Prof. Dr. Michael Hinze
Dr. Sema Coskun

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Mathematical Institute

Exercises for Optimization I
Sheet 2*

You can submit your solutions until Friday, May 6 at 23pm,
Please name your file LastnameFirstnameSheet2

Exercise 1: Let $A \in \mathbb{R}^{m\times n}$ be a matrix with $\operatorname{rank}(A) = n$ and $b \in \mathbb{R}^m$ be given. Why does a unique solution to...

a) ...the linear fitting problem

$$\min_{x\in\mathbb{R}^n} \|Ax - b\|^2$$

b) ...the restricted linear fitting problem

$$\min_{x\in M} \|Ax - b\|^2, \quad \text{with } M := \{x \in \mathbb{R}^n\,;\, x \geq 0\}$$

exist? Compute the solution x^* in the case of a).

Exercise 2: Let $f : X \to \mathbb{R}$ be defined on the convex set $X \subset \mathbb{R}^n$. Prove that f is convex, if and only if its *epigraph*

$$\operatorname{epi}(f) := \{(x,\alpha) \in X \times \mathbb{R}\,;\, f(x) \leq \alpha\}$$

is a convex set.

Exercise 3: Show that the *Rosenbrock function* $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x) := 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

has a global minimum, which is also the only stationary point of the function f.
Furthermore calculate the eigenvalues of the hessian matrix of f at the global minimum.

Exercise 4: Are the following functions (strictly) convex? Prove (strict) convexity or give a counterexample.

a) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) := x^2$

b) $g : \mathbb{R} \to \mathbb{R}, \quad g(x) := x^3$

c) $h : \mathbb{R}^n \to \mathbb{R}, \quad h(x) := \|x\|$

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*__Group 2__ will present the solutions.

Numerical Exercise 1: Implement the descent algorithm with $d^k = -\nabla f(x^k)$ and fixed steplength $t_k = 0.00125$ for the Rosenbrock function $f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x) := 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

Try at least 5 different starting points of your choice. As a termination condition use

$$\|\nabla f(x^k)\| < 10^{-5}$$

and don't iterate more than 10^5 times. Visualize the iteration process.

Descent algorithm

i) $x^0 \in \mathbb{R}^n$ given, $k := 0$ iteration counter

ii) if x^k satisfies the termination condition: STOP with $x^* = x^k$

iii) find a descent direction d^k of f in x^k

iv) find a steplength t_k such that $f(x^k + t_k d^k) < f(x^k)$

v) $x^{k+1} = x^k + t_k d^k, \quad k = k + 1, \quad$ go to ii)