CHAPTER 1

QUADRATIC EQUATIONS

An equation is an expression used to find unknown e. g x-a= 6

Quadratic equations are equations whose highest power (exponent or degree) is 2.

e. g $2x^2 - 2x - 3 = 0$, ax^2+bx+c are quadratic

FACTORISING QUADRATIC EQUATION

Factors are numbers which can be multiplied to give a product.

For example 12=4×3, in this case 4 and 3 are factors while 12 is a product

Example 1; factorize x^2+7x+6

Solution

In the expression, 6 is constant

We need **two** numbers whose **sum** is 7 and **product** is 6

The numbers we want are 1 and 6

So $x^2+7x+6 = x^2+x+6x+6$

$= x(x+1)\ 16(x+1)$

$= \underline{(x+1)\ (x+6)}$

Example 2,factorize $3x^2-8x-16$

Solution

$3x^2-8x-16$

3x16=48, so we need to find two factors that their summation should be **-8** and their product should be **48.**

Factors are -12 and 4,

$3x^2 - 12x + 4x - 16$

$3x(x - 4) + 4(x - 4)$

$(3x + 4)\ (x - 4)$

Example 3, factorize $6x^2 + x - 22$

Solution

$6x^2 + x - 22$

$6x^2 - 11x - 12x - 22$

$x(6x-11) + 2\ (6x-11)$

$(x + 2)\ (6x - 11)$

EXERCISE

Factorize the following

1. $x^2 + x - 12$
2. $2x^2 - 3x + 1$
3. $2x^2 + 17x + 30$
4. $x^2 - xy + 6y^2$

FACTOSING USING DIFFERENT OF TWO SQUARES

A square is a product found when you multiply the number by itself once.

e. g 25= 5 x 5
$x^2 = x \times x$

A general formula for factorizing using different of two squares.

$(A^2 - B^2) = (A+B)\ (A-B)$

EXAMPLES

1. Factorize; a^2-4
   Solution
   $a^2 - 2^2$
   A= a, B= 2
   $(a+2)\ (a-2)$
2. $4a^2 - 9$

Solution

$(2a)^2 - 3^2$

A=2a, B=3

$4a^2 - 9 = (2a+3)(2a-3)$

3. $2x^2-18y$

Solution

$2x^2-18y^2 = 2x^2-18y^2$

$= 2(x^2-(3y)^2)$

$A = x,\ B = 3y$

$2x^2-18y^2 = \mathbf{2(x+3y)(x-3y)}$

NB: Different of two cubes and sum of two cubes are applied sometimes. The general formula;

$A^3 - B^3 = (A-B)(A^2+AB+B^2)$

$A^3 + B^3 = (A+B)(A^2-AB+B^2)$

<u>EXAMPLE</u>

1. Factorize; $r^3-8 = r^3-2^3$

   $A=r,\ B=2$

   $r^3-8 = (r-2)(r^2+2r+2^2)$

   $= (r-2)(r^2+2r+4)$

2. Factorize; $1-27y^3 = 1^3-(3y)^3$

   $A=1,\ B=3y$

   $1-27y^3 = (1-3y)(1^2+3y+(3y)^2)$

   $= (1-3y)(1+3y+9y^2)$

<u>EXERCISE</u>

1. $16-y^2$
2. $49x^2-16$
3. $(x^2-x-8)^2-4(2x-1)^2$
4. x^3+1

<u>SOLVING QUADRATIC EQUATIONS BY FACTORIZATION</u>

To solve quadratic equations, we need to factorize first then equate each factor to zero.

EXAMPLES

1. Solve $x^2+3x=0$

   Solution

   $x^2+3x=0$

   $x(x+3)=0$

   $x=0$ or $x+3=0$

   $x=-3$

2. Solve $\mathbf{x^2-5x+6=0}$

   Solution

   $X^2-5x+6=0$

   $x^2-2x-3x+6=0$

   $x(x-2)-3(x-2)=0$

   $(x-2)(x-3)=0$

   $(x-2)=0$ or $(x-3)=0$

   $X=2$ $x=3$

3. Solve $3\mathbf{x^2-2x-8=0}$

   Solution

   $3x^2-2x-8=0$

   $3x^2-6x+4x-8=0$

   $3x(x-2)+4(x-2)=0$

   $(x-2)(3x+4)=0$

   $x-2=0$ or $3x+4=0$

   $x=2$ $3x=-4$

   $x=\frac{-4}{3}$

4. **Solve $x^2-4=0$**

   Solution

   $x^2-4=0=x^2-2^2$

   $=(x+2)(x-2)=0$

   $=x+2=0$ or $x-2=0$

   $x=-2$ $x=2$

5. Solve $\mathbf{2r^2-18=0}$

   Solution

   $2r^2-18=0$

   Divide by 2

   $r^2-9=0$

   $r^2-3^2=0$

   $(r-3)(r+3)=0$

   $r=3$ or $r=-3$

EXERCISE

Solve the following equations

1. $x^2 - 3x - 10 = 0$
2. $a^2 + 5a - 15 = 0$
3. $x(x + 4) = 0$
4. $4(x - 1)^2 = 9$

COMPLETING THE SQUARE

A square is a product we get after multiplying a number by itself once e.g

3 x 3 =9 or **6 x 6 = 36**

Completing the square refers to changing a quadratic equation to the form a $(x + b)^2 + c$.

When completing the square, multiply the coefficient of x by half, then add and subtract the result while being squared.

Brackets are introduced to prevent subtracting numbers.

EXAMPLE

Factorize the following by completing the square;

1. $x^2 - 4x + 12$
2. $3x^2 + 6x - 5$

Solutions

1. $x^2 - 4x + 12$

Divide -4 by half which is -2

$\{x^2 - 4x + (-2)^2\} - (-2)^2 + 12$

$(x - 2)^2 - 4 + 12$

$(x - 2)^2 + 8$

2. $3x^2 + 6x - 5$

$3\left(x^2 + 2x - \frac{5}{3}\right)$takeout coefficient x^2 3)

$3\{(x^2 + 2x + (1)^2\} - (1)^2 - \frac{5}{3}$

$3(x + 1)^2 - 1 - \frac{5}{3}$

$3(x + 1)^2 - \frac{3-5}{3}$

$3\left\{(x + 1)^2 - \frac{8}{3}\right\}$

$3(x + 1)^2 - \frac{8}{3} \times 3$

(To remove brackets multiply it with 3)

$\underline{\mathbf{3}\ (x + 1)^2 - 8}$

SOLVING BY COMPLETING THE SQUARE.

Solving by completing the square we factorize by completing the square first.

EXAMPLE

Solve the following by completing the square

1. $x^2 + 4x - 5 = 0$
2. $x^2 - 7x + 2 = 0$

Solutions

1. $x^2 + 4x - 5 = 0$

$\{(x^2 + 4x + (2)^2\} - (2)^2 - 5 = 0$

$(x + 4)^2 - 4 - 5 = 0$

$(x + 2)^2 - 9 = 0$

$(x + 2)^2 = 9$

$x + 2 = \pm\sqrt{9}$

$x = -2 + 3$ or $x = -2 - 3$

$x = 1$ $x = -5$

2. $\mathbf{x^2 - 7x + 2 = 0}$

$x^2 - 7x + \left(\frac{-7}{2}\right)^2 \left(\frac{-7}{2}\right)^2 + 2 = 0$

$(x - \frac{7}{2})^2 - \frac{49}{4} + 2 = 0$

$(x - \frac{7}{2})^2 - \frac{41}{4} = 0$

$(x - \frac{7}{2})^2 = \frac{41}{4}$

$x - \frac{7}{2} = \pm\sqrt{\frac{41}{4}}$

$x = \frac{7}{2} + \frac{\sqrt{41}}{2}$ **or** $x = \frac{7}{2} - \frac{\sqrt{41}}{2}$

$= \frac{7 + \sqrt{41}}{2}$ $\frac{7 - \sqrt{41}}{2}$

$= 6.70$ 0.30

EXERCISE

1. Factorize the following by completing the square
   a. $2x^2 + 8x + 3$
   b. $3x^2 + 5x + 11$
   c. $2y^2 - 7x + 4$
2. The following by completing the square
   a. $x^2 + 6x - 1 = 0$
   b. $4x^2 + 5x - 3= 0$
   c. $3a^2 - 7a + 7 =0$
   d. $x^2 + 36x - 36= 0$

SOLVING QUADRATIC EQUATIONS USING FORMULA

The general formula for solving quadratic equations is $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

EXAMPLE

1. $2x^2 + 3x - 5= 0$, solve using the formula leaving your answer to 2 decimal place.

Solution

$$2x^2 + 3x - 6 =0$$

a=2 b= 3 c= -6

$$x= \frac{-(3)\pm\sqrt{(3)^2-4(2)(-6)}}{2(2)}$$

$$= \frac{-3\pm\sqrt{9+48}}{4}$$

$$= \frac{-3 \pm \sqrt{57}}{4}$$

$$x= \frac{-3+7.55}{4} \text{ or } \frac{-3-7.55}{4}$$

$$= \frac{4.45}{4} \qquad = \frac{-10.55}{4}$$

$$= 1.14 \qquad = -2.64$$

2. Solve $3y^2- 4y -8 = 0$ using the formula, leaving the answer to 2 decimal place.

Solution

$$3y^2- 4y -8 = 0$$

A= 3 b= -4 c= - 8

$$y= \frac{-(-4)\pm\sqrt{(-4)^2-4(3)(-8)}}{2(3)}$$

$$y= \frac{4\pm\sqrt{16+96}}{6}$$

$$y= \frac{4\pm \sqrt{112}}{6}$$

$$y= \frac{4+10.56}{6} \text{ or } \frac{4-10.58}{6}$$

$$= \frac{14.56}{6} \qquad = \frac{-6.58}{6}$$

y = 2.43 or y= -1. 10

EXERCISE

Solve the following using the formula leaving your answer to 2 Dec. place

1. $x^2+7x-3=0$
2. $2x^2-7x-3=0$
3. $5x^2+8x-2=0$
4. $3x^2-12x+10=0$

Forming equations given its roots

EXAMPLES

1. The roots of quadratic equation are 4 and -3, formulate appropriate equation

Solution

The roots are 4 and -3

x= 4 or x= -3

x -4=0 or x +3 =0

(x -4) (x +3) =0

$x^2 - x - 12 = 0$

3. 5 is one root of the equation $x^2 + mx - 15$.find m and the other root

Solution

$x^2+mx-15=0$

$(5)^2+m(5) -15=0$

$25+5m-15=0$

$\frac{10}{-5} = -\frac{5m}{5}$

$m=-2$

So the equation is $x^2-2x-15=0$

$x^2-5x+3x-15=0$

$x(x-5)+3(x-5)=0$

$(x-5)(x+3)=0$

$x=5$ or $x=-3$

The second root is -3

EXERCISE

1. Find the simplest version 5of the Equation whose roots are $\frac{5}{7}$ and -1
2. If 3 is one roots of equation $x^2-mx+16=0$. Find **m** and the other root.
3. Find the value of **c** if one root of $3x^2-8x+c=0$ is three times the other.
4. The equation $2x^2-x+m=0$ has one root of $\frac{-3}{2}$**.** Find m and the other root.

<u>WORD PROBLEMS</u>

Translating the English into mathematical equation word problems is often quiet challenging.

Here is the list of some of the phrases and their mathematical translations.

ENGLISH	MATHEMATICS
Sum of	$+$
The difference between	$-$
The product of	$\times$
The quotient of	$\div$
The number increased by 3	X + 3
The number decreased by 3	X – 3
Reciprocal of a number	$\frac{1}{x}$
An even number	2x
Odd number	2x + 1
Two consecutive even or odd numbers	X and x + 2
Two consecutive numbers (They differ by one)	X and x+1 X and x – 1

NB:

i. **Even numbers** are numbers that can be exactly divided by two e.g 2, 4, 6 e. t. c
ii. **Odd numbers** are numbers that cannot be divided exactly by 2 e.g 1, 3, 5, 7, 9 e. t. c
iii. **Prime numbers** are numbers that can be dividend only by itself and 1 e.g 2, 3, 5, 7, 11 e.t. c

EXAMPLES

1. **Bertha's father** is six times as old as Bertha's age. The product of their ages is 150years. What are their respective ages.

Solution

Let **Bertha's** age be$\boldsymbol{x}$

∴ **Her father's** age is $6x$

$x \times 6x = 150$

$6\boldsymbol{x}^2 = 150$

$\boldsymbol{x}^2 = \frac{150}{6}$

$\boldsymbol{x} = 25$

$\boldsymbol{x} = \pm\sqrt{25}$

$\boldsymbol{x} = 5$ or -5

∴**Bertha's** age is 5 and his father's age is **6 × 5= 30**

2. The product of two consecutive even numbers is 168. Find the numbers

Solution

Let the smaller number be x

∴ The other number is $x + 2$

$x(x + 2) = 168$

$x^2 + 2x = 168$

$x^2 + 2x - 168 = 0$

$x^2 + 14x - 12x - 168 = 0$

$x(x + 14) - 12(x + 14) = 0$

$(x - 12)(x + 14) = 0$

$x - 12 = 0$ **or** $x + 14 = 0$

$x = 12$ $x = -14$

∴ The numbers are 12 and 14 or -12 and-14.

3. Eugenio's taxi is three years older than Geoffrey's taxi. In two years' time, the product of their ages of two taxis will be 54. How old is each taxi at present.

Solution

Let Geoffrey's taxi age be **y**

∴Eugenio's age taxi will be **y + 3**

In two years' time each taxi age will increase by **2**

Geoffrey's tax age in two years' time= **(y+2)yrs**

Eugenio's taxi age in two years =**(y+5)**

$(y+2)(y+5) = 54$

$y^2 + 7y + 10 = 54$

$y^2 + 7y - 44 = 0$

$y^2 + 11y - 4y - 44 = 0$

$y(y + 11) - 4(y + 11) = 0$

$(y - 4)(y + 11) = 0$

$y = 4$ or $y = -11$

∴ Geoffrey's taxi is 4years old and Eugenio's taxi is **4 + 3= 7** years old.

EXCERCISE

1. Lawrence is four times older than his brother Geoffrey. If the product of their ages is 96. What are their respective ages.
2. If **2n (n + 1)** and **(n - 1) (3n - 1)** are consecutive numbers. What numbers are they?
3. Find the consecutive old numbers such that the square of the smaller one added to the twice the square of greater is 211.
4. The sum of the square of two consecutive positive old numbers is 650. Find the numbers.

<u>SOLVING SIMULTANEOUS EQUATION</u>

Example

1. Solve $x^2 - y^2 = 16$
   $x + y = 8$

Solution

$x^2 - y^2 = 16$……………….i)

$x + y = 8$…………………ii)

$x = 8 - y$…………………iii)

Substitute $x = 8 - y$ in i)

$(8 - y)^2 - y^2 - = 16$

$64 - 16y + y^2 - y^2 = 16$

$64 - 16 = 16y$

$48 = 16y$

$y = 3$

Substitute y= 3 in iii)

$x = 8 - 3$

$= 5$

2. solve $x + y = 2$
   $x^2 - y^2 = 52$

Solution

$x + y = 2$……………………i)

$x^2 - y^2 = 52$……………………ii)

$x = 2 - y$……………………iii)

Substitute x = 2 – y in ii)

$(2 - y)^2 - y^2 = 52$

$4 - 4y + y^2 - y^2 = 52$

$-4y = 48$

$y = -12$

Substitute y = -12 in iii)

$x = 2 - (-12)$

$= 2 + 12$

$= 14$

EXERCISE

1. Solve $a - b = 5$, $ab = 6$
2. Solve $3x + 4y = 5$
   $x^2 + xy = y + 1$
3. Solve $y = 2(x + y - 1) = -2(x + y + 3)$
4. Solve $r + s = 2$
   $r^2 - s^2 - rs - s = 0$

CHAPTER 2

SURDS

A surd is any expression that involve a root.

MULTIPLICATION OF SURDS

When two or more surds are to be multiplied, they should be simplified first if possible, then whole numbers should be taken with whole numbers and surds with surds.

EXAMPLE

1. $\sqrt{5} \times \sqrt{10}$
2. $\sqrt{12} \times \sqrt{3}$
3. $(4\sqrt{3})^2$

NB; $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\sqrt{a} + \sqrt{b} \pm \sqrt{ab}$

Solutions

1. $\sqrt{5} \times \sqrt{10}$
$\sqrt{5} \times \sqrt{5} \times \sqrt{2}$

$\sqrt{25} \times \sqrt{2}$

$\underline{5\sqrt{2}}$

2. $\sqrt{12} \times \sqrt{3}$
$\sqrt{4} \times \sqrt{3} \times \sqrt{3}$
$2 \times \sqrt{9}$
2×3
$\underline{6}$

3. $(4\sqrt{3})^2$

$4^2 \times (\sqrt{3})^2$

$16 \times \sqrt{9}$

16×3

$\underline{\underline{48}}$

DIVISION OF SURDS

If a fraction has a surd in the denominator, it is usually best to rationalize the denominator. To rationalize the denominator means to make the denominator into a rational number. This is done by multiplying the numerator and the denominator of the fraction by a surd of the denominator.

EXAMPLES

1. **Simplify $\frac{\sqrt{18}}{\sqrt{2}}$**

Solution

$= \frac{\sqrt{18}}{\sqrt{2}} = \frac{\sqrt{18}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{36}}{4}$

$= \frac{6}{2}$

$= 3$

2. **Simplify $\frac{1}{\sqrt{2}}$**

Solution

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}}$

$= \frac{\sqrt{2}}{2}$

3. **Simplify $\frac{2\sqrt{3}}{\sqrt{6}}$**

Solution

$= \frac{2\sqrt{3}}{\sqrt{6}} = \frac{2\sqrt{3}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{2\sqrt{18}}{\sqrt{6}}$

$= \frac{2\sqrt{2}\times\sqrt{9}}{\sqrt{36}}$

$= \frac{2\sqrt{2}\times 3}{6}$

$= \sqrt{2}$

ADDITION AND SUBTRACTION OF SURDS

Usually we add or subtract like surds and only whole numbers

E. g $2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

$2\sqrt{2} - 3\sqrt{2} = -\sqrt{2}$

EXAMPLES

1. Simplify $\sqrt{12} + \sqrt{3}$

Solution

$= \sqrt{4} \times \sqrt{3} + \sqrt{3}$
$= 2\sqrt{3} + \sqrt{3}$

$= 3\sqrt{3}$

2. **Simplify $2\sqrt{150} - \sqrt{96} - \sqrt{216}$**

Solution

$= 2\sqrt{25} \times \sqrt{6} - \sqrt{16} - \sqrt{16} \times \sqrt{6} - \sqrt{36} \times \sqrt{6}$
$= 2 \times 5\sqrt{6} - 4\sqrt{6} - 6\sqrt{6}$
$= 10\sqrt{6} - 10\sqrt{6}$
$= 0$

3. **Simplify $\frac{\sqrt{3}}{1} + \frac{3}{\sqrt{3}} - \frac{1}{\sqrt{27}}$**

Solution

$= \frac{\sqrt{3}}{1} + \frac{3}{\sqrt{3}} - \frac{1}{\sqrt{27}}$

$= \frac{\sqrt{3} \times \sqrt{27} + \sqrt{9} - 1}{\sqrt{27}}$

$= \frac{\sqrt{81} + 3 - 1}{\sqrt{27}}$

$= \frac{9 + 2}{\sqrt{27}}$

$= \frac{11}{\sqrt{9}\, x\, \sqrt{3}}$

$$=\frac{11}{3\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{11\sqrt{3}}{3\ x\ \sqrt{9}}$$

$$=\frac{11\sqrt{3}}{9}$$

CONJUGATE SURDS

A pair of expressions in a surd form are said to be conjugate surds. E . g

$2\sqrt{2}$ - $3\sqrt{2}$, $3+\sqrt{2}$ and 3- $\sqrt{2}$ are examples of conjugate surds.

EXAMPLE

1. **Expand $(\sqrt{2}+\sqrt{5})\ (\sqrt{7}+\sqrt{2})$**

Solution

$$=\sqrt{14}+\sqrt{4}+\sqrt{35}+\sqrt{10}$$
$$=\sqrt{14}+2+\sqrt{35}+\sqrt{10}$$
$$=\sqrt{14}+\sqrt{35}+\sqrt{10}+2$$

2. **Expand $(\sqrt{6}+\sqrt{3})\ (\sqrt{6}+\sqrt{3})$**

Solution

$$=\sqrt{36}+\sqrt{18}+\sqrt{18}+\sqrt{9}$$
$$=6+2\sqrt{18}+3$$
$$=9+2\times\sqrt{9}\times\sqrt{2}$$
$$=9+2\times 3\sqrt{2}$$
$$=9+6\sqrt{2}$$
$$=3(3+2\sqrt{2})$$

RATIONALIZING CONJUGATE SURDS

When the denominator is a conjugate surd, multiply both the numerator and the denominator with the conjugate surds.

EXAMPLE

1. Rationalize $\frac{\mathbf{1}}{\mathbf{4+2\sqrt{7}}}$

Solution

$$=\frac{1}{4+2\sqrt{7}}\times\frac{4-2\sqrt{7}}{4-2\sqrt{7}}$$

$$=\frac{4-2\sqrt{7}}{16-4\ x\ 7}$$

$$=\frac{4-2\sqrt{7}}{16-28}$$

$$=\frac{2(2-\sqrt{7})}{-12}$$

$$=\frac{(2-\sqrt{7})}{-6}$$

$$=\frac{(\sqrt{7}-2)}{6}$$

EXERCISE

1. **Simplify**
   a. $(\sqrt{2})^2$
   b. $\sqrt{5}\times\sqrt{15}$
   c. $2\sqrt{54}$ - $\sqrt{24}-\sqrt{216}$
   d. $(2\sqrt{3}$ - $4\sqrt{2})\ (\sqrt{3}+2\sqrt{2})$
   e. $\sqrt{75}+\sqrt{48}-2\sqrt{27}$
   f. $\frac{11+\sqrt{5}}{3-\sqrt{2}}$
   g. $\frac{2+\sqrt{2}}{(2-\sqrt{2})}$
   h. $\frac{\sqrt{2}+2\sqrt{5}}{(\sqrt{5}-\sqrt{2})}$

CHAPTER 3

CIRCLE GEOMETRY

PARTS OF A CIRCLE

1. **Circumference**

It is the distance around the circle

2. **Diameter**

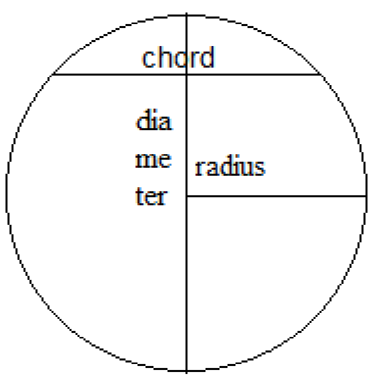
It is a line that divide a circle into two equal parts.

3. **Radius**

It is the half of the diameter

4. **Chord**

Is a line which joins two parts of the circle.



5. Arc

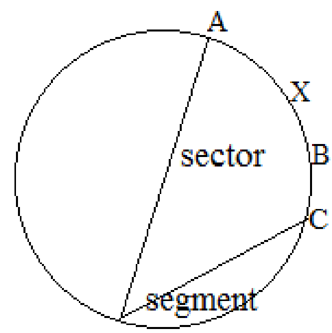
Part of circumference or fractional of the circumference.

6. Segment

Is an area between an arc and two radii.

7. A sector

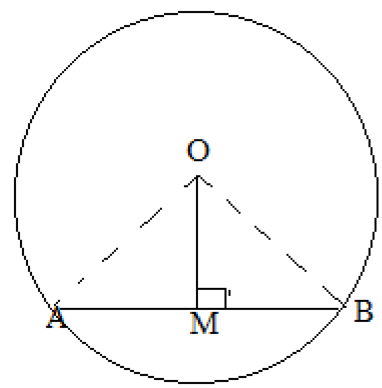
Is area between an arc and two radii



CIRLCLE PROPERTIES (PART ONE)

Theorems;

1. If a line is drawn from the center of the circle to the mid-point of the chord, it bisects the chord.
   (Reason, perpendicular bisector)



Given; circle Centre O, chord AB $\perp$OM

To prove; AM = BM

Construction: join A and B to O respectively

Proof; In Δs AMO and BMO

AO= BO, radii

MO is common

$\angle$AMO =$\angle$ BMO, given right $\angle s$

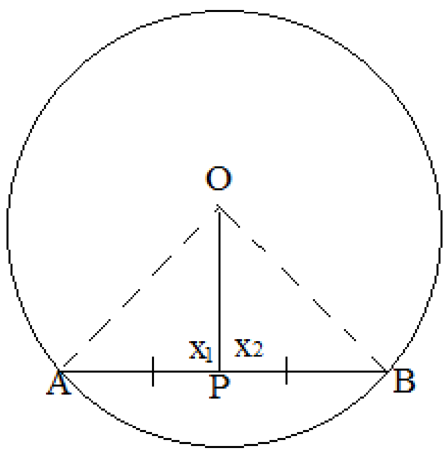
$\therefore$ ΔAMO $\equiv$ ΔBMO , RHS

$\therefore$ AM = BM

Theorem 2 converse

If a line is drawn from the circle to the midpoint of the chord, then that line is perpendicular to the chord

PROVE



Given; circle Centre O, AP=AB, mid-point P.

To prove; OP⊥AB

Construction = join O to A and B

Proof; in ΔOPA and ΔOPB

AO= BO, radii

OP is common

AP=BP, given

∴ ΔOPA ≡ ΔOPB

∴ $x_1 = x_2$

$X_1 + x_2 = 180^o$, adj $\angle s$ on a str. line

$2x = 180^0$

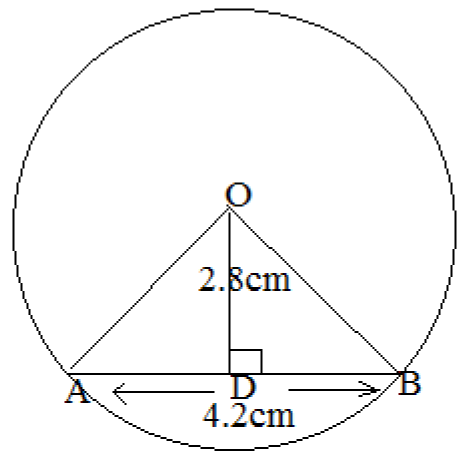
$x=90^0$

∴ $x_1 = x_2 = 90^0$

∴OP⊥AB

EXAMPLE

1. A chord 4.2cm long is 2.8cm from the Centre of the circle. Calculate radius of the circle.
   Solution



AD= BD, OD⊥bisector

$\therefore BD = \frac{4.2}{2}$

$= 2.1$cm

$BO^2 = OD^2 + DB^2$, Pythagoras theorem

$BO^2 = 2.8^2 + 2.1^2$

$BO^2 = 7.84 + 4.41$

$BO = \sqrt{12.25}$

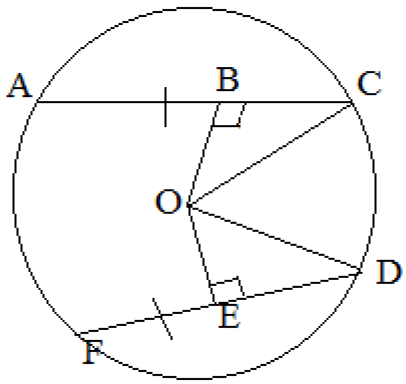
$= 3.5$cm

∴The radius is 3.5cm

Theorem 3

Equal chords are equidistant from the Centre of the circle.

PROVE



Given; Centre O, chords AC=FD, OB and OE are ⊥ *s*

To prove: OB=OE

Construction; join OC and OD

Proof; AC= FD, given

AB= BC, OB⊥bisector

FE= DE, OE⊥bisector

∴ BC= ED

In Δ**OBC** and Δ**OED**

OC=OD, given

BC= EB, proved

$\widehat{B}=\widehat{E}$, given right angles

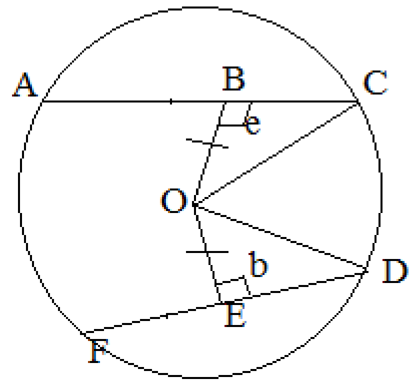
∴ Δ**OBC** ≡ Δ**OED**, (**RHS**)

∴OB=OE

Theorem 4

Chords which are equidistant from the Centre of the circle are equal to each other.

PROVE



Given; circle Centre O, equal chords, ⊥ *s* OB and OE, chords AC and FD.

To prove; AC= FD

Construction; join OC and OD

Proof: AB = BC, OB⊥ bisector

FE= BE, OE⊥ bisector

In Δ**OBC** and Δ**OED**

OB= OE, given

OC= OD, radii

e= d, given right angles

∴ Δ**BOC** ≡ Δ**OED**, (**RHS**)

∴BC= ED

∴AB= BC= FE= DE

AB+BC=AC

FE+ DE = FD

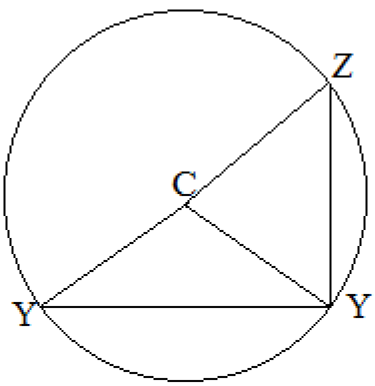
∴AC= FD

AC= FD

EXAMPLE

In a circle Centre C, if the chords XY, YZ are equal. Prove that XY bisects CY and $X\hat{Y}Z$.

Solution



Given; Centre C, XY=YZ

To prove; $X\hat{Y}C = Z\hat{Y}C$ or CY bisects$X\hat{Y}C$

Construction: join C to Z and X

Proof: in Δ**CXY** andΔ**CZY**

XY=ZY, given

CX= CZ, radii

CY is common

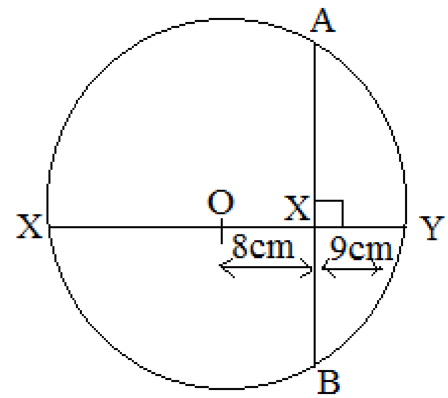
$\therefore$ Δ**CXY** $\equiv$ Δ**CZY** (**SSS**)

$\therefore X\hat{Y}C = Z\hat{Y}C$

$\therefore$ CY bisects$X\hat{Y}C$

EXERCISE

1. A chord 7cm long is drawn in a circle of radius 3.7cm long. Calculate distance of a chord from the Centre of the circle.
2. The figure below shows a circle AYBX and Centre O. XY cuts AB at C such that angle ACY=90^0

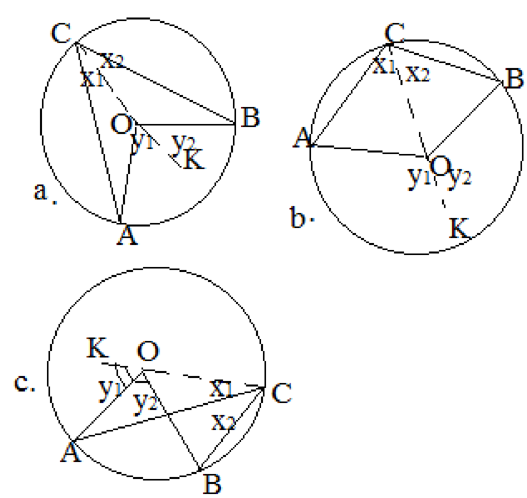


PART TWO

ANGLE PROPERTIES

Theorem 5

Angle at the center is twice angle formed at the circumference.



Given; center O, arc AB forming $A\hat{O}B$ at the Centre, $A\hat{C}B$ at the circumference

To prove: $A\hat{O}B = A\hat{C}B$

Construction: draw CO and produce it to K

Proof: in figures above

$y_1 = x_1 + \hat{A}$, ext. $\angle of\ \Delta = sum\ of\ int. opp < s$ in a Δ

But $x = \hat{A}$,

$\therefore y_1 = x_1 + x_2$

$= 2x_2$

In the figures **a** and **b**

$\therefore y_1 + y_2 = 2x_1 + 2x_2$

$y_1 + y_2 = 2(x_1 + x_2)$

$\therefore \mathbf{A\hat{O}B = 2A\hat{C}B}$

In figure c

$Y_2 - y_1 = 2(x_2 - x_1) = 2(x_2 - x_1)$

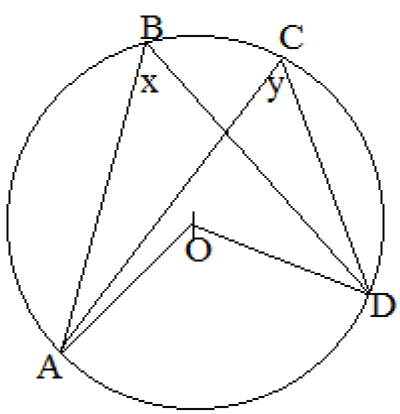
$\therefore \mathbf{A\hat{O}B = 2A\hat{C}B}$

Theorem 6

Angles in the same segment of a circle are equal.

Reason, $< s$ in same seg.

PROVE



Given; circle ABCD, $A\hat{B}D$ and $A\hat{C}D$

To prove; $\mathbf{A\hat{B}D = A\hat{C}D}$

Construction; join O to A and B respectively

Proof; $\mathbf{A\hat{O}D} = 2x$, < at the Centre= 2 ×< @ Circumference

$\mathbf{A\hat{O}D = 2y}$, < At the Centre= 2 ×< @ Circumference

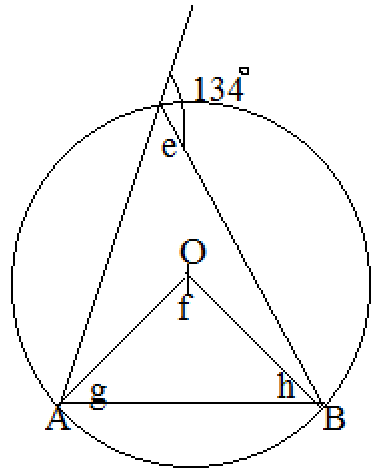
$\therefore 2x = 2y$

$\therefore x = y$

$\therefore A\hat{B}D = A\hat{C}D$

EXAMPLES

1. Find the e, f, g, and h if O is the Centre of the circle.



Solution

$e + 134° = 180°, sum\ < s\ in\ a\ \Delta$

$e = 180° - 134°$

$e = 46°$

$f = 2 \times e$, < @ the Centre= 2 ×< @ Circumference

$f = 2 \times 46°$

$= 92°$

AO= BO, radii

$\therefore \Delta$ AOB is isosceles

$\therefore g = h$

$g + h + f = 180°, sum < s\ in\ \Delta$

$2h + 96° = 180°$

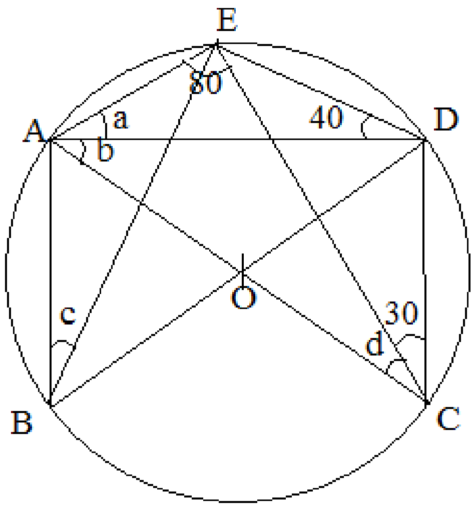
$2h = 180° - 96°$

$h = \frac{88°}{2}$

$\therefore g = 44°$

$y = 44°$

2. Find all letters in the figure below



Solution

$a = 30°, < s\ in\ the\ same\ seg$
$c = 40°, < s\ in\ the\ same\ seg$
$c = d, < s\ in\ the\ same\ seg$

∴ d= 40°

In Δ **ACE,**

$\hat{A} + \hat{C} + \hat{E} = 180°$, sum $< s\ in\ a\ \Delta$

(a + b) +d + 80^0= 180^o

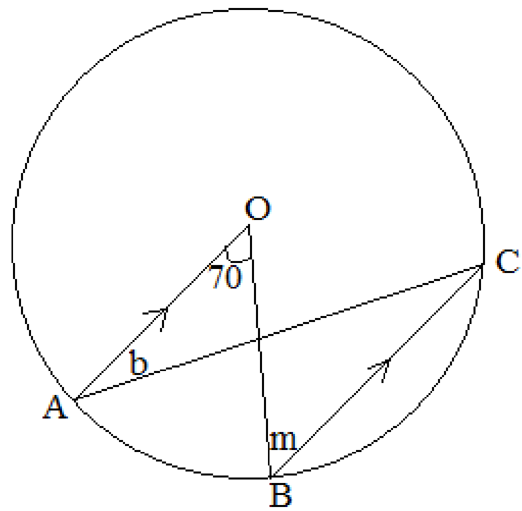
30°+b +40°+80°=180°

$b = 180° - 150°$

$b = 30°$

EXARCISE

1. Find marked angles in the circles below

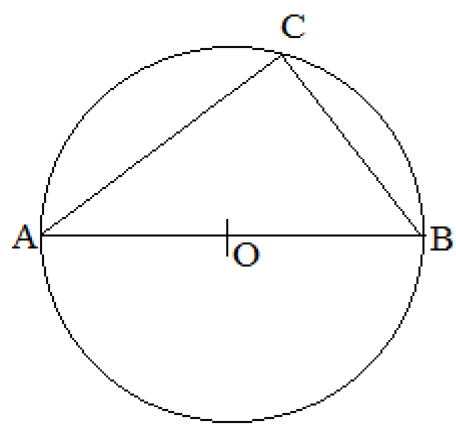


Theorem 7

Angle in a semicircle is a right angle.

Reason= <s in a semicircle

PROVE



Given; circle Centre O, diameter AB and <ACB

To prove: <ACB= 90

Proof: $\mathbf{A\hat{O}B = 2A\hat{C}B}$, < At the Centre= 2 ×<

At circumference

But $A\hat{O}B = 180°, < on\ a\ str\ line$

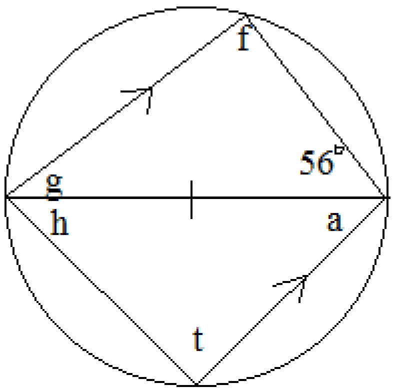
$\therefore 180° = 2A\hat{C}B$

$\frac{180°}{2} = A\hat{C}B$

$\therefore A\hat{C}B = 90°$

EXAMPLE

1. Find the marked angles in the figure below.



Solution

$f = 90°$,<s in a semicircle

$f + g + 56° = 180°, sum < s\ in\ a\Delta$

$90° + g + 56° = 180°$

$g = 180° - 146°$

$g = 34°$

$\boldsymbol{a = g, alt < s}$

$\therefore$ a= 34°

$t = 90° < s\ in\ a\ semicircle$

$h + a + 90° = 180°, sum < s\ in\ a\Delta$

$h + 34° + 90° = 180°$

$h = 180° - 126°$

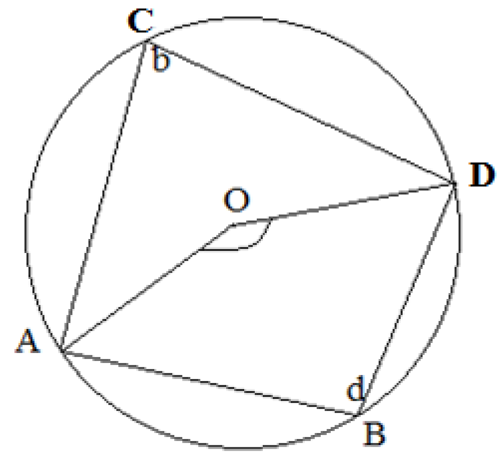
$= 56°$

CYCLIC QUADRILATERAL

A cyclic quadrilateral is the quadrilateral which has a circle passing through its vertices.

Theorem 8

The sum of interior opposite angles of a cyclic quadrilateral is 180° Or the interior opposite <s of a cyclic quadrilateral are supplementary.



Given; Centre O, cyclic quad ABCD

To prove; $A\hat{B}D + A\hat{C}D = 180°$

Construction; join O to A and C respectively

Proof; reflex $\mathbf{A\hat{O}D}$ $= 2b, < @\ a\ centre = 2 \times < @\ circum$

Obtuse $\mathbf{A\hat{O}D}$ $= 2d, < @\ a\ centre = 2 \times < @\ circum$

But 2b + 2d = 360°, <s @ a point

2(b + d) = 360°

b + d= $\frac{360°}{2}$

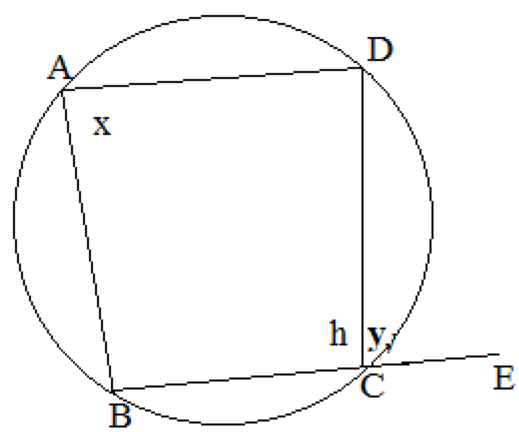
b + d= 180°

$\therefore A\hat{B}C + A\hat{B}C = 180°$

Theorem 9

The interior opposite angles of a cyclic quadrilateral is equal to the exterior angle.

PROVE



Given; cyclic quad, $B\hat{A}D$ and $D\hat{C}E$

To prove: $B\hat{A}D = D\hat{C}E$

Proof; $x + h = 180°$, sum int. opp. <s in Cyclic quad

$$\therefore x = 180° - h$$

$$y + h = 180°, < s\ on\ a\ str\ line$$

$$\therefore y = 180° - h$$

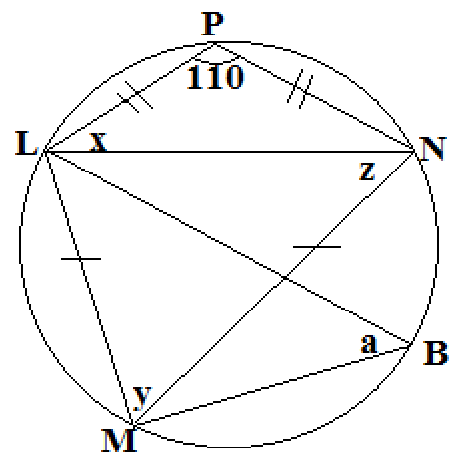
$$\therefore x = y$$

$$\therefore\ \mathbf{B\hat{A}D} = \mathbf{D\hat{C}E}$$

Reason, ext. <= inter. < in cyclic quad

EXAMPLE

Find the lettered angles.



Solution

In Δ **LPN**, LP = NP, given

$$\therefore \Delta \text{ LPN is an isoscles}$$

$$\therefore \hat{N} = x, \text{base angles}$$

$$x + \hat{N} + 110° = 180°, \text{sum} < s\ in\ \Delta$$

$$x + x + 110° = 180°$$

$$2x = 180° - 110°$$

$$2x = 70°$$

$$\therefore x = 35°$$

$$110° + y = 180°, \text{opp. int.} < s\ in\ cyclic\ q.$$

$$y = 180° - 110°$$
$$y = 70°$$

InΔ **LMN**, LM = NP, given

$$\therefore \Delta\ \mathbf{LMN} \text{ is an isoscles}$$

$$\therefore \hat{L} = z \text{ base angles}$$

$$z + \hat{L} + y = 180°, \text{sum} < s\ in\ \Delta$$

$$z + z + 70° = 180°$$

$$2z = 180° - 70°$$

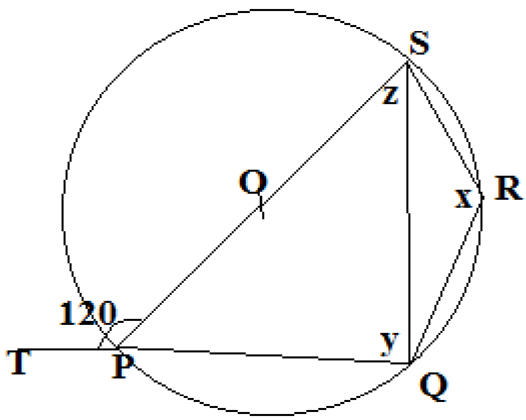
$$2z = 110°$$

$$\therefore z = 55°$$

$$z = a, < s\ in\ same\ seg$$

$\therefore \text{a} = 55°$

2. Find lettered angles



Solution

$\text{x} = 120°, \text{ext.} < \textit{of a cyclic quad}$

$\text{y} = 90°, < \textit{s in a sermicircle}$

$\text{z} + \text{y} = 120°, \text{ext.} < \textit{in } \Delta = \textit{sum int. opp.} < \textit{s}$

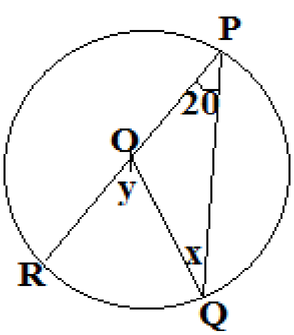
$\text{z} + 90° = 120°$

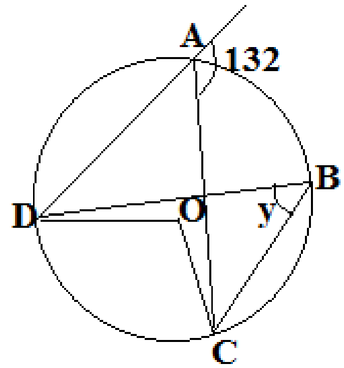
$\text{z} = 120° - 90°$

$\text{z} = 30°$

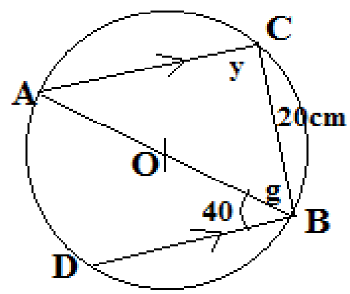
EXERCISE

1. Find all lettered angles in the figures below

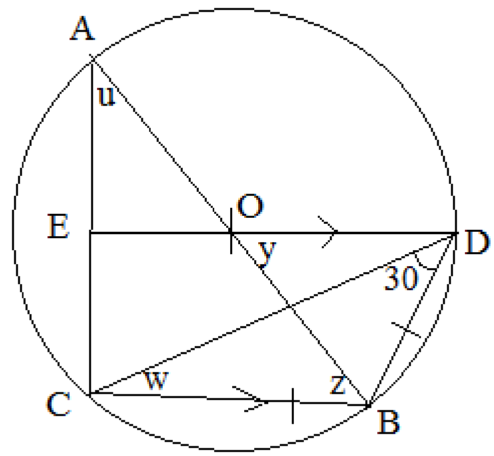




2. AOB is a diameter. Find the value of **g** and **y**



3. AB is a diameter. AC and ED are straight lines. Find all letters

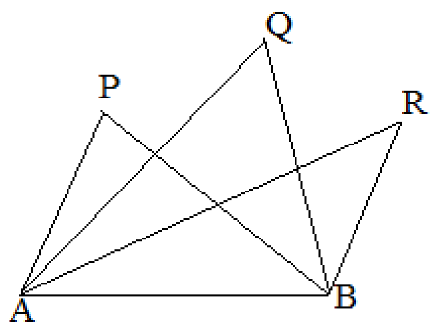


<u>CONCYCLIC POINTS</u>

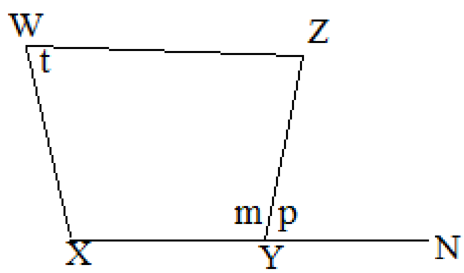
Concyclic points are points which lie on a circle.

Points are concyclic;

I. If the angles subtended by the same line such as APB, AQB, ARB.



II. If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is a cyclic



If $t + m = 180^o$, then **WXYZ** is cyclic Quad.

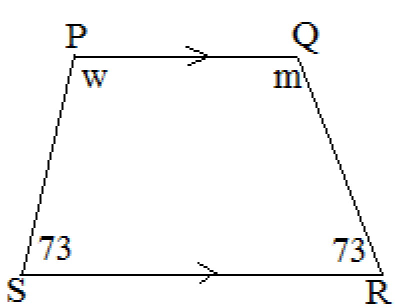
If $t = p$, then **WXYZ** is cyclic quad

EXAMPLE

PQRS is a trapezium having **PQ** parallel to **SR** and $\mathbf{P\hat{S}R = Q\hat{R}S = 73°}$.Prove that PQRS is a cyclic quadrilateral.

Solution

NB; A trapezium is a quadrilateral with two opposite sides parallel



Given; trapezium **PQRS,** $\mathbf{P\hat{S}R = Q\hat{R}S = 73°}$.

To prove; PQRS is a cyclic quad

Proof: $w + P\hat{S}R = 180^o$, allied <s

$W + 73^0 = 180^o$

$W = 180^o - 73^o$

$w = 107^o$

$m + P\hat{S}R = 180^o$, allied <s

$m + 73^0 = 180^o$

$m = 180^o - 73^o$

$m = 107^o$

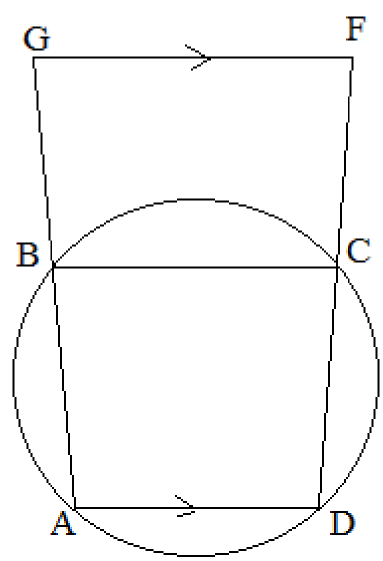
$\therefore m = w$

$\therefore w + Q\hat{R}S = 180°$,
but this is the sum of 2int opp < s of a cyclic quad

∴**PQRS** is cyclic quad.

<u>EXERCISE</u>

1. Figure below shows a circle ABCD in which AB and DC are produced to G and respectively such that GF is parallel to AD. **(MANEB 2006)**



Prove that quadrilateral **GBCF** is cyclic

CHAPER 4

ALGEBRIC FRACTIONS

This topic will deal with;

a. Adding algebraic fraction
b. subtracting algebraic fraction
c. dividing algebraic fraction
d. multiplying algebraic fraction

A. <u>ADDING ALGEBRAIC FRACTION</u>

<u>EXAMPLE</u>

Simplify the following

1. $\frac{y+2}{2}+\frac{x-2}{3}$

<u>Solution</u>

$\frac{y+2}{2}+\frac{x-2}{3}$

$\frac{3(y+2)+2(x-2)}{6}$

$\frac{3y+6+2x-4}{6}$

$\frac{3y+2x+2}{6}$

2. $\frac{n}{3}+\frac{m-1}{3}+\frac{m}{6}$

<u>Solution</u>

$\frac{n}{3}+\frac{m-1}{3}+\frac{m}{6}$

$\frac{2\times n+2(m-n)+m}{6}$

$\frac{2n+2m-2n+m}{6}$

$\frac{3m}{6}$

$\frac{m}{2}$

3. Simplify $\frac{3}{2x-4}+\frac{2}{6-3x}$

<u>Solution</u>

$\frac{2}{2(x-2)}+\frac{2}{3(2-x)}$

$\frac{3}{2(x-2)}+\frac{2}{-3(x-2)}$

NB; **a – b = - (b- a)**

Thus 2 – x = - (2 - x)

$\frac{3}{2(x-2)}-\frac{2}{3(x-2)}$

$\frac{9-4}{6\,(x-2)}$

$\frac{5}{6\,(x-2)}$

EXERCISE

Simplify the following

1. $\frac{2p+r}{r}+\frac{4p}{4r}$
2. $\frac{3mn}{2m+2n}+\frac{5mn}{3m+3n}$
3. $\frac{2r-3}{4}+\frac{2-r}{3}$
4. $3+\frac{2b}{a-b}$

EXAMPLES

1. Simplify $\frac{m+3}{m2-3n+2}+\frac{7m+5}{m2+m-2}$

<u>Solution</u>

$\frac{m+3}{(m-2)\,(m-1)}+\frac{7m+5}{(m+2)\,(m-1)}$

$\frac{(m+3)(m+2)+(7m+5)(m-2)}{(m-2)(m-1)(m+2)}$

$\frac{m^2+3m+2m+6+7m^2-14m+5m-10}{(m-2)(m-1)(m+2)}$

$$\frac{8m^2 - 4m - 4}{(m-2)\,(m-1)(m+2)}$$

$$\frac{4(2m^2 - m - 1)}{(m-2)\,(m-1)(m+2)}$$

$$\frac{4(2m+1)(m-1)}{(m-2)\,(m-1)(m+2)}$$

$$\frac{4(2m+1)}{(m-2)\,(m+2)}$$

2. **Simplify** $\frac{\boldsymbol{a+b}}{\mathbf{4}} + \frac{\boldsymbol{(b-2a)}}{\mathbf{5}}$

<u>Solution</u>

$$\frac{a+b}{4} + \frac{(b-2a)}{5}$$

$$\frac{5(a+b)-4\,(b-2a)}{20}$$

$$\frac{5a+5b-4b+8a}{20}$$

$$\frac{13a+b}{20}$$

3. Simplify $\frac{c\,(3-c)}{c2 + 3c - 10} + \frac{c-1}{c+5}$

<u>Solution</u>

$$\frac{c\,(3-c)}{c2 + 3c - 10} + \frac{c-1}{c+5}$$

$$\frac{c\,(3-c)}{(c+5)(c-2)} + \frac{c-1}{c+5}$$

$$\frac{c(3-c)+(c-1)(c-2)}{(c+5)(c-2)}$$

$$\frac{3c - c^2 + c^2 - 3c + 2}{(c+5)(c-2)}$$

$$\frac{2}{(c+5)(c-2)}$$

4. Simplify $\frac{1}{2n-3} + \frac{1}{2n+1} - \frac{1}{n-1}$

<u>Solution</u>

$$\frac{(2n+1)(n-1)+(2n-3)(n-1)-(2n-3)(2n+1)}{(2n-3)(2n+1)(n-1)}$$

$$\frac{2n^2-n-1+2n^2-5n+3-(4n^2-4n-3)}{(2n-3)(2n+1)(n-1)}$$

$$\frac{4n^2-6n+2-4n^2+4n+3}{(2n-3)(2n+1)(n-1)}$$

$$\frac{5-2n}{(2n-3)(2n+1)(n-1)}$$

EXERCISE

Simplify the following

1. $\frac{5}{d^2-16} + \frac{2}{(4d-d)^2}$
2. $\frac{5}{d^2-2d-8} + \frac{2}{d^2-6d+8}$
3. $\frac{3}{2(x+y)} - \frac{1}{3(x+y)}$
4. $\frac{2}{m-n} + \frac{3}{a+2}$
5. $\frac{2}{2t^2+3t-2} + \frac{1}{t+2} - \frac{1}{2t-1}$

<u>DIVISION OF ALGEBRAIC EXPRESSIONS</u>

EXAMPLE

1. **Simplify** $\frac{\boldsymbol{x^2-5x+6}}{\boldsymbol{x}}$ **÷ (x - 3)²**

<u>Solution</u>

$$\frac{x^2-5x+6}{x} \div (x-3)^2$$

$$\frac{(x-2)\cancel{(x-3)}}{x} \times \frac{1}{(x-3)\cancel{(x-3)}}$$

$$\frac{x-2}{x(x-3)}$$

2. Simplify $\left(\frac{1}{x} - \frac{1}{y}\right) \div \frac{x^2-y^2}{x^2y^2}$

<u>Solution</u>

$$\frac{(y-x)}{xy} \times \frac{x^2y^2}{x^2-y^2}$$

$\frac{(y-x)}{xy} \times \frac{x^2y^2}{(x+y)(x-y)}$

$\frac{-(x-y)}{xy} \times \frac{x^2y^2}{(x+y)(x-y)}$

$\frac{-xy}{x+y}$

$\frac{xy}{y-x}$

MULTIPLICATION OF ALGEBRAIC FRACTIONS

EXAMPLES

1. Simplify $\frac{x+1}{x^2-4} \times \frac{x+2}{5}$

Solution

$\frac{x+1}{x^2-4} \times \frac{x+2}{5}$

$\frac{x+1}{x^2-2^2} \times \frac{x+2}{5}$

$\frac{x+1}{(x+2)(x-2)} \times \frac{x+2}{5}$

$\frac{x+1}{5(x-2)}$

2. Simplify $\frac{n^2-9}{n^2-n} \times \frac{n^2-3n+2}{n^2+n-6}$

Solution

$\frac{n^2-9}{n^2-n} \times \frac{n^2-3n+2}{n^2+n-6}$

$\frac{n^2-3^2}{n(n-1)} \times \frac{(n-1)(n-2)}{(n-2)(n+3)}$

$\frac{(n+3)(n-3)}{n(n-1)} \times \frac{(n-1)(n-2)}{(n-2)(n+3)}$

$\frac{n-3}{n}$

SOLVING ALGEBRAIC FRACTIONS

EXAMPLE

1. Solve $\frac{4}{x^2-x-2} + \frac{3}{x^2-4} = \frac{2}{x^2+3x+2}$

Solution

$$\frac{4}{(x-2)(x+1)} + \frac{3}{(x-2)(x+2)} = \frac{2}{(x+2)(x+1)}$$

$$\frac{4(x+2)+3(x+1)=2(x-2)}{(x-2)(x+1)(x+2)}$$

$$4(x+2)+3(x+1) = 2(x-2)$$

$$4x+8+3x+3 = 2x-4$$

$$7x+11 = 2x-4$$

$$5x = -15$$

$$x = -3$$

NB: When solving algebraic fractions, if the denominator has been used in both sides (the equal to side and other side), then we disregard the denominator

2. **Solve $\frac{3}{2b+5} = \frac{2}{b+2}$**

Solution

$\frac{3}{2b+5} = \frac{2}{b+2}$

3(b+2) = 2(2b+ 5)

3b+6 = 4b+ 10

-b = 4 $\therefore$ b= -4

WORD PROBLEMS

EXAMPLE

Mada Kalima bought some fish for k4000. If the fish had each been sold for k20 less, then he would have bought 10 more. How much was one fish sold.

Solution

Total money = k4000

Let the current price be k x

:. The number of fish at x kwacha= $\frac{4000}{x}$

If the price had been each sold for k20 less, the current = (x- 20)

:. Number of fish at (x-20) kwacha = $\frac{4000}{x-20}$

Number of fish bought due to decrease in price is 10 more than that of the current price.

$\therefore \frac{4000}{x-20} - \frac{4000}{x} = 10$

$\frac{4000x-4000(x-20)=10x(x-20)}{x(x-20)}$

$4000x - 4000(x-20) = 10x(x-20)$

$4000x - 4000x + 8000 = 10x^2 - 200x$

$8000 = 10x^2 - 200x$

$10x^2 - 200x$- $8000 = 0$

Divide by 10

$x^2 - 20x - 800 = 0$

$x^2 - 100x + 80x - 800 = 0$

$x(x-100) + 80(x-100) = 0$

$(x-100)(x+80) = 0$

$(x-100) = 0 \; or \; (x+80) = 0$

$x = 100 \quad x = -80$

:. The cost of one fish was k100 each.

EXERCISE

1. Simplify $\left(xy - \frac{x^2y}{x+y}\right) \div \frac{x^2y}{x+y}$
2. $\frac{3d^2-12}{9d^2} \times \frac{6d^2}{4d+8}$
3. solve $\frac{b+1}{2b-3} = 2 - \frac{b+3}{2b+3}$
4. **Beauty** drove a distance of 40km on a return journey, she drove 20km/hour faster and took 2hrs less. What was her speed on outward journey
5. A taxi driver takes k36 in fare per trip with a full load of passengers, if he increases his fare by k1 per passenger she can make the same amount when carrying three passengers less. How many passengers did he originally carry?

CHAPTER 5

SET THEORY

A set is a collection of objects or items

Types of sets

i. equal sets
ii. Empty sets
iii. Infinite sets
iv. Finite sets

EQUAL SETS

Sets with same elements and equal elements e. g A={a, b, c}

B= {a, b, c}

:. A= B

EMPTY SET

Is a set with no elements e. g C={}

∅a symbol for an empty set.

INFINITE SET

A set which has uncountable number of elements e. g A={*negative numbers*}

FINITE SETS

Has countable number of elements e. g

A= {*whole numbers from 1 to 10*}

SET SYMBOLS

1. ∈= An element of

Example, 8∈ A means 8 is an element of A

2. ∉ = is not an element of
3. μ *or* = universal set
4. ∪= union
5. ∩= intersection
6. ⊂= subset of

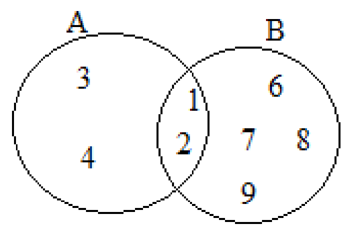
UNION OF SETS

The union of two sets is the set of all elements that are members of either set

EXAMPLE

A= {1,2,3,4} **B**={1,2,6,7,7}, find **A**∪**B**.

Solution



∴ A∪B= {1, 2,3,4,5,6,7,8}

INTERSECTION SETS

The intersection of two sets is the set of elements that are in both sets. E. g in the above example **A**∩**B**={1,2}

An intersection of sets have elements that are common to given sets

SET BUILDER NOTATION

Real numbers (R)

Are all categories of numbers e. g negative, fraction, positive numbers e. t. c

Natural numbers (N)

Are positive whole numbers excluding 0

Whole numbers (W)

Are all natural numbers including 0

Prime numbers (P)

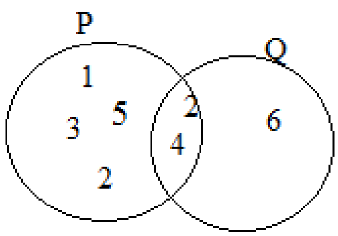
Are numbers which can be divided by one and it self e. g P={2,3,5,7,11,13….}

EXAMPLE

1. **A={x : x∈R, x^2=0}**
   Read as A is a set of values of x such that x is an element of a real number and x square is equal to zero.
   Solution
   A={x : x∈R, x^2=0}
   $X^2 = 0$
   $X = \sqrt{0}$
   X = 0
   ∴ A= {0}
2. **B={x : x ∈ R, x^2-3=0}**
   Solution
   $x^2-3=0$
   $x^2 = 3$
   $x = \pm\sqrt{3}$
   ∴ B= $\{\pm\sqrt{3}\}$
   B= $\{+\sqrt{3}, -\sqrt{3}\}$

NB: ∩ (A) means number of elements in A

3. P= {1,2,3,4,5} and Q= { 2,4,6}
   Find a. n(P∩Q), b. all subsets of Q
   Solution



a.
(P∩Q)= {2, 4}
n(P∩Q)= 2

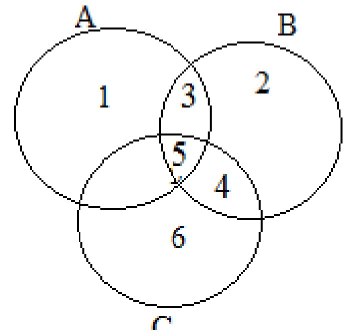
NB: a set with n elements has 2^n subsets. Every set has got an empty set

b. Q= {2,4,6}

n(Q)= 3

Number of subsets = 2^3= 8

4. Given that A= {1,3,5}, B= {2,3,4,5} and C= {2,4,5,6}
   Find a. **(A∪B)∩ C,** b. **A∪B∪C** d. **A∩C**
   Solution



a. **(A∪B)= {1,2,3,4,5}**
   C= {3, 4, 5, 6}
   ∴(A∪B)∩ C= {3, 5}

b. **A∪B∪C={1, 2, 3, 4, 5, 6}**
c. **A∩C** = {}

UNIVERSAL SET (∪)

It is a collection of all sets

COMPLEMENT OF A SET

These are elements that are absent in a set but are present in a universal set.

A' = means complements of A

EXAMPLES

1. Given that μ= {1,2,3,4,5,6,7,8,9,10}

a. If A = {1,3,5,7,9} list A'

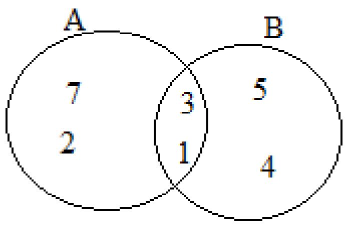
b. If P = {2,3,5,7} list P'

Solution

a. A'= {2,4,6,8,10}

b. P'= {1,4,6,8,9,10}

NUMBER OF ELEMENTS IN A TWO SET PROBREMS



n **(A∪B)= 6**

n (A∩B)= 2

n(A)=4

n (B) =4

In general, n **(A∪B)= n(A) + n(B) - n(A∩B)**

= 4 + 4 – 2

= 6

EXAMPLE

1. Given that **n (A∪B)=14, n (A)=10, n(B) =9,** find **n (A∩B).**
   Solution

n **(A∪B)= n(A) + n(B) - n(A∩B)**

14 = 10 – 9 - n(A∩B)

14 = 19 - n(A∩B)

n(A∩B) = 19 – 14

= 5

2. Given that n(A)={2,4,6,8,10}, n(B)={4,8,12,16}. Show that , n **(A∪B)= n(A) + n(B) - n(A∩B)**
   Solution

n **(A∪B)= 7**

n(A) = 5

n(B)=4

n(A∩B)= 2

By substituting;

7 = 5 + 4 – 2

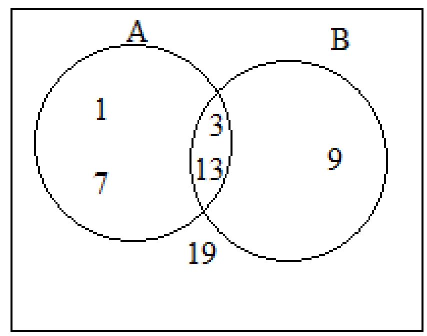
7 = 9 – 2

7 = 7

∴ n**(A∪B)= n(A) + n(B) - n(A∩B)**

EXERCISE

1. The diagram below shows sets A, B and C the universal set.



3. List **A∩B'** and **B∩A'**

   ii. What are **n(A∩B**) and**n(A∪B)**

2. Given μ= {1,2,3,4,5,6,7,8,9,10}
   A= {2,4,6,8}

   Find A' and (A')'

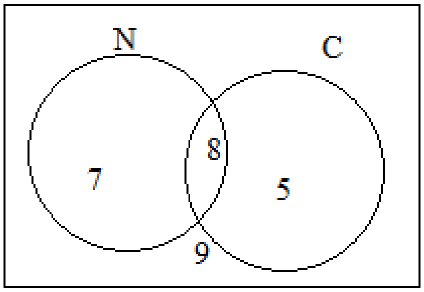
3. Given that A = B= {1,2,3,4}, show that n**(A∪B)= n(A) + n(B) - n(A∩B)**

WORD PROBLEMS

EXAMPLE

1. In a class, 15 pupils like nature studies, 13 like craft and 8 like both subjects. Use the Venn diagram to find how many pupils are there in the class
   Solution

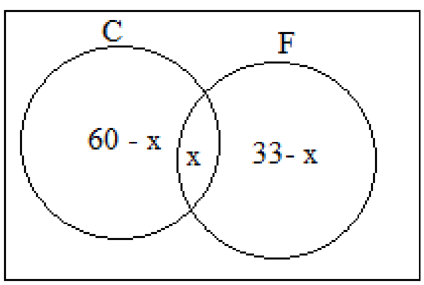
NB: always start with intersection



Number of pupils = 7 + 8 + 5 + 9
= 29
NB; Make sure the elements In N add up to 15 and in C add up to 13.
8 is subtracted from 15 and 13 because 8 pupils like both subjects.

2. In a class, 60 pupils study Chichewa and 33 study French. All pupils study at least one of the languages. If there are 70 pupils in the class, how many study both languages?

Solution



60 – x + x + 33 – x = 70
93 – x = 70
93 – 70 = x
X= 23
∴ 23 Pupils study both languages

3. In a group of 43 pupils, 29 are girls and 26 like dancing, 7 do not like dancing. How many boys do not like dancing?

Solution

Number of pupils= 43
Number of girls = 29
∴Number of boys= 43 – 29
= 22

Number of people who do not like dancing=7

Number of girls who like dancing= 29 – 7
= 22

No. of boys who like dancing= 26 – 22
= 4

But there are 14 boys

∴No. of boys who do not like dancing=14 – 4
= 10

THREE SET PROBLEMS

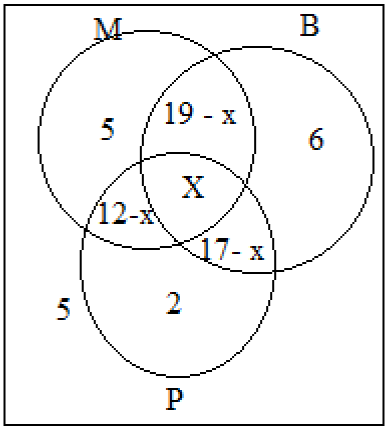
EXAMPLE

1. Class of 50 students wrote test in mathematics, biology and physical science. The results of the tests were as follows

- 12 passed maths and physical
- 19 passed maths and biology
- 17 passed biology and physical
- 2 passed physical only
- 5 passed maths only
- 6 passed biology only

If 5 students failed all the three subjects and x passed all subjects use Venn diagram to calculate the value of x. (**MANEB 2013**)

Solution



$x+5 + 19 - x + 12 - x + 17 - x + 6 + 2 + 5 = 50$

$66 - 2x = 50$

$66 - 50 = 2x$

$16 = 2x$

$x = 8$

EXERCISE

1. In a class of 50 students, each of the students ate at least one of the following types of fruits; banana, mango and orange. It was found that
    - $(x+1)$ ate all the three types of fruits
    - 9 students ate mangoes and oranges only
    - 8 students ate bananas and mangoes only
    - 5 students ate bananas and oranges only
    - X ate bananas only
    - $(x-1)$students ate mangoes only

Represent the data on a Venn diagram hence find the value of x

CHAPTER 6

MAPPING AND FUNCTIONS

Mapping and functions looks at relationships that exist among or between elements

RELATIONS

Mary "is a friend of " **mercy**

Mary...............first element

Is the friend of......defining condition

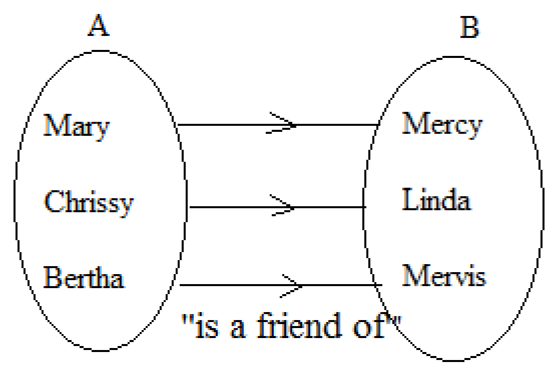
Mercy..........second element

(Mary, Mercy)............**ordered pair**

TYPES OF RELATIONS

Arrow diagrams are used to map relations

1).**ONE TO ONE RELATIONS**



The above arrow diagram **is example of one to one relation**

In this relation, every member of **A** has one image in **B**

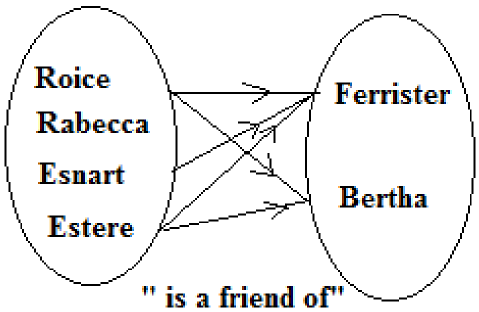
Every member of B is the image of exactly one member of A;

Mercy is the image of Mary

Linda is the image of Chrissy

Mervis is the image of Bertha

2).**MANY TO ONE RELATION**



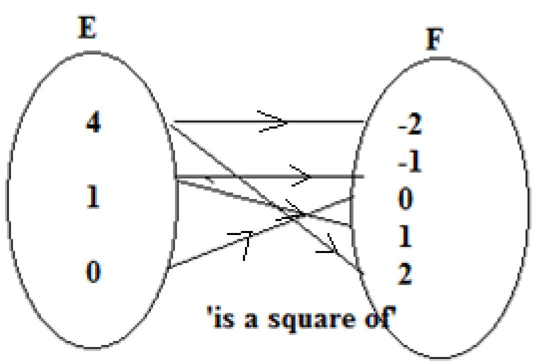
In the above diagram, many elements of C have the same elements in D

No elements of C have more than one elements in D

3).**ONE TO MANY RELATION**

If **E** = (0,1,4) **F** = (-2,-1,0,1,2)

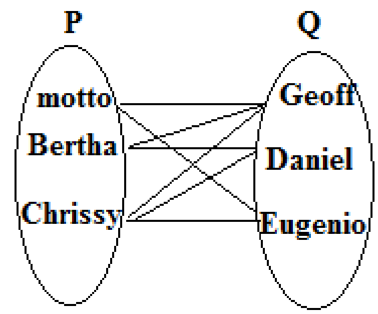
And the relation is a square of



In the above diagram,some elements of E have more than one image in F

No image of F is the image of more than one element of E

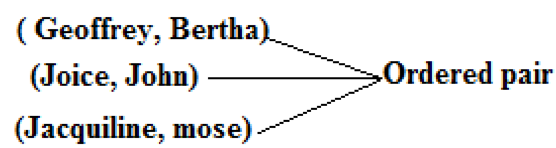
4). **MANY TO MANY RELATIONS**



In many to many to many relation, some elements of P have more than one image of Q.

Some elements of Q are more images of many elements of P.

DOMAIN AND RANGE



Domain is a set of the first elements of all ordered pair.

∴ **D**= (Geoffrey, Joyce, Jacqueline)

Range is the set of all the second elements of all ordered pair.

∴**R** = (Bertha, John, Mose)

EXAMPLES

Given the domain and range of the following relation from A to B where **A**= {1,2,3} and **B**= {1,2,3,4,5}

a. **P = {(x, y) : x ∈ A, y∈ B and y= 2x}**
   ∴ P= {(1,2),(2,4)}

   D (P) = (1, 2)

   R (P) = (2, 4)

b. **R= {(x, y) : x ∈ A, y∈ B and x= 2 }**
   ∴ R = {(2, 2)}
   ∴ D(R) = (2)

R(S) = (2)

c. **S = {(x, y) : x ∈ A, y∈ B and x= 2y}**

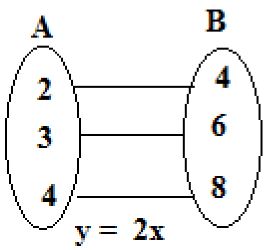
∴ S= {(2, 1)}

∴ D (S) = (2)

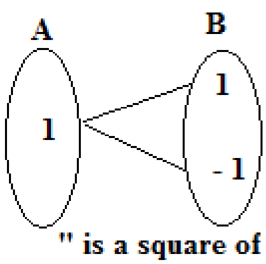
R(S) = (1)

FUNCTIONS

A function is a special kind of relation. It is a rule which assigns to each element x of a set A just one element of a set B.



one to one relation



One to many relation

FUNCTION NOTATION

We can write a function in different ways or notations e. g y= x^2, can be written as f(x)= x^2, or f : x ⟶x^2.

Where y / f(x) / f: x ⟶ is a domain and x^2 is the range.

EXAMPLES

1. A function f from P to Q is difficult by the formulae f (t) = t + 3, where t is an element of P and Q consists of the elements of the form t + 3.

Solution

f: p ⟶Q

f (p) = Q

f(t) = t + 3

P = {1,2,3,4,5}

f(1) = 1 + 3= 4

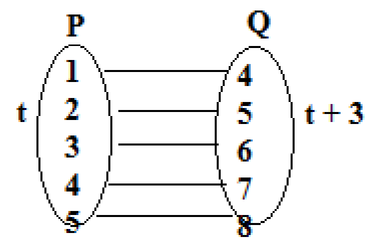
f (2) = 2 + 3= 5

f(3) = 3 + 3= 6

f(4) = 4 + 3= 7

f (5)= 5 + 3= 8

∴ Q = {4, 5, 6, 7, 8}



2. If f **(x) = 2x + 1** and **g(x) = x^2**, find

a. fg**(3)** b. gf**(3)** c. gf**(-2)**

Solution

a. g(3)= 3^2

= 9

∴ fg(3) = 2 (g (3)) + 1

= 2(9) + 1

= 19

b. f(3) = 2(3) + 1

= 7

∴ gf (3) = 7^2

= 49

c. f(-2) = 2 (-2) + 1

= - 4 + 1

= - 3

∴ gf(x) = $(-3)^2$

= $(-3)^2$

$= 9$

MANEB QUESTION

1. Given that $g(\mathrm{x}) = \frac{3x}{x+1}$ calculate the value of x if $g(x) = 2$

   Solution

   $g(\mathrm{x}) = \frac{3x}{x+1}$

   $2 = \frac{3x}{x+1}$

   $2x + 2 = 3x$

   $2 = 3x - 2x$

   $x = 2$

2. Given that $g(x) = dx - 5$, and that $g(2) = 1$. Find the value of x.

   Solution

   $g(2) = d(2) - 5$

   But $g(2) = 1$

   $\therefore \quad 1 = 2d - 5$

   $1 + 5 = 2d$

   $6 = 2d$

   $\therefore 3 = d$

3. Given that $\boldsymbol{f(x) = x^2 + 1}$and $\boldsymbol{g(x) = kx + n}$where n and k are constants. $\boldsymbol{f(0) = g(0)}$and $\boldsymbol{g(2) = 15}$. Find k and n.

   Solution

   $f(x) = x^2 + 1$

   $f(0) = (0)^2 + 1$

   $= 1$

   $g(0) = k(0) + n$

   $= 0 + n$

   $= n$

But $g(0) = f(0)$

$\therefore 1 = \mathrm{n}$

$g(2) = k(2) + 1$

$15 = 2k + 1$

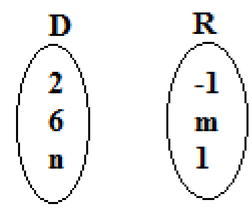
$15 - 1 = 2k$

$14 = 2k$

$\therefore k = 7$

EXERCISE

1. Figure below shows an arrow diagram for the function $\boldsymbol{y = 2x - 5}$. Find the value of **m** and **n**



2. Given that $f(x) = \frac{2x^3}{3} + 1$. Find $g(-1)$ in its simplest form.
3. Given that $f(x) = 2x + 3$ and $(x) = \frac{5}{x+2}$. find $fg(x)$
4. The function $y = 2 + x$ has range {3,6}, find its domain.

CHAPTER 7

CHANGING SUBJECT OF THE FORMULA

To make unknown variable subject, it must have coefficient and power 1and all the other terms and numbers must go to the other side of the equation

In the equation$x = 2y - 3$, x is the subject while y is not.

STEPS TO FOLLOW

i. Begin by clearing fractions, brackets and root signs
ii. Rearrange the formula so that all the terms which contain the new subject are on one side of the equals sign and the rest on the other
iii. If more than one term contain the subject, take it outside the bracket
iv. Divide both sides by the bracket and simplify as far as possible

EXAMPLES

Make x subject

1. $4xy = 2 - 2x$

$4xy + 2x = 2$

$x(4y + 2x) = 2$

$x = \frac{2}{4y+2}$

$\therefore x = \frac{2}{2(2y+1)}$

$x = \frac{1}{2y+1}$

2. $\frac{a}{x}$ +b=c

$\frac{a}{x}$ =c-b

$\frac{a}{x} = \frac{c-b}{1}$

$a = x(c - b)$

$\frac{a}{c-b} = x$

3. $a = \frac{2b+3x}{3b-2x}$

$a(3b - 2x) = 2b + 3x$

$3ab - 2ax = 2b + 3x$

$3ab - 2b = 3x + 2ax$

$3ab - 2b = x(3 + 2a)$

$\frac{3ab-2b}{3+2a} = x$

$\therefore x = \frac{b(3a-2)}{3+2a}$

EXERCISE

Make the letter in the bracket subject

1.$\frac{x}{2a} + \frac{x}{3a} = b$ … … … . . (x)

2. $a\sqrt{x} - 1 = b$ … … … . (x)

3. $x(a - b) = b(c - x)$ … . . (x)

4. $\sqrt[3]{\frac{x}{a}} = b$ … … … … … … … … . . (x)

EXAMPLES

Make x subject of the formula

1. $\frac{a}{a-x} = \frac{b}{b+x}$

Solution

$\frac{a}{a-x} = \frac{b}{b+x}$

$a(b + x) = b(a - x)$

$ab + ax = ab - bx$

$ax + bx = ab - abx(a + b) = 0$

$x = \frac{0}{a+b}$

$x = 0$

2. $Make\ x\ subject$

$a(a^2 - x) = b(b^2 - x)$

$a^3 - ax = b^3 - bx$

$bx - ax = b^3 - a^3$

$x(b - a) = (b - a)(b^2 + ab + a^2)$

$\underline{x = (b - a)(b^2 + ab + a^2)}$
$(b - a)$

$x \ = (b^2 + ab + a^2)$

3. Make **a** subject of the formula $c = \frac{ab\sqrt{2g}}{\sqrt{a^2-b^2}}$

Solution

$c = \frac{ab\sqrt{2g}}{\sqrt{a^2-b^2}}$

$c\sqrt{a^2 - b^2} = ab\sqrt{2g}$

$a^2 - b^2 = \left(\frac{ab\sqrt{2g}}{c}\right)^2$

$a^2 - b^2 = \left(\frac{a^2b^2(2g)}{c^2}\right)$

$c^2\left(a^2 - b^2\right) = 2a^2b^2g$

$c^2a^2 - c^2b^2 = 2a^2b^2g$

$c^2a^2 - 2a^2b^2g = a^2b^2$

$a^2(c^2 - 2b^2g) = c^2b^2$

$a^2 = \frac{c^2b^2}{c^2-2b^2}$

$a = \sqrt{\frac{c^2b^2}{c^2-2b^2g}}$

$a = \frac{cb}{\sqrt{c^2-2b^2g}}$

EXERCISE

Make the latter in the brackets subject of the formula

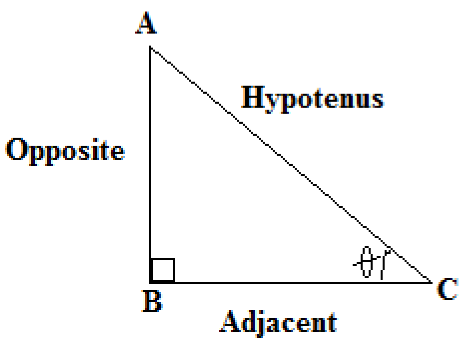
1. $H=\frac{w^2}{2g}\ (R^2 - r^2)$ (r)
2. $T=\frac{mu^2}{k} - 2mg$ (k)
3. $y^2=2a(x - 4a)$ (x)
4. $H=\frac{A}{a-4b} - R^2$ (A)
5. $D=\sqrt{\frac{2v^2d}{g} + \frac{d^2}{4}}-\frac{d}{2}$ (d)
6. $T=2\pi\sqrt{\frac{h^2+k^2}{2gh}}$ (k)

CHAPTER 8

TRIGONOMETRY

Trigono means triangle and **metron** means measures. Trigonometry is a Greek word

It deals with size and angles of triangles



TRIGONOMETRIC RATIOS

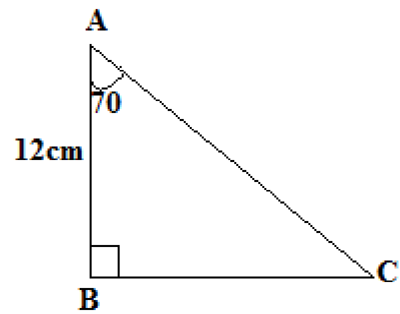
1. $\text{Sine } \theta = \dfrac{\textbf{opposite side}}{\textbf{hypotenus side}} = \dfrac{\text{AB}}{\text{AC}}$

2. $\text{Cos } \theta = \dfrac{\textbf{adjacent}}{\textbf{hypotenus side}} = \dfrac{\text{BC}}{\text{AC}}$

3. $\text{Tan } \theta = \dfrac{\textbf{opposite side}}{\textbf{Adjacent}} = \dfrac{\text{AB}}{\text{BC}}$

EXAMPLE

1. ABC is a triangle in which **AB**= 12cm, angle **B**= 90° and angle **A**= 70°. Find the length of **BC** leaving your answer to 2 significant figures.
   Solution



$\text{Tan A} = \dfrac{BC}{AB}$

$\text{Tan } 70° = \dfrac{BC}{12cm}$

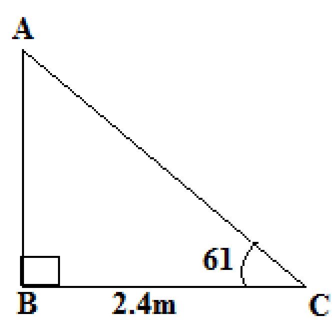
$2.7475 = \dfrac{BC}{12cm}$

$2.7475 \times 12\text{cm} = \text{BC}$

$\therefore \text{BC} = 32.9700$

$= 33\text{cm}$

2. A ladder makes 61°with the ground. The base of the ladder is 2.4m from a vertical wall. How far does the ladder reach up the wall? To 3 significant figures
   Solution



$\text{Tan } 61° = \dfrac{AB}{2.4m}$

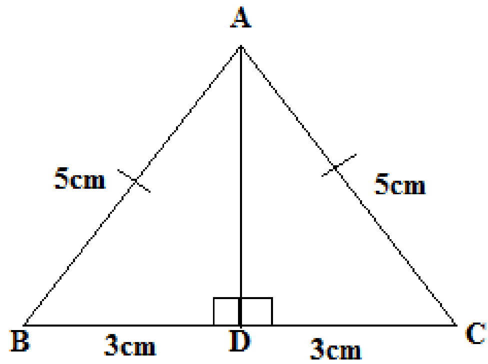
$1.8040 = \dfrac{AB}{2.4m}$

$1.8040 \times 2.4\text{cm} = \text{AB}$

$\therefore \text{AB} = 4.329$

$= 4.33\text{m}$

3. A triangle of sides 5cm, 5cm and 6cm. what are its angles?
   Solution



Construction: join a perpendicular bisector AD

In Δ **ABC**

Cos B= $\frac{3cm}{5cm}$

= 0.6000

$\cos^{-1} 0.6000$= 53.2°

∴< B= 53.2°

But angle B = angle C, (base angles)

∴< C= 53.2°

In Δ **ABC**

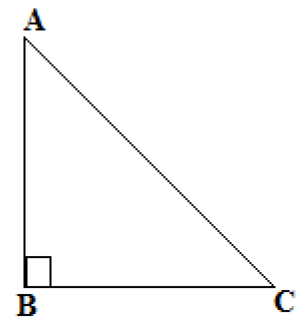
< A + < B + < C= 180°, Sum of <s in a Δ

< A + 53.2° + 53.2° = 180°

< A= 180°- 106.4°

= 73.6°

RELATIONSHIP BETWEEN ANGLES OR RATIOS



Sine A= $\frac{BC}{AC}$

Cos C= $\frac{BC}{AC}$

∴ Sin A = Cos C

But < A + < C= 90°

< A = 90°- < C

But sin A = Cos C

∴ Sin A = Cos (90°- A)

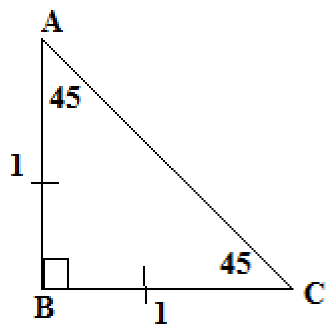
Cos A = Sin (90°- A)

In general

1. Sin θ = Cos (90°- θ)
2. Cos θ = Sin (90°- θ)
3. $\tan\theta = \frac{\sin\theta}{\cos\theta}$
4. sin θ = sin (180- θ)
5. Cos θ = -Cos (180°- θ)
6. $\sin^2\theta + \cos^2\theta = 1$

SPECIAL ANGLES

0°, 30°, 45°, 60°, 90°



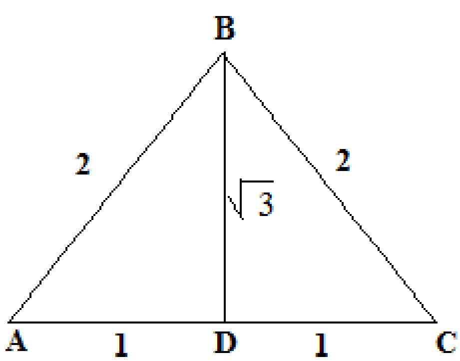
$AC^2= 1^2 + 1^2$, Pythagoras

$\therefore AC= \sqrt{2}$

In ΔABC

1. $\text{Sin } 45°=\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
2. $\text{Cos } 45°=\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
3. $\text{Tan } 45°=\frac{1}{1} = 1$

30° and **60°** uses equilateral triangle



In In ΔABD

$AB^2= BD^2+ BD^2$, Pythagoras

$2^2 = BD^2+ 1^2$

$BD^2= 4 - 1$

$AD= \sqrt{3}$

1. $\text{Sin } 30°= \frac{1}{2}$
2. $\text{Cos } 30°=\frac{\sqrt{3}}{2}$
3. $\text{Tan } 30°=\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
4. $\text{Sin } 6o°=\frac{\sqrt{3}}{2}$
5. $\text{Cos } 60°=\frac{1}{2}$
6. $\text{Tan } 60°=\sqrt{3}$

Angle	**0°**	**30°**	**45°**	**60°**	**90°**
Sin	0°	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos	1°	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
Tan	0°	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞

EXAMPLE

1. Without using a calculator or mathematical table, show that

   $\text{Sin } 30°+ \text{Cos } 45°+\text{Tan}60°=\frac{1+\sqrt{2}+2\sqrt{2}}{2}$

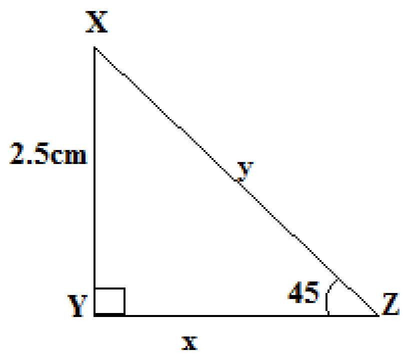
   Solution

   $\text{Sin } 30°+ \text{Cos } 45°+\text{Tan}60°=\frac{1}{2}+ \frac{\sqrt{2}}{2} +\sqrt{3}$

   $=\frac{1+\sqrt{2}+2\sqrt{2}}{2}$

$\therefore$**Sin 30°+ Cos 45°+Tan60°=**$\mathbf{\frac{1+\sqrt{2}+2\sqrt{2}}{2}}$

2. Find the value of x and y in the figure below without using a calculator or a four figure table.



Solution

Tan 45°$=\frac{2.5cm}{x}$

$1=\frac{2.5cm}{x}$

$x=2.5cm$

Sin 45°$=\frac{2.5cm}{y}$

$\frac{\sqrt{2}}{2}=\frac{2.5cm}{y}$

$y\sqrt{2}=2.5cm\times 2$

$y=\frac{5}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

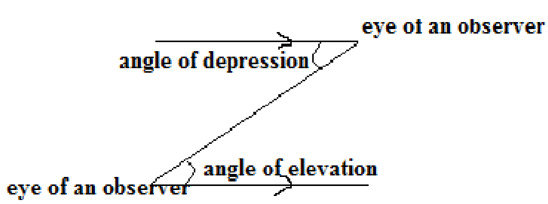
y $=\frac{5\sqrt{2}}{2}$

y$=2.5\sqrt{2}$

<u>ANGLES OF ELEVATION AND DEPRESSION</u>

I. Angles of depression and elevation are measured from eye level of an observer.

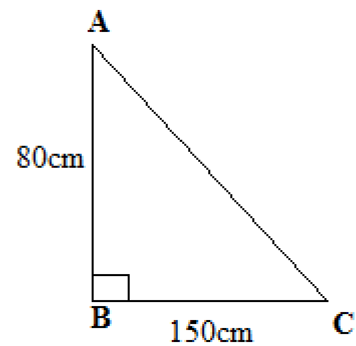
II. Angle of depression is always equal to angle of elevation.



EXAMPLE

1. A church tower is 80m high. Find the angle of elevation of its top from a point on the ground 150m away on the level ground.

Solution



Tan C $=\frac{80}{150}$

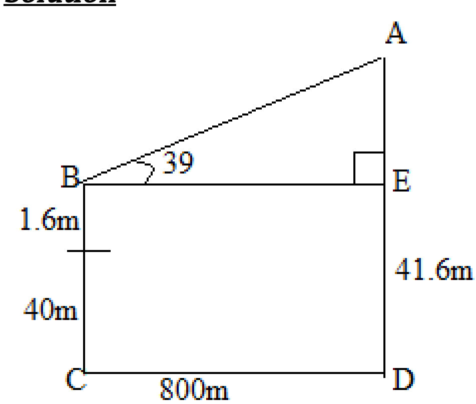
$=0.5333$

$\tan^{-1}0.5333=28.1°$

$\therefore$ C = 28.1°

2. A boy is standing on top of a building 40m tall watching an airplane in the sky. He estimates that the airplane is vertically above a point on the ground 800m from the base of the building. If the angle of elevation is 39°, and the boy is 1.6 tall. How high is the airplane above the ground?

Solution



In Δ ABC, BE = CD 800m

$$\text{Tan}=\frac{AE}{BE}$$

$$\text{Tan } 39° = \frac{AE}{800m}$$

$$0.8098 = \frac{AE}{800m}$$

AE= 0.8098cm ×800m

= 647.84m

AD = AE + ED

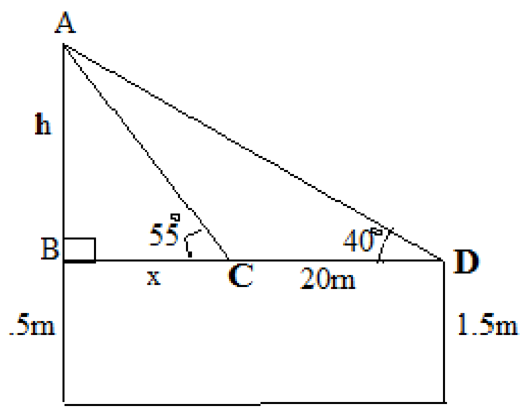
AD= 647.84 + 41.6

= 689.44m

∴The airplane is 689.44m above the ground

3. A girl is looking up at the top of the building. She measures the angle of elevation of the building as 40°. She walks 20m towards the building and finds that the angle of elevation is now 55°, the girl is 1.5m tall. How far was she from the building when she started? How tall is the building?

Solution



In Δ ABC; tan 55°$=\frac{h}{x}$

$h = x$ Tan 55°.........i)

In ΔABD; tan 40°$= \frac{h}{x+20}$

$h = (x + 20)$ Tan 40°......ii)

In i) put $h = x$ Tan 40°

x Tan 55°$= (x + 20)$ Tan 40°

$$1.4281x = 0.8391(x + 20)$$

$$1.4281x = 0.8391x + 16.7820$$

$$1.4281x - 0.8391x = 16.7820$$

$$0.5890x = 16.7820$$

$$x = \frac{16.7820}{0.5890}$$

$= 28.492$

$= 28.5$m

BD= 20m + 28.5m = 48.5m

In i) put x = 28.5m

$h = 1.4281 \times 28.5m$

= 40.7m

∴The girl was 48.5m when she started

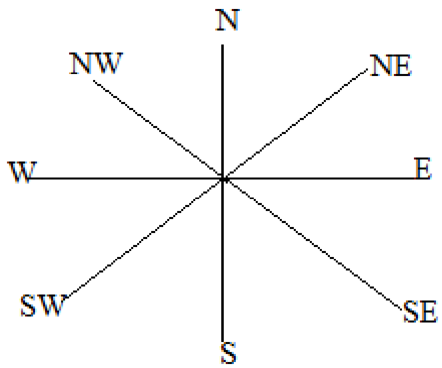
The height of the building = 40.7m + 1.5m

= 42.2m

BEARING

Bearing is the direction of something as compared to the other.

It uses cardinal points e. g North, South, East, West e. t. c



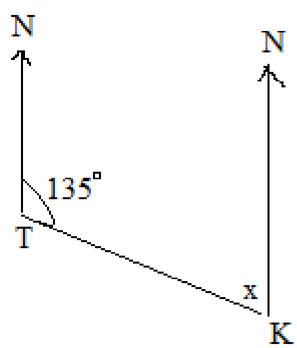
TWO TYPES OF BEARING

I. Three figure bearing
II. Campus bearing

THREE FIGURE BEARING

1. Measured in degrees
2. Uses three digits
3. If it is an acute angle, a zero may be added
4. It is measured from north clockwise

EXAMPLE



Find the bearing of

a. K from T
b. T from k

Solution

NB: we draw always north arrow at the point we need to find bearing.

∴ The bearing of K from T is 135°

b. Bearing of T from K

$$x + 135° = 180°\ allied\ angles$$

$$x = 180 - 135°$$

$$x = 45°$$

x+<K= 360° angles at a point

<K=360°-45°

= 315°

The bearing of T from K 315°

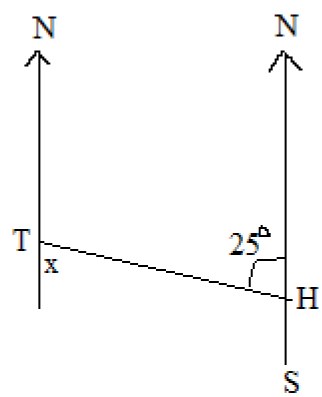
CAMPUS BEARING

1. It is based on cardinal points
2. The angle is measured from north or south(whichever is nearer) turning east or west.
3. An acute angle is always taken

EXAMPLE

Find the campus bearing in the diagram below

a. **H** from **T**
b. **T** from **H**



Solution

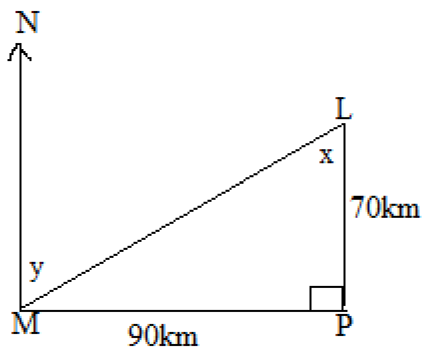
a. The bearing of **T** from **H** is **N25°W**
b. $x = 25°, alt\ angles$

∴ The bearing of **H** from T is **S25°E**

EXAMPLE

1. A ship sails 70km from Likoma Island and then 90km west. What is the bearing of Likoma Island from its new position?

   Solution

   NB: we always draw a north arrow at point of bearing.

   $$\text{Tan L} = \frac{90km}{70km}$$

   $= 1.2857$

   <L= 52.1°

   ∴ x= 52.1°

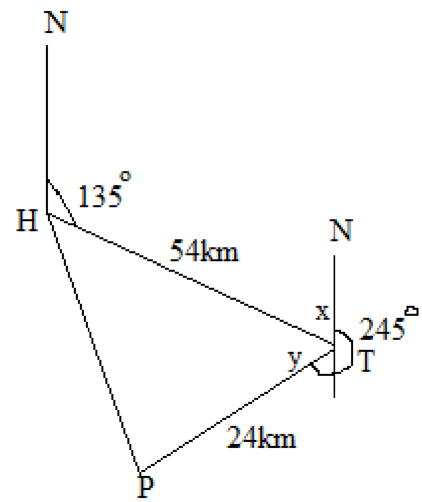
   $x = y$, alt angles

   $\therefore y = 52.1°$

   ∴ The bearing of Likoma Island from its new position is **052°** or **N52°E**

2. My mother leaves her home and travels on a bearing of 135°at a speed of 18km/h, after 3hrs she changes direction and continues on a bearing of 245° for another 24km. how far from home is she?

   Solution



$x + 155° = 180$, Allied angles

$x = 180° - 155$

$x = 25°$

$x + y + 245° = 360$, <s at a point

25°+245°+y = 360°

$y = 360° - 270°$

y=90°

∴ Δ HTP is a right angled triangle

∴ $HP^2 = HT^2 + TP^2$

$HP^2 = 24^2 + 54^2$

$HP^2 = 576 + 2916$

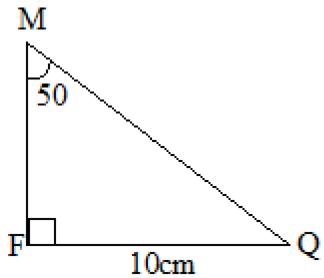
$HP = \sqrt{3492}$

$= 59.09$km

∴ She is 59.1km away from home

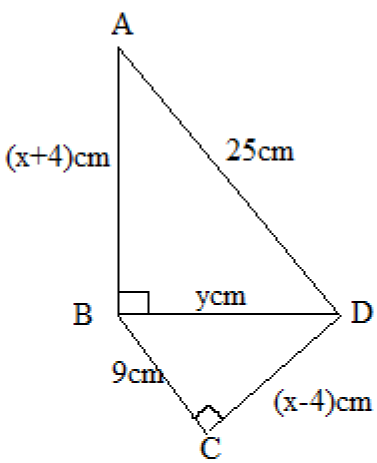
EXERCISE

1. In the figure, find **MF** and area of triangle **MFO**.



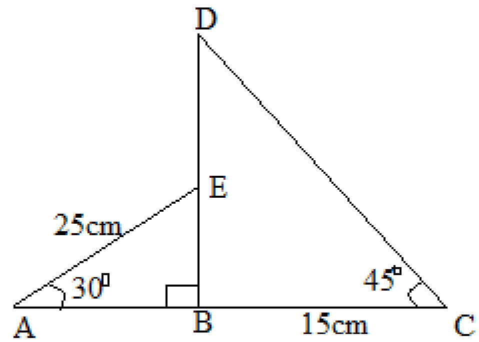
2. Town A is 56km from town B on a bearing of 205°
   a. Write down the bearing of B from A
   b. Town C is 34km from town A on a bearing of 115°
      I. Find the bearing of C from B
      II. Find the distance of C from B

3. The figure below ABD and BCD are right angled at B and C respectively. **AB= (x+4)cm, AD= 25cm, BD=y cm**, **BC=9cm** and **CD=(x-4).**



Find the value of x and y and angle CBD

4. In the figure below, DB is perpendicular to the line ABC, AE=25cm, BC= 15cm, angle EAB=30°, angle BCD=45°



Without using calculator, find the length of DE.

5. Mrs. Motto who is 1.6 tall stands 30m from a vertical tree and finds that the angle of elevation of the top of the tree is 40°. Find the height of the tree assuming the tree is perpendicular to the ground
6. Village **B** is on the bearing 135° and the distance of 40km from the village **C** is on a bearing of 225° and a distance of 62km from the village A.
   a. Show that **A,B** and **C** form a right angled triangle
   b. Calculate angle **ACB** to the nearest degree

CHAPTER 9

EXPONENTIAL AND LOGARITHM FUNCTIONS

$y = x^2$ It's an exponential function of the power 2
2 Is a power
x Is a base

LAWS OF INDICES

1. Any number to power 0 is equal to $1 a^{\circ} = 1$ e. g **$5^0 = 1$**
2. Any number to power 1 is the same as that number$a^1 = a$ e. g **$5^1 = 5$**
3. When multiplying numbers of the same bases, add powers e. g **$m^a \times m^b = m^{a+b}$**
4. When dividing numbers of the same base, subtract powers e. g **$m^a \div m^b = m^{a-b}$**
5. Any number with a negative power is the equal to the reciprocal of that number e. g **$m^{-a} = \frac{1}{m^a}$**
6. $m^{\frac{a}{b}} = (\sqrt[b]{m})^a$

SOLVING EXPONENTIAL FUNCTIONS

When solving exponential functions, it is necessary to identify an appropriate base from a given number and raise it to a certain power.

EXAMPLES

1. $2^x = 64$
2. $7^n = \sqrt{7}$
3. $8^x = 0.25$
4. $2^{3x} = 4^{-1}$

Solutions

1. $\mathbf{2^x = 64}$
   $2^x = 2^6$
   $\therefore x = 6$
2. $\mathbf{7^n = \sqrt{7}}$
   $7^n = 7^{\frac{1}{2}}$
   $\therefore n = \frac{1}{2}$
3. $\mathbf{8^x = 0.25}$
   $2^{3x} = \frac{25}{100}$
   $2^{3x} = \frac{1}{4}$
   $2^{3x} = 4^{-1}$
   $2^{3x} = 2^{-2}$
   $3x = -2$
   $x = \frac{-2}{3}$
4. $\mathbf{2^{3x} = 4^{-1}}$
   $2^{3x} = 2^{-2}$
   $3x = -2$
   $x = \frac{-2}{3}$

MORE EXAMPLES

1. Solve, $2^{2x} + 5 \times 2^x + 4 = 0$
   Solution
   $$2^{2x} - 5 \times 2^x + 4 = 0$$
   Let $2^x = y$
   $\therefore y^2 - 5y + 4 = 0$
   $(y-4)(y-4) = 0$
   $y = 4 \ \ or\ y = 1$
   But $2^x = y$
   $\therefore\ 2^x = 4$ or $2^x = 1$
   $2^x = 2^2$ $\quad$ $2^x = 2^0$
   $\therefore x = 2$ $\quad$ $x = 0$
2. Solve, $2^{2b+1} - 17\left(2^b\right) + 8 = 0$
   Solution
   $$2^{2b+1} - 17\left(2^b\right) + 8 = 0$$
   $$2^{2b}2^1 - 17\left(2^b\right) + 8 = 0$$

$$\textbf{Let } 2^b = x$$

$2x^2 - 17x + 8 = 0$

$(2x - 1)(x - 8) = 0$

$2x = 1 \quad or\ x = 8$

$x = \frac{1}{2}\, or\ x = 8$

But $2^b = x$

$\therefore 2^b = \frac{1}{2}$ **Or** $2^b = 8$

$2^b = 2^{-1} \qquad 2^b = 2^3$

$b = -1 \qquad\quad b = 3$

EXERCISE

1. Solve for m and n $2^{,m} \times 7^n = 392$
2. Solve, $3^{2x} - 6 \times 2^x + 9 = 0$
3. $(2^x)^2 - 9 \times (2^x) + 8 = 0$

LOGARITHM

$\log_a b = x$ It's a **logarithm function**

$\log_a b = x$ Is the same as $b = a^x$

$b = a^x$ It's an **exponential function**

RULES OF LOGARITHMS

1. $\mathbf{\log_a a = 1}$
2. $\mathbf{\log_x mn = \log_x m + \log_x n}$
3. $\mathbf{\log_m 1 = 0}$
4. $\mathbf{\log_x m^n = n \log_x m}$
5. $\mathbf{\log_x \frac{m}{n} = \log_x m - \log_x n}$

EXAMPLES

Express the following as a log of a single numbers

1. $\log_3 8 + \log_3 5$
   Solution

$$\log_3 8 + \log_3 5 = \log_3 8 \times 5$$
$$= \log_3 40$$

2. $\mathbf{\log_{10} 3 + \log_{10} 4 - \log_{10} 6}$
   Solution

$$\log_{10} 3 + 2\log_{10} 4 - \log_{10} 6$$
$$\log_{10} 3 + \log_{10} 4^2 - \log_{10} 6$$
$$\log_{10} \frac{3 \times 16}{6}$$
$$\log_{10} \frac{48}{6}$$
$$\log_{10} 8$$

3. Simplify $\frac{\log\sqrt{5}}{\log 25}$
   Solution

$$\frac{\log\sqrt{5}}{\log 2} = \frac{\log 5^{\frac{1}{2}}}{\log 5^2}$$
$$= \frac{\frac{1}{2}\cancel{\log 5}}{2\cancel{\log 5}}$$
$$= \frac{1/2}{2}$$
$$= \frac{1}{2} \div 2$$
$$= \frac{1}{4}$$

EVALUATING LOGARITHM

EXAMPLES

1. If $\mathbf{\log_{10} 2 = 0.301}$ and $\mathbf{\log_{10} 3 = 0.477}$. find the value of
   a. $\mathbf{\log_{10} 6}$
   Solution

$\log_{10} 6 = \log_{10} 3 \times 2$

$= \log_{10} 3 + \log_{10} 2$
$= 0.477 + 0.301$
$= 0.778$

b. $\mathbf{\log_2 5}$

Solution

$\text{Log } 5 = \log_{10} \frac{10}{2}$

$= \log_{10} 10 - \log_{10} 2$

$= 1 - 0.301$

$= 0.699$

2. Given that $\mathbf{\log_{10} 2 = 0.301}$ and $\mathbf{\log_{10} 3 = 0.477}$. without using calculator; find $\mathbf{\log_{10}(\tan 60°)}$.

Solution

In special angles, $\tan 60° = \sqrt{3}$

$\therefore \log_{10}(\tan 60°) = \log_{10} \sqrt{3}$

$= \log_{10} 3^{\frac{1}{2}}$

$= \frac{1}{2} \log_{10} 3$

$= \frac{1}{2} \times 0.477$

$= 0.2385$

3. If $\log_5 x = a$ and $\log_5 y = b$, express the following in terms of a and b. $\mathbf{\log_5 xy^3}$

Solution

$\log_5 xy^3 = \log_5 x \times y^3$

$= \log_5 x + \log_5 y^3$

$= \log_5 x + 3\log_5 y$

$= a + 3b$

4. Evaluate without using a calculator or four figure table. $\mathbf{\log_3 81}$

Solution

$\log_3 81 = y$

$\therefore 81 = 3^y$

$3^4 = 3^y$

$y = 4$

EXERCISE

Given that $\mathbf{\log_{10} 2 = 0.301}$ and $\mathbf{\log_{10} 3 = 0.477}$. evaluate

a. $\log_{10} 108$
b. $\log_{10} 0.5$
c. $\log_{10} 6$

3. Evaluate $\frac{1}{2} \log_{10} 25 - 2\log_{10} 3 + \log_{10} 180$. Without using a calculator.
4. $\log_8 \sqrt{2}$

SOLVING LOGARITHMS

EXAMPLE

Given that $\mathbf{\log_x 8 + \log_x 4 = 5}$, find the value of $\mathbf{x}$

Solution

$\log_x 8 + \log_x 4 = 5$

$\log_x 8 \times 4 = 5$

$\log_x 32 = 5$

$32 = x^5$

$2^5 = x^5$

$\therefore x = 2$

Solve the equation;

$$\mathbf{\log_{10}(2m + 6) = 1 + \log_{10}(m - 1)}$$

Solution

$$\log_{10}(2m + 6) = 1 + \log_{10}(m - 1)$$

$$\log_{10} 2m + 6 = \log_{10} 10 + \log_{10} m - 1$$

$$\text{Log}_{10} 2m + 6 = \log_{10} 10(m - 1)$$

$$2m + 6 = 10m - 10$$

$$6 + 10 = 10m - 2m$$

$$16 = 8m$$

$$\therefore m = 2$$

EXERCISE

Solve the following

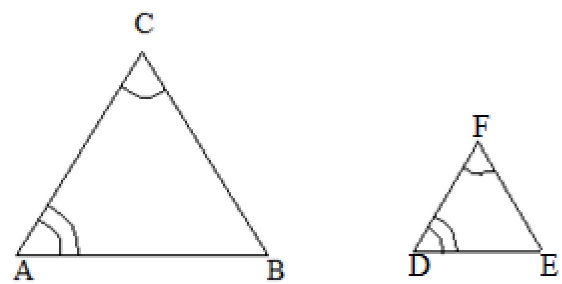
1. $\log_y 864 - \log_y 6 = 2$
2. $\log_7 343 = 2x - 5$
3. $\log_9 27^k = k + 1$
4. $\log_a 3x - \log_a x - 5 = \log_a 8$
5. Given that $\log p - \log q = 2\log r$. Find p in terms of q and r
6. Solve, $\mathbf{\log_3 w + \frac{3}{\log_3 w} = 4}$

CHAPTER 10

SIMILAR FIGURES

THEOREMS

1. If two triangles **are equiangular**, their corresponding sides **are proportional.**



Given: Δ ABC and Δ DEF, <A=<D, <C=<F

To prove: $\frac{AC}{DF} = \frac{BC}{EF} = \frac{AB}{DE}$

Proof: in Δs ABC and Δ DEF

<A=<E, given

<C=<D, given

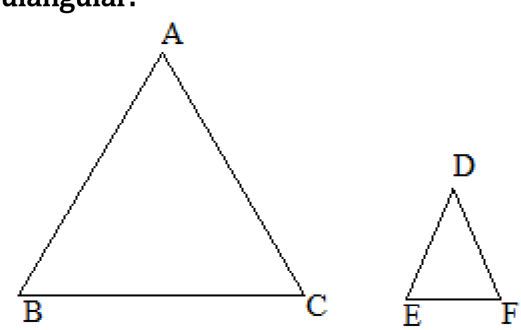
$\therefore$<B=<F, sum $< s$ in a Δ

$\therefore$ Δ ABC/// Δ DEF (a. a. a)

$\therefore \frac{AC}{DF} = \frac{BC}{EF} = \frac{AB}{DE}$

2. **Converse theorem**

If the corresponding sides of two triangles **are proportional**, then the triangles **are equiangular.**



Given: $\frac{AC}{DF} = \frac{BC}{EF} = \frac{AB}{DE}$

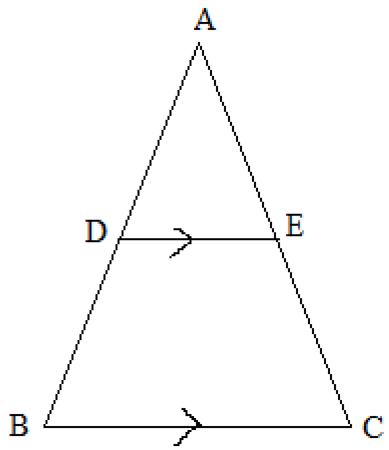
To prove=<A=<D, <B=<E,<C=<F

Proof: $\frac{AC}{DF} = \frac{BC}{EF} = \frac{AE}{DE}$, given

∴ Δ ABC///Δ DEF

∴<A=<D, <B=<E, and <C=<F

3. For any given triangle, if a parallel line is drawn from one source of the triangle to the other side, then the two triangles formed are similar.



Given: Δ ADE and Δ ABC, DE// BC

To prove: Δ ADE///Δ ABC

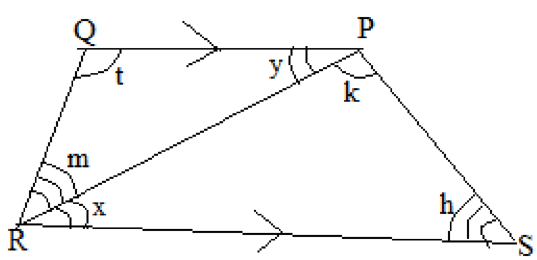
Proof: in Δ ADE and Δ ABC

: DE// BC

∴ Δ ADE///Δ ABC (a. a. a)

EXAMPLES

1. Figure below PQRS is a trapezium with PQ//SR and angle PQR=angle SPR. Prove that PQ×QR=PQ×RS



Solution

Given: Δ**PQR and** Δ**RPS**, PQ//SR, trapezium PQRS, <PQR=<SPR

To prove: PQ×QR=PQ×RS

Proof: in Δ**PQR** and Δ**RPS**

x = y, alt angles

t = k, given

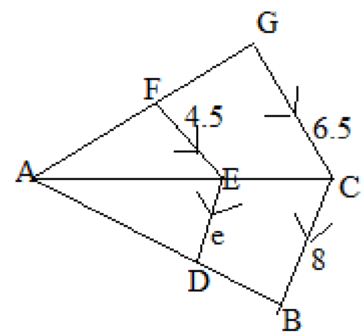
∴ m = h, sum angles in a Δ

∴ **ΔPQR /// ΔRPS, (a. a. a)**

$$\therefore \frac{QR}{PS} = \frac{PQ}{RP}$$

∴ PQ×QR=PQ×RS

2. Find the value of e in the figure below



Solution

In Δ**AGC** and Δ **AFE,**

GC// FE, given

∴ Δ AGC /// Δ AFE

$\therefore \frac{AG}{AF} = \frac{GC}{FE} = \frac{AC}{AE}$

$\frac{6.5}{4.5} = \frac{AC}{AE}$

In Δ**ABC** and Δ**ADE,**

BC// DE, given,

∴ **Δ ABC /// Δ ADE**

$\therefore \frac{AC}{AE} = \frac{8}{e}$

$\frac{6.5}{4.5} = \frac{8}{e}$

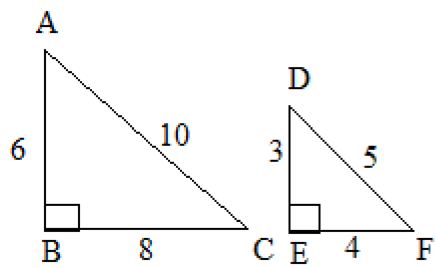
$6.5 \times e = 36$

$e = \frac{36}{6.5}$

$e = 5.5cm$

AREAS OF SIMILAR FIGURES

1. **Scale factor**
   Consider similar triangles below,



$\frac{AC}{DF} = AC: BF$

$\frac{10}{5} = 10: 5$

2:1 is a scale factor

In general the ratio of the corresponding sides of two similar figures will give the scale factor.

2. **Ratio of the areas of similar figures**

The ratio of the areas of two similar figures is equal to the ratio of the squares of their corresponding sides.

For example in the triangles ABC and DEF above;

Area of ΔABC$=\frac{1}{2} \times 8 \times 6$

$= 24cm^2$

Area in ΔDEF$=\frac{1}{2} \times 4 \times 3$

$= 6cm^2$

Area of ΔABC: area of ΔDEF$= 24: 6$

$= 4: 1$

And $AB^2: DE^2 = 6^2: 3^2$

$=36: 9$

$= 4: 1$

THUS $\frac{\text{area ABC}}{\text{Area DEF}} = \left(\frac{AB}{DE}\right)^2$

$\therefore$ Area factor $= 4: 1\ (2^2, 1)$

In general if the lengths of corresponding sides of two similar figures are in the ratio **k: 1, k²: 1** is called area factor and **k: 1** is called scale factor.

EXAMPLE

1. Triangles **ABC** and **XYZ** are similar, the size of triangle **ABC** are **6cm, 7cm**,and **8cm**. the shortest side of triangle **XYZ** is **2cm**. given that the area of triangle **XYZ** is **4.5cm²**.Calculate the area of triangle ABC
   Solution

$$\frac{\text{Area of}\Delta\text{ ABC}}{\text{Area of }\Delta\text{ XYZ}} = \left(\frac{6}{2}\right)^2$$

$$\frac{\text{Area of}\Delta\text{ ABC}}{4.5\text{cm}^2} = \frac{36}{4}$$

$$\text{Area of }\Delta\text{ ABC} = \frac{4.5cm^2 \times 366}{4}$$

$$= 40.5\text{cm}^2$$

2. The area of two similar triangles **ABC** and **XYZ** are in the ratio 1:16. If the height of the smaller triangle is **2cm**, calculate the height of the bigger triangle
   Solution

$$\frac{\text{Area of}\Delta\text{ ABC}}{\text{Area of }\Delta\text{ XYZ}} = \left(\frac{h1}{h2}\right)^2$$

$$\frac{1}{16} = \left(\frac{2}{h^2}\right)^2$$

$$\frac{\mathbf{1}}{\mathbf{16}} = \frac{\mathbf{4}}{\boldsymbol{h}^2}$$

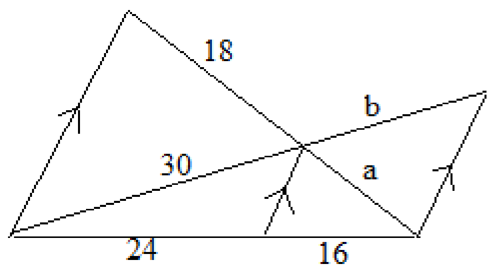
$$h^2 = 64$$

$$h = \sqrt{64}$$

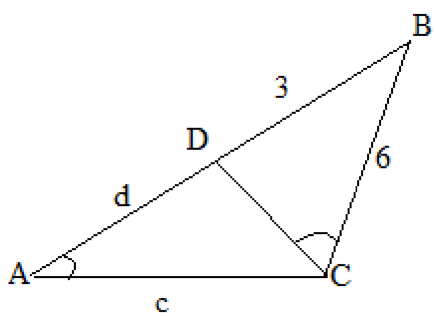
= 8cm

EXERCISE

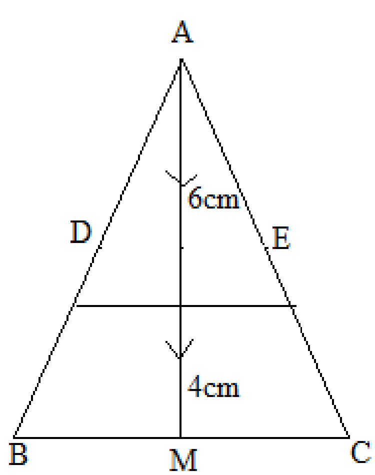
1. Find the value of the letters



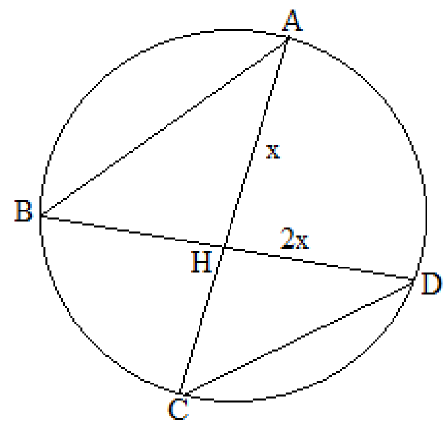
2.



3. Figure below shows two similar triangles ADE=12cm in which DE//BC. The area of triangle ADE=12cm^2. If the height of triangle ADE and trapezium DECB are 6cm and 4cm respectively. Calculate the length of BC



4. In the figure below, circle ABCD has chord AC and BD intersecting at H. length of line HD is twice that of HA.



Find the ratio of areas of triangle ABH to triangle CDH.

5. The areas of two similar triangles are **ABC** and **HKL** are **100cm²** and **256cm²** respectively. If the length of AB is 5cm, calculate the length of **HK** (MANEB).

CHAPTER 11

TRANSFORMATION

Transformation simply means complete change

TYPES OF TRANSFORMATIONS

1. Translation
2. Reflection
3. Rotation (out of syllabus)
4. Enlargement

1. TRANSLATION

To move from one place to the other.

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, this is a translation vector or displacement vector or column vector.

- 2 means move two spaces to the right (x)
- -4 means move 4 spaces downwards (y)

$$A \xrightarrow{\begin{pmatrix} 2 \\ -4 \end{pmatrix}} B$$

In general:

Coordinates + translation= coordinates

Of object of vector of image

EXAMPLE

1. A (0, 2) is a point on the Cartesian graph, find the coordinates of its image under translation vector (-3, -2).

   Solution

   C. Object + C. vector =C. image

   $$\begin{pmatrix} 0 \\ 2 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

   $$\begin{pmatrix} -3 \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

   ∴ The coordinates of image are (-3, 0)

2. A point **P (-2, 4)** is translated to point **p′**. If P is 5units down and three units to the right. Find the coordinates of **p′**

   Solution

   Object + Vector = Image

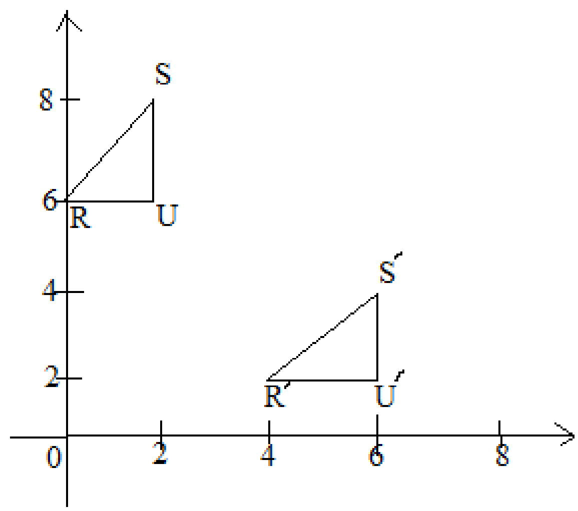
   $$\begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

   $$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

   ∴ p′ = (1, -1)

3. Triangle RSU has vertices R (0, 6), (2, 6), and (2,8). Draw the graph on the paper provided, triangle RSU and its image under translation vector $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$

   Solution



∴ R′= (4, 2), S′= (6, 4) U′= (6, 2)

EXERCISE

1. Point A (3, 4) is translated to A′= (7, 9). Calculate the translation vector
2. P (1, 2) has the image Q (-2, -3) under translation. Find the translation vector.

REFLECTION

Reflection means to turn over

To have reflection, you need to have a mirror (line of symmetry)

The distance from the object to the mirror must be equal to the distance from the image to the mirror.

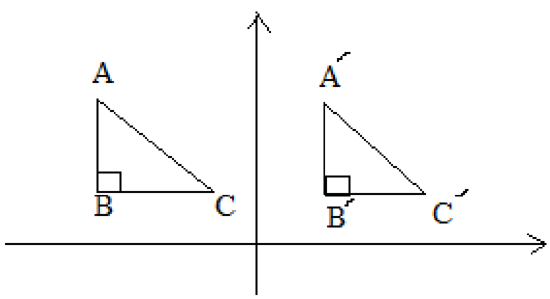
The line joining the object point and the image point must always be perpendicular to the mirror of symmetry

Reflection changes direction but not shape.

The line of symmetry is a line that divides an object into two equal parts such that the corresponding sides match each other when bent.

EXAMPLE

1. In the figure below, reflect triangle ABC in the y-axis



ENLARGEMENT

Means making something small or large

To enlarge a figure the following are used

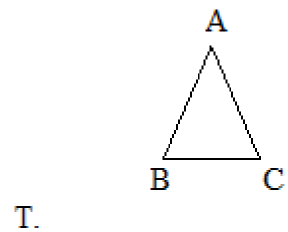
- Centre of enlargement
- Scale factor

PROCEDURES

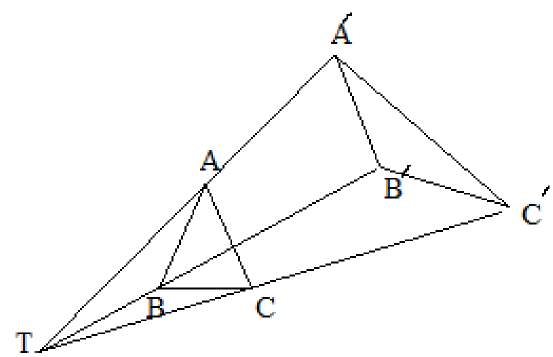
- Join Centre to each object's points
- Measure each length from the Centre to the object point
- Multiply each length by the scale factor to get the length from the Centre to image point
- When the scale factor is a fraction, the image will be smaller
- When the scale factor is negative, the image becomes upside down
- When scale factor is a whole number, the image will be larger than the object

EXAMPLES

Enlarge the figure below using a scale factor of 2. T is the center of enlargement.



Solution

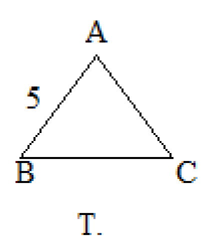


$TA=3 \quad \therefore TA'=3\times 2=6$

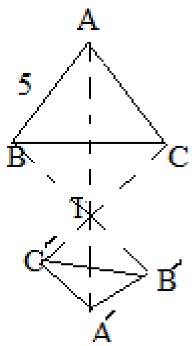
$TB=2 \quad \therefore TB'=2\times 2=4$

$TC=4 \quad \therefore TC'=4\times 2=8$

2. Enlarge triangle ABC using scale factor $-\frac{1}{2}$ and Centre T



Solution



NB: When the scale factor is negative

- The Centre is between image and object
- The image is upside down

HOW TO FIND SCALE FACTOR

Scale factor $=\frac{\text{length of image}}{\text{length of object}}$

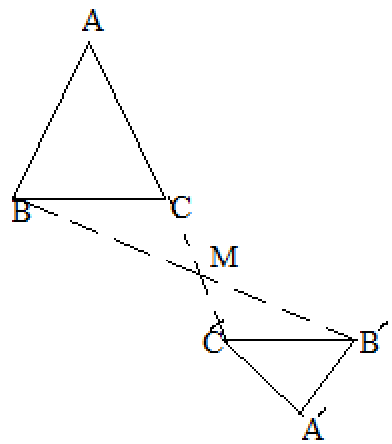
Or $=\frac{\text{line from image to the centre}}{\text{line from object to the centre}}$

NB: when finding scale factor consider the size of and shape (is it upside down or not?)

FINDING CENTRE

Corresponding points of the object and image, where the lines meet that is the Centre of enlargement.

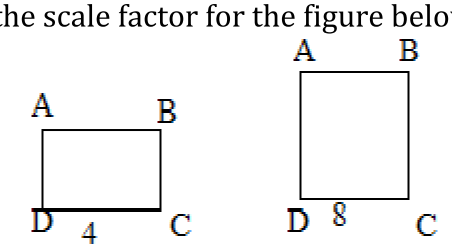
EXAMPLE



M is the Centre of enlargement and scale factor$=\frac{\mathbf{B'C'}}{\mathbf{BC}}$

EXERCISE

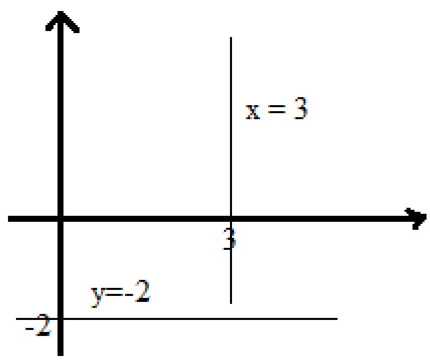
1. A triangle has vertices D(0, 2), E(4,6) and F(6, 2).
   a. Using the scale of 2cm to represent 2 units on both axes, draw triangle DEF on the graph paper provided
   b. On paper provided enlarge DEF using O (0,2) as a Centre
2. Find the Centre of the enlargement and the scale factor for the figure below.



CHAPTER 12

COORDINATE GEOMETRY

- $x -$ coordinates are those coordinates along x-axis
- $y -$ coordinates are those coordinates along y $-$axis
- All equations in terms of x $-$coordinates gives vertical lines.
- All equations in terms of y $-$coordinates gives horizontal lines



SKETCHING A STRAIGHT LINE GRAPH

When sketching a straight line graph you need to know coordinates of two points.

EXAMPLE

$y = 2x + 6$ (6 is a constant or y-intercept)

Sketching

a. You need to know where graph cuts y-axis and x-axis
b. The graph cuts the y axis where x=0
c. The graph cuts the x axis where y=0

So, $y = 2x + 6$

When x=0

$y = 2(0) + 6$

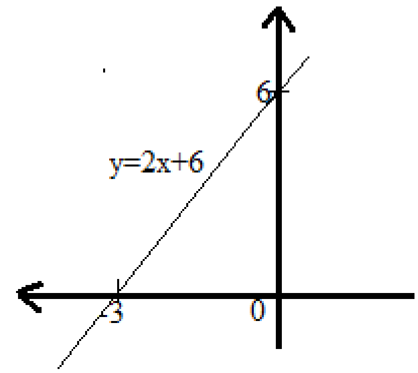
y=6

When y=0

$0 = 2x + 6$

$-6 = 2x$

$x = -3$



$\therefore$ -3Is x- intercept

GRADIENT (M)

Gradient is a slope of a line.

You can find gradient;

a. From a given equation
b. Using two coordinates
c. From a given graph

A. FROM A GIVEN EQUATION

We can find gradient from a given equation by making y subject of the formula. The coefficient of x is the gradient.

EXAMPLE

1. $2y - 3x = 0$

$2y = 3x$

$y = \frac{3}{2}x$

$\therefore$ Gradient$=\frac{3}{2}$

2. $y = 2 - x$

$\therefore$ Gradient = -1

B. GRADIENT FROM COORDINATES OF TWO POINTS

Gradient (m) $=\frac{\text{change in y}}{\text{change in x}}=\frac{y_2-y_1}{x_2-x_1}$

EXAMPLE

1. Find the gradient of a line passing through (0,-2) and (2,4)

Solution

(0,-2) (2, 4)

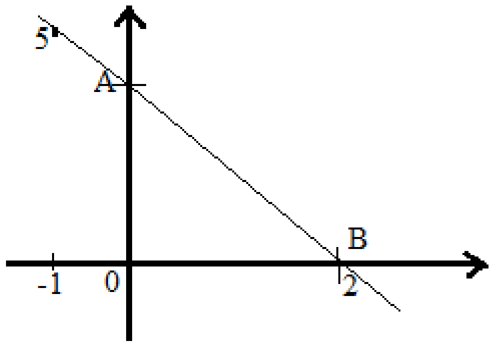
Gradient$=\frac{4-(-2)}{2-0}$

$=\frac{6}{2}$

C. **GRADIENT FROM A GIVEN GRAPH**

EXAMPLE

Find the gradient of the line AB below



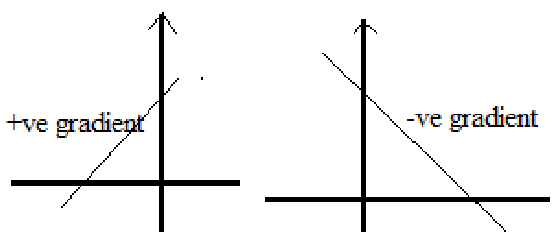
(2, 0) (-1, 5)

$\therefore$ m $=\frac{5-0}{-1-2}$

$=\frac{5}{-3}$

NB: when line slopes upwards from left to right, then the gradient is positive.

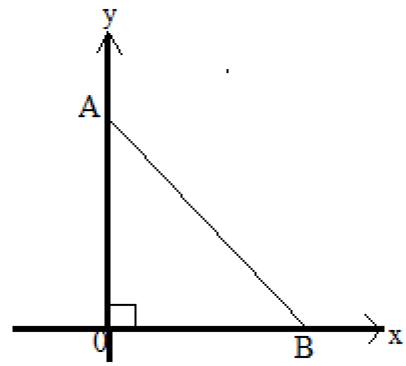
: Gradient will be negative if the line slopes from left to right.



The gradient of straight line is zero

The gradient of a vertical line is undefined

TANGENT OF AN ANGLE AND GRADIENT



Gradient AB$=\frac{A0}{B0}$

Tan B $=\frac{A0}{B0}$

$\therefore$ Gradient of AB= Tan of AB

In general, Tan θ= Gradient

FINDING EQUATION OF A STRAIGHT LINE

The equation can be found when given the following

a. **Gradient and y-intercept**
b. **Gradient and coordinate of a point**
c. **Coordinates of two points**

In general, equation for all linear equation is $y = mx + c$

Where, m = gradient

c = y-intercept

A. **Given gradient and y-intercept**

EXAMPLE

Find the equation of a line where gradient is -2 and y-intercept is 10.

Solution

$y = mx + c$

m =-2, c= 10

$y = -2x + 10$

B. Given a gradient and coordinates of a point
EXAMPLE

Find the equation of a straight line passing through (3, -4), where gradient is 2.

Solution

$\boldsymbol{y = mx + c}$

m= 2, y= -4, x=3

$\therefore -4 = 2(3) + c$

$-4 = 6 + c$

C= -10

But m = 2

$\therefore y = 2x - 10$

C. Given coordinates of two points

EXAMPLE

A straight line passes through points (3, 1) and (4, 2). Find the equation of the line

Solution

(3, 1) (4, 2)

$m = \frac{2-1}{4-3}$

$= 1$

m=1 y=1 x=3

$y = mx + c$

$1 = 1(3) + c$

$1 = 3 + c$

$1 - 3 = c$

$c = -2$

$\therefore y = x - 2$

EXERCISE

1. A straight line passes (3, 1) and (b, 2). Find the value of b if the gradient of the line is 1.
2. Find the gradient of a straight line whose equation is $\frac{y+2x}{4} = \frac{x}{3}$
3. A gradient of a straight line passing through (-2, 5) is $\frac{1}{2}$. Find the equation of the straight line.
4. Given that A (1,-1) and B (7,-9). Find the equation of the line AB

GRADIENT OF PARALLEL LINES

All parallel lines have got equal gradient.

EXAMPLE

A straight line $3y = x$ is parallel to a line which passes through a point (4, -5). Find the other equation.

Solution

$3y = x$

$y = \frac{x}{3} = \frac{1}{3}x$

$\therefore m = \frac{1}{3} \quad y = -5 \quad x = 4$

$\therefore y = mx + c$

$-5 = \frac{1(4)}{3} + c$

$-5 = \frac{4}{9} + c$

$c = \frac{-19}{3}$

$\therefore y = \frac{1}{3}x - 19$

2. A straight line $y + (b - 2)x + 10 = 0$ is parallel to $y = 2bx - 3$. find the value of b.

   Solution

$$y + (b-2)x + 10 = 0$$
$$y = -(b-2)x - 10$$
$$\therefore 2b = -(b-2)$$
$$2b = -b + 2$$
$$2b + b = 2$$
$$3b = 2$$
$$b = \frac{2}{3}$$

GRADIENT OF A PERPENDICULAR LINE

The gradient of perpendicular line is upside down negative of the other gradient.

Thus, $m_1 \times m_2 = -1$ or the product of gradient of perpendicular lines is - 1

EXAMPLE

A straight line passing through a point (7, 2) is perpendicular to line $4y = 3 - x$. Find the equation of this line, hence find the coordinates of the point of intersection of the two lines.

Solution

$4y = 3 - x$

$y = \frac{3}{4} - \frac{x}{4}$

$m = -\frac{1}{4}$

$m \times \frac{-1}{4} = -1$

$m = \frac{-1\times 4}{-1}$

$m = 4$

$y = mx + c$

Taking (7, 2)

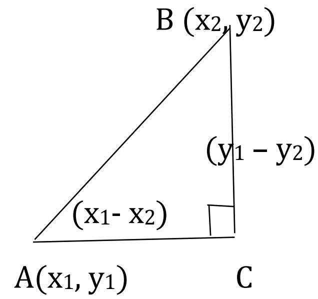
m= 4 x=7 y=2

$2 = 4(7) + c$

$c = -26$

$\therefore y = 4x - 26$

DISTANCE BETWEEN TWO POINTS



$AB^2 = AC^2 + BC^2$, Pythagoras theorem

$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

EXAMPLE

1. A straight line passes A (4, -2) and B(3, -1). Find the distance of the line AB.

Solution

$AB=\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

A (4, -2) B (3, -1)

$AB=\sqrt{(3-4)^2 + (-1-(-2))^2}$

$= \sqrt{1 + (-1+2)^2}$

$= \sqrt{1+1}$

$= \sqrt{2}$

EXERCISE

1. A straight line passes through point **A (b, -2**) and **B (3, -1).** Find the value of b if the distance of line AB is$\sqrt{\mathbf{2}}$**.**
2. Find the equation of a straight line which passes through (1,-2) perpendicular to $2x + 3y - 3 = 0$
3. Find the equation of the line

a. Through (3, 4) parallel to y=2x+5
b. Through (-2, 3) parallel to 6y + 2x – 5=0

4. The distance between M (3, 5) and N (8, a) is 13. Find the value of a.
5. Calculate the distance between x and y intercepts of the graph y = 12 – 2.4x
6. The straight line $2x + 3y + 5 = 0$ and $3x = -2y$ intersect at point M. without drawing graphs find the coordinates of M.

CHAPTER 13

LINEAR INEQUALITIES

A statement containing (=) sign is called an equation.

A statement containing a< or ≤ is called an inequality.

Symbol and their meanings

< (less than)

> (greater than)

≤ (less than or equal to)

≥ (greater than or equal to)

LINES TO USE

(<,>) Dotted line (--------------------)

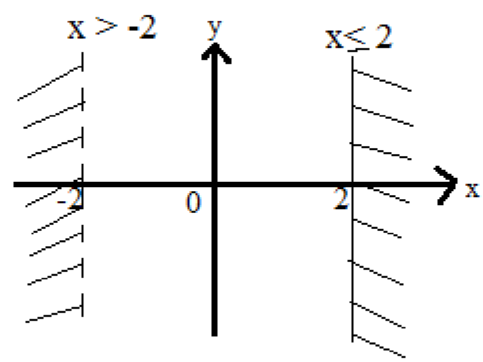
(≤, ≥) Solid line (——————)

When shading inequalities on the graph, shade the unwanted region.

EXAMPLE

Show the inequality $x > -2$ and $x \leq 2$ on the graph and shade on the unwanted region.

Solution

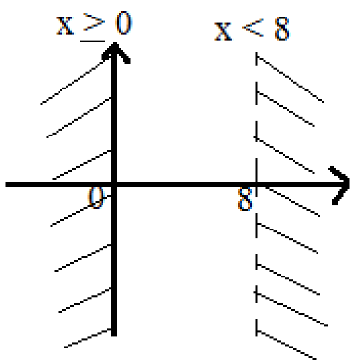


2. Sketch $0 \leq x < 8$

Solution

$$0 \leq x < 8$$

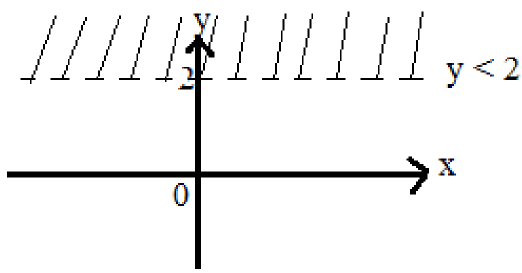
$$x < 8\,, \qquad x \leq 0$$



3. Sketch $y < 2$

Solution

$y < 2$



INEQUALITIES IN TWO VARIABLES

EXAMPLE

1. Sketch $y \leq 2x + 3$ in the graph

Solution

$y = 2x + 3$

When $x = 0$

$y = 2(0) + 3$

$y = 3$ (0, 3)

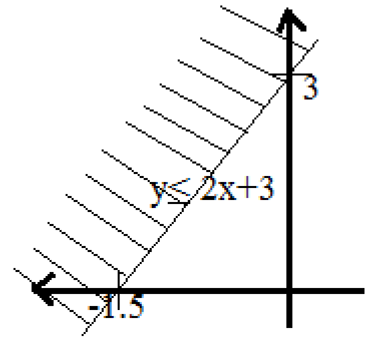
When $y = 0$

$0 = 2x + 3$

$-3 = 2x$

$x = -\frac{3}{2}$

= - 1.5 (-1.5, 0)



TWO OR MORE INEQUALITIES

Example

1. Sketch the region

Q $\{(x, y){:}3 < x + 3y \leq 6\}$

Solution

$3 < x + 3y$

$x + 3y \leq 6$

a. $\mathbf{3 < x + 3y}$

When $x = 0$

$3 = (0) + 3y$

$y = 1$ (0, 1)

When $y = 0$

$3 = x + 3(0)$

$x = 3$ (3, 0)

b. $\mathbf{x + 3y = 6}$

When $x = 0$

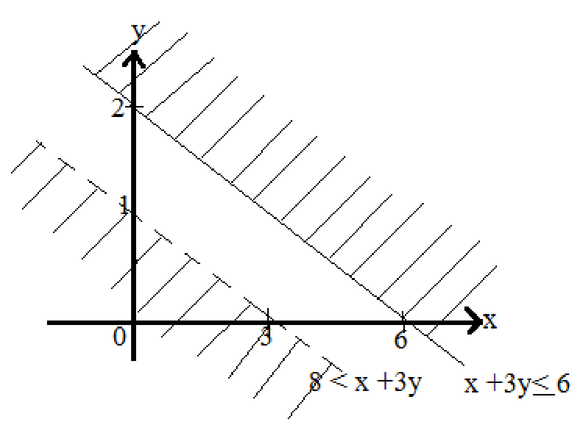
$(0) + 3y = 6$

$y = 2$ (0, 2)

When $y = 0$

$x + 3(0) = 6$

$x = 6$ (6, 0)



2. Sketch defined by the following inequalities giving coordinates and vertices
   a. $x \leq 4$
   b. $y \leq 5$
   c. $x + y \geq 3$

Solution

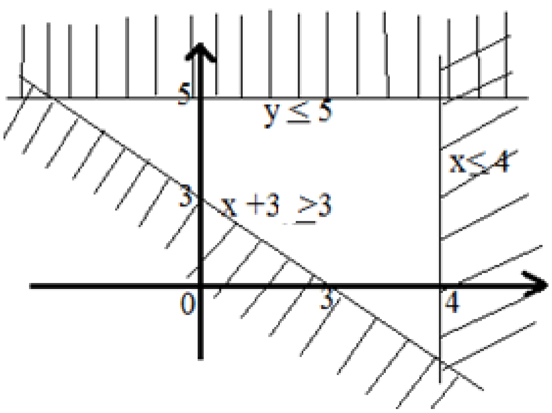
$x + y = 3$

$x = 0, (0) + y = 3$

$y = 3 \quad (0, 3)$

$y = 0\,, \; x + (0) = 3$

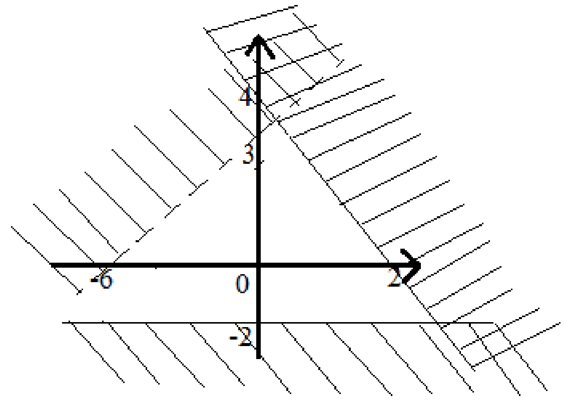
$x = 3 \quad (3, 0)$



FINDING EQUATIONS FROM A GIVEN GRAPH

EXAMPLE

Find three inequalities which describes the graph below.



Solution

1. $y \geq -2$
2. (2, 0) (0,4)
   $m = \frac{4-0}{0-2}$
   $m = \frac{4}{-2}$
   = -2
   Taking (0, 4)
   $y = mx + c$
   $4 = -2(0) + c$
   c =4
   $y = -2x + 4$
   $\therefore y \geq -2x + 4$
3. (-6,0) (0, 3)
   $m = \frac{3-0}{0-(-6)}$
   $m = \frac{3}{6} = \frac{1}{2}$

   Taking (-6, 0)

   $y = mx + c$
   $0 = \frac{1}{2}(-6) + c$
   $c = 3$
   $y = \frac{1}{2}x + 3$
   $\therefore y < \frac{1}{2}x + 3$

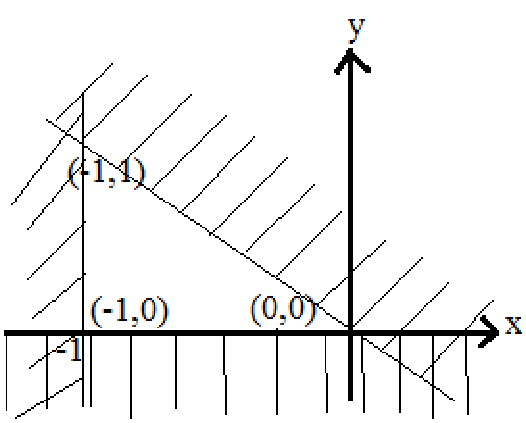
EXERCISE

1. M is the set of points (x, y) which satisfies the following inequalities

   i. $y-x\leq 1$
   ii. $2x\leq 5$
   iii. $5y> -4x$
   iv. $y\leq 2$

Show on the graph the graph the region represented by M

2. On the same axes, sketch the graphs of the regions described by the following inequalities.
   i. $x \geq 0$
   ii. $y \geq 0$
   iii. $y \leq 3x + 2$
   iv. $y + 4x < 8$
3. The fig below shows unshaded region bounded by inequalities. Write down three inequalities from the graph.



CHAPTER 14

VARIATIONS

To vary means to change.

TYPES OF VARIATIONS

1. Direct variations
2. Inverse variations(inverse)
3. Joint variations
4. Partial variations

DIRECT VARIATION

Variation in which increase in one variable results in the other variable, e. g

D ∝ T

Distance varies with time.

$\boldsymbol{D = kt}$, where $\boldsymbol{k}$ is constant.

EXAMPLES

i. If P is proportional to q and p=30 when q=6, find the constant proportional and find the value of p when q=24.

   Solution

   $P \propto q$

   $p = kq$

   p=30, q=6

   ∴30=k×6

   $\frac{30}{6} = k$

   k=5

   ∴ p=5q, when q=24

   P=5×24

   =120

ii. If P varies directly as the cube of q, and p=6 when q=2,find p when q=8

Solution

$P \propto q^3$

$p=kq^3$

p=6, q=2

$6=k\ (2)^3$

6=8k

$\frac{6}{8} = k$

$k = \frac{3}{4}$

$\therefore$ p$= \frac{3}{4}q^3$

$q = 8$

P$=\frac{3}{4}x8^3$

P$=\frac{3}{4} \times 8 \times 8 \times 8$

P=38.4

INVERSE VARIATIONS

This is the variation in which one quantity increases while the other quantity decreases.

EXAMPLES

1. If P varies inversely as q and p=4, when q=6, find p when q=8

Solution

p $\propto \frac{1}{q}$

p $=\frac{k}{q}$

p=4, q =6

$4=\frac{k}{6}$

$\therefore$ k=24

$\therefore$ p$=\frac{k}{q}$

q=8

$\therefore$ p$=\frac{24}{8} = 3$

2. The number of beats per minutes of a pendulum varies inversely as the square root of its length. An **81cm** long pendulum makes **24** beats per minutes. Calculate the number of beats per minutes of a **25cm** pendulum.

Solution

n $\propto \frac{1}{\sqrt{l}}$

n$=\frac{k}{\sqrt{l}}$

L=81cm, n= 24beats

$24=\frac{k}{\sqrt{81}}$

9x24=k

$\therefore$ k=216

$\therefore$n$=\frac{216}{\sqrt{l}}$

l=25, k=216

$\therefore\ n = \frac{216}{\sqrt{25}}$

$= \frac{216}{5}$

$= 43.2 beats$

JOINT VARIATIONS

This is the combination of variations

EXAMPLES

1. If x $\propto$ zy and x = 8 when y = 2 and z = 1. find x when y = 3 and z = 2

Solution

x $\propto zy$

x =kzy

x=8, z=1, y=2

$\therefore$ 8=k×1×2

$8 = 2k$

K=4

$\therefore$ x=4zy

$y = 3, z = 2$

$\therefore$ x=4×3×2

$= 24$

$x = 24$

2. Suppose that w varies directly as z^2 and inversely as xy, and that w=10 when x=15,y=2 and z=5.Find z when w=2,x=8 and y=27

Solution

$$w \propto \frac{z^2}{xy}$$

$$w = \frac{kz^2}{xy}$$

w =10, x=15, y=2, z =5

$$10 = \frac{k5^2}{15\times2}$$

$$10 = \frac{25k}{32}$$

300 = 25k

k=12

$$w = \frac{12z^2}{xy}$$

$$w = 2, x = 8 \ and \ y = 27$$

$$\therefore 2 = \frac{12\times z^2}{8\times27}$$

$$2 = \frac{12z^2}{216}$$

$432 = 12z^2$

$z^2 = 36$

$z = \pm\sqrt{36}$

$\therefore z = 6 \ or - 6$

PARTIAL VARIATION

Involves three set of numbers.

The first set gives first equations.

The second set gives second equations.

The third set gives answer to the equation.

EXAMPLES

1. a is partly constant and partly varies as b when **b=4**, **a=60** and when **b=12a =100**
   a) Find the relationship between a and b
   b) Find a when b=3

   Solution

   a). $a \propto c + b$

   a = c + kb

   a=60, b =4

   ∴ 6=c+4k………. (1)

   b=12, a=100

   ∴ 100=c+12k…….. (2)

   100=c+12k

   \- 60=c+4k

   40=8k

   K=5

   In equation 2 substitute with k=5

   60=c+4(5)

   60=c+20

   60-20=c

   40=c

   b). when b =3

   ∴a=40+5(3)

   a=40+15

   a=55

EXERCISE

1. The distance travelled by a lorry varies directly as the time taken for the journey. Find how far the lorry travels in 5hours if it can travel 86km in 1:15hour.
2. A varies jointly with b and with c, and a=12 when b=3 and c=2
   a) Find an equation which expresses a in terms of b and c.

c. Find the value of a when b=$\frac{1}{4}$ $and\ c = \frac{1}{2}$

3. A quantity b varies jointly with r and t, and b=108 when r=3 and t=6.Find an equation which expresses b in terms of r and t.

5. Given that x $\propto \frac{y}{z^2}$and x=12 when y=2 and z =1, Find y when z=2 and x=15.

6. The cost of running a private secondary school per is partly constant and partly varies as the square of the number of students enrolled in the school. If the school has 100 students, the cost per day is k4000 and if there are 160 students the cost per day is k7900.Calculate the cost of running the school if it has 240 students.

CHAPTER 15

STATISTICS

Statistics refers to a set of data such as average, mode, median standard deviation e. t. c

Data is the unprocessed information.

FREQUENCY TABLE

Construct a frequency table from the ages of form four students

20, 17, 15, 18, 19, 17, 17, 18, 20, 17, 19, 19, 18, 19, 17, 18, 17, 18, 18, 20, 17, 21, 20, 19, 17, 17, 18 ,17, 16, 16, 16, 16, 20, 18, 17, 19, 20, 19, 19, 18, 18, 19, 21, 18, 17 ,21, 18, 18, 19, 18, 18

Solution

Score	Tally	Frequency
15	/	1
16	///	3
17	~~////~~ ~~////~~ /	11
18	~~////~~ ~~////~~ ~~////~~ //	17
19	~~////~~ ~~////~~ /	11
20	~~////~~ //	7
21	//	2
		$\sum f = 52$

Ways of showing data from the frequency table

1. Histogram
2. Frequency polygon

HISTOGRAM

- ✓ It uses rectangles/ bars.
- ✓ The area of a triangle is proportional to frequency
- ✓ The width of rectangles are equal in each bar.

- ✓ It must have a title and the axes must be labeled

PIE CHART

A pie chart is a circular way of showing data in relation to the whole data.

The circle represents the total, each slice represents the named parts of the data.

EXAMPLE

1. At a certain political meeting, a researcher observed that out of 90 people, 30 people had black shirts, 20 had blue shirts, 10 had white shirts, 5 had no shirts. Show the information on a pie chat.

Solution

90 people represent 360°

Black shirts = 30 people

$$=\frac{30\ \times 360°}{90}$$

$= 120°$

Yellow shirts = 20 people

$$=\frac{20\ \times 360°}{90}$$

$= 80°$

Blue shirts =25 people

$$=\frac{25\ \times 360°}{90}$$

$= 100°$

White shirts = 10 people

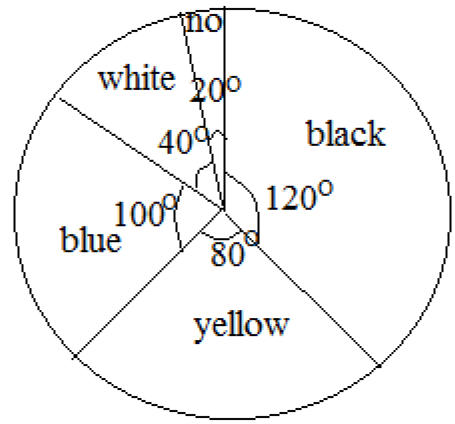
$$=\frac{10\ \times 360°}{90}$$

$= 20°$

No shirt= 5people

$$=\frac{5\times 360^{o}}{90^{o}}$$

$= 20°$



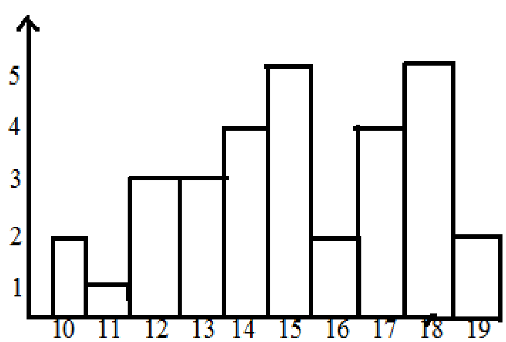
1. A teacher was concerned that her pupils were taking too long to answer examination questions. She asked them to record how long it took them to answer a certain questions. These are the times in minutes that they told her.
   10, 15, 18, 17, 15, 13, 18, 19, 12, 15, 17, 18, 13, 14, 12, 14, 14, 18, 19, 17,15, 17, 15, 13, 18, 19, 12, 11, 10, 14, 16, 16
   a. Draw up a frequency table for this information
   b. Draw a histogram to show this information
   c. Examination question is worth 10% of the 2hours paper.
      1. How long should each student be spending on the questions
      2. What percentage of the students are spending on the questions

Solution

a.

Score (x)	tally	Frequency
10	//	2
11	/	1
12	///	3
13	///	3
14	////	4
15	~~////~~	5
16	//	2
17	////	4
18	~~////~~	5
19	///	3
		$\sum f = 32$

a. TIME TAKEN FOR A STUDENT TO ANSWER A QUESTION



b. 1. 2hour = 120 minutes

$$\therefore 10\% = \frac{10}{100} \times 120$$

$\underline{= 12 \text{ minutes}}$

∴ Each student should spend 12 minutes for a question

12 minutes = 10%

2. Greater than 12 min= more

People who solve in greater than 12 minutes

$= 32 - 6 = 26$

$= \frac{26}{32} \times 100$

$\underline{= 81.25\%}$

GROUPED DATA

When we have a large amount of data, it is useful to group the data into classes

This makes data display and calculations simple.

Things to be known

I. Class (group)
II. Class limits
III. Class boundaries
IV. Class mid points
V. Class width

EXAMPLE

The following are weights of 30 pupils (in kg)

45, 62, 35, 54, 48, 55, 48, 59, 52, 40, 54, 46, 59, 51, 32, 37, 49, 42,53, 38, 37, 35, 53, 46, 48, 44, 33, 52, 54, 44

a. Using classes of 30- 34, 35- 39, 40- 44 and so on, construct a frequency table for this data.
b. For the second class (35- 39), write down the lower and upper class boundaries and hence find class width.
c. What percentage of pupils had their weight at least 52 kg?

Solution

30- 34 or 35- 39 are class limits

30 or 35 are lower class limits

34 or 39 are upper class limits

NB; To find class boundaries, subtract 0.5 from lower class limits and add 0.5 to upper class limits. E. g for 30- 39, 29.5 is a lower class boundary and 34.5 is the upper class boundary

i.e. $30 - 0.5 = 29.5$

$34 + 0.5 = 34.5$

Class width

Is the difference between the upper and lower class boundary.

I.e. for 30- 34

Class width = 34.5 – 29.5

= 5

Class mid points

Mid-point= $\frac{\text{upper class b+lower class b}}{2}$

i.e. $\frac{34.5 + 29.5}{2}$

$= \frac{64.0}{2}$

= 32

a. **Frequency table**

Score (x)	Tally	Frequency(f)	Mid-point
30- 34	//	2	32
35- 39	~~////~~ /	6	37
40- 44	////	4	42
45- 49	~~////~~ //	7	47
50- 54	~~////~~ ///	8	52
55- 59	//	2	57
60- 64	/	1	62
		$\sum f = 30$	

a. For 35- 39
   Upper class boundary = 39.5
   Lower class boundary= 34.5
   ∴ **Class width**= 39.5- 34.5
   = 5
b. At least means ≥
   11 pupils had ≥ 52 kg

$$\therefore \text{percentage} = \frac{11}{30} \times 100\%$$

= 36.7%

MEAN (AVERAGE), MODE AND MEDIAN

1. **MEDIAN**

A number in the middle of a given scores.

Odd number of scores

To find the median scores first arrange the number of scores in either ascending or descending order.

Formula for finding position for given odd numbers is $\left(\frac{n+1}{2}\right)^{\text{th}}$

EXAMPLE

Find the median for 5, 6, 7, 7, 8

Solution

5, 6, 7, 7, 8

$n = 5$ (Odd number)

Median $= \frac{5+1}{2} = 3^{\text{rd.}}$ position

∴ Median = 7

Even number of scores

Arrange scores in ascending or descending older.

Formula for finding **position** for a given even number of scores is $\frac{\left(\frac{n}{2}\right)th + \left(\frac{n}{2}+1\right)th}{2}$

EXAMPLE

Find the median given the following ages

5, 5, 6, 7, 7, 8

Solution

5, 5, 6, 7, 7, 8

$n = 6$ (Even number)

Median $=\frac{6}{2}=3^{rd.}$ position

$= 6$

Median $=\frac{6}{2}+1= 4^{th.}$ Position

$= 7$

$\therefore$ Median $=\frac{6+7}{2}$

$= 6.5$

2. **MODE**

Score with highest frequency

EXAMPLE

For the following set of data find the mode

2, 2, 5, 8, 6, 2, 3, 4, 4

Solution

Mode= 2

3. **MEAN (average)**

   The symbol for mean is $\bar{x}$

   EXAMPLE

   1. Find mean of 2, 7, 8, 3, 10

      Solution

$$\text{Mean} = \frac{2+7+10+8+3}{5}$$

$$= \frac{30}{5}$$

$$= 6$$

MEAN FOR GROUPED DATA

The formula for finding mean of grouped data.

Mean $(\bar{x}) = \frac{\sum fx}{\sum f}$

EXAMPLE

1. The following marks were obtained by class at Nguludi Secondary school in a mathematics examination marked out of 30.

   30, 26, 29, 23, 25, 26, 24, 26, 27, 26, 22, 24, 27, 28, 25, 27, 26, 28, 26, 26, 28, 25, 24, 25, 24, 28, 26, 25, 27, 26

   a. Work out mean, median and mode for this data.
   b. Students which obtained more than 87% in the test received a small prize. What percentage of students received a prize?

   Solution

First draw the frequency table

Score (x)	Tally	Freq. (f)	fx
22	/	1	22
23	/	1	23
24	////	4	96
25	~~////~~	5	125
26	~~////~~ ///	8	208
27	////	4	108
28	~~////~~	5	140
29	/	1	29
30	/	1	30
		$\sum f = 30$	$\sum fx = 781$

a. Mean $(\bar{x})=\frac{\sum fx}{\sum f}$

$= \frac{781}{30}$

$= 26.3$

Median $= \frac{26+26}{2}$

$= 26$

Mode = 26

b. 30 marks= 100%

y = 87%

$$y = \frac{30 \times 87\%}{100\%}$$

= 26.1

∴ Students who get more than 87% are 11

(Those above 26.1marks)

∴ Percentage $= \frac{11}{30} \times 100\%$

= 36.7%

EXERCISE

1. Height of students at Nguludi secondary school in form four are;

Height (cm)	Number of students
60- 65	7
65- 70	11
70- 75	17
75- 80	20
80- 85	16
85- 90	9

Find the mean height for the class

2. Find the mean from the following scores

Score	1	2	3	4	5	6
Frequency	26	16	21	15	24	18

3. Find mean, mode and median for 2, 2, 1, 2, 7, 10, 20, 11.

CHAPTER 16

QUADRATIC GRAPHS

Examples of quadratic graph expressions are;
$y = ax^2$, $y = 6x^2 - 5x - 2$

WHAT DO WE NEED TO KNOW

i. How to copy and complete the tables of the quadratic expressions
ii. How to draw graph of quadratic expressions
iii. The minimum and maximum values of quadratic equations on the graph
iv. How to find solutions of quadratic equations using the graph
v. How to solve simultaneous Equations using the graph
vi. How to find the turning points of the quadratic graph
vii. Line of symmetry
viii. How to come up with the quadratic equations from the given graph

In general if the coefficient of quadratic expression is negative the parabola curves downwards(∩-shaped) if it is positive, it curves upwards (U-shaped)

(∩-shaped) Parabola gives maximum values

(U-shaped) parabola gives minimum values

- The line of symmetry pass at the turning point.

EXAMPLE

Table below shows some values of the equation, $y = x^2 + x - 2$

x	-4	-3	-2	-1	0	1
Y	10		0	-2		0

i. Copy and complete the table

ii. Using the scale of 2cm to represent 2 units on the y-axis and 2cm to represent 1 unit on the x-axis, draw the graph
$y = x^2 + x - 2$

iii. Use your graph to solve $x^2 + 2x - 3=0$

Solutions

$y = x^2 + x - 2$
$= (-3)^2 + (-3) - 2 = 4$
$y = x^2 + x - 2$
$= (0)^2 + (0) - 2 = -2$

$$\boldsymbol{y = x^2 + x - 2}$$

x	-4	-3	-2	-1	0	1
Y	10	4	0	-2	-2	0

TURNING POINT

The turning point is found by using

i. Completing the square method

ii. Calculus

In the above equation,

$$x^2 + x - 2$$

$$\left(x^2 + x + \left(\frac{1}{2}\right)^2\right) - \left(\frac{1}{2}\right)^2$$

$$\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} - 2$$

$$\left(x + \frac{1}{2}\right)^2 - \frac{1-8}{4}$$

$$\left(x + \frac{1}{2}\right)^2 - \frac{9}{4}$$

$\therefore$ Turning point $= x = -\frac{1}{2}$

$= -0.5$

$-\frac{9}{4} = -2.25$

$(-0.5, -2.25)$

Using calculus,

NB: If $y = x^n$ then $\frac{dy}{dx} = nx^{n-1}$

$\frac{dy}{dx} = 2x + 1 = 0$

$2x = -1$

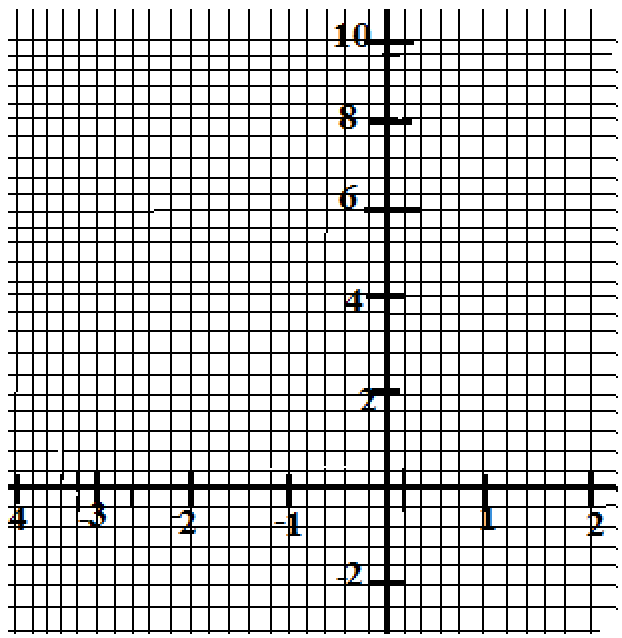
$x = -0.5$

Put $x = -0.5$ in the equation

$y = (-0.5)^2 - 0.5 - 2$

$= -2.25$

$(-0.5, -2.25)$



iii. From the graph,

$x = -2$ *or* $x = 1$

iv. We subtract the new equation from the original equation one, then we plot the new expression after finding the y-intercept and x-intercept. Where the line intersect the cubic line on the graph gives the solution.

$$y = (x^2 + 2x - 2) - (x^2 + x - 3)$$
$$= x^2 + 2x - 2 - x^2 - x + 3$$
$$y = x + 1$$

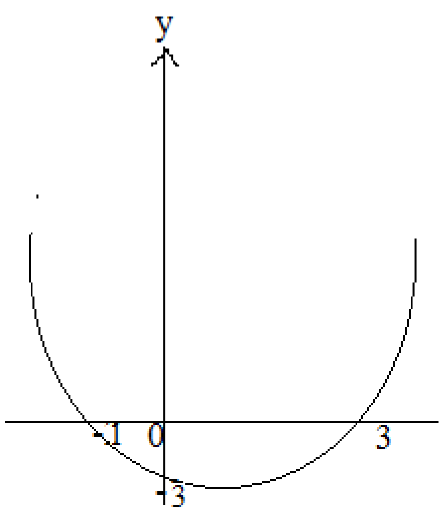
When y=0, $x + 1 = 0$
$x = -1$ (-1, 0)

When $x = 0, y = 0 + 1$
$y = 1$ (0, 1)
$\therefore$ from the graph, $x =$ $y =$

HOW TO FIND EQUATION OF THE GRAPH

EXAMPLE

i. Find the equation of the graph below in terms of $y = ax^2 + bx + c$



Solution

$x = -1 or\ x = 3$

$x + 1 = 0,\ x - 3 = 0$

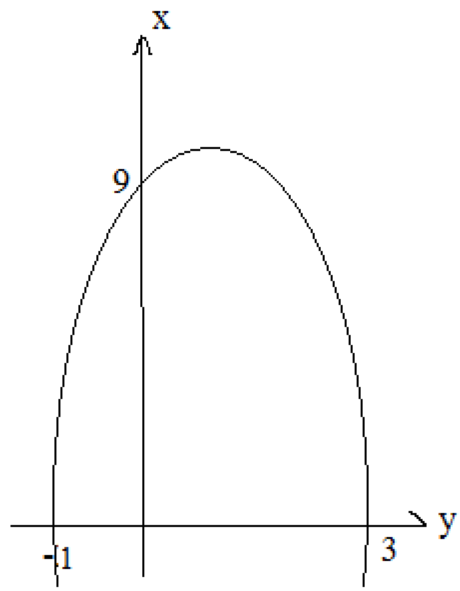
$(x + 1)(x - 3) = 0$

$x^2 - x - 3 = 0$

$\therefore y = x^2 - x - 3$

EXAMPLE 2.

The figure below shows the graph of equation, $y = ax^2 + bx + c$



Find the equation of the graph

Solution

$x = -1\ or\ x = 3$

$x + 1 = 0 \quad x - 3 = 0$

$(x + 1)(x - 3) = y$

$x^2 - 3x + x - 3 = y$

$x^2 - 2x - 3 = y$

Divide -3 by y $-$ intercept $= \frac{9}{-3}$ $=-3$ and the answer multiply the equation

$-3(x^2 - 2x - 3) = y$

$-3x^2 + 6x + 9 = y$

$9 + 6x - 3x^2 = y$

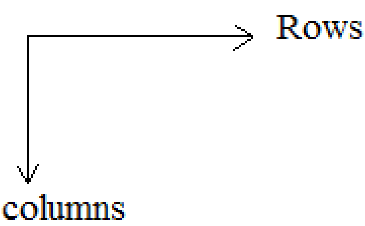
BOOK 4

MATHEMATICS IS A SYSTEMATIC WAY OF WASTING OUR TIME

CHAPTER 1

MATRIX

Matrix is an array of numbers i. e rows and columns of numbers.



e. 5 packets of sugar at k10 each

6 under wears at k30 each

8 sweets at k5 each

This data can be represented in matrices form as

$$\begin{bmatrix} 5 & 6 & 8 \end{bmatrix} \begin{bmatrix} 10 \\ 30 \\ 5 \end{bmatrix}$$

5, 6, 8, 10, 30, 5 are components or elements

<u>NAMING MATRICES</u>

When naming matrices, we start with counting number of rows then number of columns.

1. **ROW MATRIX**

$\begin{bmatrix} 2 & 4 & 7 \end{bmatrix}$, this is a 1×3 matrix

2. **COLUMN MATRIX**

$\begin{bmatrix} 10 \\ 7 \\ 0 \end{bmatrix}$, this is a 3×1 matrix

3. **ZERO MATRIX**

A zero matrix is a matrix which has zeros as its elements.

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

4. **SQUARE MATRIX**

It's a matrix which has equal number of rows and columns

e. g $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

ADDITION OF MATRICES

When adding matrices together, add corresponding elements together.

Only those matrices of the same order can be added together.

EXAMPLE

1. Given that $\mathbf{A}=\begin{bmatrix} 4 & -9 \\ 6 & 7 \end{bmatrix}\mathbf{B}=\begin{bmatrix} 8 & 22 \\ -5 & 0 \end{bmatrix}$
   Find $A + B$.
   Solution

$\mathbf{A+B}=\begin{bmatrix} 4 & -9 \\ 6 & 7 \end{bmatrix}+\begin{bmatrix} 8 & 22 \\ -5 & 0 \end{bmatrix}$

$=\begin{bmatrix} 4+8 & -9+22 \\ 6+(-5) & 7+0 \end{bmatrix}$

$=\begin{bmatrix} 12 & 13 \\ 1 & 7 \end{bmatrix}$

SUBTRACTION OF MATRICES

Corresponding elements must be subtracted together.

Only those matrices of the same order can be subtracted together.

EXAMPLE

1. Given that
   $A=\begin{bmatrix} 3 & 2 & 5 \\ 1 & -2 & 4 \end{bmatrix}B=\begin{bmatrix} -1 & 0 & -2 \\ 3 & -2 & -1 \end{bmatrix}.$
   Find A – B

Solution

$A - B=\begin{bmatrix} 3 & 2 & 5 \\ 1 & -2 & 4 \end{bmatrix}-\begin{bmatrix} -1 & 0 & -2 \\ 3 & -2 & -1 \end{bmatrix}$

$\mathbf{=}\begin{bmatrix} 3-(-1) & 2-0 & 5-(-2) \\ 1-3 & -2-(-2) & 4-(-1) \end{bmatrix}$

$=\begin{bmatrix} 4 & 2 & 7 \\ -2 & 0 & 5 \end{bmatrix}$

2. Given that
   $\mathbf{A}=\begin{bmatrix} 1 & 2 & c \\ 2 & 4 & 5 \\ 3 & d & 8 \end{bmatrix}\mathbf{B}=\begin{bmatrix} 4 & 3 & -1 \\ 3 & 2 & b \\ 2 & 2 & -4 \end{bmatrix}\mathbf{C}=\begin{bmatrix} 5 & a & 4 \\ 6 & 6 & 1 \\ 5 & 0 & e \end{bmatrix}.$
   Find **a, b, c, d** and so that $\mathbf{A+B=C}$

Solution

$$\begin{bmatrix} 1 & 2 & c \\ 2 & 4 & 5 \\ 3 & d & 8 \end{bmatrix}+\begin{bmatrix} 4 & 3 & -1 \\ 3 & 2 & b \\ 2 & 2 & -4 \end{bmatrix}=\begin{bmatrix} 5 & a & 4 \\ 6 & 6 & 1 \\ 5 & 0 & e \end{bmatrix}$$

$\begin{bmatrix} 5 & 5 & c-1 \\ 6 & 6 & 5+b \\ 5 & 2+d & 4 \end{bmatrix}=\begin{bmatrix} 5 & a & 4 \\ 6 & 6 & 1 \\ 5 & 0 & e \end{bmatrix}$

$\underline{\therefore \boldsymbol{a}=5}$

$5+\boldsymbol{b}=1$

$\boldsymbol{b}=1-5$

$\underline{\therefore \boldsymbol{b}=-4}$

$\boldsymbol{c}-1=4$

$\boldsymbol{c}=4+1$

$\underline{\therefore \boldsymbol{c}=5}$

$2+\boldsymbol{d}=0$

$\underline{\therefore \boldsymbol{d}=-2}$

$\underline{\therefore \boldsymbol{e}=4}$

MULTIPLICATION OF MATRIX BY A NUMBER (SCALAR)

If $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and k is the number then,

$KA=k\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$

EXAMPLE

1. Given the following matrices,

A=$\begin{bmatrix}3 & 1 & 0\\2 & 5 & 6\end{bmatrix}$**B**=$\begin{bmatrix}2 & -3 & 2\\1 & 2 & -3\end{bmatrix}$. Calculate matrix **5A- 3B**

Solution

5A- 3B = $5\begin{bmatrix}3 & 1 & 0\\2 & 5 & 6\end{bmatrix} - 3\begin{bmatrix}2 & -3 & 2\\1 & 2 & -3\end{bmatrix}$

$= \begin{bmatrix}15 & 5 & 0\\10 & 25 & 30\end{bmatrix} - \begin{bmatrix}6 & -9 & 6\\3 & 6 & -9\end{bmatrix}$

$= \begin{bmatrix}9 & 14 & -6\\7 & 19 & 39\end{bmatrix}$

MALTIPLYING MATRICES TOGETHER

To multiply matrices together, we multiply the first number in the row matrix by the first number in the column matrix.

A second number in the raw matrix must multiply by the second number in the column matrix and so on.

Add the products together.

NB: we can only multiply matrices together if the number of columns in the first matrix is equal to the number of rows in the second matrix.

EXAMPLE

Given that V=$\begin{bmatrix}3 & 5\\-1 & 1\end{bmatrix}$ and W= $\begin{bmatrix}-1 & 1\\0 & 2\end{bmatrix}$. Find VY.

Solution

VY= $\begin{bmatrix}3 & 5\\-1 & 1\end{bmatrix}\begin{bmatrix}-1 & 1\\0 & 2\end{bmatrix}$

$=\begin{bmatrix}3\times(-1)+5\times 0 & 3\times 1+5\times 2\\-1\times(-1)+1\times 0 & -1\times 1+1\times 0\end{bmatrix}$.

$=\begin{bmatrix}-3\times 0 & 3+10\\1+0 & -1+2\end{bmatrix}$.

$= \begin{bmatrix}-3 & 13\\1 & 1\end{bmatrix}$.

WORD PROBLEMS

EXAMPLE

1. A shop was selling skirts for K3000 each and dresses at K5000 each.
   Bertha and **Taonga** bought some clothes listed in the table.

Bertha	3	2
Taonga	4	5

   Find how much each person spent on the clothes

   Solution

   $=\begin{bmatrix}3 & 2\\4 & 5\end{bmatrix}\begin{bmatrix}3000\\5000\end{bmatrix}$.

   $= \begin{bmatrix}9{,}000 + 10000\\12{,}000 + 25000\end{bmatrix}$

   $= \begin{bmatrix}19{,}000\\37{,}000\end{bmatrix}$

   ∴ Bertha spent K19,000
   Taonga spent K 37,000

2. Mphatso wants to buy 3 pencils, 2 rulers and 4 ball pens. Bali wants to buy 1 pencil, 1 ruler and 8 ball pens. In Zomba pencils and rulers costs K60 each and ball pens cost K20 each. In Mzuzu pencils and rulers costs K80 each and ball pens cost K30 each.
   Use matrix multiplication to find how much more these items will cost Mphatso and Bali in Mzuzu

   Solution

	Pencil	ruler	pen	Zomba	Mzuzu
Mphatso	3	2	4	60	80
Bali	1	1	8	60	80
				20	30

$=\begin{bmatrix}3 & 2 & 4\\1 & 1 & 8\end{bmatrix} \begin{bmatrix}60 & 80\\60 & 80\\20 & 30\end{bmatrix}$

$$\begin{array}{cc} \textbf{Zomba} & \textbf{Mzuzu} \end{array}$$
$$= \begin{bmatrix} 180+120+80 & 240+160+120 \\ 60+60+160 & 80+80+240 \end{bmatrix}$$
$$= \begin{bmatrix} 380 & 520 \\ 280 & 400 \end{bmatrix}$$

∴ Mphatso will pay **K520 – K380 = K140**,

More in Mzuzu

∴ Bali will pay **K400 – K280 = K120**,

More in Mzuzu

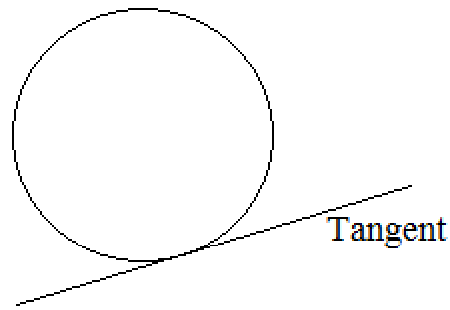
EXERCISE (MANEB papers)

1. Given that **P**=$\begin{bmatrix} 7 & 5 \\ 2 & 4 \end{bmatrix}$ and **Q**=$\begin{bmatrix} 3 & 10 \\ 5 & 1 \end{bmatrix}$, Find PQ
2. Given that Matrix **P**=$\begin{bmatrix} 3 & 4 \\ 1 & -2 \end{bmatrix}$ **Q**= $\begin{bmatrix} 1 & 3 \\ 6 & 1 \end{bmatrix}$ and **R**=$\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}$. Find 3(Q - PR).
3. T and **R** are two matrices. Given the **T**= $\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$**R**=$\begin{bmatrix} 0 & 3 \\ -1 & 1 \end{bmatrix}$. Find **3R – T²**.
4. Find **x** and **y** in the matrix below
$$\begin{bmatrix} 6 & 3 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ -4 \end{bmatrix}$$
5. Show that $\begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 5 & -1 \end{bmatrix} + \begin{bmatrix} 8 & -12 \\ -8 & -1 \end{bmatrix}$is a zero matrix
6. Given that **M** =$\begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix}$, **N**=$\begin{bmatrix} -3 & 2 \\ -1 & k \end{bmatrix}$, and **MN**$\begin{bmatrix} -3 & 6 \\ -1 & -1 \end{bmatrix}$. Find the value of **k**

CHAPTER 2

TANGENT PROPERTIES

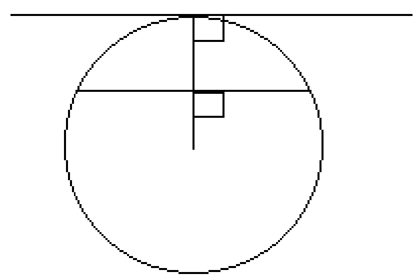
Tangent is a straight line which touches the circumference of a circle at only one point even after producing it.



Theorem 1

A tangent to a circle is perpendicular to the radius from point of contact to the Centre of the circle.

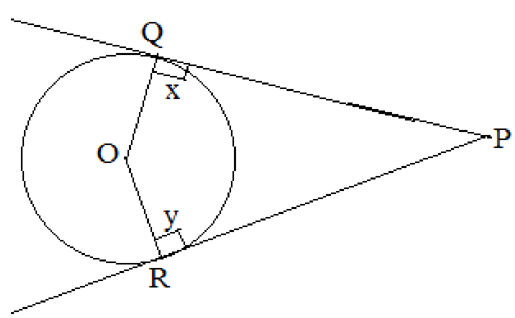
Reason, *tan ⊥ to radius*



Theorem 2

The two tangents from an external point to a circle are equal in length.

Reason, *tan from ext. pt.*



Given; two tangents PQ and PR

To prove; PQ = PR

Construction; draw ⊥s QO and RO and line PO

Proof; in Δs PQO and PRO

QO= RO, radii

x= y tan ⊥ to radius

PO is common

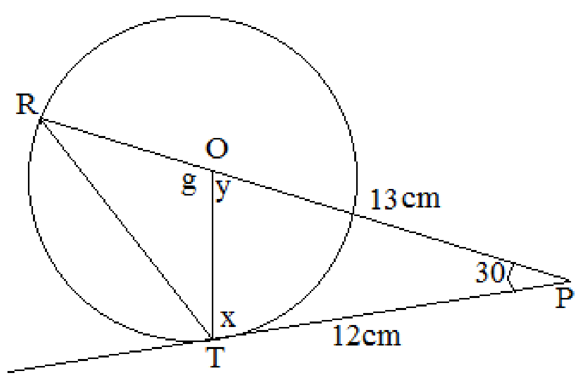
∴ **Δ PQO≡ Δ PRO (RHS)**

∴ **PQ=PR**

EXAMPLE

1. P is 13cm from the Centre O of the circle, PT is a tangent to a circle and is 12cm long. Angle OPT=30°. find
   a. OT
   b. $P\hat{O}T$
   c. $O\hat{T}R$

Solution



$x = 90°$, tan ⊥ to radius

a. In Δ**PTO**
   $PO^2=PT^2+OT^2$, Pythagoras
   $13^2=12^2+OT^2$
   $169=144+OT^2$
   $169-144=OT^2$
   $25=OT^2$
   $OT=\sqrt{25}$
   $\therefore OT = 5$

b. In Δ**OPT**
   $y + 30° + x = 180°$, sum $< s$ in Δ
   $y + 30° + 90° = 180°$
   $y = 180° - 120$
   $y = 60°$
   $\therefore P\hat{O}T = 60°$

c. $y + g = 180°$, adj. $< s$ on str.line
   $60° + g = 180°$
   $g = 120°$

OR = OT, radii

∴ ΔOTR is isosceles

M=n, base <s

M+ n+ g=180°, sum <s in a Δ

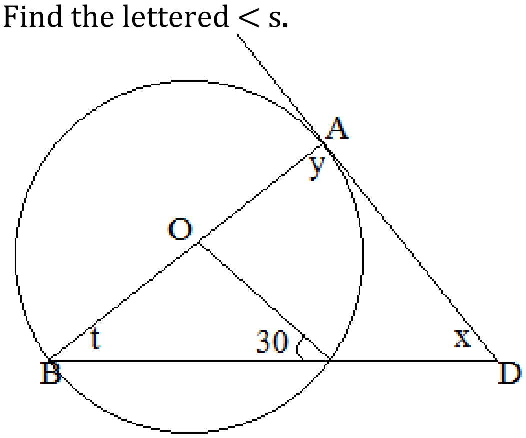
2n= 180°- 120°

2n=60°

N= 30°

$\therefore O\hat{T}R = 30°$

2. Find the lettered < s.

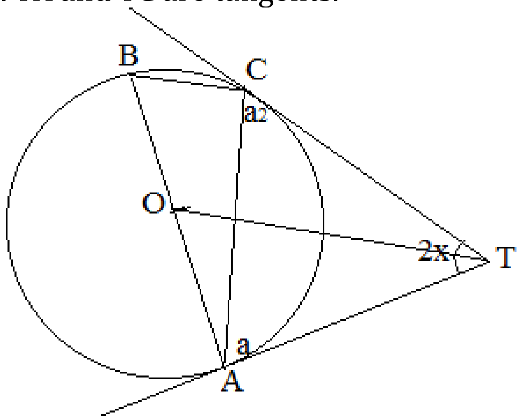


Solution

$y = 90°$, tan ⊥ to radius
In Δ**OBC**
OB=OC, radii
∴ ΔOBCIs an isosceles
∴ $t = 30°$, base $< s$
in Δ ABD
$x + y + x = 180°$, sum $< s$ in a Δ
$30° + 90° + x = 180°$

$X = 180° - 120°$
$x = 60°$

3. AB is a diameter of a circle ABCD Centre O. TA and TC are tangents.



If $A\hat{T}C = 2x$. show that $BAC = x$

Solution

In Δ**ATC**,

$AT = CT$, Tan from ext point

$\therefore \Delta$ **ACT**, is an isosceles

$\therefore a = a_2$, base <s

$a + a_2 + 2x = 180°$, sum $< s \ in \ a \ \Delta$

$$2x + 2a = 180°$$
$$2(x + a) = 180°,$$
$$x + a = \frac{180°}{2}$$
$$x + a = 90°$$
$$\therefore x = 90 - a$$

$B\hat{A}T = 90°$, Tan $\perp$ to radius

$B\hat{A}C = B\hat{A}T - a$,

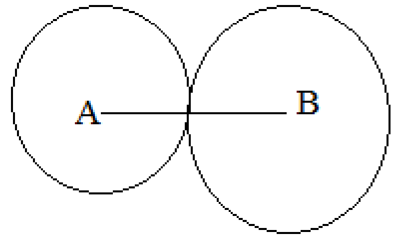
$\therefore B\hat{A}C = 90° - a$

But $90° - a = x$

$\therefore B\hat{A}C = x$

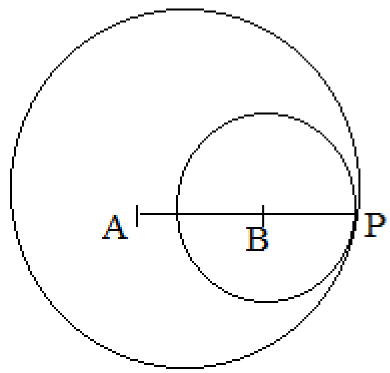
CONTACT OF CIRCLES

Circles can touch internally or externally. Where the distance between their centers is the sum of their radii.



AB=AP + PB

When they touch internally the distance between their centers is the difference between radii.



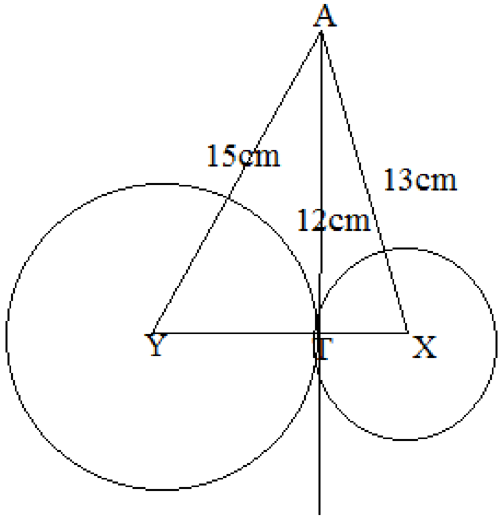
AB= AP – BP

EXAMPLE

Two circle centers X and Y touch externally at T. A is a point on their common tangent such that AT=12cm, AX=13cm and AY=15cm.

I. Calculate XY
II. If the circle touch internally instead of externally what is XY?

Solution



$A\hat{T}Y = 90°$, Tan $\perp$ to radius

$A\hat{T}X = 90°$, Tan $\perp$ to radius

$\therefore$ Δ ATY and ATX are right $< d$

In Δ ATY

$AY^2 = YT^2 + AT^2$, pythagorus theorem

$15^2 = YT^2 + 12^2$

$225 - 144 = YT^2$

$81 = YT^2$

$YT = \sqrt{81}$

YT=9cm

In Δ **ATX**,

$AX^2 = AT^2 + XT^2$, Pythagoras theorem

$13^2 = 12^2 + XT^2$

$169 - 144 = XT^2$

$25 = XT^2$

$XT = \sqrt{25}$

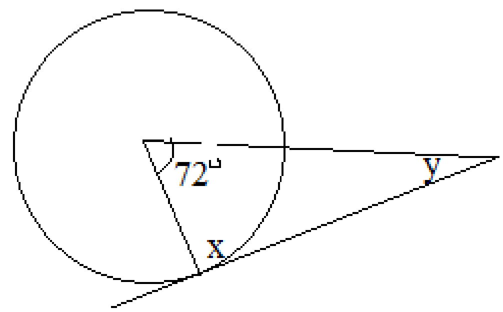
= 5cm

I. XY = YT+TX
 = 9cm + 5cm
 =14cm

II. XY= YT- TX
 = 9cm- 5cm
 = 4cm

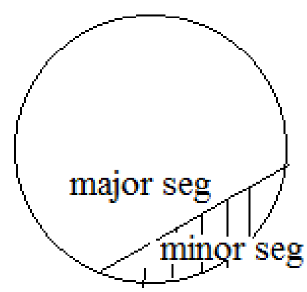
EXERCISE

1. AD is a diameter of a circle. AB is a chord and AT is a tangent.
   i. State the size of ABT
   ii. If angle BAT is an acute of x°, find the value of DAB in terms of x.
2. Calculate the marked angles



Theorem 3

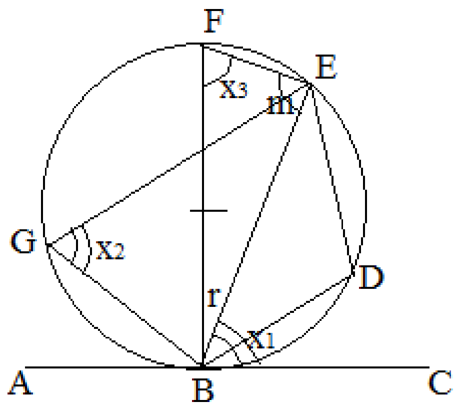
<u>**ALTERNATE SEGMENTS**</u>



If a straight line touches a circle and from the point of contact a chord is drawn, the angles which chord makes with the tangent are equal to the angles in the alternate segments

Reason, <s in alt seg. are equal.

PROVE



Given; a circle, str. line ABC, chord BC, $C\hat{B}E$ and $B\hat{G}E$, $A\hat{B}E$ and $B\hat{D}E$

To prove; $C\hat{B}E = B\hat{G}E$ and $A\hat{B}E = B\hat{D}E$

Construction: draw diameter BE, join E to F

Proof: $r + x = 90°$, tan ⊥ to radius

$x = 90° - r$

$m = 90°, < s\ in\ a\ sem\ circle$

In Δ **BEF**

$r + x_3 + m = 180°$, sum <s in a triangle

$r + x_3 + 90^o = 180^o$

$r + x_3 = 180 - 90^o$

$r + x_3 = 90^o$

$x_3 = 90^o - r$

$\therefore x_1 = x_3$

But $x_2 = x_3$, <s in same seg.

$\therefore x_2 = x_3$

$\therefore C\hat{B}E = B\hat{G}E$

$X_2 + B\hat{D}E = 180^0$, sum intr. Opp. <s in cyclic Quad.

$\therefore B\hat{D}E = 180^{\circ} - x_2$

$A\hat{B}E + x = 180°, < s\ on\ a\ str\ line$

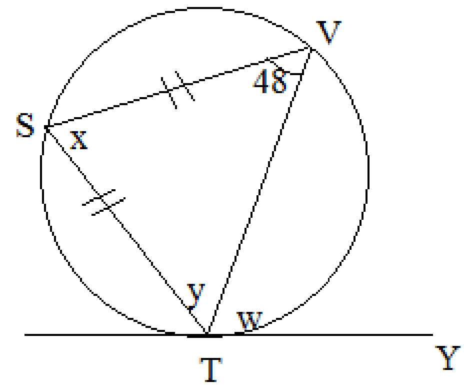
$A\hat{B}E = 180° - x$

$\therefore A\hat{B}E = 180° - x_2 (x_1 = x_2)$

$A\hat{B}E = B\hat{D}E$

EXAMPLE

1. TY is a tangent to the circle TVS. If STV= 48° and VS= ST. what is $V\hat{T}Y$?
   Solution



SV= ST, given

$\therefore$ ΔSTV, is isosceles

$\therefore y = 48°$

$x + y + 48° = 180°$, sum $< s\ in\ \Delta$

$x + 48° + 48° = 180°$

$x = 180° - 96°$

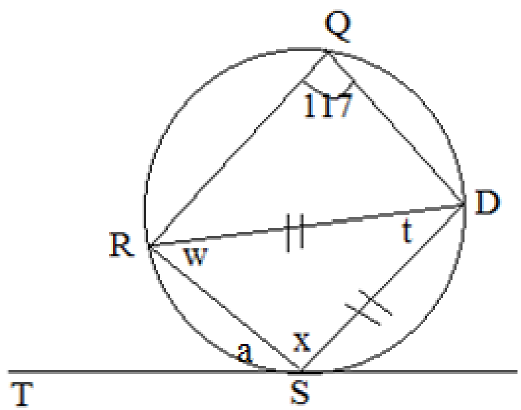
$x = 84°$

$w = x, < s\ in\ alt\ seg$

$w = 84°$

$\therefore V\hat{T}Y = 84°$

2. TS is a tangent to a circle PQDS. If **DR=PS** and angle **DQR**=117°, calculateangle RST
   Solution



$$x + 117° = 180°, \text{sum int. opp} < s \textit{ in cyclic quad}$$

$$x = 180° - 117°$$

$$= 63^0$$

In Δ **DRS**

RD= DS, given

$$\therefore \Delta DRS, \text{is an isosceles}$$

$$\therefore x = w, \text{base} < s$$

$$\therefore w = 63°$$

$$x + w + t = 180°, \text{sum} < s \textit{ in a } \Delta$$

$$63° + 63° + t = 180°$$

$$t = 180° - 126$$

$$t = 54°$$

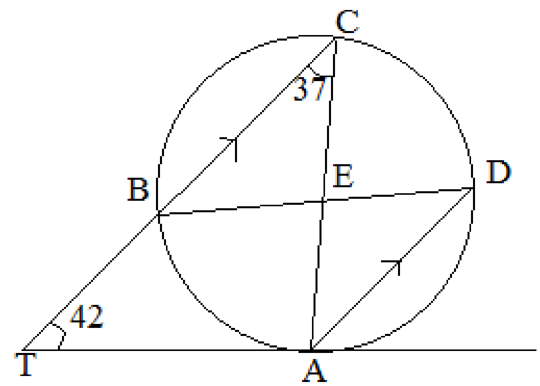
$$a = t = 54°, < s \textit{ in alt seg}$$

$$a = 54°$$

$$\therefore D\hat{S}T = 54°$$

EXERCISE

1. AT is a tangent to the circle ABCD, angle BAC = 64^0 and angle CAT=72^0. Calculate $B\hat{C}A$ and $C\hat{D}A$
2. If angle ACB=37° and angle ATB=42°, Calculate $A\hat{B}T$ and $A\hat{E}B$

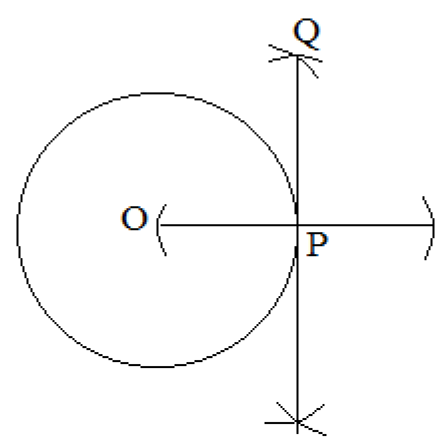


CONSTRUCTION

Construction of a tangent to a circle at a point on the circumference.

PROCEDURES

1. Draw a circle Centre O and mark P on its circumference.
2. Join O to P and produce it to K
3. Through P, construct line PQ perpendicular to OP. then PQ is the required tangent.

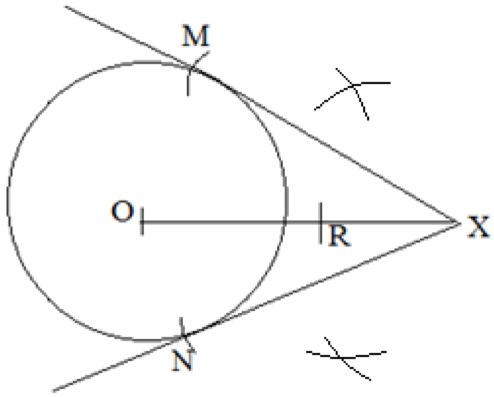


Construction of a tangent from an external point.

PROCEDURES

i. Draw a circle Centre O and mark a point X outside the circle.
ii. Join O to X
iii. Bisect OX and let the mid-point of OX be R.
iv. With radius from R, draw arcs cutting the circle at M and N. join M to X and N

to X. then XM and XN are the required tangents.



CHAPTER 3

STATISTICS

RANGE

It's the difference between the highest and the lowest score. E. g. 10, 2, 7, 100, 20

Range =100-2

= 98

EXAMPLE

1. At the end of term, the lowest in the mathematics test was 34%. The range is 52%. How many marks did the top student score?
   Solution
   Range = highest – lowest
   52%= highest -34%
   52%+ 34%= highest
   ∴ Highest = 86%

INTERQUATILE RANGE

It is the difference between lower quartile and upper quartile.

The score at the middle of the lower half of score is called lower quartile range (it is a $\frac{1}{4}$way along the distribution)

QL= $\left(\frac{n+1}{4}\right)^{\text{th}}$ score.

The score of the middle of the upper half of score is called upper quartile range (QU).

Because it is a $\frac{3}{4}$ way along the distribution.

QU=$\frac{3}{4}(n+1)$

The median is also called second quartile.

EXAMPLE

1. Find the lower, second and upper quartiles of the following data.
   10, 7, 2, 11, 15, 17, 12, hence find the interquartile and semi quartile range.

Solution

10, 7, 2, 11, 15, 17, 12

Lower quartile (QL) $= \frac{(n+1)}{4}$th score

$= \frac{(7+1)}{4}$th score

$= 2^{nd}$ score

$\underline{= 7}$

Upper quartile $= \frac{3}{4}(n+1)^{th}$score

$= \frac{3}{4}(7+1)^{th}$ score

$= \left(\frac{24}{4}\right)^{th}$ score

$= 6^{th}$ score

$\underline{= 17}$

Median $\underline{= 11}$

Interquartile range= 17- 7

$\underline{= 10}$

Semi-quartile $= \frac{1}{2} \times 10$

$\underline{= 5}$

EXERCISE

1. For the following set of numbers state,
   i. The lowest quartile
   ii. The median
   iii. The interquartile range
   iv. The upper quartile
   v. The semi-interquartile range
   8, 8, 9, 11, 14, 17, 21, 25, 29, 35, 37, 43.

STANDARD DEVIATION

VARIANCE (S^2)

It is the average of deviations from mean.

Standard deviation is the square root of variance(s)

Deviation is the value we get after subtracting mean $(\bar{x})$ from given score(x).

EXAMPLE

1. Find the standard deviation for 5, 6, 2, 9, 13, 25, 18, 10

Solution

Mean$(\bar{x}) = \frac{2+5+6+10+9+13+25+18}{8}$

$= \frac{88}{8}$

$= 11$

Score (x)	Deviation from mean(x- $\bar{x}$)	Squared deviation
2	-9	81
5	-6	36
6	-5	25
9	-2	4
10	-1	1
13	2	4
18	7	49
25	14	196
Total	0	$\sum(x-\bar{x})^2 = 396$

Variance (S^2) $= \frac{\sum(x-\bar{x})^2}{n}$

$= \frac{396}{8}$

$= 49.5$

Standard deviation(S)= $\sqrt{49.5}$

= 7.03

GROUPED DATA

EXAMPLE

1. Find correct to 1 decimal place, the mean standard deviation of the following;
   26, 29, 29, 30, 31, 33, 35, 33, 35

Solution

Score (x)	Tally	Freq.(f)	fx
26	/	1	26
29	//	2	58
30	/	1	30
31	/	1	31
33	///	3	99
35	/	1	35
Total		$\sum f = 9$	$\sum fx = 279$

$$\text{Mean}(\bar{x}) = \frac{\sum fx}{\sum f}$$

$$= \frac{279}{9}$$

$$= 31$$

$\therefore$ Mean $= 31$

X	$(x - \bar{x})$	f	$f(x - \bar{x})$	$(x - \bar{x})^2$	$f(x-x)^2$
26	-5	1	-5	25	25
29	-2	2	-4	4	8
30	-1	1	-1	1	1
31	0	1	0	0	0
33	2	3	6	4	12
35	4	1	4	16	16
Total		$\sum f = =9$			$\sum f(x - x)^2 = 62$

$$S^2 = \frac{\sum f(x-\bar{x})^2}{\sum f}$$

$$S = \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$$

$$= \sqrt{\frac{62}{9}}$$

$$= \sqrt{6.886}$$

$$= 2.6$$

2. The ages of the students in a class were as follows; 14, 15, 14, 16, 14, 14, 15,17, 15, 18, 14, 15, 15, 16, 15, 14, 13, 15, 15, 16. Find
   a. Mean
   b. Standard deviation.

Solution

Score (x)	Tally	Freq.(f)	fx
13	/	1	13
14	~~////~~ /	6	84
15	~~////~~ ///	8	120
16	///	3	48
17	/	1	17
18	/	1	18
		$\sum f = 20$	$\sum fx = 300$

$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}$$

$$= \frac{300}{20}$$

$$= 15$$

Score x	$(x-\bar{x})$	F	$f(x-\bar{x})$	$(x-\bar{x})^2$	$F(x-\bar{x})^2$
13	-2	1	-2	4	4
14	-1	6	-6	1	6
15	0	8	0	0	0
16	1	3	3	1	3
17	2	1	2	4	4
18	3	1	3	9	9
	=20		0		= 26

$$S^2 = \frac{\sum(x-\bar{x})^2}{\sum f}$$

$$= \frac{26}{20}$$

$$= 1.3$$

$$S = \sqrt{1.3}$$

$= 1.14$ years

EXERCISE

1. If the variance of 2 and 2a is 9, find the value of a
2. Find mean and standard deviation for; 7, 8, 10, 12, 14, 16, 17, 20.
3. Given the following information, find the standard deviation.

Order size (x)	Number of orders (f)
150	10
250	28
350	42
500	50
300	20

4. Find the standard deviation for the data below, using grouped data.

Ages	Frequency
100- 200	10
200-300	28
300- 400	42
400- 600	50
600- 1000	20

CHAPTER 4

PROGRESSIONS AND SERIES

- Arithmetic progression
- Sequence
- Term, first term, common sequence
- Rule
- Infinite sequence
- nth term
- sum of arithmetic progression

ARITHMETIC PROGRESSION

A sequence whereby the terms increase or decrease by the same amount is called **arithmetic progression**.

E. g 1, 3, 5, 7……. This sequence is increasing by adding 2 to the previous term.

8, 6, 4…….. This sequence is decreasing by adding -2 to the previous sequence.

SEQUENCE

It is an arrangement of numbers arranged according to their definite rule.

Two conditions are needed to come up with a sequence

i. **first term**
ii. **rule**

Rule; to previous term ***add 2*** to get the next term.

2, 4, 6, 8, 10……..sequence

INFINITE SEQUENCE

It is a sequence in which last or first term may not be known.

E. g 1, 3, 5………..

FINITE SEQUENCE

A sequence whereby last and first term are known.

E. g 1, 3, 5………..13

n^{th} TERM OF AN AP

1, 3, 5, 7……..

1 is a first term **(a)**

3-1=2

5-3=2

7-5=2

∴ 2 is a common difference **(d)**

1, 3, 5, 7……

1^{st}, 2^{rd}, 3^{th}, 4^{th} ………………. n^{th}

1, 1+2, 1+2+2, 1+2+2+2……..n^{th}

Substitute a=1 and d=2

a, a+d, a+d+d, a+d+d+d……… n^{th}

a^{st}, $a+d^{2nd}$, $a+2d^{3rd}$, $a+3d^{4th}$…………… n^{th}

a, a+ (2-1)d, a+(3-1)d, a+(4-1)d…….. n^{th}

let n = position

∴ a + (n-1) d

n = number of terms

EXAMPLE

1. For 1, 3, 5…., find the 7^{th} term.
   Solution
   7^{th} term = 1+ (7-1)2
   = 1+12
   = 13
2. How many are there in the AP 5, 12, 19…..82.
   Solution
   a=5, d=7 n=? n^{th} term =82
   n^{th} term =a+ (n-1)d
   82=5+ (n-1)7
   82= 7n-2
   82+2=7n
   84= 7n
   n=12
3. Find the formula for the n^{th} term of the sequence of old numbers 1, 3, 5, 7…….
   And find thousandth odd number.
   Solution
   1, 3, 5, 7……
   A=1, d= 2
   n^{th} term = 1 +(n-1)2
   =1 + 2n -2
   = 2n- 1
   Thousandth old number = 2(1000)-1
   = 2000-1
   = 1999

EXERCISE

1. Find the tenth term of the AP 3, 8, 15.
2. How many terms are there in an AP 45, 41, 37……...1
3. Find the formula for the n^{th}term of even numbers 2, 4, 6, 8 hence find
4. The thousandth even number.

EXAMPLES

1. The twenty fifth term of an AP is 43 and the fifty ninth term is 26. Find the first term and the common difference and the thirtieth term.
   Solution
   25^{th} term = a + (25-1) d
   43= a +24d…………….. (i)
   59^{th} term = a + (59-1) d
   26= a + 58d…………….. (ii)
   43= a + 24d
   (26=a+58d)
   17 =- 34d
   $d= -\frac{34}{17}$
   $=-\frac{1}{2}$
   In i. put $d=-\frac{1}{2}$
   $43=a+24(-\frac{1}{2})$
   43= a -12
   43+ 12= a
   a= 55
   $30^{th}= 55+ (30-1) -\frac{1}{2}$
   $=55 + 29\times-\frac{1}{2}$
   = 55- 14.5
   = 40.5
2. An AP has fourth term 8 and seventh term 17. Find the first term and the common

difference. Find an expression for the n^{th} term.

Solution

4^{th} term = a + (4-1) d

8 = a +3d……………… (i)

7^{th} term = a + (7-1) d

17 = a +6d……………(ii)

ii) - i) gives 17 = a +6d

(8 = a +3d)

9= 3d

$d=\frac{9}{3}$

d=3

In i.) Put d=3

8=a + 3(3)

8-9= a

-1 = a

a= -1

n^{th} term = a + (n -1) d

= -1 + (n-1) 3

= -1 + 3n – 3

= 3n – 4

SUM OF TERMS IN AN AP

The formula for finding sum of n^{th} term of an AP is;

$S_n = \frac{n}{2}(2a + (n-1)d)$

EXAMPLE

1. An arithmetic progression has first term 3 and common difference 2. Find the sum of the first 50 terms.

Solution

a= 3, d= 2, n= 50

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{50} = \frac{50}{2}(2(3) + (50-1)2)$

= 25 (6+98)

= 25×104

S_{50}= 2600

2. The second term of an A IS 15 and the fifth term is 21. Find the common difference, the first term and the sum of the first ten terms.

Solution

2^{nd} term = a + d (2-1)

15=a +d…………………. (i)

5^{th} term = a + d (5-1)

21= a + 4d……………… (ii)

ii) – i); gives 21= a +4d

(15=a +d)

6=3d

$d=\frac{6}{3}=2$

in i) put d= 2

15= a + 2

15-2= a

a= 13

$S_{10} = \frac{10}{2}(2(13) + (10-1)2)$

= 5(26+18)

= 220

3. How many terms an AP 1, 3, 5……..are required to make a sum of 1525?

Solution

a=1, d=2

$S_n = \frac{n}{2}(2a + (n-1)d)$

$1521 = \frac{n}{2}(1) + (n-1)2)$

$1521=\frac{n}{2}(2 + 2n - 2)$

$1521=\frac{n}{2}\times 2n$

$1521=n^2$

$n=\sqrt{1521}$

= 39

4. The sixth term of an AP is twice the third term, and the first term is 3. Find the common difference and the 10^{th} term.

Solution

6^{th} term = 3 + d (6-1)

=3+ 5d

3^{rd} term = 3 +d (3-1)
= 3 + 2d
But 6^{th} term = 2× 3^{rd} term
∴ 3+ 5d = 2(3+2d)
3 +5d= 6+ 4d
5d- 4d =6-3
d= 3
10^{th} term= 3 +3(10- 1)
= 3+3(9)
= 30

EXERCISE

1. The twentieth term of an AP is 55 and the eightieth term is 255. Find the first term, the common difference and the thirtieth term
2. How many terms are there in the following series
   200, 192, 184…………120
3. The first term of an AP is 2 and the last term is 59. If the sum of their terms is 610, find the common difference.
4. The sum of n terms of an AP for all values of n is $5n^2$- 2n. find the first term and the common difference
5. Find the value of n if the value of the first n terms of the two APs 2, 5, 8….. and 47, 45, 43 are equal

GEOMETRIC PROGRESSION (GP)

1^{1st}, 2^{2nd}, 4^{3rd}, 8^{4th}………… n^{th} term

1, 1×2, 1×2×2, 1×2×2×2………… n^{th} term

$a\times2^0$, $a\times2^1$, $a\times2^2$, $a\times2^3$……………. n^{th} term

(Position and power differs by 1)

$$\frac{a\times 2}{a}=\frac{a\times 2\times 2}{a\times 2}=\frac{a\times 2\times 2\times 2}{a\times 2\times 2}=2$$

2 is a common ratio

a, ar^1, ar^2, ar^3 …………….ar^{n-1}

∴n^{th} term of a GP = ar^{n-1}

EXAMPLE

1. Write down the sixth term and the n^{th} term of the GP. 2, 1, $\frac{1}{2}$ … …

   Solution

   2, 1,$\frac{1}{2}$…………..

   a= 2, r=$\frac{1}{2}$, n=6

   6^{th} term = $2\times\left(\frac{1}{2}\right)^{6-1}$

   = $2\times(\frac{1}{2})^5$

   = $\frac{1}{16}$

2. The sixth term of GP is 16 and the third term 2, find the first term and the common ratio

   Solution

   6^{th} term= ar^{6-1}

   16= ar^5…………. i)

   3^{rd} term =ar^{3-1}

   2= ar^2………….. ii)

   i) ÷ ii) gives $\frac{16}{2}=\frac{ar^5}{ar^2}$

   $8=r^3$

   $2^3= r^3$

   r= 2

   in ii) put r=2

   $2= a\times2^2$

   $2=a\times 4$

   2= 4a

   a=$\frac{1}{2}$

SUM OF TERMS IN A GP

The formula for finding sun of terms in a GP where **r> 1** is;

$$\mathbf{S_n = \frac{a(r^n-1)}{r-1}}$$

Where **r <1** the formula is;

$$\mathbf{S_n = \frac{a(1-r^n)}{1-r}}$$

EXAMPLE

1. A GP has first term 3 and common ratio2. Find the sum of the first ten term.
   Solution
   a=3, r= 2, n= 10
   $S_{10} = \frac{3(2^{10}-1)}{2-1}$
   $=\frac{3(1024-1)}{1}$
   $= 3\times1023$
   $= 3069$
2. What is the largest number of terms of GP, 8+24+74….. that will give less than 3,000,000
   Solution
   $A = 8$, r= 3, $s_n = 3{,}000{,}000$
   $S_n = \frac{8(3^n-1)}{3-1}$
   $\frac{8(3^n-1)}{3-1} < 3,000,000$
   $4(3^n-1) < 3,000,000$
   $3^n-1 < \frac{3,000,000}{4}$
   $3^n < 750,000+1$
   $3^n < 750,001$
   $\text{Log } 3^n < \log 750,001$
   $n \log 3 < \log 750,001$
   $n < \frac{\log 750,001}{\log 3}$
   $n < \frac{5.8751}{0.4771}$
   $n < 12.31$
   $n < 12$
   ∴ The largest number is 12

EXERCISE

1. Write down the sixth term and the n^{th} term of
   i. 2, 4, 8 ……………
   ii. 3, -6, 12 ………..
2. The first term of a GP is 5 and the common ratio is 2. Find the common ratio
3. The third term of a GP is 6 and the eighth term is 192. Find the first term of the series and the common ratio.
4. Mr. Motto saves from his profit as follows, K3 the first week, K6 the second week, K12 the third week, K24the fourth week and so on. If he manages to keep on saving under this system,
   i. How much would he save in the tenth week
   ii. What would be the total amount of his savings in the first ten weeks
5. The sum of n terms of a GP is 14. The first term is 2, find the possible values of the sum of first five terms.
6. The first and fourth terms of a GP are; $\frac{x}{y^2}$ and $\frac{y}{x^5}$. Find the second and third terms.
7. In a GP a product of the first and 7^{th} terms is equal to the 4^{th} term. Given that the sum of the first and 4^{th} term is 9. Find the 1^{st} term and common ratio.

CHAPTER 5

TRAVEL GRAPHS

We shall look at

1. Velocity- time graph
2. Acceleration – time graph
3. Deceleration – time graph

To calculate the distance covered, find the areas under the given graph.

The formula for finding the area depends on the shape of the graph.

Triangle $=\frac{2}{2}$ **base** $\times$ **hieght**

Rectangle =length× breadth

Trapezium= $\frac{1}{2}$**(sum // sides) h**

VELOCITY TIME GRAPH

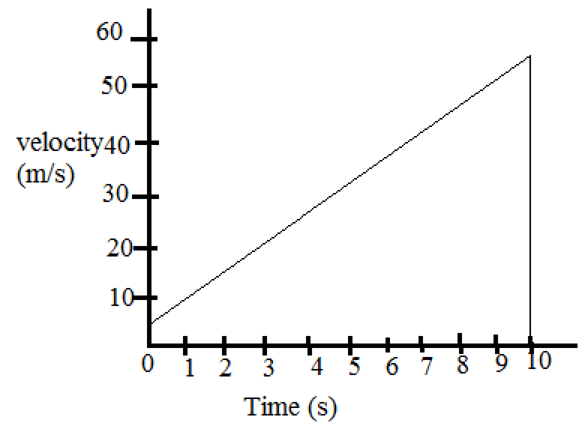
Velocity= $\frac{\text{distance}}{\text{time}}$

Velocity is the rate of change of distance for given time

EXAMPLE

1. Draw a velocity time graph for a body which starts with initial velocity of 4m/s and continues to move for a further 10s with acceleration of $5m/s^2$. Use your graph to find the distance covered by the body in this time.
   Solution

Time S	0	1	2	3	4	5	6	7	8	9	10
Velo city	4	9	14	19	24	29	34	39	44	49	54



To find distance covered, calculate the area under the graph.

Area of trapezium= $\frac{1}{2}$(Sum // sides)h

$$=\frac{1}{2}\left(\frac{4m}{s}+\frac{54m}{s}\right)\times 10s$$

$$=\frac{1}{2}\times\frac{58m}{s}\times 10s$$

$$= 290\text{m}$$

∴Distance traveled = 290m

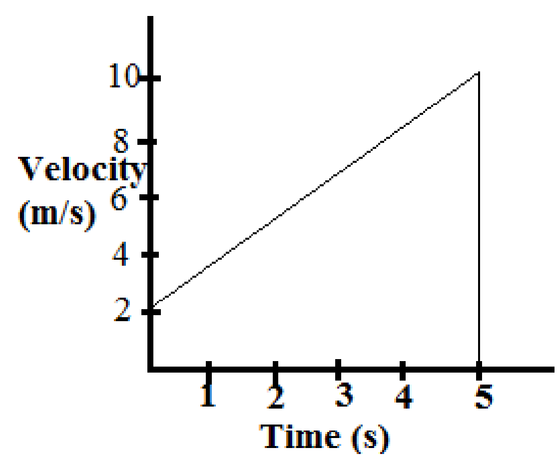
ACCELERATION

Acceleration $=\frac{\text{velocity}}{\text{time}}=\frac{m/s}{s}$

$=m/s^2$

EXAMPLE

1. Find the acceleration of the graph below



Solution

$$\text{Acceleration} = \frac{\text{velocity}}{\text{time}}$$

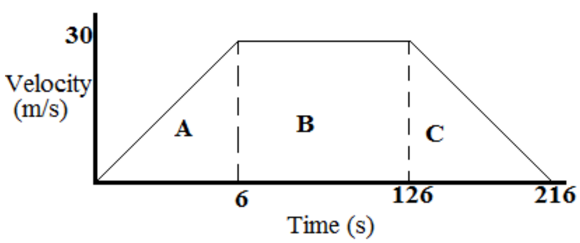
$$= \frac{(10-2)\text{m/s}}{5\text{S}}$$

$$= \frac{8\text{m/S}}{5\text{S}}$$

$$= 1.6\text{m/s}^2$$

2. A car starts from rest and is uniformly accelerated at 5m/s^2 until it attains a velocity of 30m/s. it maintains this velocity for 120s after which it is brought to the rest in another 90s. Sketch the velocity time graph for this motion and use it to find the distance covered by the car. Find the deceleration for the last part.

Solution



$$\text{Area A} = \frac{1}{2} \times 6\cancel{s} \times \frac{30m}{\cancel{s}}$$

$$= 90\text{m}$$

$$\text{Area B} = \frac{30m}{\cancel{s}} \times 120\cancel{s}$$

$$= 3600\text{m}$$

$$\text{Area C} = \frac{1}{2} \times 90\cancel{s} \times \frac{30m}{\cancel{s}}$$

$$= 1350\text{m}$$

$$\therefore \text{Distance} = 90\text{m} + 3600\text{m} + 1350\text{m}$$

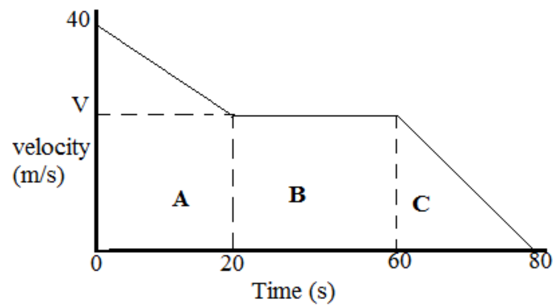
$$= 5040\text{m}$$

$$\textbf{Deceleration} = \frac{0-30m/s}{90s}$$

$$= \frac{-30m/s}{90s}$$

$$= -0.3\text{m/s}^2$$

3. Figure below is a speed time graph of a velocity. The train decelerates 40m/s to V m/s in 20s. It then maintain this speed for the next 40 seconds after which it decelerates uniformly for 20seconds and stops.



Given that the total distance travelled is 1600m. Calculate

i. Value of V
ii. Deceleration during the first 20 seconds

Solution

$$\text{Area of trapezium A} = \frac{1}{2}(\text{sum// sides})\text{x h}$$

$$= \frac{1}{2}\left(\frac{V+40m}{s} \times 20s\right)$$

$$= 10(\text{v} + 40)\text{m}$$

$$\text{Area B} = 40\text{s} \times \frac{Vm}{s}$$

$$= 40\text{Vm}$$

$$\text{Area C} = \frac{1}{2} \times 20s \times \frac{Vm}{s}$$

$$= 10\ \text{Vm}$$

$$\text{A+B+C} = 1600\text{m}$$

$$\therefore 10(\text{v} + 40) + 40\text{V} + 10\text{V} = 1600\text{m}$$

$$10\text{V} + 400 + 50\text{V} = 1600\text{m}$$

$$60\text{V} = 1600\text{m} - 400\text{m}$$

$$60\text{V} = 1200\text{m}$$

$$\text{V} = 20\text{m/s}$$

Deceleration $= \frac{\text{change in speed}}{\text{change in time}}$

$= \frac{(20-40)m/s}{20s}$

$= \frac{-20m/s}{20s}$

$= -1\ m/s^2$

EXERCISE

1. A car starts from the rest and is uniformly accelerated at $5m/s^2$ until it attains a velocity of 30m/s. It maintains this velocity for 120s after which it is brought to the rest in another 90s. Sketch the velocity time graph for this motion and use it to find the distance covered.

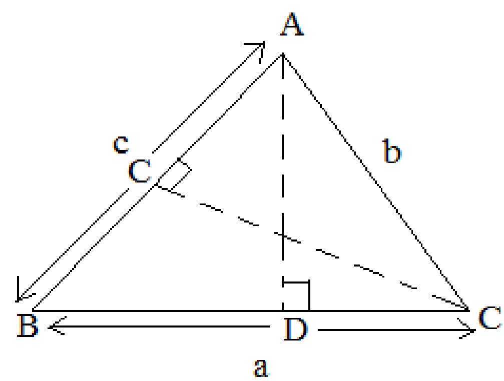
CHAPTER 6

TRIGONOMETRY

SINE AND COSINE RULES

Sine and cosine rules are used to find unknown sides and angles of triangles which are not right angled.

SINE RULE



In Δ ABD

$\text{Sin B} = \frac{AD}{AB}$

$\text{Sin B} = \frac{AD}{c}$

$c \sin B = AD$

In ΔADC

$\text{Sin C} = \frac{AD}{AC}$

$\text{Sin C} = \frac{AD}{b}$

$b \sin C = AD$

$\therefore c \sin B = b \sin C$

Divide both sides by c and b

$\therefore \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{C}$

In general; $\frac{\text{Sin A}}{a} = \frac{\text{Sin B}}{b} = \frac{\text{Sin C}}{C}$

EXAMPLE

1. Find the unknown side a

Solution

$$\frac{\text{Sin A}}{\text{a}} = \frac{\text{Sin C}}{\text{C}}$$

$$\frac{\text{Sin } 50°}{\text{a}} = \frac{\text{Sin } 60°}{12}$$

$$\frac{0.7660}{\text{a}} = \frac{0.8660}{12}$$

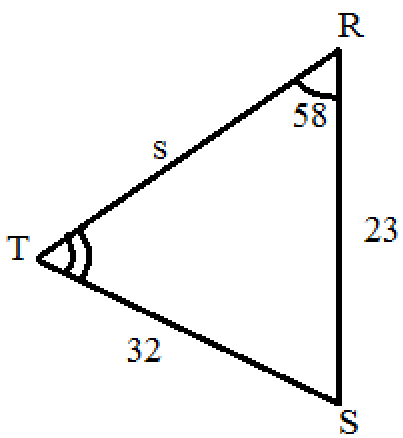
$12\times 0.7660 = \text{a}\times 0.8660$

$\frac{9.192}{0.866} = \text{a}$

a= 10.6cm

2. In triangle **RST**, **RS**=23cm, **ST**=32cm, angle **R**=58. Find angle T and RT.

Solution



$$\frac{\text{Sin T}}{23} = \frac{\text{Sin } 58°}{32}$$

$\text{Sin T} = \frac{0.8480\times 23}{32}$

$= 0.6095$

$\sin^{-1} 6095 = 37.6°$

$$\frac{\text{Sin S}}{\text{S}} = \frac{\text{Sin R}}{\text{r}}$$

$\hat{S} + 58° + 37.6° = 180°$, Sum<s in a Δ

$\hat{S} = 180° - 95.6°$

$= 84.4°$

$$\frac{\text{Sin } 84.4°}{\text{S}} = \frac{\text{Sin } 58°}{32}$$

$32 \sin 84.4° = \text{S} \sin 58°$

$$\frac{32 \sin 84.4°}{\sin 58°} = S$$

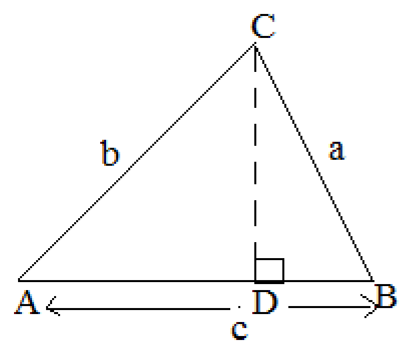
$\text{S} = \frac{32 \sin 0.9952}{\sin 58°}$

$= \frac{31.8482}{0.8480}$

$= 37.7\text{cm}$

$\therefore$ RT $= 37.7$cm

__COSINE RULE__



In Δ ABC,

$BC^2 = DC^2 + DB^2$, Pythagoras

But DB=AB- AD

$\therefore$ DB=c-AD

$\therefore BC^2 = DC^2 + (c-AD)^2$

$BC^2 = DC^2 + c^2 - 2c.AD + AD^2$

Rearranging

$BC^2 = DC^2 + AD^2 + c^2 - 2c.AD$

But $DC^2 + AD^2 = b^2$

$\therefore BC^2 = b^2 + c^2 - 2c.AD$

In ΔADC,

$\text{Cos A} = \frac{AD}{b}$

$b\text{Cos } A=AD$

$\therefore BC^2= b^2+c^2-2bc\text{Cos}A$

$a^2=b^2+c^2-2bc\text{Cos } A$

In general;

$a^2= b^2+c^2-2bc\text{Cos}A$

$b^2= a^2+c^2-2ac\text{Cos}B$

$c^2= a^2+b^2-2ab\text{Cos}C$

<u>**To find angles make Cosine subject**</u>

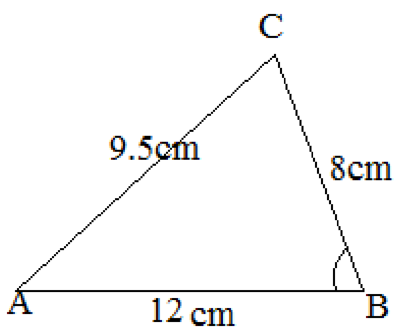
$$\therefore \text{Cos } A=\frac{b^2+c^2-a^2}{2bc}$$

$$\text{Cos } B=\frac{a^2+c^2-b^2}{2ac}$$

$$\text{Cos } C=\frac{a^2+b^2-c^2}{2ab}$$

EXAMPLE

1. Find the stated angle that in a triangle ABC given that a=8cm, b=9.5cm, c=12cm. find angle B
   Solution



Solution

$$\text{Cos } B=\frac{a^2+c^2-b^2}{2ac}$$

$$=\frac{8^2+12^2-9.5^2}{2\times8\times12}$$

$$=\frac{177.75}{192}$$

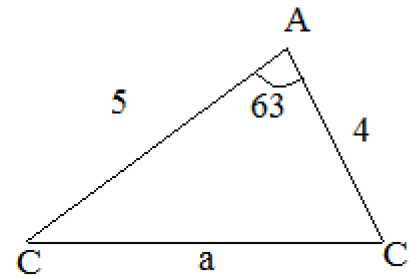
$$=0.6133$$

$\cos^{-1} 0.6133= 52.2°$

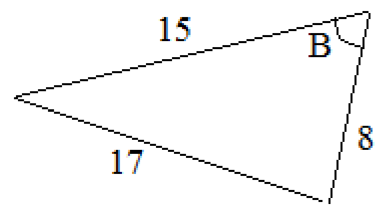
$\therefore \hat{B}= 52.2°$

EXERCISE

1. Find the unknown side in the triangle below



2. In a triangle ABC, AB=8cm, AC=12cm and $\hat{A}=36°$. Find BC
3. In triangle XYZ, x=5cm, y=6cm and $\hat{Z}=60°$, Show that $z=\sqrt{31}$
4. Find the labeled angles in these triangle

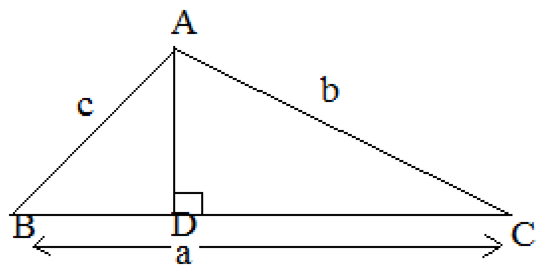


<u>**MIXED PROBLEMS**</u>

Here are tips to help you to know which rule to use (between cosine and sine rule).

1. **SSS**: if the three sides triangle are known then use **cosine rule** to find angles
2. **SAS**: if two sides of a triangle are known and the angle between them is known, then use **cosine rule** to find the other side
3. **ASA**: if one side and two angles are known then use **sine rule** to find the other side
4. **SSA**: if the two sides of a triangle are known and the angle which is not between them, then use **sine rule** to find the other angle(there may be two possible outcomes)

AREA OF A TRIANGLE



Area $\Delta = \frac{1}{2}$base $\times$ height

Area Δ ABC $= \frac{1}{2}$a $\times$ AD

In a Δ ABD

Sin B$=\frac{AD}{C}$

c sin B$=$ AD

$\therefore$ Area of Δ ABC $=\frac{1}{2}$ acSinB

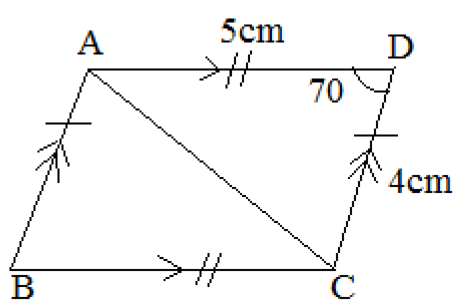
Area of Δ ABC $=\frac{1}{2}$ abSinC

Area of Δ ABC $=\frac{1}{2}$ bcSinA

NB: These formula expresses the area in terms of two sides and their included angle.

EXAMPLE

1. A parallelgram has sides of 4cm and 5cm and an angle of 70°. What is its area
   Solution



NB: diagonal bisects the //gram

$\Delta ADC \equiv \Delta CBA$

Area of Δ ADC$=\frac{1}{2}$acSin D

$=\frac{1}{2}\times 5cm\times 4cm\times Sin\ 70°$

$= 10cm\times 0.9370cm$

$= 9.4cm^2$

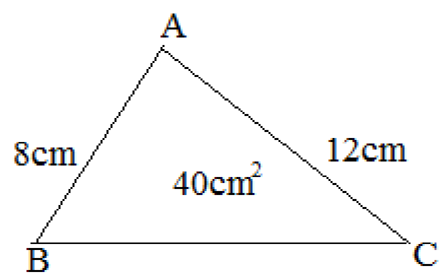
$\therefore$ Area of a //gram $= 2\times 9.4cm^2$

$= 18.8cm^2$

Finding an angle

A triangle has an area of $40cm^2$. The two adjacent sides of a triangle are 8cm and 12cm long. What is the angle between them?

Solution



Area of Δ ABC$=\frac{1}{2}$bcSin A

$40=\frac{1}{2}\times 8\times 12\times Sin\ A$

$40=4\times 12\times Sin\ A$

$\frac{40}{48}=$ Sin A

Sin A$=$ 0.8333

A$=$ 56.4°

Or A$=$ 123.6°

(Sin 56.4°)$=$Sin (180°-56.4°)

BEARING

Many problems involving bearings can be solved using the sine and cosine rules.

EXAMPLE

1. Two planes leaves an airport at the same time. One flies on a bearing of 150° at 400km/h of 230° at 600km/h. how far apart are the two planes after three hours
   Solution

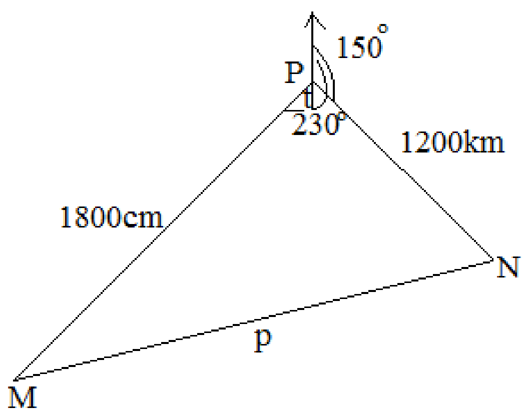
Distance= time × speed

∴ Distance (MP)= 400km/h ×3hrs

= 1200km

Distance (NP) = 600km/h×3hrs

= 1800km



150°+t = 230°

t= 230°- 150°

= 80°

$p^2=m^2+n^2-2mn \text{ Cos } P$

$p^2=1200^2+1800^2-(1200)(1800) \text{ Cos } 80°$

$= 144\ 0000+3240000-2320000\times0.1736$

$= 4680000-749952$

$p^2=3930048$

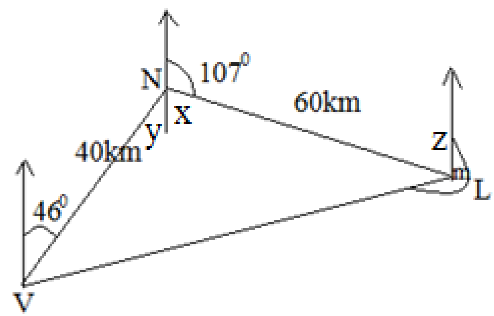
$p=\sqrt{3930048}$

$= 1982.43$

∴ p= 1980(to 3 sig. fig)

∴ They are 1980km apart

2. A man drives from his village to town, a distance of 40km on a bearing of 046°, he then drives from the town to a lake 60km away on a bearing of 107°.
   i. How far away is the lake from his house
   ii. What is the bearing of his house from the lake?
   Solution



y=46°, alt <s

$x + 107° = 180°$,<s on str. Line

$x = 180° - 107°$

=73°

∴$V\hat{O}L= x + y$

=46° + 107°

= 119°

$n^2=l^2+v^2\text{-}2lv\text{Cos}N$

$n^2=40^2+60^2\text{-}2(40)(60)\text{Cos } 119°$

$n^2= 1600+3600\text{-}\ 4800\times\text{-}0.4848$

$n^2=5200+2327.04$

$n^2=7527.04$

$n=\sqrt{7527.04}$

= 86.8km

a. ∴ They are 86.8km apart

Cos L= $\frac{V^2+n^2-l^2}{2vn}$

$=\frac{60^2+86.759^2-40^2}{2\times60\times86.759}$

$=\frac{9527.04}{10411.080}$

Cos L $= 0.9151$

$= 23.8°$

Z= x, alt angles

$\therefore z = 73^o$

23.8°+73°+m= 360°,<s @ a point.

M=360°- 96. 8°

= 263.2°

∴ The bearing of home from the lake is

263.2°

EXRCISE

1. Given that A=(0,0), B =(4, 3) and C=(4, -3).
   i. Find the area of triangle ABC
   ii. Calculate angle BAC
2. What is the area of an equilateral triangle with sides 10cm long?
3. A triangle has an area of 60cm^2. Two adjacent sides of the triangle are 8cm and 12cm long. What is the angle between them?
4. An aero plane leaves airport A on a bearing of N23°E and flies for 340km to another airport B and flies on a bearing of N60°W to another airport C. If the airport A and C are 680km apart. Calculate the bearing of airport A from airport C.
5. A boat sails 40km from point A to B east of A on a lake. Lodge P is south of point B. the bearing of Q from point A is S52°E and the bearing of lodge P from B is S39°W. if P is 60km from Q. calculate the bearing of lodge Q from lodge P.

CHAPTER 7

POLYNOMIALS

Poly means many.

$3x^2$ monomial

$3x^2 + 2x$ Polynomial

DIVISSION OF POLY NOMIAL

EXAMPLE

1. Find the quotient and the remainder of the following;

   $x^2 + 2x + 1$Divided by $x + 1$

We start with dividing the highest power of x by (x+1), dividing x² by x, give x so put x on top line, multiply x by x(x+1) and subtract the result from the first two terms of the polynomial.

I.

$$\begin{array}{r|l} & \underline{\quad x+1} \\ x+1 & x^2+2x+1 \\ & \underline{-(x^2+x)} \\ & \quad x+1 \\ & \quad \underline{-(x+1)} \\ & \qquad 0 \end{array}$$

∴ The quotient is $x + 1$ and the remainder is 0

NB; in terms of quotient, divisor and the remainder, the polynomial can be expressed as

Polynomial = devisor ×quotient+ Remainder

2. Find the quotient and the remainder when $x^3 - 3x + 4x + 1$ Is divided by $x - 2$

Solution

$$
\begin{array}{r|l}
 & x^2 - x + 2 \\
\hline
x - 2 & x^3 - 3x^2 + 4x + 1 \\
 & \underline{-(x^3 - 2x^2)} \\
 & \quad x^2 - 4x \\
 & \quad \underline{-(-x^2 + 2x)} \\
 & \qquad 2x + 1 \\
 & \qquad \underline{-(2x - 4)} \\
 & \qquad\qquad 5
\end{array}
$$

$\therefore$ The quotient is $x^2 - x + 2$ and the remainder is 5.

EXERCICE

Find the quotient and the remainder of the following,

1. $3x^3 + 5x^2 + 7x - 14$ divided by $x - 1$
2. $x^3 - 4x^2 + x + 2$, divided by $x^2 - 3$
3. $8x^3 + 6x^2 - 3x - 1$divided by $2x + 1$

<u>REMAINDER THEOREM</u>

The remainder when $f(x)$is divided by $x + a$ is $f(-a)$

EXAMPLE

Use remainder theorem to find the remainder when $5x^3 + 4x^2 + 7x - 12$ is divided by $x - 1$

Solution

$x - 1 = 0$

$x = 1$

$f(1) = 3(1)^3 + 4(1)^2 + 7(1) - 12$

$= 3+4+7+12$

$=2$

$\therefore$ 2 is the remainder

EXAMPLE 2

When $x^3 + 3x^2 + ax + b$ is divided by $(x + 1)$ the remainder is 5 and when it is divided by $x - 2$, the remainder is 8. Find a and b

Solution

$x + 1 = 0$

$x = -1$

f(-1)=(-1)^3+3(-1)^2+$a(-1) + b$

5=-1+3-$a + b$

5-2=$b - a$

3=$b - a$……………………(i)

$x - 2 = 0$

$x = 2$

f(2)=(2)^3+3(2)^2+$a(2) + b$

8=8+12+2$a + b$

8-20=2$a + b$

-12=2$a + b$…………………(ii)

In (ii) put $b = a + 3$

$\therefore$-12=2$a + (a + 3)$

-12=2$a + a + 3$

-12-3=3a

-15=3a

$a = -5$

In (i) put $a = -5$

$\therefore 3 = b - (-5)$

$b = -2$

EXERCISE

1. Use the remainder theorem to find the following expressions
   - i. $x^3 - 8x^2 - 4x + 5$ divided by $(x - 2)$
   - ii. $7x^3 - 7x - 42$ divided by $(x - 2)$
   - iii. $4x^3 - 5x + 4$ divided by $(2x - 1)$

2. When $x-3$ is divided into x^3-4x^2+5x+k find k.
3. When ax^3+bx^2+3x-4 is divided by $(x-1)$ the remainder is 3, and when it is divided by $(x+2)$, the remainder is 6. Find a and b
4. $(x-1)$ and $(x+1)$ are factors of the expression x^3+ax^2+bx+c.the expressions leaves the remainder of 12 when it is divided by $(x-2)$. What are the values of a, b and c

FACTOR THEOREM

If $f(a)=0$, then $(x+a)$ is a factor of $f(x)$

EXAMPLES

Factorize, $\boldsymbol{t^3-6t^2+11t-6}$

There are two methods of factorizing polynomials

i. Using factors of the constant
ii. By inspection

Solution

i. By using factors of the constant. We find the factors of the constant, in this case

The factors of 6 are; $\pm1, \pm2, \pm3, \pm6$

Let $t=1$

$f(1)=(1)^3-6(1)^2+11(1)-6$

$=1-6+11-6$

$=0$

$\therefore (t-1)$ Is a factor

$$
\begin{array}{r|l}
 & t^2-5t+6 \\
\hline
t-1 & t^3-6t^2+11t-6 \\
 & \underline{-(t^3-t^2)} \\
 & \quad -5t^2+11t \\
 & \quad \underline{-(-5t^2+5t)} \\
 & \qquad 6t-6 \\
 & \qquad \underline{-(6t-6)} \\
 & \qquad\qquad 0
\end{array}
$$

$\therefore t^3-6t^2+11t-6=$

$=(t-1)(t^2-5t+6)$

$=(t-1)(t-2)(t-3)$

ii. By inspection

Used when one of the factors is known

EXAMPLE

Factorize $2x^3-x^2-2x+1$

If one factor is $(x-1)$

Solution

$(x-1)(ax^2+bx+c)=2x^3-x^2-2x+1$

$=ax^3+bx^2+cx-ax^2-bx-c$

$=ax^3+bx^2-ax^2+cx-bx-c$

$=ax^3+(b-a)x^2+(c-b)x-c$

(**Factorizing**)

$\therefore a=1, b-a=-1, -c=1$

$b-2=-1, c=-1$

$b=1$

$\therefore (x-1)(2x^2+x-1)$ **We substitute**

$(x-1)(x+1)(2x-1)$

EXERCISE

1. Factorize the following polynomials by factor method
   i. x^3+3x^2-6x-8
   ii. $6y^3+5y^2-21y+10$
   iii. $2k^3+k^2+k+2$
   iv. $6x^3-13x^2+x+2$
2. Factorize the following polynomials given one factor
   i. $x^3-2x^2-5x+6, (x-1)$
   ii. $2x^3+x^2-8x-4, (x-2)$
   iii. $x^3-4x^2+x+6, \ (x+1)$
3. Show that $12x^3+16x^2-5x-3$ is divisible by $(2x-1)$ and hence find the other factors of the expression

IDENTICAL POLYNOMIALS

These are polynomials which when expanded, gives the same expression

EXAMPLE

Given that $\boldsymbol{2x^3 + kx^2 + mx + 1}$ is identical to$(\boldsymbol{2x-1})(\boldsymbol{x^2+x+1})$ find k and m.

Solution

$$2x^3 + kx^2 + mx + 1 = (2x+1)(x^2+x+1)$$

$$=2x^3 + 3x^2 + 3x + 1$$

$$\therefore k = 3, m = 3$$

EXERCISE

1. Given that $\boldsymbol{By^3 - Cy^2 - y + 3}$ is identical to$(\boldsymbol{y-1})(\boldsymbol{4y^2-1})$. Find the value of B and C
2. Given that $(\boldsymbol{3x^2+x-6})(\boldsymbol{x+1})$ and$\boldsymbol{3x^2 + kx^2 + hx - 6}$ are identical, find the value of k and h.

SOLVING POLYNOMIAL

Factor theorem is used when solving polynomial.

EXAMPLE

$x^3 - x^2 - x + 1 = 0$

Solution

Factors of $1 = \pm 1$

Let x =1

$f(1) = (1)^3 - (1)^2 - (1) + 1$

$= 0$

$f(1) = 0$

$\therefore x - 1$ is a factor

$$\begin{array}{r|l} & x^2 + 0 - 1 \\ \hline x - 1 & x^3 - x^2 - x + 1 \\ & \underline{(x^3 - x^2)} \\ & \quad 0 - x \\ & \quad \underline{(0 - 0)} \\ & \qquad -x + 1 \\ & \qquad \underline{(-x + 1)} \\ & \qquad\qquad 0 \end{array}$$

$(x+1)(x^2-1) = 0$

$(x+1)(x-1)(x+1) = 0$

$(x+1)(x+1)(x-1)=0$

$\therefore x = -1 \text{ or } x = 1$

EXERCISE

Solve the following

1. $x^3 + 3x^2 - x - 3 = 0$
2. $t^3 - 13t - 12 = 0$
3. $4y^3 - 12y^2 - y + 3 = 0$
4. $x^3 - 3x^2 + 3x - 1 = 0$
5. $x^3 - 9x + 10 = 0$

CHAPTER 8

PROBABILITY

Means a chance

$$\text{Probability} = \frac{\text{no. of ways an event can occur}}{\text{total no. of possible outcomes}}$$

P(certainity) = 1

P(impossible event) = 0

Generally, probability is between 0 and 1 inclusive

$P \geq 0$

$P \leq 1$

Thus $0 \leq p \leq 1$

p(success)+p(not success)=1

EXAMPLE

Given that the probability of having girls at a school is $\frac{7}{10}$, what is the probability of not having girls.

Solution

p(not having) + p(having)=1

p(not having)=$1-\frac{7}{10}$

p(not having)=$\frac{3}{10}$

EXAMPLE 2

What is the probability of getting an even number when a die is rolled?

Solution

The numbers are, 1,2,3,4,5,6

Even numbers are;2,4 and 6

$\therefore$p(even numbers)=$\frac{3}{6}$

$=\frac{1}{2}$

EXAMPLE

A bag contains 3 red and 7 white balls. What is the probability that when I pick one ball out, it is a red ball?

Solution

There are 3 red ball

$\therefore$p(red ball)=$\frac{3}{10}$

PARK OF CARDS

In total there are 54 cards but in mathematics we only use 52 cards

Clubs	Hearts	spades	Diamond
Ace	Ace	Ace	Ace
King	King	King	king
Queen	Queen	queen	Queen
Pack	Pack	Pack	Pack
10	10	10	10
9	9	9	9
8	8	8	8
7	7	7	7
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2

EXAMPLE

A card is picked from a park of cards. What is the following probabilities?

i. P(picking an ace)
ii. P(picking an ace of diamond)
iii. P(picking a red card)
iv. P(picking a red heart)
v. P(picking two red heart)
vi. P(a card with a prime number on it)

Solution

i. P(ace)=$\frac{4}{52}=\frac{1}{13}$

ii. P(ace of diamond)=$\frac{1}{52}$

iii. p(red card)$=\frac{26}{52}=\frac{1}{2}$

iv. p(heart)$=\frac{13}{52}=\frac{1}{4}$

v. p(2 red heart)$=\frac{1}{52}$

vi. p(card with prime no.)$=\frac{16}{52}$

$=\frac{4}{13}$

EXERCISE

1. A die is rolled once, calculate the probability of getting the following
   a. Even number
   b. Prime or odd number
   c. Score of 6
   d. Score of 10
2. A bag contains 6 red balls and 8 blue balls. A ball is removed , what is the probability that this ball is blue
3. A card is picked at random from pack of cards. What are these probabilities
   a. p(king)
   b. P(diamond)
   c. p(18 of hearts)

EXPERIMENTAL PROBABILITIES

Exp. Prob$=\frac{\text{no. of favourable outcones}}{\text{total number of trials}}$

EXAMPLE

One weekend, the number of goals scored by the teams in a football league matches were as follows.

No. of goals	0	1	2	3	4	5
No. of teams	8	10	12	2	3	5

What is the probability that a team picked at random had the following score?

a. No goals
b. More than one goal
c. Less than mean number of goals per football match.

Solutions

a. P(no goals)$=\frac{8}{40}=\frac{1}{5}$

b. P(>1goal) $=\frac{12+2+3+5}{40}$

$=\frac{22}{40}$

$=\frac{11}{20}$

c. Mean $=\frac{0(8)+1(10)+2(12)+3(2)+4(3)+5(5)}{40}$

$=\frac{77}{40}$

$=1.9$

≈ 2

$\therefore$ P (< mean no. of goals) $=\frac{8+10}{40}$

$=\frac{9}{20}$

EXERCISE

In a traffic survey, the number of people(including the driver) in each car passing a school was recorded and the results were as follows

No. of people	1	2	3	4	5
No. of cars	17	12	14	21	16

What is the probability that a car picked at random has;

i. Exactly 4 passengers
ii. Less than 2 passengers
iii. Less than the average number of passengers

In a survey, the heights of pupils in form 3 were recorded as follows

Heights of pupils	140-9	150-9	160-9	170-9	180-9
No. of pupils	2	12	14	21	16

What is the probability that a pulp picked at random is

i. More than 169cm tall
ii. Less than 160cm tall

POSSIBILITY/ PROBABILITY /SAMPLE SPACE

Possibility space show all the possible outcomes by listing them down

It is a complete set of possible results or outcomes of an experiment

EXAMPLE

Three coins are tossed. State the possibility space and use it to find the following

i. P(three heads)
ii. P(two heads)
iii. P(one head)
iv. P(0 head)
v. P(not head)

Solutions

The possible outcomes are;

HHH, HHT, HTH, THH,

HTT, THT, TTH, TTT

i. P(3heads)=p(HHH)=$\frac{1}{8}$
ii. P(2heads)=p(HHT) or P(HTH) or p(THH)
$=\frac{3}{8}$
iii. P(1head)=p(THT) or P(TTH) or p(HTT)
$=\frac{3}{8}$
iv. P(0 head)= p(TTT)
$=\frac{1}{8}$
v. P(no 0 head)=1-p(0 head)
$=1-\frac{1}{8}$
$=\frac{7}{8}$

EXAMPLE 2

Use the possibility space for tossing two dice to calculate these probabilities

i. P(a score of 10 or more)
ii. P(an even score)
iii. P(score of less than 12)
iv. P(a score of 13)

Solution

1st die

2nd die	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

NB; for score add number of die.

i. P(score of 10 or more)
$=\frac{6}{36}$
$=\frac{1}{6}$
ii. P(even score)
$=\frac{18}{36}$
$=\frac{1}{2}$
iii. P(score<12)=$\frac{35}{36}$
iv. P(score of 13)=0

EXERCISE

1. Three digits 2, 1 and 3 are written in random order to make a two digit number.
   a. list the possibility space
   b. what is the probability that a number formed is even
   c. What is the probability that the number formed is prime
2. Construct the possibility space for 4 coins and use it to calculate these probabilities
   i. P(4 heads)
   ii. P(less than 4 heads)
   iii. P(2 heads)
3. A number is selected from the set S={1,2,3…18}. Find the following
   i. P(prime number)
   ii. P(even number)
   iii. P(multiple of 6)
4. A coin is tossed and a die is rolled. Illustrate the possible outcomes on the possibility space. Find the probability that the outcomes will be;
   i. A head and a 1
   ii. A head and a 3
   iii. A tail and a 5

TREE DIAGRAMS

Use branches.

The probability is found using the branches.

It is clear and more flexible illustrations of outcomes and their probabilities other than possibility space.

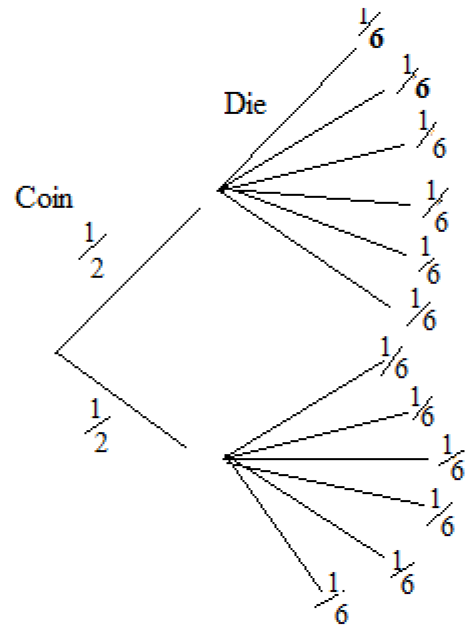
The probabilities of the individual outcomes are shown on these branches.

NB; at any intermediate stage of at final count, the sum of the branches must be equal to one

EXAMPLES

I Toss a coin and roll a die, draw the tree diagram to show all the possible outcomes. Use it to find P (having a head and an even number)

Solution



P (H and even #) =P(H,2) or p(H,4) or p(H,6)

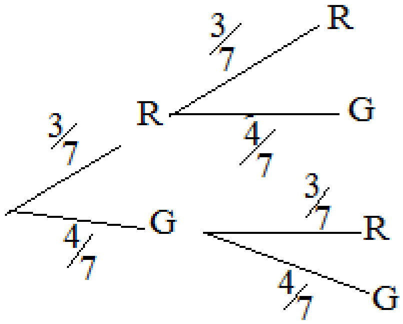
$$=\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{6}$$

$$=\frac{1}{12}+\frac{1}{12}+\frac{1}{12}$$

$$=\frac{1}{4}$$

2. A bag contains 3 red beads and 4 green beads. A bead is taken at random, color noted and a bead replaced. If two successive draws are made, find the probability that the result will be;
   i. 2 red balls
   ii. 1 red and 1 green

Solution



i. P(RR)=$\frac{3}{7}\times\frac{3}{7}$

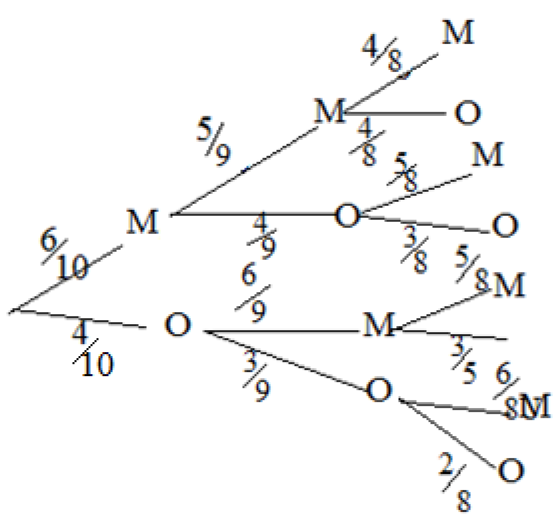
$=\frac{9}{49}$

ii. P(red and green)=$\frac{12}{49}+\frac{12}{49}$

$=\frac{24}{49}$

3. A basket contains 6 mangoes and 4 oranges. Three fruits are removed from it without replacement. Use a tree diagram to work out the following probabilities.
   a. P (three mangoes are removed)
   b. P(a mango and two oranges)
   c. P(not picking 3mangoes)

Solution



a. P(3 mangoes)= P(MMM)

$=\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}$

$=\frac{1}{6}$

b. P(MOO or OMO or OOM)=

$=\left(\frac{6}{10}\times\frac{4}{9}\times\frac{3}{8}\right)+\left(\frac{4}{10}\times\frac{6}{9}\times\frac{3}{8}\right)+\left(\frac{4}{10}\times\frac{3}{9}\times\frac{6}{8}\right)$

$=\frac{72}{720}+\frac{72}{720}+\frac{72}{720}$

$=\frac{3}{10}$

c. P(not 3)= 1- P(MMM)

$= 1 - \frac{1}{6}$

$=\frac{5}{6}$

4. A bag contains beans, g nuts and maize seed. The probability of getting at random a beam seed is $\frac{1}{5}$, a g nut seed is $\frac{x}{15}$maize is$\frac{1}{3}$. Find the value of x

Solution

$\frac{x}{15}+\frac{1}{3}+\frac{1}{5}=1$

(sum of prob = 1)

$x+5+3=15$

$\boldsymbol{x+8=15}$

$\boldsymbol{x=15-8}$

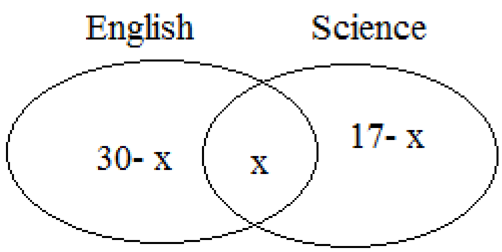
$\boldsymbol{=7}$

USING VENN DIAGRAM TO FIND PROBABILITY

EXAMPLE

In a class of 30 pupils like English and 17 like science. All 46 students say they like at least one of these subjects. What is the probability that a pupil chosen at random likes exactly one subject?

Solution



Let those taking both subjects be x

$$\therefore 30 - x + x + 17 - x = 46$$

$$47 - 46 = x$$

$$x = 1$$

$\therefore$ P(pupil like exactly one sub)= $\frac{45}{46}$

EXERCISE

1. In a plastic bag, there are x blue pens, 6 black pens and 4 red pens. If the probability of picking a red pen is $\frac{1}{5}$.Calculate number of blue pens
2. A coin A is tossed followed by coin B. The probability that a coin A shows head is $\frac{1}{2}$, while the probability that coin B shows head is $\frac{1}{4}$. Using the tree diagram, calculate the probability that both coins A and B shows tails
3. The probability of a bus arriving early at a depot is $\frac{1}{10}$and arriving rate is $\frac{3}{10}$.If 400 buses are expected at the depot during the day, calculate the number of buses that are likely to arrive at the depot in time.
4. The probability of having early lunch at a boarding is $\frac{2}{3}$, when lunch is early, the probability of having a beef is $\frac{7}{10}$and when late the probability of having a beef is$\frac{1}{8}$.

Draw a tree diagram to represent this information completing all branches.

CHAPTER 9

VECTORS

Any translation vector can be written as $\binom{x}{y} = \binom{2}{-4}$

Coordinates + translation = coordinates of
object vector of image

Thus O + V = I

EXAMPLE

1. Find the coordinates of the image of (-3, 4) under translation vector $\binom{-3}{2}$.

   Solution

   $$\binom{3}{4} + \binom{-3}{2} = \binom{x}{y}$$
   $$\binom{-6}{6} = \binom{x}{y}$$

   ∴ The coordinates of image are $(-6, 6)$

2. $\underline{p} = \binom{-2}{3}$, $\underline{q}=\binom{-3}{1}$

   a. Find the column vector which is parallel to p and six times as long.
   b. Find column vector which is four times the length of q but in opposite direction.

NB: If k is a positive // then $k\overrightarrow{AB}$, is k times the length of$\overrightarrow{AB}$, and in the same direction$\overrightarrow{AB}$.

In other words $k\overrightarrow{AB}$, is parallel to $\overrightarrow{AB}$, and has length $k|\overrightarrow{AB}|$ or k $\underline{a}$ is parallel to $\underline{a}$ and of length $k|\underline{a}|$.

<u>VECTORS ARE PARALLEL IF</u>

a. There is a common vector
b. They have the same direction

   Solution

   a. $\underline{p} = \binom{-2}{-3}$

   ∴ 6p=6$\binom{-2}{-3} = \binom{-12}{-18}$

   b. $\underline{q} = \binom{-3}{1}$

   $-4\underline{q} = -4\binom{3}{1} = \begin{matrix}12\\-4\end{matrix}$

3. P =(5, -3)

   a. If $\overrightarrow{\mathbf{PQ}}$=$\binom{2}{-1}$. Find the coordinates of Q
   b. $\overrightarrow{\mathbf{PR}}$**=3**$\overrightarrow{\mathbf{PQ}}$, find the coordinates of R

   solution

   a. P+$\overrightarrow{PQ}$=Q

   $$\binom{5}{-3} + \binom{2}{-1} = \binom{x}{y}$$
   $$\binom{7}{-4} = \binom{x}{y}$$

   ∴ Q= (7, -4)

   b. $\overrightarrow{\text{PR}} = \binom{2}{-1}$

   ∴ 3$\overrightarrow{\text{PR}}$= 3$\binom{2}{-1}$= $\binom{6}{-3}$

   P + $\overrightarrow{\text{PR}}$=R

   $$\binom{5}{-3} + \binom{6}{-3} = R$$

   $$\binom{11}{-6} = R$$

∴The coordinates of R= (11, -6)

EXERCISE

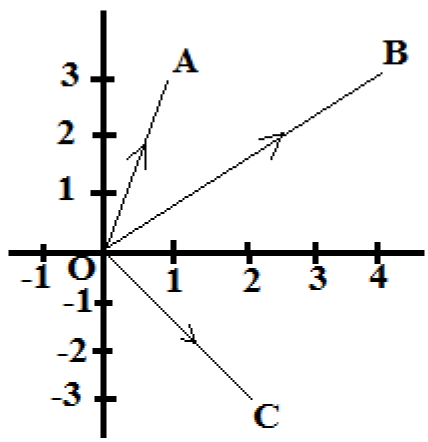
1. A =(2, 1), is mapped on to point (3, -1) under a translation .What would be the image of C=(-2, -3) under the same translation vector.
2. Vector $\underline{g}$=$\binom{3}{-2}$, vector $\underline{h}$=$\binom{-2}{-4}$.

   a. Find the image of A==(0, -1) under a translation $(2\underline{g} - 3\underline{h})$
   b. Find the image of B= (-2, -1) under the translation $3(\underline{g} - \underline{h})$

<u>POSITION VECTORS</u>

P0sitional vectors can be used to specify the position of a point with respect to a fixed point.

On a Cartesian coordinates, the fixed point is usually the origin O



In positional vectors, the coordinates of an object is (0, 0)

$\overrightarrow{OA}=\binom{1}{3}-\binom{0}{0}$

$=\binom{1}{3}$

$\overrightarrow{OB}=\binom{3}{4}-\binom{0}{0}$

$=\binom{3}{4}$

$\overrightarrow{OC}=\binom{2}{-3}-\binom{0}{0}$

$=\binom{2}{-3}$

In general, in position vectors, translation vector is equal to coordinates of image

EXAMPLE

If k =(2, 3) and M=(5, 6). Find the following

a. The positional vectors of M and K
b. $\overrightarrow{\mathrm{KM}}$
c. $\overrightarrow{\mathrm{MK}}$

Solution

a. $\overrightarrow{\mathrm{OK}}=\binom{2}{3}-\binom{0}{0}=\binom{2}{3}$

$\overrightarrow{\mathrm{OM}}=\binom{5}{6}-\binom{0}{0}=\binom{5}{6}$

b. $\overrightarrow{\mathrm{KM}}=\mathrm{M}-\mathrm{K}$

$=\binom{5}{6}-\binom{2}{3}$

$=\binom{3}{3}$

c. $\overrightarrow{\mathrm{MK}}=-\overrightarrow{\mathrm{KM}}$

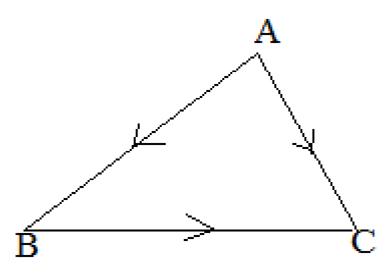
$=\binom{-3}{-3}$

EXERCISE

1. If T= (8, 0), W=(2, 0) and Z= (5, 0). Find the following.
   a. The positional vector of T and Z
   b. $\overrightarrow{\mathrm{WZ}}and\overrightarrow{\mathrm{ZT}}$.
2. $\underline{a}=\binom{4}{2}$ and $\underline{b}=\binom{-6}{-12}$. Work out $\frac{1}{2}\underline{a}+\frac{2}{3}\underline{b}$.

ADDITION OF VECTORS

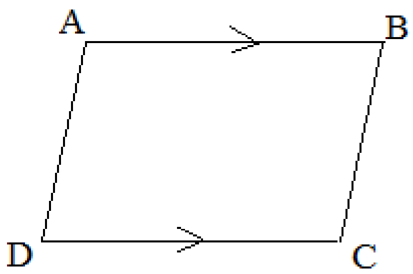
TRIANGLE LAW OF ADDITION OF VECTORS



$\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{BC}}=\overrightarrow{\mathrm{AC}}$

PARALLELGRAM

A quadrilateral is a //gram when two opposite sides are equal and parallel.



When opposite <s are equal,

$$\overrightarrow{AD} // \overrightarrow{BC}$$

$$\overrightarrow{AD} = \overrightarrow{BC}$$

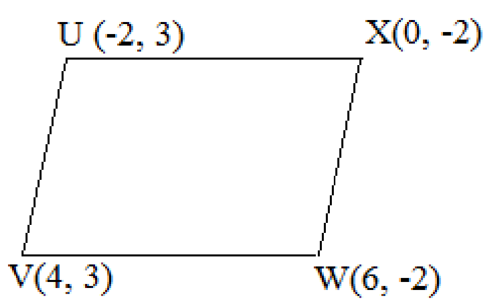
∴ ABCD is a //gram

A quadrilateral is //gram when

- One pair of the sides is equal and // to each other.
- Opposite angles are equal
- Two opposite sides are equal

EXAMPLE

1. Show that quadrilateral UVWX which has U(-2, 3), V(4, 3), W(6, -2) and X(0, -2) is a //gram.
   Solution



$\overrightarrow{UX} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

$\overrightarrow{VW} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

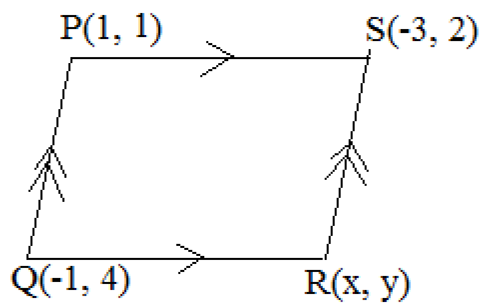
$= \begin{pmatrix} 2 \\ -5 \end{pmatrix}$

∴ $\overrightarrow{UX} = \overrightarrow{VW}$ and $\overrightarrow{UX} // \overrightarrow{VW}$

∴ UVWX is a //gram

2. In a parallelogram PQRS, P= (1,1), Q=(-1, 4), S=(-3, 2). Find the coordinates of R.

Solution



$\overrightarrow{PS} = \overrightarrow{QR}$, opp. Sides of a //gram

∴ $\overrightarrow{QR} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$$Q + \overrightarrow{QR} = R$$

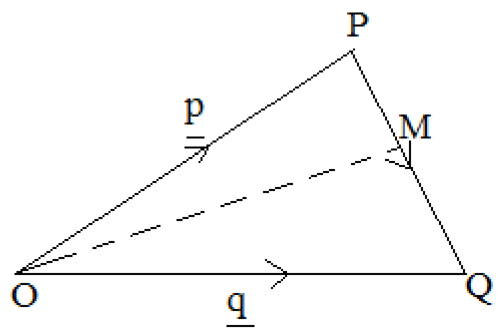
$$\begin{pmatrix} -1 \\ 4 \end{pmatrix} + \begin{pmatrix} -4 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} -5 \\ 5 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

∴ R=(-5, 5)

<u>VECTOR ALGEBRA</u>

We can apply triangle law of addition of vectors to position vectors.



$\overrightarrow{OP}+\overrightarrow{PQ}=\overrightarrow{OQ}$

$\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}$

$=\underline{q}-\underline{p}$

<u>MID POINT OF A VECTOR</u>

If M is a mid-point in the figure above.

$$\overrightarrow{PM}=\frac{1}{2}\overrightarrow{PQ}=\frac{1}{2}(\underline{q}-\underline{p})$$

$\overrightarrow{OM}=\overrightarrow{OP}+\overrightarrow{PM}$

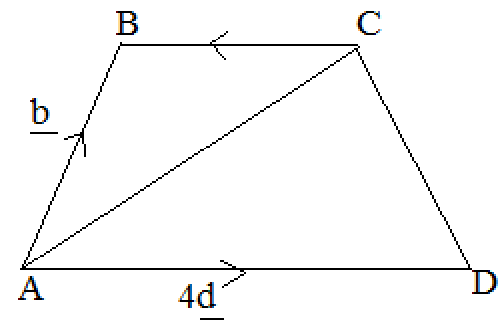
$=\underline{p}+\frac{1}{2}(\underline{q}-\underline{p})$

$=\frac{1}{2}(\underline{p}+\underline{q})$

EXAMPLE

1. ABCD is a quadrilateral in which BC // AD. $\overrightarrow{AD}=4\underline{d}, \overrightarrow{AB}=\underline{b}, \ \overrightarrow{BC}=\frac{3}{4}\overrightarrow{AD}$. Find
   a. $\overrightarrow{BC}$
   b. $\overrightarrow{AC}$
   c. $\overrightarrow{CD}$(in terms of $\underline{b}$ and $\underline{d}$)

<u>solution</u>



a. $\overrightarrow{BC}=\frac{3}{4}\overrightarrow{AD}$.

$=\frac{3}{4}\times 4\underline{d}$

$=3\underline{d}$

b. $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

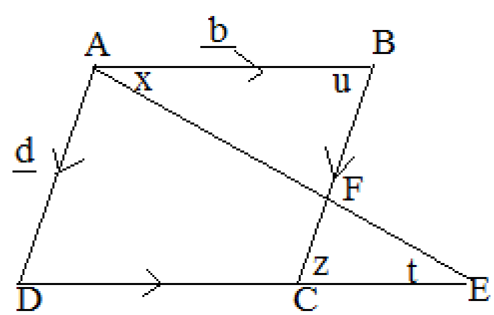
$=\underline{b}+3\underline{d}$

c. $\overrightarrow{AC}+\overrightarrow{CD}=\overrightarrow{ACD}$

$\underline{b}+3\underline{d}+\overrightarrow{CD}=4\underline{d}$

$\overrightarrow{CD}=4\underline{d}-\underline{b}-3\underline{d}$

2. In the figure, ABCD is a //gram $\overrightarrow{DC}=4\overrightarrow{CE}, \overrightarrow{AB}=\underline{b}$ and $\overrightarrow{AD}=\underline{d}$



a. Show that Δs ABC and ECF are similar
b. Show that $\overrightarrow{BF}=4\overrightarrow{FC}$
c. Show that $\overrightarrow{\mathbf{AF}}=\frac{1}{5}((\mathbf{5\underline{b}}+\mathbf{4\underline{d}})$
d. Show that $\overrightarrow{\mathbf{AF}}=\frac{1}{4}(\mathbf{5\underline{b}}+\mathbf{4\underline{d}})$

<u>Solution</u>

a. **AB //DC**, opp. Sides of //gram

In Δs ABF and ECF

$x=t, alt<s$

$u=z, alt<s$

< CFE=<AFB, vert. opp. <s

∴ **ΔABC /// ECF (a. a. a)**

$$\therefore \frac{\text{AB}}{\text{EC}} = \frac{\text{BF}}{\text{CF}} = \frac{\text{AF}}{\text{EF}}$$

b. $\therefore \frac{\overrightarrow{\text{AB}}}{\overrightarrow{\text{EC}}} = \frac{\overrightarrow{\text{BF}}}{\overrightarrow{\text{CF}}}$

But $\overrightarrow{\text{AB}} = \overrightarrow{\text{DC}}$, opp. Sides of a //gram

$\overrightarrow{\text{DC}} = 4\overrightarrow{\text{CE}}$

$\frac{\overrightarrow{\text{DC}}}{4} = \overrightarrow{\text{CE}}$

$\therefore \overrightarrow{\text{EC}} = \frac{-\underline{b}}{4}$

$\therefore \frac{\overrightarrow{\text{AB}}}{\overrightarrow{\text{EC}}} = \frac{\underline{b}}{-\underline{b}/4}$

$= -4$

$-4 = \frac{\overrightarrow{\text{BF}}}{\overrightarrow{\text{CF}}}$

$-4\overrightarrow{CF} = \overrightarrow{BF}$

$\overrightarrow{CF} = -\overrightarrow{FC}$

$\therefore \overrightarrow{BF} = -4(-\overrightarrow{FC})$

$\overrightarrow{BF} = 4\overrightarrow{FC}$

c. $\text{FC}=\frac{\underline{d}}{5}$

$\overrightarrow{\text{AF}}=\overrightarrow{\text{AB}}+\overrightarrow{\text{BP}}$

$= \underline{b} + 4(\frac{\underline{d}}{5})$

$=\frac{5\underline{b}+4\underline{d}}{5}$

$=\frac{1}{5}(5\underline{b} + 4\underline{d})$

d. $\overrightarrow{\text{AD}}+\overrightarrow{\text{DE}}=\overrightarrow{\text{AE}}$

$\underline{d} + \frac{5\underline{b}}{4}=\overrightarrow{\text{AE}}$

$\frac{4\underline{d}+5\underline{b}}{4}=\overrightarrow{\text{AE}}$

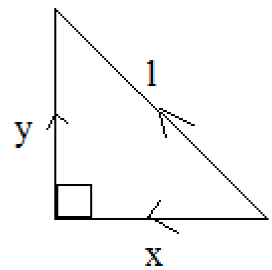
$\overrightarrow{\text{AE}}=\frac{1}{4}(4\underline{d} + 5\underline{b})$

$=\frac{1}{4}(5\underline{b} + 4\underline{d})$

LENGTH OF A VECTOR

$\overrightarrow{\text{AB}} = \binom{x}{y}$

Length of vector AB=$|AB|$



$l^2 = x^2 + y^2$, Pythagoras theorem

$l = \sqrt{x^2 + y^2}$

EXAMPLE

1. Find the length of vector $\binom{-3}{4}$

   Solution

$\left|\begin{matrix}-3\\4\end{matrix}\right| = \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9 + 16}$

$=\sqrt{25}$

$= 5$ units

2. If a $=\binom{1}{2}$ and b$=\binom{3}{1}$. find t in the following equation,
   $|2a - b| = t|a - b|$

Solution

$2a - b = 2\binom{1}{2} - \binom{3}{1}$

$=\binom{2}{4} - \binom{3}{1}=\binom{-1}{3}$

$\therefore |2a - b| = \sqrt{(-1)^2 + (3)^2}$

$=\sqrt{1 + 9}$

$=\sqrt{10}$

$a - b = \binom{1}{2} - \binom{3}{1}$

$=\binom{-2}{1}$

$\therefore |a - b| = \sqrt{(-2)^2 + (1)^2}$

$=\sqrt{5}$

$\therefore \sqrt{10} = t\sqrt{5}$

$t = \frac{\sqrt{10}}{\sqrt{5}}$

$t = \sqrt{2}$

PARALLEL LINES

Parallel lines have common vector and same direction.

EXAMPLE

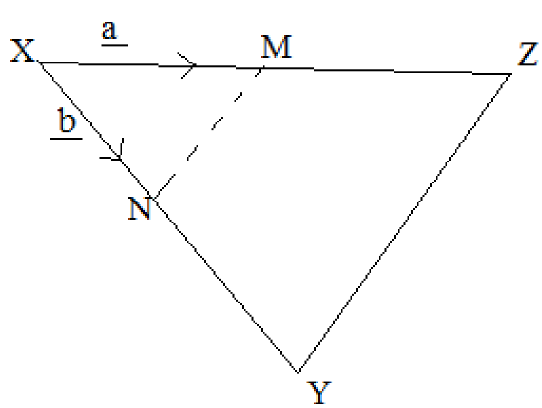
1. Show that $\binom{4}{-8}$ and $\binom{3}{-6}$ are parallel.

   Solution

   $\binom{4}{-8}$ And $\binom{3}{-6}$

   $4\binom{1}{-2}$ *and* $3\binom{1}{-2}$

   $\therefore$ They are //
2. In figure M and N are the mid points of $\overrightarrow{XZ}$ and $\overrightarrow{XY}$ respectively.



Show that $\overrightarrow{MN}$ and $\overrightarrow{ZY}$ are parallel.

Solution

XM=MZ, M is a mid-point

XN=NY, M is a mid-point

$\overrightarrow{XM}+\overrightarrow{MN}=\overrightarrow{XN}$

$\underline{a} + \overrightarrow{MN}=\underline{b}$

$\overrightarrow{MN}= \underline{b} - \underline{a}$

$\overrightarrow{XZ} = 2\underline{a}$ And $\overrightarrow{XY} = 2\underline{b}$

$\overrightarrow{XZ}+\overrightarrow{ZY}=\overrightarrow{XY}$

$2\underline{a} + \overrightarrow{ZY} = 2\underline{b}$

$\overrightarrow{ZY} = 2\underline{b} - 2\underline{a}$

$= 2(\underline{b} - \underline{a})$

Since $(\underline{b} - \underline{a})$ is common in both $\overrightarrow{MN}$ and $\overrightarrow{ZY}$

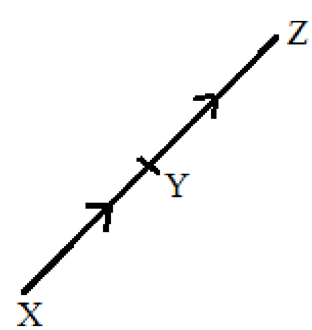
$\therefore \overrightarrow{MN}//\overrightarrow{ZY}$

COLLINEAR POINTS

Points are collinear if they lie in a straight line.

If two vectors are equal, they are parallel and have the common point.

i.e.



XY// YZ

And Y is common

∴ The points X, Y and Z are collinear (they lie in straight line)

EXAMPLE

1. Show that X(-4, 4), Y(2, 7) and Z (6,9) are collinear
   Solution

$\overrightarrow{XY}=\binom{2}{7}-\binom{-4}{4}$

$=\binom{6}{3}=3\binom{\mathbf{2}}{\mathbf{1}}$

$\overrightarrow{YZ}=\binom{6}{9}-\binom{2}{7}$

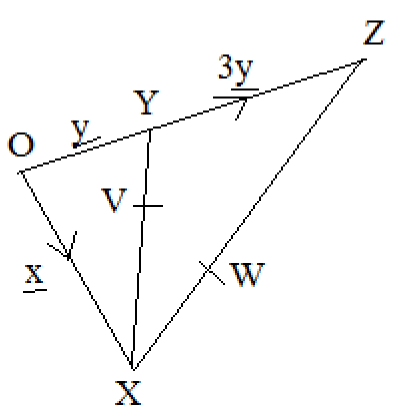
$=\binom{4}{2}=2\binom{\mathbf{2}}{\mathbf{1}}$

∴ $\overrightarrow{XY}//\overrightarrow{YZ}$ and Y is common

∴X, Y and Z are collinear

EXAMPLE 2(MENEB 2016)

Figure below shows positional vectors$\overrightarrow{\mathbf{OX}}=\underline{x}$. $\overrightarrow{OY}$ is produced to a point **Z** where **OY: YZ**=1:3. V is a point on $\overrightarrow{\mathbf{YX}}$ such that $\overrightarrow{\mathbf{YV}}:\overrightarrow{\mathbf{VX}}=$ 1: 2 and W is a point on $\overrightarrow{\mathbf{XZ}}$such that $\overrightarrow{\mathbf{XW}}:\overrightarrow{\mathbf{WX}}=\mathbf{1}:\mathbf{2}$



Show that, O, V and W are collinear

Solution

In ΔOZX, $\overrightarrow{OZ}+\overrightarrow{ZX}=\overrightarrow{OX}$

$4\underline{y}+\overrightarrow{ZX}=\underline{x}$

$\overrightarrow{ZX}=\underline{x}-4\underline{y}$

$\overrightarrow{XZ}=-(\underline{x}-4\underline{y})$

$=4\underline{y}-\underline{x}$

$\overrightarrow{XW}+\overrightarrow{WZ}=\overrightarrow{XZ}$

But $2\overrightarrow{XW}=\overrightarrow{WZ}$

$3\overrightarrow{XW}=4\underline{y}-\underline{x}$

$=\frac{4\underline{y}-\underline{x}}{3}$

$\overrightarrow{OY}+\overrightarrow{YX}=\overrightarrow{OX}$

$\overrightarrow{YX}=\underline{x}-\underline{y}$

But $2\,\overrightarrow{YV}=\overrightarrow{VX}$

∴ $\overrightarrow{YV}+2\overrightarrow{YV}=\overrightarrow{YX}$

$3\,\overrightarrow{YV}=\underline{x}-\underline{y}$

$=\frac{\underline{x}-\underline{y}}{3}$

∴ $\overrightarrow{VX}=\frac{2(\underline{x}-\underline{y})}{3}$

$=\frac{2\underline{x}-2\underline{y}}{3}$

$\overrightarrow{VX}+\overrightarrow{XW}=\overrightarrow{VW}$

$\frac{2\underline{x}-2\underline{y}}{3}+\frac{4\underline{y}-\underline{x}}{3}=\overrightarrow{VW}$

$\overrightarrow{VW}=\frac{2\underline{y}+\underline{x}}{3}$

$\overrightarrow{OY}+\overrightarrow{YV}=\overrightarrow{OV}$

$\underline{y} + \frac{\underline{x}-\underline{y}}{3} = \overrightarrow{OV}$

$\overrightarrow{OV} = \frac{2\underline{y}+\underline{x}}{3}$

$\therefore \overrightarrow{OV} // \overrightarrow{VW}$ and V is common

$\therefore$ O, V and W are collinear

CHAPTER 10

LINEAR PROGRAMMING

In inequalities, the unshaded region on the graph is the region which satisfies all the conditions

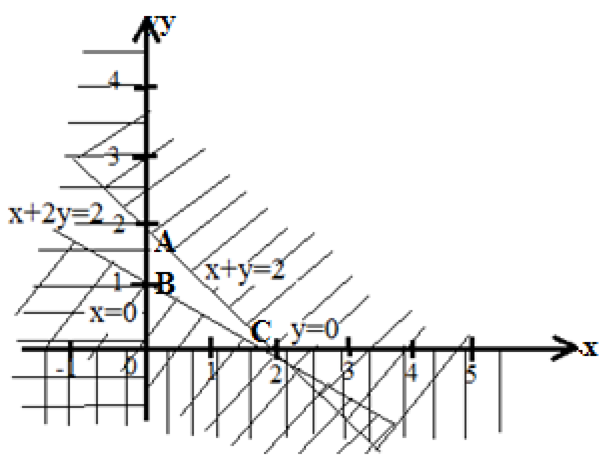
The unshaded region is called feasible or accessible region

EXAMPLE

Show the feasible region representing the following inequalities;

I. $\boldsymbol{x + 2y \geq 2, x + y \leq 2, x \geq 0, y \geq 0}$
II. Find the minimum and maximum value of $2x + 5y$

Solution



To find minimum and maximum values of an expression within the region, we only consider the values at the vertices in the feasible region and substitute in the given expression.

At A(0,2)=2(0)+5(2)=10
At B(0,1)=2(0)+5(1)=5
At C(2,0)=2(2)+5(0)=4
$\therefore$ The minimum value of $2x + 5y$ is 4 at C

The maximum value is 10

EXERCISE

1. Show the region representing the following inequalities;
    i. $x + y \leq 8, x + 3y > 12, x > 0, \ y > 0$
    ii. Find the minimum value of $x - 4y$ in the unshaded region.
2. i. show the feasible region representing the following inequalities;
    $x + y > 0, 3x + 2y \leq 9, x \geq 0, \ y \geq 0$

iii. find the maximum and minimum value of $7x + 2y$ in the feasible region

LENEAR PROGRAMMING AND ITS APPLICATIONS

In linear programming questions, we must find the maximum or minimum value of the objective function while satisfying a set of constraints function.

Points to follow when solving linear programming questions;

i. summarize the information in a table
ii. write down the objective function and all the necessary constraints
iii. graph the feasible region
iv. identify all corner points
v. find the value of the objective function at each corner points
vi. for the bounded region the solution is given by the corner point giving the optimal value(minimum a and maximum) of the objective function
vii. For the unbounded region, check that the solutions actually exists. If it does it will occur at the corner point

EXAMPLES

A shopkeeper wants to buy up to 500 necklaces for her shop. She has k9000 to spend. There are two types of necklaces available, one type costing k30 each and the other costing k10 each. She wants to buy at least 100 of each type.

i. If the shopkeeper buys$x$ of the expensive type and y of the cheaper type of necklace, show graphically the region in which points (x, y) lie, using a scale of 1cm to represent 100 units on each axis
ii. The profit is k10 on the expensive necklaces and k5 on the cheaper necklaces. How many of each type of necklaces should she buy to make the most profit

Solution

Types	Number	Cost	Profit
Type A	x	$30x$	$10x$
Type B	y	$10y$	$5y$
Total	500	9000	P

Using the table, we can have the following inequalities

$x + y \leq 500$……………………..(i)

$30x + 10y \leq 9000,$

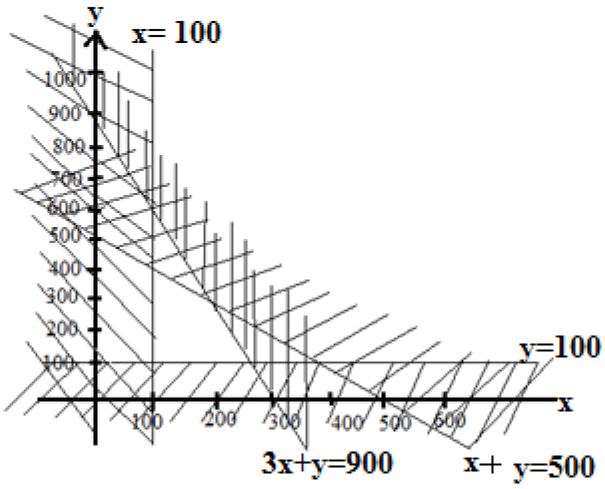
$3x + y \leq 900$……………………(ii)

$x \geq 100$……………………………(ii)

$y \geq 100$……………………………(iii)

NB. At least means greater than or equal to $(\geq)$

At most means less than or equal to $(\leq)$



$P=10x+5y$
At A(100,400)=10(100)+5(400)
=K3000
B(200,300)=10(200)+5(300)
=K3500
C(250,100)=10(250)+5(100)
=K3000
D(100,100)=10(100)+5(100)
=K1500
The maximum profit is K3500 at B(200,300)
∴ She should buy 200 0f the expensive necklaces and 300 of the cheaper necklaces

EXAMPLE 2

A transport company has three 30-passenger buses and nine 15-passenger buses. The company contracts to transport more than 120 passengers a day to the national parks. It cost k3000 per day to run each 30-passenger bus and k1000 per day to run each 15-passenger bus and the company must spend less than k12000 per day in order to meet the costs, if $x$and y are the numbers of 30-passenger and 15 passenger buses each day;

i. Show that $2x+y>8$

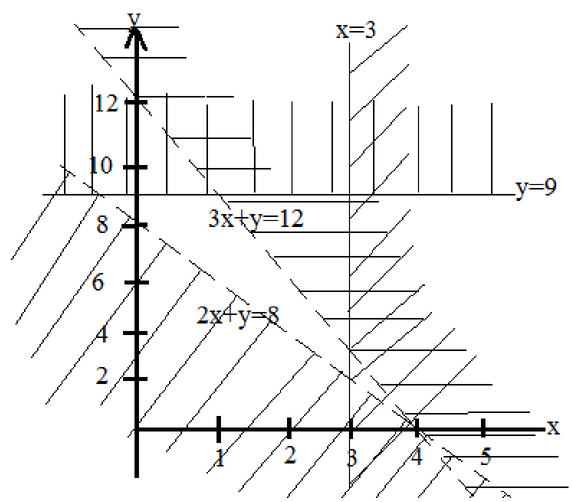
ii. Write down the other inequalities involving x and$y$

iii. Illustrate the solution set of the four inequalities on the graph paper provided and shade the unwanted region

Solution

Types	Number	cost	profit
30-pBus	x	$30x$	$3000x$
15-pBus	y	$15y$	$1000y$
Total		120	12000

$30x+15y>120$
$\therefore 2x+y>8$……………………i
$x\leq 3$…………………………….ii
$y\leq 9$…………………………….iii
$3000x+1000y<12000$
$3x+y<12$…………………..iv



EXERCISE

1. Vendor decides to spend up to k600 to buy sweets and biscuits. She does not want to buy more than 10 packets of each. A packet of sweet costs k40 and that of biscuits costs k80. The profit on each packet of sweet is k10 and on its packet of biscuits is k15.

i. By taking x to represent the number of packets of sweet and y to represent the number of packets of biscuits, write down four inequalities that satisfies the above information
ii. Using the scale of 2cm to represent 2 units on axes, draw graphs to show the region represented by the inequalities
iii. Use your graph to find the number of packets of sweets and biscuits respectively for the vendor to maximize profit

2. A business man wants to buy x meters of low quality cloth at k200 per meter and y meters of high quality cloth at k400 per meter. He decided to
   . Buy at most a total of 1000 meters of cloth
   . Spend at least a total sum of k80, 000
   a. Write down two inequalities in addition to $x \geq 0, y \geq 0$
   b. Using a scale of 2cm to represent 200 units on both axes, draw on the graph paper the regions represented by the four inequalities
   c. A business man will make a profit of k10 per meter on the cheaper cloth and k20 per meter on the expensive cloth. How many meters of cloth of each type of cloth must he buy to get the maximum profit?

CHAPTER 11;

THREE DIMESIONSFIGURES

A PRISMS; A prism is any solid object which have uniform cross-section

Examples of prisms;

i. Triangular prism
   - has cross-section of a triangle
ii. Hexagonal prism
   - Its cross-section is a hexagon
iii. Cuboid
   - Its cross-section is a rectangle
iv. L-shaped prism
   - Its cross-section has an L-shape

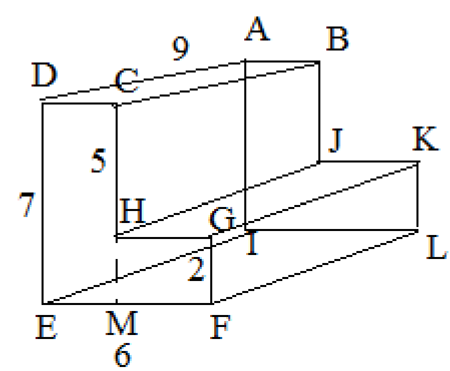
VOLUME OF A PRISM

For any prism;

V =cross-sectional area × height (length)

EXAMPLES

Calculate the volume of the prism below



Solution

Area of rect. DCME=7×2=14cm^2

Area of rect. HGFM=4×2=8cm^2

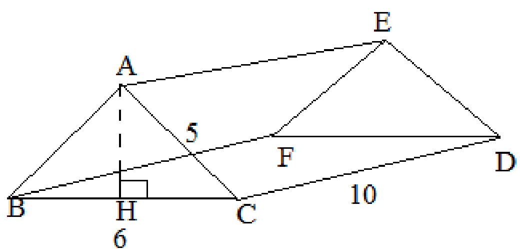
∴ Area of L-shape=(14+8) cm^2=22cm^2

∴ Volume=22cm^2×9cm=198cm^3

EXAMPLE 2

A cross-section of a prism is a triangle with two sides of 5cm and the other side 6cm long. The length of the prism is 10cm. calculate,

i. The height of the triangle
ii. The area of the triangle
iii. The volume of the prism.



Solution

i. In ΔABH, BH= $\frac{6}{2}$, AH ⊥ bisector
=3cm
$AB^2=AH^2+BH^2$, Pythagoras
$25=AH^2+9$
$25-9=AH^2$
$16=AH^2$
$AH=\sqrt{16}$
=4cm
∴ Height=4cm

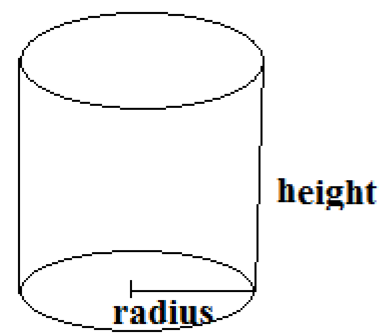
ii. Area ΔABH=$\frac{1}{2}$×6×4
=12cm^2

iii. Volume=12cm^2×10cm
=120cm^3

EXERCISE

1. Calculate the volume of the prism below. All the lengths are in cm(drawing)
2. The cross-section of prism 20cm long has an area of 25cm^2, what is the volume of the prism
3. The cross-section of a prism is a triangle ABC with angle ABC= 90°, AB=10cm and BC=6cm. if the prism is 30cm high, what is its volume?

<u>V0LUME OF THE CYLINDER</u>



Area of the circle= πr^2 (cross- sectional area)

∴ Volume=area circle×height

$$V=\pi r^2 h$$

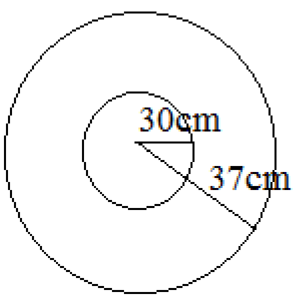
EXAMPLES

1. Figure shows the cross-section of a cylindrical water main pipe which is 10m long. The pipe has an inner radius of 30cm and outer radius 37cm. find the area of the material needed to make the pipe in litres

NB: it is necessary for the units to be the same, so 10m should be in cm too here is the table to assist in converting units,

Km	hm	dam	m	dm	cm	mm
Kg	hg	dag	g	dg	cg	mg
1	0	0	0	0	0	0
	1	0	0	0	0	0
		1	0	0	0	0
			1	0	0	0
				1	0	0
					1	0
						1

Solution



1m = 100cm

10m= more

= 100cm×10

= 1000cm

V of inner cylinder=$\frac{22}{7} \times 30 \times 30 \times 1000$

$= \frac{19800000cm^3}{7}$

$= 2828571.\ cm^3$

V of outer cylinder= $\frac{22}{7} \times 37 \times 37 \times 1000$

$=4302571.4cm^3$

∴ V of the material=2828571.4-4302571.4

Needed

$=1474000cm^3$

In litres the volume $=\frac{1474000}{1000}$

$=1474l$

In general the vol.=πR^2h-$\pi r^2 h$

$= \pi\text{h}(R^2 - r^2)$

$= \pi\text{h}(\text{R} - \text{r})(\text{R} + \text{r})$

EXAMPLE

A lead has internal radius 2.3cm and external radius 3.7cm. if 1cm^3 of lead weighs 11.4g, what is the weight of a 14cm length of wire

Solution

14m =14×100cm

= 1400cm

Volume= $\pi\text{h}(\text{R} - \text{r})(\text{R} + \text{r})$

$= \frac{22}{7} \times 1400(3.7 + 2.3)(3.7 - 2.3)$

$= 4400(6)(1.4)$

$= 36960cm^3$

1cm^3=11.4g

36960cm^3= more

$=\frac{36960cm^3 \times 11.4g}{1cm^3} = 421344g$

∴ 14m length wire weigh 421344g

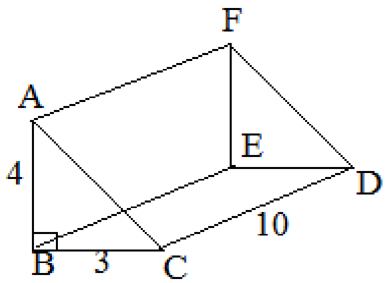
AREA OF A PRISM

The surface area of any prism is the sum of areas of all its faces.

To find surface area of the prism we work out the areas of all faces and add them up.

EXAMPLE

Find the surface area of the prism below lengths are in cm



Area of Δ ABC$=\frac{1}{2}\times4\times3 = 6\text{cm}^2$

$\therefore \Delta$ FED$=6\text{cm}^2$

Area of rect. BCDE$=10\times3=30\text{cm}^2$

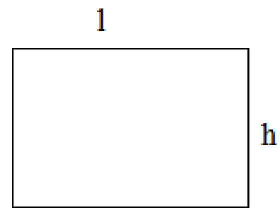
Area of a rect. ABEF$=4\times10=40\text{cm}^2$

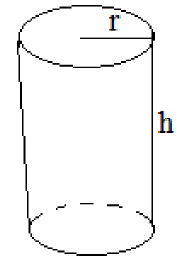
Area of rect. ACDF$=5\times10=50\text{cm}^2$

$\therefore$ Surface area $=6\text{cm}^2+6\text{cm}^2+30\text{cm}^2+40\text{cm}^2+50\text{cm}^2$

$=132\text{cm}^2$

SURFACE AREA OF A CYLINDER





Length of rect. =length of circumf. $= 2\pi r$

Area of a rect. = length $\times$height

$\therefore$ Area of a cylinder$= 2\pi r$

But the area of a circle $=\pi r^2$

$\therefore$ Surface area of a cylinder (open at one end) $=2\pi rh + \pi r^2$

Surface area of a cylinder closed at both ends$=\boldsymbol{2\pi rh} + \boldsymbol{2\pi r}^2$

$= \boldsymbol{2\pi r}(\boldsymbol{h} + \boldsymbol{r})$

EXAMPLE

1. The cartridge of a bullet is cylinder closed at one end. The cartridge is 70mm long and has a diameter of 14mm. find the surface area of the cartridge ($\frac{22}{7}$).

Solution

S. A$=2\pi rh + \pi r^2$

$h = 70mm, r = 7mm$

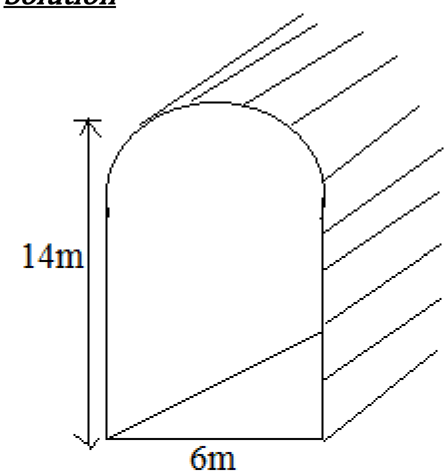
$\therefore$ S. A $=2 \times \frac{22}{7} \times 7 \times 70 + \frac{22}{7} \times 7 \times 7$

$= 3080+154$

$= 3234\text{mm}^2$

2. The figure below shows a tunnel 200m long and 14m high. The roof of the tunnel is semicircular with a diameter of 6m.
   a. It is proposed to paint inside of the tunnel (but not the floor). What is the area to be painted?
   b. Painting cost K2 per square meter. What is the total cost of painting the tunnel?

Solution



Radius=$\frac{6m}{2} = 3m$

a. ∴AB=14m-3m=11m

∴Area of a rect. (outside)= 11m×200m

=2200m^2

∴ area of two rectangles=2×2200=4400m^2

Area of the roof=$\frac{1}{2}$×$2\pi rh$ (sermcircular)

$= \pi rh$

∴Area $=\frac{22}{7} \times 3m \times 200m$

= 1885.7m^2

∴ The area to be painted= 1885.7+4400

= 6285.7m^2

b. 1m^2= 2k
6285.7m^2= x

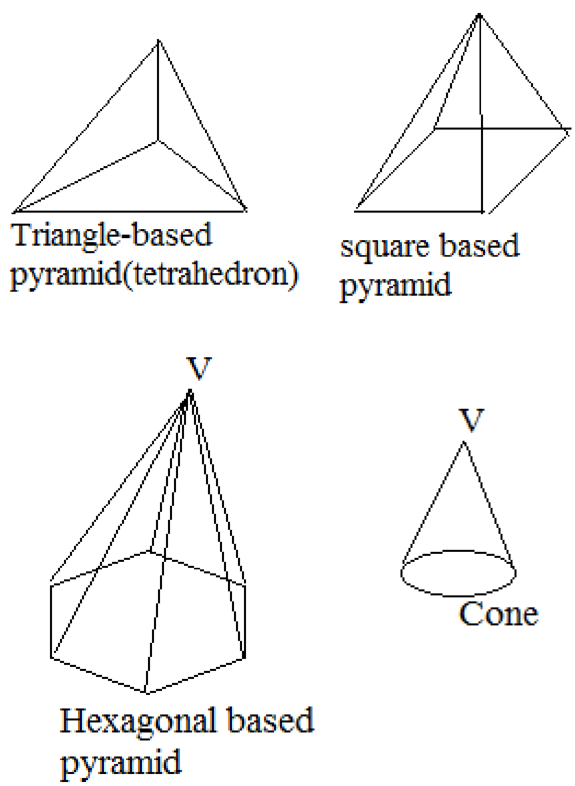
$\frac{1m^2}{6285.7m^2} = \frac{2k}{x}$
X = K12571.4
∴ Total cost= K 12571. 40

PYRAMIDS

A pyramid is a solid whose base is a plane figure and whose side faces meet in a point or vertex.

If the vertex is directly above the Centre of a regular base, it is called a right pyramid. If the base is a circle, then the pyramid is called a Cone.

Examples of pyramids



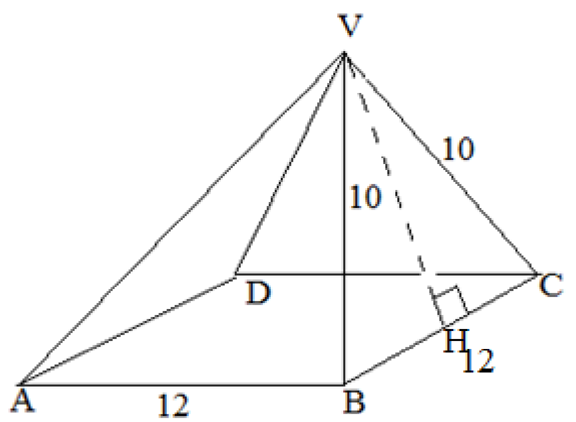
SURFACE AREA OF A PYRAMID

We work out the area of all faces and add up them.

EXAMPLE

A square based pyramid has a base side of 12cm. The other faces are isosceles triangles with equal sides of length 10cm. what is the surface area of the pyramid.

Solution



In Δ VHC, HC=$\frac{12}{2}$, VH $\perp$ bisector

=6cm

$VC^2=VH^2+HC^2$

$100-36= VH^2$

$VH^2=64$

$VH=\sqrt{64}$

= 8cm

Area of Δ VHC=$\frac{1}{2}\times 12cm\times 8cm$

$= 48cm^2$

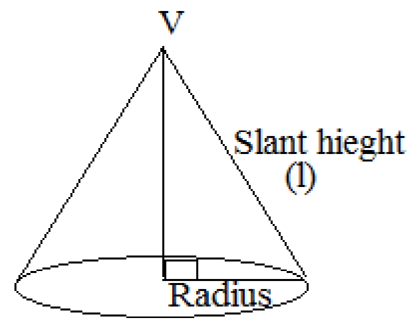
Area of rect. ABCD=12cm×12cm

$=144cm^2$

$\therefore$ Total surface area=$4\times 48cm^2+144cm^2$

$= 336cm^2$

SRFACE AREA OF A CONE

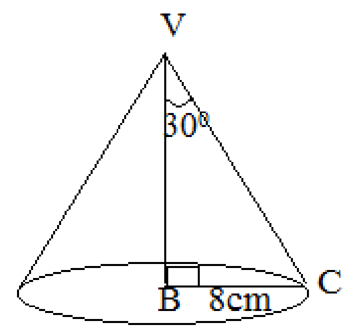


Surface area of a cone (open) $=\pi rl$

S. A of a cone when closed=$\pi rl + \pi r^2$

EXAMPLE

1. Find the total surface area of a cone shown in the figure below



Solution

Sin 30°=$\frac{8cm}{VC}$

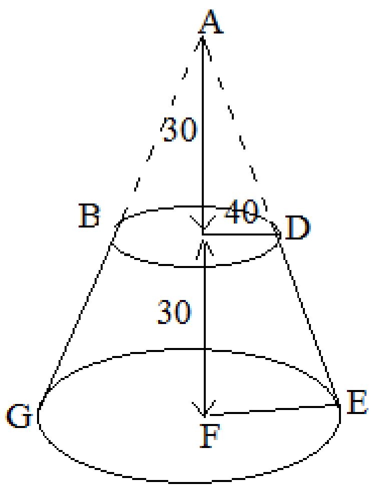
0.5×8cm=VC

VC=4cm

$\therefore$ Surface area $=\frac{22}{7}\times 8cm\times 4cm\times \frac{22}{7}\times 8^2$

$=301.43cm^2$

2. Fig below shows a lump-shade made out of the base of a cone, with the top part removed. Find area of material to make the lampshade



Solution

In ΔACD, $AD^2 = 40^2 + 30°$

$AD = \sqrt{25{,}000}$

$= 50cm$

Area of a cone $ABD = 3.14 \times 40cm \times 50cm$

$= 6280cm^2$

In ΔACD and AFE, $<F = <C$, right $<d$

A is common

$\therefore \Delta\ ACD /// \Delta AEF$

$\therefore \frac{AF}{AC} = \frac{AE}{AD}$

$\frac{60}{30} = \frac{AE}{50}$

$AE = 100$

$AE^2 = EF^2 + AF^2$

$1000 - 3600 = AF^2$

$AF = \sqrt{6400}$

$AF = 80cm$

$\therefore$ Area of a cone $AGF = 3.14 \times 80 \times 100$

$= 25120cm^2$

Area of material used to make lampshade= $25120cm^2 - 6280cm^2$

$= 18840cm^2$

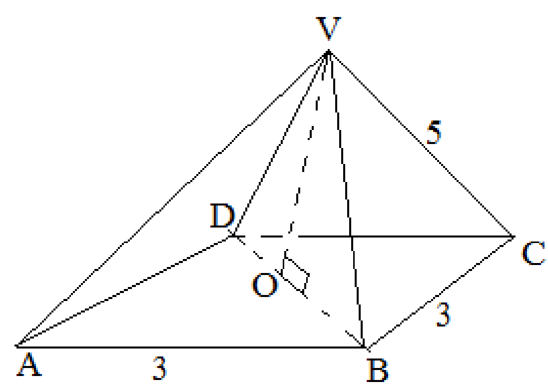
VOLUME OF A PYRAMID

V of a pyramid$= \frac{1}{3}$base area $\times$ heigth

EXAMPLE

Find the volume of a right pyramid which has a square base of side 3cm and slant edges 5cm long

Solution



In ΔABD, $DB^2 = 3^2 + 3^2$, Pythagoras

$= \sqrt{18}$

$DB = 3\sqrt{2}$

$\therefore OB = \frac{3\sqrt{2}}{2}$

In ΔVOB, $VO^2 + OB^2 = VB^2$, Pythagoras

$VO^2 = 5^2 - \left(\frac{3\sqrt{2}}{2}\right)^2$

$= 25 - \frac{9\times2}{4}$

$= \sqrt{20.5}$

$= 4.53\text{cm}$

Area of a square ABCD=3cm×3m

$= 9\text{cm}^2$

$\therefore$ Volume $= \frac{1}{3} \times 9cm^2 \times 4.53cm$

$= 13.6\text{cm}^2$

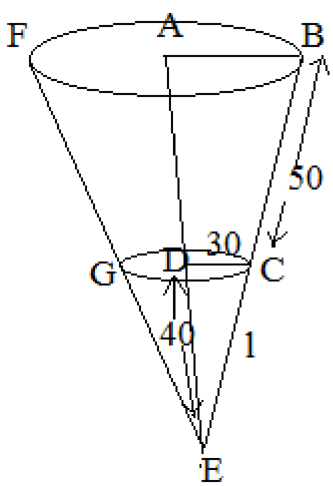
VOLUME A CONE

Since the base of the cone is a circle,

Volume of a cone $= \frac{1}{3}\pi r^2 h$

EXAMPLE

Find the volume of the basket in the diagram below,



Solution

$l^2 = 40^2 + 30^2, pythagorus$

$= \sqrt{2500}$

l =50cm

In Δ ABE and DCE

<A=<D, Right <s

<E is common

$\therefore$ ΔABE///Δ DCE

$\therefore \frac{AB}{DC} = \frac{AE}{DE}$

AB=$\frac{100}{50} \times 30$

$= 60\text{cm}$

$\frac{AB}{DC} = \frac{AE}{DE}$

$\frac{60}{30} = \frac{AE}{40}$

AE=80cm

$\therefore$ Vol. of cone FBE=$\frac{1}{3} \times \frac{22}{7} \times 60 \times 60 \times 80$

$=301{,}714.29\text{cm}^2$

Volume of GCE=$\frac{1}{3} \times \frac{22}{7} \times 30 \times 30 \times 40$

$= 37{,}714.29\text{cm}^3$

$\therefore$Volume of a bucket= 264000cm^3

VOLUME OF A SPHERE

Surface area of a sphere=$\frac{4}{3}\pi r^3$

Surface area of a sphere= $4\pi r^2$

EXAMPLE

1. Find the volume of a spherical ball with a radius of 3.14. Take $\pi = 3.14$

Solution

The vol. of a tennis ball=$\frac{4}{3} \times 3.14 \times 3.18$

$$= 134.6\text{cm}^3$$

2. A metal cylinder with length of 32cm and radius 3cm is melted down to form a sphere. What is the radius of the sphere?
   Solution

Volume of a cylinder $= \frac{22}{7} \times 3 \times 3 \times 32$

$$= 905.14\text{cm}^3$$

$\therefore$ Volume of a sphere= 905.14cm^3

$\frac{4}{3}\pi r^3 = 905.14\text{cm}^3$

$\frac{4}{3} \times \frac{22}{7} \times r^3 = 905.14$

$r^3 = 216$

$r = \sqrt[3]{216} = 6\text{cm}$

GEOMETRY IN THREE DIMESIONS

A PLANE

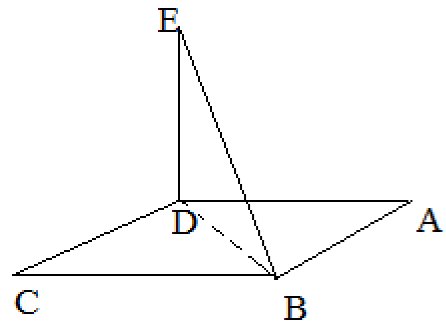
A plane is any flat surface such as a floor. The walls of the box are the planes

LINES AND PLANES

A straight line can be put in a plane. When two lines meet their line of intersection is always a straight line.

ANGLES BETWEEN ALINE AND A PLANE

It is the angle between a line and its projection on a plane.

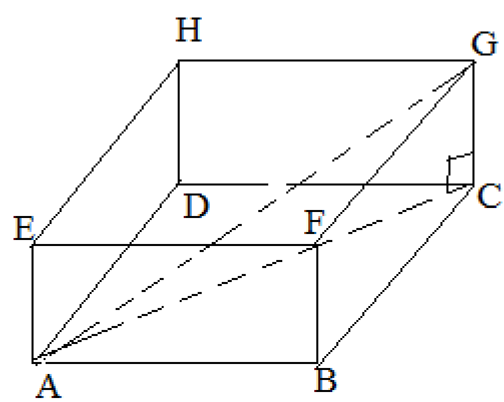


ABCD is a plane while BE is the line and BD is a line of projection

The angle between the line AB and the plane ABCD is angle EBD

EXAMPLE

Figure bellow show the water tank with rectangular faces. AB=8m, BC=5m, and CG=3m



i. Calculate the length AG leaving your answer in a simplified surd form
ii. The angle AG makes with the plane ABCD
   Solutions
   In ΔABC, $AC^2 = AB^2 + BC^2$, Pythagoras
   $= 8^2 + 5^2$
   $= \sqrt{89}$

In ΔAGC,

$AG^2 = AC^2 + GC^2$, Pythagoras

$= \sqrt{89}^2 + 3^2$

=89+9

$=\sqrt{98}$

$=7\sqrt{2}$

In general the longest diagonal can be found by the following formulae

$AC^2 = l^2 + w^2$

And $AG^2 = AC^2 + GC^2 (h)$

$\therefore AG^2 = l^2 + w^2 + h^2$

$$= \sqrt{l^2 + w^2 + h^2}$$

In ΔGAC, Sin A $= \frac{3}{7\sqrt{2}} = 0.3030$

$<A = 18°$ (To the nearest degree)

<u>**ANGLES BETWEEN TWO PLANES**</u>

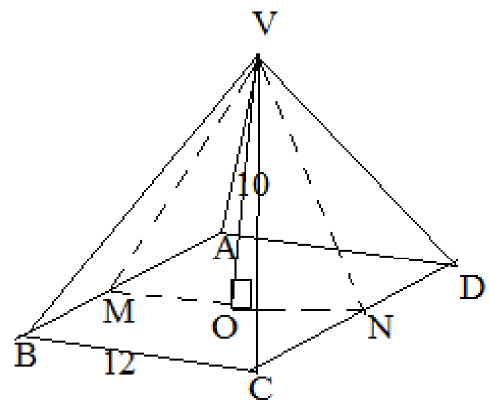
Is the angle between two lines one on the first plane and the other on the second plane that both meet at the line of intersection of the planes at right angle

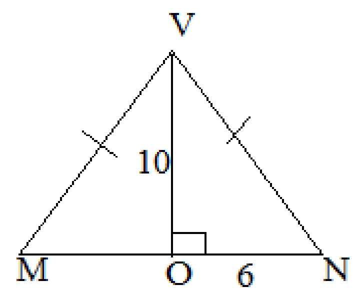
EXAMPLE

VABCD is a pyramid on a square base ABCD of side 12cm. the height of the pyramid is 10cm. calculate the angle between,

i. ABV and VCD
ii. VCD and ABCD

<u>Solution</u>





i. In ΔVON, $\tan V = \frac{6}{10}$

$= 0.6$

$\therefore <VON = 30.96° = 31°$

$\therefore <MVN = 2 \times <VON = 62°$

∴The angle between ABV and ABCD is 62°

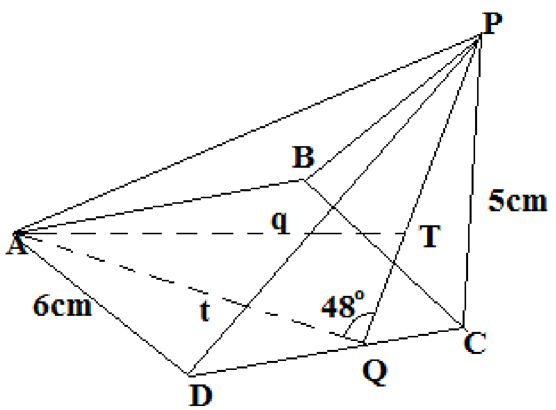
ii. Tan N$= \frac{10}{6} = 1.6667$

$<N = 59°$

∴ The angle between VCD and ABCD=59°

EXAMPLE(2016)

Figure below is a square based pyramid PABCD. T is the midpoint of the slant height OQ.



If <AQP=48°, PC=5m, and AD=6m, calculate the length of AT giving your answer to 2 decimal place

Solution

In ΔPDC, QC=DQ,

∴QC=3m

InΔPQC,$PQ^2+QC^2=PC^2$, Pythagoras

$PQ^2=5^2-3^2$

$=25-9$

$=\sqrt{16}$

$=4m$

∴TQ=2m, T midpoint

In ΔADQ, $AQ^2=AD^2+DQ^2$, Pythagoras

$AQ^2=6^2+3^2$

$=\sqrt{45}$

$=6.7m$

In ΔATQ, using cosine rule,

$q^2=a^2+t^2-2atCosQ$

$=2^2+\sqrt{45}^2-2\times2\times\sqrt{45}\times Cos48°$

$=\sqrt{31.0453}$

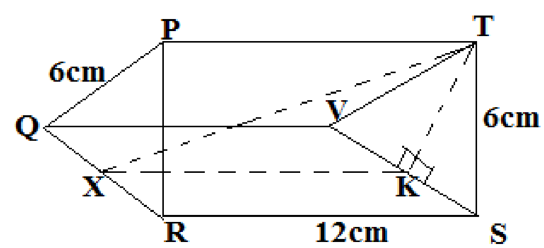
$=5.57m$

∴ AT =5.57m

<u>TRIANGULAR PRISM</u>

EXAMPLE

Figure below is a prism with a cross-section of equilateral triangles PQR. PRST, PQST, AND QRSV are rectangles, such that RS=12cm, PQ=6cm and X is the midpoint of QR.



Calculate

i. Length of TX giving your answer correct to 2 decimal place

ii. The angle TX Makes with the base QRSV

<u>SOLUTION</u>

I. In Δ VTS, VK=KS=3cm, TK ⊥ bisector

In Δ TKS

$TK^2= 6^2 – 3^2$, Pythagoras

$= 36 – 9$

$= \sqrt{27}$

In Δ TXK, $TX^2= XK^2+ TK^2$

$= 12^2+ \sqrt{27}^2$

$= 144 + 27$

$=\sqrt{171}$

$= 13.08cm$

II. $\text{Cos } x= \frac{XK}{TX}$

$=\frac{12}{13.08}$

$= 0.9174$

X= 23.4^0

CHAPTER 12

GRAPHS OF CUBIC FUNCTIONS

Cubic graphs are graphs of cubic equation or cubic expression.

A cubic equation is the one in which the highest power of x is 3. E. g $x^3, 2x^3 + 3$.

Just as in quadratic graphs, a table of values need to be drawn when drawing cubic graphs so that corresponding points are found.

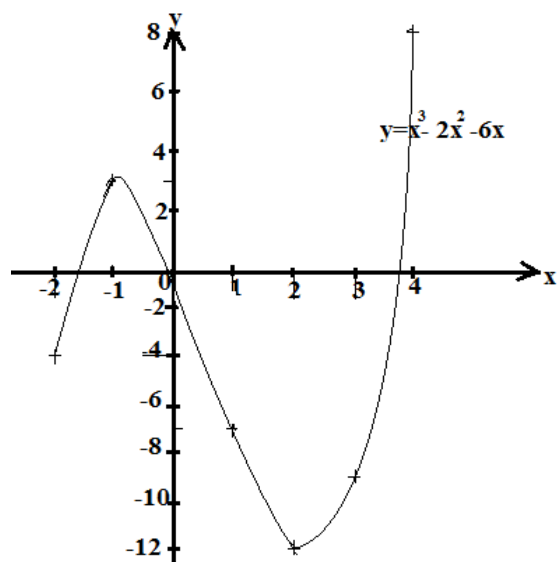
EXAMPLE

1. Complete the table below and draw the graph $y = 2x^3 - 2x^2 + 6x$

x	-2	-1	0	1	2	3	4
y							

Solution

x	-2	-1	0	1	2	3	4
y	-4	3	0	-7	-12	-9	8



SOLVING SILMUTANEOUS EQUATION GRAPHICALLY

EXAMPLE

Complete the table below and draw the graph $y = 2x^3 - 2x^2 - 6x$ and use it to solve $y = x - 2$.

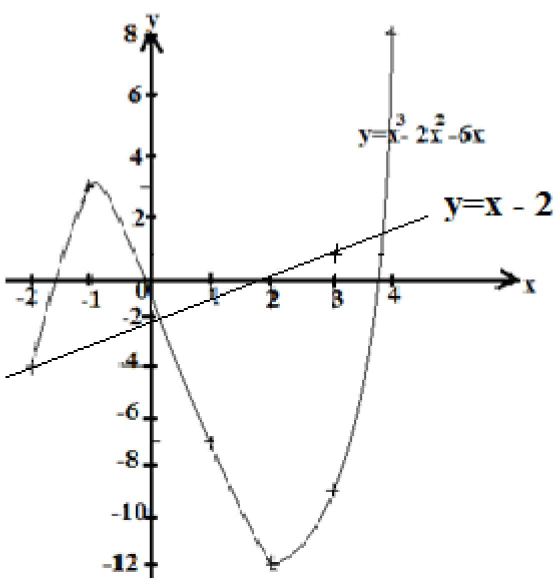
x	-2	-1	0	1	2	3	4
y							

Solution

x	-2	-1	0	1	2	3	4
y	-4	3	0	-7	-12	-9	8

To draw $y = x - 2$ the graph $y = x - 2$ we need a table like this.

x	-2	0	3
y	-4	-2	1



$\therefore x = -2, 0.25, 3.8$

SUPPLEMENTARY EXERCISE

1. Expand
$$\left(x^{\frac{1}{2}} + 1 + x^{-\frac{1}{2}}\right)\left(x^{\frac{1}{2}} + 1 + x^{-\frac{1}{2}}\right)$$
2. In a GP, the product of the 1st and 7th term is equal to the fourth term. Given that the sum of the 1st and 4th terms is 9. Find
   a. The first term
   b. Common ratio
3. Then 1st term of GP exceeds the second term by 4 and the sum of the 2nd and 3rd term is $2\frac{1}{3}$. Find the first three terms
4. If $\log_a x$, $\log_a x + 3$ and $\log_a x + 12$ are three consecutive terms of an AP. Find the value of x
5. Find the angle that is supplementary to the complement of the complementary of base angle of any isosceles triangle?
6. Factorize $(2a^2 + 3ab - 5b^2)(2a^2 - 5ab - 3b^2)$
7. At Mr. Chitopa's party, there were 240 people. There were 20 more men than women and there were 20 more adults than children. Find how many women were there.
8. Droben is 20 years older than her sister Bertha. In ten years, Bertha's age will be half of Droben. Find how old is Bertha now.
9. 8 percent of 36 is 72% of what number?
10. A committee is composed of **w** women and **m** men. 3 women and 2 men are added to the committee and if one person is selected at random from the enlarged committee, what is the probability that a person selected is a woman?
11. Given that the area of a cube is 25cm^2. Calculate its volume.
12. Solve for **w**
$$\log_3 w + \frac{3}{\log_3 w} = 4$$

13. Given the function $f(x) = 2x - 1$, find $f^{-1}(x)$
14. Function f and g are defined by $f(x) = 2x + 3$ and $g(x) = \frac{5}{x-2}, x \neq 2$. find the value of x for which $ff(x) = gf(2)$
15. Given that the function $f(x) = 3^x$ and $g(x) = 5x - 2$. What is the value of x if $gf(x) = 403$
16. There was a tea party at Mr. Motto's house. All people at the party brought their Cuts, in all they were 22 heads and 72 feet. Find how many people and cuts were there.
17. Simplify $\sqrt[4]{16t} + 3\sqrt[4]{t} - \sqrt[4]{81t}$
18. The sides of two square fields are in the ratio 4:5. The area of the larger field is 1296cm^2 greater than the smaller field. Find the area of the larger field.
19. Given that $\boldsymbol{n}(\mathbf{A}) = 7x + 13$, $\mathbf{n}(\mathbf{B}) = 6x + 17$, $\mathbf{n}(\mathbf{C}) = 8x + 20$, $\mathbf{n}(\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}) = x$, $\mathbf{n}(\mathbf{A} \cap \mathbf{B} \cap \mathbf{C^1}) = 2x$, $\mathbf{n}(\mathbf{C} \cap \mathbf{A} \cap \mathbf{B^1}) = 4x$, $\mathbf{n}(\mathbf{A^1} \cap \mathbf{B} \cap \mathbf{C}) = 3x$, $\mathbf{n}(\mathbf{A} \cup \mathbf{B} \cup \mathbf{C}) = 150$.

a. Draw a Venn diagram to show this information

b. What is $\mathbf{n}(\mathbf{A} \cap \mathbf{B} \cap \mathbf{C^1})$

20. The 1st and 4th terms of a GP are $\frac{x}{y^2}$ and $\frac{y}{x^5}$. Find the second and third terms
21. Geoffrey conducted a research on a sample about the relationship between profit and the age of a company and the following are sums he calculated from the sample.

$\sum x = 1{,}310$

$\sum x^2 = 28{,}000$

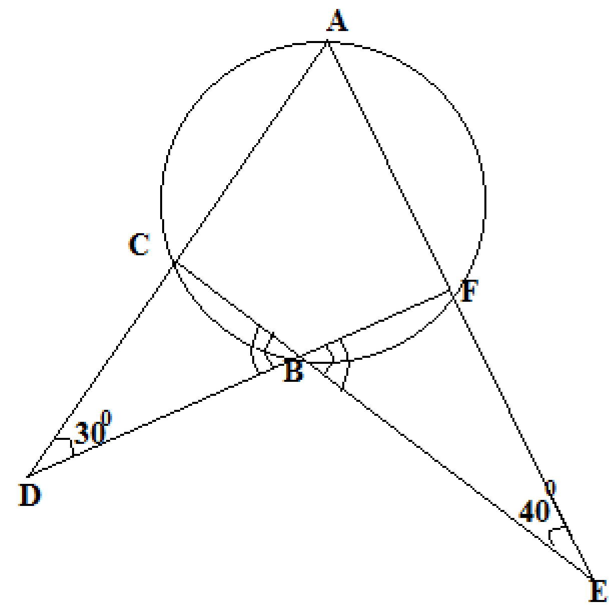
$\sum y = 5{,}700$

$\sum y^2 = 927{,}000$

$\sum xy = 65{,}500$

Where y represents annual profit and x represents the age of each firm in years. Calculate **mean** and **standard deviation** of **x** and **y**.

22. Figure below shows a circle **ABCF**, $\mathbf{C\hat{B}D} = \mathbf{F\hat{B}E}$, $\mathbf{A\hat{E}B} = \mathbf{40}°$ and $\mathbf{A\hat{D}B} = \mathbf{30}°$. Find the angle **BAC**



23. The first two terms of an AP are $\frac{1}{a+2}, \frac{1}{a+1}$; find the nth term.
24. If 4 is a solution of the equation $x^2 - 3x + k = 10$, where k is a constant, what will be the other solution
25. Simplify $\left(a^{-\frac{1}{2}}b^{\frac{3}{2}}\right)^{-2}$
26. In a multiple-choice exam paper, there are 5 questions and 4 choices per question. Find the probability of
    - iii. Guessing all right
    - iv. A pass if pass mark= 80^o%
27. In a factory production line, it is found that on average, 1 bulb in 10 is faulty. Find the probabilities that in the next 5 bulbs produced;

a. None is faulty
b. 1 is faulty
c. Less than 3 are faulty

28. A stone is dropped into the lake from a height of 5m. Acceleration in air is $10m/s^2$. How long will the stone to reach the bottom of the lake if it is 10m deep.
29. Factorize$8x^2 - x^5$
30. In a multiple choice exam paper, each question has 3 answers to choose from. What are the probabilities of
    a. Guessing a right answer
    b. A wrong answer

    If the paper consists of 4 questions, what are the probabilities of guessing correctly,

    a. None right
    b. One right
    c. Two right
    d. Three right
    e. Four right
31. The vertices of a quadrilateral are A(1,7), **B**(4,2), **C**(-3,-2), **O**(-5,4). **P, Q, R, S** are mid-points of **AB**, **BC**, **CD**, and **DA** respectively.
    a. Find the position vectors **p, q, r, s** and hence find the vectors$\widehat{\mathbf{SP}}$ and $\widehat{\mathbf{RQ}}$**.**
    b. Show that$\widehat{\mathbf{SP}}$ is equal and parallel to $\widehat{\mathbf{RQ}}$ and that **PQRS** is a parallelogram.
32. A Fair die is tossed 5times find the probability of obtaining
    a. 5 heads
    b. 2 heads and 3 tails
    c. Greater than 2 heads
    d. Greater than or equal to 2 heads
33. The probability that **Geoffrey** will beat **Lawrence** in a mathematics exam is $\frac{8}{10}$, of beating **Eugenio** is $\frac{4}{5}$, of beating **Gumzy** is $\frac{1}{4}$. Find the probability that

a. Geoffrey will beat all.
b. Geoffrey will beat Eugenio, Gumzy but not Lawrence
c. The probability that no one will beat Geoffrey

THE END

GRORY BE TO GOD