

# GroupRepresentationsForCAP

**Skeletal category of group  
representations for CAP**

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# Chapter 1

## Associators

### 1.1 Introduction

Let  $G$  be a finite group and let  $G\text{-mod}$  be a skeletal version of the monoidal category of finite dimensional complex representations of  $G$ . The purpose of these GAP methods is the computation of the associators of  $G\text{-mod}$ .

### 1.2 Quickstart

The following commands compute the associator of  $D_8$  and write all data necessary for the reproducibility of the computation to files with the prefix "D8".

Example

```
gap> G := DihedralGroup( 8 );;  
gap> ComputeAssociator( G, true, true, false );;  
gap> path := Concatenation(  
>   PackageInfo( "GroupRepresentationsForCAP" )[1].InstallationPath,  
>   "/examples/doc/D8" );;  
gap> WriteAssociatorComputationToFiles( path );
```

### 1.3 Read, Write, and Display

The following intermediate steps of the associator computation can be read from/written to files.

- Irreducible representations of a finite group given by matrices (Data 1).
- Decomposition isomorphisms of tensor products into direct sums of irreducibles (Data 2).

Furthermore, the following data can be written to files.

- A database key for the AssociatorsDatabase/DatabaseKeys.g file.
- The final result, namely the associator (Data 3).

Data 1 and Data 2 involve choices and thus are subject to changes in further versions of this package. However, the process Data 2  $\rightarrow$  Data 3 is a mathematical function and thus stable. For reproducibility, it is recommended to store all three data. To facilitate this task, use the function `WriteAssociatorComputationToFiles`.

### 1.3.1 WriteDatabaseKeysToFile (for IsString)

▷ WriteDatabaseKeysToFile(*s*) (operation)

**Returns:** nothing

The argument is a filename *s*. This operation writes the database keys computed by the last call of InitializeGroupData to the corresponding file.

### 1.3.2 WriteRepresentationsDataToFile (for IsString)

▷ WriteRepresentationsDataToFile(*s*) (operation)

**Returns:** nothing

The argument is a filename *s*. This operation writes the representations computed by the last call of InitializeGroupData to the corresponding file.

### 1.3.3 WriteSkeletalFunctorDataToFile (for IsString)

▷ WriteSkeletalFunctorDataToFile(*s*) (operation)

**Returns:** nothing

The argument is a filename *s*. This operation writes the skeletal functor data computed by the last call of SkeletalFunctorTensorData to the corresponding file.

### 1.3.4 WriteAssociatorDataToFile (for IsString)

▷ WriteAssociatorDataToFile(*s*) (operation)

**Returns:** nothing

The argument is a filename *s*. This operation writes the associator data of the initialized group to the corresponding file. You have to call AssociatorForSufficientlyManyTriples first.

### 1.3.5 WriteAssociatorComputationToFiles (for IsString)

▷ WriteAssociatorComputationToFiles(*s*) (operation)

**Returns:** nothing

Only call this function if you did a whole associator computation first (e.g. using ComputeAssociator). The argument is a string *s*. This function writes 4 files:

- *s*Key.g: A file for the database key of the associator computation.
- *s*Reps.g: A file containing the irreducible representations used for the associator computation.
- *s*Dec.g: A file for the tensor decompositions used for the associator computation.
- *s*Ass.g or *s*AssD.g: A file containing the computed associator. The suffix *D* is used if the associator was not computed for all triples.

### 1.3.6 ReadDatabaseKeys (for IsString)

▷ ReadDatabaseKeys(*s*) (operation)

**Returns:** a list

The argument is a filename *s* of a file written by WriteDatabaseKeysToFile. The output is a list [ group, conductor, position of trivial character, field, category ].

### 1.3.7 ReadRepresentationsData (for IsString, IsString)

▷ `ReadRepresentationsData( $s_1$ ,  $s_2$ )` (operation)

**Returns:** a list

The arguments are a filename  $s_1$  of a file written by `WriteDatabaseKeysToFile`, and a filename  $s_2$  of a file written by `WriteRepresentationsDataToFile`. The output is a list [ ,number of irreducibles, irreducibles, representations given by images of generators, inverses of these images, vector space objects for the irreducibles ].

### 1.3.8 ReadSkeletalFunctorData (for IsString, IsString)

▷ `ReadSkeletalFunctorData( $s_1$ ,  $s_2$ )` (operation)

**Returns:** a list

The arguments are a filename  $s_1$  of a file written by `WriteDatabaseKeysToFile`, and a filename  $s_2$  of a file written by `WriteSkeletalFunctorDataToFile`. The output is a list [ irreducibles, skeletal functor tensor data, vector space objects for the irreducibles ].

## 1.4 Computing associators

### 1.4.1 InitializeGroupData (for IsGroup)

▷ `InitializeGroupData( $G$ )` (operation)

**Returns:** a list

The argument is a group  $G$ . This method calls `InitializeGroupData( $G$ , false)`.

### 1.4.2 InitializeGroupData (for IsGroup, IsBool)

▷ `InitializeGroupData( $G$ ,  $b$ )` (operation)

**Returns:** a list

The arguments are a group  $G$  and a boolean  $b$ . The output is a list [ generators of  $G$ , number of irreducibles, irreducibles, representations given by images of generators, inverses of these images, vector space objects for the irreducibles ]. Furthermore, this method stores the database key, which can be written using `WriteDatabaseKeysToFile`. If  $b$  is true, then the id of the group in the database key is given by its string, otherwise it is given by its id in the `SmallGroupLibrary`.

### 1.4.3 InitializeGroupDataDixon (for IsGroup)

▷ `InitializeGroupDataDixon( $G$ )` (operation)

**Returns:** a list

The argument is a group  $G$ . This method calls `InitializeGroupDataDixon( $G$ , false)`.

### 1.4.4 InitializeGroupDataDixon (for IsGroup, IsBool)

▷ `InitializeGroupDataDixon( $G$ ,  $b$ )` (operation)

**Returns:** a list

The arguments are a group  $G$  and a boolean  $b$ . This method does the same as `InitializeGroupData`, but uses `IrreducibleRepresentationsDixon` for affording irreducible representations.

### 1.4.5 InitializeGroupData (for IsGroup, IsList, IsBool)

▷ InitializeGroupData(arg1, arg2, arg3) (operation)

### 1.4.6 SkeletalFunctorTensorData

▷ SkeletalFunctorTensorData() (operation)

**Returns:** a list

There is no argument. This methods calls SkeletalFunctorTensorData with the output of the last call of InitializeGroupData or InitializeGroupDataDixon.

### 1.4.7 SkeletalFunctorTensorData (for IsList)

▷ SkeletalFunctorTensorData(l) (operation)

**Returns:** a list

The argument is a list  $l$  which is the output of InitializeGroupData, InitializeGroupDataDixon, or ReadRepresentationsData. The output is a triple  $[t_1, t_2, t_3]$ .  $t_1$  is the list of all characters of  $G$ .  $t_2$  is a list such that the  $(i, j)$ -th entry, where  $i, j$  range from 1 to the number of irreducibles, is a pair of mutual inverse morphisms  $[\alpha, \alpha^{-1}]$ , and  $\alpha$  is a decomposition isomorphism  $\bigoplus_{\chi \in \text{Irr}(G)} V_{\chi}^{n_{\chi}} \rightarrow V_i \otimes V_j$ .  $t_3$  is a list of vector space objects for the irreducibles.

### 1.4.8 AssociatorDataFromSkeletalFunctorTensorData (for IsInt, IsInt, IsInt, IsList)

▷ AssociatorDataFromSkeletalFunctorTensorData(a, b, c, l) (operation)

**Returns:** a list

The arguments are integers  $a, b, c$  and a list  $l$  which is the output of SkeletalFunctorTensorData. The output is a list containing homalg matrices representing the components of the associator of  $V_a, V_b, V_c$ , where the numbers correspond to the enlisting of the irreducible characters given by  $l$ .

### 1.4.9 AssociatorForSufficientlyManyTriples

▷ AssociatorForSufficientlyManyTriples() (operation)

**Returns:** a list

There is no argument. This methods calls AssociatorForSufficientlyManyTriples with the output of the last call of SkeletalFunctorTensorData and false.

### 1.4.10 AssociatorForSufficientlyManyTriples (for IsList, IsBool)

▷ AssociatorForSufficientlyManyTriples(l, b) (operation)

**Returns:** a list

The arguments are a list  $l$  which is the output of SkeletalFunctorTensorData, and a boolean  $b$ . The output is a list of lists  $L$  such that  $L[a][b][c]$  contains the associator computed by AssociatorDataFromSkeletalFunctorTensorData(a,b,c). If  $b$  is true, then  $a, b, c$  ranges through all possible triples, otherwise,  $a, b, c$  are computed for so many triples such that the others can be obtained using braidings.

### 1.4.11 ComputeAssociator (for IsGroup, IsBool)

▷ `ComputeAssociator( $G$ ,  $b_1$ )` (operation)

**Returns:** a list

The arguments are a group  $G$ , and a boolean  $b_1$ . The output is `ComputeAssociator( $G$ ,  $b_1$ , false, true)`.

### 1.4.12 ComputeAssociator (for IsGroup, IsBool, IsBool)

▷ `ComputeAssociator( $G$ ,  $b_1$ ,  $b_2$ )` (operation)

**Returns:** a list

The arguments are a group  $G$ , and two booleans  $b_1, b_2$ . The output is `ComputeAssociator( $G$ ,  $b_1$ ,  $b_2$ , true)`.

### 1.4.13 ComputeAssociator (for IsGroup, IsBool, IsBool, IsBool)

▷ `ComputeAssociator( $G$ ,  $b_1$ ,  $b_2$ ,  $b_3$ )` (operation)

**Returns:** a list

The arguments are a group  $G$ , and three booleans  $b_1, b_2, b_3$ . The output is a list  $l$  whose  $(a, b, c)$ -th entry contains a string representing the associator of the objects  $V_a, V_b, V_c$  in a skeleton of the representation category of  $G$ , where  $V_*$  are irreducible representations corresponding to the ordering of the irreducible characters  $\text{Irr}(G)$ . If  $b_1$  is true, this method uses `IrreducibleRepresentationsDixon`, otherwise it uses `IrreducibleAffordingRepresentation`. If  $b_2$  is true, the associators are computed for all possible triples  $a, b, c$ , otherwise only for sufficiently many such that the others can be reproduced using the braiding in the representation category. If  $b_3$  is true, then the id of the group in the database key is given by its string, otherwise it is given by its id in the `SmallGroupLibrary`. This last boolean is relevant only if you want to write the computed associators to files (e.g. using `WriteAssociatorComputationToFiles`).

## 1.5 Technical functions

### 1.5.1 SetInfoLevelForAssociatorComputations (for IsInt)

▷ `SetInfoLevelForAssociatorComputations( $l$ )` (operation)

**Returns:** nothing

The argument is an integer  $l$ . If  $l > 0$ , then the functions for computing associators provide information during the computation. This is useful in cases where the computation may take a long time.

### 1.5.2 DefinedOverCyclotomicField (for IsInt, IsGroupHomomorphism)

▷ `DefinedOverCyclotomicField( $n$ ,  $f$ )` (operation)

**Returns:** a boolean

The arguments are an integer  $n$  and a group homomorphism  $f$  whose images are matrices. The output is true if the entries of the images of  $f$  lie in a cyclotomic field generated by a primitive  $n$ -th root of unity, false otherwise.

### 1.5.3 GroupReperesentationByImages (for IsGroup, IsList)

▷ `GroupReperesentationByImages( $G, L$ )` (operation)

**Returns:** a group homomorphism

The arguments are a group  $G$  with generators  $g_1, \dots, g_n$  and a list  $L = [l_1, \dots, l_n]$ . The output is the group homomorphism from  $G$  to the group generated by the elements of  $L$ , mapping  $g_i$  to  $l_i$ .

### 1.5.4 DiagonalizationTransformationOfBraiding (for IsVectorSpaceMorphism)

▷ `DiagonalizationTransformationOfBraiding( $e$ )` (attribute)

**Returns:** an invertible endomorphism in  $\text{Hom}(V, V)$

The argument is an endomorphism  $e \in \text{Hom}(V, V)$  of vector spaces whose minimal polynomial divides  $x^2 - 1$ . The output is an invertible endomorphism  $t$  such that  $t^{-1} \circ e \circ t$  is a diagonal matrix.

### 1.5.5 AffordAllIrreducibleRepresentations (for IsGroup)

▷ `AffordAllIrreducibleRepresentations( $G$ )` (operation)

**Returns:** a list

The argument is a group  $G$ . The output is a list of all irreducible representations of  $G$  using the command `IrreducibleAffordingRepresentation`.

### 1.5.6 AffordAllIrreducibleRepresentationsDixon (for IsGroup)

▷ `AffordAllIrreducibleRepresentationsDixon( $G$ )` (operation)

**Returns:** a list

The argument is a group  $G$ . The output is a list of all irreducible representations of  $G$  using the command `IrreducibleRepresentationsDixon`.

### 1.5.7 DefaultFieldForListOfRepresentations (for IsList)

▷ `DefaultFieldForListOfRepresentations( $L$ )` (operation)

**Returns:** a GAP field

The argument is a list  $L$  of representations of a group  $G$ . The output is a field over which all representations are defined simultaneously.

### 1.5.8 RewriteMatrixInCyclotomicGenerator (for IsMatrix, IsInt)

▷ `RewriteMatrixInCyclotomicGenerator( $M, n$ )` (operation)

**Returns:** a matrix

The arguments are a matrix  $M$  and an integer  $n$ . The output is a matrix  $N$  in  $Q[\varepsilon]$ . Substituting an  $n$ -th root of unity for  $\varepsilon$  in  $N$  yields  $M$ .

### 1.5.9 InternalHomToTensorProductAdjunctionMapTemp (for IsVectorSpaceObject, IsVectorSpaceObject, IsVectorSpaceMorphism)

▷ `InternalHomToTensorProductAdjunctionMapTemp( $b, c, g$ )` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, c)$ .

The arguments are objects  $b, c$  and a morphism  $g : a \rightarrow \underline{\text{Hom}}(b, c)$ . The output is a morphism  $f : a \otimes b \rightarrow c$  corresponding to  $g$  under the tensor hom adjunction.



### 1.5.10 HomalgMatrixAsString (for IsHomalgMatrix)

▷ HomalgMatrixAsString( $M$ ) (operation)

**Returns:** a string

The argument is a homalg matrix  $M$ . The output is a string consisting of the elements of  $M$ , separated by commas.

### 1.5.11 DataFromSkeletalFunctorTensorDataAsStringList (for IsList)

▷ DataFromSkeletalFunctorTensorDataAsStringList( $l$ ) (operation)

**Returns:** a list of strings and empty entries

The argument is a list  $l$  of homalg matrices. In  $l$ , empty entries are allowed. The output is a list where each non-empty entry of  $l$  is converted to a string using HomalgMatrixAsString.

### 1.5.12 AsVectorSpaceMorphism (for IsHomalgMatrix)

▷ AsVectorSpaceMorphism( $M$ ) (attribute)

**Returns:** a vector space morphism

The argument is a homalg matrix  $M$ . The output is a vector space morphism whose underlying matrix is given by  $M$ .

### 1.5.13 CreateEndomorphismFromString (for IsVectorSpaceObject, IsString)

▷ CreateEndomorphismFromString( $V$ ,  $s$ ) (operation)

**Returns:** a vector space morphism

The arguments are a vector space object  $V$  and a string  $s$  consisting of  $\dim(V)^2$  elements of the ground field of  $V$ . The output is a vector space endomorphism  $V \rightarrow V$  defined by  $s$ .

## Chapter 2

# Semisimple Categories

### 2.1 Introduction

Let  $k$  be a field and  $I$  be a totally ordered set. We denote the matrix category of  $k$  by  $k\text{-vec}$  (see the package `LinearAlgebraForCAP`). The semisimple category  $\bigoplus_{i \in I} k\text{-vec}$  associated to  $k$  and  $I$  is defined as the full subcategory of the product category  $\prod_{i \in I} k\text{-vec}$  generated by those  $I$ -indexed tuples having only finitely many non-zero entries. By  $\chi^i$ , we denote the object which is 1 at entry  $i$  and 0 otherwise. Thus, an arbitrary object in  $\bigoplus_{i \in I} k\text{-vec}$  can be written as  $\bigoplus_{i \in I} a_i \chi^i$  for non-negative numbers  $a_i$  for which only finitely many are non-zero.

### 2.2 Constructors

#### 2.2.1 SemisimpleCategory (for IsFieldForHomalg, IsFunction, IsObject, IsString, Is-Bool, IsList)

▷ `SemisimpleCategory( $k, m, u, s, b, L$ )` (operation)

**Returns:** a category

The arguments are:

- a homalg field  $k$ ,
- a membership function  $m$  sending any GAP object to a boolean,
- a GAP object  $u$ ,
- a string  $s$  containing a filename in the folder `"/gap/AssociatorsDatabase/"` of this package,
- a boolean  $b$ ,
- a list  $L$  containing 4 entries, where the first 3 are filters and the last one is a string.

The output is a CAP category modelling  $\bigoplus_{i \in I} k\text{-vec}$ , where  $I$  is the set defined by the membership function  $m$ . Note that objects in  $I$  are expected to be equipped with operations enlisted in the chapter "Irreducible Objects". Furthermore, this CAP category is a rigid symmetric closed monoidal Abelian category. Its tensor product is defined by the data of the file  $s$ , where the boolean  $b$  is true if the associator stored in  $s$  was computed for all triples, and false otherwise (cf. chapter "Associators"). Its braiding and duality comes from the additional structure required for  $I$ . Its tensor unit is modelled by

$u$ . The three filters of the  $L$  are filters for the resulting category, its objects, and its morphisms.  $L_4$  is the name of the resulting category.

### 2.2.2 SemisimpleCategory (for IsFieldForHomalg, IsFunction, IsObject, IsString, Is-Bool)

▷ SemisimpleCategory( $k, m, u, s, b$ ) (operation)

**Returns:** a category

The arguments are:

- a homalg field  $k$ ,
- a membership function  $m$  sending any GAP object to a boolean,
- a GAP object  $u$ ,
- a string  $s$  containing a filename in the folder "/gap/AssociatorsDatabase/" of this package,
- a boolean  $b$ .

This function calls SemisimpleCategory on the six arguments [  $k, m, u, s, b, [ \text{IsObject}, \text{IsObject}, \text{IsObject}, \text{automatically generated name} ] ]$

### 2.2.3 SemisimpleCategoryMorphism (for IsSemisimpleCategoryObject, IsList, IsSemisimpleCategoryObject)

▷ SemisimpleCategoryMorphism( $s, L, r$ ) (operation)

**Returns:** a morphism

The arguments are an object  $s$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ , a list of pairs  $L = [[\phi_1, i_1], \dots, [\phi_l, i_l]]$  where  $\phi_j$  are morphisms in the Matrix Category  $k\text{-vec}$  and  $i_j \in I$ , and another object  $r$  in the same semisimple category. The output is a morphism in  $\bigoplus_{i \in I} k\text{-vec}$  from  $s$  to  $r$  whose  $i$ -th component is given by  $\phi_i$ . For this morphism to be well defined, there has to be an  $\phi_i$  for every  $i$  in the support of  $s$  and  $r$ .

### 2.2.4 SemisimpleCategoryMorphismSparse (for IsSemisimpleCategoryObject, IsList, IsSemisimpleCategoryObject)

▷ SemisimpleCategoryMorphismSparse( $s, L, r$ ) (operation)

**Returns:** a morphism

The arguments are an object  $s$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ , a list of pairs  $L = [[\phi_1, i_1], \dots, [\phi_l, i_l]]$  where  $\phi_j$  are morphisms in the Matrix Category  $k\text{-vec}$  and  $i_j \in I$ , and another object  $r$  in the same semisimple category. The output is a morphism in  $\bigoplus_{i \in I} k\text{-vec}$  from  $s$  to  $r$  whose  $i_j$ -th component is given by  $\phi_{i_j}$  for  $j = 1, \dots, l$ , and by the zero morphism otherwise.

### 2.2.5 ComponentInclusionMorphism (for IsSemisimpleCategoryObject, IsObject)

▷ ComponentInclusionMorphism( $v, j$ ) (operation)

**Returns:** a morphism

The arguments are an object  $v = \bigoplus_{i \in I} a_i \chi^i$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ , and an object  $j \in I$ . The output is the canonical inclusion  $a_j \chi^j \hookrightarrow \bigoplus_{i \in I} a_i \chi^i$  in  $\bigoplus_{i \in I} k\text{-vec}$ .

### 2.2.6 ComponentProjectionMorphism (for IsSemisimpleCategoryObject, IsObject)

▷ `ComponentProjectionMorphism( $v$ ,  $j$ )` (operation)

**Returns:** a morphism

The arguments are an object  $v = \bigoplus_{i \in I} a_i \chi^i$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ , and an object  $j \in I$ . The output is the canonical projection  $\bigoplus_{i \in I} a_i \chi^i \rightarrow a_j \chi^j$  in  $\bigoplus_{i \in I} k\text{-vec}$ .

### 2.2.7 SemisimpleCategoryObject (for IsList, IsCapCategory)

▷ `SemisimpleCategoryObject( $L$ ,  $C$ )` (operation)

**Returns:** an object

The arguments are a list  $L$  and a semisimple category  $C = \bigoplus_{i \in I} k\text{-vec}$ . The list  $L$  contains pairs  $L = [[a_1, i_1], \dots, [a_l, i_l]]$  of non-negative integers  $a_j$  and objects  $i_j \in I$ . The output is the object in  $C$  given by  $\bigoplus_{j=1}^l a_j \chi^{i_j}$ .

### 2.2.8 SemisimpleCategoryObjectConstructorWithFlatList (for IsList, IsCapCategory)

▷ `SemisimpleCategoryObjectConstructorWithFlatList( $L$ ,  $C$ )` (operation)

**Returns:** an object

The arguments are a list  $L$  and a semisimple category  $C = \bigoplus_{i \in I} k\text{-vec}$ . The list  $L$  contains an even number of elements  $L = [a_1, i_1, \dots, a_l, i_l]$  of non-negative integers  $a_j$  and objects  $i_j \in I$ . The output is the object in  $C$  given by  $\bigoplus_{j=1}^l a_j \chi^{i_j}$ .

## 2.3 Attributes

### 2.3.1 MembershipFunctionForSemisimpleCategory (for IsSemisimpleCategory)

▷ `MembershipFunctionForSemisimpleCategory( $C$ )` (attribute)

**Returns:** a function

The argument is a semisimple category  $C = \bigoplus_{i \in I} k\text{-vec}$ . The output is its underlying membership function  $m$  for  $I$ .

### 2.3.2 UnderlyingCategoryForSemisimpleCategory (for IsSemisimpleCategory)

▷ `UnderlyingCategoryForSemisimpleCategory( $C$ )` (attribute)

**Returns:** a category

The argument is a semisimple category  $C = \bigoplus_{i \in I} k\text{-vec}$ . The output is its underlying category  $k\text{-vec}$ .

### 2.3.3 UnderlyingFieldForHomalgForSemisimpleCategory (for IsSemisimpleCategory)

▷ `UnderlyingFieldForHomalgForSemisimpleCategory( $C$ )` (attribute)

**Returns:** a homalg field

The argument is a semisimple category  $C = \bigoplus_{i \in I} k\text{-vec}$ . The output is its underlying field  $k$ .

### 2.3.4 GivenObjectFilterForSemisimpleCategory (for IsSemisimpleCategory)

▷ `GivenObjectFilterForSemisimpleCategory(C)` (attribute)

**Returns:** a filter

The argument is a semisimple category  $C = \bigoplus_{i \in I} k\text{-vec}$ . The output is its object filter which could be specified in the constructor of  $C$ .

### 2.3.5 GivenMorphismFilterForSemisimpleCategory (for IsSemisimpleCategory)

▷ `GivenMorphismFilterForSemisimpleCategory(C)` (attribute)

**Returns:** a filter

The argument is a semisimple category  $C = \bigoplus_{i \in I} k\text{-vec}$ . The output is its morphism filter which could be specified in the constructor of  $C$ .

### 2.3.6 SemisimpleCategoryMorphismList (for IsSemisimpleCategoryMorphism)

▷ `SemisimpleCategoryMorphismList(alpha)` (attribute)

**Returns:** a list

The argument is a morphism  $\alpha = (\alpha_i)_{i \in I}$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ . The output is the list of pairs  $[[\alpha_{i_1}, i_1], \dots, [\alpha_{i_l}, i_l]]$  where  $i_j$  ranges through the support of the source and range of  $\alpha$ .

### 2.3.7 UnderlyingFieldForHomalg (for IsSemisimpleCategoryMorphism)

▷ `UnderlyingFieldForHomalg(alpha)` (attribute)

**Returns:** a homalg field

The argument is a morphism  $\alpha = (\alpha_i)_{i \in I}$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ . The output is the homalg field  $k$ .

### 2.3.8 SemisimpleCategoryObjectList (for IsSemisimpleCategoryObject)

▷ `SemisimpleCategoryObjectList(v)` (attribute)

**Returns:** a list

The argument is an object  $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$  in a semisimple category. The output is the list  $[[a_1, i_1], \dots, [a_l, i_l]]$ .

### 2.3.9 SemisimpleCategoryObjectListWithActualObjects (for IsSemisimpleCategory-Object)

▷ `SemisimpleCategoryObjectListWithActualObjects(v)` (attribute)

**Returns:** a list

The argument is an object  $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$  in a semisimple category. The output is the list  $[[a_1, \chi^{i_1}], \dots, [a_l, \chi^{i_l}]]$ .

### 2.3.10 Support (for IsSemisimpleCategoryObject)

▷ `Support(v)` (attribute)

**Returns:** a list

The argument is an object  $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$  in a semisimple category. The output is the list  $[i_1, \dots, i_l]$ .

### 2.3.11 UnderlyingFieldForHomalg (for IsSemisimpleCategoryObject)

▷ `UnderlyingFieldForHomalg(v)` (attribute)

**Returns:** a homalg field

The argument is an object  $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ . The output is the homalg field  $k$ .

### 2.3.12 Dimension (for IsSemisimpleCategoryObject)

▷ `Dimension(v)` (attribute)

**Returns:** an integer

The argument is an object  $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ . The output is the integer  $\sum_{j=1}^l a_j \cdot \dim(i_j)$ . This functions works under the assumption that there is a notion of dimension on the objects in  $I$ .

## 2.4 Operations

### 2.4.1 Component (for IsSemisimpleCategoryMorphism, IsObject)

▷ `Component(alpha, i)` (operation)

**Returns:** a vector space morphism

The argument is a morphism  $\alpha = (\alpha_i)_{i \in I}$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$  and an object  $i \in I$ . The output is  $\alpha_i$ .

### 2.4.2 LaTeXStringOp (for IsSemisimpleCategoryMorphism)

▷ `LaTeXStringOp(m)` (operation)

**Returns:** a string

The argument is a morphism  $m$  in a semisimple category. The output is a LaTeX string (without enclosing dollar signs) that may be used to print out  $m$  nicely.

### 2.4.3 NormalizeSemisimpleCategoryObjectList (for IsList)

▷ `NormalizeSemisimpleCategoryObjectList(L)` (operation)

**Returns:** a list

The argument is a list  $L = [[a_1, i_1], \dots, [a_l, i_l]]$  of non-negative integers  $a_j$  and objects  $i_j \in I$ , where  $I$  correspond to irreducible objects in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ . The output is again a list of pairs consisting of integers and elements in  $I$ , but with the following normalization:

- Each  $a_j$  is positive,
- $i_j$  is strictly less than  $i_{j+1}$ .

### 2.4.4 Multiplicity (for IsSemisimpleCategoryObject, IsObject)

▷ `Multiplicity(v, i)` (operation)

**Returns:** an integer

The arguments are an object  $v = \bigoplus_{j=1}^l a_j \chi^{i_j}$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ , and an element  $i \in I$ . The output is the integer  $a_i$ .

### 2.4.5 Component (for IsSemisimpleCategoryObject, IsObject)

▷ `Component(v, i)` (operation)

**Returns:** a vector space object

The arguments are an object  $v = \bigoplus_{j=1}^I a_j \chi^{ij}$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ , and an element  $i \in I$ . The output is the  $k$ -vector space object  $k^{a_i}$  in Cap's Matrix Category.

### 2.4.6 TestPentagonIdentity (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `TestPentagonIdentity(v_1, v_2, v_3, v_4)` (operation)

**Returns:** a boolean

This is a debug operation. The arguments are 4 objects  $v_1, v_2, v_3, v_4$  in a category. The output is true if the pentagon identity holds for those 4 objects, false otherwise.

### 2.4.7 TestPentagonIdentityForAllQuadruplesInList (for IsList)

▷ `TestPentagonIdentityForAllQuadruplesInList(L)` (operation)

**Returns:** a boolean

This is a debug operation. The argument is a list  $L$  consisting of quadruples of objects in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ . The output is true if the pentagon identity holds for all those quadruples, false otherwise.

### 2.4.8 TestBraidingCompatability (for IsSemisimpleCategoryObject, IsSemisimpleCategoryObject, IsSemisimpleCategoryObject)

▷ `TestBraidingCompatability(v_1, v_2, v_3)` (operation)

**Returns:** a boolean

This is a debug operation. The arguments are 3 objects  $v_1, v_2, v_3$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ . The output is true if the braiding compatibilities with the associator hold, false otherwise.

### 2.4.9 TestBraidingCompatabilityForAllTriplesInList (for IsList)

▷ `TestBraidingCompatabilityForAllTriplesInList(L)` (operation)

**Returns:** a boolean

This is a debug operation. The argument is a list  $L$  consisting of triples of objects in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ . The output is true if the braiding compatibilities with the associator hold for all those triples false otherwise.

### 2.4.10 TestZigZagIdentitiesForDual (for IsSemisimpleCategoryObject)

▷ `TestZigZagIdentitiesForDual(v)` (operation)

**Returns:** a boolean

This is a debug operation. The argument is an object  $v$  in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ . The output is true if the zig zag identity for duals hold, false otherwise.

### 2.4.11 TestZigZagIdentitiesForDualForAllObjectsInList (for IsList)

▷ TestZigZagIdentitiesForDualForAllObjectsInList( $L$ ) (operation)

**Returns:** a boolean

This is a debug operation. The argument is a list  $L$  consisting of objects in a semisimple category  $\bigoplus_{i \in I} k\text{-vec}$ . The output is true if the zig zag identity for duals hold for all those objects, false otherwise.

### 2.4.12 LaTeXStringOp (for IsSemisimpleCategoryObject)

▷ LaTeXStringOp( $c$ ) (operation)

**Returns:** a string

The argument is an object  $c$  in a semisimple category. The output is a LaTeX string (without enclosing dollar signs) that may be used to print out  $c$  nicely.

## 2.5 GAP Categories

### 2.5.1 IsSemisimpleCategoryMorphism (for IsCapCategoryMorphism and IsCellOfSkeletalCategory)

▷ IsSemisimpleCategoryMorphism( $object$ ) (filter)

**Returns:** true or false

The GAP category of morphisms in a semisimple category.

### 2.5.2 IsSemisimpleCategoryObject (for IsCapCategoryObject and IsCellOfSkeletalCategory)

▷ IsSemisimpleCategoryObject( $object$ ) (filter)

**Returns:** true or false

The GAP category of objects in a semisimple category.



## Chapter 3

# Irreducible Objects

### 3.1 Introduction

For a semisimple category  $C$  of the form  $\bigoplus_{i \in I} k\text{-vec}$  to become a rigid symmetric closed monoidal skeletal category, the set  $I$  has to be equipped with extra structure. To become a skeletal category, we need:

- a total ordering on  $I$ .

To become a monoidal category, we need:

- a function `IsYieldingIdentities`, deciding whether an object yields the identity whenever it is part of an associator triple or a braiding pair,
- functions `Multiplicity` and `*`, defining the tensor product on objects,
- a function `AssociatorFromData`, defining the tensor product on morphisms.

To become a symmetric monoidal category, we need:

- a function `ExteriorPower`.

To become a rigid symmetric monoidal category, we need:

- a function `Dual`, defining duals on objects.

In the following, two families of such sets  $I$  are introduced:

- `GIrreducibleObject`: For a group  $G$ , the set  $I$  consists of the irreducible characters of  $G$ . We call the elements in  $I$  the  $G$ -irreducible objects.
- `GZGradedIrreducibleObject`: For a group  $G$ , the set  $I$  consists of the irreducible characters of  $G$  together with a degree  $n \in \mathbb{Z}$ . We call the elements in  $I$  the  $G - \mathbb{Z}$ -irreducible objects.

### 3.2 Constructors

#### 3.2.1 `GIrreducibleObject` (for `IsCharacter`)

▷ `GIrreducibleObject(c)` (attribute)

**Returns:** a  $G$ -irreducible object

The argument is a character  $c$  of a group. The output is its associated  $G$ -irreducible object.

### 3.2.2 GZGradedIrreducibleObject (for IsInt, IsCharacter)

▷ `GZGradedIrreducibleObject( $n$ ,  $c$ )` (operation)

**Returns:** a  $G - \mathbb{Z}$ -irreducible object

The argument is an integer  $n$  and a character  $c$  of a group. The output is their associated  $G - \mathbb{Z}$ -irreducible object.

## 3.3 Attributes

### 3.3.1 UnderlyingCharacter (for IsGIrreducibleObject)

▷ `UnderlyingCharacter( $i$ )` (attribute)

**Returns:** an irreducible character

The argument is a  $G$ -irreducible object  $i$ . The output is its underlying character.

### 3.3.2 UnderlyingGroup (for IsGIrreducibleObject)

▷ `UnderlyingGroup( $i$ )` (attribute)

**Returns:** a group

The argument is a  $G$ -irreducible object  $i$ . The output is its underlying group.

### 3.3.3 UnderlyingCharacterTable (for IsGIrreducibleObject)

▷ `UnderlyingCharacterTable( $i$ )` (attribute)

**Returns:** a character table

The argument is a  $G$ -irreducible object  $i$ . The output is the character table of its underlying group.

### 3.3.4 UnderlyingIrreducibleCharacters (for IsGIrreducibleObject)

▷ `UnderlyingIrreducibleCharacters( $i$ )` (attribute)

**Returns:** a list

The argument is a  $G$ -irreducible object  $i$ . The output is a list consisting of the irreducible characters of its underlying group.

### 3.3.5 UnderlyingCharacterNumber (for IsGIrreducibleObject)

▷ `UnderlyingCharacterNumber( $i$ )` (attribute)

**Returns:** an integer

The argument is a  $G$ -irreducible object  $i$ . The output is the integer  $n$  such that the  $n$ -th entry of the list of the underlying irreducible characters is the underlying irreducible character of  $i$ .

### 3.3.6 Dimension (for IsGIrreducibleObject)

▷ `Dimension( $i$ )` (attribute)

**Returns:** an integer

The argument is a  $G$ -irreducible object  $i$ . The output is the dimension of its underlying irreducible character.

### 3.3.7 Dual (for IsGIrreducibleObject)

▷ `Dual(i)` (attribute)

**Returns:** a  $G$ -irreducible object

The argument is a  $G$ -irreducible object  $i$ . The output is the  $G$ -irreducible object associated to the dual character of  $c$ , where  $c$  is the associated character of  $i$ .

### 3.3.8 UnderlyingCharacter (for IsGZGradedIrreducibleObject)

▷ `UnderlyingCharacter(i)` (attribute)

**Returns:** an irreducible character

The argument is a  $G - \mathbb{Z}$ -irreducible object  $i$ . The output is its underlying character.

### 3.3.9 UnderlyingDegree (for IsGZGradedIrreducibleObject)

▷ `UnderlyingDegree(i)` (attribute)

**Returns:** an integer

The argument is a  $G - \mathbb{Z}$ -irreducible object  $i$ . The output is its underlying degree.

### 3.3.10 UnderlyingGroup (for IsGZGradedIrreducibleObject)

▷ `UnderlyingGroup(i)` (attribute)

**Returns:** a group

The argument is a  $G - \mathbb{Z}$ -irreducible object  $i$ . The output is its underlying group.

### 3.3.11 UnderlyingCharacterTable (for IsGZGradedIrreducibleObject)

▷ `UnderlyingCharacterTable(i)` (attribute)

**Returns:** a character table

The argument is a  $G - \mathbb{Z}$ -irreducible object  $i$ . The output is the character table of its underlying group.

### 3.3.12 UnderlyingIrreducibleCharacters (for IsGZGradedIrreducibleObject)

▷ `UnderlyingIrreducibleCharacters(i)` (attribute)

**Returns:** a list

The argument is a  $G - \mathbb{Z}$ -irreducible object  $i$ . The output is a list consisting of the irreducible characters of its underlying group.

### 3.3.13 UnderlyingCharacterNumber (for IsGZGradedIrreducibleObject)

▷ `UnderlyingCharacterNumber(i)` (attribute)

**Returns:** an integer

The argument is a  $G - \mathbb{Z}$ -irreducible object  $i$ . The output is the integer  $n$  such that the  $n$ -th entry of the list of the underlying irreducible characters is the underlying irreducible character of  $i$ .

### 3.3.14 Dimension (for IsGZGradedIrreducibleObject)

▷ `Dimension(i)` (attribute)

**Returns:** an integer

The argument is a  $G - \mathbb{Z}$ -irreducible object  $i$ . The output is the dimension of its underlying irreducible character.

### 3.3.15 Dual (for IsGZGradedIrreducibleObject)

▷ `Dual(i)` (attribute)

**Returns:** a  $G - \mathbb{Z}$ -irreducible object

The argument is a  $G - \mathbb{Z}$ -irreducible object  $i$ . The output is the  $G - \mathbb{Z}$ -irreducible object associated to the degree  $-n$  and the dual character of  $c$ , where  $n$  is the underlying degree and  $c$  is the underlying character of  $i$ .

## 3.4 Properties

### 3.4.1 IsYieldingIdentities (for IsGIrreducibleObject)

▷ `IsYieldingIdentities(i)` (property)

**Returns:** a boolean

The argument is a  $G$ -irreducible object  $i$ . The output is true if the underlying character of  $i$  is the trivial one, false otherwise.

### 3.4.2 IsYieldingIdentities (for IsGZGradedIrreducibleObject)

▷ `IsYieldingIdentities(i)` (property)

**Returns:** a boolean

The argument is a  $G - \mathbb{Z}$ -irreducible object  $i$ . The output is true if the underlying character of  $i$  is the trivial one, false otherwise.

## 3.5 Operations

### 3.5.1 Multiplicity (for IsGIrreducibleObject, IsGIrreducibleObject, IsGIrreducibleObject)

▷ `Multiplicity(i, j, k)` (operation)

**Returns:** an integer

The arguments are 3  $G$ -irreducible objects  $i, j, k$ . Let their underlying characters be denoted by  $a, b, c$ , respectively. Then the output is the number  $\langle a, b \cdot c \rangle$ , i.e., the multiplicity of  $a$  in the product of characters  $b \cdot c$ .

### 3.5.2 \\* (for IsGIrreducibleObject, IsGIrreducibleObject)

▷ `\*(i, j)` (operation)

**Returns:** a list

The arguments are 2  $G$ -irreducible objects  $i, j$  with underlying irreducible characters  $a, b$ , respectively. The output is a list  $L = [[n_1, k_1], \dots, [n_l, k_l]]$  consisting of positive integers  $n_c$  and  $G$ -irreducible objects  $k_c$  representing the character decomposition into irreducibles of the product  $a \cdot b$ .

### 3.5.3 AssociatorFromData (for IsGIrreducibleObject, IsGIrreducibleObject, IsGIrreducibleObject, IsList, IsFieldForHomalg, IsList)

▷ `AssociatorFromData(i, j, k, A, F, L)` (operation)

**Returns:** a list

The arguments are

- three  $G$ -irreducible objects  $i, j, k$ ,
- a list  $A$  containing the associator on all irreducibles as strings, e.g., the list constructed by the methods provided in this package,
- a homalg field  $F$ ,
- a list  $L = [[n_1, h_1], \dots, [n_l, h_l]]$  consisting of positive integers  $n_c$  and  $G$ -irreducible objects  $h_c$  representing the character decomposition into irreducibles of the product of  $i, j, k$ .

The output is the list  $[[\alpha_{h_1}, h_1], \dots, [\alpha_{h_l}, h_l]]$ , where  $\alpha_{h_c}$  is the  $F$ -vector space homomorphism representing the  $h_c$ -th component of the associator of  $i, j, k$ .

### 3.5.4 ExteriorPower (for IsGIrreducibleObject, IsGIrreducibleObject)

▷ `ExteriorPower(i, j)` (operation)

**Returns:** a list

The arguments are two  $G$ -irreducible objects  $i, j$ . The output is the empty list if  $i$  is not equal to  $j$ . Otherwise, the output is a list  $L = [[n_1, k_1], \dots, [n_l, k_l]]$  consisting of positive integers  $n_j$  and  $G$ -irreducible objects  $k_j$ , corresponding to the decomposition of the second exterior power character  $\wedge^2 c$  into irreducibles. Here,  $c$  is the associated character of  $i$ .

### 3.5.5 Multiplicity (for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject)

▷ `Multiplicity(i, j, k)` (operation)

**Returns:** an integer

The arguments are 3  $G - \mathbb{Z}$ -irreducible objects  $i, j, k$ . Let their underlying characters be denoted by  $a, b, c$ , respectively, and their underlying degrees by  $n_i, n_j, n_k$ , respectively. The output is 0 if  $n_i$  is not equal to  $n_j + n_k$ . Otherwise, the output is the number  $\langle a, b \cdot c \rangle$ , i.e., the multiplicity of  $a$  in the product of characters  $b \cdot c$ .

Let their underlying characters be denoted by  $a, b$ , respectively, and their underlying degrees by  $n_i, n_j$ , respectively. if  $n_i = n_j$  and the underlying character number of  $j$

### 3.5.6 \\* (for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject)

▷ `\*(i, j)` (operation)

**Returns:** a list

The arguments are 2  $G - \mathbb{Z}$ -irreducible objects  $i, j$  with underlying irreducible characters  $a, b$ , respectively. The output is a list  $L = [[n_1, k_1], \dots, [n_l, k_l]]$  consisting of positive integers  $n_c$  and  $G$ -irreducible objects  $k_c$  representing the character decomposition into irreducibles of the product  $a \cdot b$ . The underlying degrees of  $k_c$  are given by the sum of the underlying degrees of  $i$  and  $j$ .

### 3.5.7 AssociatorFromData (for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject, IsList, IsFieldForHomalg, IsList)

▷ AssociatorFromData( $i, j, k, A, F, L$ ) (operation)

**Returns:** a list

The arguments are

- three  $G - \mathbb{Z}$ -irreducible objects  $i, j, k$ ,
- a list  $A$  containing the associator on all irreducibles (of  $G$ -irreducible objects) as strings, e.g., the list constructed by the methods provided in this package,
- a homalg field  $F$ ,
- a list  $L = [[n_1, h_1], \dots, [n_l, h_l]]$  consisting of positive integers  $n_c$  and  $G - \mathbb{Z}$ -irreducible objects  $h_c$  representing the character decomposition into irreducibles of the product of  $i, j, k$ .

The output is the list  $[[\alpha_{h_1}, h_1], \dots, [\alpha_{h_l}, h_l]]$ , where  $\alpha_{h_c}$  is the  $F$ -vector space homomorphism representing the  $h_c$ -th component of the associator of  $i, j, k$ .

### 3.5.8 ExteriorPower (for IsGZGradedIrreducibleObject, IsGZGradedIrreducibleObject)

▷ ExteriorPower( $i, j$ ) (operation)

**Returns:** a list

The arguments are two  $G - \mathbb{Z}$ -irreducible objects  $i, j$ . The output is the empty list if the underlying characters of  $i$  and  $j$  are unequal. Otherwise, the output is a list  $L = [[n_1, k_1], \dots, [n_l, k_l]]$  consisting of positive integers  $n_j$  and  $G - \mathbb{Z}$ -irreducible objects  $k_a$ , corresponding to the decomposition of the second exterior power character  $\wedge^2 c$  into irreducibles. Here,  $c$  is the associated character of  $i$  and  $j$ . The underlying degree of  $k_a$  is the sum of the underlying degrees of  $i$  and  $j$ .

## Chapter 4

# Representation Category of Groups

### 4.1 Introduction

For a finite group  $G$ , the following methods provide computational tools for working with  $G\text{-mod}$ , a skeletal version of the monoidal category of finite dimensional complex representations of  $G$ , and with  $G - \mathbb{Z}\text{-mod}$ , a skeletal version of the monoidal category of finite dimensional complex representations of  $G$  equipped with a degree in  $\mathbb{Z}$ .

### 4.2 Quickstart

The following commands construct the category  $D_8\text{-mod}$ , the unique object  $v$  corresponding to the irreducible character of degree 2, and perform some computations.

```
Example
gap> RepG := RepresentationCategory( 8, 3 );
The representation category of Group( [ f1, f2, f3 ] )
gap> G := UnderlyingGroupForRepresentationCategory( RepG );
<pc group of size 8 with 3 generators>
gap> StructureDescription( G );
"D8"
gap> c := First( Irr( G ), i -> Degree( i ) = 2 );
Character( CharacterTable( D8 ), [ 2, 0, 0, -2, 0 ] )
gap> v := RepresentationCategoryObject( c, RepG );
1*(x_5)
gap> Dimension( v );
2
gap> Display( AssociatorLeftToRight( v, v, v ) );
Component: (x_5)

1/2,-1/2,1/2, 1/2,
1/2,-1/2,-1/2,-1/2,
1/2,1/2, 1/2, -1/2,
1/2,1/2, -1/2,1/2

A morphism in Category of matrices over Q
-----
gap> Display( Braiding( v, v ) );
Component: (x_1)
```

```

1
A morphism in Category of matrices over Q
-----
Component: (x_2)

1
A morphism in Category of matrices over Q
-----
Component: (x_3)

1
A morphism in Category of matrices over Q
-----
Component: (x_4)

-1
A morphism in Category of matrices over Q
-----
gap> alpha := IdentityMorphism( TensorProductOnObjects( v, v ) ) + Braiding( v, v );
<A morphism in The representation category of Group( [ f1, f2, f3 ] )>
gap> CokernelObject( alpha );
1*(x_4)
gap> TensorUnit( RepG );
1*(x_1)

```

## 4.3 Constructors

### 4.3.1 RepresentationCategory (for IsGroup)

▷ RepresentationCategory( $G$ ) (attribute)

**Returns:** a Cap category

The argument is a group  $G$ . The output is the Cap category  $G$ -mod. This method uses String( $G$ ) as an identifier of  $G$ .

### 4.3.2 RepresentationCategory (for IsInt, IsInt)

▷ RepresentationCategory( $o, n$ ) (operation)

**Returns:** a Cap category

The arguments are 2 integers  $o, n$ . The output is the Cap category  $G$ -mod, where  $G$  is the group of order  $o$  corresponding to the SmallGroupLibrary identification number  $n$ .

### 4.3.3 RepresentationCategoryObject (for IsList, IsCapCategory)

▷ RepresentationCategoryObject( $L, C$ ) (operation)

**Returns:** an object in  $G$ -mod



There are 2 arguments. The first argument is a list  $L = [[n_1, c_1], \dots, [n_l, c_l]]$  consisting of non-negative integers  $n_i$  and characters  $c_i$  of the same group. Alternatively, the first argument can simply be an irreducible character  $c$ , which will be then interpreted as giving the input  $[[1, c]]$ . The second argument is the Cap category  $C = G\text{-mod}$ . The output is the unique object in  $G\text{-mod}$  having  $L$  as its character decomposition.

#### 4.3.4 RepresentationCategoryObject (for IsCharacter, IsCapCategory, IsString)

▷ `RepresentationCategoryObject(c, C, str)` (operation)

**Returns:** an object in  $G\text{-mod}$

There are 3 arguments. The first argument is an irreducible character  $c$ . The second argument is the CAP category  $C = G\text{-mod}$ . The third argument is a string used as follows: `SetString(GIrreducibleObject(c), str)`. The output is the unique object in  $G\text{-mod}$  having  $[[1, c]]$  as its character decomposition.

## Chapter 5

# Tools

### 5.1 Helper functions

#### 5.1.1 LaTeXStringOfSemisimpleCategoryObjectList

▷ `LaTeXStringOfSemisimpleCategoryObjectList(L)` (function)

**Returns:** a string

The argument is a list  $L$  defining an object  $c$  in a semisimple category. The output is a LaTeX string (without enclosing dollar signs) that may be used to print out  $c$  nicely.

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