Problem Set 3

Due: February 18th, 6pm PST on Gradescope EE483 Spring 2025

---

110 pts. You can use without proof the properties and DTFT pairs viewed during Lecture, and the tables from Mitra uploaded to the course website.

1. **LDE, LTI, DTFT. (32pts)** Consider the following LDE corresponding to an LTI system with the initial condition and $y[n] = 0$ for all $n < 0$.

$$y[n] - \left(\frac{1}{3}\right) y[n-1] = x[n]$$

   (a) (10pts) Compute the impulse response $h[n]$ by solving the LDE.

   (b) (2pts) Is the system causal? Briefly explain.

   (c) (2pts) Is the system BIBO stable? Briefly explain.

   (d) (2pts) Find $H(e^{j\omega})$, the DTFT of $h[n]$.

   (e) Find the output of the system for the following input signals, using convolution ($y[n] = h[n] \circledast x[n]$). Simplify your answer.
   (i) (5pts) $x[n] = 2^n u[n]$
   (ii) (5pts) $x[n] = \delta[n] + 2\delta[n-1] - \delta[n+1]$.

   (f) (6pts, 3pts each answer) Can you use the DTFT and its properties to compute the answers for (e). If yes, show the computations. If not, give a brief explanation.

2. **DTFT practice.(28pts, 4pts each)** Compute the DTFT and the inverse DTFT for the following sequences. It should be obvious which one to compute for each case.

   (a) $x[n] = (\frac{2}{3})^n u[n+1]$

   (b) $x[n] = (\frac{1}{2})^{2|n|}$

   (c) $x[n] = \cos\left(\frac{\pi n}{4}\right) + \sin(n)$

   (d) $x[n] = (\frac{1}{3})^n \cos\left(\frac{n}{3}\right) u[n]$

   (e) $X(e^{j\omega}) = \sin(4\omega)$ for $-\pi \leq \omega \leq \pi$

   (f) $X(e^{j\omega}) = \sin\left(\frac{\omega}{4}\right)$ for $-\pi \leq \omega \leq \pi$

   (g) $X(e^{j\omega}) = \sin^2(\omega) + \cos^2(3\omega)$ for $-\pi \leq \omega \leq \pi$

3. **DTFT Properties (14 pts)**

   (a) (2 pts) Compute $X(e^{j\omega})$, the DTFT of the signal $x[n] = \frac{\sin\left(\frac{\pi n}{8}\right)}{\pi n}$, and sketch the magnitude of the the DTFT, $|X(e^{j\omega})|$, for $\omega \in [-\pi, \pi]$ by hand.

   (b) For each of the following signals, use the properties of DTFT to find their corresponding DTFT and sketch their magnitude for $\omega \in [-\pi, \pi]$ by hand.

      i. (3pts)$y_1[n] = x[n] \cos\left(\frac{\pi n}{4}\right)$

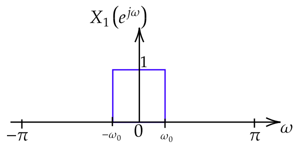
      ii. (3pts) $y_2[n] = x[n] \circledast \cos\left(\frac{\pi n}{12}\right)$

      iii. (3pts) $y_3[n] = x[n] \circledast \sin\left(\frac{\pi n}{10}\right) \circledast \cos\left(\frac{\pi n}{4}\right)$

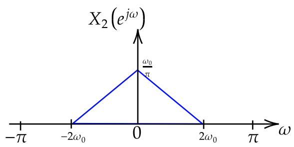
      iv. (3pts) $y_4[n] = x[n+1] + x[n-1]$

4. **Inverse DTFT and Properties. 36 pts, a)8pts, b)8pts, c)12pts, d)8pts** For each of the sketches of $X(e^{jw})$ below, find the corresponding signals $x[n]$ in the simplified form. (Hint: Express each of these as sums of sacled, shifted rectangles ($X_1(e^{j\omega})$) and triangles ($X_2(e^{j\omega})$) and use their inverse DTFTs)

<u>DTFT Pairs:</u>



$$X_1(e^{j\omega}) = \begin{cases} 1 & |\omega| \le \omega_0 \\ 0 & \omega_0 < |\omega| \le \pi \end{cases} \rightleftharpoons x_1[n] = \begin{cases} \dfrac{\sin(\omega_0 n)}{\pi n} & n \neq 0 \\ \dfrac{\omega_0}{\pi} & n = 0 \end{cases}$$



$$X_2(e^{j\omega}) = \begin{cases} \dfrac{1}{2\pi}(2\omega_0 - |\omega|) & |\omega| \le 2\omega_0 \\ 0 & 2\omega_0 < |\omega| \le \pi \end{cases} \rightleftharpoons x_2[n] = \begin{cases} \left(\dfrac{\sin(\omega_0 n)}{\pi n}\right)^2 & n \neq 0 \\ \left(\dfrac{\omega_0}{\pi}\right)^2 & n = 0 \end{cases}$$