HOMEWORK SET #4

EE 510: Linear Algebra for Engineering — Assigned: 20 September 2024
Due: No submission

Directions:

1. Introduction to Linear Algebra by Gilbert Strang (5th Edition):

   a) Problem Set 4.1: #21, #23

   b) Problem Set 4.4: #18, #22.

2. Let V be the vector space of $m \times n$ matrices over $\mathbb{R}$. Show that $\langle A, B\rangle = \mathrm{Tr}(B^T A)$ defines an inner product in V.

3. Find a basis for the subspace W of $\mathbb{R}^4$ that is orthogonal to u_1 and u_2:

$$u_1 = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \end{bmatrix} \qquad u_2 = \begin{bmatrix} 3 \\ -5 \\ 7 \\ 8 \end{bmatrix}.$$

4. Show that $U = \{u_1, u_2, u_3\}$ is an orthogonal subset of $\mathbb{R}^4$. Find the fourth vector u_4 such that $U' = \{u_1, u_2, u_3, u_4\}$ forms an orthogonal basis in $\mathbb{R}^4$.

$$u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad u_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix} \qquad u_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -3 \end{bmatrix}.$$

5. Use the Gram-Schmidt process to find an orthonormal basis for the subspace of $\mathbb{R}^n$ spanned by the following vectors:

$$u_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \qquad u_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad u_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

6. Prove that if S is a subset of $\mathbb{R}^n$, then the orthogonal complement $S^{\perp}$ is a subspace of $\mathbb{R}^n$.