Introduction to Robotics

Presented by

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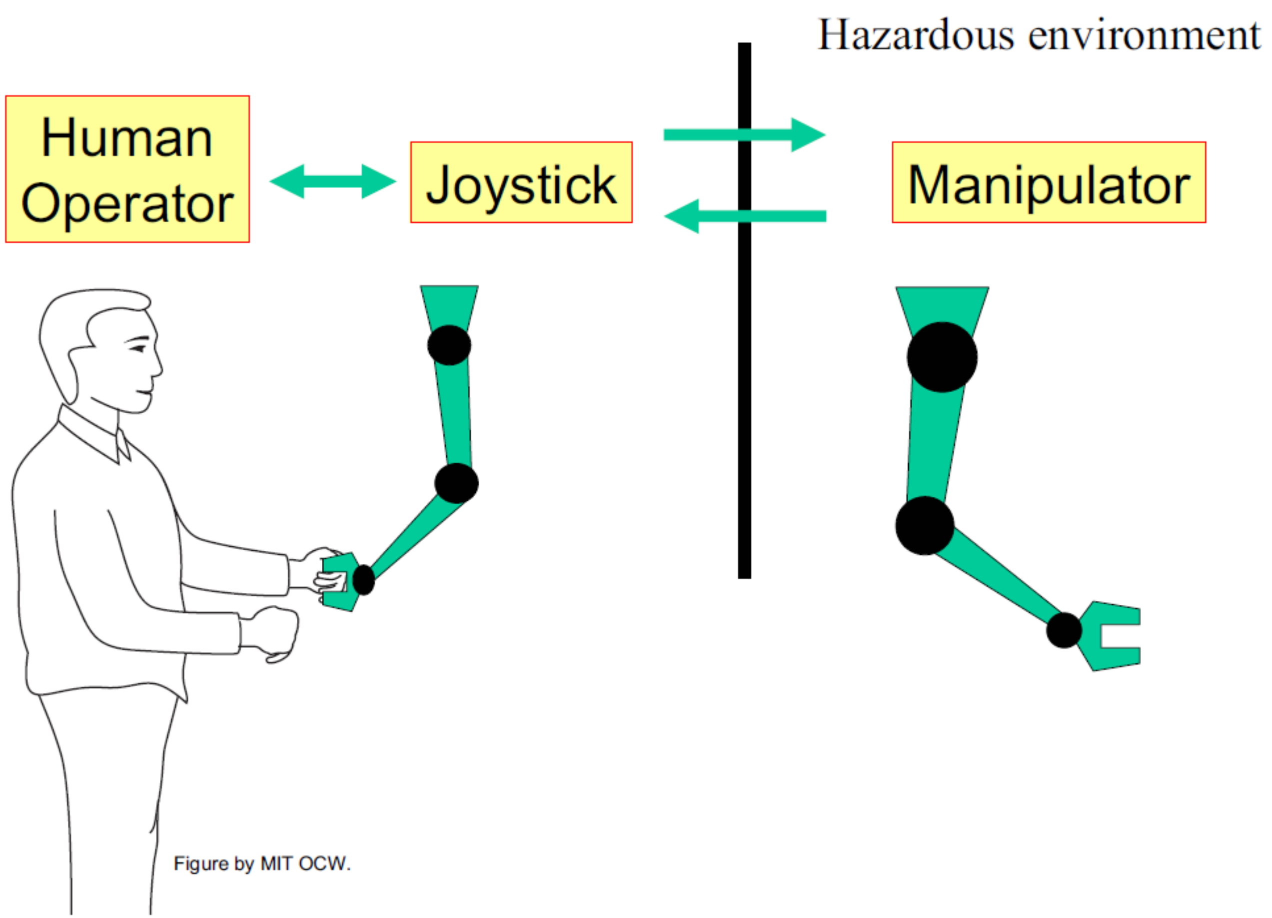
What is a Robot?

- A machine that looks and acts like a human being.
- An efficient but insensitive person.
- An automatic apparatus.
- Something guided by automatic controls.
- A computer whose primary function is to produce motion.



Characteristics of a Robot

- Repeatability
- Manual control
- Automatic control
- Speed of operation

End Effectors

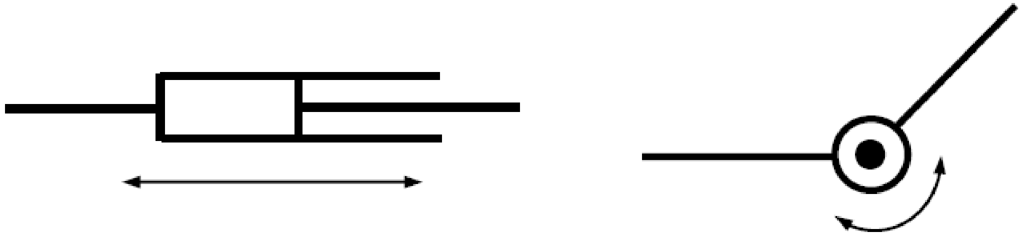
- Grippers
- Hands
- Vacuum heads
- Welding attachments
- Spray Paint attachments
- So on…

Motion types

- A robot mechanism is a multi-body system consisting of rigid bodies called links, connected to form a linkage.

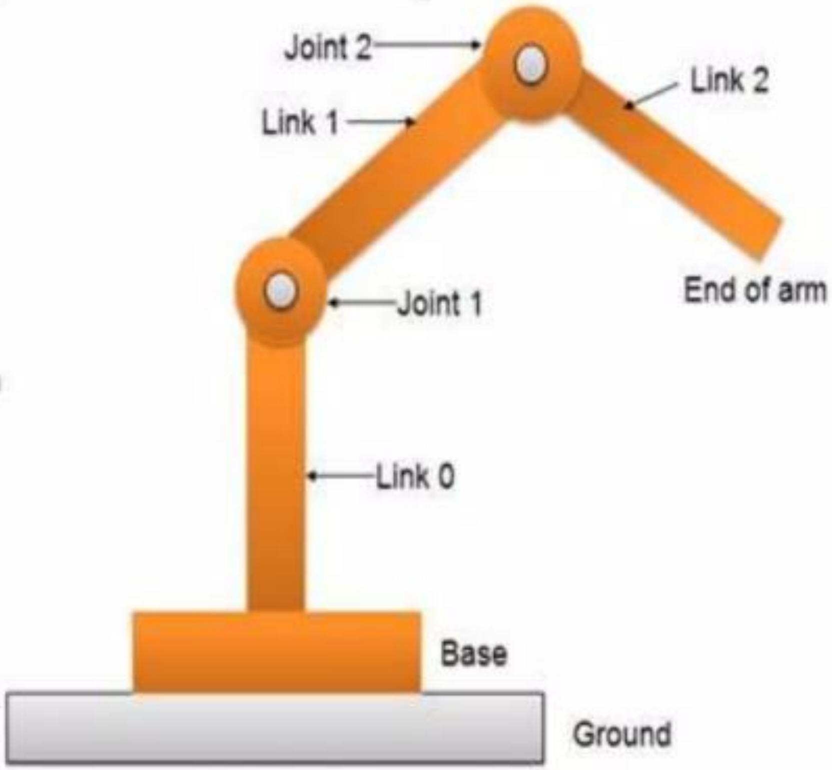
- There are two main types of joints between links:
  1. Prismatic joint, where one link slides along a straight line on the other, is called a sliding joint.
  2. Revolute joints, where one link rotates around a fixed axis, this type of joint is often referred to as a hinge, articulated, or rotational joint.

- These joints are essential for the movement and function of robotic mechanisms.



(a) Prismatic joint (b) Revolute joint

Figure 3.1.1 Primitive joint types: (a) a prismatic joint and (b) a revolute joint



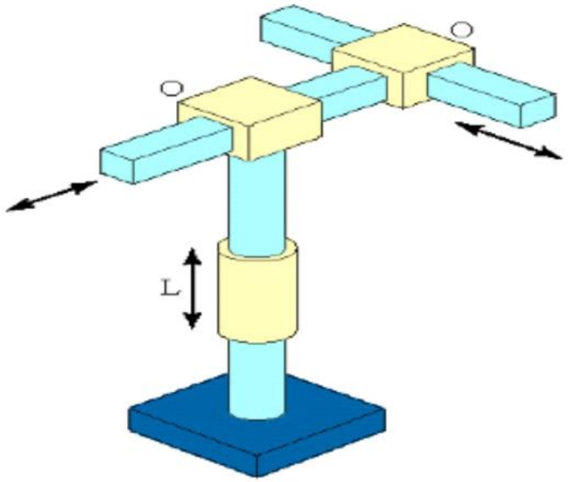
Robot mechanisms analogous to coordinate systems

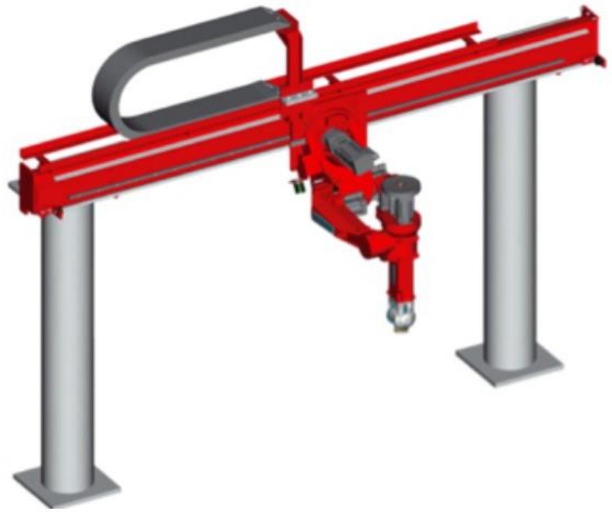
- One of the fundamental functional requirements for a robotic system is to locate its end-effecter, e.g. a hand, a leg, or any other part of the body performing a task, in three-dimensional space.

- three types of robot arm structures corresponding to the **Cartesian coordinate system**, the **cylindrical coordinate** system, **and the spherical coordinate** system, respectively.

Cartesian-coordinate robot

- The Cartesian coordinate robot has three prismatic joints, corresponding to three axes denoted *x*, *y*, and *z*.





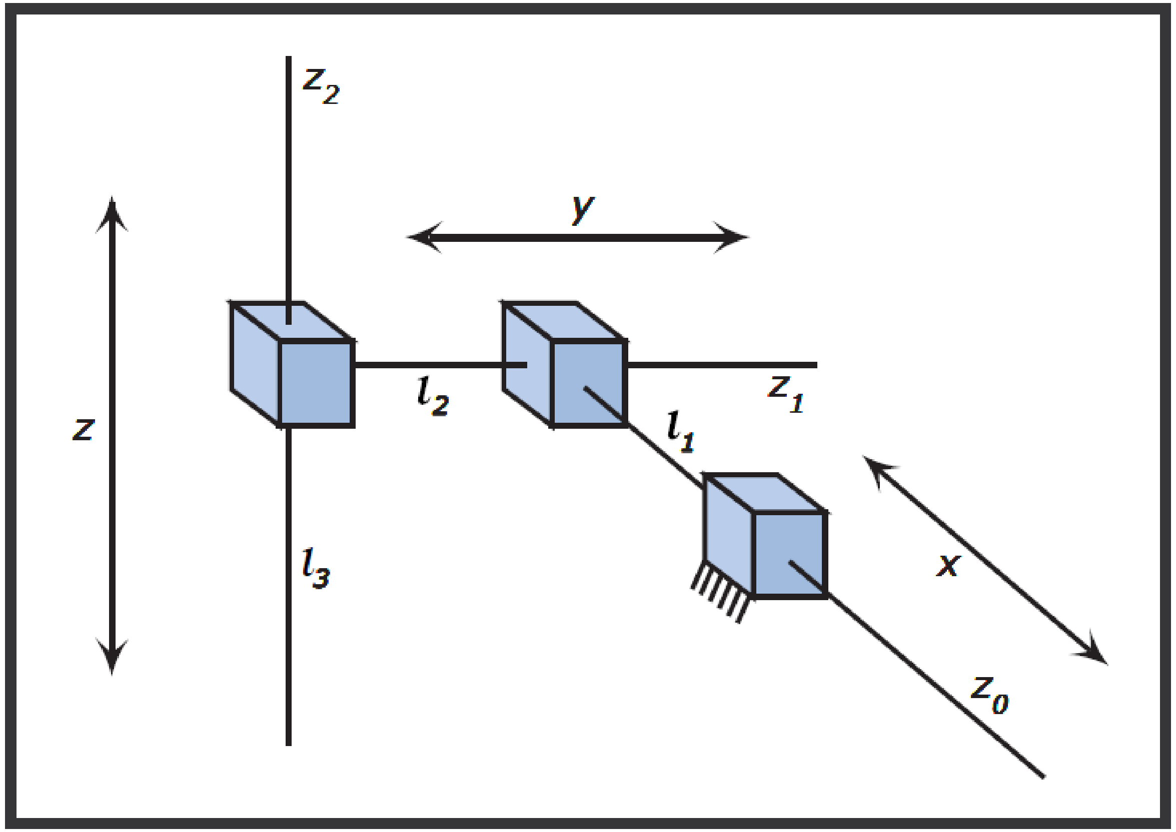
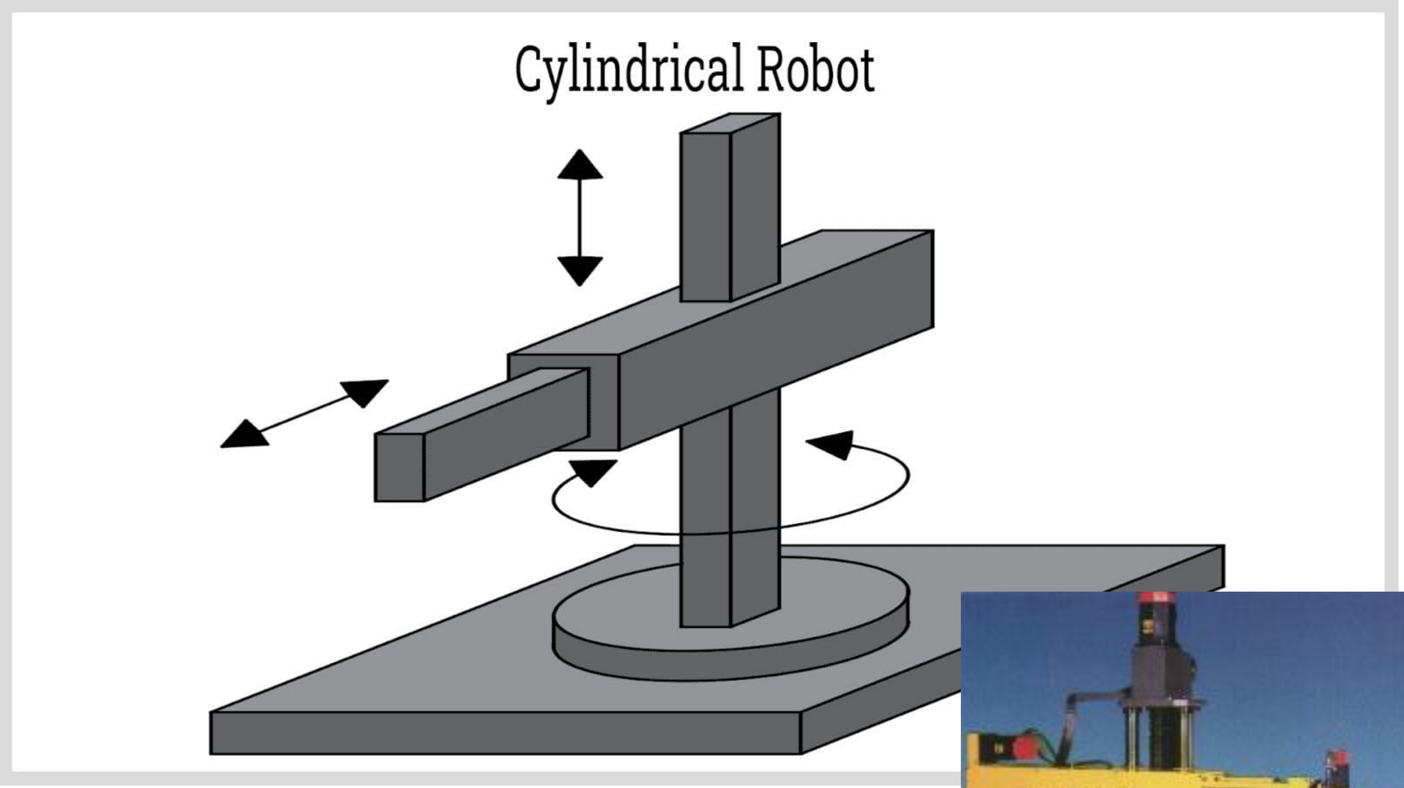
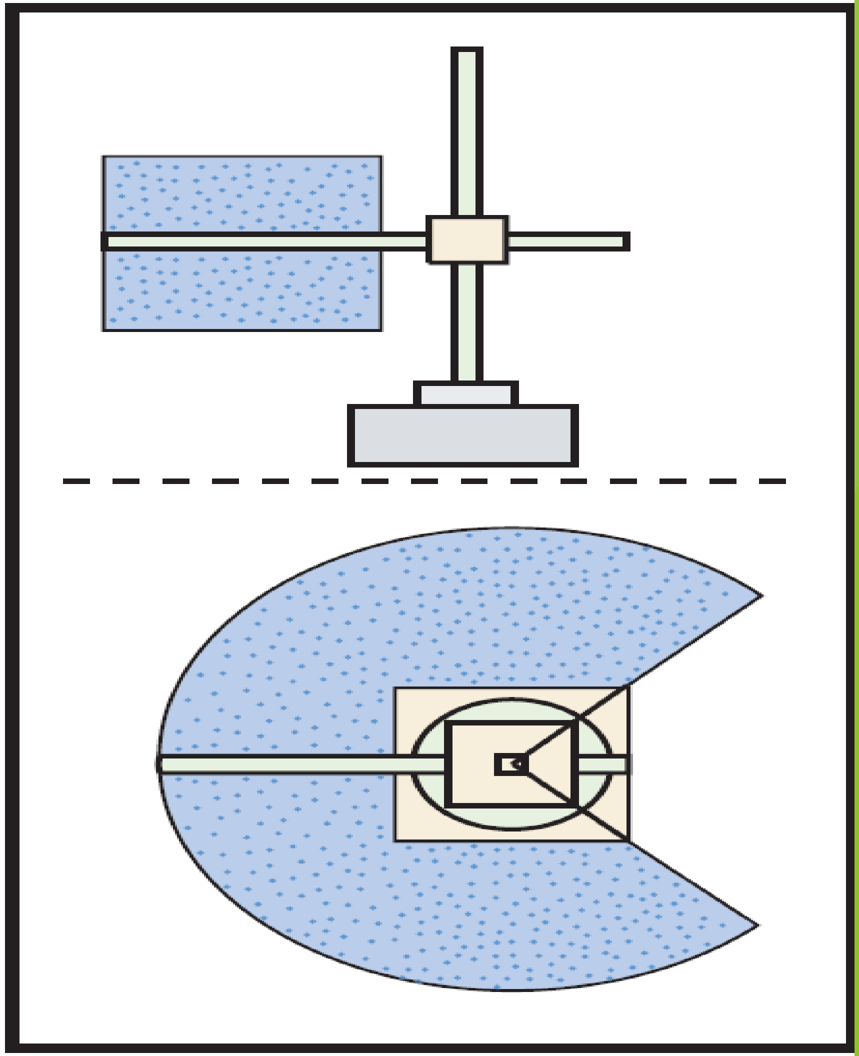


Figure by MIT OCW.

Cylindrical-coordinate robot

- The cylindrical robot consists of one revolute joint and two prismatic joints, with *r*, and *z* representing the coordinates of the end-effecter.





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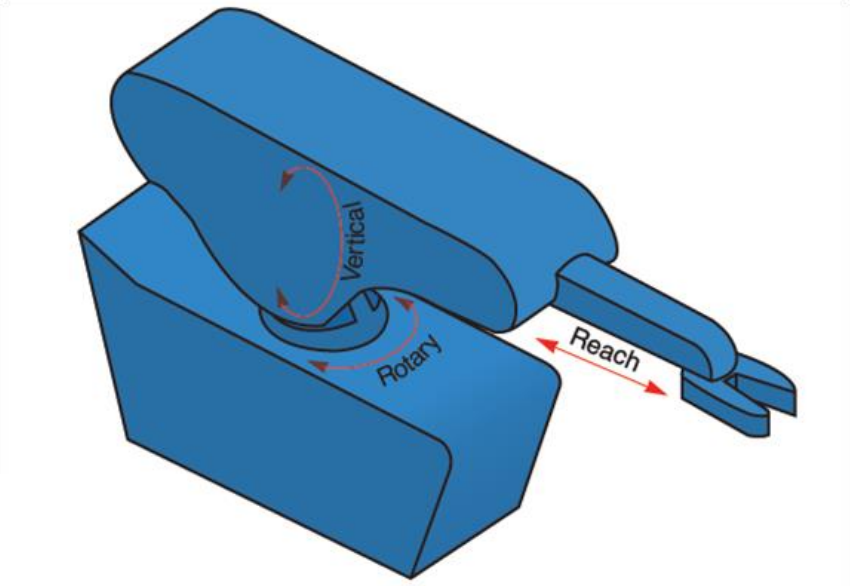
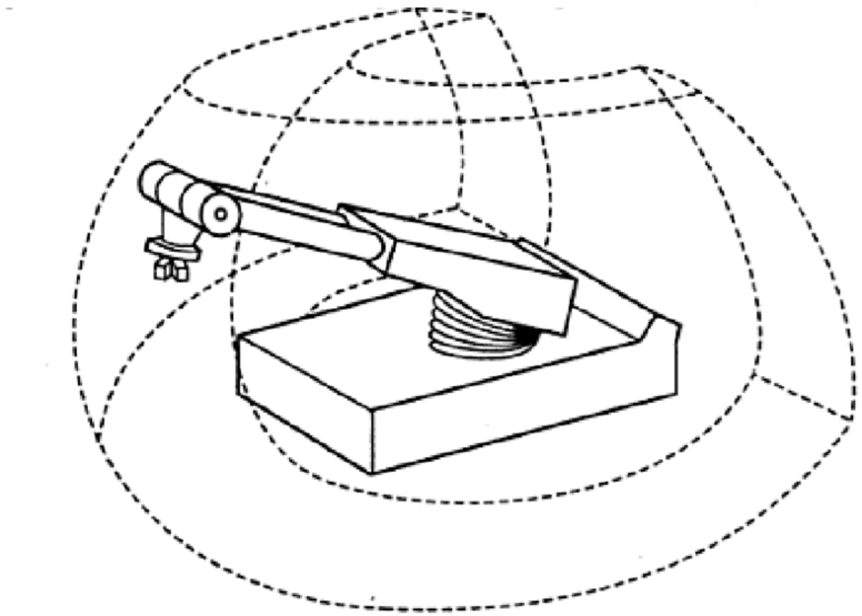
Spherical- coordinate robot

- The spherical robot has two revolute joints denoted and one prismatic joint denoted *r*.





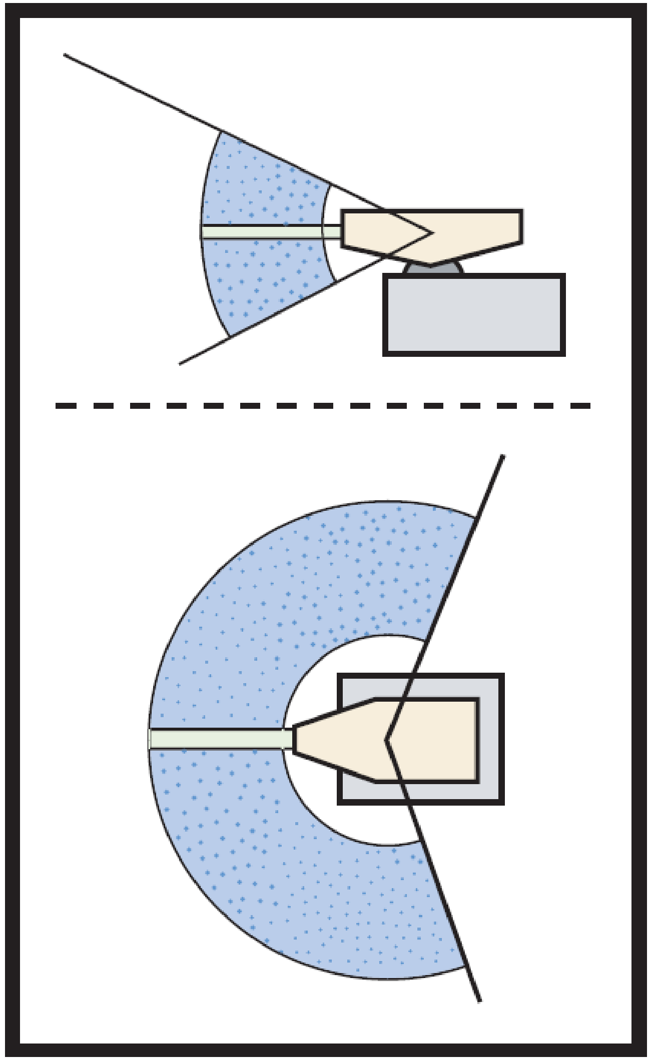
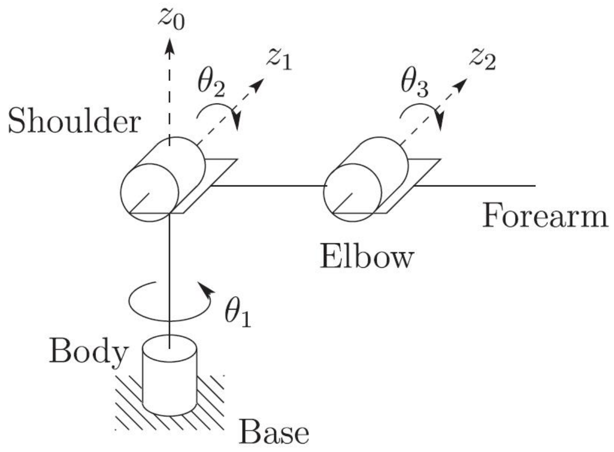


Figure by MIT OCW.

An articulated robot or Elbow robot

- It consists of all three revolute joints, like a human arm.
- This type of robot has great flexibility and versatility, being the most standard structure of robot manipulators.
- The third kinematic structure, consisting of three revolute joints, has a unique mass-balancing structure. The counter balance at the elbow eliminates gravity load for all three joints, thus reducing toque requirements for the actuators. This structure has been used for the direct-drive robots having no gear reducer.



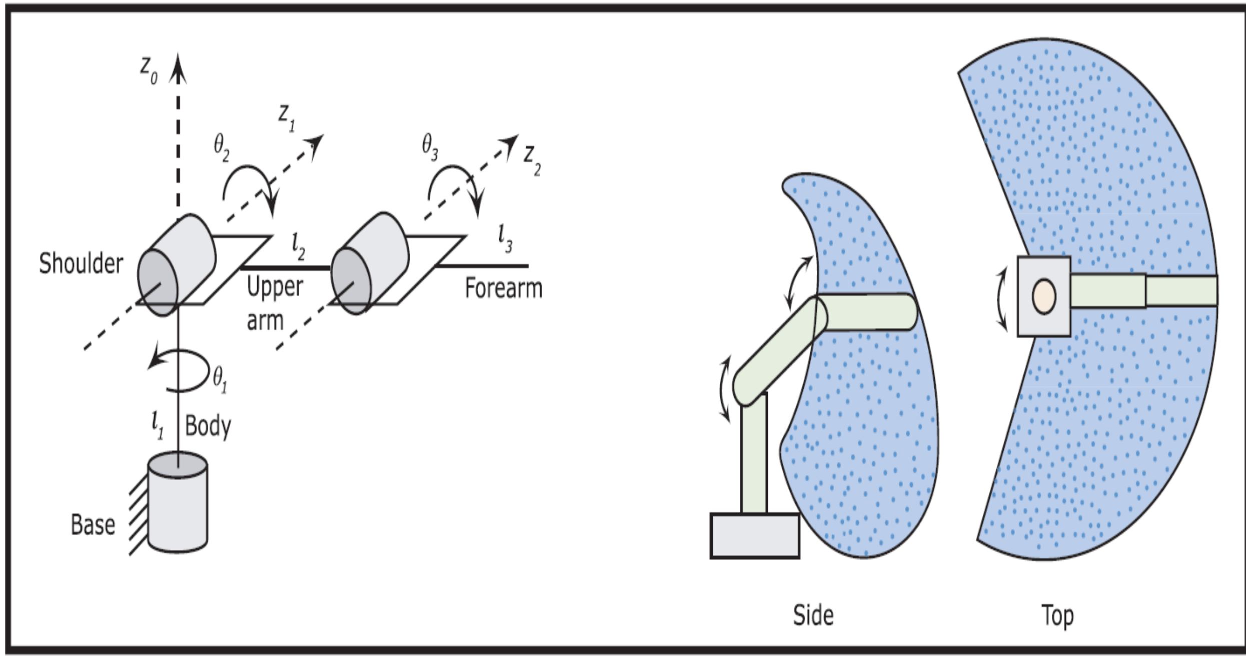
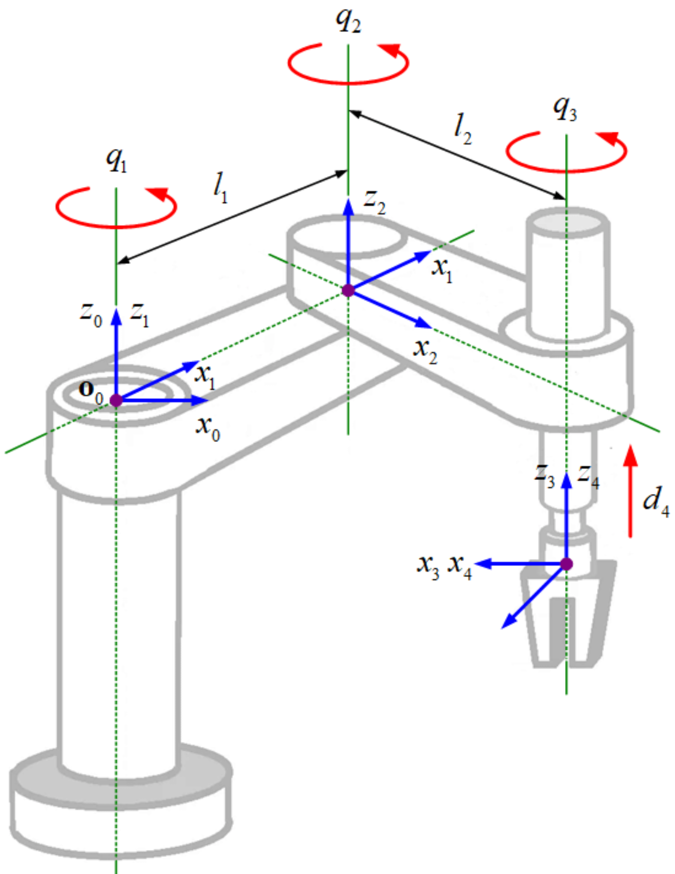


Figure by MIT OCW.

SCARA robot

- SCARA robot consisting of two revolute joints and one prismatic joint.

- This robot structure is particularly desirable for assembly automation in manufacturing systems, having a wide workspace in the horizontal direction and an independent vertical axis appropriate for the insertion of parts.



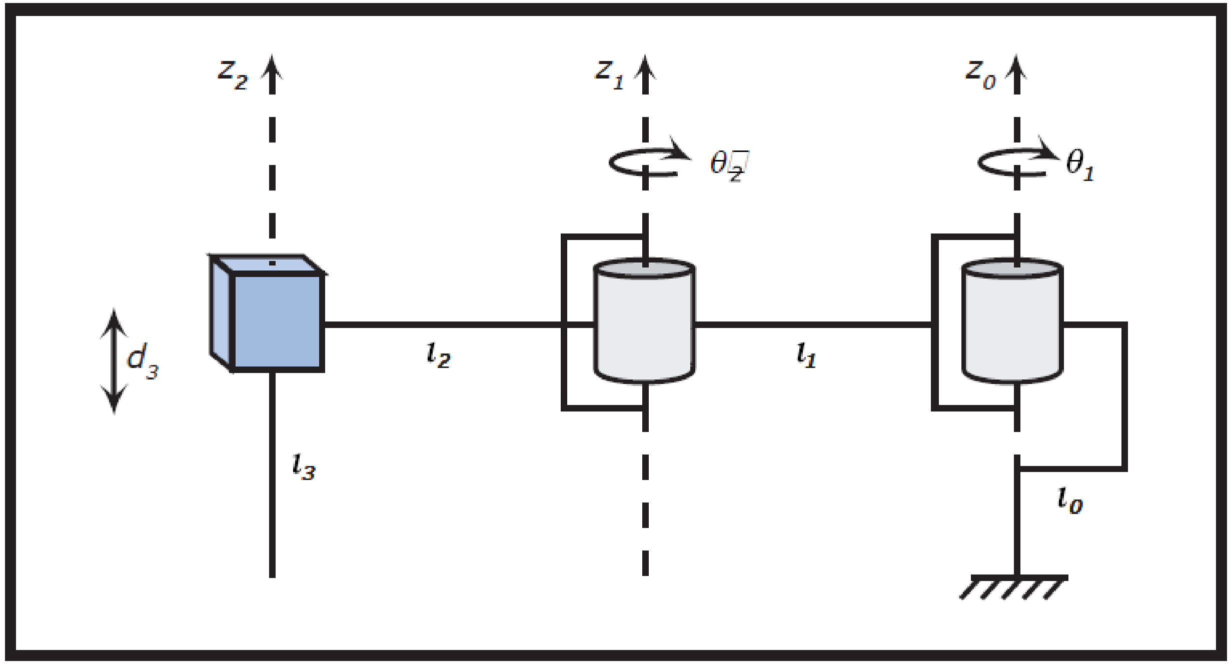


Figure by MIT OCW.

Summery

Robot	Axis 1	Axis 2	Axis 3	Total Revolute	Total Prismatic
Cartesian robot	Prismatic (P)	Prismatic (P)	Prismatic (P)	0	3
Cylindrical robot	Revolute (R)	Prismatic (P)	Prismatic (P)	1	2
Spherical robot	Revolute (R)	Revolute (R)	Prismatic (P)	2	1
Articulated robot	Revolute (R)	Revolute (R)	Revolute (R)	3	0
SCARA robot	Revolute (R)	Revolute (R)	Prismatic (P)	2	1

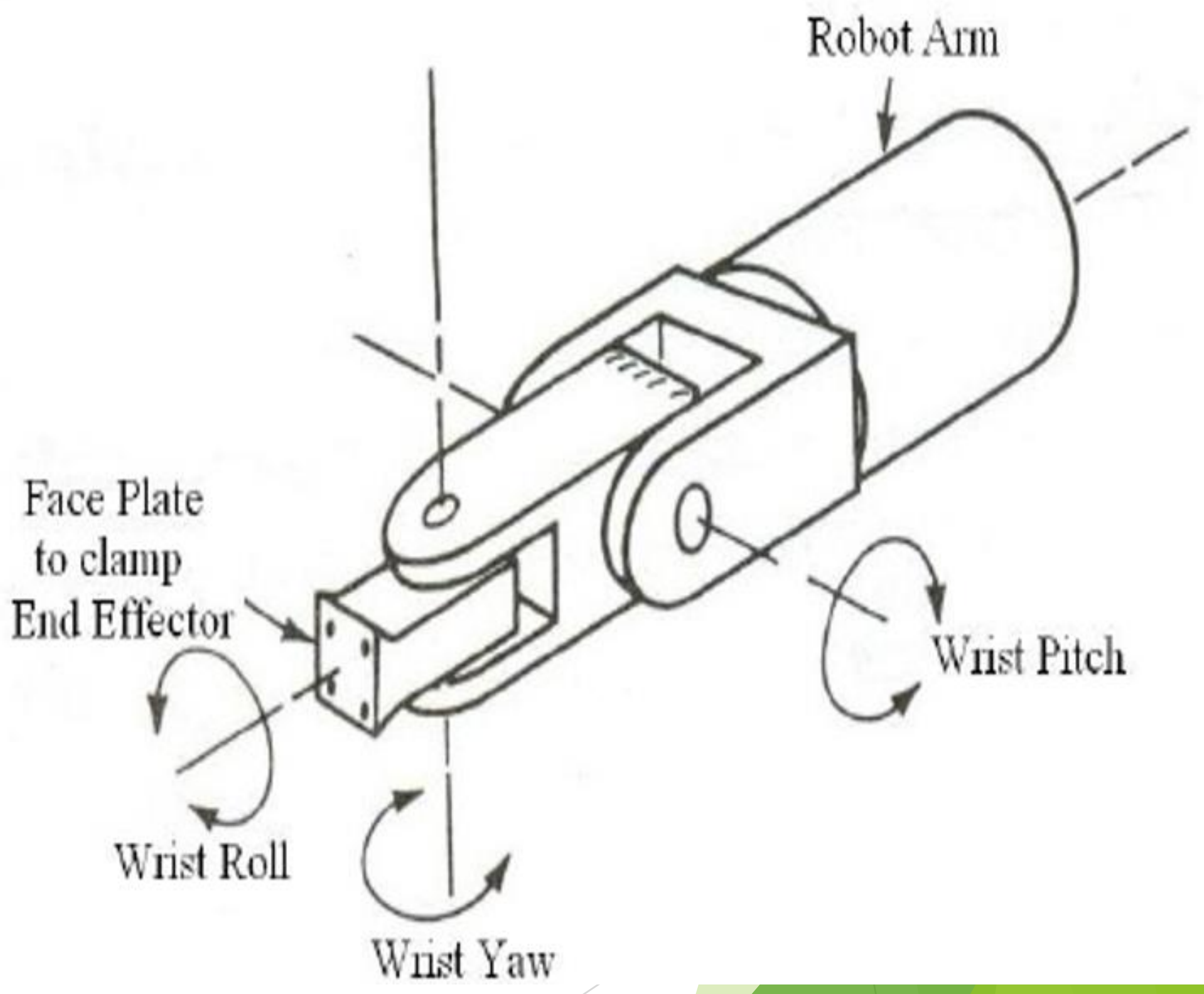
Robots Components

1. Manipulator or Rover: Main body of robot (links, joints, other structural element).
2. End Effecter: The part connected to the manipulator's last joint (hand).
3. Actuators: Muscles of the manipulators (servomotor, stepper motor, pneumatic, and hydraulic cylinder).
4. Sensors: To collect information about the robot's internal state or to communicate with the outside environment.
5. Controller: It controls and coordinates the motion of the actuators.
6. Processor: The brain of the robot. It calculates the motions and the velocity of the robot's joints, etc.
7. Software: Operating, robotic software, and the collection of routines.

Wrist motion

- Wrist assembly is attached to the end of the robot arm.
- The end effector is attached to the wrist assembly.
- The function of the wrist assembly is to orient the end effector.
- Three degrees of freedom wrist:

1. Pitch: up and down
2. Yaw: right and left
3. Roll or swivel: rotation of the hand



Matrix

Matrices introduction

Matrix algebra has at least two advantages:

•Reduces complicated systems of equations to simple expressions

•Adaptable to systematic method of mathematical treatment and well suited to computers

Definition:

A matrix is a set or group of numbers arranged in a square or rectangular array enclosed by two brackets

$$\begin{bmatrix} 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 4 & 2 \\ -3 & 0 \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Matrices introduction

Properties:

•A specified number of rows and a specified number of columns

•Two numbers (rows x columns) describe the dimensions or size of the matrix.

Examples:

3x3 matrix $\begin{bmatrix} 1 & 2 & 4 \\ 4 & -1 & 5 \\ 3 & 3 & 3 \end{bmatrix}$

2x4 matrix $\begin{bmatrix} 1 & 1 & 3 & -3 \\ 0 & 0 & 3 & 2 \end{bmatrix}$

1x2 matrix $\begin{bmatrix} 1 & -1 \end{bmatrix}$

Matrices introduction

A matrix is denoted by a bold capital letter and the elements within the matrix are denoted by lower case letters

e.g. matrix [**A**] with elements a_{ij}

$$\mathbf{A}_{mxn} = {}_m\mathbf{A}^n \quad \begin{bmatrix} a_{11} & a_{12}\ldots & a_{ij} & a_{in} \\ a_{21} & a_{22}\ldots & a_{ij} & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{ij} & a_{mn} \end{bmatrix}$$

i goes from 1 to m

j goes from 1 to n

Matrices introduction

EQUALITY OF MATRICES

Two matrices are said to be equal only when all corresponding elements are equal

Therefore their size or dimensions are equal as well

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \qquad \mathbf{A} = \mathbf{B}$$

Matrices introduction

Some properties of equality:

- If $\mathbf{A} = \mathbf{B}$, then $\mathbf{B} = \mathbf{A}$ for all $\mathbf{A}$ and $\mathbf{B}$
- If $\mathbf{A} = \mathbf{B}$, and $\mathbf{B} = \mathbf{C}$, then $\mathbf{A} = \mathbf{C}$ for all $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

If $\mathbf{A} = \mathbf{B}$ then $a_{ij} = b_{ij}$

Addition and subtraction of Matrices

The sum or difference of two matrices, **A** and **B** of the same size yields a matrix **C** of the same size

$$c_{ij} = a_{ij} + b_{ij}$$

Matrices of different sizes cannot be added or subtracted

Addition and subtraction of Matrices

Commutative Law:

$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$

Associative Law:

$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + \mathbf{B} + \mathbf{C}$

$$\underset{\substack{\mathbf{A}\\ 2\text{x}3}}{\begin{bmatrix} 7 & 3 & -1 \\ 2 & -5 & 6 \end{bmatrix}} + \underset{\substack{\mathbf{B}\\ 2\text{x}3}}{\begin{bmatrix} 1 & 5 & 6 \\ -4 & -2 & 3 \end{bmatrix}} = \underset{\substack{\mathbf{C}\\ 2\text{x}3}}{\begin{bmatrix} 8 & 8 & 5 \\ -2 & -7 & 9 \end{bmatrix}}$$

Addition and subtraction of Matrices

$\mathbf{A} + \mathbf{0} = \mathbf{0} + \mathbf{A} = \mathbf{A}$

$\mathbf{A} + (-\mathbf{A}) = \mathbf{0}$ (where $-\mathbf{A}$ is the matrix composed of $-a_{ij}$ as elements)

$$\begin{bmatrix} 6 & 4 & 2 \\ 3 & 2 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 2 \\ 2 & 2 & -1 \end{bmatrix}$$

Scalar Multiplication of Matrices

Matrices can be multiplied by a scalar (constant or single element)

Let k be a scalar quantity; then

$$kA = Ak$$

Ex. If k=4 and

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \\ 2 & -3 \\ 4 & 1 \end{bmatrix}$$

Scalar Multiplication of Matrices

$$4\times\begin{bmatrix}3 & -1\\ 2 & 1\\ 2 & -3\\ 4 & 1\end{bmatrix}=\begin{bmatrix}3 & -1\\ 2 & 1\\ 2 & -3\\ 4 & 1\end{bmatrix}\times 4=\begin{bmatrix}12 & -4\\ 8 & 4\\ 8 & -12\\ 16 & 4\end{bmatrix}$$

Properties:

- $k\,(\mathbf{A}+\mathbf{B})=k\mathbf{A}+k\mathbf{B}$
- $(k+g)\mathbf{A}=k\mathbf{A}+g\mathbf{A}$
- $k(\mathbf{AB})=(k\mathbf{A})\mathbf{B}=\mathbf{A}(k)\mathbf{B}$
- $k(g\mathbf{A})=(kg)\mathbf{A}$

Multiplication of Matrices

The product of two matrices is another matrix

Two matrices **A** and **B** must be **conformable** for multiplication to be possible

i.e. the number of columns of **A** must equal the number of rows of **B**

Example.

$$\underset{(1\text{x}3)}{\mathbf{A}} \quad \text{x} \quad \underset{(3\text{x}1)}{\mathbf{B}} \quad = \quad \underset{(1\text{x}1)}{\mathbf{C}}$$

Multiplication of Matrices

B x **A** = Not possible!

(2x1) (4x2)

A x **B** = Not possible!

(6x2) (6x3)

Example

A x **B** = **C**

(2x3) (3x2) (2x2)

Multiplication of Matrices

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$(a_{11} \times b_{11}) + (a_{12} \times b_{21}) + (a_{13} \times b_{31}) = c_{11}$$

$$(a_{11} \times b_{12}) + (a_{12} \times b_{22}) + (a_{13} \times b_{32}) = c_{12}$$

$$(a_{21} \times b_{11}) + (a_{22} \times b_{21}) + (a_{23} \times b_{31}) = c_{21}$$

$$(a_{21} \times b_{12}) + (a_{22} \times b_{22}) + (a_{23} \times b_{32}) = c_{22}$$

Successive multiplication of row i of **A** with column j of **B** – row by column multiplication

Multiplication of Matrices

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 2 & 7 \end{bmatrix}\begin{bmatrix} 4 & 8 \\ 6 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} (1\times4)+(2\times6)+(3\times5) & (1\times8)+(2\times2)+(3\times3) \\ (4\times4)+(2\times6)+(7\times5) & (4\times8)+(2\times2)+(7\times3) \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix}$$

Remember also:

IA = A

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix} = \begin{bmatrix} 31 & 21 \\ 63 & 57 \end{bmatrix}$$

Multiplication of Matrices

Assuming that matrices **A**, **B** and **C** are conformable for the operations indicated, the following are true:

1. $\mathbf{AI} = \mathbf{IA} = \mathbf{A}$
2. $\mathbf{A(BC)} = \mathbf{(AB)C} = \mathbf{ABC}$ - (associative law)
3. $\mathbf{A(B+C)} = \mathbf{AB} + \mathbf{AC}$ - (first distributive law)
4. $\mathbf{(A+B)C} = \mathbf{AC} + \mathbf{BC}$ - (second distributive law)

Caution!

1. **AB** not generally equal to **BA**, **BA** may not be conformable
2. If $\mathbf{AB} = \mathbf{0}$, neither **A** nor **B** necessarily $= \mathbf{0}$
3. If $\mathbf{AB} = \mathbf{AC}$, **B** not necessarily $= \mathbf{C}$

Multiplication of Matrices

AB not generally equal to **BA**, **BA** may not be conformable

$$T = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix}$$

$$TS = \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 15 & 20 \end{bmatrix}$$

$$ST = \begin{bmatrix} 3 & 4 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 23 & 6 \\ 10 & 0 \end{bmatrix}$$

Multiplication of Matrices

If **AB** = **0**, neither **A** nor **B** necessarily = **0**

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Transpose of a Matrix

If :

$$\underset{2x3}{A} = {}_2A^3 = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 3 & 1 \end{bmatrix}$$

Then transpose of A, denoted A^T is:

$$A^T = {}_2A^{3^T} = \begin{bmatrix} 2 & 5 \\ 4 & 3 \\ 7 & 1 \end{bmatrix}$$

$$a_{ij} = a^T_{ji} \quad \text{For all } i \text{ and } j$$

Transpose of a Matrix

To transpose:

Interchange rows and columns

The dimensions of $\mathbf{A}^T$ are the reverse of the dimensions of $\mathbf{A}$

$$A = {}_2A^3 = \begin{bmatrix} 2 & 4 & 7 \\ 5 & 3 & 1 \end{bmatrix} \qquad 2 \times 3$$

$$A^T = {}_3A^{T^2} = \begin{bmatrix} 2 & 5 \\ 4 & 3 \\ 7 & 1 \end{bmatrix} \qquad 3 \times 2$$

Transpose of a Matrix

Properties of transposed matrices:

1. $(\mathbf{A}+\mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$
2. $(\mathbf{A}\mathbf{B})^T = \mathbf{B}^T \mathbf{A}^T$
3. $(k\mathbf{A})^T = k\mathbf{A}^T$
4. $(\mathbf{A}^T)^T = \mathbf{A}$

Determinant of a Matrix

To compute the inverse of a matrix, the determinant is required

Each square matrix **A** has a unit scalar value called the determinant of **A**, denoted by det **A** or |**A**|

If $A = \begin{bmatrix} 1 & 2 \\ 6 & 5 \end{bmatrix}$

then $|A| = \begin{vmatrix} 1 & 2 \\ 6 & 5 \end{vmatrix}$

Determinant of a Matrix

For a 3 x 3 matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The cofactors of the first row are:

$$c_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} = a_{22}a_{33} - a_{23}a_{32}$$

$$c_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} = -(a_{21}a_{33} - a_{23}a_{31})$$

$$c_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = a_{21}a_{32} - a_{22}a_{31}$$

Determinant of a Matrix

The determinant of a matrix A is:

$$|A| = a_{11}c_{11} + a_{12}c_{12} = a_{11}a_{22} - a_{12}a_{21}$$

Which by substituting for the cofactors in this case is:

$$|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Determinant of a Matrix

Example 2:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}$$

$$|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

$$|A| = (1)(2-0) - (0)(0+3) + (1)(0+2) = 4$$

The inverse of a Matrices

Consider a scalar k. The inverse is the reciprocal or division of 1 by the scalar.

Example:

$k=7$ the inverse of k or $k^{-1} = 1/k = 1/7$

Division of matrices is not defined since there may be $\mathbf{AB} = \mathbf{AC}$ while $\mathbf{B} \neq \mathbf{C}$

Instead matrix inversion is used.

The inverse of a square matrix, **A**, if it exists, is the unique matrix $\mathbf{A}^{-1}$ where:

$$\mathbf{AA}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$$

The inverse of a Matrices

Example:

$$A{=}_2A^2 = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

Because:

$$\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Why will it help us solve equations?

Because if we can express a system of equations in the form

$$Ax = b$$

Then we can multiply both sides by the inverse matrix

$$A^{-1}Ax = A^{-1}b$$

And we can then know the values of X because $\boxed{A^{-1}A = I}$

$$\boxed{\mathbf{x = A^{-1}b}}$$

The inverse of a 3x3 Matrices

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

- **Step 1:** *First, verify if the matrix can be inverted. To do this, calculate the matrix's determinant. If the determinant is not zero, proceed to the next step.*
- **Step 2:** *Calculate the determinant of smaller 2 × 2 matrices within the larger matrix.*
- **Step 3:** *Create the cofactor matrix.*

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

- **Step 4:** *Obtain the matrix's Adjugate or Adjoint by making the cofactor matrix transpose.*
- **Step 5:** *Finally, divide each element in the adjugate matrix by the determinant of the original 3 by 3 matrix.*

The inverse of a Matrices

$$If\ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} and\ det(A) \neq 0, then$$

$$A^{-1} = \frac{1}{det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example 1: Find the Inverse of

$$D = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 1 & 0 \\ 1 & 4 & 2 \end{bmatrix}$$

Solution:

Det D = 3(2) + 0(-4) + 2(7)   *Det D = 20*

$$\text{Minor Matrix of } D = \begin{bmatrix} \begin{pmatrix} 1 & 0 \\ 4 & 2 \end{pmatrix} & \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} & \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix} \\ \begin{pmatrix} 0 & 2 \\ 4 & 2 \end{pmatrix} & \begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix} & \begin{pmatrix} 3 & 0 \\ 1 & 4 \end{pmatrix} \\ \begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix} & \begin{pmatrix} 3 & 2 \\ 2 & 0 \end{pmatrix} & \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix} \end{bmatrix}$$

$$\textit{Minor Matrix of } D = \begin{bmatrix} (2-0) & (4-0) & (8-1) \\ (0-8) & (6-2) & (12-0) \\ (0-2) & (0-4) & (3-0) \end{bmatrix}$$

$$\textit{Cofactor of Matrix i.e., } X = \begin{bmatrix} +2 & -(\quad 4) & +7 \\ -(-8) & +4 & -(12) \\ +2 & -(-4) & +3 \end{bmatrix}$$

$$\textit{Transpose of Matrix } X = \textit{Adj } D = \begin{bmatrix} 2 & 8 & 2 \\ -4 & 4 & 4 \\ 7 & -12 & 3 \end{bmatrix}$$

$$\textit{Det } D = 20$$

Inverse of Matrix D or D^{-1} = Adj D / Det D

$$D^{-1} = \begin{bmatrix} \frac{2}{20} & \frac{8}{20} & \frac{2}{20} \\ -\frac{4}{20} & \frac{4}{20} & \frac{4}{20} \\ \frac{7}{20} & -\frac{12}{20} & \frac{3}{20} \end{bmatrix}$$

The inverse of a Matrices

Properties of the inverse:

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(A^{-1})^{-1} = A$$

$$(A^{T})^{-1} = (A^{-1})^{T}$$

$$(kA)^{-1} = \frac{1}{k}A^{-1}$$

A square matrix that has an inverse is called a nonsingular matrix

A matrix that does not have an inverse is called a singular matrix

Square matrices have inverses except when the determinant is zero

When the determinant of a matrix is zero the matrix is singular

1. Find the Inverse of the Following Matrix

$$\begin{bmatrix} 6 & 2 & 3 \\ 3 & 1 & 1 \\ 10 & 3 & 4 \end{bmatrix}$$

Solution:

$$A^{-1} = 1/1 = \begin{bmatrix} -1 & -1 & 1 \\ 2 & 6 & -3 \\ 1 & -2 & 0 \end{bmatrix}$$