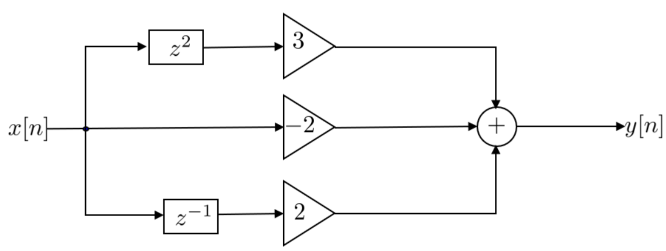
Problem Set 2

Due: February 10th, 6pm PST on Gradescope EE483 Spring 2025

Total: 110 pts

1. **Properties (24 pts, 1pt for each answer of causality, stability, time invariance)** For the following systems, determine whether they are i) causal or non causal, ii) stable or unstable, iii) time invariant or time variant. Justify your answer with a proof or a counter example.
   (a) $y[n] = x[n^2]$
   (b) $y[n] = x[n](u[n] - u[n-6])$
   (c) $y[n] = \log_{10}(1 + |x[n]|)$
   (d) $y[n] = (x[n])^2$
   (e) $y[n] = nx[-3n+2]$
   (f) $y[n] = -x[n] + 2x[n+1] + 3$
   (g) $y[n] = 2^n x[n] + (-1)^n x[2n-1]$
   (h) $y[n] = \left(\frac{1}{2}\right)^n x[n-2] + \left(\frac{1}{3}\right)^n x[n+3]$

2. **Impulse response and properties 1 (12 pts, 2pt each impulse response, 1 pt for stability, 1pt causality)** For the following LTI systems, find the inpulse response $h[n]$ of the system and determine whether they are i) causal or non causal, and ii) stable or unstable. Give a brief explanation for your answer.
   (a) $y[n] = 2x[n-2] - x[n-1] + 4x[n] + \frac{1}{2}x[n+1]$
   (b) $y[n] = \sum_{k=-M}^{M} a_k x[n-k]$ for $a_k \in \mathbb{R}$.
   (c)



3. **Impulse response and Properties 2 (20 pts, 2pts for each answer)** For the following impulse responses of the LTI systems, determine whether they are i) causal or non causal, and ii) stable or unstable. Give a brief explanation for your answer.
   (a) $h[n] = 2^n u[n]$
   (b) $h[n] = \left(\frac{1}{2}\right)^n u[n] + 3^n u[-n-1]$
   (c) $h[n] == \left\{\dots, 0, 0, 8, \underset{\uparrow}{4}, 2, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, 0, 0, \dots\right\}$
   (d) $h[n] = \left(\frac{2}{3}\right)^n u[-n+3]$
   (e) $h[n] = \delta[n] + 3\delta[n-1] + 4\delta[n-3] + 2\delta[n-3]$

4. **Composition of LTI systems. (10 pts, 5 pts each part)** Consider LTI systems with impulse responses $h_1[n]$ and $h_2[n]$. In this question you will have to prove or disprove BIBO stability. Justify your answers with a proof or a counter example.

(a) If $h_1[n]$ is BIBO stable and $h_2[n]$ is bounded, is $h[n] = h_1[n] \circledast h_2[n]$ also BIBO stable?

(b) If $h_1[n]$ and $h_2[n]$ are not BIBO stable, can $h[n] = h_1[n] + h_2[n]$ be BIBO stable?

5. **Discrete convolution. (10 pts, 2 pts each part)** For each of the following pair of $x_1[n]$ and $x_2[n]$, compute the discrete-time convolution of the signals $y[n] = x_1[n] \circledast x_2[n]$. Give your answer in the most simplified form.

   (a) $x_1[n] = \left\{\ldots, 0, 0, 2, \underset{\uparrow}{1}, 3, 0, 0, \ldots\right\}$, $x_2[n] = \left(\frac{1}{2}\right)^n u[n]$

   (b) $x_1[n] = u[n]$, $x_2[n] = u[n-5]$

   (c) $x_1[n] = \left(\frac{1}{2}\right)^n u[n-3]$, $x_2[n] = u[n]$

   (d) $x_1[n] = \left\{\ldots, 0, 0, 1, 1, 2, \underset{\uparrow}{2}, 3, 3, 0, 0, \ldots\right\}$, $x_2[n] = \left\{\ldots, 0, 0, \underset{\uparrow}{1}, 1, 2, 0, 0, \ldots\right\}$

   (e) $x_1[n] = \cos\left(\frac{\pi n}{2}\right)$, $x_2[n] = \left(\frac{1}{3}\right)^n u[n-3]$ (Hint: use Euler's formula for the cosine)

6. **Systems. (15 pts, 5 points each part)** Let $\mathcal{H}$ denote a system and let $y_1[n] = \mathcal{H}\{x_1[n]\}$. $y_2[n] = \mathcal{H}\{x_2[n]\}$ and $y_3[n] = \mathcal{H}\{x_3[n]\}$ be the output of the system for the input signals $x_1[n]$, $x_2[n]$, and $x_3[n]$ respectively. The signals $x_1[n]$, $y_1[n]$, $x_2[n]$, and $x_3[n]$ are shown in Fig.1.

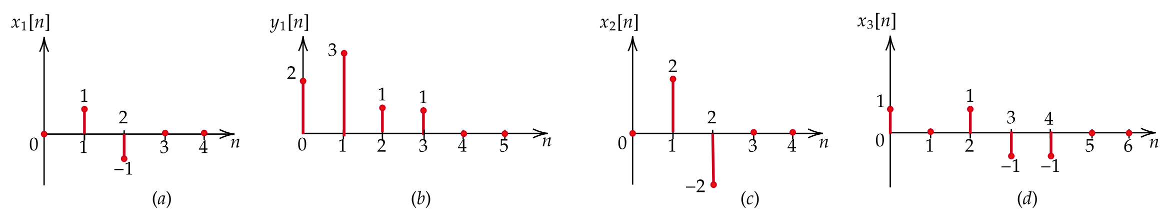


Figure 1: Plots of $x_1[n]$, $y_1[n]$, , $x_2[n]$, and $x_3[n]$.

   (a) Express $x_2[n]$ and $x_3[n]$ shown in Fig. 1 (c) and (d) respectively as a linear combination of the time-shifted versions of $x_1[n]$.

   (b) If it is given only that $\mathcal{H}$ is linear, can you find $y_2[n]$ and $y_3[n]$? If yes, find and plot the signals. If not, give a brief explanation.

   (c) If it is given that $\mathcal{H}$ is linear and time invariant, can you find $y_2[n]$ and $y_3[n]$? If yes, find and plot the signals. If not, give a brief explanation.

7. **LTI systems.(19 pts, 7,4,4,4 pts for each part)** Given that $\mathcal{H}$ is an LTI system. For a signal $x_1[n]$, the output of the system is $y_1[n] = \mathcal{H}\{x_1[n]\}$. The signals $x_1[n]$ and $y_1[n]$ are shown in Fig.2 (a) and (b) respectively.

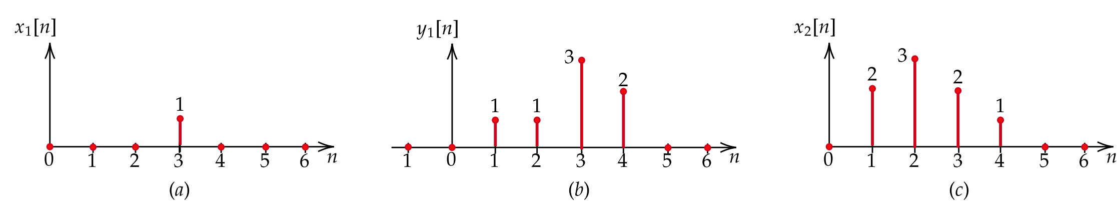


Figure 2: Plots of $x_1[n], y_1[n]$ and $x_2[n]$.

   (a) Find the impulse response of the system, $h[n]$, and plot $h[n]$ by hand.

(b) Is the system $\mathcal{H}$ causal? Give a brief explanation for your answer.

(c) Is the system $\mathcal{H}$ BIBO stable? Give a brief explanation for your answer.

(d) For the input signal $x_2[n]$ shown in Fig. 2 (c), compute $y_2[n] = \mathcal{H}\{x_2[n]\}$.