

Assignment 2

1. Derive the following Equations:

$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \quad \text{and} \quad R_{y,\theta} = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$$

2. Consider the following Sequence of rotations:

1. Rotate by ϕ about the world x-axis
2. Rotate by θ about the current z-axis
3. Rotate by ψ about the world y-axis

Write the matrix product that will give the resulting rotation matrix
(Do not perform the matrix multiplication)

3. Consider the following Sequence of rotations:

1. Rotate by ϕ about the world x-axis
2. Rotate by θ about the world z-axis
3. Rotate by ψ about the current x-axis

Write the matrix product that will give the resulting rotation matrix
(Do not perform the matrix multiplication)

4. Consider the following Sequence of rotations:

1. Rotate by ϕ about the world x-axis
2. Rotate by θ about the current z-axis
3. Rotate by ψ about the current x-axis
4. Rotate by α about the world z-axis

Write the matrix product that will give the resulting rotation matrix
(Do not perform the matrix multiplication)

5. Consider the following Sequence of rotations:

1. Rotate by ϕ about the world x-axis
2. Rotate by θ about the world z-axis
3. Rotate by ψ about the current x-axis
4. Rotate by α about the world z-axis

Write the matrix product that will give the resulting rotation matrix
(Do not perform the matrix multiplication)

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6. If the Coordinate frame $o_1x_1y_1z_1$ is obtained from the coordinate frame $o_0x_0y_0z_0$ by a rotation of $\frac{\pi}{2}$ about the x-axis followed by a rotation of $\frac{\pi}{2}$ about the fixed y-axis, find the rotation matrix R representing the composite transformation. Sketch the initial and final frames.

7. Suppose that three coordinate frames $o_1x_1y_1z_1$, $o_2x_2y_2z_2$ and $o_3x_3y_3z_3$ are given, and suppose:

$${}^1R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}, \quad {}^1R_3 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{Find the matrix } {}^2R_3$$

Use Robotics Toolbox for Matlab to solve the following problems unless otherwise is stated.

1- Consider the following sequence of rotations:

- a. Rotate by ϕ about the world x-axis.
- b. Rotate by θ about the current z-axis.
- c. Rotate by ψ about the current x-axis.
- d. Rotate by α about the world z-axis.

Write the matrix product that will give the resulting rotation matrix.

2- Find the rotation matrix representing a roll of $\pi/4$ followed by a yaw of $\pi/2$ followed by a pitch of $\pi/2$.

3- Suppose that three coordinate frames $o_1x_1y_1z_1$, $o_2x_2y_2z_2$ and $o_3x_3y_3z_3$ are given, and suppose

$$R_2^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}; R_3^1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Find the matrix R_3^2 .