Preface

Continuum mechanics is a broad advanced study of material behavior based on the assumption that matter is continuously distributed in space. Initially such studies were limited to common solids and fluids that were present in engineering and scientific applications of the day. These classical studies went under the names *elasticity, plasticity, viscoelasticity, fluid mechanics, rheology, etc*. After a rebirth in the 1960's-70's, continuum mechanics began focusing on much more general and complicated nonlinear material behavior employing higher level mathematics. Advances in the field have continued into such areas as *multi-phase composites, graded materials, granular substances, cellular and porous solids, materials with microstructure, coupled multi-field problems, biological materials, etc.*

The mechanics of continuous media is often thought of as a somewhat *special* course which is more general and theoretical than others found in engineering curricula. Normally the study is broad in nature, and seeks to capture far-reaching principles that set the foundations for more specific applications in the mechanics of solids and fluids. Such an approach requires the use of tensor notation and other more advanced mathematical tools. As reflected in the literature, the community of scholars has generally agreed that continuum mechanics texts should include:

1. Tensor algebra and calculus, field theory, and related mathematical principles
2. Kinematics, deformation and rate of deformation measures
3. Force and stress descriptions
4. Conservation or balance principles
5. Constitutive equation development
6. Formulation and example solutions of particular theories

Items 1-4 are classical topics, and many existing continuum mechanics texts do an adequate job with these. However, it is with items 5 and 6 that the author feels the need for improved presentation and hence is the primary motivation for creating the current text. In this regard extensive coverage is provided in the development of a very broad class of constitutive relations in chapters 6–9. For a single semester course, coverage of all of these different constitutive theories would likely be overly ambitious. However, particular theories within these chapters can be selected for class presentation/discussion as per instructor or student interest, while other constitutive theories can be assigned for student term paper projects. The author has used this scheme many times with good success. In this fashion students gain a broad perspective of the vast range of continuum mechanics applications especially in contemporary areas of micromechanical modeling. Since much of graduate engineering classroom education is designed to help prepare students to conduct their program research, it is hoped that this broad coverage will aid in this effort.

Much of this material is an outgrowth of the author's class notes on the subject, coming from teaching numerous courses in continuum mechanics over the past several decades. The material represents a textbook for use in a beginning graduate level course on the subject. As such, emphasis is placed on organization, clarity and breadth, rather than mathematical rigor and highly specialized theoretical aspects. The text is certainly mathematical, but certain theoretical concepts are often avoided so as to not lose student interest. Examples are generously distributed in all chapters to provide extensive applications of the theoretical concepts and to illustrate how various theories are applied within the study.

Following the author's previous elasticity text, this book integrates the use of numerics by employing MATLAB software. This greatly aids in presenting and illustrating applications of particular theory through calculation and graphical display. Although other options (e.g. Mathematica) could also be used, the author has found MATLAB to be well suited for such application and is a very popular engineering software package. This software is used in many places for applications such as: tensor transformation operations, calculation of invariants and principal values and directions, two-dimensional deformation plotting, and calculation and plotting various specific results from example application problems. With numerical and graphical evaluations, application problems become more interesting and useful for student learning. Many of the MATLAB codes are listed in Appendix C and can also be accessed through the text's companion web site. This allows both instructors and students to easily integrate the numerics into homework exercises or to pursue further application cases of their own interest.

Contents Summary

Chapter 1 provides several introductory topics that the author finds necessary to discuss before delving into the heart of the subject matter. First, an introduction to materials, the continuum hypothesis, and length scale concepts are given. The need for tensor representation is then provided along with a brief introduction to the objectivity concept. A summary of the structure of continuum mechanics is given to provide a general overall picture of the subject, and the chapter ends with a brief historical summary to illustrate how we got to now.

Chapter 2 presents much of the mathematical tools and notation necessary for a modern treatment of continuum mechanics. Although some students may have seen some of these topics, the author feels that a reasonably comprehensive presentation will best serve the general student audience. Tensor analysis is one of the key features of this chapter, and both Cartesian and general tensors are presented in detail. Both index and direct notation are employed. Related topics of Cayley-Hamilton theorem, matrix polynomials and representation theorems for isotropic functions are also covered. The principle of objectivity or frame invariance is initially introduced in this chapter. Most all of these topics are to be used in later sections of the text.

Some use of MATLAB is incorporated to evaluate particular tensor algebra and calculus applications. The author feels that this material is best done in the beginning of the study; however, depending on the mathematical background of the students, some of this material could be skipped over and introduced later in the text at points of application.

Chapter 3 discusses motion, kinematics and deformation of continuous media. Lagrangian and Eulerian descriptions are presented, and many different strain and strain rate tensors are established for both large and small deformation. Strain compatibility is explored, and the concept of objectivity is investigated for various strain and rate of strain tensors. Formulation using the current configuration as reference and Rivlin-Ericksen tensors are also developed for later use. Finally various deformation measures using curvilinear cylindrical and spherical coordinates are presented. MATLAB is used to illustrate some particular two-dimensional deformation examples.

Chapter 4 addresses external and internal forces and various stress measures. The stress or traction vector is introduced and this leads to the definition of the Cauchy stress tensor. Definitions of principal, spherical, deviatoric, octahedral and vonMises stress are made. Piola-Kirchhoff stress tensors are defined and compared to Cauchy stress. Evaluation of objectivity of the various stress tensors is presented. Finally stress tensor components using curvilinear cylindrical and spherical coordinates are established. Use of MATLAB is again incorporated to evaluate and plot various stress tensor components to illustrate the field nature of such variables.

Chapter 5 presents the development of conservation or balance principles of mass, momentum and energy that are used in continuum mechanics. Both integral and differential forms are developed. In regard to energy, new thermodynamic field variables are introduced, and both the first and second laws of thermodynamics are discussed. The chapter ends with a summary of previously developed general relations and a listing of the associated unknown field variables.

Chapter 6 begins the presentation of constitutive equation development but limits the discussion to only the classical linear theories of elasticity, fluid mechanics, viscoelasticity and plasticity. The aim of this chapter is to explore each of the specific constitutive relations, develop the complete field equations and boundary conditions, and finally to present several closed-form analytical solutions to several problems of interest. This chapter sets the stage for more complex constitutive models in the following three chapters.

Chapter 7 explores constitutive theories and formulations for continuum problems that contain more than a single independent field behavior. Examples presented combine linear elastic deformation with thermal, fluid saturated and electro behaviors; leading to the formulation of thermoelasticity, poroelasticity and electroelasticity. Within each theory new constitutive relations are coupled with linear elasticity to develop the combined material model. The solution to many example problems are provided to demonstrate how the multiple fields produce coupled response.

Chapter 8 expands constitutive modeling into nonlinear behavior for both solids and fluids. Finite deformation theory is now used along with much more general constitutive principles including material frame indifference. Noll's general theory of simple materials forms the starting point in the presentation. Non-linear elasticity, non-linear viscous fluids and non-linear viscoelastic material models are discussed in detail. Numerous examples of these cases are provided.

Chapter 9 presents several different constitutive models that incorporate material microstructure. The discussion begins with concepts of the representative volume element, homogenization and length scales. The specific chosen material models include: micropolar elasticity, elasticity with voids, doublet mechanics, higher gradient elasticity, fabric tensor theories and damage mechanics. This collection is based on the elastic response, and this allows a relatively easy introduction to this type of important and contemporary modeling. Several example problems using these extended theories provide comparisons with the classical predictions to illustrate particular aspects of the newer models.

In general the text includes many worked-out examples to demonstrate the theory, and numerous exercises are given at the end of each chapter for student engagement and may also be used for class presentations. Appendices A and B offer a convenient summary of basic field equations and transformation relations between Cartesian, cylindrical and spherical coordinate systems. Appendix C provides a listing of many MATLAB codes used throughout the text. A humorous poem written by the author's doctoral advisor ends the text in Appendix D.

Web Support

The companion and instructor sites for this text can be accessed at https://www.elsevier.com/books/continuum-mechanics-modeling-of-material-behavior/sadd/978-0-12-811474-2. The companion site includes downloadable MATLAB codes listed in Appendix C. These codes will aid both students and instructors in developing codes for their own particular use and thus allow easy integration of the numerics. Instructors who register will be able to access PowerPoint lecture slides and solutions to the exercises featured in the text. Errata (when available) will be available at the companion site.

Feedback

The author is committed to continual improvement of engineering education and welcomes feedback from users of this book. Please feel free to send comments concerning suggested improvements or corrections via email (msadd@cox.net). It is likely that such feedback will be shared with the text's user community via the publisher's website.

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Martin H. Sadd is Professor Emeritus of Mechanical Engineering at the University of Rhode Island. He received his Ph.D. in mechanics from the Illinois Institute of Technology in 1971 and then began his academic career at Mississippi State University. In 1979 he joined the faculty at Rhode Island and served as department chair from 1991–2000. He is a member of Phi Kappa Phi, Pi Tau Sigma, Tau Beta Pi, Sigma Xi, and is a Fellow of ASME. Professor Sadd's teaching background is in the area of solid mechanics with emphasis in elasticity, continuum mechanics, wave propagation, and computational methods. He has taught numerous courses in these fields at three academic institutions, several industries, and at a government laboratory.

Dr. Sadd's research has been in the area of analytical and computational modeling of materials under static and dynamic loading conditions. His recent work has involved micromechanical modeling of geomaterials including granular soil, rock, and concretes. He has authored over 75 publications and has given numerous presentations at national and international meetings. He is the author of *Elasticity: Theory, Applications and Numerics* (Third Ed.).