_1(z)$ ROC: $|z|>3$

$\Rightarrow y_1[n] = x_1[n-3] + x_1[n-2] + x_1[n-1] + x_1[n] + x_1[n+1] + x_1[n+2] + x_1[n+3]$

Similarly let $Y_2(z) = H(z)X_2(z)$ ROC: $|z|<2$

$\Rightarrow y_2[n] = x_2[n-3] + x_2[n-2] + x_2[n-1] + x_2[n] + x_2[n+1] + x_2[n+2] + x_2[n+3]$

$\therefore y = y_1[n] + y_2[n] = (x_1[n-3] + x_2[n-3]) + \cdots + (x_1[n+3] + x_2[n+3])$

$= x[n-3] + \cdots + x[n+3]$

$\therefore y[n] = x[n+3] + x[n+2] + x[n+1] + x[n] + x[n-1] + x[n-2] + x[n-3]$

② Cannot use Fourier based approach

$\because$ when $n \geq 3$, $x[n] = 3^n$

$\therefore \sum_{n=-\infty}^{\infty} |x[n]| = \infty$, $x[n]$ is unstable

$\therefore X(e^{jw})$ does not exist

$\therefore$ we cannot use $X(e^{jw}) \cdot H(e^{jw})$ to compute $Y(e^{jw})$

3. **LTI systems (10 pts)** Consider two LTI systems with impulses response $h_1[n]$ and $h_2[n]$, and define $h[n]$ as the impulse response of the cascade of $h_1[n]$ and $h_2[n]$. Suppose

$$h_1[n] = 4^n u[n-1]. \tag{2}$$

(a) Using z transform properties, find $h_2[n]$ knowing that $h[n] = 2\delta[n-2]$.

(b) Is it possible to solve (a) using a Fourier approach?. Explain.

(a) $\because h_1[n] = 4^n u[n-1]$

$\therefore H_1(z) = \sum_{n=-\infty}^{+\infty} 4^n u[n-1] z^{-n} = \sum_{n=1}^{+\infty} 4^n z^{-n} = \sum_{n=1}^{+\infty} (4z^{-1})^n$

$= \dfrac{4z^{-1}}{1-4z^{-1}}$ ROC: $|z| > 4$

$\because h[n] = 2\delta[n-2]$

$\therefore H(z) = 2z^{-2}$ ROC: $|z| > 0$

$\because H(z) = H_1(z) \cdot H_2(z)$

$\therefore H_2(z) = \dfrac{H(z)}{H_1(z)} = \dfrac{2z^{-2}(1-4z^{-1})}{4z^{-1}} = \dfrac{z^{-1} - 4z^{-2}}{2} = \frac{1}{2}z^{-1} - 2z^{-2}$

$\therefore h_2[n] = \frac{1}{2}\delta[n-1] - 2\delta[n-2]$

(b) Cannot use a Fourier approach

$\because h_1[n] = 4^n u[n-1]$

$\therefore \sum_{n=-\infty}^{+\infty} |h_1[n]| = \infty$, $h_1[n]$ is unstable

$\therefore H_1(e^{jw})$ does not exist

$\therefore$ we cannot use $H(e^{jw}) / H_1(e^{jw})$ to compute $H_2(e^{jw})$

4. **Transfer function (12 pts).** Consider an LTI system with impulse response $h[n]$ whose DTFT $H(e^{j\omega})$ converges uniformly. The transfer function is

$$H(z) = \frac{z-2}{z-1/2} \tag{3}$$

(a) Is this system causal?

(b) Is this system BIBO stable?

(c) Compute the impulse response $h[n]$ if it is possible, if not explain why.

(d) Find a linear difference equation that implements the LTI system described by $h[n]$.

(a) $\because H(z) = \dfrac{z-2}{z-\frac{1}{2}} \Rightarrow \begin{cases} \text{pole}: \lambda = \frac{1}{2} \\ \text{zero}: z = 2 \end{cases}$

$\therefore$ there are two possible ROC:

$\{z : |z| > \frac{1}{2}\}$ or $\{z : |z| < \frac{1}{2}\}$

$\because$ DTFT $H(e^{j\omega})$ exists and converges uniformly

$\therefore$ unit circle must be contained in the ROC

$\therefore$ ROC of $H(z)$ is $\{z : |z| > \frac{1}{2}\}$

$\therefore$ $h[n]$ is a right-sided sequence

$\therefore$ this system is causal

(b) $\because$ DTFT $H(e^{j\omega})$ exists and converges uniformly

$\therefore$ $h[n]$ is absolute summable

$\therefore$ this system is BIBO stable

(c) $\because H(z) = \dfrac{z-2}{z-\frac{1}{2}} = \dfrac{1-2z^{-1}}{1-\frac{1}{2}z^{-1}} = \dfrac{1}{1-\frac{1}{2}z^{-1}} - \dfrac{2z^{-1}}{1-\frac{1}{2}z^{-1}}$

$= (\frac{1}{2})^n u[n] - 2(\frac{1}{2})^{n-1} u[n-1]$

$= (\frac{1}{2})^n (u[n] - 4u[n-1])$

(d) $\because H(z) = \dfrac{z-2}{z-\frac{1}{2}} = \dfrac{Y(z)}{X(z)}$

$\therefore (z-\frac{1}{2})Y(z) = (z-2)X(z)$

$\therefore (1-\frac{1}{2}z^{-1})Y(z) = (1-2z^{-1})X(z)$

$\therefore Y(z) - \frac{1}{2}z^{-1}Y(z) = X(z) - 2z^{-1}X(z)$

$\therefore y[n] - \frac{1}{2}y[n-1] = x[n] - 2x[n-1]$