Problem Set 4

Due: Wednesday February 26th, 6pm PST on Gradescope EE483 Spring 2025

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107 pts. You can use without proof the properties and DTFT pairs viewed during Lecture, and the tables from Mitra uploaded to the course website. Include your code in the submission.

1. **DTFT practice.(16 pts, 2, 2, 2, 5, 5)** Compute the DTFT and the inverse DTFT for the following sequences using DTFT properties. It should be obvious which one to compute for each case.
   (a) $x[n] = n^2(\frac{1}{3})^n u[n]$
   (b) $x[n] = \left(\frac{j}{4}\right)^n u[n+5]$
   (c) $x[n] = n(u[n+3] - u[n-4])$
   (d) $X(e^{j\omega}) = \frac{1+e^{j\omega}}{2-e^{-j\omega}}$ for $-\pi \leq \omega \leq \pi$
   (e) $X(e^{j\omega}) = \frac{e^{j3\omega}}{1-(2/3)e^{-j(\omega-\pi/5)}} - \frac{2}{1+(1/2)e^{-j(\omega+\pi/5)}}$ for $-\pi \leq \omega \leq \pi$

2. **Frequency response (57 pts).** Consider the following input-output relationships describing different LTI causal systems. Use MATLAB for plotting for this problem.
   (a) $y[n] = -x[n-3]$
   (b) $y[n] - y[n-2] = x[n] + x[n-2]$
   (c) $y[n] = 4x[n] + 3x[n-1] + 2x[n-2] + x[n-4]$
   (d) $y[n] = x[n] + 4x[n-1] + 3x[n-2] + 4x[n-3] + x[n-4]$
   (e) $5y[n-2] + 2y[n] = 5x[n] + 2x[n-2]$
   (f) $y[n] = \frac{1}{10}\sum_{k=0}^{9} x[n-k]$
   (g) $y[n] = \frac{1}{10}\sum_{k=0}^{9}(-1)^k x[n-k]$
   (h) $y[n] = \sum_{k=0}^{41}\left(\frac{4}{7}\right)^k x[n-k]$
   (i) $y[n] = \sum_{k=0}^{41}\left(-\frac{4}{7}\right)^k x[n-k]$

   1) **(18 pts, 2pts for each)** For each system, find a closed form expression for its frequency response $H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})}$.
   2) **(18 pts, 1 pt each plot, 1pt each filter type)** Plot the magnitude of the frequency response in the interval $[-\pi, \pi]$. Based on your plots, would you say the filters are low pass (they attenuate frequencies closer to π and $-\pi$, while preserving frequencies closer to 0), high pass (the opposite of low pass), all-pass or something else?.
   3) **(18 pts, 1 pt each plot, 1pt each filter type)** Plot all the phase responses. From the phase plots of (a)-(i), do they appear to have generalized linear phase or non linear phase?
   4) (**3 points**) For the cases that appear to have linear phase, do they satisfy the sufficient conditions for GLP discussed in class?

   In MATLAB, the frequency response can be evaluated with the function `freqz`. You should call it using input-output arguments of the form: `[h,w] = freqz(b,a,n,‘whole’)` (see Matlab documentation for details). The output of freqz will be in the interval $[0, 2\pi]$, so you should use the periodicity of the DTFT and the output of freqz to obtain the plot on $[-\pi, \pi]$. To compute the magnitude and phase, see the documentation for `abs` and `angle`.

3. **Filtering, 25pts** Use MATLAB for plotting for this problem. Consider the following signal.

$$x[n] = u[n] + 2u[n-10] - 2u[n-30] - u(n-40)$$

(a) (**2pts**)Plot the signal $x[n]$ using the `stem` command.

(b) (**2pts**)Plot the magnitude and phase of the DTFT of $x[n]$ in the interval $[-\pi, \pi]$.

(c) Using the `filter` command, apply each of the filters (a)-(i) from Problem 2 and
(i) (**9pts**) plot the resulting signals using the `stem` command.
(ii) (**9pts**)plot the magnitude and phase of the DTFT of the resulting signals in the interval $[-\pi, \pi]$.

(d) (**3pts**) Do you see any significant differences between linear and non-linear phase filters that would make you prefer one over the other?

4. **Non integer delay (9 pts, 3 pts each)** For an integer M, a system that produces a delay by M samples ($\delta[n-M]$) has DTFT $H(e^{j\omega}) = e^{-j\omega M}$. Now consider a system $h[n]$ with frequency response $H(e^{j\omega}) = e^{-j\omega\sqrt{5}}$ for $-\pi \leq \omega \leq \pi$. Whether $y[n]$ can be called a delay or not is subjective so you should answer based on an interpretation of your results.

(a) Compute the output for this system $y[n]$ with the input $x[n] = \alpha\cos(\omega_0 n) + \beta\sin((\omega_0/3)n)$. Compare $y[n]$ to $x[n]$, do you think this system should be called a non integer delay?.

(b) For $\omega_0 = \pi/10$, what is the period of $x[n]$?. Plot (using stem in MATLAB) $x[n]$, $x[n-1]$, $x[n-2]$ and $y[n]$ assuming that $\omega_0 = \pi/10$, $\alpha = -1/2$ and $\beta = 1$. Make sure your plots includes at least 2 full periods. By comparing the values of $y[n]$ to the other delayed inputs, can you say that $y[n]$ is also a delayed version of $x[n]$?

(c) Compute the impulse response $h[n]$ of this system.