Introduction to Robotics

Presented by

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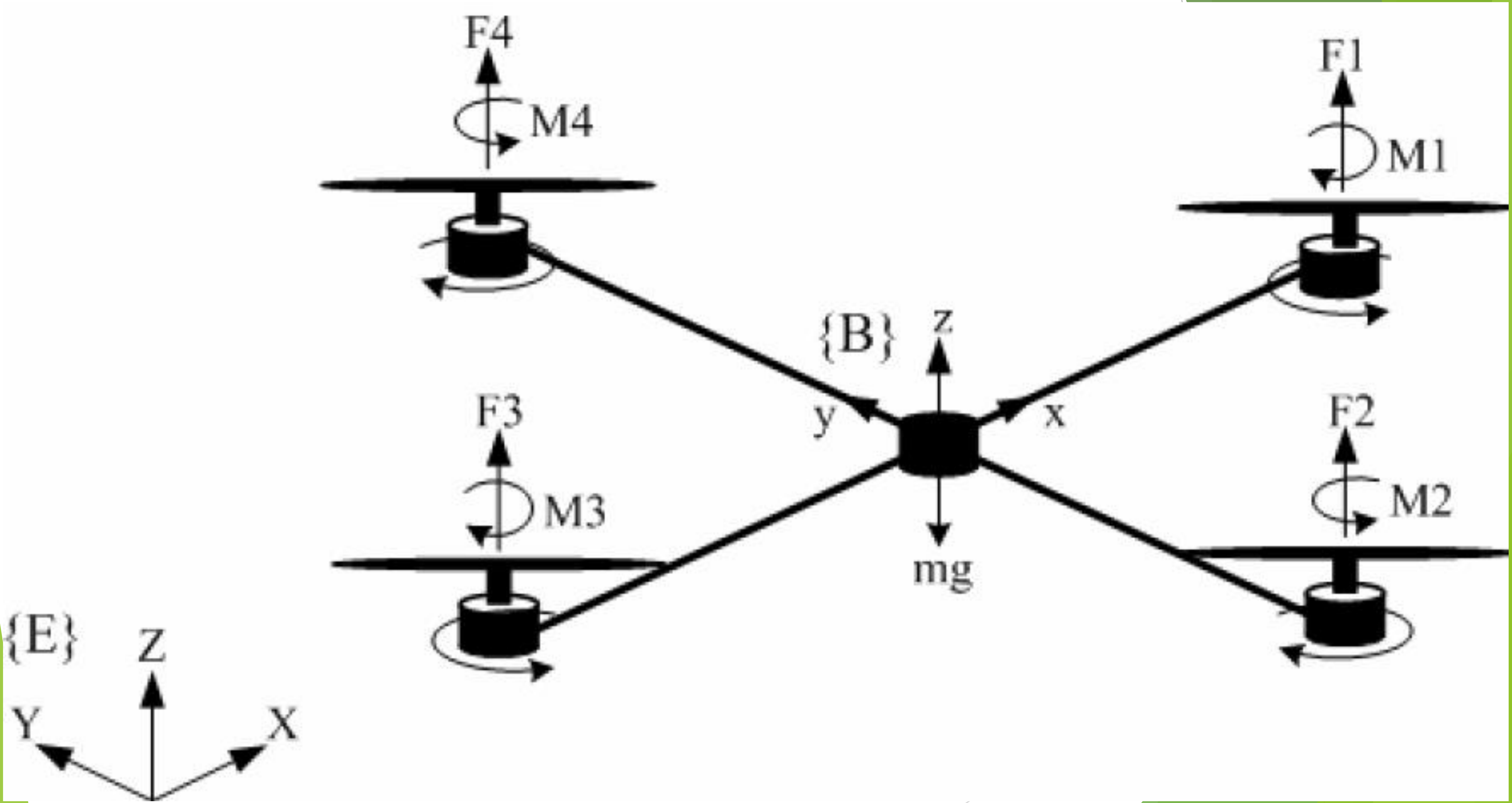
Flying Robots


F4
M4
F1
M1
{B}
z
y
x
F3
M3
F2
M2
mg
{E}
Z
Y
X

Equations of Motion

Using the formalism of Newton-Euler, with the consideration of two frames; inertial frame $E = \{\hat{X}, \hat{Y}, \hat{Z}\}$ where $\hat{Z} = [0,\ 0,\ 1]^T$ pointed upward and body-fixed frame $B = \{\hat{x}, \hat{y}, \hat{z}\}$ that is attached to the centre of mass of the 4-Rotor, then we can start modeling the vehicle as follow: Let the position of the origin of B be $p = \{p_x, p_y, p_z\}$ with respect to E, then:

$$\dot{p} = \upsilon$$

Where υ is the linear velocity of the origin of B expressed in E, then:

$$\sum F = ma$$

$$\boxed{m\dot{\upsilon} = m(g_r(Z) - g)\hat{Z} + TR\hat{Z} - K_{ft}\upsilon}$$

$$R\hat{Z} = \begin{bmatrix} \cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi \\ \sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi \\ \cos\phi\cos\theta \end{bmatrix}$$

Where T is the thrust applied to the body-frame from all four rotors, and is given by:

$$T = b\sum_{i=1}^{4}\Omega_i^2$$

Where Ω_i is the angular velocity of the rotor and b is a constant that depends on many variables such as air density, shape and radius of the rotors. K_{ft} is the translation drag coefficients matrix given by:

$$K_{ft} = \begin{bmatrix} K_{ftx} & 0 & 0 \\ 0 & K_{fty} & 0 \\ 0 & 0 & K_{ftz} \end{bmatrix}$$

The term $g_r(Z)$ represents the ground effect during landing.

$$g_r(Z) = \begin{cases} \dfrac{A}{(2Z)^2} - \dfrac{A}{(Z_0 + Z)^2} & 0 < Z \le Z_0 \\ 0 & else \end{cases}$$

where A is the ground effect constant.

We assumed that the ground effect affects the Quadrotor below certain altitude, Z_0

Equations of Motion

The equations of motion can now be rearranged and expanded. The resultant equations are:

$$\ddot{X} = \frac{1}{m}\left\{T(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi) - K_{ftx}\dot{X}\right\}$$

$$\ddot{Y} = \frac{1}{m}\left\{T(\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi) - K_{fty}\dot{Y}\right\}$$

$$\ddot{Z} = (g_r(Z) - g) + \frac{1}{m}\left\{T(\cos\phi \cos\theta) - K_{ftz}\dot{Z}\right\}$$

$$\ddot{\phi} = \frac{1}{I_{xx}}\left\{\dot{\theta}\dot{\psi}(I_{yy} - I_{zz}) - K_{fax}\dot{\phi}^2 - I_r\dot{\theta}\overline{\Omega} + \tau_a^1\right\}$$

$$\ddot{\theta} = \frac{1}{I_{yy}}\left\{\dot{\phi}\dot{\psi}(I_{zz} - I_{xx}) - K_{fay}\dot{\theta}^2 + I_r\dot{\phi}\overline{\Omega} + \tau_a^2\right\}$$

$$\ddot{\psi} = \frac{1}{I_{zz}}\left\{\dot{\phi}\dot{\theta}(I_{xx} - I_{yy}) - K_{faz}\dot{\psi}^2 + \tau_a^3\right\}$$

Where $\overline{\Omega} = \Omega_1 - \Omega_2 + \Omega_3 - \Omega_4$. Note that, small angles are assumed during the derivation of the last three equations of motion.

Simplified Equations of motion

The simplified equations of motion are obtained by neglecting translation drag, aerodynamic friction, and ground effect.

u is the thrust force $= b \sum_{i=1}^{4} \Omega_i^2$

$$m\ddot{x} = -u \sin\theta$$

$$m\ddot{y} = u \cos\theta \sin\phi$$

$$m\ddot{z} = u \cos\theta \cos\phi - mg$$

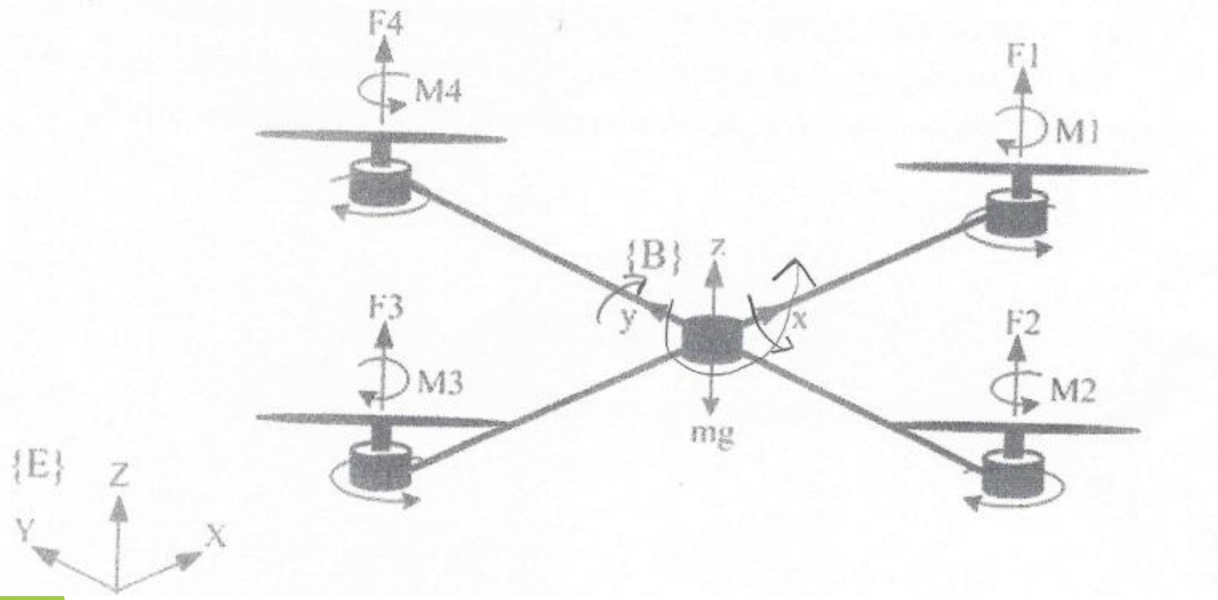
$$\ddot{\psi} = \tilde{\tau}_\psi$$

$$\ddot{\theta} = \tilde{\tau}_\theta$$

$$\ddot{\phi} = \tilde{\tau}_\phi$$

b) Consider the Quadrotor shown below. The total mass of the vehicle is 0.33 kg. The distance between vehicle mass center and the rotational axis of each propeller, l, is 0.2m. The quadrotor's moments of inertia around axes x, y and z are 0.006769, 0.006769 and 0.009937 kg m^2 respectively. The rotor moment of inertia about its rotational axis is 1.2e-06 kg m^2. The propeller lift and drag coefficients (b, k) are 1.3234e-05 N s^2 and 1.0697e-07 Nm s^2 respectively. The rated power of each DC motor is 1200 W and its maximum speed is 20 000 rpm. Neglect translation drag, aerodynamic friction, and ground effect.

- Are the specified motors capable to lift the vehicle? Provide reasons for your answer.
- The initial states of the vehicle $\{X,Y,Z,\emptyset,\theta,\psi,\dot{X},\dot{Y},\dot{Z},\dot{\emptyset},\dot{\theta},\dot{\psi}\} = \{0,0,0,0,0,0,0,0,0,0,0,0\}$, where $\emptyset,\theta$ *and* ψ are the roll, pitch and yaw angles respectively. If all propellers rotate, as shown in the figure below, with equal speeds of 2500 rpm, what will be the vehicle states after 10 seconds?



$$m\,\ddot{x} = 0$$

$$m\,\ddot{y} = 0$$

$$m\,\ddot{z} = F_{lift} - mg\ (weigh) = u\ - mg$$

$u = b * 4 * (20{,}000 * \frac{2\pi}{60})^2$=232.2 N

Then u > mg

$$m\,\ddot{z} = F_{lift} - mg\ (weigh) = u\ - mg$$

$u = b * 4 * (2500 * \frac{2\pi}{60})^2$=3.63 N

$m\,\ddot{z}$ = 3.63-0.33*9.81 = 0.33 $\ddot{z}$

$\ddot{z}$= 1.212 m/sec2

$$z = z_o + \dot{z}_o t + \frac{1}{2}\ \ddot{z}\, t^2$$

$$z = 0 + 0 + \frac{1}{2} * 1.212 * 10^2 = 60.6\,m$$