

Microfluidic technology and applications

Lec 1. Introduction of microfluidics and fluid mechanics

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Introduction of the teacher

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Course overview

1. Introduction to microfluidics and basic fluid mechanics
2. Fabrication of microfluidic chips
3. Fluid control
4. Droplet microfluidics
5. Applications: Biomedical tests and immunoassays
6. Applications: Molecular diagnostics
7. Applications: Single cell analysis
8. Challenges and outlook

Course grade

- Lecture attendance
 - Presence
 - Engagement
- Final project
 - Write a review on topics related to microfluidics

WeChat group for the course



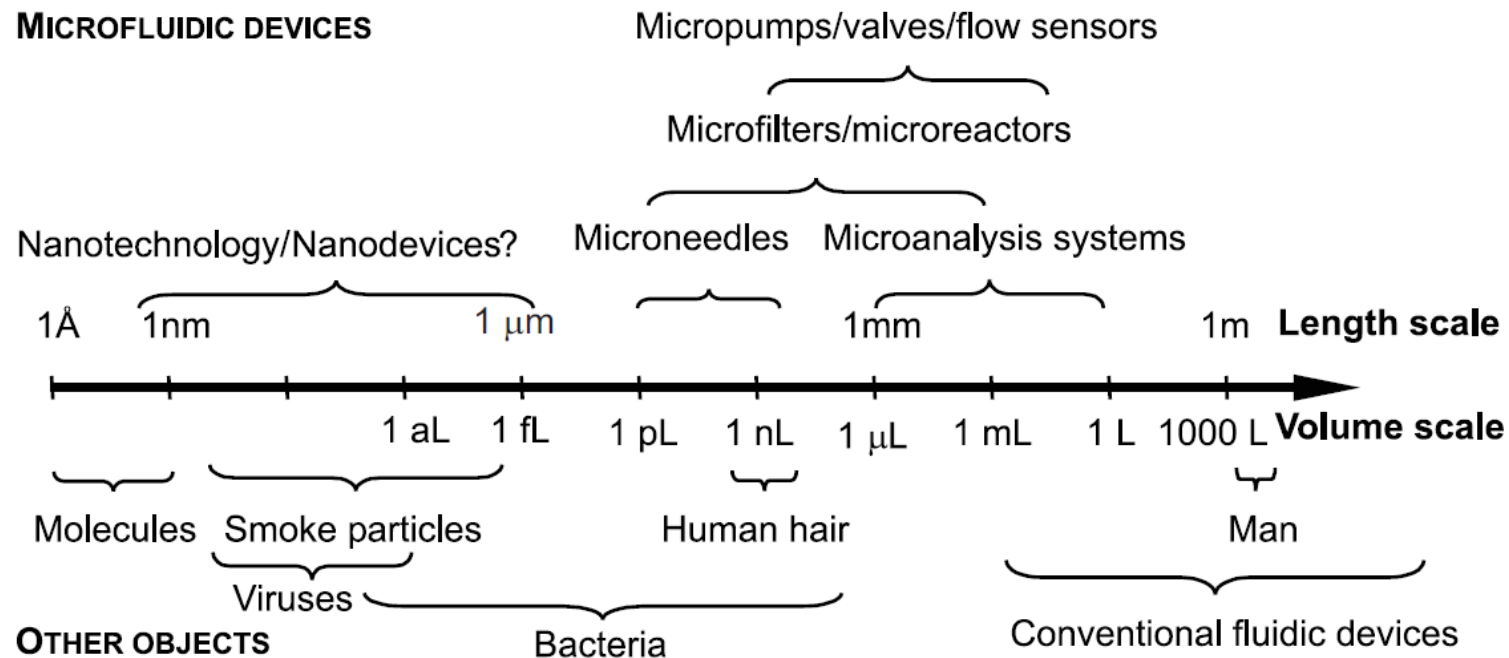
该二维码7天内(9月16日前)有效，重新进入将更新

Lecture Overview

- Lecture objectives:
 - Understand what microfluidics is
 - Explore its scientific and commercial importance
 - Connect microfluidics to fluid mechanics fundamentals
- Structure of today's session
 - What is Microfluidics → Microfluidics Milestones
 - → Fluid Mechanics

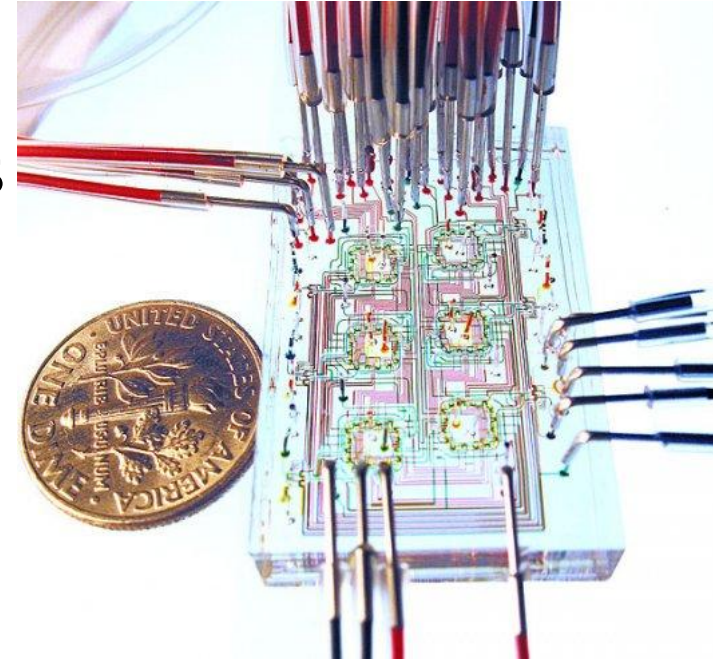
Microfluidics

- The science and engineering of systems in which fluid behavior differs from conventional flow theory primarily due to the small length scale of the system.

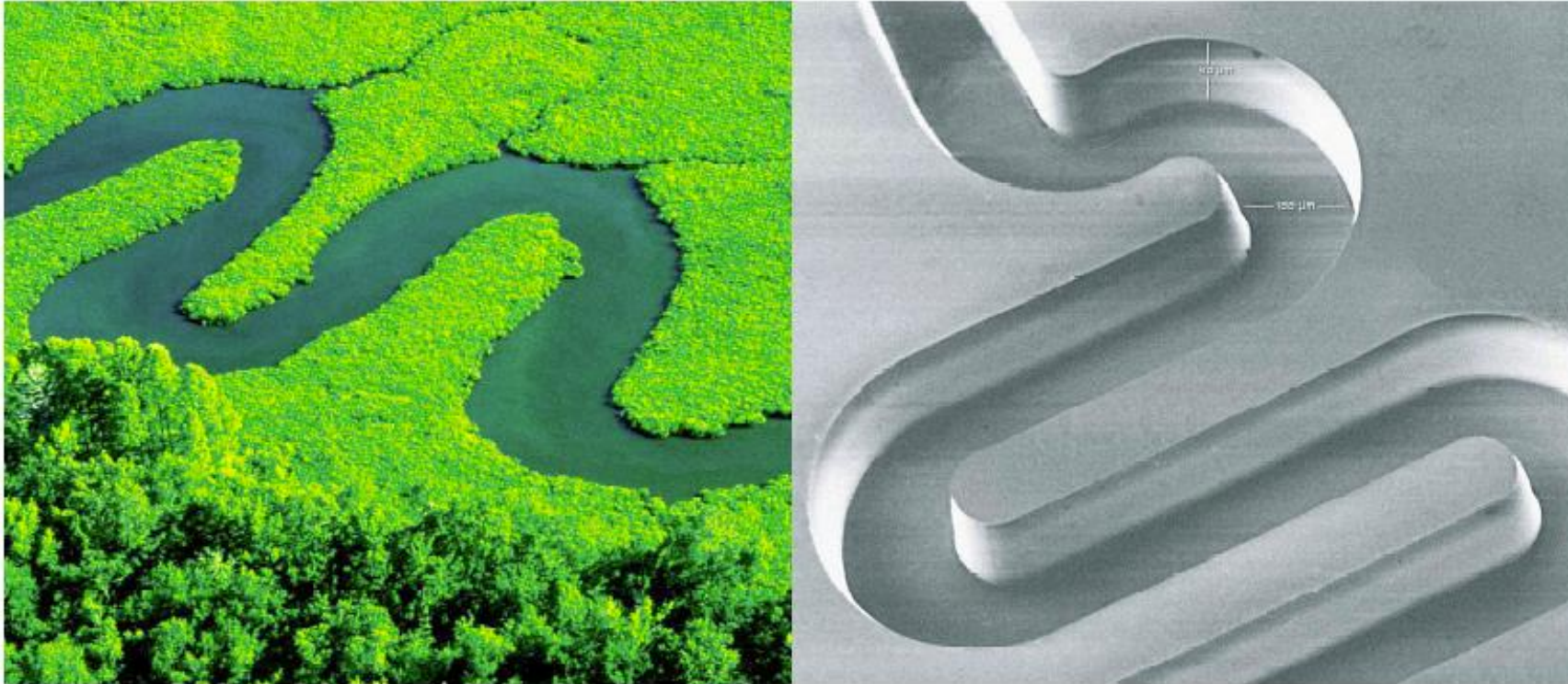


Terminologies

- Microfluidics
 - Biomicrofluidics
 - Micro-Electro-Mechanical Systems (MEMS)
 - BioMEMS
 - Lab-on-a-chip
 - Micro Total Analysis System
 - (μ TAS)
-
- Can be used interchangeably,
 - Even though not exactly synonyms



Is microfluidics different ?



Some important features of microfluidics:

1. Laminar flow
2. Small size scales
3. Scaling effects
4. Parallelization and integration

Boehringer Ingelheim microParts GmbH

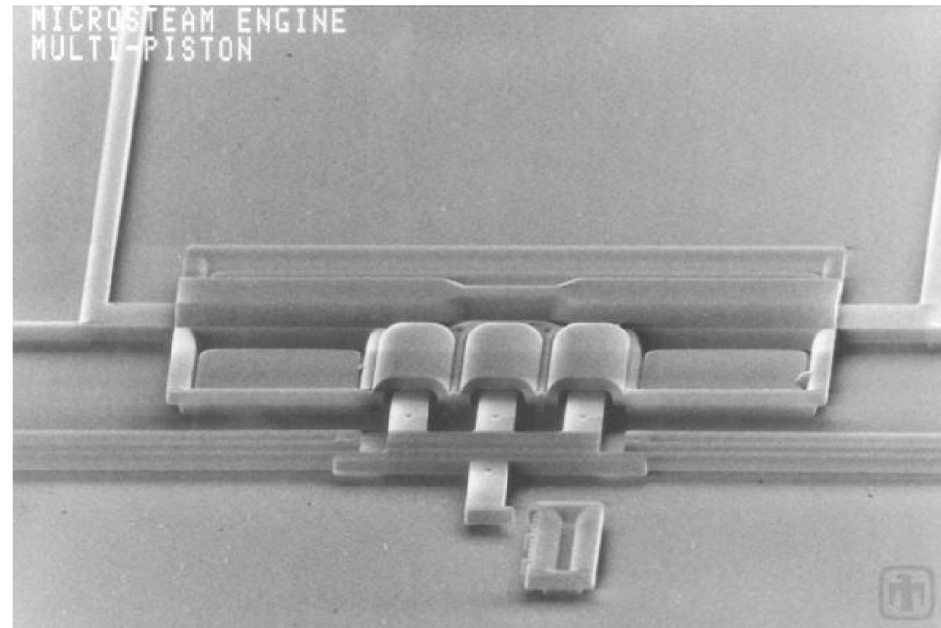
Multidisciplinary science

Microfluidics can mean many things:

1. Fundamental science relating to micro and nanoscale fluids (physics, chemistry)
2. Fabrication of microfluidic devices (engineering, materials science, etc.)
3. Applications (a plethora of fields, engineering, chemistry, biology)

Why miniaturize ?

- because it is possible?
- because it is improves performance?
- because it opens up new possibilities?



"Courtesy Sandia National Laboratories,
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Applications

- Healthcare & Diagnostics
 - Point-of-care blood tests, DNA sequencing, rapid infection detection
- Pharmaceuticals
 - High-throughput drug screening, personalized medicine
- Everyday Life
 - Inkjet printers, pregnancy tests, COVID-19 rapid tests
- Environmental & Industrial Uses
 - Water testing, chemical production in microreactors

The tradeoff of miniaturization

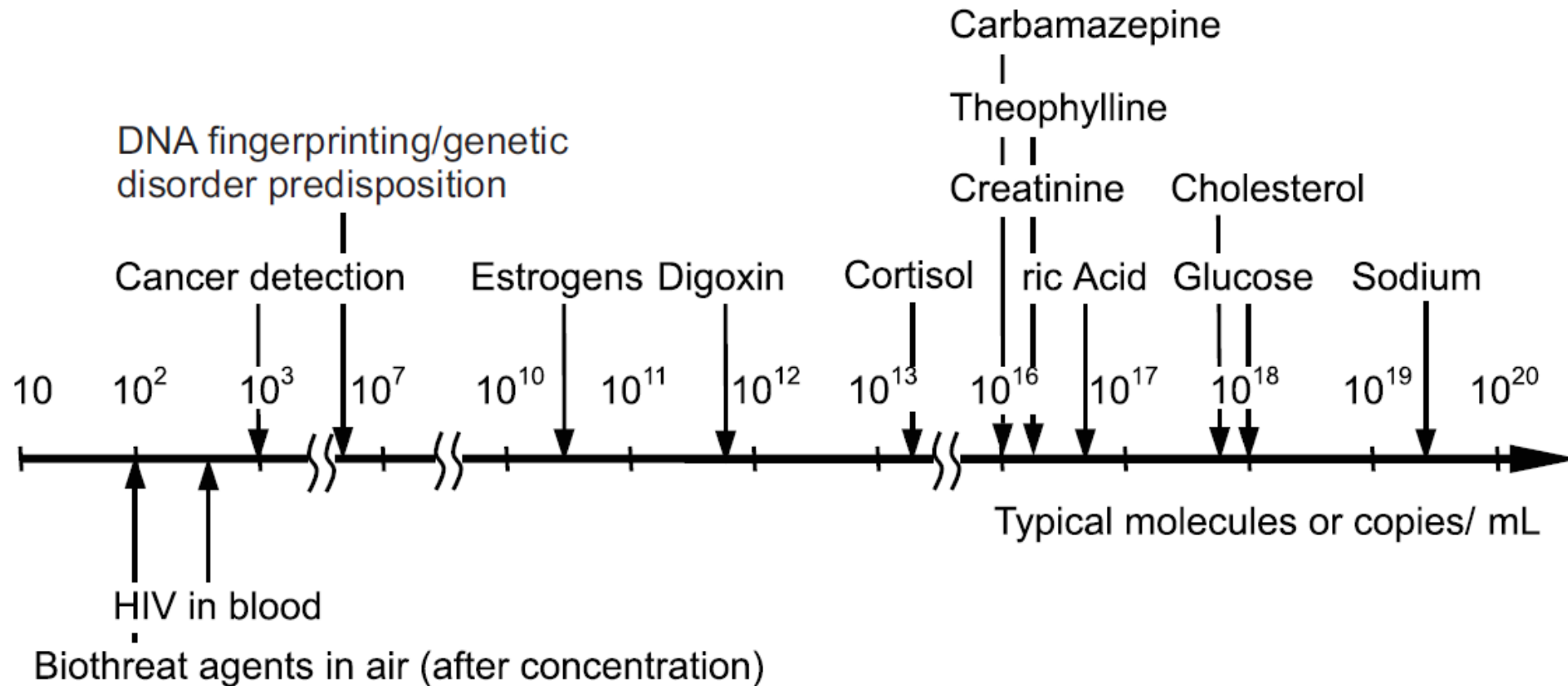
- How Many Molecules Are in My Sample?

- Detection depends on how many molecules fall into the sample volume.
- Equation:

$$N_i = \eta_s N_A A_i V$$

- If $N_i \gg 1 \rightarrow$ detection possible.
- If $N_i \ll 1 \rightarrow$ sample is too small or analyte too rare.
- Microfluidics balances *tiny volumes* with *analyte abundance*.
 - V = minimum detectable sample volume
 - η_s = sensor efficiency (between 0 and 1)
 - N_A = Avogadro's number
 - A_i = analyte concentration

Concentrations of typical diagnostic analytes in human blood or other samples.



- Case 1: HIV in Blood

- Typical concentration: 10^3 copies/mL = 10^6 molecules/L.
- Calculation:

$$N_i = (6.022 \times 10^{23})(10^{-18} \text{ mol/L})(10^{-9} \text{ L}) \approx 0.001$$

- **≈ 0 molecules expected!**
- \rightarrow Impossible to detect reliably in 1 nL without pre-concentration.



- Case 2: Glucose in Blood

- Typical concentration: $5 \text{ mM} = 5 \times 10^{-3} \text{ mol/L}$.
- Calculation:

$$N_i = (6.022 \times 10^{23})(5 \times 10^{-3})(10^{-9}) \approx 3 \times 10^{12}$$

- **≈ 3 trillion molecules!**
- \rightarrow Easily detectable in 1 nL.

Molecule Counts in 1 nL Sample

Analyte	Concentration	Molecules in 1 nL	Detectable?
HIV (rare)	$\sim 10^3$ copies/mL	~ 0.001	 Too few
Glucose (abundant)	~ 5 mM	$\sim 3 \times 10^{12}$	 Easy

Microfluidics Adventures

- See videos

MILESTONES OF MICROFLUIDICS

- Microscale Fluidic Phenomena
- Device Development
- Technology Development

Microscale Fluidic Phenomena

- At microscale, fluids show new physics not seen in macroscale flows.
- Key phenomena studied in microfluidics:
 - Electrokinetic flows → on-chip pumping & separation
 - Micromixing & microscale reactions
 - Ion concentration polarization (ICP) at nano/micro interfaces
 - Droplet & multiphase flows in microchannels
 - Opto-, magneto-, and acoustofluidics (light, magnetic, and acoustic control of fluids)
 - Centrifugal microflows & microfluidic electrochemistry
- Applications enabled:
 - Concentrators, separators, sensors, detectors
 - Micropower generators
 - Drug delivery systems & carriers
- Big idea: Understanding these effects drives innovation in lab-on-chip devices.

Device Development

1. Miniaturization Approach (1980s–mid-1990s)

- **Early phase of microfluidics:** shrinking conventional device principles.
- Enabled by **silicon micromachining**.
- Key devices: **microvalves, micropumps, microflow sensors**.
- **Scaling law limits:**
 - Power $\propto (\text{length})^3 \rightarrow$ less power at smaller scales.
 - Surface/volume $\uparrow \rightarrow$ viscous forces dominate.
- Consequence: **external actuators often needed** \rightarrow device sizes remained in **mm–cm range**.

Device Development

2. Exploration of New Effects (mid-1990s onward)

- Shift away from mechanical actuators.
- Focus on **actuation without moving parts**.
- New principles exploited at microscale:
 - **Electrokinetic pumping**
 - **Surface tension-driven flows**
 - **Electromagnetic forces**
 - **Acoustic streaming**
- These effects are **negligible macroscopically**, but powerful at microscale.
- New paradigm: mimic **cells and molecular processes** → link to **nanotechnology & biotechnology**.

Device Development

3. Application Developments (2000s–today)

- Expansion **beyond flow control & analysis** → into biology, diagnostics, industry.
- **Biomedical applications:**
 - Manipulating cells, mimicking microenvironments.
 - **Organ-on-chip** for disease modeling & drug testing.
- **Chemical/industrial applications:**
 - **Microreactors** for synthesis of new materials & pharmaceuticals.
 - Large-scale production via “**numbering up**” → running many reactors in parallel.
- Philosophy: *scalability through parallelization* (like nanotech computing).

Technology Development

- **Early Stage: Silicon Micromachining**
- First microfluidic devices fabricated in silicon.
- Integrated sensors and actuators.
- Fluids require larger channels than electrons.
- Consequences:
 - Fewer devices per wafer.
 - High costs.
 - Not suitable for disposable/commercial use.

Shift to Plastics (mid-1990s onward)

- Chemists emphasized **functionality over miniaturization** and **simplicity over complexity**.
- Devices often just microchannels with external actuators/sensors.
- Plastic micromachining became dominant (low-cost, scalable).
- Batch production methods: injection molding, hot embossing.
- Example: successful point-of-care diagnostic systems using plastics.

Emerging Manufacturing Paradigms

- **3D printing (additive manufacturing):**
 - Rapid prototyping of complex 3D architectures.
 - No need for molds.
 - Integration of functional elements still a challenge.
- **Alternative materials:** stainless steel, ceramics for corrosive applications.
- **Alternative methods:** laser machining, micro-EDM, laminating, microcutting.
- Growing flexibility in materials and fabrication methods.

Fluid Mechanics Theory

What is a Fluid?

- Everyday view: liquids and gases (water, air, oil).
- Scientific definition:
 - *A fluid is a substance that deforms continuously under an applied shear stress.*
- Unlike solids, fluids cannot resist shear forces without flowing.
- Two main categories:
 - **Liquids**: nearly incompressible, fixed volume, variable shape.
 - **Gases**: compressible, expand to fill available volume.

Solids vs. Fluids under Shear Stress

- **Solid:**

- Deforms a finite amount, then resists further deformation.
- Returns to original shape when stress is removed (elastic behavior).

- **Fluid:**

- Deforms continuously as long as shear stress is applied.
- No “restoring” shape → keeps flowing.

- **Thought experiment:**

- Push the top plate over a block of rubber vs. over a thin film of water.

Solids vs. Fluids under Shear Stress

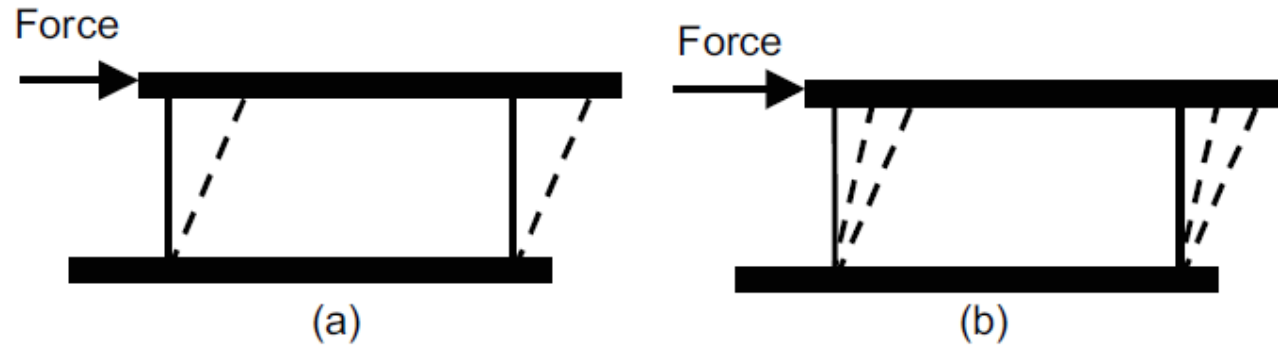
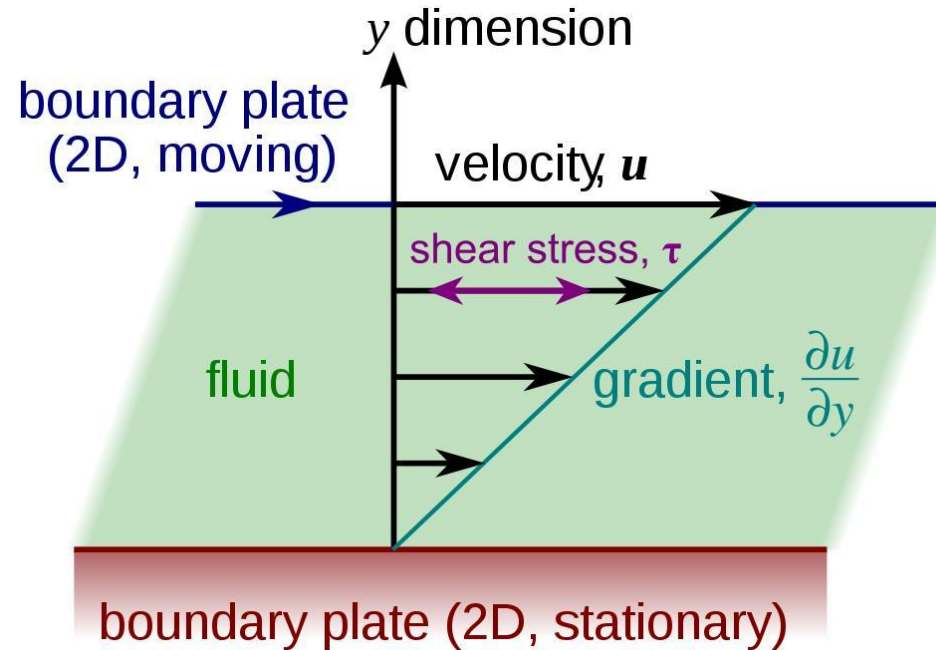


Figure 2.1 (a) A block of solid material, and (b) a fluid contained between two plates are subjected to a shearing force. Both materials are shown in an original position (solid lines) and deformed positions (dashes). When the force is removed from the solid material, it returns from its deformed position to its original or equilibrium position. The fluid remains deformed upon removal of the force.

Shear and viscosity



- Shear stress = force / area
- Newtonian fluid: shear rate is directly proportional to shear stress
- The constant of proportionality is called viscosity, μ

For a Newtonian fluid in laminar flow, there is a linear relation:

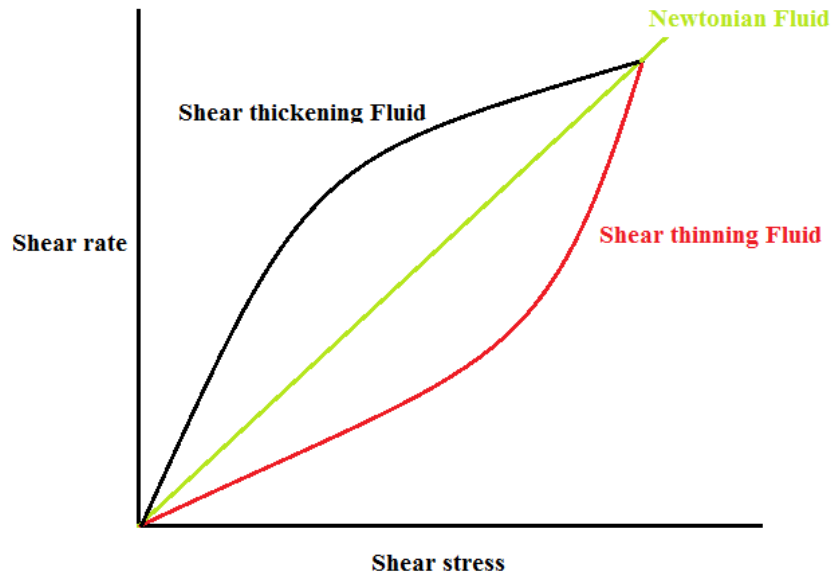
$$\tau = \mu \frac{\partial u}{\partial y} \quad (\text{contrast to a spring and Hooke's law } F = -kx)$$

Types of Fluids

- **Newtonian fluids:**
 - Shear stress \propto shear rate (linear relationship).
 - Examples: water, air, most simple liquids.
- **Non-Newtonian fluids:**
 - Shear stress \neq linear with shear rate.
 - Can be shear-thinning (ketchup), shear-thickening (cornstarch slurry), viscoelastic (blood, polymers).
- **Relevance to microfluidics:**
 - Many biological fluids (blood, mucus, cytoplasm) are non-Newtonian.
 - Device design must account for these behaviors.

Viscosity

- Shear viscosity, or dynamic viscosity μ , unit is Pa*s
- Kinematic viscosity ν ($= \mu/\rho$), unit is m²/s
- Non-Newtonian fluids common: shear thinning and shear thickening.
- Water and air are close to Newtonian. Ketchup is shear thinning. Mixture of potato starch and water is shear thickening.



Water viscosity

Temperature - t - (°C)	Dynamic Viscosity - μ - (Pa s, N s/m ²) $\times 10^{-3}$	Kinematic Viscosity - ν - (m ² /s) $\times 10^{-6}$
0	1.787	1.787
5	1.519	1.519
10	1.307	1.307
20	1.002	1.004
30	0.798	0.801
40	0.653	0.658
50	0.547	0.553
60	0.467	0.475
70	0.404	0.413
80	0.355	0.365
90	0.315	0.326
100	0.282	0.29

Properties of Fluids

- **Kinematic Properties**

How Fluids Move

- Velocity field: $v(x,y,z,t)$
- Acceleration: local + convective
- Vorticity: measure of local rotation
- Streamlines vs. pathlines

Transport Properties

How Fluids Resist and Transfer

- **Viscosity (μ):** internal friction of fluid layers
- **Thermal conductivity (k):** how well heat moves through fluid
- **Diffusivity (D):** molecular mixing by random motion
- **Analogy:** “resistance to motion, heat, and mass transport”

Thermodynamic Properties

- *Energy and State Variables*
- **Pressure (p)**: normal stress per unit area
- **Density (ρ)**: mass per unit volume
- **Temperature (T)**: molecular kinetic energy
- Relationship through state equations (ideal gas law, etc.)

Miscellaneous but Crucial Properties

- *Surface Tension and Beyond*
- **Surface tension (γ):** energy cost of creating new surface area
- Responsible for capillary rise, droplet formation
- **Vapor pressure:** tendency of a liquid to evaporate
- **Compressibility:** change in volume under pressure
- At microscale: these “secondary” properties often dominate

Surface Tension in Microfluidics

- *What Creates Surface Tension?*
- Molecules in the bulk → forces balanced in all directions.
- Molecules at the interface → missing neighbors → net inward pull.
- Creates an **energy cost** per unit surface area.
- Seen at both **liquid-gas** and **liquid-solid** interfaces.

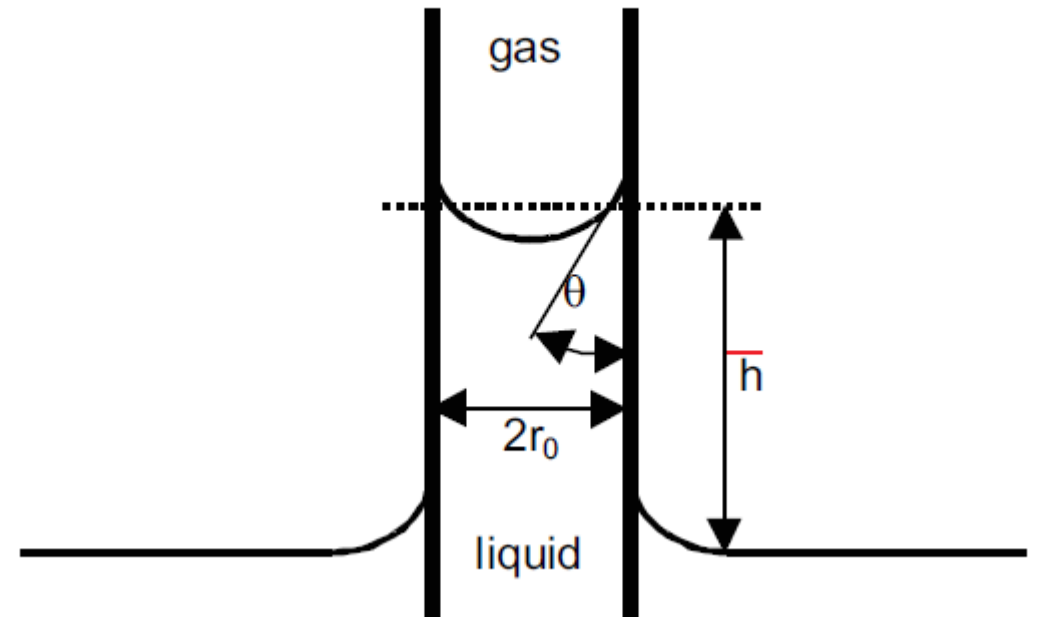
Wetting, Contact Angle & Capillarity

- How Liquids Interact with Surfaces

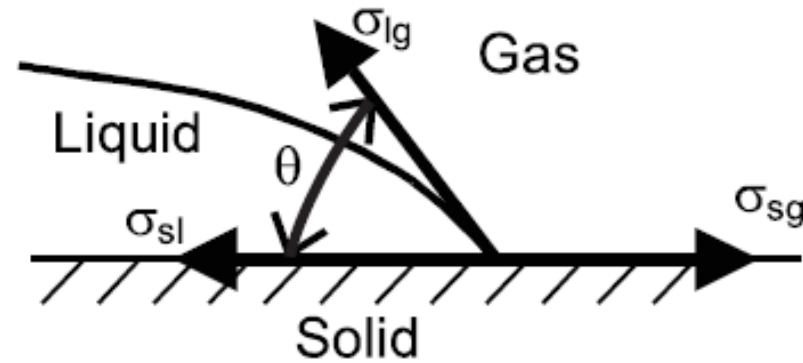
- **Contact line:** where liquid, solid, and gas meet.
- **Contact angle (θ):** balance of surface tensions.
 - $\theta < 90^\circ \rightarrow$ wetting (hydrophilic).
 - $\theta > 90^\circ \rightarrow$ non-wetting (hydrophobic).
- Capillary rise (or depression):

$$h = \frac{2\sigma \cos \theta}{\rho g r_0}$$

- Example: water climbs up a glass capillary.



Surface tension



$$\sigma_{sl} + \sigma_{lg} \cos \theta = \sigma_{sg}$$

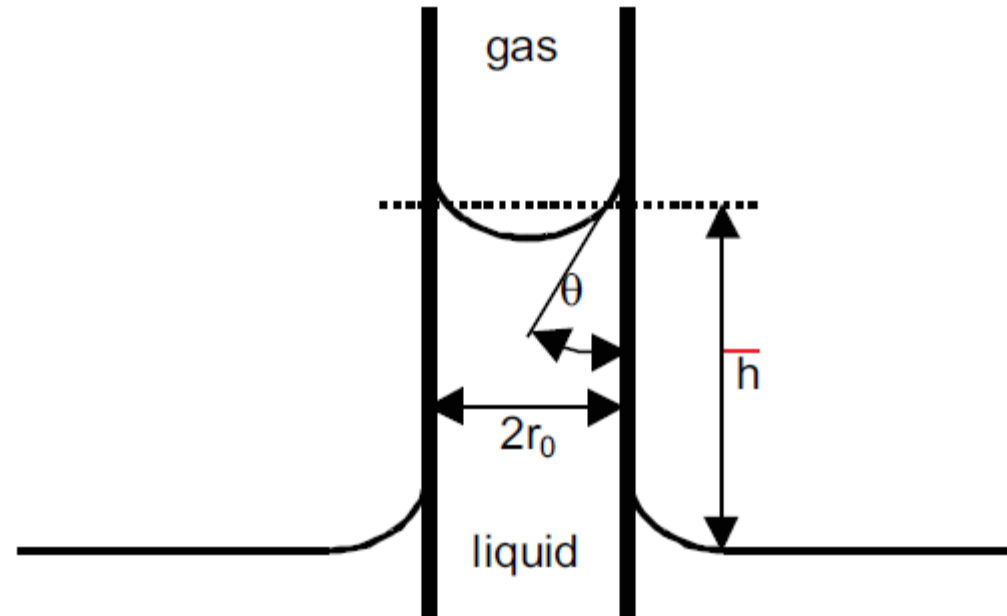
Curved Interfaces & Microfluidics Relevance

- **Laplace pressure:**

$$\Delta p = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

- Small radius of curvature \rightarrow large pressure jump.
- **Importance in microfluidics:**
 - Drives droplet formation and stability.
 - Governs filling and emptying of microchannels.
 - Can cause fabrication issues (stiction, collapsed structures).
 - Enables passive pumping and self-filling devices.

$$\Delta p = \sigma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$



$$\rho g h = \Delta p = \frac{2\sigma_{lg} \cos \theta}{r_0}$$

Molecular Forces and States of Matter

- Intermolecular Forces: *What Holds Fluids Together?*
- Molecules interact through **short-range forces**
- Attractive: van der Waals, hydrogen bonding
- Repulsive: electron cloud overlap (Pauli exclusion)
- Balance of these forces → determines fluid state
- Modeled with **Lennard-Jones potential**

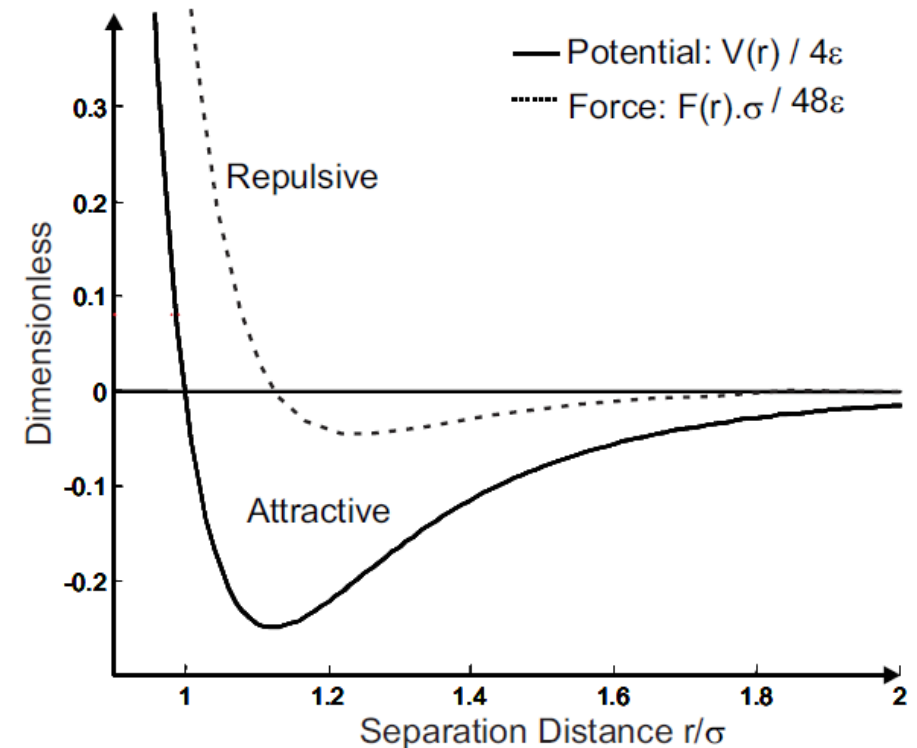
Lennard-Jones Potential

- Attraction vs. Repulsion

$$V_{ij}(r) = 4\epsilon \left[c_{ij} \left(\frac{r}{\sigma} \right)^{-12} - d_{ij} \left(\frac{r}{\sigma} \right)^{-6} \right]$$

$$F_{ij}(r) = \frac{\partial V_{ij}(r)}{\partial r} = \frac{48\epsilon}{\sigma} \left[c_{ij} \left(\frac{r}{\sigma} \right)^{-13} - \frac{d_{ij}}{2} \left(\frac{r}{\sigma} \right)^{-7} \right]$$

- Minimum of curve = equilibrium spacing between molecules
- Explains why molecules cluster but don't collapse



States of Matter (Molecular View)

- *Solids, Liquids, and Gases Compared*
- **Solids:**
 - Molecules in fixed lattice positions
 - Strong attractive forces, small vibration amplitude
- **Liquids:**
 - Molecules close, but can slide past one another
 - Weaker ordering, higher mobility
- **Gases:**
 - Molecules far apart, mostly free motion
 - Weak interactions, random collisions
- Table of density, spacing, molecular mobility

The Continuum Assumption

Treating Fluids as Continuous

- Fluid properties (ρ , p , T , v) defined at every point
- Valid if molecular scale \ll system scale
- Works for most macro- and microscale flows
- Basis of classical fluid mechanics (Navier–Stokes equations)

Limits of the Continuum Model

- **When the Continuum Assumption Breaks Down**
- If channel size \approx mean free path of molecules
- Fluid properties fluctuate \rightarrow can't define them smoothly
- Breakdown occurs in very small channels or very low pressures
- Example: micro/nanochannels in MEMS, rarefied gases in vacuum systems

Knudsen Number (Kn)

- Measuring Continuum Validity

- Defined as:

$$Kn = \frac{\lambda}{L}$$

where λ = mean free path, L = characteristic length

- Flow regimes:
 - **Kn < 0.01**: continuum valid (Navier–Stokes applies)
 - **0.01 < Kn < 0.1**: slip flow regime
 - **0.1 < Kn < 10**: transition regime
 - **Kn > 10**: free molecular flow (no continuum)

Molecular Modeling Approaches

- *Beyond the Continuum*
- When Kn is large, need molecular methods
- **Molecular dynamics (MD)**: simulate individual molecules with forces
- **Direct Simulation Monte Carlo (DSMC)**: probabilistic collisions and motions
- Applications: vacuum systems, nanofluidics, aerospace reentry flows
- Trade-off: higher accuracy vs. huge computational cost

Molecular Dynamics (MD)

- *Molecular Dynamics Simulation*
- Tracks motion of **individual molecules** using Newton's laws.
- Forces derived from **interatomic potentials** (e.g., Lennard-Jones).
- Time evolution computed in **tiny timesteps** (~fs).
- Captures:
 - Transport properties (diffusion, viscosity).
 - Interfacial phenomena (surface tension, wetting).
 - Nano-confined flows.
- **Pros:** detailed, accurate at molecular level.
- **Cons:** extremely computationally expensive → small systems, short times.

Direct Simulation Monte Carlo (DSMC)

- **Statistical method** for simulating rarefied gases.
- Particles represent “packets” of molecules.
- Algorithm alternates:
 - Free molecular motion.
 - Probabilistic collisions based on cross sections.
- Suitable for **high Knudsen number flows** (slip, transition, free molecular).
- **Applications:** vacuum systems, aerospace reentry, micro/nanoflows.
- **Pros:** efficient for large domains, handles rarefaction well.
- **Cons:** less molecular detail than MD.

Governing Equations

- Conservation of Mass: Continuity Equation

- Mass cannot be created or destroyed
- Local form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

- For incompressible fluids ($\rho = \text{const.}$):

$$\nabla \cdot \mathbf{v} = 0$$

- Meaning: whatever flows into a control volume must flow out

Conservation of Momentum

- Navier–Stokes Equation
 - Newton's 2nd law applied to a fluid element
 - General form:

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{f}$$

- Terms:
 - Inertia
 - Pressure gradient
 - Viscous forces
 - Body forces (gravity, EM forces)

Full form of the NS Equation

For each dimension when the velocity is $V(u, v, w)$:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = \rho g_x - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (15)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = \rho g_y - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (16)$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = \rho g_z - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (17)$$

Conservation of Energy

- *Energy Equation*
- First law of thermodynamics → energy balance
- Includes:
 - Internal energy changes
 - Work by stresses
 - Heat conduction
- Simplifies in many microfluidic cases (constant T or negligible heating)

Applications of the NS Equation

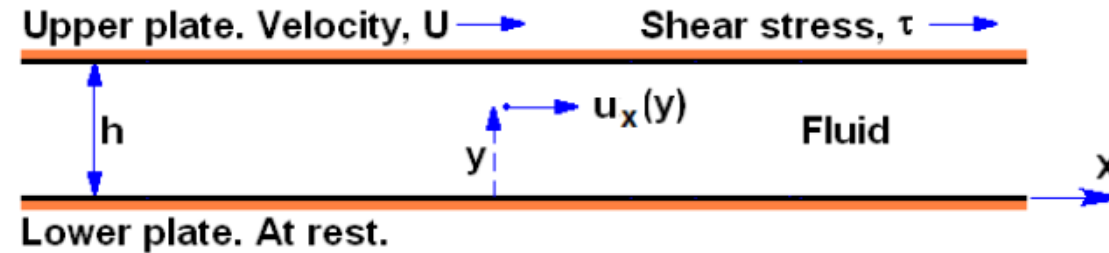


Figure 1: Couette flow.

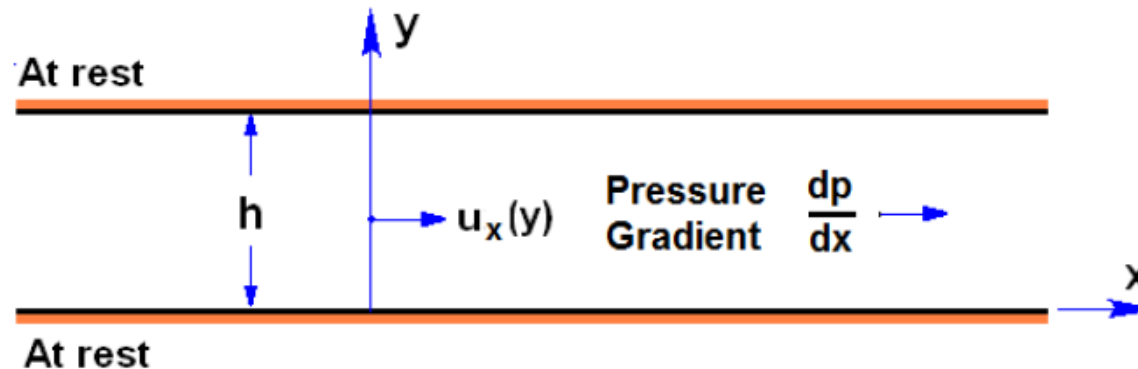


Figure 2: Planar Poiseuille flow.

Full form of the NS Equation

For each dimension when the velocity is $V(u, v, w)$:

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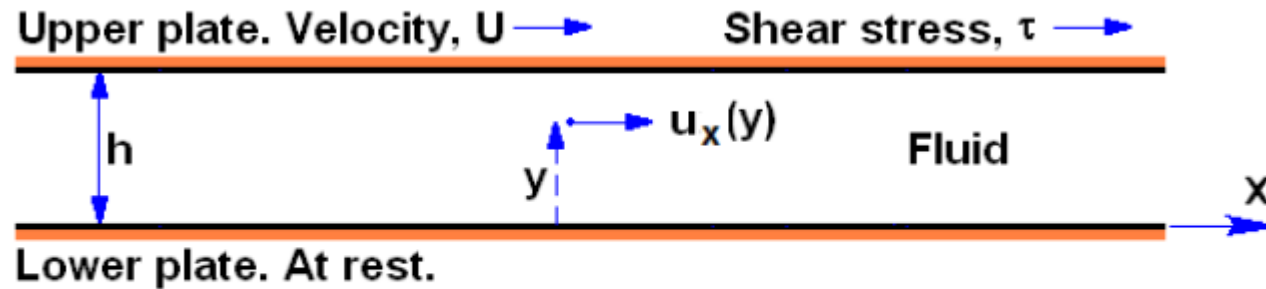
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$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u_x}{\partial y^2}$$

$$\frac{\partial p}{\partial y} = 0$$

$$u_x = \frac{1}{\mu} \left(\frac{dp}{dx} \right) \frac{y^2}{2} + C_1 y + C_2$$



$$(u_x)_{y=h} = U \quad \text{and} \quad (u_x)_{y=0} = 0$$

$$C_1 = \frac{U}{h} - \frac{C_2}{h} \quad \text{and} \quad C_2 = 0$$

Figure 1: Couette flow.

$$u_x = \frac{Uy}{h}$$

$$u_x = \frac{1}{\mu} \left(\frac{dp}{dx} \right) \frac{y^2}{2} + C_1 y + C_2$$

$$(u_x)_{y=0} = 0 \quad \text{and} \quad (u_x)_{y=h} = 0$$

$$C_2 = 0 \quad \text{and} \quad C_1 = -\frac{h}{2} \left(\frac{dp}{dx} \right)$$

$$u_x = \frac{1}{\mu} \left(-\frac{dp}{dx} \right) \frac{y}{2} (h - y)$$

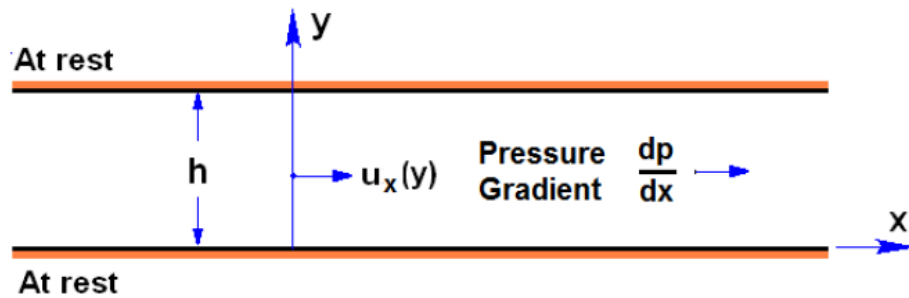
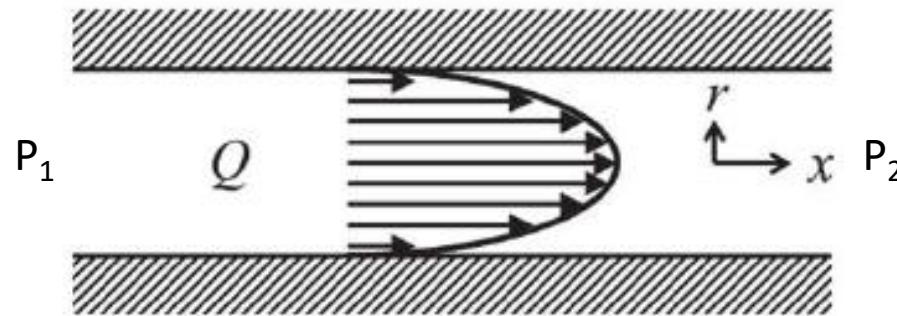


Figure 2: Planar Poiseuille flow.

Pressure driven laminar flow: Poiseuille flow

- Assumptions: Newtonian and non compressible fluid, laminar flow, cylindrical channel
- Velocity profile inside a microchannel is parabolic.
- Ideally, the velocity at channel walls is 0. (called *no-slip boundary condition*)



Parabolic velocity profile:

$$u = -\frac{\Delta P}{L} \frac{1}{4\mu} (R^2 - r^2)$$

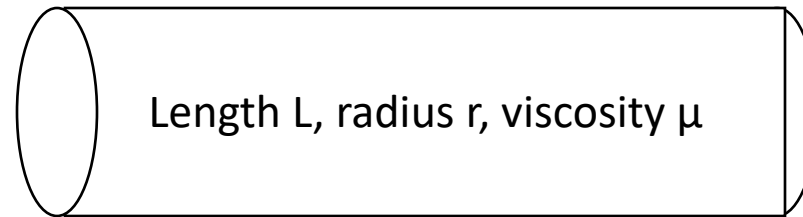
ΔP = pressure difference = $P_1 - P_2$

L = length of the channel

R = the radius of the channel

Hagen-Poiseuille's Law

If a pressure difference of ΔP is applied over a cylindrical channel, what is the volumetric flow rate Q ($\mu\text{l}/\text{min}$)?



$$\Delta P = \frac{8\mu L}{\pi r^4} Q = R_H Q$$

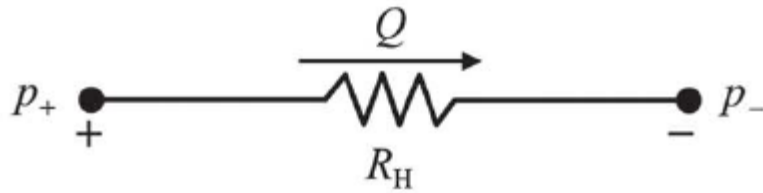
R_H = hydraulic resistance, contains all the geometrical parameters

Why inverse r^4 dependency?

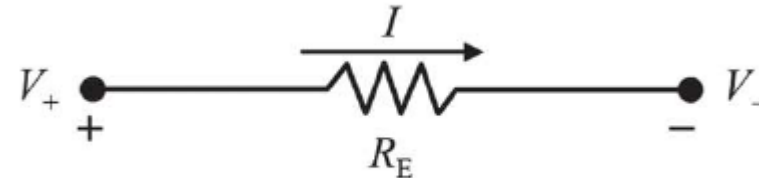
One r^2 comes from the area of the cross section a

The other r^2 comes from the average velocity of parabolic flow profile. ($Q = A \cdot V_{\text{ave}}$)

Fluidic circuits



Hagen-Poiseuille's law: $\Delta p = p_+ - p_- = Q R_H$



Ohm's law: $V = V_+ - V_- = I R_E$

- Analogous to electric circuits,
- Hydraulic resistances in series and parallel sum exactly as electrical resistors
- Analogies to Kirchhoffs laws also exist: volume is conserved and pressure drop over a loop is 0.

Series: $R_{total} = R_1 + R_2 + \dots + R_N$

Parallel: $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$

- Fluidic circuit:
1. calculate R of each component,
 2. calculate R_{total} ,
 3. insert R_{total} into Hagen-Poiseuille

An example:

1. Conservation of volume:

$$Q_1 = Q_2 + Q_3$$

2. Pressure drop across both paths is the same:

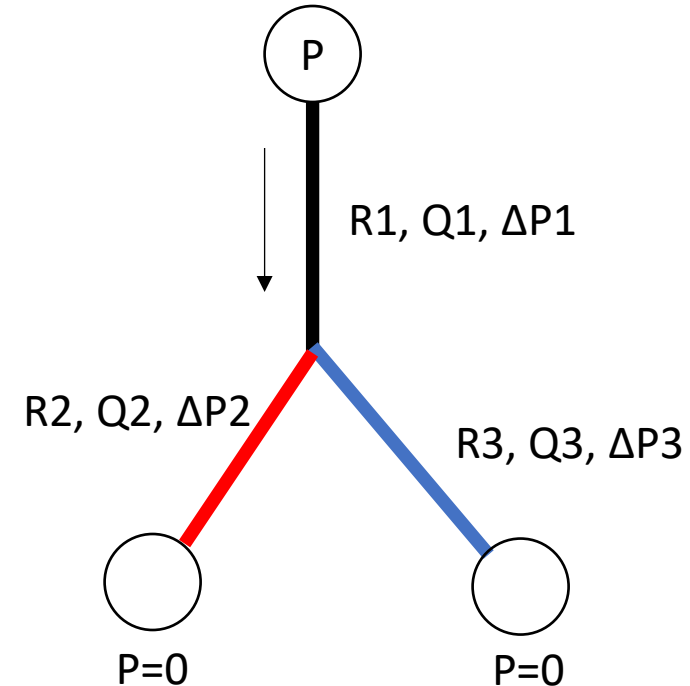
$$P = \Delta P_1 + \Delta P_2 = \Delta P_1 + \Delta P_3$$

Ratio of flow in channels 2 and 3:

$\Delta P_2 = \Delta P_3 \rightarrow$ apply Hagen Poiseuille's law

$$R_2 Q_2 = R_3 Q_3$$

$$Q_2 / Q_3 = R_3 / R_2$$



Simplifications for Microscale Flows

- Why Microflows Are Easier (and Harder)
 - **Incompressible liquids** → density nearly constant
 - **Low Reynolds number** → viscous forces dominate, inertia negligible
 - **Creeping flow (Stokes regime):**

$$-\nabla p + \mu \nabla^2 \mathbf{v} = 0$$

- Gravity often negligible compared to surface or EM forces
- Leads to **linear, more predictable equations**

Dimensionless Numbers

- Combine physical variables into **ratios without units**
- Show the **relative importance** of competing effects
- Example:
 - Force ratio (inertia vs. viscosity → Reynolds number)
 - Time scale ratio (advection vs. diffusion → Peclet number)
- Help generalize results: one experiment can apply to many systems
- Key in microfluidics: scaling laws become clear only through dimensionless numbers

Reynolds Number (Re)

- Inertia vs. Viscosity

- Definition:

$$Re = \frac{\rho UL}{\mu}$$

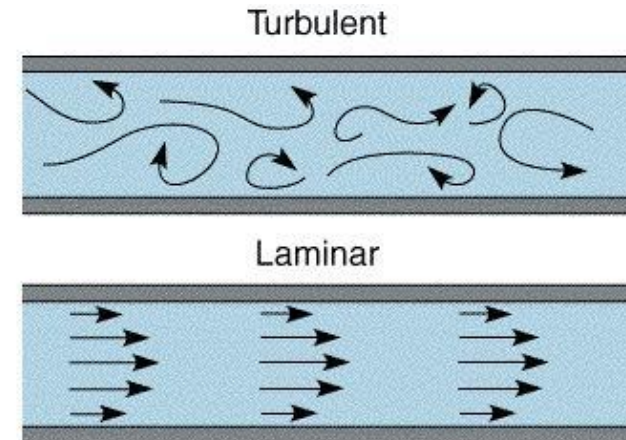
ρ = density

v = velocity

L = characteristic length scale

μ = dynamic viscosity

- Ratio of inertial forces to viscous forces
- High $Re \rightarrow$ inertia dominates (turbulent flows)
- Low $Re \rightarrow$ viscosity dominates (laminar flows)
- Microfluidics: usually $Re \ll 1 \rightarrow$ creeping flow



Mach and Knudsen Numbers

- Compressibility and Rarefaction

- **Mach number (Ma):**

$$\text{Ma} = \frac{U}{c}$$

- Speed relative to sound speed
- $\text{Ma} < 0.3 \rightarrow$ incompressible flow assumption valid

- **Knudsen number (Kn):**

$$\text{Kn} = \frac{\lambda}{L}$$

- Ratio of mean free path to system size
- $\text{Kn} \ll 0.01 \rightarrow$ continuum valid
- $\text{Kn} > 0.1 \rightarrow$ slip or molecular effects important

Peclet Number (Pe)

- Advection vs. Diffusion

- Definition:

$$Pe = \frac{UL}{D}$$

- Ratio of convective transport to molecular diffusion
- High Pe \rightarrow advection dominates (large scale flows)
- Low Pe \rightarrow diffusion dominates (microscale mixing)
- In microfluidics, diffusion often sets mixing limits

Why Dimensionless Numbers Matter

Scaling Laws in Microfluidics

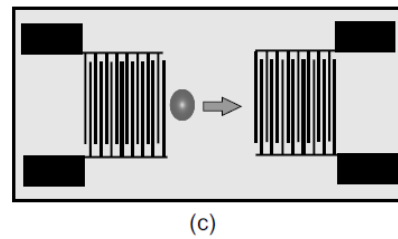
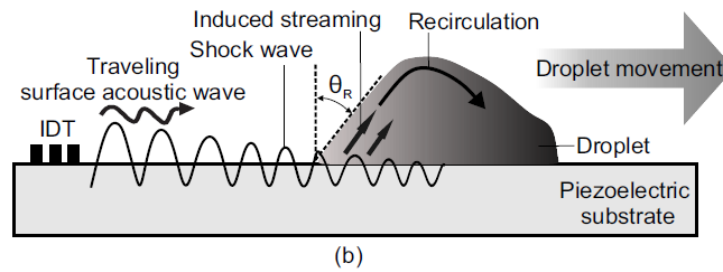
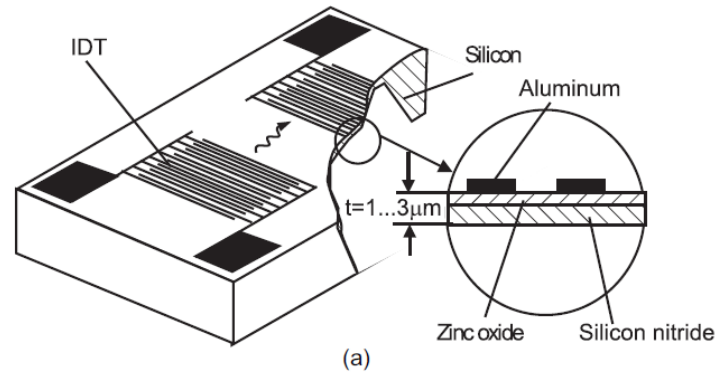
- Provide physical intuition for dominant forces
- Predict when simplifications are valid (e.g., incompressible, laminar)
- Help compare very different systems on the same scale
- Explain why microscale flows behave differently than macroscale flows
 - Always laminar, predictable
 - Diffusion-driven mixing
 - Surface forces dominate over gravity

Example Problems & Applications

- Is Flow in Microchannels Laminar?
 - Water in a $100\ \mu\text{m}$ channel at $1\ \text{mm/s}$
 - $Re = \frac{\rho UL}{\mu}$
 - For water: $\rho = 1000\ \text{kg/m}^3$, $\mu = 0.001\ \text{Pa} \cdot \text{s}$
 - Result: $Re \approx 0.1$
 - \rightarrow Flow is laminar, viscous dominated

Introduction to Microacoustofluidics

- *Acoustics Meets Microfluidics*
- Microacoustofluidics: using acoustic waves to manipulate fluids & particles at microscale.
- Relies on **surface acoustic waves (SAWs)** traveling along a solid substrate.
- Two main types:
 - **Traveling SAWs (TSAWs)**
 - **Standing SAWs (SSAWs)**
- Generated using **interdigital transducers (IDTs)** on a piezoelectric layer.
- SAWs have amplitudes of only a few nanometers, but can exert strong forces on droplets.



How SAWs Manipulate Fluids

- Mechanisms of Acoustic Control
 - SAWs travel slower than bulk sound → energy trapped near surface.
 - Can:
 - Push and move droplets on hydrophobic substrates.
 - Propel liquid plugs through enclosed microchannels.
 - Induce internal circulation inside droplets (acoustic streaming).
 - Typical vibration speeds: $\sim 1 \text{ m/s}$, but accelerations up to 10^8 m/s^2 .
 - Enables **on-chip pumping, mixing, and microcentrifugation**.

Advanced Phenomena: Jetting & Atomization

- *Harnessing High-Power Acoustics*
- At high power, TSAWs can overcome capillary stress → **jetting**.
 - Droplet surface deforms → thin liquid jet extruded.
- Even higher power: **atomization**.
 - Droplet breaks into aerosol droplets ($\sim 1 \mu\text{m}$ size).
 - Requires power $\sim 1 \text{ W}$.
- Applications:
 - Controlled droplet generation.
 - Aerosol creation for drug delivery.
 - Novel mixing and spraying methods.

