More Ordinal Outcomes Models

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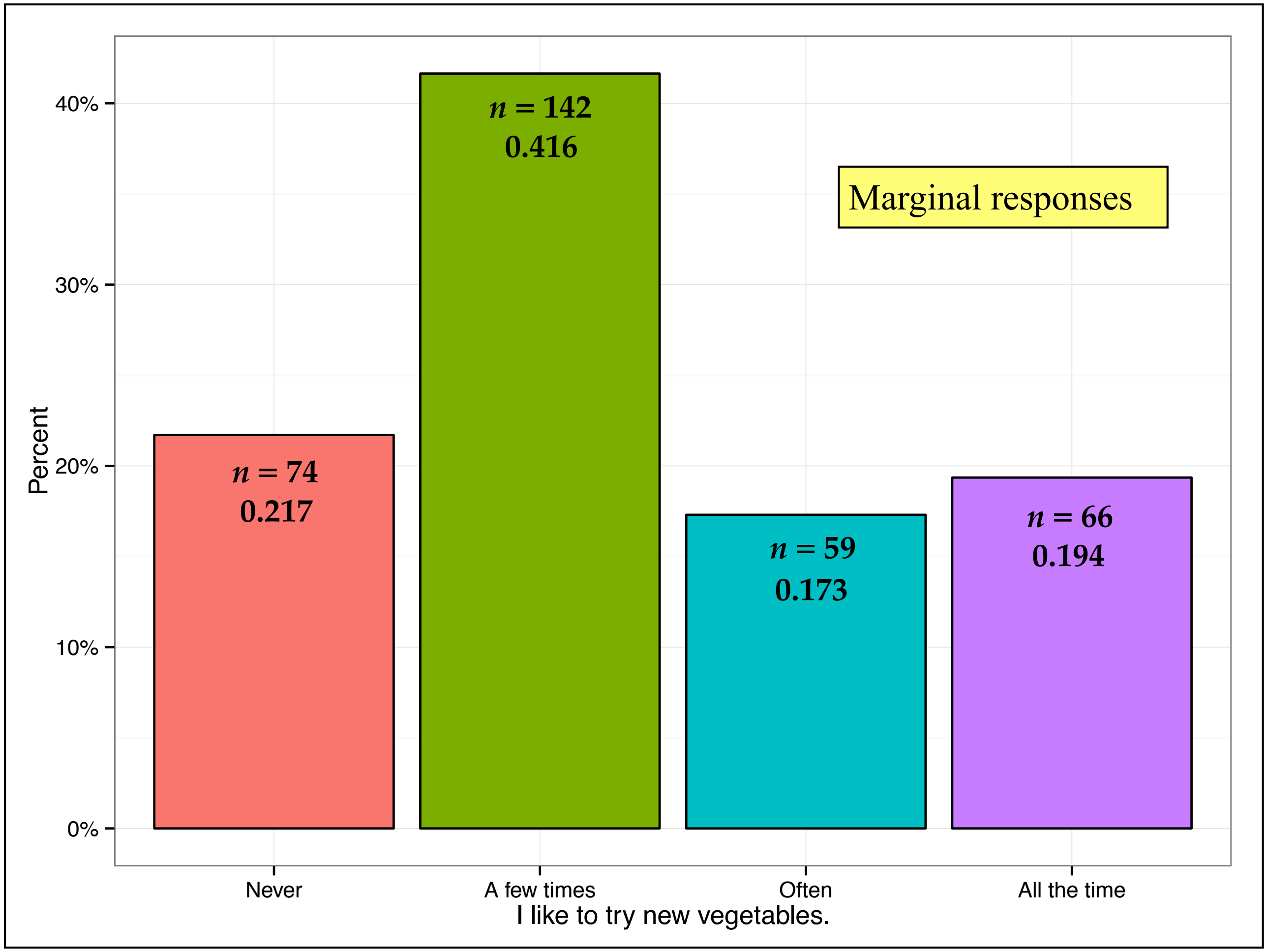
Reading and Examining the Data

```r
# Read in the data
> snap = read.csv(file = "http://www.tc.umn.edu/~zief0002/Data/SNAP.csv")



# Create factors
> snap$talk = ordered(snap$talk, levels = c("Never", "A few times", "Often", "All the time"))

> snap$vegetables = factor(snap$vegetables,
    levels = c(0, 1, 2, 3),
    labels = c("Never", "A few times", "Often", "All the time"),
    ordered = TRUE
    )

> snap$fruits = factor(snap$fruits,
    levels = c(0, 1, 2, 3),
    labels = c("Never", "A few times", "Often", "All the time"),
    ordered = TRUE
    )

> snap$treatment2 = factor(snap$treatment, labels = c("Control", "Treatment"))
```

Barplots

```
# Load libraries
> library(ggplot2)
> library(scales)


# Marginal responses
> ggplot(data = snap, aes(x = vegetables, y = (..count..)/sum(..count..), fill = vegetables)) +
    geom_bar(color = "black", show_guide = FALSE) +
    theme_bw() +
    scale_y_continuous(labels = percent, name = "Percent") +
    xlab("I like to try new vegetables.")


# Condition on treatment
> ggplot(data = snap, aes(x = vegetables, y = (..count..)/sum(..count..), fill = vegetables)) +
    geom_bar(color = "black", show_guide = FALSE) +
    theme_bw() +
    scale_y_continuous(labels = percent, name = "Percent") +
    xlab("I like to try new vegetables.") +
    facet_wrap(~treatment2)
```



Marginal responses
n = 74
0.217
n = 142
0.416
n = 59
0.173
n = 66
0.194
Percent
0%
10%
20%
30%
40%
Never
A few times
Often
All the time
I like to try new vegetables.

Continuation Ratio Model

The cumulative odds (CO) model considers the probability of being at or beyond a category relative to the probability of being below that category

- An assumption made in the CO analysis is that across all cumulative logit comparisons the odds of being in higher categories relative to being in any category below it remains constant across the categories.

- If interest lies in determining the effects of predictors on the event of being in a higher stage or category, then a comparison group that includes *all* people who failed to make it to a category may not lead us to the best conclusions or understanding of the data in terms of differences between people at a low stage versus all higher stages

- Rather than grouping together all people who failed to make it to a category at any point, an alternative ordinal approach involves **comparisons between respondents in any given category versus all those who achieved a higher category score**.

The focus of a CR analysis is to understand the factors that distinguish between those persons who have reached a particular response level but do not move on from those persons who do advance to a higher level.

$$\ln\left[\frac{P(Y>j)}{P(Y\geq j)}\right] = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \epsilon$$

Continuation Ratios as Conditional Probablities

The continuation ratio (CR) model estimates the probability of a response *continuing past* level *j*, *given* that the response is at least *j* in the first place

$$P(Y > j | Y \geq j)$$

$$\frac{P(Y > j) \text{ and } P(Y \geq j)}{P(Y \geq j)}$$

Two alternative formulations of the continuation ratio

$$\frac{P(Y > j)}{P(Y \geq j)}$$

Comparing Each Category to Higher Categories

Fitting the CR Model

```
# Load library
> library(VGAM)


# Fit model
> crm.1 = vglm(vegetables ~ 1, data = snap, family = cratio(parallel = TRUE))
> summary(crm.1)
```

```
Coefficients:
              Estimate Std. Error  z value
(Intercept):1  1.28318    0.13137  9.76749
(Intercept):2 -0.12751    0.12265 -1.03968
(Intercept):3  0.11212    0.17917  0.62577

Number of linear predictors:  3

Names of linear predictors: logit(P[Y>1|Y>=1]), logit(P[Y>2|Y>=2]), logit(P[Y>3|Y>=3])

Dispersion Parameter for cratio family:   1

Residual deviance: 898.7035 on 1020 degrees of freedom

Log-likelihood: -449.3517 on 1020 degrees of freedom

Number of iterations: 4
```

```
Coefficients:
               Estimate Std. Error  z value
(Intercept):1  1.28318    0.13137  9.76749
(Intercept):2 -0.12751    0.12265 -1.03968
(Intercept):3  0.11212    0.17917  0.62577
```

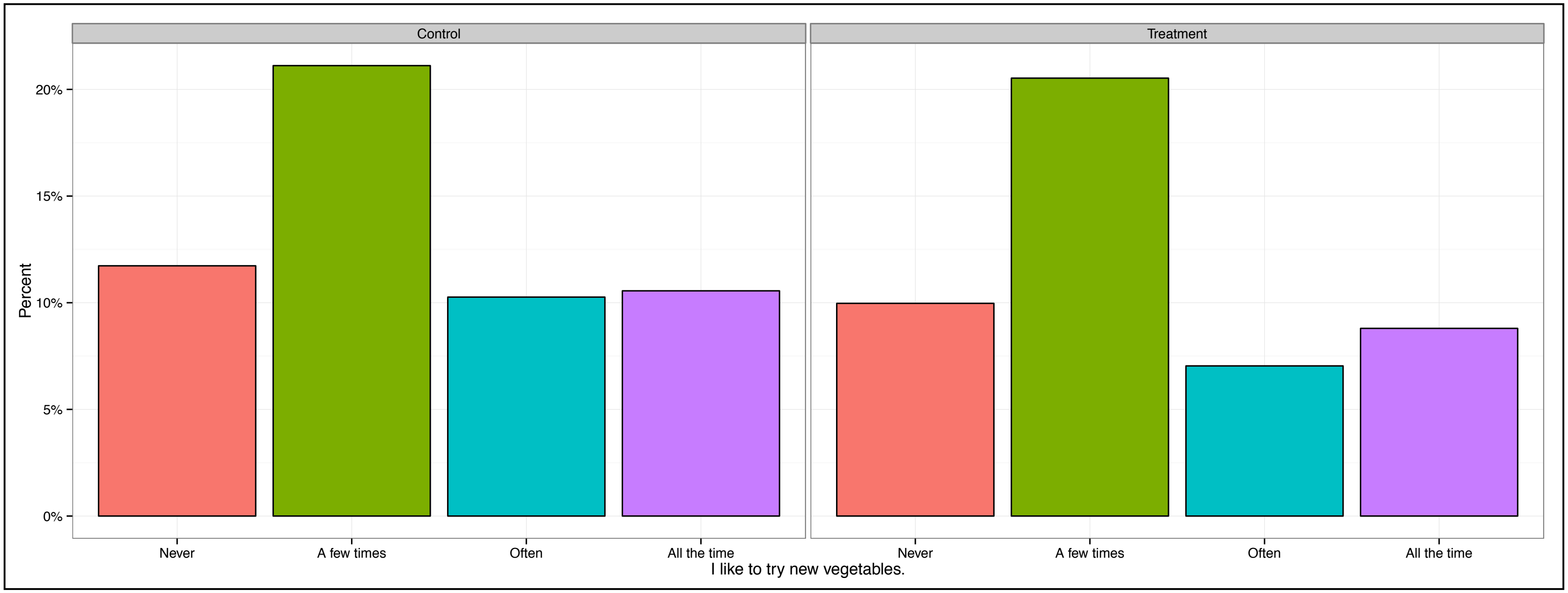
```
> exp(coef(crm.1))

(Intercept):1 (Intercept):2 (Intercept):3
    3.6081081     0.8802817     1.1186441
```

The odds of a subject responding “A few times”, “Often”, or “All the time” is 3.61 times higher than responding “Never”. This difference seems to be statistically reliable.

The odds of a subject responding “Often” or “All the time” is –0.13 times as high as responding “A few times”. This difference does not seem to be statistically reliable.

The odds of a subject responding “All the time” is 1.12 times higher than responding “Often”. This difference does not seem to be statistically reliable.

Treatment Effect?




```
# Fit model
> crm.2 = vglm(vegetables ~ treatment, data = snap, family = cratio(parallel = TRUE))
> summary(crm.2)
```

```
Coefficients:
              Estimate Std. Error  z value
(Intercept):1  1.31076    0.15171  8.64009
(Intercept):2 -0.10015    0.14318 -0.69945
(Intercept):3  0.13762    0.19342  0.71152
treatment     -0.05899    0.16083 -0.36679
```

There is a slight negative effect of treatment, although it does not seem to be a statistically reliable.

In general, subjects in the treatment group are no more or less likely to respond higher than any specific category than control subjects

```
Coefficients:
              Estimate Std. Error  z value
(Intercept):1  1.31076    0.15171  8.64009
(Intercept):2 -0.10015    0.14318 -0.69945
(Intercept):3  0.13762    0.19342  0.71152
treatment     -0.05899    0.16083 -0.36679
```

```
> exp(coef(crm.2))

(Intercept):1 (Intercept):2 (Intercept):3     treatment
    3.7089952     0.9047061     1.1475449     0.9427162
```

Control Group

The odds of a subject responding “A few times”, “Often”, or “All the time” is 3.61 times higher than responding “Never”. This difference seems to be statistically reliable.

The odds of a subject responding “Often” or “All the time” is –0.13 times as high as responding “A few times”. This difference does not seem to be statistically reliable.

The odds of a subject responding “All the time” is 1.12 times higher than responding “Often”. This difference does not seem to be statistically reliable.

Treatment Group

The odds of a subject responding “A few times”, “Often”, or “All the time” is 3.50 times higher than responding “Never”. This difference seems to be statistically reliable.

The odds of a subject responding “Often” or “All the time” is 0.85 times as high as responding “A few times”. This difference does not seem to be statistically reliable.

The odds of a subject responding “All the time” is 1.08 times higher than responding “Often”. This difference does not seem to be statistically reliable.

The odds of a subject in the treatment group responding beyond category j rather than category j is 0.94 times as high as for control subjects. This difference does not seem to be statistically reliable.

Adjacent Categories Model

Adjacent category (AC) models are a specific form of generalized logit models for multinomial outcomes.

- Simultaneously estimates the effects of explanatory variables *in pairs of adjacent categories*
- Effects are constrained to be constant for comparisons of adjacent categories (i.e., assumptions of proportionality/parallelism)

Comparisons for the AC model using the SNAP data

- Category 2 (A few times) vs. Category 1 (Never)
- Category 3 (Often) vs. Category 2 (A few times)
- Category 4 (All the time) vs. Category 3 (Often)

The logit transformation compares $\pi_{i,j}$, the probability of the *i*th subject's response being in category *j*, to the probability of response in the next successive category, $\pi_{i,j+1}$.

The purpose of this approach is to simultaneously determine the odds of response in the next highest category for each pair of adjacent categories

$$\ln\left[\frac{\pi_{i,j+1}}{\pi_{i,j}}\right] = \beta_{0j} + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \epsilon$$

Odds Ratios

Odds ratio are formed for each specific pair of adjacent categories

$$\mathrm{OR}_{j+1} = \exp\left[\hat{\beta}_{0j} + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \ldots + \hat{\beta}_k X_k\right]$$

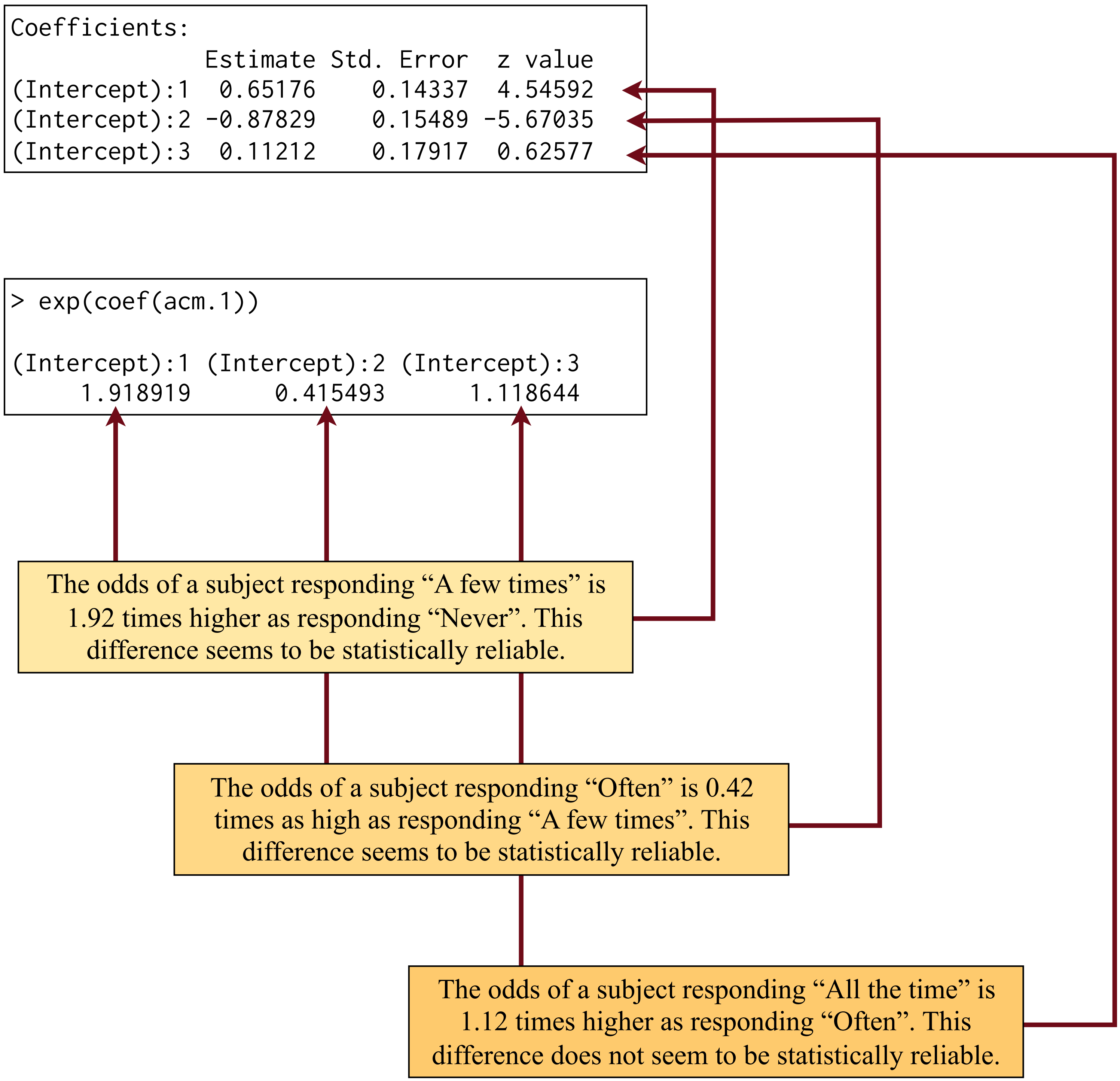
They provide the odds for choosing response j+1 as opposed to response j

The odds ratios for the AC model are referred to as **local odds ratios** because the association they describe is from a localized region in the overall table

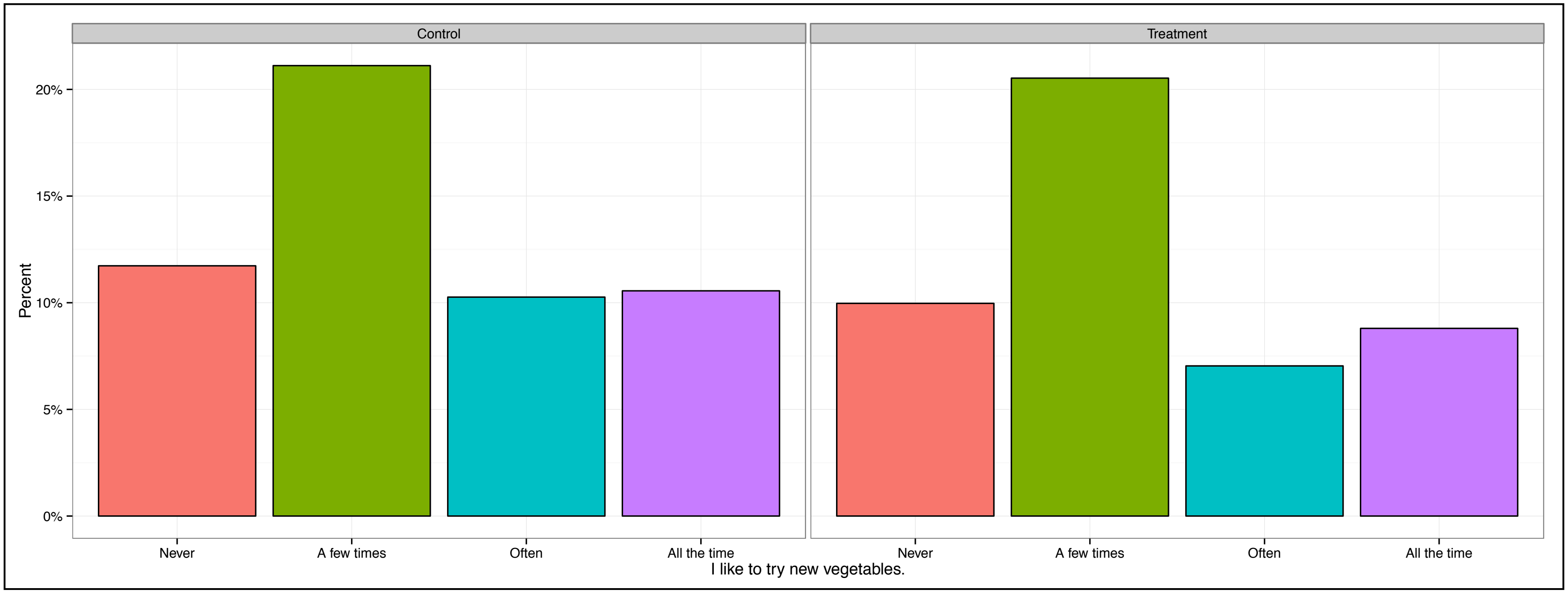
Fitting the AC Model

```
# Load library
> library(VGAM)


# Fit model
> acm.1 = vglm(vegetables ~ 1, data = snap, family = acat(parallel = TRUE))
> summary(acm.1)
```

```
Coefficients:
              Estimate Std. Error  z value
(Intercept):1  0.65176    0.14337  4.54592
(Intercept):2 -0.87829    0.15489 -5.67035
(Intercept):3  0.11212    0.17917  0.62577

Number of linear predictors:  3

Names of linear predictors: log(P[Y=2]/P[Y=1]), log(P[Y=3]/P[Y=2]), log(P[Y=4]/P[Y=3])

Dispersion Parameter for acat family:   1

Residual deviance: 898.7035 on 1020 degrees of freedom

Log-likelihood: -449.3517 on 1020 degrees of freedom
```


Coefficients:
Estimate Std. Error z value
(Intercept):1 0.65176 0.14337 4.54592
(Intercept):2 -0.87829 0.15489 -5.67035
(Intercept):3 0.11212 0.17917 0.62577
> exp(coef(acm.1))
(Intercept):1 (Intercept):2 (Intercept):3
1.918919 0.415493 1.118644
The odds of a subject responding “A few times” is 1.92 times higher as responding “Never”. This difference seems to be statistically reliable.
The odds of a subject responding “Often” is 0.42 times as high as responding “A few times”. This difference seems to be statistically reliable.
The odds of a subject responding “All the time” is 1.12 times higher as responding “Often”. This difference does not seem to be statistically reliable.

Treatment Effect?




```
# Fit model
> acm.2 = vglm(vegetables ~ treatment, data = snap, family = acat(parallel = TRUE))
> summary(acm.2)
```

There is a slight negative effect of treatment, although it does not seem to be a statistically reliable.

```
Coefficients:
               Estimate Std. Error  z value
(Intercept):1  0.674247    0.15230  4.42707
(Intercept):2 -0.856367    0.16242 -5.27265
(Intercept):3  0.133481    0.18507  0.72125
treatment     -0.047507    0.10629 -0.44696
```

```
> exp(coef(acm.2))

(Intercept):1 (Intercept):2 (Intercept):3     treatment
    1.9625543     0.4247024     1.1427992     0.9536037
```

```
Coefficients:
                Estimate Std. Error  z value
(Intercept):1  0.674247    0.15230  4.42707
(Intercept):2 -0.856367    0.16242 -5.27265
(Intercept):3  0.133481    0.18507  0.72125
treatment     -0.047507    0.10629 -0.44696
```

```
> exp(coef(acm.2))

(Intercept):1 (Intercept):2 (Intercept):3     treatment
    1.9625543     0.4247024     1.1427992     0.9536037
```

Control Group

The odds of a subject responding “A few times” is 1.96 times higher as responding “Never”. This difference seems to be statistically reliable.

The odds of a subject responding “Often” is 0.42 times as high as responding “A few times”. This difference seems to be statistically reliable.

The odds of a subject responding “All the time” is 1.14 times higher as responding “Often”. This difference does not seem to be statistically reliable.

Treatment Group

The odds of a subject responding “A few times” is 1.87 times higher as responding “Never”. This difference seems to be statistically reliable.

The odds of a subject responding “Often” is 0.40 times as high as responding “A few times”. This difference seems to be statistically reliable.

The odds of a subject responding “All the time” is 1.09 times higher as responding “Often”. This difference does not seem to be statistically reliable.

The odds of a subject in the treatment group responding in category j+1 rather than category j is 0.95 times as high as for control subjects. This difference does not seem to be statistically reliable.

Effect of BMI?

```
# Fit model
> acm.3 = vglm(vegetables ~ bmi, data = snap, family = acat(parallel = TRUE))
> summary(acm.3)
```

```
Coefficients:
               Estimate Std. Error  z value
(Intercept):1 -0.081488   0.301754 -0.27005
(Intercept):2 -1.630073   0.318364 -5.12016
(Intercept):3 -0.662820   0.345686 -1.91740
bmi            0.041031   0.015198  2.69968
```

```
> exp(coef(acm.3))

(Intercept):1 (Intercept):2 (Intercept):3           bmi
    0.9217438     0.1959152     0.5153956     1.0418843
```

There is a positive effect of BMI, which seems to be statistically reliable.

For a one unit change in subjects' BMI, the odds of a subject responding in category j+1 rather than category j is 1.04 times higher on average.

Visualize Effects of BMI