In [14]:
```python
%matplotlib inline
import matplotlib.pylab as plt
import numpy as np
```

Consider the Rosenbrock function

$$f(x,y) = 100(y - x^2)^2 + (1-x)^2$$

The only minimum is at $(x,y) = (1,1)$ where $f(1,1) = 0$.

In [15]:
```python
def rosenbrockfunction(x,y):
    return 100*(y-x**2)**2 + (1-x)**2
```

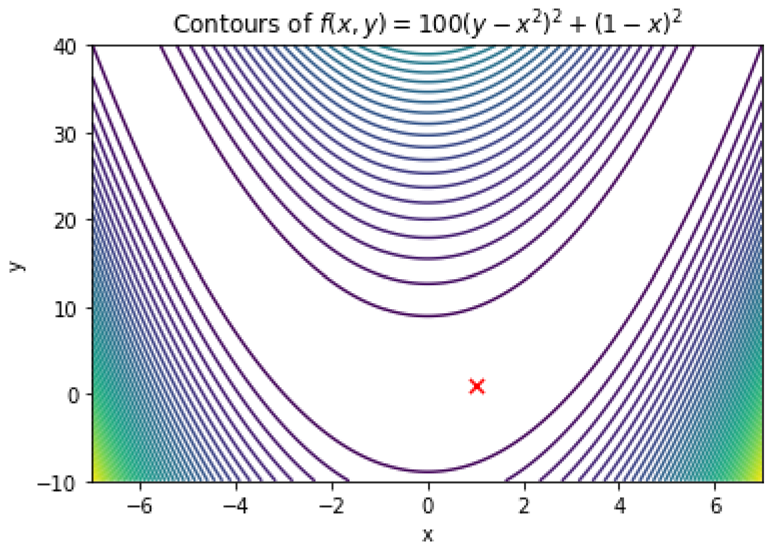
Create a utility function that plots the contours of the Rosenbrock function.

In [16]:
```python
def contourplot(rosenbrockfunction, xmin, xmax, ymin, ymax, ncontours=50, fill=True):

    x = np.linspace(xmin, xmax, 200)
    y = np.linspace(ymin, ymax, 200)
    X, Y = np.meshgrid(x,y)
    Z = rosenbrockfunction(X,Y)
    if fill:
        plt.contourf(X,Y,Z,ncontours); # plot the contours
    else:
        plt.contour(X,Y,Z,ncontours); # plot the contours
    plt.scatter(1,1,marker="x",s=50,color="r");  # mark the minimum
```

Here is a contour plot of the Rosenbrock function, with the global minimum marked with a red cross.

In [17]:
```python
contourplot(rosenbrockfunction, -7,7, -10, 40, fill=False)
plt.xlabel("x")
plt.ylabel("y")
plt.title("Contours of $f(x,y)=100(y-x^2)^2 + (1-x)^2$");
```



Steepest descent (gradient descent) with decreasing step length

First we write a function that uses the steepest descent method. Initializes the solution at position `init`, moves along the opposite of the gradient with step length $\alpha_k = \alpha_0/(k+1)^\gamma$, until the absolute difference between function values drops below `tolerance` or until the number of iterations exceeds `maxiter`.

The function returns the array of all intermediate positions, and the array of function values.

In [18]:
```python
def steepestdescent(rosenbrockfunction, gradient, init, tolerance=1e-5, maxiter=100000, alpha0=0.01, gamma=0.25
    p = init
    iterno=0
    parray = [p]
    fprev = rosenbrockfunction(p[0],p[1])
    farray = [fprev]
    while iterno < maxiter:
        steplength = alpha0 * (iterno+1)**(-gamma)
        p = p - steplength*gradient(p[0],p[1])
        fcur = rosenbrockfunction(p[0], p[1])
        if np.isnan(fcur):
            break
        parray.append(p)
        farray.append(fcur)
        if abs(fcur-fprev)<tolerance:
            break
        fprev = fcur
        iterno += 1
    return np.array(parray), np.array(farray)
```

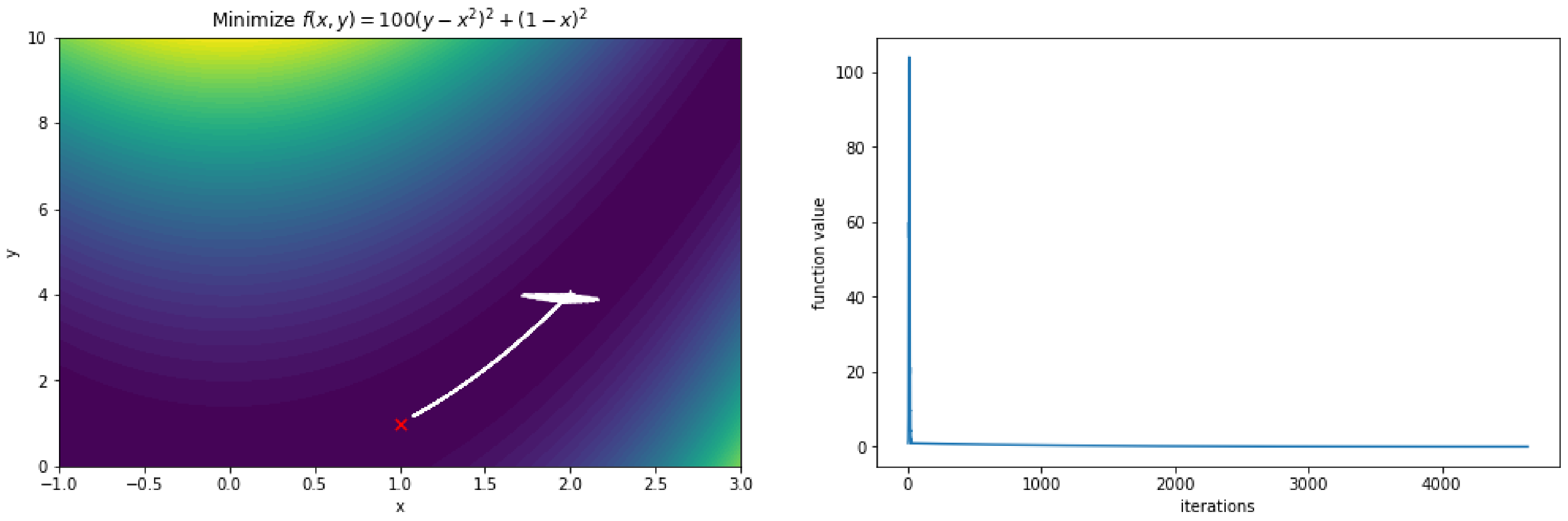
Now let's see how the steepest descent method behaves with the Rosenbrock function.

In [19]:
```python
p, f = steepestdescent(rosenbrockfunction, gradient, init=[2,4], alpha0=0.01, gamma=0.2)
```

Plot the convergence of the solution. Left: The solution points (white) superposed on the contour plot. The star indicates the initial point. Right: The objective function value at each iteration.

In [22]:
```python
plt.figure(figsize=(17,5))
plt.subplot(1,2,1)
contourplot(rosenbrockfunction, -1,3,0,10)
plt.xlabel("x")
plt.ylabel("y")
plt.title("Minimize $f(x,y)=100(y-x^2)^2 + (1-x)^2$");
plt.scatter(p[0,0],p[0,1],marker="*",color="w")
for i in range(1,len(p)):
        plt.plot( (p[i-1,0],p[i,0]), (p[i-1,1],p[i,1]) , "w");

plt.subplot(1,2,2)
plt.plot(f)
plt.xlabel("iterations")
plt.ylabel("function value");
```



In [ ]: