Homework #7

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1. Let $A =$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(a) Note $A = A^T$ by certain diagonalizing

$\det(A^T) = \det(A) \quad \det(A) = (1)(-3)(-3) = 9$

(b) Using diagonalization

$B = T\Lambda T^{-1}$

$\Rightarrow (T\Lambda T^{-1})(T\Lambda T^{-1})$

$= (T^{-1})^{-1}\Lambda^2 T^{-1}$

Eigenvalues are squared

The eigenvectors do not change.

(c) $B^T = T\Lambda^2 T^{-1}\Lambda T^{-1} = T\Lambda^3 T^{-1}$

$\Rightarrow B^T = T(\Lambda^3)T^{-1}$

$= T(\Lambda^2 \cdot \Lambda)T^{-1}$

The new eigenvalues are cubed from λ_i^2 to

$\lambda_1 = 1 \quad \lambda_1^3 = 1$

$\lambda_2 = -3 \quad \lambda_2^3 = -27$

$\lambda_3 = -3 \quad \lambda_3^3 = -27$

Note $(B+I)^{-1}$, the eigenvalues are:

$\lambda_1 = 0 \quad \lambda_1^{-1}$

$\lambda_2 = -2 \quad \lambda_2^{-1} = -\frac{1}{2}$

$\lambda_3 = -2 \quad \lambda_3^{-1} = -\frac{1}{2}$

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2. Note

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$\det(A) = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow (1)(1)(1)(1)(1)(1) = (1)(1)(1)(1)(1)(1)$

$\Rightarrow (1)(1)(1)(1)(1)(1) - (1)(1)(1)(1)(1)(1) = 0$

$\Rightarrow (1)(1)(1)(1)(1)(1) = 0$

$\Rightarrow \det(A) = 0$

$\Rightarrow \det(A^{-1}) = \det(A^{-1})^{-1} = \det(A)$

$\Rightarrow \det(A^{-1}) = 0$

$\Rightarrow (A^{-1})^{-1} = 0$

$\Rightarrow (A^{-1})(A^{-1}) = 0$

$\Rightarrow (A^{-1})(A^{-1}) - 0 = 0$

$\Rightarrow (A^{-1})(A^{-1}) - (A^{-1})(0) = 0$

$\Rightarrow (A^{-1})(A^{-1} - 0) = 0$

$\Rightarrow (A^{-1})(A^{-1}) = 0$

$\Rightarrow (A^{-1})^{-1}(