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Mathematical Institute

Exercises for Optimization I
Sheet 4*

You can submit your solutions for Numerical Exercise until Friday, May 29 until 9pm.

Exercise 1: Show that the Armijo steplength is not efficient in general.

Hint: Use $f(x) := \frac{x^2}{8}$ with the feasible directions $d^k = -2^{-k} f'(x^k)$ and show that for any starting point $x^0 > 0$ the generated sequence (x^k) is monotonically decreasing, but converges to $\bar{x} \geq \frac{x^0}{2}$. Then explain, why $\bar{x}$ can not be a stationary point and how this shows that the steplengths are not efficient.

Exercise 2: Show that if the sequence (t_k) of Armijo-Goldstein steplengths fulfills

$$t_k \geq -\alpha \frac{\nabla f(x^k)^\top d^k}{\|d^k\|^2},$$

with an $\alpha > 0$, then (t_k) is efficient.

Exercise 3: Let $f : \mathbb{R}^n \to \mathbb{R}$ be a quadratic function, i.e. $f(x) = \frac{1}{2} x^\top A x - b^\top x + c$, with $A \in \mathbb{R}^{n \times n}$ spd, $b \in \mathbb{R}^n, c \in \mathbb{R}$ and $t_{\min}$ the exact steplength $t_{\min} := \operatorname{argmin}_{t \geq 0} f(x + td)$, with $x, d \in \mathbb{R}^n$, such that $\nabla f(x)^\top d < 0$. Then show

$$t_{\min} \in T_{\mathrm{WP}}(x, d)$$

where the Wolfe-Powell steplength rule is given for $\sigma \in (0, \frac{1}{2})$ and $\rho \in [\sigma, 1)$ by

$$T_{\mathrm{WP}}(x, d) = \left\{ t > 0 \mid f(x + td) \leq f(x) + \sigma t f'(x) d \text{ and } f'(x + td) \geq \rho f'(x) d \right\}$$

with d being the descent direction of f at x, i.e. $f'(x)d < 0$.

Numerical Exercise 2: Implement the descent algorithm for the Rosenbrock function (as in Numerical exercise 1), with

a) $d^k = -\nabla f(x^k)$ and Armijo steplength t_k,

b) $d^k = -\nabla^2 f(x^k)^{-1} \nabla f(x^k)$ and Armijo steplength t_k,

c) $d^k = -\nabla^2 f(x^k)^{-1} \nabla f(x^k)$ and fixed steplength $t_k = 1$.

Start with $x^0 = (-1.2, 1)^\top$ and use the termination condition

$$\|\nabla f(x^k)\| \leq \epsilon \|\nabla f(x^0)\|, \quad \text{with } \epsilon = 10^{-3}.$$

As output, display the number of iterations, the norm of the gradient and the function value of f. Visualize the iteration process in each case and compare the different cases.

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*[*] **Group 4** will present the solutions.