Notes On Rendering

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1 Introduction

Rendering, in the most general sense, refers to the process that takes a scene as input and outputs an image. Since the work of Kajiya [1] and Veach [2], rendering has been one of the central topics in computer graphics. Despite numerous books and papers that talk about rendering in detail, I still want to write this concise summary of the key ingredients in classic rendering. It could serve as a quick lookup for my own study, as well as a casual introduction for people first studying this subject.

2 Light Model

To start off, we want a concise model that lays the common ground for discussion. Rendering has two components: a scene that we want to render, and the rendering process. A *scene* comprises a camera sensor, some 2-manifold closed surfaces, and some emitter lights. The *rendering process* refers to the simulation of the *light transport* — light photons bouncing in the scene — to predict their convergence on the camera image plane. While there is a whole other universe of modeling 3D objects, this note focuses on light transport, and this section focuses on modeling light.

Like all models require assumptions, the assumption [2] we make in classic light transport is that

- light travels along straight lines
- light is emitted, scattered, and absorbed only at surfaces

2.1 Photons

A *photon* is the fundamental unit of light. Conceptually we can think of a beam of light as numerous photons traveling in the same direction. The *phase space* ψ describes the state of a photon. It is a 6-dimensional space comprising the position, the direction of motion, and the wavelength of the photon.

$$\psi = \mathbb{R}^3 \times \mathbb{S}^2 \times \mathbb{R}$$

Generalizing to over a range of time, we have the *trajectory space*

$$\Psi = \psi \times \mathbb{R}$$

where the second parameter represents time.

2.2 Radiometry

To measure the light, we could either simulate numerous photons and add them up, or more wisely, we could define a notion of energy that is proportional to the number of photons. In rendering we care most about the converged photon equilibrium, so the notion of power is more common, and from it derives many different radiometric measurements.

- Radiant energy Q denotes the total number of photons.
- Radiant power $\Phi = \dfrac{dQ}{dt}$ denotes the amount of energy per time.
- Solid angle $\sigma = \dfrac{A}{R^2}$ gives a measure on the set of directions (i.e. the hemisphere).
- Radiant intensity $I(\omega) = \dfrac{d\Phi(\omega)}{d\sigma^\perp(\omega)}$ denotes power per solid angle, where

$$d\sigma^\perp(\omega) = \langle \omega, \mathbf{n} \rangle \; d\sigma(\omega)$$

- Irradiance $E(x) = \dfrac{d\Phi(x)}{dA(x)}$ denotes power per area.
- Radiance $L(x, \omega) = \dfrac{d^2\Phi(x, \omega)}{dA(x) d\sigma^\perp(\omega)}$ denotes power per area per solid angle.

3 Light Transport

After deriving a model for light, we continue to derive a model for light transport. We will first discuss how light bounces and introduce the notion of BSDF. We then continue to derive the rendering equation and talk about how rendering is essentially an integration.

3.1 BSDF

Recall that the power of light is the number of photons per unit of time. If we assume that upon hitting a surface, the photons all scatter in the same direction, then it is clear that the scattered number of photons is proportional to the incoming number of photons. Taking their ratio gives us the *Bidirectional Scattering Distribution Function (BSDF)* from incoming/incident direction ω_i to outgoing direction ω_o

$$f(\mathbf{x}, \omega_i \to \omega_o) = \frac{dL_o(\omega_o)}{dE(\omega_i)} = \frac{dL_o(\omega_o)}{L_i(\omega_i) d\sigma^\perp(\omega_i)}$$

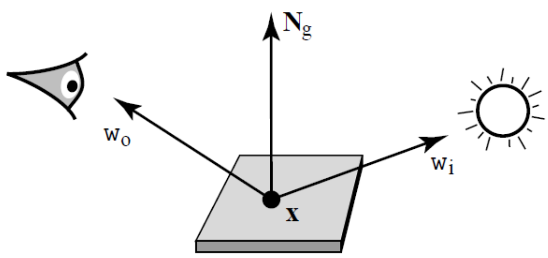

Figure 1: Illustration of bidirectional scattering. Image from [2]

Here, an incident function $L_i(\mathbf{x}, \omega)$ measures the radiance arriving at position $\mathbf{x}$ from direction ω. An exitant function $L_o(\mathbf{x}, \omega)$ measures the radiance leaving from position $\mathbf{x}$ in direction ω. We omit $\mathbf{x}$ everywhere above since it is fixed.

3.2 The Rendering Equation

Suppose we have the intersection point $\mathbf{x}$ and the outgoing direction ω_o fixed, then integrating over all possible incident directions gives us an estimation of the exitant radiance $L_o(\omega_o)$. This is the renowned *rendering equation* [1]

$$L_o(\mathbf{x}, \omega_o) = L_e(\mathbf{x}, \omega_o) + \int_{\mathcal{H}^2} f(\mathbf{x}, \omega_i \to \omega_o) \langle \mathbf{n}, \omega_i \rangle L_i(\mathbf{x}, \omega_i) \; d\omega_i$$

With the rendering equation, we now only need to solve $L_i(\mathbf{x}, \omega_i) = L_o(\mathbf{x}', -\omega_i)$ for all $\mathbf{x}$ on the camera image plane and all possible ω_i. Here $\mathbf{x}'$ is the intersection point of $\mathbf{x}$ along ω_i with the scene.

References

[1] James T. Kajiya. “The Rendering Equation”. In: *Proceedings of the 13th Annual Conference on Computer Graphics and Interactive Techniques*. SIGGRAPH ’86. New York, NY, USA: ACM, 1986, pp. 143–150. ISBN: 0-89791-196-2. DOI: `10.1145/15922.15902`.

[2] Eric Veach. “Robust Monte Carlo Methods for Light Transport Simulation”. PhD thesis. Stanford University, 1997.