# HW10-Q1

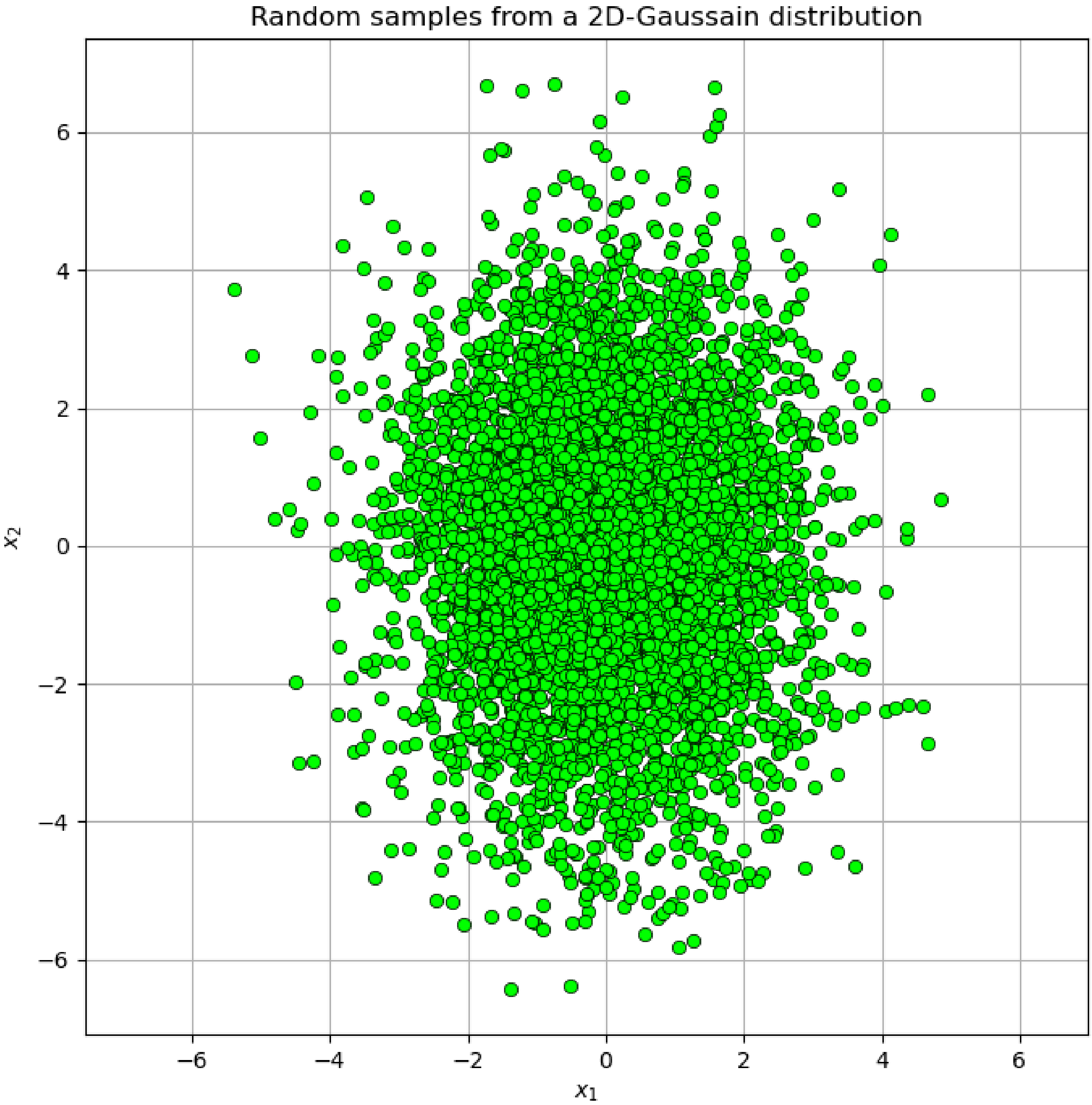
In [123…

```
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.patches import Ellipse
from scipy.stats import multivariate_normal

plt.rcParams['figure.figsize']=8,8

def generate_and_plot(kx,mu):
    distr=multivariate_normal(
        cov=kx,mean=mu,
        seed=1000
    )

    data=distr.rvs(size=5000)
    plt.grid()

    plt.plot(data[:,0],data[:,1],'o',color='lime',
                markeredgewidth=0.5,
                markeredgecolor='black')

    plt.title(r'Random samples from a 2D-Gaussain distribution')
    plt.xlabel(r'$x_1$')
    plt.ylabel(r'$x_2$')
    plt.axis('equal')

def ellipsoid(x,mu,kx):
    return ((x-mu) @ np.linalg.inv(kx) @ (x-mu).T)<=1

def plot_ellipsoid(mu,kx,ax):
    #repeat generate 5000 points
    distr=multivariate_normal(
        cov=kx,mean=mu,
        seed=1000
    )
    data=distr.rvs(size=5000)

    #filter the points which are belong to the ellipsoid and plot
    filtered_data=np.array([point for point in data if ellipsoid(point,mu,kx)])

    plt.grid()
    plt.plot(filtered_data[:,0],filtered_data[:,1],'o',color='lime',
                markeredgewidth=0.5,
                markeredgecolor='black')
    plt.title(r'filtered data and ellipsoid')
    plt.xlabel(r'$x_1$')
    plt.ylabel(r'$x_2$')
    plt.axis('equal')

    eigenvalues,eigenvectors=np.linalg.eigh(kx)
    axis_lengths=2*np.sqrt(eigenvalues)
    angle=np.degrees(np.arctan2(*eigenvectors[:, 0][::-1]))
    #print(angle)
    ell=Ellipse(xy=mu,width=axis_lengths[0],height=axis_lengths[1],angle=angle,
                edgecolor='red', facecolor='none', lw=2)
    ax.add_patch(ell)
```

when mu = $\begin{bmatrix} 0 & 0 \end{bmatrix}$ Kx = $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

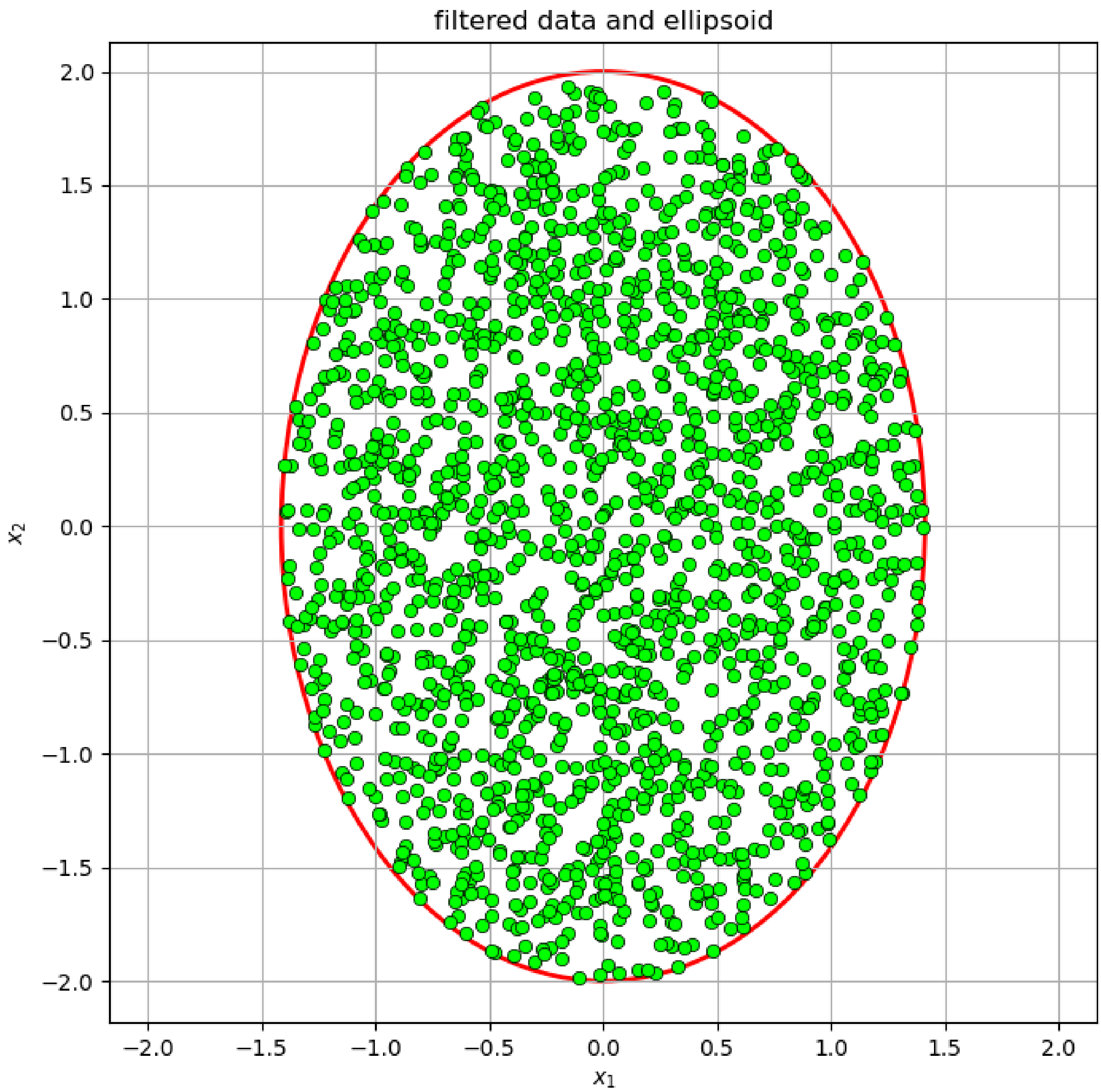
In [124…

```
Kx=np.array([[2.0,0.0],[0.0,4.0]])
mu=np.array([0,0])
random_seed=10

generate_and_plot(Kx,mu)
```




In [125…

```
ax=plt.gca()
plot_ellipsoid(mu,Kx,ax)
plt.show()
```




## Compare

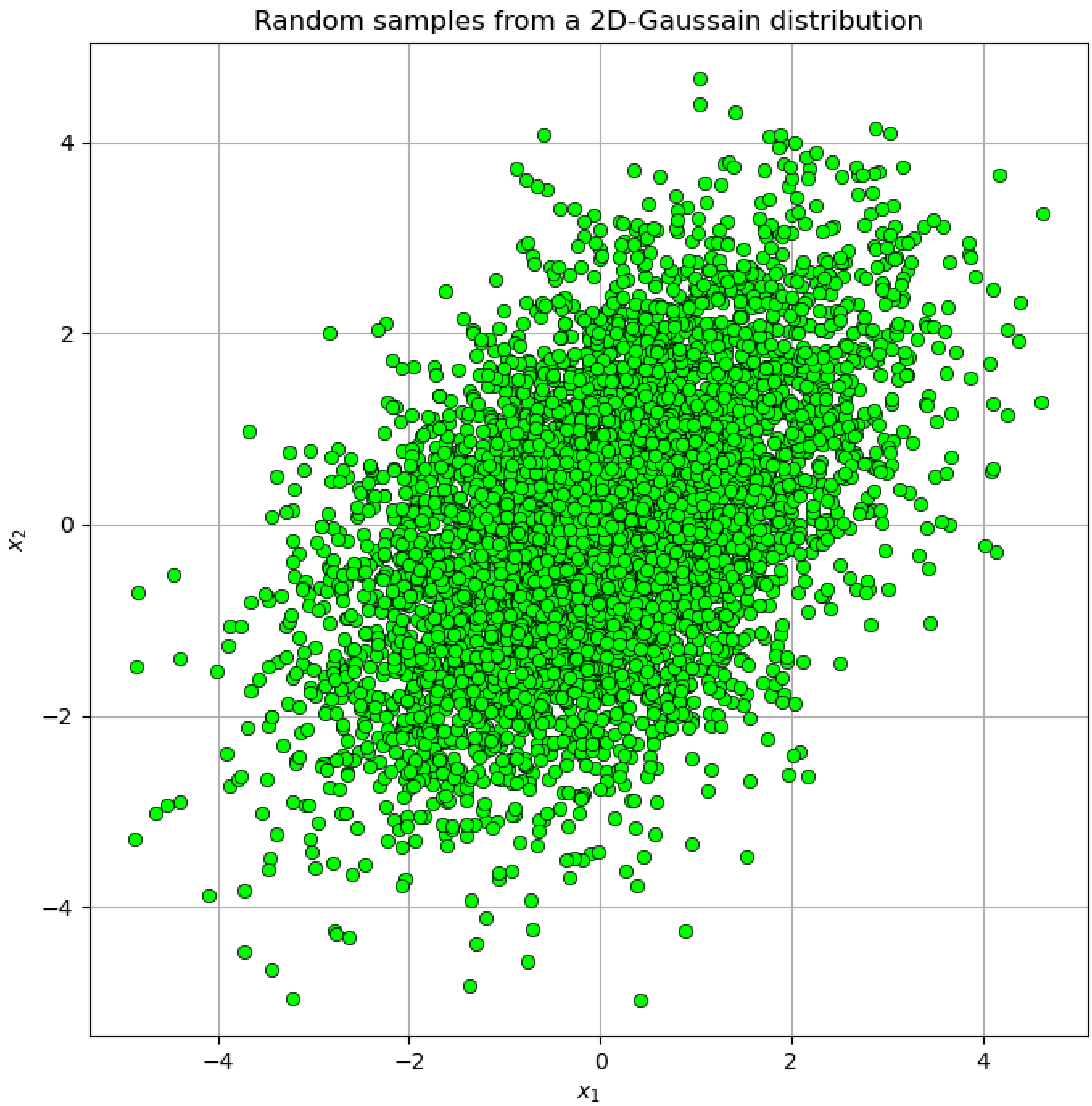
- I observe that when I use the definition of an ellipsoid to filter the random points, they lie precisely within the ellipsoid we draw based on the eigenvalues and eigenvectors.
- Additionally, the lengths of the principal axes are shorter than the range of the random points, approximately by a factor of one-third.

(The comparison results of the last two experiments are the same as this one.)

## when mu = $\begin{bmatrix} 0 & 0 \end{bmatrix}$ Kx = $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

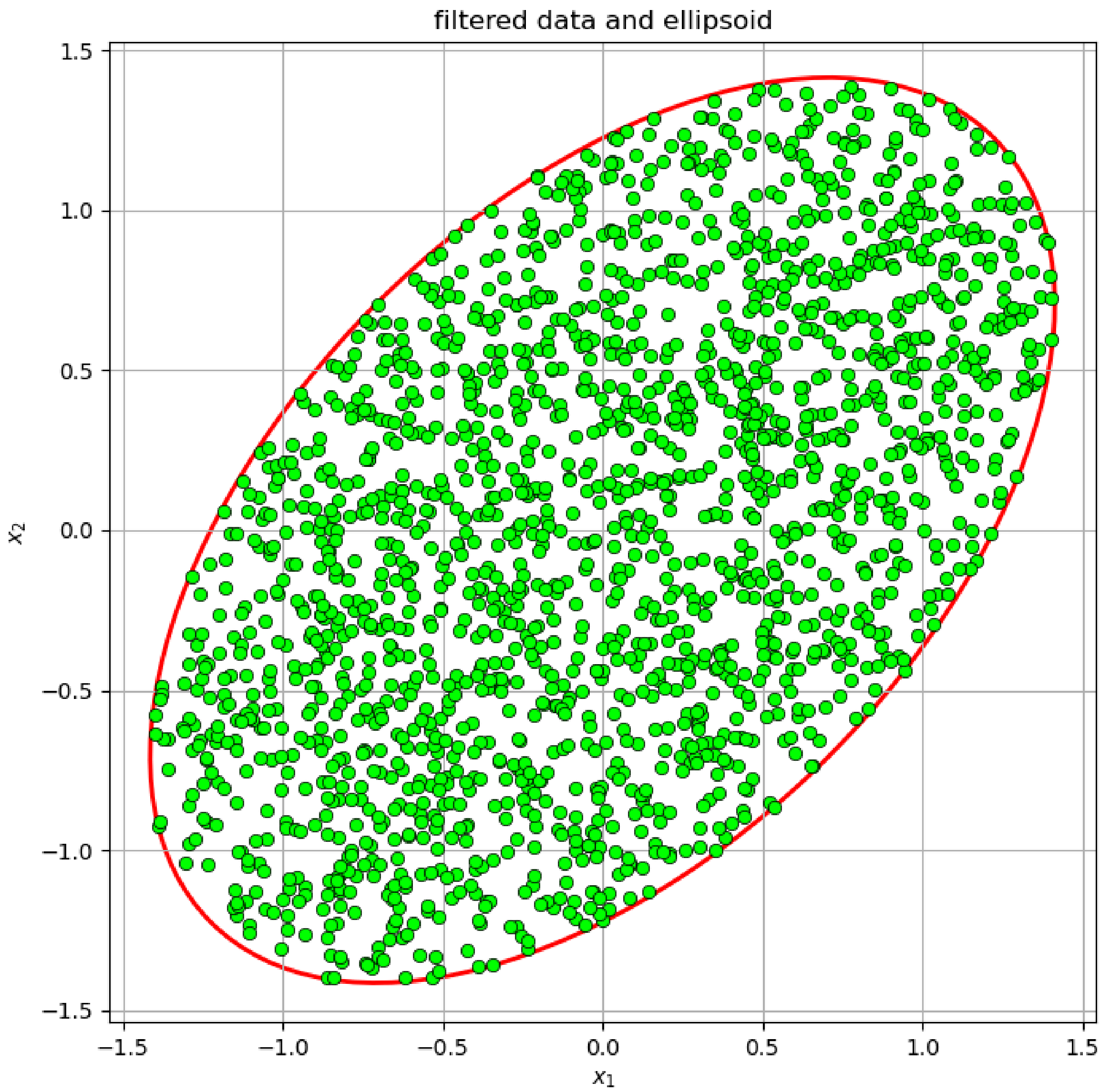
In [126…

```
Kx=np.array([[2.0,1.0],[1.0,2.0]])
mu=np.array([0,0])
random_seed=10

generate_and_plot(Kx,mu)
```




In [127…

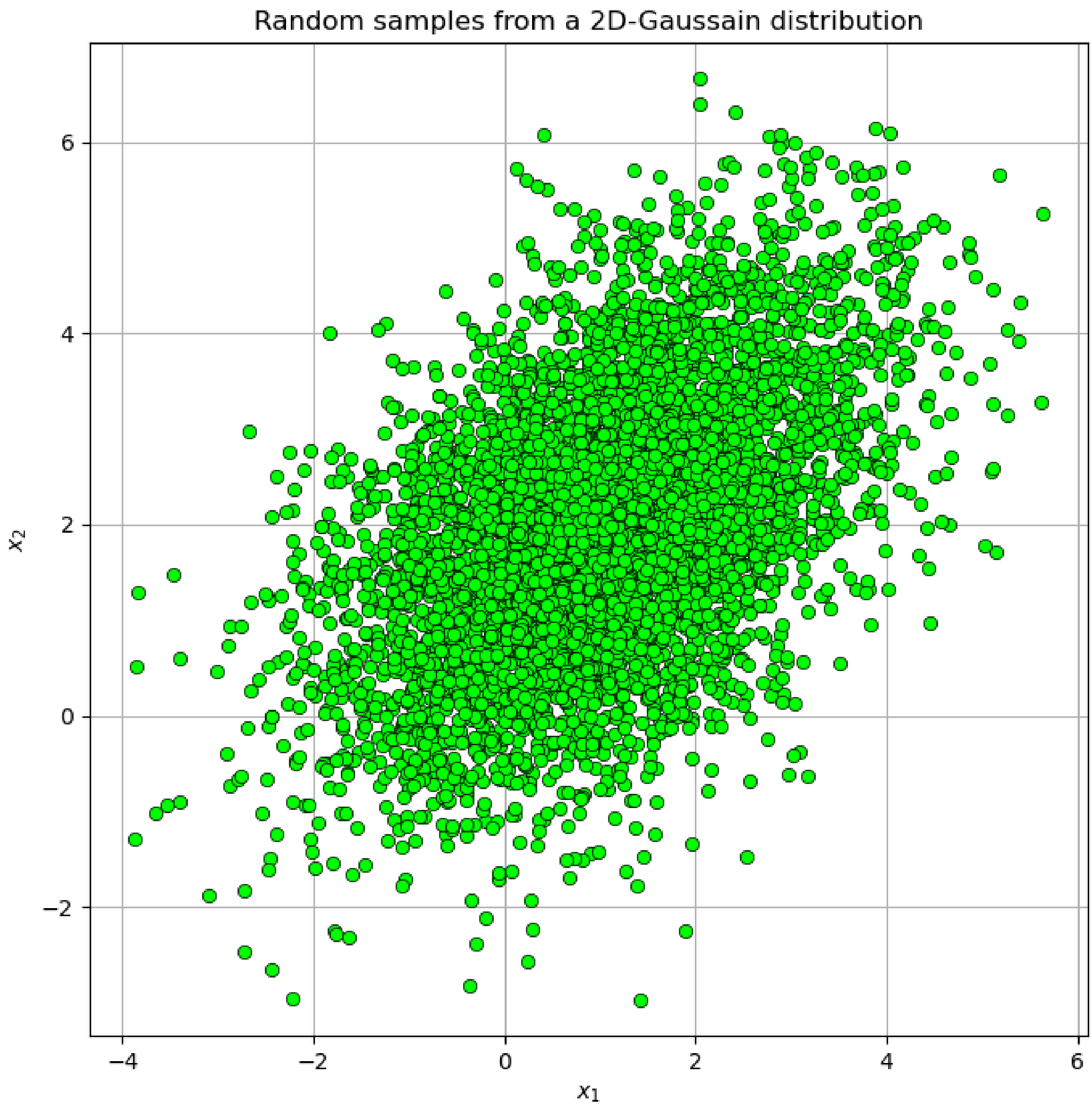
```
ax=plt.gca()
plot_ellipsoid(mu,Kx,ax)
plt.show()
```




when mu = $\begin{bmatrix} 2 & 1 \end{bmatrix}$ Kx = $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

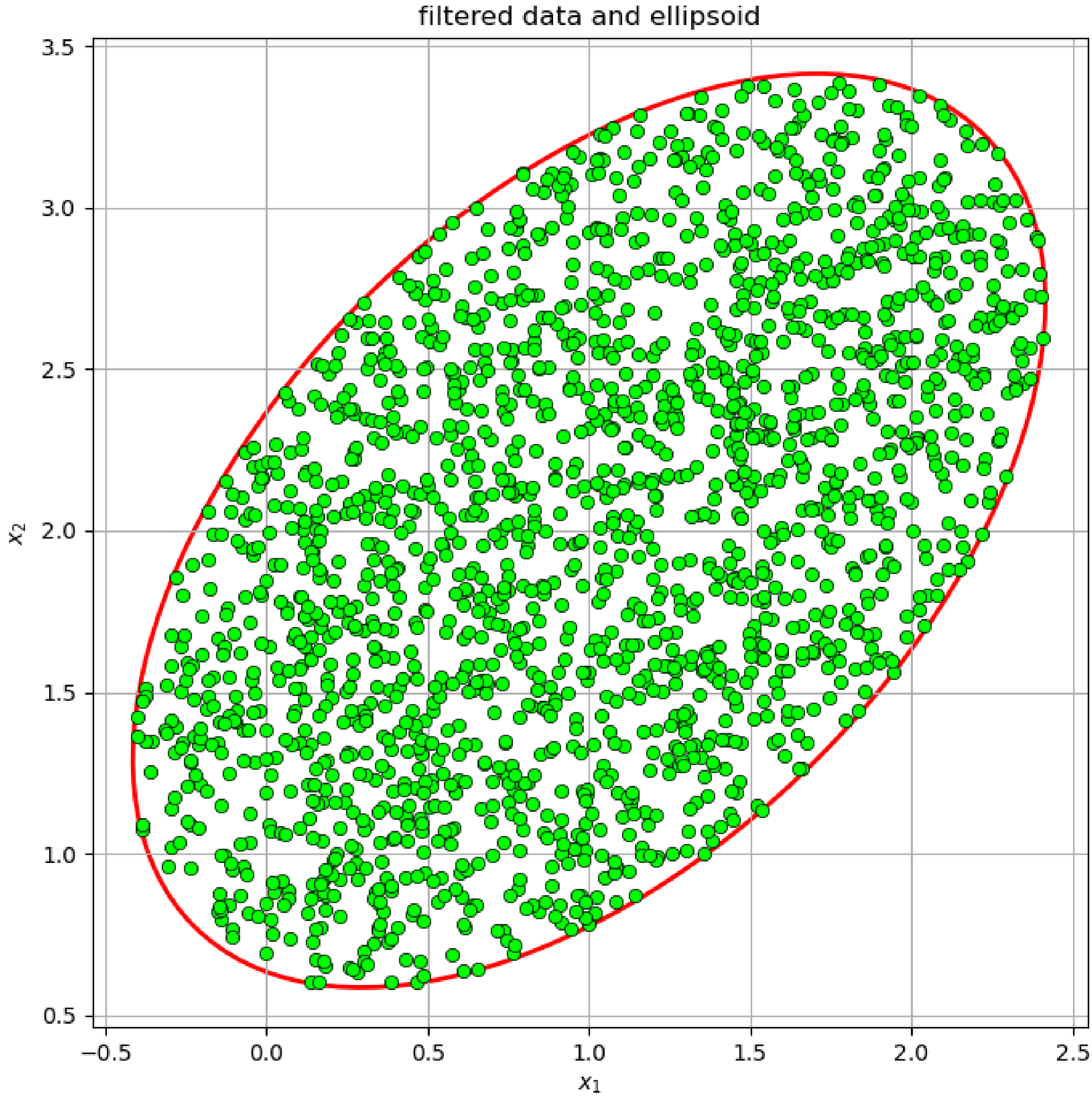
In [128…

```
Kx=np.array([[2.0,1.0],[1.0,2.0]])
mu=np.array([1.0,2.0])
random_seed=10

generate_and_plot(Kx,mu)
```




In [129…

```
ax=plt.gca()
plot_ellipsoid(mu,Kx,ax)
plt.show()
```


filtered data and ellipsoid
3.5
3.0
2.5
2.0
1.5
1.0
0.5
$x_2$
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
$x_1$