Summation Rules

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Assume the X is a random variable such that

$$X = \{x_1, x_2, x_3, \ldots, x_n\}$$

Then,

$$\sum_{i=1}^{n} X_i = x_1 + x_2 + x_3 + \ldots + x_n$$

To keep the notation simpler, we will just denote this as $\sum X$.

Summation Rules

Rule 1: When a summation is itself a sum or difference, the summation sign may be distributed among the separate terms of the sum. That is:

$$\sum(X + Y) = \sum X + \sum Y$$

Proof.

$$\begin{aligned}\sum(X + Y) &= (x_1 + y_1) + (x_2 + y_2) + \ldots + (x_n + y_n) \\ &= (x_1 + x_2 + \ldots x_n) + (y_1 + y_2 + \ldots y_n) \\ &= \sum X + \sum Y\end{aligned}$$

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Rule 2: The sum of a constant, a, is n times the value of the constant.

$$\sum(a) = na$$

Proof.

$$\begin{aligned}\sum a &= \underbrace{a + a + \ldots + a}_{n} \\ &= na\end{aligned}$$

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Examples

In the following examples X and Y are random variables and a is a constant.

1. $\sum(X + 2)$

$$\begin{aligned}\sum(X+2) &= \sum X + \sum 2 \\ &= \sum X + 2n\end{aligned}$$

2. $\sum(2X + 3)$

$$\begin{aligned}\sum(2X+3) &= \sum(2X) + \sum 3 \\ &= 2\sum X + 3n\end{aligned}$$

3. $\sum(X - Y)^2$

$$\begin{aligned}\sum(X-Y)^2 &= \sum(X^2 - 2XY + Y^2) \\ &= \sum(X^2) - \sum(2XY) + \sum(Y^2) \\ &= \sum(X^2) - 2\sum(XY) + \sum(Y^2)\end{aligned}$$