

HOMEWORK SET #5

EE 510: Linear Algebra for Engineering

Assigned: 27 September 2024

Due: 5 October 2024

Directions: Please show all work and box answers when appropriate.

1. Introduction to Linear Algebra by Gilbert Strang (5th Edition):

a) Problem Set 8.1: #10, #12, #14

2. Let $F : V \rightarrow U$ and $G : U \rightarrow V$ be linear. Prove or disprove that if F and G are nonsingular, then $G \circ F$ is nonsingular.

3. Suppose $F : V \rightarrow U$ is linear. Show that the image of any subspace of V is a subspace of U .

4. Let $V = \mathbf{P}_{10}(t)$ be the vector space of polynomials of degree ≤ 10 . Consider the linear map $\mathbf{D}^4 : V \rightarrow V$, where \mathbf{D}^4 is the fourth-order derivative. Find a basis for the image of \mathbf{D}^4 and determine whether \mathbf{D}^4 is nonsingular or not.

5. Suppose that U, V , and W are vector spaces and $T : U \rightarrow V$ and $S : V \rightarrow W$ are linear transformations. Prove or disprove that $S \circ T$ is a linear transformation.

6. Let $F : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ and $G : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be defined by $F(x, y, z) = (y, x + z)$ and $G(x, y, z) = (2z, x - y)$:

a) Find a basis for $3F + 2G$.

b) Find the kernel of $3F + 2G$.