Homework 6

Atlas42

November 2021

1 Gini score

$0 \leqslant a_i \leqslant 1$, with $\forall$ i = i,..n and $\sum_{n=1}^{N} a_i = 1$

Find a_i so that S = $\sum_{n=1}^{N} a_i^2$

a. S max
b. S min

a. $\sum_{i=1}^{N} a_i^2 \leqslant (\sum_{i=1}^{N} a_i)^2 = \sum_{i=1}^{N} a_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} a_i a_j$

with $j \neq i$

$\therefore Smax = 1 \leftrightarrow a_j = 1, a_{j\neq i} = 0$
b. Applying the Bunyakovsky inequality, we have:

$$(a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + ... + a_n^2)(b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2 + ... + b_n^2) \geqslant (a_1b_1 + a_2b_2 + a_3b_3 + ... + a_nb_n)^2$$

with $b_i = 1$:

$NS \geqslant (\sum_{i=1}^{N} a_i)^2 = 1$

$\therefore S \geqslant \frac{1}{N}$

$\therefore Smin \leftrightarrow a_1 = a_2 = a_n = \frac{1}{N}$