(1) $A = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}$ 2 rows, 2 columns $= 2\times 2$

$C = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ 2 rows, 3 columns $= 2\times 3$

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(2) C is <u>not</u> square since its dimensions are not equal

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(3) $tr(A) = tr\begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} = 3+1 = 4$

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(4) $\det(A) = \det\begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} = 3(1) - 5(-2)$

$= 3 + 10$

$= 13$

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(5) $A + B = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3+3 & -2+-1 \\ 5+-1 & 1+2 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ 4 & 3 \end{bmatrix}$

⑥ $C^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$

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⑦ $\underset{2\times \boxed{2}}{A} \quad \underset{\boxed{2}\times 3}{C}$

We can multiply AC since the inner dimensions match

$\underset{2\times \boxed{3}}{C} \quad \underset{\boxed{2}\times 2}{A}$

We cannot multiply CA since the inner dimensions do not match

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⑧ $AC = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3(1) + -2(0) & 3(2) + -2(1) & 3(3) + -2(2) \\ 5(1) + 1(0) & 5(2) + 1(1) & 5(3) + 1(2) \end{bmatrix}$

$= \begin{bmatrix} 3 & 4 & 5 \\ 5 & 11 & 17 \end{bmatrix}$

(9) $BI = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 3(1) + -1(0) & 3(0) + -1(1) \\ -1(1) + 2(0) & -1(0) + 2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \stackrel{?}{=} B$$

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(10) Many solutions here. The diagonal must sum to 10

$$\begin{bmatrix} 6 & 2 & 7 \\ 0 & 3 & 5 \\ 1 & 4 & 1 \end{bmatrix}$$

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(11) A matrix has an inverse as long as its determinant is not $\emptyset$

$$\det(B) = \det\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} = 3(2) - (-1)(-1)$$
$$= 6 - 1 = 5 \checkmark$$

(12) $B^{-1} = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}^{-1}$

from #11 $\det(B) = 5$

create new matrix where we swap main diagonal element & change signs on off-diagonal elements

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

multiply this matrix by reciprocal of det.

$$\frac{1}{5}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{3}{5} \end{bmatrix}$$

OR $\begin{bmatrix} .4 & .2 \\ .2 & .6 \end{bmatrix}$

(13) Create 2 columns (or rows) that are independent; i.e., write $\boxed{\text{col. } 1 \neq c(\text{col. } 2)}$ where c is any number

many options, here is one

$$\begin{bmatrix} 1 & 2 \\ 2 & 5 \\ 3 & 0 \end{bmatrix}$$

here $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \neq c \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

Now write col. 3 as a linear combination of col. 1 and col. 2; i.e.,

$$\boxed{\text{col. } 3 = a(\text{col. } 1) + b(\text{col. } 2)}$$

here I choose $a = 2$ and $b = 0$

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 5 & 4 \\ 3 & 0 & 6 \end{bmatrix}$$

(14) Many solutions

$$\begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$