Introduction to Longitudinal Analysis

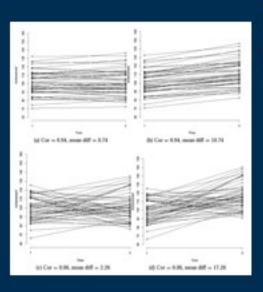
Andrew S. Zieffler

Means vs. Correlations

- Primary focus is examination of mean change of response variable over time
- Correlations are also informative
 - Indicate strength of dependency of response
 - Often more closely spaced observations are more highly correlated
- Focus should depend on research question

Relative vs. Absolute Change

- Correlations used to make inferences about relative change
 - Based on z-scores and when computed for each time point, have constant mean = 0 across time
 - Precludes meaningful inference about absolute change (i.e., change in quantity of response)
 - Focus in on change in relative position (rank order) over time or stability of individuals



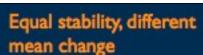
Achievement (y) measured at two time points for same cohort of individuals

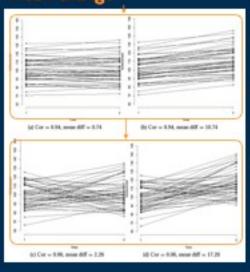
Generally high stability W Cor = 8.94, mose dif = 8.34 W Cor = 8.94, mose dif = 18.34 W Cor = 8.94, mose dif = 18.34

Generally low stability

Achievement (y) measured at two time points for same cohort of individuals

Extent of crossing lines is related to strength of stability coefficient (correlation)





Achievement (y)
measured at two time
points for same cohort
of individuals

Extent of crossing lines is related to strength of stability coefficient (correlation)

Mean difference is indifferent to between-time correlation

What Accounts for Instability

- Variability in individual rate of change
- Random measurement error
 - Scores are not perfectly reliable
 - Statistical model needed to sort out change between these two sources

Measurement Issues

- Same instrument used across time
 - Developmentally appropriate? Construct can change as individuals age—Externalizing behavior (e.g., aggression) measured from age 5 to age 30. Measurement will change (e.g., number of playground bites to ability to get along with boss at work)
 - Empirical check on validity-longitudinal IRT models and examine invariance of IRT parameters

Measurement Issues

- For analysis of mean change, construct must be measured in absolute sense
 - Measurement determined in reference to objective scale (e.g., height)
 - · Not influenced by measurement of other subjects
 - Relative measurements do not allow mean change to be studied (e.g., rating of social cooperation)

Response Variable Assumptions

- ★ Constitutes valid foundation for making inferences about mean change
 - Quantitative, continuous (only for this course)

Conceptual Overview of Linear Mixed Effects Regression (LMER)

- Used to analyze mean change, but also allows estimation of individual variation in change
- Incorporates random error term to account for measurement unreliability
- Extension of linear (multiple) regression

Conceptual Overview of Linear Mixed Effects Regression (LMER)

- Goal is to account for variation of the response variable
 - Within-subjects variation: row-to-row change for individual subjects
 - Due to changes in response variable over time (i.e., variation in repeated measures)
 - Between-subjects variation: block-to-block change in response variable
 - Due to individual differences (i.e., variation in starting values and growth rates between individuals)

Subject identifier

	subid	read	grade	gen	att
1	1	172	5	F	8.94
2	1	185	6	F	0.94
3	1 1 1	179	7	F	8.94
1 2 3 4 5 6 7 8	1	194	8	F	8,94
5	3	191	5	-	0.97
6	***	199	6	н	8.97
7	3	283	1	H	8.97
8	3	215	8	H	0.97
9	5	287	5	F	8.85
18	5 5	213	6	F	0.85
11	5	212	7	F	0.85
12	5	213	8	F	8.85
13	7	199	5	н	0.97
14	7	288	6	н	8.97
15	7 7 7	213	7	H	8.97
16	7	218	8	H	0.97

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Between-subject variation to account for in start value of response

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7	3	283	-7	_ H	8.97	
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9	5	287	5	F	8.85	rates of response
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11	5	212	1	F	8.85	
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14	7	288	10	H	8.97	
13 14 15	7	213	7		8.97	
16	ż	218			8.97	
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Predictors and Accounting for Variation

- Predictors that change over time (time predictors, dynamic predictors) account for within-subject variation
- Predictors that do not change over time (static predictors), but vary between individuals, account for between-subject variation

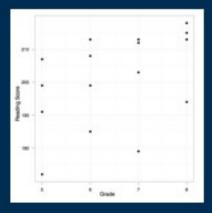
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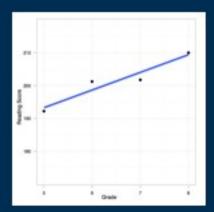
grade accounts for within-subject variation

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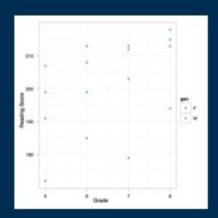
gen and att account for between-subject variation

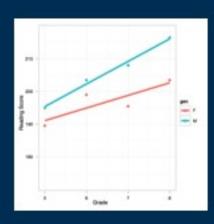
Using grade to Account for Variation in read



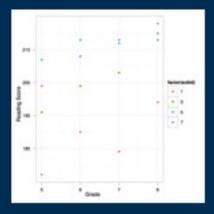


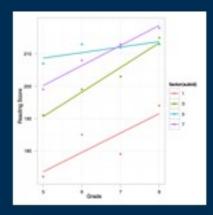
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Examining Random Effects



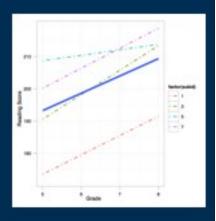


Within-Subject Variation

- Scatter of observed data around single subjects' regression line
- Variation considered to be measurement unreliability
- Typical assumptions of normality, mean = 0, constant variance

Examining Random Effects

subid	intercept	slope
ľ	143.50	6.00
3	152.60	7.60
5	200.20	1.70
7	169.20	6.20
Mean	166.38	6.20



Between-Subject Variation

- Deviation of individual lines around mean regression line (random effects)
- Variation in random intercepts and random slopes (subject-to-subject variation)
- Correlation between random intercepts and random slopes, r = -0.87

Examining Random Effects Conditioning on Gender

