Optimal Indirect Regulation of Externalities*

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Abstract

In many markets ranging from gasoline to alcohol and vaccines, individuals generate different amounts of externalities that cannot be directly taxed. I study how such externalities should be optimally regulated. I characterize the optimal policy and show that it generally requires quantity surcharges and discounts. I evaluate the gain from using the optimal indirect policy rather than a uniform tax and show that it can be significant. I apply my results to gasoline taxes to demonstrate their policy implications. Finally, I incorporate distributional concerns and show how "non-market" solutions such as quantity floors and ceilings might be required.

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1 Introduction

How should a government regulate activities that cause externalities? Textbook analyses typically focus on the Pigouvian approach, under which each individual faces a tax (or subsidy) equal to the marginal external damage (or benefit) that he causes. Consequently, individuals internalize the true social impact of their actions, and market efficiency is restored. Yet, individuals typically produce different amounts of externality in practice, which often renders infeasible the Pigouvian premise of directly pricing the externality produced by each individual. For instance, prohibitive technological and political costs prevent policymakers from monitoring vehicle emissions produced by each driver. Instead, policymakers regulate emissions indirectly via gasoline and vehicle taxes (e.g., annual registration fees). Unlike a Pigouvian tax, such indirect policies are imperfect when individuals who consume the same amount of the good produce different amounts of externality—as is often the case in reality. For example, the level of emissions per gallon of gasoline consumed differs not only across different vehicle types, but also within vehicles of the same make, model, and model year (Kahn, 1996; Knittel and Sandler, 2018).

Other examples abound. The marginal social benefit of vaccinating an individual depends on his risk of contracting and spreading the virus, which differs across individuals and is difficult to measure. Similarly, the marginal social costs of alcohol, tobacco, and over-the-counter medications containing ephedrine or pseudoephedrine—precursor chemicals used in the illicit manufacture of methamphetamine and amphetamine—depend on the individual likelihood of abuse, which might be both costly to monitor and illegal to discriminate upon. Moreover, even when the externality can be directly observed, the policymaker might be concerned about the distributional impact of externality-correcting policies, which can be challenging to estimate in practice.

In this paper, I study the optimal indirect policy to regulate individuals who produce different amounts of externality that the policymaker cannot observe. Using mechanism design methods, I characterize the optimal indirect policy and evaluate the welfare gain that it yields over a uniform tax. I apply these results to an empirical setting to show how they compare to and complement existing work on indirect taxation, as well as how they could inform policy.

Whereas existing work on indirect taxation tends to focus on uniform taxes, I show that the optimal indirect policy generally involves a nonlinear tax: it requires quantity surcharges and/or discounts. To understand why, observe that even though the policymaker is unable to directly tax the externality, she can estimate the amount of externality produced by each individual from his consumption behavior. For example, when the elasticity of driving with respect to gasoline prices is correlated with the amount of vehicle emissions produced by each driver, the policymaker can improve on a linear tax by exploiting this correlation. The sign of the correlation determines whether a quantity surcharge or discount is optimal. For instance, if more price-responsive drivers tend to have dirtier vehicles, then the optimal indirect policy sets a high initial tax rate on gasoline that decreases with the amount of gasoline consumed (or miles driven). A uniform tax is optimal only in the special case where the residual correlation between elasticity and externality, conditional on the amount of gasoline consumed, is exactly zero.

Having characterized the optimal indirect policy, I show that the gain from using the optimal indirect policy rather than a uniform tax can be significant. Under a uniform tax, deadweight loss arises because of heterogeneity in the amount of externality produced by each individual. This heterogeneity can be decomposed into two components: that which can be statistically explained by differences in individual consumption behavior, and that which cannot. No indirect policy can eliminate the deadweight loss from the latter component: it arises because the policymaker cannot directly measure the externality. By contrast, the deadweight loss from the former component can be eliminated by using the optimal indirect policy. The welfare gain is thus largest when all of the heterogeneity in externality can be statistically explained by differences in consumption behavior (e.g., if individuals with the same elasticity produce the same amount of externality).

To show how my theoretical results can be used to inform policy, I apply my results to the empirical setting of Knittel and Sandler (2018). Knittel and Sandler examine the use of uniform gasoline taxes to indirectly regulate three pollutants—carbon monoxide (CO), hydrocarbons (HCs), and nitrogen oxides (NO_x)—produced by vehicles in the state of California between 1998 and 2008. Given Knittel and Sandler's estimates of the demand for gasoline and the distribution of pollutant emissions produced per gallon, my results imply that the optimal indirect policy sets a decreasing marginal tax rate on gasoline. On average, the optimal indirect policy performs 12.4% better than the optimal uniform tax over the sample period.

This application outlines two ways in which my results could provide policy guidance, namely in deciding *how* to regulate and *what* to regulate. A natural approach to the design of indirect regulation is to rule out nonlinear pricing policies a priori due to the complexity of implementation. But this approach is not entirely satisfactory: the cost of implementation should ultimately be weighed against the benefits of optimal design. My results enable policymakers to evaluate these benefits and—if they so decide to implement it—design the optimal indirect policy. In addition, my results can inform policy choices about what proxy goods should be used to indirectly regulate the externality. For example, is it better to regulate pollution by taxing vehicle type or gasoline? My results show that the proxy good for the optimal indirect policy should be chosen so as to maximize the informativeness of consumption behavior about the amount of externality produced.

By contrast, the choice of proxy good for the optimal uniform tax might have no effect on welfare in equilibrium.

The relevance of my results is supported by recent technological advancements and policy shifts. For example, the Bipartisan Infrastructure Bill, signed into law by President Biden in November 2021, includes a $125 million pilot study for a vehicle mileage tax. By exploiting advancements in telematics technology, cumulative miles driven can be monitored and used as a proxy for how much externality a driver produces. More generally, the development of new technologies that can elicit more granular data from individuals will increase the range of proxy goods that can be used to regulate externalities indirectly. While these technologies might reduce implementation costs, they might also increase the benefits of using the optimal indirect policy relative to a uniform tax.

My results also show that "non-market" regulations, such as quantity ceilings and floors, can arise in the optimal indirect policy. This might help to explain why such regulations are observed in real-world markets. One such example is the Combat Methamphetamine Epidemic Act of 2005, a federal legislation that restricts the amount of pseudoephedrine any individual in the United States can purchase daily and within a 30-day period. Vaccine mandates provide another example. In an extension, I show how non-market regulations can be a part of the optimal policy when the correlation between externality and consumption behavior is sufficiently strong. Indeed, if price responsiveness decreases too steeply with the amount of externality generated, then a quantity surcharge might not be sufficient for regulation; instead, a quantity ceiling might be necessary.

My analysis generalizes to a setting where the policymaker takes distributional considerations into account. Motivated by policy discussions about the distributional impact of corrective taxes, I incorporate equity concerns into the policymaker's objective. This is modeled by having the policymaker assign different social welfare weights to different individuals; the policymaker then maximizes the sum of individuals' utilities weighted by their welfare weights. For example, empirical evidence suggests that lower-income households spend a larger proportion of their income on gasoline (Parry, Walls, and Harrington, 2007). The policymaker might thus assign a higher welfare weight to lower-income households in response to concerns about the regressivity of a gasoline tax. As with externalities, welfare weights are not directly observed: for instance, gasoline taxes might be designed at the county level by local government agencies, who do not have access to income data. Instead, the policymaker must infer each individual's externality and welfare weight from his consumption behavior.

This generalization shows that, even if the policymaker could directly tax the externality, a uniform tax is not optimal when the policymaker accounts for distributional considerations. Just as the correlation between consumption behavior and externality leads to the optimality of nonlinear taxes, the correlation between consumption behavior and welfare weight necessitates the same instruments in the optimal policy. In other words, even when the good is a perfect proxy for the externality, it is an imperfect proxy for each individual's welfare weight, thereby introducing a wedge between what a policymaker can implement and her most preferred outcome.

1.1 Related literature

A key feature of this paper that distinguishes it from much of the existing literature on externalities is its focus on indirect regulation. Since Pigou's (1920) seminal work, there has been a vast literature on externalities, where it is often assumed—as in Pigou's original analysis—that the policymaker is able to directly measure and tax the externality. In this literature, nonlinear taxation has been discussed in the contexts of uncertainty in abatement costs (e.g., Weitzman, 1974; Roberts and Spence, 1976; Kaplow and Shavell, 2002) and income taxation in the presence of externalities (e.g., Sandmo, 1975; Ng, 1980; Bovenberg and van der Ploeg, 1994; Cremer, Gahvari, and Ladoux, 1998; Kopczuk, 2003; Kaplow, 2012). In these contexts, the optimality of nonlinear taxation arises from asymmetric information about either abatement costs or individual wage rates and tastes. By contrast, my paper assumes that the policymaker is unable to directly measure and tax the externality, leading to a problem of indirect regulation. The optimality of nonlinear taxation arises precisely from her inability to do so.

Given its focus, this paper shares more in common with existing work on indirect regulation. Historically, this line of work stems from Diamond (1973), who first pointed out that the Pigouvian approach fails when individuals produce different amounts of externality per unit of the good consumed. Green and Sheshinski (1976) analyze a similar model with two individuals. I contribute to this line of work by using mechanism design methods to derive the optimal indirect policy and evaluate its welfare gain over a uniform tax.

This paper also complements recent empirical papers that examine the indirect regulation of externalities in various markets. For example, Anderson and Auffhammer (2014) study the control of vehicle accident externalities via a gasoline tax in various U.S. states; Knittel and Sandler (2018) study the regulation of emissions via a gasoline tax in California, on which my empirical exercise is based; and Griffith, O'Connell, and Smith (2019) study the control of problematic alcohol consumption in the U.K. by exploiting correlation between risk of problematic behavior and alcohol consumption patterns. Relatedly, Jacobsen, Knittel, Sallee, and van Benthem (2020) develop a sufficient statistics approach to analyzing imperfect pricing policies; they show that regression statistics can be used to quantify the performance of uniform taxes relative to the first-best Pigouvian benchmark. My results complement these papers by quantifying the performance of the optimal indirect policy relative to uniform taxes, thereby shedding new light on the tradeoff between using simple (but suboptimal) policies and optimal (but complex) ones.

To study the optimal indirect regulation of externalities, I draw from methods developed in the mechanism design literature. Specifically, I adopt a "large market" approach to mechanism design, pioneered by Bulow and Roberts (1989) and based on the seminal work of Mussa and Rosen (1978) and Myerson (1981). This approach has been recently used in a growing number of papers, including Dworczak ⓡ Kominers ⓡ Akbarpour (2021) and Loertscher and Muir (2022).

However, relatively few other papers have applied mechanism design to study settings with externalities. Examples include Jehiel, Moldovanu, and Stacchetti (1996), Jehiel, Moldovanu, and Stacchetti (1999), and Jehiel and Moldovanu (2001), who analyze externalities in auction environments with a finite number of bidders, each with discrete demand for the good(s) sold. I differ from these papers in both motivation and methodology: I focus on the indirect regulation of externalities in a large market, where individuals have continuous demand and produce unobserved, heterogeneous externalities. I also apply my results to an empirical setting in order to derive policy implications.

Subsequent to the first draft of this paper, a number of new papers have applied mechanism design methods to study settings with externalities. Sarkisian and Yamashita (2022) study optimal mechanisms in large market settings with externalities. They examine a discrete demand model with linear utility where each individual's marginal externality is proportional to his consumption type, and show that the optimal mechanism has a simple form. By contrast, I study a continuous demand model with a general utility function, and allow each individual's marginal externality to be imperfectly correlated with his consumption type. Akbarpour ⓡ Budish ⓡ Dworczak ⓡ Kominers (2021) and Pai and Strack (2022) study the impact of distributional concerns on the optimal regulation of externalities. Motivated by scarce supply during the COVID-19 pandemic, Akbarpour ⓡ al. analyze the application of vaccine allocation and develop a discrete demand framework for trading off efficiency and equity. By contrast, I develop a continuous demand framework that focuses on the problem indirect regulation. Pai and Strack allow the policymaker to have a richer family of objective functions and model the externality as a function of total consumption, over which individuals might have different costs. By contrast, I allow individuals to produce heterogeneous externalities so as to examine how the policymaker who is unable to directly tax these externalities can nonetheless regulate them via an imperfect proxy. In Section 5.3, I provide a more detailed discussion of the connection between the problems of indirect regulation (as in my paper) and direct regulation with distributional considerations (as in Pai and Strack's).

Finally, while I develop a mechanism design model with a general utility function, I illustrate many of my results with a quadratic utility model. While the quadratic utility model has been used widely in economics (Choné and Linnemer, 2020), I show that it is particularly simple and tractable for studying problems in mechanism design. On one hand, the canonical model of linear utility and unit demand can be viewed as a special case of a quadratic utility model, where individuals have flat demand curves with different vertical intercepts, up to some maximum quantity of the good demanded. On the other hand, while tractability from the linear utility model derives from mathematical results in linear programming, I show how tractability from the quadratic utility model derives from mathematical results in quadratic programming (Reid, 1968; Barron, 1983).

1.2 Organization of this paper

The remainder of this paper is organized as follows. Section 2 develops a model of indirect regulation. Section 3 presents the main results, while Section 4 shows how the main results can be applied to an empirical setting. Section 5 then demonstrates how quantity ceilings and floors might be optimal and how the main results extend to settings with distributional concerns; and Section 6 concludes.

2 Model

In this section, I develop a general model of indirect regulation and formalize the mechanism design problem that the policymaker faces.

2.1 Setup

There is a unit mass of risk-neutral individuals in a market for a homogeneous good. The good is supplied by a competitive industry with a constant marginal cost c. Individuals in the market have quasilinear utility over money. Each individual is privately informed of his consumption type θ for the good. Each individual also produces an externality ξ per unit of the good consumed.[1] Denote the joint distribution of (θ, ξ) by G, which is assumed to be absolutely continuous and supported on $[\underline{\theta}, \overline{\theta}] \times [\underline{\xi}, \overline{\xi}] \subset \mathbb{R}^2$.

[1] For expositional simplicity, I assume that the marginal externality per unit of good consumed is constant for each individual. My results can be extended to a more general setting where the externality produced by each individual is a function $\chi(q, \xi)$ of the quantity q of good that he consumes and his externality type ξ.

The utility that each individual derives from consuming the good is affected by the total externality produced by all individuals in the market. Formally, let $q(\theta, \xi)$ denote the quantity of the good that each individual consumes given his θ and ξ; then the total externality produced by all individuals is

$$e = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\xi}}^{\overline{\xi}} \xi q(\theta, \xi) \ \mathrm{d}G(\theta, \xi).$$

Given the total externality e produced by all individuals in the market, an individual with consumption type θ derives utility $u(q, e, \theta)$, measured in money terms. Throughout this paper, I impose the following assumption on u:

Assumption 1. *Let $A \in \mathbb{R}_+ \cup \{+\infty\}$ be the maximum quantity of good an individual can consume. The utility function $u : [0, A] \times \mathbb{R} \times [\underline{\theta}, \overline{\theta}] \to \mathbb{R}$ is bounded, twice continuously differentiable, and satisfies:* (i) $\partial u/\partial e < 0$*;* (ii) $\partial u/\partial q > 0$*;* (iii) $\partial^2 u/\partial q^2 < 0$*; and* (iv) $\partial^2 u/\partial q\, \partial \theta > 0$. *Moreover, if $A = +\infty$, then*

$$\frac{\partial u}{\partial q}(q, e, \theta) \to 0 \quad \textit{uniformly in } e \in \mathbb{R} \textit{ and } \theta \in [\underline{\theta}, \overline{\theta}].$$

The conditions in Assumption 1 are weak and relatively standard. In particular, condition (i) interprets e as a negative externality when $\xi > 0$ for each individual, such as when ξ represents the amount of pollution produced; positive externalities are accommodated by allowing $\xi < 0$. The other conditions (ii)–(iv) are familiar: Individuals derive positive but diminishing marginal utility from consuming the good. Furthermore, the utility function satisfies the strict single-crossing property, so that the marginal utility from consuming the good increases with consumption type. Finally, when $A = +\infty$, then the marginal utility for each consumption type is required to converge to zero uniformly in e and θ as $q \to +\infty$. Notice that this requirement of uniform convergence in e is vacuous when the externality is additively separable (in which case the marginal utility function is independent of e); and uniform convergence in θ is equivalent to pointwise convergence in θ when $\overline{\theta} < +\infty$.

To obtain explicit solutions, my analysis also considers the special case of a quadratic utility function with an additively separable externality:

$$u(q, e, \theta) = \theta A q - \frac{\theta}{2} q^2 - e \qquad \text{for } q \in [0, A]. \tag{Q}$$

This generates a linear inverse demand curve for the good, $P(q, \theta) = \theta\,(A - q)$, which has been used in empirical work (see Figure 1). As such, the results that I derive for this special case can also be calibrated to such empirical settings, as I show in Section 4.

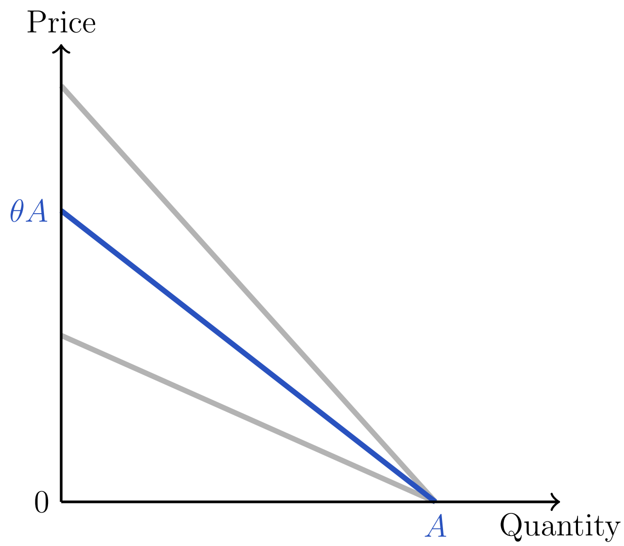


Figure 1: Linear inverse demand curves of individuals given by $P(q,\theta) = \theta\,(A-q)$.

2.2 Mechanism design

I now formalize the mechanism design problem faced by a policymaker who wishes to maximize total surplus. If the policymaker could measure and directly tax the externality produced by each individual, then she would simply design the Pigouvian tax—which is proportional to the amount ξ of externality that each individual produces per quantity of good consumed, and independent of his consumption type θ. However, this assumption is strong and difficult to satisfy in practice, as argued in Section 1.

When the policymaker cannot measure and directly tax the externality produced by each individual, a mechanism design problem arises: she has to regulate the externality by screening individuals based on their consumption of the good, which is an imperfect proxy. The policymaker chooses a direct mechanism (q,t) to maximize total social surplus, which consists of an allocation function $q : [\underline{\theta}, \overline{\theta}] \times [\underline{\xi}, \overline{\xi}] \to [0, A]$ and a payment function $t : [\underline{\theta}, \overline{\theta}] \times [\underline{\xi}, \overline{\xi}] \to \mathbb{R}$.

In this formulation, the mechanism that the policymaker chooses is allowed to depend on both θ and ξ. This implicitly assumes that individuals are privately informed of both θ and ξ. This is the case when individuals have private information over how much externality they produce. For example, drivers might have preferences over driving routes and times. These affect the amount of congestion that they contribute to—and, by extension, the amount of pollution that they emit.

An alternative—and, a fortiori, equivalent—formulation supposes that the policymaker chooses a mechanism that can depend on θ but not ξ. This is the case when neither the policymaker nor the individual observes how much externality he produces. For example, heterogeneity in rates of normal wear and tear might not be observed by either the policymaker or the individual, but will result in heterogeneity over the amount of pollution that vehicles produce. As I show below in Claim 1, my analysis applies regardless of whether the individual observes his own ξ.

By the revelation principle, the policymaker can restrict attention to incentive-compatible policies without any loss of generality. That is, for any (θ, ξ), the policy (q, t) satisfies:

$$(\theta, \xi) \in \underset{(\hat{\theta}, \hat{\xi})}{\arg\max} \left[u\left(q(\hat{\theta}, \hat{\xi}), \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\xi}}^{\overline{\xi}} \xi' q(\theta', \xi') \, \mathrm{d}G(\theta', \xi'), \theta \right) - t(\hat{\theta}, \hat{\xi}) \right]. \tag{IC}$$

Because of the large market assumption, individual reports $(\hat{\theta}, \hat{\xi})$ do not affect the total amount of externality in the market. Consequently, as in the standard Pigouvian analysis, each individual fails to internalize how much his own consumption affects others.

The (IC) constraint also reveals an important difference between the present model and most other mechanism design models. In particular, the policymaker's choice of allocation function q affects each individual's utility through not only his allocation, but also the total externality in the market. This complicates the analysis of the present model: the policymaker has to account for how her choice of the *entire* allocation function might affect a given individual, rather than just the *part* of the allocation function that she intends the individual to consume at. Nevertheless, I show in Section 3 how the optimal mechanism can be characterized in spite of this complication.

In principle, the policymaker faces a multidimensional mechanism design problem as individuals have two dimensions of private information. But the following observation simplifies the analysis:

Claim 1. *It is without loss of generality for the policymaker to consider incentive-compatible policies that elicit information only about each individual's θ.*

The intuition behind Claim 1 is straightforward.[2] Each individual's consumption decision is entirely determined by his θ, rather than his externality ξ. Formally, this can be seen from (IC): even though individuals can report ξ, their reports do not directly affect their payoffs. As such, no policy can truthfully elicit information about ξ; instead, the policymaker must form beliefs about ξ based on each individual's revealed preferences. As such, I abuse notation henceforth and write $q(\cdot)$ and $t(\cdot)$ as functions of only θ.

Let F denote the marginal distribution of θ induced by the joint distribution G. Using Claim 1, the mechanism design problem can be simplified by making the following standard observation:

[2] A similar intuition underlies analogous characterization results in the mechanism design literature, such as those by Jehiel and Moldovanu (2001), Che, Dessein, and Kartik (2013), and Dworczak ⓡ al. (2021).

Claim 2. *A direct mechanism (q,t) is incentive-compatible if and only if the allocation function $q : [\underline{\theta}, \overline{\theta}] \to \mathbb{R}_+$ is non-decreasing.*

Claim 2 is an adaptation of Myerson's lemma to the present setting. As individual utility for the good satisfies the strict single-crossing property at every level of total externality (Assumption 1), Myerson's lemma implies that any implementable allocation function must be non-decreasing in θ (cf. Theorem 1 of Rochet, 1987). Denote the set of all implementable allocation functions by

$$\mathcal{Q} := \left\{ q : [\underline{\theta}, \overline{\theta}] \to [0, A] \text{ is non-decreasing} \right\}.$$

I conclude this section by formally stating the policymaker's mechanism design problem. The observations above imply that the policymaker chooses an allocation function $q(\cdot)$ from $\mathcal{Q}$ to solve

$$\max_{q \in \mathcal{Q}} \int_{\underline{\theta}}^{\overline{\theta}} \left[u\left(q(\theta), \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\xi \mid \theta = s] q(s) \, \mathrm{d}F(s), \theta \right) - c q(\theta) \right] \, \mathrm{d}F(\theta). \tag{P}$$

As transfers between individuals and the policymaker are welfare-neutral, this problem (P) is independent of the payment function $t(\cdot)$.

3 Optimal indirect regulation

In this section, I present the main results of the paper. First, I characterize the optimal indirect policy by determining when discounts and surcharges are optimal (Theorem 1). Second, I quantify the welfare gain from using the optimal indirect policy rather than the optimal uniform tax, and determine when it is substantial (Theorems 2 and 3).

For the sake of simplicity, I rule out the possibility of bunching throughout this section, so that the optimal allocation function is increasing. Instead, I defer the analysis of bunching to Section 5.1. Appendix A formally states the required assumptions to rule out bunching. For the quadratic model (Q), these reduce to the assumption that

$$\theta \mapsto A - \frac{c + \mathbf{E}[\xi \mid \theta]}{\theta} \quad \text{is increasing and in } [0, A].$$

Observe that the conditional expectation $\mathbf{E}[\xi \mid \theta]$ captures the amount of correlation between θ and ξ. For example, when $\mathbf{E}[\xi \mid \theta]$ is increasing, then individuals with higher consumption types produce more negative externality per unit of consumption in expectation, which captures the case of positive correlation between ξ and θ. Thus the above assumption simply requires that θ and ξ are not too positively correlated in order to rule out bunching.

3.1 Characterization of the optimal indirect policy

I begin by characterizing when surcharges and discounts are required.

Theorem 1. *Suppose that no bunching is required under the optimal indirect policy. Then an individual with consumption type $\theta = \hat{\theta}$ faces:*

(i) an increasing marginal price (i.e., a quantity surcharge) if the conditional expectation $\mathbf{E}[\xi \,|\, \theta]$ is increasing at $\theta = \hat{\theta}$; and

(ii) a decreasing marginal price (i.e., a quantity discount) if the conditional expectation $\mathbf{E}[\xi \,|\, \theta]$ is decreasing at $\theta = \hat{\theta}$.

Theorem 1 demonstrates one way in which a uniform tax is a very special indirect policy: a uniform tax is optimal only when $\mathbf{E}[\xi \,|\, \theta]$ is constant for all θ. The intuition underlying this result is that the optimal indirect policy improves on a uniform tax by exploiting the correlation between each individual's consumption behavior and amount of externality produced. As Theorem 1 shows, whether a surcharge or a discount is optimal depends on the sign of this correlation.

The logic of Theorem 1 is simplest to understand in the case of the quadratic model (Q), where the optimal allocation function q^* can be found explicitly:

$$q^*(\theta) = A - \frac{c + \mathbf{E}[\xi \,|\, \theta]}{\theta}.$$

To induce the individual to consume this amount of the good, the optimal indirect policy sets a marginal price equal to his marginal utility:

$$\frac{\partial u}{\partial q}(q^*(\theta), e, \theta) = \theta \,[A - q^*(\theta)] = c + \mathbf{E}[\xi \,|\, \theta].$$

If $\mathbf{E}[\xi \,|\, \theta]$ is increasing at $\theta = \hat{\theta}$, then the marginal price is increasing (since the optimal allocation function q^* is increasing); thus a quantity surcharge is optimal. Conversely, if $\mathbf{E}[\xi \,|\, \theta]$ is decreasing at $\theta = \hat{\theta}$, then the marginal price is decreasing; thus a quantity discount is optimal.

While this logic appears to rely heavily on the fact that the externality in the quadratic model (Q) is additively separable, I now extend it to general utility functions. To do so, I reformulate the policymaker's problem (P) view as a two-stage problem: she first chooses the total amount of externality in the market, and then chooses an allocation function that produces that amount of externality in aggregate. Mathematically, this can be expressed by writing (P) as the equivalent nested optimization problem

$$\max_{e\in\mathbb{R}}\left[\max_{q\in\mathcal{Q}}\left\{\int_{\underline{\theta}}^{\overline{\theta}}[u(q(\theta),e,\theta)-cq(\theta)]\ \mathrm{d}F(\theta):\int_{\underline{\theta}}^{\overline{\theta}}\mathbf{E}[\xi\,|\,\theta]q(\theta)\ \mathrm{d}F(\theta)=e\right\}\right].$$

Notice that the inner problem is convex in q under Assumption 1. Consequently, it can be solved by assigning a Lagrange multiplier $\mu\in\mathbb{R}$ to the constraint. The resulting Lagrangian is

$$\mu e+\int_{\underline{\theta}}^{\overline{\theta}}[u(q(\theta),e,\theta)-(c+\mu\,\mathbf{E}[\xi\,|\,\theta])\,q(\theta)]\ \mathrm{d}F(\theta).$$

When the optimal indirect policy does not require bunching, the policymaker's problem can be solved by pointwise maximization of this Lagrangian, which yields the first-order condition

$$\frac{\partial u}{\partial q}(q^*(\theta),e,\theta)=c+\mu\,\mathbf{E}[\xi\,|\,\theta].$$

In particular, the shadow cost of the externality is $\mu=1$ when the externality enters the utility function additively. Therefore, this extends the logic for the quadratic model (Q).

A final subtlety needs to be addressed in order to complete the proof of Theorem 1: a priori, the sign of μ is ambiguous, in which case it is unclear how the monotonicity of $\mathbf{E}[\xi\,|\,\theta]$ affects the monotonicity of the marginal price. This is done by the following lemma, the proof of which is deferred to Appendix A:

Lemma 1. *Under Assumption 1, the optimal Lagrange multiplier μ^* satisfies $\mu^*>0$.*

By Lemma 1, monotonicity of $\mathbf{E}[\xi\,|\,\theta]$ implies that the marginal price faced by the individual must also be monotone with the same sign. As this holds for all feasible e, it must also hold for the optimal e^* that the policymaker chooses in the outer problem. This proves Theorem 1.

3.2 Welfare gain from the optimal indirect policy

While Theorem 1 demonstrates how different the optimal indirect policy is from a uniform tax, I now determine when there is a substantial welfare gain by using the optimal indirect policy rather than a uniform tax. To this end, I proceed in two steps: First, I determine the *marginal* welfare gain from using a nonlinear indirect policy that—unlike a uniform tax—is able to exploit the correlation between ξ and θ. Second, I determine the *equilibrium* welfare gain from using the optimal indirect policy rather than a uniform tax. Below, I explain the intuitions underlying these two results and defer their proofs to Appendix A.

The first result shows the marginal welfare gains relative to the laissez-faire benchmark from using a nonlinear indirect policy and a uniform tax. The thought experiment underlying this result considers two alternative, small increases in the marginal price that each individual faces: *(i)* a uniform increase and *(ii)* a non-uniform increase proportional to $\mathbf{E}[\xi \mid \theta]$. I then compare the resulting gain in welfare under each alternative.

Theorem 2. *Let* $\mathrm{d}W^{\mathrm{SB}}$ *and* $\mathrm{d}W^{\mathrm{U}}$ *respectively denote the marginal welfare gain (relative to the welfare attained under laissez-faire) from a change in the marginal price proportional to* $\mathbf{E}[\xi \mid \theta]$ *and a uniform change in the marginal price. Moreover, let* $e^{\varnothing}$ *denote the total amount of externality under laissez-faire, and denote by* $D(p, e, \theta)$ *the quantity of the good demanded at a price of* p *by an individual with consumption type* θ *when the total amount of externality is* e*. Then*

$$\frac{\mathrm{d}W^{\mathrm{SB}}}{\mathrm{d}W^{\mathrm{U}}} = \frac{\int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}^2[\xi \mid \theta] \cdot \frac{\partial D}{\partial p}(c, e^{\varnothing}, \theta) \ \mathrm{d}F(\theta)}{\mathbf{E}[\xi] \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\xi \mid \theta] \cdot \frac{\partial D}{\partial p}(c, e^{\varnothing}, \theta) \ \mathrm{d}F(\theta)}.$$

Theorem 2 shows that the marginal welfare gain from using a nonlinear indirect policy depends on how well an individual's consumption behavior predicts the amount of externality that he generates (i.e., $\mathbf{E}[\xi \mid \theta]$), as well as the gradient of his demand curve (i.e., $\partial D/\partial p$). One extreme is when there is maximal dispersion in $\mathbf{E}[\xi \mid \theta]$, so that $\mathbf{E}[\xi \mid \theta] = \mathbf{E}[\xi]$, such as when θ and ξ are independent. In this case, the two marginal welfare gains are equal: as consumption behavior is uninformative of externality production, a uniform tax is second-best. More generally, however, less-than-maximal dispersion in $\mathbf{E}[\xi \mid \theta]$ results in a difference in the marginal welfare gain.

Theorem 2 also shows that the marginal welfare gain from using a nonlinear indirect policy is large relative to that from using a uniform tax precisely when more variation in externalities can be explained by variation in consumption behavior. To see why, hold constant the denominator of the expression in Theorem 2, namely the product of the average externality, $\mathbf{E}[\xi]$, and the average externality weighted by the gradient of demand, $\mathbf{E}[\xi \cdot \partial D/\partial p]$. Then the numerator is proportional to

$$\widetilde{\mathbf{E}}\left[\mathbf{E}^2[\xi \mid \theta]\right] = \widetilde{\mathbf{Var}}\left(\mathbf{E}[\xi \mid \theta]\right) + \left(\mathbf{E}[\xi \cdot \partial D/\partial p]\right)^2,$$

where $\widetilde{\mathbf{E}}$ and $\widetilde{\mathbf{Var}}$ respectively denote the expectation and variance operators when the distribution of θ is weighted by $\partial D/\partial p$ and normalized accordingly. In turn, $\widetilde{\mathbf{Var}}\left(\mathbf{E}[\xi \mid \theta]\right)$ can be interpreted as the component of variation in ξ that can statistically explained by variation in θ, weighted by $\partial D/\partial p$. Thus, the greater the explained variation in ξ, the larger the marginal welfare gain from using a nonlinear indirect policy relative to that from using a uniform tax.

The second result shows that this intuition applies not just to marginal welfare gains, but also to equilibrium welfare gains. While it is possible to compute equilibrium welfare gains in the general case, doing so leads to a complicated formula that is not particularly insightful. Instead, I pursue the alternative route of computing equilibrium welfare gains for the quadratic model (Q). In this case, a remarkably simple formula is obtained that is comparable to that in Theorem 2.

Theorem 3. *In the quadratic model* (Q)*, suppose that no bunching is required under the optimal indirect policy and that every individual consumes a positive quantity of the good under the optimal indirect uniform tax. Let* W^{SB}*,* W^{U}*, and* $W^{\varnothing}$ *respectively denote the welfare attained under the optimal indirect policy, the optimal indirect uniform tax, and laissez-faire. Then*

$$\frac{W^{\mathrm{SB}} - W^{\varnothing}}{W^{\mathrm{U}} - W^{\varnothing}} = \frac{\mathbf{E}[1/\theta]}{\mathbf{E}^2[\xi/\theta]} \cdot \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{s}\, \mathbf{E}^2[\xi \mid s] \ \mathrm{d}F(s).$$

Theorem 3 reinforces the intuition provided in the discussion above. As Appendix A shows, the welfare gap between the optimal indirect policy and the optimal uniform tax can be computed explicitly:

$$W^{\mathrm{SB}} - W^{\mathrm{U}} = \frac{1}{2}\widetilde{\mathbf{E}}\left(\mathbf{E}^2[\xi \mid \theta]\right) - \frac{1}{2}\widetilde{\mathbf{E}}^2\left(\mathbf{E}[\xi \mid \theta]\right) = \frac{1}{2}\widetilde{\mathbf{Var}}\left(\mathbf{E}[\xi \mid \theta]\right),$$

where $\widetilde{\mathbf{E}}$ and $\widetilde{\mathbf{Var}}$ denote the expectation and variance operators when the distribution of θ is weighted by $\partial D/\partial p = 1/\theta$ and normalized accordingly. This further reinforces the interpretation provided by Theorem 2, namely that the welfare gain from using a nonlinear indirect policy is large relative to that from using a uniform tax precisely when more variation in externalities can be explained by variation in consumption behavior.

Theorem 3 can also be employed to understand how much deadweight loss arises from the policymaker's inability to directly measure and tax the externality generated by each individual. In addition to the assumptions of Theorem 3, assume that every individual also consumes a positive quantity of the good under the first-best outcome. Then one can compute that

$$\frac{W^{\mathrm{FB}} - W^{\varnothing}}{W^{\mathrm{SB}} - W^{\varnothing}} = \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{s}\, \mathbf{E}[\xi^2 \mid s] \ \mathrm{d}F(s)}{\int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{s}\, \mathbf{E}^2[\xi \mid s] \ \mathrm{d}F(s)}.$$

This formula offers a complementary intuition to that developed above: while the welfare gained by moving from the optimal indirect uniform tax to the optimal indirect policy increases with the amount of explained variation in ξ (by variation in θ), the welfare gained by moving from the optimal indirect policy to the first-best Pigouvian benchmark increases with the amount of unexplained variation in ξ. Indeed, the difference between the first-best Pigouvian benchmark and the optimal indirect policy is equal to the total conditional variance in ξ on θ, appropriately weighted:

$$W^{\mathrm{FB}} - W^{\mathrm{SB}} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{2s} \mathbf{Var}[\xi \mid s] \, \mathrm{d}F(s) = \frac{1}{2} \widetilde{\mathbf{E}} \left(\mathbf{Var}[\xi \mid \theta] \right).$$

In turn, this determines the potential social gains from incurring technological and political costs of directly measuring and taxing the externality.

4 Empirical and policy applications

I now show how the theoretical findings of Section 3 can be applied to inform policy. To do so, I apply my results to the setting of Knittel and Sandler (2018), who empirically examine the use of a gasoline tax to indirectly regulate CO, HCs, and NO_x pollutant emissions in California between 1998 and 2008. While Knittel and Sandler consider only a uniform gas tax, I show how my results can be used in conjunction with Knittel and Sandler's estimates to *(i)* characterize the optimal indirect policy in their setting and *(ii)* evaluate the welfare gain from the optimal indirect policy. Overall, I find that the optimal indirect policy requires a quantity surcharge on gas and would have performed an average of 12.4% better than a uniform tax each year over the sample period.

Knittel and Sandler consider a setting with a unit mass of risk-neutral individuals, each of whom has linear demand for gasoline but produces a different amount of pollution (measured in money terms) for every gallon of gas consumed. The total amount of pollution shifts individual utility levels. As such, their model is equivalent to my quadratic model (Q). In addition, Knittel and Sandler introduce a lognormal distribution specification[3] for individual consumption types θ and pollution amounts ξ:

$$\begin{bmatrix} \log \theta \\ \log \xi \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_\theta \\ \mu_\xi \end{bmatrix}, \begin{bmatrix} \sigma_\theta^2 & \rho\sigma_\theta\sigma_\xi \\ \rho\sigma_\theta\sigma_\xi & \sigma_\xi^2 \end{bmatrix} \right).$$

[3] While Knittel and Sandler do not assume a lognormal distribution in their baseline specification, they adopt a lognormal distribution specification in an online appendix and show that resulting welfare estimates corroborate well with results from their baseline specification.

As it turns out, the correlation ρ, as well as the standard deviations σ_θ and σ_ξ, will play a crucial role in determining the form of the optimal indirect policy and its welfare gain over a uniform tax.

Finally, I defer the derivation of all results in this section to Appendix A.

4.1 Characterization of the optimal indirect policy

I begin by characterizing the optimal indirect policy in the context of Knittel and Sandler's model. Specifically, I show that the optimal indirect policy might require quantity surcharges and discounts (Proposition 1) and quantity ceilings (Proposition 2), and discuss how these results relate to my Theorem 1.

Proposition 1. *When $\rho < \sigma_\theta/\sigma_\xi$, the optimal indirect policy sets a marginal price for the q^{th} unit of the good equal to*

$$p(q) = \exp\left[\frac{\sigma_\theta\mu_\xi - \rho\sigma_\xi\mu_\theta + \frac{1}{2}\left(1-\rho^2\right)\sigma_\theta\sigma_\xi^2}{\sigma_\theta + \rho\sigma_\xi}\right](A-q)^{-\frac{\rho\sigma_\xi}{\sigma_\theta - \rho\sigma_\xi}}.$$

In particular:

(i) when $0 < \rho < \sigma_\theta/\sigma_\xi$, all individuals face an increasing marginal price (a quantity surcharge);

(ii) when $\rho = 0$, all individuals face a constant marginal price (a uniform tax); and

(iii) when $\rho < 0$, all individuals face a decreasing marginal price (a quantity discount).

As Proposition 1 shows, the correlation ρ plays a crucial role in determining whether a quantity surcharge or discount is required. To interpret the proposition, observe that individuals' price elasticities are given by

$$\varepsilon(p,\theta) = \frac{\partial \log D}{\partial \log p}(p,e,\theta) = -\frac{p}{A\theta - p}.$$

This shows that individuals with higher consumption types θ are less price-responsive. As such, ρ determines the relationship between individuals' elasticities and externalities. When more price-responsive drivers tend to have cleaner vehicles (i.e., $\rho > 0$), the optimal indirect policy sets an increasing tax rate on gasoline. Conversely, when more price-responsive drivers tend to have dirtier vehicles (i.e., $\rho < 0$), the optimal indirect policy sets a decreasing tax rate on gasoline.

Proposition 1 fully characterizes the optimal indirect policy when $\sigma_\theta > \sigma_\xi$. This condition is equivalent to the assumption in Theorem 1 that no bunching is required under the optimal indirect policy. To relate the conclusions of Theorem 1 to those of Proposition 1, observe that $\mathbf{E}[\xi \,|\, \theta]$ is

Year	σ_θ	σ_ξ	ρ
1998	1.210	1.186	−0.322
1999	1.213	1.187	−0.299
2000	1.219	1.199	−0.308
2001	1.223	1.207	−0.311
2002	1.227	1.221	−0.283
2003	1.239	1.232	−0.283
2004	1.237	1.235	−0.265
2005	1.241	1.214	−0.265
2006	1.241	1.217	−0.251
2007	1.244	1.219	−0.247
2008	1.238	1.224	−0.252
Average	1.230	1.213	−0.281

Table 1: Estimates from Knittel and Sandler's (2018) Table A.8.[4]

increasing when $\rho > 0$ and decreasing when $\rho < 0$ (see Appendix A). Thus Proposition 1 can be viewed as a restatement of Theorem 1, albeit with a closed-form expression for the marginal price afforded by the lognormal distribution specification.

As Table 1 shows, Knittel and Sandler estimate that $\rho < 0$ for all years in their sample period. Thus individuals with dirtier vehicles tend to be more price-responsive. In turn, my results imply that quantity discounts are optimal. To illustrate the optimal indirect policy characterized in Proposition 1, Figure 2 plots the normalized marginal price $p(q)$ with the average σ_θ, σ_ξ, and ρ.

While Proposition 1 is sufficient given Knittel and Sandler's estimates, it turns out that the optimal indirect policy can be easily characterized even when $\rho \geq \sigma_\theta/\sigma_\xi$. In this case, a quantity ceiling is required to control the amount of externality that individuals produce:

Proposition 2. *When $\rho \geq \sigma_\theta/\sigma_\xi$, the optimal indirect policy sets a quantity ceiling that binds for all individuals.*

As Proposition 2 shows, quantity controls (in the form of quantity ceilings) might be necessary when there is sufficient heterogeneity in externality. In particular, quantity ceilings are required whenever there is sufficient heterogeneity in externality and a high positive correlation between individuals' elasticities and externalities. Intuitively, this occurs because the difference in marginal tax rates required to regulate the most price-responsive drivers and the least price-responsive

[4] Knittel and Sandler's Table A.8 reports the correlation between $\log(1/\theta) = -\log\theta$ and $\log\xi$, while Table 1 reports the correlation between $\log\theta$ and $\log\xi$. This accounts for the sign difference between the reported correlations.

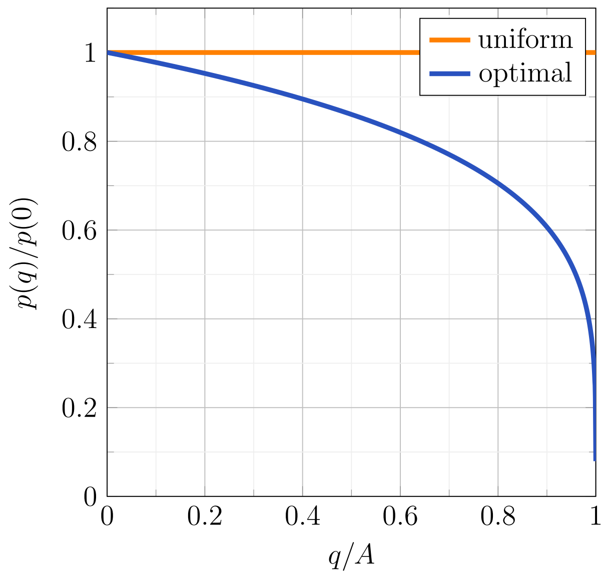


Figure 2: Plot of the normalized marginal price $p(q)$ based on Knittel and Sandler's estimates.

drivers is too large. This makes it necessary for the policymaker to use quantity ceilings to limit the amount of externality produced instead.

Taken together, Propositions 1 and 2 paint a complete picture of how correlation between θ and ξ affects the optimal indirect policy, as shown in Figure 3. In turn, this justifies longstanding skepticism about the optimality of uniform taxes in the regulation of externalities, first expressed by Arrow (1951):

> "The general feeling is that in these cases, optimal allocation can be achieved by a price system, accompanied by a suitable system of taxes and bounties. However, the problem has only been discussed in simple cases; and no system has been shown to have, in the general case, the important property possessed by the price system. . . "

Figure 3 provides an answer to Arrow by showing that a uniform tax is generally suboptimal when externalities must be indirectly regulated.

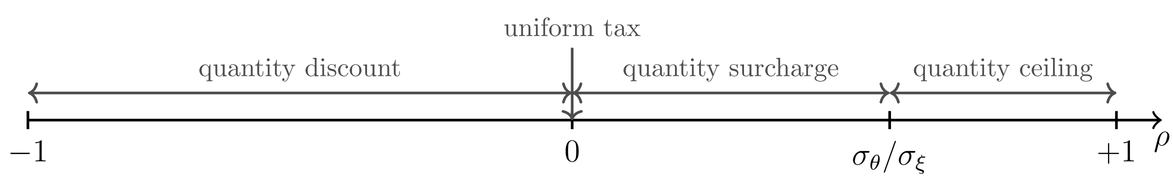


Figure 3: How the optimal indirect policy changes with the correlation ρ between θ and ξ.

4.2 Welfare gain from the optimal indirect policy

I now quantify the gain in welfare from using the optimal indirect policy rather than the optimal uniform tax. I focus on the case where $\rho \leq 0$, which is the empirically relevant regime for Knittel and Sandler's setting (cf. Table 1).

Proposition 3. *Suppose that $\rho \leq 0$, and let W^{SB}, W^{U}, and $W^{\varnothing}$ respectively denote the welfare attained under the optimal indirect policy, the optimal indirect uniform tax, and laissez-faire. In the limit where $A \to +\infty$, the ratio of the welfare gain from the optimal indirect policy to the welfare gain from the optimal indirect uniform tax is*

$$\frac{W^{\mathrm{SB}} - W^{\varnothing}}{W^{\mathrm{U}} - W^{\varnothing}} = \exp\left(\rho^2 \sigma_\xi^2\right).$$

Proposition 3 can be proven in direct analog to Theorem 3. Here, the lognormal distribution specification allows for the explicit computation of the ratio of the welfare gain from the optimal indirect policy to the welfare gain from the optimal indirect uniform tax.

The assumption of $A \to +\infty$ is maintained by both Diamond (1973) and Knittel and Sandler, and I preserve it here to make my result directly comparable to theirs. This assumption is important only to the extent that, in this limit, all individuals consume an interior amount of the good. In turn, this allows for a closed-form expression to be derived. As such, the expression in Proposition 3 should be understood as an approximation that holds well when A is large enough.

Like Theorem 3, Proposition 3 quantifies when a uniform tax can be expected to perform well or poorly. Unsurprisingly, when $\rho = 0$, the ratio of welfare gains is exactly equal to 1 because the optimal indirect policy is a uniform tax. However, Proposition 3 also shows that uniform tax performance worsens when elasticity and externality become more correlated. This is because the optimal indirect policy exploits this correlation, whereas a uniform tax does not.

As noted in the discussion following Theorem 3, Proposition 3 is also informative about how much deadweight loss arises from the policymaker's inability to directly measure and tax the pollution produced by each vehicle. If this measurement were possible, the policymaker would be able to achieve the first-best welfare W^{FB}. Knittel and Sandler show (in their Proposition 5) that

$$\frac{W^{\mathrm{FB}} - W^{\varnothing}}{W^{\mathrm{U}} - W^{\varnothing}} = \exp\left(\sigma_\xi^2\right).$$

Combining Proposition 3 with their result yields

$$\frac{W^{\mathrm{FB}} - W^{\varnothing}}{W^{\mathrm{SB}} - W^{\varnothing}} = \exp\left[\left(1 - \rho^2\right) \sigma_{\xi}^2\right] .$$

This shows that the performance of the optimal indirect policy relative to a direct Pigouvian tax improves with the amount of heterogeneity in externality that can be statistically explained by the heterogeneity in elasticity. Consequently, this measure determines whether the policymaker should incur technological and political costs of directly measuring and taxing the externality.

In addition, Proposition 3 sheds light on *which* good the policymaker should use to indirectly regulate the externality if she had a choice. By the formula above, the policymaker should regulate the good for which individuals' elasticities are most correlated with the amounts of externality that they produce. By contrast, if the policymaker was restricted to a uniform tax, she would be indifferent between the goods as the performance of a uniform tax depends only on σ_{ξ}.

Next, using Proposition 3 in conjunction with Knittel and Sandler's estimates (Table 1), I show that the optimal indirect policy performs 12.4% better than the optimal indirect uniform tax, which is equivalent to an average gain of \$2.50 million each year over the sample period. The results of this calibration exercise are reported in Table 2.

In percentage terms, the welfare gains from the optimal indirect policy are relatively modest as the strength of the correlation between θ and ξ is moderate, with an average $\rho = -0.281$ (Table 1).

Year	W^{U}	W^{SB}	$\frac{W^{\mathrm{SB}}-W^{\varnothing}}{W^{\mathrm{U}}-W^{\varnothing}}$	$\Delta W = W^{\mathrm{SB}} - W^{\mathrm{U}}$
1998	\$48.11 M	\$55.67 M	1.157	\$7.56 M
1999	\$38.68 M	\$43.87 M	1.134	\$5.19 M
2000	\$31.15 M	\$35.71 M	1.146	\$4.55 M
2001	\$23.39 M	\$26.93 M	1.151	\$3.54 M
2002	\$17.25 M	\$19.44 M	1.127	\$2.19 M
2003	\$12.91 M	\$14.58 M	1.129	\$1.67 M
2004	\$9.28 M	\$10.33 M	1.113	\$1.05 M
2005	\$6.28 M	\$6.97 M	1.109	\$0.69 M
2006	\$4.72 M	\$5.18 M	1.098	\$0.46 M
2007	\$3.52 M	\$3.86 M	1.095	\$0.33 M
2008	\$2.76 M	\$3.03 M	1.100	\$0.28 M
Average	\$18.0 M	\$20.5 M	1.124	\$2.50 M

Table 2: Welfare gains from optimal indirect policy.

As such, Proposition 3 indicates that, while there are welfare gains to be realized from using the optimal indirect policy, a uniform tax should be expected to perform relatively well. Moreover, as σ_ξ stays relatively constant while the strength of the correlation $|\rho|$ decreases throughout the sample period, it can be seen from Table 2 how the performance of a uniform tax decreases with $|\rho|$.

While these gains might be modest in percentage terms, the implied magnitudes can be large. The maximum W^{SB} attained by the optimal indirect policy in Knittel and Sandler's sample period is \$55.67 million in 1998, meaning that the 15.7% gain from using the optimal indirect policy in that year yields an improvement of \$7.56 million. However, as Knittel and Sandler note, W^{SB} decreases over the sample period as the fleet of vehicles in California became cleaner.[5] Nevertheless, the average annual gain from using the optimal indirect policy is \$2.50 million over the sample period.

These gains can increase if the policymaker finds a better proxy good for pollutant emissions than gasoline. For example, one might wonder if taxing mileage under the Bipartisan Infrastructure Bill performs better than taxing gasoline. I therefore examine the average welfare gain from using the optimal indirect policy with a counterfactual ρ. The resulting plot is shown in Figure 4. At the average $\rho = -0.281$, a 1% increase in ρ results in a 1.89% (or \$34,000) gain in welfare per year.

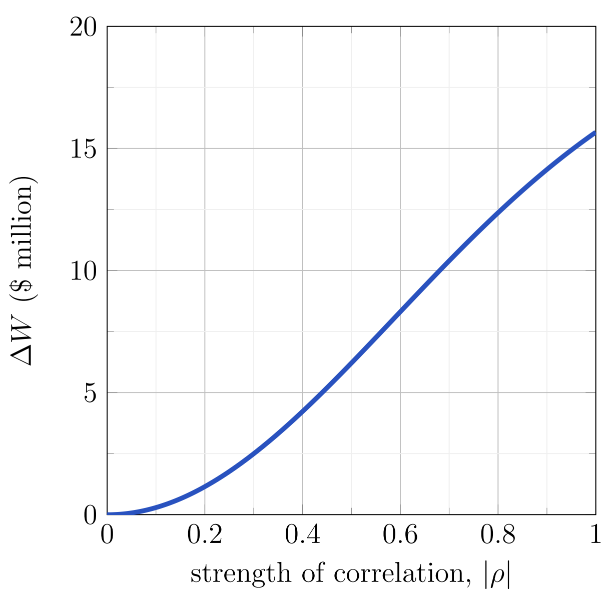


Figure 4: Counterfactual average welfare gains from optimal indirect policy.

[5] As Knittel and Sandler observe, this is primarily because the U.S. Environmental Protection Agency tightened limits on the emissions per mile of new vehicles, whose overall share in the fleet grew over time.

4.3 Discussion

In summary, the results presented in this section suggest four ways that my theoretical findings can be applied to empirical and policy settings.

First, my results show how the optimal indirect policy can be characterized using empirical estimates. A full characterization of the optimal indirect policy can always be obtained with non-parametric estimates of each individual's utility function u, the distribution F of consumption types, and the conditional expectation $\mathbf{E}[\xi \mid \theta]$ of the externality produced. In parametric settings, such as the application presented in this section, the form of the optimal indirect policy (i.e., whether it requires a quantity discount or surcharge) depends only on the correlation between ξ and θ, while a full characterization of the optimal indirect policy requires estimating all other parameters of the model.

Second, my results quantify the welfare gains from employing the optimal indirect policy. This allows the policymaker to quantify the tradeoff between using optimal—but complicated—policies rather than simple—but suboptimal—policies such as a uniform tax. Implementing the optimal indirect policy in practice can be complicated, as it requires policymakers to track the total amount of good consumed by the same individual. In the present application, this might entail deploying telematics technology as in the vehicle mileage tax pilot as part of the Bipartisan Infrastructure Bill, which would allow the policymaker to track total vehicle mileage as a proxy for the amount of pollution produced. Deploying telematics technology might be costly to the policymaker and should be traded off with the welfare benefits of externality reduction that it would allow. My results provide a framework according to which these welfare benefits can be quantified.

Third, my results quantify the value of new technologies that can elicit more granular data from individuals in the regulation of externalities. In other settings with asymmetric information, the development of new monitoring technologies has reduced information asymmetry—and hence the gap between second-best and first-best outcomes (Jin and Vasserman, 2021). In the present application, it is conceivable that technological advancement might reduce the cost of monitoring how much pollution is produced by each vehicle, which would allow implementation of the first-best Pigouvian benchmark. My results thus allow the cost of developing such technology to be traded off with cheaper ways of indirectly regulating the externality.

Fourth, my results provide guidance on which imperfect proxy good to tax in order to regulate externalities. In the present application, levying the optimal uniform tax on *any* proxy good yields the same amount of externality reduction—and hence the same welfare in equilibrium. But a higher equilibrium welfare obtains if the optimal indirect policy is applied on a good for which demand is more correlated with the amount of externality produced. More generally, the policymaker should consider regulating proxy goods for which demand is statistically most informative of the amount of externality produced, as measured by the conditional expectation $\mathbf{E}[\xi \mid \theta]$.

Finally, while this section has focused on a single application of my theoretical findings, they have potential policy implications to a number of other settings as well. These are discussed in Appendix B.

5 Extensions

In this section, I consider three extensions of my theoretical findings. First, I show how my results extend when bunching occurs; in this case, ironing is required and the optimal indirect policy entails quantity ceilings and/or floors, as in Proposition 2. Second, motivated by policy discussions about the distributional impact of corrective taxes, I demonstrate how distributional considerations can be incorporated into the policymaker's objective. Third, I present the implications of my results for the optimal direct regulation of externalities with distributional considerations.

Rather than show how my theoretical findings extend in the most general setting, I focus on the quadratic model (Q) in this section due to its elegance and simplicity. The quadratic model is not only of empirical interest, as shown in Section 4, but also yields simplest possible demand system (namely the linear demand system) to study settings with continuous demand. Moreover, as I show below, the quadratic model is particularly well-suited for analyzing mechanism design problems due to the availability of mathematical tools from quadratic programming. This permits closed-form solutions to be obtained, which aids in the interpretation of the results.

5.1 Quantity ceilings and floors

I begin by showing how a characterization of the optimal indirect policy can be obtained even when bunching occurs. I then demonstrate how the required ironing procedure implies the optimality of non-market regulations, such as quantity ceilings and floors.

To state these results, I define the target allocation function q^{T}, which plays a central role to my analysis:

$$q^{\mathrm{T}}(\theta) := A - \frac{c + \mathbf{E}[\xi \mid \theta]}{\theta}.$$

The target allocation function q^{T} is the solution to the policymaker's relaxed problem—that is, if the policymaker were not required to satisfy individuals' (IC) constraints. In particular, if q^{T}

is nonnegative and increasing (as assumed in Section 3), then the optimal allocation function is simply $q^* = q^{\mathrm{T}}$. Moreover, let h represented the density of individuals weighted by the slopes θ of their inverse demand curve (normalized by $\mathbf{E}[\theta]$), and let H denote its cumulative distribution function:

$$h(\theta) := f(\theta) \cdot \frac{\theta}{\mathbf{E}[\theta]} \quad \text{and} \quad H(\theta) := \int_{\underline{\theta}}^{\theta} h(s)\ \mathrm{d}s.$$

Finally, let the concave closure of a function ϕ be defined as the pointwise smallest concave function that lies above ϕ, and denote the concave closure of ϕ by $\operatorname{co}\phi$.

Theorem 4. *Define the* ironed target allocation function *as*

$$\overline{q}^{\mathrm{T}}(\theta) := \frac{\mathrm{d}}{\mathrm{d}t}\left(\operatorname{co}\int_{1-t}^{1} q^{\mathrm{T}}(H^{-1}(s))\ \mathrm{d}s\right)\Bigg|_{t=1-H(\theta)}.$$

Then the unique optimal allocation function q^* in the quadratic model (Q) *is given by*

$$q^*(\theta) = \begin{cases} 0 & \text{if } \overline{q}^{\mathrm{T}}(\theta) \leq 0, \\ A & \text{if } \overline{q}^{\mathrm{T}}(\theta) \geq A, \\ \overline{q}^{\mathrm{T}}(\theta) & \text{otherwise.} \end{cases}$$

The proof of Theorem 4 (given formally in Appendix A) exploits an equivalence between the policymaker's problem for the quadratic model and an L^2 regression problem. The policymaker solves

$$\max_{q\in\mathcal{Q}} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ [\theta A - c - \mathbf{E}[\xi \mid \theta]]\, q(\theta) - \frac{\theta}{2}\, [q(\theta)]^2 \right\}\ \mathrm{d}F(\theta),$$

which—by completing the square—can be written as

$$\min_{q\in\mathcal{Q}} \int_{\underline{\theta}}^{\overline{\theta}} \left\{ q(\theta) - \left[A - \frac{c + \mathbf{E}[\xi \mid \theta]}{\theta} \right] \right\}^2\ \mathrm{d}H(\theta) = \min_{q\in\mathcal{Q}} \int_{\underline{\theta}}^{\overline{\theta}} \left[q(\theta) - q^{\mathrm{T}}(\theta) \right]^2\ \mathrm{d}H(\theta).$$

This is an L^2 regression problem: the policymaker chooses an implementable allocation function $q \in \mathcal{Q}$ that minimizes the L^2 distance to the target allocation function q^{T}.

In analogy with standard least-squares linear regression problems, this L^2 regression problem can be solved by applying an appropriate projection operator to the target allocation function q^{T}. In linear regression, the best-fit parameters are obtained by projecting the data onto the linear subspace spanned by the covariates; thus the solution to a linear regression problem can be found by applying a projection matrix to the data. Similarly, the optimal allocation function q^* can be found by projecting the target allocation function q^{T} onto the set of all implementable allocation functions $\mathcal{Q}$. Theorem 4 solves the L^2 regression problem by characterizing the projection operator.

Next, I show how Theorem 4 implies that quantity floors and ceilings might be required in the optimal indirect policy. To state the result, define a quantity floor (or ceiling) as a minimum (or maximum) quantity that binds for a positive mass of individuals with different consumption types. Moreover, let $\phi(t)$ denote the total quantity of the good allocated to individuals in the highest t^{th} percentile (measured with respect to the distribution H) under the target allocation function q^{T}:

$$\begin{aligned}\phi(t) &:= \int_{H^{-1}(1-t)}^{1} q^{\mathrm{T}}(\theta)\ \mathrm{d}H(\theta)\\ &= \int_{H^{-1}(1-t)}^{1} \left[A - \frac{c + \mathbf{E}[\xi \mid \theta]}{\theta}\right]\ \mathrm{d}H(\theta).\end{aligned}$$

Corollary 1. *Consider an individual whose $\hat{\theta}$ is such that ϕ is not concave at $t = 1 - H^{-1}(\hat{\theta})$, and let $(\theta_*, \theta^*) \subset [\underline{\theta}, \overline{\theta}]$ be the largest open interval containing $\hat{\theta}$ such that*

$$\phi(t) \neq \operatorname{co}\phi(t) \quad \textit{for all } t \in (\theta_*, \theta^*).$$

Denote the quantity consumed by the individual by $\hat{q} > 0$. Then the individual faces:

(i) a quantity floor of $\hat{q}$ if $\theta_* = \underline{\theta}$; and

(ii) a quantity ceiling of $\hat{q}$ if $\theta^* = \overline{\theta}$.

Corollary 1 thus shows that quantity discounts and surcharges might not be sufficient for the optimal indirect policy, and that quantity ceilings and floors are potentially required. This applies in the context of Proposition 2, where individuals face a quantity ceiling when their externality and consumption type are sufficiently strongly correlated.

While Theorem 4 and Corollary 1 might seem technical, they might explain why non-market regulations are sometimes used to indirectly regulate externalities in real-world markets. One example is the Combat Methamphetamine Epidemic Act, which restricts quantities of ephedrine and pseudoephedrine (found in over-the-counter medications such as Claritin and Sudafed) that individuals may purchase. This can be viewed as a quantity ceiling to limit the illicit use of these drugs to produce methamphetamine. Another example is a vaccine mandate, which can be viewed as a quantity floor to limit the spread of viruses. Appendix B discusses these examples in detail.

5.2 Optimal indirect regulation with distributional concerns

I now incorporate distributional considerations into the policymaker's objective. This is motivated by policy discussions about the distributional impact of corrective taxes. For example, uniform gasoline taxes are generally viewed as regressive because lower-income households spend a larger proportion of their income on gasoline (Parry et al., 2007). The policymaker might therefore wish to trade off efficiency with equity in the optimal indirect policy.

To do so, I model the policymaker's distributional preferences via different welfare weights on individual utilities. This modeling technique draws on an extensive literature in public finance (e.g., Diamond and Mirrlees, 1971; Atkinson and Stiglitz, 1976; and Saez and Stantcheva, 2016). This modeling technique has also been recently applied to mechanism design by Condorelli (2013) and Dworczak Ⓡ al. (2021).

Formally, rather than just θ and ξ as in Section 2, each individual is now also endowed with a welfare weight ω. The welfare weight ω has a natural interpretation: it is the amount of utility that the policymaker realizes by giving an additional \$1 to the individual. However, just as θ and ξ are the individual's private information, so is ω. This arises because ω represents information about the individual's socioeconomic status that the policymaker cannot observe. For example, ω might depend on income, which a policymaker designing a gasoline tax or a congestion toll on a county level does not have access to. Alternatively, ω might represent wealth, which might not be observed even if the policymaker has access to individuals' incomes.[6] I also assume that the welfare weight that the policymaker assigns to revenue is 1, and that

$$\int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\omega \mid \theta] \, \mathrm{d}F(\theta) = 1.$$

This implies that uniform transfers between the policymaker and all individuals are welfare-neutral, so that the policymaker faces no additional incentive to redistribute in cash.

Next, I impose an additional regularity assumption to ensure the existence of an optimal indirect policy and to help characterize it. To state the assumption, I define

$$\Delta(\theta) := \frac{\int_{\theta}^{\overline{\theta}} \left[1 - \mathbf{E}[\omega \mid s]\right] \, \mathrm{d}F(s)}{f(\theta)}.$$

[6] In this case, ω represents the residual uncertainty in the individual's welfare weight, conditional on observable characteristics such as income. Thus, a more complete model would specify ω as a function of both observable and unobservable characteristics of individuals. I do not pursue this line of analysis here, but instead note that Akbarpour Ⓡ Dworczak Ⓡ Kominers (2020) show how this additional layer of modeling complexity can be handled at little cost to tractability.

Notice that Δ can be expressed as the average deviation of the welfare weights (from $\mathbf{E}[\omega] = 1$) of individuals with slopes above θ, weighted by the inverse hazard rate:

$$\Delta(\theta) = \underbrace{\frac{\int_\theta^{\overline{\theta}} [1 - \mathbf{E}[\omega \mid s]] \ \mathrm{d}F(s)}{1 - F(\theta)}}_{=\mathbf{E}[\omega \mid s \geq \theta]} \times \frac{1 - F(\theta)}{f(\theta)}.$$

Thus Δ provides a local measure of how different the policymaker's objective function is from that of a utilitarian welfare objective (in which each individual has equal welfare weight).

Assumption 2. *The function $\theta \mapsto \theta - \Delta(\theta)$ is strictly quasiconcave.*

In particular, Assumption 2 implies that $\theta - \Delta(\theta) > 0$ for every $\theta \in [\underline{\theta}, \overline{\theta}]$, which (as I show below) ensures the existence of an optimal indirect policy.

A sufficient condition for Assumption 2 is that Δ is strictly concave. In turn, strict concavity of Δ is guaranteed when $\mathbf{E}[\omega \mid \theta]$ is non-decreasing in θ—that is, when more price-responsive drivers have higher welfare weights—and when the distribution F is locally regular. When F is uniform, for example, Assumption 2 is implied by the assumption that $\mathbf{E}[\omega \mid \theta]$ is non-decreasing in θ.

One might notice that Assumption 2 is similar to Assumption 1 of Dworczak ⓡ al. (2021), but this connection is superficial beyond the mathematical roles that these assumptions play. In particular, the model presented here assumes linear individual demand curves and is therefore different from Dworczak ⓡ al.'s linear utility model; in addition, individuals here are privately informed about the slopes of their demand curves, while they are privately informed about their marginal utility for the good (i.e., the intercept) in Dworczak ⓡ al.'s model. As I show below, however, the assumptions play similar mathematical roles in that they allow simple conditions to be derived under which $\theta - \Delta(\theta)$ is guaranteed to be non-decreasing, as well as a characterization of the shape of $\theta - \Delta(\theta)$ even when it fails to be non-decreasing.

I now state the generalization of Theorem 4 that incorporates distributional considerations into the policymaker's objective. Let

$$H(\theta) := \left[\int_{\underline{\theta}}^{\overline{\theta}} [s - \Delta(s)] \ \mathrm{d}s\right]^{-1} \int_{\underline{\theta}}^{\theta} [s - \Delta(s)] \ \mathrm{d}s.$$

Here, H^{-1} is well-defined because $\theta - \Delta(\theta) > 0$ for all θ, as discussed after Assumption 2.

Theorem 5. *Define the* target allocation function *and the* ironed target allocation function *as*

$$\begin{cases} q^{\mathrm{T}}(\theta) & := A - \dfrac{c + \mathbf{E}[\xi \,|\, \theta]}{\theta - \Delta(\theta)}, \\ \overline{q}^{\mathrm{T}}(\theta) & := \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{co} \displaystyle\int_{1-t}^{1} q^{\mathrm{T}}(H^{-1}(s)) \, \mathrm{d}s \right)\Bigg|_{t=1-H(\theta)} \end{cases}.$$

Then the unique optimal allocation function q^* is given by

$$q^*(\theta) = \begin{cases} 0 & \text{if } \overline{q}^{\mathrm{T}}(\theta) \leq 0, \\ A & \text{if } \overline{q}^{\mathrm{T}}(\theta) \geq A, \\ \overline{q}^{\mathrm{T}}(\theta) & \text{otherwise.} \end{cases}$$

The interpretation of q^{T} in Theorem 5 is identical to that in Section 5.1: it is the allocation that the policymaker wishes to implement absent individuals' incentive constraints. The derivation of Theorem 5 is similar to that of Theorem 4 and is left to Appendix A.

It is therefore apparent that distributional considerations result in additional incentives for the policymaker to use quantity surcharges, discounts, floors, and ceilings. As in Section 5.1, one can define the function

$$\phi(t) := \int_{H^{-1}(1-t)}^{1} \{[\theta - \Delta(\theta)]\, A - [c + \mathbf{E}[\xi \,|\, \theta]]\} \ \mathrm{d}H(\theta).$$

As before, $\phi(t)$ denotes the total quantity of the good allocated to individuals in the highest t^{th} percentile (measured with respect to the distribution H) under the target allocation function q^{T}. When ϕ is concave, the marginal price that each individual faces is

$$\frac{\theta}{\theta - \Delta(\theta)} \left[c + \mathbf{E}[\xi \,|\, \theta]\right].$$

This formula shows that distortions away from uniform taxation arise from two distinct sources: correlation between ξ and θ so that $\mathbf{E}[\xi \,|\, \theta] \neq \mathbf{E}[\xi]$; and correlation between ω and θ so that $\mathbf{E}[\omega \,|\, \theta] \neq \mathbf{E}[\omega] = 1$, resulting in $\Delta(\theta) \neq 0$. These distortions are further exacerbated when ϕ is not concave, so that the policymaker must iron the target allocation function q^{T}.

5.3 Optimal direct regulation with distributional concerns

Finally, I conclude this section by showing how uniform taxation is no longer optimal once the policymaker incorporates distributional considerations into her objective—even when the policymaker can directly measure and tax the externality.

For simplicity, I assume that the amount of externality produced per unit of the good consumed is constant and identical across individuals, so that $\mathbf{E}[\xi \mid \theta] = \mathbf{E}[\xi] \geq 0$. This is the case of a congestion externality: for example, each driver at peak hour causes traffic to be backed up by the same amount, namely one unit of vehicle space. In principle, however, the amount of externality produced per unit of the good consumed could differ across individuals; then the policymaker can simply design a policy conditional on both θ and ξ, and the following analysis would hold for each ξ (see also footnote 6). By Theorem 5, the target allocation function is thus given by

$$q^{\mathrm{T}}(\theta) = A - \frac{c + \mathbf{E}[\xi]}{\theta - \Delta(\theta)}.$$

Proposition 4. *Suppose that $\mathbf{E}[\omega \mid \overline{\theta}] \leq 2$, and consider an individual with $\theta = \hat{\theta}$ who consumes a positive quantity of the good under the optimal policy. Then the marginal price that he faces is*

$$\frac{\hat{\theta}}{\hat{\theta} - \Delta(\hat{\theta})} \left(c + \mathbf{E}[\xi] \right).$$

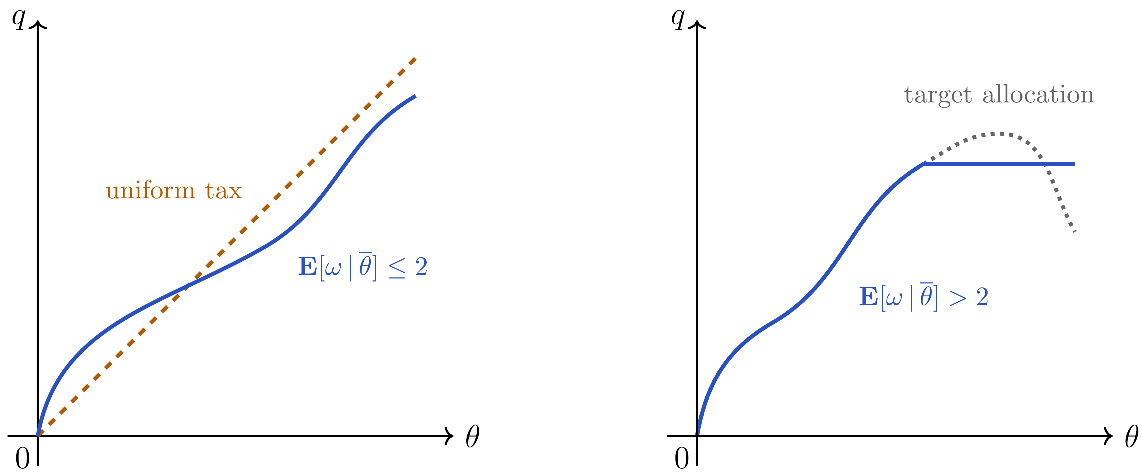
In particular, this individual faces:

(i) an increasing marginal price (i.e., a quantity surcharge) if $\Delta(\theta)/\theta$ is increasing at $\theta = \hat{\theta}$;

(ii) a decreasing marginal price (i.e., a quantity discount) if $\Delta(\theta)/\theta$ is decreasing at $\theta = \hat{\theta}$.

Just as how $\mathbf{E}[\xi \mid \theta]$ determines whether a quantity surcharge or quantity discount is optimal in Theorem 1, Proposition 4 shows how $\mathbf{E}[\omega \mid \theta]$ determines whether a quantity surcharge or quantity discount is optimal via $\Delta(\theta)$. This is summarized by Figure 5(a).

The assumption that $\mathbf{E}[\omega \mid \overline{\theta}] \leq 2$ can be interpreted in at least two ways. First, it is implied by the stronger condition that $\mathbf{E}[\omega \mid \theta] \leq 2$ for every θ, which in turn could be taken as a statement of how the policymaker has relatively minor concerns about the distributional impact of regulation in the market in question. For example, car ownership is relatively expensive in cities such as London, New York, and Singapore, where lower-income individuals tend to commute by public transit; so there might be little variation in the ω that the policymakers assigns to individuals



(a) Proposition 4 *(b) Proposition 5*

Figure 5: Results on optimal direct regulation with distributional considerations.

who drive.[7] Second, it could be understood as a weak correlation between θ and ω, so that each individual's consumption behavior (and, in particular, that of individuals with $\theta = \overline{\theta}$) is not very informative about his welfare weight. This might occur because higher-income and lower-income individuals are both price-inelastic, but for different reasons: in the case of a peak-hour congestion toll, higher-income individuals might be price-inelastic because they have a relatively low value for money, but lower-income individuals might be price-inelastic because they have relatively little flexibility over their work hours.

Proposition 5. *Suppose that $\mathbf{E}[\omega \mid \overline{\theta}] > 2$. Then there exists $\theta^* < \overline{\theta}$ such that each individual with $\theta > \theta^*$ who consumes a positive quantity of the good faces a binding quantity ceiling.*

Proposition 5 is analogous to Corollary 1 in that it presents sufficient conditions under which a quantity ceiling is necessary. It can be shown that the target allocation function is quasiconcave, which implies that a quantity ceiling is optimal and that it binds for individuals who consume a positive quantity of the good. This is summarized by Figure 5(b).

The assumption that $\mathbf{E}[\omega \mid \overline{\theta}] > 2$ is the opposite of the assumption maintained in Proposition 4, and requires significant variation in welfare weights such that the highest welfare weight exceeds 2. Under the additional assumption that $\mathbf{E}[\omega \mid \theta]$ is non-decreasing in θ, the assumption that $\mathbf{E}[\omega \mid \overline{\theta}] > 2$ holds as long as $\mathbf{E}[\omega \mid \hat{\theta}] > 2$ holds for some $\hat{\theta}$. Recall that $\mathbf{E}[\omega \mid \theta]$ is non-decreasing in θ when more price-responsive individuals tend to have higher welfare weights, which might hold

[7] Dworczak ⓡ al. refer to this assumption as the case of "low inequality" and adopt a similar interpretation.

for example if drivers who are more price-responsive with respect to gasoline—who tend to have dirtier vehicles, as Knittel and Sandler (2018) show—also tend to be poorer.

The results above thus indicate an equivalence between the problems of direct regulation with distributional considerations (as in Propositions 4 and 5) and indirect regulation (as in Theorem 1 and Corollary 1). Indeed, the key intuition underlying both sets of results is the same: the optimal policy should exploit the correlation between each individual's consumption behavior and other aspects of his private information—either his welfare weight or his externality—that enter the policymaker's objective function. Therefore, in order for a uniform tax to be optimal, the good must be a perfect proxy for both the externality and each individual's welfare weight.

6 Conclusion

Many externality-correcting policies are indirect. Individuals often generate externalities that are difficult or infeasible to measure, leading policymakers to regulate goods that are at best imperfect proxies for these externalities.

When externality-correcting policies are indirect, a uniform tax is rarely optimal. Instead, the optimal indirect policy exploits the correlation between each individual's consumption behavior and externality, which generally entails quantity surcharges and/or discounts. The welfare gain from using the optimal indirect policy instead of a uniform tax can be significant, and increases with the informativeness of an individual's consumption behavior about his externality.

These insights have policy implications. In conjunction with empirical estimates produced by Knittel and Sandler (2018), my results show that the optimal indirect policy would have performed an average of 12.4% better than a uniform tax on gasoline for the purpose of regulating vehicle emissions in California between 1998 and 2008. Policymakers can therefore trade off this estimate of welfare gain against the costs of implementation. By performing a similar analysis on other candidate proxy goods, policymakers can also use my results to determine which proxy good to tax in order to indirectly regulate the same externality.

Even when externality-correcting policies are direct, a uniform tax also fails to be optimal when distributional considerations are incorporated into their design. Just as the optimal indirect policy exploits the correlation between each individual's consumption behavior and externality, the optimal policy with distributional considerations exploits the correlation between each individual's consumption behavior and welfare weight. In general, a uniform tax is thus optimal only when the good is a perfect proxy for each individual's externality and welfare weight.

The methodology employed in this paper coheres with a broader research agenda that studies the effect of incomplete information in classical economic models. Following early efforts by Bulow and Roberts (1989), recent work applies mechanism design methods to analyze questions in public finance and industrial organization (see, e.g., Loertscher and Marx, 2021). My paper contributes by applying mechanism design methods to study the canonical problem of regulating externalities.

Finally, this paper offers various avenues for future research. While I focus on the optimal design of externality-correcting policies by a policymaker, similar questions arise in optimal pricing by a monopolist who produces a good with either positive externalities (such as network effects) or negative externalities (such as congestion), or even both (such as a platform). More generally, a similar line of analysis can be applied to other settings with imperfect pricing beyond externalities, where outcomes depend on a number of characteristics, but only a subset of which can be priced. These include environments with internalities, coarse pricing, and adverse selection. The results of this paper suggest that the correlation between priceable and non-priceable characteristics is a key determinant of the optimal policy, but more work remains to be done to understand the gains from optimal design in specific contexts and how priceable characteristics are selected.

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Appendix A Omitted proofs

This appendix supplies omitted proofs, which are presented in chronological order of results in the main text of the paper.

A.1 Proofs from Section 3

I begin by stating the required assumption to rule out bunching in Section 3. I then provide the proofs of Theorems 1, 2, and 3.

A.1.1 Assumption to rule out bunching in Section 3

Let e^* denote the optimal total amount of externality that the policymaker wishes to effect in the market (the existence of e^* follows from the existence of an optimal allocation function q^* under Assumption 1, which is formally demonstrated below). Moreover, let

$$D(p, e, \theta) := \left(\frac{\partial u}{\partial q}\right)^{-1}(p, e, \theta).$$

Note that D is well-defined because $\partial u/\partial q > 0$ by Assumption 1. Then the following assumption rules out bunching in the optimal solution:

Assumption M. *The function* $\theta \mapsto D(c+\mu\,\mathbf{E}[\xi \mid \theta], e^*, \theta)$ *is increasing and in* $[0, A]$ *for any* $\mu > 0$ *that satisfies*

$$\mu = -\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial u}{\partial e}\left(D(c+\mu\,\mathbf{E}[\xi \mid \theta], e^*, \theta), e^*, \theta\right)\, \mathrm{d}F(\theta).$$

To see that Assumption M rules out bunching, write the policymaker's problem (P) as a nested optimization problem as in Section 3.1:

$$\begin{aligned}
&\max_{q\in\mathcal{Q}} \int_{\underline{\theta}}^{\overline{\theta}} \left[u\left(q(\theta), \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\xi \mid \theta = s] q(s)\, \mathrm{d}F(s), \theta\right) - cq(\theta)\right]\, \mathrm{d}F(\theta) \\
&= \max_{e\in\mathbb{R}} \left[\max_{q\in\mathcal{Q}} \left\{\int_{\underline{\theta}}^{\overline{\theta}} [u(q(\theta), e, \theta) - cq(\theta)]\, \mathrm{d}F(\theta) : \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\xi \mid \theta] q(\theta)\, \mathrm{d}F(\theta) = e\right\}\right].
\end{aligned}$$

Let μ^* be the optimal Lagrange multiplier associated with the constraint in the inner maximization problem at $e = e^*$. Observe that μ^* represents the marginal value of increasing the constraint of

e in the inner maximization problem; at $e = e^*$, this must equate the marginal cost of increasing e, namely

$$\mu^* = -\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial u}{\partial e} \left(D(c + \mu^* \, \mathbf{E}[\xi \mid \theta], e^*, \theta), e^*, \theta \right) \, \mathrm{d}F(\theta).$$

Now, pointwise maximization of the Lagrangian of the inner maximization problem at the optimal Lagrange multiplier μ^* yields (as in Section 3.1):

$$q^*(\theta) = D(c + \mu^* \, \mathbf{E}[\xi \mid \theta], e^*, \theta).$$

By Assumption M, q^* is increasing and in $[0, A]$, hence no bunching occurs.

I now explicitly derive the assumption for the case of the quadratic model (Q). The marginal utility of each individual is

$$\frac{\partial u}{\partial q}(q, e, \theta) = \theta \, (A - q) \implies D(p, e, \theta) = A - \frac{p}{\theta}.$$

On the other hand, $\partial u / \partial e = -1$, so Assumption M is simply required to hold for $\mu = 1$. This is equivalent to the condition that

$$\theta \mapsto A - \frac{c + \mathbf{E}[\xi \mid \theta]}{\theta} \quad \text{is increasing and in } [0, A],$$

which is the condition stated at the beginning of Section 3.

A.1.2 Proof of Theorem 1

I start by proving the existence of the solution q^* to the policymaker's problem (P) when $A < +\infty$. To this end, observe that $\mathcal{Q}$ is compact with respect to the L^1 topology. Indeed, since L^1 is a metric space, it suffices to show that $\mathcal{Q}$ is sequentially compact. To this end, let $\{q_n\}_{n=1}^{\infty} \subset \mathcal{Q}$ be a sequence of functions; by Helly's selection theorem, there is a subsequence of $\{q_n\}_{n=1}^{\infty}$ that converges pointwise to some $q \in \mathcal{Q}$. By the dominated convergence theorem, this subsequence must also converge to q in the L^1 topology; hence $\mathcal{Q}$ is compact. As the policymaker's objective function is continuous in q and bounded above, it must attain its maximum over $\mathcal{Q}$.

Next, I extend the above argument to the case where $A = +\infty$. By Assumption 1, there exists $\kappa > 0$ such that

$$\frac{\partial u}{\partial q}(q, e, \theta) < c \quad \text{for all } e \in \mathbb{R} \text{ and } \theta \in [\underline{\theta}, \overline{\theta}] \text{ whenever } q > \kappa.$$

Thus the policymaker can restrict attention to allocation functions q such that $\operatorname{im} q \subset [0, \kappa]$ without any loss of generality. The existence of the optimal allocation function q^* then follows from the previous argument.

Now, I formally prove Lemma 1, which states that the optimal Lagrange multiplier μ^* satisfies $\mu^* > 0$ in the inner maximization problem. To this end, the envelope theorem (Milgrom and Segal, 2002) implies that

$$\begin{aligned} 0 &= \frac{\partial}{\partial e}\left[\max_{q\in\mathcal{Q}}\left\{\int_{\underline{\theta}}^{\overline{\theta}} [u(q(\theta), e^*, \theta) - cq(\theta)]\ \mathrm{d}F(\theta) : \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\xi \,|\, \theta] q(\theta)\ \mathrm{d}F(\theta) = e^*\right\}\right] \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial u}{\partial e}(q^*(\theta), e^*, \theta)\ \mathrm{d}F(\theta) + \mu^* \implies \mu^* = -\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial u}{\partial e}(q^*(\theta), e^*, \theta)\ \mathrm{d}F(\theta) > 0. \end{aligned}$$

The inequality follows from the fact that $\partial u/\partial e < 0$ (by Assumption 1).

Finally, I solve the policymaker's problem (P) under Assumption M and shows how it implies Theorem 1. Indeed, consider the relaxed inner problem

$$\max_{q\in\mathcal{F}}\left\{\int_{\underline{\theta}}^{\overline{\theta}} [u(q(\theta), e^*, \theta) - cq(\theta)]\ \mathrm{d}F(\theta) : \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\xi \,|\, \theta] q(\theta)\ \mathrm{d}F(\theta) = e^*\right\},$$

where

$$\mathcal{F} := \left\{q : [\underline{\theta}, \overline{\theta}] \to [0, A] \text{ is measurable}\right\}.$$

Then pointwise maximization of the Lagrangian (as in Section 3.1) yields

$$q^*(\theta) = D(c + \mu^*\, \mathbf{E}[\xi \,|\, \theta], e^*, \theta),$$

where the optimal Lagrange multiplier $\mu^* > 0$ satisfies

$$\mu^* = -\int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial u}{\partial e}\left(D(c + \mu^*\, \mathbf{E}[\xi \,|\, \theta], e^*, \theta), e^*, \theta\right)\ \mathrm{d}F(\theta).$$

Assumption M implies that $\operatorname{im} q^* \subset [0, A]$ and that q^* is increasing; hence q^* must solve the policymaker's inner problem as well (with $\mathcal{F}$ replaced by $\mathcal{M}$). In particular, the marginal price faced by each individual is $c + \mu^*\, \mathbf{E}[\xi \,|\, \theta]$, which shares the same monotonicity as $\mathbf{E}[\xi \,|\, \theta]$ by Lemma 1. This concludes the proof of Theorem 1.

A.1.3 Proof of Theorem 2

Let $\mathrm{d}e^{\mathrm{SB}}$ and $\mathrm{d}e^{\mathrm{U}}$ respectively denote the marginal change in total externality due to a change in the marginal price proportional to $\mathbf{E}[\xi \mid \theta]$ and a uniform change in the marginal price. Moreover, parametrize the changes in marginal price by $\mathrm{d}p^{\mathrm{SB}} = \mathbf{E}[\xi \mid \theta]\ \mathrm{d}p$ and $\mathrm{d}p^{\mathrm{U}} = \mathbf{E}[\xi]\ \mathrm{d}p$ respectively. By the envelope theorem, observe that

$$\mathrm{d}W^j = \frac{\partial u}{\partial e}(c, e^{\varnothing}, \theta)\ \mathrm{d}e^j \quad \text{for } j \in \{\mathrm{SB}, \mathrm{U}\}.$$

The total amount of externality is

$$\begin{aligned} e^{\varnothing} + \mathrm{d}e^j &= \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\xi \mid \theta] D(c + \mathrm{d}p^j, e^{\varnothing} + \mathrm{d}e^j, \theta)\ \mathrm{d}F(\theta) \\ &= e^{\varnothing} + \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\xi \mid \theta] \left[\frac{\partial D}{\partial p}(c, e^{\varnothing}, \theta)\ \mathrm{d}p^j + \frac{\partial D}{\partial e}(c, e^{\varnothing}, \theta)\ \mathrm{d}e^j \right]\ \mathrm{d}F(\theta). \end{aligned}$$

Consequently,

$$\mathrm{d}e^j = \frac{\int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\xi \mid \theta] \cdot \frac{\partial D}{\partial p}(c, e^{\varnothing}, \theta)\ \mathrm{d}p^j\ \mathrm{d}F(\theta)}{1 - \mathbf{E}\left[\xi \cdot \frac{\partial D}{\partial e}(c, e^{\varnothing}, \theta)\right]}.$$

It follows that

$$\frac{\mathrm{d}W^{\mathrm{SB}}}{\mathrm{d}W^{\mathrm{U}}} = \frac{\mathrm{d}e^{\mathrm{SB}}}{\mathrm{d}e^{\mathrm{U}}} = \frac{\int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}^2[\xi \mid \theta] \cdot \frac{\partial D}{\partial p}(c, e^{\varnothing}, \theta)\ \mathrm{d}F(\theta)}{\mathbf{E}[\xi] \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\xi \mid \theta] \cdot \frac{\partial D}{\partial p}(c, e^{\varnothing}, \theta)\ \mathrm{d}F(\theta)}.$$

A.1.4 Proof of Theorem 3

I begin by computing the total social surpluses $W^{\varnothing}$, W^{U}, W^{SB}, and W^{FB} separately for the quadratic model (Q). I then combine each of these expressions to get the desired formula reported in Theorem 3.

Computing $W^{\varnothing}$

Under laissez-faire, each individual's allocation is

$$q^{\varnothing}(\theta) = A - \frac{c}{\theta}.$$

Consequently, the total social surplus under laissez-faire is

$$\begin{aligned} W^{\varnothing} &= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ [\theta A - c - \mathbf{E}[\xi \,|\, \theta]]\, q^{\varnothing}(\theta) - \frac{\theta}{2} \left[q^{\varnothing}(\theta)\right]^2 \right\} \, \mathrm{d}F(\theta) \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{\theta}{2} \left(A - \frac{c}{\theta}\right)^2 - \left(A - \frac{c}{\theta}\right) \mathbf{E}[\xi \,|\, \theta] \right\} \, \mathrm{d}F(\theta). \end{aligned}$$

Computing W^{U}

Diamond (1973) showed that the optimal uniform tax is

$$\tau^* = \frac{\mathbf{E}[\xi/\theta]}{\mathbf{E}[1/\theta]}.$$

Since $\theta A \geq \tau^*$ for all θ, under the optimal uniform tax, each individual's allocation is

$$q^{\mathrm{U}} = A - \frac{c + \tau^*}{\theta}.$$

Consequently, the total social surplus under the optimal uniform tax is

$$\begin{aligned} W^{\mathrm{U}} &= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ [\theta A - c - \mathbf{E}[\xi \,|\, \theta]]\, q^{\mathrm{U}}(\theta) - \frac{\theta}{2} \left[q^{\mathrm{U}}(\theta)\right]^2 \right\} \, \mathrm{d}F(\theta) \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ \frac{\theta}{2} \left(A - \frac{c}{\theta}\right)^2 - \left(A - \frac{c + \tau^*}{\theta}\right) \mathbf{E}[\xi \,|\, \theta] - \frac{(\tau^*)^2}{2\theta} \right\} \, \mathrm{d}F(\theta). \end{aligned}$$

Computing W^{SB}

Under the assumptions of Theorem 3, the optimal indirect policy involves giving each individual an allocation of

$$q^*(\theta) = A - \frac{c + \mathbf{E}[\xi \,|\, \theta]}{\theta}.$$

Consequently, the total social surplus under the optimal indirect policy is

$$\begin{aligned} W^{\mathrm{SB}} &= \int_{\underline{\theta}}^{\overline{\theta}} \left\{ [\theta A - c - \mathbf{E}[\xi \,|\, \theta]]\, q^*(\theta) - \frac{\theta}{2} \left[q^*(\theta)\right]^2 \right\} \, \mathrm{d}F(\theta) \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta}{2} \left[A - \frac{c + \mathbf{E}[\xi \,|\, \theta]}{\theta}\right]^2 \, \mathrm{d}F(\theta). \end{aligned}$$

Computing W^{FB}

Assume that every individual consumes a positive quantity of the good. Then each individual's allocation is

$$q^{\mathrm{FB}}(\theta, \xi) = A - \frac{c + \xi}{\theta}.$$

Consequently, the total social surplus under the first-best outcome is

$$\begin{aligned} W^{\mathrm{FB}} &= \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\xi}}^{\overline{\xi}} \left\{ (\theta A - c - \xi)\, q^{\mathrm{FB}}(\theta, \xi) - \frac{\theta}{2} \left[q^{\mathrm{FB}}(\theta, \xi) \right]^2 \right\} \, \mathrm{d}G(\theta, \xi) \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta}{2} \, \mathbf{E} \left[\left(A - \frac{c + \xi}{\theta} \right)^2 \,\middle|\, \theta \right] \, \mathrm{d}F(\theta). \end{aligned}$$

Combining expressions

I now combine the different expressions derived above. First, observe that

$$\begin{aligned} W^{\mathrm{U}} - W^{\varnothing} &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{\tau^*}{\theta} \, \mathbf{E}[\xi \,|\, \theta] - \frac{1}{2\theta} \, (\tau^*)^2 \right] \, \mathrm{d}F(\theta) \\ &= \tau^* \, \mathbf{E}[\xi/\theta] - \frac{1}{2} \, (\tau^*)^2 \, \mathbf{E}[1/\theta] = \frac{1}{2} \cdot \frac{\mathbf{E}^2[\xi/\theta]}{\mathbf{E}[1/\theta]}. \end{aligned}$$

Next, observe that

$$W^{\mathrm{SB}} - W^{\varnothing} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{2\theta} \, \mathbf{E}^2[\xi \,|\, \theta] \, \mathrm{d}F(\theta).$$

It follows that

$$\frac{W^{\mathrm{SB}} - W^{\varnothing}}{W^{\mathrm{U}} - W^{\varnothing}} = \frac{\mathbf{E}[1/\theta]}{\mathbf{E}^2[\xi/\theta]} \cdot \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{s} \, \mathbf{E}^2[\xi \,|\, s] \, \mathrm{d}F(s).$$

Finally, observe that

$$W^{\mathrm{FB}} - W^{\varnothing} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{2\theta} \, \mathbf{E}[\xi^2 \,|\, \theta] \, \mathrm{d}F(\theta).$$

This means that

$$W^{\mathrm{FB}} - W^{\mathrm{SB}} = \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{2\theta} \left\{ \mathbf{E}[\xi^2 \,|\, \theta] - \mathbf{E}^2[\xi \,|\, \theta] \right\} \, \mathrm{d}F(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{2\theta} \, \mathbf{Var}[\xi \,|\, \theta] \, \mathrm{d}F(\theta).$$

Moreover, it follows that

$$\frac{W^{\mathrm{FB}} - W^{\varnothing}}{W^{\mathrm{SB}} - W^{\varnothing}} = \frac{\int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{s}\, \mathbf{E}[\xi^2 \,|\, s]\ \mathrm{d}F(s)}{\int_{\underline{\theta}}^{\overline{\theta}} \frac{1}{s}\, \mathbf{E}^2[\xi \,|\, s]\ \mathrm{d}F(s)}.$$

A.2 Proofs from Section 4

Next, I derive closed-form expressions for the application to Knittel and Sandler (2018) presented in Section 4. I begin by stating a few useful facts about the lognormal distribution. I then use these facts to derive the closed-form expressions reported in Propositions 1, 2, and 3.

A.2.1 Preliminaries

First, I document a list of well-known facts about the lognormal distribution that will be useful for deriving the closed-form expressions in Propositions 1, 2, and 3. Recall that individuals have θ and ξ distributed according to a joint lognormal distribution, so that

$$\begin{bmatrix} \log\theta \\ \log\xi \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_\theta \\ \mu_\xi \end{bmatrix}, \begin{bmatrix} \sigma_\theta^2 & \rho\sigma_\theta\sigma_\xi \\ \rho\sigma_\theta\sigma_\xi & \sigma_\xi^2 \end{bmatrix} \right).$$

For simplicity, I state some results below in terms of θ, but it should be understood that they also apply symmetrically to ξ (and, more generally, any lognormally distributed random variable).

Fact 1. *The mean and variance of θ are given by*

$$\mathbf{E}[\theta] = \exp(\mu_\theta) \quad \textit{and} \quad \mathbf{Var}[\theta] = \left[\exp\left(\sigma_\theta^2\right) - 1\right] \exp\left(2\mu_\theta + \sigma_\theta^2\right).$$

Fact 2. *For any $\alpha \in \mathbb{R}$, the α^{th} moment of θ is*

$$\mathbf{E}[\theta^\alpha] = \exp\left(\alpha\mu_\theta + \frac{1}{2}\alpha^2\sigma_\theta^2 \right).$$

Fact 3. *The conditional random variable $\xi \,|\, \theta$ is also lognormally distributed with*

$$\log\xi \,|\, \theta \sim \mathcal{N}\left(\mu_\xi + \frac{\rho\sigma_\xi}{\sigma_\theta}(\log\theta - \mu_\theta), \left(1 - \rho^2\right)\sigma_\xi^2 \right).$$

A.2.2 Proof of Proposition 1

To economize on notation, let F denote the marginal lognormal distribution function of θ. Then the policymaker's problem can be written as

$$\min_{q(\cdot)\text{ non-decreasing}} \int_0^\infty \frac{\theta}{2}\left[q(\theta)-\left(A-\frac{1}{\theta}\,\mathbf{E}[\xi\mid\theta]\right)\right]^2 \,\mathrm{d}F(\theta).$$

Thus, by Facts 1 and 3, the target allocation function is

$$q^{\mathrm{T}}(\theta)=A-\frac{1}{\theta}\,\mathbf{E}[\xi\mid\theta]=A-\exp\left[\mu_\xi-\frac{\rho\sigma_\xi}{\sigma_\theta}+\frac{1}{2}\left(1-\rho^2\right)\sigma_\xi^2\right]\cdot\theta^{\rho\sigma_\xi/\sigma_\theta-1}.$$

When $\rho<\sigma_\theta/\sigma_\xi$, observe that q^{T} is a non-decreasing function; hence the optimal allocation function is simply $q^*=\max\{q^{\mathrm{T}},0\}$. Under the optimal indirect policy, an individual who consumes a quantity $q\in(0,A)$ has a slope of inverse demand curve given by

$$\theta(q)=\left[\frac{A-q}{\exp\left[\mu_\xi-\frac{\rho\sigma_\xi}{\sigma_\theta}\mu_\theta+\frac{1}{2}\left(1-\rho^2\right)\sigma_\xi^2\right]}\right]^{-\frac{1}{1-\rho\sigma_\xi/\sigma_\theta}}.$$

Consequently, the marginal price that the individual faces is equal to his utility for the marginal unit of the good consumed:

$$p(q)=(A-q)\cdot\theta(q)=\exp\left[\frac{\sigma_\theta\mu_\xi-\rho\sigma_\xi\mu_\theta+\frac{1}{2}\left(1-\rho^2\right)\sigma_\theta\sigma_\xi^2}{\sigma_\theta+\rho\sigma_\xi}\right](A-q)^{-\frac{\rho\sigma_\xi}{\sigma_\theta-\rho\sigma_\xi}}.$$

Differentiating the marginal price yields

$$p'(q)=\frac{\rho\sigma_\xi}{\sigma_\theta-\rho\sigma_\xi}\exp\left[\frac{\sigma_\theta\mu_\xi-\rho\sigma_\xi\mu_\theta+\frac{1}{2}\left(1-\rho^2\right)\sigma_\theta\sigma_\xi^2}{\sigma_\theta+\rho\sigma_\xi}\right](A-q)^{-\frac{\sigma_\theta}{\sigma_\theta-\rho\sigma_\xi}}.$$

Therefore, the marginal price is strictly increasing when $\rho>0$, in which case a quantity surcharge is optimal; and strictly decreasing when $\rho<0$; in which case a quantity discount is optimal.

A.2.3 Proof of Proposition 2

Recall from the proof of Proposition 1 that the policymaker's target allocation function is

$$q^{\mathrm{T}}(\theta) = A - \frac{1}{\theta}\,\mathbf{E}[\xi \,|\, \theta] = A - \exp\left[\mu_\xi - \frac{\rho\sigma_\xi}{\sigma_\theta} + \frac{1}{2}\left(1-\rho^2\right)\sigma_\xi^2\right] \cdot \theta^{\rho\sigma_\xi/\sigma_\theta - 1}.$$

When $\rho \geq \sigma_\theta/\sigma_\xi$, observe that q^{T} is a non-increasing function. Hence, to satisfy individuals' incentive compatibility (IC) constraints, the policymaker sets a constant allocation function $q^*(\theta) \equiv \hat{q} < A$ for all individuals. This can be implemented by setting a marginal price of zero for each unit of the good, together with a binding quantity ceiling of $\hat{q}$.

A.2.4 Proof of Proposition 3

When $\rho < \sigma_\theta/\sigma_\xi$, recall from Proposition 1 that each individual's allocation under the optimal indirect policy is

$$q^*(\theta) = \max\left\{A - \frac{1}{\theta}\,\mathbf{E}[\xi \,|\, \theta], 0\right\} = \max\left\{A - \exp\left[\mu_\xi - \frac{\rho\sigma_\xi}{\sigma_\theta} + \frac{1}{2}\left(1-\rho^2\right)\sigma_\xi^2\right] \cdot \theta^{\rho\sigma_\xi/\sigma_\theta - 1}, 0\right\}.$$

In the limit where $A \to +\infty$, the nonzero quantity constraint binds for a vanishingly small measure of individuals. Thus the assumptions of Theorem 3 hold in that limit. It follows that

$$\frac{W^{\mathrm{SB}} - W^{\varnothing}}{W^{\mathrm{U}} - W^{\varnothing}} = \frac{\mathbf{E}[1/\theta]}{\mathbf{E}^2[\xi/\theta]} \cdot \mathbf{E}\left[\frac{1}{\theta}\,\mathbf{E}^2[\xi \,|\, \theta]\right].$$

To compute $\mathbf{E}[\xi \,|\, \theta]$, use Facts 1 and 3 to obtain

$$\mathbf{E}[\xi \,|\, \theta] = \exp\left[\mu_\xi - \frac{\rho\sigma_\xi}{\sigma_\theta} + \frac{1}{2}\left(1-\rho^2\right)\sigma_\xi^2\right] \cdot \theta^{\rho\sigma_\xi/\sigma_\theta}.$$

Then, use Fact 2 to obtain

$$\begin{aligned}
\mathbf{E}\left[\frac{1}{\theta}\,\mathbf{E}^2[\xi \,|\, \theta]\right] &= \exp\left[2\mu_\xi - \frac{2\rho\sigma_\xi}{\sigma_\theta} + \left(1-\rho^2\right)\sigma_\xi^2\right] \cdot \mathbf{E}\left[\theta^{2\rho\sigma_\xi/\sigma_\theta - 1}\right] \\
&= \exp\left[2\mu_\xi - \frac{2\rho\sigma_\xi}{\sigma_\theta} + \left(1-\rho^2\right)\sigma_\xi^2\right] \cdot \exp\left[\left(\frac{2\rho\sigma_\xi}{\sigma_\theta} - 1\right)\mu_\theta + \frac{1}{2}\left(\frac{2\rho\sigma_\xi}{\sigma_\theta} - 1\right)^2 \sigma_\theta^2\right].
\end{aligned}$$

To compute $\mathbf{E}[1/\theta]$, use Fact 2 to obtain

$$\mathbf{E}[1/\theta] = \exp\left(-\mu_\theta + \frac{1}{2}\sigma_\theta^2\right).$$

To compute $\mathbf{E}[\xi/\theta]$, use the law of iterated expectations and Fact 2 to obtain

$$\begin{aligned}\mathbf{E}[\xi/\theta] &= \mathbf{E}\left[\frac{1}{\theta}\,\mathbf{E}[\xi \,|\, \theta]\right] \\ &= \exp\left[\mu_\xi - \frac{\rho\sigma_\xi}{\sigma_\theta} + \frac{1}{2}\left(1-\rho^2\right)\sigma_\xi^2\right] \cdot \mathbf{E}\left[\theta^{\rho\sigma_\xi/\sigma_\theta - 1}\right] \\ &= \exp\left[\mu_\xi - \frac{\rho\sigma_\xi}{\sigma_\theta} + \frac{1}{2}\left(1-\rho^2\right)\sigma_\xi^2\right] \cdot \exp\left[\left(\frac{\rho\sigma_\xi}{\sigma_\theta} - 1\right)\mu_\theta + \frac{1}{2}\left(\frac{\rho\sigma_\xi}{\sigma_\theta} - 1\right)^2 \sigma_\theta^2\right].\end{aligned}$$

Finally, combining these results yields the desired expression:

$$\frac{W^{\mathrm{SB}} - W^{\varnothing}}{W^{\mathrm{U}} - W^{\varnothing}} = \exp\left(\rho^2\sigma_\xi^2\right).$$

Note that Knittel and Sandler show in their Proposition 5 that

$$\frac{W^{\mathrm{FB}} - W^{\varnothing}}{W^{\mathrm{U}} - W^{\varnothing}} = \exp\left(\sigma_\xi^2\right).$$

Therefore, it also follows that

$$\frac{W^{\mathrm{FB}} - W^{\varnothing}}{W^{\mathrm{SB}} - W^{\varnothing}} = \exp\left[\left(1-\rho^2\right)\sigma_\xi^2\right].$$

A.3 Proofs from Section 5

Finally, I prove the results presented in Section 5.

A.3.1 Proof of Theorem 4

Recall from Section 5.1 that the policymaker's problem (P) reduces to the following quadratic programming problem:

$$\min_{q\in\mathcal{Q}} \int_{\underline{\theta}}^{\overline{\theta}} \left[q(\theta) - q^{\mathrm{T}}(\theta)\right]^2 \, \mathrm{d}H(\theta), \quad \text{where } q^{\mathrm{T}}(\theta) := A - \frac{c + \mathbf{E}[\xi \,|\, \theta]}{\theta}.$$

To characterize the solution to this problem, I employ the following lemma that extends the argument given in Barron (1983). To state the lemma, define the set

$$\mathcal{X} := \{x : [0,1] \to [0, A] \text{ is non-decreasing}\}.$$

Lemma 2. *Let $\chi : [0,1] \to \mathbb{R}$ be a square-integrable function. Then there exists a unique solution to the problem*

$$\min_{x \in \mathcal{X}} \int_0^1 [x(t) - \chi(t)]^2 \, \mathrm{d}t.$$

Moreover, the unique solution is given by

$$x^*(t) := \begin{cases} 0 & \text{if } \overline{\chi}(t) \leq 0, \\ A & \text{if } \overline{\chi}(t) \geq A, \\ \overline{\chi}(t) & \text{otherwise,} \end{cases} \quad \text{where } \overline{\chi}(t) := -\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{co} \int_t^1 \chi(s) \, \mathrm{d}s\right)$$

Proof of Lemma 2. Observe that $\mathcal{X} \subset L^2([0,1])$ is nonempty, closed, and convex. Therefore, by the Hilbert projection theorem, a unique projection exists for any $\chi \in L^2([0,1])$ onto $\mathcal{X}$.

Next, to show that its unique solution is given by x^*, begin by assuming that $\overline{\chi}(t) > 0$ for all $t \in [0,1]$. Define

$$\mathcal{I} := \{x : [0,1] \to \mathbb{R} \text{ is non-decreasing}\}.$$

Let $\Pi_{\mathcal{I}}\chi$ denote the projection of χ onto $\mathcal{I}$. Since $\mathcal{I} \subset L^2([0,1])$ is also nonempty, closed, and convex, the projection is unique, hence $\Pi_{\mathcal{I}}\chi$ is well-defined. Since $\mathcal{I}$ is a convex cone,

$$\int_0^1 [\Pi_{\mathcal{I}}\chi(s) - \chi(s)] \, x(s) \, \mathrm{d}s \geq 0 \quad \text{for any } x \in \mathcal{I}.$$

In particular, choose $x(s) = \mathbf{1}_{s>t}$ for some $t \in [0,1]$. Then the above implies that

$$\overline{X}(t) := \int_t^1 \Pi_{\mathcal{I}}\chi(s) \, \mathrm{d}s \geq \int_t^1 \chi(s) \, \mathrm{d}s =: X(t).$$

Since $\Pi_{\mathcal{I}}\chi$ is non-decreasing on $[0,1]$, $\overline{X}$ must be concave; hence $\overline{X}$ is a concave majorant of X. Now, if $\overline{X}$ is not the *least* concave majorant of X, then there exist $0 < s_1 < s_2 < 1$ and a separating linear function $\ell(s)$, such that $\ell(s) \geq X(s)$ for $s \in [0,1]$; $\overline{X}(s) > \ell(s)$ for $s \in (s_1, s_2)$; and $\overline{X}(s_i) = \ell(s_i)$ for $i = 1, 2$. Define $\widetilde{\chi}$ by $\widetilde{\chi}(s) = \Pi_{\mathcal{I}}\chi(s)$ for $s \notin (s_1, s_2)$ and $\widetilde{\chi}(s) = \ell'(s)$ for

$s \in (s_1, s_2)$. Then integration by parts yields the following contradiction:

$$\begin{aligned} 0 &\leq \int_0^1 \left[\Pi_{\mathcal{I}}\chi(s) - \chi(s)\right]\left[\widetilde{\chi}(s) - \Pi_{\mathcal{I}}\chi(s)\right]\, \mathrm{d}s \\ &= -\int_0^1 \left[X(s) - \overline{X}(s)\right]\, \mathrm{d}\left[\widetilde{\chi}(s) - \Pi_{\mathcal{I}}\chi(s)\right] = \int_{s_1}^{s_2} \left[X(s) - \overline{X}(s)\right] \mathrm{d}\Pi_{\mathcal{I}}\chi(s) < 0. \end{aligned}$$

Therefore $\overline{X}$ is the least concave majorant of X. It follows that $\Pi_{\mathcal{I}}\chi = -\left(\mathrm{co}\, X\right)' = \overline{\chi}$.

It remains to consider the case where either $\overline{\chi}(t) \leq 0$ or $\overline{\chi}(t) \geq A$ for some $t \in [0, 1]$. Define κ so that $\overline{\chi}(t) \leq 0$ for all $t \in [0, \kappa)$; and $0 < \overline{\chi}(t)$ for all $t \in (\kappa, 1]$. Such a $\kappa \in [0, 1]$ exists since $\overline{\chi}$ is non-decreasing by construction. The argument above shows that $\overline{\chi}$ minimizes the integral between κ and 1:

$$\overline{\chi} \in \arg\min_{x \in \mathcal{X}} \int_\kappa^1 \left[x(s) - \chi(s)\right]^2\, \mathrm{d}s.$$

Since $\mathcal{X} \subset \mathcal{I}$, the argument above also shows that the constraint $x(t) \geq 0$ must bind for $t \in [0, \kappa]$. The case where $\overline{\chi}(t) \geq A$ can be similarly dealt with. This yields the solution x^* as claimed. $\square$

I now use the result of Lemma 2 to prove Theorem 4. To do so, I employ the following change of variables:

$$x(t) := q(H^{-1}(t)) \quad \text{and} \quad \chi(t) := q^{\mathrm{T}}(H^{-1}(t)) \quad \text{for every } t \in [0, 1].$$

Then, using the result of Lemma 2, define

$$\begin{aligned} \overline{\chi}(t) &:= -\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{co} \int_t^1 \chi(s)\, \mathrm{d}s\right) \\ &= -\frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{co} \int_t^1 q^{\mathrm{T}}(H^{-1}(s))\, \mathrm{d}s\right) \quad \text{for every } t \in [0, 1]. \end{aligned}$$

Consequently,

$$\overline{\chi}(H(\theta)) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{co} \int_{1-t}^1 q^{\mathrm{T}}(H^{-1}(s))\, \mathrm{d}s\right)\bigg|_{t = 1 - H(\theta)} \quad \text{for every } \theta \in [\underline{\theta}, \overline{\theta}].$$

Reversing the change of variables, define $\overline{q}^{\mathrm{T}} := \overline{\chi} \circ H$ and observe that

$$\min_{x \in \mathcal{X}} \int_0^1 [x(t) - \chi(t)]^2 \, \mathrm{d}t = \min_{q \in \mathcal{Q}} \int_0^1 \left[q(H^{-1}(t)) - q^{\mathrm{T}}(H^{-1}(t))\right]^2 \, \mathrm{d}t$$
$$= \min_{q \in \mathcal{Q}} \int_{\underline{\theta}}^{\overline{\theta}} \left[q(\theta) - q^{\mathrm{T}}(\theta)\right]^2 \, \mathrm{d}H(\theta).$$

Therefore, the result of Lemma 2 implies that the unique optimal allocation function for the policymaker's problem (P) q^* is given by

$$q^*(\theta) = \begin{cases} 0 & \text{if } \overline{q}^{\mathrm{T}}(\theta) \leq 0, \\ A & \text{if } \overline{q}^{\mathrm{T}}(\theta) \geq A, \\ \overline{q}^{\mathrm{T}}(\theta) & \text{otherwise.} \end{cases}$$

A.3.2 Proof of Theorem 5

I start by deriving the policymaker's problem that incorporates distributional considerations. By the envelope theorem (Milgrom and Segal, 2002), each individual's utility under any incentive-compatible policy is given by

$$u(\theta) = u(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \left[Aq(s) - \frac{1}{2}[q(s)]^2\right] \, \mathrm{d}s.$$

Without loss of generality, set $u(\underline{\theta}) = 0$ since uniform transfers between the policymaker and all individuals are welfare-neutral. Hence the total weighted surplus of all individuals (i.e., weighted consumer surplus) is

$$\int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\omega \,|\, \theta] u(\theta) \, \mathrm{d}F(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \mathbf{E}[\omega \,|\, \theta] \int_{\underline{\theta}}^{\theta} \left[Aq(s) - \frac{1}{2}[q(s)]^2\right] \, \mathrm{d}s \, \mathrm{d}F(\theta)$$
$$= \int_{\underline{\theta}}^{\overline{\theta}} \int_{\theta}^{\overline{\theta}} \mathbf{E}[\omega \,|\, s] \, \mathrm{d}F(s) \left[Aq(\theta) - \frac{1}{2}[q(\theta)]^2\right] \, \mathrm{d}\theta.$$

Meanwhile, the policymaker's revenue is

$$\int_{\underline{\theta}}^{\overline{\theta}} [t(\theta) - cq(\theta)] \, \mathrm{d}F(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left[(\theta A - c) \, q(\theta) - \frac{\theta}{2} [q(\theta)]^2 \right] \, \mathrm{d}F(\theta) \\ - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\theta}^{\overline{\theta}} \, \mathrm{d}F(s) \left[Aq(\theta) - \frac{1}{2} [q(\theta)]^2 \right] \, \mathrm{d}\theta.$$

Thus the policymaker's objective function is

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[(\theta A - c - \mathbf{E}[\xi \mid \theta]) \, q(\theta) - \frac{\theta}{2} [q(\theta)]^2 - \Delta(\theta) \left[Aq(\theta) - \frac{1}{2} [q(\theta)]^2 \right] \right] \, \mathrm{d}F(\theta),$$

where, as in Section 5.2,

$$\Delta(\theta) := \frac{\int_{\theta}^{\overline{\theta}} [1 - \mathbf{E}[\omega \mid s]] \, \mathrm{d}F(s)}{f(\theta)}.$$

Thus, by completing the square, the policymaker's problem is equivalent to

$$\min_{q \in \mathcal{Q}} \int_{\underline{\theta}}^{\overline{\theta}} [\theta - \Delta(\theta)] \left[q(\theta) - \left[A - \frac{c + \mathbf{E}[\xi \mid \theta]}{\theta - \Delta(\theta)} \right] \right]^2 \, \mathrm{d}F(\theta) \\ = \min_{q \in \mathcal{Q}} \int_{\underline{\theta}}^{\overline{\theta}} \left[q(\theta) - \left[A - \frac{c + \mathbf{E}[\xi \mid \theta]}{\theta - \Delta(\theta)} \right] \right]^2 \, \mathrm{d}H(\theta).$$

Consequently, as in the statement of Theorem 5, define the target allocation function to be

$$q^{\mathrm{T}}(\theta) := A - \frac{c + \mathbf{E}[\xi \mid \theta]}{\theta - \Delta(\theta)}.$$

The result of Theorem 5 follows from Lemma 2, as in the proof of Theorem 4.

A.3.3 Proof of Proposition 4

As in Section 3.1, define

$$\phi(t) := \int_{H^{-1}(1-t)}^{1} \left[A - \frac{c + \mathbf{E}[\xi]}{s - \Delta(s)} \right] \, \mathrm{d}H(s).$$

I begin by showing that, if $\mathbf{E}[\omega \mid \overline{\theta}] \leq 2$, then $\phi = \operatorname{co}\phi$. Indeed, observe that

$$\begin{aligned}\frac{\mathrm{d}q^{\mathrm{T}}}{\mathrm{d}\theta}(\overline{\theta}) &= \frac{(c+\mathbf{E}[\xi])\left[1-\Delta'(\overline{\theta})\right]}{\left[\overline{\theta}-\Delta(\overline{\theta})\right]^2} \\ &= \frac{c+\mathbf{E}[\xi]}{\left[\overline{\theta}-\Delta(\overline{\theta})\right]^2}\cdot\left[2-\mathbf{E}[\omega \mid \overline{\theta}]\right] \geq 0.\end{aligned}$$

Recall that $\theta \mapsto \theta - \Delta(\theta)$ is strictly quasiconcave by Assumption 2, so there exists some $\theta^* \in [\underline{\theta}, \overline{\theta}]$ such that $\theta \mapsto \theta - \Delta(\theta)$ is increasing on $[\underline{\theta}, \theta^*]$ and decreasing on $[\theta^*, \overline{\theta}]$. But q^{T} is increasing if and only if $\theta \mapsto \theta - \Delta(\theta)$ is increasing. Since q^{T} is increasing at $\theta = \overline{\theta}$, thus $\theta^* = \overline{\theta}$: that is, $\theta \mapsto \theta - \Delta(\theta)$ is increasing on the entire interval $[\underline{\theta}, \overline{\theta}]$. It follows that $\phi = \operatorname{co}\phi$, as claimed. As such, $q^* = \max\{q^{\mathrm{T}}, 0\}$ by Theorem 5.

Next, consider an individual with $\theta = \hat{\theta}$, and suppose that the individual consumes a positive amount of the good. Similar to Theorem 1, this individual faces a marginal price of

$$U'(q^*(\hat{\theta});\hat{\theta}) = \hat{\theta}\left[A(\hat{\theta}) - q^*(\hat{\theta})\right] = \frac{\hat{\theta}}{\hat{\theta}-\Delta(\hat{\theta})}\left(c+\mathbf{E}[\xi]\right).$$

Since q^* is increasing, hence the marginal price is increasing when $\theta \mapsto \theta/\left[\theta - \Delta(\theta)\right]$ is increasing; conversely, the marginal price is decreasing when $\theta \mapsto \theta/\left[\theta - \Delta(\theta)\right]$ is decreasing. Finally, note that

$$\frac{\theta}{\theta-\Delta(\theta)} = \frac{1}{1-\Delta(\theta)/\theta}.$$

Hence $\theta \mapsto \theta/\left[\theta - \Delta(\theta)\right]$ is increasing if and only if $\theta \mapsto \Delta(\theta)/\theta$ is increasing. It follows that the individual experiences a increasing marginal price (and hence a quantity surcharge) when $\theta \mapsto \Delta(\theta)/\theta$ is increasing at $\theta = \hat{\theta}$, and a decreasing marginal price (and hence a quantity discount) when $\theta \mapsto \theta/\left[\theta - \Delta(\theta)\right]$ is decreasing at $\theta = \hat{\theta}$.

A.3.4 Proof of Proposition 5

Similar to the proof of Proposition 4, observe that if $\mathbf{E}[\omega \mid \overline{\theta}] > 2$, then

$$\begin{aligned}\frac{\mathrm{d}q^{\mathrm{T}}}{\mathrm{d}\theta}(\overline{\theta}) &= \frac{(c+\mathbf{E}[\xi])\left[1-\Delta'(\overline{\theta})\right]}{\left[\overline{\theta}-\Delta(\overline{\theta})\right]^2} \\ &= \frac{c+\mathbf{E}[\xi]}{\left[\overline{\theta}-\Delta(\overline{\theta})\right]^2}\cdot\left[2-\mathbf{E}[\omega \mid \overline{\theta}]\right] < 0.\end{aligned}$$

Because $\theta \mapsto \theta - \Delta(\theta)$ is strictly quasiconcave by Assumption 2, there exists some $\theta_*, \theta^* \in [\underline{\theta}, \overline{\theta}]$ such that $\theta \mapsto \theta - \Delta(\theta)$ is strictly increasing on $[\underline{\theta}, \theta_*]$ and strictly decreasing on $[\theta^*, \overline{\theta}]$. But q^{T} is (strictly) decreasing if and only if $\theta \mapsto \theta - \Delta(\theta)$ is (strictly) decreasing. Since q^{T} is strictly decreasing at $\theta = \overline{\theta}$, thus $\theta^* < \overline{\theta}$. By Theorem 5, each individual with $\theta > \theta^*$ who consumes a positive quantity of the good faces a binding quantity ceiling.

Appendix B Other policy applications

In this appendix, I discuss how my results can be applied to the indirect regulation of externalities in different markets. This supplements Section 4, where I apply my results to Knittel and Sandler's (2018) study on the use of gasoline taxes to regulate emissions.

B.1 Combat Methamphetamine Epidemic Act

One setting that my modeling assumptions and results apply to is the market for drugs in the United States that contain ephedrine or pseudoephedrine. These drugs include common over-the-counter allergy, cold, and cough medications such as Claritin and Sudafed. Since 2006, these drugs have been regulated under the Combat Methamphetamine Epidemic Act, which places restrictions on quantities that can be purchased daily and within a 30-day period.

The premise of my model—that individual elasticity and externality are correlated—is likely to hold in this setting. Individuals who buy these medications can be crudely classified into one of two categories: actual users and illicit peddlers who resell to clandestine labs. Actual users are likely to be much more price-inelastic on average than illicit peddlers, and all of the externality associated with the sale of these medications are generated by illicit peddlers.

As the correlation between elasticity and externality is likely to be substantial and such that the average externality conditional on elasticity is steeply increasing, my results support the use of quantity ceilings in this setting. By contrast, a uniform tax is likely to perform poorly: actual users of the drug would be subject to the same marginal price as illicit peddlers despite generating none of the externality. In the extreme, if the average externality was high enough relative to the value of actual users, actual users could be driven out of the market under a uniform tax, akin to an adverse selection "death spiral."

B.2 Alcohol taxes

My modeling assumptions and results also provide some intuitions for alcoholic beverage markets. Griffith et al. (2019) show empirically with U.K. data that problematic alcohol consumption is correlated with alcohol consumption patterns: heavy drinkers who create the majority of the alcohol-associated external costs tend to prefer cheaper beverages with higher alcoholic content. Given their findings, my results suggest that taxes on alcoholic beverages should vary nonlinearly with alcohol proof. However, there are also important aspects of alcoholic beverage markets that my model does not capture. One such example is product differentiation, which induces substitution between different kinds of alcoholic beverages and between different alcoholic and non-alcoholic beverages. Individuals thus have multi-dimensional preferences that a single elasticity parameter might not adequately capture. Regardless, intuitions from my results—that the optimal indirect policy exploits these correlations between individual preferences and externalities, for instance—nonetheless extend to this setting.

B.3 Vaccine subsidies

Another setting that fits the outlines of my modeling assumptions and results is vaccine markets. To see how, imagine a vaccine that both protects the vaccinated individual and makes them less contagious to others. Individuals vary in their fear of the disease: more fearful individuals are more willing to pay to be vaccinated, more likely to wear masks that protect themselves and others, and more likely to avoid crowds where the disease can spread. In this way, individual value for the vaccine is negatively correlated with the positive externality that arises from vaccination. Therefore, at first blush, my results appear to suggest applying a nonlinear subsidy for vaccines.

However, more careful consideration reveals that there are important differences between my model and vaccine markets. Crucially, given that individuals have discrete demand for vaccines, it is unclear what nonlinear subsidies might entail. In fact, individual value for vaccines might be more appropriately modeled by a linear utility model with unit demand, rather than my model which assumes that individuals have diminishing marginal utility for the good.

These differences motivate a slightly different model to describe markets which, similar to vaccine markets, have discrete demand. Other examples include housing, health insurance, and vehicle ownership. My results on the optimality of quantity controls (cf. Section 5.1) have a direct application to such markets in the form of rationing. In the context of vaccines, this means that the optimal vaccine subsidy might require awarding a larger subsidy than the market-clearing subsidy when the negative correlation between value and externality is sufficiently strong, resulting in the rationing of vaccines. This is in contrast to policy proposals for a uniform vaccine subsidy (Mankiw, 2020), as well as work that shows how rationing might be optimal because of redistributive concerns (Akbarpour ⓡ al., 2021).