

## HOMework SET #12

EE 510: Linear Algebra for Engineering

Assigned: 23 November 2024

Due: 1 December 2024

### Directions:

1. Introduction to Linear Algebra by Gilbert Strang (5th Edition):

a) Problem Set 10.4: #1, #2.

2. Derive the optimal strategies for the two person game with the payoff matrix  $A$  where

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = [a_{ij}]$$

and the matrix represents the payoff from player 2 to player 1. The term  $a_{ij}$  is the payoff when player 1 makes the  $i^{th}$  move and player 2 makes the  $j^{th}$  move.

3. Matrix  $A$  is the payoff matrix for a two-player *zero-sum* game:

$$A = \begin{bmatrix} -5 & 2 & -7 & 8 & 5 \\ 3 & 6 & 2 & 2 & 10 \\ -1 & 0 & -9 & 0 & 3 \\ 10 & 3 & 8 & 9 & 4 \\ 3 & 2 & 4 & 5 & -3 \end{bmatrix}.$$

Entry  $a_{ij}$  is the *payoff* for *Player 1* when Player 1 makes move  $i$  and Player 2 makes move  $j$ . Find the value  $v(A)$  of the game for *stochastic vectors*  $\mathbf{x}$  and  $\mathbf{y}$ :

$$v(A) = \max_{\mathbf{x}} \left( \min_{\mathbf{y}} \mathbf{x}^T A \mathbf{y} \right).$$