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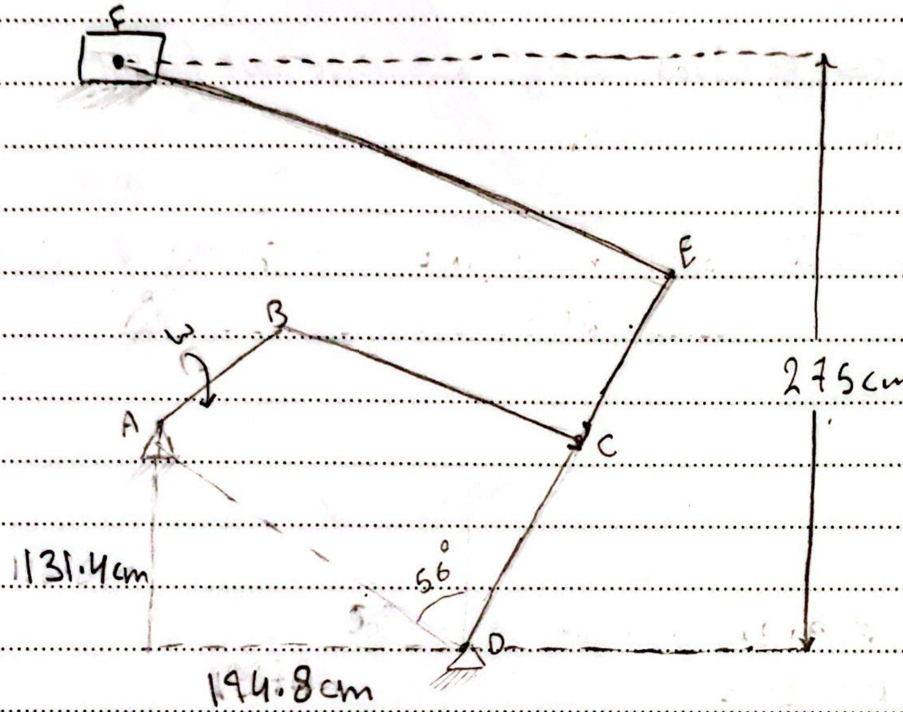
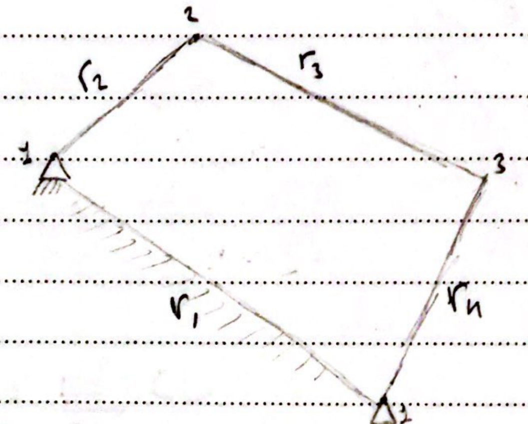
Machine design Milestone 1



Investigating the rotatability

4 Bar mechanism



ID = 49-10359

$r_2 = 59$ cm

$r_1 = \sqrt{131.4^2 + 194.8^2} = 235$ cm

let $r_3 = 208$ cm

& $r_4 = 131.75$ cm

r_1 fixed ∴ crank rocker mechanism ∴ r_2 should make full revolution

→ To prove that, we use Grashof

$S + l \leq P + Q \rightarrow 59 + 235 \leq 208 + 131.75 \rightarrow \boxed{294 \leq 339.75}$

∴ the crank (r_2) will make full revolution

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Now after simulating on linkage the dimensions are as follows

AB = 59 cm BC = 208 cm DC = 131.75 cm CE = 131.75 EF = 372 cm

Vector analysis

The vector analysis will be done in 2 parts 1 for the 4 bar mechanism to get its outputs which act as the inputs to the slider mechanism

Note that $\vec{r} = r e^{j\theta}$

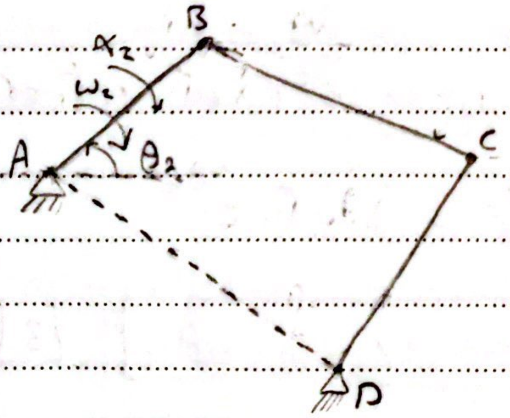
$\dot{\vec{r}} = (\dot{r} + j\dot{\theta} r) e^{j\theta}$

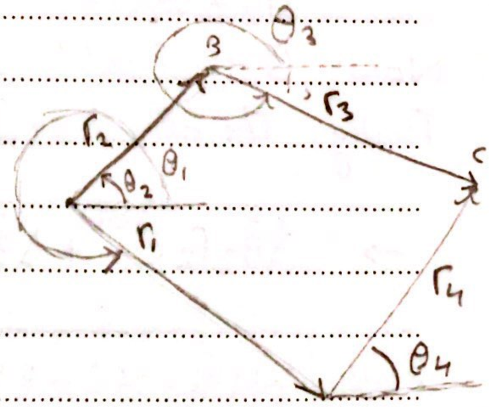
$\ddot{\vec{r}} = [(\ddot{r} - \dot{\theta}^2 r) + j(\ddot{\theta} r + 2\dot{\theta}\dot{r})] e^{j\theta}$

$e^{j\theta} = \cos\theta + j\sin\theta$

$j e^{j\theta} = j\cos\theta - \sin\theta$

4 bar mechanism





$\therefore \boxed{\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4}$ (i)

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Position Analysis

$$\vec{r}_2 + \vec{r}_3 = \vec{r}_1 + \vec{r}_4$$

$$r_2 e^{j\theta_2} + r_3 e^{j\theta_3} = r_1 e^{j\theta_1} + r_4 e^{j\theta_4}$$

Real: $\boxed{r_2 \cos\theta_2 + r_3 \cos\theta_3 = r_1 \cos\theta_1 + r_4 \cos\theta_4}$ [1]

Im: $\boxed{r_2 \sin\theta_2 + r_3 \sin\theta_3 = r_1 \sin\theta_1 + r_4 \sin\theta_4}$ [2]

Known parameters: $r_1, r_2, r_3, r_4, \theta_1, \theta_2$
unknown parameters: θ_3, θ_4

Velocity Analysis

$$\dot{\vec{r}}_2 + \dot{\vec{r}}_3 = \dot{\vec{r}}_1 + \dot{\vec{r}}_4$$

$$(\dot{r}_2 + j\dot{\theta}_2 r_2) e^{j\theta_2} + (\dot{r}_3 + j\dot{\theta}_3 r_3) e^{j\theta_3} = (\dot{r}_1 + j\dot{\theta}_1 r_1) e^{j\theta_1} + (\dot{r}_4 + j\dot{\theta}_4 r_4) e^{j\theta_4}$$

Now θ_1 is fixed $\therefore \frac{d\theta_1}{dt} = 0$, also $\frac{d\theta_i}{dt} = \omega_i$

Finally the magnitude of $r_1, r_2, r_3 \& r_4$ is fixed $\therefore \dot{r}_1 = \dot{r}_2 = \dot{r}_3 = \dot{r}_4 = 0$

$$\rightarrow \cancel{j}\dot{\theta}_2 r_2 e^{j\theta_2} + \cancel{j}\dot{\theta}_3 r_3 e^{j\theta_3} = \cancel{j}\dot{\theta}_4 r_4 e^{j\theta_4}$$

Real: $\boxed{\omega_2 r_2 \cos\theta_2 + \omega_3 r_3 \cos\theta_3 = \omega_4 r_4 \cos\theta_4}$ [3]

Im: $\boxed{\omega_2 r_2 \sin\theta_2 + \omega_3 r_3 \sin\theta_3 = \omega_4 r_4 \sin\theta_4}$ [4]

Known parameters: $\omega_2, \theta_2, r_2, \theta_3, r_3, \theta_4, r_4$
unknown parameters: ω_3, ω_4

A

Scanned with CamScanner

Acceleration Analysis

$$\ddot{\vec{r}}_2 + \ddot{\vec{r}}_3 = \ddot{\vec{r}}_1 + \ddot{\vec{r}}_4$$

now from the velocity analysis, we found that $\dot{\vec{r}}_1 = 0$ ∴ $\ddot{\vec{r}}_1 = 0$

$$[(\ddot{r}_2 - \dot{\theta}_2^2 r_2) + j(\ddot{\theta}_2 r_2 + \cancel{2\dot{\theta}_2 \dot{r}_2})] e^{j\theta_2} + [(\ddot{r}_3 - \dot{\theta}_3^2 r_3) + j(\ddot{\theta}_3 r_3 + \cancel{2\dot{\theta}_3 \dot{r}_3})] e^{j\theta_3}$$
$$= [(\ddot{r}_4 - \dot{\theta}_4^2 r_4) + j(\ddot{\theta}_4 r_4 + \cancel{2\dot{\theta}_4 \dot{r}_4})] e^{j\theta_4}$$

again the magnitude of r_1, r_2, r_3 & r_4 is fixed ∴ $\ddot{r}_1 = \dot{r}_1 = \ddot{r}_2 = \dot{r}_2 = \ddot{r}_3 = \dot{r}_3 = \ddot{r}_4 = \dot{r}_4 = 0$
& $\frac{d\dot{\theta}_i}{dt} = \alpha_i$

$$\rightarrow -\omega_2^2 r_2 e^{j\theta_2} + j\alpha_2 r_2 e^{j\theta_2} - \omega_3^2 r_3 e^{j\theta_3} + j\alpha_3 r_3 e^{j\theta_3} = -\omega_4^2 r_4 e^{j\theta_4} + j\alpha_4 r_4 e^{j\theta_4}$$

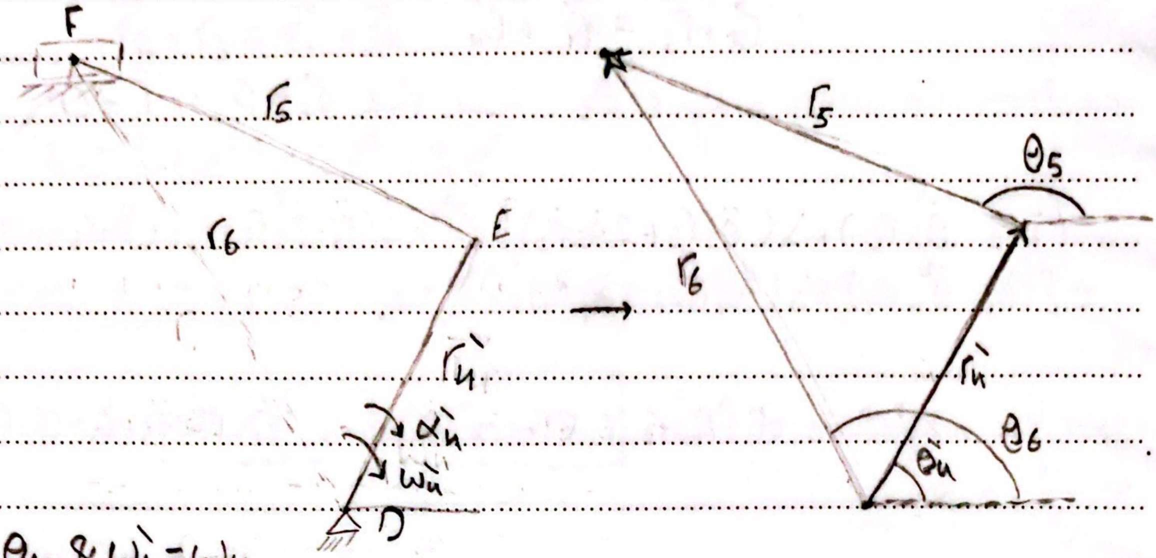
Real: $\boxed{-\omega_2^2 r_2 \cos\theta_2 - \alpha_2 r_2 \sin\theta_2 - \omega_3^2 r_3 \cos\theta_3 - \alpha_3 r_3 \sin\theta_3 = -\omega_4^2 r_4 \cos\theta_4 - \alpha_4 r_4 \sin\theta_4}$ [5]

Im: $\boxed{-\omega_2^2 r_2 \sin\theta_2 + \alpha_2 r_2 \cos\theta_2 - \omega_3^2 r_3 \sin\theta_3 + \alpha_3 r_3 \cos\theta_3 = -\omega_4^2 r_4 \sin\theta_4 + \alpha_4 r_4 \cos\theta_4}$ [6]

Known parameters : $\omega_2, \alpha_2, r_2, \omega_3, r_3, \omega_4, r_4$
Unknown parameters : α_3, α_4

This analysis was imprtat since we need it to analyze our output. This Analysis was for the 4 bar mechanism ABCD

Vector Analysis of the slider



Now $\theta_4' = \theta_4$ & $\omega_4' = \omega_4$

& $\alpha_4' = \alpha_4$

& from the previous analysis, we know $\theta_4, \omega_4,$ & α_4

the vector equation would be $\boxed{\vec{r_4'} + \vec{r_5} = \vec{r_6}}$ (ii)

Position Analysis

$$\vec{r_4'} + \vec{r_5} = \vec{r_6}$$

$$r_4' e^{j\theta_4'} + r_5 e^{j\theta_5} = r_6 e^{j\theta_6}$$

Real: $\boxed{r_4' \cos\theta_4' + r_5 \cos\theta_5 = r_6 \cos\theta_6}$ [7]

Im: $\boxed{r_4' \sin\theta_4' + r_5 \sin\theta_5 = r_6 \sin\theta_6}$ [8]

Known parameters: $r_4', \theta_4', r_5, r_6$

unknown parameters : θ_5, θ_6

Velocity Analysis

$$\vec{r_4'} + \vec{r_5} = \vec{r_6}$$

$$(\dot{r}_4' + j\dot{\theta}_4' r_4')e^{j\theta_4'} + (\dot{r}_5 + j\dot{\theta}_5 r_5)e^{j\theta_5} = (\dot{r}_6 + j\dot{\theta}_6 r_6)e^{j\theta_6}$$

Now $\dot{r}_4' = \dot{r}_5 = 0$ since its a fixed magnitude, but r_6 is not fixed

$$\rightarrow \quad j\dot{\theta}_4' r_4' e^{j\theta_4'} + j\dot{\theta}_5 r_5 e^{j\theta_5} = \dot{r}_6 e^{j\theta_6} + j\dot{\theta}_6 r_6 e^{j\theta_6}$$

Real: $\boxed{-\omega_4' r_4' \sin\theta_4' - \omega_5 r_5 \sin\theta_5 = \dot{r}_6 \cos\theta_6 - \omega_6 r_6 \sin\theta_6}$ [9]

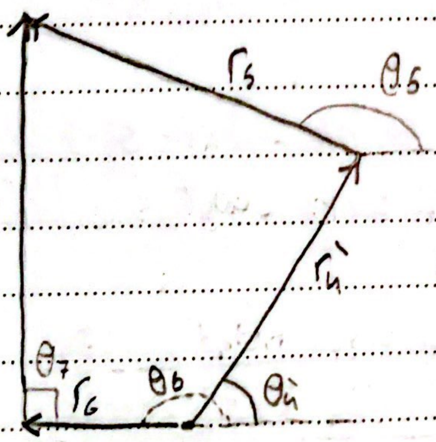
Im: $\boxed{\omega_4' r_4' \cos\theta_4 + \omega_5 r_5 \cos\theta_5 = \dot{r}_6 \sin\theta_6 + \omega_6 r_6 \cos\theta_6}$ [10]

Known parameters: $\omega_4', \theta_4', r_4', \theta_5, r_5, \theta_6, r_6$

Unkown parameters: $\omega_5, \omega_6, \dot{r}_6$ ← 2 equations 3 unknowns ∴ we need a different representation

We shall breakdown $\vec{r_6}$ into vertical & horizontal

∴



∴ our new vector equation is:

$$\boxed{\vec{r_6} + \vec{r_7} = \vec{r_4'} + \vec{r_5}} \quad \text{(iii)}$$

Now again $\theta_4' = \theta_4$, $\omega_4' = \omega_4$ & $\theta_6 = 180°$, $\theta_7 = 90°$, $\omega_7 = \omega_6 = 0$

& $\alpha_4' = \alpha_4$

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Position Analysis

$$\vec{r_6} + \vec{r_7} = \vec{r_4} + \vec{r_5}$$

$$r_6 e^{j\theta_6} + r_7 e^{j\theta_7} = r_4 e^{j\theta_4} + r_5 e^{j\theta_5}$$

Real: $r_6 \cos\theta_6 + r_7 \cos\theta_7 = r_4 \cos\theta_4 + r_5 \cos\theta_5$, & $\theta_7 = 90^\circ$ & $\theta_6 = 180^\circ$

$$\boxed{-r_6 = r_4 \cos\theta_4 + r_5 \cos\theta_5} \quad [7.1]$$

Im: $r_6 \sin\theta_6 + r_7 \sin\theta_7 = r_4 \sin\theta_4 + r_5 \sin\theta_5$

$$\boxed{r_7 = r_4 \sin\theta_4 + r_5 \sin\theta_5} \quad [8.1]$$

Known parameters: r_4, r_5, r_7, θ_4
Unknown parameters: θ_5, r_6

Velocity Analysis

$$\dot{\vec{r_6}} + \dot{\vec{r_7}} = \dot{\vec{r_4}} + \dot{\vec{r_5}}$$

$$(\dot{r}_6 + j\dot{\theta}_6 r_6) e^{j\theta_6} + (\dot{r}_7 + j\dot{\theta}_7 r_7) e^{j\theta_7} = (\dot{r}_4 + j\dot{\theta}_4 r_4) e^{j\theta_4} + (\dot{r}_5 + j\dot{\theta}_5 r_5) e^{j\theta_5}$$

$\dot{\theta}_6 = \dot{\theta}_7 = 0$, $\dot{r}_7 = \dot{r}_4 = \dot{r}_5 = 0$

$$\rightarrow \quad \dot{r}_6 e^{j\theta_6} = j\dot{\theta}_4 r_4 e^{j\theta_4} + j\dot{\theta}_5 r_5 e^{j\theta_5}$$

Real: $\dot{r}_6 \cos\theta_6 = -\dot{\theta}_4 r_4 \sin\theta_4 - \dot{\theta}_5 r_5 \sin\theta_5$ $\theta_6 = 180$ & $\dot{\theta}_x = \omega_x$

$$\rightarrow \boxed{-\dot{r}_6 = -\omega_4 r_4 \sin\theta_4 - \omega_5 r_5 \sin\theta_5} \quad [9.1]$$

Im: $\dot{r}_6 \sin\theta_6 = \dot{\theta}_4 r_4 \cos\theta_4 + \dot{\theta}_5 r_5 \cos\theta_5$

$$\boxed{0 = \omega_4 r_4 \cos\theta_4 + \omega_5 r_5 \cos\theta_5} \quad [10.1]$$

Known parameters: $\omega_4, \theta_4, \theta_5, r_4, r_5$
Unknown parameters: $\dot{r}_6, \omega_5$

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Acceleration Analysis

$$\ddot{\vec{r}}_6 + \ddot{\vec{r}}_7 = \ddot{\vec{r}}_4 + \ddot{\vec{r}}_5$$

Since $\dot{r}_7 = 0 \therefore \ddot{r}_7 = 0$

$$\ddot{\vec{r}}_6 = \ddot{\vec{r}}_4 + \ddot{\vec{r}}_5$$

$$[(\ddot{r}_6 - \dot{\theta}_6^2 r_6) + j(\ddot{\theta}_6 r_6 + 2\dot{\theta}_6 \dot{r}_6)]e^{j\theta_6} = [(\ddot{r}_4 - \dot{\theta}_4^2 r_4) + j(\ddot{\theta}_4 r_4 + 2\dot{\theta}_4 \dot{r}_4)]e^{j\theta_4} + [(\ddot{r}_5 - \dot{\theta}_5^2 r_5) + j(\ddot{\theta}_5 r_5 + 2\dot{\theta}_5 \dot{r}_5)]e^{j\theta_5}$$

$\ddot{r}_4 = \ddot{r}_5 = 0 \quad \omega_6 = 0 \quad \ddot{\theta}_6 = 0$

Real: $\ddot{r}_6 \cos\theta_6 = -\omega_4^2 r_4 \cos\theta_4 - \ddot{\theta}_4 r_4 \sin\theta_4 - \omega_5^2 r_5 \cos\theta_5 - \ddot{\theta}_5 r_5 \sin\theta_5$

$\ddot{\theta}_x = \alpha_x$ & $\theta_6 = 180$

$$\boxed{-\ddot{r}_6 = -\omega_4^2 r_4 \cos\theta_4 - \alpha_4 r_4 \sin\theta_4 - \omega_5^2 r_5 \cos\theta_5 - \alpha_5 r_5 \sin\theta_5} \quad [11]$$

Im: $$\boxed{0 = -\omega_4^2 r_4 \sin\theta_4 + \alpha_4 r_4 \cos\theta_4 - \omega_5^2 r_5 \sin\theta_5 + \alpha_5 r_5 \cos\theta_5} \quad [12]$$

because $\ddot{r}_6 \sin\theta_6$ with $\theta = 180 = 0$

Known parameters: $\omega_4, \theta_4, r_4, \alpha_4, \omega_5, \theta_5, r_5$
Unknown parameters: $\ddot{r}_6, \alpha_5$

This would conclude the motion study of the mechanism. The position, velocity, & acceleration of the slider can now be found. (Denote slider as S)

where $$\boxed{\begin{aligned} r_S &= r_6 \\ V_S &= \dot{r}_6 \\ a_S &= \ddot{r}_6 \end{aligned}}$$