2 LDPC codes

(2.2.1) Question: Print the outputs of the function for the matrix.

Answer: For the given parity check matrix H

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

The systematic form of this H, $\hat{H}$ is:

$$\hat{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

The systematic encoding matrix of this H, G is:

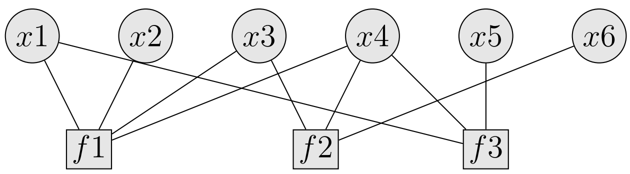
$$G = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(2.2.2) Question: Draw a factor-graph for the parity check matrix H shown in question 2.2.1:

Answer: The 3 factors $f1$, $f2$ and $f3$, each corresponding to a parity check (i.e. the 3 rows of the parity check matrix H) are shown in squares as is the convention.

The edges connecting each factor to specific variables ($x1$, $x2$, $x3$, $x4$, $x5$, $x6$) indicate which variables of the input vector X are involved in the constraint of a given factor.

So for $f1$, the first four variables should sum to 0 because the first four bits along the first row are 1, the last two are 0, and so on.



2 LDPC codes contin.

(2.2.2) Question: Write the distribution corresponding to this factor-graph and the updates used for the messages.

Answer: The probability distribution of the constraints defined by H from 2.2.1. are three conditionally-independent joint distributions, so the whole distribution is their product:

$p(x_1, x_2, x_3, x_4) \times p(x_3, x_4, x_6) \times p(x_1, x_4, x_5)$

The updates of the values used for the messages occurs in steps 2 and 3 (from the slides), (with step 1 as the initialisation and step 4 as the evaluation of whether we have reached convergence).

Step 2 involves recomputing (hence updating) factor-to-variable messages:

$$\mu_{f_m \to \mathrm{x}_n}(\mathrm{x}_n) = \sum_{\mathrm{x}_{n'}, n' \in N(m) \setminus n} \left[I \left[\mathrm{x}_n + \sum_{n'} \mathrm{x}_{n'} = 0 \right] \right] \prod_{\mathrm{x}_{n'}, n' \in N(m) \setminus n} \mu_{\mathrm{x}_{n'} \to f_m}(\mathrm{x}_{n'})$$

Step 3 involves recomputing (hence updating) variable-to-factor messages:

$$\mu_{\mathrm{x}_n \to f_m} \propto p(\mathrm{y}_n | \mathrm{x}_n) \prod_{m' \in N(n) \setminus m} \mu_{f_{m'} \to \mathrm{x}_n}(\mathrm{x}_n)$$

2 LDPC codes contin.

(2.2.3) Question: Print the result of the decoding for a given parity check matrix H_1 and vector y_1.

Answer: The Python script 'ldpc.py' has been written to adhere as much as possible to the description in the ldpc slides and talks. It is well documented and written to be readable in an object-oriented format. Unfortunately though it is not optimised and seems to be not possible to run to completion on a laptop, which becomes overheated. As such, it is not possible to decode the vector $y1$.

(2.2.4) Question: Recover the original English message. Answer: NA