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Sound transmission through a curved sandwich panel

A solution to the coincidence and ring frequency effect

Zibo Liu, Wuzhou Yu, Qi Li

Content

- Introduction
- Sandwich panel and coincidence effect
- Curved panel and ring frequency effect
- Curved sandwich and design criteria
- Conclusion

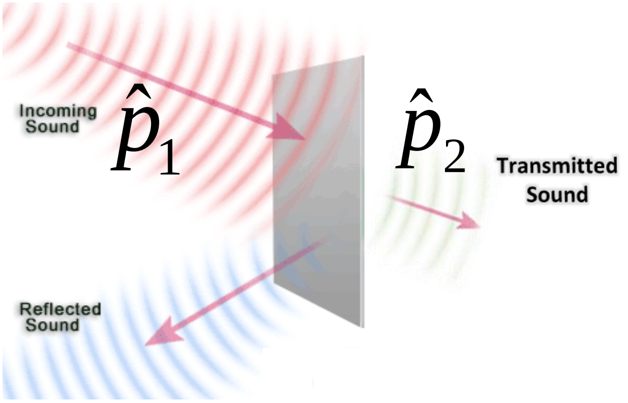
Sound insulation

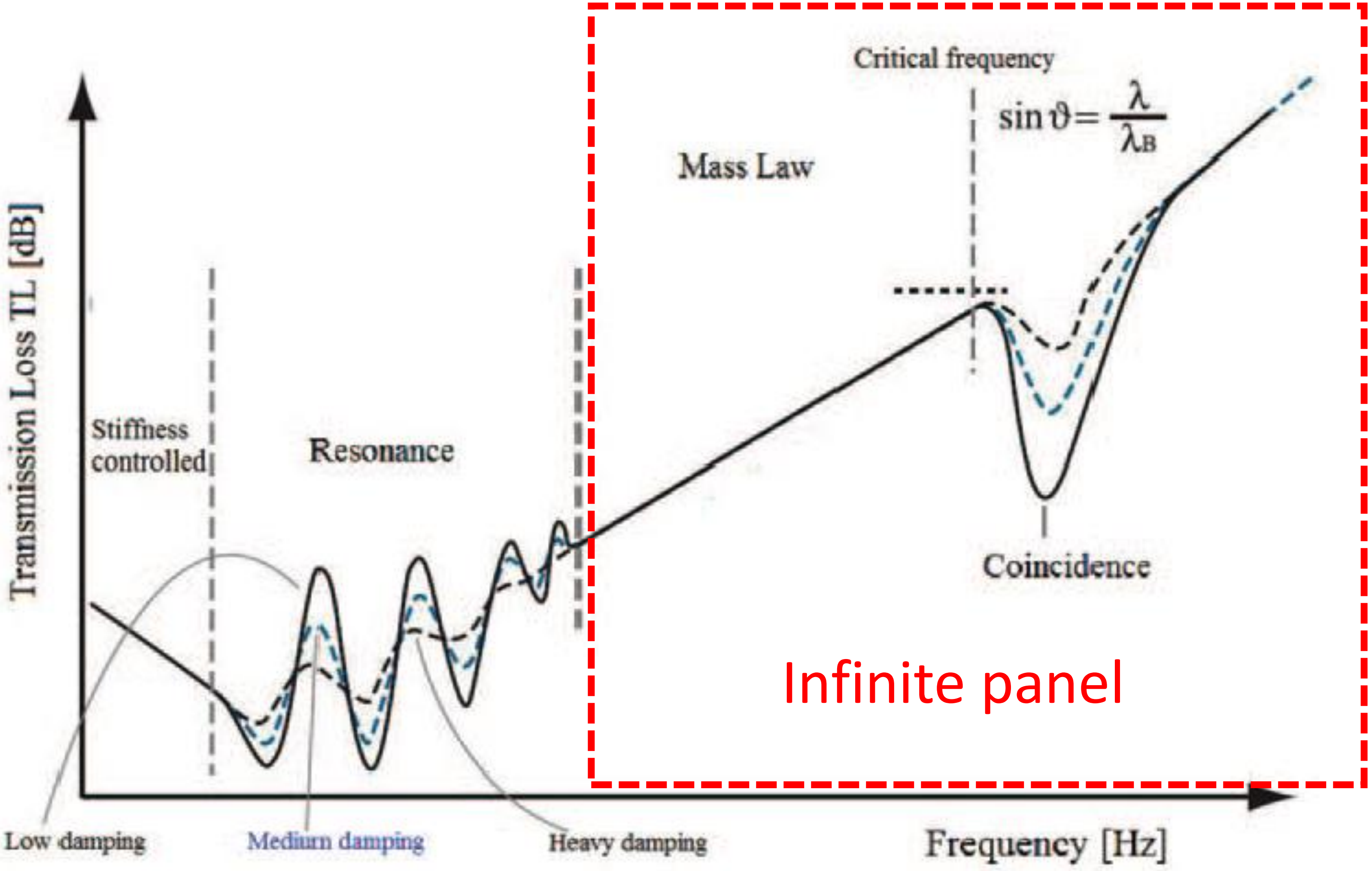
Noise reduction engineering

Isolation for sound transmission path

Sound insulation/sound transmission loss properties

Introduction to the sound transmission through panels

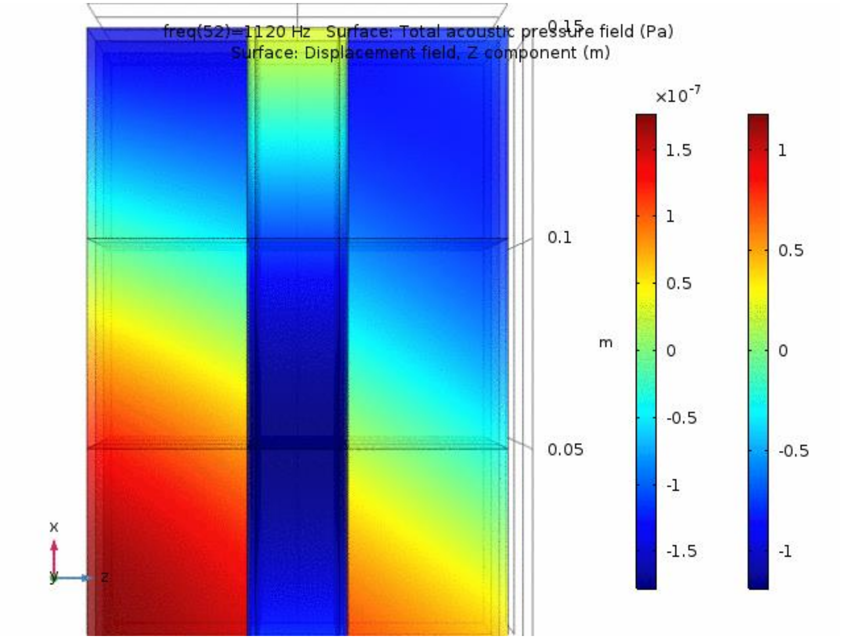




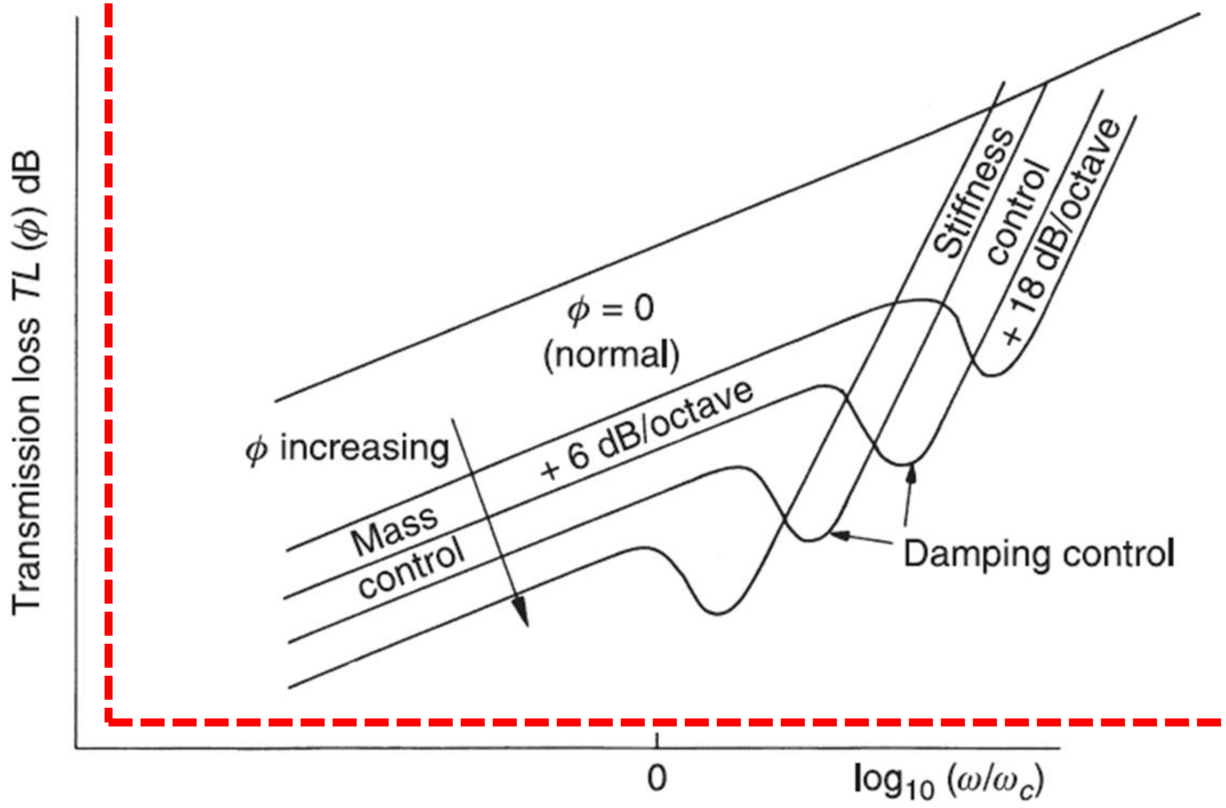
D'Alessandro, V., Petrone, G., Franco, F., & De Rosa, S. (2013). A review of the vibroacoustics of sandwich panels: Models and experiments. *Journal of Sandwich Structures & Materials*, *15*(5), 541-582.

Coincidence effect

$$\lambda_{\mathbf{trace}} = \lambda_{\mathbf{bending}}$$



Infinite panel



Variation of TL with frequency for a single angle of incidence (Fahy, 1987).

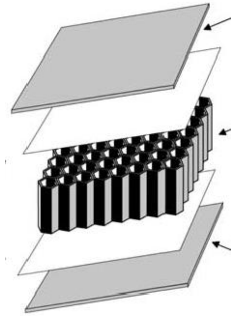
Sandwich panels

Advantage:

- low mass
- high stiffness

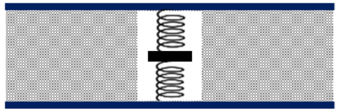
Disadvantage:

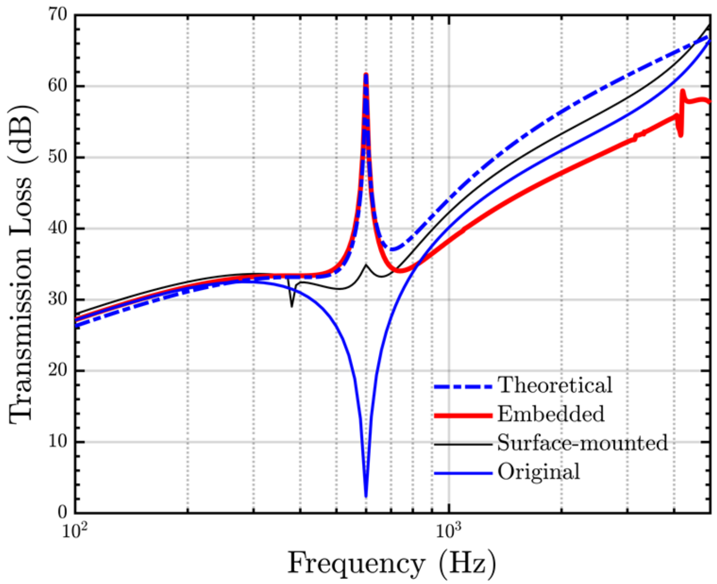
- bad acoustic properties (sound insulation)
  - ➢ Broad coincidence region
  - ➢ often drops to the low frequency range



Early works:

- move coincidence out of frequency range of interest
- metamaterial design





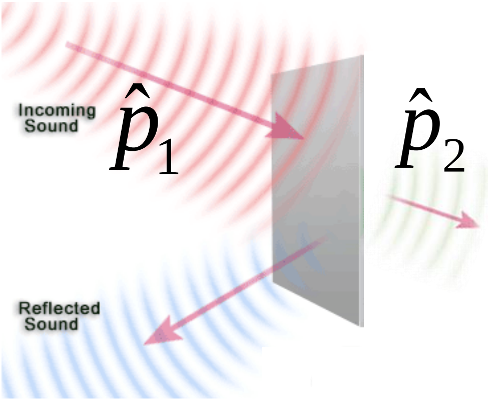
New solution:

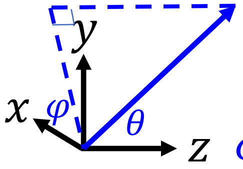
Curved sandwich→ stiffness to stiffness

Moore, J. A., & Lyon, R. H. (1991). Sound transmission loss characteristics of sandwich panel constructions. The Journal of the Acoustical Society of America, 89(2), 777-791.

Liu, Z., Rumpler, R., & Feng, L. (2018). Broadband locally resonant metamaterial sandwich plate for improved noise insulation in the coincidence region. Composite Structures, 200, 165-172.

Sound transmission through a single-layer panel

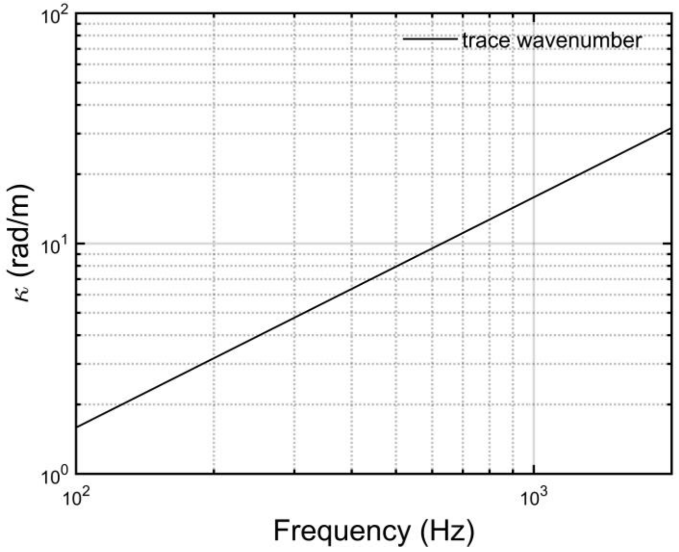




$$\hat{p}_1 = \hat{p}_{\text{inc}} + \hat{p}_{\text{ref}}$$

$$\hat{p}_2 = \hat{p}_{\text{trans}}$$

$$\varphi = 0; \theta = \frac{\pi}{3}$$

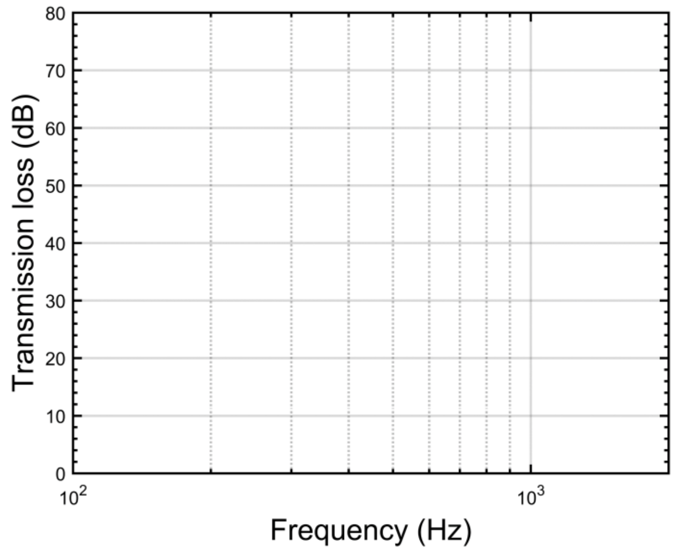


- Continuity of Velocity: $\rho_0 \dfrac{\partial \vec{v}_z}{\partial t} = -\vec{\nabla}_z \hat{p}_1 = -\vec{\nabla}_z \hat{p}_2$

- Newton's second law:

For a limp wall: $\hat{p}_1 - \hat{p}_2 = \mathrm{j}\omega m \hat{v}$

For a thin plate: $\hat{p}_1 - \hat{p}_2 = \left(D\nabla^4 - \omega^2 m\right)\hat{v} / \mathrm{j}\omega$



Under the Thin Plate Assumption:

Sound transmission through a single-layer panel

- Transmission coefficient: $\tau = \left| \frac{\hat{p}_{\text{trans}}}{\hat{p}_{\text{inc}}} \right|^2 = \left| 1 + \frac{Z\cos\theta}{2\rho_0 c_0} \right|^{-2}$ $\quad \text{STL} = 10\log\left(\frac{1}{\tau}\right)$

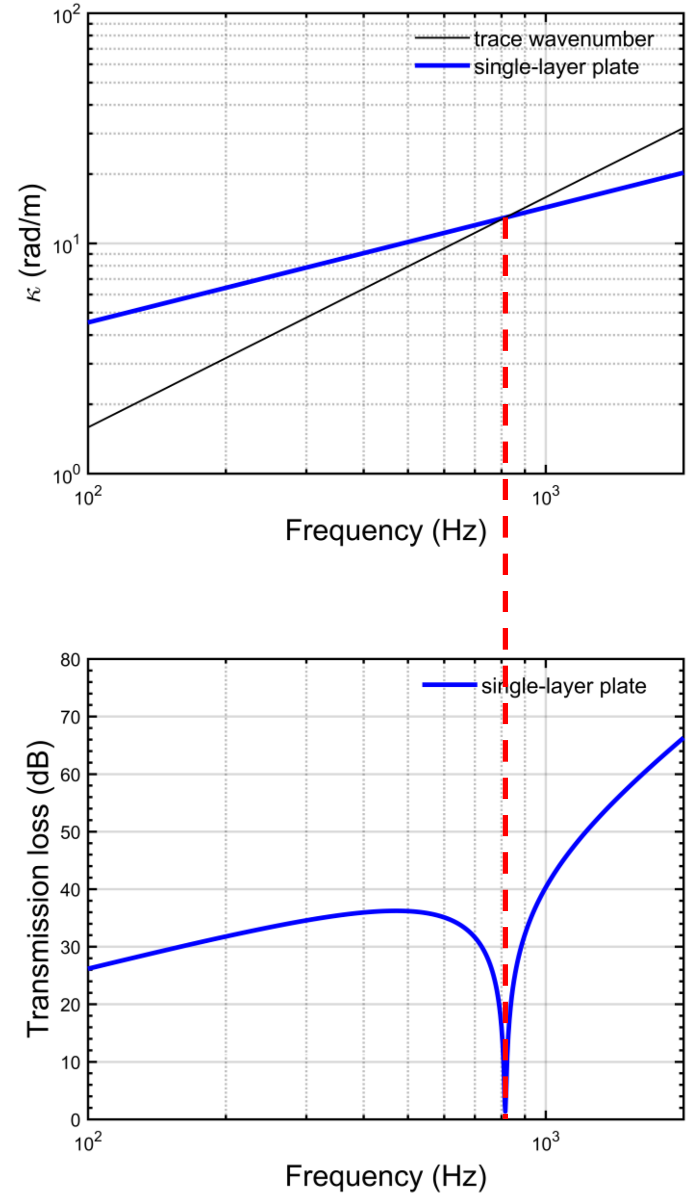
$Z = \frac{\hat{p}_1 - \hat{p}_2}{\hat{v}}$ is the corresponding plate impedance.

$$Z_0 = \mathrm{j}\omega m\left(1 - k^4 \frac{D}{\omega^2 m}\sin^4\theta\right) = \mathrm{j}\omega m\left(1 - \frac{k^4}{\kappa^4}\sin^4\theta\right)$$

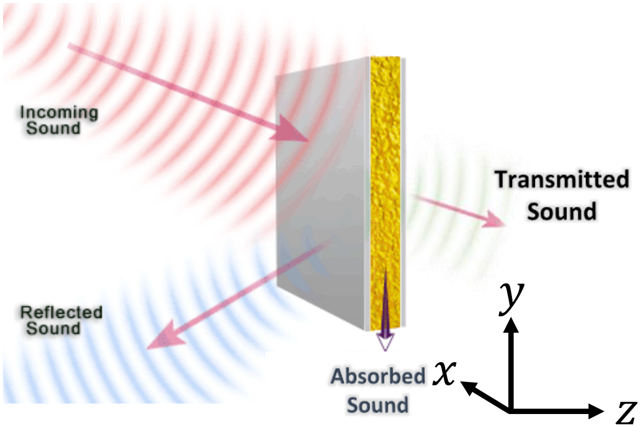
- Coincidence effect: $\kappa = k\sin\theta$ $\quad f_{\text{co}} = \frac{1}{2\pi}\frac{c_0^{\,2}}{\sin^2\theta}\sqrt{\frac{m}{D}}$

$$Z_0 = \mathrm{j}\omega m\left(1 - \frac{f^2}{f_{\text{co}}^{\,2}}\right)$$

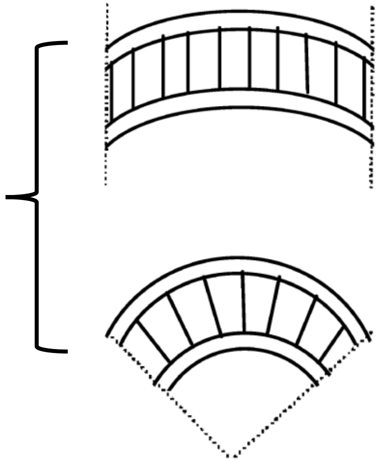
When Z = 0, coincidence effect occurs, total transmission is induced.

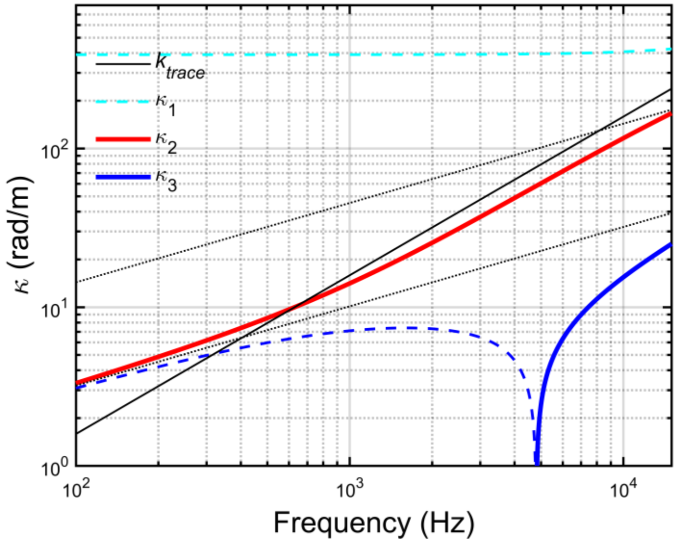


Sound transmission through a sandwich panel

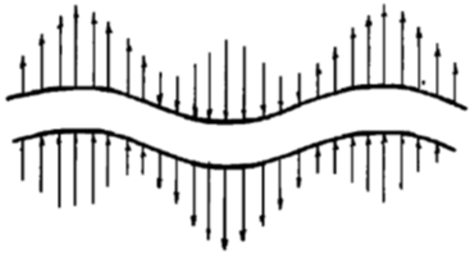


- **Equation of in-phase motion:**



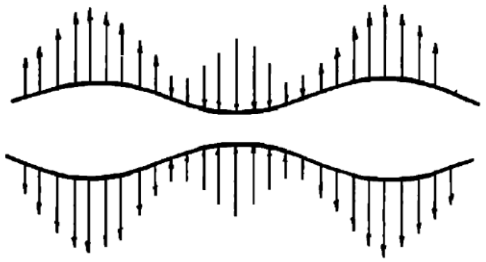


In-phase Mode:



- $-2D_S D_f \frac{\partial^6 w}{\partial x^6} + 2D_f I_S \frac{\partial^6 w}{\partial t^2 \partial x^4} - (D_S m + 2D_f m + I_S G_c t_c) \frac{\partial^4 w}{\partial t^2 \partial x^2} + G_c t_c \left(D_S \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} \right) + I_S m \frac{\partial^4 w}{\partial t^4} = 0$

Anti-phase Mode:



- $2D_S D_f {k_x}^6 - 2D_f I_S \omega^2 {k_x}^4 - (D_S m + 2D_f m + I_S G_c t_c)\omega^2 {k_x}^2 + G_c t_c (D_S {k_x}^4 - m\omega^2) + I_S m \omega^2 = 0$

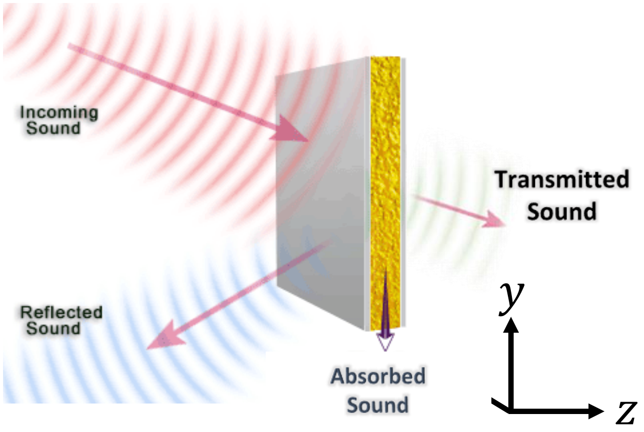
$$D_f = \frac{E_f {t_f}^3}{12\left(1-{\nu_f}^2\right)}$$

$$D_S = \frac{E_c {t_c}^3}{12\left(1-{\nu_c}^2\right)} + \frac{E_f}{\left(1-{\nu_f}^2\right)}\left(\frac{{t_c}^2 t_f}{2} + t_c {t_f}^2 + \frac{2}{3}{t_f}^3\right)$$

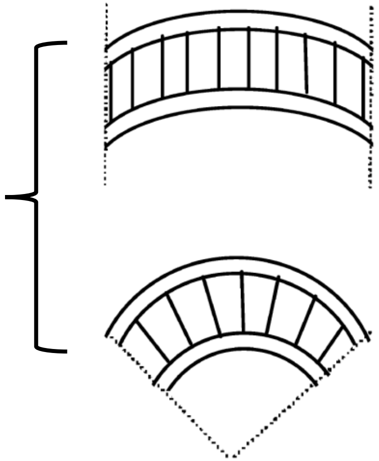
$$G_c = \frac{E_c}{2\left(1+\nu_c\right)}$$

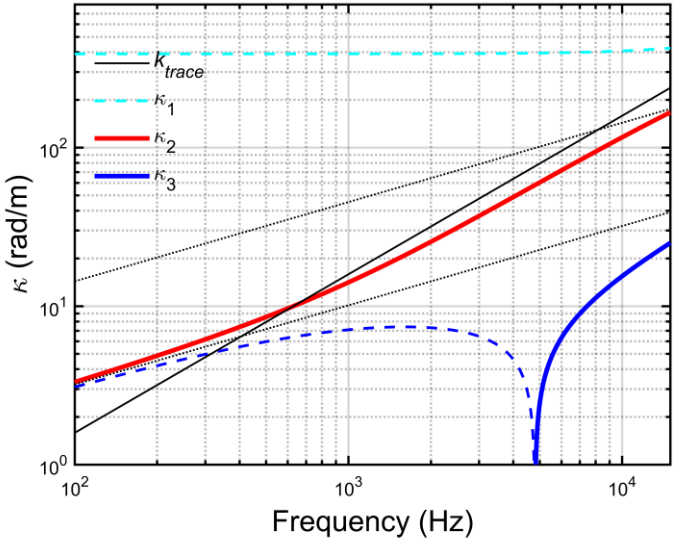
$$I_S = \frac{1}{12}\rho_c {t_c}^3 + \rho_f \left(\frac{{t_c}^2 t_f}{2} + t_c {t_f}^2 + \frac{2}{3}{t_f}^3\right)$$

Sound transmission through a sandwich panel

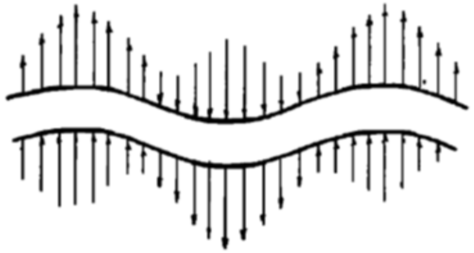


- **Equation of in-phase motion:**



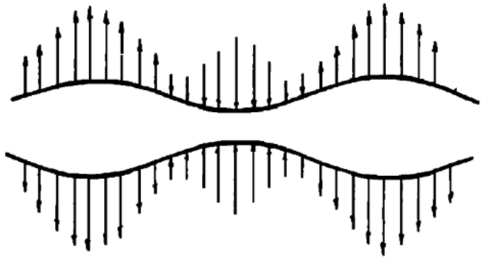


In-phase Mode:

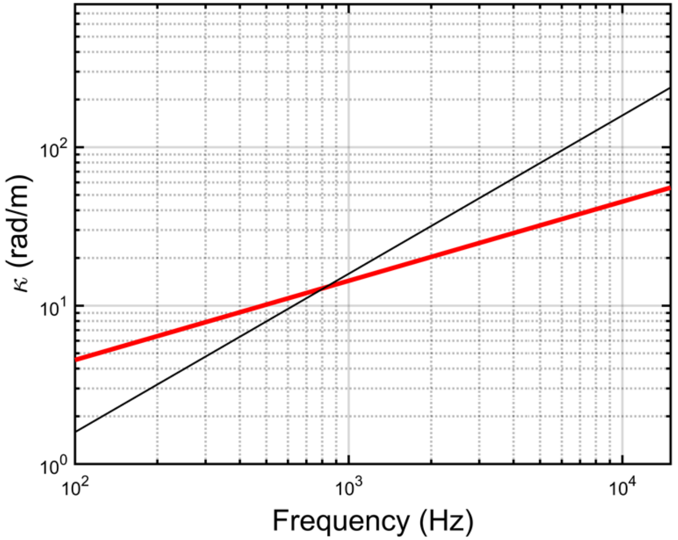


- $-2D_S D_f \frac{\partial^6 w}{\partial x^6} + 2D_f I_S \frac{\partial^6 w}{\partial t^2 \partial x^4} - (D_S m + 2D_f m + I_S G_c t_c) \frac{\partial^4 w}{\partial t^2 \partial x^2} + G_c t_c \left(D_S \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} \right) + I_S m \frac{\partial^4 w}{\partial t^4} = 0$

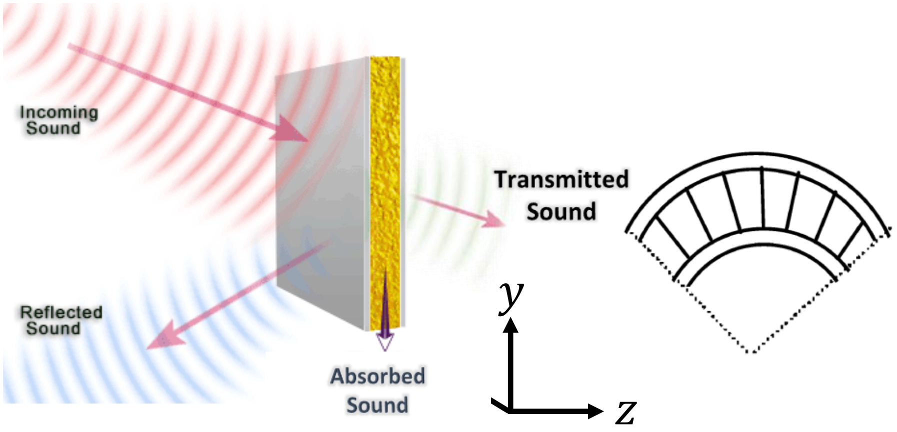
Anti-phase Mode:



- ➢ $2D_S D_f {k_x}^6 - 2D_f I_S \omega^2 {k_x}^4 - (D_S m + 2D_f m + I_S G_c t_c)\omega^2 {k_x}^2 + G_c t_c (D_S {k_x}^4 - m\omega^2) + I_S m \omega^2 = 0$

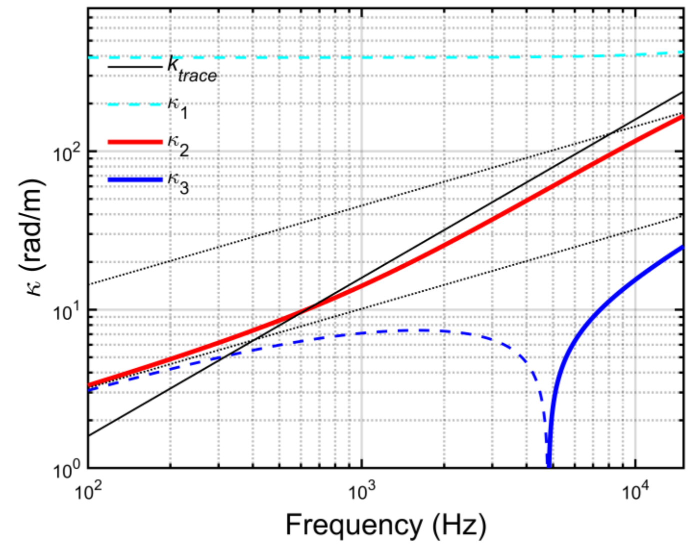


Sound transmission through a sandwich panel



Thin plate assumption

Bending wavenumber κ_2

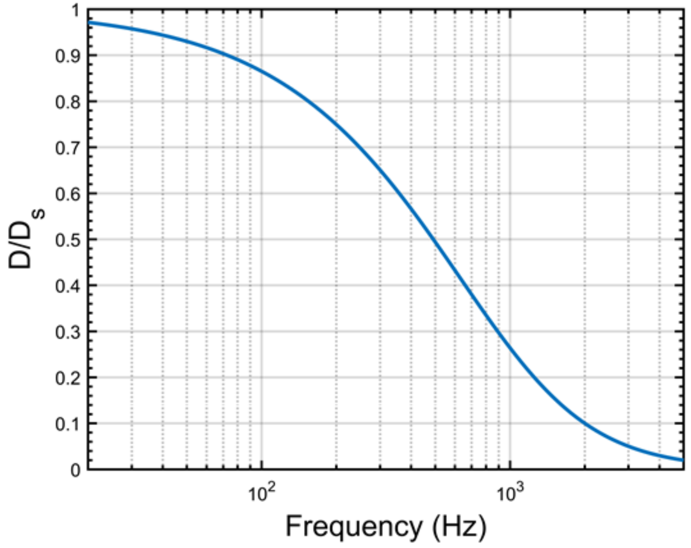


- Sandwich plate impedance

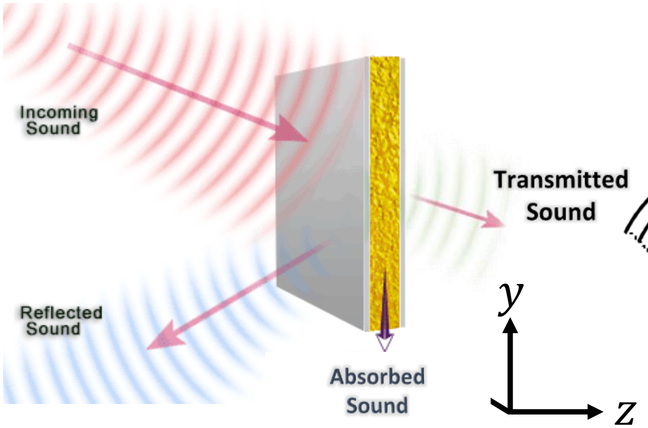
$$Z_{\mathrm{Sa}} = \mathrm{j}\omega m\left(1-\frac{k^4}{\kappa_2{}^4}\sin^4\theta\right) = \mathrm{j}\omega m\left(1-\frac{f^2}{f_{\mathrm{Sco}}{}^2}\right)$$

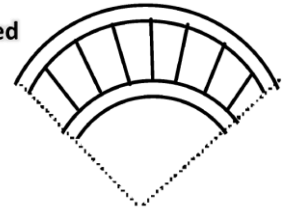
$$f_{\mathrm{Sco}} = \frac{1}{2\pi}\frac{c_0}{\sin^2\theta}\frac{\kappa_2{}^2}{k}$$

is a symbolic expression, and is a function of frequency.



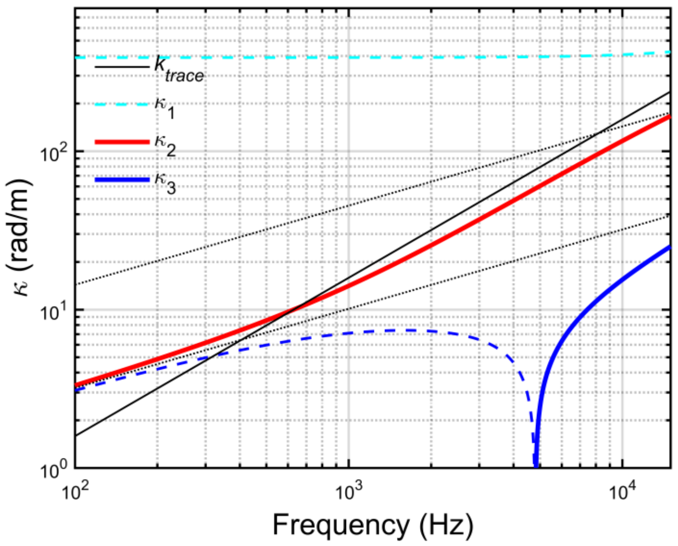
Sound transmission through a sandwich panel





Thin plate assumption

Bending wavenumber κ_2



- Sandwich plate impedance

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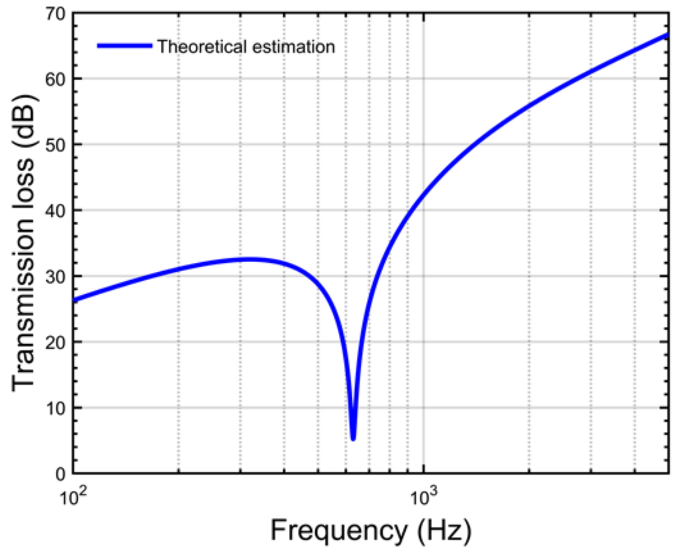
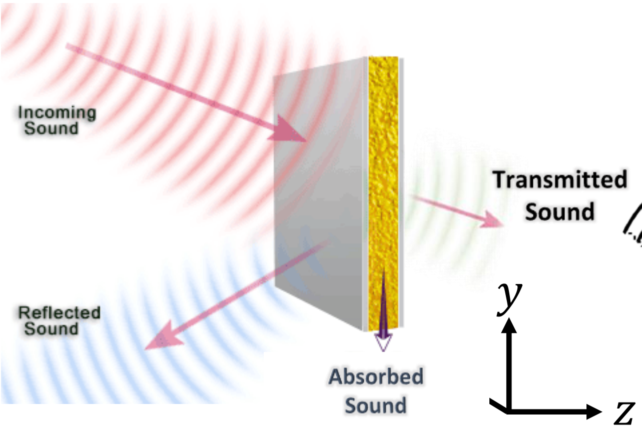
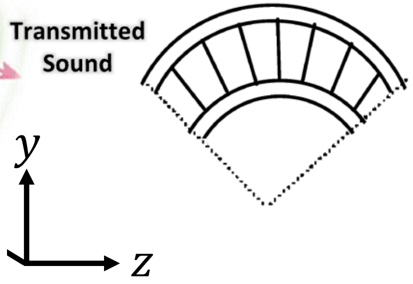


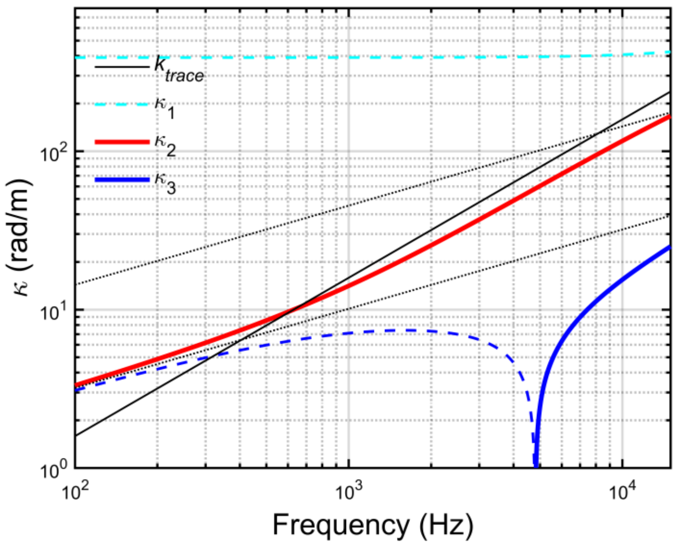
Sound transmission through a sandwich panel





Thin plate assumption

Bending wavenumber κ_2



- Sandwich plate impedance

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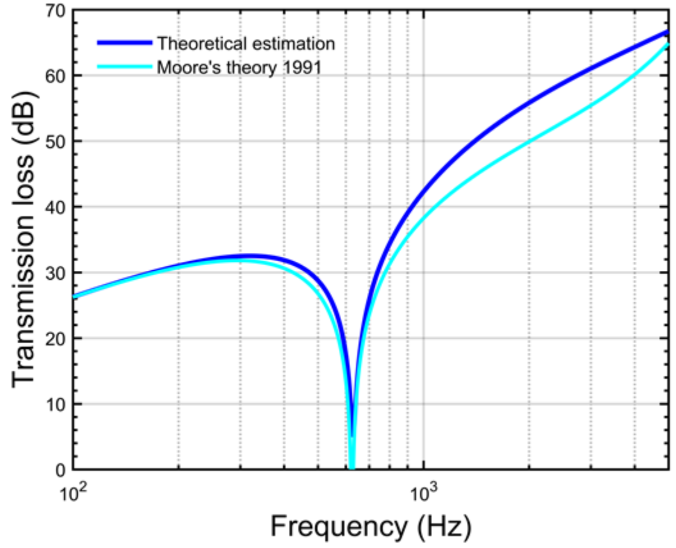
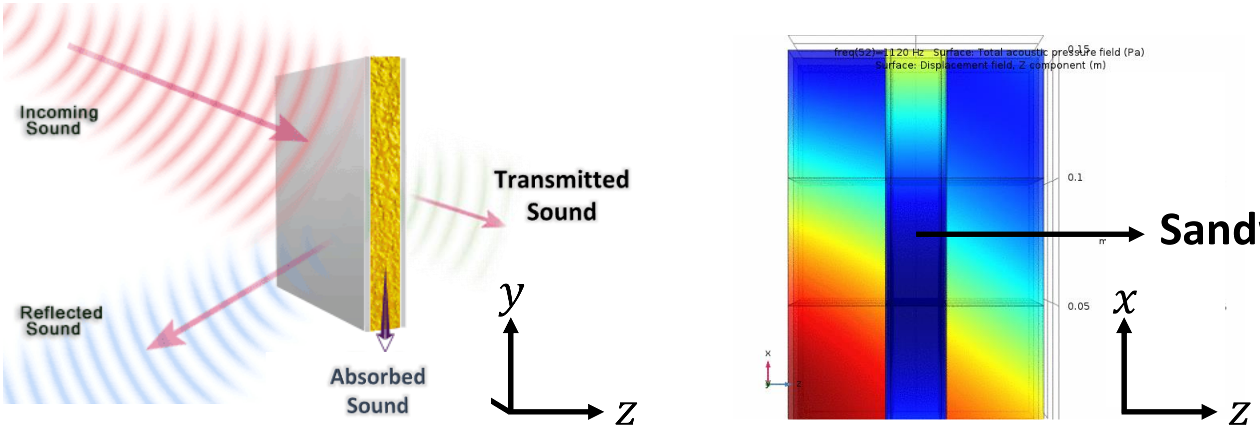
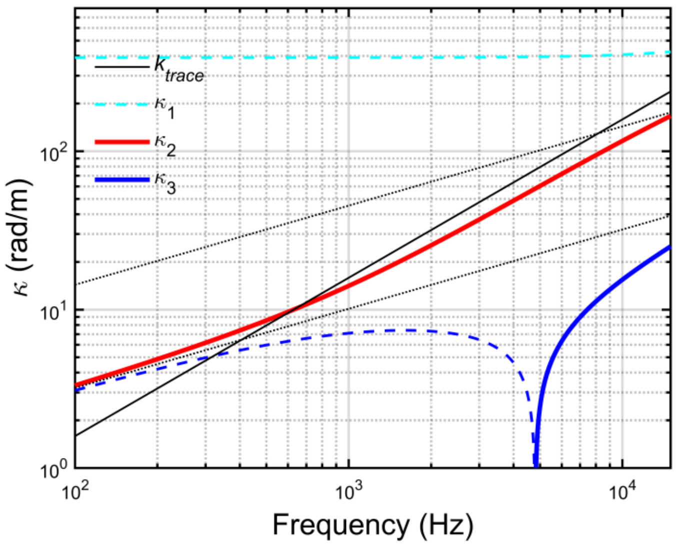


Sound transmission through a sandwich panel



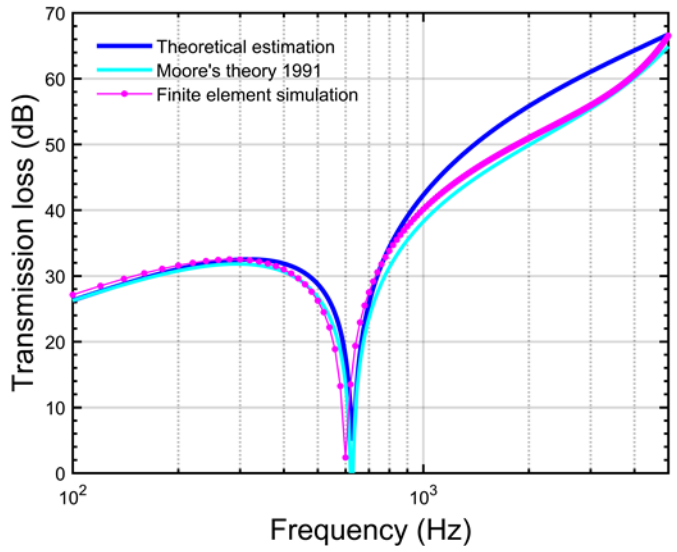


Single-leaf:

$$Z_0 = \mathrm{j}\omega m\left(1 - \frac{f^2}{f_{\mathrm{co}}^2}\right)$$

Sandwich:

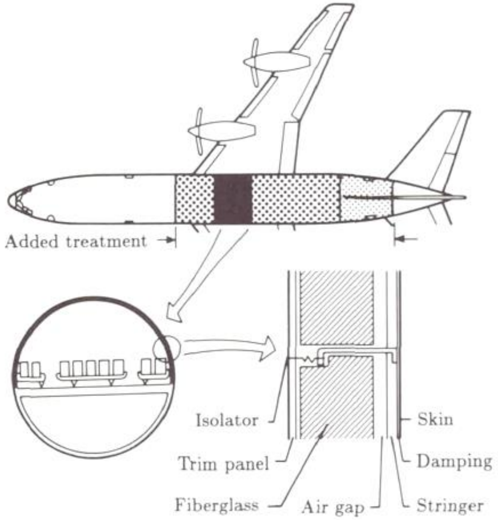
$$Z_{\mathrm{Sa}} = \mathrm{j}\omega m\left(1 - \frac{f^2}{f_{\mathrm{Sco}}^2}\right)$$

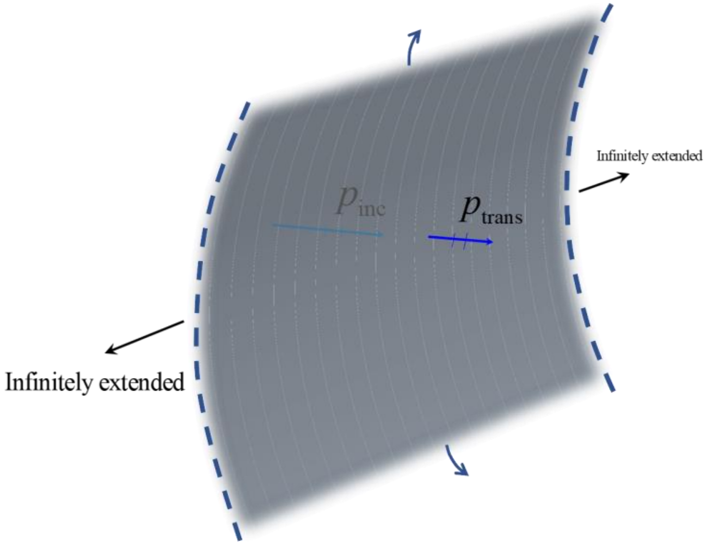


Intention:

- **Integrate the ring frequency effect**

Curved panels





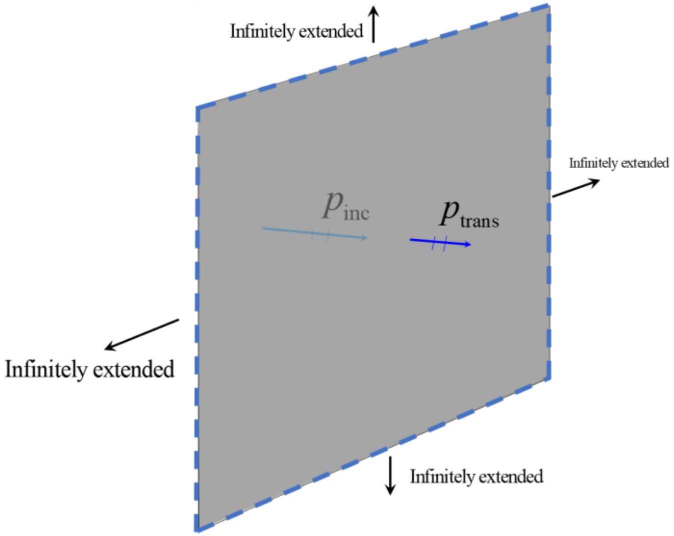
Curved panel:

- Aeronautical/aerospace engineering

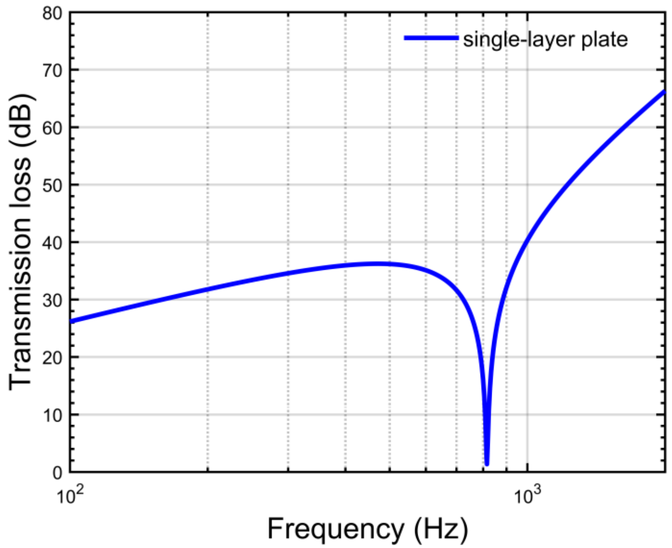
Ring frequency effect

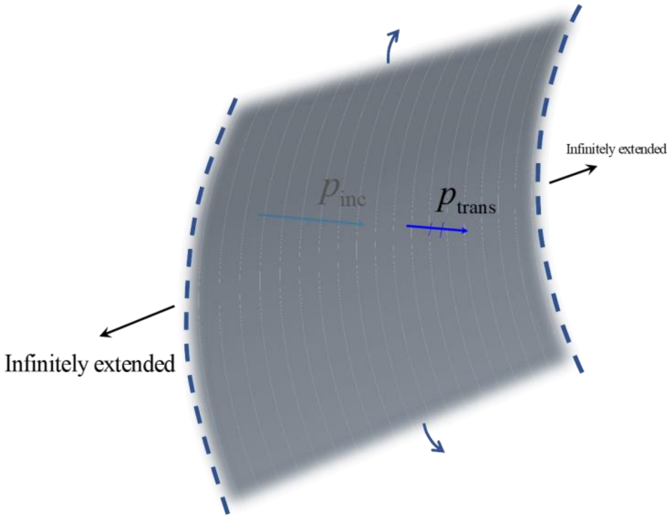
$$\mathbf{circumference} = \lambda_{\mathbf{longitudinal}}$$

Ring frequency effect



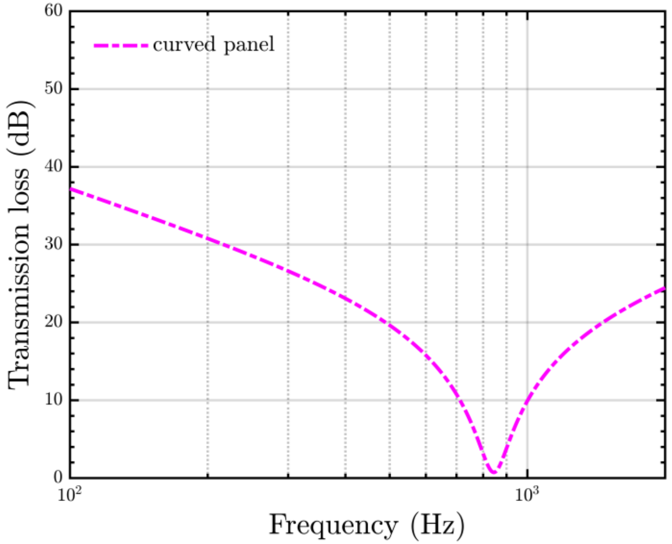
$$\lambda_{\mathbf{trace}} = \lambda_{\mathbf{bending}}$$



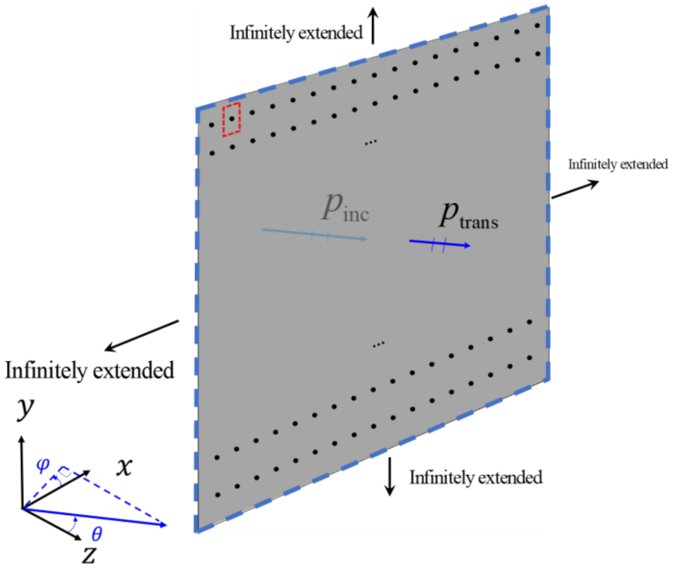


$$\mathbf{circumference} = \lambda_{\mathbf{longitudinal}}$$

Hard to overcome

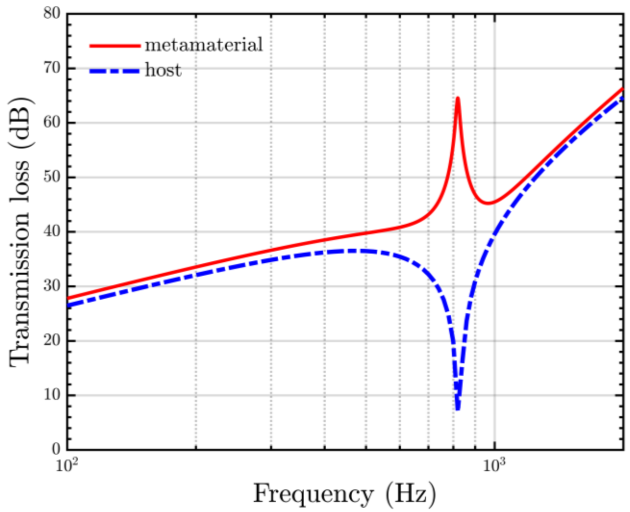


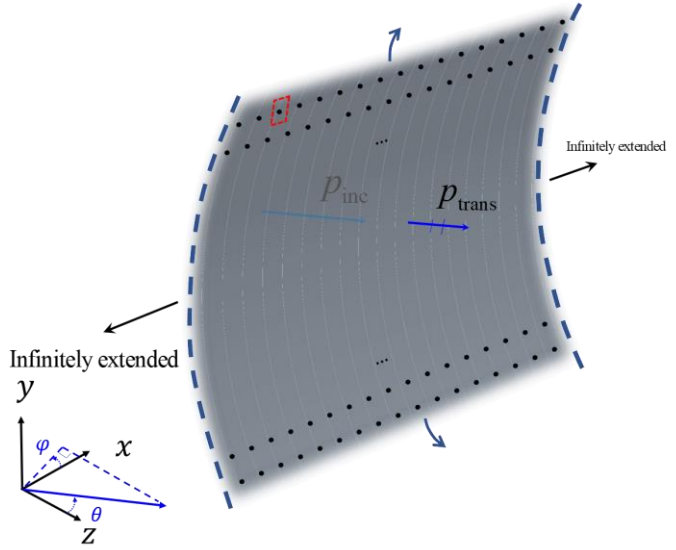
Ring frequency effect



$$Z = j\omega m\left(1-\frac{f^2}{f_{\text{co}}^2}\right)$$

$$Z_{\text{eff}} = Z + Z_{\text{eq}}^{\text{r}}$$

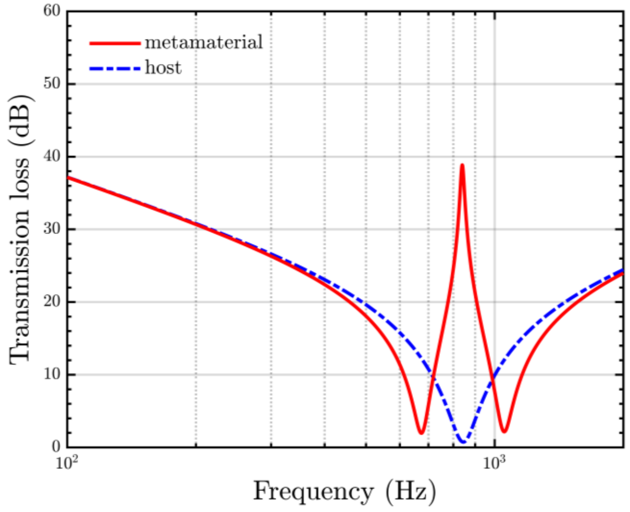




$$Z = j\omega m\left(1-\frac{f^2}{f_{\text{co}}^2}-\frac{f_{\text{ri}}^2}{f^2}\right)$$

$$Z_{\text{eff}} = Z + Z_{\text{eq}}^{\text{r}}$$

- *Unlike for the coincidence effect*
- *'Side effects' are observed in the ring frequency region*

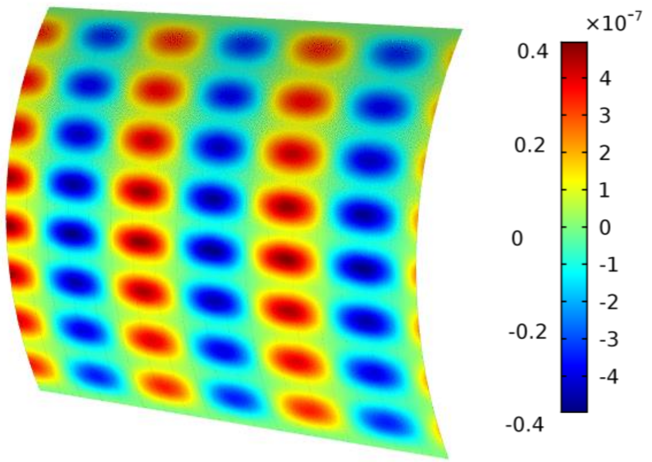


Impedance of curved panels

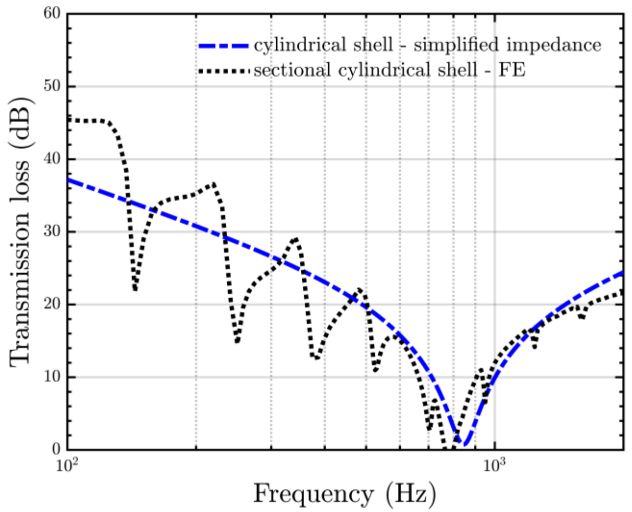
Impedance of a slightly curved shell:

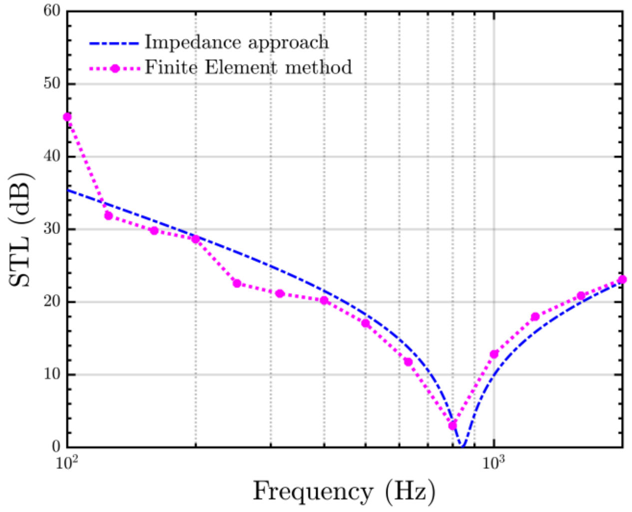
$$Z = \mathrm{j}\omega m\left(1 - \frac{f^2}{f_{\mathrm{co}}^2} - \frac{f_{\mathrm{ri}}^2}{f^2}\right) \qquad f_{\mathrm{ri}} = \frac{c_l}{2\pi R}$$

Finite Element model of the section of the shell at the frequency where the worst sound transmission loss occurs:

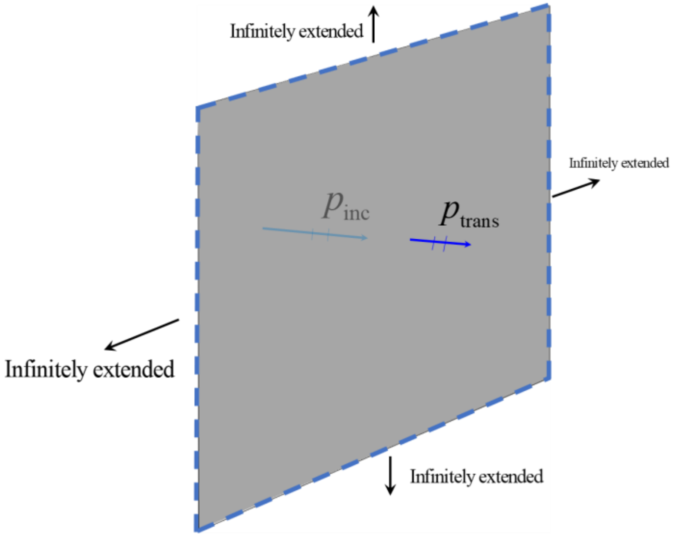


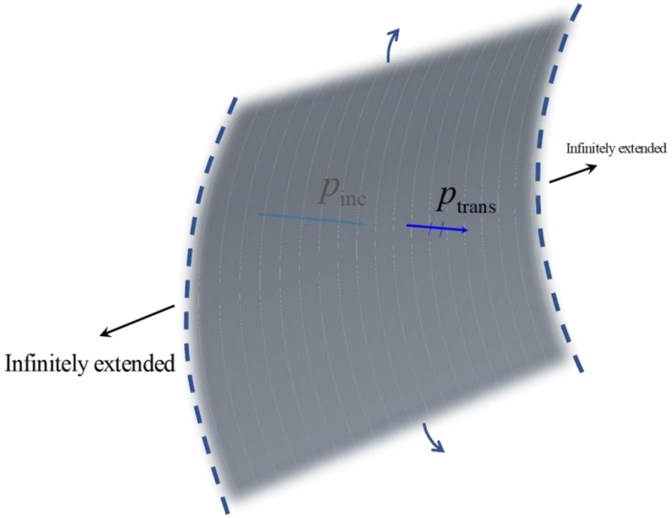
- Mathematically: minimum impedance
- Physically: maximum radiation efficiency





Ring frequency effect

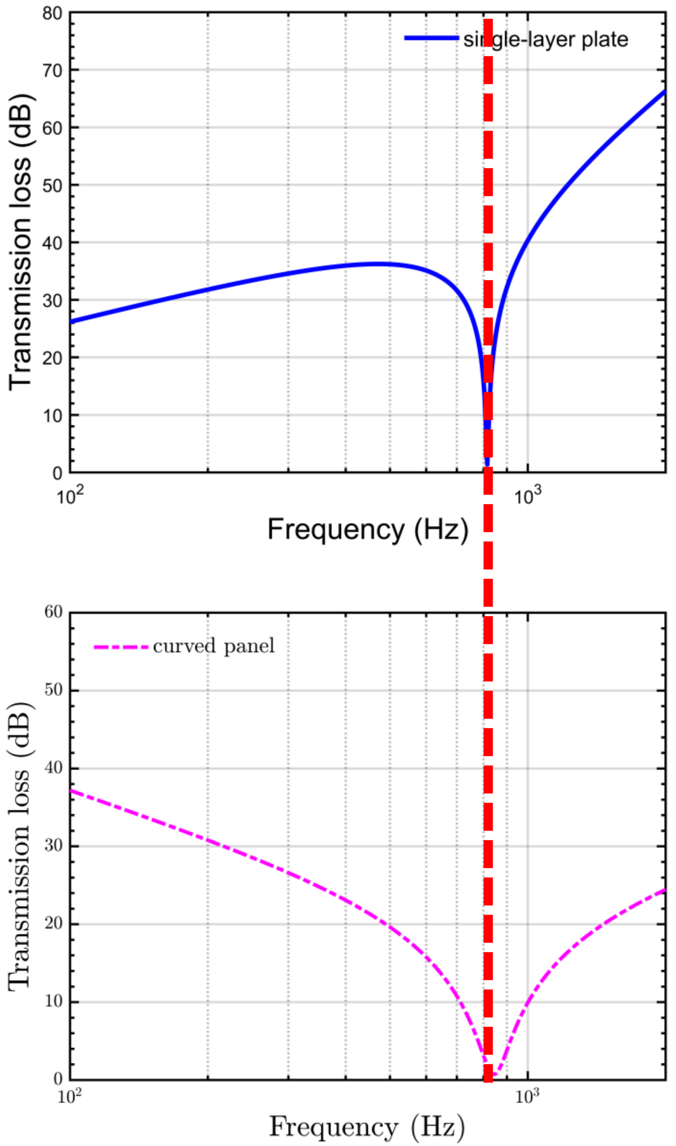




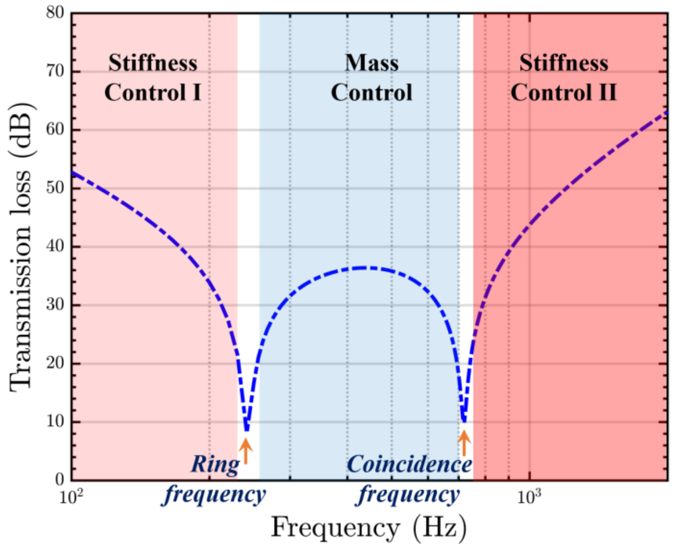
Recall: control region

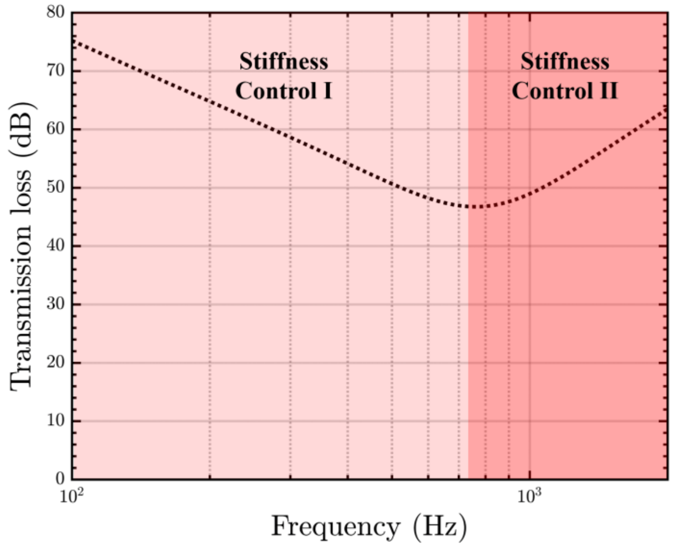
Mass to stiffness

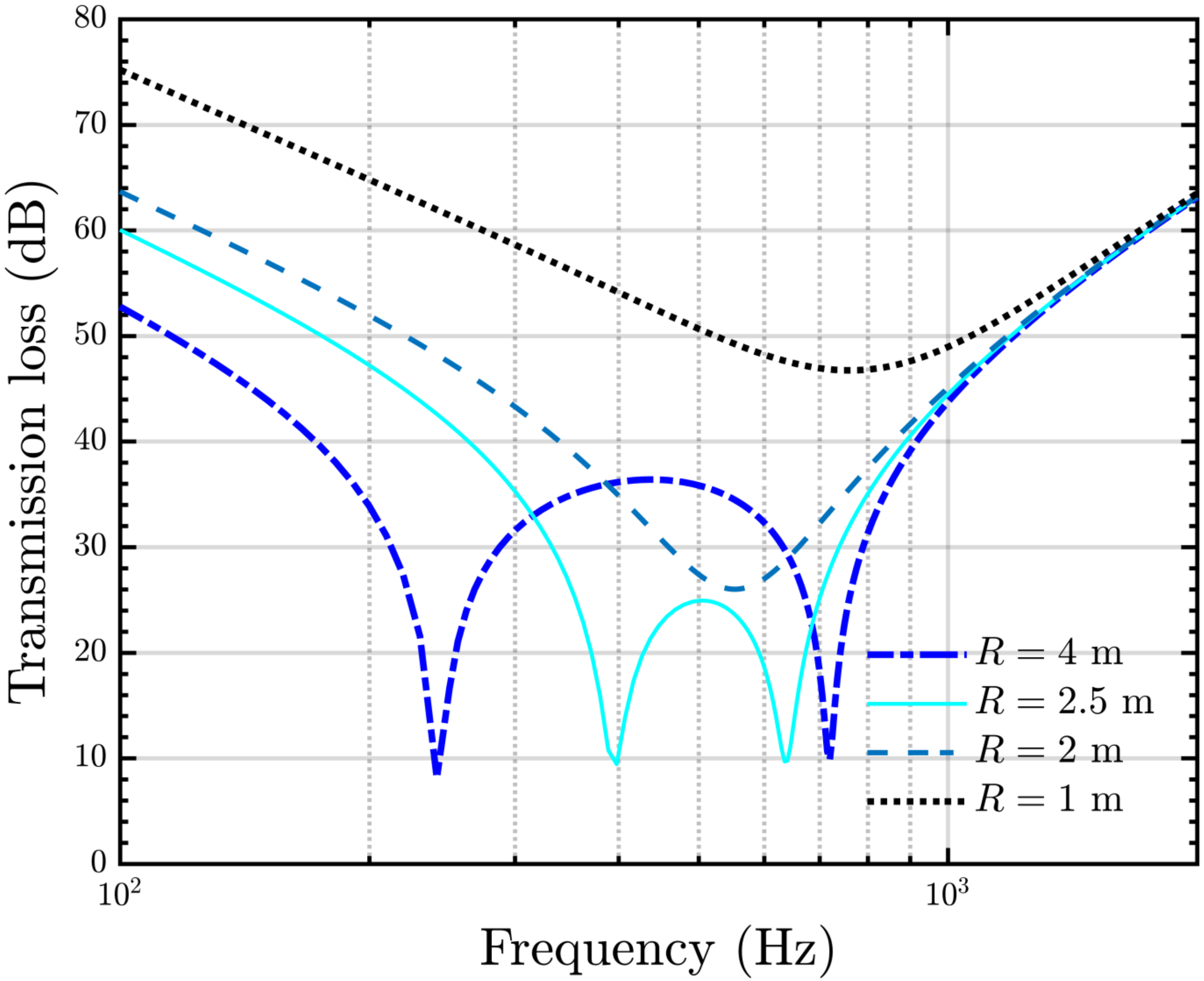
Stiffness to mass



Overcome the ring frequency effect







Overcome the ring frequency effect

Condition:

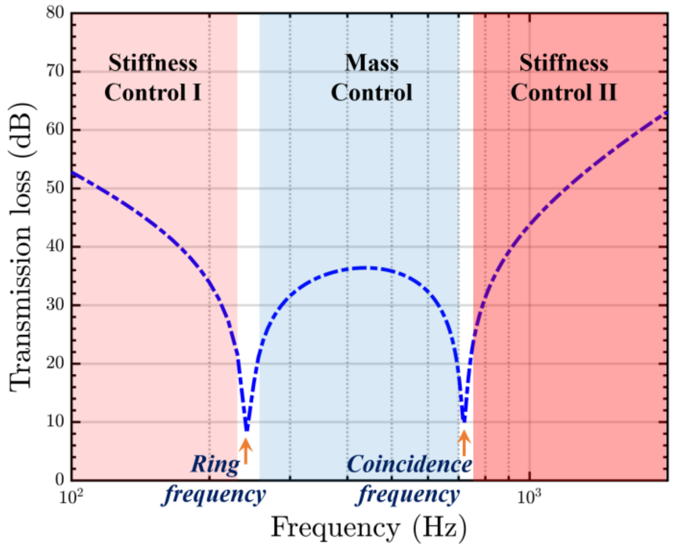
- *The impedance is enforced not equal to zero*

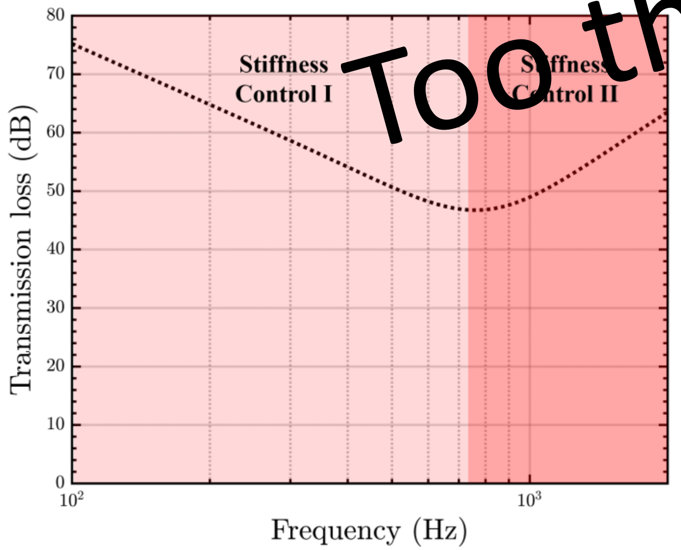
$$Z = \mathrm{j}\omega m\left(1-\frac{f^2}{f_{\mathrm{co}}{}^2}-\frac{f_{\mathrm{ri}}{}^2}{f^2}\right)\neq 0$$

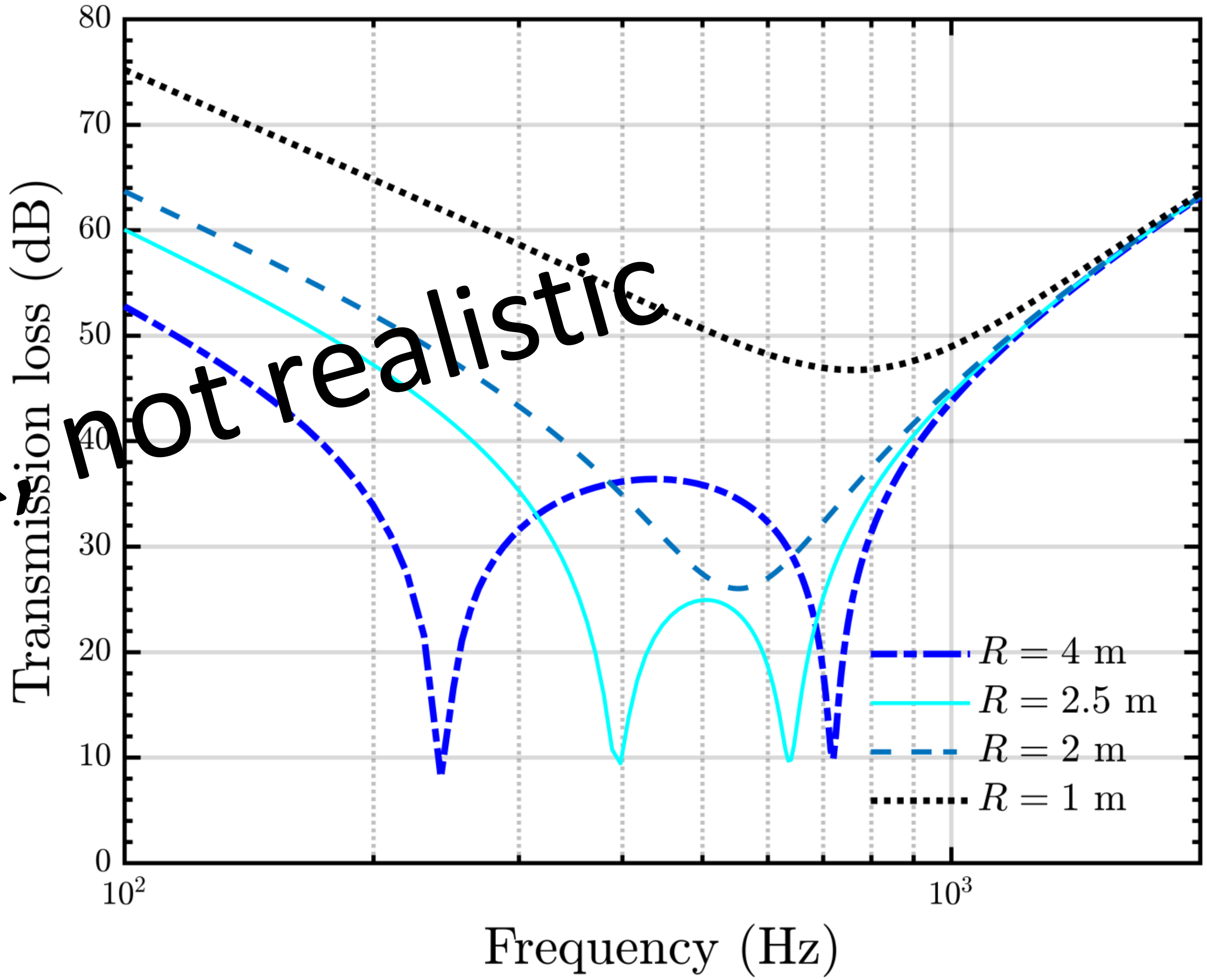
Such that

$$f_{\mathrm{ri}} > \frac{1}{2} f_{\mathrm{co}}$$ → **Design criterion**

Overcome the ring frequency effect







Too thick, not realistic

Impedance of curved sandwich panels

recall

Single-leaf:

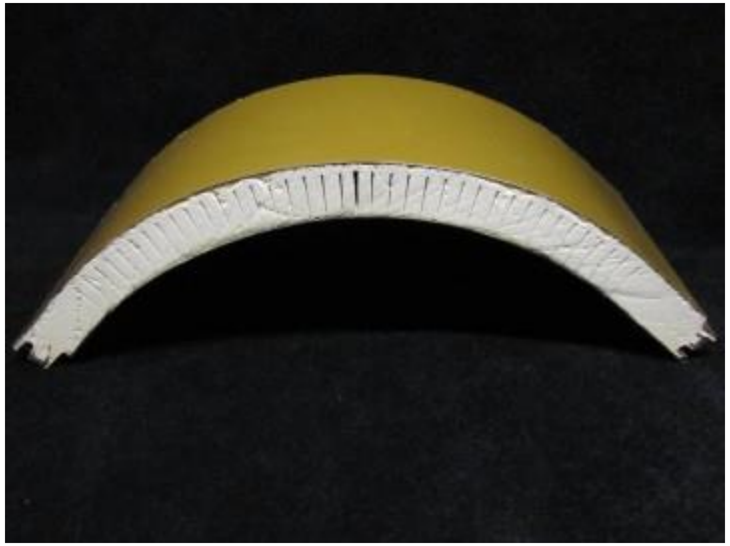
$$Z_0 = \mathrm{j}\omega m\left(1-\frac{f^2}{f_{\mathrm{co}}^{\;2}}\right)$$

Sandwich:

$$Z_{\mathrm{Sa}} = \mathrm{j}\omega m\left(1-\frac{f^2}{f_{\mathrm{Sco}}^{\;2}}\right)$$

Curved:

$$Z = \mathrm{j}\omega m\left(1-\frac{f^2}{f_{\mathrm{co}}^{\;2}}-\frac{f_{\mathrm{ri}}^{\;2}}{f^2}\right)$$



Therefore:

Curved sandwich:

$$Z = \mathrm{j}\omega m\left(1-\frac{f^2}{f_{\mathrm{Sco}}^{\;2}}-\frac{f_{\mathrm{Sri}}^{\;2}}{f^2}\right)$$

$$f_{\mathrm{Sco}} = \frac{1}{2\pi}\frac{c_0}{\sin^2\theta}\frac{\kappa^2}{k}$$

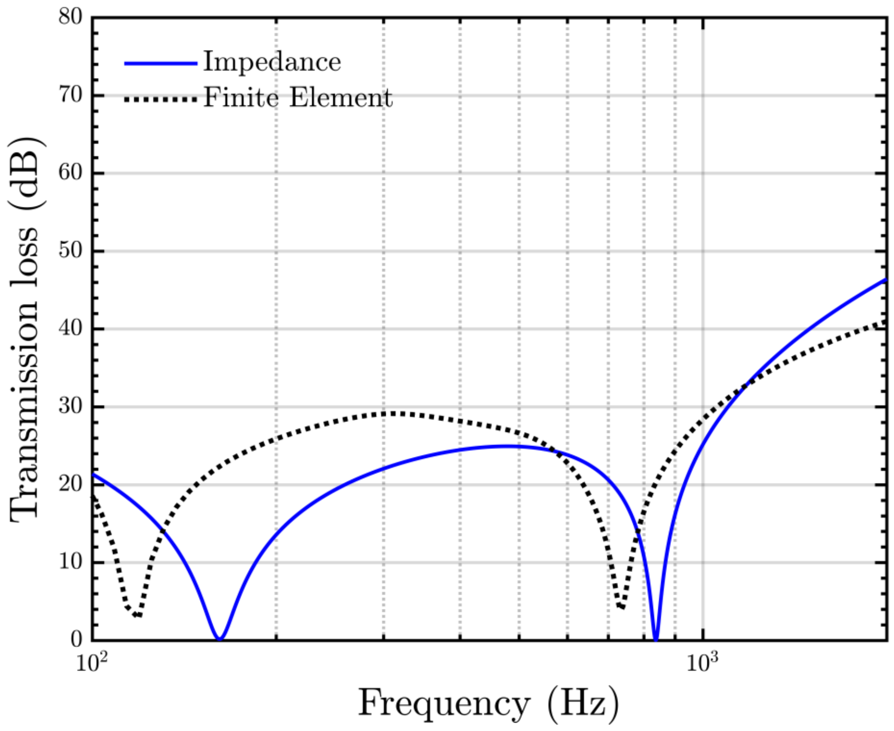
$$f_{\mathrm{Sri}} = \frac{1}{2\pi R}\sqrt{\frac{2E_{\mathrm{f}}^{\;*}t_{\mathrm{f}} + E_{\mathrm{c}}^{\;*}t_{\mathrm{c}}}{m}}$$

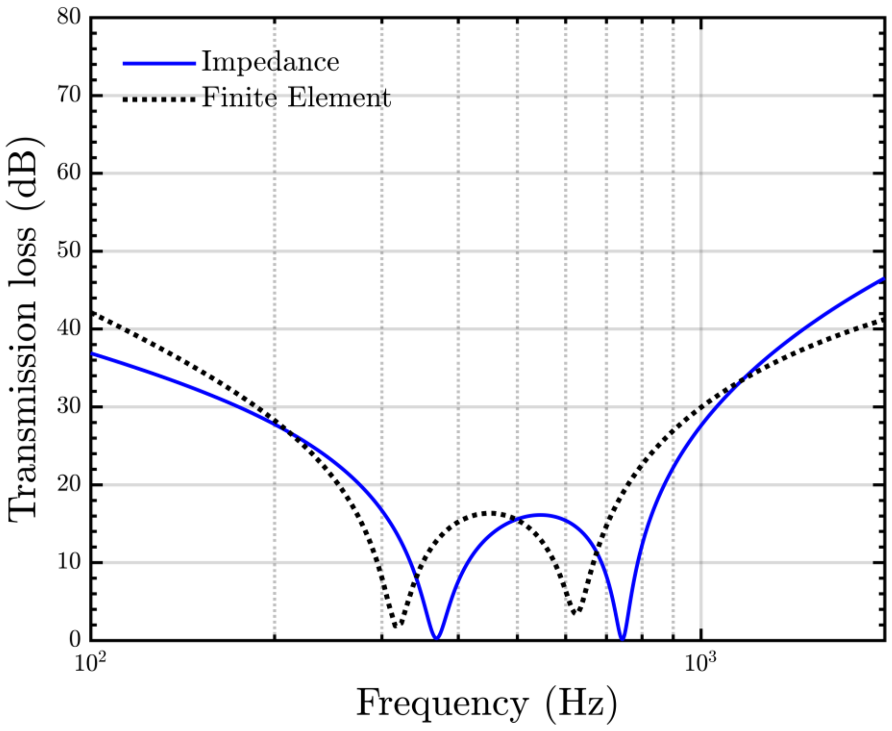
→ **Design criterion applicable to sandwich** $f_{\mathrm{Sri}} > \frac{1}{2} f_{\mathrm{Sco}}$

Curved sandwich panels

Ef	nuf	rhof	Ec	nuc	rhoc	tf	tc	fco
6.9e10	0.3	2700	8e8	0.3	500	2mm	2cm	760 Hz

when $f_{\mathrm{ri}} < \frac{1}{2} f_{\mathrm{co}}$

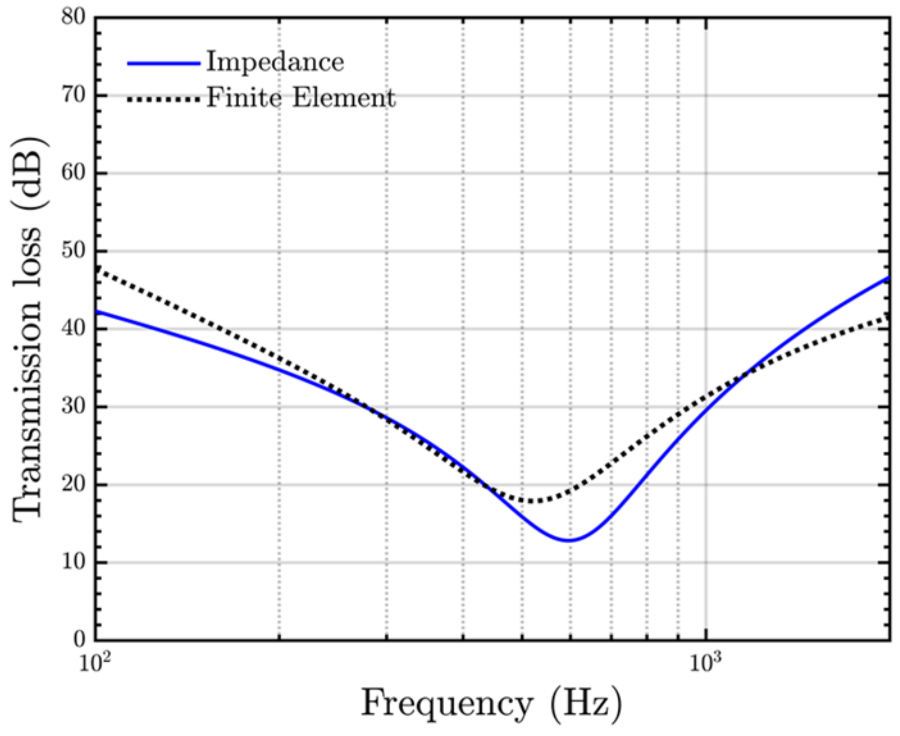


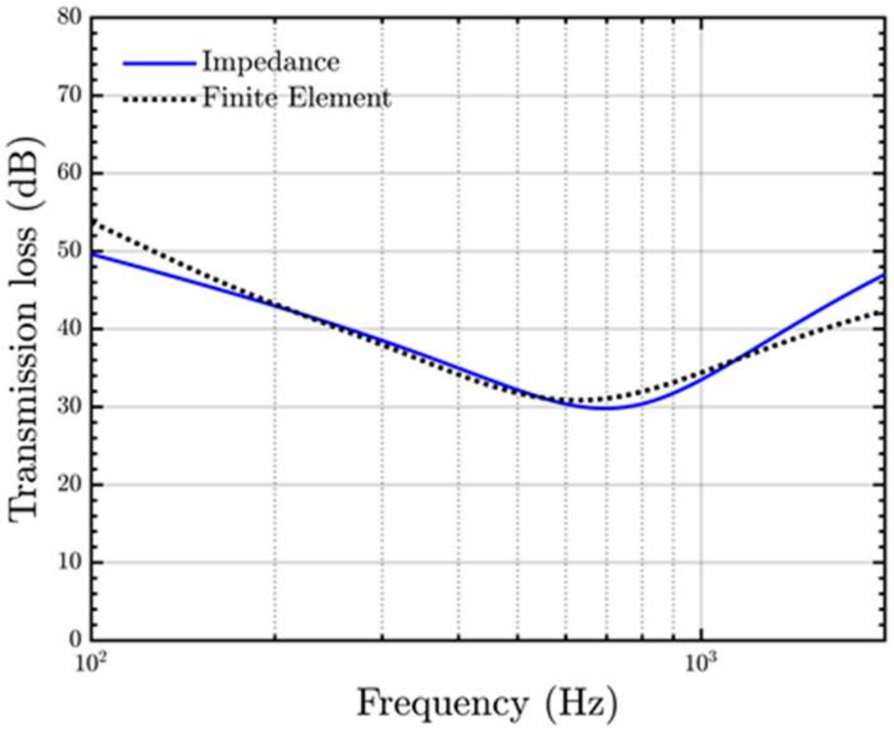


Curved sandwich panels

Ef	nuf	rhof	Ec	nuc	rhoc	tf	tc	fco
6.9e10	0.3	2700	8e8	0.3	500	2mm	2cm	760 Hz

when $f_{ri} > \frac{1}{2} f_{co}$





Conclusion

- An impedance approach is developed

$$Z = j\omega m\left(1 - \frac{f^2}{f_{Sco}{}^2} - \frac{f_{Sri}{}^2}{f^2}\right)$$

- A design criterion is proposed to overcome the coincidence and ring frequency effects

$$f_{ri} > \frac{1}{2} f_{co}$$

- Physical insights into coincidence and ring frequency effect is illustrated