Introduction to Robotics

Presented by

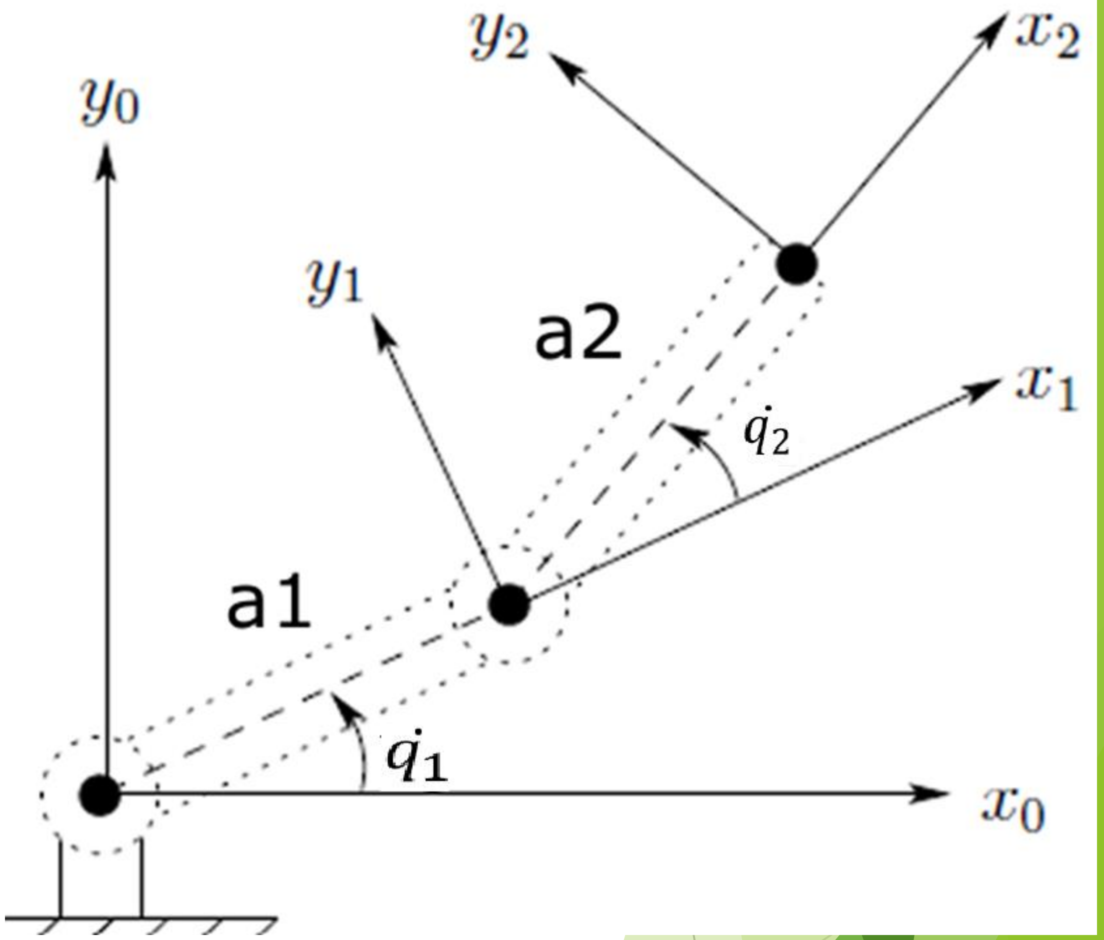
Dr. Mohamed Magdy

Ph.D. in Mechatronics Engineering, Faculty of Engineering, Ain Shams University

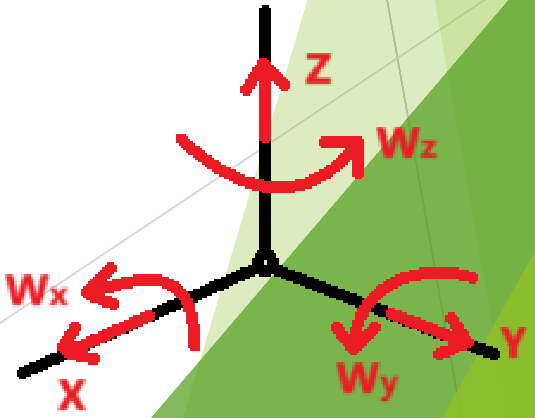
Velocity Kinematics

$$\begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \boldsymbol{J} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \vdots \\ \vdots \\ \dot{q}_6 \end{bmatrix}$$

6x1 6xn nx1



$$J_{6\,x\,n} = \begin{bmatrix} \cdots \\ \cdots \\ \cdots \\ \vdots \\ \cdots \end{bmatrix}$$, n : number of joints



Jacobian Matrix

$J_{6\,x\,n}$ =$[J_1 \quad J_2 \quad J_3 \quad \dots \quad J_n]$, n : number of joints

If Joint i is prismatic $J_{Prismatic}$ = $\begin{bmatrix} Z_{i-1} \\ 0 \end{bmatrix}$ 3x1

If Joint i is revolute $J_{Revolute}$ = $\begin{bmatrix} Z_{i-1}(O_n - O_{i-1}) \\ Z_{i-1} \end{bmatrix}$ 3x1

$$T_a^0 = \begin{array}{cccc} x_a & y_a & z_a & O_a \\ \end{array}$$
$$T_a^0 = \begin{bmatrix} \boxed{\begin{matrix} r_{11} \\ r_{21} \\ r_{31} \end{matrix}} & \boxed{\begin{matrix} r_{12} \\ r_{22} \\ r_{32} \end{matrix}} & \boxed{\begin{matrix} r_{13} \\ r_{23} \\ r_{33} \end{matrix}} & \boxed{\begin{matrix} O_x \\ O_y \\ O_z \end{matrix}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note: $z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

How to get the Jacobian Matrix

Steps:

- D-H parameters.
- Get ***A*** Matrix.
- Get $T_1^0 = A_1$,...., $T_n^0 = A_1 A_2 \ldots\ldots A_n$
- $O_1, z_1 \rightarrow T_1^0$, $O_2, z_2 \rightarrow T_2^0$, ..., $O_n, z_n \rightarrow T_n^0$
- $\boldsymbol{if\ \ joint\ i\ \ is\ a\ revolute\ joint\ \ Z_{i-1}(O_n - O_{i-1})\ \rightarrow it\ a\ cross\ product}$
- Get J

Cross Product

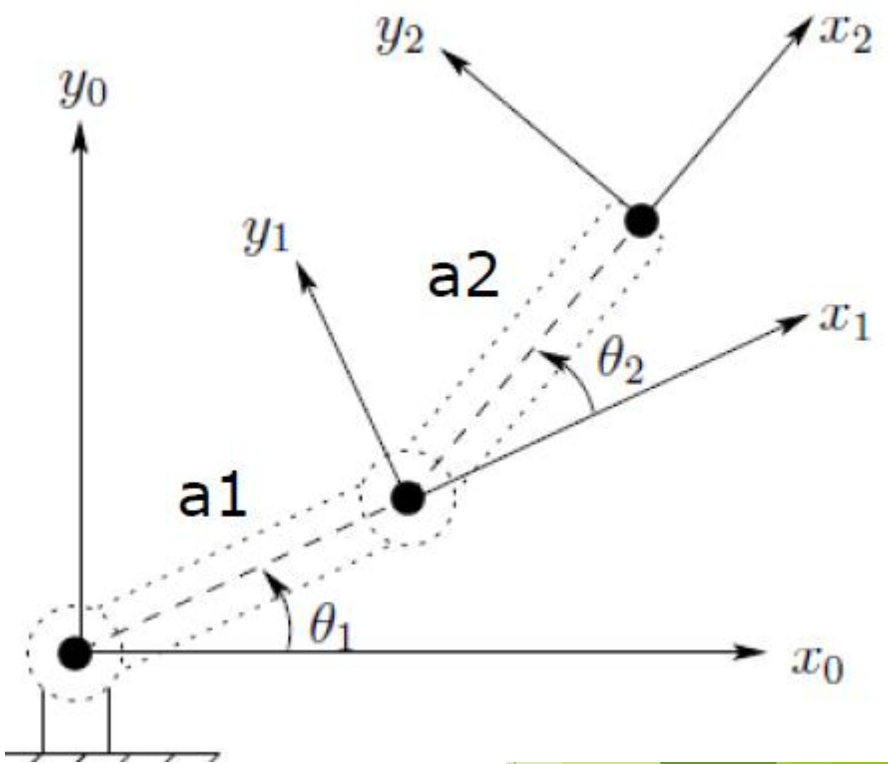
$$= \begin{bmatrix} + & - & + \\ z_x & z_y & z_z \\ O_x & O_y & O_z \end{bmatrix}$$

$$= \begin{bmatrix} z_y O_z - O_y z_z \\ -(z_x O_z - O_x z_z) \\ z_x O_y - O_x z_y \end{bmatrix}$$

Example

There are two revolute joints only, therefore, the jacobian matrix is 6x2

$$J(q) = \begin{bmatrix} z_0 \times (o_2 - o_0) & z_1 \times (o_2 - o_1) \\ z_0 & z_1 \end{bmatrix}$$



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	θ_1
2	a_2	0	0	θ_2

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = A_1$$

$$T_2^0 = A_1A_2 = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad o_1 = \begin{bmatrix} a_1c_1 \\ a_1s_1 \\ 0 \end{bmatrix} \quad o_2 = \begin{bmatrix} a_1c_1 + a_2c_{12} \\ a_1s_1 + a_2s_{12} \\ 0 \end{bmatrix}$$

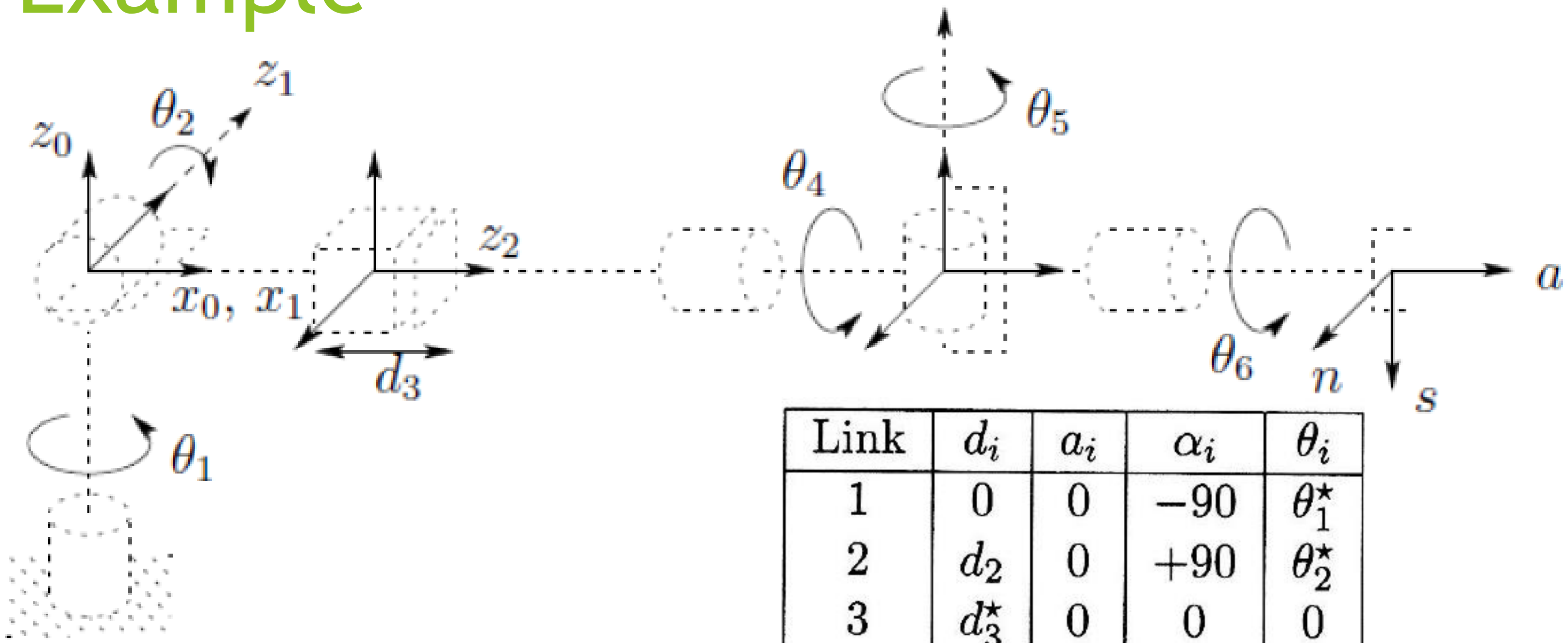
$$z_0 = z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad o_1 = \begin{bmatrix} a_1c_1 \\ a_1s_1 \\ 0 \end{bmatrix} \quad o_2 = \begin{bmatrix} a_1c_1 + a_2c_{12} \\ a_1s_1 + a_2s_{12} \\ 0 \end{bmatrix}$$

$$\boldsymbol{Z_0}(\boldsymbol{O_2} - \boldsymbol{O_0}) = \begin{bmatrix} + & - & + \\ 0 & 0 & 1 \\ a_1c_1 + a_2c_{12} & a_1s_1 + a_2s_{12} & 0 \end{bmatrix} = \begin{bmatrix} -a_1s_1 - a_2s_{12} \\ a_1c_1 + a_2c_{12} \\ 0 \end{bmatrix}$$

$$\boldsymbol{Z_1}(\boldsymbol{O_2} - \boldsymbol{O_1}) = \begin{bmatrix} + & - & + \\ 0 & 0 & 1 \\ a_2c_{12} & a_2s_{12} & 0 \end{bmatrix} = \begin{bmatrix} -a_2s_{12} \\ a_2c_{12} \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -a_1s_1 - a_2s_{12} & -a_2s_{12} \\ a_1c_1 + a_2c_{12} & a_2c_{12} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

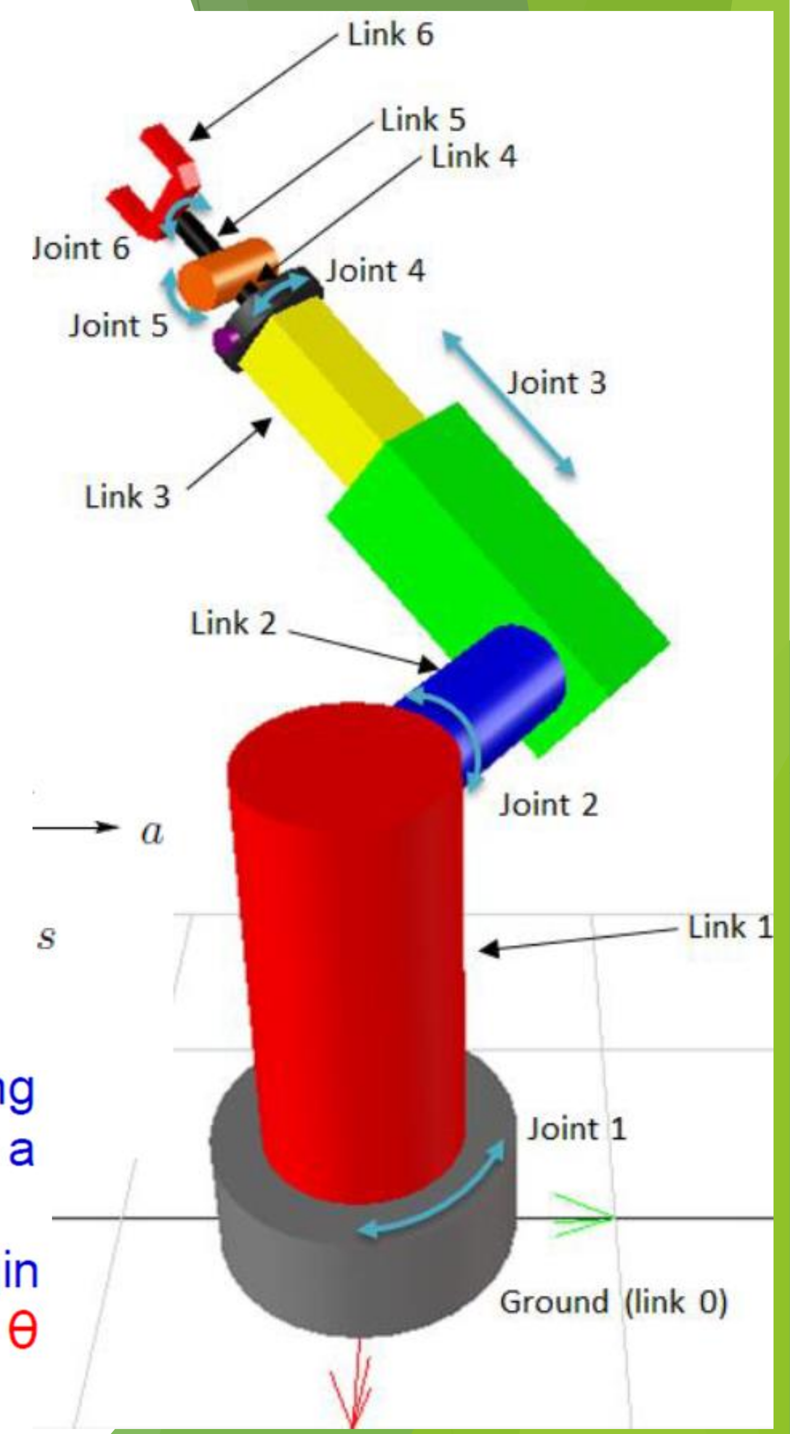
Example



D-H Parameters for Stanford Robot

Link	d_i	a_i	α_i	θ_i
1	0	0	-90	$\theta_1^\star$
2	d_2	0	$+90$	$\theta_2^\star$
3	$d_3^\star$	0	0	0
4	0	0	-90	$\theta_4^\star$
5	0	0	$+90$	$\theta_5^\star$
6	d_6	0	0	$\theta_6^\star$

* joint variable



parameter a is the distance between the axes z_0 and z_1, and is measured along the axis x_1. The angle α is the angle between the axes z_0 and z_1, measured in a plane normal to x_1. The positive sense for α is determined from z_0 to z_1 by the right-handed rule. The parameter d is the perpendicular distance from the origin o_0 to the intersection of the x_1 axis with z_0 measured along the z_0 axis. Finally, θ is the angle between x_0 and x_1 measured in a plane normal to z_0.

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & -s_4 & 0 \\ s_4 & 0 & c_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$J_i = \begin{bmatrix} z_{i-1} \times (o_6 - o_{i-1}) \\ z_{i-1} \end{bmatrix} \quad i = 1, 2$$

$$J_3 = \begin{bmatrix} z_2 \\ 0 \end{bmatrix}$$

$$J_i = \begin{bmatrix} z_{i-1} \times (o_6 - o) \\ z_{i-1} \end{bmatrix} \quad i = 4, 5, 6$$

The z_i are given as

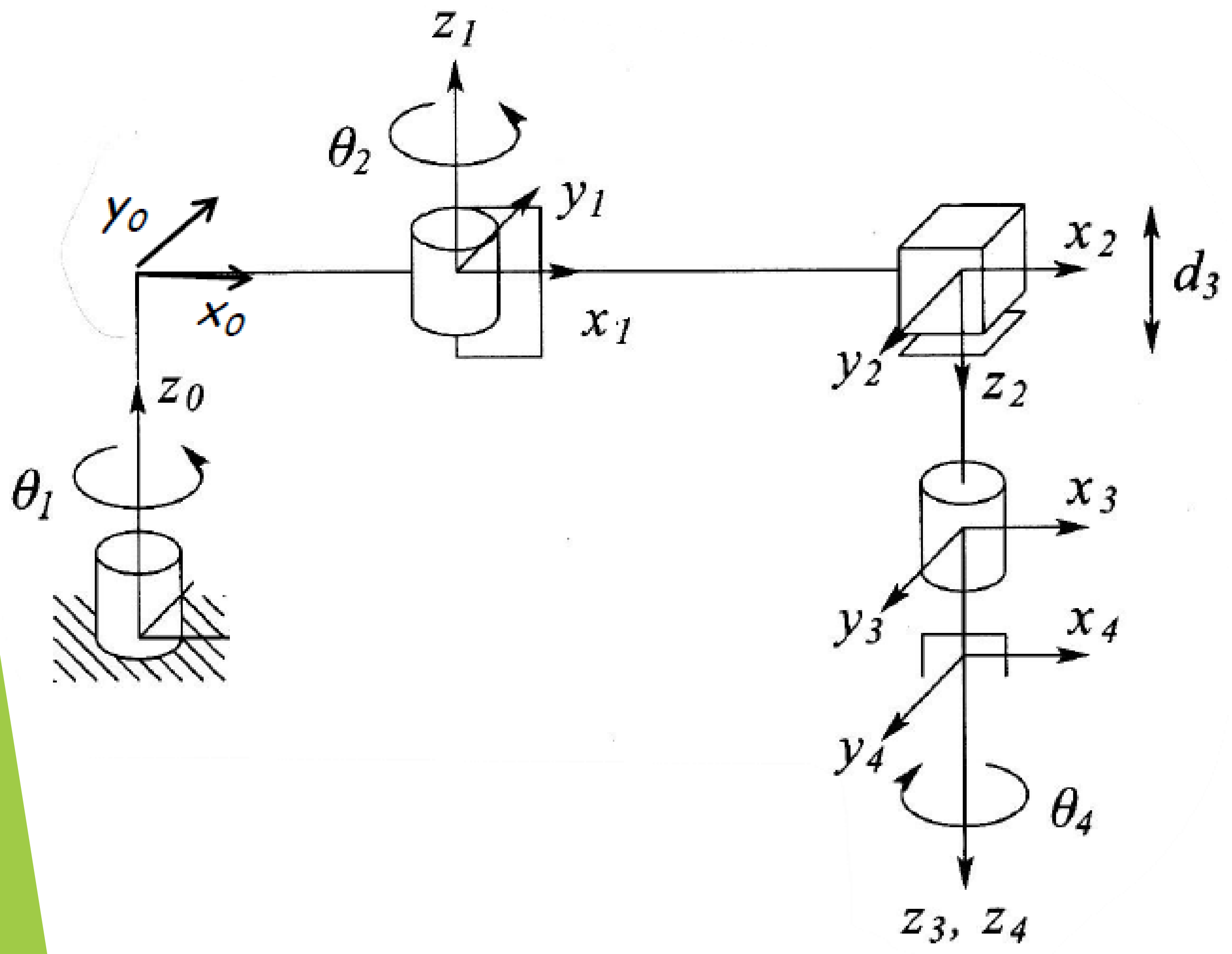
$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_1 = \begin{bmatrix} -s_1 \\ c_1 \\ 0 \end{bmatrix}$$

$$z_2 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix} \qquad z_3 = \begin{bmatrix} c_1 s_2 \\ s_1 s_2 \\ c_2 \end{bmatrix}$$

$$z_4 = \begin{bmatrix} -c_1 c_2 s_4 - s_1 c_4 \\ -s_1 c_2 s_4 + c_1 c_4 \\ s_2 s_4 \end{bmatrix}$$

$$z_5 = \begin{bmatrix} c_1 c_2 c_4 s_5 - s_1 s_4 s_5 + c_1 s_2 c_5 \\ s_1 c_2 c_4 s_5 + c_1 s_4 s_5 + s_1 s_2 c_5 \\ -s_2 c_4 s_5 + c_2 c_5 \end{bmatrix}.$$

SCARA Robot



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\theta_1^\star$
2	a_2	180	0	$\theta_2^\star$
3	0	0	$d_3^\star$	0
4	0	0	d_4	$\theta_4^\star$

* joint variable

parameter a is the distance between the axes z_0 and z_1, and is measured along the axis x_1. The angle α is the angle between the axes z_0 and z_1, measured in a plane normal to x_1. The positive sense for α is determined from z_0 to z_1 by the right-handed rule. The parameter d is the perpendicular distance from the origin o_0 to the intersection of the x_1 axis with z_0 measured along the z_0 axis. Finally, θ is the angle between x_0 and x_1 measured in a plane normal to z_0.

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} T_4^0 &= A_1 \cdots A_4 \\ &= \begin{bmatrix} c_{12}c_4 + s_{12}s_4 & -c_{12}s_4 + s_{12}c_4 & 0 & a_1c_1 + a_2c_{12} \\ s_{12}c_4 - c_{12}s_4 & -s_{12}s_4 - c_{12}c_4 & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$o_1 = \begin{bmatrix} a_1c_1 \\ a_1s_1 \\ 0 \end{bmatrix}$$

$$o_2 = \begin{bmatrix} a_1c_1 + a_2c_{12} \\ a_1s_1 + a_2s_{12} \\ 0 \end{bmatrix}$$

Similarly $z_0 = z_1 = k$, *and* $z_2 = z_3 = -k$. *Therefore the Jacobian of the SCARA Manipulator is*

$$J = \begin{bmatrix} -a_1s_1 - a_2s_{12} & -a_2s_{12} & 0 & 0 \\ a_1c_1 + a_2c_{12} & a_2c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$