Personal Notes

Tensors

Foundations of Continuum Mechanics

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Contents

1 Acoustic Tensor

Definition. There are two definitions for the acoustic (localisation, polarisation, or characteristic stiffness) tensor (**Etse.1999**; **Ottosen.2005**):

$$\underline{\underline{\boldsymbol{A}}} \coloneqq \hat{\boldsymbol{n}} \cdot \boldsymbol{\mathcal{C}} \cdot \hat{\boldsymbol{n}} \quad \overset{\text{not}}{=\!=} \quad A_{jk} \coloneqq C_{ijkl} \hat{n}_i \hat{n}_l, \tag{1a}$$

$$\underline{\underline{\boldsymbol{A}}} \coloneqq \hat{\boldsymbol{n}} \cdot \boldsymbol{\mathcal{C}}^{\mathrm{RT}} \cdot \hat{\boldsymbol{n}} \quad \overset{\text{not}}{=\!=} \quad A_{jk} \coloneqq C_{ijkl} \hat{n}_i \hat{n}_k, \tag{1b}$$

where $\hat{\boldsymbol{n}}$ is the direction vector.

Criteria for material instability. Material instability is detected under the singularity of the acoustic tensor, i.e., vanishing either the determinant or eigenvalues of the acoustic tensor. Criteria for detecting localisation are (**Staber.2021**):

1. loss of ellipticity (equivalent to non-singularity and is Rice's criterion for localisation): any zero eigenvalues for the acoustic tensor,
2. loss of strong ellipticity: loss of positive-definiteness of the symmetrical tensor for all directions.

Example 1.1 — Acoustic tensor in elasticity. Acoustic tensor in isotropic elasticity is (**Bigoni.2012**):

$$\underline{\underline{\boldsymbol{A}}}(\hat{\boldsymbol{n}}) = (\lambda + \mu)\hat{\boldsymbol{n}} \otimes \hat{\boldsymbol{n}} + \mu \underline{\underline{\boldsymbol{I}}}, \tag{2}$$

where instability criteria will be:

1. ellipticity (non-singularity): $\mu \neq 0 \wedge \lambda + 2\mu \neq 0$, or
2. strong ellipticity (positive-definiteness): $\mu > 0 \wedge \lambda + 2\mu > 0$.