Problem Set 9

Due: May 2nd 11.59pm PST on Gradescope EE483 Spring 2025

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150 points

1. **Transfer function (30 pts, 5 each)** For the following transfer functions

   (a) $\frac{z-4}{(z-2)^2}$

   (b) $\frac{(z-1/4)^4}{(z+1/2)^5}$

   (c) $\frac{2-z^{-1}}{z-2}$

   (d) $\frac{(z-2)^2}{z+2}$

   (e) $\frac{(z-1/4)^5}{(z+1/2)^4}$

   (f) $\frac{z^2-z-2}{(z-4)^2(z-2)}$

   Sketch the zero/pole locations and identify their possible ROCs, and for each of them indicate whether the system is stable, unstable, causal or non causal. You do not have to compute the impulse response to answer this question.

2. **Filter type (30 pts, 5 each)** For the systems of the previous problem, indicate when does the DTFT converges uniformly. If the DTFT exists, use the pole/zero locations to indicate whether the system is low-pass, high-pass, or something else. Plot the magnitude response using MATLAB to verify your results.

3. **GLP filters (35 pts)**

   A) (5 pts) Prove that $(1 - az^{-1})(1 - a^*z^{-1}) = 1 - 2\text{Re}(a)z^{-1} + |a|^2z^{-2}$.

   B) You are provided with the following information about the filter $h[n]$.

   - $h[n]$ is a causal FIR filter and has a generalized linear phase (GLP).
   - All the zeros of $H(z)$ are located on the unit circle and have multiplicity 1 (simple).
   - For $\omega \in [0, \pi]$, $H(e^{j\omega}) = 0$ only at the frequencies $\omega = \pi/3$, $\omega = 2\pi/3$, and $\omega = \pi$.
   - $h[0] = 1$.

   (a) (10 pts) Find all the zeros of $H(z)$ and indicate the order of the filter $h[n]$

   (c) (5pts) Roughly sketch by hand the magnitude response of the filter $h[n]$ and state whether the filter is low-pass, high-pass, band-pass, or any other?

   (d) (10 pts) Find an expression for $H(z)$ and hence determine $h[n]$.

   (e) (5 pts) (i) Is $h[n]$ real? (ii) Does $h[n]$ satisfy the symmetric/antisymmetric property for GLP learned in the class?

4. **Transfer function (30 pts)**. Consider an LTI system with impulse response $h[n]$ whose DTFT $H(e^{j\omega})$ converges uniformly. The transfer function is

$$H(z) = \frac{2z - 3}{z - \frac{1}{3}} \tag{1}$$

   (a) (5 pts) Is this system (i) causal, (ii) BIBO stable?

   (b) (10 pts)Let $x[n] = \left(\frac{3}{2}\right)^n u[n]$. Find $X(z)$ and then find the output of the LTI system when $x[n]$ is given as input using $z-$transform approach.

   (c) (10 pts) Derive a new transfer function $G(z)$ that has the same magnitude response as $H(z)$, but with minimum group delay. Derive the impulse response $g[n]$ of such system.

(d) (5 pts)Plot the magnitude and phase response of $H(z)$ and $G(z)$ using MATLAB.

5. **Causality and minimum phase (25 pts)**. The pole-zero plot and the ROC of a transfer function $H(z)$ are shown in Figure 1.

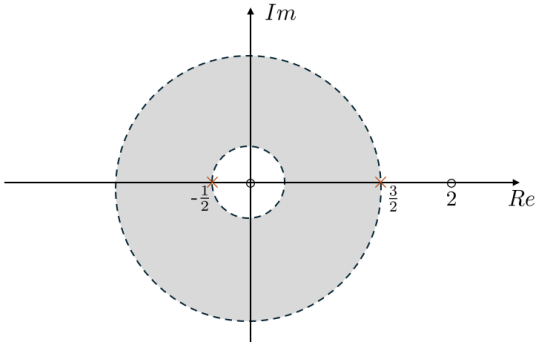

Figure 1: Pole-zero plot and ROC of $H(z)$.

(a) (10 pts)If $\lim_{z\to\infty} H(z) = 1$, (i) find $H(z)$, (ii) find $h[n]$, and (iii) plot the magnitude and phase response of $H(z)$ using MATLAB.

(b) (15 pts)Does this system have minimum group delay? If not, (i) find a new transfer function $G(z)$ that has the same magnitude response as $H(z)$ with minimum group delay, (ii) plot the pole-zero plot of such system and show the ROC, (iii) find $g[n]$, and (iv) plot the magnitude and phase response of $G(z)$ using MATLAB.