ELECTRO-STATICS

BASIC TERMINOLOGIES

- Electric field intensity (E):
- Line of force:
- Electric flux (Φ):
- Permitivity(ε):
- Charge density (λ σ ρ):

RELATION BETWEEN FLUX AND INTENSITY AND GAUSS'S THRM

- $\Phi = \oint \vec{E}.\overrightarrow{ds} = E.ds.cos\Theta$

- Gauss's theorem:
- Φ = q/εo

APPLICATION OF GAUSS'S THEOREM

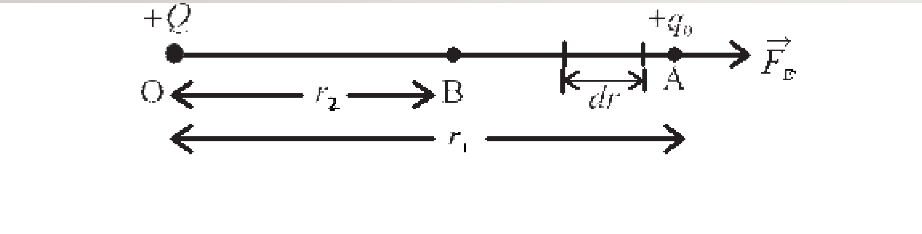
- **Electric Field Intensity due to Uniformly Charged Spherical Shell or Hollow Sphere:** $E = q/4\Pi\varepsilon r^2$ or $E = \sigma R^2/r^2\varepsilon_o$ or $E = \sigma/\varepsilon_o$ (if point is near and out side of sphere)
- **Electric Field Intensity due to an Infinitely Long Straight Charged Wire:**
  $E = \lambda / 2\Pi\varepsilon_o r$
- **Electric Field due to a Charged Infinite Plane Sheet:** $E = \sigma/2\varepsilon_o$

ELECTRIC POTENTIAL AND POTENTIAL ENERGY

- Electric potential : Electric work done to bring unit point charge at a point in an electric field is called Electric potential at that point of the field.
- If charge q displace in electric field is dr, the force acting on charge q by the field is F, then energy increases dU=dW and V at point is = dU/q =dW/q

- Electric potential energy: Electric work done to bring point charge at point in electric field, equivalent energy stored in system is called Electric potential energy.
- If charge q displace in electric field is dr, the force acting on charge q by the field is F, then energy increases dU=dW=F.dr

DERIVATION FOR POTENTIAL ENERGY IN THE SYSTEM OF TWO POINT CHARGE:



Let us consider the electrostatic field due to a source charge +Q placed at the origin O.
Let a small charge + q0 be brought from point A to point B at respective distances r1 and r2 from O, against the repulsive forces on it.

- $F_E$is the force acting on q_o at point A, during displacement dr the small work done is given by, dw= -F_E.dr
- Change in potential energy of system, dU=dW = -F_E.dr
- Change in potential energy, when q_o displace from A to B is ΔU = $\int_{r1}^{r2} dU$
- ΔU = $\int_{r1}^{r2} -F_E$.dr

- But $\vec{F}_E = \frac{1}{4\Pi\varepsilon_o}\frac{Qq_o}{r^2}\hat{r}$
- $\Delta U = \int_{r1}^{r2} -(\frac{1}{4\Pi\varepsilon_o}\frac{Qq_o}{r^2})\hat{r}.\overrightarrow{dr}$

  $= - \frac{Qq_o}{4\Pi\varepsilon_o}\left[\frac{-1}{r}\right]_{r1}^{r2}$

  $= \frac{Qq_o}{4\Pi\varepsilon_o}\left[\frac{1}{r2} - \frac{1}{r1}\right]$
- for $r1$ = ∞ then 1/ $r1$ =0
- $\Delta U = \frac{Qq_o}{4\Pi\varepsilon_o}\left[\frac{1}{r2}\right]$
- Thus energy stored in system of two charges q1 and q2 separated by distance r is given by
- $U = \frac{1}{4\Pi\varepsilon_o}\frac{q1\,q2}{r}$
- SI unit joule (J)
- Another convenient unit is eV (electron volt)

CONCEPT OF ELECTRIC POTENTIAL

- Since potential energy of system of two charges q_1 and q_2 separated by distance r is given by, U(r)= $\frac{1}{4\Pi\mathcal{E}_o}\frac{q1\ q2}{r}$ but V(r) = $\frac{1}{4\Pi\mathcal{E}_o}\frac{q}{r}$
- Hence U(r) = $V_1(r)q_2$ = $V_2(r)q_1$ where V_1 and V_2 are electric potential due charge q_1 and q_2 at the distance r.
- Thus, electric potential energy U = electric potential V $*$ charge q

  or Electric potential V = potential energy/charge q

RELATIONSHIP BETWEEN ELECTRIC POTENTIAL AND ELECTRIC FIELD INTENSITY

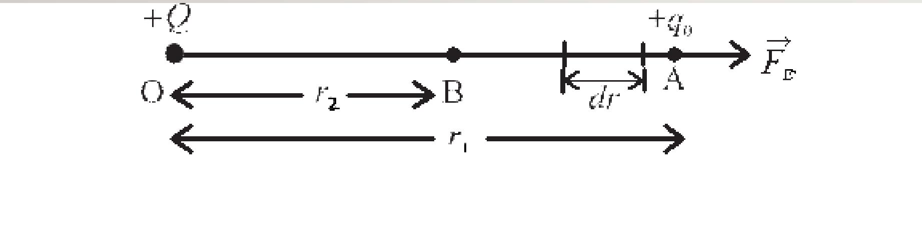
- To displace point charge q under the influence of electrostatic force F in electric field with very small displacement dx, the small work done $dw = - F.dx$ but $F = E/q$
- Hence $dw = - E.q.dx$ and $dw=dV.q$

$$\Rightarrow -E.q.dx = dV.q$$

$$\Rightarrow dV= -E.dx$$

or $E= -dV/dx$

ELECTRIC POTENTIAL DUE TO POINT CHARGE



Let us consider the electrostatic field due to a source charge +Q placed at the origin O.
Let a small charge + q0 be brought from point A to point B at respective distances r1 and r2 from O, against the repulsive forces on it.

- $F_E$is the force acting on q_o at point A, during displacement dr the small work done is given by, dw= -F_E.dr
- Change in potential energy of system, dU=dW = -F_E.dr
- Change in potential energy, when q_o displace from A to B is ΔW = $\int_{r1}^{r2} dU$
- ΔW = $\int_{r1}^{r2} -F_E$.dr

- But $\vec{F}_E = \frac{1}{4\Pi\mathcal{E}_o}\frac{Qq_o}{r^2}\hat{r}$
- $\Delta W = \int_{r1}^{r2} -(\frac{1}{4\Pi\mathcal{E}_o}\frac{Qq_o}{r^2})\hat{r}.\overrightarrow{dr}$

$$= -\frac{Qq_o}{4\Pi\mathcal{E}_o}\left[\frac{-1}{r}\right]_{r1}^{r2}$$

$$= \frac{Qq_o}{4\Pi\mathcal{E}_o}\left[\frac{1}{r2} - \frac{1}{r1}\right]$$

- for $r1$ = ∞ then 1/ $r1$ =0 and r2 = r, ΔW=W
- $W = \frac{Qq_o}{4\Pi\mathcal{E}_o}\left[\frac{1}{r}\right]$
- Since V(r)=W/q_o

$$\Rightarrow V(r) = \frac{Qq_o}{4\Pi\mathcal{E}_o}\left[\frac{1}{r}\right]/q_o$$

$$\Rightarrow V(r) = \frac{1}{4\Pi\mathcal{E}_o}\left[\frac{Q}{r}\right]$$

- A positive electric charge produces positive electric potential
- A negative point charge produces, negative electric potential
- At r= ∞,V= $\frac{1}{4\Pi\mathcal{E}_o}\left[\frac{Q}{\infty}\right]$ = 0, this shows electric potential is zero at infinite

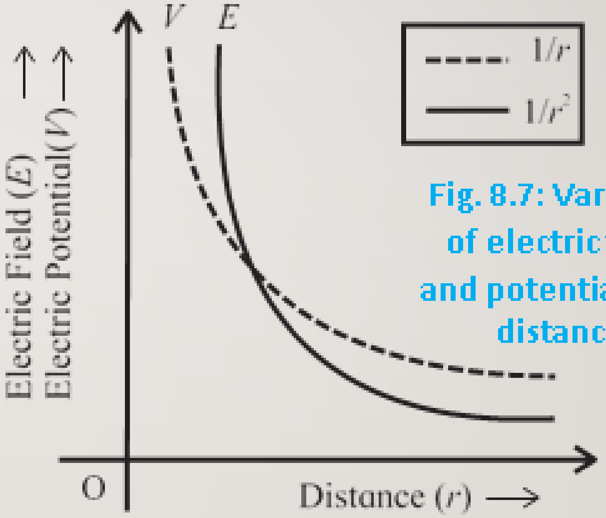
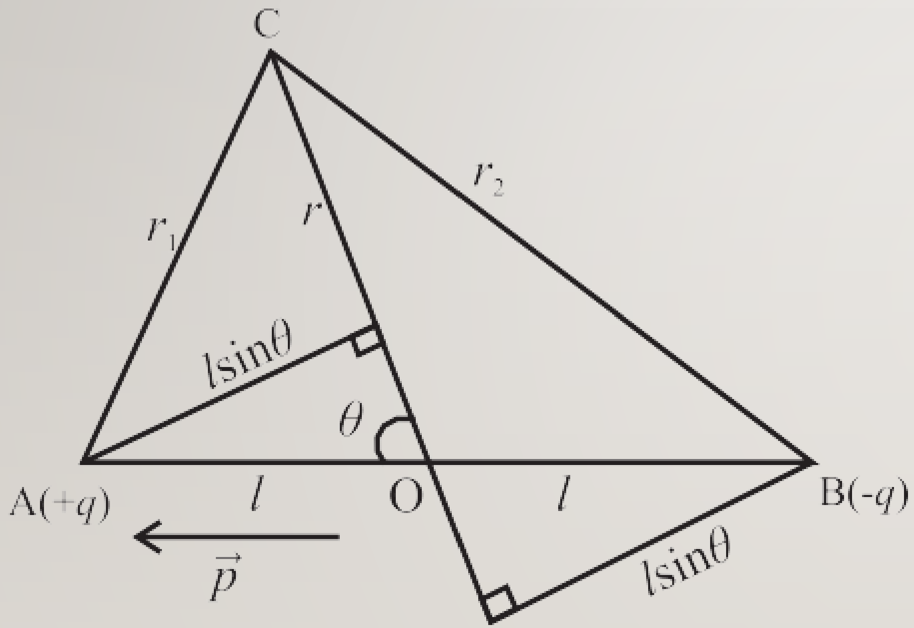


Fig. 8.7: Variation of electric field and potential with distance

Remember this

Due to a single charge at a distance *r*,
Force (*F*) ∝ 1/ r^2, Electric field (*E*) ∝ 1/ r^2
but Potential (*V*) ∝ 1/ *r*.

ELECTRIC POTENTIAL DUE TO DIPOLE:



- In order to determine the electric potential due to a dipole, let the origin be at the centre (O) of the dipole.
- Let C be any point near the electric dipole at a distance *r* from the centre O inclined at an angle *q* with axis of the dipole.
- $r1$ and $r2$ are the distances of point C from charges +*q* and -*q*, respectively.
- Potential at C due to charge +*q* at A is, V1 $= \frac{1}{4\Pi\mathcal{E}_o}\left[\frac{+q}{r1}\right]$ and potential at C due to charge –q at B is, V2 $= \frac{1}{4\Pi\mathcal{E}_o}\left[\frac{-q}{r2}\right]$
- Total electric potential at point C, V= V1+V2

  V $= \frac{1}{4\Pi\mathcal{E}_o}\left[\frac{+q}{r1}\right] + \frac{1}{4\Pi\mathcal{E}_o}\left[\frac{-q}{r2}\right] = \frac{q}{4\Pi\mathcal{E}_o}\left[\frac{1}{r1} - \frac{1}{r2}\right]$ -----------(1)

But from the diagram,

$r_1^2 = (r - lcos\theta)^2 + (lsin\theta)^2$ and $r_2^2 = (r + lcos\theta)^2 + (lsin\theta)^2$

$r_1^2 = r^2 + l^2cos^2\theta - 2rlcos\theta + l^2sin^2\theta$ and $r_2^2 = r^2 + l^2cos^2\theta + 2rlcos\theta + l^2sin^2\theta$

$r_1^2 = r^2 + l^2 - 2rlcos\theta$ and $r_2^2 = r^2 + l^2 + 2rlcos\theta$

$r_1^2 = r^2(1 + \frac{l^2}{r^2} - \frac{2lcos\theta}{r})$ and $r_2^2 = r^2(1 + \frac{l^2}{r^2} + \frac{2lcos\theta}{r})$

$\frac{1}{r_1^2} = \frac{1}{r^2}\left[1 + \frac{l^2}{r^2} - \frac{2lcos\theta}{r}\right]^{-1}$ and $\frac{1}{r_2^2} = \frac{1}{r^2}\left[1 + \frac{l^2}{r^2} + \frac{2lcos\theta}{r}\right]^{-1}$

As l<<r, neglecting higher order term of l

$\frac{1}{r_1^2} = \frac{1}{r^2}\left[1 - \frac{2lcos\theta}{r}\right]^{-1}$ and $\frac{1}{r_2^2} = \frac{1}{r^2}\left[1 + \frac{2lcos\theta}{r}\right]^{-1}$

Hence $\frac{1}{r_1} = \frac{1}{r}\left[1 - \frac{2lcos\theta}{r}\right]^{-1/2}$ and $\frac{1}{r_2} = \frac{1}{r}\left[1 + \frac{2lcos\theta}{r}\right]^{-1/2}$

Since $(1 + x)^n \approx$ 1+nx after neglecting higher order term using binomial expansion

$\frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{2lcos\theta}{2r}\right]$ and $\frac{1}{r_2} = \frac{1}{r}\left[1 - \frac{2lcos\theta}{2r}\right]$

$\frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{lcos\theta}{r}\right]$ and $\frac{1}{r_2} = \frac{1}{r}\left[1 - \frac{lcos\theta}{r}\right]$

Using above expression in equation (1)

$$V = \frac{q}{4\Pi\mathcal{E}_o}\left[\frac{1}{r}\left[1 + \frac{lcos\theta}{r}\right] - \frac{1}{r}\left[1 - \frac{lcos\theta}{r}\right]\right]$$

$$= \frac{q}{4\Pi\mathcal{E}_o}\left[\frac{1}{r} + \frac{lcos\theta}{r^2} - \frac{1}{r} + \frac{lcos\theta}{r^2}\right]$$

$$V = \frac{q}{4\Pi\mathcal{E}_o}\left[\frac{2lcos\theta}{r^2}\right]$$

But electric dipole moment p=q.(2l)

$$V = \frac{1}{4\Pi\mathcal{E}_o}\left[\frac{p.cos\theta}{r^2}\right]$$

$$V = \frac{1}{4\Pi\mathcal{E}_o}\left[\frac{\vec{p}.\hat{r}}{r^2}\right]$$

$$V = \frac{1}{4\Pi\mathcal{E}_o}\left[\frac{\vec{p}.\vec{r}}{r^3}\right] \text{ as } \hat{r} = \frac{\vec{r}}{r}$$

Cases:

1) Potential at point on axis, $\theta = 0 \; or \; 180$

$$V = \frac{\pm 1}{4\Pi\mathcal{E}_o}\left[\frac{p}{r^2}\right]$$

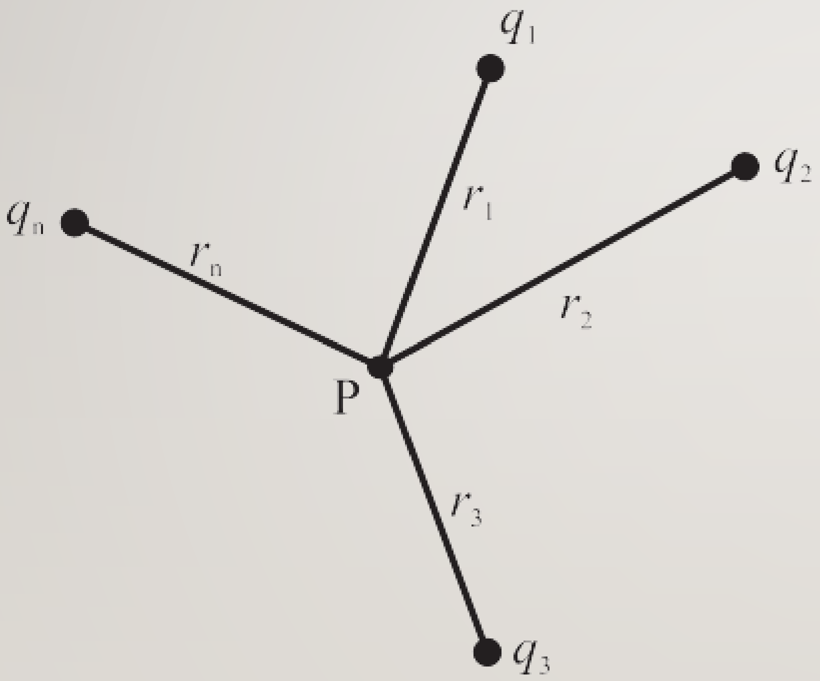
this is maximum value of potential due to dipole

2) Potential at point on equator, $\theta = 90$

V=0

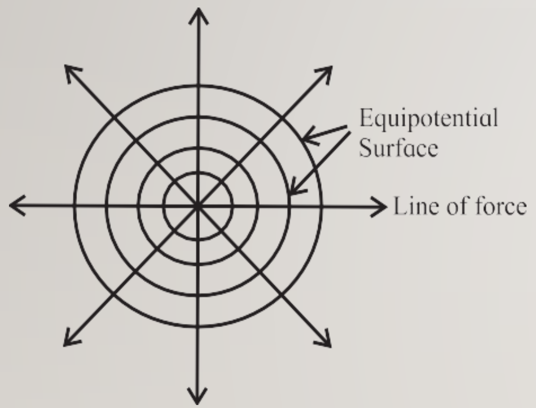
thus, potential at any point on equator is zero, this minimum potential due to dipole.

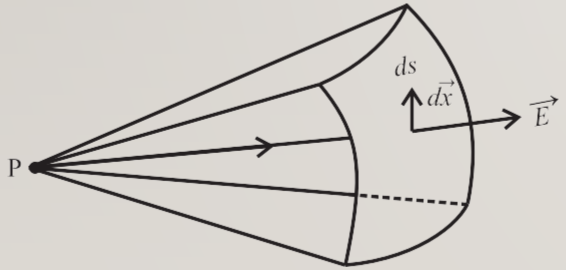
ELECTROSTATICS POTENTIAL DUE TO A SYSTEM OF CHARGES:



- Consider a system of charges $q1$, $q2$ .... qn at distances $r1$, $r2$ ...... rn respectively from point P. The potential $V1$ at P due to the charge q1 is V1 = $\frac{1}{4\Pi\mathcal{E}_o}\left[\frac{q1}{r1}\right]$
- Similarly potentials V2, V3, ….Vn are V2 = $\frac{1}{4\Pi\mathcal{E}_o}\left[\frac{q2}{r2}\right]$,V3 = $\frac{1}{4\Pi\mathcal{E}_o}\left[\frac{q3}{r3}\right]$ ….Vn = $\frac{1}{4\Pi\mathcal{E}_o}\left[\frac{qn}{rn}\right]$
- By principal of superposition, total potential at point P is V=V1+V2+V3+……+Vn
- V= $\frac{1}{4\Pi\mathcal{E}_o}\left[\frac{q1}{r1}+\frac{q2}{r2}+\frac{q3}{r3}+\cdots+\frac{qn}{rn}\right] = \frac{1}{4\Pi\mathcal{E}_o}\sum_{i=1}^{n}\frac{q_i}{r_i}$

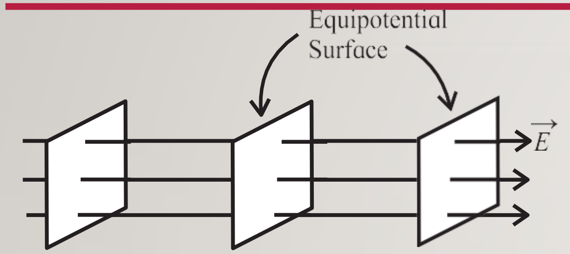
Equipotential Surfaces



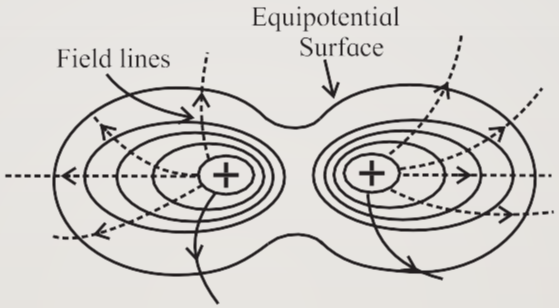


- An equipotential surface is that surface, at every point of which the electric potential is the same.
- The potential (V) for a single charge q is given by, V= $\frac{1}{4\Pi\mathcal{E}_o}\left[\frac{q}{r}\right]$, if r is constant, V will also be constant.
- Hence, equipotential surfaces of single point chargeare concentric spherical surfaces centered at the charge.
- For a line charge, the shape of equipotential surface is cylindrical.
- By definition the potential difference between two points P and Q is the work done per unit positive charge displaced from Q to P,
  i.e. Vp-VQ = W_{QP}
- For equipotential surface W_{QP}=0 and $V_Q = V_P$
- If dx is the small distance over the equipotential surface through which unit positive charge is carried then $dW = E$ .d $x = E\ dx$ cosθ
- if θ=90 then E is $\perp$ to equipotential surface.

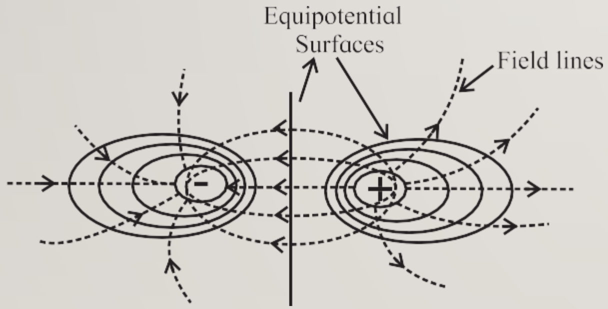
SOME EQUIPOTENTIAL SURFACE



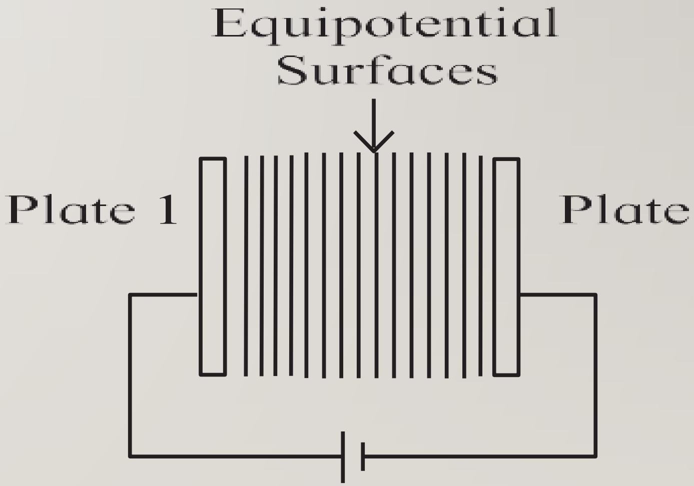
Equipotential surface of uniform Electric field



Equipotential surfaces for two identical positive charges

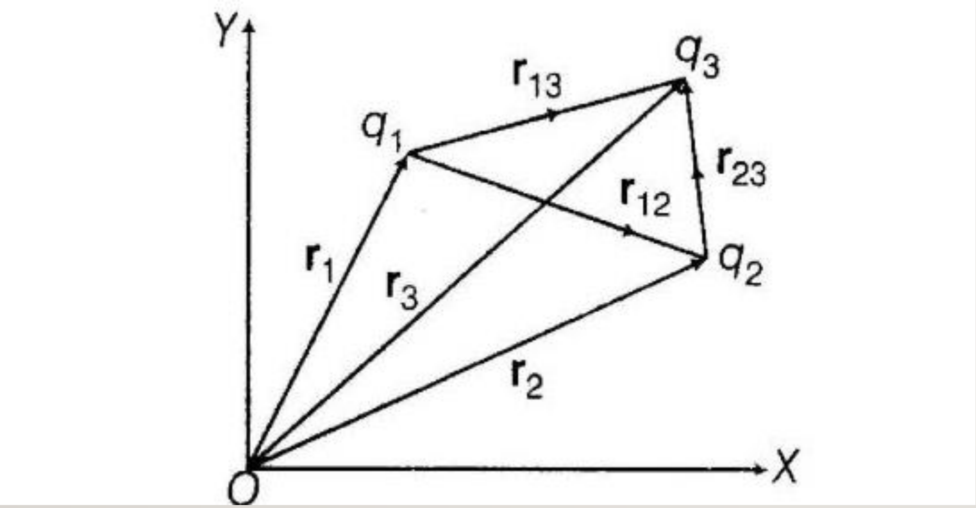


Equipotential surfaces for a dipole



Between 2 plane metallic sheets

POTENTIAL ENERGY OF SYSTEM OF THREE CHARGES



- Consider system of three charges q_1, q_2, q_3 in Cartesian coordinate system at distance $r_1, r_2,$ and $r_3$respectively.
- No work require to bring first charge charge q_1 at position r_1, hence W_1=0.

- This charge produces a potential in space given by $V_1 = \frac{1}{4\Pi\varepsilon_o}\left[\frac{q1}{r}\right]$
- When we bring charge q_2 from infinity to point at distance r_2 as shown in diagram, the work done against force applied by $q_1$is, W_{12}=$V_1 q_2$.
- i.e. W_{12}=$\frac{1}{4\Pi\varepsilon_o}\left[\frac{q1}{r_{12}}\right] q_2$,

  r_{12} is the distance between both the charges.

- To bring charge q_3 from infinity, work have to against the force applied by both the charges q_1 and q_2 , which are w_{13} and w_{23} respectively.
- i.e w_{13}= $V_1 . q_3$ and w_{23}= $V_2 . q_3$
- w_{13}=$\frac{1}{4\Pi\mathcal{E}_o} \left[\frac{q1}{r_{13}}\right] . q_3$ and $w_{23} = \frac{1}{4\Pi\mathcal{E}_o} \left[\frac{q2}{r_{23}}\right] q_3$

  hence total work to assemble all three charges in the system W=$w_1 + w_{12} + w_{13} + w_{23}$

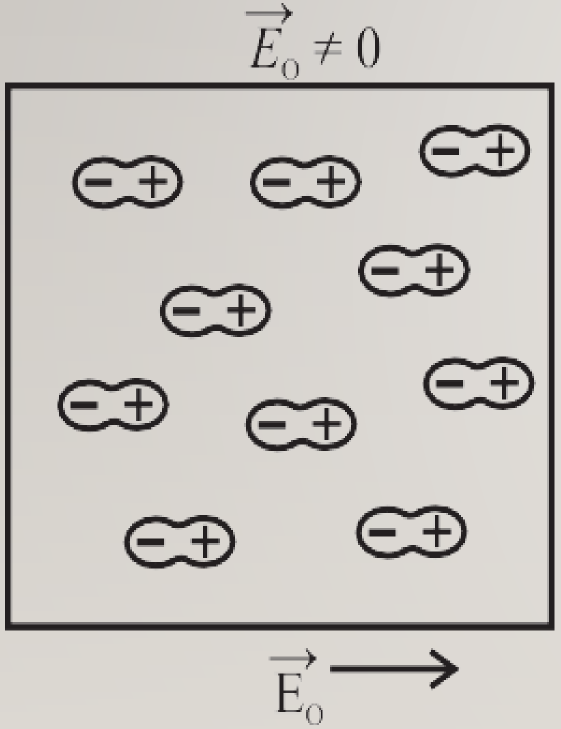
$$= 0+ \frac{1}{4\Pi\mathcal{E}_o} \left[\frac{q1}{r_{12}}\right] . q_2 + \frac{1}{4\Pi\mathcal{E}_o} \left[\frac{q1}{r_{13}}\right] . q_3 + \frac{1}{4\Pi\mathcal{E}_o} \left[\frac{q2}{r_{23}}\right] . q_3$$

$$W= \frac{1}{4\Pi\mathcal{E}_o} \left[\frac{q1.q_2}{r_{12}} + \frac{q1.q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

thus, potential energy stored in this system is, U=W

$$U=\frac{1}{4\Pi\mathcal{E}_o} \left[\frac{q1.q_2}{r_{12}} + \frac{q1.q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

DIELECTRICS AND ELECTRIC POLARIZATION



Dielectrics:

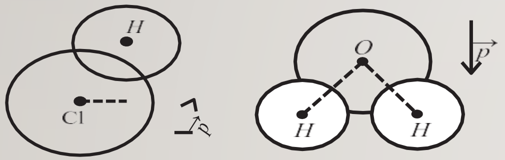
- Dielectrics are insulates which can be used to store electrical energy
- Eg. glass, wax, water, wood , mica, rubber, stone, plastic etc.

Polarization:

- when dielectric substances are placed in an external field, the center of positive and center of negative charges get displaced in opposite directions and the molecules develop a net dipole moment. This is called polarization

TYPES OF DIELECTRICS

Polar dielectric

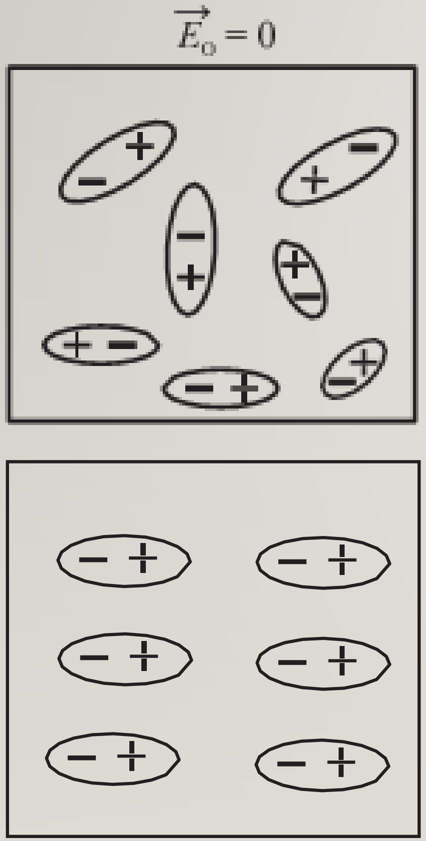


- A molecule in which the centre of mass of positive charges (protons) does not coincide with the centre of mass of negative charges (electrons), because of the asymmetric shape of the molecules is called polar molecule
- They have permanent dipole moments of the order of 10^{-30} Cm
- The dielectrics like HCl, water, alcohol, NH3 etc are made of polar molecules and are called polar dielectrics

Non-Polar dielectric

- A molecule in which the centre of mass of the positive charges coincides with the centre of mass of the negative charges is called a non polar molecule
- These have symmetrical shapes and have zero dipole moment in the normal state
- The dielectrics
- like hydrogen, nitrogen, oxygen, CO2, benzene, methane are made up of nonpolar molecules and are called non polar dielectrics

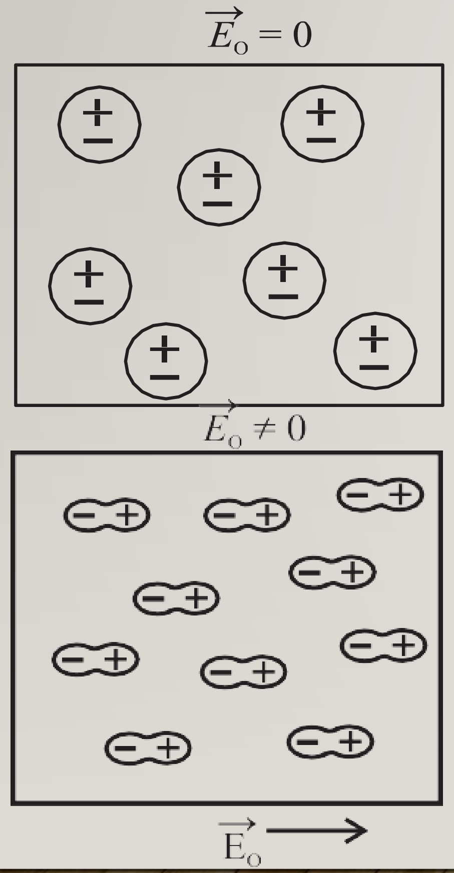
POLARIZATION OF A POLAR DIELECTRIC



- The molecules of a polar dielectric have tiny permanent dipole moments
- Due to thermal agitation in the material in the absence of any external electric field, these dipole moments are randomly oriented
- Hence the total dipole moment of dielectric material is zero.
- When an external electric field is applied the dipole moments of different molecules tend to align with the field
- As a result the dielectric develops a net dipole moment in the direction of the external field and dielectric material is polarized
- The extent of polarization depends on the relative values of the two opposing energies

1) The applied external electric field which tends to align the dipole with the field.
2) Thermal energy tending to randomise the alignment of the dipole

POLARIZATION OF A NON-POLAR DIELECTRIC

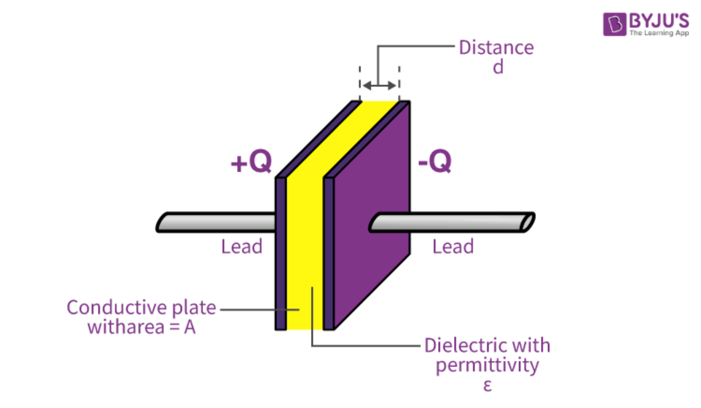


- In the presence of an external electric field *Eo*, the centres of the positive charge in each molecule of a non-polar dielectric is pulled in the direction of *Eo*, while the centres of the negative charges are displaced in the opposite direction
- The displacement of the charges stops when the force exerted on them by the external field is balanced by the restoring force between the charges in the molecule
- Each molecule becomes a tiny dipole having a dipole moment. The induced dipole moments of different molecules add up giving a net dipole moment to the dielectric in the presence of the external field

POLARIZATION

- Both polar and nonpolar dielectric develop net dipole moment in the presence of an electric field
- The dipole moment per unit volume is called polarization and is denoted by *P*
- For linear isotropic dielectrics $P = \chi_e E$, *where* χ_e is a constant called electric susceptibility of the dielectric medium
- It describes the electrical behavior of a dielectric
- For vacuum $\chi_e = 0$.

CAPACITORS AND CAPACITANCE



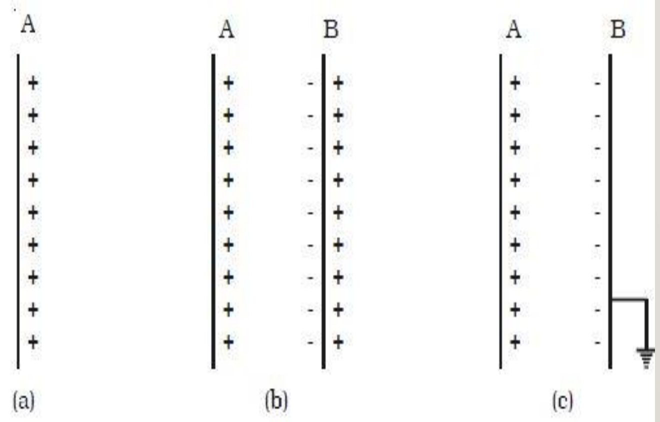
Capacitor:

- The most common arrangement for this consists of a set of conductors (conducting plates) having charges on them and separated by a dielectric material is called capacitor

Capacitance:

- The capacity to store charge by capacitor is called capacitance.
- SI unit of capacitance is farad (F).
- The conductors 1 and 2 have charges $+Q$ and $-Q$ with potential difference, $V = V1 - V2$ between them.
- The electric field in the region between them is proportional to the charge Q
- The potential difference V is the work done to carry a unit positive test charge from the conductor 2 to conductor 1 against the field
- As this work done will be proportional to Q, then $V \alpha Q$ and the ratio Q/V is constant, called capacitance(C).
- Hence, C = Q/V

PRINCIPLE OF A CAPACITOR



- Consider a metal plate A having positive charge *+Q and potential on plate be V, hence capacity of plate C= Q/V.*
- Now consider another insulated metal plate B held near the plate A.
- By induction a negative charge is produced on the nearer face and an equal positive charge develops on the farther face of B.
- The induced negative charge lowers the potential of plate A, while the induced positive charge raises its potential
- If the outer surface of B is connected to earth, the induced positive charges on B being free and flows to earth, This greatly reduces the potential of B.
- If *V*1 is the potential on plate B due to charge (- *Q*) then the net potential of the system will now be *+V-V*1.
- *Hence, Capacity (C1) = Q/(V-V1) and C1 > C.*
- Thus capacity of metal plate A, is increased by placing an identical earth connected metal plate B near it