Bargaining, Efficiency, and Pareto Improvements

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Motivation: The Bulow Puzzle

Alice and Bob are bargaining over a perfectly divisible pie.

Both are privately informed about value for the pie: $v_A, v_B \overset{\text{iid}}{\sim} \mathcal{U}(0, 1)$.

Both pay the same waiting cost per unit time, $c > 0$.

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When are such Pareto improvements possible? What protocol maximizes ex ante surplus?

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$\rightsquigarrow$ the mechanism can offer **discounts** and/or **make-whole payments**.

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#1. convexity of demand determines when strict Pareto improvements are possible; and

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$\rightsquigarrow$ Our results also identify optimal bargaining protocols that Pareto-improve on WoA.

Model

Setup

- There is a mass of risk-neutral consumers with unit demand for an indivisible good. Consumers differ in value, $v \in [\underline{v}, \overline{v}]$; CDF is F with density $f > 0$, demand is $1 - F$.

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- There is a principal with mass $Q < 1$ of the good; chooses a mechanism (x, t), with:
  - the allocation function $x : [\underline{v}, \overline{v}] \to [0, 1]$, denoting probability of allocation; and
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- The principal's objective is a weighted sum of consumer surplus and revenue:

$$(1-\alpha)\cdot \underbrace{\int_{\underline{v}}^{\overline{v}} [vx(v) - t(v)]\, \mathrm{d}F(v)}_{\text{consumer surplus}} + \alpha \cdot \underbrace{\int_{\underline{v}}^{\overline{v}} t(v)\, \mathrm{d}F(v)}_{\text{revenue}}, \qquad \alpha \in [0, 1].$$

Special cases: consumer surplus ($\alpha = 0$), total surplus ($\alpha = 1/2$), revenue ($\alpha = 1$).

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Special cases: $\underbrace{\text{consumer surplus } (\alpha = 0)}_{\text{alt. interpretation: money burning}}$, total surplus ($\alpha = 1/2$), revenue ($\alpha = 1$).

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When make-whole payments are not allowed, we impose the constraint:

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#2. Pareto Improvement

There is a status quo mechanism (x_0, t_0).

The mechanism (x, t) is required to be a Pareto improvement over (x_0, t_0):

$$vx(v) - t(v) \geq vx_0(v) - t_0(v) \qquad \forall\, v \in [\underline{v}, \overline{v}].$$

Mechanism Design

In summary, the principal maximizes a weighted sum of consumer surplus and revenue:

$$\max_{(x,t)} \left[(1-\alpha) \cdot \int_{\underline{v}}^{\overline{v}} [vx(v) - t(v)] \, \mathrm{d}F(v) + \alpha \cdot \int_{\underline{v}}^{\overline{v}} t(v) \, \mathrm{d}F(v) \right],$$

subject to

- incentive compatibility, $\theta \in \arg\max_{\hat{v} \in [\underline{v}, \overline{v}]} [vx(\hat{v})) - t(\hat{v})] \quad \forall\, v \in [\underline{v}, \overline{v}]; \quad \text{(IC)}$
- capacity, $\int_{\underline{v}}^{\overline{v}} x(v) \, \mathrm{d}F(v) \leq Q; \quad \text{(C)}$
- no make-whole payments (sometimes relaxed), $t(v) \geq 0 \quad \forall\, v \in [\underline{v}, \overline{v}]; \quad \text{(MW)}$
- individual rationality, $vx(v) - t(v) \geq vx_0(v) - t_0(v) \quad \forall\, v \in [\underline{v}, \overline{v}]. \quad \text{(PI)}$

Related Work

- **Mechanism Design.**
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- **Application: Bargaining.**
  - Generalized war of attrition: Bulow and Klemperer (1999).
  - Mechanisms and status quos as offers: Strulovici (2017); Pęski (2022, 2024).

  $\rightsquigarrow$ **This paper:** use mechanism design to analyze optimal protocol instead of solving for equilibrium.

General Results

Optimal Pareto-Improving Allocation

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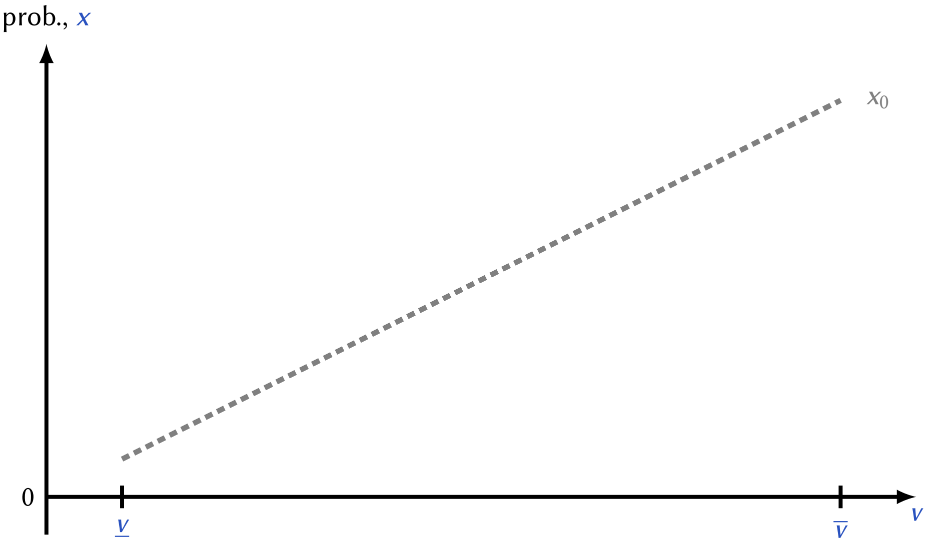

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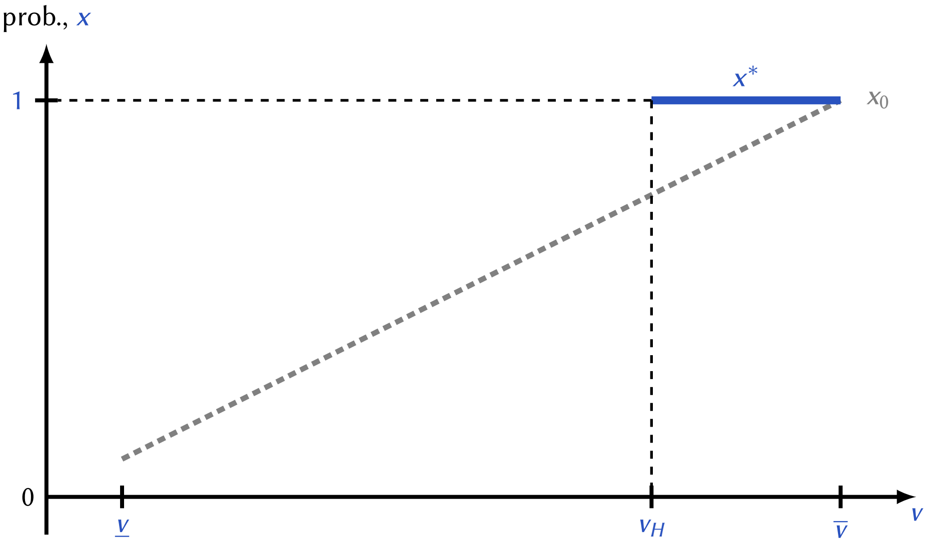

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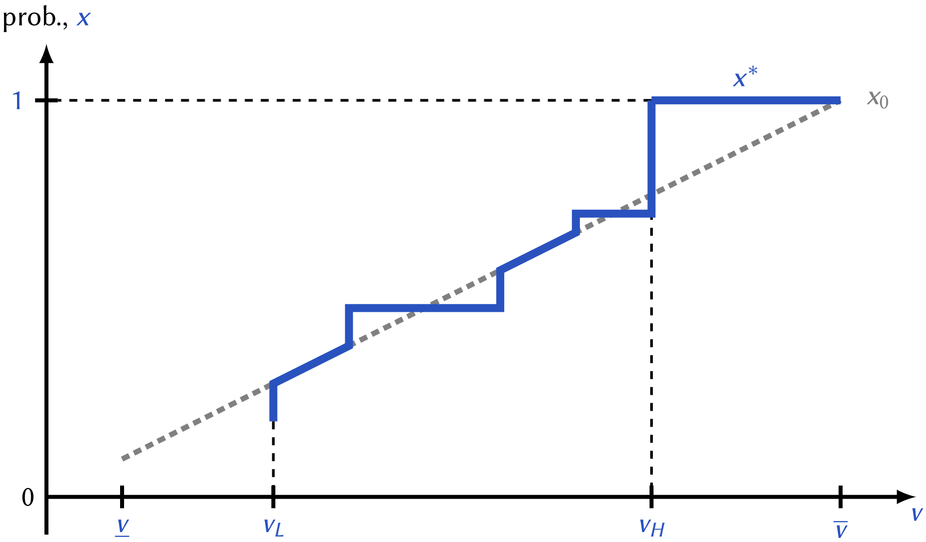

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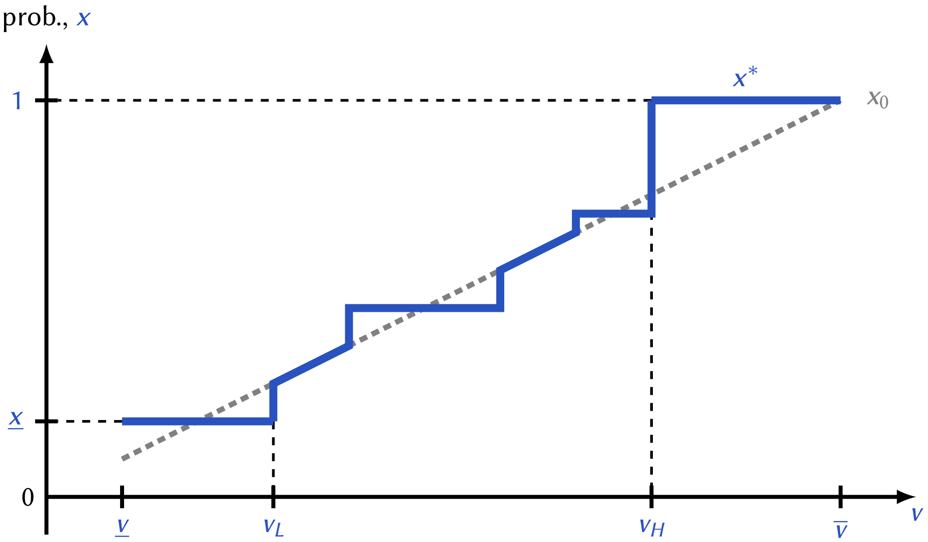

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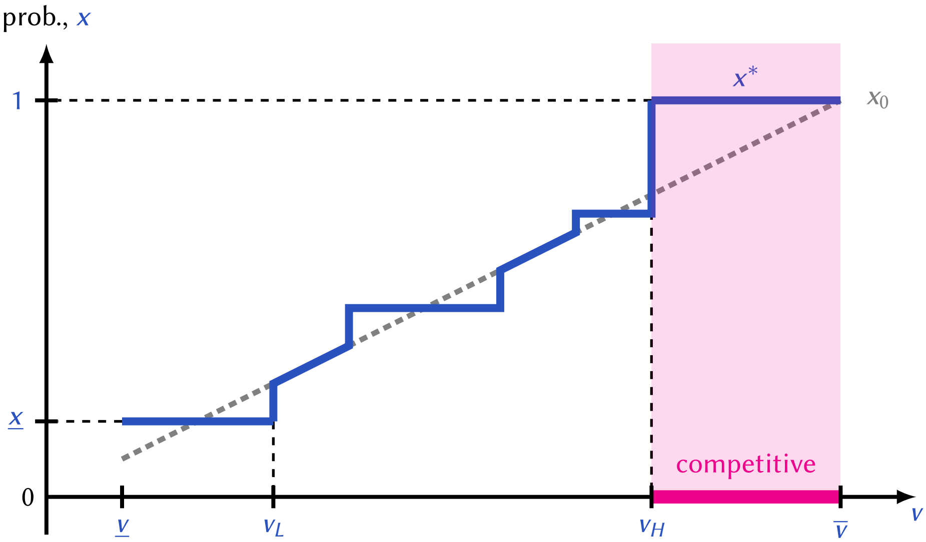

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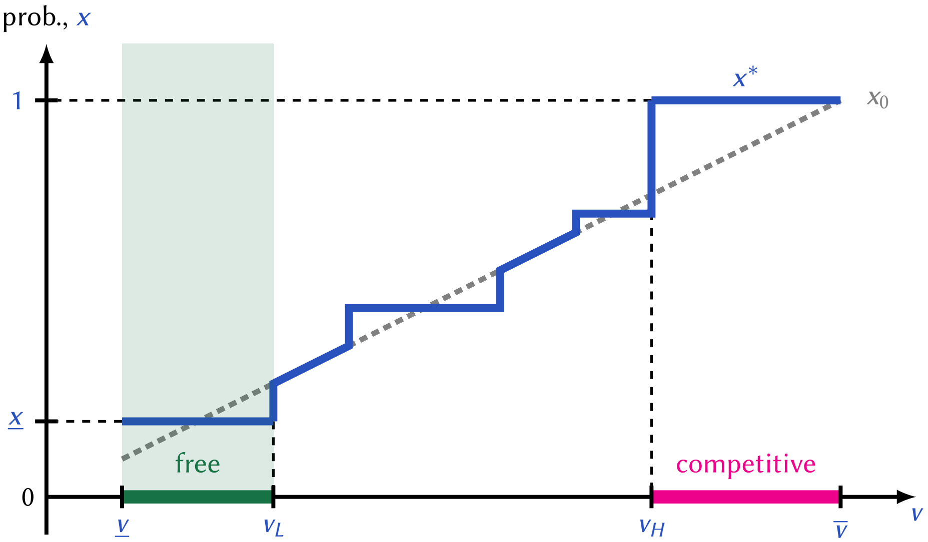

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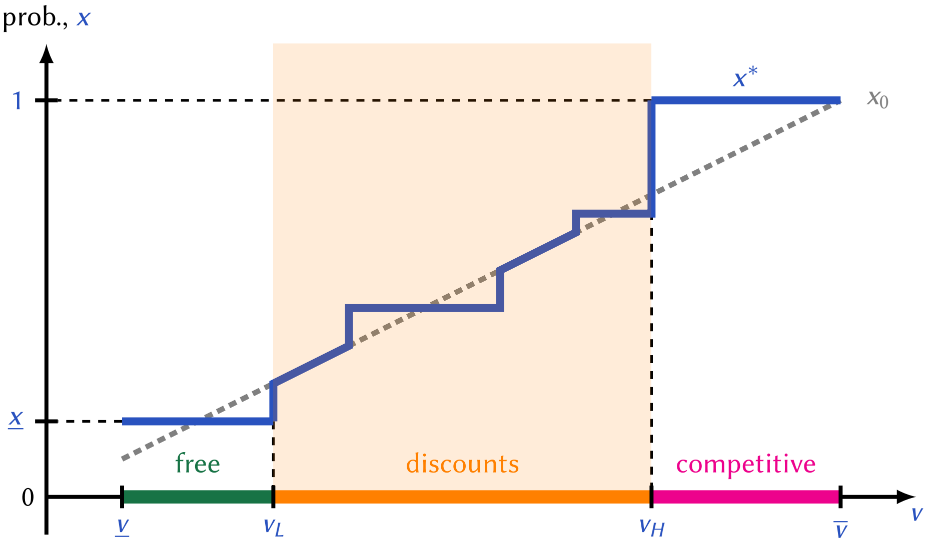

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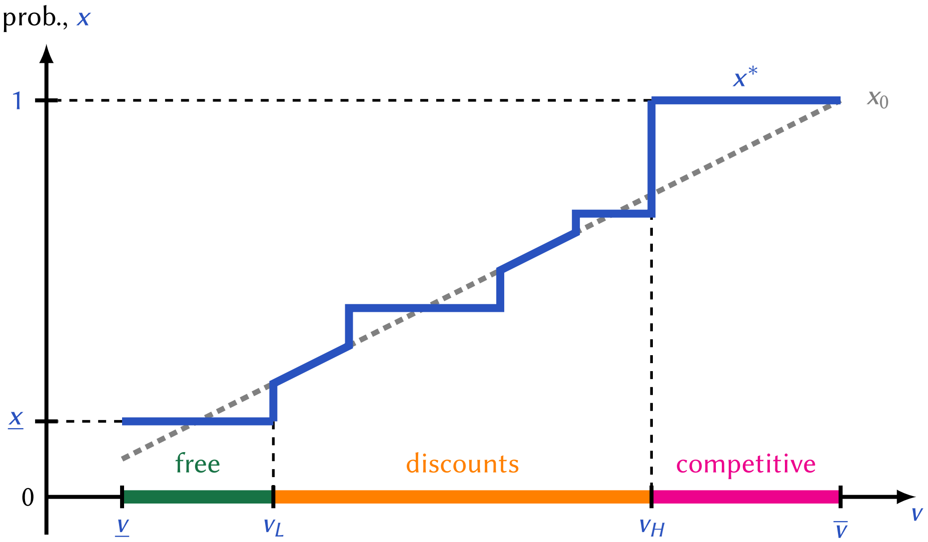

Proof Remarks

To prove this technical result, we adapt Lagrangian approach of Amador and Bagwell (2013):

#1. Guess optimal Lagrange multipliers for (C), (MW), and (PI) constraints.

#2. Solve for optimal mechanism and verify that these constraints are satisfied.

#3. Verify that complementary slackness conditions are satisfied.

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- $\rightsquigarrow$ For $\alpha \geq 1/2$, we derive a similar characterization without the (MW) constraint.

Dworczak and Muir (2024) prove a related characterization with only (PI) constraints.

- $\rightsquigarrow$ (MW) introduces possibility of free allocation; but (C) restricts free allocation.
- $\rightsquigarrow$ (MW) and (C) interact with (PI); our characterization solves this fixed-point problem.

Money Burning ($\alpha = 0$)

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Proposition 1. If demand is concave, given any status quo, the CS-maximizing Pareto-improving mechanism sets a **price of zero** and **allocates goods uniformly at random**.

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Interpretation:

- Without Pareto improvement constraint, random allocation maximizes aggregate consumer surplus if demand is log-concave (Bulow and Klemperer, 2012).
- With Pareto improvement constraint, concavity (rather than log-concavity) matters.

Proposition 2. If $\underline{v} > 0$, given any status quo, the CS-maximizing Pareto-improving mechanism allocates a **positive mass** of goods at a price of zero.

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Interpretation:

- (MW) constraint always binds: principal wishes to pay consumers to increase CS.
- Since principal cannot pay consumers, the next-best instrument is free allocation.

Proposition 3. Given any status quo mechanism (x_0, t_0), the CS-maximizing Pareto-improving mechanism offers certain allocation to a positive mass of consumer with values $v \notin x_0^{-1}(1)$ if and only if

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Necessity of Non-Competitive Allocation

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Interpretation:

- If Q remains same as status quo, then (C) constraint prevents competitive allocation.
- (C) constraint always binds: principal can increase CS by allocating more of the good.

We are currently working on comparative statics:

#1. Capacity expansions.

Suppose (x_0, t_0) is an optimal mechanism for quantity Q_0.

What is the Pareto-improving mechanism when capacity expands to $Q > Q_0$?

Comparative Statics

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What is the Pareto-improving mechanism when capacity expands to $Q > Q_0$?

#2. Shifts in demand.

Suppose (x_0, t_0) is an optimal mechanism for the demand curve $1 - F_0$.

What is the Pareto-improving mechanism when demand shifts to $1 - F \prec_{\text{FOSD}} 1 - F_0$?

Discussion

The Bulow Puzzle Revisited

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Both are privately informed about value for the pie: $v_A, v_B \overset{\text{iid}}{\sim} F$, density f with support $[0, 1]$.

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Nevertheless, our analysis can be applied to resolve the Bulow puzzle.

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Ongoing work: generalized war of attrition (N pies, $N + K$ players); suggestions welcome!

The Bulow Puzzle Resolved

Regularity assumption: the density f is log-concave.

This is common in economics: see, e.g., An (1998) and Bagnoli and Bergstrom (2005).

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Theorem. Under the regularity assumption:

(i) When F is convex, the optimal Pareto-improving bargaining mechanism is an immediate 50–50 split between both players.

(ii) When F is strictly concave, no bargaining mechanism strictly Pareto-dominates the unique symmetric equilibrium of the war of attrition.

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In this sense, Bulow's original uniform distribution example is the knife-edge case!

Concluding Remarks

Status quo allocations can restrict feasibility of institutional changes.

- In bargaining, non-Pareto-improving changes to protocol might signal weakness.
- In political economy, non-Pareto-improving changes may face holdouts from some.

This paper: mechanism design + Pareto improvement constraints.

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Alternative interpretation: characterization of Pareto frontier for mechanisms.

- "Pareto undominatedness" seems like a weak criterion to impose on mechanisms.
- We show that such a criterion can sometimes impose considerable structure.

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