MACT 3223:Statistical Inference
Practice Sheet for the Second Midterm

1- Consider a random sample of size n from a gamma distribution with parameters $\alpha = 2$ and θ and has the following p.d.f

$$f(x) = \frac{1}{\theta^2} \times e^{-\frac{x}{\theta}} \quad x > 0$$

(a) Find the likelihood function.

(b) Find a sufficient statistics for θ.

(c) Is $\hat{\theta} = \bar{X}$ an unbiased estimator for θ ?

(d) Is $\hat{\theta} = \bar{X}$ a consistent estimator for θ ?

(e) Find a MVUE estimator for θ and explain why it is an MVUE.

2. Find a sufficient statistic for the unknown parameter θ for the following Pareto distribution:

$$f(y \mid \theta) = \begin{cases} \frac{2^\theta \theta}{y^{\theta+1}} & y \geq 2 \\ 0 & \text{elsewhere} \end{cases}$$

3. Let $X_1, \ldots, X_n$ be a random sample of size n from the uniform distribution on the interval $(0, \theta)$,

(a) Find the expected value and variance of the estimator $\hat{\theta} = 2\bar{X}$.

(b) Find the expected value of the estimator $\tilde{\theta} = X_{(n)}$

(c) Find an unbiased estimator of the form $\breve{\theta} = C X_{(n)}$ and calculate its variance.

(d) Compare the mean square error of $\hat{\theta}$ and $\breve{\theta}$.

(e) Which estimator is consistent?

4. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a normal distribution with **mean zero** and variance $\theta, 0 < \theta < \infty$. Show that $\sum_1^n X_i^2/n$ is an unbiased estimator of θ and has variance $2\theta^2/n$.

5. Let Y1, . . . , Yn be independent uniform (0,θ) distributed random variables If we define a quantity W=Y(n)/ θ. Such that the cdf of W is given by

$$F(w) = w^n \qquad 0 < w < 1$$

(a) Can we consider W as a pivotal quantity for estimating θ? Why?
(b) Derive an exact 95%confidence interval for θ.

6. Let X1,...,Xn be a random sample from a Uniform(0,θ) distribution, where θ is unknown. Define the estimator H to be the maximum of the sample vaules, X(n). Is H a consistent estimator of θ ?

7. A random sample X1,...,X16 is given from a normal distribution with unknown mean μ and unknown variance. For the observed sample, the sample mean is 16.7, and the sample variance is 7.5.

a. Find a 95% confidence interval for μ.
b. Find a 95% confidence interval for σ2.

8. Let X1,X2,...,Xn; be independently distributed according to N(μ,σ2) respectively. Find the joint MVUE for (μ,σ2).

9. Bacteria in water samples are something difficult to count, but their presence can easily be detected by culturing. We select n1 water samples from a certain lake to study the proportion of water samples containing certain harmful bacteria. After adding a chemical to the lake water, another n2 water samples are selected from this lake to study the proportion of water samples containing this harmful bacteria. If we want to estimate the true difference between proportions of harmful bacteria before and after a chemical is added to within 0.1 with a 95% confidence coefficient, how many water samples should we select (assume the sample sizes n1 and n2 are equal.)