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Rewriting the Mahanalobis distance for a conditional vector: This derivation uses a matrix inversion formula that uses the Schur complement $\Sigma_* \equiv \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$. We first use the blockwise inversion formula to write the inverse-variance matrix as:

$$\Sigma^{-1} = \Sigma_{11}\Sigma_{12}\Sigma_{21}\Sigma_{22}{}^{-1} = \Sigma_{11}^{*}\Sigma_{12}^{*}\Sigma_{21}^{*}\Sigma_{22}^{*}, \tag{1}$$

where:

$$\Sigma_{11}^{*} = \Sigma_{*}^{-1} \qquad \Sigma_{12}^{*} = -\Sigma_{*}^{-1}\Sigma_{12}\Sigma_{22}^{-1}, \qquad \Sigma_{21}^{*} = -\Sigma_{22}^{-1}\Sigma_{12}\Sigma_{*}^{-1}\Sigma_{22}^{*} = \Sigma_{22}^{-1} + \Sigma_{22}^{-1}\Sigma_{12}\Sigma_{*}^{-1}\Sigma_{12}\Sigma_{22}^{-1}. \tag{2}$$

Using this formula we can now write the Mahanalobis distance as:
(y - $\mu)^T\Sigma^{-1}(y-\mu)$

$= (y_1 - \mu_*)^T\Sigma_*^{-1}(y_1 - \mu_*) + (y_2 - \mu_2)^T\Sigma_{22}^{-1}(y_2 - \mu_2),$

(2)

where $\mu_* \equiv \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(y_2 - \mu_2)$ is the conditional mean vector. Note that this result is a general result that does not assume normality of the random vectors involved in the decomposition. It gives a useful way of decomposing the Mahanalobis distance so that it consists of a sum of quadratic forms on the marginal and conditional parts. In the conditional part the conditioning vector y_2 is absorbed into the mean vector and variance matrix. To clarify the form, we repeat the equation with labelling of terms:
$(y-\mu)^T\Sigma^{-1}(y-\mu) = \underbrace{(y_1 - \mu_*)^T\Sigma_*^{-1}(y_1 - \mu_*)}_{ConditionalPart} + \underbrace{(y_2 - \mu_2)^T\Sigma_{22}^{-1}(y_2 - \mu_2)}_{MarginalPart}.$
Now that we have the above form for the Mahanalobis distance, the rest is easy. We have:

$p(y_1|y_2,\mu,\Sigma) y_1 \propto p(y_1,y_2|\mu,\Sigma)$

$= N(y|\mu,\Sigma)$

$y_1 \propto \exp\Big(-\frac{1}{2}(y-\mu)^T\Sigma^{-1}(y-\mu)\Big)$

$y_1 \propto \exp\Big(-\frac{1}{2}(y_1-\mu_*)^T\Sigma_*^{-1}(y_1-\mu_*)\Big)$

$y_1 \propto N(y_1|\mu_*,\Sigma_*).$