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4                                  **DEVELOPMENT OF SCALABLE APPROACHES TO NEUTRINO MASS**  
5                                  **MEASUREMENT WITH THE PROJECT 8 EXPERIMENT**

6                                  A Thesis in  
7                                  The Physics Department  
8                                  by  
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<sup>20</sup> **Abstract**

<sup>21</sup> Some shit goes here.

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<sup>146</sup> **Acknowledgments**

<sup>147</sup> Shout out to all the haters.

<sup>148</sup> **Dedication**

<sup>149</sup> Something heartfelt.

<sup>150</sup> **Chapter 1** |  
<sup>151</sup> **Introduction and Summary**

<sup>152</sup> Neutrinos are one of the fundamental particles that comprise the standard model of  
<sup>153</sup> particle physics and account for a significant fraction of the matter in the universe.  
<sup>154</sup> Neutrinos are the most abundant fermions in the universe, but due to their weak  
<sup>155</sup> interactions neutrinos seldom interact with other particles. Regardless, neutrinos play a  
<sup>156</sup> unique role in the evolution of the early-universe, therefore, a detailed understanding  
<sup>157</sup> of the properties of the neutrino is important to understanding the cosmology of the  
<sup>158</sup> universe as well as understanding the universe at the fundamental particle physics scale.

<sup>159</sup> Unlike other fermions it was unclear that neutrinos had nonzero mass until neutrino  
<sup>160</sup> flavor oscillations were definitively observed in the late 90's and early 00's. Flavor  
<sup>161</sup> oscillations require that neutrinos experience time so that when acted upon by the  
<sup>162</sup> time-evolution operator the initial neutrino state can evolve to a new flavor state. This  
<sup>163</sup> implies that the neutrino flavor states are really a superposition of at least three separate  
<sup>164</sup> neutrino states with well-defined masses. Measurements of neutrino oscillations that have  
<sup>165</sup> taken place over the past couple of decades have measured the differences between  
<sup>166</sup> neutrino mass eigenstates with increasing precision. However, oscillation measurements  
<sup>167</sup> cannot tell us the mass scale of the neutrinos, which is required in order to measure the  
<sup>168</sup> absolute neutrino masses.

<sup>169</sup> The neutrino mass scale remains an unknown quantity in the standard model of  
<sup>170</sup> particle physics. The value of the neutrino mass influences the evolution of the early  
<sup>171</sup> universe and is likely relevant to the energy-scale of new physics responsible for the factor  
<sup>172</sup> of  $10^{-6}$  difference between the neutrino and electron masses. A model-independent way  
<sup>173</sup> to measure the neutrino mass is to measure the tritium beta-decay spectrum near its  
<sup>174</sup> endpoint. Energy conservation requires that the neutrino mass carry away some kinetic  
<sup>175</sup> energy from the beta-decay electron in the form of its mass, which causes a distortion in  
<sup>176</sup> the shape of the tritium beta-decay spectrum near the endpoint. The isotope tritium has  
<sup>177</sup> many advantages for this measurement, and has been used by the KATRIN collaboration

178 to perform the most sensitive direct neutrino mass measurement to date.

179 KATRIN represents the state-of-the-art experiment in the current generation of  
180 neutrino mass direct measurement experiments and has a final projected sensitivity to  
181 neutrino masses  $m_\nu < 200$  meV. This sensitivity does not fully exhaust the allowed  
182 parameter space of neutrino masses under the normal and inverted neutrino mass  
183 ordering scenarios, which motivates the development of a next generation of neutrino  
184 mass measurement experiments.

185 The Project 8 collaboration is developing a next-generation neutrino mass direct  
186 measurement experiment designed to be sensitive to  $m_\nu < 40$  meV. This sensitivity  
187 is sufficient to exhaust the range of neutrino masses allowed under the inverted mass  
188 ordering regime. Project 8 intends to achieve its sensitivity goal utilizing two technologies  
189 that are novel to the space of direct neutrino mass measurement — atomic tritium and  
190 cyclotron radiation emission spectroscopy (CRES). Atomic tritium is required in order to  
191 avoid systematic broadening the tritium beta-decay spectrum caused by the final state  
192 of the  ${}^3\text{He}^+ \text{-T}$  molecule, and the CRES technique enables a differential measurement of  
193 the tritium spectrum that is background-free and able to be directly integrated with the  
194 atomic tritium source.

195 The Project 8 collaboration is currently engaged in a research and development  
196 program intended to simultaneously develop the atomic tritium and CRES technologies  
197 so that they can be combined in a next-generation experiment. This past year (2022)  
198 Project 8 has used the CRES technique to measure the molecular tritium beta-decay  
199 spectrum and place an upper limit on the neutrino mass:  $m_\beta \leq 152$  eV. This measurement,  
200 while not competitive scientifically, represents the first proof-of-principle that the CRES  
201 technique can be used to measure the neutrino mass.

202 The future goals of the Project 8 collaboration are to develop the technologies  
203 and techniques necessary to scale-up the volumes in which CRES measurements can  
204 be performed. Project 8's first neutrino mass measurement with CRES utilized a  
205 measurement volume on the cubic-centimeter scale, however, sensitivity calculations  
206 estimate that an experiment sensitive to neutrino masses of 40 meV will require several  
207 tens of cubic-meters of experiment volume filled with atomic tritium. Developing a new  
208 approach to performing CRES measurements that can be successfully scaled to these  
209 volumes is a necessary step towards Project 8's neutrino mass measurement goal, and is  
210 the primary topic of my dissertation research.

211 A parallel development is the technology necessary to produce, cool, trap, and  
212 recirculate a supply of atomic tritium that is compatible with CRES measurements. The

213 atomic tritium system is equally important as the large-volume CRES measurement  
214 technology, but it will not be the focus of this dissertation since I did not contribute  
215 significantly to this effort.

216 The Project 8 collaboration has identified two scalable approaches to neutrino mass  
217 measurement using the CRES technique. One approach is to use an array of antennas  
218 that surrounds a volume of trapped atomic tritium that can perform CRES measurements  
219 by collection the cyclotron radiation emitted by beta-decay electrons into free-space. The  
220 other approach uses a resonant cavity filled with atomic tritium to perform CRES by  
221 measuring the

- 222 In Chapter 2...
- 223 In Chapter 3...
- 224 In Chapter 4...
- 225 In Chapter 5...
- 226 In Chapter 6...
- 227 In Chapter 7...

<sup>228</sup> **Chapter 2 |**

<sup>229</sup> **Neutrinos and Neutrino Masses**

<sup>230</sup> **2.1 Introduction**

<sup>231</sup> In this chapter I provide a cursory overview of background information relevant to  
<sup>232</sup> neutrinos and neutrino mass measurements.

<sup>233</sup> In Section 2.2 I provide some background information on the history of neutrinos and  
<sup>234</sup> beta-decay. In Section 2.3 I describe the discover of neutrino oscillations, which proved  
<sup>235</sup> unambiguously that neutrinos have non-zero masses. In Section 2.4 I discuss the current  
<sup>236</sup> state of the theoretical understanding of neutrino masses in the standard model. Lastly,  
<sup>237</sup> in Section 2.5 I discuss methods for measuring the absolute scale of the neutrino mass.

<sup>238</sup> **2.2 Neutrinos and Beta-decay**

<sup>239</sup> Late in the 19th century the phenomena of radioactivity was first observed in experiments  
<sup>240</sup> performed by Henri Becquerel with uranium, and further studied using thorium and  
<sup>241</sup> radium by Marie and Pierre Curie. Early work in radioactivity classified different forms  
<sup>242</sup> of radiation based on it's ability to penetrate different materials. Rutherford was the first  
<sup>243</sup> to separate radioactive emissions into two types, alpha and beta radiation. Alpha rays  
<sup>244</sup> can be easily stopped by a piece of paper or thin foil of metal, whereas beta radiation  
<sup>245</sup> could penetrate metals several millimeters thick. Later a third form of radiation was  
<sup>246</sup> identified by Villard, which was still more penetrating, and was eventually termed gamma  
<sup>247</sup> radiation by Rutherford.

<sup>248</sup> When these forms of radioactivity were first discovered it was unclear what physically  
<sup>249</sup> constituted an alpha, beta, or gamma particle. Experiments with radioactivity in  
<sup>250</sup> magnetic fields was eventually able to identify the charge composition of different forms  
<sup>251</sup> of radiation. In particular, experiments by Becquerel identified that beta radiation had

252 an identical charge-to-mass ratio to the electron discovered by Thompson in his work on  
253 cathode rays. This was strongly suggestive that beta particles were indeed electrons.

254 Further studies of beta radiation lead to the discovery that radioactivity resulted  
255 in the transmutation of elements caused by the decay of a heavier nucleus to a lighter  
256 species. One feature of beta radiation, which we now properly call beta-decay, that  
257 was different from alpha-decays and gamma radiation is that the electrons produced by  
258 beta-decay have a continuous spectrum of kinetic energies, whereas, alpha and gamma  
259 particles are emitted with discrete energies. This feature of beta-decay was first observed  
260 by Chadwick in 1914, and was extremely puzzling at the time since the continuous  
261 spectrum apparently violates energy conservation.

262 Famously, in 1930 Pauli proposed the existence of a new neutral particle, which he  
263 termed the "neutron", that was also produced during beta-decay in order to resolve the  
264 missing energy problem posed by the beta-decay spectrum. Because this particle carried  
265 no charge, it was hypothesized at the time that it had simply not been observed in any  
266 experiments up to that time. This "neutron", which was initially estimated to have a  
267 mass no larger than that of an electron, was eventually renamed the "neutrino" by Fermi  
268 after the discovery of the neutron by Chadwick in 1932. Later, in 1933, Fermi developed  
269 a quantum mechanical theory for beta-decay in which both an electron and neutrino are  
270 produced by the decay of a neutron to a proton inside the radioactive nucleus.

271 Little more than a speculation when first introduced, indirect evidence for the existence  
272 of neutrinos was obtained in 1938 by the simultaneous observation of the electron and  
273 recoiling nucleus in cloud chambers by Crane and Halpern. However, it wasn't until the  
274 Cowan-Reines experiment in 1956 that direct evidence for the existence of neutrinos  
275 was observed by detecting the inverse beta-decays caused by neutrinos from a nuclear  
276 reactor interacting with protons contained in water molecules. The difficulty in detecting  
277 neutrinos is caused by their weak interactions with other particles. Further, experiments  
278 revealed that different types of neutrinos existed based on the nature of the leptons  
279 produced in neutrino charged-current interactions, but the existence of a neutrino mass  
280 remained an open question that would take more than 40 year to resolve.

## 281 **2.3 Neutrino Oscillations**

282 The first hint of neutrino flavor transitions or neutrino oscillations was indicated by  
283 the solar neutrino problem, which referred to discrepancies between the predicted flux  
284 of  $\nu_e$  from the standard solar model and measurements of the solar neutrino flux such

as the famous experiment at the Homestake mine by Ray Davis Jr. and collaborators in the 1960's. Essentially, fewer electron-type neutrinos than expected were being observed from the sun. Finally, in the early 2000's the SNO experiment was able to resolve the solar neutrino problem by identifying neutrino oscillations as the cause of the observed deficit. Furthermore, measurements of the atmospheric flux of neutrinos by the Super-Kamiokande experiment and others revealed that fewer muon-type neutrinos survived passage through the earth than expected providing strong evidence for neutrino oscillations for both flavors.

The origin of neutrino oscillations is that the weak eigenstates are distinct from the mass eigenstates. The neutrino mass eigenstates represent physical particles in the sense that they are solutions to the free-particle Hamiltonian, whereas, the neutrino weak eigenstates correspond to the neutrino states that interact via the weak charged-current interaction. The neutrino weak eigenstates are a linear superposition of the neutrino mass eigenstates

$$\nu_\ell = \sum_i U_{\ell i} \nu_i, \quad (2.1)$$

where  $\ell = e, \mu, \tau$  and  $i = 1, 2, 3$ . The matrix elements  $U_{\ell i}$  are the elements of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix that describes the mixing between the neutrino flavor and mass states.

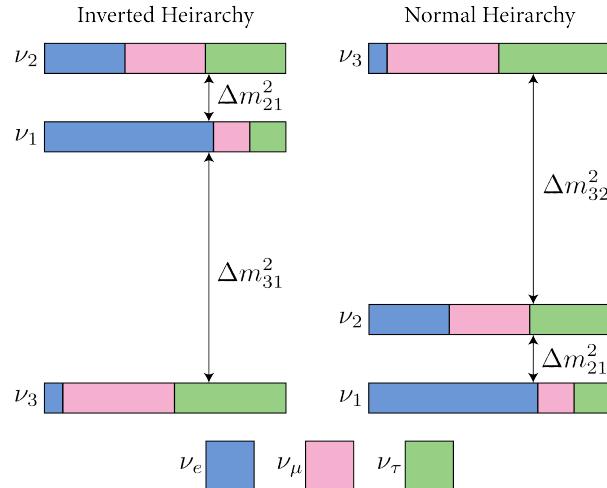


Figure 2.1: A diagram of two different neutrino mass ordering scenarios. In the inverted hierarchy (inverted mass ordering) the lightest neutrino mass is  $m_3$ , whereas, in the normal hierarchy (normal mass ordering)  $m_1$  is the lightest neutrino. What cannot be measured by neutrino oscillations is the neutrino absolute mass scale, which is essentially the mass of the lightest neutrino mass eigenstate.

302 One standard parameterization of the PMNS matrix is

$$U_{PMNS} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (2.2)$$

303 where  $c_{ij} = \cos \theta_{ij}$  and  $s_{ij} = \sin \theta_{ij}$ . The parameters  $\alpha_1$  and  $\alpha_2$  are only included in the  
 304 PNMS matrix if neutrinos are Majorana particles, something which represents a current  
 305 area of research in neutrino physics. The phase  $\delta$  quantifies the degree of CP-violation  
 306 in the neutrino sector. Including the Majorana phases the PMNS matrix contains six  
 307 independent parameters. In addition, neutrino oscillation probabilities depend on the  
 308 squared mass differences between neutrino mass eigenstates

$$\Delta m_{ij}^2 = m_i^2 - m_j^2, \quad (2.3)$$

309 where  $ij = 12, 32, 31$  respectively. Because  $\Delta m_{32}^2 = \Delta m_{31}^2 - \Delta m_{21}^2$ , this adds an additional  
 310 two parameters that must be constrained by neutrino oscillations.

311 A giant experimental effort over the past couple of decades has greatly contained the  
 312 majority of parameters in the PMNS matrix, many to relative uncertainties of only a  
 313 few percent. However, some parameters still remain relatively unconstrained, which is  
 314 the origin of the current uncertainty in the ordering of the neutrino masses (see Figure  
 315 2.1). The neutrino masses can be organized by their relative mass. The current neutrino  
 316 oscillation data can confirm that  $m_2 > m_1$ , however, the sign of  $\Delta m_{32}^2$  is still unknown.  
 317 This leads to two scenarios where neutrino masses follow the ordering  $m_3 > m_2 > m_1$ ,  
 318 which is called the normal mass ordering (NMO), or alternatively neutrino masses may  
 319 be ordered  $m_2 > m_1 > m_3$ , which is called the inverted mass ordering (IMO). Next-  
 320 generation neutrino oscillation experiments such as JUNO, Hyper-Kamiokande, and  
 321 DUNE are poised to resolve this ambiguity in the coming years.

322 Neutrino oscillation probabilities are only sensitive to the neutrino masses via the  
 323 squared mass differences. Therefore oscillation probabilities are unaffected by the absolute

scale of the neutrino mass. However, oscillations can be used to obtain a lower bound on the neutrino masses by setting the mass of the lightest neutrino mass state to zero. This results in different lower limits depending on the ordering of the neutrino mass states. Current best-fit values with  $1\sigma$ -uncertainties for the squared mass differences are

$$\Delta m_{21}^2 = (7.42^{+0.21}_{-0.20}) \times 10^{-5} \text{ eV}^2, \quad (2.4)$$

$$\Delta m_{31}^2 = (2.5176^{+0.026}_{-0.028}) \times 10^{-3} \text{ eV}^2 \text{ (NMO)}, \quad (2.5)$$

for the normal mass ordering, and in the case of the inverted ordering we have

$$\Delta m_{32}^2 = (-2.498^{+0.028}_{-0.028}) \times 10^{-3} \text{ eV}^2 \text{ (IMO).} \quad (2.6)$$

By letting the lightest neutrino mass in each ordering scenario ( $m_{\text{least}}$ ) take on a range of values one can visualize the relative masses of the neutrinos as a function of  $m_{\text{least}}$  (see Figure 2.2).

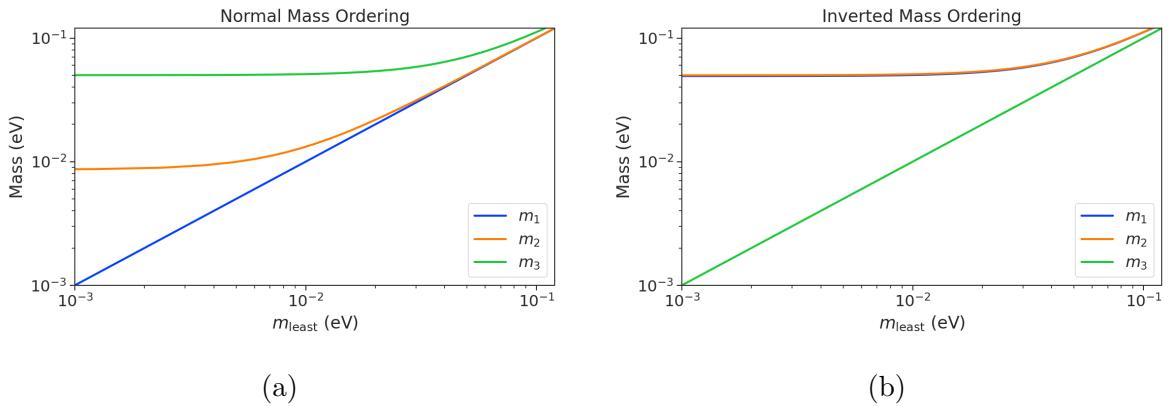


Figure 2.2: The masses of the neutrinos as a function of the lightest neutrino mass in both the normal (a) and inverted (b) mass ordering regimes.

## 2.4 Neutrino Masses in the Standard Model

Neutrinos are spin 1/2 particles and in modern quantum field theory spin-1/2 particles, or fermions, are described using the Dirac equation.

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi(x) = 0, \quad (2.7)$$

335 where the field that describes the particle is denoted as  $\psi(x)$ . In the standard fermions ac-  
 336 quire mass through the Yukawa interaction, which add to the standard model Lagrangian  
 337 terms of the form

$$\mathcal{L}_{\text{Yukawa}} = -Y_{ij}^\ell \bar{L}_{Li} \phi E_{Rj} + \text{h.c.}, \quad (2.8)$$

338 where  $Y_{ij}^\ell$  is an element of the  $3 \times 3$  Yukawa coupling matrix for leptons,  $L_{Li}$  is the  
 339 left-handed lepton doublet for generation  $i$ ,  $\phi$  is the Higgs doublet, and  $E_{Rj}$  is the  
 340 right-handed lepton field for generation  $j$ . In the standard model neutrinos are only  
 341 represented as left-handed neutrinos and right-handed antineutrinos, consistent with  
 342 experimental observations. Since there are no right-handed neutrino singlet fields and  
 343 no Yukawa interaction terms for neutrinos are strictly massless, and non-zero neutrino  
 344 masses is evidence for physics beyond the standard model. For the charged leptons, the  
 345 Yukawa interaction leads to masses of the form

$$m_{ij}^\ell = Y_{ij}^\ell \frac{v}{\sqrt{2}}, \quad (2.9)$$

346 where  $v$  is the Higgs vacuum expectation value.

347 The observation of massive neutrinos motivates the extension of the standard model  
 348 to explain the origin of neutrino masses, which can be approached in different way, but  
 349 all methods add additional degrees of freedom to the standard model. One approach  
 350 is to introduce to the standard model a right-handed neutrino field that allows one to  
 351 introduce Yukawa terms of the form

$$\mathcal{L}_{\nu \text{Yukawa}} = -Y_{ij}^\ell \bar{L}_{Li} \phi \nu_{Rj} + \text{h.c.} \quad (2.10)$$

352 where  $\nu_{Rj}$  is the right-handed neutrino singlet. Because experimental evidence strongly  
 353 predicts only three active neutrinos these additional neutrinos are sterile and do not in-  
 354 teract via the strong, weak, or electromagnetic interactions. After spontaneous symmetry  
 355 breaking, the Yukawa interaction leads to mass terms given by

$$\mathcal{L}_D = -M_{Dij} \bar{\nu}_{Ri} \nu_{Lj} + \text{h.c.}, \quad (2.11)$$

356 which is called a Dirac mass term. One of the issues with constructing neutrino masses  
 357 in this way is that the required Yukawa couplings are at least a factor of  $10^6$  smaller than  
 358 that of an electron, which begs the question: why are the Yukawa couplings so small for  
 359 the neutrinos?

360 An alternative approach is to allow the neutrinos to have a Majorana mass, which is

<sup>361</sup> possible because neutrinos are electrically neutral particles. The Majorana mass terms  
<sup>362</sup> for the neutrino have the form

$$\mathcal{L}_M = -\frac{1}{2}(M_{Rij}\bar{\nu}_{Ri}\nu_{Rj}^c M_{Lij}\bar{\nu}_{Li}\nu_{Lj}^c) + \text{h.c.}, \quad (2.12)$$

<sup>363</sup> where  $M_{Rij}$  and  $M_{Lij}$  are right-handed and left-handed Majorana mass matrices. A  
<sup>364</sup> consequence of neutrinos being Majorana particles is lepton number violation, which  
<sup>365</sup> predicts the occurrence of neutrino-less double beta-decay at a rate proportional to the  
<sup>366</sup> neutrino mass.

<sup>367</sup> In the most general case neutrinos have both Dirac and Majorana mass terms, which  
<sup>368</sup> allows one to generate neutrino masses with Yukawa couplings similar to the rest of  
<sup>369</sup> the standard model. Considering just one generation of neutrinos for illustration, the  
<sup>370</sup> combined Lagrangian can be written as

$$\mathcal{L}_{D+M} = -m_D\bar{\nu}_R\nu_L - \frac{1}{2}(m_L\bar{\nu}_L\nu_L^c + m_R\bar{\nu}_R\nu_R^c) + \text{h.c.}, \quad (2.13)$$

<sup>371</sup> or equivalently,

$$\mathcal{L}_{D+M} = -\frac{1}{2} \begin{bmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \nu_L^c \\ \nu_R \end{bmatrix} + \text{h.c..} \quad (2.14)$$

<sup>372</sup> An example mass generation mechanism with this approach is the Type-I see-saw  
<sup>373</sup> mechanism, in which we take  $m_L = 0$  and  $m_R \gg m_D$ . By diagonalizing Equation 2.14  
<sup>374</sup> one obtains the mass eigenvalues that represent the physical masses of the neutrinos.  
<sup>375</sup> The light neutrino mass eigenstate, which represents the observed neutrino mass, has a  
<sup>376</sup> mass given by

$$m_1 \approx \frac{m_D^2}{m_R}, \quad (2.15)$$

<sup>377</sup> and the heavy neutrino mass eigenstate, which represents the unobserved sterile neutrino,  
<sup>378</sup> has a mass

$$m_2 \approx m_R. \quad (2.16)$$

<sup>379</sup> For  $m_D$  similar to the other quark or lepton masses, one obtains physical neutrino masses  
<sup>380</sup> consistent with observations from sterile neutrino masses of  $m_R \approx O(10^{15})$  GeV. This  
<sup>381</sup> mass scale is well beyond the capabilities of modern particle accelerators.

## <sup>382</sup> 2.5 Neutrino Absolute Mass Scale

<sup>383</sup> The neutrino absolute mass scale or simply "neutrino mass" cannot be probed with  
<sup>384</sup> neutrino oscillations, since oscillation probabilities are determined by the squared mass  
<sup>385</sup> differences between neutrino mass eigenstates, therefore, alternative techniques are needed  
<sup>386</sup> to perform an effective measurement of the neutrino mass.

### <sup>387</sup> 2.5.1 Limits from Cosmology

<sup>388</sup> In the  $\Lambda$ CDM model, which summarizes our current cosmological understanding of our  
<sup>389</sup> universe, the mass-energy content of the universe is composed of approximately 27%  
<sup>390</sup> dark matter and only 5% normal matter including neutrinos. From this observation, a  
<sup>391</sup> rough limit on the neutrino mass can be obtained from the condition that neutrinos are  
<sup>392</sup> not responsible for the entirety of the matter content of the universe. Using only this  
<sup>393</sup> condition one can constrain the neutrino mass to be ...

<sup>394</sup> A prediction of the  $\Lambda$ CDM model is that the universe originated from a single  
<sup>395</sup> expansion event colloquially called the "Big Bang". In the Big Bang scenario, our  
<sup>396</sup> universe originated as a hot spacetime singularity, which abruptly experience rapid  
<sup>397</sup> expansion in a process called inflation. After the inflationary epoch the universe entered  
<sup>398</sup> the reheating phase where the potential energy responsible for inflation decays into  
<sup>399</sup> standard model particles such as electrons, quarks, and gluons. The universe continued to  
<sup>400</sup> expand in size resulting in a decrease in energy density and lower temperature. Eventually  
<sup>401</sup> the temperature of the universe decreased enough to allow the formation of protons,  
<sup>402</sup> neutrons, and other baryons from quarks and gluons produced from the decays of the  
<sup>403</sup> inflationary fields.

<sup>404</sup> Also produced during the Big Bang are electrons, neutrinos and other leptons as  
<sup>405</sup> well as a population of photons. These particles are kept in thermal equilibrium with  
<sup>406</sup> the rest of the quark-gluon plasma through interactions that take place at the high  
<sup>407</sup> temperatures and densities of the early universe. However, as the universe continues  
<sup>408</sup> to expand it's density and temperatures decreases leading to the eventual decoupling  
<sup>409</sup> of photons and leptons from the quarks and gluons. A prediction of inflation is that  
<sup>410</sup> this population of photons produced during the Big Bang should still be present, but  
<sup>411</sup> with a significantly reduced temperature due to the expansion of the universe. This is  
<sup>412</sup> consistent with the observation of the CMB (cosmic microwave background), which is a  
<sup>413</sup> population of microwave radiation with a blackbody temperature of 2.7 K. The CMB  
<sup>414</sup> is extremely uniform in all directions with slight anisotropies that can be analyzed to

study the evolution of the early universe. A series of experiments have measured the CMB with increasing levels of precision, which has lead to a significant increase in our current understanding of cosmology.

In addition to the CMB, inflation predicts the existence of a  $C\nu B$  (cosmic neutrino background), which are the remnant neutrinos produced during the Big Bang. Since neutrinos only interact via the weak force, they decouple from the hot Big Bang plasma at an earlier time than the CMB radiation. The temperature at which the  $C\nu B$  decouples depends on the neutrino rest mass. Neutrinos play a somewhat unique role in the  $\Lambda$ CDM model due to the fact that neutrinos act as radiation early in the universe but as matter in the late universe. This leads to unique signatures that impact anisotropies of the CMB as well as the distribution of matter in the universe. By combining measurements of the CMB with measurements of the large-scale structure (LSS) of the universe one can constrain the neutrino mass scale by fitting these datasets with the  $\Lambda$ CDM model. This analysis results in some of the most stringent constraints on the neutrino mass. A recent analysis was able to constrain the neutrino mass scale to

$$\Sigma_{m_\nu} \equiv \sum_i m_i < 0.12 \text{ eV}, \quad (2.17)$$

where  $m_i$  are the neutrino mass eigenstates.

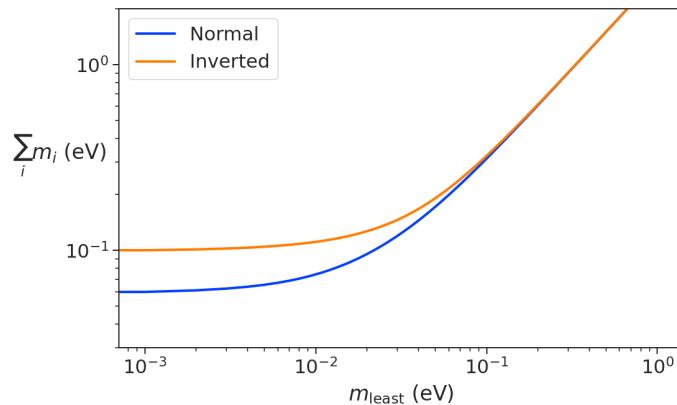


Figure 2.3: The neutrino mass observable measured by cosmology as a function of the lightest neutrino mass eigenstate.

The observable  $\Sigma_{m_\nu}$  constrains the neutrino mass by setting the mass of the lightest neutrino mass eigenstate ( $m_{\text{least}}$ ). In the normal mass ordering  $\Sigma_{m_\nu}$  can be rewritten in

<sup>433</sup> the form

$$\Sigma_{m_\nu} = m_{\text{least}} + \sqrt{\Delta m_{21}^2 + m_{\text{least}}^2} + \sqrt{\Delta m_{32}^2 + m_{\text{least}}^2}, \quad (2.18)$$

<sup>434</sup> where it is clear that a measurement of  $\Sigma_{m_\nu}$  effectively sets the neutrino mass scale  
<sup>435</sup> through  $m_{\text{least}}$ . The analogous formula for the inverted mass ordering is

$$\Sigma_{m_\nu} = m_{\text{least}} + \sqrt{-\Delta m_{32}^2 + m_{\text{least}}^2} + \sqrt{-\Delta m_{31}^2 + m_{\text{least}}^2}. \quad (2.19)$$

<sup>436</sup> In figure 2.3 we plot the observable  $\Sigma_{m_\nu}$  as a function of  $m_{\text{least}}$ .

<sup>437</sup> Upcoming experiments are planned to refine measurements of the CMB, LSS, and  
<sup>438</sup> other cosmological observables. With this additional data it is possible that in the  
<sup>439</sup> near future cosmological measurements will be able to positively constrain the neutrino  
<sup>440</sup> absolute mass scale. However, the strength of these limits strictly depend on the accuracy  
<sup>441</sup> of the  $\Lambda$ CDM model, which highlights the need for direct experimental measurements of  
<sup>442</sup> the neutrino mass to confirm the predictions of cosmology and to fix the neutrino mass  
<sup>443</sup> parameter in future cosmological analyses.

#### <sup>444</sup> 2.5.2 Limits from Neutrinoless Double Beta-decay Searches

<sup>445</sup> If neutrinos are Majorana fermions then the neutrino is equivalent to its own antiparticle  
<sup>446</sup> and lepton conservation is not an exact law of nature. Searches for lepton number  
<sup>447</sup> violation, specifically the neutrinoless double beta-decay ( $0\nu\beta\beta$ ) process, are some of the  
<sup>448</sup> most powerful tests of lepton number conservation, which depend on the neutrinos being  
<sup>449</sup> Majorana fermions. In double beta-decay two neutrons contained in the decay species  
<sup>450</sup> nucleus spontaneously decay into two protons resulting in the production of two electrons  
and two neutrinos (see Figure 2.4). However, for  $0\nu\beta\beta$  the two neutrinos self-annihilate

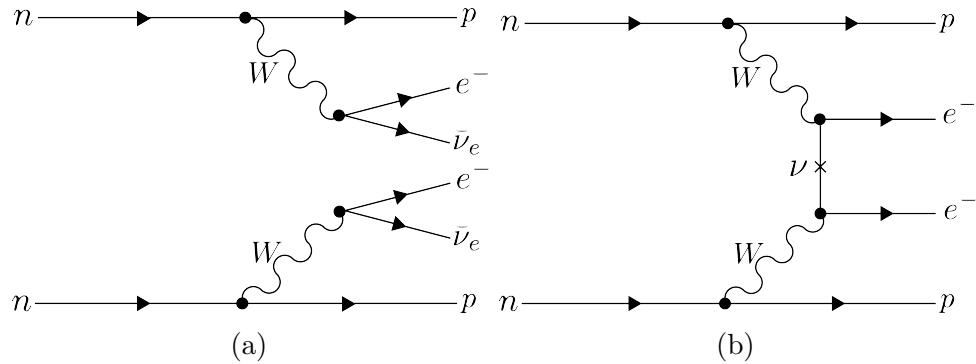


Figure 2.4: Feynman diagrams for double beta-decay (a) and  $0\nu\beta\beta$  (b).

<sup>451</sup>

452 during the decay resulting only in the production of two electrons and a violation of  
 453 lepton number by two.

454 Assuming that the exchange of two Majorana neutrinos is the dominant channel for  
 455  $0\nu\beta\beta$ , then a measurement of the  $0\nu\beta\beta$  half-life for a particular isotope can be used to  
 456 set the neutrino absolute mass scale. The half-life is written in terms of the effective  
 457 neutrino mass for  $0\nu\beta\beta$  ( $m_{\beta\beta}$ ) using the equation

$$T_{1/2}^{0\nu} = \frac{1}{G|\mathcal{M}|^2 m_{\beta\beta}^2}, \quad (2.20)$$

458 where  $G$  is the phase-space factor for the decay and  $\mathcal{M}$  is the relevant nuclear matrix  
 459 element.  $m_{\beta\beta}$  is given by an incoherent sum of the neutrino mass eigenstates weighted  
 460 by the PMNS mixing matrix parameters,

$$m_{\beta\beta} = \left| \sum_i U_{ei}^2 m_i \right|. \quad (2.21)$$

461 The information provided from  $0\nu\beta\beta$  on the neutrino mass scale can be visualized  
 462 by expressing the value of  $m_{\beta\beta}$  in terms of  $m_{\text{least}}$  and two relative Majorana phases.  
 463 The allowed regions for  $m_{\beta\beta}$  as a function of  $m_{\text{least}}$  are shown in Figure 2.5 as the  
 464 regions bounded by the black curves overlayed with the discovery probabilities of future  
 $0\nu\beta\beta$  decay experiments based on current neutrino data. Because of the possibility

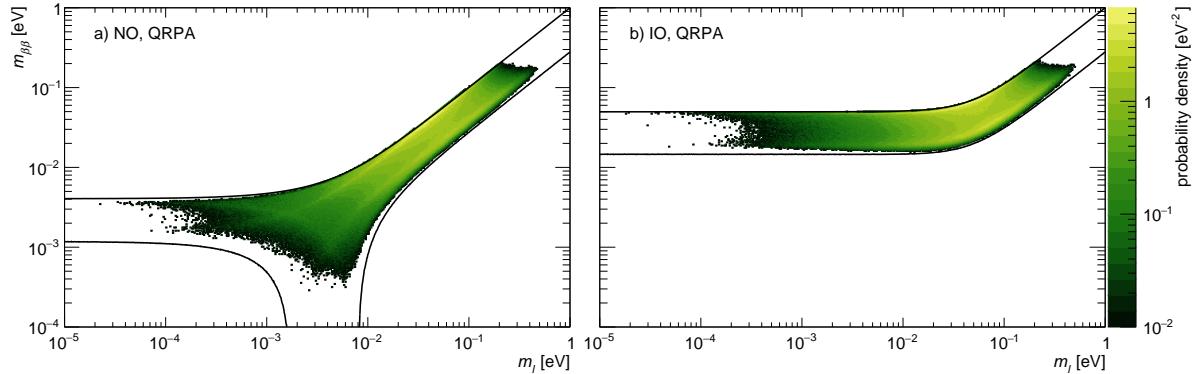


Figure 2.5: The discovery probabilities for the future generation of  $0\nu\beta\beta$  experiments as a function of  $m_{\beta\beta}$  and  $m_{\text{least}}$ .

465  
 466 of cancellation due to the unknown Majorana phases included in the sum specified by  
 467 Equation 2.21, the information gained is necessarily imperfect. Additionally, theoretical  
 468 uncertainty in the calculation of the nuclear matrix elements complicates the calculation  
 469 of  $m_{\beta\beta}$  from a measurement of  $0\nu\beta\beta$  half-life. Similar to cosmology there is a high degree

<sup>470</sup> of complementarity between direct measurements of the neutrino mass and  $0\nu\beta\beta$ . In  
<sup>471</sup> particular, a measurement of  $m_{\text{least}}$  to less than than 0.1 eV sensitivity provides significant  
<sup>472</sup> information for  $0\nu\beta\beta$  searches based of the discovery probabilities of Figure 2.5.

### <sup>473</sup> 2.5.3 Limits from Beta-decay

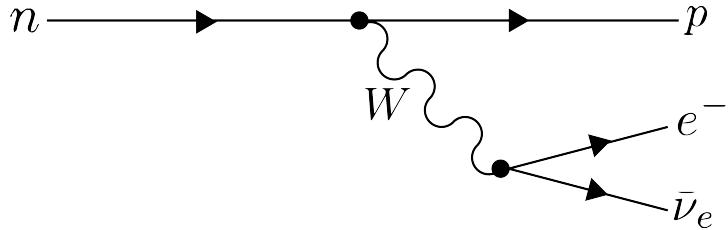


Figure 2.6: A Feynman diagram of beta decay

<sup>474</sup> Certain processes involving neutrinos, in particular beta-decay (see Figure 2.6), have  
<sup>475</sup> initial states with well-defined total energies and final states where the kinetic energies  
<sup>476</sup> of each of the particles can be measured with high accuracy and precision. Beta-decay  
<sup>477</sup> involves the decay of an unstable isotope where a neutron spontaneously converts to  
<sup>478</sup> a proton and emits and electron and anti-neutrino ("neutrino" for brevity) to conserve  
<sup>479</sup> charge and lepton number. Therefore, by applying the principles of energy and momentum  
<sup>480</sup> conservation a measurement of the kinematics of the final state can be used to constrain  
<sup>481</sup> the neutrino mass as proposed by Fermi in his 1934 description of nuclear beta-decay  
 (see Figure 2.7). Because the constraint on the neutrino mass from beta-decay depends

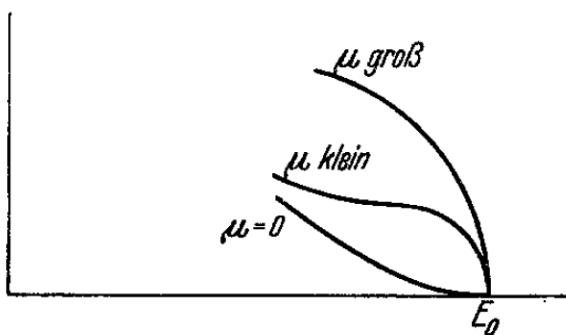


Figure 2.7: A figure from Fermi's 1934 paper on a theory of beta-decay depicting the kinetic energy spectrum of the emitted electron. The effect of the neutrino mass, written as  $\mu$ , is to distort the shape of the spectrum near the endpoint from the zero-mass spectrum.

483 only on the final state measurement capabilities, such measurements of the neutrino mass  
 484 are often referred to as model-independent or direct in contrast to constraints on the  
 485 neutrino mass from cosmology and  $0\nu\beta\beta$ .

486 The beta-decay isotope of choice for direct neutrino mass measurements with beta-  
 487 decay has been tritium ( $^3H_2$ ) for many decades, because it conveniently fulfills many  
 488 experimental requirements. Of upmost importance is a decay with a low Q-value, which  
 489 is the available kinetic energy based on the mass difference between the initial and final  
 490 states. The effect of a massive neutrino on the shape of the spectrum is magnified for  
 491 low Q-values and tritium decays have an unusually low Q-value of 18.6 keV. Additionally,  
 492 tritium beta-decay is a super-allowed decay, which means that it has a relatively short  
 493 half-life of 12.3 years making it easy to obtain a high-activity source with a relatively  
 494 small source mass. High-activity is desireable due to the low-activity near the tritium  
 495 spectrum endpoint. For tritium beta-decays only a factor of  $3 \times 10^{-13}$  of the decays  
 496 occur in the last 1 eV of the spectrum. Isotopes with Q-values lower than tritium are  
 497 known, but this is outweighed by exceedingly long half-lives leading to unobtainable  
 498 source masses.

499 Neutrino mass measurements using beta-decay measure the effect of the neutrino's  
 500 mass on shape of the electron's kinetic energy spectrum near the endpoint. The kinetic  
 501 energy spectrum (see Figure ??) is given by

$$\frac{d\Gamma}{dE} = \frac{G_F^2 |V_{ud}|^2}{2\pi^3} (G_V^2 + 3G_A^2) F(Z, \beta) \beta (E + m_e)^2 (E_0 - E) \\ \times \sum_{i=1,2,3} |U_{ei}|^2 [(E_0 - E)^2 - m_i^2]^{1/2} \Theta(E_0 - E - m_i), \quad (2.22)$$

502 where  $G_F$  is the Fermi coupling constant,  $V_{ud}$  is an element of the CKM matrix,  $E$  is  
 503 the kinetic energy of the electron,  $\beta$  is the velocity of the electron divided by the speed  
 504 of light,  $E_0$  is the endpoint energy assuming zero neutrino mass,  $F(Z, \beta)$  is the Fermi  
 505 function, and  $\Theta(E_0 - E - m_i)$  is the Heaviside function, which enforces energy conservation.  
 506 One can see that the decay spectrum is actually a combination of three spectra with  
 507 different endpoints based on the actual values of the neutrino mass eigenstates,  $m_i$ . This  
 508 results in "kinks" in the spectrum shape due to the overlapping spectra but such an  
 509 effect would be nearly impossible to resolve given the finite energy resolution of a real  
 510 experiment.

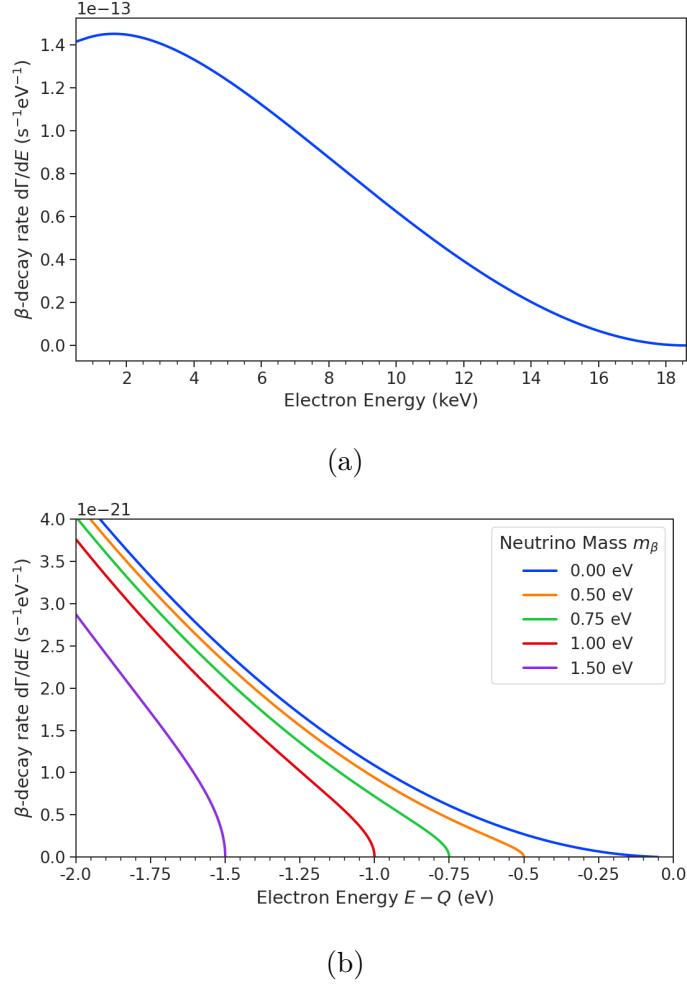


Figure 2.8: The tritium beta-decay spectrum. The affect of a massive neutrino on the spectrum is to change it's shape near the endpoint by an amount proportional to the size of the neutrino mass. This suggests that a sufficiently high-statistic and high-resolution measurement of the spectrum endpoint would be able to measure the neutrino mass.

511      The neutrino mass scale variable measured by beta-decay is given by

$$m_\beta^2 = \sum_i |U_{ei}|^2 m_i^2, \quad (2.23)$$

512    where  $m_{beta}$  is the effective mass of the neutrino in beta-decay or simply neutrino mass  
 513    for brevity. By assuming unitarity, the neutrino mass can be expressed in terms of  
 514    the PMNS matrix elements, squared mass differences, and the lightest neutrino mass

<sup>515</sup> eigenstate. For the normal mass ordering the equation is

$$m_\beta^2 = m_{\text{least}}^2 + |U_{e2}|^2 \Delta m_{21}^2 + |U_{e3}|^2 \Delta m_{31}^2, \quad (2.24)$$

<sup>516</sup> and for the inverted ordering the equation is

$$m_\beta^2 = m_{\text{least}}^2 + |U_{e1}|^2 (-\Delta m_{32}^2 - \Delta m_{21}^2) + |U_{e2}|^2 (-\Delta m_{32}^2). \quad (2.25)$$

<sup>517</sup> Therefore, a measurement of the neutrino mass in combination with neutrino mixing  
<sup>518</sup> parameters is effectively a measurement of  $m_{\text{least}}$ .

<sup>519</sup> Since the neutrino mass is small ( $< 1$  eV), it's effect on the spectrum is limited to  
<sup>520</sup> the endpoint region. The affect of a non-zero neutrino mass on the endpoint spectrum is  
<sup>521</sup> plotted for the reader in Figure 2.8. Resolving the small changes in the spectrum shape  
<sup>522</sup> requires an experimental technique with high statistics, excellent energy resolution, and  
<sup>523</sup> low background activity.

<sup>524</sup> The KATRIN collaboration, utilizing a large MAC-E (magnetic adiabatic collimation  
<sup>525</sup> with electrostactic) filter spectrometer recently obtained the best direct measurement of  
<sup>526</sup> the neutrino mass, with a 90% confidence upper limit of 0.8 eV. With more statistics the  
<sup>527</sup> KATRIN collaboration estimates an ultimate sensitivity to neutrino masses of 0.2 eV.

528 **Chapter 3**

529 **Direct Measurement of the Neutrino Mass**  
530 **with Cyclotron Radiation Emission Spec-**  
531 **troscopy**

532 **3.1 Introduction**

533 A promising technique for direct measurements of the neutrino mass beyond the projected  
534 limit of the ongoing KATRIN experiment is tritium beta-decay spectroscopy with an  
535 atomic tritium source [1]. Atomic tritium, combined with a large-volume, high-resolution  
536 energy measurement technique, is capable of measuring the neutrino mass with sensitivity  
537 below the 50 meV limit allowed by neutrino oscillations.

538 Cyclotron Radiation Emission Spectroscopy or CRES is a high-resolution energy  
539 measurement technique compatible with atomic tritium production and storage that can  
540 enable the next-generation of neutrino mass direct measurement experiments [2]. The  
541 Project 8 collaboration is currently engaged in a program of research and development  
542 (R&D) aimed at developing the technology necessary for a 40 meV sensitivity measurement  
543 of the neutrino mass using CRES and atomic tritium [3].

544 In Section 3.2 we provide an introduction to the basics of the CRES technique  
545 as well as the goals of the Project 8 experiment. Additionally, we sketch out the  
546 phased experiment development plan being implemented by Project 8 to build towards a  
547 next-generation neutrino mass experiment.

548 In Section 3.3 we give a brief overview of Phase II of the Project 8 experiment [4, 5],  
549 which completed early in 2023. Although the bulk of the work presented in this thesis is  
550 relevant to designs of future Project 8 experiments, a description of the work in Phase II  
551 provides useful context for the rest of the work.

552 In Section 3.4 we introduce a CRES measurement concept based on antenna arrays [6],

553 which could be the basis for the ultimate Project 8 neutrino mass experiment. A significant  
554 portion of the R&D efforts of Project 8 in Phase III were directed towards simulating  
555 and modeling this experimental concept in order to understand the achievable sensitivity  
556 to the neutrino mass.

557 Lastly, in Section 3.5 we introduce conceptual designs of pilot-scale experiments that  
558 combine atomic CRES with a large-volume CRES detection technique. This includes a  
559 design concept for an antenna array based experiment, but also a design for a resonant  
560 cavity based experiment. Resonant cavities are discussed in more depth in Chapter 6  
561 and have become the preferred choice for future CRES experiments in Project 8 over  
562 antenna arrays.

## 563 **3.2 CRES and Project 8**

### 564 **3.2.1 Cyclotron Radiation Emission Spectroscopy — CRES**

565 Of the standard physical quantities the one that can be measured with the highest  
566 precision is time and the inversely related quantity frequency. In fact it is often advan-  
567 tageous to convert measurements of other physical quantities like mass or length into  
568 frequency measurements due to the digital nature of frequency measurements that make  
569 them immune to many sources of noise. Atomic clocks, which operate by measuring the  
570 frequencies of various atomic transitions, have been used to measure time with astounding  
571 relative uncertainties of  $10^{-18}$  seconds. The extreme precision possible with frequency  
572 measurements is often summarized using the a quote from the Physicist Arthur Schawlow  
573 who said advise his students to "Never measure anything but frequency!".

574 Neutrino mass measurements using tritium beta-decay require us to measure a  
575 perturbation of the 18600 eV tritium endpoint to precisions as low as 0.1 eV, therefore, a  
576 spectroscopic technique with extremely high resolution is required for this measurement.  
577 Part of the reason that frequency measurements are capable of such high resolutions is  
578 that they are essentially counting measurements, which average the number of oscillations  
579 of a physical system over time. By observing a rapidly oscillating system over a sufficient  
580 length of time one can obtain essentially arbitrary precision on a frequency limited only  
581 by the time available for measurement and the SNR of the system.

582 In order to perform frequency-based high-resolution spectroscopy of the tritium beta-  
583 decay spectrum one needs to translate the kinetic energy of the electron into a frequency.  
584 The simplest way to accomplish this is to place a gaseous supply of tritium into a magnetic

585 field. When one of the atoms decays the resulting electron will immediately begin to  
 586 orbit around a magnetic field line at the cyclotron frequency which is proportional to  
 587 its kinetic energy (see Figure 3.1). The acceleration caused by the orbit leads to the  
 588 emission of cyclotron radiation that can be detected using an array of antennas or a  
 589 different RF sensor such as a resonant cavity. The frequency of the radiation gives the  
 590 electron's kinetic energy, which is used to build the beta-decay spectrum and measure  
 591 the neutrino mass. The name for this measurement technique is Cyclotron Radiation  
 592 Emission Spectroscopy or CRES.

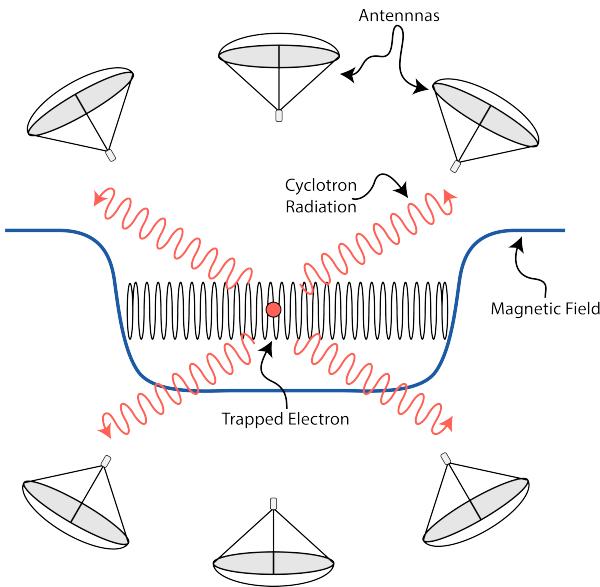


Figure 3.1: A cartoon illustration of the CRES technique. An electron is contained in a magnetic trap so that it's cyclotron radiation can be detected by an array of antennas. Detecting the cyclotron radiation allows us to measure its cyclotron frequency and determine its kinetic energy.

593 For non-relativistic particles the cyclotron frequency is only a function of the charge-  
 594 to-mass ratio of the particle, however, from the relativistic form of the cyclotron frequency

$$f_c = \frac{qB}{2\pi m_e \gamma} = \frac{1}{2\pi} \frac{qB}{m_e + E_{\text{kin}}/c^2}, \quad (3.1)$$

595 one can see that the kinetic energy ( $E_{\text{kin}}$ ) of the electron is directly proportional to the  
 596 inverse of the cyclotron frequency ( $f_c$ ). Electrons with kinetic energies of 18.6 keV are in  
 597 the weakly relativistic regime with  $\beta = \frac{v}{c} = 0.263$  and  $\gamma = 1.036$ .

598 The required frequency resolution needed for neutrino mass measurement can be

599 obtained by differentiating Equation 3.1,

$$\frac{df_c}{dE_{\text{kin}}} = \frac{1}{2\pi} \frac{-qBc^2}{(m_e c^2 + E_{\text{kin}})^2}, \quad (3.2)$$

600 from which we can obtain the relationship between fractional differences in energy and  
601 frequency,

$$\frac{df_c}{f_c} = \frac{1 - \gamma}{\gamma} \frac{dE_{\text{kin}}}{E_{\text{kin}}}. \quad (3.3)$$

602 Therefore, an energy precision of 1 eV for an 18.6 keV electron requires a frequency  
603 precision of approximately 2 ppm.

604 The minimum observation time required to achieve this resolution can be estimated  
605 using the uncertainty principle as formulated by Gabor. Electron's from tritium beta-  
606 decay experience random collisions with the background gas particles, which limits the  
607 uninterrupted radiation lifetime. The time between collision events, referred to as track  
608 length in the context of CRES measurements, is an exponentially distributed variable.  
609 Differences in the track lengths of a population of mono-energetic electrons leads to  
610 uncertainty or broadening in the distribution of measured frequencies proportional to  
611 the mean track length,  $\tau_\lambda$ . The resulting frequency distribution has a Lorentzian profile,  
612 whose width is given by the Gabor limit,

$$\tau_\lambda \Delta f_c = \frac{1}{2\pi} \implies \Delta f_c = \frac{1}{2\pi\tau_\lambda}. \quad (3.4)$$

613 The cyclotron frequency for a 18.6-keV electron in a 1 T field is approximately  
614 27 GHz, from which one can estimate the minimum observation time for 2 ppm frequency  
615 resolution at approximately 3  $\mu$ sec. The Gabor limit is not the true lower bound on the  
616 frequency resolution for a CRES signal, since it is based on the details of the Fourier  
617 representation of a time-series with a fixed length. If one takes the approach of fitting the  
618 CRES signal in the time-domain, then one finds that the limit on frequency precision is  
619 given by the Cramér-Rao lower bound (CRLB), which depends on both the track length  
620 as well as the SNR. In general, the CRLB allows for better precision on the cyclotron  
621 frequency, however, the Gabor limit provides an illustrative limit with the correct order  
622 of magnitude.

623 Ensuring that an electron remains under observation long enough so that it's frequency  
624 can be properly measured requires a magnetic trap. A magnetic trap is a local minimum  
625 in a background magnetic field generated an appropriate configuration of electromagnetic  
626 coils. Since magnetic fields can do no work, there is no danger of the magnetic trap

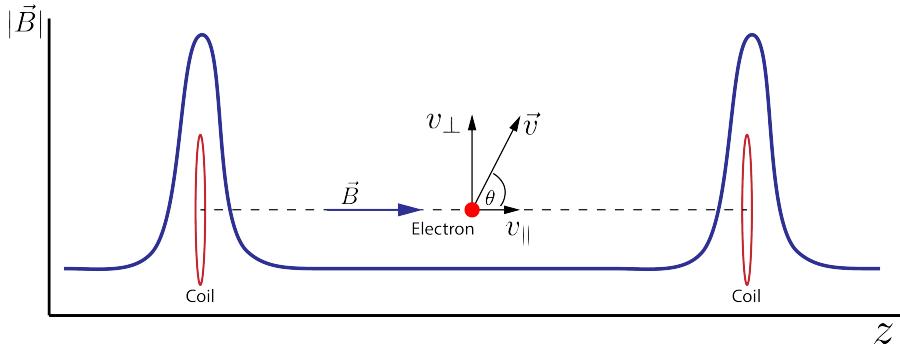


Figure 3.2: An illustration of an electron in a bathtub magnetic trap generated by two well-separated coils.

affecting the kinetic energy electron after it is emitted from the beta-decay. One common approach to creating a magnetic trap is the "bathtub" trap configuration, which in its simplest form consists of two high magnetic field pinch coils aligned on a central axis that are well separated (see Figure 3.2). This configuration produces a trap with a flat uniform bottom and relatively steep walls, which is ideal for CRES measurements.

Electrons produced in the trap oscillate back and forth between the trap walls at a frequency that depends upon the pitch angle, unless they are produced with pitch angles too small to be contained in the trap. Pitch angle is defined as the angle between the component of the electron's velocity perpendicular to the magnetic field and the component parallel to the magnetic field,

$$\tan \theta = \frac{v_{\perp}}{v_{\parallel}}. \quad (3.5)$$

The axial motion of the electron leads to variation in the cyclotron frequency due to the changing value of the magnetic fields. This leads to frequency modulation that generates sidebands in the cyclotron radiation spectrum. Resolving these sideband frequency components is necessary for a complete reconstruction of the CRES signal in the experiment.

Electrons trapped in a cylindrically symmetric trap have three primary components of motion (see Figure 3.3). The dominant component, typically with the highest frequency, is the electron's cyclotron orbit, which encodes information on the electron's kinetic energy. Axial motion from the electron's pitch angle leads to frequency modulation but also a shift in the average magnetic field experienced by an electron. This leads to a correlation between the kinetic energy of the electron and the pitch angle depending on the particular shape of the magnetic trap, which can negatively impact energy resolution.

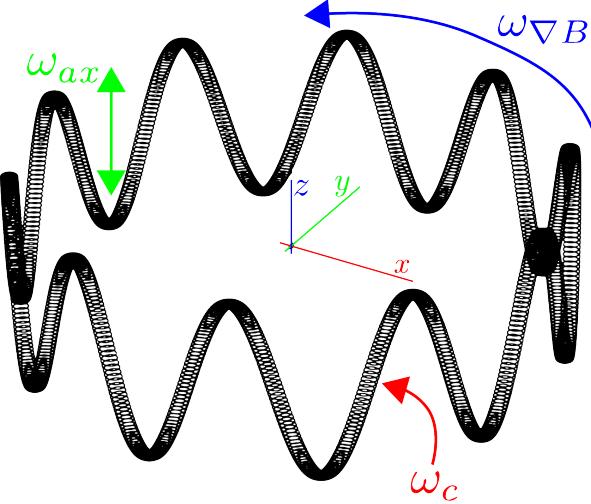


Figure 3.3: A plot of the main components of an electron's trajectory in a cylindrically symmetric trap.

649 To reduce this correlation one must engineer the trap to have a flat bottom with very  
 650 steep wall both of which are more easily achieved with a small aspect ratio bathtub  
 651 trap. Radial gradients in the trap oftentimes leads to a third component of motion called  
 652 grad-B drift. The equation for the drift velocity is

$$\mathbf{v}_{\nabla B} = \frac{m_e v_\perp^2}{2qB} \frac{\mathbf{B} \times \nabla B}{B^2}. \quad (3.6)$$

653 These additional components of motion all influence the shape of the CRES signal so  
 654 modeling their effects is critical to proper measurement of the kinetic energy.

655 The total power of the radiation emitted by an electron in a free-space environment  
 656 is given by the Larmor equation

$$P(\gamma, \theta_p) = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{q^2 \omega_c^2}{c} (\gamma^2 - 1) \sin^2 \theta_p, \quad (3.7)$$

657 where  $\omega_c$  is the cyclotron frequency multiplied by  $2\pi$  and  $\theta_p$  is the pitch angle to distinguish  
 658 it from the spherical angle coordinate. A single electron with a  $90^\circ$  pitch angle and  
 659 18.6 keV of kinetic energy in a 1 T magnetic field emits a total radiation power of 1.2 fW,  
 660 which is quite small compared with typical RF systems, furthermore, one is typically  
 661 only able to receive a fraction of this total power with an antenna or other detection  
 662 system. Therefore, RF systems in CRES experiments must be operated at cryogenic  
 663 temperatures to limit the noise power such that adequate SNR can be achieved for signal  
 664 detection and reconstruction. Alternatively, longer tracks enable detection of weaker

665 signals due to the increase in the total signal energy available for the detection algorithm.

### 666 **3.2.2 The Project 8 Collaboration**

667 The Project 8 collaboration is a group of institutions in the United States and Germany  
668 aiming to measure the neutrino mass by developing a novel spectrometer technology  
669 based on CRES. In the ultimate Project 8 experiment the CRES technique will be used  
670 to measure the beta-decay spectrum using a large source of atomic tritium sufficient to  
671 achieve the required statistics in the last  $O(10)$  eV of the decay spectrum. Project 8 is  
672 targeting a neutrino mass sensitivity below 50 meV, which exhausts the range of possible  
673 neutrino masses under the inverted hierarchy and is a factor of four less than sensitivity  
674 projections for the ongoing KATRIN experiment.

675 Project 8's proposed experiment requires the development of two novel technologies:  
676 the production and trapping of a source of atomic tritium on cubic-meter scales and  
677 technology to enable CRES measurements of individual electrons in the same volume.

### 678 **Atomic Tritium**

679 Previous measurements of the tritium beta-decay spectrum for neutrino mass measure-  
680 ments have all relied on a sources of molecular tritium for their measurements due to the  
681 numerous practical and technical challenges associated with the production and storage  
682 of hydrogen isotopes.

683 To produce atomic hydrogen one must supply sufficient energy to the tritium molecule  
684 to break the molecular bond between. Common approaches to this include the use of hot  
685 coaxial filament atom crackers as well as plasma atom sources. Both approaches heat the  
686 tritium atoms to temperatures  $> 2500$  K, which must then be cooled to temperatures  
687 on the order of a few mK so that the tritium atoms can be trapped. Cooling the atoms  
688 requires the construction of a large tritium infrastructure and cooling system that can  
689 supply a source of cold atoms to the trap.

690 Once cold tritium atoms are produced they cannot make contact with any surfaces  
691 to avoid recombination of the atoms to molecules. Therefore, a magnetic trap is required  
692 to store the atoms for a sufficient length of time that they have a chance to decay before  
693 escaping the trap. Trapping the atoms at this scale requires the construction of a large  
694 and complex magnet system that must be cooled to cryogenic temperatures to avoid  
695 heating of the atoms, which leads to their escape from the trap.

696 The significant experimental complexity caused by atomic tritium makes a molecular

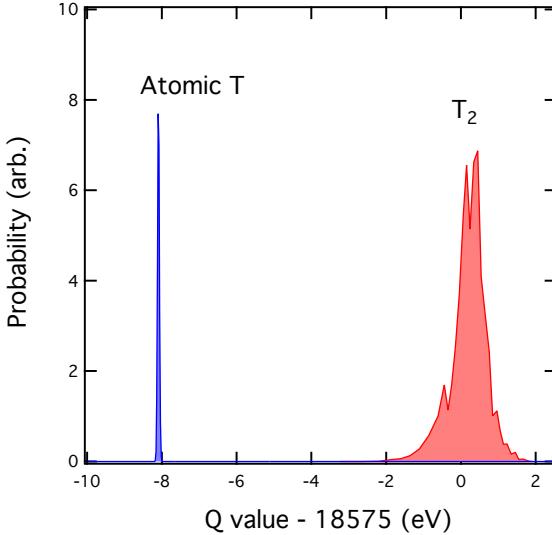


Figure 3.4: A plot of the final state distributions of atomic and molecular tritium. The final state distribution provides the primary contribution to the width of the molecular spectrum whereas thermal doppler broadening is responsible for the width of the atomic spectrum.

source the obvious choice from practical considerations. However, the drawback of molecular tritium for neutrino mass measurement is the irreducible broadening in the electron's kinetic energy due to the final state spectrum of molecular tritium (see Figure 3.4). The broadening of the final state spectra has a RMS amplitude of 436 meV caused by variation in the final vibrational state of the daughter molecule. For atomic tritium the primary sources of broadening in the final state spectrum are magnetic hyperfine splittings ( $O(10^{-5})$  eV) and thermal Doppler broadening caused by the motion of the trapped atom. For atomic tritium at a temperature of 1 mK thermal broadening is the dominant contribution, providing about 1 meV RMS of broadening to the electron's kinetic energy.

The larger energy broadening with molecular tritium leads to an irreducible statistical uncertainty that limits the achievable sensitivity to approximately 100 meV at 90% confidence. For previous direct measurements of the neutrino mass this uncertainty is an insignificant contribution to the overall uncertainty budget, however, for experiments like Project 8 atomic tritium is a key component to the success of the experiment.

## 712 CRES for Neutrino Mass Measurement

713 Several promising features of the CRES technique make it a particularly attractive choice  
714 for a next generation neutrino mass measurement experiment. For example, with a CRES

715 experiment the volume of the source gas can be the same as the volume of the CRES  
716 spectrometer. This is due to the fact that CRES is a remote-sensing technique that can  
717 observe the energy of the electron without altering its trajectory or directly interacting  
718 with the electron. Given that tritium gas is transparent to cyclotron radiation the kinetic  
719 energies of electrons can be measured with an appropriate sensing technology, such as a  
720 cavity or antenna array, located directly outside the atom trapping volume.

721 The current state-of-the-art tritium beta-decay spectroscopy experiment, KATRIN,  
722 utilizes the magnetic adiabatic collimation with an electrostatic filter (MAC-E filter)  
723 technique to measure the beta-decay spectrum of molecular tritium. In this approach, a  
724 source of molecular tritium is located outside of the spectrometer. When a beta-decay  
725 occurs the electron must exit the tritium source and travel through the MAC-E filter  
726 before it can be detected on the other side of the filter using a charge sensor. With this  
727 approach the measurement statistics are limited by the transverse areas of the tritium  
728 source and MAC-E filter due to the need to travel through the detector without scattering.  
729 This scaling is less favorable than the volumetric scaling that one has with CRES due to  
730 the ability to co-locate source and detector.

731 Another promising aspect of the CRES technique is the inherently high precision  
732 of frequency based measurements. The endpoint of the molecular tritium beta-decay  
733 spectrum is approximately 18.6 keV, which dwarfs the neutrino mass scale of  $< 1 \text{ eV}/c^2$   
734 by at least a factor of  $10^5$ . Measuring the effect of such a small mass on a high energy  
735 electron requires excellent energy resolution. Since frequency measurements are essentially  
736 counting measurements they are intrinsically quite accurate due to the ability to measure  
737 the cyclotron frequency by effectively averaging over millions of cyclotron orbits. Using  
738 off-the-shelf RF components its is possible to achieve part-per-million accuracy on the  
739 kinetic energy with the CRES technique.

740 A final aspect of the CRES technique that is attractive for a next-generation experi-  
741 ment is the relative immunity to backgrounds. Since CRES operates via non-destructive  
742 measurements of the electron's cyclotron frequency potential sources of background elec-  
743 trons are effectively filtered out by limiting the frequency bandwidth of the measurement.  
744 The fiducial volume of the experiment is free from any surfaces that could introduce  
745 stray electrons and electrons from sources outside the fiducial volume can be prevented  
746 from entering the experiment.

747 **Neutrino Mass Sensitivity Goals**

748 Project 8's ultimate goal is to combine CRES with atomic tritium to measure the neutrino mass with 40 meV sensitivity at the 90% confidence level (see Figure 3.5). This sensitivity

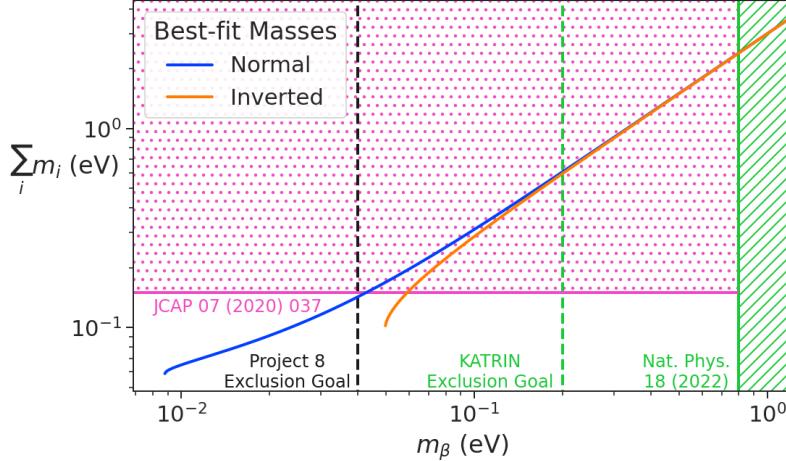


Figure 3.5: Neutrino mass exclusion plot including limits from cosmological measurements and the KATRIN experiment. Allowed ranges for neutrino masses under the normal and inverted hierarchies are shown as the blue and orange lines respectively. The black dashed line shows Project 8's goal neutrino mass sensitivity for the Phase IV experiment.

749  
 750 is sufficient to fully exhaust the range of allowable neutrino masses under the inverted  
 751 neutrino mass ordering regime and is approximately an order of magnitude less than the  
 752 projected final sensitivity of the KATRIN experiment. Excluding the full neutrino mass  
 753 parameter space would require a sensitivity an order of magnitude lower than what is  
 754 proposed by Project 8, which would require an experiment whose size and complexity  
 755 are currently well beyond proposals for the next-generation of neutrino mass direct  
 756 measurement experiments.

757 **3.2.3 Project 8 Phased Development Plan**

758 Reaching 40 meV sensitivity will require the simultaneous development and eventually  
 759 combination of two novel technologies. The first is the technology required to supply a  
 760 source of atomic tritium of the appropriate size, density, purity, and temperature along  
 761 so that the atoms can be trapped and their beta-decays measured in the spectrometer.  
 762 The second is a CRES measurement technology that is both compatible with the tritium  
 763 atom trap and is capable of reconstructing CRES events with sufficient energy resolution

764 to achieve the required sensitivity.

765 These technologies require a significant up-front research and development (R&D)  
766 investment to build-out the required capabilities for a 40 meV CRES experiment. There-  
767 fore, Project 8 is following a phased experiment plan in which incremental progress can  
768 be made towards the ultimate goal of a 40 meV neutrino mass measurement with CRES.

## 769 **Phase I and II: Proof of Principle and First Tritium Measurements**

770 The earlier phases of the Project 8 experiment, Phase I and II, were focused on demon-  
771 stration and development of the CRES technique itself as well as a proof-of-principle  
772 measurement of the neutrino mass using the CRES technique.

773 In Phase I, Project 8 performed a proof-of-principle measurement of the  $^{83m}\text{Kr}$   
774 spectrum using CRES, which marked the first ever energy spectrum measurement with  
775 CRES. The experiment included all of the main components expected for the full-scale  
776 version of the experiment. An electron source consisting of a gas of  $^{83m}\text{Kr}$  was supplied  
777 to a waveguide gas cell constructed out of a segment of WR-42 waveguide and sealed  
778 with Kapton windows at the top and bottom. A magnetic trapping region was created  
779 in the waveguide cell using a single electromagnetic coil wrapped around the waveguide  
780 which provided a trapping volume on the order of a few cubic-millimeters. Detection of  
781 the cyclotron radiation was performed by connecting the waveguide cell to an additional  
782 segment of waveguide that transmitted the radiation to a cryogenic amplifier.

783 Success in Phase I was achieved with the 2014 publication of the measured  $^{83m}\text{Kr}$   
784 conversion spectrum, which contains a mono-energetic 17.8-keV as well as several other  
785 conversion lines at higher energies. Publication of this result marked the official end of  
786 Phase I and the start of Phase II in which Project 8 shifted its focus to the demonstration  
787 of the first tritium beta-decay spectrum using CRES. Phase II successfully concluded  
788 in 2023 with the submission of the papers demonstrating the first tritium beta-decay  
789 spectrum endpoint and neutrino mass measurement using CRES. For more information  
790 on Phase II please see Section 3.3.

## 791 **Phase III: Research and Development and a Pilot-scale Experiment**

792 With the completion of Phase II Project 8 has shifted into a phase focused on the  
793 construction of an experiment that demonstrates all the technologies of the final experi-  
794 ment in Phase IV. The goal for this pilot-scale experiment is to successfully retire all  
795 technological and engineering risks associated with the Phase IV experiment, while being

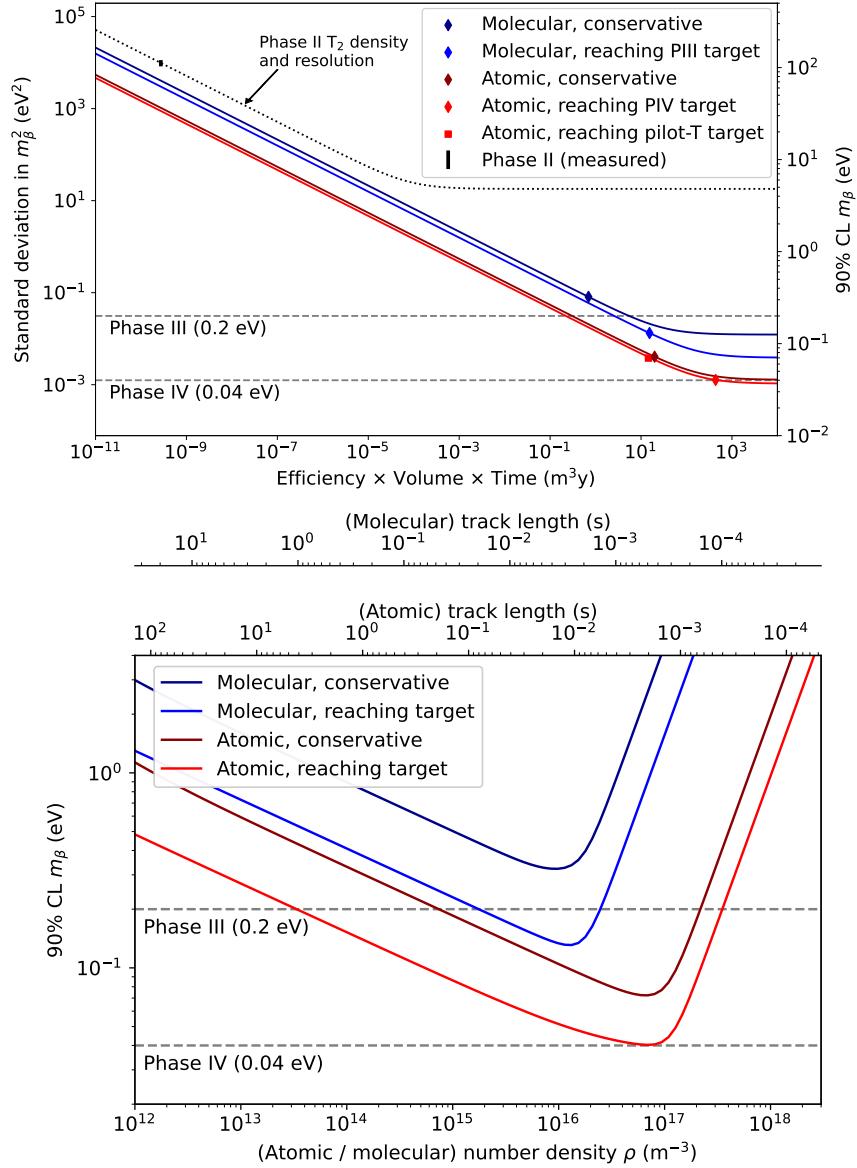


Figure 3.6: Sensitivity calculations for a cavity based CRES experiment that demonstrate the neutrino mass measurement goals of the Project 8 collaboration throughout the phased development plan. The blue tinged curves indicate molecular tritium sources and the red tinged curves indicate atomic tritium sources. In the current plan Phase III contains two tritium experiments. The first is the Low-frequency Apparatus (LFA) which is a molecular tritium experiment and the second is the atomic tritium pilot-scale experiment that ends Phase III. The sensitivity of these experiments is primarily a function of statistics, however, there is a critical density beyond which CRES electrons do not have enough time to radiate between collisions for a high-resolution frequency measurement leading to worse sensitivity.

796 a scientifically interesting experiment in it's own right that has sensitivity to neutrino  
797 masses on par with KATRIN's final projected sensitivity.

798 Phase III R&D is divided into two equally important efforts — atomic tritium and  
799 CRES detection techniques. Atomic tritium development in Phase III includes the  
800 development of all aspects of the tritium system required for the pilot-scale experiment.  
801 This includes the production of tritium atoms, atomic cooling and recirculation systems,  
802 purity and isotope concentration monitoring, and trapping. Currently, Project 8 is  
803 operating small scale demonstrator systems developing atom crackers to show that atom  
804 production at the estimated rates needed for Phase IV is achievable. Future efforts  
805 will continue the current developments on atom production and expand to include  
806 demonstrations of atomic cooling with an evaporative beam line as well as atom trapping  
807 using Halbach magnet arrays.

808 The need for new CRES detection techniques is driven by the drastic increase in scale  
809 from Phase II to the Phase IV and the pilot-scale experiments. The physical volume  
810 used for CRES in Phase II was on the order of a few cubic-centimeters, and achieving  
811 Project 8's sensitivity target of 40 meV requires an experiment volume on the multi-cubic  
812 meter scale. Therefore, the waveguide gas cell CRES detection technique used in Phase  
813 II is not a feasible option for the future of Project 8 due to it's inability to scale to the  
814 required size.

815 Two alternative CRES detection techniques have been proposed for the pilot-scale  
816 experiment — antenna arrays and resonant cavities (see Section 3.4 and Chapter 6  
817 respectively). Both approaches have relative advantages and disadvantages, however,  
818 the improved understanding of the antenna array and cavity approaches to CRES in  
819 the recent years has led to cavities being the preferred technology for the pilot-scale  
820 experiment due to the estimated reduced cost and complexity of this approach. Since a  
821 large degree of the work presented in this thesis is focused on the development of the  
822 antenna array CRES technique as well as the design of demonstrator experiments, we  
823 described the proposed R&D plan for antenna array CRES in Phase III in Section 3.4.

824 Cavity CRES R&D in Phase III consists of a series of demonstrator experiments  
825 intended to demonstrate cavity CRES at a variety of scales and magnetic fields using  
826 electrons from  $^{83m}\text{Kr}$ , an electron gun, and potentially molecular tritium sources. The  
827 near-term cavity effort in Project 8 is the cavity CRES apparatus (CCA), which is a  
828 small-scale cavity experiment operating near 26 GHz, that will perform the first CRES  
829 measurements using a small cavity. This experiment will pave the way towards larger  
830 scale cavity experiments in preparation for the eventual pilot-scale tritium experiment.

831        The pilot-scale experiment is the first experiment, which will combine atomic tritium  
832        and large-volume CRES detection in the same experiment. It will directly demonstrate  
833        all the technologies required for Phase IV such that no technical risks remain for scaling  
834        the experiment to required scale. A robust approach to scaling the pilot-scale experiment  
835        is to simply build multiple copies of it for the Phase IV experiment.

836        **Phase IV: Project 8's Ultimate Neutrino Mass Experiment**

837        The design of Phase IV should be a direct extension of the pilot-scale CRES experiment  
838        that marks the official end of Phase III (see Section 3.5). The Phase IV experiment  
839        represents the final experiment in the Project 8 neutrino mass measurement experiment  
840        plan and will have sensitivity to neutrino masses of 40 meV.

841        **3.3 Phase II: First Tritium Beta Decay Spectrum and**  
842        **Neutrino Mass Measurement with CRES**

843        In Phase II Project 8 demonstrate the first ever measurement of the tritium beta-decay  
844        spectrum endpoint using the CRES technique, which lead to the first neutrino mass  
845        measurement by the Project 8 collaboration. This milestone was made possible by  
846        many improvements in the CRES technique and more developed understanding of CRES  
847        systematics, which takes an important first step towards larger scale measurements of  
848        the tritium beta-decay spectrum with CRES. In this section, I shall briefly describe some  
849        the important elements of the Phase II experiment, with the goal of contextualizing the  
850        research and development efforts for Phases III and IV of Project 8. For more complete  
851        descriptions of the work that lead to Project 8's Phase II results please refer to the many  
852        Phase II papers produced by the collaboration.

853        **3.3.1 The Phase II CRES Apparatus**

854        **Magnet and Cryogenics**

855        The magnetic field for the the Phase II experiment is provided by a nuclear magnetic  
856        resonance (NMR) spectroscopy magnet with a central bore diameter of 52 mm (see Figure  
857        3.7). The magnet produces a background magnetic field with an average value of 0.959 T  
858        and a 10 ppm variation across the bore diameter achieved using several shim coils built  
859        into the magnet. Using an external NMR field probe the variation of the magnetic field

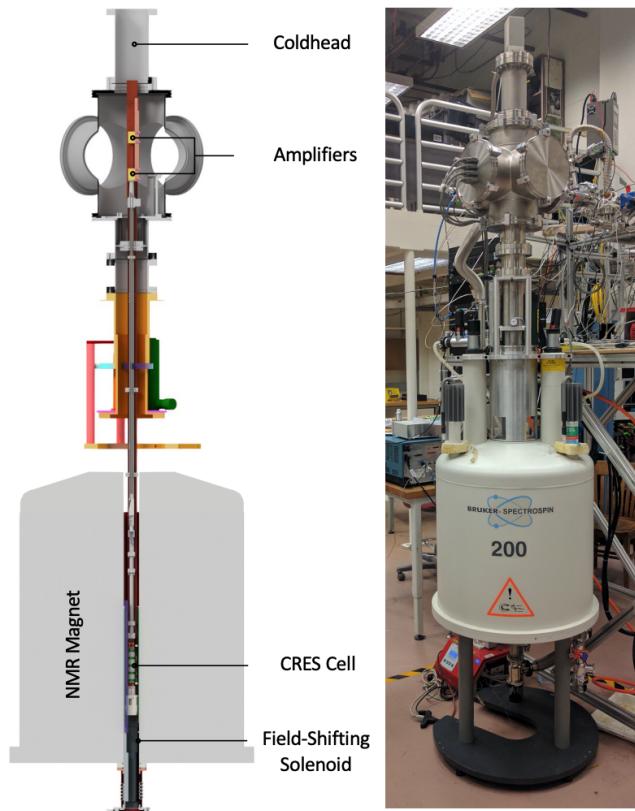


Figure 3.7: The Phase II CRES apparatus used to perform the first measurement of the tritium beta-decay spectrum using CRES.

860 along the vertical axis of the magnet bore was measured to obtain an accurate model of  
 861 the magnetic field so that the CRES cell could be positioned for optimal magnetic field  
 862 uniformity.

863 An external solenoid magnet was installed inside the magnet bore to provide the  
 864 ability to shift the magnitude of the background magnetic field by values on the order of  
 865 a few mT. The solenoid has inside diameter of 46 mm and a length of 350 mm, which  
 866 terminates in a vacuum flange that allows it to be inserted into the NMR magnet bore  
 867 from the bottom. By shifting the value of the magnetic field by a few mT, the cyclotron  
 868 frequencies of electrons produced by the 17.8 keV  $^{83m}\text{Kr}$  internal-conversion line can be  
 869 shifted over a range of frequencies on the order of 100 MHz. This allows one to study the  
 870 frequency dependent behavior of multiple CRES systematics such as detection efficiency  
 871 that directly affect the measured shape of the tritium spectrum.

872 The inside of the magnet bore diameter was pumped down to a vacuum of less than  
 873 10  $\mu\text{torr}$  using a turbomolecular pump, which allows for cryogenic cooling of the CRES

cell and RF system. Cooling power was supplied to the Phase II apparatus using a cryopump with its coldhead mounted above the primary magnet and CRES cell. This arrangement allowed for sufficient cooling power to be delivered to the amplifiers to cool them to a temperature of  $\approx 40$  K, while keeping the amplifiers far enough from the magnet so as not to be damaged by the large field strength. Thermal contact between the coldhead, amplifiers, RF system, and CRES cell is achieved using a copper bar that runs the full length of the apparatus. To prevent freeze-out of  $^{83m}\text{Kr}$  on the walls of the CRES cell a separate heater was installed to keep the CRES cell near a temperature of 85 K during the operation of the experiment.

### 883 CRES Cell

884 Located in the most uniform region of the magnetic field is the CRES cell, which is the  
 885 region of the apparatus where radioactive decays of  $^{83m}\text{Kr}$  and  $T_2$  emit electrons that can  
 be trapped and measured using CRES (see Figure 3.8). The CRES cell is manufactured

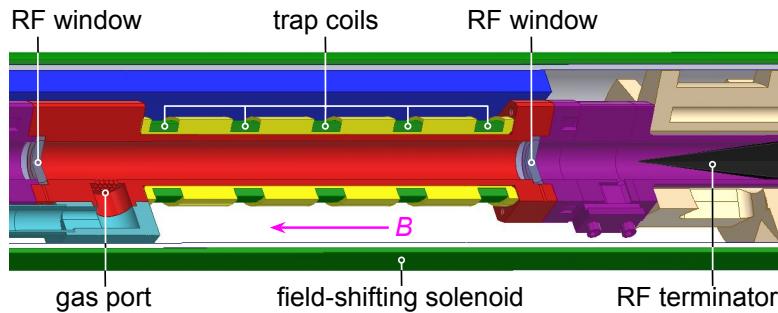


Figure 3.8: Diagram of the CRES cell portion of the Phase II apparatus.

886  
 887 from a segment of cylindrical waveguide designed to operate at K-band frequencies  
 888 near 26 GHz. The diameter of the waveguide determines which resonant modes of the  
 889 waveguide will couple to the electron and transmit its radiation to the amplifiers. For  
 890 Phase II a waveguide diameter of 1 cm was selected, which allows electrons to couple to  
 891 the TE<sub>11</sub> and TM<sub>01</sub> cylindrical waveguide modes. To reduce complexity in modeling and  
 892 analyzing the CRES data, it is ideal to select a diameter that prevents electrons from  
 893 coupling to higher-order waveguide modes beyond the fundamental TE and TM modes.

894 Around the exterior of the cylindrical waveguide are several magnetic coils used  
 895 to produce magnetic traps inside the CRES cell volume. Without a magnetic trap  
 896 electrons produced from decays inside the CRES cell quickly impact the cell wall, which  
 897 prevents a measurement of their cyclotron frequency using CRES. Each coil along the  
 898 length of the waveguide produces a separate trap that is approximately harmonic in

shape. By independently controlling the currents provided to each coil the traps could be configured to have equal values of the magnetic field at the trap bottom despite a variable background magnetic field from the NMR magnet.

Two primary magnetic trap configurations were used during the Phase II experiment. The first was a shallow trap configuration used primarily for it's high energy resolution to study systematics using  $^{83m}\text{Kr}$  decays, and the second was a deeper trap that could trap a higher percentage of pitch angles. The trade-off with this trap is that the higher trapping efficiency comes at the cost of lower energy resolution due to the greater variation in pitch angle. The deep trap was the trap used to measure the tritium beta-decay spectrum in Phase II.

The source gases were delivered into the CRES cell through a gas port located near the top end of the cylindrical waveguide. To prevent the gases from escaping the cell, vacuum tight RF transparent windows are needed to contain the tritium and krypton source gas across a 1 atm pressure differential, while still transmitting the cyclotron radiation without distortion. The crystalline material,  $\text{CaF}_2$ , which has a thermal expansion coefficient similar to that of copper, was used for this purpose in the CRES cell. Two windows, each 2.4 mm thick, were used to seal off the ends of the CRES cell. The thickness of 2.4 mm corresponds to half of a cyclotron wavelength when one accounts for the permittivity of  $\text{CaF}_2$ .

## RF System

The RF system in the Phase II apparatus transferred the cyclotron radiation from the CRES cell to the receiver chain. The receiver chain performs the down-conversion and digitization required to obtain signals that can be analyzed to determine the cyclotron frequencies of electrons in the CRES cell (see Figure 3.9).

Below the CRES cell, at the bottom of the Phase II apparatus, is a tickler port and waveguide terminator. The tickler port is used to inject signals into the CRES cell and RF system for testing and calibration purposes. The waveguide terminator is designed to absorb cyclotron radiation emitted by electrons that transmits out of the bottom of the CRES cell. This lowers the total power received from electrons in the CRES cell, since all the energy radiated downwards is absorbed into the terminator. Earlier iterations of the Phase II apparatus used an RF short in this location that reflected this power up towards the amplifiers, however, interference between the upward traveling and reflected radiation led to a disappearance in the signal carrier that made reconstruction impossible.

Radiation traveling upward passes through the  $\text{CaF}_2$  window passes through a  $\lambda/4$

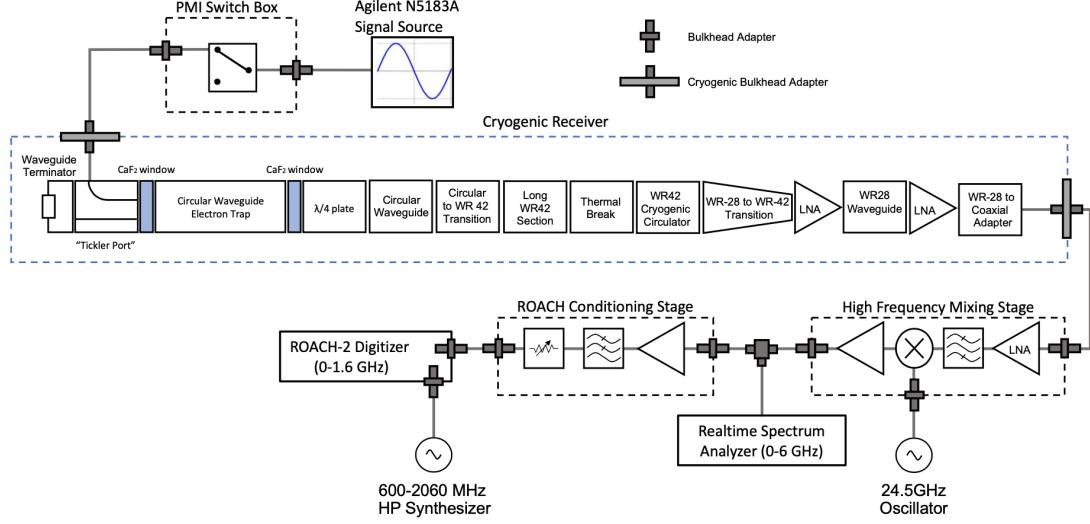


Figure 3.9: RF system diagram for the Phase II apparatus.

plate, which transforms the circularly polarized cyclotron radiation into linear polarization. The linearly polarized fields next travel through a segment of circular waveguide that transitions into a long segment of WR-42 waveguide that carries the fields out of the high magnetic field region. A thermal break segment is included, which consists of a segment of gold-plated stainless steel WR-42 waveguide, to help thermally isolate the relatively warm CRES cell from the colder amplifiers. The radiation then passes through a cryogenic circular, which prevents signals reflected from the amplifiers from interfering with the CRES cell before a WR-42 to WR-28 transition connects the waveguide to the first of the cryogenic amplifiers. The radiation passes through two cryogenic amplifiers before being coupled to a coaxial termination at the top of the Phase II apparatus.

The coaxial cable transfers the cyclotron radiation signals to a high-frequency mixing stage that performs an analog frequency down-conversion using a 24.5 GHz LO. Two forms of digitization can be used at this stage to readout the CRES data. One is a real-time spectrum analyzer that digitizes the CRES signal data in time-domain and computes the frequency spectrum in real-time, which allows for direct visualization of CRES signal spectrograms as the experiment is running. The real-time spectrum analyzer is most useful for taking small amount of streamed data for debugging and analysis of the system. The other method, which was used to collect the majority of the CRES data in Phase II, is a ROACH-2 FPGA and digitizer system. The ROACH system consists of a fast ADC that samples the CRES signal data at 3.2 GSps. Internal digital down-conversion stages implemented in the FPGA perform a mixing operation that reduces the bandwidth of the

954 CRES signals to 100 MHz. The FPGA implements a 8192 sample FFT and packetizes  
955 time and frequency domain records in parallel. The packetized data is then transferred  
956 from the ROACH to be analyzed by the data-processing pipeline.

### 957 3.3.2 CRES Track and Event Reconstruction

#### 958 Time-Frequency Spectrogram

959 The online data-processing is intended to identify interesting data that could contain  
960 CRES signals using a software real-time triggering algorithm. Interesting segments of  
961 data identified by this algorithm are collected into files that are transferred to a server for  
962 offline processing and analysis. The data files contain a continuous series of time-domain  
963 samples, broken into a set of records, which are 4096 samples long. The time-series is  
964 made up of 8-bit IQ samples acquired at 100 MHz.

965 Each time-series record is accompanied by an associated frequency spectrum consisting  
966 of 4096 frequency bins approximately 24.4 kHz wide, which is represented as a power  
967 spectral density. The individual frequency spectra can be organized temporally to create  
968 a time-frequency spectrogram that represents the evolution of the cyclotron frequency  
spectrum over the course of the CRES event (see Figure 3.10). The time-frequency

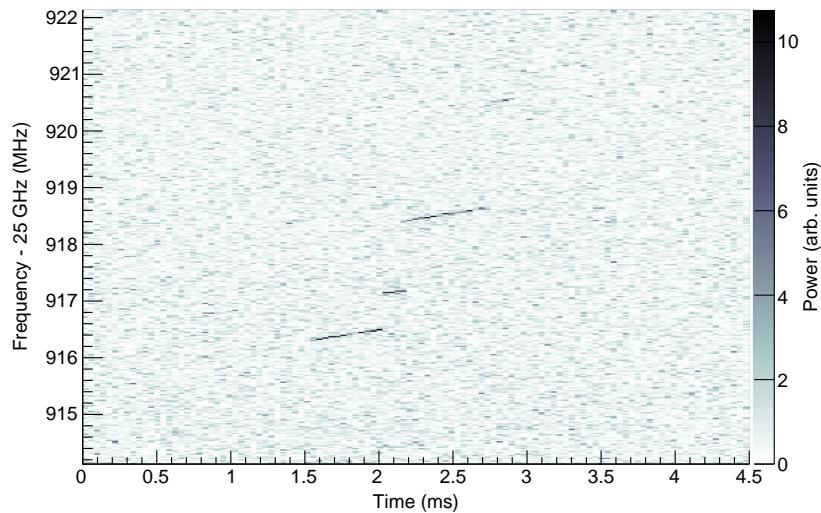


Figure 3.10: The time-frequency spectrogram of a tritium CRES event in the Phase II apparatus.

969 spectrogram is represented as a two-dimensional image where the color of each pixel is  
970 proportional to the power spectral density. Each vertical slice of pixels in the image  
971

972 represents a frequency spectrum, therefore, each horizontal bin represents the data  
973 obtained over a duration of  $4096 \times 0.01 \text{ MHz}^{-1} = 40.96 \mu\text{sec}$ .

## 974 CRES Event Data Features

975 Phenomenologically, a CRES signal appears as a sinusoidal signal whose frequency slow  
976 increases ("chirps") over time. Axial motion of the electron in the trap leads to the  
977 formation of frequency sidebands that surround the more powerful carrier frequency, due  
978 to doppler modulation of the electron's frequency as it bounces between the walls of the  
979 magnetic trap. The critical piece of information that must be extracted from the track  
980 and event reconstruction procedure is the carrier frequency, since it is this frequency  
981 that gives the cyclotron frequency and thus the kinetic energy. While axial motion from  
982 non- $90^\circ$  pitch angles does change the average magnetic field experienced by an electron  
983 and, therefore, changes the cyclotron frequency. We were not able to resolve sidebands  
984 in Phase II, so a correction for the effect of the pitch angle on the cyclotron frequency  
985 was not possible.

986 In the time-frequency spectrogram representation the chirping carrier frequency  
987 appears as a linear track of high-power frequency bins (see Figure 3.10). The vertical  
988 slope of the tracks is caused by the emission of energy from the electron in the form  
989 of cyclotron radiation, therefore, the size of the slope parameter is directly proportional  
990 to the Larmor power. The continuous track is periodically interrupted by random  
991 jumps to higher frequency and lower energy caused by random inelastic collisions with  
992 background gas molecules. The length of a track is an exponentially distributed variable  
993 whose mean value is inversely proportional to the gas density. The size of the frequency  
994 discontinuities is directly proportional to the energies of the rotational and vibrational  
995 states of background gas species such as  $\text{CO}_2$ .

996 A CRES event refers to the collection of tracks produced by a trapped electron until  
997 it inevitably scatters into a pitch angle that can no longer be trapped. The goal of track  
998 and event reconstruction is to first identify the set of tracks present in a time-frequency  
999 spectrogram that represents a segment of data acquired in the Phase II apparatus. These  
1000 tracks must then be clustered into events from which we can determine the first track  
1001 produced by the electron and thus estimate its starting cyclotron frequency and kinetic  
1002 energy.

1003 **Track Reconstruction**

1004 The first step in this process is the identification of tracks in the time-frequency spectro-  
1005 gram, which is essentially an image processing feature identification task. The first step  
1006 in the track finding procedure is to normalize the power spectral density based on the  
1007 average noise power to obtain the time-frequency spectrogram in the form of normal-  
1008 ized, unitless power. Next a power threshold is applied to the normalized spectrogram  
1009 where only bins that have a signal-to-noise ratio greater than five are selected to build  
1010 tracks. In this case signal-to-noise ratio is defined as the ratio between the normalized,  
1011 unitless power of a bin divided by the average normalized power across the full frequency  
1012 spectrum.

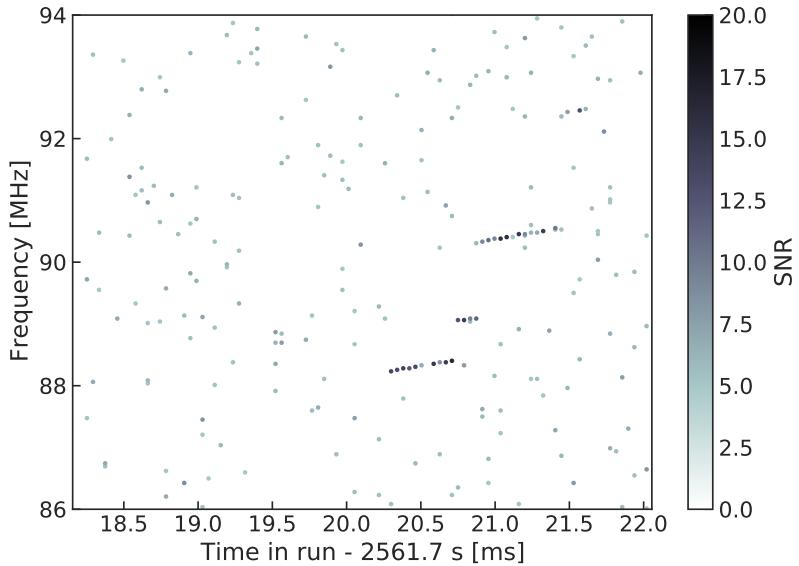


Figure 3.11: The sparse spectrogram obtained by placing a power cut on the raw spectrogram shown in Figure 3.10.

1013 The spectrogram produced by this power cut, termed the sparse spectrogram, consists  
1014 only of a sparse collection of high-power frequency bins that could be part of a CRES  
1015 signal track (see Figure 3.11). In this form is it much easier to identify tracks "by eye",  
1016 however, for the Phase II analysis Project 8 developed it's own custom-made track finding  
1017 algorithm, called the sequential track finder (STF).

1018 The STF algorithm processes the sparse spectrogram in sequential fashion, processing  
1019 each time-slice one-by-one until the end of the spectrogram is reached. Tracks are found  
1020 by searching for points in the sparse spectrogram that appear to fall on a straight line.  
1021 Multiple configurable parameters are built into the STF algorithm that allow the user to

1022 tune the criteria for adding a point to an existing track or creating a brand new track.  
1023 These include parameters such as maximum time and frequency differences between  
1024 subsequent points in a track as well as minimum SNR values for the start and endpoints  
1025 of the track. Additionally, tracks are required to have a minimum length and slope to be  
1026 considered potential CRES tracks rather than random noise fluctuations.

1027 The resulting output of the STF is a collection of track objects that consist of all of the  
1028 points that make up the track and their properties. The final step in track reconstruction  
1029 is to calculate the track properties and apply final cuts to reject the majority of false  
1030 tracks found by the STF. This involves the fitting of a line to the collection of track  
1031 points as well as the total and average power of the track obtained by computing the  
1032 sum and mean of the points powers. The starting frequency of the track is determined by  
1033 calculating the time coordinate that intersects with the linear fit. A cut is performed  
1034 to remove all tracks that do not have a specified average power over their duration, which  
1035 helps to remove the majority of noise fluctuations that have passed all previous cuts up  
1036 to this point.

## 1037 Event Reconstruction

1038 The final step is event reconstruction where the identified tracks are grouped into events  
1039 that contain all tracks likely caused by the same electron. This procedure simply attempts  
1040 to match tracks head to tail by checking if the start and end times of a pair of tracks  
1041 falls withing a certain tolerance. This tolerance is an additional configurable parameter  
1042 that can be tuned to an optimal value using monte carlo simulations of events in the  
1043 Phase II apparatus.

1044 After the event building procedure has completed there is still a small likelihood that  
1045 false tracks have made it through to this stage in the reconstruction. Typically, cuts at  
1046 the track level are able to remove 95% of the false tracks identified by the STF, which  
1047 leads to a significant number of false tracks at the event building stage. However, the  
1048 additional event-level information makes it possible to reject events that contain these  
1049 false tracks with a high degree of confidence.

1050 Two event level features are associated with events caused by real electrons — the  
1051 duration of the first track as well as the number of tracks in the event. Real electrons  
1052 tend to have event structures with longer first tracks and a higher number of total tracks.  
1053 Based on the values of these two criteria, a minimum threshold on the average power in  
1054 the first track was configured to reject false events. The average power in the first track  
1055 was chosen due to the critical nature of the starting frequency of the first track in an

1056 event to the krypton and tritium spectrum analyses.

### 1057 3.3.3 Results from Phase II

1058 The primary result from Phase II is the first-ever measurement of the tritium beta-decay  
1059 spectrum using CRES, which lead to the first neutrino mass limit using the CRES  
1060 technique. However, Phase II also included a significant  $^{83m}\text{Kr}$  measurement campaign  
1061 to understand important systematics relevant to the tritium spectrum measurement, but  
1062 also to understanding the fundamentals of the CRES technique itself. This required  
1063 high-resolution measurements of the  $^{83m}\text{Kr}$  internal-conversion spectrum, which is an  
1064 interesting science result in its own right.

1065 The results from Phase II represents a significant effort from the entire Project 8  
1066 collaboration over several years. Because the focus of my contributions to Project 8 is  
1067 directed towards the research and development efforts for the Phase III experiments, the  
1068 goal in this section is not to provide a detailed description of the the analyses that lead to  
1069 the Phase II results. Rather, I will provide brief descriptions of a few plots representative  
1070 of the main results from Phase II and direct the interested reader to the relevant Phase  
1071 II papers.

#### 1072 Measurements with Krypton

1073 Measurements with krypton were a key calibration tool for Phase II of the experiment  
1074 and will most likely continue to be useful in future Phases of Project 8. In the context of  
1075 Project 8 krypton measurements refers to CRES measurements of the internal-conversion  
1076 spectrum of the metastable state of krypton-83,  $^{83m}\text{Kr}$ , produced by electron capture  
1077 decays of  $^{83}\text{Rb}$ . A supply of  $^{83}\text{Rb}$  was built into the Phase II apparatus gas system that  
1078 supplied the CRES cell with  $^{83m}\text{Kr}$  via emanation.

1079 The  $^{83m}\text{Kr}$  internal-conversion spectrum consists of several lines based on the orbital  
1080 of the electron ejected during the decay. The conversion lines useful to Project 8 are  
1081 those that emit electrons with kinetic energies that fall inside the detectable frequency  
1082 bandwidth of the Phase II apparatus. These are the K; L2 and L3; M2 and M3; and N2  
1083 and N3 lines with kinetic energies of 17.8 keV,  $\approx$  30.4 keV,  $\approx$  31.9 keV, and  $\approx$  32.1 keV,  
1084 respectively. The different energies of the lines allow us to test the linearity of the  
1085 relationship between kinetic energy and frequency across the range of frequencies covered  
1086 by the continuous tritium spectrum.

1087 By measuring the shape of the krypton spectrum we can characterize the effects of

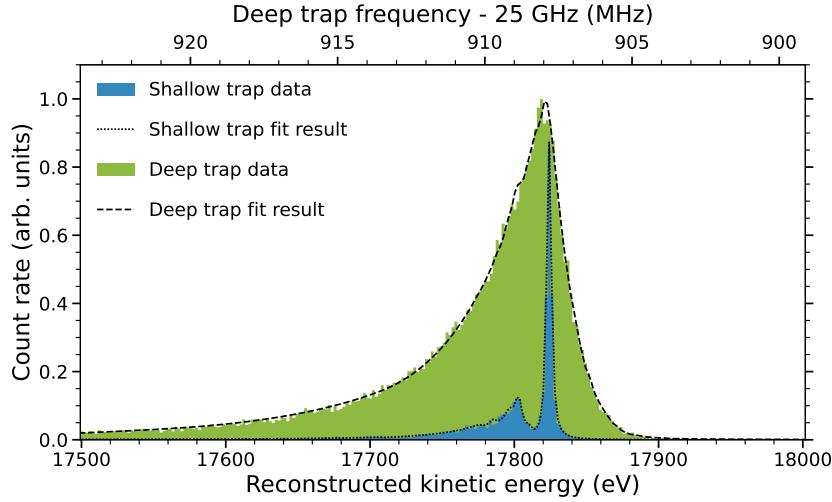


Figure 3.12: Fits to the measured 17.8-keV  $^{83m}\text{Kr}$  conversion line using the deep and shallow trap configurations.

numerous detector related effects relevant to the tritium analysis. Specific examples include the variation in the magnetic field as a function of the radial position of the electron, variation in the magnetic field caused by the trap shape, variation in the average magnetic field for electron of different pitch angles, the effect of missing tracks due to scattering, among others. These spectrum shape measurements focused on the 17.8-keV krypton line and utilized different trap geometries based on the particular goal of the dataset (see Figure 3.12).

Krypton measurements with a shallow trap allow for high energy resolution, since variation in frequency due to pitch angle differences is sharply reduced in the shallow trap configuration. With this trap the main 17.8-keV peak of the conversion spectrum is clearly visible along with additional satellite peaks at lower energy, which correspond to the shakeup/shakeoff spectrum of the decay. The high accuracy of the fit demonstrates a high degree of understanding of the CRES systematics.

The broadening of the krypton spectrum seen for the deeper track is due to the higher range of electron pitch angles that can be trapped. Furthermore, with a deeper trap there is a larger parameter space of electron that could be produced with pitch angles that are trappable but not visible in the time-frequency spectrogram. These electrons live in the trap and can scatter multiple times before randomly scattering to a pitch angle that is now visible. This causes us to miss one to several of the electron's tracks earlier in the event, which leads us to mis-reconstruct the true starting frequency of an event. By measuring the krypton spectrum shape in the same deep trap used to detect tritium

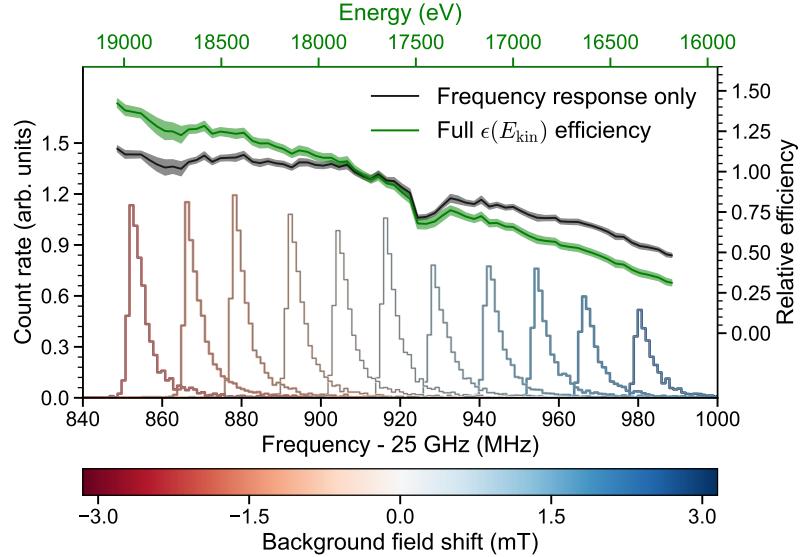


Figure 3.13: Measurements of the 17.8-keV  $^{83m}\text{Kr}$  line using the deep trap configuration for different values of the magnetic field from the field shifting solenoid.

events we can characterize the affect that this has on the spectrum shape to mitigate it's impact on the tritium measurements.

An additional systematic characterized with krypton is the calibration of the detection efficiency of the Phase II apparatus as a function of frequency. Variations in the detection efficiency as a function of frequency directly changes the measured shape of the continuous tritium spectrum, which can lead to errors in the neutrino mass estimate if not modeled appropriately. Using the field shifting solenoid the cyclotron frequency of the krypton 17.83 keV line was shifted across the full frequency range of the tritium spectrum data (see Figure 3.13). Variations in the deep trap krypton spectrum shape can be used to infer the detection efficiency as a function of frequency and correct for this affect in the tritium measurements.

## 1120 Tritium Spectrum and Neutrino Mass Results

The tritium measurement campaign resulted in the collection of 82 days of detector live time during which 3770 total tritium events were detected. The track and event reconstruction analysis extracted the starting frequencies of these tritium events, which were used to build a frequency spectrum of tritium beta-decays. The resulting frequency spectrum was then converted to an energy spectrum using the information gleaned from the krypton measurement campaign to obtain the tritium beta-decay spectrum (see

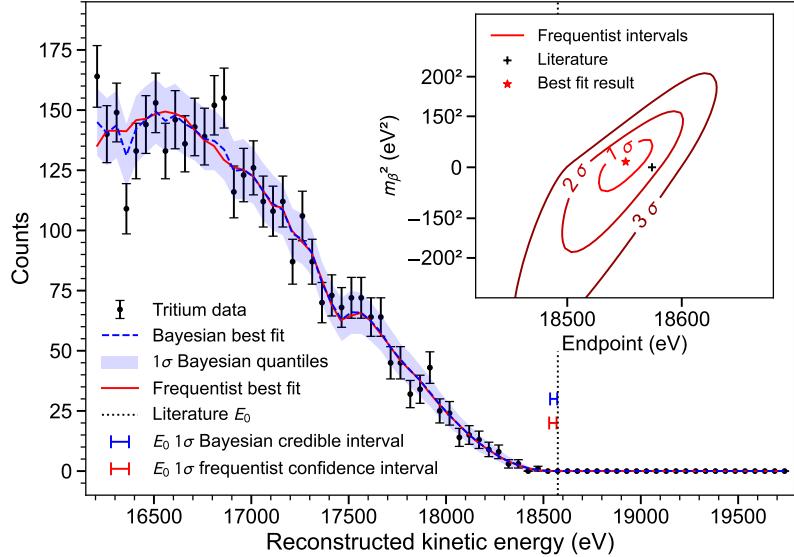


Figure 3.14: The measured tritium spectrum from Phase II with Bayesian and frequentist fits.

1127 Figure 3.14).

1128 CRES is inherently a very low background technique with the dominant source of noise  
 1129 being random RF fluctuations. Monte carlo simulations backed up by measurements  
 1130 of the RF noise background were used to set track and event characteristic cuts to  
 1131 guarantee that zero false events would occur over the duration of the experiment with  
 1132 90% confidence. Notably, the measured spectrum has zero events beyond the tritium  
 1133 spectrum endpoint, which allows us to constrain the background rate in the Phase II  
 1134 apparatus to less than  $3 \times 10^{-10}$  counts/ev/s. Achieving a low background is critical for  
 1135 future neutrino mass experiments that seek to measure the neutrino mass with less than  
 1136 100 meV sensitivity.

1137 Bayesian and frequentist based fits to the measured tritium spectrum, incorporating  
 1138 information gained about CRES systematics from the krypton measurements, were  
 1139 performed to extract upper limits on the tritium beta-decay spectrum endpoint as well as  
 1140 the neutrino mass. The estimated spectrum endpoints are  $18553^{+18}_{-19}$  eV for the Bayesian  
 1141 analysis and  $18548^{+19}_{-19}$  eV for the frequentist analysis. The quoted uncertainties are  
 1142 1- $\sigma$ , and both results are within 2- $\sigma$  of the literature endpoint value of 15574 eV. The  
 1143 estimated neutrino mass for both results is consistent with  $m_\beta^2 = 0$ . The 90% confidence  
 1144 upper limits for the Bayesian analysis is  $m_\beta < 155$  eV/c<sup>2</sup> and  $m_\beta < 152$  eV/c for the  
 1145 frequentist analysis.

Though the neutrino mass results from Phase II are not competitive with KATRIN it is a promising first step towards the development of more precise neutrino mass measurements using CRES. The low background and demonstrated high resolution with krypton measurements are promising features of the technique that were able to be demonstrated with the Phase II apparatus. As new technologies are developed to enable CRES measurements in larger volume, many of the lessons learned from Phase II will continue to influence the operation and design of the detectors.

### 3.4 Phase III R&D: Antenna Array CRES

The goal of Phase III in the Project 8 experimental program is to develop the technologies and expertise required to build an experiment that uses CRES to measure the neutrino mass with a target sensitivity of 40 meV. One of the key technologies is a method for performing high resolution CRES measurements in a large volume, which allows one to observe a sufficient quantity of tritium to measure the low-activity endpoint region of the tritium spectrum.

#### 3.4.1 The Basic Approach

One possible approach, suggested in the original CRES publication, is to use many antennas to surround a volume of tritium gas in a magnetic field (see Figure 3.15). When a decay occurs the electron will begin to emit cyclotron radiation that can be collected by the array and used to perform CRES. Each antenna in the array collects only a small

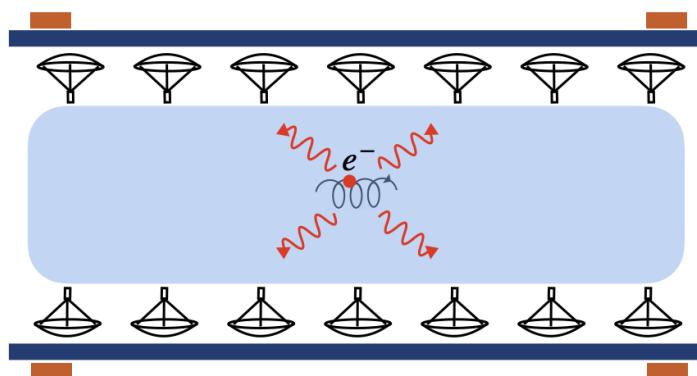


Figure 3.15: A cartoon illustration of the basics of the antenna array CRES technique.

fraction of the electron's signal power, which is less than 1 fW for a 18.6 keV kinetic energy electron in a 1 T magnetic field. Scaling to large volumes with the antenna

array approach is accomplished by increasing the number of antennas in the array, which increases the volume under observation proportionally, so that a sufficient population of tritium atoms can be observed to measure the tritium spectrum endpoint shape.

Several features of the antenna array approach make it an attractive candidate technology for a large volume experiment. One example is the accurate position reconstruction made possible by the multichannel nature of the array. Using techniques like digital beamforming it is possible to estimate the radial and azimuthal positions of the electron in the magnetic trap with a precision significantly less than the size of the cyclotron wavelength. This capability allows one to perform event-by-event estimations of the magnetic field experienced by an electron, which is crucial to achieving high energy resolution with the CRES technique.

The easy availability of position information with the antennas array approach is potentially a unique advantage that provides significant flexibility in the magnetic field uniformity requirements compared to other proposed approaches to large volume CRES (see Chapter 6). Spatial discrimination using digital beamforming leads to pileup reduction, which helps to reduce the potential of background events caused by missing tracks or by incorrectly clustering a group of tracks into an event. Limits on the background rate for a neutrino mass measurement with 40 meV sensitivity are stringent and the total activity of the tritium source for such an experiment is gigantic relative to the activity near the endpoint. Thus, pileup discrimination could be an important tool for a large scale CRES experiment.

Another beneficial quality of the antenna array approach is that the volume of the experiment can be scaled independent of frequency by simply adding more antennas to the array (see Figure 3.19). Resonant cavities, the proposed alternative large volume CRES technology, are ideally operated in magnetic fields that cause electrons to move with cyclotron frequencies near the fundamental cavity resonance, to avoid complex coupling of the electron to many cavity modes simultaneously. This leads to a coupling between the cavity volume and the magnetic field magnitude, which forces one to lower the magnetic field in order to increase the experiment scale. Whereas, for antenna arrays, in principle there is no physical limitation on the size of the antenna array that can be used at a particular magnetic field. However, the nature of scaling an antenna array based experiment leads to rapidly increasing cost and complexity due to the large number of antennas, amplifiers, and data streams that require substantial computer processing power to effectively analyze.

### **3.4.2 The FSCD: Free-space CRES Demonstrator**

The complex collection of new experimental techniques and methods that come together in the antenna array CRES technique require the construction of a small scale demonstration experiment designed to develop an understanding of the principles of antenna array CRES measurements and the relevant systematics. Without operating such an experiment it is not possible to develop a design for a large scale CRES experiment with sufficient confidence that the experiment is capable of measuring the shape of the tritium spectrum endpoint to the degree of accuracy required for 40 meV sensitivity to the neutrino mass. Therefore, Phase III of the Project 8 experimental program is primarily focused on the development and operation of demonstrator experiments to inform the design of the final Phase IV experiment.

Specifically for antenna array CRES, the associated demonstrator experiment in Phase III is called the Free-space CRES Demonstrator or FSCD. The goals of the FSCD include not only the development of antenna array CRES itself, but is also a capable neutrino mass measurement experiment in it's own right, with a target neutrino mass sensitivity of a few eV using a molecular tritium source.

### **Magnetic Field**

The background magnetic field for the FSCD experiment is provided by a hospital-grade MRI magnet (see Figure 3.16). The magnet produces a magnetic field of approximately 0.958 T, which corresponds to a tritium spectrum endpoint frequency of approximately 25.86 GHz. The magnet is installed in the Project 8 laboratory located at the University of Washington, Seattle, and is shimmed to produce a uniform magnetic field with variations on the ppm scale. Measurements of the magnetic field non-uniformities were performed using a NMR probe and rotational gantry to capture measurements of the magnetic field around an elliptical surface in the center of the MRI magnet. During the operation of the FSCD an array of Hall or NMR magnetometers could be used to periodical measure the magnetic field in order to quantify its time stability.

Inside the main magnetic field of the MRI magnet are additional magnets that provide the capability to shift the value of the background magnetic field as well as the magnets that produce the magnetic trap. Shifting the background value of the magnetic field on a scale of  $O(\mu T)$  allows one to control the cyclotron frequencies of electrons with a fixed kinetic energy, which is key to effectively calibrating the FSCD. The preferred calibration method for the FSCD is a mono-energetic electron gun that can inject electrons into



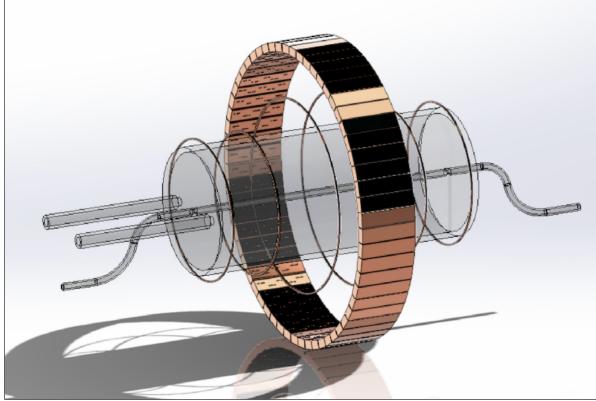
Figure 3.16: An image of the MRI magnet installed in the Project 8 laboratory at the University of Washington, Seattle.

1234 the magnetic trap with a known kinetic energy. In combination with the field shifting  
 1235 magnet one can vary the cyclotron frequencies of the electrons to measure the response  
 1236 of the antenna array as a function of the radiation frequency and electron position. This  
 1237 procedure not only characterizes the response of the antenna array but also provides  
 1238 further information on magnetic field uniformity, which important to achieving optimal  
 1239 energy resolution.

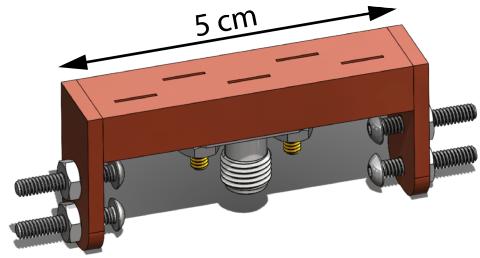
1240 Several additional magnetic coils will need to included inside the MRI magnet to  
 1241 produce the magnetic trap. The ideal trap shape for CRES is the perfect magnetic box,  
 1242 which has a flat bottom and step function walls. Any variation in the average magnetic  
 1243 field experienced by an electron leads to changes in the cyclotron frequency that can  
 1244 make determining the true starting kinetic energy more difficult. This includes changes  
 1245 in the magnetic field caused by the walls of the magnetic trap as well as radial magnetic  
 1246 field variations. The perfect box trap is completely uniform and has infinitely steep walls  
 1247 that cause no change in the electron's cyclotron frequency as it is reflected from the  
 1248 trap wall, however, such a trap cannot be made from any combination of magnetic coils  
 1249 since it violates Maxwell's equations. The goal of magnetic trap design is to identify the  
 1250 configuration of coils that produces a trap that approximates the perfect box trap as  
 1251 closely as possible.

1252 **Antenna Array**

1253 The canonical antenna array design for a CRES experiment is a uniform cylindrical array  
1254 of antennas that surrounds the magnetic trap volume. Since the FSCD is a demonstrator  
1255 experiment, the antenna array design is the simplest form of the uniform cylindrical  
1256 array, which is a single circular ring of antennas with a diameter of 20 cm (see Figure  
3.17). Along this circle are sixty slotted waveguide antennas that fully populate the



(a)



(b)

Figure 3.17: (a) A model of the FSCD antenna array, magnetic trap, and tritium containment vessel design.(b) A more detailed model of a prototype design for the 5-slot waveguide antenna design.

1257  
1258 available space around the array circumference. In order to maximize the power collected  
1259 from each electron it is optimal to cover as large a fraction of the solid angle around the  
1260 magnetic trap as possible.

1261 The distance between antennas around the circumference of the array is proportional  
1262 to the wavelength of the cyclotron radiation. Therefore, maximizing the solid angle  
1263 coverage of the array, while minimizing channel count to keep the hardware and data  
1264 acquisition costs manageable, biases one towards smaller array diameters. Antenna  
1265 near-field effects limit the minimum diameter of the array for a given antenna design  
1266 since the radiation from electrons that are too close to the array cannot be detected  
1267 due to destructive interference caused by path-length differences from the electron to  
1268 different points on the antenna surface.

1269 Slotted waveguide antennas are used in the FSCD antenna array due to their high  
1270 efficiency and low loss, which comes from the lack of dielectric materials in the antenna  
1271 structure. Coupling to the waveguide can be performed with a coaxial cable connected  
1272 at the center or on either end of the waveguide. One of the drawbacks of waveguide

1273 antennas is the large amount of space required to fit them inside the limited MRI magnet  
1274 volume. Alternative antenna designs, constructed from microstrip printed circuit boards  
1275 require significantly less space at the cost of slightly higher energy loss in the antenna  
1276 structure.

1277 The FSCD antenna design is a 5 cm long segment of WR-34 waveguide with 5 vertical  
1278 slots cut into the side. The distance between slots along the length of the waveguide is  
1279 a half wavelength for optimal power combination between the individual antenna slots.  
1280 Each slot is offset from the center of the antenna face a small distance in order to most  
1281 effectively couple the slot to waveguide modes inside the antenna.

1282 The passive power combination achieved by placing 5 slots in a single waveguide is a  
1283 compromise intended to reduce the cost and complexity of the antenna array system.  
1284 Each additional channel in the array requires it's own cryogenic amplifier and also increase  
1285 the required computer power to process the raw data collected by digitizing each channel.  
1286 Passive summation, achieved by combining antennas into arrays axially, reduces the array  
1287 channel count at the cost of losses from imperfect passive combination. Imperfect passive  
1288 combination is caused by effects such as re-radiation of energy from and destructive  
1289 interference between slots in the waveguide antenna.

1290 Interference and re-radiation eventually limit the achievable the axial extent of passive  
1291 power combination. The 5-slot designed developed for the FSCD is optimized to minimize  
1292 the impact of these losses while achieving the maximum amount of axial coverage with a  
1293 single ring of antennas. Scaling beyond the volume covered by a single ring of antennas is  
1294 achieved by stacking additional rings of antennas together to cover a larger trap volume  
1295 for a higher statistics measurement of the tritium spectrum endpoint region. A likely  
1296 scenario for the FSCD experiment involves a staged experiment approach, where first  
1297 a series of measurements is performed using only a single ring of antennas followed by  
1298 experiments that add additional rings to the FSCD. The goal would be to first understand  
1299 the principles of antenna array CRES using the simplest possible experiment, before  
1300 attempting to scale the technique by expanding the antenna array size.

## 1301 **Tritium Source**

1302 While the primary purpose of the FSCD is as a technology demonstrator, it is unlikely  
1303 for the collaboration to gain the required confidence in the antenna array CRES tech-  
1304 nique to perform neutrino mass measurements at the 40 meV sensitivity level without  
1305 an intermediate scale measurement of the neutrino mass using antenna array CRES.  
1306 Therefore, the FSCD has an additional scientific goal of measuring the neutrino mass

1307 with a rough sensitivity goal of a few eV. This level of precision is achievable using a  
1308 source of molecular tritium with a volume of approximately 1 L at a density comparable  
1309 to potential Phase IV scenarios.

1310 Unlike previous CRES experiments, where the tritium source could be co-located  
1311 with the receiving antenna inside a waveguide transmission line, the tritium source  
1312 in the FSCD is thermally isolated from the antenna array to avoid freeze-out of the  
1313 tritium molecules. The tiny radiation power emitted by electrons requires a system noise  
1314 temperature of  $\approx 10$  K or less, in order to detect events at a high enough efficiency to  
1315 reach the neutrino mass sensitivity goals of the experiment. Achieving a system noise of  
1316 10 K requires that the antenna array and amplifiers operate at cryogenic, liquid helium  
1317 temperatures of  $\approx 4$  K, which significantly lowers the vapor pressure of molecular tritium.  
1318 By keeping the molecular tritium isolated in an RF-transparent vessel the tritium gas can  
1319 be kept at a relatively warmer temperature in the range of 30 K to avoid the accumulation  
1320 of tritium on the experiment surfaces.

## 1321 Data Acquisition and Reconstruction

1322 A fundamental change in the data acquisition system for the FSCD is the shift from  
1323 single to multi-channel reconstruction. This transition results in a significant increase in  
1324 the data-generation rate, which is linearly related to the number of independent channels  
1325 in the array. The larger data volume coincides with an increased demand for computer  
1326 processing power based on the need for more precise signal reconstruction algorithms  
1327 driven by the FSCD and Phase IV sensitivity goals. Therefore, the data acquisition  
1328 system for the FSCD is likely to represent a significantly larger fraction of the experiment  
1329 cost and complexity than previous CRES experiments.

1330 Each antenna in the array is connected to a cryogenic amplifier and down-converted  
1331 from the 26 GHz CRES frequency using an IQ-mixer to reduce the size of the analysis  
1332 window in which the tritium spectrum is measured. Using an LO with a frequency of  
1333 approximately 25.80 GHz the antenna array signals can be digitized at a rate of 200 MHz,  
1334 which is sufficient bandwidth to resolve the complete sideband spectrum produced by  
1335 axial oscillations of electrons in the FSCD magnetic trap.

1336 Direct storage of the raw FSCD antenna array data is undesirable, since the estimated  
1337 amount of raw data generated is  $O(1)$  exabyte per year. The management and storage  
1338 of such a large dataset is infeasible for a demonstrator experiment on the scale of the  
1339 FSCD and would represent a large fraction of the budget for a Phase IV scale antenna  
1340 array based CRES experiment. Therefore, a sub-goal of the FSCD experiment is the

1341 development of real-time reconstruction methods that could reduce the raw data volume  
1342 by detecting and reconstructing CRES events in real-time. The ultimate goal would be  
1343 a complete real-time reconstruction pipeline that takes raw voltages samples from the  
1344 antenna array and returns estimates for the starting kinetic energies of CRES events in  
1345 the data.

1346 The feasibility of a real-time reconstruction pipeline rests on the development of  
1347 computationally efficient algorithms that can be implemented without the need for  
1348 enormous computing resources. One challenge with the antenna array approach is that  
1349 the small radiation power of a single electron is distributed between each channel in  
1350 the array, such that reconstruction using only the information in a single channel is not  
1351 possible. Therefore, the simply performing the initial step in reconstruction — signal  
1352 detection — requires orders of magnitude more computational power than previous CRES  
1353 experiments. This operation will then be followed by other, potentially more expensive,  
1354 reconstruction steps that are required in order to determine the kinetic energy of the  
1355 electron.

## 1356 **3.5 Pilot-scale Experiments**

### 1357 **3.5.1 Choice of Frequency**

1358 The optimal CRES frequency for Project 8 is that which can reach our target sensitivity  
1359 of 40 meV, while minimizing the cost and complexity of the overall experiment. Since the  
1360 size of the background magnetic field determines the cyclotron frequency, which affects  
1361 the entirety of the CRES detection system design, specifying the operating frequency of  
1362 the CRES experiments is one of the first steps towards developing a full design.

### 1363 **Scaling Laws**

1364 In Phases I and II the background magnetic field was provided by an NMR magnet with  
1365 a 0.959 T magnetic field. This magnetic field was selected primarily for convenience,  
1366 however, the cyclotron frequencies for electrons near the tritium endpoint in a 0.959 T  
1367 field ranges from 25 to 26 GHz, which is within the standard RF Ka-band. Therefore,  
1368 microwave electronics specialized for these frequencies are easily obtainable for relatively  
1369 low cost. Frequency choice for the upcoming large-scale experiments must be selected  
1370 in a more rigorous manner than in the earlier phases due to the increasing scale and  
1371 complexity of the systems and the 40 meV neutrino mass science goal.

1372      Naturally, for a larger volume experiment there is a bias towards lower frequencies, due  
1373      to the direct relationship between wavelength and the physical size of the compatible RF  
1374      components like antennas and cavities. With a longer wavelength a larger volume can be  
1375      surrounded by an array with fewer antennas, which reduces hardware and data-processing  
1376      costs. On the other hand, for a cavity experiment, the volume of the experiment is  
1377      directly proportional to the wavelength since this sets the physical dimensions of the  
1378      cavity. Furthermore, it is easier to engineer a magnet that provides a uniform magnetic  
1379      field across several cubic-meters of space at a lower magnetic fields, which provides  
1380      advantages in terms of cost-reduction as well as more uniform magnetic fields for CRES.

1381      A concern with lower magnetic fields and frequencies is the scaling of the Larmor  
1382      power equation, which is proportional to the square of the frequency. Naively, one would  
1383      predict that the SNR would decrease with lower fields, however, two additional scaling  
1384      laws that affect the noise power also come into play. Noise power is directly proportional  
1385      to the required bandwidth, which decreases linearly with the magnetic field. Furthermore,  
1386      at lower frequencies it is possible to purchase amplifiers with lower noise temperatures  
1387      until approximately 300 MHz at which point this relationship tends to flatten. Therefore,  
1388      it is expected that the SNR remains approximately constant as the frequency decreases.

1389      The SNR directly impacts the overall efficiency of the experiment through its affects  
1390      on CRES signal detection probabilities as well as energy resolution. Thus, the expectation  
1391      that SNR remains the same at lower frequencies clearly biases large-scale experiments  
1392      in this direction. One drawback of lower magnetic fields is the increased influence of  
1393      external magnetic fields on the experiment. This includes magnetic fields from the  
1394      building materials as well as variations in the earth's magnetic field. To deal with these  
1395      affects a suitable magnetic field correction system will need to be devised, which includes  
1396      constant monitoring of external fields.

## 1397      **Atomic Tritium Considerations**

1398      The pilot-scale experiments will be the first Project 8 experiments to combine CRES  
1399      with atomic tritium, therefore, the optimal frequency should take into account the affect  
1400      of the background magnetic field size on atom trapping. The primary influence of the  
1401      background field magnitude is through the rate of dipolar spin-flips caused by a spin  
1402      exchange interaction between trapped atoms.

1403      Atomic tritium is a simple quantum system with a hyperfine structure given by the  
1404      addition of the nuclear and atomic spins. The addition of two spins leads to a hyperfine  
1405      structure with four states in the  $(m_s, m_I)$  basis. The states with atomic spins directed

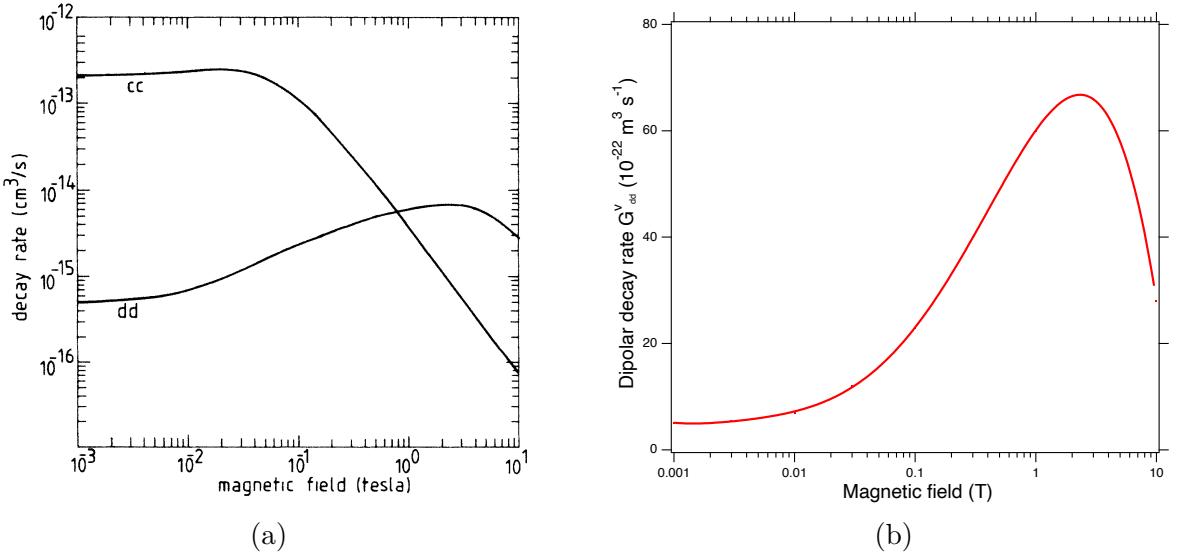


Figure 3.18: (a) A plot of the decay rate for the two-body dipolar spin exchange interaction for  $c+c$  and  $d+d$  state. (b) A plot of the decay rate of the dipolar spin exchange interaction for  $d+d$  states as a function of magnetic field magnitude. Lowering the magnetic field is key for reducing the losses from this interaction.

1406 anti-parallel to the magnetic field have  $m_s = -1/2$  and are labeled as the a and b states.  
 1407 The a and b states are colloquially known as high-field seeking states, since their energy is  
 1408 minimized when in regions of higher magnetic field. This leads to losses in the magnetic  
 1409 trap as these atoms are drawn to higher fields away from the trap center. Alternatively,  
 1410 the c and d states, with atomic spin  $m_s = +1/2$ , minimize their energy in low magnetic  
 1411 fields because of the parallel alignment between spin and the magnetic field. Therefore,  
 1412 these low-field seeking states tend to stay trapped significantly longer than the high-field  
 1413 seeking states.

1414 Project 8 would do well to prepare the tritium atoms in purely c and d states before  
 1415 trapping, however, even in this case losses still occur due to dipolar interactions between  
 1416 pairs of c and d states leading to a flipped atomic spins and subsequent losses due  
 1417 to high-field seeking atoms. The rate of these interactions depends on the magnitude  
 1418 of the background magnetic field and is maximal for dd interactions around 1 T (see  
 1419 Figure 3.18). The rate of losses from these interactions at 1 T requires atomic tritium  
 1420 production at a rate two orders of magnitude larger than at 0.1 T, thus, requirements  
 1421 on the whole atomic tritium system are significantly relaxed at lower magnetic fields,  
 1422 which provides an additional argument for transitioning to lower frequencies with the  
 1423 pilot-scale experiments.

1424 **3.5.2 Pilot-scale Experiment Concepts**

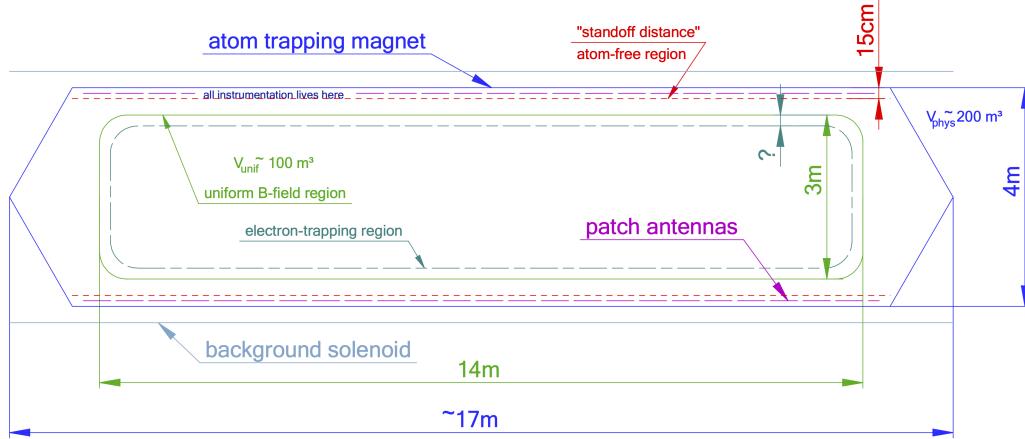


Figure 3.19: A conceptual sketch of a large-volume antenna array based CRES experiment to measure the neutrino mass.

1425 While the pilot-scale experiments are still in the early stages, enough is known to  
1426 sketch the general features of these experiments at the cartoon level.

1427 **Pilot-scale Antenna Array CRES Experiment Concept**

1428 A conceptual design for an antenna-based CRES experiment is shown in Figure 3.19.  
1429 A large solenoid magnet provides a uniform background magnetic field less than 0.1 T  
1430 in magnitude. Inside this region is the atom trapping magnet that generates a high  
1431 magnetic field at the walls, which decays exponentially towards the central region. Known  
1432 magnet designs that produce suitable atom trapping fields include Ioffe-Prichard traps,  
1433 which use conducting coils, as well as a Halbach array made from permanent magnets.  
1434 Either magnet choice produces a region of high magnetic fields, which excludes atoms  
1435 and allows for the placement of antennas inside the experiment.

1436 Inside this region an array of microstrip patch antennas is inserted to collect the  
1437 cyclotron radiation without providing a surface for atomic tritium recombination. Due  
1438 to the lower frequency of cyclotron radiation antennas of a larger size can be used,  
1439 which lowers the total number of antennas required to observe the experiment volume.  
1440 Because of this scaling, the lower frequency experiment uses a similar number of antennas  
1441 compared to a much smaller demonstrator experiment with a 1 T magnetic field.

1442 The atomic tritium beamline that supplies fresh tritium atoms to the experiment is  
1443 not shown in the figure. The general configuration would matches the one shown for the

<sup>1444</sup> pilot-scale cavity experiment (see Figure 3.20).

### <sup>1445</sup> Pilot-scale Cavity CRES Experiment Concept

<sup>1446</sup> The pilot-scale cavity experiment includes both an atomic tritium system and cavity  
<sup>1447</sup> CRES system. The atomic system consists of a thermal atom cracker located at the  
<sup>1448</sup> start of an evaporatively cooled atomic beamline. The atomic tritium system provides a  
<sup>1449</sup> supply of tritium atoms to the trap with temperatures on the order of a few mK. Atoms  
<sup>1450</sup> at this temperature can be trapped magneto-gravitationally, which is the reason for the  
<sup>1451</sup> vertical orientation of the cavity. At these low magnetic fields the trapping requirements  
<sup>1452</sup> for electrons and atoms differ enough such that it is advantageous to decouple the the  
<sup>1453</sup> trapping potentials to avoid radioactive heating of the tritium atoms from excess trapped  
<sup>1454</sup> electrons. Electron trapping is provided by a set of magnetic pinch coils at the top and  
<sup>1455</sup> bottom of the cavity and a multi-pole Ioffe or Halbach magnet serves to contain the  
<sup>1456</sup> atoms.

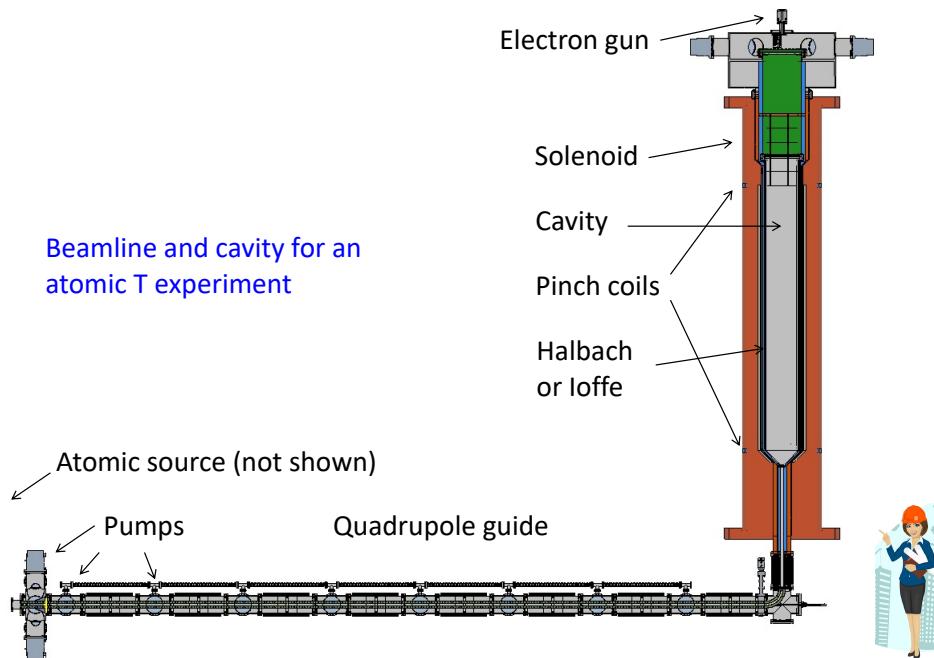


Figure 3.20: A conceptual sketch of a pilot-scale cavity CRES experiment with an atomic tritium beamline.

<sup>1457</sup> The cavity design for the pilot-scale experiment consists of a large cylindrical cavity  
<sup>1458</sup> with a TE011 resonance of 325 MHz. Such a cavity is truly enormous, with a diameter  
<sup>1459</sup> of approximately 1.2 m and a height of 10 m. When an electron is produced inside  
<sup>1460</sup> the cavity with a cyclotron frequency that matches the TE011 resonant frequency it's

<sup>1461</sup> cyclotron orbit couples the electron to the TE011, which drives a resonance in the cavity.  
<sup>1462</sup> These resonant fields can be read-out using an appropriate cavity coupling mechanism  
<sup>1463</sup> located at the center of the cavity. For more information on the cavity approach to  
<sup>1464</sup> CRES see Chapter 6.

<sup>1465</sup> The bottom of the cavity has a cone termination to match the contour of the atom  
<sup>1466</sup> trapping magnet. This shape still allows for TE011 resonances with high internal Qs,  
<sup>1467</sup> which are required for good SNR in the cavity experiment. A small opening in the bottom  
<sup>1468</sup> of the cone serves as an entry point for the tritium atoms. To allow for calibration of  
<sup>1469</sup> the magnetic field inhomogeneities with an electron gun, the top of the cavity is left  
<sup>1470</sup> nearly completely open. Normally, this would drastically lower the Q-factor of the TE011  
<sup>1471</sup> mode, but a specially configured coaxial partition is inserted at the top. This termination  
<sup>1472</sup> scheme is designed to act as a perfect short for the TE011 mode since the circular shape  
<sup>1473</sup> of the partition matches the electric field boundary conditions for the TE011 mode.  
<sup>1474</sup> Simulations with HFSS have confirmed that this design results in a high quality TE011  
<sup>1475</sup> resonance despite the nearly completely open end.

## <sup>1476</sup> 3.6 Phase IV

<sup>1477</sup> The baseline CRES technology being pursued by the Project 8 collaboration are resonant  
<sup>1478</sup> cavities, which, due to their geometric properties, simple CRES signal structure, and low  
<sup>1479</sup> channel count, appear to be the better option for Phase IV. The current knowledge of the  
<sup>1480</sup> antenna array CRES approach reveals no technical obstacles that would preclude it as a  
<sup>1481</sup> baseline technology for Phase IV though it would most certainly be significantly more  
<sup>1482</sup> expensive. Therefore, antenna arrays represent a fallback approach if resonant cavities  
<sup>1483</sup> prove infeasible.

<sup>1484</sup> The sensitivity of the pilot-scale atomic tritium experiment is estimated to be on  
<sup>1485</sup> the order of 0.1 eV, which means that increasing the sensitivity to reach the Phase IV  
<sup>1486</sup> goal will require a larger volume experiment. Because of the direct coupling between the  
<sup>1487</sup> RF characteristics of a cavity and its geometry, the baseline plan is to build multiple  
<sup>1488</sup> copies of the pilot-scale experiment (see Figure 3.21) to obtain the required amount of  
<sup>1489</sup> volume rather than increase the size of the cavity beyond the pilot-scale. The built-in  
<sup>1490</sup> redundancy of this approach is attractive in that the experiment has no single point of  
<sup>1491</sup> failure, additionally, building several copies of the a pilot-scale experiment will require  
<sup>1492</sup> minimal new engineering and design.

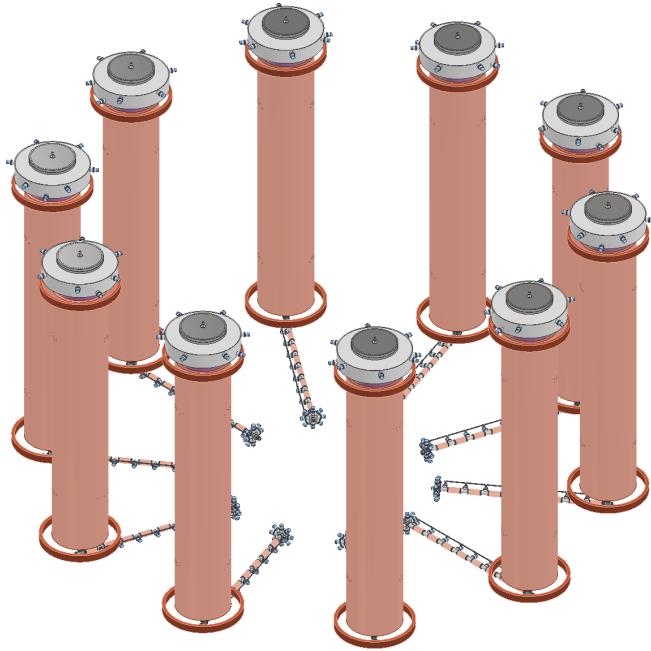


Figure 3.21: An illustration of a possible arrangement of ten pilot-scale cavity experiments for Phase IV. The experiments are arranged in a circle with an approximate diameter of 50 meters. Each atomic beamline connected to the bottom of each cavity is approximately 10 m in length. The cavities themselves are designed to operate at 325 MHz and are approximately 11 m tall. The circular arrangement of cavities has some advantages when it comes to cancellation of fringe fields from neighboring magnets, which is important due to the small magnetic field magnitudes consistent with these CRES frequencies. The advantage of ten independent atomic sources and cavities is that there is no single point of failure for the experiment. If an experiment goes down for repairs the other nine may continue running. Figure courtesy of Michael Huehn at UW-Seattle.

# **Chapter 4**

## **Signal Reconstruction Techniques for Antenna Array CRES and the FSCD**

### **4.1 Introduction**

The transition from a waveguide CRES experiment to an antenna array CRES experiment introduces new challenges related to data acquisition, signal detection, and signal reconstruction caused by the multi-channel nature of the data. The development of signal reconstruction algorithms is crucial to the design of antenna array based experiments like the FSCD, because these algorithms directly influence the detection efficiency and energy resolution of the CRES experiment. In this Chapter I summarize my contributions to the development and analysis of signal reconstruction and detection algorithms for the FSCD experiment.

In Section 4.2 I discuss the primary tool for this work, which is the Locust simulations package developed by the Project 8 experiment. Locust is used to simulate CRES events in the detector. Locust uses Kassiopeia to calculate particle trajectory solutions for electrons in the magnetic trap. The trajectories are then used to calculate the response of the antenna array to the cyclotron radiation produced by the electron, which results in signals that can be used to analyze the performance of different signal reconstruction algorithms. More recently, Project 8 has developed CREsana, which is a new simulations package that takes a more analytical approach to CRES signal simulations for antenna arrays. Although CREsana signals were not used for the signal reconstruction algorithm development detailed here, we introduce the software as it plays a role in the antenna array measurements presented in Section 5.4.

In Section 4.3 I discuss the signal reconstruction and detection approaches analyzed for the FSCD experiment. In general there are two steps to signal reconstruction — detection and parameter estimation. With signal detection one is primarily concerned

only with distinguishing between data that contains a signal versus data that contains only noise, whereas, with parameter estimation one is interested in extracting the kinematic parameters of the electron encoded in the cyclotron radiation signal shape. Due to the low signal power of electrons near the spectrum endpoint in the FSCD experiment, signal detection is a non-trivial problem. This is magnified by the need to maximize the detection efficiency of the experiment in order to achieve the neutrino mass sensitivity goals. My contributions to signal reconstruction analysis for the FSCD are focused on this signal detection component of reconstruction.

After the discussion of various signal detection approaches, in Section 4.4 I present a more detailed analysis of the detection performance of three algorithms, which could be used to signal detection in the FSCD. This section was originally prepared for publication in JINST as a separate paper. The algorithms include a digital beamforming algorithm, a matched filter algorithm, and a neural network algorithm, which I analyze in terms of classification accuracy and estimated computational cost.

## 4.2 FSCD Simulations

Antenna array CRES and the FSCD requires a combination of different capabilities not often found in a single simulation tool. First of all, accurate calculations of the magneto-static fields produced by current-carrying coils are required in order to accurately model the magnetic trap and background magnets. The resulting magnetic fields must then be used to calculate the exact relativistic trajectory of electrons, which is required in order to calculate the electro-magnetic (EM) fields produced by the acceleration of the electron. Finally, the simulation has to model the interaction of the antenna and RF receiver chain with these EM-fields in order to produce the simulated voltage signals produced by the antenna array during the CRES event. At the time when Project 8 was developing this simulation capability, no single available simulation tool was known to adequately perform this suite of calculations, which prompted the development of custom simulation framework to simulate the FSCD. This simulation framework includes custom simulation tools developed by Project 8 as well as other open-source and proprietary software developed by third-parties.

### 1548 4.2.1 Kassiopeia

1549 Kassiopeia<sup>1</sup> is a particle tracking and static EM-field solver developed by the KATRIN  
1550 collaboration for simulations of their spectrometer based on magnetic adiabatic collimation  
1551 with an electrostatic filter [7]. Due to the measurement technique employed by the  
1552 KATRIN collaboration, Kassiopeia is not designed to solve for the EM-fields produced by  
1553 electrons in magnetic fields. However, it does provide efficient solvers for static electric  
1554 and magnetic fields and charged particle trajectory solvers. Because of this, Project 8  
1555 has incorporated parts of Kassiopeia into its own simulation framework.

### 1556 Magnetostatic Field Solutions

1557 The solutions to the electric and magnetic fields generated by a static configuration of  
1558 charges and currents is given by Maxwell's equations in the limit where the time-dependent  
1559 terms go to zero. In their static form Maxwell's equations [8] are

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (4.1)$$

$$\nabla \times \mathbf{E} = 0 \quad (4.2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (4.3)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \quad (4.4)$$

1560 where we can see that the electric and magnetic fields are now completely decoupled  
1561 from each other. The solution for the magnetic field in this boundary value problem is  
1562 given by the Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int dr' \frac{r'^3 \mathbf{J}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r}' - \mathbf{r}|^3}, \quad (4.5)$$

1563 which Kassiopeia uses a variety of numeric integration techniques to solve for a user  
1564 defined current distribution.

### 1565 Kassiopeia Simulation of the FSCD Magnetic Trap

1566 The trap developed for the FSCD experiment utilizes six current carrying coils, which  
1567 surround a cylindrical tritium containment vessel (see Figure 4.1). Some critical aspects  
1568 of the trap design include the total trapping volume, the maximum trap depth, the

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<sup>1</sup><https://github.com/KATRIN-Experiment/Kassiopeia>

steepness of the trap walls, as well as the radial and azimuthal uniformity of the magnetic fields.

The volume of the FSCD trap is a cylindrically shaped region with a radius of 5 cm and a length of 15 cm resulting in a roughly 1 L total trap volume. The trap volume is an important design feature, because it sets the volume of the experiment that is potentially usable for CRES measurements. Trapping a larger volume allows one to observe a larger number of tritium atoms, which increases the statistical power and sensitivity of the neutrino mass measurement. Due to the cost of constructing magnets with large and uniform magnetic fields it is important that the trap use as much of the available volume as possible to limit the overall cost of the experiment.

Coil	Radius (mm)	Z Pos. (mm)	Current (A × Turns)
1	50.0	-92.3	750.0
2	50.1	-56.9	-220.3
3	68.5	-19.5	-250.0
4	68.5	19.5	-250.0
5	50.1	56.9	-220.3
6	50.0	92.3	750.0

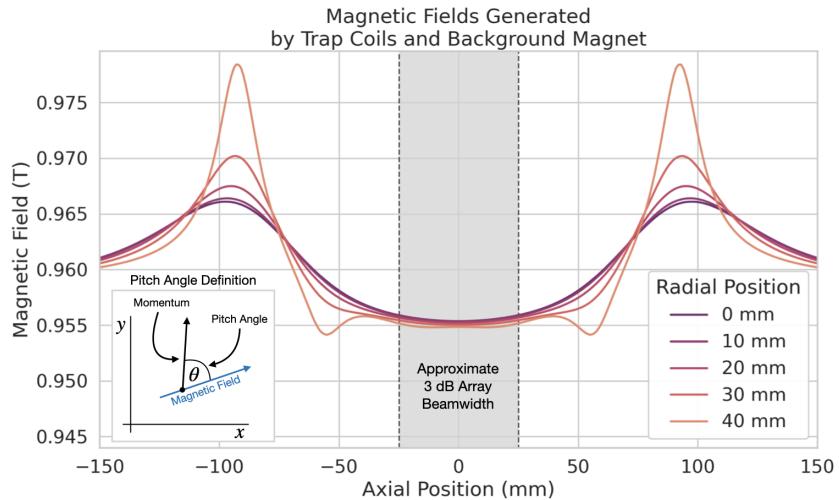
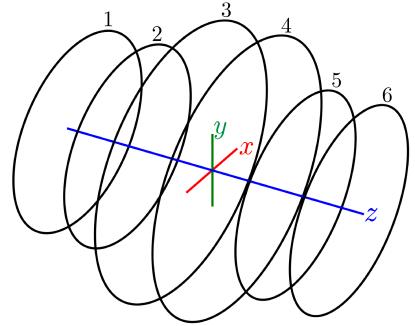


Figure 4.1: The geometry and parameters of the coils used to simulate the FSCD magnetic trap in Kassiopeia. Some axial profiles of the magnetic trap at different radial positions are show to demonstrate the shape of the magnetic field and trap depth as a function of position. Calculation of the magnetic field profiles was graciously done by René Reimann.

The depth of the FSCD trap is approximately 10 mT when measured along the central axis, which is sufficient to trap electrons with pitch angles as small as  $84^\circ$ . The

trap depth factors into the efficiency of the experiment by directly controlling the range of electron pitch angles that can be trapped. If a higher fraction of pitch angles are trapped then, in principle, more decay events can be observed. However, the signals from electrons with small pitch angles are typically significantly harder to detect than larger pitch angles when using an antenna array, which increases the likelihood of not detecting the first track of the CRES event and harms the energy resolution of the experiment.

The steepness of the trap walls as well as any non-uniformities in the magnetic field contribute to the total energy resolution of the CRES measurement by causing uncertainty in the relationship between an electron's kinetic energy and it's cyclotron frequency. When an electron is trapped, it oscillates back and forth along the trap z-axis (see Figure 4.1) unless it is produced with a pitch angle of exactly 90° [9]. As the electron is reflected from the trap walls it experiences a change in the total magnetic field, which causes a modulation in the cyclotron frequency. This change in magnetic field from the trap introduces a correlation between the pitch angle and kinetic energy parameters of the electron that can reduce energy resolution. In order to mitigate this effect it is important to make the trap walls as steep as possible.

## Particle Trajectory Solutions

The magnetic fields solved by direct integration of the electron's current density can be used by Kassiopeia to solve for the trajectory of electrons based on user specified initial conditions. Various distributions are available within Kassiopeia that can be sampled in order to replicate realistic event statistics, including uniform, Gaussian, and Lorentzian among others. In general, an electron has six kinematic parameters that define its trajectory, which are the three-dimensional coordinates of the initial position and the three components of the electron's momentum vector. However, when simulating CRES events it is more common to parameterize the electron's trajectory in terms of it's initial position, the kinetic energy, the pitch angle, and the initial direction of the component of the electron's momentum perpendicular to the magnetic field. This parameterization is completely equivalent to specify each component of the electrons initial position and momentum vectors.

From the initial parameters of the electron and the magnetic field, Kassiopeia solves for the trajectory of the electron. The direct approach proceeds by solving the motion of the electron using the Lorentz force equation, which takes the form of a set of differential

1613 equations

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}}{\gamma m} \quad (4.6)$$

$$\frac{d\mathbf{p}}{dt} = e(\mathbf{E} + \frac{\mathbf{p} \times \mathbf{B}}{\gamma m}), \quad (4.7)$$

1614 where  $\mathbf{r}$  is the position of the electron,  $\mathbf{p}$  is the electron's momentum,  $e$  is the charge of  
1615 the electron,  $m$  is the electron's mass, and  $\gamma$  is the relativistic Lorentz term. To account  
1616 for kinetic energy losses from radiation Kassiopeia includes an additional term in the  
1617 momentum differential equation, which calculates the change in the electron's momentum  
1618 induced by synchrotron radiation. Kassiopeia solves this pair of differential equations  
1619 using numerical integration, however, the exact trajectory can be computationally  
1620 intensive to solve. If the adiabatic approximation can be applied, then Kassiopeia can  
1621 make use of a simpler set of equations that can be more readily solved numerically.

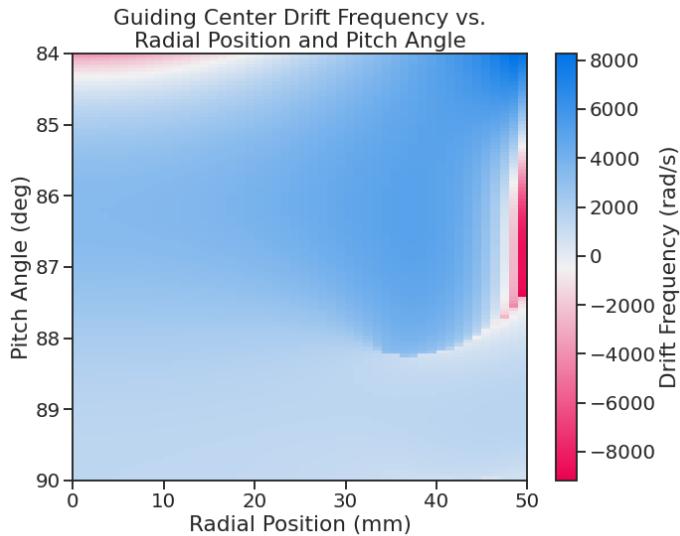


Figure 4.2: A map of the average  $\nabla B$ -drift frequency for electrons trapped in the prototype FSCD trap shown in Figure 4.1. Negative drift frequencies indicate electrons that are drifting opposite to the standard direction, which means that they are close to escaping the magnetic trap.

1622 Even though Kassiopeia is not directly capable of simulating the cyclotron radiation,  
1623 it is still an invaluable CRES simulation tool, due to the accurate trajectory solutions  
1624 for electrons in magnetic traps. With Kassiopeia it is possible to test the efficiency of a  
1625 particular trap design and analyze features of the electron trajectories that are important  
1626 to the position, track, and event reconstruction algorithms (see Section 4.3). One example

of this for the FSCD is the analysis of the average  $\nabla B$ -drift frequency as a function of  
 the electrons radial position and pitch angle in the magnetic trap (see Figure 4.2). Radial  
 gradients in the trap cause the guiding center of the electron to drift around the center of  
 the magnetic trap with an average frequency on the order of  $10^3$  rad/s. This frequency,  
 while slow compared to the length of a typical CRES time-slice, is large enough to cause  
 a significant loss in efficiency of certain signal reconstruction algorithms. Therefore, it is  
 important to model the drift of the electron in the reconstruction algorithm in order to  
 mitigate the effects of this motion on the reconstruction.

### 4.2.2 Locust

The Locust<sup>2</sup> software package [10] is the primary simulation tool developed and used  
 by the Project 8 collaboration for CRES experiments. Locust simulates the responses  
 of antennas and receiver electronics chain to rapidly time-varying electric fields using  
 a flexible approach that allows one to choose from a variety of electric field sources  
 and antennas. Similarly, one can simulate the receiver chain using a series of modular  
 generators that include standard signal processing operations such as down-mixing and  
 fast Fourier transforms (FFT). Since the primary focus of this chapter is the application  
 of Locust to analyses of the FSCD, we shall describe only the most relevant aspects of  
 the software rather than provide a comprehensive description.

### Cyclotron Radiation Field Solutions

Simulating CRES events in the FSCD requires that we calculate the electric fields  
 produced by the acceleration of the electron. In the general case, this can be a complicated  
 question to answer, due to back-reaction forces on the electron from its own electric fields  
 that occur when the electron is surrounded by conductive material such as a waveguide  
 or cavity. However, in the case of the FSCD it is possible to ignore such effects and  
 approximate the electron as radiating into a free-space environment.

The equations that describe the electromagnetic fields from a relativistic moving  
 point particle are the Liénard-Wiechert field equations [11, 12], which are obtained by  
 differentiating the Liénard-Wiechert potentials. In their full form the Liénard-Wiechert  
 field equations are

$$\mathbf{E} = e \left[ \frac{\hat{n} - \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta} \cdot \hat{n})^3 |\mathbf{R}|^2} \right]_{t_r} + \frac{e}{c} \left[ \frac{\hat{n} \times [(\hat{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{n})^3 |\mathbf{R}|} \right]_{t_r} \quad (4.8)$$

---

<sup>2</sup>[https://github.com/project8/locust\\_mc/tree/master](https://github.com/project8/locust_mc/tree/master)

$$\mathbf{B} = [\hat{n} \times \mathbf{E}]_{t_r}, \quad (4.9)$$

where  $e$  is the charge of the particle,  $\hat{n}$  is the unit vector pointing from the particle to the position where the fields are calculated,  $\beta$  and  $\dot{\beta}$  are the velocity and acceleration of the particle divided by the speed of light ( $c$ ),  $\mathbf{R}$  is the distance from the particle to the field calculation position, and  $\gamma$  is the relativistic Lorentz term. The subscript  $t_r$  indicates that the equations must be evaluated at the retarded time so that the time-delay from the travel time of the electromagnetic radiation is correctly accounted for.

The only required input to calculate the electric field at the position of an FSCD antenna is the velocity and acceleration of the electron, which can be obtained from Kassiopeia simulations. Therefore, when simulating a CRES event Locust first runs a Kassiopeia simulation of the electron and calculates the electric field incident on the antenna. The only difficulty with this approach is the determination of the retarded time. The retarded time corresponds to the time that a photon, which has just arrived at an antenna at the space-time position  $(t, \mathbf{r})$ , was actually emitted by the electron at the space-time position of  $(t_r, \mathbf{r}_e(t_r))$ . Defined in this way, finding the retarded time requires solving

$$c(t - t_r) = |\mathbf{r} - \mathbf{r}_e(t_r)|, \quad (4.10)$$

where the distance traveled by the photon between the measurement and retarded times is equal to the distance between the antenna and the electron at the retarded time. Locust solves Equation 4.10 using a built-in root finding algorithm to find the retarded time, and thus the electric field produced by the electron at the position of each antenna in the FSCD array.

## Antenna Response Modeling

With the electric field it is possible, in principle, to calculate the resulting voltages produced in the antenna. However, direct simulation of the antenna itself is computationally expensive since it would require the modeling of complex interactions of the electron's electric fields with charge carriers in the conductive elements of the antenna. Direct simulation of the antenna in Locust can be avoided by modeling the antenna response using the antenna factor, or antenna transfer function, approach. The antenna factor defines the voltage produced in the antenna terminal for an incident electric field [13],

$$A_F = \frac{V}{|\mathbf{E}|}, \quad (4.11)$$

1684 where  $V$  is the voltage and  $|\mathbf{E}|$  is the magnitude of the incident electric field. To obtain the  
 1685 antenna factor for the antennas developed for the FSCD Project 8 employs Ansys HFSS.  
 1686 HFSS is a commercially available finite element method electromagnetic solver widely  
 1687 used throughout the antenna engineering industry [14]. HFSS is capable of calculating  
 1688 the antenna factor and gain patterns for complex antenna designs and outputting the  
 1689 resulting quantities in the form of a text file that can be used as an input to the Locust  
 1690 simulation.

1691 The antenna factor defines the steady-state response of the antenna to electromagnetic  
 1692 plane waves and is a function of the frequency of the radiation. Therefore, in order to  
 1693 apply the transfer function for the calculation of the antenna voltage response in the  
 1694 time domain, Locust models the antenna as a linear time-invariant system [15]. In this  
 1695 formalism the response of the system to the driving force is given by

$$y[n] = h * x = \sum_k h[k]x[n - k], \quad (4.12)$$

1696 where  $y[n]$  is the discretely sampled response,  $x$  is the driving force stimulus, and  $h$  is  
 1697 the finite impulse response (FIR) filter. When applied to the FSCD array, this formalism  
 1698 calculates the voltage time-series produced in each antenna by convolving the electric  
 1699 field time-series with the antenna FIR filter, which is obtained by performing a inverse  
 1700 Fourier transform on the transfer function from HFSS.

## 1701 Radio-frequency Receiver and Signal Processing

1702 After obtaining the voltage time-series by computing the electron trajectory and antenna  
 1703 response, Locust simulates the signal processing associated with the radio-frequency  
 1704 receiver chain. The standard receiver chain used in Locust simulations of the FSCD  
 1705 attempts to mimic the operations that would actually occur in hardware (see Figure 4.3).

1706 Frequency down-conversion is used in the FSCD to reduce the digitization bandwidth  
 1707 required to read-out CRES data. According to the Nyquist sampling theorem [16], the  
 1708 minimal sampling rate that guarantees no information loss for a signal with a bandwidth  
 1709  $\Delta f$  is given by

$$f_{\text{Nyq}} = 2\Delta f. \quad (4.13)$$

1710 The total bandwidth of CRES signal frequencies from tritium beta-decay ranges from 0  
 1711 to 26 GHz in a 0.95 T magnetic field, therefore, direct digitization of CRES signals from  
 1712 the FSCD would require sampling frequencies greater than 50 GHz, which is infeasible for

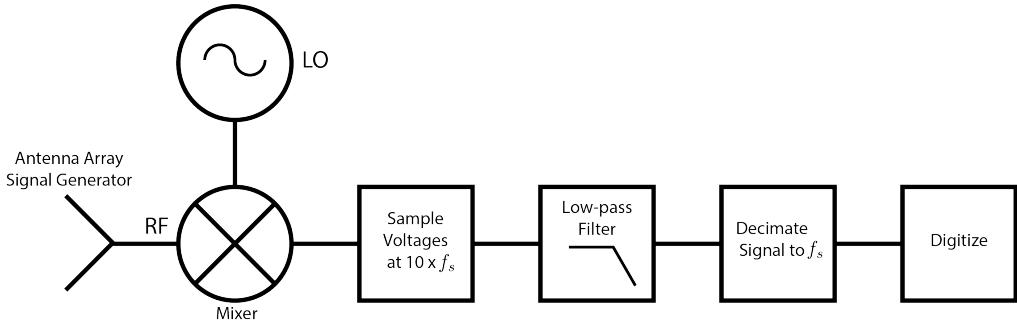


Figure 4.3: The receiver chain used by Locust when simulating CRES events in the FSCD.

1713 a real experiment. However, for the purposes of neutrino mass measurement we are only  
 1714 interested in measuring the shape of the spectrum in the last 100 eV, which corresponds  
 1715 to a frequency bandwidth of 5 MHz. Down-conversion is a technique for reducing the  
 1716 base frequencies of signals in a bandwidth given by  $[f_{\text{LO}}, f_{\text{LO}} + \Delta f]$  to the bandwidth  
 1717  $[0, \Delta f]$ , by performing the following multiplication

$$x(t) \rightarrow x(t)e^{-2\pi f_{\text{LO}} t}. \quad (4.14)$$

1718 In down-conversion the signal ( $x(t)$ ) is multiplied by a sinusoidal signal with frequency  
 1719  $f_{\text{LO}}$  to reduce the absolute frequencies of the signals in the bandwidth. In the FSCD this  
 1720 allows us to detect events in the last 100 eV of the tritium spectrum while sampling the  
 1721 data far below 50 GHz. The standard bandwidth used in the FSCD is 200 MHz, which  
 1722 allows for higher frequency resolution than the minimum sampling frequency for 100 eV  
 1723 of energy bandwidth.

1724 Trying to directly simulate down-conversion with a frequency multiplication in Locust  
 1725 would require the sampling of the electric fields at each antenna in the FSCD array with  
 1726 a period of  $\approx 20$  ps, which is extremely slow computationally. To avoid this Locust  
 1727 performs the down-conversion by intentionally under-sampling the electric fields with  
 1728 a frequency of 2 GHz. Sampling below the Nyquist limit causes the higher frequency  
 1729 components of the CRES signal to alias, however, Locust can remove these aliased  
 1730 frequency peaks using a combination of low-pass filtering and decimation to recreate  
 1731 frequency down-conversion. After filtering and decimation, Locust simulates digitization  
 1732 by an 8-bit digitizer at a sampling frequency of 200 MHz to recreate the conditions of  
 1733 the FSCD. The voltage offset and the digitizer range must be configured by the user  
 1734 based on the characteristics of the simulation.

1735 **Data**

1736 The output of Locust simulations for the FSCD primarily consists of two data files. The  
1737 first is the electron trajectory information calculated by Kassiopiea, which is output in  
1738 the form of a `.root` file [17]. This file contains important kinematic information about  
1739 the electron such as it's position and pitch angle as a function of time. The other file is  
1740 produced by Locust and it contains the digitized signals acquired from each antenna in  
1741 the FSCD array. The Locust output files conform to the Monarch specification developed  
1742 by Project 8, which is based on the commonly used HDF5 file format, and matches the  
1743 format of the files produced by the Project 8 data acquisition software. This makes it  
1744 possible to use the same data analysis code to analyze both simulated and real data.

1745 **4.2.3 CRESana**

1746 Locust is the primary simulation tool used by Project 8 in the development and simulation  
1747 of the FSCD. However, simulations of CRES events in larger antenna arrays ( $\geq 100$   
1748 antennas) using Locust can take several hours to complete, which is prohibitively long  
1749 when one is performing a sensitivity analysis for a large scale antenna experiment. One  
1750 of the reasons for Locust's slow operation is that the electric fields from the electron  
1751 must be solved numerically for each time-step for each of the antennas in the array.  
1752 These numerical solutions allow Locust to accurately simulate the electric fields from  
1753 arbitrarily complicated electron trajectories at the cost of more computations and slower  
1754 simulations. Therefore, an additional simulation tool that sacrifices some accuracy for  
1755 computational efficiency would be extremely useful simulations and sensitivity analyses  
1756 of larger antenna array experiments.

1757 To fill this need, Project has developed a new simulations package called CRESana<sup>3</sup>,  
1758 specifically designed to perform analytical simulations of antenna array based CRES  
1759 experiments. CRESana is not as flexible as Locust, but it provides a significant increase  
1760 in simulation speed. It does this by using well-justified analytical approximations of the  
1761 electrons motion in the magnetic field and the resulting electric fields from the electron's  
1762 acceleration. The electric fields and signals generated by CRESana are consistent with  
1763 theoretical calculations of the electron's radiation, and are test for accuracy using  
1764 well-known test-case simulations and consistency checks.

---

<sup>3</sup><https://github.com/MCFlowMace/CRESana>

## **4.3 Signal Detection and Reconstruction Techniques for Antenna Array CRES**

### **1767 Antenna Array CRES Signal Reconstruction**

**1768** A robust set of FSCD simulation tools are vital to the development of the analysis  
**1769** algorithms necessary for antenna array CRES to succeed. In order to perform CRES  
**1770** measurements using an antenna array, one must develop an algorithm that uses the  
**1771** multi-channel time-series obtained by digitizing the array to estimate the starting kinetic  
**1772** energies of electrons produced in the magnetic trap. This procedure consists of a multi-  
**1773** stage process of detecting a CRES signal then estimating the parameters of the electron  
**1774** that produced and is often referred to as simply CRES signal reconstruction.

**1775** Compared with the signal reconstruction approaches of the Phase I and II CRES  
**1776** experiments, antenna array CRES requires a significantly different approach to signal  
**1777** reconstruction. In Phase I and II, CRES was performed using a waveguide gas cell that  
**1778** could be directly connected to a waveguide transmission line. The transmission line  
**1779** efficiently transmits the cyclotron radiation along its length to an antenna at either end  
**1780** of the waveguide. However, with an antenna array the electron is essentially radiating  
**1781** into free-space, therefore, the cyclotron radiation power collected by the array is directly  
**1782** proportional to the solid angle surrounding the electron that is covered with antennas.  
**1783** Because it is not practical to fully surround the magnetic trap with antennas, some of the  
**1784** cyclotron radiation power that would have been collected by the waveguide escapes into  
**1785** free-space. Furthermore, the power that is collected by the antenna array is split between  
**1786** every channel in the antenna array, which significantly lowers the signal-to-noise ratio  
**1787** (SNR) of CRES signals in a single antenna channel compared to a waveguide apparatus.  
**1788** Therefore, a suite of completely new signal reconstruction techniques are needed in order  
**1789** to perform CRES in the FSCD.

**1790** Changes to the approach to CRES signal reconstruction are also motivated by the  
**1791** more ambitious scientific goals of the FSCD experiment. A measurement of the tritium  
**1792** beta-decay spectrum that is sensitive to neutrino masses as small as 40 meV requires that  
**1793** we measure the kinetic energies of individual electrons with a total energy broadening  
**1794** of 115 meV [18]. This resolution includes all sources of uncertainty in the electron's  
**1795** kinetic energy such as magnetic field inhomogeneities. This level of energy resolution is  
**1796** compatible only with an event-by-event signal reconstruction approach where the kinetic  
**1797** energies, pitch angles, and other parameters of the CRES events are estimated before

1798 constructing the beta-decay spectrum.

1799 The event-by-event approach is distinct from the analysis done for the Phase I and  
1800 Phase II experiments where only the starting cyclotron frequency of the event was  
1801 estimated by analyzing the tracks formed by the carrier frequency in the time-frequency  
1802 spectrogram. These frequencies were then combined into a frequency spectrogram, which  
1803 was converted to the beta-decay energy spectrum using an ensemble approach that  
1804 averaged over all other event parameters. The ensemble approach to signal reconstruction  
1805 results in poor energy resolution because other kinematic parameters such as pitch angle  
1806 change the cyclotron carrier frequency due to changes in the average magnetic field  
1807 experience by the electron, and it is therefore incompatible with the future goals of the  
1808 Project 8 collaboration.

### 1809 Components of Reconstruction: Signal Detection and Parameter Estimation

1810 CRES signal reconstruction can be viewed as a two-step procedure consisting of signal  
1811 detection followed by parameter estimation. In the former, one is concerned with  
1812 identifying CRES signals in the data regardless of the signal parameters, whereas, in the  
1813 latter one operates under the assumption that a signal is present and then estimates its  
1814 parameters.

1815 More formally, signal detection is essentially a binary hypothesis test between the  
1816 signal and noise data classes and parameter estimation describes a procedure of fitting a  
1817 model to the observed data. While both of these processes are required for a complete  
1818 reconstruction (see Figure 4.4), the focus of my work and this chapter is on the signal  
1819 detection aspect of antenna array CRES signal reconstruction.

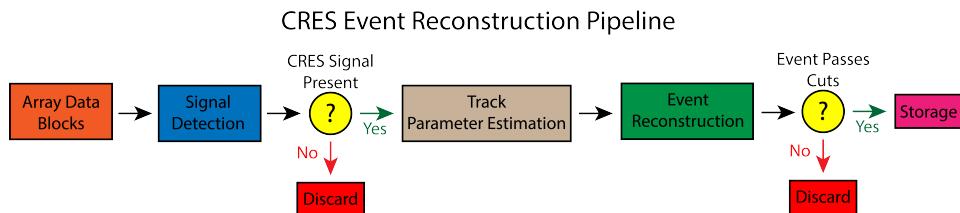


Figure 4.4: A high-level diagram depicting the process of CRES event reconstruction. The first step consists of identifying the presence of a signal in the data. This step is necessary to avoid the danger of performing a reconstruction of a false event, which would constitute a background contribution to the tritium spectrum measured by CRES.

1820 **Detection Theory**

1821 The problem of signal detection can be posed as a statistical hypothesis test [19]. For  
1822 CRES signals, which are essentially vectors with added white Gaussian noise (WGN),  
1823 one needs to choose between two hypotheses

$$\mathcal{H}_0 : \mathbf{y} = \boldsymbol{\nu} \quad (4.15)$$

$$\mathcal{H}_1 : \mathbf{y} = \mathbf{x} + \boldsymbol{\nu}, \quad (4.16)$$

1824 where  $\mathbf{y}$  is the CRES data vector,  $\boldsymbol{\nu}$  is a sample of WGN, and  $\mathbf{x}$  represents the CRES  
1825 signal. The hypothesis that the data contains only noise is labeled  $\mathcal{H}_0$  and the hypothesis  
1826 that the data contains a signal is labeled  $\mathcal{H}_1$ .

1827 For illustrative purposes one can examine the case where one the first sample of  
1828 data is used to distinguish between  $\mathcal{H}_0$  and  $\mathcal{H}_1$ . The value of the first data sample is  
1829 distributed according to two gaussian distributions corresponding to  $\mathcal{H}_0$  and  $\mathcal{H}_1$  (see  
1830 Figure 4.5). By setting a decision threshold on the value of this sample, one can choose  
1831 the correct hypothesis with a probability given by the areas underneath the probability  
1832 distribution curves. A true positive corresponds to correctly identifying that the data  
1833 contains signal, whereas, a true negative means that one has correctly identified the data  
1834 as noise. The rate at which the detector performs a true positive classification is given  
1835 by the green region underneath  $p(\mathbf{y}[0]; \mathcal{H}_0)$ , and the rate at which the detector performs  
1836 a true negative classification is given by the orange region underneath  $p(\mathbf{y}[0]; \mathcal{H}_1)$ . Two  
1837 types of misclassifications are possible. Either we declare noise data as signal, which is  
1838 call a false positive, or we declare signal data as noise, which is a false negative. Note  
1839 that it is only possible to trade off these two types of errors by tuning the detection  
1840 threshold. One cannot simultaneously reduce the rate of false positives without also  
1841 increasing the rate of false negatives.

1842 The approach taken with CRES signals is to fix the rate of false positives by setting  
1843 a minimum value for a detection threshold. The rate of false positives that is acceptable  
1844 at the detection stage depends upon the rate of background events compatible with the  
1845 sensitivity goals of the experiment. The ultimate goal of a neutrino mass measurement  
1846 with 40 meV sensitivity in general has strict requirements on the number of background  
1847 events, which requires a relatively high detection threshold to achieve. Consequently,  
1848 the ideal signal detection algorithm is the one that achieves the maximum rate of true  
1849 positives for a fixed rate of false positives, so that the detection efficiency of the experiment  
1850 is maximized and potential sources of background are kept to a minimum.

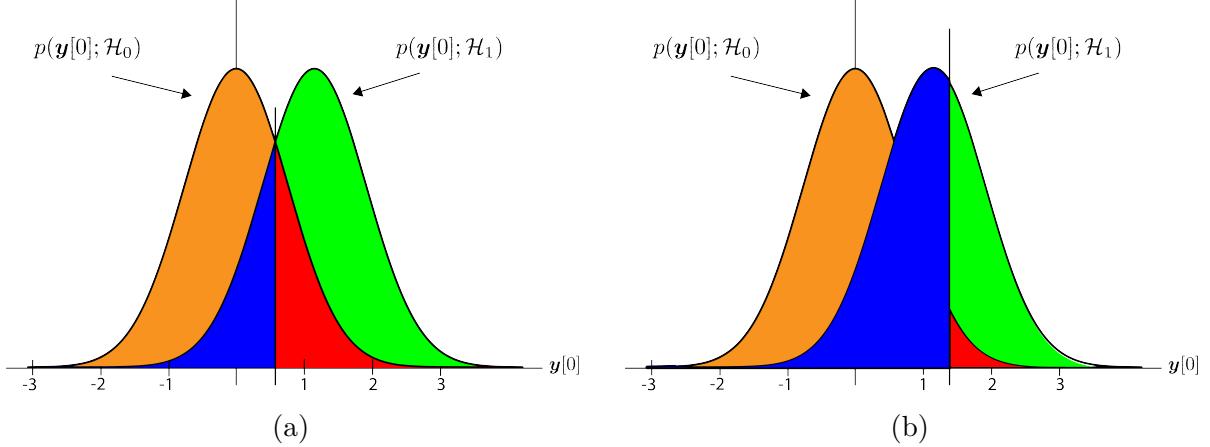


Figure 4.5: An illustration of two PDFs associated with a binary hypothesis test. The decision threshold is represented by the vertical line that partitions both distributions. The orange and red areas correspond to the true negative and false positive probabilities and the blue and green areas correspond to the false negative and true positive probabilities respectively. To decide between the two hypotheses we perform the likelihood ratio test specified by the Neyman-Pearson theorem. This approach achieves the highest true positive probability for a given false positive probability.

According to the Neyman-Pearson theorem [20], the statistical hypothesis test that maximizes the probability of detection for a fixed rate of false positives is the likelihood ratio test, which is formed by computing the ratio of the signal likelihood to the noise likelihood,

$$L(x) = \frac{P(\mathbf{y}; \mathcal{H}_1)}{P(\mathbf{y}; \mathcal{H}_0)} > \gamma. \quad (4.17)$$

Here, the likelihood of the hypotheses  $\mathcal{H}_0$  and  $\mathcal{H}_1$  are described by the probability distributions  $P(\mathbf{y}; \mathcal{H}_0)$  and  $P(\mathbf{y}; \mathcal{H}_1)$  respectively, and  $\gamma$  is the threshold for deciding  $\mathcal{H}_1$ . The decision threshold is determined by integrating  $P(\mathbf{y}; \mathcal{H}_0)$  such that

$$P_{\text{FP}} = \int_{\gamma}^{\infty} P(\tilde{\mathbf{y}}; \mathcal{H}_0) d\tilde{\mathbf{y}} = \alpha, \quad (4.18)$$

where  $\alpha$  is the desired false positive detection rate given by the red colored areas shown in Figure 4.5. The true positive detection rate is given by the similar integral

$$P_{\text{TP}} = \int_{\gamma}^{\infty} P(\tilde{\mathbf{y}}; \mathcal{H}_1) d\tilde{\mathbf{y}}, \quad (4.19)$$

which corresponds to the green areas in Figure 4.5.

Changing the decision threshold allows one to trade-off between  $P_{\text{TP}}$  and  $P_{\text{FP}}$  as

appropriate for the given situation. It is common to summarize the relationship between  $P_{\text{TP}}$  and  $P_{\text{FP}}$  using the receiver operating characteristic (ROC) curve, which is obtained by evaluating the true positive and false positive probabilities as a function of the decision threshold value (see Figure 4.6). The ROC curve provides a convenient way to compare

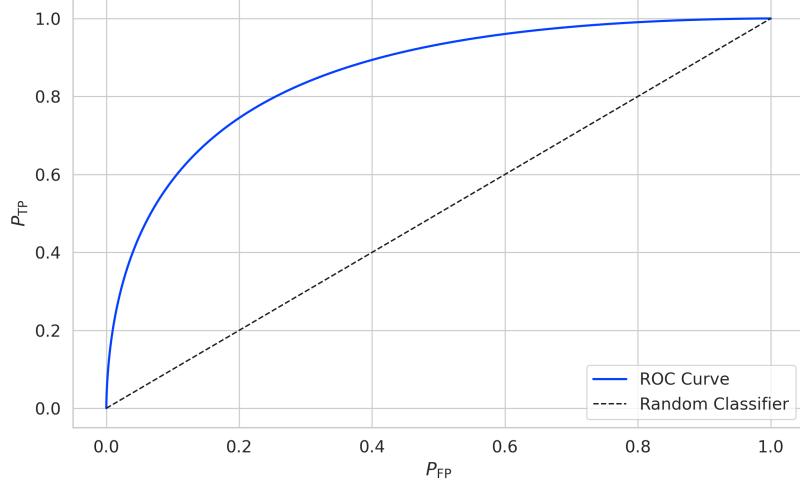


Figure 4.6: An example ROC curve formed by computing the  $P_{\text{FP}}$  and the  $P_{\text{TP}}$  for a given likelihood ratio test. As the decision threshold is increased  $P_{\text{FP}}$  decreases at the expense of a lower  $P_{\text{TP}}$ . The black dashed line indicates the lower bound ROC curve obtained by randomly deciding between  $\mathcal{H}_0$  and  $\mathcal{H}_1$ .

the performance of different signal detection algorithms. In general, a classifier with a higher the  $P_{\text{TP}}$  as a function of  $P_{\text{FP}}$  is desirable, which corresponds to a larger area underneath the respective ROC curve. A perfect classifier has an area underneath the curve of 1.0, however, such a classifier is almost never achievable in practice.

### 4.3.1 Digital Beamforming

#### Introduction to Beamforming

Beamforming refers to a suite of antenna array signal processing techniques that are designed to enhance the radiation or gain of the array in certain directions and suppress it in other direction [13]. Beamforming is of interest to Project 8 as a first level of signal reconstruction for the FSCD and other antenna array CRES experiments, which operates at the signal detection stage of reconstruction.

Beamforming is accomplished by performing a phased summation of the signals received by the antenna array. The beamforming phases are chosen such that the signals

1879 emitted by the array will constructively interfere at the point of interest (see Figure  
 1880 4.7). As a consequence of the principle of reciprocity [21], when the array is operating in  
 1881 receive mode, the signals emitted from a source at the same point will constructively  
 interfere when summed. The origin of the phase delays in beamforming is the path-

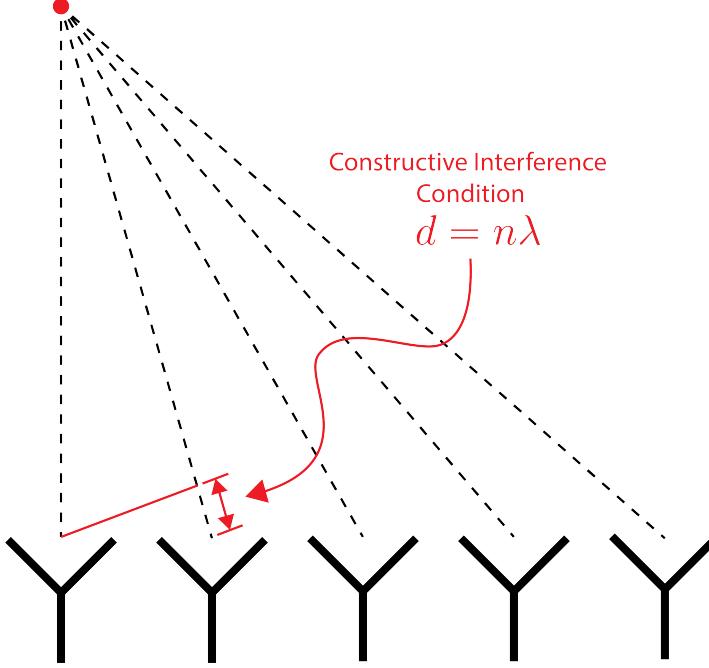


Figure 4.7: An illustration of the constructive interference condition which is the operating principle of digital beamforming using a uniform linear array as an example.

1882  
 1883 length difference to the beamforming point between different antennas in the array. The  
 1884 relationship between the phase delay and the path-length difference is given by the  
 1885 familiar equation

$$\phi = \frac{2\pi d}{\lambda}, \quad (4.20)$$

1886 where  $\phi$  is the phase delay,  $d$  is the path-length difference, and  $\lambda$  is the wavelength of  
 1887 the radiation. In practice, one chooses the values of  $d$  by specifying the beamforming  
 1888 positions of interest and then calculates the beamforming phases using Equation 4.20,  
 1889 which is guaranteed to follow the constructive interference condition shown in Figure 4.7.

1890 Beamforming can be neatly expressed mathematically using the vector equation

$$y[n] = \Phi^T[n] \mathbf{x}[n], \quad (4.21)$$

1891 where  $\mathbf{x}[n]$  is the array snapshot vector,  $\Phi[n]$  is a vector of beamforming shifts, and  
 1892  $y[n]$  is the resulting summed signal. The beamforming shifts consist of a set of complex

1893 numbers that contain the beamforming phase shift and an amplitude weighting factor,

$$\Phi[n] = [A_0[n]e^{-2\pi i \phi_0[n]}, A_1[n]e^{-2\pi i \phi_1[n]}, \dots, A_{N-1}[n]e^{-2\pi i \phi_{N-1}[n]}], \quad (4.22)$$

1894 where the set of magnitudes  $A_i[n]$  are amplitude weighting factors and  $\phi_i[n]$  are the phase  
1895 shifts from the path-length differences. The index  $i$  is used to denote the antenna channel  
1896 number. The amplitude weighting factor is the relative magnitude of the signal received  
1897 by a particular antenna to the other antennas in the array, such that the antennas that  
1898 receive signals with higher amplitude, due to being closer to the source, have more  
1899 weight in the beamforming summation. The input and outputs signals beamforming  
1900 are naturally expected to be functions of time as indicated by the index  $[n]$ , however, it  
1901 is also possible to use time dependent beamforming phases that shift the beamforming  
1902 position of the array over time.

1903 Digital beamforming is the type of beamforming algorithm of interest to Project 8 for  
1904 CRES. Specifically, digital beamforming means that the beamforming phases are applied  
1905 to the array signals in software rather than employing fixed beamforming phase shifts in  
1906 the receiver chain hardware. The advantage of digital beamforming is that for a given  
1907 series of array snapshots one can specify a large number of beamforming positions and  
1908 effectively search for electrons by performing the beamforming summation associated  
1909 with each point and applying a signal detection algorithm to identify the presence of a  
1910 CRES signal.

1911 One of the most attractive features of digital beamforming is the spatial filtering  
1912 effect, which is a direct consequence of the constructive interference condition used to  
1913 define the beamforming phases. Spatial filtering allows for signals from multiple electrons  
1914 at different positions in the trap to be effectively separated, because the constructive  
1915 interference condition will force the signals from electrons at positions different from the  
1916 beamforming position to cancel. This helps to reduce signal pile-up that could become  
1917 an issue for large scale CRES experiments using a dense tritium source.

1918 The digital beamforming positions can be specified with arbitrary densities limited  
1919 only by the available computational resources. This provides a very straight-forward way  
1920 to estimate the position of the electron in the trap by using a dense grid of beamforming  
1921 positions and maximizing the output power of the beamforming summation over this  
1922 grid. This natural approach to position reconstruction is attractive due the requirements  
1923 of an event-by-event signal reconstruction, which needs an accurate estimation of the  
1924 exact magnetic field experienced by the electron in order to correctly estimate it's kinetic

1925 energy. Combined with an accurate map of the magnetic field inhomogeneities of the  
1926 trap obtained from calibrations, beamforming allows one to apply this magnetic field  
1927 correction with a spatial resolution that is a fraction of the cyclotron wavelength.

1928 **Laboratory Beamforming Demonstrations**

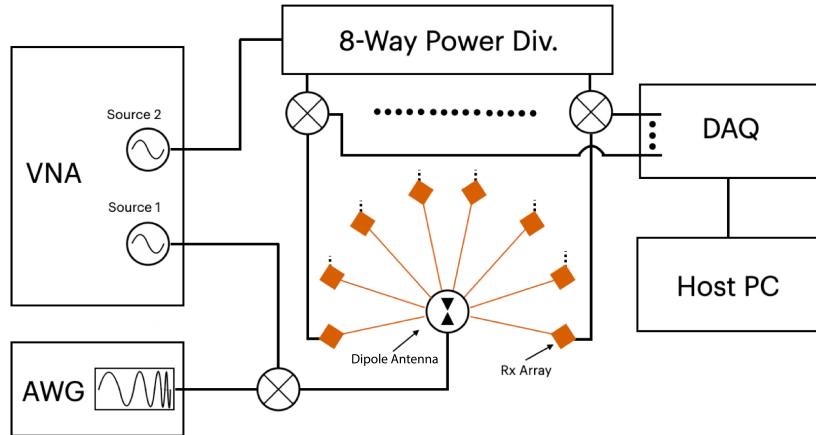


Figure 4.8

1929 **FSCD Beamforming Simulations**

1930 Using Locust simulations of the FSCD one can perform beamforming reconstruction  
1931 studies using the simulated CRES signal data. As we mentioned in the previous section,

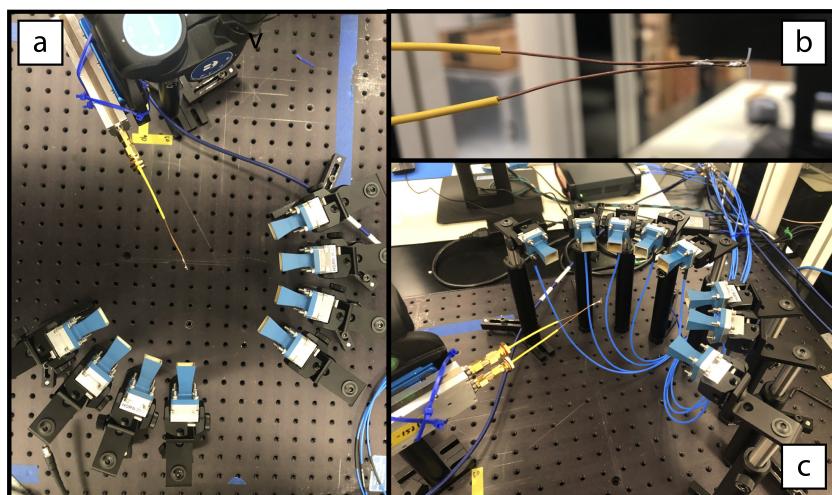


Figure 4.9

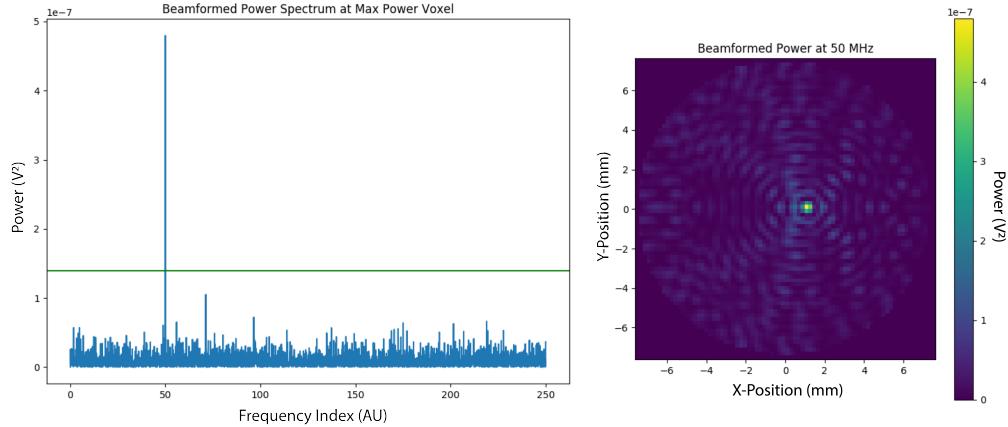


Figure 4.10

1932 the beamforming procedure begins by specifying a set of beamforming positions and  
 1933 corresponding beamforming shifts. The beamforming positions form a grid that covers the  
 1934 region of interest in the field of view of the antenna array. There are effectively an infinite  
 1935 number of ways to specify the grid positions, however, uniform square grids are the most  
 1936 commonly used due to their simplicity. In the FSCD experiment the number and pattern  
 1937 of the grid positions would be optimized to cover the most important regions of the trap  
 1938 volume to maximize detection efficiency while minimizing superfluous calculations.

1939 The beamforming grids used for signal reconstruction with the FSCD consist of a  
 1940 set of points that cover a region of the two-dimensional plane formed by the perimeter  
 1941 of the antenna array. The axial dimension is left out of the beamforming grid because  
 1942 the electrons are assumed to occupy only an average axial position, which corresponds  
 1943 to the center of the magnetic trap. This is because it is impossible to resolve the axial  
 1944 position of the electron as a function of time due to the rapid axial oscillation frequencies  
 1945 of trapped electrons relative to the FSCD time-slice duration.

1946 After beamforming, a summed time-series is obtained for each beamforming position  
 1947 that can be evaluated for the presence of a signal using a detection algorithm. A beam-

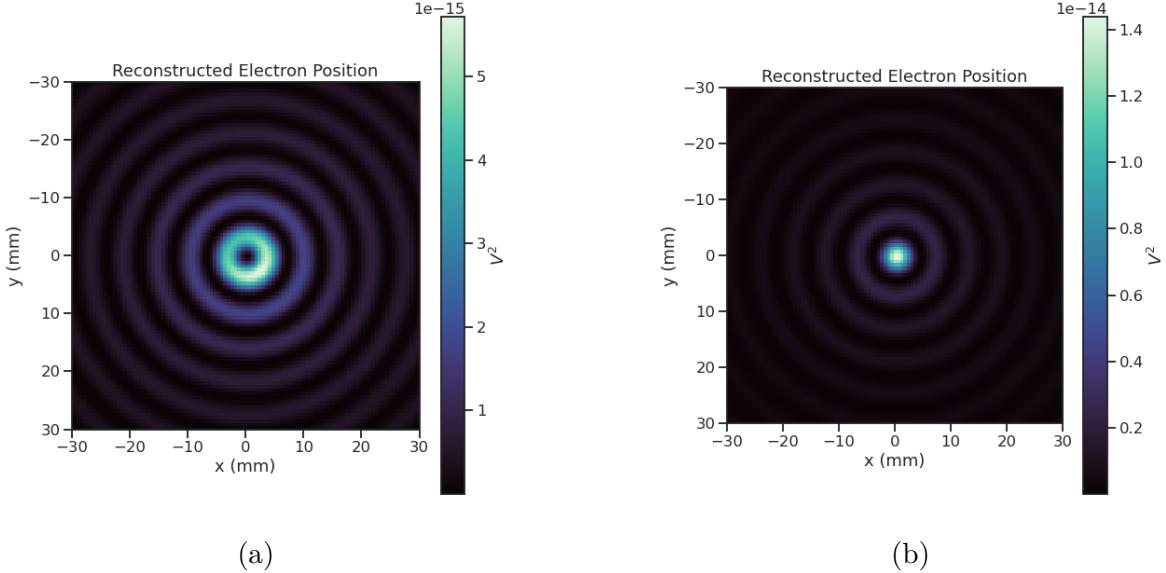


Figure 4.11: Beamforming images visualizing the reconstruction of an electron without (a) and with (b) the cyclotron phase correction. The images were generated using data from Locust simulations. The cyclotron phase refers to a phase offset equal to the relative azimuthal position of an antenna in the array. This phase offset is caused by the circular electron orbit and must be corrected for during reconstruction.

1948 forming image is a visualization method that is equivalent to arranging the beamforming  
 1949 grid points according to their physical locations to form a three-dimensional matrix  
 1950 where the first two dimensions encode the XY-position of the beamforming point and  
 1951 the third dimension contains the summed time-series. The image is formed by taking the  
 1952 time-averaged power (see Figure 4.11). Beamforming images are purely for the purposes  
 1953 of visualization and are not particularly useful for signal detection or reconstruction.

1954 If the beamforming phases consist only of the spatial phase component from Equation  
 1955 4.20, then the resulting beamforming image contains a relatively high-power ring-shaped  
 1956 region that is centered on the position of the electron (see Figure 4.11a). The origin  
 1957 of this shape is an additional phase offset particular to a cyclotron radiation source.  
 1958 Essentially, the circular motion that produces the cyclotron radiation introduces a relative  
 1959 phase offset to the electric fields that is equal to the azimuthal position of the field  
 1960 measurement point. For example, if we have two antennas, one located at an azimuthal  
 1961 position of  $0^\circ$  and another located at an azimuthal position of  $90^\circ$ , then the CRES signals  
 1962 received by these antennas will be out of phase by  $90^\circ$ , which is the difference in their  
 1963 azimuthal positions. This phase offset can be corrected by adding an additional term to  
 1964 the beamforming phase equation that is equal to the azimuthal position of the antenna

1965 relative to the electron,

$$\phi_i[n] = \frac{2\pi d_i[n]}{\lambda} + \Delta\varphi_i[n], \quad (4.23)$$

1966 where  $\Delta\varphi_i$  is difference between the azimuthal position of the electron and the  $i$ -th  
 1967 antenna channel. Using the updated beamforming phases in the summation changes the  
 1968 ring feature into a Bessel function peak whose maximum corresponds to the position of  
 1969 the electron. Including this cyclotron phase correction significantly improves the signal  
 1970 detection and reconstruction capabilities of beamforming by more than doubling the  
 1971 summed signal power and shrinking the beamforming maxima feature size.

1972 The beamforming image examples in Figure 4.11 were produced using an electron  
 1973 located on the central axis of the magnetic trap, which do not experience  $\nabla B$ -drift.  
 1974 However, for electrons produced at non-zero radial position the beamforming phases  
 1975 must be made time-dependent in order to track the position of the electron's guiding  
 1976 center over time. Without this correction the  $\nabla B$ -drift causes the electron to move  
 1977 between beamforming positions, which effectively spreads the cyclotron radiation power  
 over a wider area in the beamforming image (see Figure 4.12). This effect significantly

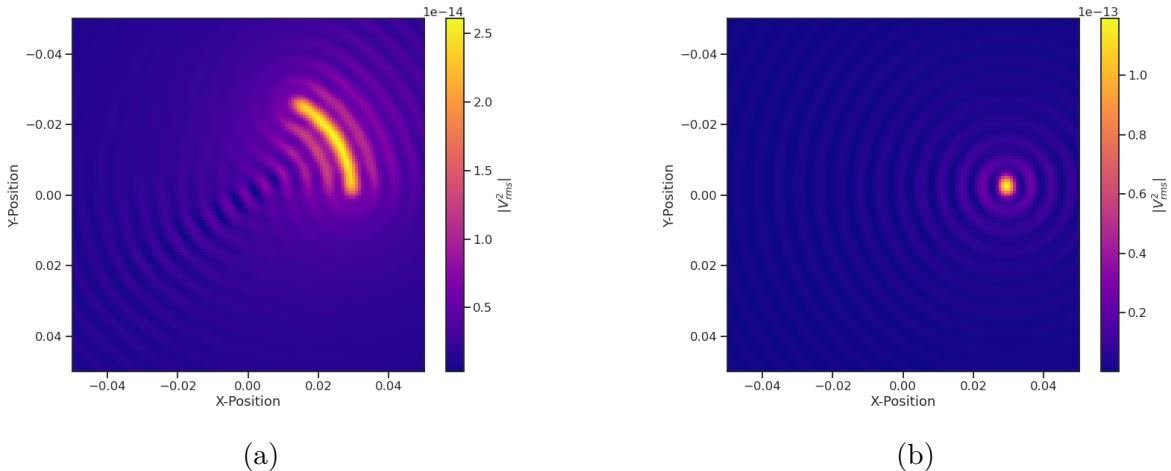


Figure 4.12: Beamforming images visualizing the reconstruction of an electron located off the central axis of the FSCD trap. In (a) we performing beamforming without the  $\nabla B$ -drift correction, and in (b) we include the  $\nabla B$ -drift correction.

1978  
 1979 reduces the power of the beamforming maxima and increases the size of the beamforming  
 1980 features, simultaneously harming detection efficiency and position reconstruction.

1981 The  $\nabla B$ -drift correction simply adds a circular time-dependence to the beamforming  
 1982 positions as a function of time,

$$r[n] = r_0 \quad (4.24)$$

$$\varphi[n] = \varphi_0 + \omega_{\nabla B} t[n], \quad (4.25)$$

where  $\omega_{\nabla B}$  is the drift frequency and  $t[n]$  is the time vector. In the ideal case the  $\nabla B$ -drift frequencies from Figure 4.2 for the correct pitch angle and radial position would be used, however, it is not possible to know the electron's pitch angle a priori. In principle, one could perform multiple beamforming summations for a given beamforming position using different drift frequencies and choose the one that maximizes the summed power, but this approach leads to a huge computational burden that would be impractical for a real FSCD experiment. A compromise is to use an average value of  $\omega_{\nabla B}$  obtained by averaging over the drift frequencies for electrons of different pitch angle at a particular radius. This approach keeps the computational cost of time-dependent beamforming to a minimum while still providing a significant increase in the detection efficiency of digital beamforming.

#### Signal Detection with Beamforming and a Power Threshold

Up to this point we have neglected any specific discussion of how digital beamforming is used for signal detection and reconstruction. This is because, strictly speaking, digital beamforming consists only of the phased summation of the array signals and cannot be used alone for signal detection. The example beamforming images shown in Figure 4.11 and Figure 4.12 were produced using simulated data that contained no noise, which significantly degrades the utility of analyzing the beamforming images for signal detection and reconstruction.

Digital beamforming as a detection algorithm is understood to mean digital beamforming plus a detection threshold placed on the amplitude of the frequency spectrum obtained by applying a fast Fourier transform (FFT) to the summed time-series (see Figure 4.13). This approach is most similar to the time-frequency spectrogram analysis employed in previous CRES experiments, however, in principle any signal detection algorithm could be used after the beamforming procedure. In Section 4.4 I analyze the signal detection performance of the power threshold approach in detail.

From the example frequency spectra in Figure 4.13 it is clear that without a reconstruction technique that coherently combines the signals from the full antenna our ability to detect CRES signals will be drastically reduced. Because the CRES signals are in-phase at the correct beamforming position the summed power increases as a function of  $N^2$  compared to a single antenna channel, where  $N$  is the number of antennas. It is true that the noise power is also increased by beamforming, but, because the noise

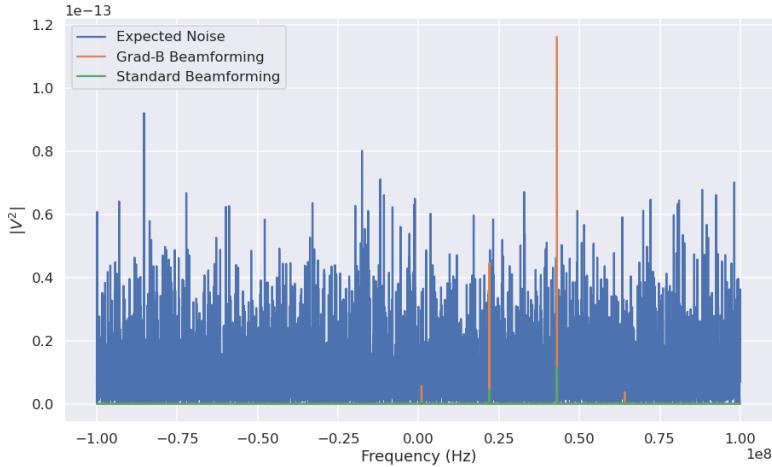


Figure 4.13: A plot of a typical frequency spectrum obtained by applying a Fourier transform to the time-series obtained from beamforming. The frequency spectra are plotted without noise on top of an example of a typical noise spectrum to visualize a realistic signal-to-noise ratio. In the example we see that without beamforming it would not be possible to detect anything since the signal amplitudes would be reduced by a factor of sixty relative to the noise. Additionally, we see that the  $\nabla B$ -drift correction is needed to detect this electron since it comes from a simulation of an electron with a significant off-axis position.

2015 is incoherent, it's power only increases linearly. Consequently, the signal-to-noise ratio  
 2016 (SNR) of the CRES signal increases linearly with the number of antennas, which greatly  
 2017 improves detection efficiency compared to using only the information in a single antenna.

2018 The power threshold detection algorithm searches for high-power frequency bins that  
 2019 should correspond to a frequency component of the CRES signal. In order to prevent  
 2020 random noise fluctuations from being mistaken as CRES signals the power threshold  
 2021 must be set high enough so that it is unlikely that random noise could be responsible. A  
 2022 consequence of this is that many electrons that can be trapped will go undetected because  
 2023 the modulation caused by axial oscillations leads to the cyclotron carrier power to falling  
 2024 below the decision threshold. The time-dependent beamforming used to correct for the  
 2025  $\nabla B$ -drift increases the volume of the magnetic trap where electrons can be detected,  
 2026 but it is ineffective at increasing the range of detectable pitch angles (see Figure 4.14).  
 2027 Fundamentally, this is because the power threshold only uses a fraction of the signal  
 2028 power to detect electrons and ignores the power present in the frequency sidebands. In  
 2029 the subsequent sections I examine two other signal detection algorithms that seek to  
 2030 improve the detection efficiency of the FSCD by utilizing the more of the signal shape to  
 2031 compute the detection test statistics.

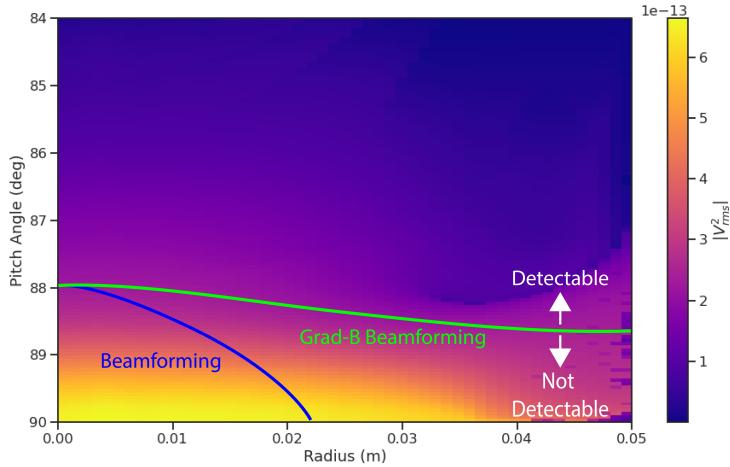


Figure 4.14: A plot of the total signal power received by the FSCD array from trapped electrons with different radial positions and pitch angles generated using Locust simulations. The lines on the plot indicate a 10 dB detection threshold above the mean value of the noise in the frequency spectrum. With static beamforming electrons with radial positions larger than about two centimeters are undetectable due to the change in the electron's position over time causing losses from beamforming phase mismatch. This is corrected by including  $\nabla B$ -drift frequencies in the beamforming phases. Both beamforming techniques fail to detect electrons below  $\approx 88.0^\circ$ , since these signal are composed of several relatively weak sidebands that are comparable to the noise.

### 4.3.2 Matched Filtering

#### Introduction to Matched Filtering

The problem of CRES signal detection is the problem of detecting a signal buried in WGN, which has been examined at great depth in the signal processing literature [19]. For a fully known signal in WGN the optimal detector is the matched filter, which means that it achieves the highest true positive rate for a fixed rate of false positives. The matched filter test statistic is calculated by taking the inner product of the data with the matched filter template

$$\mathcal{T} = \left| \sum_n h^\dagger[n] y[n] \right|, \quad (4.26)$$

where  $h[n]$  is the matched filter template and  $y[n]$  is the data. The matched filter test statistic defines a binary hypothesis test in which the data vector is assumed to be an instance of two possible data classes. By setting a decision threshold on the value of  $\mathcal{T}$ , one can classify a given data vector as belonging to two distinct hypotheses. Under the first hypothesis the data is composed of pure WGN, and under the second hypothesis the

2045 data is composed of the known signal with additive WGN. The matched filter template  
 2046 is obtained by rescaling the known signal in the following way

$$h[n] = \frac{x[n]}{\sqrt{\tau \sum_n x^\dagger[n]x[n]}}, \quad (4.27)$$

2047 where  $\tau$  is the variance of the WGN and  $x[n]$  is the known signal. Strictly speaking,  
 2048 Equation 4.27 is only true for noise with a diagonal covariance matrix, however, in the  
 2049 context of the FSCD we are justified in assuming this to be true. Defining the matched  
 2050 filter templates in this way guarantees that the expectation value of  $\mathcal{T}$  is equal to one  
 2051 when the data contains only noise, which is the standard matched filter normalization in  
 2052 the signal processing literature.

2053 Although matched filters are canonically formulated in terms of a perfectly known  
 2054 signal, it is still possible to apply the matched filter technique given imperfect information  
 2055 about the signal provided that the signal is deterministic. From our discussion of CRES  
 2056 simulation tools for the FSCD (see Section 4.2) we know that the shape of CRES signals  
 2057 are completely determined by the initial parameters of the electron. The random collisions  
 2058 with background gas molecules which cause the formation of signal tracks are the only  
 2059 stochastic component of the CRES event after the initial beta-decay, therefore, it is  
 2060 possible to develop a matched filter for the detection of CRES signal tracks which are fully  
 2061 determined by the parameters of the electron after the initial beta-decay or subsequent  
 2062 collision events.

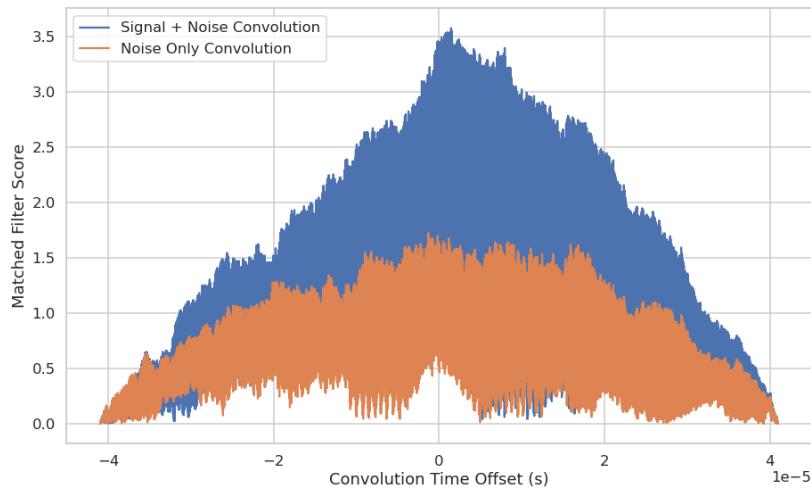


Figure 4.15

2063 The matched filter test statistic for CRES signals is a modified version of Equation

2064 4.26

$$\mathcal{T} = \max_{\mathbf{h}, m} |\mathbf{h} * \mathbf{y}| = \max_{\mathbf{h}, m} \left| \sum_k h^\dagger[k] x[m - k] \right|, \quad (4.28)$$

2065 where the matched filter inner product has been replaced with a convolution operation  
 2066 and a maximization over the template and convolution delay ( $m$ ). Replacing the inner  
 2067 product with a convolution accounts for the fact that the start time of the CRES signal is  
 2068 now an unknown parameter, in addition, we now perform a maximization of the matched  
 2069 filter convolution over a number of different templates. Because the shape of the signal is  
 2070 unknown we are forced to guess a number of different signal shapes to create a template  
 2071 bank with which we can identify unknown signals by performing an exhaustive search.

2072 The template bank approach to matched filtering, while quite powerful, can quickly  
 2073 become computationally intractable. This is especially true in the case of the FSCD  
 2074 because of the large amount of raw data produced by the array that must be analyzed.  
 2075 Specifically, the time-domain convolution specified by Equation 4.28 is particularly  
 2076 computationally intensive and is a major barrier towards the implementation of a  
 2077 matched filter for signal detection in an experiment like the FSCD. This can be avoided  
 2078 by using the convolution theorem to replace the time-domain convolution with an inner  
 2079 product in the frequency domain.

2080 The convolution theorem states that

$$\mathbf{f} * \mathbf{g} = \mathcal{F}^{-1}(\mathbf{F} \cdot \mathbf{G}) \quad (4.29)$$

2081 where  $\mathbf{f}$  and  $\mathbf{g}$  are discretely sampled time-series,  $\mathbf{F}$  and  $\mathbf{G}$  are the respective discrete  
 2082 Fourier transforms, and  $\mathcal{F}^{-1}$  is the inverse discrete Fourier transform operator. The  
 2083 convolution theorem allows us to perform the matched filter convolution by first com-  
 2084 puting the Fourier transform of the template and data, then performing a point-wise  
 2085 multiplication of the two frequency series, and finally performing the inverse Fourier  
 2086 transform to obtain the convolution output. Because discrete Fourier transforms can be  
 2087 performed extremely efficiently, the convolution theorem is almost always used in lieu of  
 2088 directly computing the convolution.

2089 One thing to note here is that the convolution theorem for discrete sequences shown  
 2090 here, is technically valid only for circular convolutions, which is not directly specified  
 2091 in Equation 4.28. However, because typical CRES track lengths are much longer than  
 2092 the Fourier analysis window and also that the frequency chirp rates are small compared  
 2093 to the time-slice duration, it is relatively safe to use circular convolutions to evaluate

2094 matched filter scores for CRES signals, which allows us to apply the convolution theorem  
2095 to compute matched filter scores using the frequency representation of the data and  
2096 matched filter template.

2097 **Matched Filter Analysis of the FSCD**

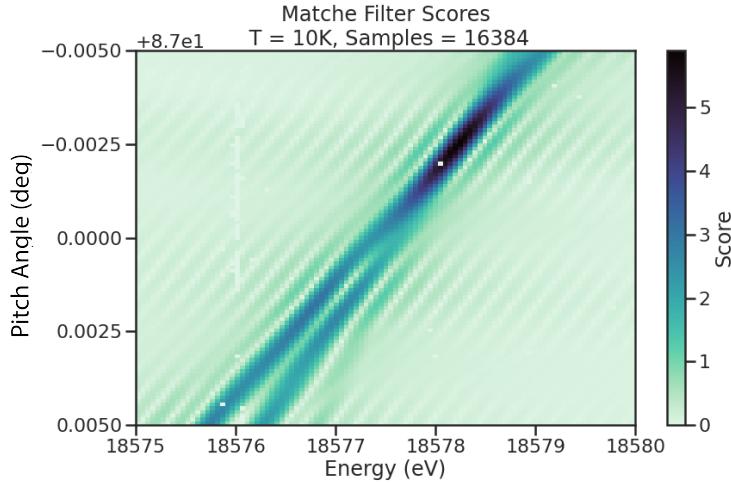


Figure 4.16

2098 The optimality provided by the matched filter makes it a useful algorithm for analysis  
2099 of CRES experiment designs for sensitivity analyses, since it indicates the best possible  
2100 detection efficiency achievable by an experiment configuration. The standard approach to  
2101 performing these studies involves generating a large number of simulated electron signals  
2102 that span the kinematic parameter space of electrons in the magnetic trap. In general,  
2103 electrons have six kinematic parameters along with an additional start time parameter.

2104 In order to limit the number of simulations required to evaluate the detection efficiency  
2105 the standard approach is to fix the starting axial position, starting azimuthal position,  
2106 starting direction of the perpendicular component of the electron's momentum, and event  
2107 start time to reduce the parameter space to starting radial position, starting kinetic  
2108 energy, and starting pitch angle. The fixed variables are true nuisance parameters that do  
2109 not affect the detection efficiency estimates for the FSCD design, because they manifest  
2110 as phases which are marginalized during the calculation of the matched filter score.

2111 Across radial position, kinetic energy, and pitch angle we can define a regular grid of  
2112 parameters and use Locust to simulate the corresponding signals. This grid of simulated  
2113 signals can be used to estimate the likelihood of detecting signals in the FSCD by using  
2114 the same set of signals as both the data and templates when evaluating the matched

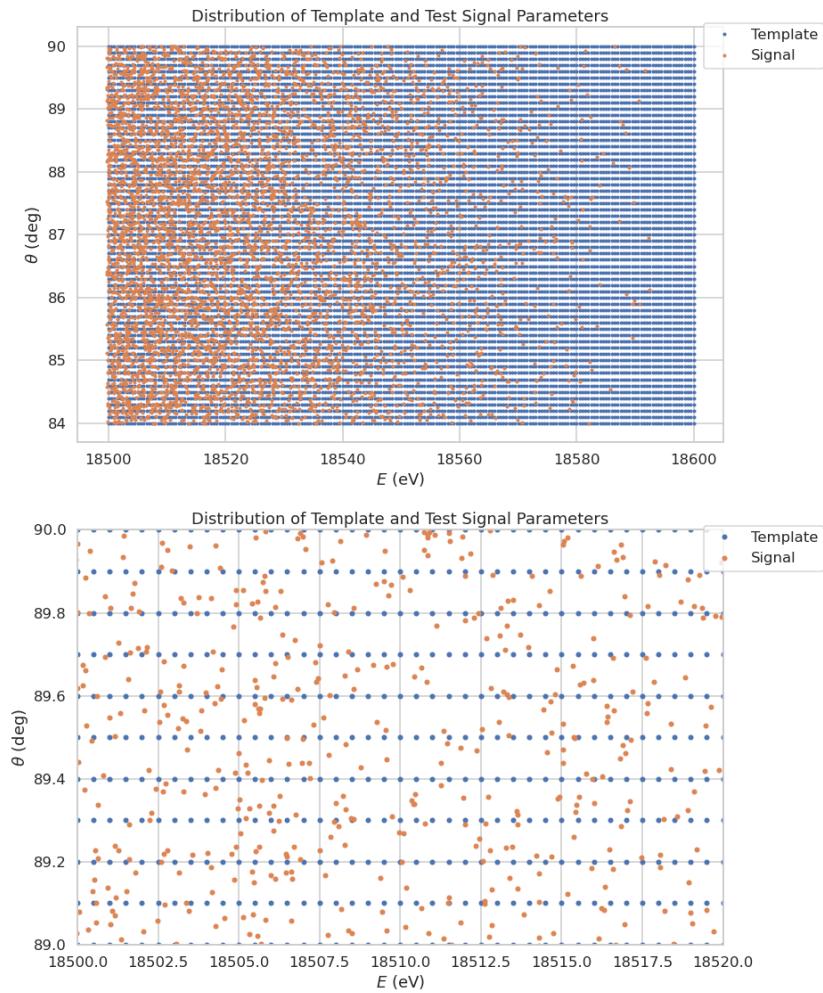


Figure 4.17

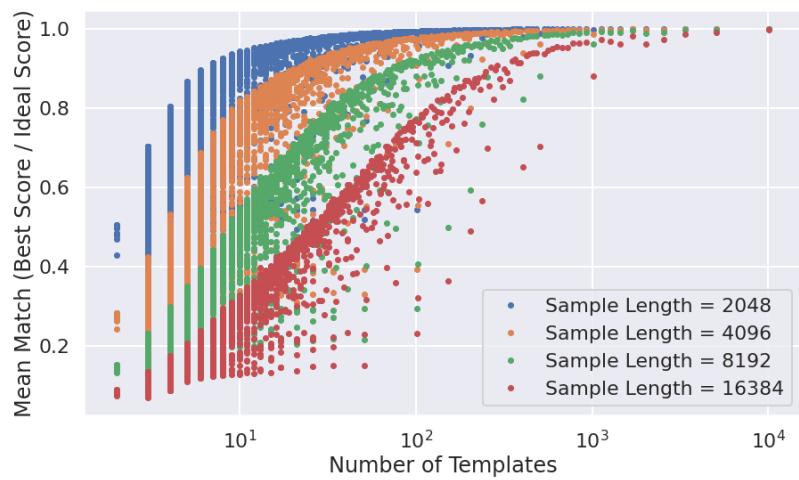


Figure 4.18

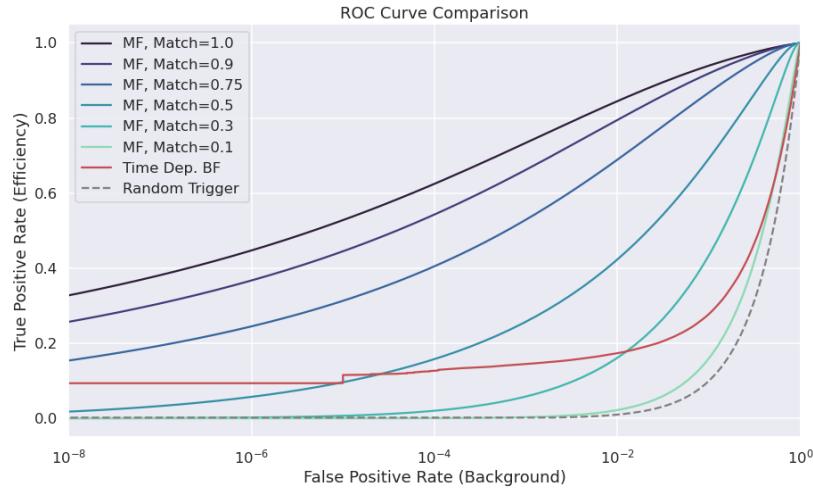


Figure 4.19

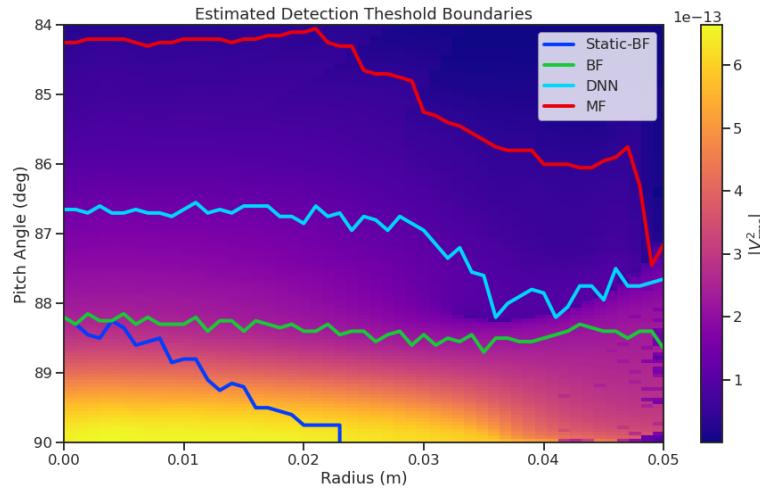


Figure 4.20

filter scores. The matched filter score specifies the shape of the PDF that defines the detection probability.

## Optimized Matched Filtering Implementation for the FSCD

The biggest practical obstacle to the implementation of a matched filter template bank detection approach is oftentimes the computational cost associated with exhaustively calculating the matched filter scores of the template bank, and the FSCD is no exception in this regard. At a basic level computing a matched filter score requires the convolution of two vectors, which can be performed very efficiently by computers if the convolution

theorem and fast Fourier transforms (FFT) are utilized. Furthermore, one can consider applying digital beamforming as a pre-processing step to reduce the dimensionality of the data before the matched filter is applied. In order to understand the relative gain in computational efficiency offered by these optimizations we analyze the total number of floating-point operations (FLOP) of several matched filter implementations in big  $O$  notation that utilize different combinations of optimizations.

A direct implementation of a matched filter as specified by Equation 4.28 involves the convolution of  $N_{\text{ch}}$  signals of length  $N_s$  with template signals of length  $N_t$ . As a uniform metric we shall compare the FLOP of the various matched filter implementations on a per-template basis, since each implementation scales linearly with the number of templates. The direct convolution approach to matched filtering costs

$$O(N_{\text{ch}}) \times O(N_s \times N_t) \quad (4.30)$$

FLOP per-template, whose cost is dominated by the  $O(M \times N)$  convolution operation.

The computational cost of the direct matched filter approach can be significantly reduced by exploiting the convolution theorem and FFT algorithms. If we restrict ourselves to signals and templates that contain equal numbers of samples then the convolution can be calculated by Fourier transforming both vectors, performing the point-wise multiplication, and then performing the inverse Fourier transform to obtain the convolution result. The FFT algorithm is able to compute the Fourier transform utilizing only  $O(N \log N)$  operations compared to  $O(N^2)$  for a naive Fourier transform implementation. This optimization results in a computational cost per-template of

$$O(N_{\text{ch}}) \times O(N_s \log N_s) \quad (4.31)$$

A typical signal vector in the FSCD contains  $O(10^4)$  samples in which case the FFT reduces the computational cost of the matched filter by a factor of  $O(10^3)$ . This large reduction in computational cost implies that a direct implementation of a matched filter is completely infeasible in the FSCD due to resource constraints.

Rather than relying solely on the matched filter it is tempting to consider using digital beamforming as an initial step in the signal reconstruction for the purposes of data reduction. The primary motivation is to reduce the dimensionality of the data by a factor of  $N_{\text{ch}}$  by combining the array outputs coherently into a single channel. One can view the beamforming operation as a partial matched filter, in the sense that the matched filter convolution contains the beamforming phased summation along with a

2153 prediction of the signal shape. By separating beamforming from the signal shape one  
2154 hopes to reduce the overall computational cost by effectively shrinking the number of  
2155 templates and reducing the number of operations required to check each one.

2156 The nature of this optimization requires that we account for the number of templates  
2157 used for pure matched filtering versus the hybrid approach. To first order, the total  
2158 number of templates at the trigger stage is a product of the number of guesses for each  
2159 of the electron's parameters

$$N_T = N_E \times N_\theta \times N_r \times N_\varphi, \quad (4.32)$$

2160 where  $N_E$  is the number of kinetic energies,  $N_\theta$  is the number of pitch angles,  $N_r$  is the  
2161 number of starting radial positions, and  $N_\varphi$  is the number of starting azimuthal positions.  
2162 The starting axial position and cyclotron motion phase are not necessary to include in  
2163 the template bank since these parameters manifest themselves as the starting phase of  
2164 the signal, which is effectively marginalized when using a FFT to compute the matched  
2165 filter convolution. Therefore, the total number of operations required by a matched filter  
2166 to detect a signal in a segment of array data is on the order of

$$O(N_T) \times O(N_{\text{ch}}) \times O(N_s \log N_s) \quad (4.33)$$

2167 With the hybrid approach we attempt to remove the spatial parameters from the  
2168 template bank by using beamforming to combine the array signals into a single channel.  
2169 Beamforming explicitly assumes a starting position, which allows us to only use matched  
2170 filter templates that span the two-dimensional space of kinetic energy and pitch angle.  
2171 The total computational cost of the hybrid method is directly proportional to the number  
2172 of beamforming positions. For the time-dependent beamforming defined in Section 4.3.1,  
2173 the number of beamforming positions is given by

$$N_{\text{BF}} = N_r \times N_\varphi \times N_{\omega_{\nabla B}}, \quad (4.34)$$

2174 where  $N_r$  and  $N_\varphi$  are the same spatial parameters encountered in the pure matched  
2175 filter template bank and  $N_{\omega_{\nabla B}}$  is the number of  $\nabla B$ -drift frequency assumptions. If a  
2176 unique drift frequency is used for each pitch angle then the hybrid approach is effectively  
2177 equivalent to a pure matched filter in the number of operations. The key efficiency gain  
2178 of the hybrid approach is to exploit the relatively small differences in  $\omega_{\nabla B}$  for electrons  
2179 of different pitch angles by using only a small number of average drift frequencies.

2180        The total number of operations for the hybrid approach can be expressed as a sum of  
 2181        the operations required by the beamforming and matched filtering steps,

$$O(N_{\text{BF}}) \times O(N_{\text{ch}}N_s) + O(N_{\text{BF}}) \times O(N_E N_\theta) \times O(N_s \log N_s). \quad (4.35)$$

2182        The first product in the sum is the number of operations required by beamforming,  
 2183        which is simply the number of beamforming points times the computational cost of the  
 2184        beamforming matrix multiplication, and the second product is the computational cost  
 2185        of matched filtering the summed signal generated by each beamforming position. To  
 2186        compare this to pure matched filtering we take the ratio of Equations 4.33 and 4.35 to  
 2187        obtain

$$\Gamma_{\text{BFFM}} = \frac{O(N_{\omega_{\nabla B}})}{O(N_E N_\theta) \times O(\log N_s)} + \frac{O(N_{\omega_{\nabla B}})}{O(N_{\text{ch}})}. \quad (4.36)$$

2188        This expression can be simplified by observing that  $O(N_E N_\theta) \times O(\log N_s) \gg O(N_{\text{ch}})$ ,  
 2189        which means that the ratio of computational cost for the two methods can be reduced to

$$\Gamma_{\text{BFFM}} \approx \frac{O(N_{\omega_{\nabla B}})}{O(N_{\text{ch}})}. \quad (4.37)$$

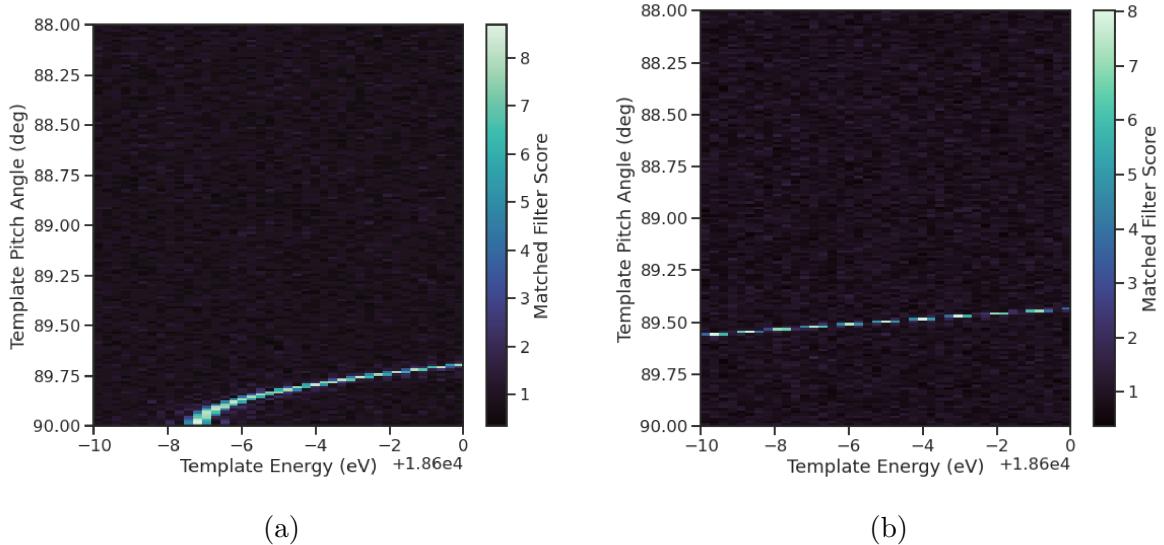
2190        If we limit ourselves to a number of estimated drift frequencies of  $O(1)$  then we see that  
 2191        the estimated computational cost reduction of the hybrid approach is of  $O(N_{\text{ch}})$ . This is  
 2192        quite a large reduction considering that the FSCD antenna array contains sixty antennas  
 2193        in the baseline design.

2194        The main drawback of the hybrid approach is that the limited number of allowed  
 2195        drift frequency guesses can lead to detection efficiency loss due to phase mismatch. The  
 2196        degree of phase error from an incorrect drift frequency is proportional to the length of  
 2197        the array data vector used by the signal detection algorithm. For signals with lengths  
 2198        equal to the baseline FSCD Fourier analysis window of 8192 samples, typical phase errors  
 2199        from using an average versus the exact  $\nabla B$ -drift frequency are on the order of a few  
 2200        percent in terms of the signal energy. This has a relatively small impact on the overall  
 2201        detection efficiency, however, future experiments with antenna array CRES will want to  
 2202        balance optimizations such as these during the design phase to keep experiment costs to  
 2203        a minimum while still achieving scientific goals.

## 2204        **Kinetic Energy and Pitch Angle Degeneracy**

2205        More accurate modeling of a matched filter requires that we consider the effects of  
 2206        mismatched signals and template, since this more accurately reflects the real-world usage

2207 of a matched filter where many incorrect templates are convolved with the data until the  
 2208 matching template is found. One way to study this is to use the grid of simulated signals  
 2209 to compute the matched filter scores between mismatched signals and templates and  
 2210 evaluate the matched filter scores under this scenario. What one finds when performing  
 2211 this analysis is that templates for kinetic energies and pitch angles that do not match  
 2212 the underlying signal can have matched filter scores that are indistinguishable from the  
 matched filter score of the correct template (see Figure 4.21 and Figure 4.21).



2213 Figure 4.21: Two example illustrations of the correlation between kinetic energy and  
 2214 pitch angle imparted by the shape of the FSCD magnetic trap. The correlations manifest  
 themselves as degeneracies in the matched filter score where multiple matched filter  
 2215 templates have the same matched filter for a particular signal. These degeneracies are a  
 sign that the magnetic trap must be redesigned in order to break the correlation between  
 2216 pitch angle and kinetic energy.

2217 This degeneracy in matched filter score is the result of correlations between the kinetic  
 2218 energy of the electron and the pitch angle caused by changes in the average magnetic field  
 2219 experienced by an electron for different pitch angles. While in principle helpful for the  
 2220 purposes of signal detection these correlations are unacceptable since they greatly reduce  
 2221 the energy resolution of the experiment by causing electrons with specific kinetic energy  
 2222 to templates across a wide range of energies. It is important to emphasize that this  
 2223 degeneracy cannot be fixed by implementing a different signal reconstruction algorithm.  
 As revealed by the matched filter scores the shapes of the signals for different parameters  
 are identical. Resolving this degeneracy between pitch angle and energy requires the  
 design of a new magnetic trap with steeper walls so that the average magnetic field

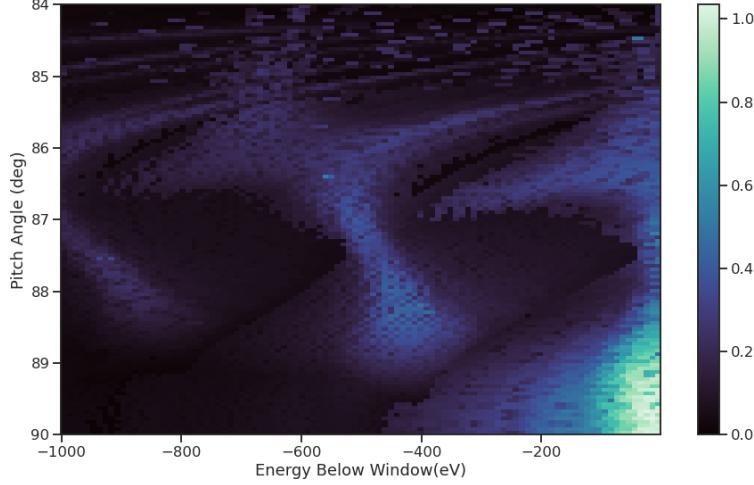


Figure 4.22: A visualization of the correlation between energy and pitch angle in the FSCD magnetic trap. The image is formed by computing the match of the best template from a grid consisting of pitch angles from 84 to 90 degrees in steps of 0.05 degrees, kinetic energies from 17574 to 18574 eV, located at 2 cm from the central axis, and simulated for a length of three FSCD time-slices. The signals used to compute the best matching template consisted of a grid from 84 to 90 degrees in steps of 0.05 degrees, kinetic energies from 18550 to 18575 eV in steps of 0.25 eV, located 2 cm from the central axis, and simulated for three FSCD time-slices. The colored regions of the plot show how well signals with lower energy can match those of higher energy for the FSCD magnetic trap, which is proportional to the achievable energy resolution of the FSCD design.

2224 experienced by an electron is less dependent on pitch angle.

### 2225 4.3.3 Machine Learning

2226 Machine learning is a vast and rapidly developing field of research [22]. In this Section  
2227 we shall attempt to provided a brief introduction to some of the concepts and techniques  
2228 of machine learning that were applied to CRES signal detection rather than attempt a  
2229 comprehensive overview.

#### 2230 Introduction to Machine Learning

2231 Digitization of the FSCD antenna array generates large amounts of data that must be  
2232 rapidly processed to enable real-time signal detection and reconstruction. While digital  
2233 beamforming combined with a power threshold is relatively computationally inexpensive,  
2234 it is relatively ineffective at detecting CRES signal with small pitch angles, since it relies  
2235 on a visible frequency peak above the noise. On the other hand, a matched filter is able

2236 to detect signals with a significantly larger range of parameters, however, the exhaustive  
2237 search of matched filter templates can be computationally expensive. Machine learning  
2238 based triggering algorithms have been used successfully in many different high-energy  
2239 physics experiments [23] and recent developments have shown success in the detection  
2240 of gravitational wave signals using machine learning techniques [24, 25] in place of the  
2241 more traditional matched filtering method. This motivates the exploration of machine  
2242 learning as a potential CRES signal detection algorithm.

2243 There are several different approaches to machine learning, but the one most important  
2244 to our discussion here is the supervised learning approach. In supervised machine learning  
2245 one uses a differentiable model or function that is designed to map the input data to the  
2246 appropriate label [22]. The data is represented as a multidimensional matrix of floating  
2247 point values such as an image or a time-series, and the label is generally a class name  
2248 such as signal or noise for classification problems or a continuous value like kinetic energy  
2249 in the case of regression problems.

2250 In supervised learning the model is trained to map from the data to the correct label  
2251 by evaluating the output of the model using a training dataset consisting of a set of  
2252 paired data and labels. To evaluate the difference between the model output and the  
2253 correct label a loss function is used to quantify the error between the model prediction  
2254 and the ground truth. For example, a common loss function in regression problems is the  
2255 squared error loss function, which quantifies error using the squared difference between  
2256 the model output and label.

2257 Using the outputs of the loss function the next step in supervised learning is to  
2258 compute the gradient of error with respect to the model parameters in a process called  
2259 backpropagation. Using the model parameter gradients the last step in the supervised  
2260 learning process is to perform an update of the parameter values in order to minimize  
2261 the error in the model predictions across the whole dataset. This loop is performed many  
2262 times while randomly shuffling the dataset until the error converges to a minimum value  
2263 at which point the training procedure has finished. It is standard practice to monitor  
2264 the training procedure by evaluating the performance of the model using a separate  
2265 validation dataset that matches the statistical distribution of the training data and to  
2266 check the performance of the model after training using yet another dataset called the  
2267 test dataset. These practices help to guard against overtraining which is a concern for  
2268 models with many parameters.

2269 **Convolutional Neural Networks**

2270 A popular class of machine learning models are neural networks. A neural network is  
2271 essentially a function composed of a series of linear operations called layers which take a  
2272 piece of data typically represented as a matrix, multiplies the elements of the data by a  
2273 weight, and then sums these products to produce an output matrix. Neural networks  
2274 composed of purely linear operations are unable to model complex non-linear behavior,  
2275 therefore, non-linear activation functions are applied to the outputs of each of the layers  
2276 to increase the ability of the neural network to model complex relationships between the  
2277 data.

2278 Neural networks are typically composed of at least three layers, but with the present  
2279 capabilities of computer hardware they more often contain many more than this. The  
2280 first layer in a neural network is called the input layer, because it takes the data objects  
2281 as input, and the last layer in a neural network is known as the output layer. The  
2282 output layer is trained by machine learning to map the data to a desired output using  
2283 the supervised learning procedure described in Section 4.3.3. In between the input and  
2284 the output layer are typically several hidden layers that receive inputs from and transmit  
2285 outputs to other layers in the neural network model. The term deep neural network  
2286 (DNN) refers to those neural networks that have at least one hidden layer, which have  
2287 proven to be extremely powerful tools for pattern recognition and function approximation.

2288 An important type of DNN are convolutional neural networks (CNN) that typically  
2289 contain several layers which perform a convolution of the input with a set of filters. These  
2290 convolution operations are typically accompanied by layers that attempt to down-sample  
2291 the data along with the standard neural network activation functions. A standard CNN  
2292 is composed of several convolutional layers at the beginning of the network and ends  
2293 with a series of fully-connected neural network layers at the output. Intuitively, one  
2294 can imagine that the convolutional layers are extracting features from the data that  
2295 fully-connected layers use to perform the classification or regression task.

2296 **Deep Filtering for Signal Detection in the FSCD**

2297 CNNs have been extremely influential in the field of computer vision, particularly tasks  
2298 such as image segmentation and classification, but have also been applied in numerous  
2299 experimental physics contexts. Given the particular challenge posed by signal detection  
2300 and reconstruction in the FSCD we are interested in exploring the potential of machine  
2301 learning as an effective algorithm for real-time signal detection, since this application

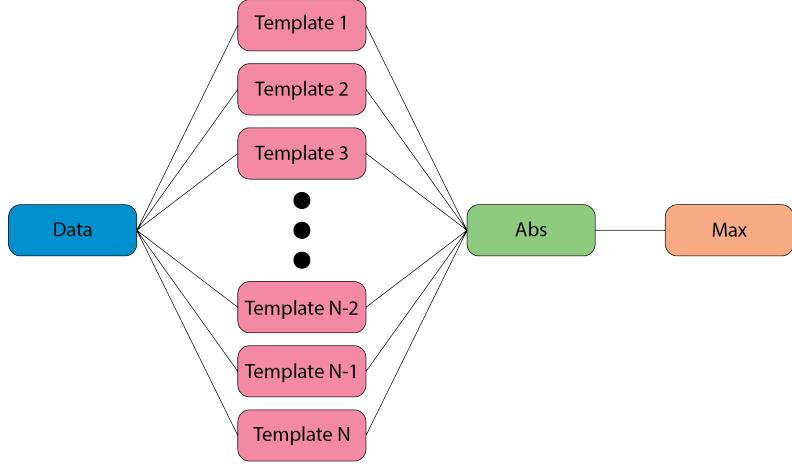


Figure 4.23: A representation of a matched filter template bank as a convolutional neural network. The network has a single layer composed of the templates, which act as convolutional filters. The activation of the neural network is an absolute value followed by a max operator.

2302 requires both high efficiency and fast evaluation.

2303 In the machine learning paradigm signal detection is equivalent to a binary classification  
 2304 problem between the signal and noise data classes, and my investigation focuses  
 2305 specifically on the application of CNNs to signal detection in the FSCD, which is moti-  
 2306 vated by relatively recent demonstrations of CNNs achieving classification accuracies for  
 2307 gravitational wave time-series signals comparable to a matched filter template bank. In  
 2308 this framework it is possible to interpret the matched filter as a type of CNN composed  
 2309 of a single convolutional layer with the templates making up the layer filters (see Figure  
 2310 4.23). Since this neural network has no hidden layers, it is not a DNN like we have  
 2311 been discussing so far, but we can attempt to construct a proper CNN that attempts to  
 2312 reproduce the classification performance of the matched filter network.

2313 The name deep filtering refers to this scheme of replacing a matched filter template  
 2314 bank with a DNN. The reason why one might want to do this is that it may be possible to  
 2315 exploit redundancies and correlations between templates that may allow one to perform  
 2316 signal detection with similar accuracy but with fewer computations, which is important  
 2317 for real-time detection scenarios like the FSCD experiment. In Section 4.4 we perform a  
 2318 detailed comparison of the signal detection performance of a CNN to beamforming and a  
 2319 matched filter template bank.

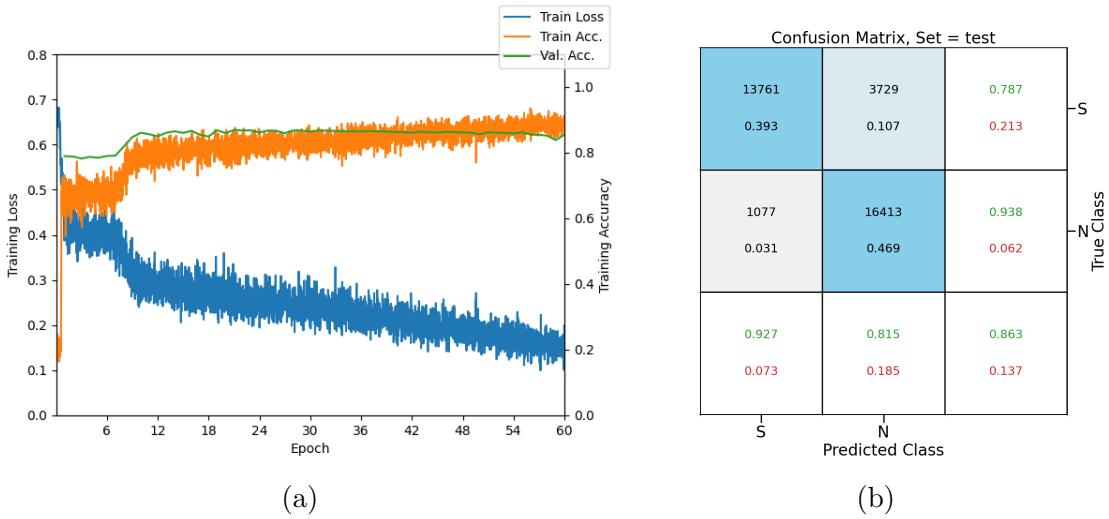


Figure 4.24

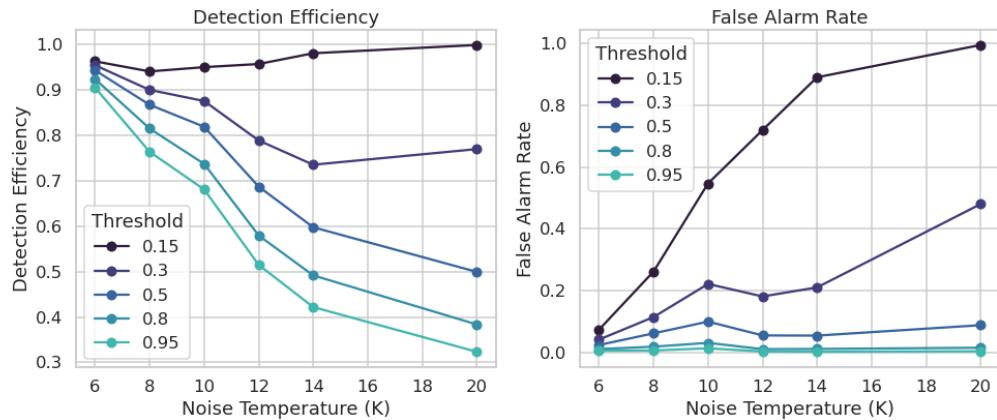


Figure 4.25

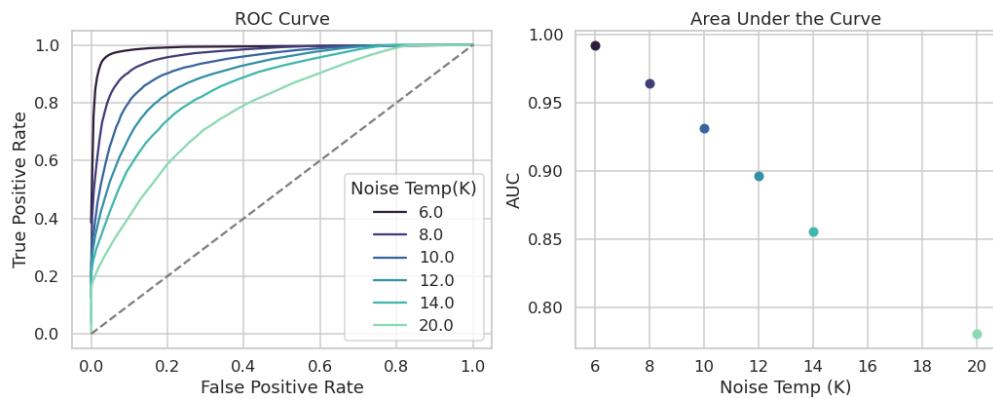


Figure 4.26

## **4.4 Analysis of Signal Detection Algorithms for the Antenna Array Demonstrator**

This section contains an early version of the manuscript for the triggering paper prepared for publication in JINST. In it I present a relatively detailed analysis of the signal detection performance of the three signal detection approaches discussed so far using a population of simulated CRES signals generated with Locust. The focus of the paper is on the performance of the signal detection algorithms for pitch angles below 88.5° where the beamforming power threshold begins to fail.

### **4.4.1 Introduction**

Cyclotron Radiation Emission Spectroscopy (CRES) is a technique for measuring the kinetic energies of charged particles by observing the frequency of the cyclotron radiation that is emitted as they travel through a magnetic field [2]. The Project 8 Collaboration is developing the CRES technique as a next-generation approach to tritium beta-decay endpoint spectroscopy for neutrino mass measurement. Recently, Project 8 has used CRES to perform the first ever tritium beta-decay energy spectrum and neutrino mass measurement [4, 5].

Previous CRES measurements have utilized relatively small volumes of gas that are directly integrated with a waveguide transmission line, which transmits the cyclotron radiation emitted by the trapped electrons to a cryogenic amplifier. While this technology has had demonstrable success, it is not a feasible option for scaling up to significantly larger measurement volumes. In particular, the goal of the Project 8 Collaboration is to use CRES combined with atomic tritium to measure the neutrino mass with a 40 meV sensitivity. Achieving this sensitivity goal will require a multi-cubic-meter scale measurement volume in order to obtain the required event statistics in the tritium beta-spectrum endpoint region; hence, there is a need for new techniques to enable large volume CRES measurements for future experiments.

One approach is to surround a large volume with an array of antennas that together collect the cyclotron radiation emitted by trapped electrons [3, 26]. A promising array design is an inward-facing uniform cylindrical array that surrounds the tritium containment volume. Increasing the size of the antenna array, by adding additional rings of antennas along vertical axis, allows one to grow the experimental volume until a sufficient amount of tritium gas can be observed by the array. A challenging aspect of this approach is

2352 that the total radiated power emitted by an electron near the tritium spectrum endpoint  
2353 is on the order of 1 fW or less, which is then distributed between all the antennas in  
2354 the array. Consequently, detecting the presence of a CRES signal and determining the  
2355 electron's kinetic energy requires reconstructing the entire antenna array output over the  
2356 course of the CRES event, posing a significant data acquisition and signal reconstruction  
2357 challenge.

2358 Project 8 has developed a triggering system to enable real-time identification of CRES  
2359 events using an antenna array [27]. Previous measurements with the CRES technique  
2360 have utilized a threshold on the frequency spectrum formed from a segment of CRES  
2361 time-series data. This algorithm relies on the detection of a frequency peak above the  
2362 thermal noise background, which limits the kinematic parameter space of detectable  
2363 electrons. Due to the limitations of this power threshold, Project 8 has been investigating  
2364 alternative signal identification approaches, including both matched filtering and machine  
2365 learning based classifiers, to improve the detection efficiency of the experiment. In  
2366 order to evaluate the relative gains in detection efficiency that come from utilizing  
2367 these alternative algorithms, we develop analytical models for the power threshold and  
2368 matched filter signal classifier performance applicable to an antenna array based CRES  
2369 detector. In addition, we implement and test a basic convolutional neural network (CNN)  
2370 as a first step towards the development of neural-network based classifiers for CRES  
2371 measurements. These results allow us to compare the estimated detection efficiencies of  
2372 each of these methods, which we weigh against the associated computational costs for  
2373 real-time applications.

2374 The outline of this paper is as follows. In Section 4.4.2 we give an overview of a  
2375 prototypical antenna array CRES experiment, and describe the major steps involved  
2376 in the proposed approach to real-time signal identification. In Section 4.4.3 we develop  
2377 models for the power threshold and matched filter algorithms, and introduce the machine  
2378 learning approach and CNN architecture. In Section 4.4.4 we describe our process for  
2379 generating simulated CRES signal data and the details of training the CNN. Finally,  
2380 in Section 4.4.5 we perform a comparison of the signal classification accuracy of the  
2381 three approaches and discuss the relevant trade-offs in terms of detection efficiency and  
2382 computational cost.

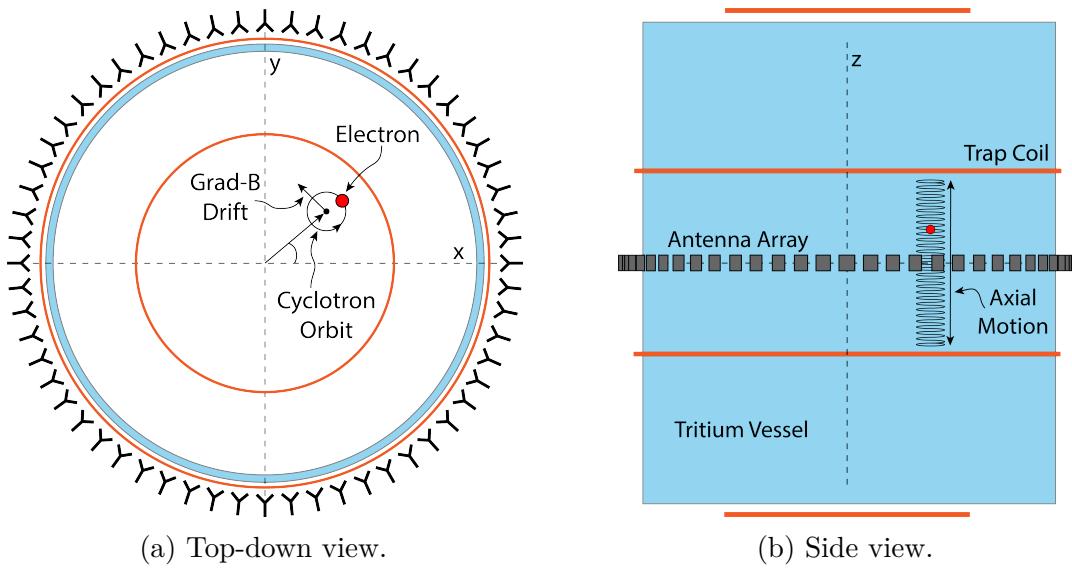


Figure 4.27: An illustration of the conceptual design for an antenna array CRES tritium beta-decay spectrum measurement. The antenna array geometry consists of a 20 cm interior diameter with 60 independent antenna channels arranged evenly around the circumference. The nominal antenna design is sensitive to radiation in the frequency range of 25-26 GHz, which corresponds to the cyclotron frequency of electrons emitted near the tritium beta-spectrum endpoint in a 1 T magnetic field. The array is located at the center of the magnetic trap produced by a set of current-carrying coils. The nominal magnetic trap design is capable of trapping electrons up to 5 cm away from the central axis of the array and traps electrons within an approximately 6 cm long axial region centered on the antenna array.

#### **4.4.2 Signal Detection with Antenna Array CRES**

##### **4.4.2.1 Antenna Array and DAQ System**

In order to explore the potential of antenna array CRES for neutrino mass measurement, the Project 8 Collaboration has developed a conceptual design for a prototype antenna array CRES experiment [3, 26], called the Free-space CRES Demonstrator or FSCD, which could be used as a demonstration of the antenna array measurement technique (see Figure 4.27). The FSCD utilizes a single ring of antennas, which is the simplest form of a uniform cylindrical array configuration, to surround a radio-frequency (RF) transparent tritium gas vessel. A prototype version of this antenna array has been built and tested by the Project 8 collaboration to validate simulations of the array radiation pattern and beamforming algorithms [6]. In the FSCD the antenna array is positioned at the center of the magnetic trap formed by a set of electro-magnetic coils that are designed to produce a magnetic trap with flat central region and steep walls both radially

2396 and axially.

2397 When a beta-decay electron is trapped its motion consists of three primary components.  
2398 The component with the highest frequency is the cyclotron orbit whose frequency is  
2399 determined by the size of the background magnetic field. The FSCD design assumes  
2400 a background magnetic field value of approximately 0.96 T, which results in cyclotron  
2401 frequencies for electrons with kinetic energies near the tritium beta-spectrum endpoint  
2402 from 25 to 26 GHz. The component with the next highest frequency is the axial oscillation  
2403 experienced by electrons with pitch angles of less than 90° [9]. The flat region of the  
2404 FSCD magnetic trap extends approximately 3 cm above and below the antenna array  
2405 plane causing electrons to move back and forth as they are reflected from the trap walls.  
2406 Typical oscillation frequencies are on the order of 10's of MHz, which results in an  
2407 oscillation period that is  $O(10^3)$  smaller than the duration of a typical CRES event.  
2408 Therefore, when reconstructing CRES events we treat the electron as occupying only an  
2409 average axial position at the center of the magnetic trap, since we are not able to resolve  
2410 the axial position as a function of time. The component of motion with the smallest  
2411 frequency is  $\nabla B$ -drift caused by radial field gradients in the trap, producing an orbit of  
2412 the electron around the central axis of the trap with a frequency on the order of a few  
2413 kHz, dependent on the pitch angle and the radial position of the electron.

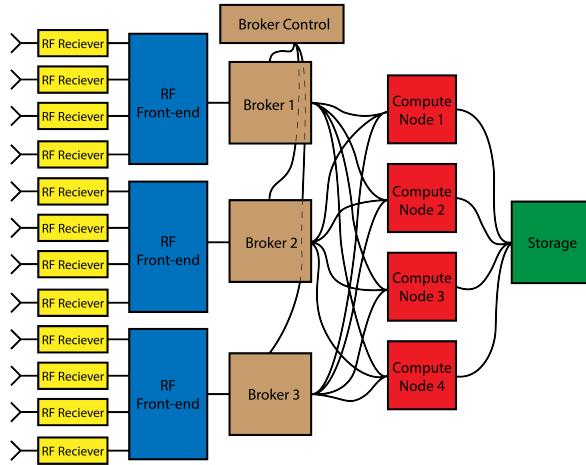


Figure 4.28: A high-level diagram of the DAQ system architecture envisioned for the FSCD.

2414 The data acquisition (DAQ) system digitizes the signals from the antenna array and  
2415 combines three data streams into a time-ordered matrix of array snapshots that can be  
2416 used by the reconstruction algorithms. The FSCD DAQ system design [27] is divided into  
2417 three layers 4.28. The first layer is the RF front-end, which includes the antenna array,

2418 the RF receiver boards, and the digitization electronics. The receiver board contains an  
 2419 amplifier, RF mixer, and bandpass filter to enable down-conversion, and is followed by  
 2420 the digitization electronics that sample the CRES signals at 200 MHz. In order to achieve  
 2421 an adequate signal-to-noise ratio to detect CRES events, the DAQ system for the antenna  
 2422 array demonstrator must have a total system noise temperature of  $\approx 10$  K, which we  
 2423 can achieve by using low-noise amplifiers and operating at cryogenic temperatures. After  
 2424 digitization, the array data must be reorganized from individual data streams sorted  
 2425 by channel into array snapshots sorted by time. In order to solve this data transfer  
 2426 and networking problem the second layer of the DAQ system consists of a set of broker  
 2427 computer nodes that reorganize the array data into time-ordered chunks. This approach  
 2428 allows us accommodate different data transfer requirements by scaling the number of  
 2429 broker nodes in this layer accordingly. Next, the broker layer distributes these chunks  
 2430 of array data to the final layer of the DAQ system, which consists of a set of identical  
 2431 reconstruction nodes that perform the calculations required for CRES reconstruction.  
 2432 Similar to the broker layer, the number of reconstruction nodes can be increased or  
 2433 decreased depending on the amount of computer power required for real-time CRES  
 2434 reconstruction.

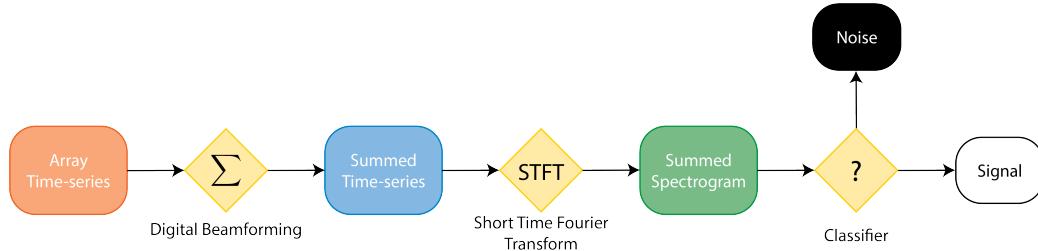


Figure 4.29: A block diagram illustration of the real-time triggering algorithm proposed for antenna array CRES reconstruction.

2435 The design of the FSCD DAQ system is intended to enable a significant portion of  
 2436 the CRES event reconstruction to occur in real-time. The motivation for this comes from  
 2437 the fact that the FSCD antenna array generates approximately 1 exabyte of raw data  
 2438 per year of operation. Therefore, in order to reduce the data-storage requirements, it is  
 2439 ideal to perform at least some of the CRES event reconstruction in real-time so that it  
 2440 is possible to save a reduced form of the data for offline analysis. The first step of the  
 2441 real-time reconstruction would be a real-time signal detection algorithm, which is the  
 2442 focus of this paper. Our approach consists of three main operations performed on the  
 2443 time-series data blocks including digital beamforming, a short time Fourier transform

2444 (STFT), and a binary classification algorithm to distinguish between signal and noise  
2445 data (see Figure 4.29).

#### 2446 4.4.2.2 Real-time Signal Detection

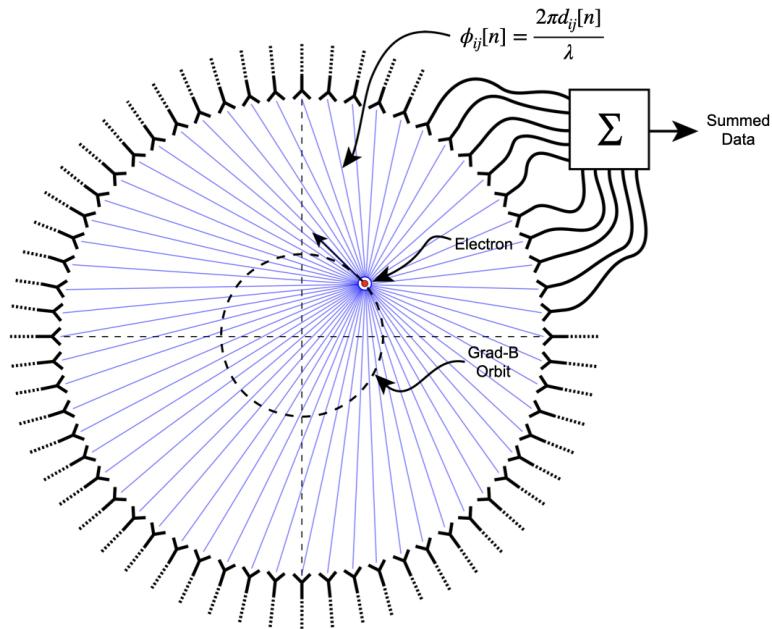


Figure 4.30: An illustration of the digital beamforming procedure. The blue lines indicate the various distances from the beamforming position to the antenna. In the situation depicted the actual position of the electron matches the beamforming position, so we should expect constructive interference when the phase shifted signals are summed. To prevent the electron's  $\nabla B$ -motion from moving the electron off of the beamforming position, the beamforming phase include a time-dependence to follow the trajectory of the electron in the magnetic trap.

2447 The first step in the real-time detection algorithm is digital beamforming, which is  
2448 a phased summation of the signals received by individual antennas in the array (see  
2449 Figure 5.21). The phase shifts correspond to the path length differences between a spatial  
2450 position and each individual antenna such that, when there is an electron located at  
2451 the beamforming position, all the signals received by the array constructively interfere.  
2452 Since we do not know ahead of time where an electron will be produced in the detector,  
2453 we define a grid of beamforming positions that cover the entire region where electrons  
2454 can be trapped and perform a phased summation for each of these points for every  
2455 time-step in the array data block. As we saw in Section 4.4.2.1, the axial oscillation  
2456 of the electrons prevents us from resolving it's position along the Z-axis of the trap,

<sup>2457</sup> therefore our beamforming grid need only cover the possible positions of the electron in  
<sup>2458</sup> the two-dimensional plane defined by the antenna array.

<sup>2459</sup> The equation defining digital beamforming can be expressed as

$$\mathbf{y}[n] = \Phi^T[n]\mathbf{x}[n], \quad (4.38)$$

<sup>2460</sup> where  $\mathbf{x}[n]$  is array snapshot vector at the sampled time  $n$ ,  $\Phi[n]$  is the matrix of  
<sup>2461</sup> beamforming phase shifts, and  $\mathbf{y}[n]$  is summed output vector that contains the voltages  
<sup>2462</sup> for each of the summed channels that correspond to a particular beamforming position.  
<sup>2463</sup> The elements of the beamforming phase shift matrix can be expressed as a weighted  
<sup>2464</sup> complex exponential

$$\Phi_{ij}[n] = A_{ij}[n] \exp(2\pi i \phi_{ij}[n]), \quad (4.39)$$

<sup>2465</sup> where the indices  $i$  and  $j$  label the beamforming and antenna positions respectively. The  
<sup>2466</sup> weight  $A_{ij}$  accounts for the relative power increase for antennas that are closer to the  
<sup>2467</sup> position of the electron, and  $\phi_{ij}$  is the total beamforming phase shift for the  $j$ -th antenna  
<sup>2468</sup> at the  $i$ -th beamforming position.

<sup>2469</sup> The beamforming phase shift is a sum of two terms

$$\phi_{ij}[n] = \frac{2\pi d_{ij}[n]}{\lambda} + \theta_{ij}[n], \quad (4.40)$$

<sup>2470</sup> where the first term is the phase shift originating from the path length difference ( $d_{ij}[n]$ )  
<sup>2471</sup> between the beamforming and antenna positions, which are represented by the vectors  
<sup>2472</sup>  $(r_j, \theta_j)$  and  $(r_i, \theta_i[n])$ , and the second term is the angular separation ( $\theta_{ij}[n]$ ) of the two  
<sup>2473</sup> positions. The angular separation enters into the beamforming phase due to an effect  
<sup>2474</sup> caused by the circular orbit of the electron that produces radiation whose phase is linearly  
<sup>2475</sup> dependent on the relative azimuthal position of the antenna [28, 29]. The time-dependence  
<sup>2476</sup> of the beamforming phases is intended to correct for the effects of  $\nabla B$ -drift, which cause  
<sup>2477</sup> the guiding centers of electrons to orbit the center of the magnetic trap. By including a  
<sup>2478</sup> linear time-dependence in the azimuthal beamforming position,

$$\theta_i[n] = \omega_{\nabla B} t[n] + \theta_{i,0}, \quad (4.41)$$

<sup>2479</sup> where  $\omega_{\nabla B}$  is the azimuthal grad-B drift frequency,  $t[n]$  is the time vector and,  $\theta_{i,0}$  is the  
<sup>2480</sup> starting azimuthal position, we can configure the beamforming phases to effectively track  
<sup>2481</sup> the XY-position of the guiding center over the event duration. Predicting accurate values  
<sup>2482</sup> of  $\omega_{\nabla B}$  for a specific trap and set of kinematic parameters will be done by simulations,

2483 which are performed using the Kassiopeia software package [7] by Project 8.

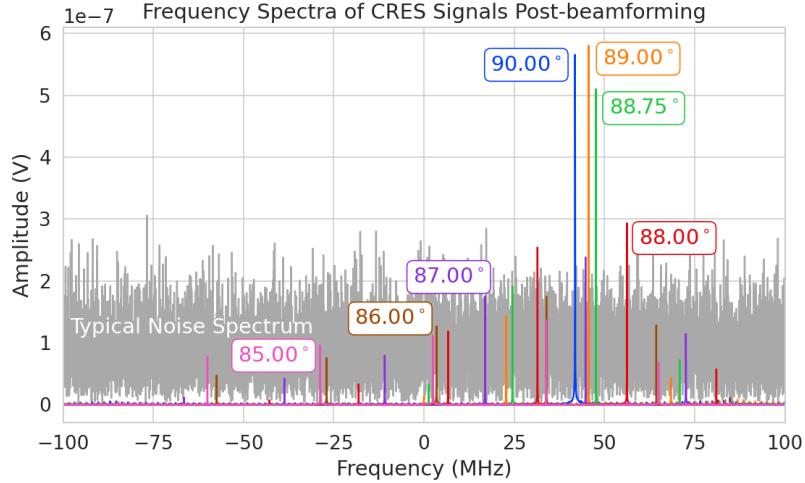


Figure 4.31: Frequency spectra of simulated CRES signals post-beamforming. The signal of a  $90^\circ$  electron consists of a single frequency component that is easy to detect with a power threshold on the frequency spectrum. This power threshold is still effective for signals with relatively large pitch angles such as  $89.0^\circ$  and  $88.75^\circ$ , which are composed of a main carrier and a few small sidebands. Signals with smaller pitch angles, below about  $88.5^\circ$ , tend to be dominated by sidebands such that no single frequency component can be reliably distinguished from the noise with a power threshold.

2484 After digital beamforming, we apply a short-time Fourier transform (STFT) to the  
2485 summed time-series to obtain the frequency spectrum representation of the signals (see  
2486 Figure 4.31). From the detection perspective, the frequency representation of the CRES  
2487 data is advantageous compared to the time domain, because the frequency spectra of  
2488 CRES signals are well-approximated by a frequency and amplitude modulated sinusoidal  
2489 whose carrier frequency increases as a linear chirp. The modulation is caused by the axial  
2490 oscillation of the electron in the magnetic trap and produce frequency spectra that are  
2491 well-described by a small number of frequency components. The linear chirp is caused  
2492 by the energy loss due to cyclotron radiation, which results in a relatively slow increase  
2493 in the frequency components of the CRES signal over time. During the standard Fourier  
2494 analysis window for the FSCD of  $40.96 \mu\text{sec}$ , we expect a typical CRES signal to increase  
2495 in frequency by approximately 15 kHz, which is smaller than the frequency bin width  
2496 given the 200 MHz sample rate. Therefore when considering a single frequency spectrum  
2497 it is justifiable to neglect the effects of the linear frequency chirp.

2498 In the cases where the electron's pitch angle is  $\gtrsim 88.5^\circ$ , the majority of the signal  
2499 power is contained in a single frequency component, with the remaining signal power

contained in a small number of sidebands proportional to the electron's axial modulation  
 (see Figure 4.31). In these cases detection is relatively straight-forward by implementing  
 a power threshold on the STFT, since the amplitude of the main signal peak is distinct  
 from the thermal noise spectrum. However, as the pitch angle of the electron is decreased  
 below  $88.5^\circ$ , the modulation index of the signal increases causing the maximum amplitude  
 of the frequency spectrum to be comparable to typical noise fluctuations. At this point,  
 the power threshold trigger is no longer able to distinguish between signal and noise  
 leading to a reduction in detection efficiency. The neutrino mass sensitivity of the FSCD  
 is directly linked to the overall detection efficiency. And, because the distribution of  
 electron pitch angles is effectively uniformly distributed across the range of pitch angles  
 that can be trapped, the overall detection efficiency is directly influenced by the range of  
 pitch angles that have detectable signals. Therefore, utilizing a signal detection algorithm  
 that can more effectively identify signals with pitch angles less than  $88.5^\circ$  will improve  
 both detection efficiency and ultimately the neutrino mass sensitivity of the FSCD and  
 other CRES experiments.

Modeling the detection performance of alternative signal detection algorithms for  
 the FSCD requires that we pose the signal detection problem in a consistent manner.  
 The approach we take is to perform a binary hypothesis test on the frequency spectra  
 generated by the STFT. Mathematically, this is expressed as,

$$\mathcal{H}_0 : y[n] = \nu[n] \quad (4.42)$$

$$\mathcal{H}_1 : y[n] = x[n] + \nu[n]. \quad (4.43)$$

Where under hypothesis  $\mathcal{H}_0$ , the vector representing the frequency spectrum ( $y[n]$ ) is  
 composed of pure white Gaussian noise (WGN) represented by  $\nu[n]$ , and under hypothesis  
 $\mathcal{H}_1$  the frequency spectrum is composed of a CRES signal ( $x[n]$ ) with added WGN. The  
 dominant source of noise in a FSCD-like experiment is expected to be thermal Nyquist-  
 Johnson noise, which is well approximated by a WGN distribution. In order to decide  
 between these two hypotheses we follow the standard Neyman-Pearson approach by  
 performing a log-likelihood ratio test between the probability distributions of the signal  
 classifier output under  $\mathcal{H}_1$  and  $\mathcal{H}_0$  [19]. The output of the log-likelihood ratio test is  
 called the test statistic, which is used to assign the data as belonging to the noise ( $\mathcal{H}_0$ )  
 or signal ( $\mathcal{H}_1$ ) classes by setting a decision threshold on the value of the test statistic.

In practice, we select the decision threshold by finding the value of the test statistic  
 that guarantees an acceptable rate of false positives and then attempt to maximize

the signal detection probability under that fixed false positive rate. Because the signal classifier will be used to evaluate the spectra of  $O(10^2)$  beamforming positions every 40.96  $\mu$ sec, we will require the signal classifiers to operate with decision thresholds that provide false positive rates significantly smaller than 1%. This reduces the burden placed on later stages of the CRES reconstruction chain to reject these false positives and decreases the overall likelihood of reconstructing a false event. Below, we calculate the probability distributions that allow us characterize how different detection algorithms will perform for CRES signals in an FSCD experiment.

### 4.4.3 Signal Detection Algorithms

#### 4.4.3.1 Power Threshold

The power threshold detection algorithm uses the maximum amplitude of the frequency spectra as the detection test statistic. To model the performance of this approach, consider first the case where the signal is pure WGN. For a single bin in the frequency spectrum, the probability that the amplitude falls below a specific threshold value is given by the Rayleigh cumulative distribution function (CDF),

$$\text{Ray}(x; \tau) = 1 - \exp(-|x|^2/\tau), \quad (4.44)$$

where the complex amplitude of the frequency bin is  $x$ , and  $\tau$  is the WGN variance. Because the noise samples for each frequency bin are independent and identically distributed (IID), the probability that every bin in the frequency spectrum falls below the threshold is the joint CDF formed by the product of each individual frequency bin CDF,

$$F_0(x; \tau, N_{\text{bin}}) = \text{Ray}(x; \tau)^{N_{\text{bin}}}. \quad (4.45)$$

The PDF for the power threshold classifier can then be obtained by differentiating the CDF.

The probability distribution for the power threshold classifier under  $\mathcal{H}_1$  is formed in a similar way, but the frequency bins that contain signal must be treated separately. For a frequency bin that contains both signal and noise we can describe the probability that the amplitude of the bin will fall below our threshold using the Rician CDF,

$$\text{Rice}(x; \tau, \nu) = 1 - Q_1 \left( \frac{|\nu|}{\sqrt{2\tau}}, \frac{|x|}{\sqrt{2\tau}} \right), \quad (4.46)$$

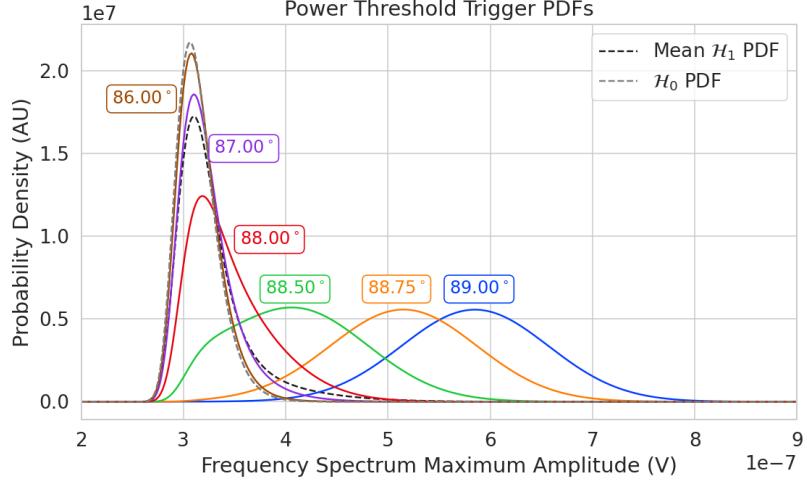


Figure 4.32: PDFs of the power threshold test statistic for CRES signals with various pitch angles as well as the PDF for the noise-only signal case. The average PDF computed for pitch angles ranging from 85.5 to 88.5° is also shown. As the pitch angle is decreased the signal PDF converges towards the noise PDF which indicates that the power threshold trigger is unable to distinguish between small pitch angle signals and noise.

where the parameter  $|\nu|$  defines the noise-free amplitude of the signal and  $Q_1$  is the Marcum Q-function. This time the CDF that describes the probability that the entire spectrum falls below the decision threshold is the product of both signal and noise CDFs,

$$F_1(x; \tau, \nu, N_{\text{bin}}, N_s) = \text{Ray}(x; \tau)^{N_{\text{bin}} - N_s} \prod_{k=0}^{N_s} \text{Rice}(x; \tau, \nu_k). \quad (4.47)$$

The first half of Equation 4.47 is the contribution from the bins in the frequency spectrum that contain only noise, and the second half is the product of the Rician CDFs for the frequency bins that contain signal peaks with a noise-free amplitude of  $|\nu_k|$ . In Figure 4.32 we show plots of example PDFs under  $\mathcal{H}_1$  and  $\mathcal{H}_0$ .

#### 4.4.3.2 Matched Filtering

The shape of a CRES signal is completely determined by the initial conditions of the electron as it is emitted from beta-decay, which implies that it is possible to apply matched filtering as a signal detection algorithm. With a matched filter one uses the shape of the known signal, which is called a template, to filter the incoming data by computing the convolution between the signal and the data [19]. For cases where the signal is buried in WGN, the matched filter is the optimal detector in that it achieves

2570 the maximum probability of a true detection for a fixed false positive rate. Since CRES  
 2571 signals have an unknown shape but are deterministic, we can apply a matched filter by  
 2572 using simulations to generate a large number of signal templates called a template bank,  
 2573 which spans the parameter space of possible signals. Then at detection time, we use the  
 2574 template bank to identify signals by performing the matched filter convolution for each  
 2575 template in an exhaustive search.

2576 As we saw from the frequency spectra in Figure 4.31, CRES signals are highly periodic  
 2577 in nature. In such cases, it is advantageous to utilize the convolution theorem to replace  
 2578 the matched filter convolution with an inner product in the frequency-domain. With the  
 2579 convolution theorem, the matched filter test statistic that describes the detection of a  
 2580 signal buried in WGN using a matched filter template bank is given by

$$\mathcal{T} = \max_{\mathbf{h}} \left| \sum_{n=0}^{N_{\text{bin}}} h^\dagger[n] y[n] \right|, \quad (4.48)$$

2581 where  $h^\dagger[n]$  is the complex conjugate of the signal template. For the case when our  
 2582 template bank consists of only a single template it is possible to derive an exact analytical  
 2583 form for the PDF describing the matched filter test statistic. First, we derive PDF under  
 2584 the signal hypothesis, where the equation describing the matched filter test statistic, also  
 2585 known as the matched filter score, becomes

$$\mathcal{T} = \left| \sum_{n=0}^{N_{\text{bin}}} h^\dagger[n] y[n] \right|. \quad (4.49)$$

2586 Each noisy frequency bin represented by  $y[n]$  is the sum between value of the signal  
 2587 at that bin and complex WGN, which means that  $y[n]$  is itself Gaussian distributed.  
 2588 Therefore, the value of the inner product between the template and the data is also a  
 2589 complex Gaussian variable; and, since the matched filter score is the magnitude of this  
 2590 inner product, it must follow a Rician distribution.

2591 We can derive the equation for the Rician PDF by expressing the matched filter  
 2592 template  $\mathbf{h}$  in terms of the corresponding simulated signal, which we write as  $\mathbf{x}_h$  to  
 2593 distinguish from the signal in the data. Using the standard normalization and assuming  
 2594 uncorrelated WGN, the matched filter templates can be written as

$$\mathbf{h} = \frac{\mathbf{x}_h}{\sqrt{\tau |\mathbf{x}_h|^2}} \quad (4.50)$$

2595 where  $\tau$  is the noise variance. Inserting this into Equation 4.48 and expressing the data  
 2596 as a sum between a signal and a WGN vector yields,

$$\mathcal{T} = \frac{1}{\sqrt{\tau|\mathbf{x}_h|^2}} \left| \sum_{n=1}^{N_{\text{bin}}} x_h[n] (x[n] + \nu[n]) \right|. \quad (4.51)$$

2597 Next, we transform the expression by isolating the randomly distributed components  
 2598 giving

$$\mathcal{T} = \frac{\left| \sum_{n=1}^{N_{\text{bin}}} x_h[n] x[n] \right|}{\sqrt{\tau|\mathbf{x}_h|^2}} + \frac{1}{\sqrt{\tau|\mathbf{x}_h|^2}} \left| \sum_{n=1}^{N_{\text{bin}}} x_h[n] \nu[n] \right|. \quad (4.52)$$

2599 The first term of 4.52 can be simplified by using the Cauchy-Schawrz inequality to express  
 2600 the magnitude of the inner product in terms of the magnitudes of the signal and template  
 2601 as well as an orthogonality constant which we call "match" ( $\Gamma$ ). Using this we obtain,

$$\mathcal{T} = |\mathbf{h}| |\mathbf{x}| \Gamma + \frac{1}{\sqrt{\tau|\mathbf{x}_h|^2}} \left| \sum_{n=1}^{N_{\text{bin}}} x_h[n] \nu[n] \right|. \quad (4.53)$$

2602 The second term is a sum of Gaussian distributed variables, which we should expect also  
 2603 follows a Gaussian distribution. Each of the samples  $\nu[n]$  is described by

$$\nu[n] \sim \mathcal{N}(0, \tau), \quad (4.54)$$

2604 where  $\mathcal{N}(0, \tau)$  is a complex Gaussian distribution with zero mean and variance  $\tau$ . There-  
 2605 fore,

$$\frac{x_h[n]}{\sqrt{\tau|\mathbf{x}_h|^2}} \nu[n] \sim \mathcal{N}\left(0, \frac{x_h[n]^2}{|\mathbf{x}_h|^2}\right), \quad (4.55)$$

$$\sum_{n=1}^{N_{\text{bin}}} \frac{x_h[n]}{\sqrt{\tau|\mathbf{x}_h|^2}} \nu[n] \sim \mathcal{N}\left(0, \frac{\sum_{n=1}^{N_{\text{bin}}} x_h[n]^2}{|\mathbf{x}_h|^2}\right) = \mathcal{N}(0, 1), \quad (4.56)$$

$$|\mathbf{h}| |\mathbf{x}| \Gamma + \sum_{n=1}^{N_{\text{bin}}} \frac{x_h[n]}{\sqrt{\tau|\mathbf{x}_h|^2}} \nu[n] \sim \mathcal{N}(|\mathbf{h}| |\mathbf{x}| \Gamma, 1). \quad (4.57)$$

2606 We see that  $\mathcal{T}$  is magnitude of a complex variable with mean  $|\mathbf{h}| |\mathbf{x}| \Gamma$  and variance one. In  
 2607 order to simply the expression a bit further, we define the quantity  $\mathcal{T}_{\text{ideal}} = |\mathbf{h}| |\mathbf{x}| \Gamma$ , which  
 2608 we call the ideal matched filter score, because it represents the value of the matched  
 2609 filter inner product that we would expect if no noise was present in the signal. We can

2610 write the matched filter test statistic as the magnitude of a two-dimensional vector in  
2611 the complex plane

$$\mathcal{T} = |(\mathcal{T}_{\text{ideal}} + n_r, n_i)|, \quad (4.58)$$

2612 where  $n_r$  and  $n_i$  are the real and imaginary components of the noise each with variance  
2613  $1/2$ , which is modeled by a Rician distribution with shape factor  $\mathcal{T}_{\text{ideal}}$ . Therefore, the  
2614 probability distribution of the matched filter test statistic is given by,

$$P_1(x; \mathcal{T}_{\text{ideal}}) = 2x \exp(-x^2 + \mathcal{T}_{\text{ideal}}^2) I_0(2x\mathcal{T}_{\text{ideal}}), \quad (4.59)$$

2615 where  $I_0$  is the zeroth-order modified Bessel function.

2616 The shape of the matched filter score distribution is controlled by the parameter  
2617  $\mathcal{T}_{\text{ideal}}$ , which is effectively the value of the matched filter score if the data contained no  
2618 noise. Without noise, the data vector reduces to the signal,  $\mathbf{x}$ , in which case Equation  
2619 4.49 becomes the magnitude of an inner product between two vectors. We can write  
2620 the magnitude of an inner product in terms of the lengths of the individual vectors and  
2621 a constant that describes the degree of orthogonality between them. Applying this to  
2622 Equation 4.49, we obtain

$$\mathcal{T}_{\text{ideal}} = |\mathbf{h}^\dagger \cdot \mathbf{x}| = |\mathbf{h}| |\mathbf{x}| \Gamma \quad (4.60)$$

2623 where  $\Gamma$  describes the orthogonality between  $\mathbf{h}$  and  $\mathbf{x}$ . From the point of view of matched  
2624 filtering, we can interpret  $\Gamma$  as describing how well the template matches the underlying  
2625 signal in the data.

2626 The matched filter score PDF under the noise hypothesis can be readily obtained  
2627 from Equation 4.59 by setting the value of  $\mathcal{T}_{\text{ideal}}$  to zero, since the data contains no signal  
2628 in the noise case. Doing this, we obtain the Rayleigh distribution that describes the  
2629 matched filter score under  $\mathcal{H}_0$ ,

$$P_0(x) = 2x \exp(-x^2). \quad (4.61)$$

2630 Equations 4.59 and 4.61 describe the behavior of the matched filter test statistic  
2631 under  $\mathcal{H}_0$  and  $\mathcal{H}_1$  for a single template. However, defining a PDF that describes the  
2632 matched filter test statistic in the case of multiple templates is in general a mathematically  
2633 intractable problem, since there is no guarantee of orthogonality between matched filter  
2634 templates. This leads to correlations between the matched filter scores of different  
2635 templates because only one sample of noise is used to compute the matched filter scores  
2636 of the template bank. In order to proceed, we need to make the simplifying assumption

that we can treat the matched filter scores as IID variables, which allows to ignore correlations between templates. The overall effect of this will be an underestimate of the performance of the matched filter, since we are under counting the number of templates that could contribute a detectable score.

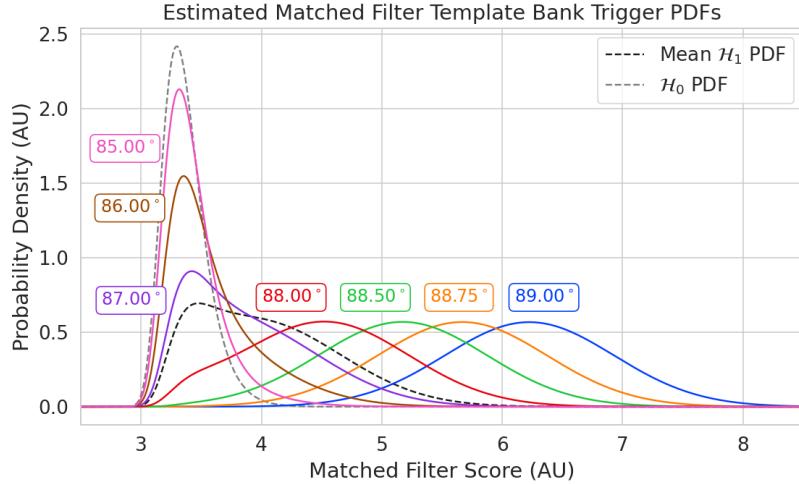


Figure 4.33: Plots of the estimated PDFs for the matched filter template bank test statistic for CRES signals with various pitch angles as well as the estimated PDF for the noise only signal case. We assume an estimated number of templates of  $10^5$  and perfect match between signal and template i.e.  $\Gamma_{\text{best}} = 1$ . The mean PDF includes signals ranging from  $85.5 - 88.5^\circ$  in pitch angle. There is a much larger distinction between the signal PDFs at small pitch angle compared to the power threshold indicating a higher detection efficiency for these signals.

For  $\mathcal{H}_0$  we model the probability that the matched filter score falls below our threshold using the CDF obtained by integrating Equation 4.61. Because we are assuming that the matched filter scores using different templates are independent, the probability that the matched filter score for all templates falls below a threshold value is the joint CDF formed by multiplying the CDF for each template. Under  $\mathcal{H}_0$  this is

$$F_0(x) = \left(1 - e^{-x^2}\right)^{N_t}, \quad (4.62)$$

where  $x$  is the matched filter score threshold and  $N_t$  is the number of templates. We should expect that the distribution describing the matched filter template bank maximum score depends on  $N_t$ , because with more templates there is a greater chance of a random match between the template and data.

For  $\mathcal{H}_1$ , we start by denoting the CDF of the best matching template as  $F_{\text{best}}(x; \mathcal{T}_{\text{best}})$ ,

and treat the matched filter scores for all other templates as negligible ( $\mathcal{T}_{\text{ideal}} \approx 0$ ). Then we form the joint CDF by combining the distributions for all templates used during detection. Since we are exhaustively checking the matched filter scores, the number of templates checked will be a randomly distributed variable that ranges from zero to the total number of available templates. If we assume that signals are uniformly distributed across the parameter space spanned by the template bank then on average we check  $(N_t - 1)/2 \approx N_t/2$  templates for each inference. Therefore, the estimated CDF under  $\mathcal{H}_1$  is

$$F_1(x; \mathcal{T}_{\text{best}}) = F_{\text{best}}(x; \mathcal{T}_{\text{best}}) \left(1 - e^{-x^2}\right)^{N_t/2}. \quad (4.63)$$

In Figure 4.33 we show plots of the estimated matched filter template bank classifier PDFs under both  $\mathcal{H}_0$  and  $\mathcal{H}_1$ .

#### 4.4.3.3 Machine Learning

In this paper we focus on Convolutional Neural Networks (CNN) as an example of a machine learning based signal classifier. CNNs are constructed using a series of convolutional layers, each composed of a set of filters that are convolved with the input data. The individual convolutional filters can be viewed as matched filter templates that are learned from a set of simulated data rather than being directly generated. This opens the possibility of finding a more efficient representation of the matched filter templates during the training process that can potentially reduce computational cost at inference time while still offering good classification performance.

The machine learning approach is distinct from both the power threshold and matched filtering in that we do not attempt to manually engineer a test statistic that is computed from the data for classification. Instead, we attempt calculate the test statistic by constructing a differentiable function that maps the complex frequency series generated by the STFT to a binary classification as either signal or noise. The test statistic for the machine learning classifier can be expressed as

$$\mathcal{T} = G(\mathbf{y}; \boldsymbol{\Omega}) \quad (4.64)$$

where  $\mathbf{y}$  is the noisy data vector and  $G(\mathbf{y}; \boldsymbol{\Omega})$  is the machine learning model parameterized by the weights  $\boldsymbol{\Omega}$ . By using supervised learning on a labeled set of training signals, we can modify the function parameters to learn the mapping from the data to the likelihood of  $\mathbf{y}$  belonging to either  $\mathcal{H}_1$  or  $\mathcal{H}_0$ .

The CNN architecture used for this work is summarized by Table 4.1. No strategic

Table 4.1: A summary of the CNN model layers and parameters. The output of each 1D-Convolution and Fully Connected layer is passed through a LeakyReLU activation function and re-normalized using batch normalization before being passed to the next layer in the model. The output of the final Fully Connected layer in the model is left without activation so that the model outputs can be directly passed to the Binary Cross-entropy loss function used during training.

Layer	Type	Input Channels	Output Channels	Parameters
1	1D-Convolution	2	15	$(N_{\text{kernel}} = 4, N_{\text{stride}} = 1)$
2	Maximum Pooling	15	15	$(N_{\text{kernel}} = 4, N_{\text{stride}} = 4)$
3	1D-Convolution	15	20	$(N_{\text{kernel}} = 4, N_{\text{stride}} = 1)$
4	Maximum Pooling	20	20	$(N_{\text{kernel}} = 4, N_{\text{stride}} = 4)$
5	1D-Convolution	20	25	$(N_{\text{kernel}} = 4, N_{\text{stride}} = 1)$
6	Maximum Pooling	25	25	$(N_{\text{kernel}} = 4, N_{\text{stride}} = 4)$
7	Fully Connected	3200	512	NA
8	Fully Connected	512	64	NA
9	Fully Connected	64	2	NA

hyper-parameter optimization approach was implemented beyond the manual testing of different CNN architecture variations, so this particular model is best viewed as a proof-of-concept rather than a rigorously optimized design. Numerous model variations were tested, some with significantly more layers and convolutions filters per layer, as well as others that were even smaller than the architecture in Table 4.1. Ultimately, the model architecture choice was driven by the motivation to find the minimal model whose classification performance was still comparable to the larger CNN’s tested, because of the importance of minimizing computational cost in real-time applications. It is possible that more sophisticated machine learning models could improve upon the classification results achieved here, but we leave this investigation for future work.

#### 4.4.4 Methods

##### 4.4.4.1 Data Generation

To test the triggering performance of the classifiers, simulated CRES signals were generated using the Locust simulations package [10, 28] developed by the Project 8 collaboration. Locust uses the separately developed Kassiopeia package to calculate the magnetic fields produced by a user defined set of current carrying coils along with any specified background magnetic fields, resulting in a magnetic trap. Next, Kassiopeia calculates the trajectory of an electron in this magnetic field starting from a set of user

2699 specified initial conditions. The Locust software then uses the electron trajectories from  
2700 Kassiopeia to calculate the resulting electromagnetic fields using the Liénard-Wiechert  
2701 equations, and determine the voltages generated in the antenna array with the antenna  
2702 transfer function. Locust then simulates the down-conversion, filtering, and digitization  
2703 steps resulting in the simulated CRES signals for an electron.

2704 The shape of the received CRES signal is determined by the initial kinematic param-  
2705 eters, including the starting position of the electron, the starting kinetic energy of the  
2706 electron, and the pitch angle. For the studies performed here we constrain ourselves to a  
2707 single initial electron position located at  $(x, y, z) = (5, 0, 0)$  mm, and using this starting  
2708 position we generate two datasets by varying the initial kinetic energy and the starting  
2709 pitch angle. The first dataset consists of a two-dimensional square grid of kinetic energy  
2710 and pitch angle spanning an energy range from 18575-18580 eV with a spacing of 0.1 eV,  
2711 and pitch angles from 85.5-88.5° with a spacing of 0.001°, resulting in 153051 signals with  
2712 a unique energy-pitch angle combination. This dataset is intended to represent a matched  
2713 filter template bank. The second dataset was generated by randomly sampling uniform  
2714 probability distributions covering the same parameter space to produce approximately  
2715 50000 signals randomly parameterized in energy and pitch angle. This dataset provides  
2716 the training and test data for the machine learning approach, and acts as a representative  
2717 sample of signals to evaluate the performance of the matched filter template bank.

2718 Each signal was simulated for a duration of 40.96  $\mu$ s, which is equivalent to 8192  
2719 samples at the FSCD digitization rate, and begins at time  $t = 0$  s for all simulations.  
2720 This duration represents a single frequency spectrum generated by the STFT. The output  
2721 of the Locust simulation is a matrix of array snapshots with size given by the number of  
2722 channels times the event length ( $N_{\text{ch}} \times N_{\text{sample}}$ ), which we pre-process using the digital  
2723 beamforming summation and STFT described in Section 4.4.2.2. The  $\nabla B$ -drift correction  
2724 uses the exact value of  $\omega_{\nabla B}$ , obtained from the Kassiopeia simulation of that electron.  
2725 In practice, an average value for  $\omega_{\nabla B}$  could be used, because there is limited variation in  
2726 drift frequency across this parameter space.

#### 2727 **4.4.4.2 Template Number and Match Estimation**

2728 The estimated PDF for the matched filter template bank depends on the score of the  
2729 best matching template or equivalently the match of the best template ( $\Gamma_{\text{best}}$ ) as well  
2730 as the number of templates. One expects that with a higher number of templates the  
2731 average value of  $\Gamma_{\text{best}}$  will increase, however, there is a point of diminishing returns at  
2732 which more templates will not significantly increase match, but will still increase the

2733 likelihood of false positives. Therefore, it is desirable to use the minimum number of  
templates that provide an acceptable mean value of  $\Gamma_{\text{best}}$ .

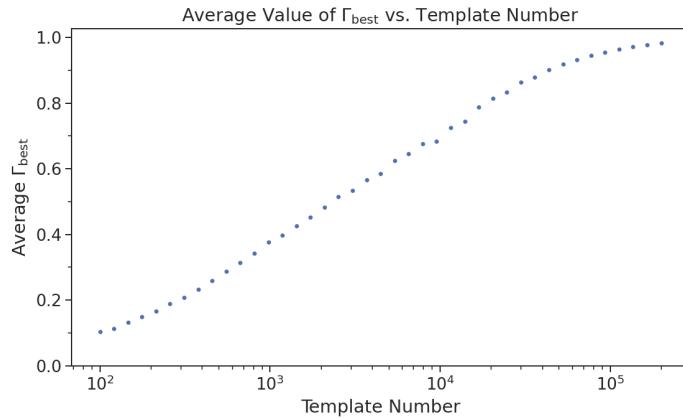


Figure 4.34: The mean match of the matched filter template bank to a test set of randomly parameterized signals as a function of the number or density of templates. The parameter space includes pitch angles from  $85.5 - 88.5^\circ$  and energies from 18575 – 18580 eV.

2734  
2735 To quantify the relationship between match and template number, we calculated  
2736 the mean match of the random dataset to a selection of templates obtained from the  
2737 regularly spaced dataset. The results are shown in Figure 4.34, where we find that the  
2738 average value of  $\Gamma_{\text{best}}$  is an exponential function of the number of templates. From this  
2739 plot we select the desired value of mean match at which we would like to evaluate the  
2740 matched filter PDF and can infer the required number of templates.

2741 **4.4.4.3 CNN Training and Data Augmentation**

2742 To prepare the data for training the model, we split the random dataset in half to create  
2743 distinct training and test datasets. Additionally, a randomly selected 20% of the training  
2744 data is isolated for use as a validation set during the training loop. The size of the  
2745 training, validation, and test datasets are then tripled by appending two additional copies  
2746 of the data to increase the sample size of the dataset after data augmentation. The  
2747 data is loaded with no noise, which is added to each data batch during the training  
2748 phase by generating a new noise sample from a complex WGN distribution. In order to  
2749 ensure an even split between signal and noise data we append to the noise-free signals an  
2750 equal number of empty signals composed of all zeros. Therefore, as the data is randomly  
2751 shuffled during training, on average an equal number of empty signals will be included  
2752 with the training signals. After adding the sample of WGN to the data batch, the empty

signals represent the noise-only data that the model must distinguish from signal data.

As the training signals are loaded we apply a unique random phase shift as the first form of data augmentation. Since the data is generated using the same initial axial position and cyclotron orbit phase, the randomization is an attempt to prevent overtraining on these features. During each training epoch the data is randomly shuffled and split into batches of 2500 signals. Each batch of signals is then circularly shifted by a random number of frequency bins to simulate a kinetic energy shift from  $-75$  to  $20$  eV to simulate a training dataset with a larger energy range. Next, a sample of complex WGN, consistent with the expected 10 K Nyquist-Johnson noise expected for the FSCD, is generated and added to the signal, which prevents overtraining on noise features. As a final step, the data is renormalized by the standard deviation of the noise so that the range of values in the data is close to  $[-1, 1]$ , which helps ensure well-behaved back-propagation.

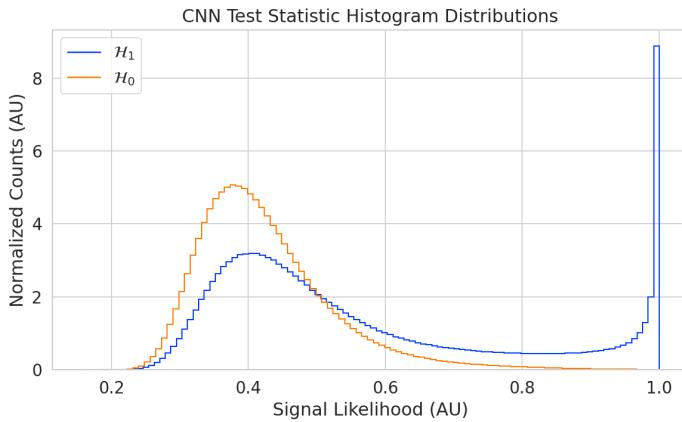


Figure 4.35: Histograms of the trained CNN model output from the test dataset. The blue histogram shows the model outputs for signal data. The oddly shaped peak near the end is the result of the softmax function mapping the long tail of the raw output distribution to the range  $[0, 1]$ .

The Binary Cross-entropy loss function is used to compute the loss for each batch of data and the model weights are updated using the ADAM optimizer with a learning rate of  $5 \times 10^{-3}$ . After each training epoch, the loss and classification accuracy of the validation dataset are computed to monitor for overtraining. It was noticed that the relatively high noise power and the fact that a new sample of noise was used for each batch together provided a strong form of regularization, since no evidence of over-training was observed even after several thousand epochs. Typically, the loss and classification accuracy of the model converged after a few hundred training epochs, but the training loop was

2774 extended to 3000 epochs to attempt to achieve the best possible performance. The  
2775 training procedure generally took about 24 hrs using a single NVIDIA V100 GPU [30].

2776 After training the model, we use it to classifying the test dataset and generate  
2777 histograms of the model outputs for both classes of data. The data augmentation  
2778 procedure for the evaluation of the test data mirrors the training procedure without  
2779 the validation split. Since a random circular shift and a new sample of WGN is added  
2780 to each batch, the testing evaluation loop is run for 100 epochs to get a representative  
2781 sample of noise and circular shifts. The model outputs for each batch are passed through  
2782 a softmax activation and then combined into histograms, which we show in Figure 4.35.

## 2783 4.4.5 Results and Discussion

### 2784 4.4.5.1 Trigger Classification Performance

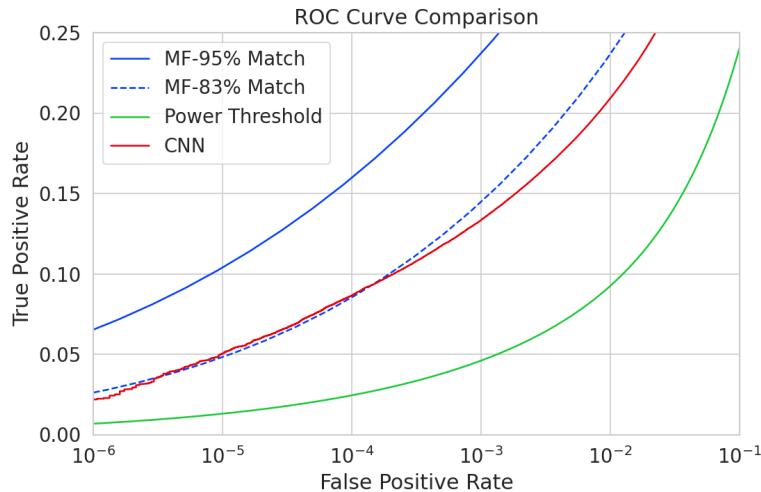


Figure 4.36: ROC curves describing the detection efficiency or true positive rates for the three signal classification algorithms examined in this paper.

2785 Using the matched filter and power threshold CDFs, along with the classification  
2786 results from the CNN, we compare detection performance by computing receiver operating  
2787 characteristic (ROC) curves. Specifically, we compare the detection performance averaged  
2788 over the full signal parameter space in order to get a measure of the overall detection  
2789 efficiency achieved by each algorithm. For the power threshold and matched filter  
2790 algorithms, we obtain the mean ROC curve by taking the average over all signals in the  
2791 regularly spaced dataset. In the case of the matched filter, we examine two cases using  
2792 different numbers of templates, which have different values of mean match. The ROC

2793 curve describing the CNN is obtained by forming a histogram of the network outputs  
2794 for each class of signal and from this computing the estimated CDFs and ROC curve.  
2795 In Figure 4.36, we show the ROC curves obtained for each of the detection algorithms,  
2796 visualized in terms of true positive rate and false positive rate.

2797 The true positive rate of a signal classifier is equivalent to its detection efficiency, and  
2798 we see that for the population of signals with pitch angles  $< 88.5^\circ$  the power threshold  
2799 has a consistently lower detection efficiency than the CNN and the matched filter. This  
2800 result could have been predicted from the visualization of signal spectra in Figure 4.31,  
2801 where we see that there is no way to distinguish between a noise peak and a signal peak  
2802 with high confidence at small pitch angles. The CNN offers a significant and consistent  
2803 increase in detection efficiency over the power threshold approach, with the relative  
2804 improvement in detection efficiency increasing as the false positive rate decreases. If  
2805 we compare the CNN to the matched filter, we see that the performance of the tested  
2806 network is roughly equivalent to a matched filter detector with an average match of  
2807 about 83%, which uses approximately 20000 matched filter templates. The overall best  
2808 detection efficiency is achieved by the matched filter classifier if a large enough template  
2809 bank is used. We show in the plot the ROC curve for a matched filter template bank  
2810 with 95% average match, which is achieved with approximately 100000 templates. Since  
2811 the matched filter is known to be statistically optimal for detecting a known signal in  
2812 WGN, it is somewhat expected that this algorithm has the highest detection efficiency.

2813 A potentially impactful difference between the matched filter and CNN algorithms is  
2814 that the CNN relies upon convolutions as its fundamental calculation mechanism, whereas  
2815 our implementation of a matched filter utilizes an inner product. Since convolution is  
2816 a translation invariant operation, the detection performance of CNN can be extended  
2817 to a wider range of CRES event kinetic energies with less cost than the matched filter,  
2818 a feature that we exploited during the CNN training by including circular translations  
2819 of the CRES frequency spectra in the training loop. Increasing the range of kinetic  
2820 energies detectable by a matched filter requires a proportional increase in the number of  
2821 templates, which directly translates into increased computational and hardware costs.  
2822 From a practical perspective, the detection algorithm is always limited by the available  
2823 computational hardware, so estimating the relative costs is a key factor in determining  
2824 their feasibility. Below we perform a more detailed analysis of the relative costs of each  
2825 of the detection algorithms.

2826 **4.4.5.2 Computational Cost and Hardware Requirements**

2827 In the process of investigating triggering approaches for an antenna array CRES exper-  
2828 iment, we have uncovered a strong tension between detection efficiency and computational  
2829 resources. To relate the computational cost estimates to actual costs, we compare the  
2830 theoretical amount of computer hardware required to implement the signal classifiers  
2831 for real-time detection in an FSCD experiment. To do this we shall utilize order of  
2832 magnitude estimates of the theoretical peak performance values for currently available  
2833 Graphics Processing Units (GPUs) as a metric. This approach will underestimate the  
2834 amount of required hardware, since it is unlikely that any CRES detection algorithm  
2835 could reach the theoretical peak performance of the hardware.

2836 Of the three detection algorithms tested, the power threshold classifier is the least  
2837 expensive. It requires that we check whether the amplitude of each frequency bin in  
2838 the STFT is below or above our decision threshold. The STFT combined with digital  
2839 beamforming produces  $N_{\text{bin}}N_b$  frequency bins that must be checked every  $N_{\text{bin}}/f_s$  seconds.  
2840 This requires approximately  $O(10^{10})$  FLOPS to check in real-time. Current generations of  
2841 GPUs have peak theoretical performances in the range of  $O(10^{13}) - O(10^{14})$  FLOPS [31],  
2842 dependent on the required floating-point precision of the computation. Therefore, the  
2843 entire computational needs of a real-time triggering system using a power threshold  
2844 classifier, including digital beamforming and generation of the STFT, could be met by a  
2845 single high-end GPU or a small number of less powerful GPUs. Since triggering is only  
2846 one step of the full real-time signal reconstruction approach, limiting the computational  
2847 cost of this stage is ideal. However, we have seen that the power threshold classifier does  
2848 not provide sufficient detection efficiency across the entire range of possible signals,  
2849 which is the primary motivation for exploring more complicated triggering solutions.

2850 As discussed, the computational cost of the matched filter approach requires counting  
2851 the number of templates that must be checked for each frequency spectra produced by the  
2852 STFT. Computing the matched filter scores requires  $O(N_bN_tN_{\text{bin}})$  operations, since for  
2853 each of the  $N_b$  beamforming positions we must multiply  $N_t$  templates with a data vector  
2854 that has length  $N_{\text{bin}}$ . The time within which we must perform this calculation is equal  
2855 to  $N_{\text{bin}}/f_s$  to keep up with the data generation rate. To cover the 5 eV kinetic energy  
2856 range spanned by the template bank, we saw that  $10^4$  to  $10^5$  templates are required in  
2857 order to match or exceed the detection efficiency of the CNN. If the number of templates  
2858 scales linearly with the kinetic energy range of interest as expected, then we would  
2859 require  $10^5$  to  $10^6$  matched filter templates with this more realistic range of energies.  
2860 Considering this, the estimated computational cost of the matched filter is between

2861  $O(10^{15})$  to  $O(10^{16})$  FLOPS, which is  $O(10^2)$  to  $O(10^3)$  high-end GPUs.

2862 Lastly, we have the CNN classifier. To estimate the computational cost we simply  
2863 sum the number of convolutions and matrix multiplications specified by the network  
2864 architecture shown in Table 4.1. Each convolutional layer consists of  $N_{\text{in}}N_{\text{out}}N_{\text{kernel}}L_{\text{input}}$   
2865 floating-point operations, where  $N_{\text{in}}$  is the number of input channels,  $N_{\text{out}}$  is the number  
2866 of output channels,  $N_{\text{kernel}}$  is the size of the convolutional kernel, and  $L_{\text{input}}$  is the length  
2867 of the input vector, and the fully connected layers each contribute  $N_{\text{in}}N_{\text{out}}$  operations.  
2868 Summing all the neural network layers we estimate that the CNN would require  $O(10^6)$   
2869 floating point operations for each frequency spectra; therefore, the total computation  
2870 cost of the CNN trigger is this cost times the number of beamforming positions per the  
2871 data acquisition time, which is  $O(10^{13})$  FLOPS or  $O(10^0)$  GPUs.

2872 Compared with the matched filter approach the CNN requires  $O(100)$  to  $O(1000)$   
2873 fewer GPUs to implement, dependent on the exact number of templates used in the  
2874 template bank. The 100 eV kinetic energy range is motivated by the application of these  
2875 detection algorithms to an FSCD-like neutrino mass measurement experiment. However,  
2876 if a significantly larger range of kinetic energies is required, a CNN may be the preferred  
2877 detection approach despite the lower average detection efficiency due to computational  
2878 cost considerations. The low estimated computational cost of the CNN is directly related  
2879 to the small network size.

2880 Additional experiments with larger CNNs, generated by increasing the depth and  
2881 width of the neural network, and we observed that these changes provided minimal  
2882 ( $\lesssim 1\%$ ) improvement in the classification accuracy of the model. A potential reason  
2883 for this could be the sparse nature of the signals in the frequency domain and the low  
2884 SNR which makes for a challenging dataset to learn from. Future work could investigate  
2885 modifications to the neural network architecture such as sparse convolutions, which may  
2886 improve the classification accuracy of the model or further reduce the computational  
2887 costs of this approach. Alternatively, more complicated CNN architectures such as a  
2888 ResNet [32] or VGG model [33] may provide improved classification performance over a  
2889 basic CNN. An additional promising area of investigation are recurrent neural networks,  
2890 which may be able to exploit the time-ordered features of the STFT for more accurate  
2891 signal detection if the electron signals last for multiple Fourier transform windows.

2892 Our estimate of the computational cost of the matched filter is somewhat naive if  
2893 we notice that the majority of the values that make up a CRES frequency spectra are  
2894 zero (see Figure 4.31). Therefore, the majority of operations in the matched filter inner  
2895 product are unnecessary, and we could instead evaluate the matched filter inner product

2896 using only the  $\lesssim 10$  frequency peaks that make up CRES signal. This optimization  
2897 reduces the number of operations required to check each template by a factor of  $O(100)$   
2898 to  $O(1000)$ , which brings the estimated computational cost of the matched filter in  
2899 line with the CNN. Although this level of sparsity results in a multiplication with very  
2900 low arithmetic complexity, the resulting sparse matched filter algorithm is still likely  
2901 to be constrained by memory access speed rather than compute speed. Ultimately, the  
2902 comparison of the relative computational and hardware costs between the matched filter  
2903 and CNN will depend on the efficiency of the software implementation and hardware  
2904 support for neural network and sparse matrix calculations.

#### 2905 **4.4.6 Conclusion**

2906 Increasing the detection efficiency and overall event rate of the CRES technique represents  
2907 a key developmental path towards new scientific results and broader applications of the  
2908 CRES technique. It is what motivates both the antenna array detection approach and  
2909 the development of real-time signal reconstruction algorithms. We have demonstrated  
2910 that significant gains in the detection efficiency of the CRES technique are achievable  
2911 by utilizing triggering algorithms that account for the specific shape of CRES signals in  
2912 the detector. These algorithms emphasize the need for accurate and fast methods for  
2913 CRES simulation, since they directly contribute to the success of matched filter methods  
2914 by providing a way to generate expected signal templates and also serve as a source of  
2915 training data for machine learning approaches.

2916 The improvements in detection efficiency offered by these alternative approaches to  
2917 triggering are crucial to the success of efforts to develop scalable technologies for CRES  
2918 measurement, since they provide a significant increase in the detectable parameter space  
2919 of CRES events, which allows for a better utilization of the larger detection volume.  
2920 While we have focused on the real-time detection of CRES signals from antenna arrays,  
2921 these same signal classifiers could be used in CRES experiments utilizing a different  
2922 detector technologies, since the same principles of signal detection will apply. For example,  
2923 previous CRES measurements by the Project 8 collaboration that utilized a waveguide  
2924 gas cell, could have improved their detection efficiency by employing a matched filter  
2925 or neural network classifier to identify trapped electrons with pitch angles that are too  
2926 small to be detected by the power threshold approach. Furthermore, alternative CRES  
2927 detector technologies such as resonant cavities [3] could also see similar improvements  
2928 in detection efficiency, which is of crucial importance to future efforts by the Project 8  
2929 collaboration to utilize CRES to measure the neutrino mass.

## **Chapter 5**

# **Antenna and Antenna Measurement System Development for the Project 8 Experiment**

### **5.1 Introduction**

The FSCD and antenna array CRES represent an innovative approach to beta-decay spectroscopy. While much can be learned from simulations about the systematics of CRES with antenna arrays, laboratory measurements and demonstrations provide critical inputs to sensitivity and simulation models as well as provide a means for calibration and commissioning of the experiment. Therefore, a robust program of antenna and antenna measurement hardware development is important to the success of the FSCD and the development of antenna array CRES more broadly.

In this chapter we summarize the development of an antenna measurement system at Penn State to implement and test the techniques of antenna array CRES on the bench-top, in order to support the efforts of the Project 8 collaboration. In Section 5.2 we provide an introduction to some fundamental parameters and concepts related to antenna measurements as well as an overview of the Penn State antenna measurement system hardware. In Section 5.3 we include the manuscript of a paper [29] which details the design and characterization of a specialized antenna developed to mimic the electric fields emitted by an electron in a CRES experiment. This antenna, called the Synthetic Cyclotron Antenna (SYNCA), is intended as a calibration tool for antenna arrays developed for CRES measurements. Lastly, in Section 5.4 we summarize a set of prototype FSCD antenna array measurements with the SYNCA [6], which we use to validate the simulated performance of the antenna array and estimate systematic errors associated with the antenna array.

2955 **5.2 Antenna Measurements for CRES experiments**

2956 **5.2.1 Antenna Parameters**

2957 Antenna characterization measurements are intended to validate simulations of the  
2958 antenna array performance, which ultimately informs the neutrino mass sensitivity of  
2959 the experiment. In this section, I shall summarize a few fundamental concepts relating  
2960 to antennas and antenna measurement, before introducing how Project 8 uses antenna  
2961 measurement for the development of antenna array CRES.

2962 **5.2.1.1 Radiation Patterns**

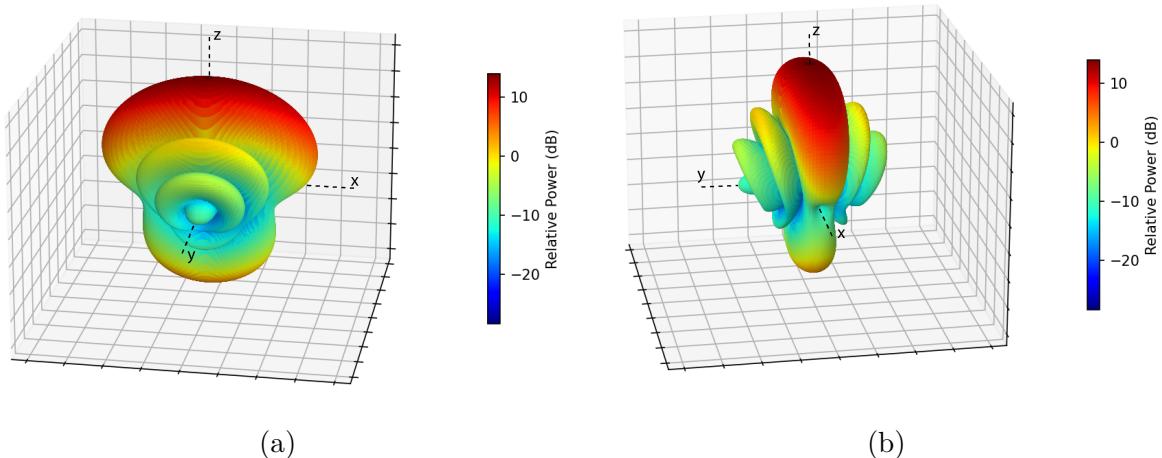


Figure 5.1: An example radiation pattern generated using HFSS simulations. The color and radial distance of the surface from the origin indicate the relative magnitude of radiation power emitted by the antenna in that direction. The primary goal of most antenna measurements is typically to measure the antenna pattern, which is used to derive many useful antenna performance parameters.

2963 Antennas are conductive structures designed to carry alternating electric currents  
2964 in order to transmit energy in the form of electro-magnetic (EM) waves [13]. Perhaps  
2965 the most fundamental way to characterize an antenna, is to map out the radiated power  
2966 density as a function of position, which is called the radiation pattern (see Figure 5.1).  
2967 We find the radiation power density by calculating the time-averaged Poynting vector for  
2968 all positions surrounding the antenna, which in equation form is

$$\mathbf{W}(x, y, z) = \langle \mathbf{E}(x, y, z, t) \times \mathbf{H}^*(x, y, z, t) \rangle_t, \quad (5.1)$$

2969 where  $\mathbf{E}(x, y, z, t)$  and  $\mathbf{H}(x, y, z, t)$  are the time-dependent electric and magnetic fields  
 2970 produced by the antenna [8]. The radiation power density has units of  $\text{W/m}^2$  and is  
 2971 more typically called the energy flux density in physics applications, since it is a measure  
 2972 of the amount of energy passing through a unit area over time.

2973 Because the radiation power density is a measure of power per unit area, its value  
 2974 in a particular direction will depend on the distance from the antenna at which we are  
 2975 measuring. This is undesirable for practical applications A related quantity, which is  
 2976 distance independent, is the energy flux per unit solid angle or radiation intensity, which  
 2977 is computed directly from the radition power density by multiplying by the squared  
 2978 distance from the antenna. Specifically,

$$U = r^2 W(x, y, z), \quad (5.2)$$

2979 where  $r$  is the distance from the antenna to the field measurement point. The radiation  
 2980 intensity is typically defined in regions where the Poynting vector consists only of a radial  
 2981 component where it is safe to treat as a scalar quantity.

### 2982 5.2.1.2 Directivity and Gain

2983 Since the radiation intensity is a measure of average power per unit solid angle, it is  
 2984 independent of distance and more useful as feature for antenna measurement. However,  
 2985 most antenna measurements are performed in terms of the directly related directivity  
 2986 and gain quantities. Directivity is defined as the ratio between the radiation intensity at  
 2987 particular point on the radiation pattern to the average radiation intensity computed  
 2988 over all solid angles [13]. The equation that relates the radiation intensity to directivity  
 2989 is

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}}, \quad (5.3)$$

2990 where  $U_0$  is the average radiation intensity over all solid angles, which simply the total  
 2991 radiated power ( $P_{\text{rad}}$ ) divided by  $4\pi$ . Closely related to directivity is concept of gain,  
 2992 which accounts for energy losses that occur inside then antenna when attempting to  
 2993 transmit or receive a signal. The antenna gain is given by

$$G = \frac{4\pi U}{P_{\text{in}}}, \quad (5.4)$$

2994 where  $P_{\text{in}}$  is the total power delivered to the antenna. Gain can be thought of as the ratio  
 2995 of the antenna's radiation intensity to that of a hypothetical isotropic, lossless radiator.

2996 The maximum values of gain and directivity exhibited by the main lobe of the antenna  
 2997 pattern as well as the ratio between the gain of the main lobe and any side-lobes are  
 2998 important figures of merit used to evaluate antenna designs.

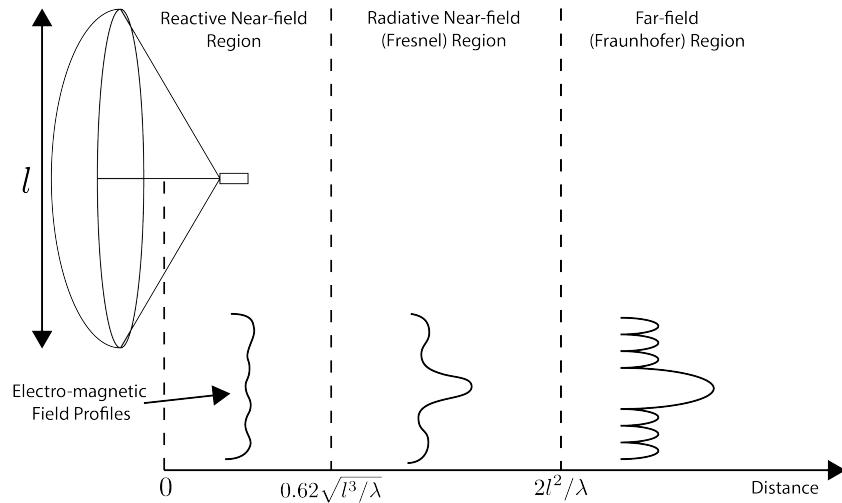


Figure 5.2: An illustration of the three field regions important for the analysis of an antenna system. Very close to the antenna the electric fields are primarily reactive so there is no radiation. If a receiving antenna were placed in this region most of the energy would be reflected back to the transmitter. Outside of the reactive near-field is the radiative near field. At these distances the antenna does radiate, but the radiation pattern is not well-defined since it changes based on the distance of the receiving antenna. It is only in the far-field region where the radiation pattern becomes constant as a function of distance, which is where the majority of antenna engineering is assumed to take place. The antenna arrays developed by Project 8 for CRES measurements operate in the radiative near-field due to the importance of limiting power loss from free-space propagation, which complicates the design of the antenna system.

### 2999 5.2.1.3 Far-field and Near-field

3000 Radiation patterns are only well-defined in regions where the shape of the radiation  
 3001 pattern is independent of distance. The region where this approximation is valid is called  
 3002 the "far-field", and in this region we can treat the EM fields from the antenna as spherical  
 3003 plane waves. A rule of thumb for antennas is that the far-field approximation is valid  
 3004 when the condition

$$R > \frac{2l^2}{\lambda} \quad (5.5)$$

3005 is met. In this expression,  $R$  is the distance from the antenna,  $l$  is the largest characteristic  
 3006 dimension of the antenna, and  $\lambda$  is the wavelength of the radiation (see Figure 5.2).

3007        The region very close to the antenna is called the reactive near-field, because in this  
3008      region the reactive component of the EM field is dominant. Unlike radiative electric  
3009      fields, the reactive electric and magnetic fields are out of phase from each other by  
3010      90°, since they are the result of electrostatic and magnetostatic effects coming from the  
3011      self-capacitance and self-inductance of the antenna. The reactive fields are unable to  
3012      transfer energy a significant distance from the antenna and are thus completely negligible  
3013      for most antenna applications. The limit of the reactive near-field for an electrically-large  
3014      antenna is typically taken to be

$$R < 0.62\sqrt{l^3/\lambda}. \quad (5.6)$$

3015        The unique application of antennas by Project 8 is somewhat limited by reactive near-  
3016      field effects in the form of a maximum radial position for electrons inside the uniform  
3017      cylindrical antenna array. If electrons are too close to the edge of the array than reactive  
3018      near-field effects leads to a large reduction in the received power and consequently  
3019      detection efficiency. This leads to a significant volume inside of the antenna array that  
3020      is unsuitable for CRES lowering the volumetric efficiency of the antenna array CRES  
3021      technique relative to a cavity experiment.

3022        In between the reactive near-field and the far-field is the radiative near-field region.  
3023        In this region the fields are primarily radiative, however we are still too close to the  
3024      antenna for the spherical plane wave approximation to apply. Therefore, interference  
3025      effects between EM waves emitted from different points on the antenna occur causing the  
3026      shape of the radiation pattern to change as a function of distance from the antenna. If we  
3027      evaluate the far-field distance limit for the FSCD one finds an estimated far-field distance  
3028      of 43 cm, which is a factor of four larger than the radius of the antenna array designed for  
3029      the experiment. Consequently, we expect near-field effects to influence the performance  
3030      of the antenna array highlighting the importance of calibration and characterization  
3031      measurements.

### 3032      **5.2.1.4    Polarization**

3033        The polarization of an EM wave defines the spatial orientation of the electric field  
3034      oscillations in the plane perpendicular to the direction of the propagation, and is defined  
3035      in terms of orthogonal polarization components. In our application, one analyzes the  
3036      properties of radiation propagating along the radial ( $\hat{r}$ ) direction away from the antenna,  
3037      which implies that the electric fields can be described as a linear combination of orthogonal

3038 polarization components

$$\mathbf{E}_{\text{tot}} = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}, \quad (5.7)$$

3039 in Cartesian coordinates, or

$$\mathbf{E}_{\text{tot}} = E_\theta \hat{\theta} + E_\phi \hat{\phi}, \quad (5.8)$$

3040 in spherical coordinates.

3041 In general, one defines partial radiation patterns, directivities, and gains so that the  
3042 performance of the antenna for the desired polarization can be analyzed. The radiation  
3043 pattern defined in terms of partial patterns is

$$U_{\text{tot}} = U_\phi + U_\theta, \quad (5.9)$$

3044 where  $U_\phi$  and  $U_\theta$  are the radiation intensities in a particular direction for the respective  
3045 polarization components. Similarly, a quantity such as gain can be written in terms of  
3046 partial gains,

$$G_{\text{tot}} = G_\phi + G_\theta = \frac{2\pi U_\phi}{P_{\text{in}}} + \frac{2\pi U_\theta}{P_{\text{in}}}. \quad (5.10)$$

3047 If we view an electron performing a circular orbit in the XY-plane from the side, that  
3048 is, along the X or Y axes, then we would observe the electron to be performing a linear  
3049 oscillation perpendicular to the viewing axis. From this intuitive picture, we can predict  
3050 that the primary polarization of electric fields from CRES events to be linearly polarized  
3051 in the  $\hat{\phi}$  direction when viewed with an antenna positioned in the XY-plane.

### 3052 5.2.1.5 Antenna Factor and Effective Aperture

3053 A useful way to characterize the performance of an antenna is to measure the electric  
3054 field magnitude required to produce a signal with an amplitude of one volt in the antenna  
3055 terminals. This ratio between the magnitude of the incoming electric field and the  
3056 magnitude of the signal produced by the antenna is called the antenna factor, which is  
3057 written as

$$A_F = \frac{|\mathbf{E}_{\text{in}}|}{V_{\text{ant}}}, \quad (5.11)$$

3058 where  $A_F$  is the antenna factor,  $E_{\text{in}}$  is the incoming electric field, and  $V_{\text{ant}}$  is the magnitude  
3059 of the voltage produced by the antenna.

3060 The antenna factor can be expressed in terms of the antenna's gain through a related  
3061 quantity called the effective aperture. The effective aperture defines for a given incident  
3062 radiation power density ( $\text{W/m}^2$ ) the power that is received by the antenna. Therefore,

3063 the effective aperture gives the equivalent area of the antenna,

$$A_{\text{eff}} = \frac{P_{\text{rec}}}{P_{\text{in}}} = \frac{\lambda^2}{4\pi} G, \quad (5.12)$$

3064 where the received power is  $P_r$  and the total incoming power is  $P_{\text{in}}$ .

3065 If we express the incident power in terms of the magnitude of the Poynting vector,  
3066 then

$$|\mathbf{S}_{\text{in}}| = |\mathbf{E}_{\text{in}}|^2 / \eta_0, \quad (5.13)$$

3067 where  $\eta_0$  is the impedance of free-space, which relates the magnitudes of the electric and  
3068 magnetic fields in a vacuum, and is defined by

$$\eta_0 = \frac{|\mathbf{E}|}{|\mathbf{H}|} = \sqrt{\frac{\epsilon_0}{\mu_0}}. \quad (5.14)$$

3069 The total received power by the antenna can therefore be expressed as

$$P_{\text{rec}} = |\mathbf{S}_{\text{in}}| A_{\text{eff}} = |\mathbf{S}_{\text{in}}| \frac{\lambda^2}{4\pi} G = \frac{|\mathbf{E}_{\text{in}}|^2 \lambda^2 G}{4\pi \eta_0}. \quad (5.15)$$

3070 To relate this to the antenna factor recall that we can relate the voltage produced by  
3071 the antenna to the received power with

$$P_{\text{rec}} = \frac{V_{\text{ant}}^2}{Z} = \frac{|\mathbf{E}_{\text{in}}|^2}{A_F^2 Z}, \quad (5.16)$$

3072 where  $Z$  is the system impedance. Setting Equations 5.15 and 5.16 equal to each other,  
3073 we obtain the following expression for antenna factor in terms of gain

$$A_F = \sqrt{\frac{4\pi\eta_0}{ZG\lambda^2}} = \frac{9.73}{\lambda\sqrt{G}}. \quad (5.17)$$

3074 The second expression in Equation 5.17 is obtained by evaluating the constant terms  
3075 assuming a system impedance of  $50 \Omega$ .

3076 We have gone through the effort of expressing the antenna factor in terms of gain  
3077 to highlight that the majority of antenna parameters that we care to measure for a  
3078 CRES experiment can be obtained from the radiation or gain pattern of the antenna.  
3079 The antenna factor is a particularly important parameter for CRES measurements  
3080 due to its relevance to antenna array simulations with the Locust software [10, 28].  
3081 Specifically, Locust simulates the trajectory of an electron in a magnetic trap by running

3082 the Kassiopeia software package [7] and then uses the Liénard-Wiechert equations [11, 12]  
3083 to calculate the electric fields that are incident on the antenna.

3084 To compute the response of the antenna to the electric field, Locust relies upon linear  
3085 time-invariant system theory [15], which computes the response of the antenna (i.e. the  
3086 voltage time series generated by the antenna) using a convolution between the electric field  
3087 time-series and the antenna impulse response. This approach is necessary for correctly  
3088 modeling the antenna response to the electric field due to the broadband and non-  
3089 stationary nature of the electric fields from CRES events. Since antenna measurements  
3090 take place under steady-state conditions, parameters such as the radiation pattern, gain,  
3091 and antenna factor are defined in the frequency domain. However, by performing an  
3092 inverse Fourier transform on the antenna factor we can obtain the antenna impulse  
3093 response, which allows us to simulate CRES events in the antenna array demonstrator  
3094 experiment.

### 3095 **5.2.2 Antenna Measurement Fundamentals**

#### 3096 **5.2.2.1 Friis Transmission Equation**

3097 The antenna factor, sometimes called the antenna transfer function, is used to model  
3098 how the antenna will respond to electric fields emitted from a CRES event. Therefore,  
3099 being able to measure the antenna transfer function of the antenna array is a key step  
3100 in the commissioning and calibration phases of an antenna array CRES experiment. A  
3101 common approach to antenna characterization is to perform a two antenna transmit-  
3102 receive measurement where an antenna with a known gain is used to characterize the  
unknown gain of the antenna under test (see Figure 5.3).

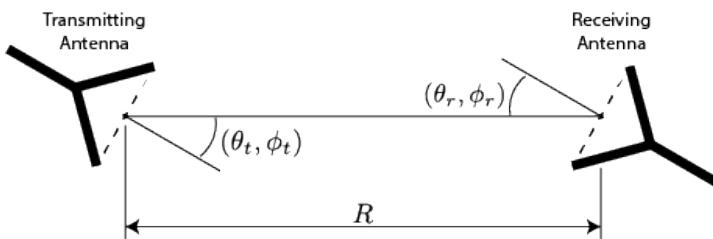


Figure 5.3: An illustration of the Friis measurement technique commonly used for antenna characterization measurements.

3103  
3104 To analyze this two antenna setup we seek to calculate the amount of power from  
3105 the transmitting antenna that we will detect with the receiving antenna. Using our  
3106 understanding of antenna gain, we can calculate the power density transmitted by an

3107 antenna in a direction  $(\theta_t, \phi_t)$  at frequency  $f$  and distance  $R$ , which is given by

$$w_t = \frac{P_t}{4\pi R^2} G_t(\theta_t, \phi_t, f). \quad (5.18)$$

3108 Here,  $P_t$  is the total power delivered to the transmitting antenna and  $G_t(\theta_t, \phi_t, f)$  is  
3109 the value of the transmitting antenna gain. The power density is the power per unit  
3110 area, so to calculate the total power delivered to the receiving antenna we multiply the  
3111 transmitted power density by the effective area of the receiving antenna,

$$P_r = w_t A_{eff,r} = P_t \frac{G_t(\theta_t, \phi_t, f) G_r(\theta_r, \phi_r, f) c^2}{(4\pi R f)^2}, \quad (5.19)$$

3112 where  $G_r(\theta_r, \phi_r, f)$  is the gain of the receiving antenna. Equation 5.19 is called the  
3113 Friis transmission equation [34, 35], which is of fundamental importance for antenna  
3114 measurements, since it allows one to measure the gain of an unknown antenna by  
3115 measuring the power received from an antenna with a known gain pattern. Alternatively,  
3116 if no antenna with a known gain pattern is available, two identical antennas with unknown  
3117 gain patterns can be used.

### 3118 5.2.2.2 S-Parameters and Network Analyzers

3119 Instead of directly measuring the power received by the antenna under test, it is more  
3120 common to measure the ratio of the received power to the transmitted power,

$$\frac{P_r}{P_t} = \frac{G_t(\theta_t, \phi_t, f) G_r(\theta_r, \phi_r, f) c^2}{(4\pi R f)^2}. \quad (5.20)$$

3121 This power ratio can be easily measured using a vector network analyzer (VNA), which  
3122 automates a significant fraction of the measurement process. Network analyzers are  
3123 used to measure the scattering or S-parameters of a multi-port RF device [36], which  
3124 describes how waves are scattered between the device ports. The antenna measurements  
3125 we have been considering can be modeled as a two-port microwave device that we can  
3126 characterize by measuring how incident voltage waves are transmitted or reflected (see  
3127 Figure 5.4). We can write the scattered waves ( $V_1^-$  and  $V_2^-$ ) in terms of the incident ( $V_1^+$   
3128 and  $V_2^+$ ) waves using the scattering matrix

$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}, \quad (5.21)$$

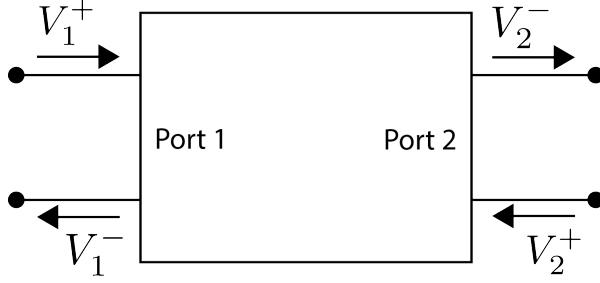


Figure 5.4: Illustration of a two-port S-parameter measurement setup. S-parameters characterize how incoming waves of voltage or power scatter off of the RF device under test. This allows you to measure important properties of the device. In particular, we can use this framework to model a two antenna radiation pattern measurement, which we can then automate using a VNA.

where the elements of the matrix are the device S-parameters. It is assumed that, when exciting the device from a particular port, that all other ports in the network are terminated at the system impedance. This ensures that the incident waves from other ports in the network are zero. Therefore, the S-parameters are the ratios between the scattered and incident waves,

$$S_{ij} = \frac{V_i^-}{V_j^+}. \quad (5.22)$$

Alternatively, S-parameters can be defined as the ratio of the scattered and incident power, which is proportional to the ratio of the squared voltage waves. Returning to our antenna measurement setup, we see that measuring the ratio of the received to the transmitted power is equivalent to measuring the ratio of power being scattered from port 1 to port 2 in a RF network. Therefore, measuring an antenna's gain can be accomplished quite easily, by using a VNA to perform a two port  $S_{21}$  measurement.

### 5.2.2.3 Antenna Array Commissioning and Calibration Measurements

Up to this point we have been discussing calibration and commissioning measurements as they apply to a single antenna. While these measurements play an important role in validating the radiation patterns of the individual array elements, the ultimate goal is to use a phased array of these antennas. Therefore, we must also consider antenna measurement techniques that apply to the whole array system.

By measuring the gain of each individual array element we can predict the features of the signals received during a CRES event using the antenna factor (see Section 5.2.1.5). However, unpredictable changes to the antenna performance can be introduced by the

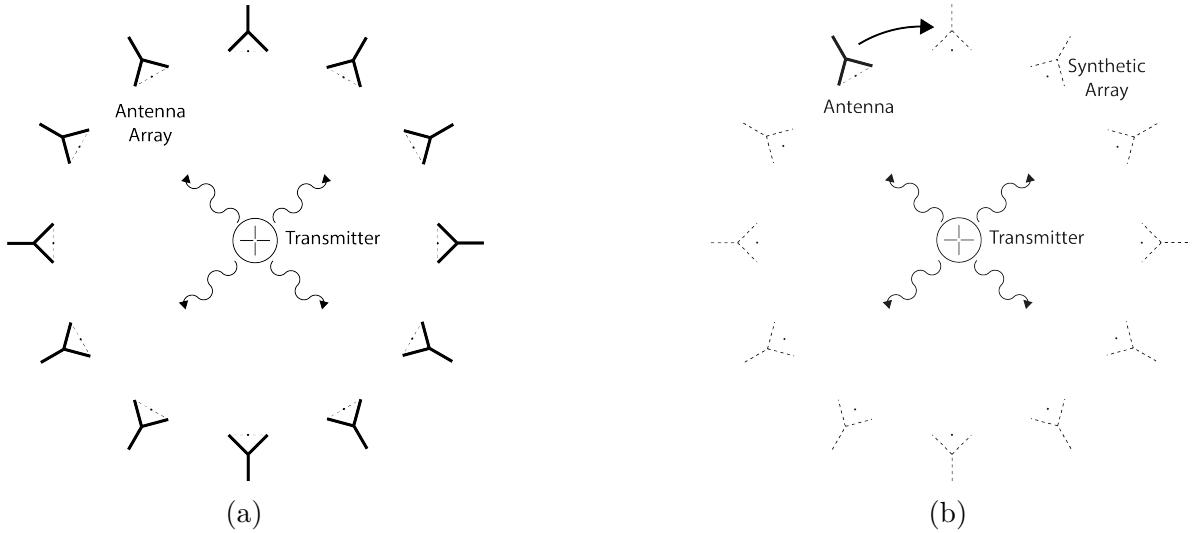


Figure 5.5: Two measurement approaches to characterizing an antenna array for CRES measurements. The full-array approach (a) requires a complete antenna array with all the associated hardware. The synthetic array approach (b) utilizes a single antenna and a set of rotation/translation stages to reposition the transmitter or the receiving antenna to synthesize the signals that would be received by the full-array. This approach reduces the cost and complexity of array measurements. A down-side of the synthetic array approach is that multi-channel effects such as reflections cannot be measured. Utilizing both the full-array and the synthetic array is a powerful way to quantify the impact of errors from the multi-channel array.

incorporation of the antennas into the circular array geometry, therefore, we employ both individual antenna and full-array measurements in the commissioning of the FSCD to account for these effects.

There are two main approaches to array measurements that could be used for characterization and calibration (see Figure 5.5). One approach is to construct the complete array and use an omni-directional transmitting antenna to measure the power received by each channel in the antenna array. In Section 5.3 we describe the development of an omni-directional transmitter that also mimics the radiation phase characteristics of a CRES event, which is useful because the entire array can be tested without repositioning. Alternatively, a full antenna array can be synthesized by repeatedly moving and measuring a single array element. This approach is ideal for identifying if different channels in the antenna array are affecting each other through multi-path interference by comparing the measurement results of the synthetic array to the real array.

### 5.2.3 The Penn State Antenna Measurement System

The development of antenna array based CRES requires the capability to test and calibrate different antenna array designs to validate the performance of the as-built antenna array before and during the experiment. With these aims in mind we developed an antenna measurement system at Penn State specifically designed to mimic the characteristics of the antenna experiment designed for demonstration of the antenna array CRES technique by the Project 8 collaboration.

The Penn State antenna measurement system utilizes a two antenna measurement configuration with a stationary reference antenna and a test antenna mounted on a set of motorized translation and rotation stages (see Figure 5.6). The antenna measurement system can be operated in two distinct modes, one focused on the characterization of the radiation patterns of prototype antennas and the other focused on the validation of data-acquisition (DAQ) and CRES signal reconstruction techniques to bridge the gap between real measurements and simulation. In both measurement configurations it is critical to isolate the antennas from the environment so that multi-path reflections do not negatively influence the measurement results. For this reason we surround the measurement volume with microwave absorber foam (AEMI AEC-1.5) [37] specifically designed to attenuate microwave radiation near the 26 GHz measurement range of the system.

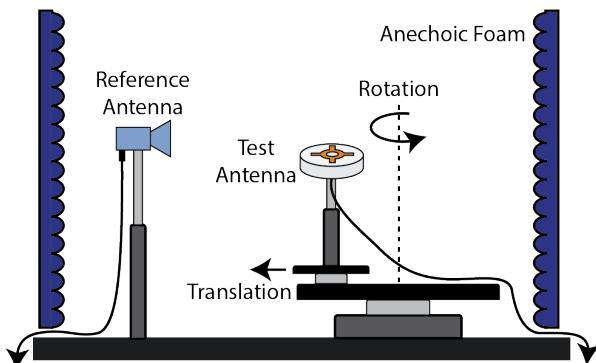


Figure 5.6: Illustration of the antenna measurement system developed for the Project 8 Collaboration. The reference and test antennas can be connected to different data acquisition configurations depending on the measurement goals. The reference antenna is typically a standard horn antenna and the test antenna is mounted on a set of translation stages for positioning. Automated translation stages allow for relatively painless data-taking enabling synthetic antenna array measurements using only a single receiving antenna. Anechoic form designed to mitigate RF reflections surrounds the setup.

In the first measurement configuration the reference antenna is typically a well-

3182 characterized horn antenna as pictured, since horn antennas have well-known and stable  
 3183 radiation patterns making them ideal as standard references. For characterization  
 3184 measurements, the test antenna represents the antenna-under-test whose pattern we wish  
 3185 to characterize. Mounting the test antenna on motorized rotation and translation stages  
 3186 allows us to automate the procedure significantly speeding up the radiation pattern  
 3187 measurement process.

3188 In the second measurement configuration one is interested in recreating the conditions  
 3189 of an antenna array CRES experiment as it concerns the antenna array and DAQ system.  
 3190 In this case, the reference antenna is a prototype FSCD antenna, which will be used to  
 3191 construct the antenna array in the FSCD experiment, and the test antenna is a specially  
 3192 designed synthetic cyclotron antenna (SYNCA) as picture in Figure 5.6. The SYNCA is  
 3193 designed such that the radiation pattern mimics that of a CRES electron so that the  
 3194 signals received by the prototype CRES array antenna mimic what is expected for a real  
 3195 CRES experiment.

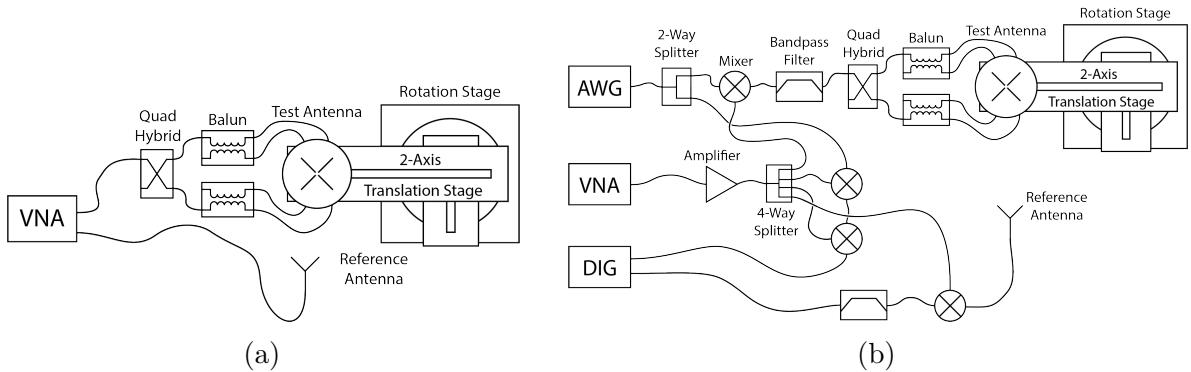


Figure 5.7: Diagrams of two measurement system configurations. Configuration (a) utilizes a VNA and is more suited to antenna characterization. Configuration (b) utilizes an AWG and VNA as a signal generation system and digitizer to collect measurement data, which is more suited to simulating CRES measurements. The transmission chain utilizes a quadrature hybrid and a pair of baluns to drive the cross-dipole variant test antenna developed for synthetic CRES measurements.

3196 In Figure 5.7 we show two high-level system diagrams of the Penn State antenna  
 3197 measurement system that depict the important system components and the connections  
 3198 between them. The two configurations of the measurement system utilize different  
 3199 hardware. For characterization and radiation pattern measurements, one prefers the  
 3200 configuration shown in Figure ???. In this case a vector network analyzer (VNA) is used  
 3201 as both the transmission source and data acquisition system and it is relatively easy to  
 3202 calibrate over a wide range of frequencies. Whereas, if one is more interested in recreating

3203 what would take place in the FSCD experiment then the configuration shown in Figure  
3204 ?? is preferable, since this system effectively mimics the receiver chain envisioned for the  
3205 FSCD experiment.

3206 The characterization configuration utilizes a network analyzer (Keysight N5222A)  
3207 [38, 39] with two independent sources and four measurement ports as the primary  
3208 measurement tool. A standard reference antenna is connected to one measurement port,  
3209 and the test antenna is connected to a separate port. The typical reference antenna  
3210 used for these studies is a Pasternack PF9851 horn antenna [40]. In the measurement  
3211 shown, the test antenna represents a SYNCA antenna, which requires a transmission  
3212 chain consisting of quadrature hybrid coupler [41, 42] (Marki QH-0226) connected to  
3213 two baluns [43] (Marki BAL-0026) to generate feed signals with the appropriate phases.  
3214 The VNA measures the radiation pattern by performing a transmission S-parameter  
3215 measurement, which can be used with the knowledge of the reference antenna’s radiation  
3216 pattern to determine the radiation pattern of the test antenna (see Section 5.2.1).

3217 The second configuration is more complicated and incorporates more hardware  
3218 components in order to more closely mimic the DAQ system envisioned for the FSCD  
3219 experiment. The basic approach is to produce CRES-like radiation and use an antenna  
3220 combined with a realistic RF receiver chain to acquire the signals. On the transmit side,  
3221 an arbitrary waveform generator [44] (AWG, RIGOL DG5252) is used to generate a  
3222 waveform that mimics a CRES signal at a baseband frequency up to 250 MHz. This  
3223 frequency is then up-converted to the CRES signal frequency band of 25.8 to 26.0 GHz  
3224 using a mixer [45] (Marki MM1-0832L) and a bandpass filter (K&L Microwave 3C62-  
3225 25900/T200-K/K) to reject unwanted mixing components outside out of the 200 MHz  
3226 CRES signal band. The local oscillator signal for mixing is provided by one of the VNA  
3227 sources configured to run in a continuous wave setting. On the receive side, a prototype  
3228 antenna is used to detect the radiation emitted by the test antenna, which is down-  
3229 converted and filtered using the same mixer and bandpass filter as the transmission chain.  
3230 Lastly, data acquisition is performed using a 14-bit ADC sampling at 500 MSa/s [46]  
3231 (CAEN DT530) to digitize the down-converted signals.

3232 In order to distribute the LO to all mixers a 4-way power splitter (MiniCircuits  
3233 ZC4PD-18263-S+) along with an amplifier (Marki APM-6848) is used to drive the four  
3234 mixers used in the measurement system. A limitation of using the VNA as an LO source  
3235 is that there is no control of the LO phase when a measurement is triggered by the  
3236 control script, which leads to a random phase offset between acquisitions. This makes it  
3237 impossible to perform synthetic array measurements, which require strict control over

3238 the starting phase of the transmitted signal. In order to monitor the random phase of the  
3239 LO, a 2-way power splitter (MiniCircuits Z99SC-62-S+) is used to split the signal from  
3240 the AWG between the transmission path and a LO monitoring path. The LO monitoring  
3241 path consists of an up-conversion and down conversion using two mixers connected by a  
3242 coaxial cable, and monitors the relative phase of the LO using a channel on the digitizer  
3243 to sample this path. A phase shift in the LO will lead to a proportional phase shift in  
3244 the mixed signal, which is measured and removed from the received signals.

3245 The test antenna is mounted on a set of motorized stages, which are identical for  
3246 both measurement configurations. A rotational stage (ThorLabs PRMTZ8) is used as  
3247 the base layer with additional translation stages mounted on top of this. The rotational  
3248 stage is ideal for measuring a complete azimuthal scan of the test antenna's radiation  
3249 pattern as well as for moving a SYNCA antenna in circular motion to recreate the  
3250 symmetry of the FSCD antenna array. On top of the rotational stage we mount two  
3251 linear translation stages (ThorLabs MTS50-Z8 and MTS25-Z8) in a cross-wise manner  
3252 so that the test antenna can be moved along two perpendicular axes. Using the linear  
3253 stages in combination with the rotational stage allows one to fine-tune the positioning of  
3254 the test antenna so that it can be perfectly aligned with the central axis of the array.  
3255 A LabView script was developed to automate the measurement of a full 360° radiation  
3256 pattern and control the measurement electronics. Data from these acquisitions is stored  
3257 on university provided cloud storage.

## 3258 **5.3 Development of a Synthetic Cyclotron Antenna (SYNCA) 3259 for Antenna Array Calibration**

3260 This section is the manuscript of the publication [29] detailing the development of a  
3261 Synthetic Cyclotron Antenna (SYNCA) for antenna array characterization measurements  
3262 by the Project 8 collaboration.

### 3263 **5.3.1 Introduction**

3264 Neutrinos are the most abundant standard model fermions in our universe, but due to  
3265 weak interaction cross-sections with other particles, neutrinos are particularly difficult  
3266 to study. Consequently, many fundamental properties of neutrinos are still unknown  
3267 including the absolute scale of the neutrino mass [47]. Direct, kinematic measurements of  
3268 the neutrino mass are particularly valuable due to their model independent nature [48].

3269 To date the most sensitive direct neutrino mass measurements have been performed by  
 3270 the KATRIN collaboration [49], which measures the molecular tritium  $\beta$ -decay spectrum  
 3271 to infer the neutrino mass. Current data from neutrino oscillation measurements [47]  
 3272 allow for neutrino masses significantly smaller than the design sensitivity of the KATRIN  
 3273 experiment; therefore, there is a need for new technologies for performing direct neutrino  
 3274 mass measurements to probe lower neutrino masses.

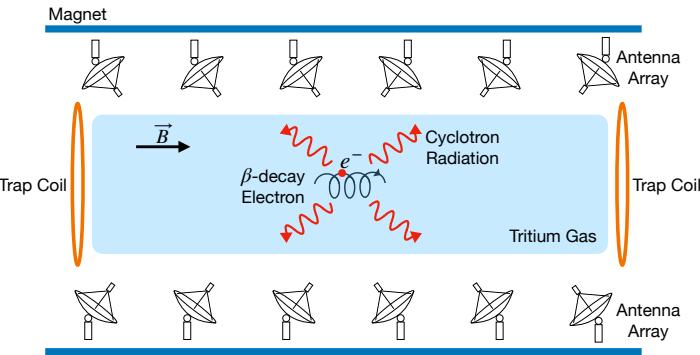


Figure 5.8: A sketch of an antenna array large-volume CRES experiment. Electrons from  $\beta$ -decays are confined in a magnetic field using a set of trap coils. The cyclotron radiation produced by the motion of the trapped electrons can be detected by a surrounding antenna array to determine the electron energies. Measuring the energies of many electrons produces a  $\beta$ -decay spectrum.

3275 The Project 8 collaboration is developing new methods for neutrino mass measurement  
 3276 based on Cyclotron Radiation Emission Spectroscopy (CRES) [50–53], with the goal of  
 3277 measuring the absolute scale of the neutrino mass with a 40 meV/c<sup>2</sup> sensitivity [?, 48].  
 3278 This sensitivity goal will require the development of two separate technical capabilities.  
 3279 First is the development of an atomic tritium source, which avoids significant spectral  
 3280 broadening due to molecular final states [54]. Second is the technology for performing  
 3281 CRES in a multi-cubic-meter experimental volume with high combined detection and  
 3282 reconstruction efficiency, which is required in order to obtain sufficient event statistics  
 3283 near the tritium spectrum endpoint.

3284 One approach for a large-volume CRES experiment is to use an array of antennas,  
 3285 which surrounds a volume of tritium gas, to detect the cyclotron radiation produced  
 3286 by the  $\beta$ -decay electrons when they are trapped in a background magnetic field using a  
 3287 set of magnetic trapping coils (see Figure 5.8). Project 8 has developed a conceptual  
 3288 experiment design to study the feasibility of this approach. The design consists of a  
 3289 single circular array of antennas with a radius of 10 cm and 60 independent channels  
 3290 positioned around the center of the magnetic trap. The motivation behind this antenna

array design is to first develop an understanding of the antenna array approach to CRES  
 with a small scale experiment before attempting to scale the technique to large volumes  
 by using multiple antenna rings to construct the full cylindrical array. The development  
 of the antenna array approach to CRES has largely proceeded through simulations using  
 the Locust software package [28, 55], which is used to model the fields emitted by CRES  
 events and predict the signals received by the surrounding antenna array. To validate  
 these simulations, a dedicated test stand is being constructed to perform characterization  
 measurements of the prototype antenna array developed by Project 8 (see Figure 5.9)  
 and benchmark signal reconstruction methods using a specially designed transmitting  
 calibration probe antenna.

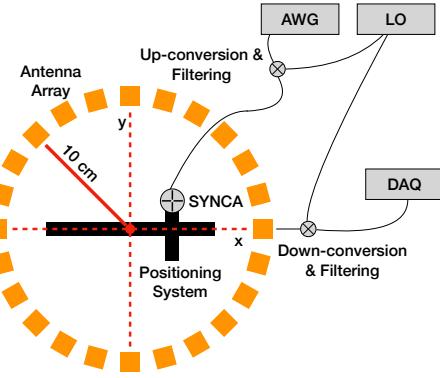


Figure 5.9: A schematic of the antenna array test stand. The circular antenna array has a radius of 10 cm with 60 independent channels (limited number shown for clarity). The test stand includes an arbitrary waveform generator (AWG), local oscillator (LO), and data acquisition (DAQ) hardware. Finally, a specialized Synthetic Cyclotron Antenna (SYNCA) is used to inject signals to test the antenna array.

We call this probe antenna the Synthetic Cyclotron Antenna or SYNCA. The SYNCA  
 is a novel antenna design that mimics the cyclotron radiation generated by individual  
 charged particles trapped in a magnetic field, which will be used in the antenna test  
 stand to perform characterization measurements, simulation validation, and reconstruc-  
 tion benchmarking. This paper provides an overview of the design, construction, and  
 characterization measurements of the SYNCA performed in preparation for its usage as  
 a transmitting calibration probe.

In Section 5.3.2 we provide a description of the cyclotron radiation field characteristics  
 that we recreate with the SYNCA. In Section 5.3.3 we give an overview of the simulations  
 performed to develop an antenna design that mimics the characteristics of cyclotron  
 radiation. In Section 5.3.4 we outline characterization measurements to validate that

3312 the fields generated by the SYNCA match simulation, and finally in Section 5.3.5 we  
 3313 demonstrate an application of the SYNCA to test phased array reconstruction techniques  
 3314 on the bench-top.

### 3315 5.3.2 Cyclotron Radiation Phenomenology

3316 To understand the cyclotron radiation phenomenology that the SYNCA should mimic,  
 3317 we consider a charged particle moving at relativistic speed in the presence of an external  
 3318 magnetic field (see Figure 5.10). In the special case we shall examine, the entirety of  
 3319 the electron's momentum is directed perpendicular to the magnetic field; therefore, the  
 3320 trajectory of the electron is confined to the cyclotron orbit plane. Because the momentum  
 3321 vector is oriented perpendicular to the magnetic field, electrons with these trajectories  
 3322 are said to have pitch angles of 90°.

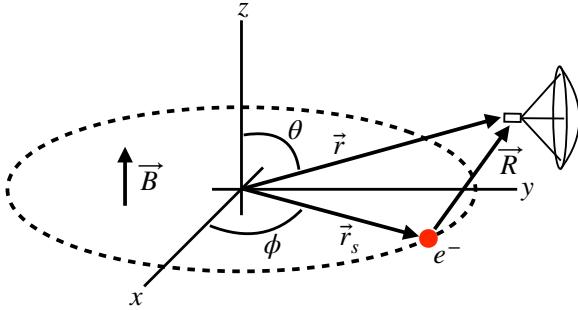


Figure 5.10: An electron (red dot) performing cyclotron motion in the x-y plane. The resulting cyclotron radiation is observed by an antenna located at the field point of interest.

3323 The cyclotron radiation fields generated by this circular trajectory are those which  
 3324 we aim to reproduce with the SYNCA. We can describe the electromagnetic (EM) fields  
 3325 using the Liénard-Wiechert equations [8, 28], which in non-covariant form express the  
 3326 electric field as

$$\vec{E} = e \left[ \frac{\hat{n} - \vec{\beta}}{\gamma^2 (1 - \vec{\beta} \cdot \hat{n})^3 |\vec{R}|^2} \right]_{t_r} + \frac{e}{c} \left[ \frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \vec{\beta} \cdot \hat{n})^3 |\vec{R}|} \right]_{t_r}, \quad (5.23)$$

3327 where  $e$  is the particle's charge,  $\hat{n} = (\vec{r} - \vec{r}_s)/|\vec{r} - \vec{r}_s|$  is the unit vector pointing from the  
 3328 electron to the field measurement point,  $\vec{\beta} = \dot{\vec{r}}_s/c$  is the velocity of the particle divided  
 3329 by the speed of light, and  $\gamma$  is the relativistic Lorentz factor. The equation is meant to  
 3330 be evaluated at the retarded time as indicated by  $t_r = t - |\vec{R}|/c$ , which accounts for the

3331 time delay due to the finite speed of light between the point where the field was emitted  
 3332 and the point where the field is detected.

3333 We would like to simplify Equation 5.23 it at all possible. As a first step we analyze  
 3334 the relative magnitudes of the electric field polarization components. Consider an electron  
 3335 following a circular cyclotron orbit in a uniform magnetic field whose guiding center  
 3336 is positioned at the origin of the coordinate system. The equation of motion can be  
 3337 expressed as

$$\vec{r}_s = (r_c \cos \omega_c t_r) \hat{x} + (r_c \sin \omega_c t_r) \hat{y}. \quad (5.24)$$

3338 For single antenna located along the y-axis at position  $\vec{r} = r_a \hat{y}$  we are interested in the  
 3339 incident electric fields from the electron. The electric field is given by Equation 5.23,  
 3340 which we evaluate in the regime where  $r_a \gg r_c$ . This limit can be justified by comparing  
 3341 the radius of the cyclotron orbit for an electron with the tritium beta-spectrum endpoint  
 3342 energy of 18.6 keV in a 1 T magnetic field to the typical ( $r_a \simeq 100$  mm) radial position  
 3343 of the receiving antenna. We find that the cyclotron orbit has a radius of 0.46 mm which  
 3344 is approximately a factor of 200 smaller than the typical antenna radial position. In this  
 3345 regime we can make the approximation  $\vec{R} \simeq r_a \hat{y}$  and the expression for the electric field  
 3346 at the antenna's position becomes

$$\vec{E} = \frac{e}{\gamma^2 r_a^2} \frac{\hat{x} \left( \frac{r_c \omega_c}{c} \sin \omega_c t_r \right) + \hat{y} \left( 1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r \right)}{\left( 1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r \right)^3} - \frac{e}{cr_a} \frac{\hat{x} \left( \frac{r_c^2 \omega_c^3}{c^2} - \frac{r_c \omega_c^2}{c} \cos \omega_c t_r \right)}{\left( 1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r \right)^3}. \quad (5.25)$$

3347 Since the receiving antenna is part of a circular array of antennas, it is useful to rewrite  
 3348 Equation 5.25 in terms of the azimuthal ( $\hat{\phi}$ ) and radial ( $\hat{r}$ ) polarizations. Making use of  
 3349 the fact that for an antenna located at  $R = r_a \hat{y}$  that  $\hat{\phi} = -\hat{x}$  and  $\hat{r} = \hat{y}$  we find

$$\vec{E} = \hat{\phi} E_\phi + \hat{r} E_r \quad (5.26)$$

$$E_\phi = \frac{e}{\left( 1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r \right)^3} \left[ -\frac{\frac{r_c \omega_c}{c} \sin \omega_c t_r}{\gamma^2 r_a^2} + \frac{\omega_c \left( \frac{r_c^2 \omega_c^2}{c^2} - \frac{r_c \omega_c}{c} \cos \omega_c t_r \right)}{cr_a} \right] \quad (5.27)$$

$$E_r = \frac{e \left( 1 - \frac{r_c \omega_c}{c} \sin \omega_c t_r \right)}{\gamma^2 r_a^2 \left( 1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r \right)^3}. \quad (5.28)$$

3350 For the purposes of designing a synthetic cyclotron radiation antenna we are interested  
 3351 in the dominant electric field polarization emitted by the electron. The antenna is being  
 3352 designed to mimic the cyclotron radiation produced by electrons with kinetic energies of  
 3353 approximately 18.6 keV in a 1 T magnetic field [54]. Since the relativistic beta factor for

3354 an electron with this kinetic energy is  $|\vec{\beta}| \simeq \frac{1}{4}$ , the approximations  $\gamma \simeq 1$  and  $\frac{r_c \omega_c}{c} \simeq \frac{1}{4}$  are  
 3355 justified. Inserting these expressions into the equations for the electric field components  
 3356 above simplifies the comparison of the magnitudes of the two components. Additionally,  
 3357 we compare the time-averaged magnitudes to evaluate the root mean squared electric  
 3358 field ratio. The time-averaged ratio of the radial and azimuthally polarized electric fields  
 3359 with the above simplifications is given by

$$\frac{\langle |E_r| \rangle}{\langle |E_\phi| \rangle} = \frac{8 - \sqrt{2}}{\left| 1 - \frac{r_a}{r_c} \frac{1-2\sqrt{2}}{8} \right|} \simeq \frac{r_c}{r_a} \frac{8(8 - \sqrt{2})}{2\sqrt{2} - 1} = 0.13, \quad (5.29)$$

3360 where we have made use of the fact that for these magnetic fields and kinetic energies  
 3361 the cyclotron radius is much smaller than the radius of the antenna array.

3362 From Equation 5.29 we see that the time-averaged azimuthal polarization is larger than  
 3363 the radial polarization by about a factor of 8, which makes it the dominant contribution  
 3364 to the electric fields at the position of the antenna. We must also consider the directivity  
 3365 of the receiving antenna which can have a gain that is disproportionately large for a  
 3366 specific polarization component. Because the  $E_\phi$  component is dominant, the receiving  
 3367 antenna array is designed with an azimuthal polarization, which negates the voltages  
 3368 induced in the antenna from the radially polarized fields. Therefore, we conclude that  
 3369 for the purpose of designing the SYNCA antenna it is acceptable to approximate the  
 3370 electric fields from Equation 5.23 as purely azimuthally or  $\phi$ -polarized. The simplified  
 3371 expression for the electric field received by an antenna becomes

$$\vec{E} = E_\phi \hat{\phi} = \frac{e \frac{r_c \omega_c}{c}}{4r_a r_c} \left[ \frac{\frac{r_c \omega_c}{c} - \cos \omega_c t - \frac{4r_c}{r_a} \sin \omega_c t}{(1 - \frac{r_c \omega_c}{c} \cos \omega_c t)^3} \right]_{t_r} \hat{\phi}, \quad (5.30)$$

3372 where the radius of the cyclotron orbit is called  $r_c$ , the cyclotron frequency is called  $\omega_c$ ,  
 3373 and the radial position of the receiving antenna is called  $r_a$ . Equation 5.30 has been  
 3374 evaluated in the non-relativistic limit where  $\gamma \simeq 1$ , which is justified by the fact that  
 3375  $|\vec{\beta}| \simeq \frac{c}{4}$  for an electron with an 18.6 keV kinetic energy in a 1 T magnetic field.

3376 This rather complicated expression can be simplified using Fourier analysis. Assuming  
 3377 a background magnetic field of 1 T and a kinetic energy of 18.6 keV we calculate  
 3378 numerically the electric field using Equation 5.30 and apply a discrete Fourier Transform  
 3379 to visualize the frequency spectrum (see Figure 5.11).

3380 We observe that the azimuthally polarized electric field is periodic with a base cyclotron  
 3381 frequency of 25.898 GHz corresponding to the highest power frequency component in

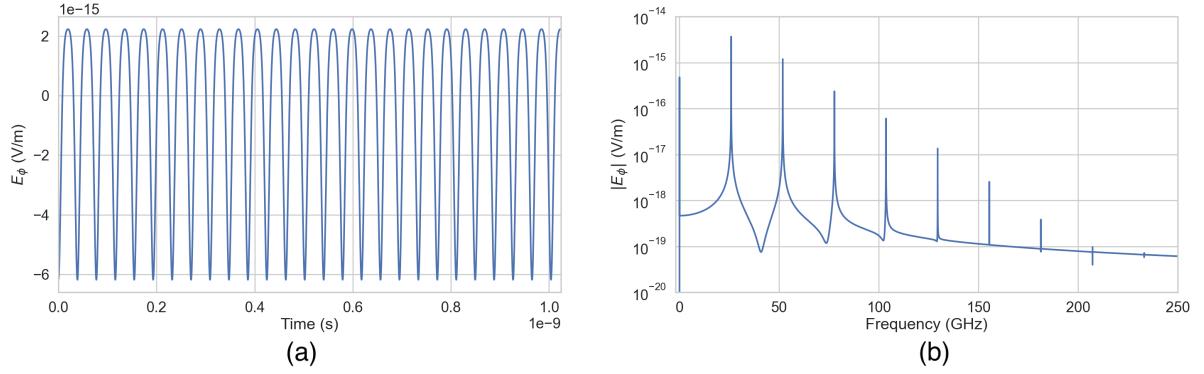


Figure 5.11: A plot of the numeric solution to Equation 5.31. The time-domain representation of the signal (a) is composed of a zero frequency term and a series of harmonics separated by the main cyclotron frequency as shown in the plot of the frequency spectrum (b). We can see that the relative amplitude of the harmonics beyond  $k = 7$  are smaller than the main carrier by a factor of about  $10^{-5}$  and are completely negligible.

3382 Figure 5.11. The frequency spectrum reveals that the signal is composed of a constant  
 3383 term with zero frequency and a series of harmonics separated by 25.898 GHz. Therefore,  
 3384 we can represent the azimuthal electric fields from the electron as a linear combination  
 3385 of pure sinusoids with frequencies given by  $\omega_k = k\omega_c$  ( $k \in 0, 1, 2, \dots$ ) and amplitudes  
 3386 extracted from the Fourier representation. Using this representation we can transform  
 3387 the equation for the azimuthally polarized electric fields in Equation 5.30 into

$$E_\phi = \frac{e^{\frac{r_c \omega_c}{c}}}{4r_a r_c} \sum_{k=0}^7 A_k e^{i\omega_k t_r}, \quad (5.31)$$

3388 where we have truncated the sum over harmonics at the 7th order for completeness. The  
 3389 amplitudes  $A_k$  are dimensionless complex numbers, which encode the relative powers of  
 3390 the harmonics as well as the starting overall phase of the cyclotron radiation. Because  
 3391 magnitude of the relative amplitudes exponentially decreases for higher harmonics, it is  
 3392 usually sufficient to consider only the terms up to  $k = 4$  where the relative amplitude  
 3393 of the harmonics has decreased from the main carrier by a factor of approximately 100.  
 3394 However, for completeness we include harmonics up to 7th order in Equation 5.31. The  
 3395 range of frequencies to which the receiving antenna array in the antenna test stand is  
 3396 sensitive is defined by the antenna's transfer function. The receptive bandwidth for  
 3397 the antennas used in the test stand is a range of frequencies with a bandwidth on the  
 3398 order of a few GHz centered around the main cyclotron carrier frequency of 25.898 GHz.  
 3399 Therefore, the higher order harmonics as well as the zero frequency term can be ignored

<sup>3400</sup> when considering only the signals that will be received by the antenna array.

<sup>3401</sup> Considering only the 1st order harmonic term from Equation 5.31, which represents  
<sup>3402</sup> the portion of the electric field that will be detected by the array, and evaluating this at  
<sup>3403</sup> the retarded time we obtain the following for the  $\phi$ -polarized electric fields

$$E_\phi \propto \cos \left( \omega_c \left( t - |\vec{R}|/c \right) - \Delta \right), \quad (5.32)$$

<sup>3404</sup> where the arbitrary phase  $\Delta$  is defined by  $A_k = |A_k|e^{i\Delta}$ . We are interested in the  
<sup>3405</sup> characteristics of the amplitude of the electric field as a function of the radial distance  
<sup>3406</sup> component ( $|\vec{R}|$ ) of the retarded time. In particular, the maximum of  $E_\phi$  occurs when  
<sup>3407</sup> the argument of the cosine function is equal  $n\pi$  where  $n \in \{0, \pm 2, \pm 4, \dots\}$ ; however, the  
<sup>3408</sup> solutions where  $n$  is negative can be discarded since they represent unphysical negative  
<sup>3409</sup> overall phases. Applying this condition to Equation 5.32 gives a condition on the radial  
<sup>3410</sup> position of the maximum of  $E_\phi$

$$\omega_c(t - |\vec{R}|/c) - \Delta = n\pi, \quad (5.33a)$$

$$|\vec{R}| = \frac{c}{\omega_c} ((\omega_c t - \Delta) - n\pi), \quad (5.33b)$$

<sup>3411</sup> which is a function of time in the frame of the moving electron ( $t$ ). Equation 5.33 can  
<sup>3412</sup> be further simplified by noticing that the azimuthal position of the electron ( $\phi_e(t)$ ) as a  
<sup>3413</sup> function of time is defined by  $\phi_e(t) = \omega_c t - \Delta$  which reduces Equation 5.33 to

$$|\vec{R}| = \frac{c}{\omega_c} (\phi_e(t) - n\pi). \quad (5.34)$$

<sup>3414</sup> Equation 5.34 represents an archimedean spiral which is formed when plotting the  
<sup>3415</sup> amplitude of  $E_\phi$  in the x-y plane. The solution where  $n = 0$  represents the leading edge  
<sup>3416</sup> of the radiation spiral which propagates outward from the electron at the speed of light.  
<sup>3417</sup> The additional solutions for  $n > 0$  represent the persistent spiral at radii inside the  
<sup>3418</sup> leading edge of the radiated fields that have not yet been detected by the receiver at the  
<sup>3419</sup> current time. In Figure 5.12a we show the expected spiral pattern for the maxima of the  
<sup>3420</sup> cyclotron radiation.

<sup>3421</sup> In particular, we note that for the circular array geometry of the test stand, depicted  
<sup>3422</sup> as the series of circles in Figure 5.12a, each antenna receives a linearly polarized wave  
<sup>3423</sup> with a phase offset that corresponds to the azimuthal angle for that antenna element.  
<sup>3424</sup> Therefore, as we show in Figure 5.12b, when the relative phase of the received signal is  
<sup>3425</sup> plotted as a function of the receiving antenna's azimuthal position the result is also an

3426 Archimedean spiral.

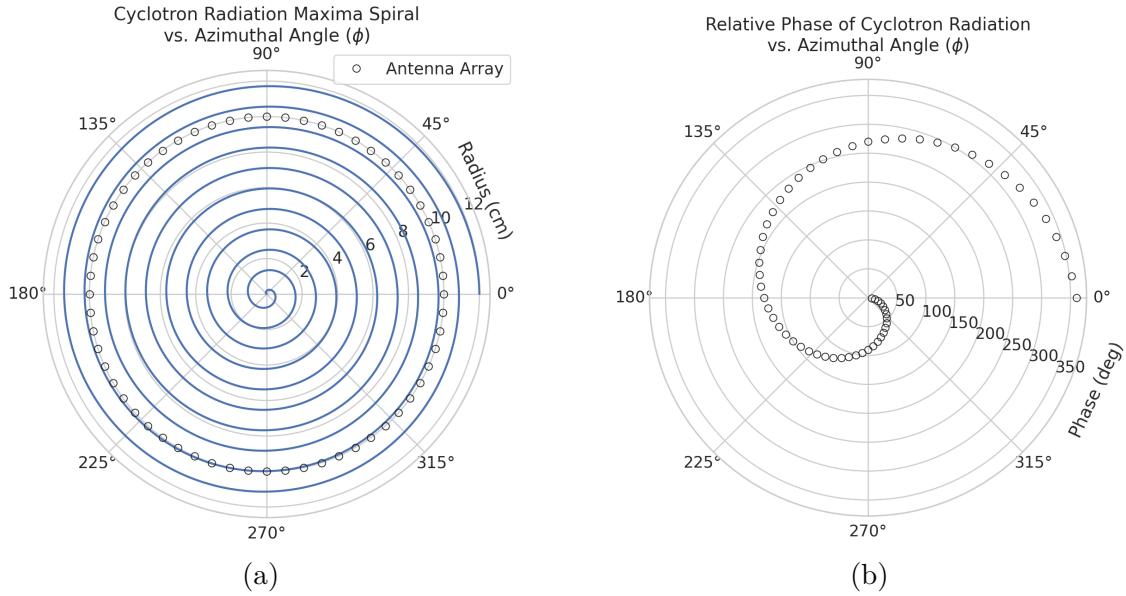


Figure 5.12: The amplitude maxima of the cyclotron radiation form an Archimedean spiral as the radiation propagates outward from the cyclotron orbit center (a). A circular antenna array located at a fixed radius from the orbit center will receive electric fields with equal magnitude in each of its channels, but the phase of the electric field incident on each array channel will be linearly out of phase from its neighbor antennas by an amount equal to the angular separation of the two channels (b).

3427 Based on these analytical calculations we can characterize the magnitude, polarization,  
3428 and phase of the signals received by the antenna array using three criteria. These criteria  
3429 are the basis of comparison for the radiation produced by the SYNCA and cyclotron  
3430 radiation emitted by electrons and will be used to evaluate the performance of antenna  
3431 designs. The criteria are:

- 3432 1. Electric fields that are  $\phi$ -polarized near  $\theta = 90^\circ$
- 3433 2. Uniform time-averaged electric field magnitudes around the circumference of a  
3434 circle centered on the antenna
- 3435 3. Electric fields whose phase is equal to the azimuthal angle at the point of measure-  
3436 ment plus a constant

3437 The Locust simulation package [55] can be used to directly simulate the EM fields  
3438 generated by electrons performing cyclotron motion to validate the analytical calculations.  
3439 Locust simulates the EM fields by first calculating the trajectory of the electrons in

3440 the magnetic trap using the Kassiopeia software package [56]. The trajectory can then  
 3441 be used to solve for the EM fields using the Liénard-Wiechert equations directly with  
 3442 no approximations. The resulting electric field solutions drive a receiving antenna by  
 3443 convolving the time-domain fields with the finite-impulse response filter of the antenna  
 3444 or they can be examined directly to study the field characteristics that the SYNCA must  
 3445 reproduce. In the next section we compare the radiation field patterns for electrons  
 3446 simulated with Locust to patterns from a SYNCA antenna design.

### 3447 5.3.3 SYNCA Simulations and Design

3448 One potential SYNCA design is the crossed-dipole antenna [57]. A crossed-dipole antenna  
 3449 consists of two dipole antennas, one of which is rotated 90° with respect to the other,  
 3450 which are fed with signals that are out of phase from the opposite dipole by 90° (see  
 Figure 5.13). This arrangement causes the signals fed to each arm of the dipole to be

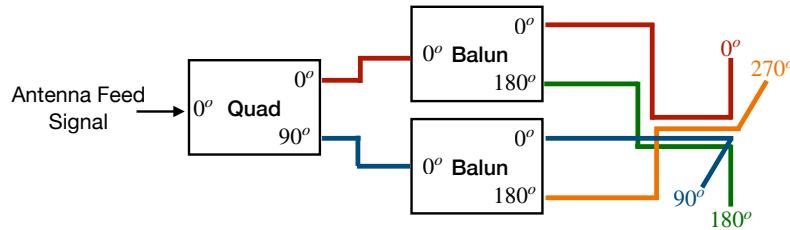


Figure 5.13: An idealized crossed-dipole antenna consists of two electric dipole antennas oriented perpendicular to each other and is fed with four signals with a quadrature phase relationship. An example antenna feed circuit is shown which is composed of a chained combination of a quadrature hybrid-coupler (Quad) and two baluns.

3451  
 3452 out of phase from each of the neighboring arms by 90°, which mirrors the spatial phase  
 3453 relationship of cyclotron radiation fields.

3454 A potential drawback of this design is that standard crossed-dipole antennas do not  
 3455 radiate uniform electric fields near the  $\theta = \pi/2$  plane. Typical crossed-dipole antennas  
 3456 use dipole arm lengths equal to  $\lambda/4$  or larger [57], where  $\lambda$  is the wavelength at the  
 3457 desired operating frequency. Such large arm lengths cause the electric field magnitude  
 3458 to vary significantly around the circumference of the antenna. However, making the  
 3459 antenna electrically small by shrinking the arm length can improve the antenna pattern  
 3460 uniformity.

3461 In general, the criterion for an electrically small antenna is that the largest dimension  
3462 of the antenna ( $D$ ) obey  $D \lesssim \lambda/10$  [13]. In our application, we are attempting to mimic  
3463 the cyclotron radiation emitted by electrons produced from tritium  $\beta$ -decay with energies  
3464 near the spectrum endpoint. For a background magnetic field of 1 T, the corresponding  
3465 cyclotron frequency of tritium endpoint electrons is approximately 26 GHz. Therefore, the  
3466 electrically small condition would require that the largest dimension of the crossed-dipole  
3467 antenna be smaller than 1.2 mm.

3468 A crossed-dipole antenna with an overall size of 1.2 mm is challenging to fabricate due  
3469 to the small dimensions of the dipole arms that, in practice, are fragile and unsuitable  
3470 for use as a calibration probe. To mitigate some of the challenges with the fabrication  
3471 of such a small antenna, a variant crossed-dipole antenna design using printed circuit  
3472 board (PCB) technology (see Figure 5.14) was developed in partnership with an antenna  
prototyping company, Field Theory Consulting <sup>1</sup>.

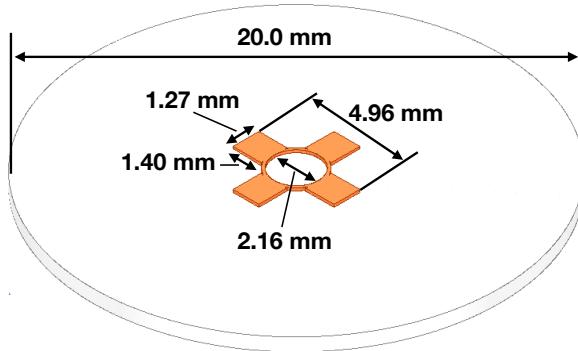


Figure 5.14: A model of the PCB crossed-dipole antenna with dimensions. The design has an inside diameter of 2.16 mm for the central circular trace, which is 0.13 mm wide. The dipole arms each have a width of 1.27 mm and protrude beyond the circular trace by 1.40 mm, which gives an overall width of 4.96 mm for the length of the antenna PCB trace from end-to-end. The overall size of the antenna is 20.0 mm the majority of which is the PCB dielectric material. This design was observed in simulation to maintain the field characteristics of the idealized crossed-dipole while being simpler to fabricate due to the increased size of the antenna.

3473  
3474 The PCB crossed-dipole design uses four rectangular pads to represent the dipole arms,  
3475 which are connected by a thin circular trace. The circular trace both adds mechanical  
3476 stability to the antenna and improves the azimuthal uniformity of the electric fields  
3477 compared to a more standard crossed-dipole geometry. Furthermore, the circular trace  
3478 allows for a greater separation between dipole arms than standard crossed-dipoles, which

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<sup>1</sup><https://fieldtheoryinc.com/>

3479 is required to accommodate the coaxial connections to each pad. The pads each contain  
 3480 a through-hole solder joint to connect coaxial transmission lines using hand soldering.  
 3481 The antenna PCB has no ground plane on the bottom layer as this was observed in  
 3482 simulation to significantly distort the radiation pattern in the plane of the PCB. The  
 3483 only ground planes present in the model are the outer conductors of the four coaxial  
 3484 transmission lines which feed the antenna. These are left unterminated on the bottom of  
 3485 the PCB dielectric material.

3486 The antenna design development utilized a combination of Locust electron simula-  
 3487 tions and antenna simulations using ANSYS HFSS [14], a commercial finite-element  
 3488 electromagnetic simulation software. Two antenna designs were simulated: an idealized  
 3489 electrically small crossed-dipole antenna with an arm length of 0.40 mm and an arm  
 3490 separation of 0.05 mm, as well as a PCB crossed-dipole antenna with the dimensions  
 3491 shown in Figure 5.14. Plotting the magnitude of the electric fields generated by the  
 3492 antennas across a 10 cm square located in the same plane as the respective antennas  
 3493 reveals the expected cyclotron spiral pattern (see Figure 5.15) which closely matches  
 3494 the prediction for simulated electrons. The spiral pattern demonstrates that the electric  
 3495 fields have the appropriate phases to mimic cyclotron radiation, which fulfills SYNCA  
 criterion 3 identified in Section 5.3.2.

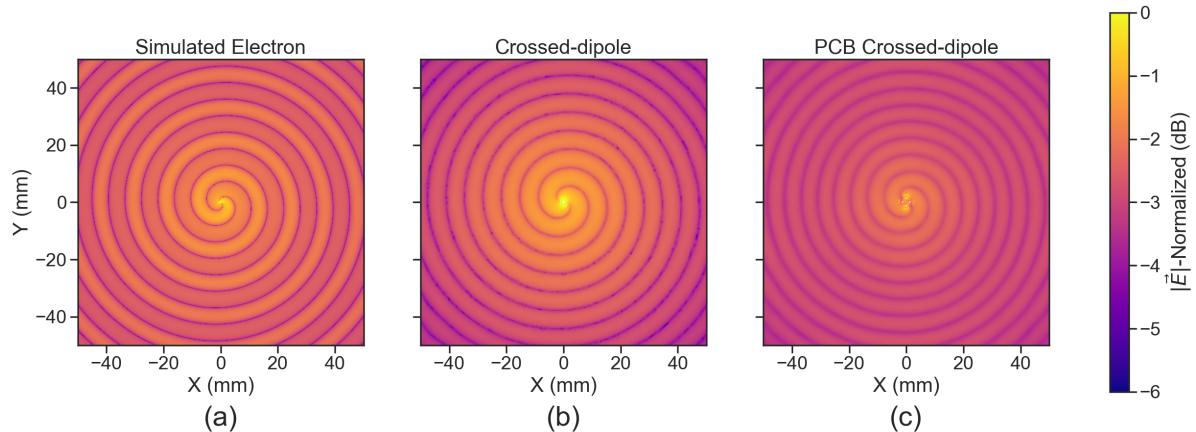


Figure 5.15: A comparison of the electric field magnitudes, normalized by the maximum value of the electric field in each simulation, plotted on a 10 cm square to visualize the Archimedean spirals formed by the electron (a), the crossed-dipole antenna (b), and a PCB crossed-dipole antenna (c). The matching patterns indicate that the electric fields have similar phase characteristics. These images were generated using Locust simulations for the electron and ANSYS HFSS for both antennas.

3496

3497 As we can see from Figure 5.16, the crossed-dipole antenna, which uses an idealized

3498 geometry, exhibits good agreement with simulation. The antenna has a maximum  
 3499 deviation from a simulated electron of approximately 0.5 dB in the total electric field, 1  
 3500 dB for the  $\phi$ -polarized electric field and 1 dB for the  $\theta$ -polarized electric field.

3501 In comparison, the pattern of the PCB crossed-dipole antenna, because the simulation  
 3502 incorporates the geometry of the coax transmission lines, exhibits some distortion from  
 3503 the idealized cross-dipole simulations. The vertically oriented ground planes of the coax  
 3504 lines introduce more  $\theta$ -polarized electric fields than are observed for simulated electrons  
 3505 near  $\theta = 90^\circ$ . The significant  $\theta$ -polarized field minimum is still present but shifted  
 to approximately  $\theta = 65^\circ$ . The  $\theta$ -polarized field deviations of the PCB crossed-dipole

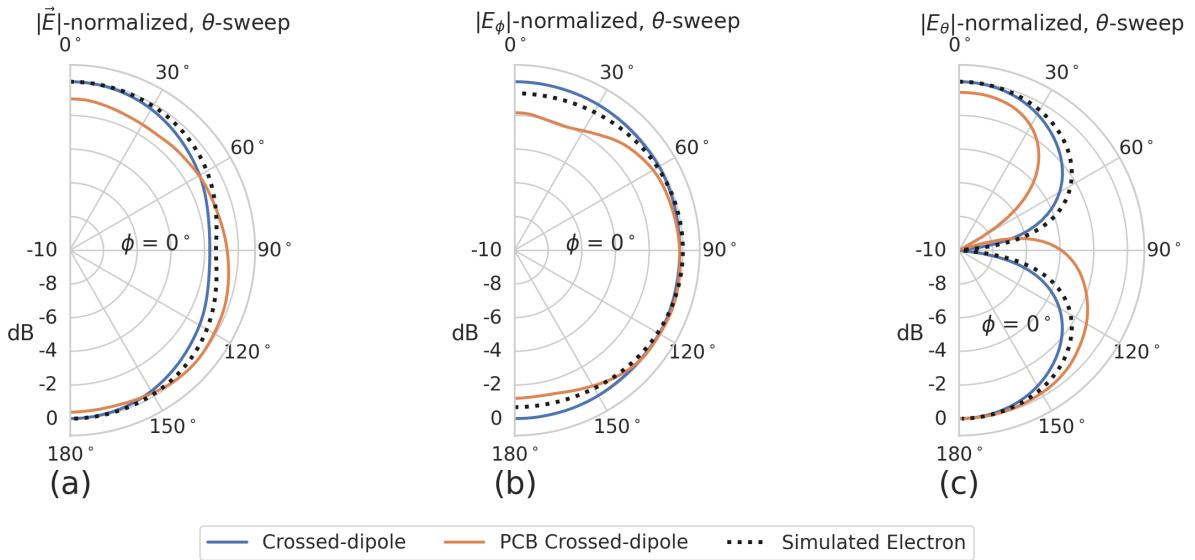


Figure 5.16: A comparison of the normalized electric field magnitudes for the ideal crossed-dipole, PCB crossed-dipole, and a simulated electron as a function of the polar angle ( $\theta$ ). (a) Shows the total electric field, (b) shows the  $\phi$ -polarized electric field component, and (c) shows the  $\theta$ -polarized electric field component. These images were generated using Locust simulations for the electron and ANSYS HFSS for both antennas.

3506  
 3507 antenna should not greatly impact the performance of the antenna because the receiving  
 3508 antenna array is primarily  $\phi$ -polarized. Therefore deviations in the  $\theta$ -polarized fields  
 3509 will be suppressed due to the polarization mismatch. More importantly, the  $\phi$ -polarized  
 3510 electric field pattern generated by the PCB crossed-dipole closely matches simulated  
 3511 electrons across the polar angle range of  $50^\circ < \theta < 150^\circ$ . In this region the PCB crossed-  
 3512 dipole differs by less than 0.5 dB from simulated electrons. This range greatly exceeds  
 3513 the beamwidth of the receiving antenna array which is designed to be most sensitive  
 3514 to fields produced near  $\theta = 90^\circ$ . Therefore, we conclude that the PCB crossed-dipole

3515 antenna generates a  $\phi$ -polarized radiation pattern that fulfills SYNCA criterion 1 from  
3516 Section 5.3.2.

3517 The final SYNCA criterion is related to the uniformity of the electric fields when  
3518 measured azimuthally around the antenna. As we saw for real electrons in Section 5.3.2  
3519 it is expected that the magnitude of the electric field be completely uniform as a function  
3520 of the azimuthal angle due to the symmetry of the cyclotron orbit. In Figure 5.17 we plot  
3521 the total electric field as a function of azimuthal angle for an electron, the crossed-dipole  
antenna, and the PCB crossed-dipole antenna. The crossed-dipole antenna exhibits

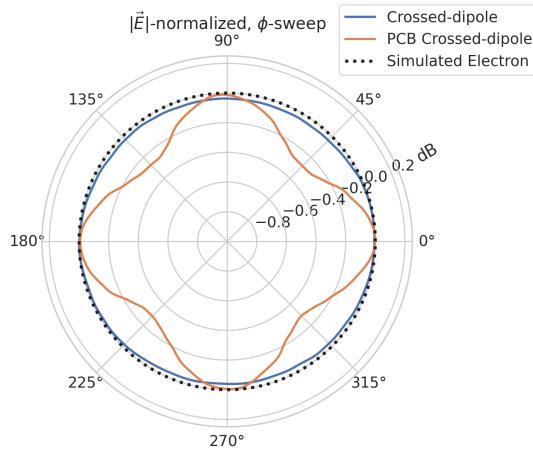


Figure 5.17: A comparison of the normalized electric field magnitudes for the crossed-dipole, PCB crossed-dipole, and a simulated electron as a function of the azimuthal angle ( $\phi$ ) evaluated at  $\theta = 90^\circ$ . This image was generated using Locust simulations for the electron and ANSYS HFSS for both antennas.

3522  
3523 perfect uniformity around the azimuthal angle, whereas the PCB crossed-dipole has a  
3524 small periodic deviation with a maximum difference of 0.3 dB caused by the coaxial  
3525 transmission lines below the PCB. Such a small deviation from uniformity is acceptable  
3526 since it is smaller than the expected variation in uniformity caused by imperfections in  
3527 the antenna fabrication process, which modifies the antenna shape in an uncontrolled  
3528 manner by introducing solder blobs with a typical size of a few tenths of a millimeter on  
3529 the dipole arms (see Figure 5.18). Additionally, the SYNCA will be separately calibrated  
3530 to account for azimuthal differences in the electric field magnitude. Therefore we see  
3531 from the simulated performance of the PCB crossed-dipole antenna that this antenna  
3532 design meets all three of the SYNCA criteria.

### 5.3.4 Characterization of the SYNCA

Two SYNCAs were manufactured using the PCB crossed-dipole design (see Figure 5.18). The antenna PCB (Matrix Circuit Board Materials, MEGTRON 6) is connected to four 2.92 mm coaxial connectors (Fairview Microwave, SC5843) using semi-rigid coax (Fairview Microwave, FMBC002), which also physically support the antenna PCB. The antenna PCB consists only of two layers which correspond to the copper antenna trace and the PCB dielectric. Each coax line is connected to the associated dipole arm using through-hole soldering and phase matched to ensure that the electrical length of each of the transmission lines is identical at the operating frequency. The antenna PCB is further reinforced using custom cut polystyrene foam blocks, which have an electrical permittivity nearly identical to air. A custom 3D printed mount is included at the base of the antenna to support the coax connectors and to provide a sturdy mounting base.

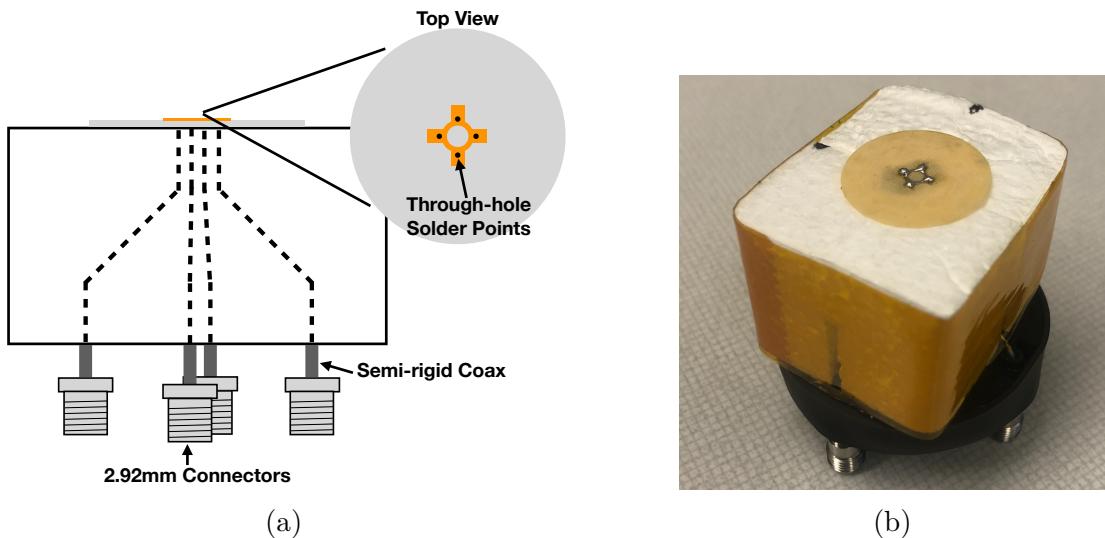


Figure 5.18: (a) A cartoon schematic which highlights the routing of the semi-rigid coax transmission lines. (b) A photograph of a SYNCA constructed using the modified crossed-dipole PCB antenna design. Visible in the photograph of the SYNCA are four blobs of solder which are an artifact of the SYNCA's hand-soldered construction. These solder blobs are the most significant deviation from the SYNCA design shown in Figure 5.14 and are responsible for a significant fraction of the irregularities seen in the antenna pattern.

Characterization measurements were performed using a Vector Network Analyzer (VNA) to measure the electric field magnitude and phase radiated by the SYNCA to verify the radiation pattern (see Figure 5.19). The VNA is connected to the SYNCA

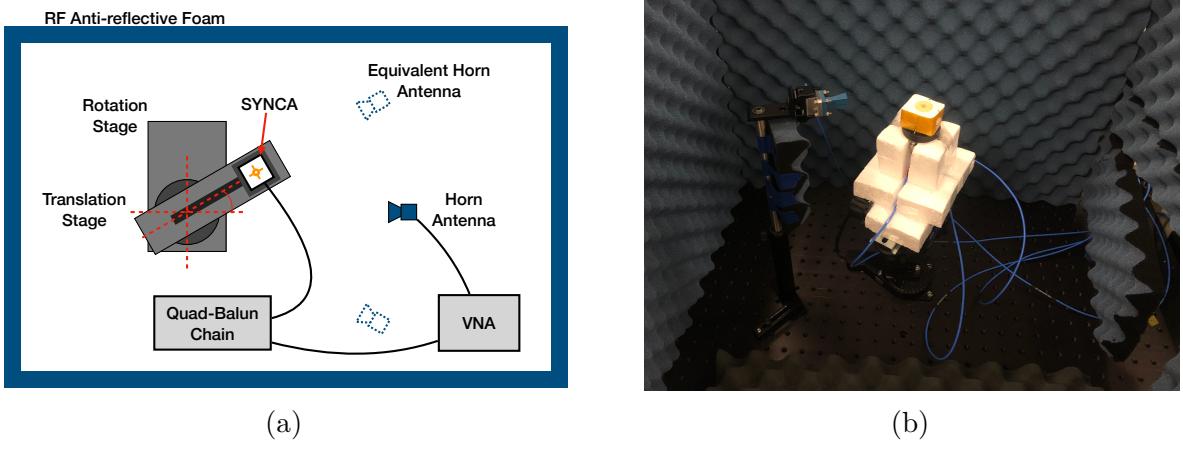


Figure 5.19: A schematic of the VNA characterization measurements (a). This setup allows for antenna gain and phase measurements across a full  $360^\circ$  of azimuthal angles using a motorized rotation stage and control of the radial position of the SYNCA using a translation stage. A photo of the setup in the lab is shown in (b).

3548 at one port through a hybrid-coupler whose outputs are connected to two baluns to  
 3549 generate the signals with the appropriate phases to feed the SYNCA (see Figure 5.13).  
 3550 The other port of the VNA is connected to a single reference horn antenna that serves  
 3551 as a field probe. To position the SYNCA, a combination of translation and rotation  
 3552 stages are used to characterize the antenna's fields across the entire radiation pattern  
 3553 circumference. This measurement scheme is equivalent to measuring the fields generated  
 3554 by the SYNCA using a full circular array of probe antennas.

3555 The antenna measurement space is surrounded by RF anti-reflective foam to isolate  
 3556 the measurements from the lab environment (see Figure 5.19b) and remaining reflections  
 3557 are removed using the VNA's time-gating feature. The SYNCA is affixed to the stages  
 3558 by a custom RF transparent mount made of polystyrene foam. The coaxial cables deliver  
 3559 the antenna feed signals generated by the VNA to the SYNCA while still allowing  
 3560 unrestricted rotation. The horn antenna probe is nominally positioned in the plane  
 3561 formed by the antenna PCB ( $\theta = 90^\circ$  or  $z = 0$  mm) at a distance of 10 cm from the  
 3562 SYNCA, to match the expected position of the antenna array relative to the SYNCA in  
 3563 the antenna array test stand. The horn antenna can be manually raised or lowered to  
 3564 different relative vertical positions to characterize the radiation pattern at different polar  
 3565 angles.

3566 Several  $360^\circ$  scans were performed with probe vertical offsets of -10.0 mm, -5.0 mm,  
 3567 0.0 mm, 5.0 mm, and 10.0 mm relative to the antenna PCB plane. These probe offsets

3568 cover a 2 cm wide vertical region centered on the SYNCA PCB, approximately equal to  
 3569  $\pm 6$  degrees of polar angle. The measurements show that the SYNCA is generating fields  
 3570 with nearly isotropic magnitude across the probed region. The standard deviation of the  
 3571 electric field magnitude measured around the antenna circumference is approximately  
 3572 2.9 dB for a typical rotational scan. The presence of a significant pattern null is noted  
 3573 near 45° (see Figure 5.20), which we attribute to small imperfections in the antenna  
 3574 PCB that could be introduced from the hand soldered terminations connecting the coax  
 3575 cables to the antenna. There is no significant difference in the radiation pattern when  
 3576 measured across the 2 cm vertical range. The measured relative phases closely follow  
 3577 the expectation for an electron, being linear with the measurement rotation angle and  
 3578 forming the expected spiral pattern. Other than the small phase imperfections there is  
 3579 a slight sinusoidal bias to the phase data, which we determined is the result of a small  
 3580 ( $\lesssim 1$  mm) offset of the antenna's phase center from the rotation axis of the automated  
 3581 stages.

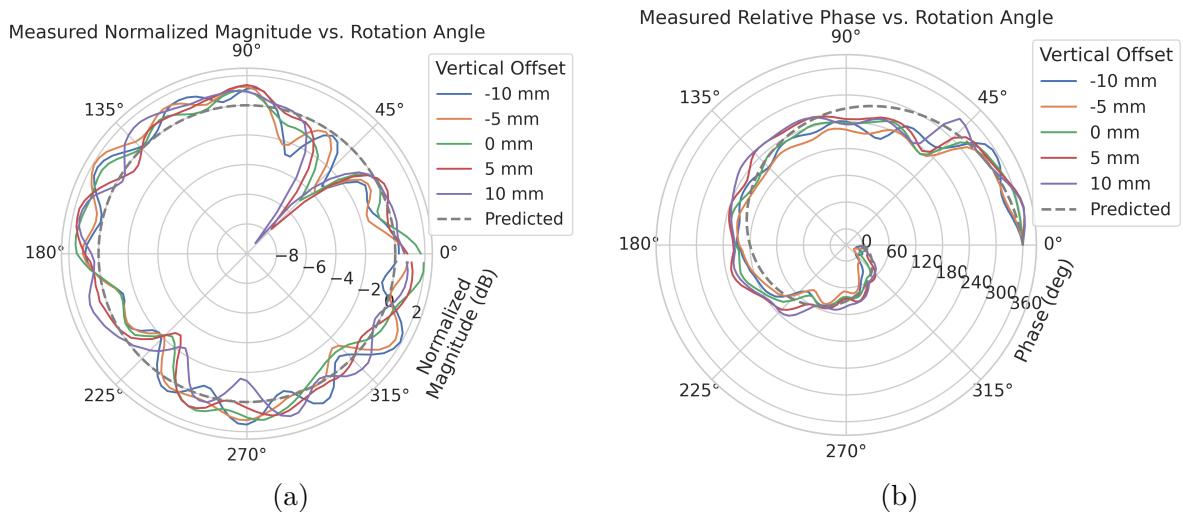


Figure 5.20: Linear interpolations of the measured electric field magnitude (a) and phase (b). The data was acquired using a VNA at 120 points spaced by 3 degrees from 0 to 357 degrees of azimuthal angle. The different color lines indicate the vertical offset of the horn antenna relative to the SYNCA PCB and the dashed line shows the expected shape from electron simulations. No significant difference in the antenna pattern is observed for the measured vertical offsets.

3582 The characterization measurements confirm the simulated performance of the SYNCA.  
 3583 As expected the fields generated by the antenna are nearly isotropic in magnitude,  $\phi$ -  
 3584 polarized, and are linearly out of phase around the circumference of the antenna as

3585 predicted for cyclotron radiation in Section 5.3.2. Small imperfections in the magnitude  
 3586 and phase of the antenna are expected, particularly at the antenna's high operating  
 3587 frequency of 26 GHz where small geometric changes can have significant impacts on  
 3588 electrical properties. However, calibration through careful characterization measurements  
 3589 can be used to remove the majority of these pattern imperfections, including the relatively  
 3590 large pattern null near 45°, which will allow for the usage of the SYNCA as a test source  
 3591 for free-space CRES experiments utilizing antenna arrays. In the next section we use the  
 3592 VNA measurements obtained here as a calibration for signal reconstruction using digital  
 3593 beamforming.

### 3594 **5.3.5 Beamforming Measurements with the SYNCA**

3595 Digital beamforming is a standard technique for signal reconstruction using a phased  
 3596 array [58]. The SYNCA, since it exhibits the same cyclotron phases as a trapped electron,  
 3597 can be used to perform simulated CRES digital beamforming reconstruction experiments  
 3598 on the bench-top without the need for the magnet, cryogenics, and vacuum systems  
 3599 required by a full CRES experiment. The fields received by the individual elements  
 3600 of the antenna array will have phases dependent on the spatial position of the source  
 3601 relative to the antennas. Therefore, a simple summation of the received signals will fail  
 3602 to reconstruct the signal due to destructive interference between the individual channels  
 3603 in the array. However, applying a phase shift associated with the source's spatial position

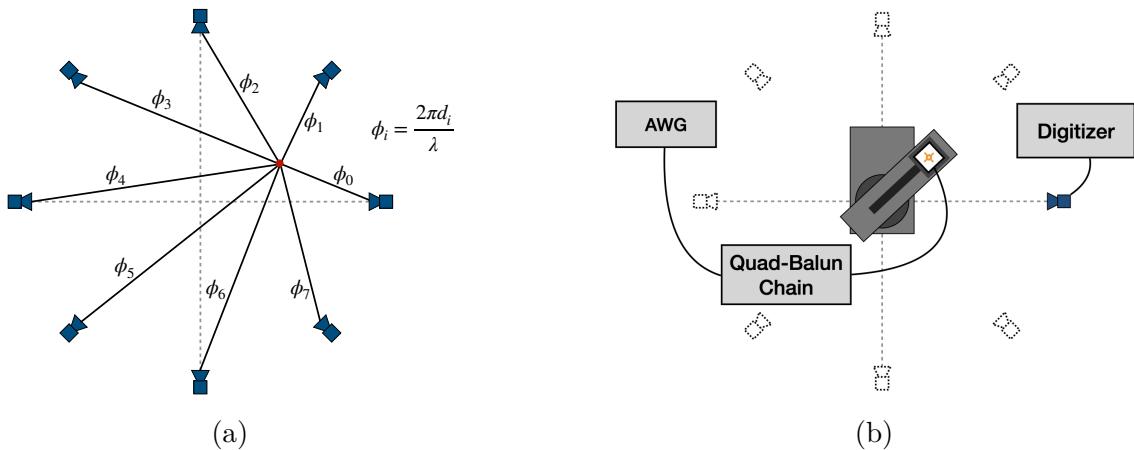


Figure 5.21: (a) A depiction of the relative phase differences for signals received by a circular antenna array from an isotropic source. The phases correspond to a unique spatial position. (b) A schematic of the setup used to perform digital beamforming.

3604 removes phase differences and results in a constructive summation of the channel signals  
 3605 (see Figure 5.21). We can summarize the digital beamforming operation succinctly using  
 3606 the following equation

$$y[t_n] = \sum_{m=0}^{N-1} x_m[t_n] A_m e^{i\phi_m}, \quad (5.35)$$

3607 where  $y[t_n]$  represents the summed array signal at time  $t_n$ ,  $x_m[t_n]$  is the signal received  
 3608 by channel  $m$  at time  $t_n$ ,  $\phi_m$  is the phase shift applied to the signal received at channel  
 3609  $m$ , and  $A_m$  is an amplitude weighting factor that accounts for the different signal power  
 3610 received by individual channels. By changing the digital beamforming phases, the point  
 3611 of constructive interference can be scanned across the sensitive region of the array to  
 3612 search for the location of a radiating source, which is identified as the point of maximum  
 3613 summed signal power above a specified threshold. The digital beamforming phases consist  
 3614 of two components,

$$\phi_m = 2\pi d_m / \lambda + \theta_m, \quad (5.36)$$

3615 where  $d_m$  is the distance from the  $m$ -th array element to the source, and  $\theta_m$  is the  
 3616 relative angle between the source position and the  $m$ -th antenna. The first component is  
 3617 the standard digital beamforming phase that corresponds to the spatial position of the  
 3618 source, and the second component is the cyclotron phase that corresponds to the relative  
 3619 azimuthal phase offset.

3620 With a small modification to the hardware used to characterize the SYNCA (see  
 3621 Figure 5.19), we can perform a digital beamforming reconstruction of a synthetic CRES  
 3622 event. By replacing the VNA with an arbitrary waveform generator (AWG), the SYNCA  
 3623 can be used to generate cyclotron radiation with an arbitrary signal structure, which  
 3624 can then be detected by digitizing the signals received by the horn antenna. Rotational  
 3625 symmetry allows us to use the rotational stage of the positioning system to rotate the  
 3626 SYNCA to recreate the signals that would have been received by a complete circular  
 3627 array of antennas.

3628 Using this setup, signals from a 60 channel circular array of equally spaced horn  
 3629 antennas were generated with the SYNCA positioned 10 mm off the central array axis,  
 3630 reconstructed using digital beamforming, and compared to Locust simulation (see Figure  
 3631 5.22). When the cyclotron spiral phases are not used, which is equivalent to setting  $\theta_m$   
 3632 in Equation 5.36 to zero, the SYNCA's position is reconstructed as a relatively faint ring  
 3633 as predicted by simulation. However, when the appropriate cyclotron phases are used  
 3634 during the beamforming procedure, both the simulated electron and the SYNCA appear

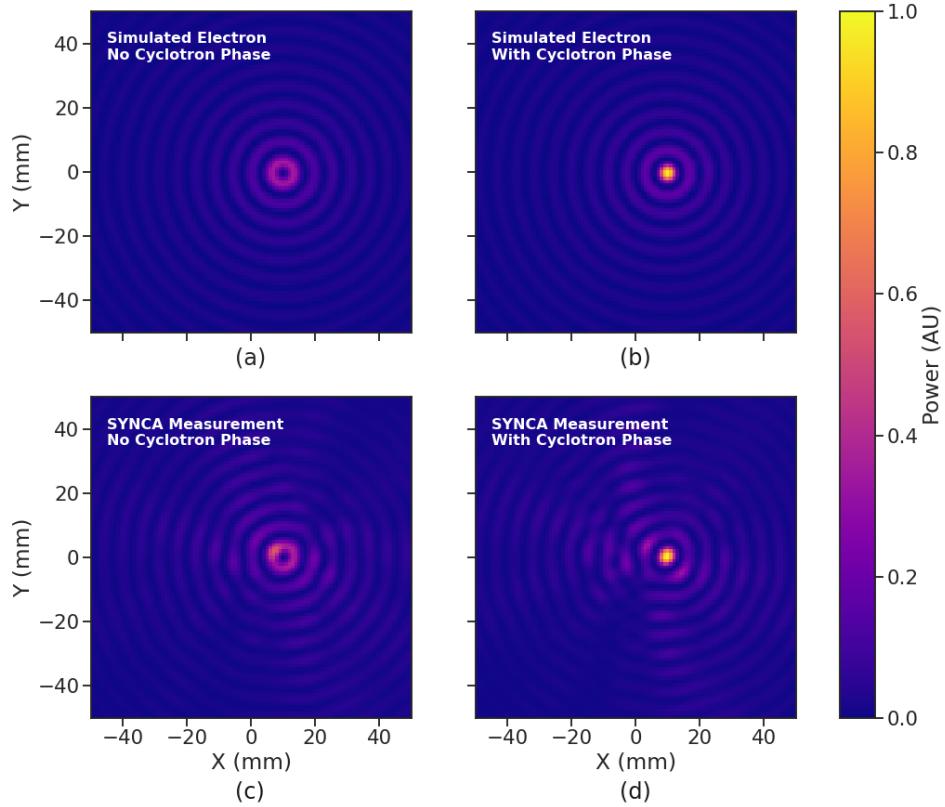


Figure 5.22: Digital beamforming maps generated using a simulated 60 channel array and electron simulated using the Locust package. (a) and (b) show the beamforming maps for simulated electrons without the cyclotron spiral phases and with the cyclotron spiral phases respectively. (c) and (d) show the beamforming maps produced from SYNCA measurements. We observe good agreement between simulated electrons and the SYNCA measurements.

as a single peak of high relative power corresponding to the source position. Therefore, we observe good agreement between the simulated and SYNCA reconstructions. While it may seem that for the case with no cyclotron phase corrections the ring reconstructs the position of the electron as effectively as beamforming with the cyclotron phase corrections, it is important to note that the simulations and measurements were generated without a realistic level of thermal noise. The larger maxima region and lower signal power, which occurs without the cyclotron phase corrections, significantly reduce the probability of detecting an electron in a realistic noise background.

To bound the beamforming capabilities of the synthetic array of horn antennas, we performed a series of beamforming reconstructions where the SYNCA was progressively moved off the central axis of the array (see Figure 5.23). To extract an estimate of the

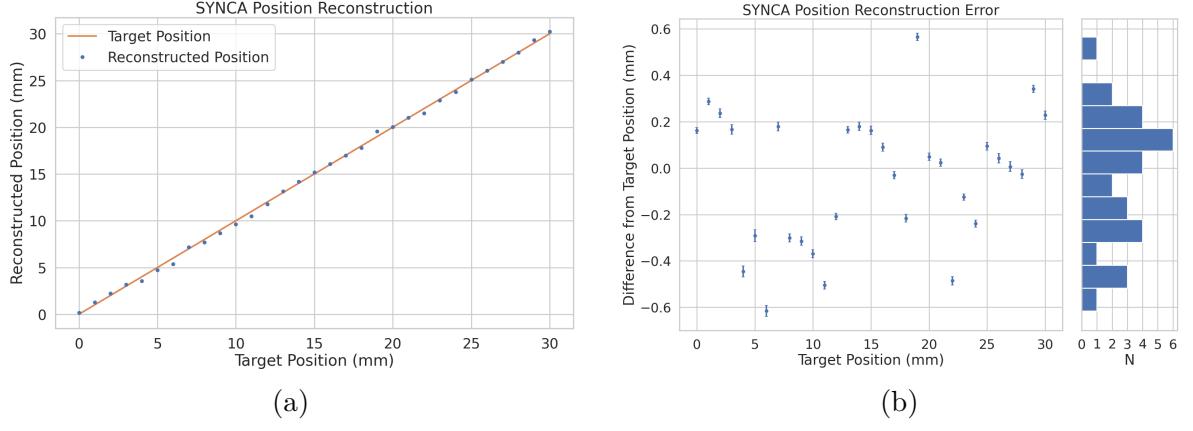


Figure 5.23: A plot of the SYNCA’s reconstructed position using the synthesized horn-antenna array and digital beamforming. (a) Shows the reconstructed position of the SYNCA compared with the target position indicated by the positioning system readout. (b) Shows the reconstruction error, which is the difference between the target and reconstructed positions. The error bars in (b) are the uncertainty in the mean position of the 2D Gaussian used to fit the digital beamforming reconstruction peak obtained from the fit covariance matrix. The mean fit position uncertainty of 0.02 mm is an order of magnitude smaller than the typical reconstruction error of 0.3 mm obtained by calculating the standard deviation of the difference between the reconstructed and target position.

position of the SYNCA using the digital beamforming image we apply a 2-dimensional (2D) Gaussian fit to the image data and extract the estimated centroid value. We find that the synthetic horn antenna array reconstructs the position of the SYNCA with a  $1\sigma$ -error of 0.3 mm with no apparent trend across the 30 mm measurement range. This reconstruction error is an order of magnitude larger than mean fit position uncertainty of 0.02 mm indicating that systematic effects related to the SYNCA positioning system could be contributing additional uncertainty to the measurements. Note that the current mean reconstruction error of 0.3 mm is a factor of 20 smaller than the full width at half maximum of the digital beamforming peak (6 mm), which could be interpreted as a naive estimate of the position reconstruction performance of this technique. Because these measurements are intended as a proof-of-principle demonstration, we do not investigate potential sources of systematic errors further; however, we expect that a similar and more thorough investigation will be performed using the Project 8 antenna array test stand, where typical reconstruction errors can be used to estimate the energy resolution limits of antenna array designs.

### **3661 5.3.6 Conclusions**

**3662** In this paper we have introduced the SYNCA, which is a novel antenna design that  
**3663** emits radiation that mimics the unique properties of the cyclotron radiation generated by  
**3664** charged particles moving in a magnetic field. The characterization measurements of the  
**3665** SYNCA validated the simulated performance of the PCB crossed-dipole antenna design.  
**3666** Additionally, the SYNCA was used to estimate the position reconstruction capabilities  
**3667** of a synthesized array of horn antennas and experimentally reproduced the simulated  
**3668** digital beamforming reconstruction of electrons.

**3669** While the SYNCA performs well, there exist discrepancies in the phase and magnitude  
**3670** of the radiation pattern compared to the simulated SYNCA design that are related to  
**3671** the small geometric differences in the soldered connections. Future design iterations that  
**3672** replace the soldered connections with a fully surface mount design could improve the  
**3673** radiation pattern at the cost of some complexity and expense. Furthermore, improving  
**3674** the design of the antenna PCB and mounting system would allow the antenna to be  
**3675** inserted into a cryogenic and vacuum environment where in-situ antenna measurement  
**3676** calibrations could be performed.

**3677** The discrepancies in the radiation pattern and phases exhibited by the as-built  
**3678** SYNCA should not greatly impact its performance as a calibration probe. Both magni-  
**3679** tude and phase variations can be accounted by applying the SYNCA characterization  
**3680** measurements as a calibration to the data collected by the antenna array test stand. The  
**3681** separate calibration of the SYNCA radiation does not impact the primary goals for the  
**3682** antenna array test stand which are array calibration and signal reconstruction algorithm  
**3683** performance characterization, because it can be performed with standard reference horn  
**3684** antennas with well understood characteristics.

**3685** The SYNCA antenna technology advances the CRES technique by providing a  
**3686** mechanism to characterize free-space antenna arrays for CRES measurements without  
**3687** the need for a magnet and cryogenics system, which would be required for calibration  
**3688** using electron sources. Both the Project 8 collaboration as well as future collaborations  
**3689** which are developing antenna array based CRES experiments can make use of SYNCA  
**3690** antennas as an important component of their calibration and commissioning phases.

3691 **5.4 FSCD Antenna Array Measurements with the SYNCA**

3692 **5.4.1 Introduction**

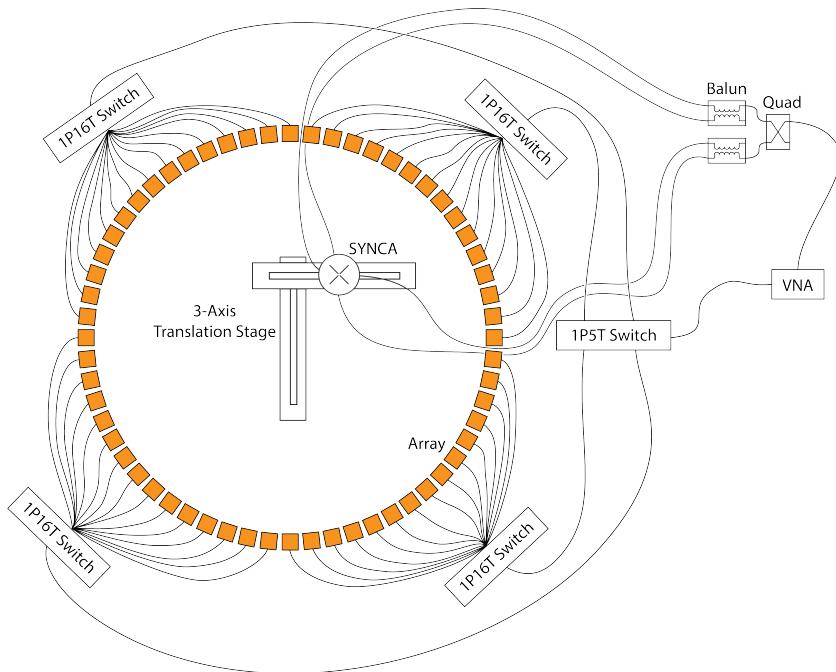


Figure 5.24: A diagram of the array measurement system used to test the prototype FSCD antenna array. A VNA is used as the primary measurement tool, which is connected to the array through a series of switches. The other port of the VNA connects to the SYNCA through the quad-balun chain used to provide the SYNCA feed signals. During measurements the SYNCA is positioned inside the center of the antenna array and translated to different radial and axial positions using a 3-axis manual translation stage setup.

3693 Using the SYNCA we can perform full-array measurements of prototype versions  
3694 of the FSCD antenna array to test its performance with a realistic cyclotron radiation  
3695 source (see Figure 5.24). The goal is to check how the measured power received by  
3696 the array compares to FSCD simulations as a function of the radial and axial position  
3697 of the SYNCA. These measurements are intended to validate the antenna research  
3698 and development by Project 8, which has been driven primarily by simulations with  
3699 Locust [10] and CREsana (see Section 4.2.3), and identify any discrepancies with these  
3700 simulations tools. This knowledge will provide confidence in the simulations necessary  
3701 for the analysis of the sensitivity of larger antenna array based CRES experiment designs  
3702 to the neutrino mass.

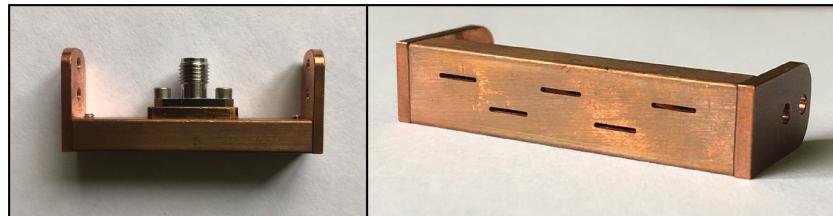
3703 As shown in Section 5.3, the SYNCA does have some radiation pattern imperfections  
3704 that complicate the comparison between measurement and simulation data. One way to  
3705 disentangle some of the effects of these imperfections is to perform an additional set of  
3706 measurements using a synthetic antenna array setup along with the SYNCA antenna.  
3707 Since the synthetic array setup uses only a single array antenna, the data should be  
3708 free of errors associated with individual antenna differences and multi-path interference,  
3709 which are two error sources being tested with the full-array setup. By comparing the  
3710 synthetic array data to the FSCD array data and to simulation data one can evaluate the  
3711 significance of these effects relative to the errors introduced by SYNCA imperfections.

### 3712 **5.4.2 Measurement Setups**

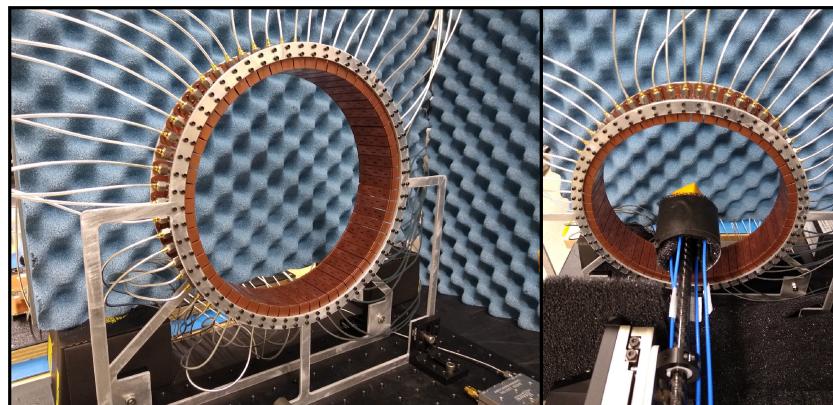
#### 3713 **5.4.2.1 FSCD Array Setup**

3714 The antenna design that composes the array is the 5-slot waveguide antenna developed  
3715 for the FSCD experiment (see Figure 5.25a). The antenna is 5 cm long and is constructed  
3716 out of WR-34 waveguide with a 2.92 mm coax connector located at the center of the  
3717 antenna. Copper flanges located on both ends of the antenna are used to mount the  
3718 antenna in the array support structure. The antennas are supported by two circular steel  
3719 brackets that can be bolted to both ends of the waveguide to construct the circular array  
3720 (see Figure 5.25b). The antenna array consists of sixty identical waveguide antennas  
3721 with a radius of 10 cm. The array is mounted perpendicular to an optical breadboard  
3722 surface using a pair of the steel brackets, which provide sufficient space for the coaxial  
3723 cable connections and allows for easy positioning of the SYNCA antenna. The SYNCA is  
3724 mounted on the end of a carbon fiber rod attached to a set of manual translation stages,  
3725 which are used to move the SYNCA antenna to different positions inside the array (see  
3726 Figure 5.25c). The stages allow for independent motion in three different axes and can  
3727 position the SYNCA at radial distances up to 5 cm from the center.

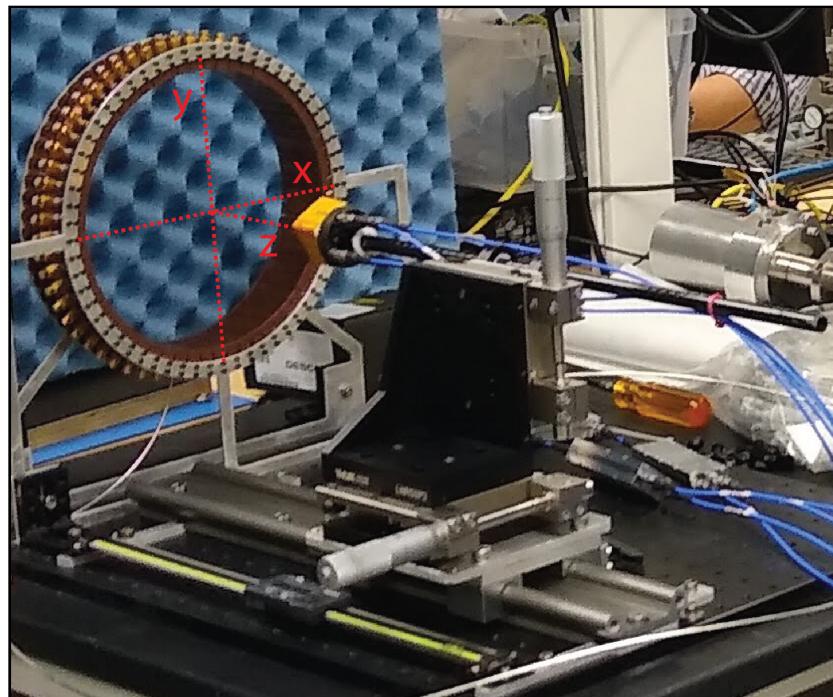
3728 Data acquisition is accomplished using a two-port VNA in combination with a series  
3729 of microwave switches that allow the VNA to connect to each channel in the array . The  
3730 first port of the VNA is connected to the quad-balun chain used to feed the SYNCA (see  
3731 Section 5.3), and the second port of the VNA connects to a 1P5T microwave switch. The  
3732 1P5T switch is connected to four separate 1P16T switch boards that connect directly  
3733 to the array. The data acquisition is controlled by a python script running on a lab  
3734 computer, which is connected to the VNA and an Arduino board programmed to control  
3735 the microwave switches. The script uses the switches to iteratively connect each of the



(a)



(b)



(c)

Figure 5.25: Photos of the prototype FSCD antenna (a), the FSCD array and SYNCA (b), and the translation stages and coordinate system used to position the SYNCA (c).

3736 antennas in the array to the VNA. The VNA is configured to load a specific calibration  
3737 file for each antenna channel and performs the measurements of all available S-parameters.  
3738 The separate calibration files is an attempt to remove phase and magnitude errors caused  
3739 by different propagation through the RF switches. Array measurements were performed  
3740 for the set of SYNCA positions consisting of radial (x-axis) positions from 0 to 50 mm in  
3741 5 mm steps and axial (z-axis) positions from 0 to 50 mm in 5 mm steps resulting in 121  
3742 array measurements. At each SYNCA position we measured the two-port S-parameter  
3743 matrix using a linear frequency sweep from 25.1 to 26.1 GHz with 101 discrete frequencies.

3744 **5.4.2.2 Synthetic Array Setup**

3745 A photograph of the setup used to perform the synthetic array measurements is shown  
3746 in Figure 5.26. One important difference between this setup and the FSCD array setup  
3747 is that the synthetic array measurements were performed with a waveform generator and  
3748 digitizer instead of a VNA. The electronics configuration is identical to the diagram in  
3749 Figure 5.7b. Despite the differences, one is still able to compare the measured phases of  
3750 the synthetic array and the relative magnitude of the power, since the digitized signal  
3751 power is directly proportional to S21.

3752 The arbitrary waveform generator in the setup is configured to produce a 64 MHz  
3753 sine wave signal that is up-converted to 25.864 GHz using a mixer and the VNA source.  
3754 This signal is passed through a bandpass filter and fed to the SYNCA quad-balun chain.  
3755 A single FSCD antenna is positioned 10 cm from the SYNCA and aligned vertically so  
3756 that the center of the 5-slot waveguide is in the plane of the SYNCA PCB (see Figure  
3757 5.26). This position corresponds to  $z = 0$  in Figure 5.25c. The SYNCA is rotated  
3758 in three degree steps to synthesize an antenna array with 120 channels. This channel  
3759 count is more than could physically fit in a 10 cm radius array, but there is no cost to  
3760 over-sampling. Additionally, over-sampling allows for a check of the smoothness of the  
3761 antenna array radiation pattern. The signals from the FSCD antenna are down-converted  
3762 using the second mixer connected to the VNA source before being digitized at 250 MHz  
3763 and saved to disk. Several synthetic array measurement scans were performed by using  
3764 the linear translation stage to change the radial position of the SYNCA. In total eight  
3765 scans were taken from 0 to 35 mm using a radial position step size of 5 mm.

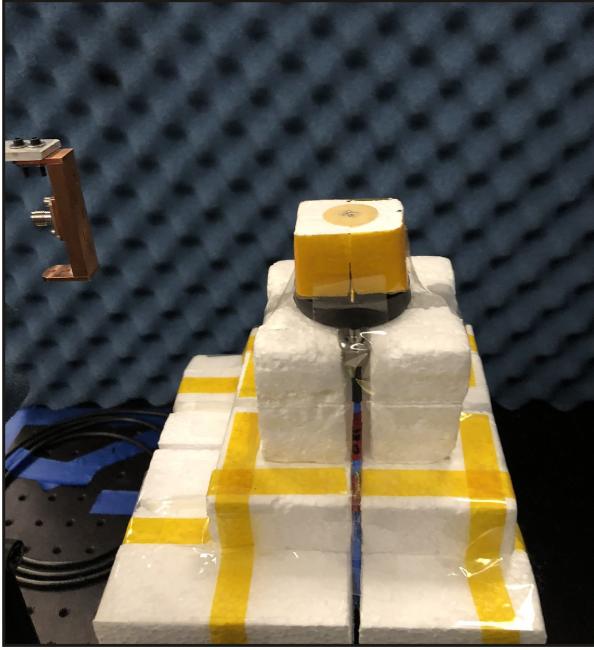


Figure 5.26: A photo of the FSCD antenna and the SYNCA in the synthetic array measurement setup at Penn State.

### **5.4.3 Simulations, Analysis, and Results**

The Locust and CRESana simulation packages utilize the antenna transfer functions to calculate the power that would be received by each antenna from a CRES electron. The equivalent quantity in the measurement setup is the S21 matrix element, which indicates the ratio of the power received by an antenna in the array to the amount of power delivered to the SYNCA. Therefore, the analysis focuses on comparing the relative magnitudes and phase of the S21 parameters measured by the VNA as a function of the array channel and the SYNCA position. Additionally, we apply a beamforming reconstruction to the S21 data to evaluate how the summed power and beamforming images change as a function of the position of the SYNCA.

#### **5.4.3.1 Simulations**

Simulations for the FSCD array measurements were performed using CRESana, which performs analytical calculations of the EM-fields produced by an electron at the position of the antennas. At each sampled time CRESana computes the electric field vector at the antenna positions, which is projected onto the antenna polarization axis to obtain the co-polar electric field. The magnitude of the co-polar electric field is then multiplied by

3782 a flat antenna transfer function to calculate the corresponding voltage signal. CRESana  
 3783 simulations exploit the flat transfer functions of the FSCD antennas, which allows the  
 3784 electric field to be multiplied by the antenna transfer function rather than performing  
 3785 the full FIR calculation. These calculations produce a voltage time-series for each of the  
 3786 antennas in the array that can be compared to the laboratory measurements.

3787 CRESana was configured to simulate a  $90^\circ$  electron in a constant background magnetic  
 3788 field of  $\approx 0.958$  T with a kinetic energy of 18.6 keV. These parameters were chosen  
 3789 in order to mimic a CRES event near the tritium beta-decay spectrum endpoint in  
 3790 the FSCD experiment. The constant background magnetic field guarantees that the  
 3791 guiding center of the electron is stationary across the duration of the simulation which is  
 3792 consistent with the SYNCA in the laboratory measurements. Simulations were performed  
 3793 with the electron's guiding center at radial positions from 0 to 45 mm in steps of 1 mm  
 3794 and axial positions from 0 to 30 mm in steps of 1 mm. The simulations generated time  
 3795 series consisting of 8192 samples at 200 MHz for the sixty channel FSCD antenna array  
 3796 geometry.

### 3797 5.4.3.2 Phase Analysis

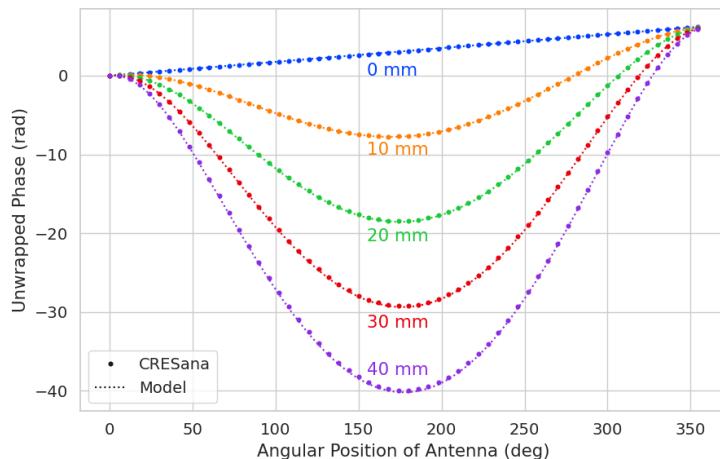


Figure 5.27: The unwrapped phases of signals received by the FSCD antenna array from an electron with a  $90^\circ$  pitch angle located in the plane of the antenna array. The data points indicated the phases extracted from simulation and the dashed lines show the model predictions.

3798 Correct modeling of the signal phases is fundamental to reconstruction for both  
 3799 beamforming and matched filter approaches. The beamforming reconstruction relies on

3800 a signal phase model developed from Locust simulations, which allows one to predict the  
3801 relative signal phases for a specific magnetic trap and electron position. The equation  
3802 for the model is

$$\phi_{ij}(t) = \frac{2\pi d_{ij}(t)}{\lambda} + \theta_{ij}(t), \quad (5.37)$$

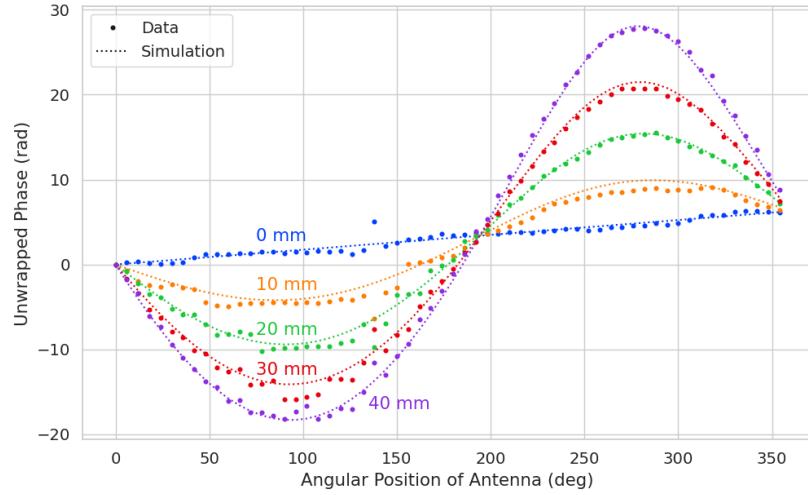
3803 where  $d_{ij}(t)$  is distance between the assumed electron position and the antenna position,  
3804 and  $\theta_{ij}(t)$  is the angular separation between the electron and antenna positions. For  
3805 details on the components of the phase model see Section 5.3.2. In Figure 5.27 we  
3806 compare the phases predicted by Equation 5.37 to phases extracted from CREsana  
3807 simulations of an electron located in the plane of the antenna array at a series of radial  
3808 positions. One observes excellent agreement between the model and simulation.

3809 The measured signal phases from the FSCD array and synthetic array are shown  
3810 in Figures 5.28a and 5.28b compared to the signal phase model. The axial position of  
3811 the SYNCA in both plots is  $z = 0$  mm, such that the plane of the PCB is aligned with  
3812 the center of the FSCD antenna. The data shown in Figure 5.28a corresponds to the  
3813 S-parameters measured at 25.80 GHz which is the frequency closest to the one used in  
3814 the synthetic array setup. The different slope and sinusoidal phases exhibited by Figure  
3815 5.28a and 5.28b reflects differences in the coordinate system for each setup. In general,  
3816 we see that the phase model predicts the large scale features of the phases quite well,  
3817 but there are some small scale deviations or errors from the phase model that do not  
3818 appear to be present in simulation.

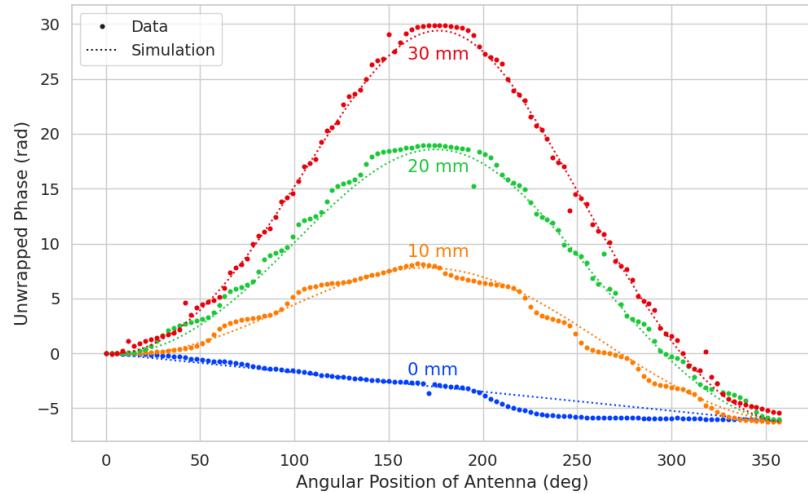
3819 A comparison of the phase errors, which are the difference between measurement and  
3820 model is shown in Figure 5.29. The FSCD array data is referred to as the JUGAAD  
3821 data in the plot legend, which is an alternative name for the FSCD array setup.

3822 The phase error at  $R = 0$  in Figure 5.29 forms a smooth curve, with the exception of  
3823 an outlier data point caused by a bug in the data acquisition script. One can attribute  
3824 the observed phase error at this position to imperfections in the antenna pattern of the  
3825 SYNCA. As the SYNCA is moved away from  $R = 0$  mm one observes that the phase  
3826 error exhibits oscillations whose frequency increases as a function of the radial position  
3827 of the SYNCA. These oscillations have the appearance of a diffraction pattern, which  
3828 is particularly clear for the radii  $\geq 15$  mm, due to the bilateral symmetry of the phase  
3829 error peaks around  $180^\circ$ .

3830 One can observe a higher average variance in the phase errors measured for the FSCD  
3831 array compared to the synthetic array. This is best seen by comparing the curves at  
3832  $R \leq 15$  mm where the smooth synthetic array curves are distinct from the relatively  
3833 noisy FSCD array errors. The extra noise in the FSCD array is most likely caused by



(a)



(b)

Figure 5.28: Plots of the measured unwrapped phases from the FSCD array (a) and the synthetic array (b) compared to the model predictions for a series of radial positions. The different phases of the sinusoidal phase oscillations in the two plots reflects differences in the coordinate systems of the measurements.

3834 differences in the radiation patterns of the antennas that make up the array as well as  
 3835 differences in the transmission lines through the switch network that introduce additional  
 3836 phase errors into the measurement. Since the synthetic array measurements use only  
 3837 a single antenna, these extra error terms are not present, which explains the relatively  
 3838 smoother phase error curves. Despite the extra phase errors in the FSCD array, it is still  
 3839 possible to observe a similar phase error oscillation effect as the SYNCA is moved away

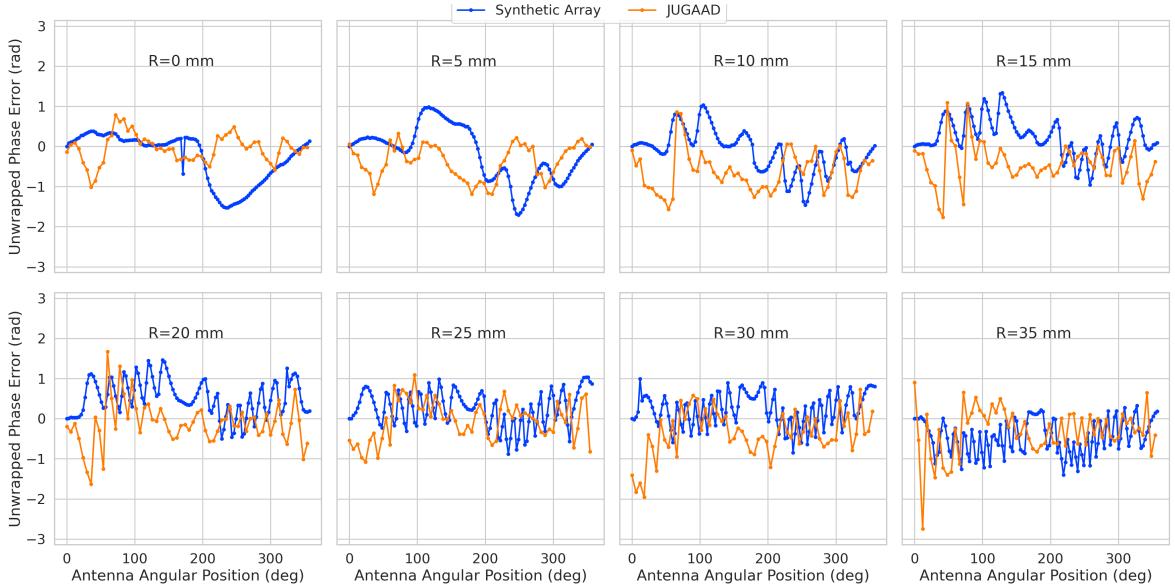


Figure 5.29: The phase errors between the measurement and model for the synthetic array (blue) and the FSCD array (orange) for a series of radial positions. The label JUGAAD refers to an alternative name for the FSCD array setup. As the SYNCA is translated off-axis phase errors with progressively higher oscillation frequency enter into the measurements.

3840 from  $R = 0$  mm.

3841 The diffraction pattern exhibited by the phase error oscillations is more easily observed  
 3842 by plotting the phase errors in a two-dimensional map, which is done in Figures 5.30a and  
 3843 5.30b. For the synthetic array data ones observes a relatively clear diffraction pattern  
 3844 that emerges as the SYNCA is moved radially. The bilateral symmetry of the diffraction  
 3845 patterns is due to the bilateral symmetry of the circular synthetic array around the  
 3846 translation axis of the SYNCA. A similar pattern is also visible in the FSCD array data,  
 3847 although, it is obscured by the additional phase error that results from the multi-channel  
 3848 array.

3849 The physical origin of the phase error diffraction pattern is attributed to interference  
 3850 effects arising from path-length differences between the individual slots in the FSCD  
 3851 antenna and the SYNCA transmitter. Since we are operating in the radiative near-field of  
 3852 the FSCD antenna, the path length differences between the slots introduces a significant  
 3853 change in the summation of the signals that occurs inside the waveguide, which causes  
 3854 the radiation pattern of the antenna to change as a function of distance. Therefore, when  
 3855 the SYNCA is positioned off-axis the different path-lengths from the SYNCA to each  
 3856 antenna results in different radiation patterns leading to the observed diffraction pattern.

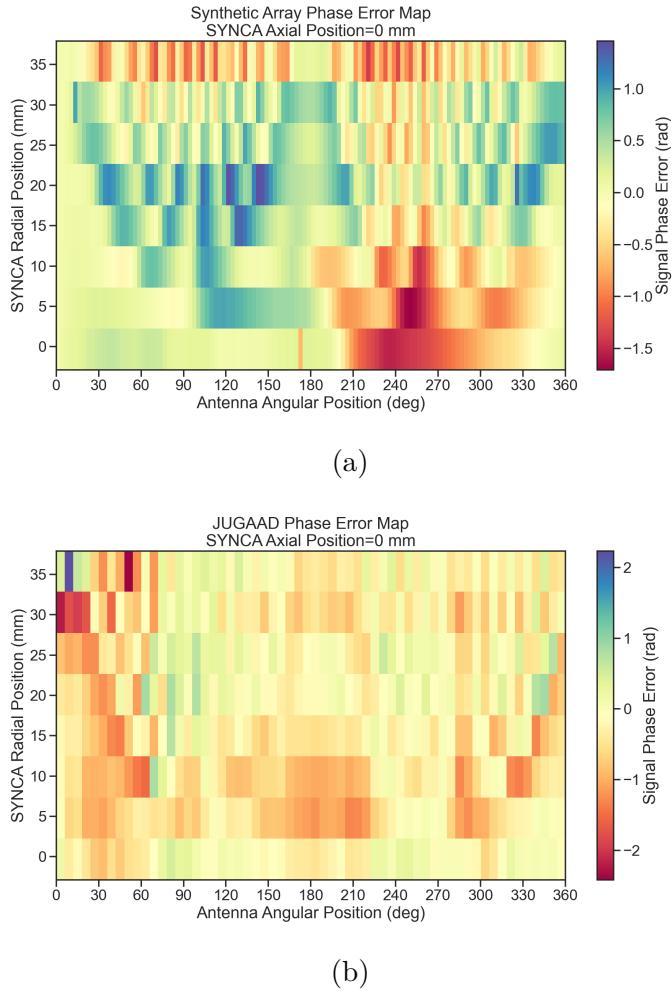


Figure 5.30: Two dimensional plots of the phase errors for the synthetic array (a) and the FSCD (JUGAAD) array (b). In both plots we observe evidence of a similar diffraction pattern with bilateral symmetry, but the FSCD array measurements have an additional phase error contribution from the different antennas and paths through the switch network.

This near-field effect is not present in simulations, because in order to simplify the calculations we assume that the far-field approximation can be applied to the FSCD antennas. This means that the radiation pattern and antenna transfer functions are independent of the distance between the transmitter and the receiving antenna. In principle, we can account for these near-field effects with a more detailed simulation of the FSCD antennas either in CRESana or Locust, which would result in an additional term in the beamforming phase model. However, this would increase the computational intensity of the simulation software. In the next section we briefly discuss the impact of

3865 these near-field effects on the measured magnitude of the signals.

### 3866 5.4.3.3 Magnitude Analysis

3867 Exactly as for the signal phase, one can use simulations to construct a model that  
3868 describes the magnitude of the signals received by each channel in the antenna array.  
3869 By examining the results of simulations or by analyzing the Liénard-Wiechert equation  
3870 one can show that radiation pattern from a  $90^\circ$  pitch angle electron in a magnetic field  
3871 is omni-directional. Therefore the relative magnitudes of the signals received by each  
3872 channel will be determined by the free-space power loss, which is proportional to the  
3873 inverse distance between the assumed electron position and the antenna.

3874 A consequence of this is that the signals produced in the array for electrons off the  
3875 central axis will have larger amplitudes for the antennas closer to the electron compared  
3876 to those which are further away. The amplitudes of the signals received by the array  
from an electron located at a series of radial positions are shown in Figure 5.31.

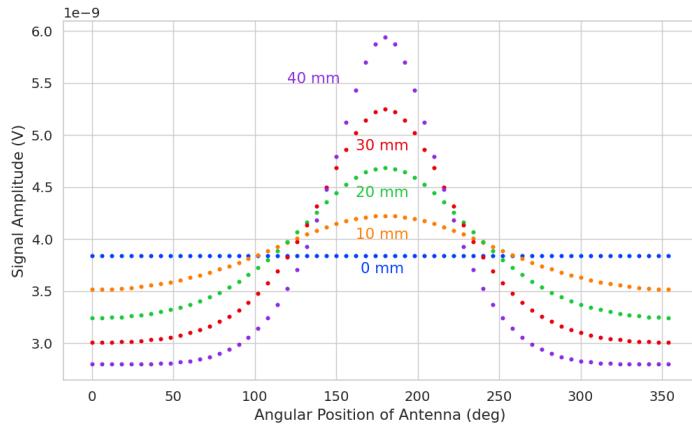


Figure 5.31: The amplitude of the signals from CREsana for the FSCD array from a  $90^\circ$  electron. As the electron is moved from  $R = 0$  the signals begin to have unequal amplitudes depending on the distance from the electron to the antenna.

3877  
3878 One expects to see a similar trend in the signal magnitudes in both the FSCD and  
3879 synthetic arrays. The normalized signal magnitudes extracted from the full and synthetic  
3880 array setups for a series of radial SYNCA positions are shown in Figure 5.32. The data  
3881 corresponds to a SYNCA axial position of  $z = 0$  mm and at a frequency 25.86 GHz. One  
3882 complication is that the radiation pattern of the SYNCA is not perfectly omni-directional,  
3883 which causes the measured magnitudes at  $R = 0$  mm to diverge from the perfectly flat  
3884 behavior exhibited by electrons.

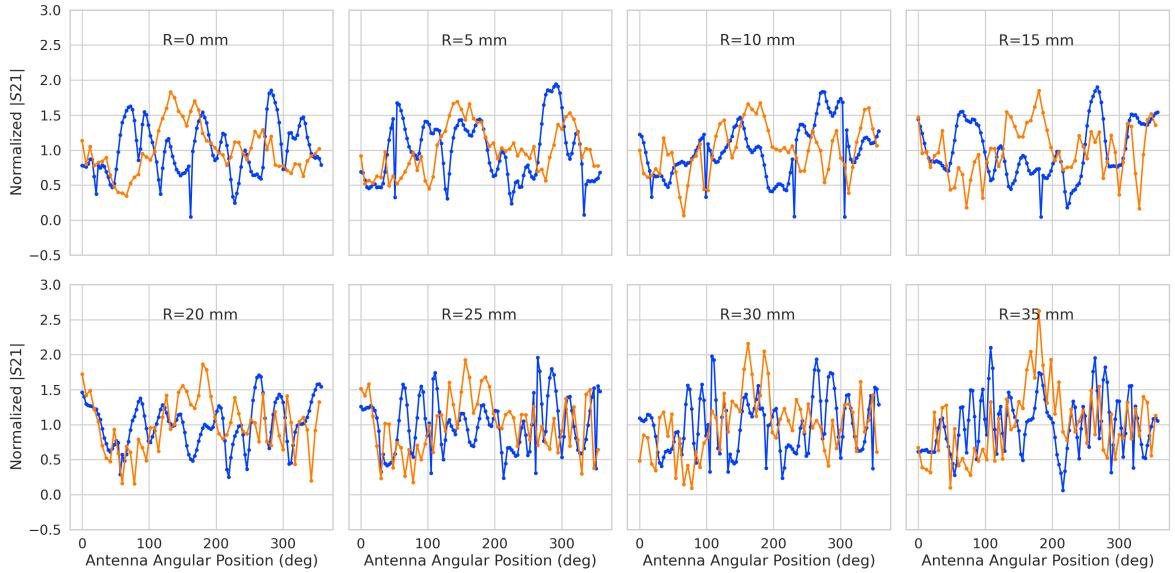
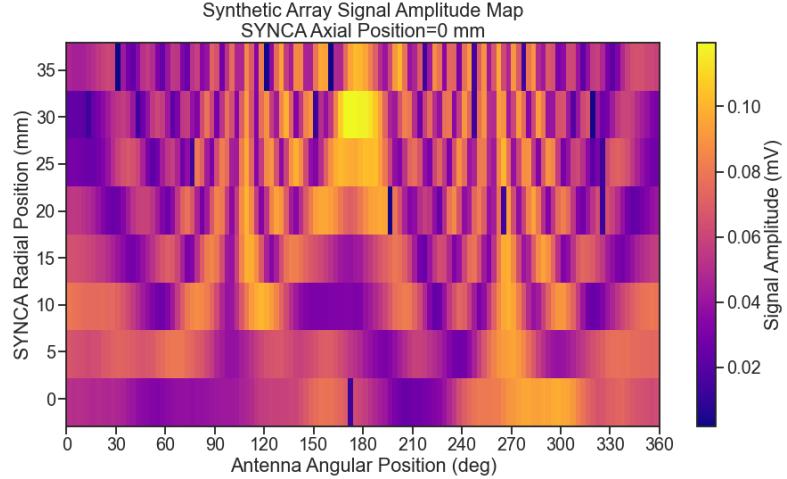


Figure 5.32: The normalized magnitudes of the S21 parameters measured in the FSCD (orange) and synthetic array (blue) setups. The dominant observed behavior as a function of radius is the increase in the number of magnitude peaks, which was noted in the phase error curves. There does not appear to be a strong change in the relative amplitude of a group of antennas as predicted by CREsana.

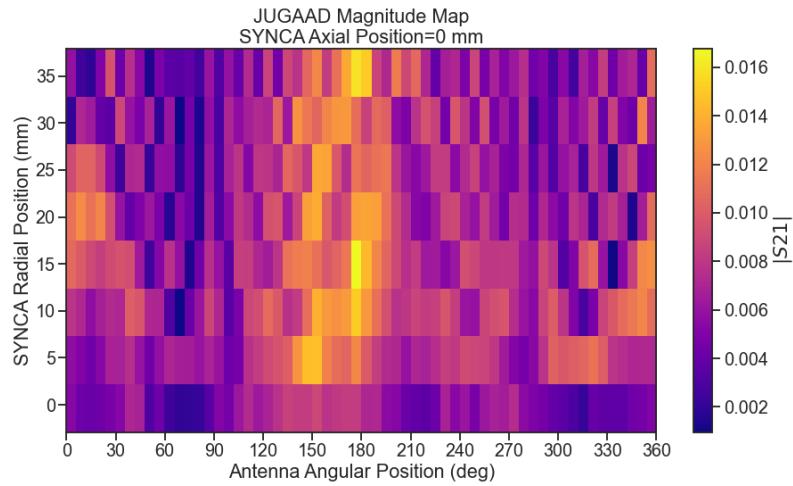
As the SYNCA is moved off-axis one observes a similar increase in the number of magnitude peaks in the synthetic array data that one would expect from a diffraction pattern, although this trend is not as stark compared to the phase data. Noticeably, there does not appear to be a set of channels with disproportionately larger amplitude at large  $R$ , which would be expected based on the trends from CREsana.

Comparing the magnitudes of the synthetic array to the FSCD array in Figure 5.32 we see that there is a similar amount of variability in the magnitudes at  $R = 0$  mm, although there is potentially more small scale error in the magnitude curve caused by channel differences in the FSCD array. We observe a similar trend in the number of magnitude error peaks in the FSCD array data to the synthetic array data, which mirrors the diffraction effect observed in the phase data. The diffraction effect can be visualized more clearly by plotting a similar two-dimensional map of the magnitudes (see Figure 5.33).

The fact that one observes a similar diffraction pattern in the signal magnitudes as a function the SYNCA position reinforces the conclusions from the phase analysis that near-field effects are having a significant impact on the radiation pattern of the FSCD array. These near-field effects lead to changes in the magnitude and phase of the



(a)



(b) The two-dimensional maps showing the diffractive pattern exhibited by the FSCD and synthetic array signal magnitudes.

Figure 5.33

3902 radiation pattern of the FSCD antenna as a function of distance. If left uncorrected these  
 3903 errors reduce detection efficiency by causing power loss in the beamforming or matched  
 3904 filter reconstruction due to phase mismatch. We explore the impact of these phase and  
 3905 magnitude errors on beamforming in the next section.

#### 3906 5.4.3.4 Beamforming Characterization

3907 Errors in the signal magnitudes and phases lead to errors in signal reconstruction. For  
 3908 example, a matched filter reconstruction requires accurate knowledge of the signals in

3909 each channel to achieve optimal performance. Uncorrected errors leads to mismatches  
 3910 between the template and signal, which reduces detection efficiency and introduces  
 3911 uncertainty in the parameter estimation. In this section, we analyze the beamformed  
 3912 signal amplitude as a function of the position of the SYNCA to quantify the impact of  
 3913 the phase and magnitude errors on signal reconstruction. Because of the imperfections  
 3914 in the SYNCA source, it is inappropriate to directly compare the beamformed signal  
 3915 amplitude of the FSCD array or synthetic array. Such a comparison would not allow  
 3916 one to disentangle losses that occur because of the antenna array from those that occur  
 3917 because of the source. Therefore, we focus on comparing the beamforming of the FSCD  
 3918 array to the synthetic array.

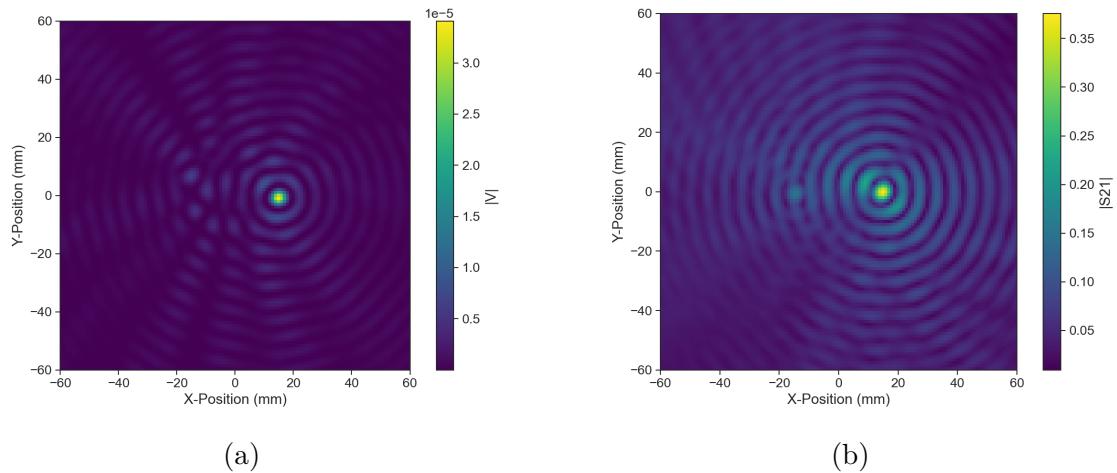


Figure 5.34: Beamforming images from the synthetic array (a) and FSCD array (b) setups with the SYNCA positioned 15 mm off the central axis. In both images we see a clear maxima that corresponds to the true SYNCA position. However, in the FSCD array there is an additional faint peak located at the opposite position of the beamforming maximum. This additional peak is the mirror of the true peak and is the result of reflections between antennas in the FSCD array.

3919 The first method of comparison is to analyze the images generated by applying the  
 3920 beamforming reconstruction specified in Section 4.3.1 to the FSCD and synthetic array  
 3921 data (see Figure 5.34). The beamforming grid consisting of a square  $121 \times 121$  grid  
 3922 spanning a range of -60-mm to 60 mm in the x and y dimensions. The beamforming  
 3923 images formed from the synthetic array produces a three-dimensional matrix where each  
 3924 grid position contains a summed time series. A single beamforming image is formed from  
 3925 this data matrix by taking the mean over the time dimension. In the case of the FSCD  
 3926 array, the VNA generates frequency domain data such that each grid position contains a  
 3927 summed frequency series produced by the VNA sweep. For this data a single image is

3928 formed by averaging in the frequency domain.

3929 There is a clear difference between the synthetic and FSCD array beamforming images,  
 3930 which is the additional faint beamforming maxima located directly opposite the maxima  
 3931 corresponding to the SYNCA position. The images in Figure 5.34 were generated with  
 3932 data collected at a SYNCA radial position of 15 mm, which agrees well with the observed  
 3933 beamforming maximum in both images. We observe that the faint beamforming peak is  
 3934 located directly opposite of the true beamforming maximum similar to a mirror image.  
 3935 Therefore, the origin of this additional feature appears to be reflections between the two  
 3936 sides of the circular antenna array that are not present for the synthetic array since only  
 3937 a single physical antenna is used.

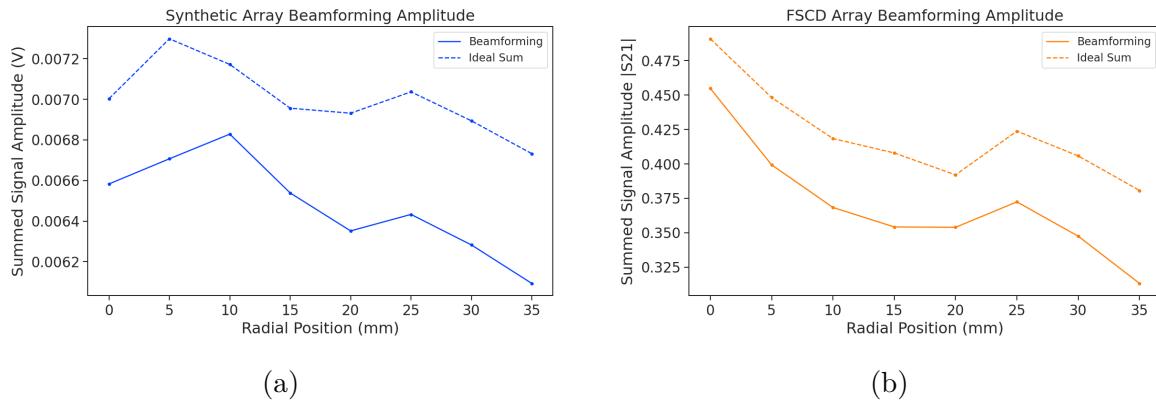


Figure 5.35: A comparison of the maximum signal amplitude obtained by beamforming to the signal amplitude obtained with an ideal summation as a function of the radial position of the SYNCA. The amplitudes for the synthetic array are shown in (a) and the FSCD array are shown in (b). In both setups we observe that the signal amplitudes obtained from beamforming are smaller than the signal amplitude that could be attained with the ideal summation without phase mismatch.

3938 From the beamforming images we extract the maximum amplitude, which we plot  
 3939 as a function of the radial position of the SYNCA (see Figure 5.35). The phase errors  
 3940 we observed in the FSCD and synthetic arrays leads to power loss at the beamforming  
 3941 stage due to phase mismatches between the signals at different channels. This power  
 3942 loss can be quantified by comparing the signal amplitude obtained from beamforming to  
 3943 the amplitude which would be obtained from an ideal summation. We perform the ideal  
 3944 summation by phase shifting each array channel to the same phase and then summing.  
 3945 The comparison between the beamforming and ideal sums is shown in Figure 5.35, where  
 3946 we observe that both the synthetic and FSCD arrays experience power losses from the  
 3947 beamforming summation.

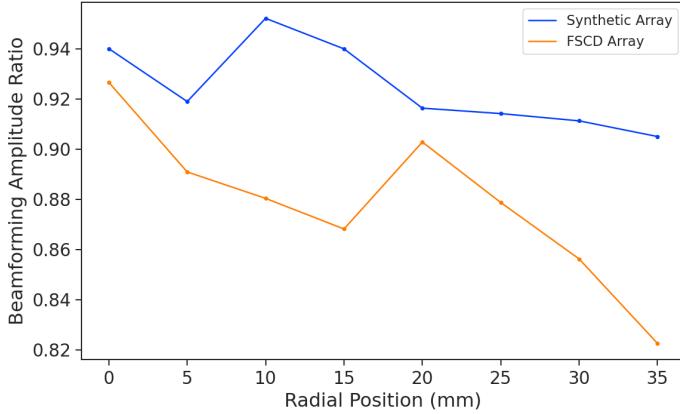


Figure 5.36: The ratio of the beamforming signal amplitude to the ideal signal amplitude for the FSCD and synthetic arrays. We see that the FSCD array has a larger power loss from phase error compare to the synthetic array which indicates that calibration errors associated with the multiple channels as well as reflections are impacting the signal reconstruction.

3948 The beamforming power loss can be quantified using the ratio of the beamforming to  
 3949 ideal signal amplitudes. Computing this ratio as a function of SYNCA radial position  
 3950 radius for the FSCD and synthetic arrays we find that the FSCD array has a uniformly  
 3951 smaller beamforming amplitude ratio, which means that the FSCD array has a larger  
 3952 beamforming power loss (see Figure 5.36). The primary contributions to the beamforming  
 3953 power loss in the synthetic array are phase errors from the SYNCA and phase errors  
 3954 from the FSCD antenna near-field. Both of these phase errors contribute to beamforming  
 3955 losses in the FSCD array, but there are clearly additional phase errors in the FSCD array  
 3956 measurements contributing to the smaller ratio. Two potential error sources include phase  
 3957 differences in the different antenna channels that could not be corrected by calibration as  
 3958 well as reflections between antennas in the array. The total effect of these additional phase  
 3959 errors is to reduce the beamforming amplitude ratio by about 5% from the beamforming  
 3960 ratio of the synthetic array. Therefore, we estimate that if no effort is made to correct  
 3961 these phase errors in an FSCD-like experiment, then we expect approximately a 10%  
 3962 total signal amplitude loss from a beamforming signal reconstruction.

#### 3963 **5.4.4 Conclusions**

3964 The estimated power loss of a beamforming reconstruction obtained from this analysis  
 3965 provides valuable inputs to sensitivity calculations of a FSCD-like antenna array exper-  
 3966 iment to measure the neutrino mass, since it helps to bound systematic uncertainties

3967 from the antenna array and reconstruction pipeline. This power loss lowers the estimated  
3968 detection efficiency of the experiment since some of the signal power is lost due to  
3969 improper combining between channels and also increases the uncertainty in the electron's  
3970 kinetic energy by contributing to errors in the estimation of the electron's cyclotron  
3971 frequency.

3972 If these reconstruction losses prove unacceptable there are steps that can be taken  
3973 to mitigate their effects. Some examples include the development of a more accurate  
3974 antenna simulation approach that can reproduce the observed near-field interference  
3975 patterns of the FSCD antennas and the implementation of a calibration approach that  
3976 allows for the relative phase delays of the array to be measured without changing or  
3977 disconnecting the antenna array configuration.

## **Chapter 6**

# **Development of Resonant Cavities for Large Volume CRES Measurements**

### **6.1 Introduction**

The cavity approach was originally an alternative CRES measurement technology under consideration by the Project 8 collaboration for the Phase IV experiment. After pursuing an antenna array based CRES demonstrator design for several years the increasing costs and complexity of the antenna arrays led to a reconsideration of the baseline technology for the ultimate CRES experiment planned by Project 8. Currently, a cavity based CRES experiment is the preferred technology choice for future experiments by the Project 8 collaboration including the Phase IV experiment.

In this chapter I provide a brief summary of resonant cavities and sketch out the key features of a cavity based CRES experiment. In Section 6.2 I provide a brief introduction to cylindrical resonant cavities and the solutions for the electromagnetic fields in the cavity volume.

In Section 6.3 I describe the main components of a cavity based CRES experiment, including the background and trap magnets, cavity geometry and design, and cavity coupling considerations. I also discuss some relevant trade-offs between an antenna array and cavity CRES experiment, and highlight some reasons for the transition of Project 8 to the development of a cavity based experiment.

Finally, in Sections 6.4 and 6.5, I present the design and development of an open mode-filtered cavity that could be used in a cavity based CRES experiment with atomic tritium. The results of the cavity simulations are confirmed by laboratory measurements of a proof-of-principle prototype that demonstrates key features of the design.

## 6.2 Cylindrical Resonant Cavities

Resonant cavities are sealed conductive containers, which allows us to describe the electromagnetic (EM) fields contained in the cavity volume as a superposition of resonant modes [36]. The field shapes of the resonant modes are determined by Maxwell's equations and the boundary conditions enforced by the cavity geometry. Of interest to Project 8 for CRES measurements are cylindrical cavities due to their ease of construction and integration with atom and electron trapping magnets.

### 6.2.1 General Field Solutions

Consider a long segment of conducting material with a cylindrical cross-section (see Figure 6.1). A geometry such as this can be used as a waveguide transmission line to transfer EM energy from point to point, or, if conducting shorts are inserted on both ends of the cylinder, the waveguide becomes a resonant cavity.

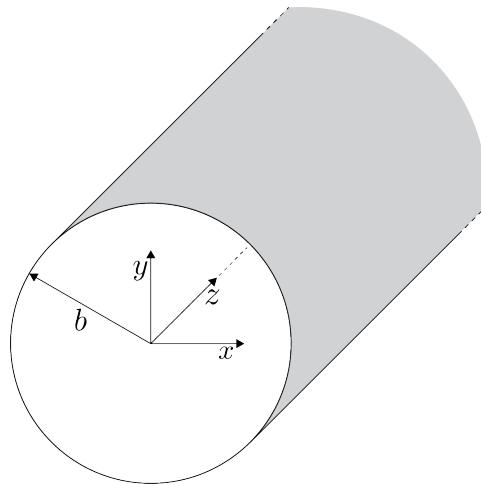


Figure 6.1: Geometry of a cylindrical waveguide with radius  $b$ .

The fields allowed inside a cylindrical cavity are determined by the boundary conditions of the cylindrical geometry. The general approach to solving the fields begins by assuming solutions to Maxwell's equations of the form

$$\mathbf{E}(x, y, z) = (\mathbf{e}(x, y) + \hat{z}e_z(x, y))e^{-i\beta z}, \quad (6.1)$$

$$\mathbf{H}(x, y, z) = (\mathbf{h}(x, y) + \hat{z}h_z(x, y))e^{-i\beta z}. \quad (6.2)$$

The solutions assume a harmonic time dependence of the form  $e^{i\omega t}$  and propagation

4018 along the positive z-axis. The functions  $\mathbf{e}(x, y)$  and  $\mathbf{h}(x, y)$  represent the transverse  
 4019 ( $\hat{x}, \hat{y}$ ) components of the electric and magnetic fields respectively, and  $e_z(x, y)$ ,  $h_z(x, y)$   
 4020 represent the longitudinal components. The version of Maxwell's equations in the case  
 4021 where there are no source terms can be written as a pair of coupled differential equations,

$$\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}, \quad (6.3)$$

$$\nabla \times \mathbf{H} = i\omega\epsilon\mathbf{E}, \quad (6.4)$$

4022 where  $\epsilon$  and  $\mu$  are the permittivity and permeability of the material inside the waveguide  
 4023 or cavity. Using the field solutions from Equations 6.1 and 6.2 one can solve for the  
 4024 transverse components of the fields in terms of the longitudinal fields. Because we  
 4025 are interested in cylindrical cavities it is advantageous to write the field solutions in  
 4026 cylindrical coordinates. After performing this transformation the set of four equations  
 4027 for the transverse field components are,

$$H_\rho = \frac{i}{k_c^2} \left( \frac{\omega\epsilon}{\rho} \frac{\partial E_z}{\partial\phi} - \beta \frac{\partial H_z}{\partial\rho} \right), \quad (6.5)$$

$$H_\phi = \frac{-i}{k_c^2} \left( \omega\epsilon \frac{\partial E_z}{\partial\rho} + \frac{\beta}{\rho} \frac{\partial H_z}{\partial\phi} \right), \quad (6.6)$$

$$E_\rho = \frac{-i}{k_c^2} \left( \beta \frac{\partial E_z}{\partial\rho} + \frac{\omega\mu}{\rho} \frac{\partial H_z}{\partial\phi} \right), \quad (6.7)$$

$$E_\phi = \frac{i}{k_c^2} \left( -\beta \frac{\partial E_z}{\partial\phi} + \omega\mu \frac{\partial H_z}{\partial\rho} \right), \quad (6.8)$$

4028 where  $k_c$  is the cutoff wavenumber defined by  $k_c^2 = k^2 - \beta^2$  with  $k = \omega\sqrt{\mu\epsilon}$  being the  
 4029 wavenumber of the EM radiation.

4030 This set of equations can be used to solve for a variety of different modes that can be  
 4031 obtained by setting conditions on  $E_z$  and  $H_z$ . For cylindrical cavities two types of modes  
 4032 are allowed, which correspond to solutions where  $E_z = 0$  and  $H_z = 0$  respectively.

### 4033 6.2.2 TE and TM Modes

4034 The TE family of modes corresponds to the case where  $E_z = 0$ . This implies that  $H_z$  is  
 4035 a solution to the Helmholtz wave equation

$$(\nabla^2 + k^2)H_z = 0. \quad (6.9)$$

4036 For solutions of the form  $H_z(\rho, \phi, z) = h_z(\rho, \phi)e^{-i\beta z}$ , Equation 6.9 can be solved using  
 4037 the standard technique of separation of variables. Rather than reproduce the derivation  
 4038 here we shall simply quote the solutions for the transverse fields [36], which are

$$H_\rho = \frac{-i\beta}{k_{c_{nm}}} (A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.10)$$

$$H_\phi = \frac{-i\beta n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.11)$$

$$E_\rho = \frac{-i\omega\mu n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.12)$$

$$E_\phi = \frac{i\omega\mu}{k_{c_{nm}}} (A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}. \quad (6.13)$$

4039 One can observe that the solutions have a periodic dependence on  $\phi$ , and radial profiles  
 4040 given by the Bessel functions of the first kind. The integer indices  $n$  and  $m$  arise from  
 4041 continuity conditions on the EM fields in the azimuthal and radial directions. For the  
 4042 TE modes  $n \geq 0$  and  $m \geq 1$ .  $k_{c_{nm}}$  is the cutoff wavenumber for the  $\text{TE}_{nm}$  mode given by

$$k_{c_{nm}} = \frac{p'_{nm}}{b}, \quad (6.14)$$

4043 where  $b$  is the radius of the cavity or waveguide and  $p'_{nm}$  is the  $m$ -th root of the derivative  
 4044 of the  $n$ -th order Bessel function (see Table 6.1).

Table 6.1: A table of the values of  $p'_{nm}$ .

$n$	$p'_{n1}$	$p'_{n2}$	$p'_{n3}$
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.970

4045 The TM mode family corresponds to the case where  $H_z = 0$ , and  $(\nabla^2 + k^2)E_z = 0$ .  
 4046 Again, we assume solutions of the form  $E_z(\rho, \phi, z) = e_z(\rho, \phi)e^{-i\beta z}$ , for which the general  
 4047 form of the solutions is the same as for the TE modes. However, the different boundary  
 4048 conditions for the TM modes results in particular solutions with a different form, which  
 4049 we shall quote here without derivation. The transverse fields of the TM modes are given  
 4050 by

$$H_\rho = \frac{-i\omega\epsilon n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.15)$$

$$H_\phi = \frac{-i\omega\epsilon}{k_{c_{nm}}}(A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z} \quad (6.16)$$

$$E_\rho = \frac{-i\beta}{k_{c_{nm}}}(A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.17)$$

$$E_\phi = \frac{-i\beta n}{k_{c_{nm}}^2 \rho}(A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.18)$$

which one may notice are the same solutions as the TE modes with  $H$  and  $E$  flipped.  
 The cutoff wavenumber for the TM modes is given by,  $k_{c_{nm}} = p_{nm}/b$ , where the values of  $p_{nm}$  correspond to the  $m$ -th zero of the  $n$ -th order Bessel function (see Table 6.2).

Table 6.2: A table of the values of  $p_{nm}$ .

$n$	$p_{n1}$	$p_{n2}$	$p_{n3}$
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

### 6.2.3 Resonant Frequencies of a Cylindrical Cavity

A cylindrical cavity is constructed by taking a section of cylindrical waveguide and shorting both ends with conductive material. This means that the electric fields inside a cylindrical cavity are exactly those we derived in Section 6.2.2 with the additional condition that the electric fields must go to zero at  $z = 0$  and  $z = L$  (see Figure 6.2).

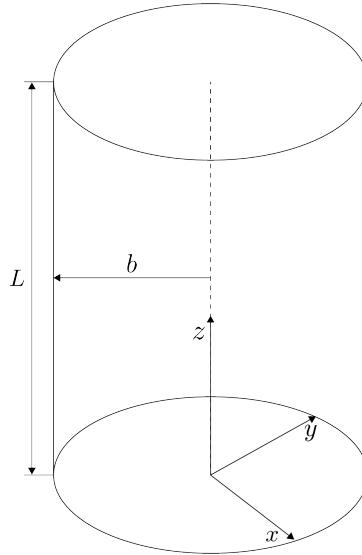


Figure 6.2: The geometry of a cylindrical cavity with length  $L$  and radius  $b$ .

4058

4059 The transverse electric field solutions for a cylindrical waveguide are of the form

$$\mathbf{E}(\rho, \phi, z) = \mathbf{e}(\rho, \phi) (A_+ e^{-i\beta_{nm}z} + A_- e^{i\beta_{nm}z}), \quad (6.19)$$

4060 where  $A_+$  and  $A_-$  are arbitrary amplitudes of forward and backward propagating waves.

4061 In order to enforce that  $\mathbf{E}$  is zero at both ends of the cavity we require that

$$\beta_{nm}L = 2\pi\ell, \quad (6.20)$$

4062 where  $\ell = 0, 1, 2, 3, \dots$ . Using this constraint on the propagation constant we can solve

4063 for the resonant frequencies of the  $\text{TE}_{nml}$  and the  $\text{TM}_{nml}$  modes in a cylindrical cavity.

4064 For the TE modes the resonant frequencies are

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{p'_{nm}}{b}\right)^2 + \left(\frac{\ell\pi}{L}\right)^2}, \quad (6.21)$$

4065 and the frequencies of the TM modes are

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{p_{nm}}{b}\right)^2 + \left(\frac{\ell\pi}{L}\right)^2}. \quad (6.22)$$

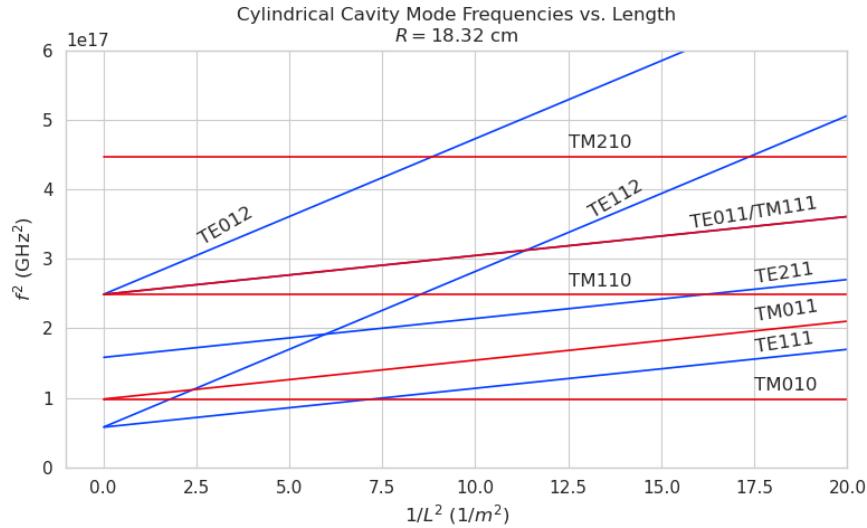


Figure 6.3: Relation of mode frequency to cavity length for a cylindrical cavity with a radius of 18.32 cm.

## 4066 6.2.4 Cavity Q-factors

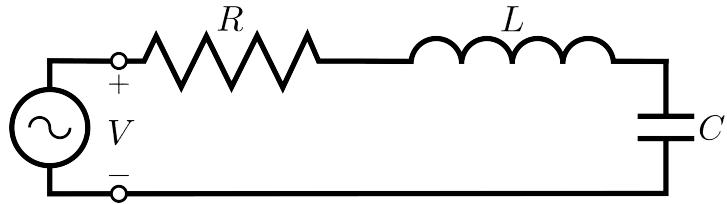


Figure 6.4: A series RLC circuit.

4067 The resonant behavior of cylindrical cavities can be modeled as a series RLC circuit  
 4068 (see figure 6.4). The input impedance of the circuit can be obtained by applying  
 4069 Kirchhoff's laws to calculate the impedance of the equivalent circuit. For a series RLC  
 4070 circuit the input impedance is

$$Z_{\text{in}} = \left( \frac{1}{R} + \frac{1}{i\omega L} + i\omega C \right). \quad (6.23)$$

4071 The resistance in the circuit represents all sources of loss in the cavity, which is primarily  
 4072 caused by the finite conductivity of the cavity walls. The inductor and capacitor represent  
 4073 the energy stored in the cavity in the form of electric and magnetic fields. If the circuit  
 4074 is being driven by an external power source we can write the input power in terms of the  
 4075 circuit input impedance and the source voltage

$$P_{\text{in}} = \frac{1}{2} Z_{\text{in}} |I|^2 = \frac{1}{2} |I|^2 \left( \frac{1}{R} + \frac{1}{i\omega L} + i\omega C \right). \quad (6.24)$$

4076 The resistor introduces a loss into the system with a power given by

$$P_{\text{loss}} = \frac{1}{2} |I|^2 R, \quad (6.25)$$

4077 and the capacitor and inductor store energies given by

$$W_e = \frac{1}{4} \frac{|I|^2}{\omega^2 C}, \quad (6.26)$$

$$W_m = \frac{1}{4} |I|^2 L, \quad (6.27)$$

4078 respectively. Using these expressions we can write the input power and input impedance

4079 expressions in terms of the lost power and stored energy

$$P_{\text{in}} = P_{\text{loss}} + 2i\omega(W_m - W_e), \quad (6.28)$$

$$Z_{\text{in}} = \frac{P_{\text{loss}} + 2i\omega(W_m - W_e)}{\frac{1}{2}|I|^2}. \quad (6.29)$$

4080 The condition for resonance in the RLC circuit is that the stored magnetic energy  
 4081 is equal to the stored electric energy ( $W_e = W_m$ ). When this occurs  $Z_{\text{in}} = R$ , which is a  
 4082 purely real impedance, and  $P_{\text{in}} = P_{\text{loss}}$ . The resonant frequency of the circuit can be  
 4083 determined from the condition  $W_e = W_m$  from which one finds that

$$\omega_0 = \frac{1}{\sqrt{LC}}. \quad (6.30)$$

4084 An important performance parameter for any resonant system is the Q-factor, which  
 4085 quantifies the quality of the resonator as the ratio of the stored energy multiplied by the  
 4086 resonant frequency to the average energy lost per second. For the series RLC circuit, the  
 4087 Q-factor is given by the expression

$$Q_0 = \omega \frac{W_e + W_m}{P_{\text{loss}}} = \frac{1}{\omega_0 RC}, \quad (6.31)$$

4088 from which one observes that as the resistance of the RLC circuit is decreased the quality  
 4089 factor of the resonator increases. From the perspective of cylindrical cavities this implies  
 4090 that as one decreases the resistance of the cavity walls it is expected that the Q-factor of  
 4091 the cavity should increase, which is indeed the case. In certain applications where a high  
 4092 Q is desireable it is possible to manufacture a cavity out of superconducting materials in  
 4093 order to minimize the power losses of the system.

4094 The Q-factor of the resonator also determines with bandwidth (BW) of the system.  
 4095 A cavity with a high Q-factor will resonant with a smaller range of frequencies than a  
 4096 cavity with a low Q-factor. To see this we can examine the behavior of the RLC circuit  
 4097 when driven by frequencies near the resonance. For a frequency  $\omega = \omega_0 + \Delta\omega$ , where  
 4098  $\Delta\omega = \omega - \omega_0 \ll \omega_0$ , we can write the input impedance as

$$Z_{\text{in}} = R + i\omega L \left( \frac{\omega^2 - \omega_0^2}{\omega^2} \right), \quad (6.32)$$

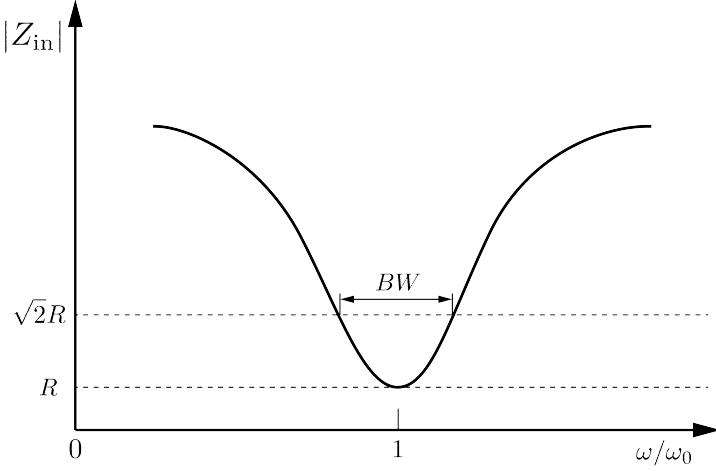


Figure 6.5: Illustration of the behavior of the input impedance of the series RLC circuit as a function of the driving frequency. The BW is proportion to the width of the resonance, which is inversely proportional to Q.

and by expanding  $(\omega^2 - \omega_0^2)/\omega^2$  to first order in  $\Delta\omega$ , we obtain

$$Z_{\text{in}} \approx R + i \frac{2RQ_0\Delta\omega}{\omega_0}. \quad (6.33)$$

Therefore, the magnitude of the input impedance near the resonance is given by

$$|Z_{\text{in}}| = R \sqrt{1 + 4Q_0^2 \frac{\Delta\omega^2}{\omega^2}}, \quad (6.34)$$

from which we observe that for the series RLC circuit the input impedance is minimized at the resonant frequency, which corresponds to the maximum input power (see Figure 6.5). The half-power BW is the range of frequencies over which the input power drops to half the input power on resonance. This occurs when  $|Z_{\text{in}}| = \sqrt{2}R$ , which corresponds to  $\Delta\omega/\omega = \text{BW}/2$ . Using Equation 6.34 one can find that

$$2R^2 = R^2(1 + Q_0^2\text{BW}^2), \quad (6.35)$$

which implies

$$\text{BW} = \frac{1}{Q_0} \quad (6.36)$$

It is important to emphasize that the Q-factor defined here,  $Q_0$ , is technically the unloaded Q. It reflects the quality of the cavity or resonant circuit without the influence of any external circuitry. In practice, however, a cavity is invariably coupled to an

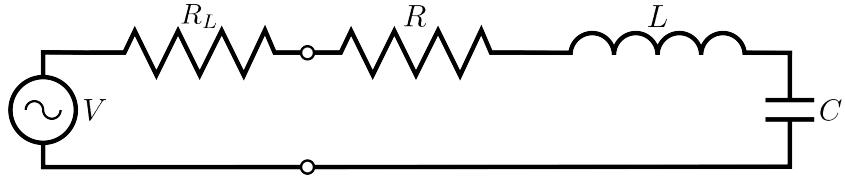


Figure 6.6: A series RLC circuit coupled to an external circuit with input impedance  $R_L$ .

external circuit to drive a cavity resonance or to measure the energy of a resonant mode. Coupling a cavity to an external circuit changes the Q by loading the equivalent cavity RLC circuit (see Figure 6.6). The Q-factor of the cavity when it is loaded by an external circuit is called the loaded Q, which is the quantity that one actually measures when exciting a resonance in the cavity. Using the series RLC circuit model one can see that the load resistor in Figure 6.6 will add in series with the resistor in the circuit for a total equivalent resistance of  $R + R_L$ . Therefore, the loaded Q is given by

$$Q_L = \frac{1}{\omega_0(R + R_L)C}, \quad (6.37)$$

from which one observes that the loaded Q is always less than the intrinsic Q of the cavity.

The amount of coupling that is desireable depends on the specific application of the resonator. If one wants a resonator that is particular frequency selective than it makes sense to limit the amount of coupling to the cavity to maintain a small BW, alternatively, if a larger BW is need one can increase the cavity coupling by tuning the input impedance of the external circuit. The critical point, where maximum power is transferred between the cavity and the external circuit, occurs when the input impedance of the cavity matches the input impedance of the external transmission line. For the series RLC circuit on resonance, this matching condition corresponds to

$$Z_0 = Z_{in} = R, \quad (6.38)$$

where  $Z_0$  is the impedance of the transmission line. The loaded Q at this critical point is, therefore,

$$Q_L = \frac{1}{2\omega_0 Z_0 C} = \frac{Q_0}{2}. \quad (6.39)$$

One can described the degree of coupling between the cavity and an external circuit by

<sup>4130</sup> defining a coupling factor,  $g$ , such that,

$$g = \frac{Q_0}{Q_L} - 1. \quad (6.40)$$

<sup>4131</sup> When  $g = 1$  then  $Q_L = Q_0/2$ , and the cavity is said to be critically coupled as we  
<sup>4132</sup> described. If  $Q_L < Q_0/2$ , then the cavity is undercoupled to the transmission line,  
<sup>4133</sup> corresponding to  $g < 1$ . Alternatively, if  $Q_L > Q_0/2$ , then  $g > 1$ , and the cavity is  
<sup>4134</sup> overcoupled to the transmission line. Various specialized circuits can be used to tune the  
<sup>4135</sup> input impedance of the external circuit as seen by the cavity to achieve a wide range of  
<sup>4136</sup> different coupling factors based on the desired application of the cavity.

## <sup>4137</sup> 6.3 The Cavity Approach to CRES

### <sup>4138</sup> 6.3.1 A Sketch of a Molecular Tritium Cavity CRES Experiment

<sup>4139</sup> Resonant cavities can be used to perform CRES measurements, and they represent the  
<sup>4140</sup> current preferred technology by the Project 8 collaboration. The basic approach to a  
<sup>4141</sup> neutrino mass measurement using a resonant cavity and molecular tritium beta-decay  
source is illustrated by Figure 6.7.

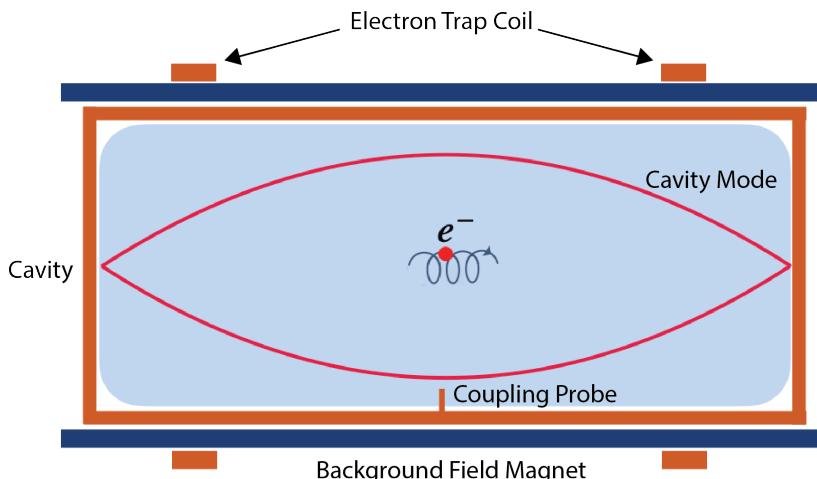


Figure 6.7: A cartoon depiction of a cavity CRES experiment. A metallic cavity filled with tritium gas is inserted into a uniform background magnetic field to perform CRES measurements. Electrons from beta-decays inside the cavity can be trapped and used to excite a resonant mode(s). By coupling to the cavity mode with a suitable probe one can measure the cyclotron frequency of the electron and perform CRES.

<sup>4142</sup>

4143 At the core of the experiment is a large resonant cavity filled with tritium gas. The  
4144 filled cavity is then placed in a uniform magnetic field provided by a primary magnet  
4145 that provides the background magnetic field. The value of the background magnetic field  
4146 sets the range of cyclotron frequencies for electrons emitted near the tritium spectrum  
4147 endpoint. When a beta-decay electron is produced in the cavity it is trapped using a set  
4148 of magnetic pinch coils that keep electrons inside the cavity volume.

4149 Electrons trapped inside the cavity do not radiate in the same way as electrons  
4150 in free-space. Effectively, the same boundary conditions that were used to derive the  
4151 resonant modes of a cylindrical cavity in Section 6.2 apply to the radiation of the electron  
4152 as well. The coupling of an electron performing cyclotron motion in a cavity has been  
4153 studied in detail for measurements of the electron’s magnetic moment [59–61] If an  
4154 electron is emitted with a kinetic energy that corresponds to a cyclotron frequency that  
4155 matches a resonant frequency of the cavity, then energy radiated by the electron excites  
4156 a corresponding resonance in the cavity. The strength of the electron’s coupling to the  
4157 cavity is given to first order by the dot product between the electrons trajectory and  
4158 the electric field vector of the resonant mode. Additional effects, such as the Purcell  
4159 enhancement [62], alter the emitted power from the free-space Larmor equation [63]. If an  
4160 electron is moving with a cyclotron frequency that is far from any resonant modes in the  
4161 cavity, then radiation from the electron is suppressed. One can interpret this somewhat  
4162 surprising effect as the metallic walls of the cavity reflecting the radiated energy back to  
4163 the electron.

4164 Detecting an electron in the cavity is accomplished by coupling the cavity to an  
4165 external transmission line that leads to an amplifier and RF receiver chain [64]. The  
4166 coupling of the cavity resonance to the amplifier occurs through a coupling probe or  
4167 aperture designed to read-out the excitation of the mode(s) excited by the electron. For  
4168 CRES measurements, the placement of a wire antenna coupling probe inside the cavity  
4169 volume leads to unacceptable losses of tritium atoms due to recombination to molecular  
4170 tritium on the antenna surface, therefore, apertures are the preferred coupling method  
4171 for cavity CRES experiments.

4172 One of the attractive features of the CRES technique for neutrino mass measurement  
4173 is the gain in statistics that comes from the differential nature of the tritium spectrum  
4174 measurement. Initially, this seems incompatible with cavities, due to the narrow reso-  
4175 nances of cavity modes giving relatively small bandwidth. However, by intentionally  
4176 over-coupling to a single cavity mode one can achieve bandwidths of a few 10’s of MHz  
4177 (see Section 6.2), which is sufficient for a measurement of the tritium spectrum endpoint

4178 region.

### 4179 **6.3.2 Magnetic Field, Cavity Geometry, and Resonant Modes**

#### 4180 **Magnetic Field and Volume Scaling**

4181 For a CRES experiment, cylindrical cavities are a natural choice since they match  
4182 the geometry of standard solenoid magnets, which are needed in order to produce the  
4183 background magnetic field for CRES measurements. Furthermore, the cylindrical shape is  
4184 compatible with a Halbach array, which is the leading choice of atom trapping technology  
4185 for future atomic tritium experiments by the Project 8 collaboration. Cylindrical  
4186 cavities also benefit from well-established machining practices that are able to achieve  
4187 high geometric precision at large lengths scales. More exotic cavity designs are under-  
4188 consideration and there are on-going efforts to investigate the potential advantages these  
4189 may have over the standard cylindrical geometry.

4190 As we saw in Section 6.2, the physical dimensions of the cavity are directly coupled  
4191 to the resonant frequencies of the cavity. This dependency links the size of the cavity to  
4192 the magnitude of the background magnetic field, because the magnetic field determines  
4193 the cyclotron frequencies of trapped electrons. Specifically, as the size of the cavity is  
4194 increased to accommodate larger volumes of tritium gas, the frequencies of the resonant  
4195 modes decrease proportionally. This requires that the magnetic field also decrease in  
4196 order to maintain coupling between electrons and the desired cavity mode.

4197 The required cavity size is ultimately determined by the required statistics in the  
4198 tritium spectrum endpoint region. Because the gas density must be kept below a certain  
4199 level to ensure that electrons have sufficient time to radiate before scattering, larger  
4200 volumes become the only way to achieve higher event statistics. To achieve the sensitivity  
4201 goals of Phase III and IV cavity volumes on the order of several cubic-meters are required,  
4202 which pushes one towards frequencies in the range of 100's of MHz.

#### 4203 **Single-mode Cavity CRES**

4204 It is tempting to consider maintaining a high magnetic field, while still increasing the size  
4205 of the cavity, in order to increase the radiated power from trapped electrons for better  
4206 SNR. However, if one were to maintain the same magnetic field while increasing the  
4207 size of the cavity, the electrons would begin to couple to higher order modes with more  
4208 complicated transverse geometries. The danger with this approach is that a complicated  
4209 mode structure could introduce systematic errors into the CRES signals. Example

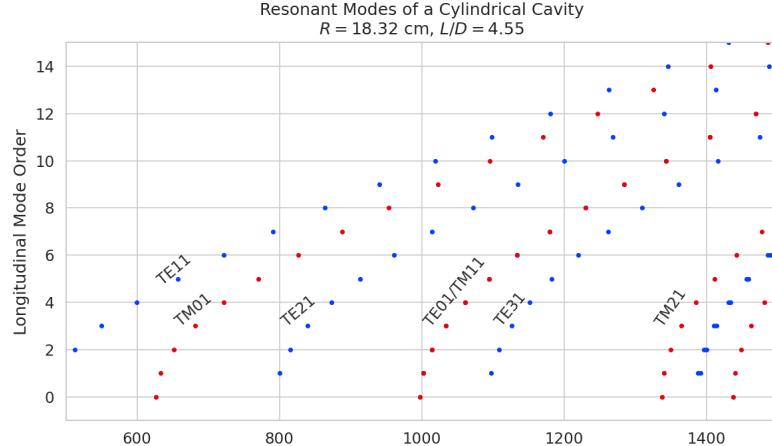
4210 systematics include unpredicted mode hybridization or changes in the mode shapes from  
4211 imperfections in the cavity construction, which would prevent reconstruction of the  
4212 electron's starting kinetic energies with adequate resolution. For this reason, it is ideal  
4213 to operate with magnetic fields that give cyclotron frequencies near the fundamental  
4214 frequency of the cavity, where the mode structure is relatively simple (see Figure 6.8).  
4215 In this frequency region it is possible to perform CRES by coupling to only a single  
4216 resonant mode, however, it is currently an open question if a single mode measurement  
4217 will provide enough information about an individual electron's position to reconstruct  
4218 the full event. Regardless, developing a solid understanding of the CRES phenomenology  
4219 when an electron is coupling to a single mode will be a necessary step towards a future  
4220 multi-mode cavity experiment.

#### 4221 Considerations for Resonant Mode Selection

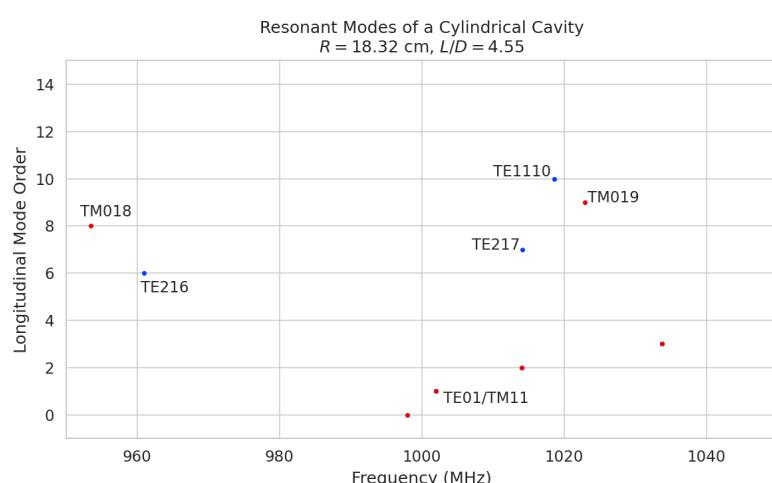
4222 A single-mode cavity experiment begs the question, which resonant mode is best for  
4223 CRES measurements? There is an immediate bias towards low order  $TE_{nm}$  and  $TM_{nm}$   
4224 modes due to the multi-mode considerations discussed above. Additionally, there is a  
4225 preference towards modes with longitudinal index  $\ell = 1$  with a single antinode along the  
4226 vertical axis of the cylindrical cavity. The reason for this is that there is a phase change  
4227 in the electric fields between antinodes that leads to modulation effects that destroy the  
4228 carrier frequency signal information.

4229 A second consideration for mode selection is the volumetric efficiency of the mode.  
4230 Volumetric efficiency can be thought of as an integral over the volume of the cavity  
4231 weighted by the relative amplitude of the mode. From the perspective of simply maximiz-  
4232 ing the volume useable for CRES measurements this integral would be as close to unity  
4233 as possible. However, there is a requirement to reconstruct the position of the electrons  
4234 inside the cavity volume so that the local magnetic fields can be used to convert the  
4235 measured cyclotron frequency to a kinetic energy. With a single mode this necessarily  
4236 requires a variable transverse mode amplitude, which lowers the volumetric efficiency, so  
4237 that position of the electron in the cavity can be estimated from the average amplitude  
4238 of the CRES signal. Longitudinal indices of  $\ell = 1$  have an advantage in volumetric  
4239 efficiency over higher order  $\ell$  modes, since there are only two longitudinal nodes, one at  
4240 each end of the cavity. Therefore, the average coupling strength of trapped electrons as  
4241 they oscillate axially is higher for  $\ell = 1$  modes.

4242 The longitudinal variation in the mode strength is ultimately critical for achieving the  
4243 energy resolution required for neutrino mass measurements. Correcting for the change in



(a)



(b)

Figure 6.8: Examples of the resonant mode frequencies of a cylindrical cavity. This cavity has a radius of 18.32 cm and a length to diameter ratio of 4.55.

4244 the average magnetic fields experienced by electrons with different pitch angles requires  
 4245 that information on the axial motion of the electron be encoded into the CRES signal.  
 4246 The longitudinal variation in the mode amplitude leads to amplitude modulation of the  
 4247 CRES signal with a frequency proportional to the electron's pitch angle.

4248 An additional factor for mode selection is the intrinsic or unloaded Q of the mode. In  
 4249 terms of SNR it is advantageous to use a mode with a very high  $Q_0$ , which is then highly  
 4250 overcoupled to achieve the necessary bandwidth to cover the tritium endpoint spectrum.  
 4251 This scheme leads to a decoupling of the physical cavity temperature from the effective  
 4252 noise temperature after the amplifier, which allows us to achieve adequate SNR without

4253 the requirement of cooling the entire cavity to single Kelvin temperatures.

4254 An example of a resonant mode that exhibits these traits is the TE<sub>011</sub> mode. At present  
4255 the TE<sub>011</sub> mode is the preferred resonance for a single-mode cavity CRES experiment  
4256 by the Project 8 collaboration. TE<sub>011</sub> is a low order mode located in a region relatively  
4257 far from other cavity modes. Furthermore, the separation of the TE<sub>011</sub> mode can be  
4258 improved by various mode-filtering techniques discussed in Section 6.4.2 below. TE<sub>011</sub>  
4259 consists of a single longitudinal antinode that can provide pitch angle information in the  
4260 form of amplitude modulation, and has an electric field with a radial profile given by the  
4261  $J'_0$  Bessel function allowing for radial position estimation. Lastly, the TE<sub>011</sub> mode has a  
4262 relatively high intrinsic Q compared to nearby modes, which helps with SNR. Unloaded  
4263 Q's greater than 80000 are achievable for a 1 GHz TE<sub>011</sub> resonance using a copper walled  
4264 cavity.

### 4265 **6.3.3 Trade-offs Between the Antenna and Cavity Approaches**

4266 The choice between cavities and antennas for large-scale CRES measurements is not  
4267 without trade-offs. Both the antenna array and cavity approaches are relatively immature  
4268 techniques, at present there are no known obstacles that would prevent either approach  
4269 from being used for a large scale neutrino mass experiment. The preference for cavities  
4270 is largely driven by important practical considerations that could make a cavity based  
4271 experiment significantly cheaper than an antenna experiment of similar size and scope.  
4272 However, the switch to cavities also introduces new challenges less relevant to the  
4273 antenna array, which must be solved in order for Project 8 to achieve its neutrino mass  
4274 measurement goals.

4275 One of the major relative drawbacks of the antenna array approach is the size and  
4276 complexity of the data-acquisition system. A large-scale antenna array experiment  
4277 requires  $O(100)$  antennas independently digitized at rates of  $O(10)$  to  $O(100)$  MHz. Since  
4278 there is insufficient information in a single antenna channel to detect or reconstruct the  
4279 CRES signal, the entire array output must be processed during the signal reconstruction.  
4280 Because data storage becomes an issue with these data volumes, there is a real-time  
4281 signal reconstruction requirement that allows one to detect CRES signals buried in the  
4282 thermal noise. As we discuss in Section 4.4, the computational cost of these real-time  
4283 detection algorithms are potentially quite large for even a small scale antenna array  
4284 experiment. However, the operating principle of a cavity experiment allows the CRES  
4285 signal to be detected using only a single read-out channel digitized at rates of  $O(10)$  MHz,  
4286 which reduces the cost of the data acquisition system by many orders of magnitude.

4287 From an engineering perspective, the simple geometry and thin-walls of a cylindrical  
4288 cavity are simpler to interface with the cryogenic and magnetic subsystems needed for a  
4289 CRES experiment. Whereas, the antenna array requires careful design and engineering  
4290 to accommodate the antenna array and receiver electronics in proximity to the trapping  
4291 magnets. Additionally, due to near-field interference effects, the antenna array is unable  
4292 to reconstruct CRES events within the reactive near-field distance of the antennas.  
4293 Because atom trapping requirements require magnetic fields which correspond to cyclotron  
4294 frequencies for endpoint electrons less than 1 GHz, the required stand-off distance leads to  
4295 a significant loss in useable experiment volume, necessitating larger and more expensive  
4296 magnets.

4297 Another advantage to the cavity approach is the relatively compact sideband structure,  
4298 which is a result of the low modulation index for cavity CRES signals. The axial motion  
4299 in an antenna array experiment leads to frequency modulation and sidebands. The shape  
4300 of the sideband structure is determined by the modulation index,  $h = \frac{\Delta f}{f_a}$ , where  $\Delta f$   
4301 is the size of the frequency deviation and  $f_a$  is the axial frequency. The large electron  
4302 traps required for a cubic-meter-scale experiment leads to high modulation indices, which  
4303 causes the signal spectrum to be made up of numerous low power sidebands that make  
4304 reconstruction and detection challenging. This behavior was observed in simulations  
4305 of the FSCD in which carrier power decreased with pitch angle due to the increase in  
4306 modulation index (see Figure 4.31). For cavities, however, the modulation index remains  
4307 near  $h = 1$  even for very long magnetic traps due to the high phase velocity in cavities  
4308 relative to the axial velocity of the electron. This results in an almost ideal spectrum  
4309 shape that has a strong carrier frequency with a few sidebands whose relative amplitudes  
4310 encode pitch angle information.

4311 A downside of the cavity approach is the apparent difficulty of estimating the position  
4312 of the electron using only the coupling of the electron to a single mode. The amplitude of  
4313 the TE<sub>011</sub> mode is completely independent of the azimuthal coordinate, therefore, position  
4314 reconstruction using the TE<sub>011</sub> mode is only able to estimate the radial position of the  
4315 electron. This position degeneracy may lead to magnetic field uniformity requirements  
4316 that are too challenging to meet due to mechanical uncertainties in cavity and magnet  
4317 construction, as well as uncertainties caused by nuisance external magnetic fields such  
4318 as the Earth's field and magnetic fields from building materials. A multi-mode cavity  
4319 experiment may provide a way to extract more precise information on the position of  
4320 the electron by analyzing the coupling of the electron to several modes that overlap in  
4321 different ways.

## 4322 **6.4 Single-mode Resonant Cavity Design and Simulations**

4323 The single-mode cylindrical cavities envisioned for the Phase III and IV experiments must  
4324 be carefully engineered in order to measure the neutrino mass with the desired sensitivity.  
4325 In this section I summarize some simulation studies performed to analyze early design  
4326 concepts for a single-mode cavity. The primary tool for these investigations was Ansys  
4327 HFSS, which was also used for the development of the SYNCA antenna described in  
4328 Section 5.3.

### 4329 **6.4.1 Open Cylindrical Cavities with Coaxial Terminations**

#### 4330 **Design Concept**

4331 A basic cavity design question relevant to Project 8's ultimate goal of an atomic tritium  
4332 CRES experiment is how to build a cavity that can be efficiently filled with atomic  
4333 tritium. To keep the rate of atom loss from recombination on surfaces it is ideal if the  
4334 ends of the cylindrical cavity are as open as possible so that tritium atoms can flow  
4335 inside unimpeded. Additionally, one of the primary calibration techniques planned for  
4336 future CRES experiments involves CRES measurements using electrons injected from  
4337 an electron gun source, which also requires an opening at the cavity end. Cylindrical  
4338 cavities with open ends can be manufactured, however, the intrinsic Q-factors of these  
4339 cavities are orders of magnitude less than their sealed counterparts, which reduces the  
4340 signal-to-noise ratio when that cavity is used for CRES measurement.

4341 Cylindrical cavities with mostly open ends that also exhibit Q values for the  $TE_{01\ell}$   
4342 modes similar to sealed cavities can be built by using coaxial endcaps to terminate the  
4343 cavity. Cavities of this type have been manufactured for specialized applications related  
4344 to the measurements of the dielectric constants of liquefied gasses (see Figure 6.9) [65, 66].  
4345 This cavity design leaves the ends of the cavity wide open, but retains high Q-values for  
4346 the  $TE_{01\ell}$  modes due to the coaxial endcap, which are designed to perfectly reflect the  
4347 electric fields of  $TE_{01\ell}$  modes. Coupling to the  $TE_{01\ell}$  mode is achieved via an aperture  
4348 located at the center of the cavity wall.

4349 A cavity similar to Figure 6.9 is a candidate design for the future CRES experiments  
4350 by Project 8, since it appears to elegantly solve many practical issues that arise when  
4351 combining cavity CRES and atomic tritium. The coaxial endcaps leave significant regions  
4352 of the cavity ends completely open, which allows for the entrance of atomic tritium as  
4353 well as the pumping away of molecular tritium that has recombined on the cavity walls.

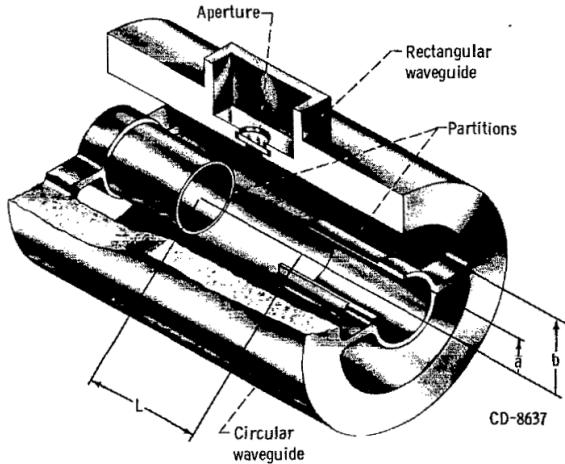


Figure 6.9: An image of an open cavity with coaxial terminations used for dielectric constant measurements. Figure from [66].

4354 These open ends are achieved while preserving the high Q-values of the  $TE_{01\ell}$  modes,  
 4355 which is important for extracting as much signal power from the electron as possible. In  
 4356 subsequent sections we shall analyze this cavity design in more detail, primarily by using  
 4357 HFSS simulations to analyze the resonant mode structure of this cavity geometry.

#### 4358 **Coaxial Terminator Constraints**

4359 The reason that coaxial endcaps can be used to achieve high Q-values for the  $TE_{01\ell}$   
 4360 modes is that the electric fields for these modes are purely azimuthally polarized (see  
 4361 Equations 6.12 and 6.13). Therefore, the boundary conditions that require the electric  
 4362 field to go to zero at the cavity ends can be supplied using a coaxial partition of the  
 4363 correct radius (see Figure 6.10). Because the cylindrical shape enforced by the partition  
 4364 does not match the boundary conditions of other cavity modes, these terminations also  
 4365 significantly suppress the Q-factors of non- $TE_{01\ell}$  modes, which is potentially beneficial  
 4366 for a single-mode cavity CRES experiment.

4367 The correct radius of the cylindrical partition is derived by setting up the boundary  
 4368 value problem in Figure 6.10, and analyzing the reflection and transmission coefficients  
 4369 for waves incident on the coaxial terminators. The basic problem is to identify the radius  
 4370  $a$  where the reflection coefficient for the  $TE_{01\ell}$  modes becomes equal to 1. One can show  
 4371 that if the coaxial partitions are made sufficiently long relative to the wavelength of the  
 4372  $TE_{01}$  modes than perfect reflection can be achieved. This derivation is quite lengthy  
 4373 and complex and is presented in full in [65]. Here, we shall simply explain the resulting

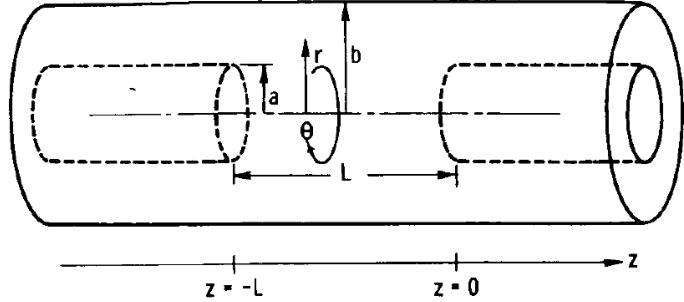


Figure 6.10: The simplified geometry of an open cavity with coaxial terminations. Figure from [65].

4374 conditions on the partition radius for perfect reflection.

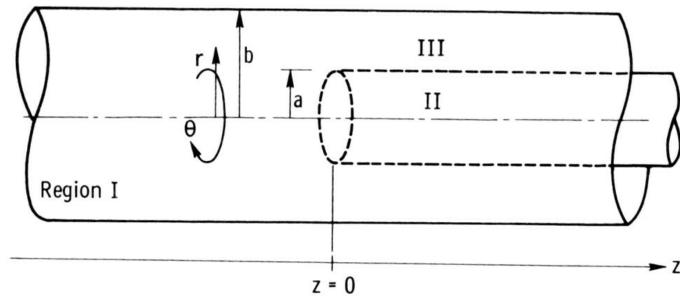


Figure 6.11: Electric field regions for the open cavity boundary value problem. Figure from [65].

4375 The open cavity boundary value problem is solved by expressing the forms of the  
 4376 electric fields in the different regions of the cavity and requiring that the electric fields are  
 4377 continuous. There are effectively three distinct regions in the open cavity corresponding  
 4378 to the central cavity volume, the inner coaxial volume, and the outer coaxial volume (see  
 4379 Figure 6.11).

4380 In Region I, the boundary conditions are those of a cylindrical waveguide, and we  
 4381 require that  $E_\phi$  for the  $TE_{0m}$  modes go to zero at the cavity wall ( $r = b$ ). This requires  
 4382 that  $J'_{0m}(k_{c0m} b) = 0$ . We aim to solve for the radius  $a$  in the specific situation where the  
 4383  $TE_{01}$  mode can propagate but all other  $TE_{0m}$  modes are below the cutoff frequency for  
 4384 the circular waveguide. This is equivalent to requiring

$$3.832 < k_{c0m} b < 7.016, \quad (6.41)$$

4385 where the numbers 3.832 and 7.016 correspond to the first and second zeros of the Bessel

4386 function (see Table 6.1).

4387 In Region II the boundary conditions are those of a cylindrical waveguide, but with  
4388 a smaller radius. The condition that  $E_\phi = 0$  at the cylindrical partition radius is that  
4389  $J'_{0m}(k_{c0m}a) = 0$ . To ensure perfect reflection, we want all modes in Region 1 of the cavity  
4390 to be below the cutoff frequency of the circular waveguide formed by the inner volume of  
4391 the coaxial terminator. Therefore, we consider the solutions where

$$k_{c0m}a < 3.832. \quad (6.42)$$

4392 Finally, in Region III the boundary condition are those of a coaxial waveguide. We  
4393 need to guarantee that  $E_\phi = 0$  at both  $r = b$  and  $r = a$ , which involves finding the  
4394 eigenvalues of the following equation

$$J'_0(k_{c0m}a)Y'_0(k_{c0m}b) - J'_0(k_{c0m}b)Y'_0(k_{c0m}a) = 0, \quad (6.43)$$

4395 where  $Y'_0$  the zeroth-order derivatives of the Bessel function of the second kind. The  
4396 solutions to this equation depend on the value of the ratio  $b/a$ . The approximate solution  
4397 is given by

$$\delta_n a \simeq \frac{n\pi}{b/a - 1}, \quad (6.44)$$

4398 where  $\delta_n$  are eigenvalues of Equation 6.43. Similar to Region II, we are interested in  
4399 solutions for which the TE<sub>01</sub> modes of Region I are below the cutoff of Region III.  
4400 Therefore, we require that

$$k_{c0m} < \delta_1. \quad (6.45)$$

4401 In general, one has some freedom in specifying the value of  $b/a$ . A value typically used  
4402 in practice is  $b/a = 2.082$ , which corresponds to positioning the radius of the cylindrical  
4403 partition at the maxima of the TE<sub>01</sub> electrical fields.

4404 Using the constraints from the three field regions one can develop a coaxial terminator  
4405 that acts as a virtual perfectly conducting surface for the TE<sub>01</sub> modes. The only required  
4406 inputs are the desired frequency of the TE<sub>011</sub> mode and a choice for the value of  $b/a$ .

#### 4407 **6.4.2 Mode Filtering**

4408 The general case of an electron coupling to a resonant cavity is complicated. This is  
4409 because cavities contain an infinite number of resonant modes, which for higher order  
4410 modes, have couplings to the electron with a complex spatial dependence. The danger is

4411 that improper modeling of the electron's coupling to the cavity can lead to systematic  
4412 errors in the CRES measurements that prevent a high-resolution measurement of the  
4413 electron's kinetic energy. This in part drives the preference for a single-mode cavity  
4414 experiment that uses only the electron's coupling to the TE<sub>011</sub> mode to perform CRES,  
4415 assuming that sufficient information on the electron's position can be obtained with a  
4416 single mode.

4417 The TE<sub>011</sub> mode is in a region where there are relatively few other modes to which  
4418 the electron could couple(see Figure 6.8). However, one can see that the frequency of  
4419 the TE<sub>011</sub> is perfectly degenerate with the TM<sub>111</sub> mode, which means that electrons will  
4420 inevitably couple to both modes if they have the correct cyclotron frequency.

4421 The magnitude of the impact of the electron coupling to both TE<sub>011</sub> and TM<sub>111</sub> is  
4422 currently unknown. To first order an electron coupling to more both modes will lose more  
4423 energy overtime, which can be measured by observing the frequency chirp rate of the  
4424 signal. This effect may be small enough to be negligible or simple enough to model that  
4425 the cavity can be treated as an effective single-mode cavity. Alternatively, the one could  
4426 consider devising a coupling scheme that is sensitive to both the TE<sub>011</sub> and the TM<sub>111</sub>  
4427 modes. By measuring the coupling of the electron to both modes more information on  
4428 the position of the electron could be obtained, which could improve the position and  
4429 energy resolution of the CRES measurements.

4430 A different approach is the mode filtering approach, which seeks to obtain a single  
4431 TE<sub>011</sub> mode cavity using perturbations to the cavity walls that selectively impede the  
4432 TM modes, while leaving the TE modes mostly unperturbed. The type of perturbations  
4433 required can be determined by visualizing the surface currents induced in the cavity  
4434 walls by each type of mode (see Figure 6.12). By definition, all TM have electric fields  
4435 directed along the vertical axis of the cylindrical cavity, which means that perturbations  
4436 that impede currents in this direction will modify TM resonances. On the other hand,  
4437 the TE<sub>01</sub> modes induce azimuthal currents in the cavity walls, therefore, it is possible to  
4438 break the degeneracy between TE<sub>01</sub> and TM<sub>11</sub> using a cavity perturbation that impedes  
4439 axial currents, but does not affect the flow of azimuthal currents.

4440 Figure 6.12 shows two cavity design concepts that achieve this selective current  
4441 perturbation. The resistive approach inserts a series of thin dielectric rings into the walls  
4442 of the cavity that introduces a resistive and capacitive impedance to the longitudinal  
4443 currents, while leaving azimuthal current paths intact. Cavities of this type with high  
4444 TE<sub>01</sub> Q's have also been constructed by tightly wrapping a thin, dielectric coated wire  
4445 around a mold to form the cavity wall. An alternative method is to introduce an inductive

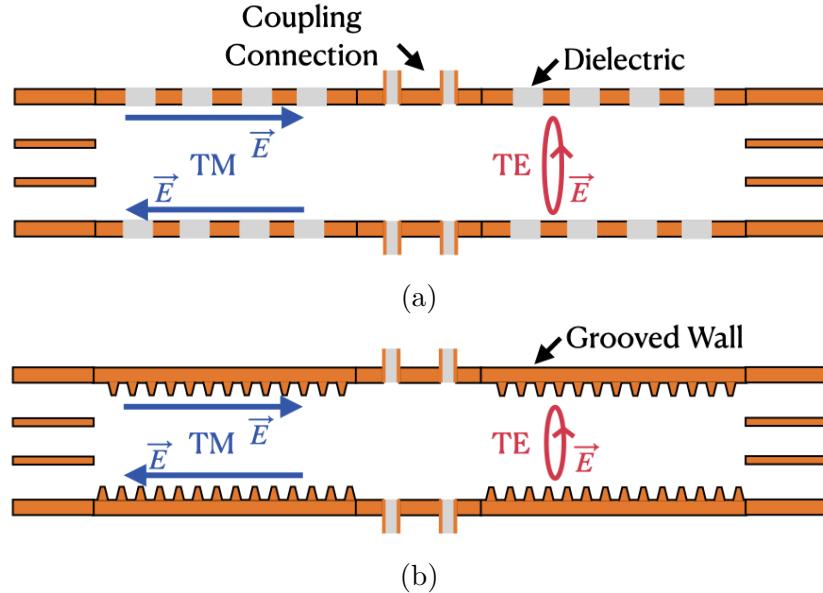


Figure 6.12: Two mode filtering concepts to break the degeneracy of  $TE_{01}$  and  $TM_{11}$  modes. The resistive approach uses dielectric materials to impede currents that travel vertically along the cavity while leaving azimuthal currents unperturbed. An alternative approach is to impede the currents using grooves cut into the cavity wall, which achieve the same effect with an inductive impedance.

4446 impedance by cutting grooves or a thread pattern on the inside wall of the cavity. For  
 4447 reasons of manufacturability and compatibility with tritium the grooved cavity approach  
 4448 is the preferred method for mode-filtered cavity construction by Project 8.

#### 4449 **6.4.3 Simulations of Open, Mode-filtered Cavities**

4450 A candidate design for a single  $TE_{011}$  mode CRES experiment is a cavity that utilizes  
 4451 the coaxial terminations combined with a mode-filtering wall. The first step towards  
 4452 validating that a cavity that combines these two design features will operate as expected  
 4453 is a thorough simulation effort for which finite element method (FEM) simulation software  
 4454 is invaluable. The primary tool for electromagnetic FEM calculations inside Project 8 is  
 4455 Ansys HFSS, which has a robust and well-established eigenmode solver that can identify  
 4456 the resonant frequencies and associated Q-factors for given structure.

4457 Four variations of a cavity design with a  $\sim 1$  GHz  $TE_{011}$  resonance were implemented  
 4458 in HFSS (see Figure 6.13). The four designs include a standard cylindrical cavity, an  
 4459 open cavity with smooth walls, an open cavity with resistive walls, and an open cavity  
 4460 with grooved walls. The relevant design parameters are summarized in Table 6.3. All

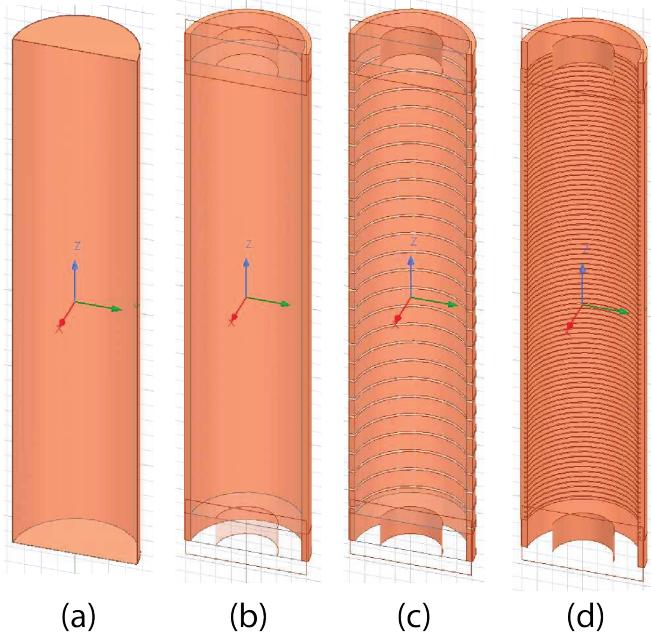


Figure 6.13: Four cavity design variations. (a) is a standard sealed cylindrical cavity, (b) is an open cavity with smooth walls, (c) is an open cavity with resistive walls, and (d) is an open cavity with grooved walls. The main cavity and coaxial terminator parameter are identical for all four cavities.

<sup>4461</sup> cavities were simulated using copper walls and filled with a vacuum dielectric. The  
<sup>4462</sup> identities of the resonant modes found by HFSS were validated by visual inspection of  
<sup>4463</sup> the electric and magnetic field patterns and by comparison to analytical calculations of  
<sup>4464</sup> the mode frequencies.

Table 6.3: A table of cavity design parameters used for HFSS simulations.

Name	Qty.	Unit	Description
$D_{\text{cav}}$	326.4	mm	Cavity diameter
$L_{\text{cav}}$	1668.0	mm	Cavity length
$D_{\text{term}}$	200.2	mm	Inner diameter of coaxial terminator
$L_{\text{term}}$	100.0	mm	Terminator length
$l_{\text{die}}$	8.3	mm	Dielectric spacer thickness
$\Delta l_{\text{die}}$	66.7	mm	Distance between dielectric spacers
$l_{\text{groove}}$	3.0	mm	Groove height
$d_{\text{groove}}$	9.0	mm	Groove depth
$\Delta l_{\text{groove}}$	18.3	mm	Distance between grooves

<sup>4465</sup> The results of the HFSS simulations validate our predictions of the resonant behavior  
<sup>4466</sup> of an open, mode-filtered cavity developed in the preceding sections (see Figure 6.14) One

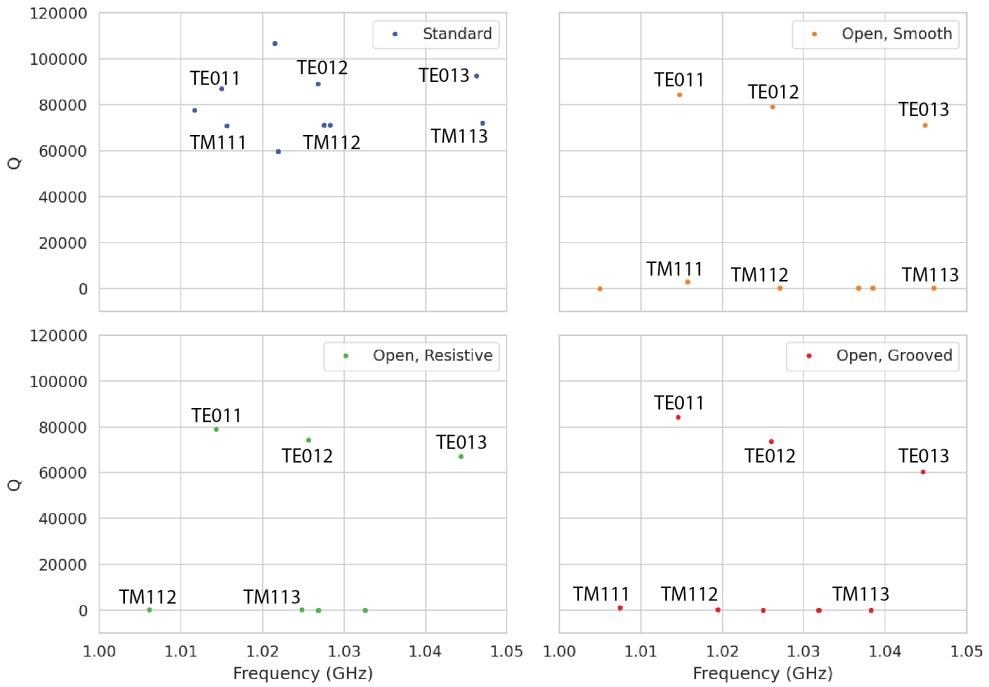


Figure 6.14: The frequencies and Q-factors of the resonant modes identified by HFSS for the cavity variations shown in Figure 6.13. The fully-sealed cavity with smooth walls has several high-Q modes near the TE<sub>011</sub> resonance. Introducing the open-termination preserves the Q-factors of the TE<sub>01 $\ell$</sub>  modes and suppresses the Q-factors of the modes whose boundary conditions do not match the cylindrical partition. Both the resistive and grooved wall perturbations shift the resonant frequencies of the TM modes away from the TE<sub>011</sub> mode. By properly tuning the geometry of the grooves or the resistive spacers several MHz of frequency separation can be achieved.

can see that for a standard cavity the TE<sub>01</sub> and the TM<sub>11</sub> are degenerate in frequency with relatively high Q-factors. The open-ended cavity preserves the high Q-factors of the TE<sub>01</sub> modes, while the other modes, since their boundary conditions do not match the coaxial geometry, have their Q-factors suppressed. One can see that the effect of the resistive and inductive mode-filtering schemes is to effectively shift the resonant frequencies of the TM<sub>11</sub> modes below those of the associated TE<sub>01</sub> modes, which breaks the degeneracy. Optimization of the dielectric spacer or groove parameters can ensure that the TE<sub>011</sub> mode is isolated from other modes by  $O(10)$  MHz, which provides sufficient bandwidth for a measurement of the tritium spectrum endpoint.

Further optimization of the cavity design requires a more detailed cavity simulation that includes the cavity coupling mechanism as well as other geometry modifications required for integration into the magnetic and tritium gas subsystems. Perhaps more

4479 important is the development of the capability to simulate the interaction of electrons  
4480 with the cavity so that simulated CRES signals can be generated using cavities designed  
4481 for CRES measurements. Simulated CRES signals can then be used to estimate the  
4482 neutrino mass sensitivity of the experiment, which allows for the optimization of the cavity  
4483 design towards the configuration that provides the best measurement of the neutrino  
4484 mass.

## 4485 **6.5 Single-mode Resonant Cavity Measurements**

4486 Measurement test stands play an important role in the research and development process  
4487 that cannot be replaced by simulations. For example, constructing a prototype CRES  
4488 cavity forces one to consider important practical issues such as manufacturability and  
4489 machine tolerances that may require modifications to the design. Furthermore, by  
4490 comparing laboratory measurements of a real cavity to simulations, one can quantify  
4491 the impact of imperfections and real-life measurement systematics, which allows for  
4492 more accurate sensitivity estimates of the experiment. Lastly, the development of these  
4493 prototypes helps to build the necessary experience and expertise within the collaboration  
4494 required for more complicated experiments to succeed.

4495 In this spirit a prototype cavity was constructed to demonstrate the open, mode-  
4496 filtered cavity concept explored in the previous sections. The primary goal of the  
4497 measurements was to validate that an open, mode-filtered cavity suppressed the  $\text{TM}_{11}$   
4498 modes as predicted by HFSS simulations.

### 4499 **6.5.1 Cavities and Setup**

4500 Two rudimentary, cavities were constructed using segments of copper pipe available from  
4501 McMaster-Carr (see Figure 6.15). The design consists of copper pipes of two diameters.  
4502 The larger diameter pipe forms the main cavity wall and the smaller diameter pipe is  
4503 used to create a coaxial termination. The diameter of the outer pipe was chosen to  
4504 produce a  $\text{TE}_{011}$  resonance of approximately 6 GHz, while the diameter of the smaller  
4505 pipe was selected based on the open termination criteria introduced in Section 6.4.1. The  
4506 approximate diameters and lengths of the copper pipe are summarized in Table 6.4.

4507 Coupling to the cavity was achieved using a hand-formable segment of coaxial cable  
4508 stripped at one end to form a loop antenna. This was inserted into a small hole located  
4509 at the center of the main cavity wall. The coaxial terminators were supported inside the

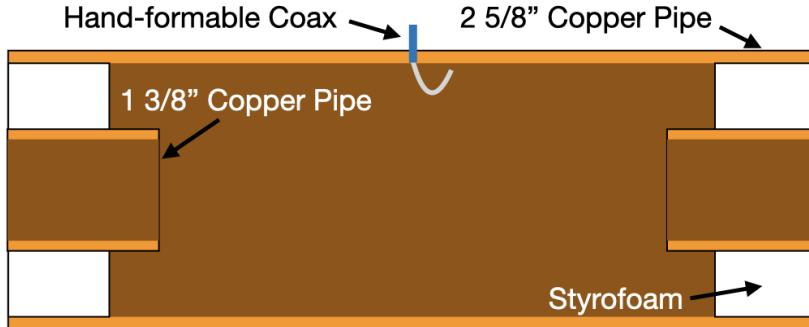


Figure 6.15: A cartoon depicting the design of the open-ended cavity prototype designed to operate at approximately 6 GHz. The main cavity wall was composed of a single copper pipe. A mode-filtered version of this cavity was constructed by

4510 main cavity by carving a spacer from polystyrene foam (styrofoam) so that they could  
 4511 be easily inserted into the cavity and repositioned. The dielectric constant of styrofoam  
 4512 is quite close to air at microwave frequencies so this is expected to have minimal impact  
 4513 on the resonant properties of the cavity.

Table 6.4: A table of parameters describing the cavity prototypes. Certain values such as the cavity length and the distance between dielectric spacers are approximate due to variation in the machining of the copper. In particular, the filtered cavity was constructed from conducting copper segments that varied in size from 1.50" to 1.85".

Name	Qty.	Unit	Description
$D_{\text{cav}}$	2.625	in	Cavity diameter
$L_{\text{cav}}$	$\approx 13$	in	Cavity length
$D_{\text{term}}$	1.375	in	Inner diameter of coaxial terminator
$L_{\text{term}}$	1.575	in	Terminator length
$l_{\text{die}}$	0.75	in	Dielectric spacer thickness
$\Delta l_{\text{die}}$	$\approx 1.50$ to $1.85$	in	Distance between dielectric spacers

4514 The actual length of the cavity is given by the distance between the inner edges of the  
 4515 coaxial terminators. The length of the outer section of pipe that forms the main wall of  
 4516 the cavity is approximately 16" in length which leads to a cavity length of  $\approx 13"$  when  
 4517 both terminators are inserted in the cavity. Because the terminators were not rigidly  
 4518 mounted this distance is only approximate, however, the uncertain length of the cavity  
 4519 will not prevent us from validating the open cavity design.

4520 Along with the smooth-walled open cavity a resistively mode-filtered cavity was  
 4521 constructed by creating dielectric spacers out of segments of clear PVC pipe (see Figure  
 4522 6.16). The spacers were machined such that the conductive segments of the cavity would

4523 be separated by 0.75" when the cavity was fully assembled. Due to variations in the  
 4524 lengths of the copper segments that make up the cavity wall the distance between spacers  
 4525 has significant variation with average value of about 1.7". Eight total spacers were used  
 4526 to build the cavity, which when assembled was approximately 16" in total length similar  
 to the non-filtered cavity.

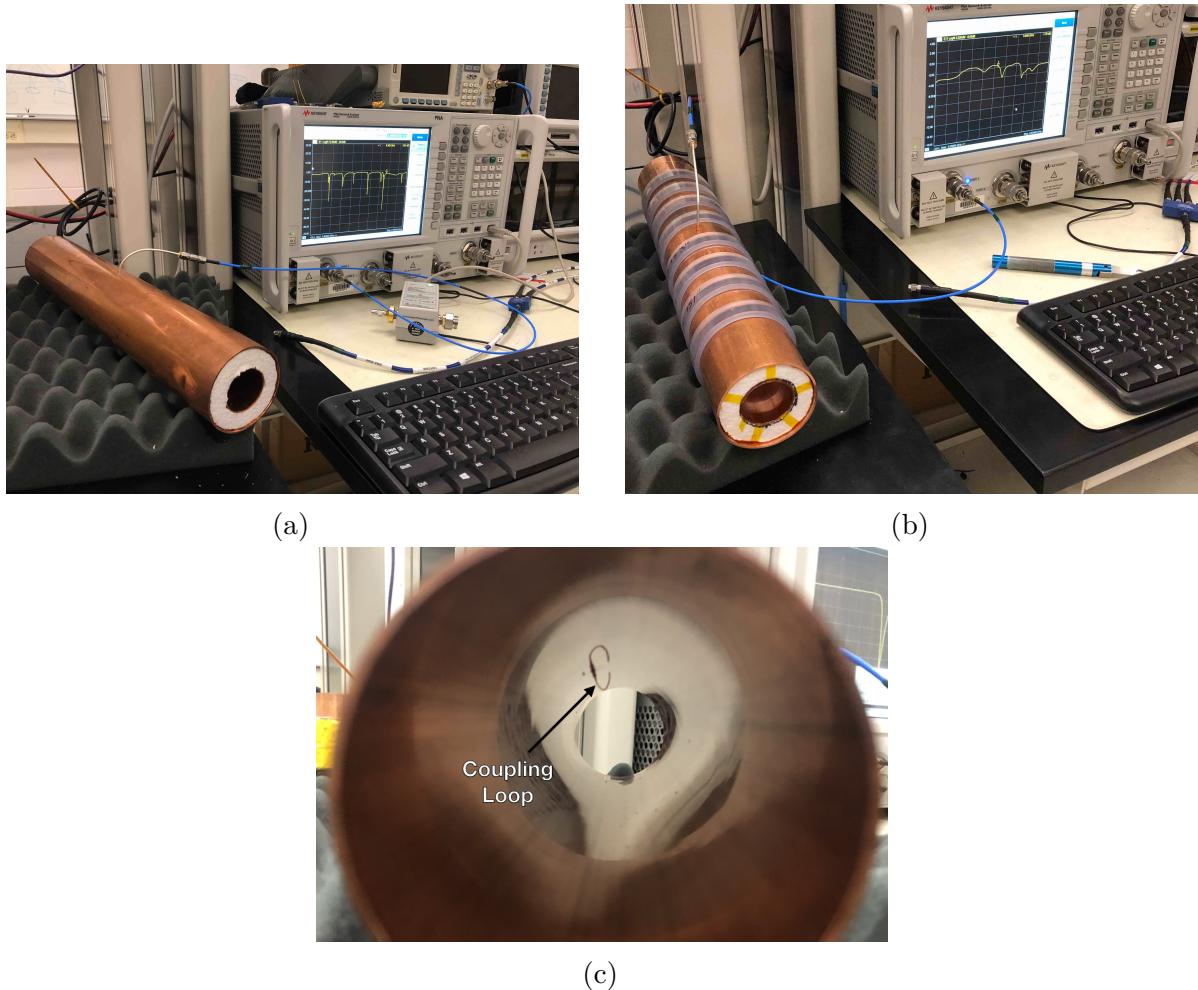


Figure 6.16: Images depicting the measurement of the filtered and non-filtered open cavities using the VNA. The coupling loop in the figure is shown in the TE orientation.

4527 Measurements of both cavities were performed using a VNA connected to the cavity  
 4528 coupling probe (see Figure 6.16). By measuring the return loss over a range of frequencies  
 4529 one can measure the frequencies and relative Q-factors of the resonant modes in the  
 4530 cavity. Due to the opposite polarity of the electric fields for the TE and TM modes,  
 4531 the loop coupling probe must be rotated 90° to change the polarity of the loop antenna.  
 4532 When the antenna is oriented such that the loop opening faces the ends of the cavity, it  
 4533

4534 couples primarily to the TE modes which have magnetic fields directed along the long  
 4535 axis of the cavity (see Figure 6.16). If the coupling loop is turned by  $90^\circ$  from where  
 4536 it is shown in the image then it will couple to the TM modes which have azimuthally  
 4537 directed magnetic fields. In this way both the TE and TM resonances can be measured  
 4538 independently.

### 4539 **6.5.2 Results and Discussion**

4540 The primary analysis method for the prototype cavities involved simply visualizing the  
 4541 return loss measured by the VNA and comparing between the filtered and non-filtered  
 4542 cavities. Since the resonances measured by the VNA are not labeled, there is some  
 4543 uncertainty about the true identities of the modes measured by the VNA. To help with  
 4544 this we performed a simulation of the simplest possible cavity that could be created from  
 4545 the prototype components, which is a fully open cavity created by simply removing the  
 4546 coaxial inserts from the non-filtered cavity configuration. The fully open cavity with the  
 4547 as-built dimensions was simulated in HFSS to get estimates on the positions of the  $\text{TE}_{011}$   
 4548 and  $\text{TM}_{111}$  modes (see Figure 6.17).

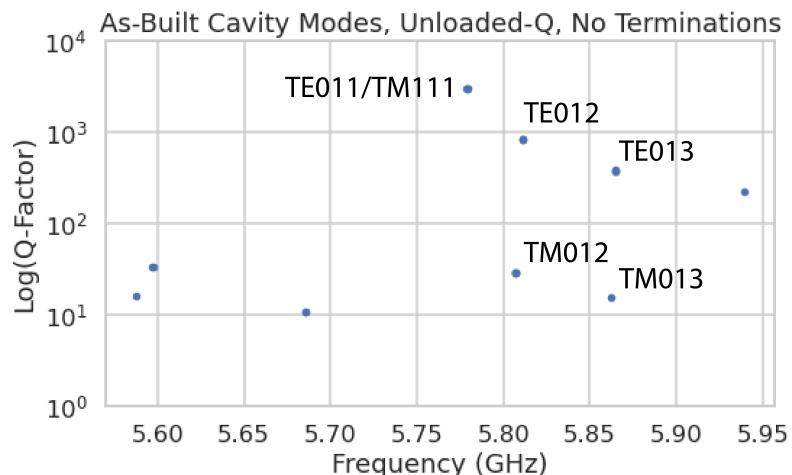


Figure 6.17: HFSS simulation results for a the as-built cavity with the coaxial terminators removed. The  $\text{TE}_{011}/\text{TM}_{111}$  frequency is approximately 5.78 GHz.

4549 Simulation of the fully open cavity shows that the  $\text{TE}_{011}/\text{TM}_{111}$  modes have a  
 4550 frequency of approximately 5.78 GHz in the fully open cavity. If the frequency of this  
 4551 mode is compared to the measurments of the fitered and non-filtered cavities with the  
 4552 terminators removed we can easily identify the  $\text{TE}_{011}$  mode at approximately 5.75 GHz

4553 (see Figure 6.18).

4554 For the non-filtered cavity one sees that the  $TE_{011}$  mode is degenerate in frequency  
 4555 with what appears to be a doublet of TM modes located at the  $TM_{111}$  frequency position.  
 4556 This doublet is actually the  $TM_{111}$  mode, which has two polarizations with opposite  
 4557 polarizations. Because the pipe used to construct the cavity is not perfectly round, the  
 4558 frequency degeneracy between the two polarizations is broken producing the doublet  
 peak. In the case of the filtered cavity with no terminators there is an isolated TE

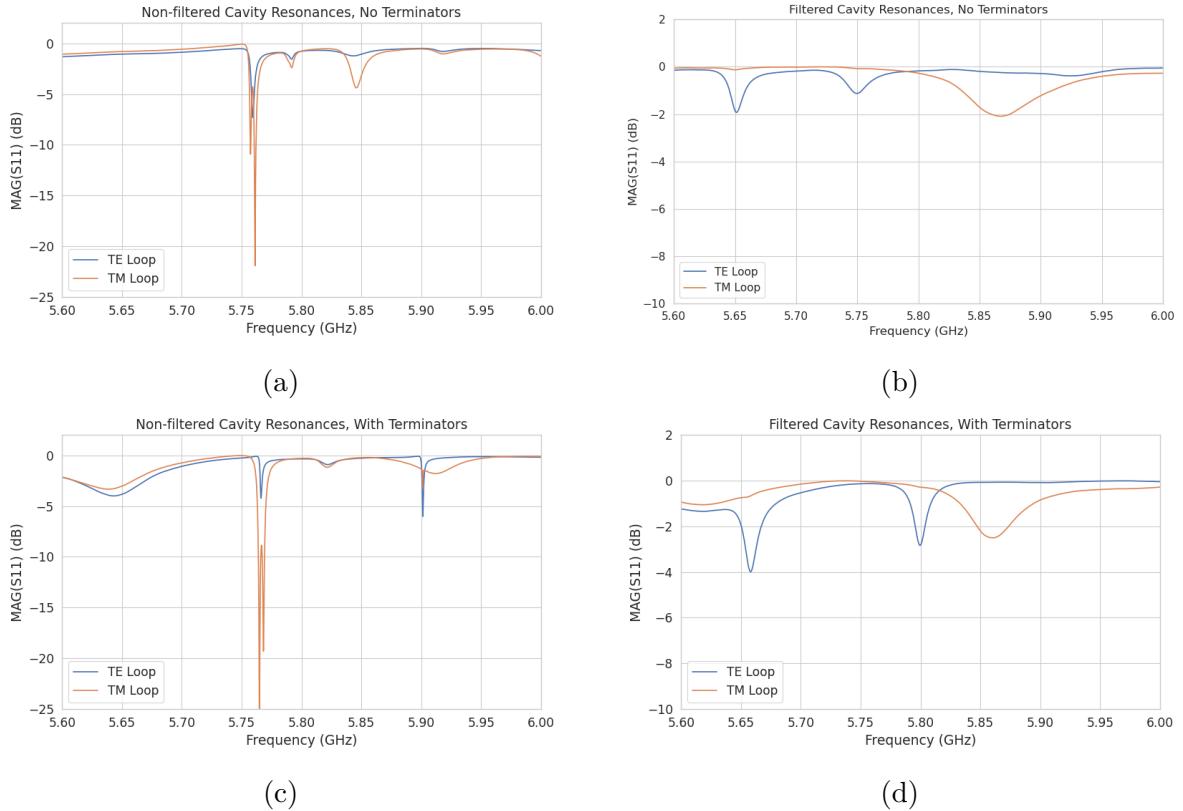


Figure 6.18: Measurements of the filtered and non-filtered prototype cavities acquired with the VNA.

4559  
 4560 resonance at 5.75 GHz that appears to be the  $TE_{011}$ , however, there is no apparent  $TM_{111}$   
 4561 doublet at the same frequency. This is what one would expect if the mode-filtering was  
 4562 effective at suppressing the TM modes. There is a notable difference in the Q of the  
 4563  $TE_{011}$  resonance for the non-filtered and filtered cavities indicated by the relative widths  
 4564 of the resonances. This is likely caused by the large width of the dielectric spacers that  
 4565 are partially impeding the TE modes. When the terminators are inserted into the cavity  
 4566 one sees that Q-factors of the modes improves as expected, by noticing the narrowing of  
 4567 the peaks compared to the no terminator plots.

4568 In conclusion, one see from these cavity measurements that, in principle, mode-  
4569 filtering can be used to separate the TE<sub>011</sub> resonance from the degenerate TM<sub>111</sub> mode in  
4570 combination with the an open cavity design. The ideal next step would be to construct a  
4571 open, mode-filtered cavity that could be used to perform CRES measurements. In order  
4572 to study the coupling of an electron to the isolated TE<sub>011</sub> mode.

4573 **Chapter 7 |**

4574 **Conclusion and Future Prospects**

4575 In this dissertation we have discussed research and development efforts towards the  
4576 development of a scalable CRES measurement technology that can be used to build a  
4577 CRES experiment at cubic-meter scales with sensitivity to neutrino masses of 40 meV.  
4578 The primary contributions of my dissertation are the development and analysis of signal  
4579 reconstruction algorithms for an antenna array based CRES experiment [67], which leads  
4580 to estimates of the neutrino mass sensitivity; the development of a synthetic cyclotron  
4581 radiation antenna (SYNCA) [29], which allowed for laboratory validation of antenna  
4582 array CRES simulation models [6]; and the development of an open-ended cavity design  
4583 compatible with atomic tritium for a cavity based CRES experiment. A measurable  
4584 impact of this work is the transition of the Project 8 collaboration's experimental plan  
4585 from an antenna array based approach to a cavity based approach, where my work played  
4586 a key role in demonstrating the significantly higher cost and complexity of the antenna  
4587 array experiment.

4588 The transition from antenna arrays to cavities requires a new set of demonstrator  
4589 experiments to make incremental progress towards a 40 meV measurement of the neutrino  
4590 mass. At the time of writing, the near-term plan of Project 8 is to design and construct a  
4591 small-scale cavity CRES experiment utilizing the 1 T magnet installed in the UW-Seattle.  
4592 This cavity is designed to have a TE011 resonance with a frequency of about 26 GHz with  
4593 a length-to-diameter ratio that mimics the larger cavities intended for the pilot-scale and  
4594 Phase IV experiments. The goal of this experiment is to demonstrate cavity CRES as  
4595 well as validate models of CRES systematics using electrons from  $^{83m}\text{Kr}$  and an electron  
4596 gun. Though the primary goal is demonstration, near-term physics measurements are  
4597 available in the form of high-resolution measurements of the  $^{83m}\text{Kr}$  conversion spectrum  
4598 of interest to the KATRIN collaboration.

4599 Furthermore, Project 8 is currently constructing a low-frequency CRES setup located  
4600 at Yale University to better understand the principles of cavity based CRES at lower

4601 magnetic fields. The Low, UHF Cavity Krypton Experiment at Yale (LUCKEY) is  
4602 a 1.5 GHz cavity CRES experiment the will use conversion electrons from  $^{83m}\text{Kr}$  to  
4603 perform CRES measurements at the lowest frequencies ever attempted with the technique.  
4604 LUCKEY will validate frequency scaling models developed by Project 8 and will pave  
4605 the way for the future Low-Frequency Apparatus (LFA), which will be a larger, 1 GHz  
4606 cavity CRES experiment that includes a molecular tritium source. The target for the  
4607 LFA is a measurement of the neutrino mass with a sensitivity of approximately 0.2 eV,  
4608 which will build towards the atomic pilot-scale CRES experiment.

4609 In parallel to the development of cavity CRES is the development of the atomic  
4610 tritium source. Recent demonstrations of the production of atomic hydrogen are excellent  
4611 steps towards the atomic tritium production needed for the pilot-scale experiment. One  
4612 area of future study includes the development of a more detailed unstanding of the  
4613 efficiency of atomic hydrogen production. Near-term plans include the development of a  
4614 magnetic, evaporatively cooled beamline, as well as the prototyping of a Halbach array  
4615 atoms trap. Nearly all of the components of the atomic tritium system will require  
4616 demonstration before the complete system can be built. The long-term goal of the  
4617 atomic tritium work is to construct a full atomic tritium prototype that demonstrates  
4618 the production, cooling, trapping, and recycling of tritium at the rates needed for the  
4619 pilot-scale experiment.

4620 More broadly, the long-term goal of the Project 8 collaboration is to fully develop  
4621 both the atomic tritium and cavity CRES technologies so that both can be combined in  
4622 a pilot-scale CRES experiment. It is envisioned that this process will take approximately  
4623 10 years for both atomic tritium and cavity CRES. After these developments comes  
4624 the pilot-scale experiment which will be the first CRES experiment that simultaneously  
4625 demonstrates all the required technologies for Phase IV. Scaling to Phase IV with cavity  
4626 CRES will require the construction of multiple copies (approximately 10) of the pilot-scale  
4627 experiment to obtain sufficient statistics for 40 meV sensitivity.

4628 Development of the CRES experimental technique by Project 8 has led to new  
4629 experiments utilizing the CRES technique for basic physics research, such as the  $^6\text{He}$ -  
4630 CRES collaboration [68], and has also found applications as a new approach to x-ray  
4631 spectroscopy [69]. Recently, a new experimental effort called CRESDA has begun in  
4632 the UK to develop new quantum technologies applied to CRES measurements for the  
4633 neutrino mass [70]. This flourishing of new experimental efforts based on the CRES  
4634 technique is likely to continue as Project 8 continues to develop the technique towards  
4635 its neutrino mass measurement goal.

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