Pareto-Improving Priority Pricing

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Motivation

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- Internet traffic;
- vehicular traffic; and
- wait lists (e.g., public housing, healthcare, DMV).

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Public debates on **priority systems** often highlight the **equity-efficiency tradeoff**:

- poorer agents might be harmed while richer agents benefit, or vice versa.

Equity-Efficiency Tradeoff Illustrated

Why Some New York City Residents Are Suing Over Congestion Pricing

Their lawsuits argue that the tolling program would shift traffic and pollution to poor and minority neighborhoods and hurt small businesses.

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Congestion pricing in Manhattan is scheduled to start on June 30, but a slate of lawsuits seek to thwart those plans. Andrea Mohin/The New York Times

By **Winnie Hu**

Published May 16, 2024 Updated May 19, 2024

This Paper

We study priority systems that **Pareto-improve** on an **unpriced** status quo.

Pareto improvements $\rightarrow$ all agents are at least weakly better off + budget balance.

Possibility of Pareto improvements depends on what priority systems are feasible.

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Main Results:

#1. When there is "friction," no Pareto improvement with 2 priority tiers exists.

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#1. When there is "friction," no Pareto improvement with 2 priority tiers exists.

$\rightsquigarrow$ 1 fast lane + 1 slow lane are not enough; some agents will be worse off.

#2. When there is "richness," a Pareto improvement with 3 priority tiers exists.

Setup

There is a unit mass of risk-neutral agents waiting in line; waiting is costly.

Agents have heterogeneous MRS between time and money, $r \in [\underline{r}, \overline{r}]$:

$$\text{utility} = V - r \cdot \mathbf{E}[\tau] - p, \quad \text{where} \begin{cases} \tau & = \text{(random) wait time,} \\ p & = \text{expected payment.} \end{cases}$$

In applications, we allow $\mathbf{E}[c(\tau)]$ instead of $\mathbf{E}[\tau]$ by a change of variables.

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Assumption. r is distributed according to G, which has positive density g on $[\underline{r}, \overline{r}]$.

Setup: Principal

There is a principal who chooses a distribution (CDF) $F : \mathbb{R}_+ \to [0, 1]$ of wait times, τ.

Wait times are allocated to agents in a **(possibly) stochastic**, **IC** mechanism with transfers.

- Agents with higher MRS are allocated a lower average wait time with higher payment.
- **Envelope theorem**: payments are pinned down up to constant by average wait times.

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Approach:

#1. impose axioms on $\mathbb{F}$ that hold in applications (not necessary altogether);

#2. derive general results under different axioms on $\mathbb{F}$ (in different combinations); and

#3. interpret these general results in the context of specific applications.

Theoretical Results

Laissez-Faire Benchmark

Axiom LF (feasibility of laissez-faire).

There exists $t_0 \in \mathbb{R}_+$ such that:

(i) $F_0(\tau) = \mathbf{1}_{\tau \geq t_0} \in \mathbb{F}$; and

(ii) $\mathbf{E}_F[\tau] \geq t_0$ for every $F \in \mathbb{F}$.

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- Under the **unpriced LF benchmark**, each agent has the same expected wait time, t_0.
  t_0 minimizes the average waiting time among all feasible distributions of wait times.
- **Axiom LF** states that the laissez-faire benchmark is feasible.

This paper: we seek to Pareto-improve on the laissez-faire benchmark.

Axiom F (infeasibility in environments with friction).

If $F \in \mathbb{F}$ and $\hat{F} \succeq_{\text{MPS}} F$ (with $\hat{F} \neq F$), then $\hat{F} \notin \mathbb{F}$.

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  There is a tradeoff (due to some “friction”) between average wait time and dispersion.

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- In applications, **Axiom F** holds when:
  - each agent's disutility from waiting is strictly convex in wait times (**Application 1**); or
  - each agent's wait time depend on others' allocations due to congestion (**Application 2**).

Priority Tiers

Definition (priority tiers).

A distribution of wait times F has n **priority tiers** if there exist $0 \leq t_1 < t_2 < \cdots < t_n$ such that

$$F(\tau) = y_1 \cdot \mathbf{1}_{\tau \geq t_1} + y_2 \cdot \mathbf{1}_{\tau \geq t_2} + \cdots + y_n \cdot \mathbf{1}_{\tau \geq t_n},$$

where $0 \leq y_1, y_2, \ldots, y_n \leq 1$ and $y_1 + y_2 + \cdots + y_n = 1$.

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Question. How many priority tiers are required to achieve a Pareto improvement?

Impossibility of Two-Tiered Pareto Improvements

Theorem 1.

Under **Axioms LF** and **F**, there is no feasible, deterministic, and budget-balanced Pareto improvement over the laissez-faire benchmark with 2 priority tiers.

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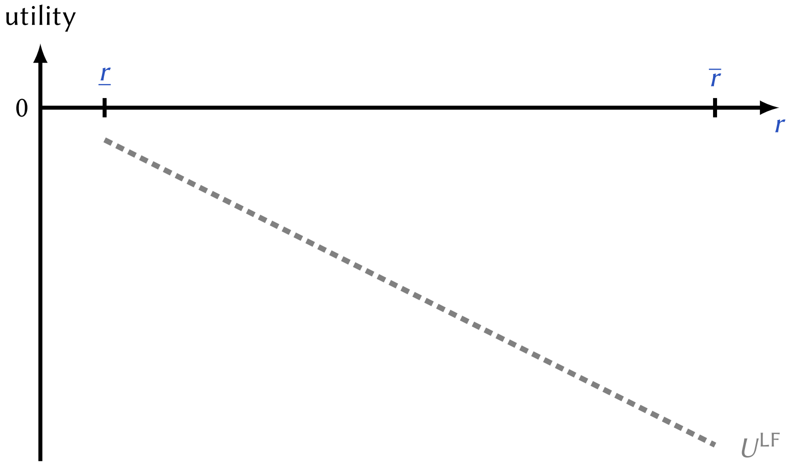
- **Proof idea:** with 2 priority tiers, BB + PI $\Rightarrow$ MPS of F_0, which is infeasible.

Proof of Theorem 1



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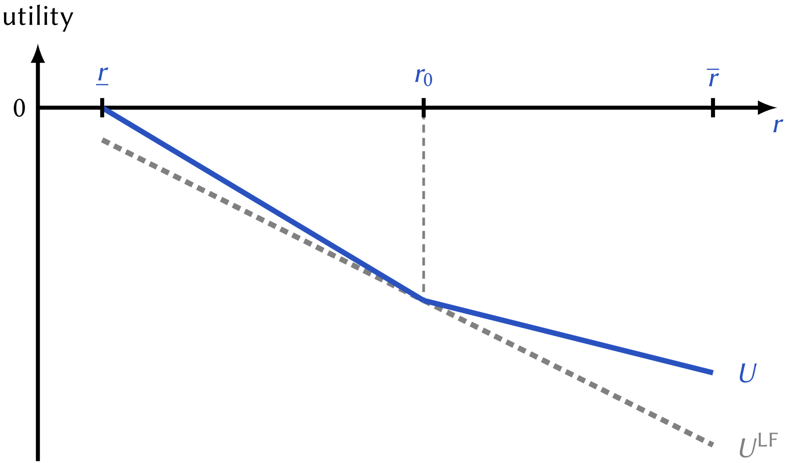

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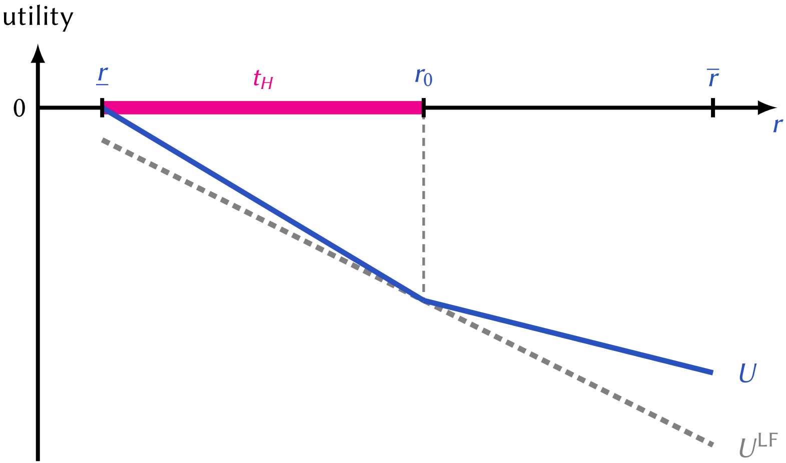

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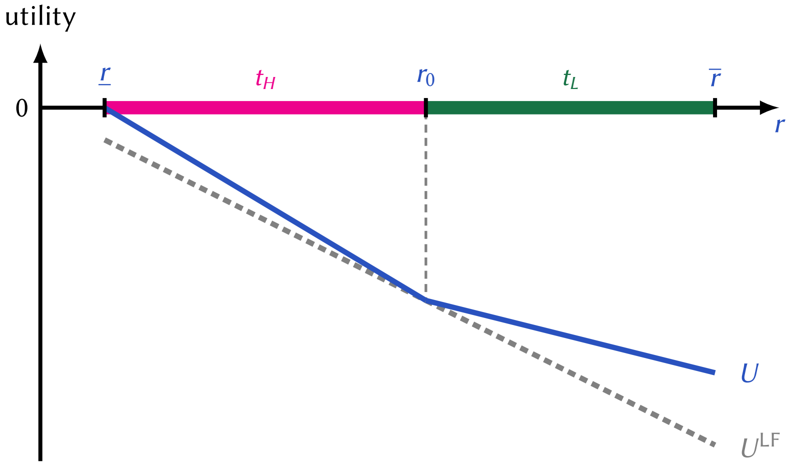

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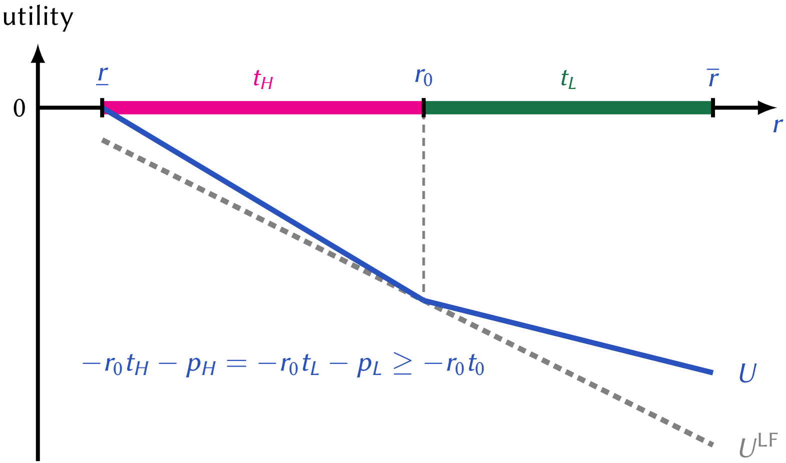

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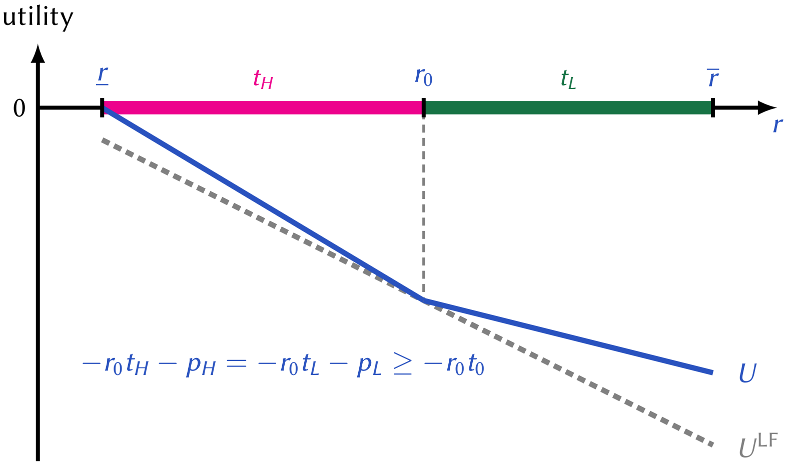

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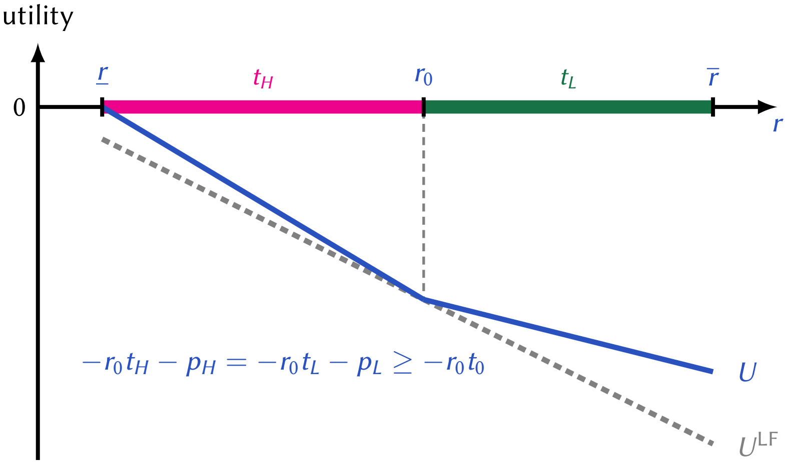

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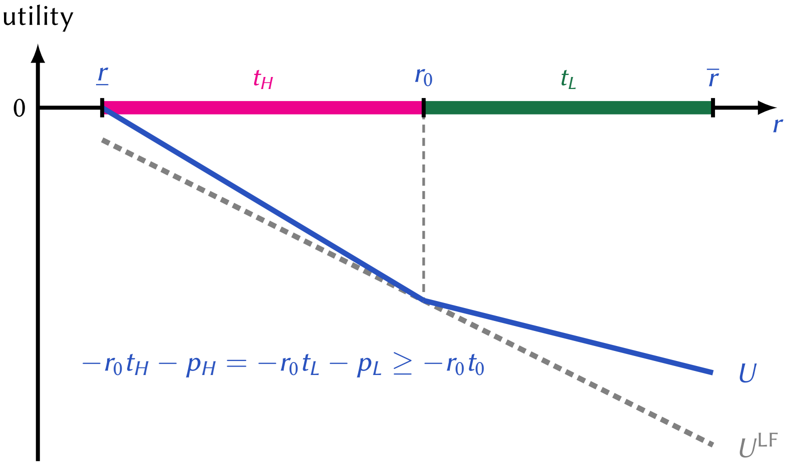

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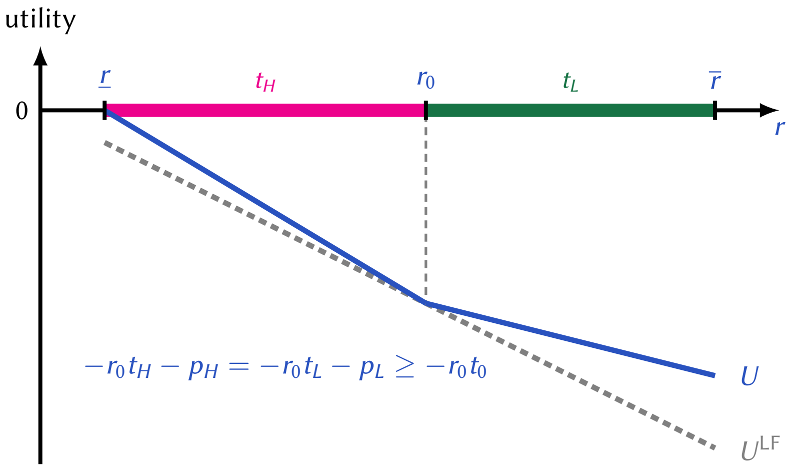

$$0 \leq G(r_0)\,(t_0 - t_H) + [1 - G(r_0)]\,(t_0 - t_L) \implies G(r_0)t_H + [1 - G(r_0)]\,t_L = t_0.$$

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Question. What condition ensures that a deterministic Pareto improvement is possible?

Environments Without Friction

Axiom NF (feasibility in environments without friction).

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  This is the opposite of **Axiom F**, where any MPS of a feasible distribution is infeasible.
- There is no tradeoff (due to no "friction") between average wait time and dispersion.
- Under **Axioms LF** + **NF**, there's a deterministic, budget-balanced Pareto improvement.

  However, **Axiom F** often holds in applications (**Axiom NF** is too strong).

Axiom R (feasibility in environments with richness).

There exists $\delta > 0$ such that, for any sequence $\varepsilon \to 0$, there exists a sequence $F_\varepsilon \in \mathbb{F}$ such that

$$F_\varepsilon^{-1}(q) = \begin{cases} t_H, & \text{if } q \geq 1 - m_H, \\ t_0, & \text{if } q \in (m_L, m_H), \\ t_L, & \text{if } q \leq m_L, \end{cases}$$

for some $t_L < t_0 < t_H$ and $m_L, m_H, 1 - m_H - m_L > \delta$, satisfying

$$\lim_{\varepsilon \to 0} \frac{\mathbf{E}_{F_\varepsilon}[\tau] - t_0}{t_0 - t_L} = 0.$$

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- **Axiom R** is a relaxation of **Axiom NF** and can hold when there is a "friction."
  Intuitively, the tradeoff between average wait time and dispersion is not "too bad."

Theorem 2.

Under **Axioms LF** and **R**, there is a feasible, deterministic, and budget-balanced Pareto improvement over the laissez-faire benchmark.

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- When **Axiom F** holds, **Theorem 1** implies that at least 3 priority tiers are required. (Converse is clearly false: not every priority system with ≥ 3 tiers is Pareto-improving.)

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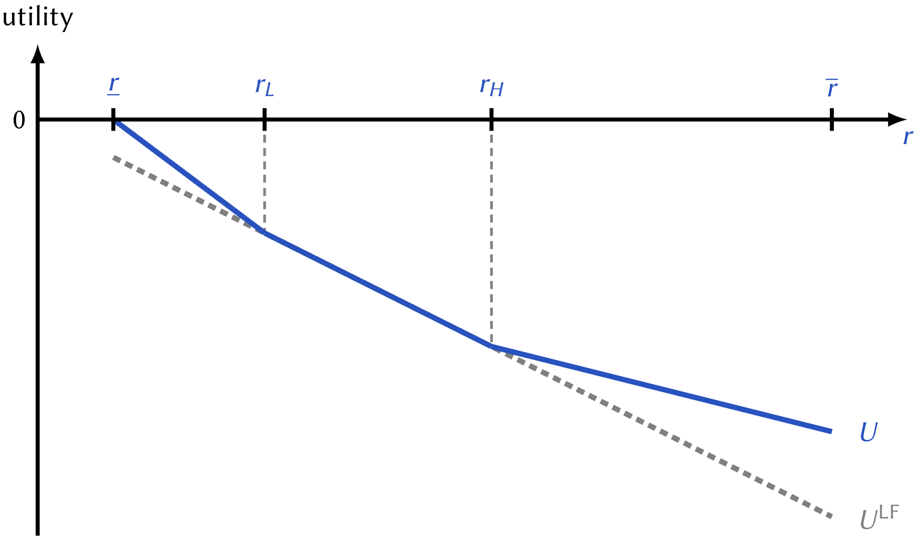
- When **Axiom F** holds, **Theorem 1** implies that at least 3 priority tiers are required. (Converse is clearly false: not every priority system with ≥ 3 tiers is Pareto-improving.)
- **Proof idea:** by construction with, 3 priority tiers.

Construction with 3 Priority Tiers



Application #1: Processing Lines

Setup: Agents

There is a unit mass of risk-neutral agents waiting in line; waiting is costly.

Agents have heterogeneous MRS between waiting and money, $r \in [\underline{r}, \bar{r}]$:

$$\text{utility} = V - r \cdot \mathbf{E}\left[c(\tau)\right] - p, \quad \text{where } \begin{cases} \tau & = \text{(random) wait time,} \\ p & = \text{expected payment.} \end{cases}$$

Here, $c : \mathbb{R}_+ \to \mathbb{R}_+$ is a increasing and strictly convex function.

Assumption. r is distributed according to G, which has positive density g on $[\underline{r}, \bar{r}]$.

Setup: Principal

There is a principal who chooses a distribution (CDF) $F : \mathbb{R}_+ \to [0, 1]$ of costs, $\mathbf{E}\left[c(\tau)\right]$.

The principal can process agents at a rate of $\rho > 0$ per unit time.

- She needs a total time of $T = 1/\rho$ to process all agents.

The principal's processing rate implies a feasible set $\mathbb{F}_\rho$ that constrains the choice of F.

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Examples. Processing applications for visas, driving licenses, business permits, etc.

Lemma. $\mathbb{F}_\rho$ satisfy **Axioms LF**, **F**, and **R**.

Implications

Corollary 1.

There is no feasible, deterministic, and budget-balanced Pareto improvement with 2 priority tiers.

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  This harms individuals who have intermediate MRS between waiting and money!
- Introducing a "gold track" with even faster service could solve this problem.

Welfare-Maximizing Pareto Improvements

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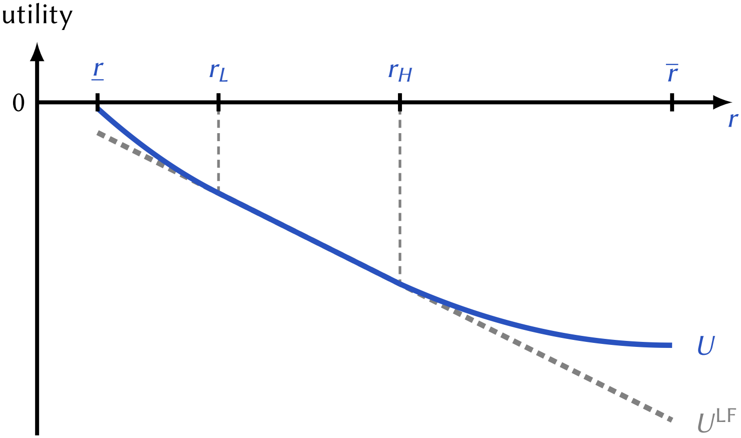

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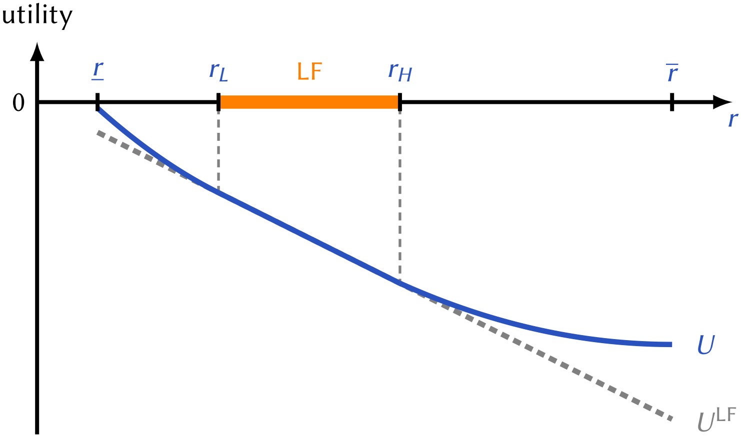

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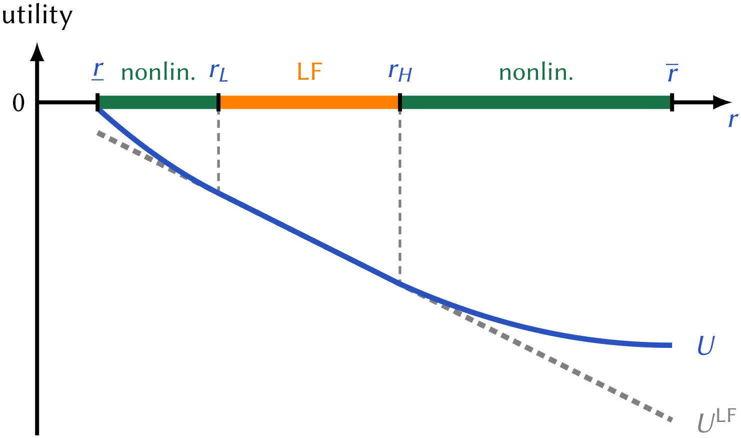

Application #2: Lane Pricing

Setup: Agents

There is a unit mass of risk-neutral agents waiting in n lanes.

Agents have heterogeneous MRS between waiting and money, $r \in [\underline{r}, \bar{r}]$:

$$\text{utility} = V - r \cdot c_i(m) - p, \quad \text{where} \begin{cases} m & = \text{mass of agents in the same lane } i, \\ p & = \text{expected payment.} \end{cases}$$

Here, $c_i : \mathbb{R}_+ \to \mathbb{R}_+$, $i = 1, 2, \ldots, n$, are increasing and strictly convex functions.

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The number of lanes n implies a feasible set $\mathbb{F}_n$ that constrains the choice of F.

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Example: express lane pricing on highways.

Implications

Corollary 3.

$\mathbb{F}_2$ satisfies **Axioms LF** and **F** $\implies$ no deterministic, budget-balanced Pareto improvement exists.

Corollary 4.

$\mathbb{F}_n$ ($n \geq 3$) satisfies **Axioms LF** and **R** $\implies$ deterministic, budget-balanced Pareto improvement exists.

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  So, having only a single toll lane (and leaving other lanes free) harms some agents.

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  So, having only a single toll lane (and leaving other lanes free) harms some agents.
- Introducing a third option (e.g., public transit, intertemporal substitution) could help.

Concluding Remarks

This paper: priority systems that **Pareto-improve** on an **unpriced** status quo.

Main Results:

#1. When there is “friction,” no Pareto improvement with 2 priority tiers exists.

$\rightsquigarrow$ 1 fast lane + 1 slow lane are not enough; some agents will be worse off.

#2. When there is “richness,” a Pareto improvement with 3 priority tiers exists.

References