

# Statistical Thinking 4.0



A Simulation Approach  
to Modeling Uncertainty

Catalysts for Change

CATALYSTS FOR CHANGE

# Statistical Thinking

A Simulation Approach to Modeling

Uncertainty

CATALYST PRESS



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Zieffler, A., & Catalysts for Change. (2017). *Statistical Thinking: A simulation approach to uncertainty* (4th ed.). Minneapolis, MN: Catalyst Press.

The work to create the material appearing in the book was made possible by the National Science Foundation (DUE-0814433).

Printed in the United States of America

ISBN 978-0615691305

Catalyst Press  
Minneapolis, MN 55455

<http://catalystsumn.blogspot.com>

*Fourth printing, June 2017*

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# INTRODUCTION



The materials in this lab manual and on the accompanying website (<http://zief0002.github.io/statistical-thinking/>) will introduce you to the seminal ideas underlying the discipline of statistics. In addition, they have been designed with your learning in mind. For example, many of the in-class activities were developed using pedagogical principles, such as small group activities and discussion, that have been shown in research to improve student learning.

**Bring a copy of this lab manual  
(either physically or electronically)  
with you to class each day.**

## COURSE READINGS

The course readings (available at <http://zief0002.github.io/statistical-thinking/>) should be completed outside of class and are intended to help you learn and extend the ideas, skills, and concepts you learn in the classroom.

## TINKERPLOTS™ SOFTWARE

Much of the material presented in the lab manual requires the use of TinkerPlots™. This software can be downloaded (for Mac or PC), and a license can be purchased from <http://www.tinkerplots.com/>.



## DATA AND OTHER RESOURCES FOR THE BOOK

The data sets used in the materials, as well as other materials that accompany the lab manual are available at <http://www.zieffler.com/resources/data.zip>. Clicking this link will download a ZIP file to your computer. Double-click on the ZIP file to view all the materials.

## PARTICIPATION IN THE LEARNING PROCESS

The lab manual, instructors, and teaching assistants are all resources that are at your disposal to help you learn the material. In the end, however, you will have to do all of the hard work associated with actually learning that material. To successfully navigate this process, it is vital that you be an active participant in the learning process. Coming to class, participating in the activities and discussions, reading, completing the assignments, and asking questions are essential to successful learning.

Learning anything new takes time and effort and this is especially true of learning statistics, as you are not just learning a set of methods, but rather a disciplined way of thinking about the world. Changing your habits of mind will take continual practice. It will also take a great deal of patience and persistence.

As you engage in and use the skills, concepts and ideas introduced in the material, you will find yourself thinking about data and evidence in a different way. This may lead you to make different decisions or choices. But, even if this course does not change your world overnight, you will at the very least be able to critically think about inferences and conclusions drawn from data.

## IPOD SHUFFLE PLAYLISTS



**Share and discuss your responses to each of the following questions with your group.**

1. Do you have an iPod or some other digital music player? Have you used the shuffle feature? If you have used the shuffle feature, have you ever wondered how truly random it is?
2. What comes to mind when you hear the word, 'random'?
3. If the iPod shuffle feature is not producing a random sequence of songs, then what might the sequence of songs look like? What would you expect to see?



4. Do you think you can be 100% certain that a sequence of songs was not randomly generated? Explain your answer.

### GROUP TASK

Albert Hoffman, an iPod owner, has written a letter to Apple to complain about the iPod shuffle feature. He writes that every day he takes an hour-long walk and listens to his iPod using the shuffle feature. **He believes that the shuffle feature is producing playlists in which some artists are played too often and others are not played enough.**

He has claimed that the iPod Shuffle feature is not generating random playlists. As evidence, Mr. Hoffman has provided both his music library (8 artists with 10 songs each) and three playlists (20 songs each) that his iPod generated using the shuffle feature.

Tim Cook, the CEO of Apple, Inc., has contacted your group to respond to Mr. Hoffman's complaint. He has provided your group with several playlists of 20 songs each using the same songs as Mr. Hoffman's library but generating them using a genuine random number generation method.

To help your group respond to Mr. Hoffman, the next four sections of the problem are designed to help your group explore properties of the randomly generated lists to develop rules that could help determine whether a set of playlists provide evidence that the shuffle feature is not producing randomly selected songs.

### EXPLORE AND DESCRIBE

Examine the randomly generated playlists (your group will be given 25) to get an idea of the characteristics of these lists. Write down at least two characteristics about the randomly generated playlist that help you address Mr. Hoffman's concern.

## DEVELOP RULES

Use the set of characteristics that your group wrote down to describe randomly generated playlists in the previous section to create a set of one or more rules that flag playlists that **do not appear to have been randomly generated**. (Be sure that each of the characteristics in the previous section is included in a rule.) *These rules should be clearly stated so that another person could easily use them.*

## TEST RULES

Your group will be given five additional randomly generated playlists on which to test your rules. Let your instructor know that you are ready to receive these playlists. See whether the set of rules your group generated would lead someone to (incorrectly) question whether these playlists are not randomly generated. Based on the performance of your group's set of rules, adapt or change the rules as your group feels necessary.

## EVALUATE

Your group will be provided with Mr. Hoffman's original three playlists. Apply your group's rules to these three playlists to judge whether there is convincing evidence that Mr. Hoffman's iPod Shuffle feature is producing playlists which do **not** seem to be randomly generated.

## SUMMARIZE

Your group will now write a letter to Mr. Hoffman that includes the following:

- Your group's set of rules, used to judge whether a playlist does not appear to have been randomly generated. In your letter the rules need to be clearly stated so that another person could apply them to a playlist of 20 songs from Mr. Hoffman's music library;
- A response to Mr. Hoffman's claim that the shuffle feature is **not random** *because it produces playlists in which some artists are played too often and others are not played enough.*

*Type the letter in a word-processed document and email it to each of your group members and the instructor.*

## DISCUSSION

As a group, discuss your responses to each of the following questions.

5. What made it difficult to come up with a rule to determine whether a sequence of data had been randomly generated? Explain.



# GENERATING RANDOM DATA—CAT FACTORY



Social scientists are increasingly using simulation methods to help them understand the social processes they study. One method they use, called Monte Carlo simulation, is to generate many samples from a specified population or model. Then they can study the patterns that emerge from these samples. In this activity you will learn how to set up a defined model (population) using TinkerPlots.

## EXPLORING A PRE-BUILT SAMPLER

Before building your own data factory, you will explore a pre-built sampler that simulates data about cats.

Open the file *Simulating Data.tp*. This is a built-in file that comes with TinkerPlots™. From the File menu select **Open Sample Document > Tutorials > Simulating Data.tp**.

Take a few moments to understand what is in this document. The sampler at the top of the screen can be used to generate a data set of cats with four attributes: *Gender*, *Name*, *Length*, and *EyeColor*. The case table and plot at the bottom of the screen show 500 cases—in this case, cats—that were generated by the sampler.

Now, you will create a new data set using the built-in sampler.

- At the top left corner of the sampler, click the **Repeat** value, *500*, and change it to *5* to generate data for five cats.
  - Now, click the **Run** button at the top left corner of the sampler.
  - Watch as the sampler generates data for five cats. As the data for a cat is completed, a new case appears in the results table, and a new case icon appears on the plot.
1. Describe the data generating process (i.e., how are the simulated data being produced; describe what is happening).
  2. What characteristics are dependent on the cat's sex?
  3. What characteristics are independent of the cat's sex?
  4. Examine the plot. What characteristics are being plotted?
- Click a case icon in the graph. Notice that the cat is also highlighted in the results table.
  - Now click on a cat in the results table (click on the cat's row number). The cat's case icon will also be highlighted.

## BUILDING A DATA FACTORY

Now you will build your own cat factory. You may want to leave the sample document open for reference.

- Open a new document in TinkerPlots™ by selecting **File > New**.
- Drag a new Sampler from the object toolbar into your blank document.
- Click and drag the samplers lower right-hand corner to make it larger.

At the bottom of the sampler, you will see six sampling devices that can be used to generate attributes.



**Mixers** and **Stacks** draw from a set of discrete elements. For example, the Name attribute in the cat factory was chosen from a mixer. If you have many repeats of the same value, such as choosing from a set of 30 boys and 45 girls, stacks are a better option than a mixer.

**Spinners** and **Bars** draw from discrete elements, but can have different probabilities for each value. The Gender and EyeColor attributes in the cat factors were determined by spinners, and the Length attribute was determined by bars.

**Curves** draw from a continuous range of numerical values, which can have different probabilities.

**Counters** select values systematically, rather than randomly.

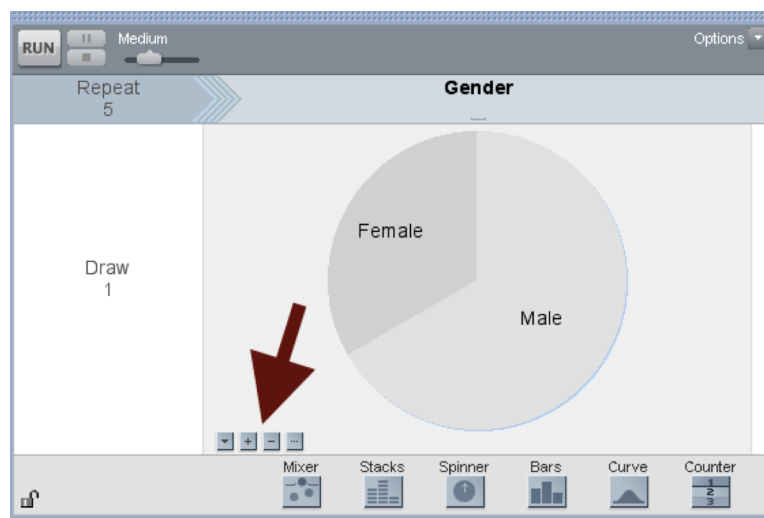
To model a cat factory, you will need to model both categorical (discrete) attributes, such as gender, and numerical (continuous) attributes, such as length.

## MODELING CAT GENDERS: SPINNER

The first attribute is *Gender*. Gender can be modeled best using a mixer or a spinner. Although the default device given in a new sampler is a mixer, we are going to use a spinner.

- Change the mixer to a spinner by dragging a spinner from the sampler's bottom toolbar into the sampler, and releasing it above the pink dot that appears in the center of the current mixer. (Pink dots show places where you can drop the new device. A black rectangle also highlights where you can drop the new device.)
- Select the text *Attr1* above the spinner and relabel it *Gender*.
- Click the *a* text in the spinner and change it to *Male*. Click the *b* text and change it to *Female*.
- Change the **Draw** value from 2 to 1. The draw value will indicate the number of sampling devices included in your sampler. Currently there is only one sampling device, a spinner.

Note the four buttons in the lower left corner of the spinner device.



Clicking the first icon shows the **Device options** menu; clicking the + and - buttons adds and subtracts values to the device; and clicking the ... button allows you to enter a range of values into the device. Click each button to see the result.



Currently one of the genders is more likely than the other. To make female cats as likely as male cats, you'll need to change the position of the divider in the spinner.

- Click the **Device options** menu and choose **Show Percent**.
- Change the percentages so that each is 50%.

## MODELING CAT NAMES: MIXER

The next attribute we want to model is *Name*. To model the cat names we will use a mixer. Furthermore, because male and female cats tend to have different names, you will use two mixers, one for male cat names and one for female cat names.

- Drag a mixer from the lower sampler toolbar into the sampler, and drop it on the pink dot to the right of the Gender device. (A black rectangle will highlight when you're in a position to drop the mixer.)
- Change the attribute name from *Attr2* to *Name*.
- Drag a second mixer into the sampler but do not release it yet. You will see four pink dots that represent locations where you can place the mixer. If you drop the mixer on the pink dot in the Gender spinner or in the first mixer, it will replace that device. Because you want this to be a second Name mixer, drop it on the pink dot attached to the Gender mixer, and directly below the first Name mixer. Notice the Male and Female labels on the lines connecting the Gender device to the Name devices.

Now we need to add the potential names into each mixer. First we will add the male names. The 10 male names we will add into the mixer are:

Charlie	Shadow	Spot	Jack	Max
Smokey	Oliver	Buddy	Simbar	Tiger

- Click the **+** (add element) button below the Male mixer. This will add an element called *a* into the sampling device.
- Change the name of the element from *a* to *Charlie*.
- Add nine more elements to the mixer, changing their names to each of the nine other male cat names. When you have finished, there should be 10 male cat names in the Male mixer.

Next we will add the 10 potential female cat names. Rather than have you enter the data manually, this time, you will use **Copy and Paste** to input the data.

- Open the file *female-cat-names.tp*.
- Highlight the Female attribute by clicking on the column header.
- Copy the list of names by selecting **Edit > Copy Attribute** (or use appropriate keyboard shortcuts).
- Go back to your cat factory document. Select the female name mixer by clicking on it, and paste the values by choosing **Edit > Paste Cases** (or use appropriate keyboard shortcuts).

## MODELING CAT LENGTH: BARS

The next attribute we want to generate data for is the cats' *Length*. This is a numeric attribute with varying probability for each value, so you can use a bars or curve device. We will use the bars device in this activity.

- Drag a bars device into the sampler and drop it just to the right of the Male Name mixer.
- Then drag a second bars device and drop it to the right of the Female Name mixer. (Males and females have different distributions of potential lengths, so their lengths will be chosen from different devices.)
- Change the name from *Attr3* to *Length*.

Cats typically have lengths between 10 and 30 inches. There are several ways to specify a range.

- Hold your mouse over the top bars device, and click the **+** (add element) button. Then change *a* to 10.
- Click the **+** (add element) button again. Notice that it automatically adds the next number to the device (e.g., 11). You can continue to click the **+** (add element) button until you have all the values from 10 to 30, but there is a faster way.
- Click the **...** button and enter the range "10 to 30."
- Enter the same range in the lower bars device.

Since all the bars are the same height, this indicates that each length, from 10 to 30, is equally likely.

5. Are all these lengths really equally likely? For example, are 30-inch-long cats just as common as 17-inch long cats?

It is more likely that cats have lengths in the middle of this range, and that lengths at the extreme values in this range are less likely. To model this, we will shape the *Length* distribution so it has a bump (is higher) in the middle. (remember, the height indicates the relative probability of each element, in this case, cat length).

- Position your cursor at the top of the far-left bar in the *Length* device for male cats (a plus-sign should appear). Click and drag the cursor over the bars in the shape of the desired distribution.
- Repeat for the Length of female cats. Since female cats tend to be a little shorter than male cats, you should place your bump a little to the left of the bump in the distribution for male cats.

### MODELING CAT EYE COLOR: SPINNER

The final attribute we want to generate is *Eye Color*. Because this is a categorical (discrete) attribute, we can use either a spinner or stacks. Here, we will again use a spinner.

- Drag a spinner into the sampler and drop it to the right of the bars device for male cats.
- Drag a second spinner into the sampler and drop it to the right of the bars device for female cats.

Since eye color does not vary for male and female cats, we do not need a separate device for males and females. To join these two devices together:

- Hold your mouse over the upper spinner, and click the **Device options** menu (down arrow). Choose **Merge Device > Merge with Device Below**.
- Change *Attr4* to *EyeColor*. (Note that attribute names cannot contain spaces so we use bumpyCase.)
- Click the **+** (add element) button to add three eye colors to the mixer. Label the values *Yellow*, *Green*, and *Blue*.
- Blue-eyed cats are less common than yellow or green-eyed cats, so make the Blue section smaller than the other two by changing the percentages.

You should now have a cat factory that resembles the one in *Simulating Data.tp*. Click **Run** to generate five cats with randomized attributes. Notice that a results table automatically appears and is filled in.

6. Save the TinkerPlots™ document and email it to all of your group members so they have a copy.

## PRACTICE 1: POPULATION OF STUDENTS

7. How would you set up a sampler to generate data for 25 students from a population of students, where 40% of the population are freshmen, 30% are sophomores, 15% are juniors, and 15% are seniors? Sketch a picture of the sampler below. Don't forget to indicate both the **Draw** and **Repeat** values in your sketch.

8. Open a new TinkerPlots™ document and implement the sampler you just sketched.
9. Generate data for the 25 students by clicking Run. How many of the 25 students generated were seniors? What percentage is that?
10. Generate data for another set of 25 students by again clicking Run. How many of the 25 students generated were seniors? What percentage is that?
11. If you run the simulation many times, will there always be exactly 15% of the generated data that are seniors? Explain.
12. Save the TinkerPlots™ document and email it to all of your group members so they have a copy.





# INTRODUCTION TO MONTE CARLO SIMULATION



Carsey and Harden<sup>1</sup> define Monte Carlo simulation as,

any computational algorithm that randomly generates multiple samples of data from a defined population based on an assumed data generating process (DGP). The DGP is the mechanism that characterizes the population from which simulated samples of data are drawn. Then the researcher explores patterns that emerge across those simulated samples.

In this activity you will learn the process of carrying out a Monte Carlo simulation and how to do so using TinkerPlots™.

## MODEL—SIMULATE—EVALUATE

Looking back at the definition of a Monte Carlo simulation above, the process encompasses (1) defining a population or model, (2) randomly generating several samples of data from the population or model, and (3) exploring the patterns that emerge across the simulated samples. In simpler terms, (1) model, (2) simulate, and (3) evaluate.

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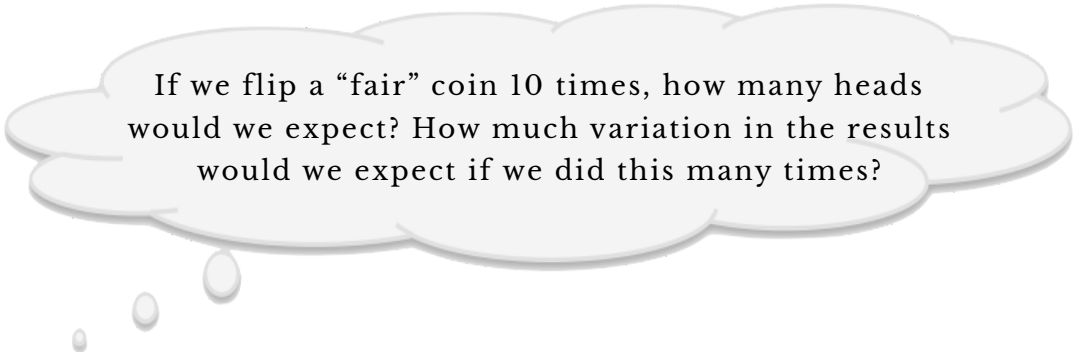
<sup>1</sup> Carsey, T. M., & Harden, J. J. (2014). *Monte Carlo simulation and resampling methods for social science*. Thousand Oaks, CA: Sage.



In the previous course activity, you created several models using TinkerPlots™ and used them to randomly generate data. The key to Monte Carlo simulation is to generate many, many randomly generated samples. The catch is that we need to collect some information from each of these samples so that we can examine this information across the many samples. The information we collect is often a quantifiable summarization of the sample, for example the mean value, a count, or a proportion. The summary we choose is based on our research question.

### **Monte Carlo Simulation 1: Coin Flips**

In the first Monte Carlo simulation you will be exploring the following questions:



If we flip a “fair” coin 10 times, how many heads would we expect? How much variation in the results would we expect if we did this many times?

*Questions 1–3 are asking for your intuitions. You do not have to calculate exact values. We will explore these questions in more detail later in this activity.*

1. Imagine that you flip a fair coin ten times and count the number of heads. How many heads would you expect to see on average? Why?

2. Now, imagine repeating this process 100 times: flipping that fair coin ten times, and counting the number of heads. Would you expect to see the same result in all 100 trials?
3. How variable would the results be? What do you think the smallest and largest number of heads would be? What do you think the range would be for most results?

## MODELING AND SIMULATING

To save time and to gather data quickly, you will use TinkerPlots™ to model tossing a coin 10 times.

4. How would you set up a sampler to toss a fair coin 10 times? Sketch a picture of the sampler below. Don't forget to indicate both the **Draw** and **Repeat** values in your sketch.

- Open a new TinkerPlots™ document and implement the sampler you just sketched.
  - After you have set up the model, click the **Run** button.
  - A *case table* displaying the 10 outcomes for the “coin flips” should have been produced.
  - Plot the 10 outcomes. Fully separate the cases and vertically stack them.
  - With the plot highlighted, click the **Case Count (N)** icon in the upper toolbar. This should display counts of the number of heads and tails in the plot.
5. Record the **number of heads** from your randomly generated data below.

In a simulation, each time the model is used to produce a sample of data, it is referred to as a **trial**. A trial can consist of one or many outcomes depending on the simulation. In this simulation, the trial consisted of 10 outcomes (flips). In TinkerPlots™, the summary, or how we quantify the sample is referred to as the trial's **result**. In this simulation, the trial result would be the number of heads. In order to study any patterns that might emerge, we need to generate many trials and record the result from each of them.

6. Re-click the **Run** button in the sampler to generate another random sample of data. Since the plot and the case counts are linked to the outcomes from the sampler, these should update automatically. Record the **number of heads** from this new sample below.
  
7. Generate 23 more samples. For each sample generated, record the number of heads below.

## EVALUATING THE RESULTS FROM MANY TRIALS

At this point, we have completed two of the three parts of the Monte Carlo simulation process, namely (1) model and (2) simulate. In order to study any patterns in the trials' results, we need to plot the results from the 25 samples of data you generated.

- Open a new TinkerPlots™ document.
- Drag a new **Case Table** from the object toolbar into your blank document.
- Click on **<new>** to change the attribute name. Rename this attribute *Results*.
- Enter the results from the 25 trials into the results column in the case table.
- Plot the 25 results. Fully separate the case icons in the plot and vertically stack the cases.

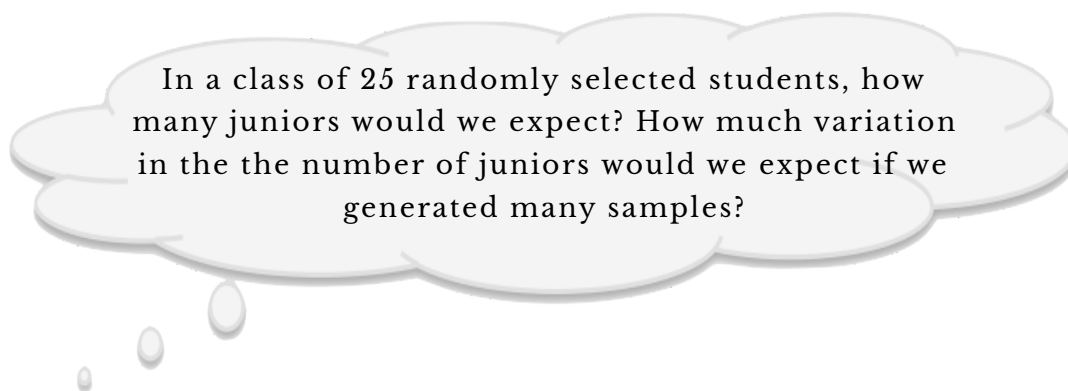




## MONTE CARLO SIMULATION 2: GENERATING A SAMPLE OF STUDENTS

In the previous course activity, you set up a sampler to generate data for 25 students from a population of students, where 40% of the population are freshmen, 30% are sophomores, 15% are juniors, and 15% are seniors.

In this Monte Carlo simulation you will be exploring the following questions:



*Questions 14 and 15 are asking for your intuitions. You do not have to calculate exact values. We will explore these questions in more detail later in this activity.*

14. Imagine generating 100 random samples of 25 students from the defined population. What do you think the typical number of juniors in a class of 25 would be? Explain your reasoning.
  
15. How variable would the results be? What do you think the smallest and largest number of juniors would be? What do you think the range would be for most classes?

## MODELING AND SIMULATING

- Open the saved TinkerPlots™ document from the previous activity where you set up this model. If you didn't save the TinkerPlots™ sampler from the previous activity, re-create the sampler.
- After you have set up the model, click the **Run** button.
- A *case table* displaying the 25 outcomes for the first trial.
- Plot the 25 outcomes. Fully separate the cases and vertically stack them.
- With the plot highlighted, click the **Case Count (N)** icon in the upper toolbar.

16. Record the **number of juniors** from your randomly generated data below.

17. Generate 24 more samples. For each sample generated, record the number of juniors below.



## EVALUATING THE RESULTS FROM MANY TRIALS

- Open a new TinkerPlots™ document.
- Enter the 25 results into a **Case Table**.
- Plot the 25 results. Fully separate the case icons in the plot and vertically stack the cases.

18. Sketch the plot below.

19. Based on the plot of the simulation results, what was a typical number of juniors in a class of 25 students? Explain how you decided this from the plot.

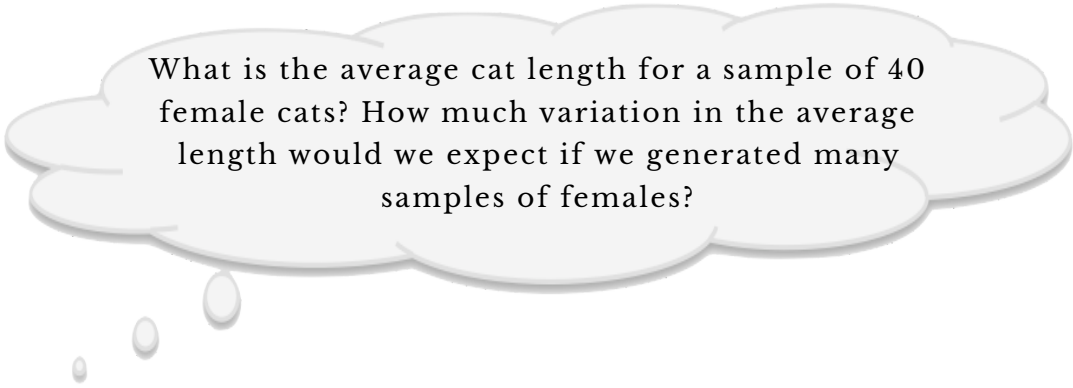
20. Based on the plot of the simulation results, how variable are the results? What are the smallest and largest number of juniors that you observed? What is the range of values where most of results lie?

21. Based on the plot of the simulation results, would 10 juniors in a class be a likely or unlikely result? Explain.

22. Save all TinkerPlots™ documents and email them to all of your group members so they have a copy.

### **MONTE CARLO SIMULATION 3: GENERATING CAT LENGTHS**

In the previous course activity, you set up a sampler to generate data for female cat lengths. In this Monte Carlo simulation you will be exploring the following questions:



What is the average cat length for a sample of 40 female cats? How much variation in the average length would we expect if we generated many samples of females?

*Question 23 is asking for your intuition. You do not have to calculate exact values. We will explore these questions in more detail later in this activity.*

23. Imagine generating 100 random samples of 40 female cats lengths from the defined population (the population was defined in the last course activity) and computing the average for each of these samples. How variable would the averages be? What do you think the smallest and largest averages would be? What do you think the range would be for most averages

## MODELING AND SIMULATING

Although you previously set up a sampler to generate female cat lengths in the previous activity, it isn't as useful for this simulation since it is connected to other samplers. It will be better to re-create the female cat lengths sampler in a new TinkerPlots™ document. (Plus, it is good practice.)

- Open a new TinkerPlots™ document.
- Set up a sampler to model female cat lengths. If you've forgotten how to do this, look at the instructions in the previous activity. You will need to modify the sampler so it generates data for 40 cats.
- After you have set up the model, click the **Run** button.
- A *case table* displaying the 40 outcomes for the first trial.
- Plot the 40 outcomes. Fully separate the cases and vertically stack them.
- With the plot highlighted, click the **Averages (Mean)** icon (triangle) in the upper toolbar. Then click on the **Averages Options** icon (down-facing triangle) in the upper toolbar, and select **Show Numeric Value(s)**. This should display the value of the mean into the plot.

24. Record the **mean female cat length** for the sample.

25. Generate 24 more samples. For each sample generated, record the mean female cat length below.

### EVALUATING THE RESULTS FROM MANY TRIALS

- Open a new TinkerPlots™ document.
- Enter the 25 results into a **Case Table**.
- Plot the 25 results. Fully separate the case icons in the plot and vertically stack the cases.

26. Sketch the plot below.

27. Based on the plot of the simulation results, what was a typical average female cat length? Explain how you decided this from the plot.
28. Based on the plot of the simulation results, how variable are the results? What are the smallest and largest mean values that you observed? What is the range of values where most of results lie?
29. Based on the plot of the simulation results, would an average of 17 inches be a likely or unlikely result? Explain.
30. Save any TinkerPlots™ documents and email them to all of your group members so they have a copy.

## AUTOMATING THE SIMULATION PROCESS



In previous activities and assignments, you have learned how to set up a model to run a simulation experiment using TinkerPlots™. In these simulations, you ran many trials from which you collected a particular outcome (e.g., the number of heads when flipping a coin 10 times). You also learned how to create a case table to collect the results from each trial into, and how to plot those results.

In this activity, you are going to be introduced to the **Collect** function in TinkerPlots™. This will automate the collecting of trial results in a simulation. It will also make carrying out several trials easier.

### MODELING COIN FLIPS

Recall in the previous activity you modeled flipping a coin 10 times. Suppose you wanted to simulate 100 more trials of 10 flips In TinkerPlots™.

- Set up a model to simulate tossing a single coin 10 times.
- After you have set up the model, click the **Run** button.

## AUTOMATING THE COLLECTION OF TRIAL RESULTS

Rather than having you record the number of heads that occurred in the 10 flips, we will automate this using TinkerPlots™. The general process for having TinkerPlots™ record and collect the trial results is: (1) plot the outcomes from the trial, and (2) collect the numerical result you are using to summarize the trial.

- Plot the 10 outcomes from the trial. Fully separate the outcomes and stack them vertically.
- Highlight the plot of the trial outcomes and click on the **Case Counts (N)** button in the upper plot toolbar. This should display a count of the number of heads and tails in the trial.

Note that **Case Counts (N)** and **Case Counts (%)** will count the number of cases **within each section of a plot**. If there are not multiple sections (no bin lines), the number of total cases in the plot will be displayed. This is why we need to fully separate the cases when we plot them.

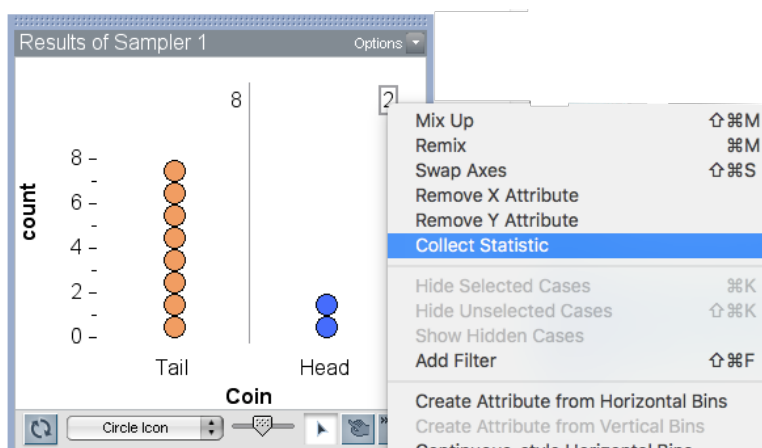
## COLLECTING THE RESULTS FROM MANY TRIALS

You can also use TinkerPlots™ to automatically collect the summarized result from the trial into a case table.

- Use TinkerPlots™ to automatically collect the result from your simulated trial into a case table (see instructions below and figure on next page).

### Collecting the Results from a Trial

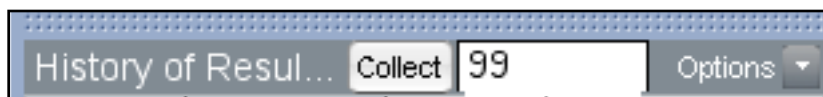
- Right-click the summary result in your plot.
- Select **Collect Statistic**.



It is important that you right-click on the *actual value of the result* in the plot since you want TinkerPlots™ to collect the value. For example, in the plot displayed above, you would right-click on the value 2 to collect the number of heads.

The result is then collected in a new case table. This case table, which is called *History of Results*, has a single row with the collected result, in this case two, displayed in a new attribute. The window next to the **Collect** button indicates the number of results that were collected, in this case one result was collected. This value can be changed to add the results of additional trials into the case table. In this case, the result collected from each trial is stored in a row of the *History of Results* case table.

- Change the value in the *History of Results* case table to 99 to add the results from an additional 99 trials of the simulation (see figure below).
- Click the **Collect** button.



*Change the value to 99 in the History of Results table.*

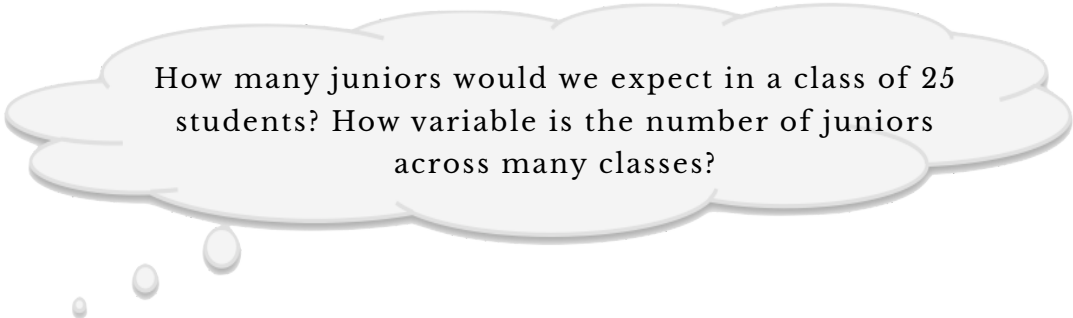


1. Record the result from the 87<sup>th</sup> trial.
2. Plot the results from your 100 simulated trials. Don't forget to fully separate the case icons, and vertically stack them.
3. Based on the plot of the simulation results, what was a typical number of heads from 10 flips? Explain how you decided this from the plot.
4. Based on the plot of the simulation results, how variable are the results? What are the smallest and largest number of heads that you observed? What is the range of values where most of results lie?

5. Based on the plot of the simulation results, would two heads out of 10 flips be a likely or unlikely result? Explain.
  
  
  
  
  
  
  
  
  
  
6. Based on the plot of the simulation results, would seven heads out of 10 flips be a likely or unlikely result? Explain.

## NUMBER OF JUNIORS

Set up a sampler to generate data for 25 students from a population of students, where 40% of the population are freshmen, 30% are sophomores, 15% are juniors, and 15% are seniors.



How many juniors would we expect in a class of 25 students? How variable is the number of juniors across many classes?

- Open a new document in TinkerPlots™.
- Set up the model of sampling students.
- Carry out a single trial of the simulation.
- Plot the 25 outcomes from the simulated trial.
- Stack and separate the cases into groups.
- Use **Case Counts (N)** to summarize the number of cases in each group.
- Collect the number of juniors from the trial into a *History of Results* case table.
- Carry out an additional 99 trials.
- Plot the results from your 100 simulated trials.

Use the plot of the results from your 100 simulated trials to answer each of the following questions.

7. Sketch a plot of the results. Be sure to label the axis.
8. Based on the plot of the simulation results, what was a typical number of juniors in a class of 25 students? Explain how you decided this from the plot.

9. Based on the plot of the simulation results, how variable are the results?  
What are the smallest and largest number of juniors that you observed?  
What is the range of values where most of results lie?
  
  
  
  
  
  
  
  
  
  
10. Based on the plot of the simulation results, would 10 juniors in a class be a likely or unlikely result? Explain.

### GENERATING CAT LENGTHS

Set up a sampler to randomly generate 40 female cat lengths (see previous activity). Use TinkerPlots™ to carry out 100 trials of a simulation to determine how much the **average female cat length** varies across samples. After plotting the 100 averages, answer the following questions:

11. Based on the plot of the simulation results, what was a typical average female cat length? Explain how you decided this from the plot.

12. Based on the plot of the simulation results, how variable are the results? What are the smallest and largest mean values that you observed? What is the range of values where most of results lie?
13. Based on the plot of the simulation results, would an average of 17 inches be a likely or unlikely result? Explain.

## MONDAY BREAKUPS



Facebook is a social networking website. One piece of data that members of Facebook often report is their relationship status: single, in a relationship, married, it's complicated, etc.

With the help of Lee Byron of Facebook, David McCandless—a London-based author, writer, and designer—examined changes in peoples' relationship status, in particular, breakups. A plot of the results showed that there were repeated peaks on Mondays. Based on this initial examination of data, McCandless speculated that breakups are reported at a higher frequency on Mondays. This is his **research hypothesis**.

To test this research hypothesis, McCandless collected a random sample of 50 breakups reported on Facebook within the last year. Of these sampled breakups, 13 occurred on a Monday.

In this activity, you will be exploring the following research question:

Is 13 (out of 50) breakups reported on Mondays consistent with the model where breakups are equally likely during the week? Or is it more consistent with a model where Mondays have a higher frequency of breakups?

## DISCUSS THE FOLLOWING QUESTIONS

1. What does the observed data (13 out of 50 breakups on Monday) suggest about the answer to the research question? Explain.

## ‘EQUALLY LIKELY’ MODEL

Suppose for the moment that the researchers’ conjecture is wrong, and breakups *are not* reported on Monday more than any other day. In other words, breakups are reported uniformly throughout the week. This, is a statistical hypothesis. Namely,

*Hypothesis:* Breakups are reported at the same frequency/percentage on each day of the week.

This statistical hypothesis specifies an “equally likely” breakup reporting model for each day of the week. We can use TinkerPlots™ to create this model and generate random outcomes.

### Statistical Hypothesis

A statistical hypothesis is a statement **specifying a model** that explains variation in a particular outcome.

Wait a minute...if McCandless believes that breakups are reported more frequently on Mondays, why wouldn't that be his statistical hypothesis? It could be, but remember, the statistical hypothesis has to specify a model that can be used to generate outcomes. One such model that fits his research hypothesis is that 30% of breakups are reported on Mondays. Another is that 50% of breakups are reported on Mondays. How many others are there? In order to answer his research question, McCandless would have to examine the results from every single one of those models to see if 13 out of 50 breakups is consistent with one of those models.

**#protip:** Rather than examine each of those models, statisticians often use the 'equally likely' model. Mathematically, this model acts as a "lower bound" for all the models where Mondays has more reported breakups than other days. If 13 out of 50 breakups is an extreme (high) result given this model, we can rule out this model and suggest that the observed data are inconsistent with the model. This points toward a model in which the reported frequency of breakups is higher on Mondays, without having to know the exact percentage of breakups that occur on Monday!

2. Draw a picture of the sampler (model) that you will use to generate outcomes from the model specified in the statistical hypothesis. In the picture, be sure to (1) indicate the type of sampling device used (mixer, spinner, etc.); (2) label all the elements in your sampling device; (3) label the probability associated with each element; and (4) indicate the **Repeat** and **Draw** values you will use.

- Set up the model/sampler in TinkerPlots™.



## SIMULATING THE DATA

- Carry out a single trial of the simulation in TinkerPlots™.
  - Plot the outcomes from the trial.
3. Sketch a plot of the outcomes from this trial. Add all labels and summary measures (counts, percentage, etc.) to your plot.

Remember what we are ultimately interested in is the number of breakups (out of 50) that are reported on Mondays.

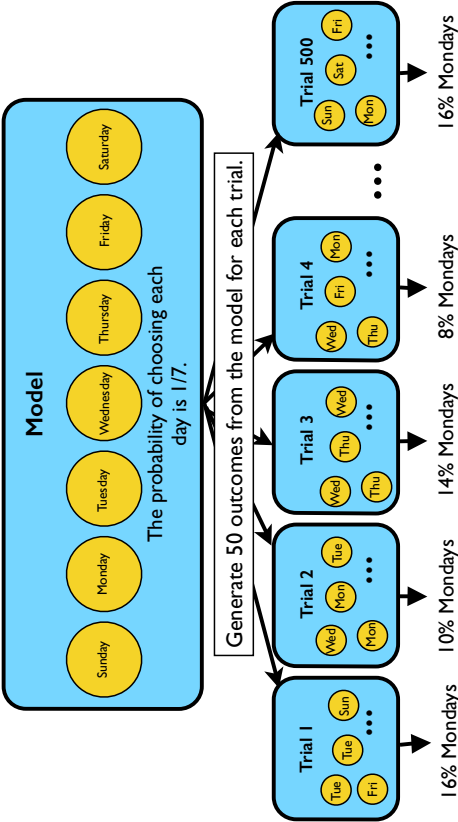
- Collect the appropriate summary measure from the plot of your first trial.
- Carry out 499 more trials (500 trials total) of the simulation in TinkerPlots™.



# Modeling the Monday Breakups Problem

**MODEL**

The model has seven outcomes—one for each day of the week. Each is equally likely.

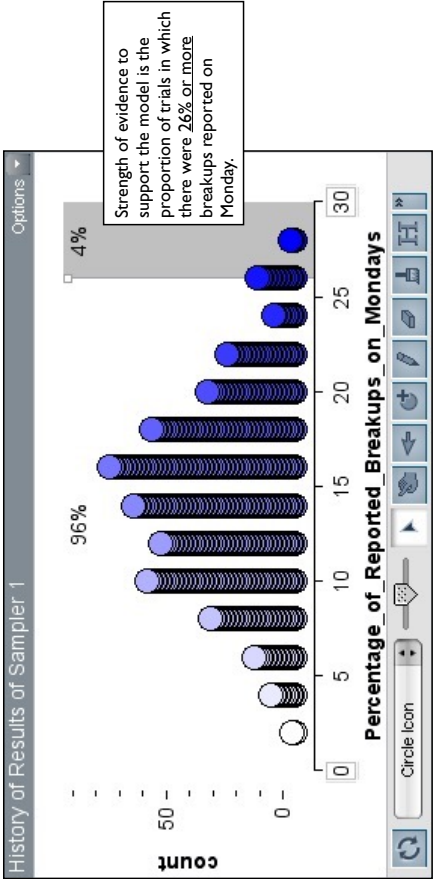


**SIMULATE**

A trial ends when 50 outcomes have been randomly generated from the model. For each trial, compute the percentage of breakups reported on Monday. Generate many trials.

**EVALUATE**

Compile all of the numerical summary measures into a single distribution. Evaluate the initial questions by quantifying the strength of evidence to support the "just-by-chance" model—the proportion of trials in which there were 26% or more breakups reported on Monday.

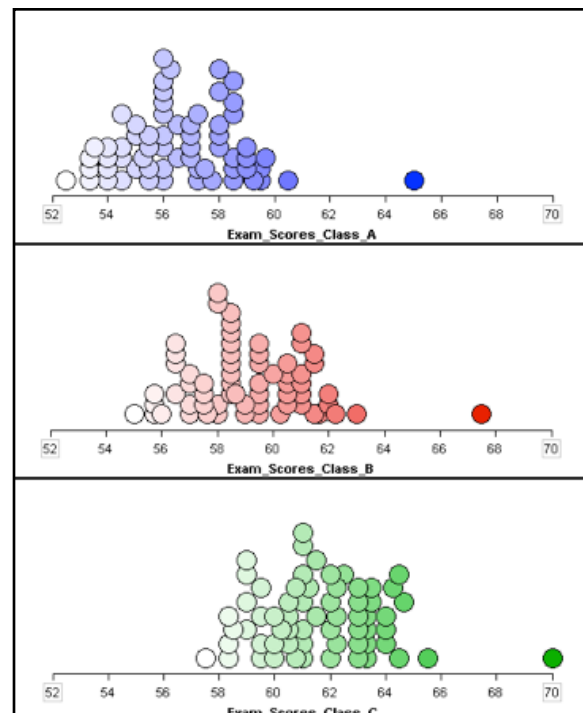


# FEATURES OF DISTRIBUTIONS

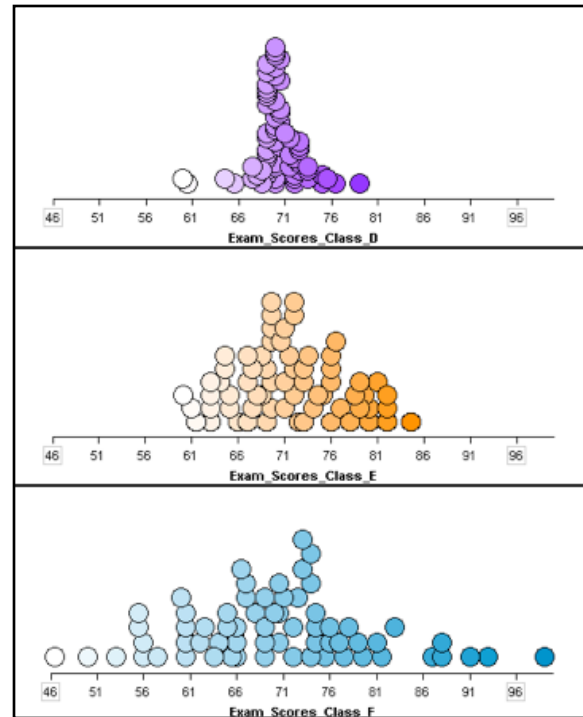


Imagine multiple sections of the same college course, taught by different instructors. Below are a series of plots that depict the distributions of hypothetical exam scores in various sections.

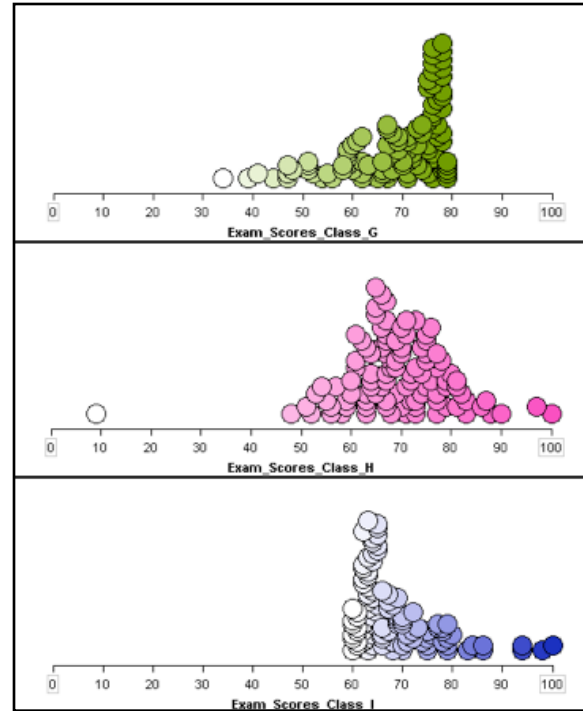
1. Examine the three distributions of exam scores for classes  $A$ ,  $B$ , and  $C$ . What are the primary differences between these three distributions? What are potential factors that might explain the differences?



2. Examine the three distributions of exam scores for classes *D*, *E*, and *F*. What are the primary differences between these three distributions? What are potential factors that might explain the differences?

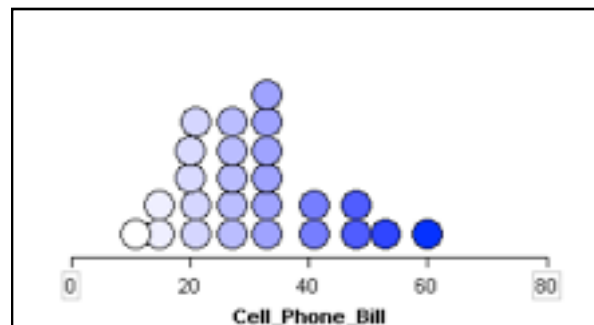


3. Examine the three distributions of exam scores for classes *G*, *H*, and *I*. What are the primary differences between these three distributions? What are potential factors that might explain the differences?



## CELL PHONE BILLS

Consider a survey study conducted on a random sample of 25 University of Minnesota students. One survey item asked students to self-report the amount of his or her last cell phone bill (in dollars). The plot of the bill amounts is shown below.

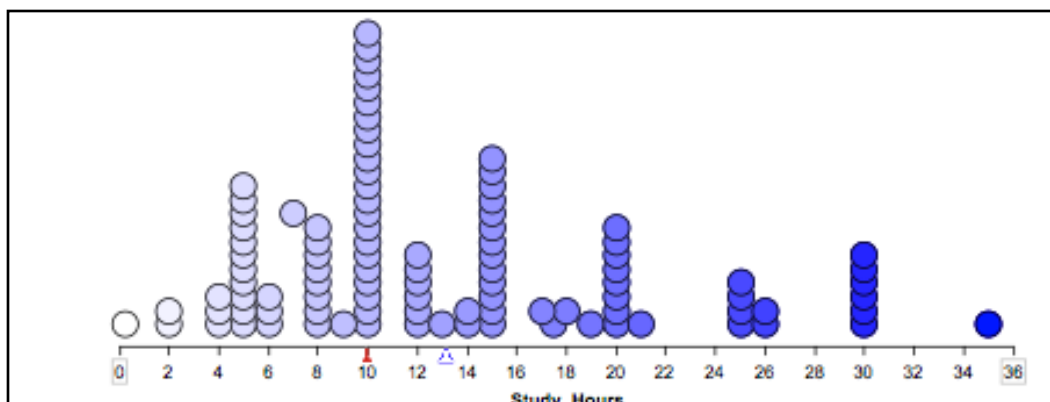


4. If you wanted to tell someone the amount of a “typical” cell phone bill for these students, what would you say?
5. How would you describe (quantify) the *overall* amount of variation in the distribution (i.e., for all 25 cell phone bills)?
6. How far do cases typically vary from the value that you identified in Question 4?

7. What is a potential factor(s) that might explain the variation in these bills?
  
8. Using the typical cell phone bill you identified previously as a reference point, consider the amount of variation in the distribution on both sides of this point. Is the variation roughly the same on the left- and right-hand side of this point? Is there more or less variation on either side of this value?

### NUMBER OF HOURS STUDIED

The plot below contains responses from 100 EPsy 3264 students who responded to the survey question: “How many hours per week do you typically study?” These students’ responses are a random sample from all responses obtained from all classroom sections of EPsy 3264 taught from 2004–2010. Examine the plot of these data.





9. What does each dot (i.e., case) in the distribution represent?
  
  
  
  
  
  
  
  
  
  
10. Summarize the features of the distribution. Be sure to identify the “**typical**” amount of time spent studying and the **variation** in the amount of studying. (When describing the variation, you should quantify the “average” amount of deviation from the typical value.) You should also indicate the **shape** of the distribution.
  
  
  
  
  
  
  
  
  
  
11. What is a potential factor(s) that might explain the variation in these data?

## HELPER OR HINDERER



Most college students recognize the difference between naughty and nice, right? What about children less than a year old—do they recognize the difference and show a preference for nice over naughty? In a study reported in the November 2007 issue of *Nature*<sup>1</sup>, researchers investigated whether infants take into account an individual's actions towards others in evaluating that individual as appealing or aversive, perhaps laying for the foundation for social interaction. In one component of the study, 10-month-old infants were shown a “climber” character (a piece of wood with “google” eyes glued onto it) that could not make it up a hill in two tries. Then they were alternately shown two scenarios for the climber's next try, one where the climber was pushed to the top of the hill by another character (*helper*) and one where the climber was pushed back down the hill by another character (*hinderer*). The infant was alternately shown these two scenarios several times. Then the child was presented with both pieces of wood (the helper and the hinderer) and asked to pick one to play with. The researchers found that 14 of the 16 infants chose the helper over the hinderer.

In this activity, you will be exploring the following research question:

Are infants able to notice and react to helpful or hindering behavior observed in others?

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<sup>1</sup> J. K. Hamlin, K. Wynn, & P. Bloom. (2007). Social evaluation by preverbal infants. *Nature*, 450, 557–559.

Before you begin, we would like you to watch the videos that were shown to the infants in the experiment. You can view them here:

- <http://www.yale.edu/infantlab/socialevaluation/Helper-Hinderer.html>



*Helping and hindering habituation events. On each trial, the climber (red circle) attempts to climb the hill twice, each time falling back to the bottom of the hill. On the third attempt, the climber is either bumped up the hill by the helper (left panel) or bumped down the hill by the hinderer (right panel).*

### DISCUSS THE FOLLOWING QUESTIONS

1. What proportion of the infants in the observed data chose the helper toy?
2. What does that suggest about the answer to the research question? Explain.



## SUMMARY OF THE SIMULATION PROCESS

The key to answering the research question in this activity is to determine the likelihood of the observed result (14 of 16 infants choosing the helper) under the assumption that infants have no preference for either the helper or the hinderer. The “no preference” model is again the “just-by-chance” model—infants randomly select either the helper or hinderer.

To determine this likelihood, you will model the process of 16 hypothetical infants making their selections using random chance. Then, you can count how many of these “infants” choose the helper toy. This process can be repeated many times to obtain a distribution of results that would be expected under the “no preference” or “just-by-chance” model.

The observed result of 14 of 16 infants choosing the helper can then be evaluated in light of this distribution to determine how likely it would be to obtain such a result (or a more extreme result) under the assumption of random chance. As such, the observed result can provide evidence to help answer the research question.

5. Draw a picture of the sampler (model) that you will use to generate outcomes from the “just-by-chance” model. In the picture, be sure to (1) indicate the type of sampling device used (mixer, spinner, etc.); (2) label all the elements in your sampling device; (3) label the probability associated with each element; and (4) indicate the **Repeat** and **Draw** values you will use.

## SIMULATING DATA FROM THE HYPOTHESIZED MODEL

In this study, a trial represents each of the 16 infants choosing a toy. The trial ends when 16 toys have been chosen randomly.

- Carry out a single trial of the simulation in TinkerPlots™.
- Plot the outcomes from the trial.

6. What is the summary measure from the plot that you will be **collecting**?

- Collect the appropriate summary measure.
- Carry out 499 more trials (500 trials total) of the simulation in TinkerPlots™.
- 

## EVALUATING THE HYPOTHESIZED MODEL

- Plot the results from the simulation.

7. Sketch a plot of the results below.

8. Describe the shape, center, and variation for the distribution of results.
9. Is the observed result from the original experiment likely or unlikely under the hypothesized model? Explain.
10. Does the observed result support the hypothesized “no preference” model? Explain.





## COMPARING HAND SPANS



In this activity, you will learn about the standard deviation, a common measure of variability.

How can you quantify variability and summarize it into a single measure?

1. Measure and record the hand span for each person in your group.
2. Enter the data into a TinkerPlots™ case table. Create a plot of the hand spans for your group. Sketch the plot below. Be sure to appropriately label the  $x$ -axis.

3. Compute the mean hand span for your group using TinkerPlots™. Record the mean.

## THE STANDARD DEVIATION

Recall that the mean is a single number that can be used to summarize the data. In this context, it is a description of the typical hand span measurement for your group. Of course, not every student in the sample is at the typical value (in fact all of them might be different from the typical value). Thus, it is also useful to have a single number description of how different the data tends to be from this typical value.

One single number description of the variability in a sample of data is called the **standard deviation** or *SD*. If the word “typical” is substituted for the word “standard” in its name, the name standard deviation (typical deviation) makes more sense. This measure quantifies variability by determining how far data cases typically deviate from the mean value.

- Use TinkerPlots™ to create a new attribute in the table, called *Deviations*, that contains the difference between the observed data (hand spans) and the mean of your group members’ hand spans. Use a formula to compute this difference (you can compute these by subtracting the mean from each observation).
  - Create a plot of the *Deviations* attribute.
4. How would you interpret the values of the *Deviations* attribute?



8. How does the distribution of the squared deviations compare to the distributions of the original hand spans? How does it compare to the distribution of *deviations*?
9. Compute the mean of the squared distributions and record its value below. Because the deviations have been squared, this value represents the **typical squared deviation**.
10. Find the square root of the mean value and record its value below.

11. What does this new value represent (i.e., interpret its value)?

### COMPUTING THE STANDARD DEVIATION USING TINKERPLOTS™

Now use TinkerPlots™ to find the standard deviation of the original data directly.

- In the case table where you entered the original hand spans, create a new attribute called standardDeviation.
- Use the **Formula Editor** to compute the standard deviation of the hand spans by using the `stdDev()` function.

The value computed using TinkerPlots™ will be similar, albeit higher, than the value you obtained in Question 10. This is because there is a slight adjustment made to the denominator when a standard deviation is computed from a sample of data. From this point forward, you should always use the `stdDev()` function to compute the standard deviation.

### COMPUTING THE STANDARD DEVIATION OF A PLOT OF RESULTS

Open the TinkerPlots™ file you saved from the *Helper or Hinderer* activity. (If you didn't save your TinkerPlots™ file from this activity, re-run the simulation.)

12. Describe the shape, center, and variation for the distribution of results. This time, rather than giving a more informal description of the variation, compute the standard deviation using the `stdDev()` function.

13. From statistical theory, we know that most observations in a distribution are within one standard deviation of the mean. Add and subtract one standard deviation from the mean to complete the following sentence:

Most simulated means will be between \_\_\_\_\_ and \_\_\_\_\_.

- Highlight the plot of results..
- Click the **Divider** tool in the upper toolbar. This will add a shaded rectangle to the plot.
- Click the **Counts (%)** button in the upper toolbar. This will show the percentage of cases that are included in the shaded area and those on both sides of the shaded area.
- Now move the ends of the shaded area to one standard deviation above and below the mean (to the values you computed in Question 13). To do this drag the white rectangles in the upper corners of the shaded rectangle. You can also double-click each of the white rectangles and enter in the value you want to move the line to.

14. What percentage of the results are within one standard deviation of the mean?

15. Now move the ends of the shaded area to two standard deviations above and below the mean. What percentage of the results are within two standard deviation of the mean?

16. How many standard deviations above the mean is the observed result of 14?

Most statisticians define *likely* results as those that are within two standard deviations of the mean. Anything more than two standard deviations from the mean would be called *unlikely*.

## RACIAL DISPARITIES IN POLICE STOPS



In the 1990s, the U.S. Justice Department and other groups studied possible abuse by Philadelphia police officers in their treatment of minorities. One study conducted by the American Civil Liberties Union analyzed whether black drivers were more likely than others in the population to be targeted by police for traffic stops.

Researchers studied the results of 262 police car stops during one week in 1997. Of those, 207 of the drivers were black. At that time Philadelphia's population was 42.2% black.

In this activity, you will be exploring the following research question:

Does the percentage of black drivers being stopped provide evidence of possible racial disparities (i.e., higher than what we would expect because of chance variation)?



## DISCUSS THE FOLLOWING QUESTIONS

1. What percentage of the drivers that were stopped were black?
2. What does that suggest about the answer to the research question? Explain

Suppose for the moment that there are *no* racial disparities in police stops. In other words, the percentage of blacks stopped by the police should reflect the percentage of blacks in the population.

3. What percentage of the drivers that were stopped would you expect to be black, given the population?

In both the *Helper or Hinderer* course activity and the *Monday Breakups* course activity, all of the elements in the “just-by-chance” model had the same probability. When the probabilities of each element in the model are exactly the same, we say the “just-by-chance” model is a **uniform probability model**.

The “just-by-chance” model does not have to be a uniform probability model. The elements can have differing probabilities. As long as the elements being selected are still random, this is still a “just-by-chance” model.

4. Using the value from the previous question, write the hypothesis for the “just-by-chance” model.
  
  
  
  
  
  
  
  
  
  
  5. Draw a picture of the sampler (model) that you will use to generate outcomes from the “just-by-chance” model. In the picture, be sure to (1) indicate the type of sampling device used (mixer, spinner, etc.); (2) label all the elements in your sampling device; (3) label the probability associated with each element; and (4) indicate the **Repeat** and **Draw** values you will use.
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- Set up the model/sampler in TinkerPlots™.

## **SIMULATING THE DATA**

- Carry out a single trial of the simulation in TinkerPlots™.
- Plot the outcomes from the trial.

6. What is the summary measure from the plot that you will be collecting?

- Collect the appropriate summary measure from the plot of your first trial.
- Carry out 499 more trials (500 trials total) of the simulation in TinkerPlots™.

## **EVALUATING THE HYPOTHESIZED MODEL**

- Plot the results from the simulation.

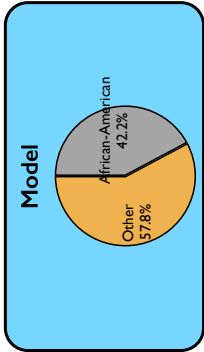
7. Sketch a plot of the results below.

8. Describe the shape of the distribution. Also compute the mean and standard deviation.
9. Use the mean and standard deviation to compute the range of *likely* results.
10. Given the range of *likely* results you just computed, is the observed result likely under the hypothesized model or not? Explain.
11. Use the evidence from the simulation to answer the research question.

# Racial Disparities in Police Stops?

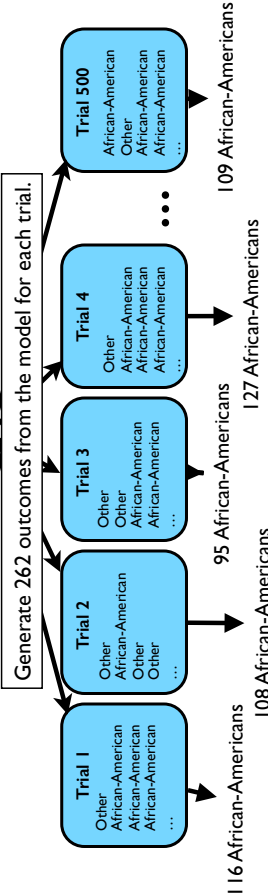
## MODEL

The model has two outcomes—one for African-Americans and one for other races. Each is **not** equally likely.



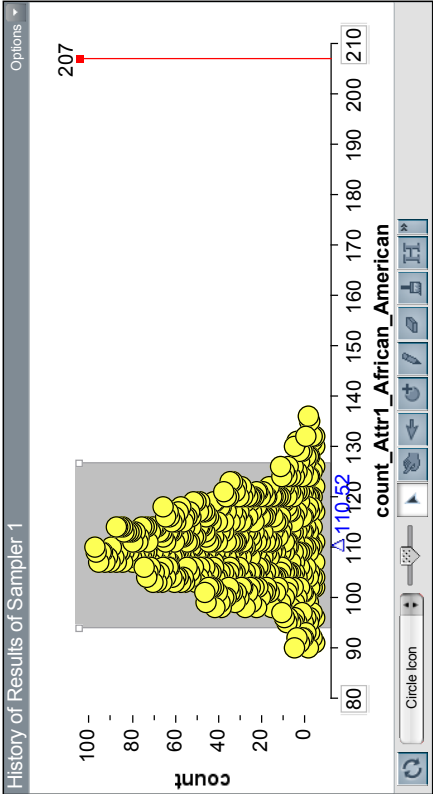
## SIMULATE

Randomly generate 262 outcomes from the model. For each trial, compute the number of African-Americans. Generate many trials.



## EVALUATE

Compile all of the numerical summary measures into a single distribution. Evaluate the initial questions by computing the range of likely values under the hypothesized model—two standard deviations from the mean. Determine whether or not the result from the observed data is within the range of likely results.



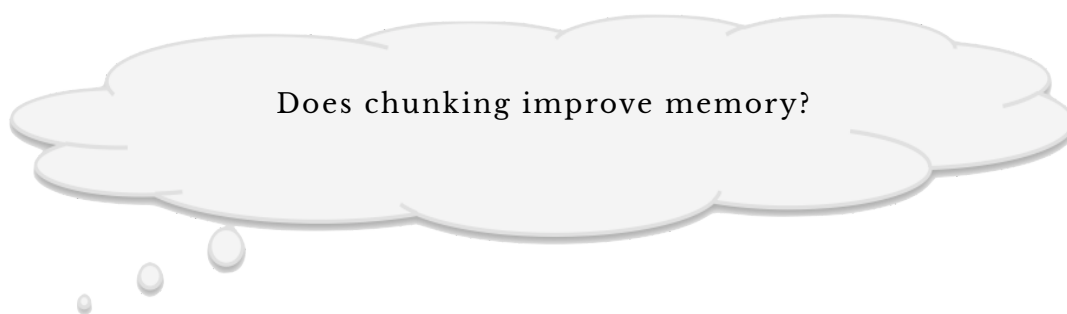
## MEMORIZATION



Many times during the semester, you may feel like your brain just cannot hold all of the information you are learning in classes. Are there ways to improve our memories so that we can comprehend even more information? Research in cognitive psychology has suggested that the answer to that question is a resounding “yes”. This literature has suggested several strategies to improve memory, enhance recall and increase retention of information.

One of the strategies identified by cognitive psychologists is that of chunking. Chunking refers to the process of taking individual units of information and grouping them into larger units (chunks). One common example of chunking occurs when we write and recall phone numbers. For example, a sequence of digits in a phone number, say 8-6-7-5-3-0-9, would be chunked into 867-5309.

In this activity, you will be exploring the following research question:



To examine this research question, you will use the data collected from the memory experiment your class just partook in.

## EXAMINING THE OBSERVED DATA

The first part of any analysis is to examine the observed data. These are the *data that are actually observed* in the research study. In this study we have data on two attributes for each participant in the study.

- The first attribute we have information about is the participant's score (i.e., the number of letters recalled) from the memory experiment. This is called the **response variable** since it contains data on the subjects' responses to the experiment.
  - The second attribute we have information about indicate the treatment condition that the subject was assigned to. This is called a **treatment variable**. In this research study the two *levels of the treatment variable* (the two conditions) are the **experimental condition** (chunking) and the **control condition** (no chunking).
- 
1. Based on the scores, does it seem like there is an effect of chunking? In other words, does it seem like the scores are higher for the chunking group than for the non-chunking group? Explain.

## SUMMARIZING THE DIFFERENCE BETWEEN THE TWO CONDITIONS

In order to answer the research question, you need to summarize the difference between the treatment and control conditions into a single number.

When the response variable is quantitative, it is conventional to do this by finding the mean value of the response variable for each condition, and then *compute the difference between the two means*. The difference in means satisfies the need for a single number summary. It also has another very nice quality, and that is the difference in means is interpretable. The difference in means indicates *how much better* the typical subject in the experimental condition does than a typical subject in the control condition.

2. Compute and record the mean score for each of the two conditions.
  
  
  
  
  
  
  
  
  
  
3. Compute the difference in means by subtracting the mean score for the non-chunking condition from the mean score for the chunking condition.

*Note that this difference is the difference in means for the observed data because we used the observed data (the data from our study) to compute it.*



4. Interpret this difference using the context of the memory study.
5. Does the difference you found in the observed data *suggest there is an effect* of chunking on memory or not? Explain.

### CONSIDERING EXPERIMENTAL VARIATION AS AN EXPLANATION FOR THE DIFFERENCE IN MEANS

Before you conclude that chunking has an effect on memory, consider another alternative: *the difference in means you saw in the observed data is solely attributable to experimental (chance) variation*. Under this model, the difference in means is not because chunking works, but rather because **the random assignment to conditions/groups introduces variation into the results**.

6. If there is *not* an effect of chunking on memory, what would you expect the difference in means to be? Explain.

### THE NO-EFFECT MODEL

To examine whether a result obtained in the observed data is solely due to chance (i.e., all the variation is due to the random assignment), one approach is to imagine the *scenario under which the chunking had no effect*, whatsoever. Under this assumption or scenario, evidence would be collected to determine if the difference in means that was observed in the data is too large to probabilistically believe that there is no effect of chunking. This statement or assumption of no effect of chunking is called the **null hypothesis** and is written as,

$H_0$ : There is no difference in the mean number of letters recalled between the control and experimental conditions.

If chunking is truly ineffective, then each subject's score on the memory test is only a function of that person and not a function of anything systematic, such as the chunking. The implication of this is that, had a subject been assigned to the other condition (through a different random assignment), her score on the memory test would have been identical since, in a sense, both conditions are doing nothing in terms of affecting the memory test scores.

### RE-RANDOMIZATION: INSPECTING OTHER POSSIBLE RANDOM ASSIGNMENTS OF THE SUBJECTS

A researcher can take advantage of the idea that each subject's score on the memory test would be identical whether she was assigned to treatment or control and examine other possible random assignments of the subjects to conditions that could have occurred. To do this, you will carry out a physical simulation (not using TinkerPlots<sup>TM</sup>).

### PHYSICAL SIMULATION OF THE RE-RANDOMIZATION

To aid you in creating these “new” random assignments of conditions, fill in the following:

In the original experiment, \_\_\_\_\_ subjects were randomly assigned to the experimental (chunking) condition and \_\_\_\_\_ subjects were assigned to the control (no chunking) condition.

- You will be given several index cards. Each index card represents a single subject. On each card you will write an *E* (for experimental) or a *C* (for control). When you are done, you should have the same number of *E* cards as subjects originally assigned to the experimental condition and the same number of *C* cards as subjects originally assigned to the control condition. Set the *E* and *C* cards to the side.
- Now, record the first subject’s name and score (number of letters correct) on new card. Continue with the other subjects’ names and scores, recording each subject on a different card. At this point you should have  $n$  subject cards (with names and scores), and  $n$  condition cards (with an *E* or a *C*), where  $n$  is the total number of subjects in the combined control and experimental groups.
- Shuffle the *E* and *C* index cards together several times.
- Shuffle the index cards with the scores several times.
- Deal the shuffled *E* and *C* index cards out one at a time. Now deal the score cards out one at a time, placing each score card you deal on one of the *E* or *C* index cards.

This represents one possible randomization of subjects to either the experimental or control conditions. It is another possible way the subjects could have been assigned to conditions. This random assignment likely has different subjects in the control and experimental conditions than the observed data. Because of this, the mean memory score for the two conditions will also likely differ from the observed data. This, in turn, implies that the difference in means will also be different.

7. Record the subjects' scores based on this possible randomization below. Record subjects assigned to the *E* condition under *Experimental* heading, and those assigned to the *C* condition under the *Control* heading.

Experimental

Control

8. Compute the means for the data from this random assignment for each condition and record them below.
  
  
  
  
  
  
  
  
  
  
9. Compute and record the **difference in means** for this random assignment of the data. Be sure that the order you use when subtracting is consistent with the order you subtracted to obtain the original observed result. (Note: You may obtain a negative number here.)

- Repeat the random assignment process four more times (five total). Each time, record the data, compute and record the mean score for each condition, and compute and record the difference between the means of the two groups. (Remember to subtract in the same order each time.)
- Record each of the five differences you obtained on the board.

### EXAMINING THE DISTRIBUTION OF THE DIFFERENCE IN MEANS

10. Enter all the groups' mean differences into TinkerPlots™. Create a plot of the difference in means. Sketch the plot of the difference in means below.

11. Does it look like it centers around zero? Explain why the distribution should be centered at zero. (Hint: Think back to what the null hypothesis was.)



### Overview of the Inferential Process for Comparing the Two Conditions in the Memorization Experiment

If there really were no effect of the grouping of letters, is it possible that random chance alone could have resulted in such an extreme observed difference between the two conditions? Once again, the answer is yes, this is indeed possible. Also once again, the key question is *how likely would it be for random chance alone to produce experimental data that favor the chunking condition by at least as much as the observational data do*. You will aim to answer that question using the following simulation analysis strategy:

- **Model:** Assume that there is no effect of the grouping of letters on the scores (the “no effect” model).
- **Simulate:** Replicate the random assignment of these subjects and their memory scores between the two conditions. You will repeat this random assignment a large number of times. Each time you will calculate a measure of how different the conditions are, in order to get a sense for what is expected and what is surprising.
- **Evaluate:** Using the observed result, evaluate the level of support the data offer to the hypothesized “no effect” model.

# MEMORIZATION USING TINKERPLOTS™



In this activity, you will learn how to use TinkerPlots™ to carry out the randomization test.

## EXAMINING THE OBSERVED DATA

The first part of any analysis is to examine the observed data. These are the *data that are actually observed* in the research study.

- Enter the data collected in the study into a TinkerPlots™ case table.

### Setting up the Case Table for the Randomization Test

- Drag a **Table** from the object toolbar into your document.
- Create a new attribute called *Score* in the first column of the case table.
- Create another new attribute called *Condition* in the second column of the case table.



Each row in the table will comprise a subject in the research study. Each column will comprise an attribute of the subject. For our purposes, you will need to enter data for two attributes. The first attribute will indicate the subject's score (i.e., the number of letters recalled) from the memory experiment. This is called the **response variable** since it contains data on the subjects' responses to the experiment. The second attribute will indicate the treatment condition that the subject was assigned to. This is called a **treatment variable**. In this research study the two treatment conditions are the **experimental condition** (chunking) and the **control condition** (no chunking).

- Enter the observed data from your class experiment into a TinkerPlots™ case table.
- Plot the observed data (see instructions on next page).

#### Plotting Data to Compare Groups

- Drag a **Plot** from the object toolbar into your document.
- Drag the response variable from the case table to the *x*-axis of the plot.
- Drag a case icon to the right until the cases in the plot are fully separated (e.g., no vertical bin lines). You can also double-click on one of the endpoints and change **Bin width = 0**.
- Drag the treatment variable from the case table to the *y*-axis of the plot.
- Click the **Vertical Stack** button in the upper plot toolbar to organize the icons.

1. Sketch the plot below that you just created in TinkerPlots™.

## MODEL THE EXPERIMENTAL VARIATION DUE TO RANDOM ASSIGNMENT

In order to carry out a randomization test using TinkerPlots™, you need to include multiple sampling devices in the sampler. The first sampling device will include the observed response data for all of the subjects. The second device will contain the experimental conditions.

### MODELING A SET OF FIXED RESPONSES UNDER THE “NO EFFECT” MODEL

Under the null hypothesis of no difference between the two experimental conditions, the response values for the subjects are **fixed**—they will always be the same for the subjects, regardless of which experimental condition the subject is assigned. To produce simulated data that are fixed, you can use a **Mixer**, but the values need to be *selected without replacement*.

- Open the *sleep-deprivation.tp* data set.
- Set up a **Mixer** that will produce the fixed responses for the subjects under the “no effect” model (see instructions below).
- Run the model a couple times.

#### Setting Up the Model: Fixed Responses

- Drag a new **Sampler** from the object toolbar into your blank document.
- The default device in the sampler is a **Mixer** with three elements. Add elements to the mixer until you have the same number of elements as there are responses. (Each element represents a participant in the experiment.)
- Change the values of the elements so that they represent the response values (i.e., the number of correctly memorized letters).
- Change the mixer to sample values without replacement. Do this by clicking on the **Device Options** button for the stacks device (upside-down triangle) and selecting **Replacement**.
- Change the name of the device from *Attr1* to *Responses*.
- Change the **Draw** value to *1*. Change the **Repeat** value to reflect the total number of participants in the experiment.

2. Do you get different response values in the outcomes when you run the model? Use your answer to explain why we refer to these as **fixed** outcomes.

### MODELING THE RANDOM ASSIGNMENT OF THE TREATMENT CONDITION LABELS BY LINKING MULTIPLE DEVICES

3. Write a detailed explanation describing the process you used to physically re-randomized the notecards in the previous activity. Be specific enough in this description that another student could replicate what you did.

To model the random assignment of the treatment condition labels that might have occurred, you need to produce simulated data from another model that generates labels of *JFK* and *JFKC*. To do this you will use the **Stacks** sampling device. We also need to include this sampling device in the same Sampler as the outcomes. To do this, you *link* multiple sampling devices in the same sampler the same way you did in the *Cat Factory* course activity.

You need to replicate the original experiment and have the same number of *JFK* labels and *JFKC* labels as were in the original experiment.

- Link another sampling device that includes the **fixed group/condition labels** to the sampling device containing the outcomes. (See instructions on next page.)
- Run the model.

### Linking a Sampling Device to Model the Random Assignment of Conditions

- Drag a **Stacks** sampling device from the device menu to the right-hand side of the existing *Responses* device. The sampler should now contain two devices linked by a grey line.
- Change the device name from *Attr2* to *Conditions*.
- Click the **Add Element** button (+) twice to add two elements to the stacks. These elements will indicate the condition/group labels.
- Change the label of the first bar from *a* to *JFK*. Change the label of the second bar from *b* to *JFKC*.
- Click on the **Device Options** button for the stacks device (upside-down triangle) and select **Show Count**.
- Change the count value for the *JFK* label to reflect the number of participants originally assigned to the *JFK* condition. Change the count value for the *JFKC* label to reflect the number of participants originally assigned to the *JFKC* condition.
- Change the device to sample values without replacement.

When you add linked devices, remember that the value for **Draw** changes automatically to the number of devices included in the sampler. A TinkerPlots™ sampler showing two linked devices modeling the random assignment of responses to conditions is shown below.

The outcomes from both linked devices are recorded in the case table, each in their own attribute. In addition, an attribute called **Join** is also created that includes the outcomes of both linked devices separated by a comma.

Each trial represents *what might have occurred* under another random assignment of subjects to conditions *if there was no difference between the conditions*.

- Plot the trial data to obtain the difference in means. (Remember the response attribute from the trial's case table is dragged to the  $x$ -axis of the plot and the condition is dragged to the  $y$ -axis of the plot.)

4. Sketch the plot below.

### **SIMULATE: COLLECT RESULTS FROM MANY TRIALS**

As you have done in previous simulations, you will numerically summarize the trial results. We summarized the observed data by computing,

$$\bar{X}_{JFK} - \bar{X}_{JFKC}$$

5. Compute the value (by hand) for this difference. Be sure you are subtracting the *JFKC* mean from the *JFK* mean.

To use TinkerPlots™ to collect the difference in means, we need to collect both the *JFK* and the *JFKC* means. Fortunately, we can collect multiple summaries in TinkerPlots™.

- Use TinkerPlots™ to collect the mean score for the *JFK* condition.
- Similarly, collect the mean score for the *JFKC* condition.

Now you should have a case table of results that includes the *JFK* and the *JFKC* mean in the same row (but in different columns).

### COMPUTING THE DIFFERENCE IN MEANS

To compute the difference in means we will use the **Formula Editor**. The **Formula Editor** allows us to compute new measures from existing information in a case table.

- Use the **Formula Editor** to compute the difference in the trial's means (see instructions below).
- Check that the difference in means is the same as the difference you computed in the previous question. (If the difference calculated by TinkerPlots™ is correct, but has a reversed sign, you need to re-open the **Formula Editor** and re-compute the mean difference.)

#### Computing the Difference in Means

- Create a third attribute (column) in the case table by clicking the column name, **<new>**. Rename this attribute *Difference*
- Select the *Difference* attribute to highlight it and then right-click the attribute and select **Edit Formula**.
- Select the *Attribute* triangle to display the names of the case table's attributes in the **Formula Editor**.
- Double-click the attribute for the *JFK* mean value. Then click the subtraction key (**-**) in the **Formula Editor** calculator. Finally, double-click the attribute for the *JFKC* mean value.
- Click the **Apply** button and then click **OK**.

### COLLECT MANY RESULTS

- In the *History of Results* table, collect an additional 499 measures. (See instructions below for speeding up the simulation.)
- Plot the differences in means from the 500 simulated trials.

#### Speeding Up the Simulation

- Minimize all of the objects (sampler, results table, plot of the results) except for the collection window.
- Select the collection window and from the **Objects** menu select **Inspect Collection**.
- Uncheck the **Animation On** option.
- Close the inspector window.
- In the **History of Results** table, change the number of samples to collect to **499**.
- Click the **Collect** button.

6. Sketch the plot of the results (i.e., mean differences) from the 500 simulated trials below.
7. What are the cases in the plot? (Hint: Ask yourself what each individual dot represents.)

8. Where is the plot of the results centered (at which value)? Explain why this makes sense. (Hint: Think about what the hypothesis for the “no effect” model is.)
9. Use TinkerPlots™ to compute the standard deviation of the differences in means. Record that value below.
10. Using the mean and standard deviation, provide a range of likely values under the model that assumes the difference in means is due completely to random chance.
11. Now include a vertical line at the difference in means for the original (observed) data. Is the observed difference in means consistent with results produced by the model specified in the null hypothesis?



12. Based on your response to the previous question, is the “no effect” model supported by the observed data or not? What does this suggest about the answer the research question? Explain.

## SLEEP DEPRIVATION



Sleep deprivation has been shown to have harmful effects such as fatigue, daytime sleepiness, clumsiness and weight loss or weight gain. Researchers have also established that sleep deprivation has a harmful effect on learning. But do these effects linger for several days, or can a person “make up” for sleep deprivation by getting a full night’s sleep in subsequent nights?

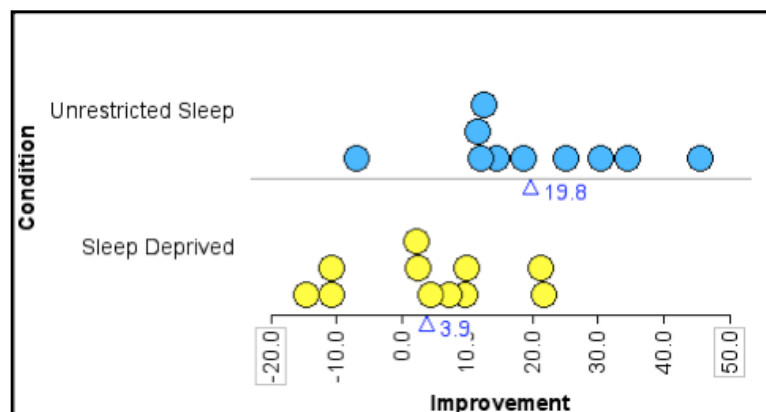
Stickgold, James, and Hobson (2000), in a recent study, investigated this question by randomly assigning 21 subjects (volunteers between the ages of 18 and 25) to one of two groups: One group was deprived of sleep on the night following training and pre-testing with a visual discrimination task, and the other group was permitted unrestricted sleep on that first night. Both groups were then allowed as much sleep as they wanted on the following two nights. All subjects were then re-tested on the third day.

In this activity, you will be exploring the following research question:

Does the effect of sleep deprivation last, or can a person “make up” for sleep deprivation by getting a full night’s sleep in subsequent nights?

Subjects' performance on the test was recorded as the minimum time (in milliseconds) between stimuli appearing on a computer screen for which they could accurately report what they had seen on the screen. The sorted data and plots presented here are the improvements in those reporting times between the pre-test and post-test (a negative value indicates a decrease in performance):

Sleep Deprived ( <i>n</i> = 11)	Unrestricted Sleep ( <i>n</i> = 10)
-14.7	-7.0
-10.7	11.6
-10.7	12.1
2.2	12.6
2.4	14.5
4.5	18.6
7.2	25.2
9.6	30.5
10.0	34.5
21.3	45.6
21.8	



*Observed data and plot of the observed data for the sleep deprivation study. The triangle under each plot indicates the mean improvement score for the respective group.*

**Discuss the following questions.**

1. Does it appear that subjects who got unrestricted sleep on the first night tended to have higher improvement scores than subjects who were sleep deprived on the first night? Explain briefly.
  
  
  
  
  
  
  
  
  
  
2. Is the mean improvement higher for those who got unrestricted sleep? Calculate the difference in the means of the improvement scores.
  
  
  
  
  
  
  
  
  
  
3. Is it possible that there is really no harmful effect of sleep deprivation, and random chance alone produced the observed differences between these two groups?

## MODEL THE EXPERIMENTAL VARIATION DUE TO RANDOM ASSIGNMENT

- Set up a sampling device that will produce the **fixed responses** for the subjects under the “no effect” model.
- Link another sampling device that includes the **fixed group/condition labels**.
- Run the model.

## SIMULATE AND EVALUATE THE RESULTS

- Use TinkerPlots™ to plot the randomized data and collect the mean for each condition. Then use the **Formula Editor** to compute the difference in means. Simulate an additional 499 randomizations (trials) of the data (500 total).
  - Plot the results (difference in means) from the 500 randomizations.
4. Sketch the plot of the results (i.e., mean differences) from the 500 simulated trials below.

5. What are the cases in the plot? (Hint: Ask yourself what each individual dot represents.)
6. Where is the plot of the results centered (at which value)? Explain why this makes sense. (Hint: Think about what the hypothesis for the “no effect” model is.)
7. Use TinkerPlots<sup>TM</sup> to compute the standard deviation of the differences in means. Record that value below.
8. Using the mean and standard deviation, provide a range of likely values under the model that assumes the difference in means is due completely to random chance.



## CONTAGIOUS YAWNS



Conventional wisdom says yawns are contagious; when you see someone else yawn, you are prone to feel sleepy and let out a yawn yourself. How many times have you caught yourself in this situation, or noticed it in someone else? But will this hypothesis withstand a scientific test? Will data support this claim?

The folks at [MythBusters](#), a popular television program on the Discovery Channel, investigated this issue by using a two-way mirror and a hidden camera. Fifty subjects sat in a booth, accompanied only by an experimental attendee. For some of the subjects, the attendee yawned (planting a yawn “seed”), while for other subjects the attendee did not yawn. The researchers decided in advance, with a random mechanism, which subjects would receive the yawn seed and which would not. As time passed, the researchers watched to see which subjects yawned. In this activity, you will answer the following research question:



Are yawns contagious?



**EXPLORE THE OBSERVED DATA**

1. Based on the research question, specify the treatment variable.
2. Based on the research question, specify the response variable. Also, identify whether it is a quantitative or categorical variable.

**Observed Data:** The researchers found that ten of 34 subjects who had been given a yawn seed actually yawned, compared with four of 16 subjects who had not been given a yawn seed.

3. Organize these data/results (i.e., frequencies) into a 2x2 table. This table is sometimes referred to as a **contingency table**.

	Yawn	No Yawn	Total
Yawn Seed			
No Yawn Seed			
Total			

4. Of the 34 subjects assigned to the yawn seed condition, what proportion yawned?
5. Of the 16 subjects assigned to the no yawn seed condition, what proportion yawned?

[illegible]

6. Find the difference between the proportion of subjects assigned to the yawn seed condition that yawned and the proportion of subjects assigned to the no yawn seed condition that yawned.
7. Write a few sentences summarizing the results in the sample. This summary should include a summary of what the data suggest about: (1) the *overall percentage of participants who yawned*; (2) the differences between the two treatment groups; and (3) whether or not the data appear to support the claim that yawns are contagious.

## DUMMY CODING THE OUTCOME

One trick that statisticians use to deal with categorical outcomes, is to *dummy code* the variable. Dummy coding is a way to turn the categories into numbers, so that the outcome becomes “quantitative”. Once the outcome is quantitative, we can compute means, etc.

The idea of dummy coding is that each category of the outcome gets a numerical value of either “1” or “0”. For example, consider the 16 subjects who did not receive the yawn seed. In the data, four of them yawned; give those folks a “1”. Twelve of them did not yawn; assign them a “0”. Their data looks like this:

In this table the column “Yawn?” is the dummy coded outcome. You can think of the the variable name, “Yawn?”, like the question, “Did the participant yawn?” The two responses are “1” (Yes), or “0” (No).

Why choose “0” and “1” as the numbers for our categories? Why not choose “5” and “10”; or “2” and “3”? The reason we choose “0” and “1” is that these values give meaning to interpretations of the mean and other summaries.

8. Compute the mean of the **Yawn?** dummy coded values for the 16 participants in the “No Yawn Seed” condition. Compare that value to your response to Question 5.
  
9. Re-code the outcome for the 16 participants to create a variable called **Didn't yawn?**. Use dummy coding to do this. This time participants who didn't yawn would get a “1” and the participants who yawned would get a “0”. Now compute the mean of the **Didn't yawn?** dummy coded values for the 16 participants in the “No Yawn Seed” condition. What does this value tell you?
  
10. Explain how to interpret the mean of a dummy coded variable generally.

## MODELING THE EXPERIMENTAL VARIATION

You will answer the research question by using TinkerPlots™ to conduct a **randomization test** in order to account for experimental variation (variation in the difference of means just because of random assignment) *under the assumption that there is no effect of the yawn seed*.

- Set up a model that will produce the **fixed dummy coded responses** for all 34 of the subjects (use Yawn = 1 and No yawn = 0). You can use a **Mixer**, but since there are only two values for the response variable, it can be quicker to use a **Stacks** device. (If you have forgotten how to do this, refer back to the instructions in the *Sleep Deprivation* course activity.)
  - Add a linked **Stacks** device that includes the **condition labels**. (If you have forgotten how to do this, refer back to the instructions in the *Sleep Deprivation* course activity.)
  - Run the model.
11. Should the sampling device containing the outcome values be sampled *with* or *without* replacement? What about the sampling device containing the condition labels? Explain why.

## PLOTTING AND COLLECTING THE RESULTS

- Use TinkerPlots™ to plot the results for the trial.
- Collect the results from the trial (see instructions on the next page).

## SIMULATE AND EVALUATE THE RESULTS

- Carry out 500 randomized trials of the simulation in TinkerPlots™.
- Plot the differences in percentages for the 500 simulated trials.

12. Sketch the plot of the distribution of simulated differences.
13. Based on the hypothesized model, what is the *expected* difference in means? Explain.
14. Compute and report the standard deviation of the differences in means.
15. Using the expected difference in means and the standard deviation, provide a range of likely results under the hypothesized model.

16. Is the observed difference within the range of likely values you computed? What does this suggest about the answer the research question? Explain.

*QUANTIFYING THE LIKELIHOOD OF THE OBSERVED RESULT GIVEN THE MODEL: P-VALUE*

17. Compute and report the  $p$ -value for the observed difference in the *Yawn* study. (If you have forgotten how to do this, consult the  $p$ -value reading.)

18. Interpret the  $p$ -value you computed.

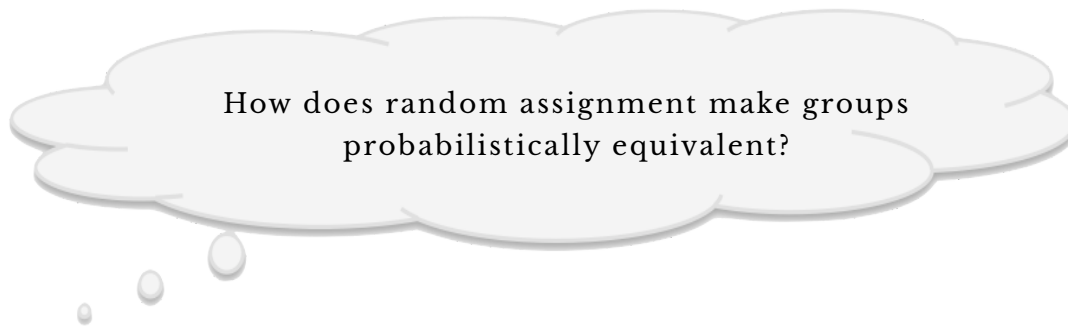
The  $p$ -value of \_\_\_\_ is the probability of ...

19. Use the  $p$ -value you computed to evaluate the tenability of the hypothesized model and help provide an answer to the research question.

## STRENGTH SHOE®



The Strength Shoe® is a modified athletic shoe with a 4-cm platform attached to the front half of the sole. Its manufacturer claims that this shoe can increase a person's jumping ability. A 1993 study published in the *American Journal of Sports Medicine* investigated the Strength Shoe® claim using 12 intercollegiate track and field athletes as study participants<sup>1</sup>. In this activity you will be replicating this investigation using data collected from 12 other subjects. to examine the following question:



Remember, when investigating whether or not one variable **causes an effect** on another, researchers seek to exert control by creating a comparison group and then assigning subjects to either the treatment group or the comparison group. An **experiment** is a study in which the experimenter actively imposes the treatment condition on the subjects. Ideally, the groups of subjects are identical in all respects other than the condition, so the researcher can then see the variable's direct effects on the response variable.

### STUDY DESIGN

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<sup>1</sup> Cook, S. D., Schultz, G., Omev, M. L., Wolf, M. W., & Brunet, M. F. (1993). Development of lower leg strength and flexibility with the strength shoe. *American Journal of Sports Medicine*, 21, 445–448.



To determine the efficacy of training in Strength Shoes®, you are planning a study design in which 6 of your subjects will train in regular shoes, and 6 will train in Strength Shoes®. Then all subjects vertical jump height will be measured.

The following subjects have volunteered to take part in your study.

Andreas	Jasmine	Mary
Antonio	John	Paul
Davieon	Ka Nong	Ringo
George	Keyaina	Tong

1. Assign the 12 subjects to two groups (ordinary shoe group and Strength Shoe® group) by assigning every other subject to the Strength Shoe® group, alphabetically. (Andreas = Ordinary Shoe group; Antonio = Strength Shoe® group, etc.) Write the names of the participants for the two groups below.

Strength Shoe® Group		Ordinary Shoe Group
Name		Name
Andreas		Antonio

Now imagine that you carried out the study and found that the participants you assigned to the Strength Shoe® group jumped 5” higher on average than the participants in the Ordinary Shoe group. Furthermore, a simulation showed that this difference was more than would be expected because of chance.

2. Are you willing to make a **cause-and-effect inference** about the efficacy of the Strength Shoe®? In other words, are you willing to attribute the mean difference in jumping height to training in Strength Shoes®? Explain.

### **OBSERVED CONFOUNDING VARIABLES**

In every study, there are potentially many factors (aside from the treatment) that may be related to the response variable and, in turn, affect the results of the study. Statisticians refer to these variables as **confounding (lurking) variables**.

3. Are there confounding factors that might affect the results? Identify a few.



6. Describe how you might re-assign the participants to group so that sex and height do not bias the results?
7. Try to carry out a new assignment of subjects to make the two groups “equivalent” on sex and height.

[illegible]

## UNOBSERVED CONFOUNDING VARIABLES

While some confounding variables may be identified and controlled in a study, others may not be identified initially by the researcher. These unidentified confounding variables may also mislead researchers into thinking that a treatment is effective (or not effective), when in reality, all of the difference in the response variable is a function of differences in the confounding variable, not differences in the treatment. In practice, erroneous results because of unobserved confounding variables are prevalent in every field. Even the smartest and most experienced researchers will probably not identify all of the confounding factors related to differences in the response variable need to be controlled prior to the study.

One unobserved confounding factor that might affect jumping distance are the participant's genetics. It turns out that there is a jumping gene, called Gene X.

- Open the *strength-shoe.tp* TinkerPlots™ file.
  - Click **Run** in the sampler.
  - To see whether the participant has Gene X, right click anywhere in the case table and select **Show Hidden Attribute**.
8. Based on your re-assignment of subjects from Question #7, compute the proportion of participants with Gene X in both groups. Based on these proportions, does it seem that Gene X will bias the results? Or does it seem like the two groups are “equivalent” when it comes to Gene X?
  9. Without having identified Gene X as a confounding variable prior to the study, how would we know if our results were biased because of Gene X?

## RANDOM ASSIGNMENT

Bias could result because the groups are not “equivalent” on any confounding variable that is related to the outcome, in this case, jumping height. It turns out that the key to making groups “equivalent” on *all* confounding variables (both observed and unobserved) is to use random assignment in forming experimental groups.

For the remainder of this course activity, you will examine how random assignment “equalizes” not only the observed confounding variables (e.g., sex), but also unobserved confounding variables (factors we haven’t yet thought of).

Remember that statisticians consider groups to be “equivalent” if the difference in means between the experimental groups have an expected value of zero (i.e., the average difference in means across all possible randomizations of the participants is zero). We will use TinkerPlots™ to simulate the randomization of subjects to experimental conditions many times to explore the expected difference in means for many confounding variables.

## CONFOUNDING VARIABLE: SEX

To explore whether random assignment makes the groups “equivalent” on sex, we are going to simulate the randomization of subjects to experimental conditions many times, and examine the expected difference in means for the dummy coded sex variable. (Note: We are thinking about the confounding variable of sex as the “outcome” in this simulation.)

Set up a sampler to randomly assign participants to the experimental conditions. To do this:

- Set up a sampling device that includes the 12 fixed values for sex. Dummy code these values so that 0 = male and 1 = female. Name this device **Female**.
- Set up a sampling device that includes the 12 fixed values for condition; 6 ordinary shoe labels, and 6 Strength Shoe labels. Name this device **Group**.
- Click **Run** to randomly assign the participants to groups. (Note that since we are only interested in the participant’s sex, their name is just superfluous information and is not needed for the simulation.)

- Plot the attributes **Female** ( $y$ -axis) and **Group** ( $x$ -axis) in a single plot;
  - Compute and display the mean for each group.
10. Report the **proportion of females** in each group. Also subtract these two proportions (taking the Strength Shoe® group's proportion minus the ordinary shoe group's proportion).

Proportion of females in Strength Shoe® Group:

Proportion of females in Ordinary Shoe Group:

Difference in proportions (Strength Shoe® – Ordinary Shoe):

This is just a single random assignment (trial). Whether the groups are statistically equivalent asks: Is the expected difference zero across many, many trials?

- Collect the proportion of females in both the Ordinary Shoe group and the Strength Shoe® group.
- Use the **Formula Editor** to compute the difference in the proportion of females between the two groups. (Note: Subtract the Ordinary Shoe group from the Strength Shoe® group.)
- Collect 499 more trials.
- Plot the 500 differences.
- Organize and fully separate the results (no bin lines) for the plot.
- Compute and display the average for the 500 randomized differences.

11. Sketch the plot below.

12. Where is this plot centered? What does this imply about the expected difference in the proportion of females in the two groups? Explain.



## CONFOUNDING VARIABLE: HEIGHT

To explore whether random assignment makes the groups “equivalent” on height, we are going to simulate the randomization of subjects to experimental conditions many times, and examine the expected difference in mean height. (Note: We are thinking about the confounding variable of height as the “outcome” in this simulation.)

Set up a sampler to randomly assign participants to the experimental conditions. To do this:

- Set up a sampling device that includes the 12 fixed values for height. Name this device **Height**.
  - Set up a sampling device that includes the 12 fixed values for condition; 6 ordinary shoe labels, and 6 Strength Shoe labels. Name this device **Group**.
  - Click **Run** to randomly assign the participants to groups.
  - Plot the attributes **Height** (*x*-axis) and **Group** (*y*-axis) in a single plot;
  - Organize and separate the cases based on both attributes.
  - Compute and display the average height for both groups.
13. Report the average height for each group. Also find the difference in these two averages (taking the Strength Shoe® group's average minus the ordinary shoe group's average).

Average height in Strength Shoe® Group:

Average height in Ordinary Shoe Group:

Difference in average height (Strength Shoe® – Ordinary Shoe):

Again, this is just a single random assignment and we want to get a sense of the difference in the average height across many random assignments.

- Collect the average height in both the Ordinary Shoe group and the Strength Shoe® group.
  - Use the **Formula Editor** to compute the difference in the means between the two groups. (Note: Subtract the Ordinary Shoe group from the Strength Shoe® group.)
  - Collect 499 more trials.
- 
- Plot the 500 differences.
  - Organize and fully separate the results (no bin lines) for the plot.
  - Compute and display the average for the 500 randomized differences.

14. Sketch the plot below.

15. Where is this plot centered? What does this imply about the expected difference in average heights between the two groups? Explain.

## UNOBSERVED CONFOUNDING VARIABLES

Although it is great that random assignment will tend to make the groups “equivalent” on the observed variables (e.g., proportion of females; average height), what about confounding variables that we did not consider or even think about.

To explore whether random assignment makes the groups “equivalent” on Gene X (an unobserved confounding variable), we are going to simulate the randomization of subjects to experimental conditions many times, and examine the expected difference in the proportion of participants with Gene X. (Note: We are thinking about the confounding variable of Gene X as the “outcome” in this simulation.)

Set up a sampler to randomly assign participants to the experimental conditions. To do this:

- Set up a sampling device that includes the 12 fixed values for Gene X. Dummy code these values so that 0 = Does not have Gene X and 1 = Has Gene X. Name this device **GeneX**.
- Set up a sampling device that includes the 12 fixed values for condition; 6 ordinary shoe labels, and 6 Strength Shoe labels. Name this device **Group**.
- Click **Run** to randomly assign the participants to groups.
- Plot the attributes **GeneX** (x-axis) and **Group** (y-axis) in a single plot;
- Organize and separate the cases based on both attributes.
- Compute and display the average for both groups.

Again, this is just a single random assignment and we want to get a sense of the difference in the average height across many random assignments.

- Collect the proportion of participants having Gene X in both the Ordinary Shoe group and the Strength Shoe® group.
- Use the **Formula Editor** to compute the difference in the proportion of participants with Gene X between the two groups. (Note: Subtract the Ordinary Shoe group from the Strength Shoe® group.)
- Collect 499 more trials.

- Plot the 500 differences.
- Organize and fully separate the results (no bin lines) for the plot.
- Compute and display the average for the 500 randomized differences.

16. Sketch the plot below.

17. Where is this plot centered? What does this imply about the expected value of the difference in proportion of participants with the Gene X in the two groups? Explain.

## LATIN AMERICAN IMMIGRANT ACHIEVEMENT



The *Center for Immigration Studies* at the *United States Census Bureau* has reported that despite shifts in the ethnic makeup of the immigrant population, Latin America—and Mexico specifically—remains this country's greatest source of immigrants. Although the average immigrant is approximately 40-years-old, large numbers are children who enroll in U.S. schools upon arrival. Their subsequent educational achievement affects not only their own economic prospects but also those of their families, communities and the nation as a whole.

Katherine Stamps and Stephanie Bohon studied the educational achievement of Latin American immigrants by examining a random sample of the 2000 decennial Census data; a subset of which will be used in this activity<sup>1</sup>. One interesting research question that has emerged from their research is whether there is a link between where the immigrants originated and their subsequent educational achievement.

Do immigrants from Mexico have lower average educational achievement scores than immigrants from other Latin American countries?

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<sup>1</sup> Stamps, K., & Bohon, S. A. (2005). Educational attainment in new and established Latino metropolitan destinations. *Social Science Quarterly*, 87(5), 1225–1240.

**DISCUSS THE FOLLOWING QUESTIONS.**

1. Based on the research question, identify each of the groups/conditions for the treatment variable.
2. Based on the research question, identify the response variable.

**Observed Data:** The data in *latin-american-achievement.tp* contains a random sample of 150 Latin American immigrants. One-hundred sixteen of these immigrants are from Mexico and 34 are from other Latin American countries. The response variable is an educational achievement score, which ranges from 1 to 100. Higher values indicate higher levels of educational achievement.

3. Identify whether the response variable is categorical or quantitative in nature.

**EXAMINE THE OBSERVED DATA**

- Plot the observed achievement scores for both groups of immigrants in the same plot. Also compute and display the average achievement score for both groups.

4. Sketch the plot of the observed data.
5. Calculate the difference in the means of the educational achievement levels for the observed data (i.e., report the observed result). What does the sample result suggest about the answer to the research question.

## MODELING THE SAMPLING VARIATION

You will now use TinkerPlots™ to conduct a **bootstrap test** in order to account for sampling variation (variation in the difference of means just because of random sampling) *under the assumption that there is no difference in means between of Mexican immigrants and other Latin American immigrants.*

- Set up a sampling device that includes all 150 **observed responses**. You can copy-and-paste the responses from the observed data into a **Mixer**, but don't forget to first remove the default elements so that you paste the results into an empty **Mixer**. Set the **Mixer** to sample *with replacement*.
- Add a linked **Stacks** device that includes the **group/condition labels**.
- Run the model.

### PLOTTING AND COLLECTING THE RESULTS

- Use TinkerPlots™ to plot the results for the trial.
- Collect the results (difference in means) from the trial.

### SIMULATE AND EVALUATE THE RESULTS

- Carry out 500 bootstrap trials of the simulation in TinkerPlots™.
  - Plot the differences in means for the 500 bootstrap trials.
6. Sketch the plot of the distribution of simulated differences.

### QUANTIFYING THE LIKELIHOOD OF THE OBSERVED RESULT GIVEN THE MODEL: $p$ -VALUE

7. Compute and report the  $p$ -value for the observed difference in the *Yawn* study. (If you have forgotten how to do this, consult the reading.)

8. Interpret the  $p$ -value you computed.

The  $p$ -value of \_\_\_\_ is the probability of ...



9. Use the  $p$ -value you computed to evaluate the tenability of the hypothesized model and help provide an answer to the research question.

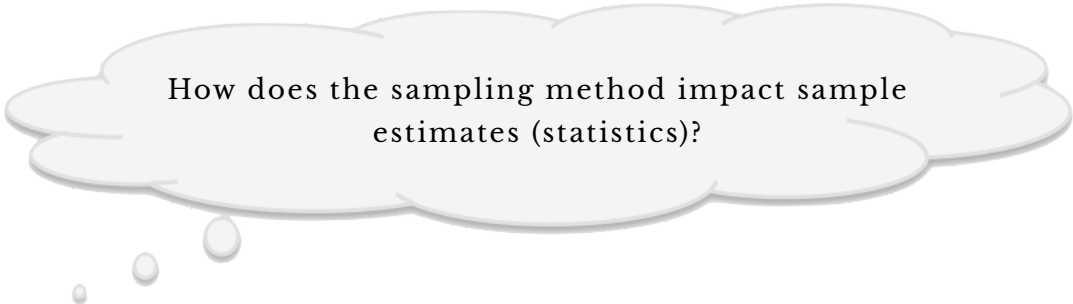
## DESIGN AND INFERENCE

10. How would you rate the level of internal validity evidence based on the study design? Explain.
11. Based on your response to the previous question, are you willing to draw a causal association between where a person immigrated from (Mexico or another Latin American country) and educational achievement? If not, offer at least two other possible explanations for the difference in mean achievement.

## GETTYSBURG ADDRESS



In statistical inference, generalization refers to the process of using sample data to draw conclusions about the larger population from which the sample was drawn. Statisticians are typically concerned with making inferences about some population parameter using a sample statistic (*Remember:* Population summary measures are called **parameters**. Sample estimates of parameters are referred to as **statistics**.) Whether that sample statistic is a statistically good estimate of the population parameter depends on whether the sampling method used is biased. In this activity you will begin by exploring the following question:



How does the sampling method impact sample estimates (statistics)?

To help answer this research question, you are going to compare two different sampling methods using the population of 268 words in the passage on the following page. The passage is, of course, Lincoln's *Gettysburg Address*, given November 19, 1863 on the battlefield near Gettysburg, PA.

Four score and seven years ago, our fathers brought forth upon this continent a new nation: conceived in liberty, and dedicated to the proposition that all men are created equal.

Now we are engaged in a great civil war, testing whether that nation, or any nation so conceived and so dedicated, can long endure. We are met on a great battlefield of that war.

We have come to dedicate a portion of that field as a final resting place for those who here gave their lives that that nation might live. It is altogether fitting and proper that we should do this.

But, in a larger sense, we cannot dedicate, we cannot consecrate, we cannot hallow this ground. The brave men, living and dead, who struggled here have consecrated it, far above our poor power to add or detract. The world will little note, nor long remember, what we say here, but it can never forget what they did here.

It is for us the living, rather, to be dedicated here to the unfinished work which they who fought here have thus far so nobly advanced. It is rather for us to be here dedicated to the great task remaining before us, that from these honored dead we take increased devotion to that cause for which they gave the last full measure of devotion, that we here highly resolve that these dead shall not have died in vain, that this nation, under God, shall have a new birth of freedom, and that government of the people, by the people, for the people, shall not perish from the earth.

The goal in many studies is to provide information about some characteristic of a population. For example, you may want to say something about the percentage of Americans who would support a particular piece of legislation. Or, you may want to provide information about the average amount of time University of Minnesota students take to graduate. One potential solution to obtain such information would be to collect the necessary data from every member of the target population.

In many studies, however, it may not be feasible given time and money constraints to collect data from each member of the population. In these cases it is only possible to consider data collected for a smaller subset, or **sample** from that population. In these cases, the characteristic of the population would be estimated from the sample data and inferences would be drawn about the population. The key is then to carefully select the sample so that the results estimated from the sample are representative of the characteristic in the larger population.

The **population** is the entire collection of who or what (e.g., the observational units) that you would like to draw inferences about. A **sample** is a subset of observational units from the population.

*Circle a sample of ten words in the text of the Gettysburg Address (the population) such that the sample you select is representative (i.e., has the same characteristics) of the population.*

12. Describe how the ten words in your sample are representative of the 268 words in the population.

13. Record the ten sampled words and their lengths:

Word	Length

14. Determine the average (mean) word length for your sample. This sample average (a statistic) is an estimate of the average word length in the population.

*Add your sample estimate to the case table on the instructor's computer.*



When the sampling method produces characteristics of the sample that systematically differ from those characteristics of the population, you say that the **sampling method is biased**. To try to eliminate potential biases, it is better to take a random sample. This should create a representative sample, no matter what variable is focused on. Humans are not very good “random samplers”, so it is important to use other techniques to do the sampling for us.

## SIMPLE RANDOM SAMPLING

A **simple random sample** (SRS) is a specific type of random sample. It gives every observational unit in the population the same chance of being selected. In fact, it gives every sample of size  $n$  the same chance of being selected. In this example you want every possible subset of ten words that could be sampled to have the same probability of being selected.

The first step in drawing a simple random sample is to obtain a **sampling frame** or a list of each member of the population. Then, you can use software to randomly select a sample from the sampling frame. We have already prepared a sampling frame of the words in the Gettysburg Address for you and saved it in a Tinkerplots™ file.

### USE TINKERPLOTS™ TO DRAW A SRS

- Open the file *gettysburg.tp*.
- Draw a simple random sample of ten words from the sampler.

19. Record the ten randomly sampled words and their lengths:

Word	Length

Use TinkerPlots™ to automatically compute the length of each word in your sample. To do this,

- Create a new attribute in the case table called *wordLength*.
- Right-click the attribute name *wordLength* and select the **Formula Editor**.
- Select `stringLength()` from the **Text** functions, and add the sampled words attribute between the parentheses.

20. Use TinkerPlots™ to plot and compute the mean word length for your ten randomly sampled words. Record the mean below.

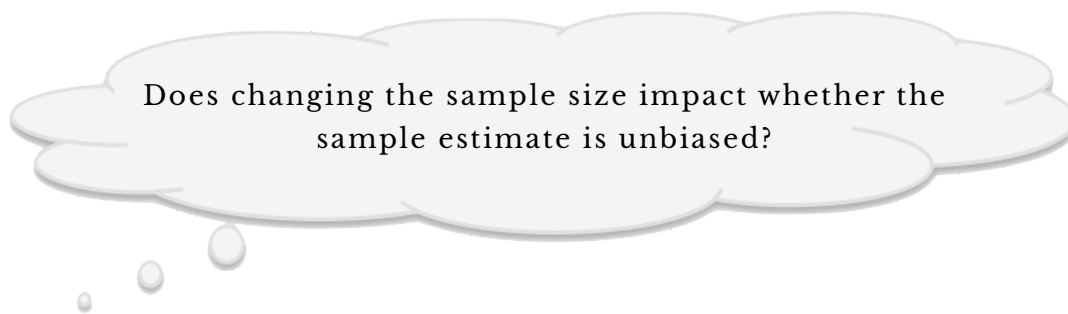


21. Use **Collect** to carry out 200 trials of this simulation in which you randomly sample ten words and compute their mean length. Sketch the plot of these means. Make sure to label the axis appropriately.
22. If the **sampling method is unbiased** the sample statistics should be centered at the population average word length of 4.3. Does simple random sampling produce an unbiased estimate of the population average? Explain.

## SAMPLE SIZE

Even when an unbiased sampling method, such as simple random sampling, is used to select a sample, you do not expect the estimate from each individual sample drawn to match the population average exactly. You should see, however, that the estimates are just as likely to over- or underestimate the population parameter. Because of this predictability to the variation in the possible sample estimates, inferences drawn about the population are said to be valid.

On the other hand, if the sampling method is biased, any inferences made about the population based on a sample estimate may not be valid. In such cases the estimate of the parameter is more likely to be too large or too small compared to the parameter. It is therefore very important to determine how a sample was selected before believing inferences drawn from sample results.



- Change the sample size from 10 to 25.
- Use TinkerPlots™ to draw 200 random samples of 25 words, and collect the average word length for each sample.

23. Sketch the plot of the sample estimates based on the 500 samples drawn. Make sure to label the axis appropriately.

24. Record the average value for the estimate of the average word length.

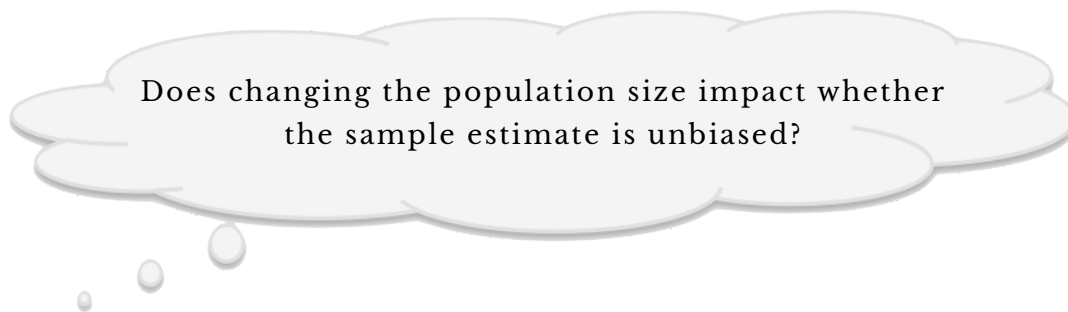
25. Does the sampling method still appear to be unbiased? Explain.

26. Compare and contrast the distribution of sample estimates for  $n = 10$  and the distribution of sample estimates for  $n = 25$ . How are they the same? How are they different?

27. Using the evidence from your simulations, answer the research question:  
Does changing the sample size impact whether the sample estimates are unbiased?

## POPULATION SIZE

It is clear that changing the size of the sample does not affect whether or not an unbiased estimate is produced. Now we examine another question:



To examine this we will now sample from a population that is quadruple the size of the original population (size = 1072) while keeping the population characteristics the same (e.g., mean word length is still 4.3 letters).

- Open the file *gettysburg-larger-population.tp*.
- Draw a simple random sample of ten words from the sampler.
- Compute the word length for each randomly sampled word.
- Plot and compute the mean word length for the ten randomly sampled words.
- Collect the mean word length for 200 random samples.

28. Sketch the plot of the sample estimates based on the 200 samples drawn. Make sure to label the axis appropriately.
29. Record the average value for the estimate of the average word length.
30. Does the sampling method still appear to be unbiased? Explain.
31. Compare and contrast the distribution of sample estimates for  $n = 10$  now that you are sampling from a larger population to the distribution of sample estimates for  $n = 10$  from before. How are they the same? How are they different?

32. Use the evidence collected from the simulation to answer the research question: Does changing the size of the population impact whether the sample estimates are unbiased?

A rather counterintuitive, but very crucial, fact is that when determining whether or not a sample estimate produced is unbiased **the size of the population does not matter!** Even more counterintuitive might be that the precision of the sample estimate is unaffected by the size of the population! (You will learn about the precision of a sample estimate in Unit 3.) This is why organizations like Gallup can state poll results about the entire country based on samples of just 1,000–2,000 respondents as long as those respondents are randomly selected.

In summary, it is important to note some caveats about random sampling:

- One still gets the occasional “unlucky” sample whose results are not close to the population even with large sample sizes.
- Second, the sample size means little if the sampling method is biased. As an example, in 1936 the *Literary Digest* magazine had a huge sample of 2.4 million people, yet their predictions for the Presidential election did not come close to the truth about the population.
- The size of the population does not affect the bias of the estimate, even if a small sample size is used.

## MURDEROUS NURSE



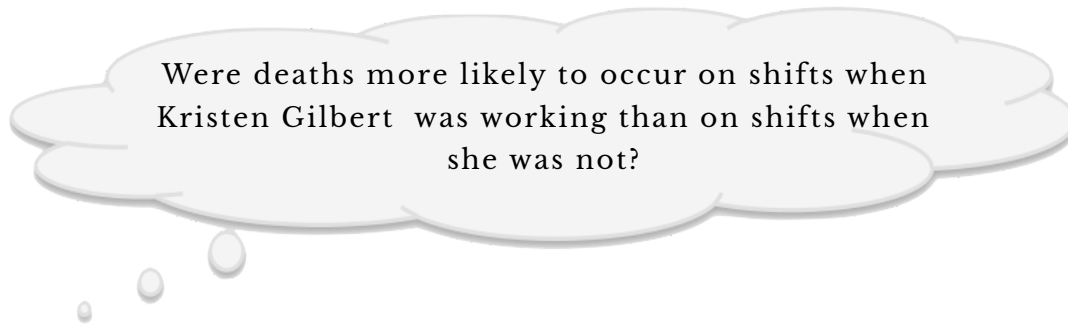
For several years in the 1990s, Kristen Gilbert worked as a nurse in the intensive care unit (ICU) of the Veteran's Administration hospital in Northampton, Massachusetts. Over the course of her time there, other nurses came to suspect that she was killing patients by injecting them with the heart stimulant epinephrine.

Part of the evidence against Gilbert was a statistical analysis of more than one thousand 8-hour shifts during the time Gilbert worked in the ICU<sup>2</sup>. Here are the data presented during her trial:

	Gilbert working on shift	Gilbert not working on Shift	Total
Death occurred on Shift	40	34	74
No death occurred on shift	217	1350	1567
Total	257	1384	1641

<sup>2</sup> Cobb, G. W., & Gehlbach, S. (2006). Statistics in the courtroom: United States vs. Kristen Gilbert. In R. Peck, G. Casella, G. Cobb, R. Hoerl, D. Nolan, R. Starbuck and H. Stern (Eds.), *Statistics: A guide to the unknown* (4th Edition), pp. 3–18. Duxbury: Belmont, CA.

You will use these data to answer the following research question:



### DISCUSS THE FOLLOWING QUESTIONS

1. Among all 1,641 shifts, what percentage of shifts had a death occur?
2. Among the 257 shifts when Gilbert was working, what percentage of shifts had a death occur?
3. Among the 1,384 shifts when Gilbert was not working, what percentage of shifts had a death occur?
4. Compute the difference between the percentage of shifts in which a death occurred when Gilbert was working and the percentage of shifts in which a death occurred when Gilbert was not working.



5. Based on the research question, specify the treatment variable.
  6. Based on the research question, specify the response variable. Also, identify whether it is a quantitative or categorical variable.
  7. The sample data indicates that shifts that Gilbert worked had a higher percentage of deaths occur than shifts when that she didn't work. Does the difference in percentages convince you that Gilbert was giving lethal injections of epinephrine to patients? Why or why not?
- Read the section *Observational Studies and the Bootstrap Test* from the online book.

## MODELING SAMPLING VARIATION

You will conduct a bootstrap test using TinkerPlots™ to find out how likely it would be, assuming there is no difference between the percent of shifts in which a death occurred when Gilbert was working and those in which she was not working.

- Set up a sampling device that includes all 1,640 **observed (dummy) responses**. Code the responses so that 1 = death occurred and 0 = no death occurred.
- Link another sampling device to produce the **fixed group/condition labels**.

8. Should the sampling device containing the outcome values be sampled *with* or *without* replacement? What about the sampling device containing the condition labels? Explain why.

### **SIMULATE AND EVALUATE THE RESULTS**

- Use TinkerPlots™ to bootstrap 500 resamples (trials) of the data.
- Collect and plot the results from these trials.

9. Sketch the plot below.

10. Compute and report the  $p$ -value.
11. Use the  $p$ -value you computed to evaluate the tenability of the hypothesized model and help provide an answer to the research question.

### DESIGN AND INFERENCE

12. How would you rate the level of internal validity evidence based on the study design? Explain.
13. Based on your response to the previous question, are you willing to draw a causal association between shifts Gilbert worked and increased death rates? If not, offer at least two other possible explanations for the difference in percentage in the data.

14. How would you rate the level of external validity evidence based on the study design? Explain.
  
15. Based on your response to the previous question, are you willing to draw a generalization about potential “future shifts”. Explain.

## MOVIE SEQUELS



As of February 1, 2017, there were 28 major motion pictures in history that earned over \$1 billion at the box office, worldwide. Many of these box office winners were sequels to other movies. While these sequels clearly earned a lot of money, many movie sequels are widely panned by critics. In this activity you will examine the following research question:

Are high-earning movies that are sequels rated more harshly by critics than non-sequels?

*Rotten Tomatoes* is a popular service that aggregates critics' reviews of movies into a "Rotten Tomatoes Score". Each of the critics' reviews are rated as positive or negative. The Rotten Tomatoes Score indicates the percentage of positive reviews that the movie received. On the *Rotten Tomatoes* website, movies are given a *Tomatometer* rating based on their score.

Another rating system used by some movie watchers is the *Meg Classification System* (MCS). Any movie having a Rotten Tomatoes Score at or above 80 is classified as "Ripe", while those films having a score below 80 are classified as "Moldy".

**Observed Data:** Of the 28 movies that earned over \$1 billion, 17 were given a MCS rating of "Ripe". Seven of the 11 non-sequels received an MCS rating of "Ripe".

## EXPLORE THE OBSERVED DATA

1. Organize the observed data (i.e., frequencies) into a 2x2 contingency table.
2. Compute and report: (a) the percentage of movie sequels that were rated as “Ripe”; (b) the percentage of non-sequels that were rated as “Ripe”, and (c) the observed difference in percentages.
3. Write a few sentences summarizing the results in the sample. This summary should include a summary of what the data suggest about: (1) the overall percentage of movies rated as “Ripe”; (2) the differences between the two groups of movies; and (3) whether or not the data appear to support the claim that sequels are rated more harshly by critics than non-sequels.

## MODEL THE CHANCE VARIATION

Consider a sampler that you could simulate from in order to explore the chance variation that would be expected in the difference of percentages if there really was no difference in how harshly critics reviewed sequels and non-sequels.

4. Based on the study design, should the sampler model experimental variation or sampling variation? Explain.
  
5. In the space below draw a picture of your sampler that you will use to generate outcomes. Be sure to clearly indicate whether each device in your sampler is sampling with or without replacement.

## SIMULATE AND EVALUATE THE RESULTS

- Use TinkerPlots™ to carry out 500 trials of the simulation.
  - Collect and plot the results from these trials.
6. Sketch the plot of the distribution of simulated differences into your word-processed document. Also give the *expected mean* based on the model and compute and report the standard deviation.
  7. Compute and report the  $p$ -value based on the observed result.
  8. Use the  $p$ -value you computed to evaluate the tenability of the hypothesized model and help provide an answer to the research question.





## KISSING THE 'RIGHT' WAY



A German bio-psychologist, Onur Güntürkün, was curious whether the human tendency for right-sightedness (e.g., right-handed, right-footed, right-eyed), manifested itself in other situations as well. In trying to understand why human brains function asymmetrically, with each side controlling different abilities, he investigated whether kissing couples were more likely to lean their heads to the right than to the left<sup>1</sup>. He and his researchers observed 124 couples (estimated ages 13 to 70 years, not holding any other objects like luggage that might influence their behavior) in public places such as airports, train stations, beaches, and parks in the United States, Germany, and Turkey.

In this activity, you will be exploring the following research question:

What percentage of couples lean their heads to the right when kissing?

**Observed Data:** Of the 124 couples observed, 80 leaned their heads to the right when kissing.

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<sup>1</sup> Güntürkün, O. (2003). Human behaviour: Adult persistence of head-turning asymmetry. *Nature*, 421, 711.

**DISCUSS THE FOLLOWING QUESTIONS.**

1. Based on only the observed data, answer the research question.
2. Consider if Güntürkün had observed a **different sample of 124 couples**. Would the data for these couples provide the same estimate of the percentage of couples who lean their heads to the right when kissing? Explain.

**MODELING SAMPLING VARIATION**

if Güntürkün had observed a different sample of 124 couples the one-number best guess for the percentage of couples lean their heads to the right when kissing would differ (i.e., there is sampling variation). Because of sampling variation, when answering research questions like Güntürkün's that ask for an estimate, it is important that we acknowledge that there is uncertainty in the estimate we provide because we know that sample estimates will vary from sample to sample.

## BOOTSTRAPPING

To model the sampling variation, you will bootstrap from the observed data. Bootstrapping to estimate the sampling variation works whether you have one sample of data or two (as in Unit 4). To carry out a nonparametric bootstrap analysis using TinkerPlots™, you:

- Set up a sampling device to bootstrap the observed data. You can either dummy code these data, or leave them as categorical labels.
  - Be sure that you set the device to sample with replacement. The Repeat value should be set to the same sample size as the observed data. (You want to replicate the sampling of 124 couples.)
3. Carry out 500 bootstrap trials, each time collecting the percentage of people who turn their heads to the right when kissing. Plot the results from the 500 trials and sketch the plot below. Make sure to label the axis. This distribution is referred to as the *bootstrap distribution*.

## THE BOOTSTRAP DISTRIBUTION

4. Find the mean of the bootstrap distribution. Explain why you could expect the bootstrap distribution to be centered at this value by referring to the model from your TinkerPlots™ sampler.

5. Compute the standard deviation of the bootstrapped percentages.

The standard deviation of a plot of results (e.g., means or proportions) is referred to as the **standard error (SE)**. To compute the SE using TinkerPlots™ continue to use the `stdDev()` function that you have been using. When you report that value from now on, you just will refer to it using its technical name, *standard error*

## MARGIN OF ERROR AND INTERVAL ESTIMATE

6. Use the standard error from the bootstrap simulation to compute the margin of error.
7. Compute (by hand) the interval estimate for the true percentage of couples that lean to the right when kissing. Use the interval estimate to provide an answer to the research question.

## CUDDLING PREFERENCES



A recent poll of British pet owners shed light on the extent to which pet owners like to cuddle with their pets. In many cases, pet owners appear to prefer cuddling with their pets more than with their partners. In this activity, you will be exploring the following research question:

What percentage of women prefer cuddling with their dog rather than with their partner?

### EXAMINE THE OBSERVED DATA

1. Use the data in the file *women-cuddle-20.tp* to provide an answer to the research question based on the observed data.

## BOOTSTRAPPING AN INTERVAL ESTIMATE

2. Carry out 500 bootstrap trials. Plot the results from the 500 trials and sketch the plot below. Make sure to label the axis.

## EVALUATING THE BOOTSTRAP DISTRIBUTION

3. Find the mean of the bootstrap distribution. Explain why you could expect the bootstrap distribution to be centered at this value by referring to the model from your TinkerPlots™ sampler.
4. Compute the standard error (use the `stDev()` function) based on this simulation.
5. Using the standard error, compute the margin of error.

6. Compute the interval estimate for the percentage of women who prefer to cuddle with their dog rather than their partner.

### EXPLORING THE EFFECT OF SAMPLE SIZE

The interval estimates you computed were both based on a sample size of 20 women. What happens to the interval if you have a different sample size? You will explore this by computing interval estimates for two other sample sizes.

7. Fill in the first row of the table below with the information from the interval estimate for women based on the sample size of 20.

Sample Size	Observed Percentage	Standard Error	Interval Estimate

8. Open the file *women-cuddle-100.tp*. Use these data to provide an answer to the research question: What percentage of women prefer cuddling with their dog rather than with their partner? To do this, estimate the percentage from the observed data, and then carry out 500 bootstrap trials to estimate the uncertainty in the estimate. Fill in the information from this analysis in the second row of the table.
9. Open the file *women-cuddle-500.tp*. Use these data to provide an answer to the research question: What percentage of women prefer cuddling with their dog rather than with their partner? To do this, estimate the percentage from the observed data, and then carry out 500 bootstrap trials to estimate the uncertainty in the estimate. Fill in the information from this analysis in the second row of the table.





## MINNESOTA COLLEGE DEBT



The Department of Education recently released its College Scorecard data. One of the variables in this data is the typical amount of loan debt accumulated at the institution by student borrowers. In this activity, you will be exploring the following research question:

What is the average amount of loan debt accumulated by student borrowers who attend public colleges/universities in Minnesota?

To answer this question, you will use the data in the file *mn-colleges.tp*. This data set contains a sample of 25 colleges/universities randomly selected from the larger population of all public colleges/universities in Minnesota. The variable *debt* provides the average student loan debt for students who attend that college/university.

### EXAMINE THE OBSERVED DATA

1. Plot the sample debt data using TinkerPlots™. Describe the sample distribution. Be sure to describe the shape of the distribution and give a measure of center and variability.

### BOOTSTRAPPING AN INTERVAL ESTIMATE

You can also carry out a bootstrap simulation to estimate the standard error when you have quantitative data.

2. Carry out 500 bootstrap trials. Plot the results from the 500 trials and sketch the plot below. Make sure to label the axis.

### *EVALUATING THE BOOTSTRAP DISTRIBUTION*

3. Compute the standard error (use the `stDev()` function) based on this simulation.
4. Using the standard error, compute the margin of error.
5. Compute the interval estimate for the average amount of loan debt for ALL students who attend public college/university in Minnesota.

### **DESIGN AND INFERENCE**

6. Based on the validity evidence for this study, is the interval an unbiased estimate for the average amount of loan debt for ALL students who attend college/university in Minnesota? Explain.

**EFFECT OF SAMPLE SIZE**

7. Consider a second sample of Minnesota colleges/universities that had the same sample mean debt as the data in the *mn-colleges.tp* file. However, this second sample is twice as large; it includes 50 observations. How would the uncertainty in the interval estimate from this second sample compare to the uncertainty in the interval estimate you computed in Question #5? Explain.
  
8. The *mn-colleges-02.tp* file contains data from 50 public colleges/universities in Minnesota. Use these data to compute a bootstrap interval estimate for the average amount of loan debt for ALL students who attend public college/university in Minnesota.
  
9. Compare and contrast the interval estimate you computed in Question #5 with the interval estimate you computed in Question #8.

## COMPARING CUDDLING PREFERENCES



In the *Cuddling Preferences* activity, you computed interval estimates to infer the percentage of women who prefer cuddling with their dog rather than with their partner. The same poll of British pet owners also collected data about the extent to which male pet owners preferred cuddling with their pets more than their partners. In this activity, you will be exploring the following research question:

Is the percentage of women who prefer cuddling with their dog higher than the percentage of men who prefer cuddling with their dog?

## BOOTSTRAP TEST

To answer this question, you will carry out a bootstrap test using the data from the files *men-cuddle-20.tp* and *women-cuddle-20.tp*.

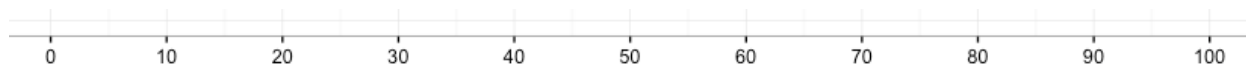
1. Compute the sample estimates for: (1) the percentage of men who prefer to cuddle with their dog rather than their partner, (2) the percentage of women who prefer to cuddle with their dog rather than their partner, and (3) the difference between the two percentages.
2. Carry out 500 trials of the bootstrap test assuming no differences between the percentage of men and women who prefer to cuddle with their dog rather than their partner. Sketch the distribution of bootstrapped differences.
3. Compute and report the  $p$ -value based on the observed result.

4. Use the  $p$ -value you computed to evaluate the tenability of the hypothesized model and help provide an answer to the research question.

### INTERVAL ESTIMATES

Interval estimates can also be used to evaluate whether there are statistical differences between two groups. In the *Cuddling Preferences* activity, you computed the interval estimate for the percentage of women who prefer cuddling with their dog rather than with their partner using the *women-cuddle-20.tp* data.

5. Draw the range of the interval estimate for women using the axis below.





6. Use the data from the file *men-cuddle-20.tp* to provide an answer to the research question: What percentage of men prefer cuddling with their dog rather than with their partner? To do this, estimate the percentage from the observed data, and then carry out 500 bootstrap trials to estimate the uncertainty in the estimate. Fill in the information from this analysis in the table.

Sample Size	Observed Percentage	Standard Error	Interval Estimate

7. Draw the range of the interval estimate for women on the axis in Question #5.

Remember that the interval estimate for women gives likely values for the percentage of women who prefer cuddling with their dogs. Similarly the interval estimate for men gives likely values for the percentage of men who prefer cuddling with their dogs. If both intervals include some of the same likely values (the intervals overlap), it provides evidence that the two groups could have the same level of preference (i.e., evidence of no difference)...or at least the uncertainty is too great for us to differentiate which group has the higher percentage.

8. Do the two intervals you drew in Question #5 overlap each other?
9. Explain using your drawing whether there is evidence that the percentage of women who prefer cuddling with their dog is higher than the percentage of men who prefer cuddling with their dog.

**EFFECT OF SAMPLE SIZE: N = 200 (100 PER GROUP)**

How would our results change if we had used a different sample size?

**BOOTSTRAP TEST**

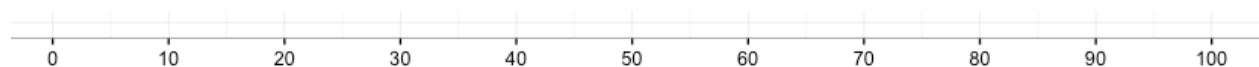
Carry out a bootstrap test using the data from the files *men-cuddle-100.tp* and *women-cuddle-100.tp*.

10. Compute the sample estimates for: (1) the percentage of men who prefer to cuddle with their dog rather than their partner, (2) the percentage of women who prefer to cuddle with their dog rather than their partner, and (3) the difference between the two percentages.
  
  
  
  
  
  
  
  
  
  
11. Carry out 500 trials of the bootstrap test assuming no differences between the percentage of men and women who prefer to cuddle with their dog rather than their partner. Sketch the distribution of bootstrapped differences.
  
  
  
  
  
  
  
  
  
  
12. Compute and report the  $p$ -value based on the observed result.

13. Use the  $p$ -value you computed to evaluate the tenability of the hypothesized model and help provide an answer to the research question.
14. How does the  $p$ -value for the bootstrap test based on a sample size of 40 (20 in each group) compare to the  $p$ -value for the bootstrap test based on a sample size of 200 (100 in each group)?

### BOOTSTRAP INTERVALS

15. In the *Cuddling Preferences* activity, you computed the interval estimate for the percentage of women who prefer cuddling with their dog rather than with their partner using the *women-cuddle-100.tp* data. Draw the range of the interval estimate for women using the axis below.



16. Use the data from the file *men-cuddle-100.tp* to provide an answer to the research question: What percentage of men prefer cuddling with their dog rather than with their partner? To do this, estimate the percentage from the observed data, and then carry out 500 bootstrap trials to estimate the uncertainty in the estimate. Fill in the information from this analysis in the table.

Sample Size	Observed Percentage	Standard Error	Interval Estimate

17. Draw the range of the interval estimate for women on the axis in Question #15.
18. Based on whether or not the two intervals you drew in Question #15 overlap each other, explain whether there is evidence that the percentage of women who prefer cuddling with their dog is higher than the percentage of men who prefer cuddling with their dog.

### EFFECT OF SAMPLE SIZE: N = 1000 (500 PER GROUP)

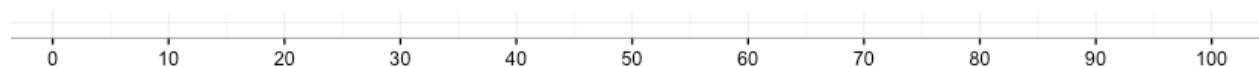
19. How would you expect the  $p$ -value for the bootstrap test to change if we used a larger sample size?
20. How would you expect the interval estimates for men and women to change if we used a larger sample size?



24. Use the  $p$ -value you computed to evaluate the tenability of the hypothesized model and help provide an answer to the research question.
25. How does the  $p$ -value for the bootstrap test based on a sample size of 1000 (500 in each group) compare to the  $p$ -value for the bootstrap test based on a sample size of 200 (100 in each group)?

### BOOTSTRAP INTERVALS

26. In the *Cuddling Preferences* activity, you computed the interval estimate for the percentage of women who prefer cuddling with their dog rather than with their partner using the *women-cuddle-500.tp* data. Draw the range of the interval estimate for women using the axis below.



27. Use the data from the file *men-cuddle-500.tp* to provide an answer to the research question: What percentage of men prefer cuddling with their dog rather than with their partner? To do this, estimate the percentage from the observed data, and then carry out 500 bootstrap trials to estimate the uncertainty in the estimate. Fill in the information from this analysis in the table.

Sample Size	Observed Percentage	Standard Error	Interval Estimate

28. Draw the range of the interval estimate for women on the axis in Question #26.
29. Based on whether or not the two intervals you drew in Question #26 overlap each other, explain whether there is evidence that the percentage of women who prefer cuddling with their dog is higher than the percentage of men who prefer cuddling with their dog.
30. How is the result of the bootstrap test (i.e., the  $p$ -value) effected by sample size? Explain by comparing the  $p$ -values for the three bootstrap tests.

31. You learned how sample size effects statistical uncertainty (i.e., the range of the interval) in the *Cuddling Preferences* activity. Based on your answer to the previous question, how is statistical uncertainty related to  $p$ -value?