Gaussian minimax lowerbounds

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April 28, 2025

Chapter 1

Minimax lowerbounds

Definition 1.0.1. Let $(\mathcal{X}_n, \mathcal{F}_n, (\mathbb{P}_{\vartheta,n})_{\vartheta\in\Theta})$ be a statistical model and (Θ, d) a metric space[1]. Suppose that $(v_n)_{n\in\mathbb{N}}$ is a null sequence. Then, $(v_n)_{n\in\mathbb{N}}$ is called optimal (minimax) convergence rate over Θ if

1. There exists an estimator $\hat{\vartheta}_n^*$ such that

$$\limsup_{n\to\infty} v_n^{-2} \sup_{\vartheta\in\Theta} \mathbb{E}_{\vartheta,n}[d(\hat{\vartheta}_n^*, \vartheta)^2] < \infty.$$

2. We have the uniform lowerbound

$$\liminf_{n\to\infty} v_n^{-2} \inf_{\hat{\vartheta}_n} \sup_{\vartheta\in\Theta} \mathbb{E}_{\vartheta,n}[d(\hat{\vartheta}_n, \vartheta)^2] > 0,$$

   where the infimum is taken over all measurable functions (estimators) in model n.

[1]In general $d : \Theta \times \Theta \to [0, \infty)$ is also allowed to be a semi-distance.

Bibliography