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4 DEVELOPMENT OF SCALABLE APPROACHES TO NEUTRINO MASS 5 MEASUREMENT WITH THE PROJECT 8 EXPERIMENT

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by
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²⁰ Abstract

²¹ Neutrinos are fundamental particles in the standard model and play an important role
²² in the current understanding of the universe, however, the masses of the neutrinos, one
²³ of the most fundamental parameters for any particles, is currently unknown. This fact
²⁴ represents a gaping hole in our current knowledge of the universe that may provide clues
²⁵ to the energy scale of possible physics beyond the standard model. This dissertation
²⁶ summarizes research and development as a member of the Project 8 collaboration towards
²⁷ an experiment to measure the neutrino mass to a sensitivity below $50 \text{ meV}/c^2$, which
²⁸ is an order of magnitude below the most sensitive direct measurements of the neutrino
²⁹ mass to date. Project 8 will perform this measurement using Cyclotron Radiation
³⁰ Emission Spectroscopy (CRES) to measure the beta-decay endpoint spectrum of atomic
³¹ tritium. I present an analysis of the signal reconstruction performance of an antenna
³² array system designed to perform large-scale CRES measurements. Next, I discuss an
³³ approach to calibrating an antenna array CRES experiment using a unique probe antenna
³⁴ designed to mimic radiation from CRES events. Finally, I present design studies for a
³⁵ resonant cavity that could be used to perform a CRES experiment with atomic tritium
³⁶ at multi-cubic-meter scales.

Table of Contents

38	List of Figures	viii
39	List of Tables	xxvii
40	Acknowledgments	xxviii
41	Chapter 1	
42	Introduction	1
43	1.1 Summary	1
44	1.2 Outline	4
45	Chapter 2	
46	Neutrinos and Neutrino Masses	6
47	2.1 Introduction	6
48	2.2 Neutrinos and Beta-decay	6
49	2.3 Neutrino Oscillations	8
50	2.4 Neutrino Masses in the Standard Model	11
51	2.5 Neutrino Absolute Mass Scale	13
52	2.5.1 Limits from Cosmology	13
53	2.5.2 Limits from Neutrinoless Double Beta-decay Searches	15
54	2.5.3 Limits from Beta-decay	17
55	Chapter 3	
56	Direct Measurement of the Neutrino Mass with Project 8	21
57	3.1 Introduction	21
58	3.2 Project 8 and Cyclotron Radiation Emission Spectroscopy	22
59	3.2.1 Cyclotron Radiation Emission Spectroscopy — CRES	22
60	3.2.2 Project 8	26
61	3.2.3 The Project 8 Phased Development Plan	30
62	3.3 Phase II: First Tritium Beta Decay Spectrum and Neutrino Mass Measurement with CRES	34
63	3.3.1 The Phase II CRES Apparatus	34
64	3.3.2 CRES Track and Event Reconstruction	38
65	3.3.3 Results from Phase II	42
66	3.4 Phase III R&D: Antenna Array CRES	46

68	3.4.1	The Basic Approach	47
69	3.4.2	The FSCD: Free-space CRES Demonstrator	48
70	3.5	Pilot-scale Experiments	53
71	3.5.1	Choice of Frequency	54
72	3.5.2	Pilot-scale Experiment Concepts	56
73	3.6	Phase IV	59

74	Chapter 4		
75	Signal Reconstruction Techniques for Antenna Array CRES and the		
76	FSCD		61
77	4.1	Introduction	61
78	4.2	FSCD Simulations	62
79	4.2.1	Kassiopeia	63
80	4.2.2	Locust	67
81	4.2.3	CRESana	71
82	4.3	Signal Detection and Reconstruction Techniques for Antenna Array CRES	71
83	4.3.1	Digital Beamforming	76
84	4.3.2	Matched Filtering	85
85	4.3.3	Machine Learning	97
86	4.4	Analysis of Signal Detection Algorithms for the FSCD	102
87	4.4.1	Introduction	103
88	4.4.2	Signal Detection with Antenna Array CRES	104
89	4.4.2.1	Antenna Array and Data Rate Estimates	104
90	4.4.2.2	Real-time Signal Detection	106
91	4.4.3	Signal Detection Algorithms	109
92	4.4.3.1	Power Threshold	110
93	4.4.3.2	Matched Filtering	112
94	4.4.3.3	Machine Learning	116
95	4.4.4	Methods	118
96	4.4.4.1	Data Generation	118
97	4.4.4.2	Template Number and Match Estimation	119
98	4.4.4.3	CNN Training and Data Augmentation	120
99	4.4.5	Results and Discussion	121
100	4.4.5.1	Trigger Classification Performance	121
101	4.4.5.2	Computational Cost and Hardware Requirements	123
102	4.4.6	Conclusion	125

103	Chapter 5		
104	Antenna and Antenna Measurement System Development for the		
105	Project 8 Experiment		127
106	5.1	Introduction	127
107	5.2	Antenna Measurements for CRES experiments	128
108	5.2.1	Antenna Parameters	128
109	5.2.1.1	Radiation Patterns	128

110	5.2.1.2	Directivity and Gain	129
111	5.2.1.3	Far-field and Near-field	130
112	5.2.1.4	Polarization	131
113	5.2.1.5	Antenna Factor and Effective Aperture	132
114	5.2.2	Antenna Measurement Fundamentals	134
115	5.2.2.1	Friis Transmission Equation	134
116	5.2.2.2	S-Parameters and Network Analyzers	135
117	5.2.2.3	Antenna Array Commissioning and Calibration Measurements	136
119	5.2.3	The Penn State Antenna Measurement System	137
120	5.3	Development of a Synthetic Cyclotron Antenna (SYNCA) for Antenna Array Calibration	140
121	5.3.1	Introduction	141
123	5.3.2	Cyclotron Radiation Phenomenology	143
124	5.3.3	SYNCA Simulations and Design	149
125	5.3.4	Characterization of the SYNCA	155
126	5.3.5	Beamforming Measurements with the SYNCA	159
127	5.3.6	Conclusions	163
128	5.4	FSCD Antenna Array Measurements with the SYNCA	164
129	5.4.1	Introduction	164
130	5.4.2	Measurement Setups	165
131	5.4.2.1	FSCD Array Setup	165
132	5.4.2.2	Synthetic Array Setup	167
133	5.4.3	Simulations, Analysis, and Results	167
134	5.4.3.1	Simulations	168
135	5.4.3.2	Phase Analysis	169
136	5.4.3.3	Magnitude Analysis	173
137	5.4.3.4	Beamforming Characterization	174
138	5.4.4	Conclusions	176

139	Chapter 6		
140	Development of Resonant Cavities for Large Volume CRES Measurements	182	
141	6.1	Introduction	182
142	6.2	Cylindrical Resonant Cavities	183
143	6.2.1	General Field Solutions	183
144	6.2.2	TE and TM Modes	184
145	6.2.3	Resonant Frequencies of a Cylindrical Cavity	186
146	6.2.4	Cavity Q-factors	188
147	6.3	The Cavity Approach to CRES	192
148	6.3.1	A Sketch of a Molecular Tritium Cavity CRES Experiment	192
149	6.3.2	Magnetic Field, Cavity Geometry, and Resonant Modes	194
150	6.3.3	Trade-offs Between the Antenna and Cavity Approaches	197
152	6.4	Single-mode Resonant Cavity Design and Simulations	199

153	6.4.1	Open Cylindrical Cavities with Coaxial Terminations	199
154	6.4.2	Mode Filtering	203
155	6.4.3	Simulations of Open, Mode-filtered Cavities	204
156	6.5	Single-mode Resonant Cavity Measurements	207
157	6.5.1	Cavities and Setup	207
158	6.5.2	Results and Discussion	210
159	Chapter 7		
160	Conclusion and Future Prospects		214
161	Bibliography		217

List of Figures

162	2.1	A diagram of two different neutrino mass ordering scenarios [1]. In the inverted hierarchy (inverted mass ordering) the lightest neutrino mass is m_3 , whereas, in the normal hierarchy (normal mass ordering) m_1 is the lightest neutrino. What cannot be measured by neutrino oscillations is the neutrino absolute mass scale, which is essentially the mass of the lightest neutrino mass eigenstate.	9
163	2.2	The masses of the neutrinos as a function of the lightest neutrino mass in both the normal (a) and inverted (b) mass ordering regimes.	10
164	2.3	The neutrino mass observable measured by cosmology as a function of the lightest neutrino mass eigenstate.	14
165	2.4	Feynman diagrams for double beta-decay (a) and $0\nu\beta\beta$ (b).	15
166	2.5	The discovery probabilities for the future generation of $0\nu\beta\beta$ experiments as a function of $m_{\beta\beta}$ and m_{least} . Figure from [2].	16
167	2.6	A Feynman diagram of beta decay	17
168	2.7	A figure from Fermi's 1934 paper on a theory of beta-decay depicting the kinetic energy spectrum of the emitted electron. The effect of the neutrino mass, written as μ , is to distort the shape of the spectrum near the endpoint from the zero-mass spectrum.	17
169	2.8	The tritium beta-decay spectrum. The effect of a massive neutrino on the spectrum is to change its shape near the endpoint by an amount proportional to the size of the neutrino mass. A sufficiently high-statistic and high-resolution measurement of the spectrum endpoint would be able to measure the neutrino mass.	19
170			
171			
172			
173			
174			
175			
176			
177			
178			
179			
180			
181			
182			
183			
184			
185			

186	3.1	A cartoon illustration of the CRES technique. An electron is contained in 187 a magnetic trap, which is a local minimum in the magnetic field, so that 188 its cyclotron radiation can be detected by an array of antennas. Detecting 189 the cyclotron radiation allows one to measure its cyclotron frequency and 190 determine its kinetic energy.	23
191	3.2	An illustration of an electron in a bathtub magnetic trap generated by 192 two well-separated coils.	24
193	3.3	A plot of the main components of an electron's trajectory in a cylindrically 194 symmetric trap.	25
195	3.4	A plot of the final state distributions of atomic and molecular tritium. The 196 final state distribution provides the primary contribution to the width of 197 the molecular spectrum whereas thermal doppler broadening is responsible 198 for the width of the atomic spectrum.	27
199	3.5	Neutrino mass exclusion plot including limits from cosmological measure- 200 ments and the KATRIN experiment. Allowed ranges for neutrino masses 201 under the normal and inverted hierarchies are shown as the blue and 202 orange lines respectively. The black dashed line shows Project 8's goal 203 neutrino mass sensitivity for the Phase IV experiment.	30
204	3.6	Sensitivity calculations for a cavity based CRES experiment that demon- 205 strate the neutrino mass measurement goals of Project 8 throughout the 206 phased development plan. The blue curves indicate molecular tritium 207 sources and the red curves indicate atomic tritium sources. In the current 208 plan, Phase III contains two tritium experiments. The first is the Low- 209 frequency Apparatus (LFA), which is a molecular tritium experiment, and 210 the second is the atomic tritium pilot-scale experiment that officially ends 211 Phase III. The sensitivity of these experiments is primarily a function 212 of statistics, however, there is a critical density beyond which CRES 213 electrons do not have enough time to radiate between collisions for a 214 high-resolution frequency measurement leading to worse sensitivity. . . .	31
215	3.7	The Phase II CRES apparatus used to perform the first measurement of 216 the tritium beta-decay spectrum using CRES.	35
217	3.8	Diagram of the CRES cell portion of the Phase II apparatus.	36
218	3.9	RF system diagram for the Phase II apparatus.	37

219	3.10 The time-frequency spectrogram of a tritium CRES event in the Phase II 220 apparatus.	39
221	3.11 The sparse spectrogram obtained by placing a power cut on the raw 222 spectrogram shown in Figure 3.10.	41
223	3.12 Fits to the measured 17.8-keV ^{83m}Kr conversion line using the deep and 224 shallow trap configurations.	43
225	3.13 Measurements of the 17.8-keV ^{83m}Kr line using the deep trap configuration 226 for different values of the magnetic field from the field shifting solenoid. .	44
227	3.14 The measured tritium spectrum from Phase II with Bayesian and frequen- 228 tist fits.	45
229	3.15 A cartoon illustration of the basics of the antenna array CRES technique.	47
230	3.16 An image of the MRI magnet installed in the Project 8 laboratory at the 231 University of Washington, Seattle.	49
232	3.17 (a) A model of the FSCD antenna array, magnetic trap, and tritium 233 containment vessel design.(b) A more detailed model of a prototype 234 design for the 5-slot waveguide antenna design.	50
235	3.18 (a) A plot of the decay rate for the two-body dipolar spin exchange 236 interaction for cc and dd state. (b) A plot of the decay rate of the dipolar 237 spin exchange interaction for d+d states as a function of magnetic field 238 magnitude. Lowering the magnetic field is key for reducing the losses 239 from this interaction.	55
240	3.19 A conceptual sketch of a large-volume antenna array based CRES experi- 241 ment to measure the neutrino mass.	56
242	3.20 A conceptual sketch of a pilot-scale cavity CRES experiment with an 243 atomic tritium beamline.	58

244	3.21 An illustration of a possible arrangement of ten pilot-scale cavity experiments for Phase IV. The experiments are arranged in a circle with an approximate diameter of 50 meters. Each atomic beamline connected to the bottom of each cavity is approximately 10 m in length. The cavities themselves are designed to operate at 325 MHz and are approximately 11 m tall. The circular arrangement of cavities has some advantages when it comes to cancellation of fringe fields from neighboring magnets, which is important due to the small magnetic field magnitudes consistent with these CRES frequencies. The advantage of ten independent atomic sources and cavities is that there is no single point of failure for the experiment. If an experiment goes down for repairs the other nine may continue running. Figure courtesy of Michael Huehn at UW-Seattle.	60
256	4.1 The geometry and parameters of the coils used to simulate the FSCD magnetic trap in Kassiopeia. Some axial profiles of the magnetic trap at different radial positions are show to demonstrate the shape of the magnetic field and trap depth as a function of position. Calulation of the magnetic field profiles was graciously done by René Reimann.	64
261	4.2 A map of the average ∇B -drift frequency for electrons trapped in the prototype FSCD trap shown in Figure 4.1. Negative drift frequencies indicate electrons that are drifting opposite to the standard direction, which means that they are close to escaping the magnetic trap.	66
265	4.3 The receiver chain used by Locust when simulating CRES events in the FSCD.	69
267	4.4 A high-level diagram depicting the process of CRES event reconstruction. The first step consists of identifying the presence of a signal in the data. This step is necessary to avoid the danger of performing a reconstruction of a false event, which would constitute a background contribution to the tritium spectrum measured by CRES.	73
272	4.5 An illustration of two PDFs associated a binary hypothesis test. The decision threshold is represented by the vertical line that partitions both distributions. The orange and red areas correspond to the true negative and false positive probabilities and the blue and green areas correspond to the false negative and true positive probabilities respectively. To decided between the two hypotheses the likelihood ratio test specified by the Neyman-Pearson theorem is applied. This approach achieves the highest true positive probability for a given false positive probability.	74

280	4.6 An example ROC curve formed by computing the P_{FP} and the P_{TP} for 281 a given likelihood ratio test. As the decision threshold is increased P_{FP} 282 decreases at the expense of a lower P_{TP} . The black dashed line indicates 283 the lower bound ROC curve obtained by randomly deciding between \mathcal{H}_0 284 and \mathcal{H}_1	75
285	4.7 An illustration of the constructive interference condition which is the 286 operating principle of digital beamforming using a uniform linear array 287 as an example.	77
288	4.8 A system level diagram of the laboratory setup used for beamforming 289 demonstrations at Penn State. For more information on this system see 290 Chapter 5. Signals near 26 GHz are fed to a dipole antenna using an 291 arbitrary waveform generator (AWG) and vector network analyzer (VNA), 292 which drive a mixer. The dipole radiation is collected by an array of 293 antennas connected to the digitizer data acquisition (DAQ) system.	78
294	4.9 Photographs of the beamforming demonstration setup. In (a) I show a 295 top-down view of the dipole antenna and the array of eight horn antennas. 296 Manual repositioning of the horn antennas allows one to synthesize a full- 297 circular antenna array. The dipole antenna is mounted on a camera tripod 298 mount that allows for manual position tuning. (b) is a close up image of 299 the dipole, which is manufactured from two segments of semi-rigid coaxial 300 cable. (c) is another image of the dipole and array.	79
301	4.10 An example of digital beamforming reconstruction of a dipole antenna 302 using a synthetic array of horn antennas. The beamforming image on 303 the right is constructed by computing the time-averaged power of the 304 summed signals for a two-dimensional grid of beamforming positions. In 305 the image, one can see a clear maximum that corresponds to the position 306 of the dipole antenna. On the left I show the frequency spectrum of the 307 time-series at the maximum power pixel. White Gaussian noise is added 308 to the signal to mimic a more realistic signal-to-noise-ratio. The signal 309 emitted by the dipole is clearly visible as the high power peak in the 310 frequency spectrum.	80
311	4.11 Beamforming images visualizing the reconstruction of an electron without 312 (a) and with (b) the cyclotron phase correction. The images were generated 313 using data from Locust simulations. The cyclotron phase refers to a phase 314 offset equal to the relative azimuthal position of an antenna in the array. 315 This phase offset is caused by the circular electron orbit and must be 316 corrected for during reconstruction.	81

317	4.12 Beamforming images visualizing the reconstruction of an electron located 318 off the central axis of the FSCD trap. In (a) beamforming is being 319 performed without the ∇B -drift correction, and in (b) it is included.	82
320	4.13 A plot of a typical frequency spectrum obtained by applying a Fourier 321 transform to the time-series obtained from beamforming. The frequency 322 spectra are plotted without noise on top of an example of a typical noise 323 spectrum to visualize a realistic signal-to-noise ratio. In the example, 324 without beamforming it would not be possible to detect anything since 325 the signal amplitudes would be reduced by a factor of sixty relative to 326 the noise. Additionally, it is clear the ∇B -drift correction is needed to 327 detect this electron in the presence of noise.	84
328	4.14 A plot of the total signal power received by the FSCD array from trapped 329 electrons with different radial positions and pitch angles generated using 330 Locust simulations. The lines on the plot indicate a 10 dB detection 331 threshold above the mean value of the noise in the frequency spectrum. 332 With static beamforming electrons with radial positions larger than about 333 two centimeters are undetectable due to the change in the electron's 334 position over time causing losses from beamforming phase mismatch. This 335 is corrected by including ∇B -drift frequencies in the beamforming phases. 336 Both beamforming techniques fail to detect electrons below $\approx 88.0^\circ$, since 337 these signals are composed of several relatively weak sidebands that are 338 comparable to the noise.	85
339	4.15 Example of a convolution of a CRES signal template with a segment of 340 noisy data. A simulated CRES signal was simulated using Locust and 341 normalized to create a matched filter template. When this template is 342 convolved with noisy data the convolution output increases dramatically compared to data with only noise. 343 The decreasing convolution output as the time offset of the convolution 344 increases is caused by zero-padding of the data and template.	86
345	4.16 An example two-dimensional parameter distribution of a matched filter 346 template bank and random test signals. θ refers to the pitch angle of the 347 electron and E is the kinetic energy. The template bank forms a regular 348 grid in pitch angle and energy, whereas, the test signals are uniformly 349 distributed in pitch angle and follow the tritium beta-decay kinetic energy 350 distribution. This is why there are fewer test signals at higher energies. 351 The need for high match across the full parameter space prevents one from 352 reducing the density of templates in this low activity region. A zoomed in 353 version of the template bank illustrates the relative density of templates 354 and signals needed for match $> 90\%$	89

356	4.17 The matched filter scores of a dense grid of templates in pitch angle energy 357 space. Dense template grids allow one to estimate the kinetic energy of 358 the electron by identifying the best matching template. The uncertainty 359 on this value is proportional to the space of templates that also match 360 the test signal well. In the worst case matched filter templates can be 361 completely degenerate where templates with different parameters match 362 a signal with equal likelihood.	90
363	4.18 The mean match of the dense template grid shown in Figure 4.17 for 364 different numbers of templates. Grids of different sizes were obtained by 365 decimating a dense grid of templates and the average match for each grid 366 was computed using the same set of randomly distributed test signals. 367 Plotting the mean match against the size of the grid allows one to visualize 368 the exponential relationship between match and template bank size. The 369 noise in each curve is caused by sampling effects from the decimation 370 algorithm. In general, longer templates are harder to match than shorter 371 templates.	91
372	4.19 Matched filter template bank ROC curves as a function of mean match. 373 One can see that for low match a matched filter is on average worse than 374 the more straight forward beamforming detection approach.	91
375	4.20 Boundaries of detectable electrons in pitch angle kinetic energy space 376 for a series of different signal detection algorithms. A detectable signal 377 is defined as a signal that is above a consistent decision with at least 378 50% probability. This non-rigorous treatment of detection probability is 379 primarily useful for the visualization the relative increases in detection 380 performance provided by the different algorithms. The static beamforming 381 (Static-BF) algorithm is the digital beamforming algorithm introduced 382 above without the ∇B -drift correction. The DNN algorithm refers to a 383 convolutional neural network classifier trained to detect CRES signals 384 (see Section 4.3.3).	92
385	4.21 Two example illustrations of the correlation between kinetic energy and 386 pitch angle imparted by the shape of the FSCD magnetic trap. The 387 correlations manifest themselves as degeneracies in the matched filter 388 score where multiple matched filter templates have the same matched filter 389 for a particular signal. These degeneracies are a sign that the magnetic 390 trap must be redesigned in order to break the correlation between pitch 391 angle and kinetic energy.	96

392	4.22 A visualization of the correlation between energy and pitch angle in the FSCD magnetic trap. The image is formed by computing the match of the best template from a grid consisting of pitch angles from 84 to 90 degrees in steps of 0.05 degrees, kinetic energies from 17574 to 18574 eV, located at 2 cm from the central axis, and simulated for a length of three FSCD time-slices. The signals used to compute the best matching template consisted of a grid from 84 to 90 degrees in steps of 0.05 degrees, kinetic energies from 18550 to 18575 eV in steps of 0.25 eV, located 2 cm from the central axis, and simulated for three FSCD time-slices. The colored regions of the plot show how well signals with lower energy can match those of higher energy for the FSCD magnetic trap, which is proportional to the achievable energy resolution of the FSCD design.	97
404	4.23 A representation of a matched filter template bank as a convolutional neural network. The network has a single layer composed of the templates, which act as convolutional filters. The activation of the neural network is an absolute value followed by a max operator.	100
408	4.24 A graphical depiction of CRES signal detection using a CNN. A noisy segment of data is converted to a frequency series using digital beamforming and a FFT. The complex-valued frequency series is input into a trained CNN model that classifies the data as signal or noise using a decision threshold on the CNN output.	101
413	4.25 The detection efficiency and false alarm rate (false positive rate) as a function of the decision threshold for different values of the noise temperature. The model is trained to output a value close to one for data that contains a signal and outputs a value near zero when the data contains only noise. One sees that a lower decision threshold will have a high detection efficiency at the cost of a high rate of false alarms.	102
419	4.26 ROC curves for a CNN model classifying CRES signals. One can see that the area under the curve, which is a figure of merit that describes the performance of the classifier, is roughly linearly dependent with the noise temperature.	103

423	4.27 An illustration of the conceptual design for an antenna array CRES	
424	tritium beta-decay spectrum measurement. The antenna array geometry	
425	consists of a 20 cm interior diameter with 60 independent antenna channels	
426	arranged evenly around the circumference. The nominal antenna design	
427	is sensitive to radiation in the frequency range of 25-26 GHz, which	
428	corresponds to the cyclotron frequency of electrons emitted near the	
429	tritium beta-spectrum endpoint in a 0.96 T magnetic field. The array is	
430	located at the center of the magnetic trap produced by a set of current-	
431	carrying coils. The nominal magnetic trap design is capable of trapping	
432	electrons up to 5 cm away from the central axis of the array and traps	
433	electrons within an approximately 6 cm long axial region centered on the	
434	antenna array.	105
435	4.28 A block diagram illustration of the real-time triggering algorithm proposed	
436	for antenna array CRES reconstruction.	106
437	4.29 An illustration of the digital beamforming procedure. The blue lines	
438	indicate the distances from the beamforming position to each antenna.	
439	In the situation depicted the actual position of the electron matches the	
440	beamforming position, therefore, one expects constructive interference	
441	when the phase shifted signals are summed. To prevent the electron's	
442	∇B -motion from moving the electron off of the beamforming position,	
443	the beamforming phases include time-dependence to follow the trajectory	
444	of the electron in the magnetic trap.	107
445	4.30 Frequency spectra of simulated CRES events in the FSCD magnetic	
446	trap after beamforming. The signal of a 90° electron consists of a single	
447	frequency component that is clearly detectable using a power threshold	
448	on the frequency spectrum. This power threshold remains effective for	
449	signals with relatively large pitch angles such as 89.0° and 88.75°, which	
450	are composed of a main carrier and a few small sidebands. Signals with	
451	smaller pitch angles, below about 88.5°, are dominated by sidebands such	
452	that no single frequency component can be reliably distinguished from	
453	the noise with a power threshold.	109
454	4.31 PDFs of the power threshold test statistic for CRES signals with various	
455	pitch angles as well as the PDF for the noise-only signal case. The average	
456	PDF computed for pitch angles ranging from 85.5 to 88.5° is also shown.	
457	As the pitch angle is decreased the signal PDF converges towards the	
458	noise PDF which indicates that the power threshold trigger is unable to	
459	distinguish between small pitch angle signals and noise.	112

460	4.32 Plots of PDFs that describe the matched filter template bank test statistic for CRES signals with various pitch angles, as well as the estimated PDF for the noise only case. 10^5 matched filter templates are used and perfect match between signal and template i.e. $\Gamma_{\text{best}} = 1$ is assumed. The mean PDF includes signals ranging from $85.5 - 88.5^\circ$ in pitch angle. There is a larger distinction between the signal PDFs at small pitch angles compared to the power threshold, which indicates a higher detection efficiency for these signals.	116
468	4.33 The mean match of the matched filter template bank to a test set of randomly parameterized signals as a function of the number or density of templates. The parameter space includes pitch angles from $85.5 - 88.5^\circ$ and energies from $18575 - 18580$ eV.	119
472	4.34 Histograms of the trained CNN model output from the test dataset. The blue histogram shows the model outputs for signal data. The oddly shaped peak near the end is the result of the softmax function mapping the long tail of the raw output distribution to the range $[0, 1]$	121
476	4.35 ROC curves describing the detection efficiency or true positive rates for the three signal classification algorithms examined in this paper. The matched filter (MF) and Power Threshold curves are computed analytically using the distribution functions introduced in Section 4.4.3, and the CNN curve is computed numerically using the classification results on the test dataset. The percent match indicated in the legend refers to the mean match of the classifier.	122
483	5.1 An example radiation pattern generated using HFSS simulations. The color and radial distance of the surface from the origin indicate the relative magnitude of radiation power emitted by the antenna in that direction. The primary goal of most antenna measurements is typically to measure the antenna pattern, which is used to derive many useful antenna performance parameters.	128

489	5.2 An illustration of the three field regions important for the analysis of 490 an antenna system. Very close to the antenna the electric fields are 491 primarily reactive so there is no radiation. If a receiving antenna were 492 placed in this region most of the energy would be reflected back to the 493 transmitter. Outside of the reactive near-field is the radiative near field. 494 At these distances the antenna does radiate, but the radiation pattern is 495 not well-defined since it changes based on the distance of the receiving 496 antenna. It is only in the far-field region where the radiation pattern 497 becomes constant as a function of distance, which is where the majority 498 of antenna engineering is assumed to take place. The antenna arrays 499 developed by Project 8 for CRES measurements operate in the radiative 500 near-field due to the importance of limiting power loss from free-space 501 propagation, which complicates the design of the antenna system.	130
502	5.3 An illustration of the Friis measurement technique commonly used for 503 antenna characterization measurements.	134
504	5.4 Illustration of a two-port S-parameter measurement setup. S-parameters 505 characterize how incoming waves of voltage or power scatter off of the RF 506 device under test. This allows you to measure important properties of the 507 device. In particular, this framework can be used to model a two antenna 508 radiation pattern measurement, which can be automated using a VNA. . .	135
509	5.5 Two measurement approaches to characterizing an antenna array for CRES 510 measurements. The full-array approach (a) requires a complete antenna 511 array with all the associated hardware. The synthetic array approach 512 (b) utilizes a single antenna and a set of rotation/translation stages to 513 reposition the transmitter or the receiving antenna to synthesize the signals 514 that would be received by the full-array. This approach reduces the cost 515 and complexity of array measurements. A down-side of the synthetic 516 array approach is that multi-channel effects such as reflections cannot 517 be measured. Utilizing both the full-array and the synthetic array is a 518 powerful way to quantify the impact of errors from the multi-channel array.	136
519	5.6 Illustration of the antenna measurement system developed for the Project 520 8 Collaboration. The reference and test antennas can be connected to 521 different data acquisition configurations depending on the measurement 522 goals. The reference antenna is typically a standard horn antenna and 523 the test antenna is mounted on a set of translation stages for positioning. 524 Automated translation stages allow for relatively painless data-taking enabling 525 synthetic antenna array measurements using only a single receiving 526 antenna. Anechoic form designed to mitigate RF reflections surrounds 527 the setup.	138

528	5.7	Diagrams of two measurement system configurations. Configuration (a) utilizes a VNA and is more suited to antenna characterization. Configuration (b) utilizes an AWG (arbitrary waveform generator) and VNA as a signal generation system and digitizer to collect measurement data, which is more suited to simulating CRES measurements. The transmission chain utilizes a quadrature hybrid and a pair of baluns to drive the cross-dipole variant test antenna developed for synthetic CRES measurements.	139
535	5.8	A sketch of an antenna array large-volume CRES experiment. Electrons from β -decays are confined in a magnetic field using a set of trap coils. The cyclotron radiation produced by the motion of the trapped electrons can be detected by a surrounding antenna array to determine the electron energies. Measuring the energies of many electrons produces a β -decay spectrum.	141
541	5.9	A schematic of the antenna array test stand. The circular antenna array has a radius of 10 cm with 60 independent channels (limited number shown for clarity). The test stand includes an arbitrary waveform generator (AWG), local oscillator (LO), and data acquisition (DAQ) hardware. Finally, a specialized Synthetic Cyclotron Antenna (SYNCA) is used to inject signals to test the antenna array.	142
547	5.10	An electron (red dot) performing cyclotron motion in the x-y plane. The resulting cyclotron radiation is observed by an antenna located at the field point of interest.	143
550	5.11	A plot of the numeric solution to Equation 5.30. The time-domain representation of the signal (a) is composed of a zero frequency term and a series of harmonics separated by the main cyclotron frequency as shown in the plot of the frequency spectrum (b). We can see that the relative amplitude of the harmonics beyond $k = 7$ are smaller than the main carrier by a factor of about 10^{-5} and are completely negligible.	146
556	5.12	The amplitude maxima of the cyclotron radiation form an Archimedean spiral as the radiation propagates outward from the cyclotron orbit center (a). A circular antenna array located at a fixed radius from the orbit center will receive electric fields with equal magnitude in each of its channels, but the phase of the electric field incident on each array channel will be linearly out of phase from its neighbor antennas by an amount equal to the angular separation of the two channels (b).	148

563	5.13 Images of an early prototype crossed-dipole antenna manufactured by hand and the first measurement setup. The antenna was constructed by hand using four stripped coaxial cables. The antenna was connected to one port of the VNA, and the remaining three ports on the VNA were connected to horn antenna arranged with 90 deg offsets around the crossed-dipole. The measured unwrapped S-parameter phases exhibit the desired relative phase behavior for a SYNCA. These early measurements were the first laboratory proof-of-principle for the crossed-dipole SYNCA.	150
571	5.14 An early iteration of a crossed-dipole SYNCA antenna simulated in HFSS. The antenna is electrically small at 26 GHz, which requires dipole arms on the order of 1 mm long. This design is limited by the minimum achievable distance between the dipole arms caused by the available diameters of coaxial cables. The assumed termination scheme for the coaxial cables to the antenna is hand-soldering, which introduces random variation in the antenna pattern from the inevitable blobs of solder left on the surface of the PCB.	151
579	5.15 An idealized crossed-dipole antenna consists of two electric dipole antennas oriented perpendicular to each other and is fed with four signals with a quadrature phase relationship. An example antenna feed circuit is shown which is composed of a chained combination of a quadrature hybrid-coupler (Quad) and two baluns.	152
584	5.16 A model of the PCB crossed-dipole antenna with dimensions. The design has an inside diameter of 2.16 mm for the central circular trace, which is 0.13 mm wide. The dipole arms each have a width of 1.27 mm and protrude beyond the circular trace by 1.40 mm, which gives an overall width of 4.96 mm for the length of the antenna PCB trace from end-to- end. The overall size of the antenna is 20.0 mm the majority of which is the PCB dielectric material. This design was observed in simulation to maintain the field characteristics of the idealized crossed-dipole while being simpler to fabricate due to the increased size of the antenna.	153
593	5.17 A comparison of the electric field magnitudes, normalized by the maximum value of the electric field in each simulation, plotted on a 10 cm square to visualize the Archimedean spirals formed by the electron (a), the crossed- dipole antenna (b), and a PCB crossed-dipole antenna (c). The matching patterns indicate that the electric fields have similar phase characteristics. These images were generated using Locust simulations for the electron and ANSYS HFSS for both antennas.	154

600	5.18 A comparison of the normalized electric field magnitudes for the ideal	
601	crossed-dipole, PCB crossed-dipole, and a simulated electron as a function	
602	of the polar angle (θ). (a) Shows the total electric field, (b) shows the	
603	ϕ -polarized electric field component, and (c) shows the θ -polarized electric	
604	field component. These images were generated using Locust simulations	
605	for the electron and ANSYS HFSS for both antennas.	155
606	5.19 A comparison of the normalized electric field magnitudes for the crossed-	
607	dipole, PCB crossed-dipole, and a simulated electron as a function of the	
608	azimuthal angle (ϕ) evaluated at $\theta = 90^\circ$. This image was generated using	
609	Locust simulations for the electron and ANSYS HFSS for both antennas.	156
610	5.20 (a) A cartoon schematic which highlights the routing of the semi-rigid	
611	coax transmission lines. (b) A photograph of a SYNCA constructed	
612	using the modified crossed-dipole PCB antenna design. Visible in the	
613	photograph of the SYNCA are four blobs of solder which are an artifact	
614	of the SYNCA's hand-soldered construction. These solder blobs are the	
615	most significant deviation from the SYNCA design shown in Figure 5.16	
616	and are responsible for a significant fraction of the irregularities seen in	
617	the antenna pattern.	157
618	5.21 A schematic of the VNA characterization measurements (a). This setup	
619	allows for antenna gain and phase measurements across a full 360° of	
620	azimuthal angles using a motorized rotation stage and control of the radial	
621	position of the SYNCA using a translation stage. A photo of the setup in	
622	the lab is shown in (b).	158
623	5.22 Linear interpolations of the measured electric field magnitude (a) and	
624	phase (b). The data was acquired using a VNA at 120 points spaced	
625	by 3 degrees from 0 to 357° of azimuthal angle. The different	
626	color lines indicate the vertical offset of the horn antenna relative to the	
627	SYNCA PCB and the dashed line shows the expected shape from electron	
628	simulations. No significant difference in the antenna pattern is observed	
629	for the measured vertical offsets.	159
630	5.23 (a) A depiction of the relative phase differences for signals received by a	
631	circular antenna array from an isotropic source. The phases correspond to	
632	a unique spatial position. (b) A schematic of the setup used to perform	
633	digital beamforming.	160

634	5.24 Digital beamforming maps generated using a simulated 60 channel array and electron simulated using the Locust package. (a) and (b) show the beamforming maps for simulated electrons without the cyclotron spiral phases and with the cyclotron spiral phases respectively. (c) and (d) show the beamforming maps produced from SYNCA measurements. We observe good agreement between simulated electrons and the SYNCA measurements.	161
641	5.25 A plot of the SYNCA's reconstructed position using the synthesized horn- antenna array and digital beamforming. (a) Shows the reconstructed position of the SYNCA compared with the target position indicated by the positioning system readout. (b) Shows the reconstruction error, which is the difference between the target and reconstructed positions. The error bars in (b) are the uncertainty in the mean position of the 2D Gaussian used to fit the digital beamforming reconstruction peak obtained from the fit covariance matrix. The mean fit position uncertainty of 0.02 mm is an order of magnitude smaller than the typical reconstruction error of 0.3 mm obtained by calculating the standard deviation of the difference between the reconstructed and target position.	162
652	5.26 A diagram of the array measurement system used to test the prototype FSCD antenna array. A VNA is used as the primary measurement tool, which is connected to the array through a series of switches. The other port of the VNA connects to the SYNCA through the quad-balun chain used to provide the SYNCA feed signals. During measurements the SYNCA is positioned inside the center of the antenna array and translated to different radial and axial positions using a 3-axis manual translation stage setup.	164
660	5.27 Photos of the prototype FSCD antenna (a), the FSCD array and SYNCA (b), and the translation stages and coordinate system used to position the SYNCA (c).	166
663	5.28 A photo of the FSCD antenna and the SYNCA in the synthetic array measurement setup at Penn State.	168
665	5.29 The unwrapped phases of signals received by the FSCD antenna array from an electron with a 90° pitch angle located in the plane of the antenna array. The data points indicated the phases extracted from simulation and the dashed lines show the model predictions.	169

669	5.30 Plots of the measured unwrapped phases from the FSCD array (a) and the 670 synthetic array (b) compared to the model predictions for a series of radial 671 positions. The different phases of the sinusoidal phase oscillations in the 672 two plots reflects differences in the coordinate systems of the measurements.	170
673	5.31 The phase errors between the measurement and model for the synthetic 674 array (blue) and the FSCD array (orange) for a series of radial positions. 675 The label JUGAAD refers to an alternative name for the FSCD array 676 setup. As the SYNCA is translated off-axis phase errors with progressively 677 higher oscillation frequency enter into the measurements.	171
678	5.32 Two dimensional plots of the phase errors for the synthetic array (a) 679 and the FSCD (JUGAAD) array (b). In both plots there is evidence 680 of a similar diffraction pattern with bilateral symmetry, but the FSCD 681 array measurments have an additional phase error contribution from the 682 different antennas and paths through the switch network.	177
683	5.33 The amplitude of the signals from CREsana for the FSCD array from a 684 90° electron. As the electron is moved from $R = 0$ the signals begin to 685 have unequal amplitudes depending on the distance from the electron to 686 the antenna.	178
687	5.34 The normalized magnitudes of the S21 parameters measured in the FSCD 688 (orange) and synthetic array (blue) setups. The dominant observed 689 behavior as a function of radius is the increase in the number of magnitude 690 peaks, which was noted in the phase error curves. There does not appear 691 to be a strong change in the relative amplitude of a group of antennas as 692 predicted by CREsana.	178
693	5.35	179
694	5.36 Beamforming images from the synthetic array (a) and FSCD array (b) 695 setups with the SYNCA positioned 15 mm off the central axis. In both 696 images, there is a clear maxima that corresponds to the true SYNCA 697 position. However, in the FSCD array there is an additional faint peak 698 located at the opposite position of the beamforming maximum. This 699 additional peak is the mirror of the true peak and is the result of reflections 700 between antennas in the FSCD array.	180

701	5.37 A comparison of the maximum signal amplitude obtained by beamforming 702 to the signal amplitude obtained with an ideal summation as a function of 703 the radial position of the SYNCA. The amplitudes for the synthetic array 704 are shown in (a) and the FSCD array are shown in (b). In both setups, 705 the signal amplitudes obtained from beamforming are smaller than the 706 signal amplitude that could be attained with the ideal summation without 707 phase mismatch.	180
708	5.38 The ratio of the beamforming signal amplitude to the ideal signal ampli- 709 tude for the FSCD and synthetic arrays. The FSCD array has a larger 710 power loss from phase error compared to the synthetic array which indi- 711 cates that calibration errors associated with the multiple channels as well 712 as reflections are impacting the signal reconstruction.	181
713	6.1 Geometry of a cylindrical waveguide with radius b	183
714	6.2 The geometry of a cylindrical cavity with length L and radius b	186
715	6.3 Relation of mode frequency to cavity length for a cylindrical cavity with 716 a radius of 18.32 cm.	187
717	6.4 A series RLC circuit.	188
718	6.5 Illustration of the behavior of the input impedance of the series RLC 719 circuit as a function of the driving frequency. The BW is proportion to 720 the width of the resonance, which is inversely proportional to Q.	190
721	6.6 A series RLC circuit coupled to an external circuit with input impedance 722 R_L	191
723	6.7 A cartoon depiction of a cavity CRES experiment. A metallic cavity filled 724 with tritium gas is inserted into a uniform background magnetic field 725 to perform CRES measurements. Electrons from beta-decays inside the 726 cavity can be trapped and used to excite a resonant mode(s). By coupling 727 to the cavity mode with a suitable probe one can measure the cyclotron 728 frequency of the electron and perform CRES.	192

729	6.8 Examples of the resonant mode frequencies of a cylindrical cavity. This 730 cavity has a radius of 18.32 cm and a length to diameter ratio of 4.55. Sev- 731 eral families of resonant modes are relevant in the ≈ 800 MHz bandwidth 732 of (a); however, after zooming in to a ~ 80 MHz bandwidth centered 733 on TE011 one sees that only a handful of resonant modes have frequen- 734 cies close to TE011. Since the bandwidth required for a cavity CRES 735 experiment is $O(10)$'s MHz, a significant number of resonant modes can 736 be safely ignored since their resonant frequencies are far from the CRES 737 bandwidth.	196
738	6.9 An image of an open cavity with coaxial terminations used for dielectric 739 constant measurements. Figure from [3].	200
740	6.10 The simplified geometry of an open cavity with coaxial terminations. 741 Figure from [4].	201
742	6.11 Electric field regions for the open cavity boundary value problem. Figure 743 from [4].	201
744	6.12 Two mode filtering concepts to break the degeneracy of TE ₀₁ and TM ₁₁ 745 modes. The resistive approach uses dielectric materials to impede currents 746 that travel vertically along the cavity while leaving azimuthal currents 747 unperturbed. An alternative approach is to impede the currents using 748 grooves cut into the cavity wall, which achieve the same effect with an 749 inductive impedance.	204
750	6.13 Four cavity design variations. (a) is a standard sealed cylindrical cavity, 751 (b) is an open cavity with smooth walls, (c) is an open cavity with walls 752 made of alternating conductor and dielectric, and (d) is an open cavity 753 with grooved walls. The main cavity and coaxial terminator parameter 754 are identical for all four cavities.	205
755	6.14 The frequencies and Q-factors of the resonant modes identified by HFSS 756 for the cavity variations shown in Figure 6.13. The fully-sealed cavity with 757 smooth walls has several high-Q modes near the TE ₀₁₁ resonance. Intro- 758 ducing the open-termination preserves the Q-factors of the TE _{01ℓ} modes 759 and suppresses the Q-factors of the modes whose boundary conditions do 760 not match the cylindrical partition. Both the resistive and grooved wall 761 perturbations shift the resonant frequencies of the TM modes away from 762 the TE ₀₁₁ mode. By properly tuning the geometry of the grooves or the 763 resistive spacers several MHz of frequency separation can be achieved. . .	206

764	6.15 A cartoon depicting the design of the open-ended cavity prototype designed 765 to operate at approximately 6 GHz. The main cavity wall was composed of 766 a single copper pipe. A mode-filtered version of this cavity was constructed 767 by	208
768	6.16 Images depicting the measurement of the filtered and non-filtered open 769 cavities using the VNA. The coupling loop in the figure is shown in the 770 TE orientation.	209
771	6.17 HFSS simulation results for a the as-built cavity with the coaxial termi- 772 nators removed. The TE_{011}/TM_{111} frequency is approximately 5.78 GHz.	210
773	6.18 Measurements of the filtered and non-filtered prototype cavities acquired 774 with the VNA.	211

List of Tables

775	4.1	A summary of the CNN model layers and parameters. The output of each 1D-Convolution and Fully Connected layer is passed through a LeakyReLU activation function and re-normalized using batch normalization before being passed to the next layer in the model. The output of the final Fully Connected layer in the model is left without activation so that the model outputs can be directly passed to the Binary Cross-entropy loss function used during training. The first layer in the network has two input channels for the real and imaginary components of the spectrum. . .	117
784	6.1	A table of the values of p'_{nm}	185
785	6.2	A table of the values of p_{nm}	186
786	6.3	A table of cavity design parameters used for HFSS simulations.	205
787	6.4	A table of parameters describing the cavity prototypes. Certain values such as the cavity length and the distance between dielectric spacers are approximate due to variation in the machining of the copper. In particular, the filtered cavity was constructed from conducting copper segments that varied in size from 1.50" to 1.85".	208

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⁷⁹⁹ my dissertation and ability as a scientist. I am honored to have been a member of this
⁸⁰⁰ team.

⁸⁰¹ **Dedication**

⁸⁰² To my parents, Chris and Doug, and my sister, Hannah. And to all those friends and
⁸⁰³ mentors who have shared this strange and beautiful thing called "life" with me.

804 **Chapter 1** |
805 **Introduction**

806 **1.1 Summary**

807 Neutrinos are one of the fundamental particles that comprise the standard model of
808 particle physics and account for a significant fraction of the matter in the universe.
809 Neutrinos are the most abundant fermions in the universe, but due to their weak
810 interactions neutrinos seldom interact with other particles. Regardless, neutrinos play a
811 unique role in the evolution of the early-universe, and a detailed understanding of the
812 properties of the neutrino is key to understanding the universe at the cosmological scale
813 as well as the smallest particle physics regime.

814 It was uncertain that neutrinos had nonzero mass until vacuum neutrino flavor
815 oscillations were observed in the late 90's and early 00's. A simple relativistic argument
816 as to why oscillations are evidence for neutrino masses is that oscillations imply neutrinos
817 experience time, which means that they do not propagate at the speed of light, therefore
818 the masses of the neutrinos must be non-zero. Current neutrino oscillation data supports
819 that neutrino flavor states are actually a superposition of three separate neutrino states
820 with well-defined masses. Measurements of neutrino oscillations that have taken place
821 over the past couple of decades have measured the differences between neutrino mass
822 eigenstates with increasing precision. However, oscillation measurements cannot tell
823 us the mass scale of the neutrinos, which is required in order to measure the absolute
824 neutrino masses.

825 The neutrino mass scale remains an unknown quantity in the standard model of
826 particle physics. The value of the neutrino mass influences the evolution of the early
827 universe and is likely relevant to the energy-scale of new physics responsible for the factor
828 of 10^{-6} difference between the neutrino and electron masses. A model-independent way
829 to measure the neutrino mass is to measure the tritium beta-decay spectrum near its
830 endpoint. Energy conservation requires that the neutrino mass carry away some kinetic

831 energy from the beta-decay electron in the form of its mass, which causes a distortion in
832 the shape of the tritium beta-decay spectrum near the endpoint. The isotope tritium has
833 many advantages for this measurement, and has been used by the KATRIN collaboration
834 to perform the most sensitive direct neutrino mass measurement to date.

835 KATRIN represents the state-of-the-art in the current generation of neutrino mass
836 direct measurement experiments with a projected neutrino mass sensitivity of $m_\nu < 200$ meV.
837 This sensitivity does not fully exhaust the allowed parameter space of neutrino
838 masses under the normal and inverted neutrino mass ordering scenarios, which motivates
839 the development of a next generation of neutrino mass measurement experiments.

840 The Project 8 collaboration is developing a next-generation neutrino mass experiment
841 with a goal neutrino mass sensitivity of $m_\nu < 40$ meV. This sensitivity is sufficient to
842 exhaust the range of neutrino masses allowed under the inverted mass ordering regime.
843 Project 8 intends to achieve its sensitivity goal utilizing two technologies that are novel
844 to the space of direct neutrino mass measurements — atomic tritium and cyclotron
845 radiation emission spectroscopy (CRES). Atomic tritium is required in order to avoid
846 systematic broadening the tritium beta-decay spectrum caused by the final state of the
847 $^3\text{He}^+ - \text{T}$ molecule, and the CRES technique enables a differential measurement of the
848 tritium spectrum that is background-free and able to be directly integrated with the
849 atomic tritium source.

850 The Project 8 collaboration is currently engaged in a research and development
851 program intended to simultaneously develop the atomic tritium and CRES technologies
852 so that they can be combined in a next-generation experiment. This past year (2022)
853 Project 8 has used the CRES technique to measure the molecular tritium beta-decay
854 spectrum and place an upper limit on the neutrino mass: $m_\beta \leq 152$ eV. This measurement,
855 while not competitive scientifically, represents the first proof-of-principle that the CRES
856 technique can be used to measure the neutrino mass.

857 The future goals of the Project 8 collaboration are to develop the technologies
858 and techniques necessary to scale-up the volume in which CRES measurements can
859 be performed. Project 8's first neutrino mass measurement with CRES utilized a
860 measurement volume on the cubic-centimeter scale, however, sensitivity calculations
861 estimate that an experiment sensitive to neutrino masses of 40 meV will require several
862 tens of cubic-meters of experiment volume filled with atomic tritium. Developing a new
863 approach to performing CRES measurements that can be successfully scaled to these
864 volumes is a necessary step towards Project 8's neutrino mass measurement goal, and is
865 the primary topic of my dissertation research.

866 A parallel development is the technology necessary to produce, cool, trap, and
867 recirculate a supply of atomic tritium that is compatible with CRES measurements. The
868 atomic tritium system is equally important as the large-volume CRES measurement
869 technology, but will not be discussed at depth here.

870 The Project 8 collaboration has identified two scalable approaches to neutrino mass
871 measurement using the CRES technique. One approach is to use an array of antennas
872 that surrounds a volume of trapped atomic tritium that can perform CRES measurements
873 by collection the cyclotron radiation emitted by beta-decay electrons into free-space. The
874 other approach uses a resonant cavity filled with atomic tritium to perform CRES by
875 measuring the excitation of resonant cavity modes caused by the motion of electrons
876 trapped inside the cavity volume.

877 The cavity and antenna approaches to CRES have been studied in detail over the past
878 five years, and, while both approaches offer a physically viable path towards a 40 meV
879 neutrino mass measurement, the collaboration has elected to pursue the cavity approach
880 for the foreseeable future. The major advantage of the cavity approach is a significant
881 reduction in the cost and complexity of the experiment design and data analysis, which
882 provides a lower risk path to Project 8’s scientific goals.

883 In this dissertation I summarize my most impactful contributions to the research and
884 development of antenna array and cavity CRES. In short these contributions are

- 885 • the development and analysis of signal reconstruction algorithms for antenna array
886 CRES, which provide key inputs to sensitivity analyses of antenna array CRES
887 experiments.
- 888 • The development of a specialized antenna, designed to synthesize fake CRES
889 radiation, which enables bench-top testing and validation of the antenna array
890 CRES technique.
- 891 • The development of an open-cavity design for CRES measurement, whose mode
892 structure can be tuned using perturbations that modify the impedance of the cavity
893 walls. The development of this cavity concept was one of many developments that
894 eventually lead to the adoption of cavities as the CRES technology of choice for
895 the future of Project 8.

896 1.2 Outline

897 The outline of this dissertation is as follows. In Chapter 2 I provide an introduction to
898 the basic physics of neutrinos and beta-decay, which provides context for a discussion of
899 various methods to measure the neutrino absolute mass scale.

900 Chapter 3 is an overview of the CRES technique and the Project 8 collaboration.
901 Project 8's experimental program is organized into four phases. The first phase completed
902 in 2015 before I began my dissertation work, so begin by highlighting the Project 8's first
903 measurement of the tritium beta-decay spectrum with CRES. Next, I discuss the planned
904 research and development for an antenna array CRES experiment for the upcoming phase
905 of Project 8's experimental program. I end Chapter 3 with a discussion of Project 8's
906 pilot-scale and final phase experiments, that will combine a scalable CRES measurement
907 technology with atomic tritium and measure the neutrino mass with 40 meV sensitivity.

908 Chapter 4 discusses the first of my contributions mentioned above, which is the
909 development of signal reconstruction techniques for antenna array CRES and an antenna
910 array demonstrator experiment called the FSCD. I discuss the key tools that Project 8
911 uses to simulate antenna array CRES before introducing signal reconstruction algorithms
912 that can be used to detect CRES signals using the array. I end Chapter 4 with a
913 detailed analysis and comparison of the signal detection performance of each algorithm,
914 as reported in a paper I have authored.

915 Chapter 5 describes my contributions to the development of antennas and an antenna
916 measurement system for Project 8, which is the second major contribution of this
917 dissertation. I begin with a general overview of basic principle of antennas and antenna
918 measurements, and describe the development, as reported in another paper I have
919 authored, of unique antenna designed to mimic the cyclotron radiation emitted by
920 electrons in free-space. I call this antenna the synthetic cyclotron radiation antenna
921 (SYNCA) and its main purpose is to serve as a fake electron for laboratory validation
922 measurements of Project 8's antenna array CRES simulations. Chapter 5 ends with an
923 overview of laboratory measurements of a prototype antenna array using the SYNCA,
924 which were compared with simulations to provide upper bounds on reconstruction errors
925 caused by imperfections in real-life measurements.

926 Chapter 6 discusses the cavity approach to CRES, which was adopted as the preferred
927 CRES technology for Phase IV late into my dissertation work. The chapter starts by
928 discussing resonant cavities in general before introducing the operating principles of the
929 cavity approach to CRES. I end the chapter by discussing a study of and open-cavity

⁹³⁰ design that could be used for CRES measurements and integrated with atomic tritium
⁹³¹ and an electron gun calibration source for the pilot-scale and Phase IV experiments.

⁹³² Finally, in Chapter 7 I conclude by briefly discussing the future directions of Project
⁹³³ 8 as development proceeds towards a direct measurement of the neutrino mass.

934 **Chapter 2 |**

935 **Neutrinos and Neutrino Masses**

936 **2.1 Introduction**

937 In this chapter I provide a cursory overview of background information relevant to
938 neutrinos and neutrino mass measurements.

939 In Section 2.2 I provide background information on the history of neutrinos and beta-
940 decay. In Section 2.3 I describe the discovery of neutrino oscillations, which demonstrated
941 unambiguously that neutrinos have non-zero masses. In Section 2.4 I discuss the current
942 state of the theoretical understanding of neutrino masses in the standard model. Lastly,
943 in Section 2.5 I discuss a few methods for measuring the absolute scale of the neutrino
944 mass.

945 **2.2 Neutrinos and Beta-decay**

946 Late in the 19th century the phenomena of radioactivity was first observed in experiments
947 performed by Henri Becquerel with uranium, and further studied using thorium and
948 radium by Marie and Pierre Curie [5,6]. Early work in radioactivity classified different
949 forms of radiation based on it's ability to penetrate different materials. Rutherford was
950 the first to separate radioactive emissions into two types, alpha and beta radiation [7].
951 Alpha rays were easily stopped by a piece of paper or thin foil of metal, whereas beta
952 radiation could penetrate metal several millimeters thick. Later a third form of radiation
953 was identified by Villard [8], which was still more penetrating, later termed gamma
954 radiation by Rutherford.

955 When these forms of radioactivity were first discovered, it was unclear what physically
956 constituted an alpha, beta, or gamma particle. Experiments with radioactivity in magnetic
957 fields were eventually able to identify the charge composition of the different forms of

radiation. In particular, experiments by Becquerel identified [9] that beta radiation had an identical charge-to-mass ratio to the electron. This was strongly suggestive that beta particles were indeed electrons.

Studies of beta radiation lead to the discovery that radioactivity resulted in the transmutation of elements [10] caused by the decay of a heavier nucleus to a lighter species. A decay that produces beta-radiation is called a beta-decay. One feature of beta radiation that differentiated it from alpha and gamma radiation is that the electrons produced by beta-decay have a continuous spectrum of kinetic energies, whereas, alpha and gamma particles are emitted with discrete energies. This feature of beta-decay was first observed by Chadwick in 1914 [11], and was extremely puzzling at the time, since the continuous spectrum apparently violates energy conservation [12].

Famously, in 1930 Pauli proposed the existence of a new neutral particle, which he termed the "neutron", that was also produced during beta-decay to resolve the missing energy problem posed by the beta-decay spectrum [13]. Because this particle carried no charge, it was hypothesized that it had simply not been observed in any previous experiments. This "neutron", which was initially estimated to have a mass no larger than that of an electron, was eventually renamed the "neutrino" by Fermi [14] after the discovery of the neutron by Chadwick in 1932 [15]. Later, in 1933, Fermi developed a quantum mechanical theory for beta-decay in which an electron and neutrino are produced by the decay of a neutron to a proton inside the radioactive nucleus [16].

Little more than a speculation when first introduced, indirect evidence for the existence of neutrinos was obtained in 1938 by the simultaneous observation of the electron and recoiling nucleus in cloud chambers by Crane and Halpern [17]. However, it wasn't until the Cowan-Reines experiment [18] in 1956 that direct evidence for the existence of neutrinos was observed through the observation of inverse beta-decays caused by neutrinos from a nuclear reactor interacting with protons contained in water molecules. The difficulty in detecting neutrinos is caused by their weak interactions with other particles. Later experiments revealed the existence of different types or flavors of neutrinos based on the nature of the leptons produced in neutrino charged-current interactions [19], but the existence of a neutrino mass remained an open question that would take more than 40 years to resolve.

989 2.3 Neutrino Oscillations

990 One of the first clues that neutrino flavor transitions or neutrino oscillations were occurring
 991 was the solar neutrino problem. The solar neutrino problem is a discrepancy between
 992 the measured and predicted flux of ν_e from the sum. The solar neutrino problem was
 993 famously observed by Ray Davis Jr. and collaborators in the 1960's [20] at the Homestake
 994 mine in South Dakota. In the early 2000's, the SNO experiment was able to resolve the
 995 solar neutrino problem by identifying neutrino oscillations as the cause of the observed
 996 deficit [21]. Furthermore, measurements of the atmospheric flux of neutrinos by the
 997 Super-Kamiokande experiment and others revealed that fewer muon-type neutrinos
 998 survived passage through the earth than expected providing strong evidence for neutrino
 999 oscillations for both flavors [22].

1000 Neutrino oscillations occur because the neutrino flavor eigenstates are distinct from
 1001 the mass eigenstates [23]. The neutrino mass eigenstates represent physical particles in
 1002 that they are solutions to the free-particle Hamiltonian, whereas, the neutrino flavor
 1003 eigenstates correspond to the neutrino states that interact via the weak charged-current
 1004 interaction. The neutrino flavor eigenstates are a linear superposition of the neutrino
 1005 mass eigenstates

$$1006 \nu_\ell = \sum_i U_{\ell i} \nu_i, \quad (2.1)$$

1007 where $\ell = e, \mu, \tau$ and $i = 1, 2, 3$. The matrix elements $U_{\ell i}$ are the elements of the
 1008 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix that describes the mixing between
 the neutrino flavor and mass states.

1009 A standard parameterization [24] of the PMNS matrix is

$$1010 U_{PMNS} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \\ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.2) \\ \times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

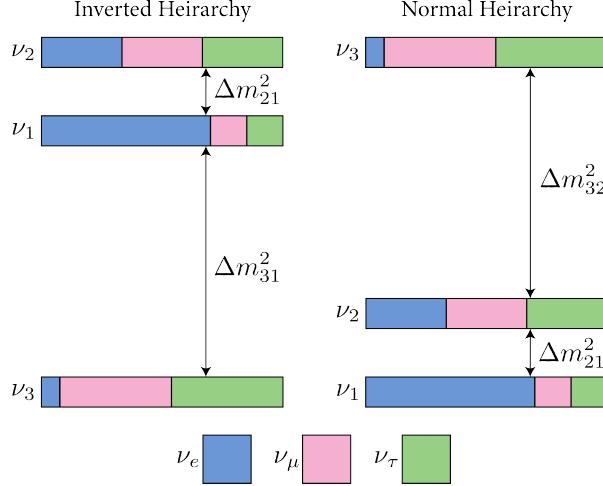


Figure 2.1. A diagram of two different neutrino mass ordering scenarios [1]. In the inverted hierarchy (inverted mass ordering) the lightest neutrino mass is m_3 , whereas, in the normal hierarchy (normal mass ordering) m_1 is the lightest neutrino. What cannot be measured by neutrino oscillations is the neutrino absolute mass scale, which is essentially the mass of the lightest neutrino mass eigenstate.

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. The parameters α_1 and α_2 are only included in the PNMS matrix if neutrinos are Majorana particles, something which represents a current area of research in neutrino physics. The phase δ quantifies the degree of CP-violation in the neutrino sector. Including the Majorana phases the PMNS matrix contains six independent parameters. Neutrino oscillation probabilities also depend on the squared mass differences between neutrino mass eigenstates

$$\Delta m_{ij}^2 = m_i^2 - m_j^2, \quad (2.3)$$

where $ij = 12, 32, 31$ respectively. Because $\Delta m_{32}^2 = \Delta m_{31}^2 - \Delta m_{21}^2$, this adds an additional two parameters that must be constrained by neutrino oscillations.

A large experimental effort over the past couple decades has greatly contained the majority of parameters in the PMNS matrix, many to relative uncertainties of only a few percent. However, certain ambiguities remain, which is the origin of the current uncertainty in the ordering of the neutrino masses (see Figure 2.1). The neutrino masses can be arranged by their relative masses. Current neutrino oscillation data supports that $m_2 > m_1$, however, the sign of Δm_{32}^2 is still unknown. Therefore, two mass-ordering scenarios are allowed, one where neutrino masses are arranged $m_3 > m_2 > m_1$, which is called the normal mass ordering (NMO), or alternatively neutrino masses may be ordered $m_2 > m_1 > m_3$, which is called the inverted mass ordering (IMO). Next-

1027 generation neutrino oscillation experiments such as JUNO [25], Hyper-Kamiokande [26],
1028 and DUNE [27] are poised to resolve this ambiguity in the coming years.

1029 Neutrino oscillation probabilities are sensitive to the neutrino masses via the squared
1030 mass differences. Therefore, oscillation probabilities are unaffected by the absolute scale
1031 of the neutrino mass. However, oscillations can be used to obtain a lower bound on the
1032 neutrino masses by setting the mass of the lightest neutrino mass state to zero. This
1033 results in different lower limits depending on the ordering of the neutrino mass states.
1034 Current best-fit values [24] with 1σ -uncertainties for the squared mass differences are

$$\Delta m_{21}^2 = (7.42^{+0.21}_{-0.20}) \times 10^{-5} \text{ eV}^2, \quad (2.4)$$

$$\Delta m_{31}^2 = (2.5176^{+0.026}_{-0.028}) \times 10^{-3} \text{ eV}^2 \text{ (NMO)}, \quad (2.5)$$

1035 for the normal mass ordering, and for the inverted ordering the limit is

$$\Delta m_{32}^2 = (-2.498^{+0.028}_{-0.028}) \times 10^{-3} \text{ eV}^2 \text{ (IMO).} \quad (2.6)$$

1036 The parameter Δm_{21}^2 is the same in the NMO and the IMO. Allowing the lightest neutrino
1037 mass in each ordering scenario (m_{least}) to take on a range of values, one can visualize the
1038 relative masses of the neutrinos as a function of m_{least} (see Figure 2.2). The absolute
1039 neutrino mass scale is effectively the value of this m_{least} parameter.

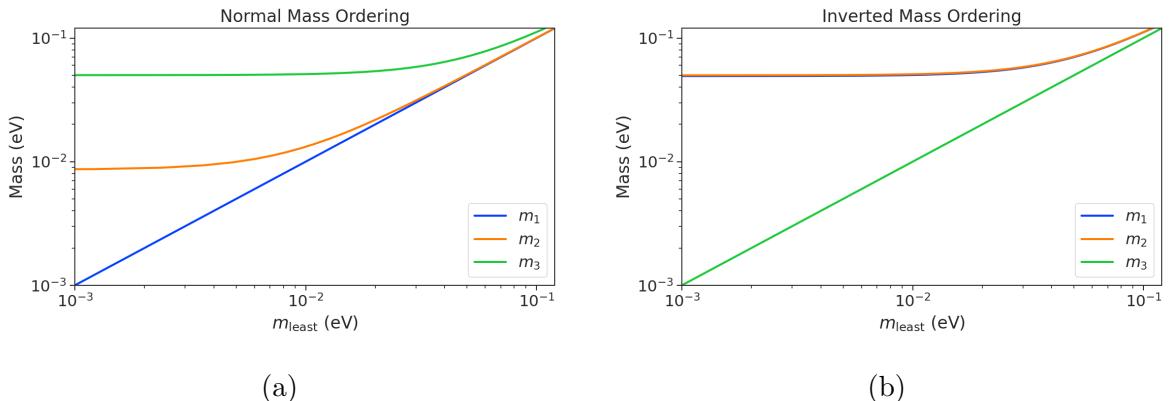


Figure 2.2. The masses of the neutrinos as a function of the lightest neutrino mass in both the normal (a) and inverted (b) mass ordering regimes.

2.4 Neutrino Masses in the Standard Model

In this section, I briefly summarize the current theoretical understanding of neutrino masses in the standard model [28–30]. Neutrinos are spin 1/2 particles, which are described using the Dirac equation.

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi(x) = 0, \quad (2.7)$$

where the field that describes the particle is denoted as $\psi(x)$. In the standard model fermions acquire mass through the Yukawa interaction, which add to the standard model Lagrangian terms of the form

$$\mathcal{L}_{\text{Yukawa}} = -Y_{ij}^\ell \bar{L}_{Li} \phi E_{Rj} + \text{h.c.}, \quad (2.8)$$

where Y_{ij}^ℓ is an element of the 3×3 Yukawa coupling matrix for leptons, L_{Li} is the left-handed lepton doublet for generation i , ϕ is the Higgs doublet, and E_{Rj} is the right-handed lepton field for generation j . Neutrinos are represented only as left-handed neutrinos and right-handed antineutrinos in the standard model, which is consistent with experimental observations. Since there are no right-handed neutrino singlet fields, there are no Yukawa interaction terms, thus neutrinos in the standard model are strictly massless. Therefore, non-zero neutrino mass is evidence for physics beyond the standard model.

For the charged leptons, the Yukawa interaction leads to masses of the form

$$m_{ij}^\ell = Y_{ij}^\ell \frac{v}{\sqrt{2}}, \quad (2.9)$$

where v is the Higgs vacuum expectation value. The observation of massive neutrinos motivates the extension of the standard model to explain the origin of neutrino masses, which can be approached in different ways, but all approaches add additional degrees of freedom to the standard model.

One approach is to introduce to the standard model a right-handed neutrino field that allows one to include Yukawa terms of the form

$$\mathcal{L}_{\nu \text{Yukawa}} = -Y_{ij}^\ell \bar{L}_{Li} \phi \nu_{Rj} + \text{h.c.} \quad (2.10)$$

where ν_{Rj} is the right-handed neutrino singlet. Because experimental evidence strongly

1063 predicts only three active neutrinos, these additional neutrinos are "sterile", in that they
1064 do not interact via the strong, weak, or electromagnetic interactions. After spontaneous
1065 symmetry breaking, the Yukawa interaction leads to mass terms given by

$$\mathcal{L}_D = -M_{Dij}\bar{\nu}_{Ri}\nu_{Lj} + \text{h.c.}, \quad (2.11)$$

1066 which is called a Dirac mass term. One of the issues with constructing neutrino masses
1067 in this way is that the required Yukawa couplings are at least a factor of 10^6 smaller than
1068 that of an electron, which begs the question: why are the Yukawa couplings so small for
1069 the neutrinos?

1070 An alternative approach is to allow the neutrinos to have a Majorana mass, which is
1071 possible because neutrinos are electrically neutral particles. The Majorana mass terms
1072 for neutrinos have the form

$$\mathcal{L}_M = -\frac{1}{2}(M_{Rij}\bar{\nu}_{Ri}\nu_{Rj}^c M_{Lij}\bar{\nu}_{Li}\nu_{Lj}^c) + \text{h.c.}, \quad (2.12)$$

1073 where M_{Rij} and M_{Lij} are right-handed and left-handed Majorana mass matrices. A
1074 consequence of neutrinos being Majorana particles is lepton number violation, which
1075 predicts the occurrence of neutrino-less double beta-decay at a rate proportional to the
1076 neutrino mass.

1077 In the most general case neutrinos have both Dirac and Majorana mass terms, which
1078 allows one to generate neutrino masses with Yukawa couplings similar to the rest of the
1079 standard model. Considering a single generation of neutrinos for demonstration, the
1080 combined neutrino mass Lagrangian can be written as

$$\mathcal{L}_{D+M} = -m_D\bar{\nu}_R\nu_L - \frac{1}{2}(m_L\bar{\nu}_L\nu_L^c + m_R\bar{\nu}_R\nu_R^c) + \text{h.c.}, \quad (2.13)$$

1081 or equivalently,

$$\mathcal{L}_{D+M} = -\frac{1}{2} \begin{bmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \nu_L^c \\ \nu_R \end{bmatrix} + \text{h.c..} \quad (2.14)$$

1082 An example mass generation mechanism with this approach is the Type-I see-saw
1083 mechanism [31], in which one takes $m_L = 0$ and $m_R \gg m_D$. By diagonalizing Equation
1084 2.14 one obtains the mass eigenvalues that represent the physical masses of the neutrinos.
1085 The light neutrino mass eigenstate, which represents the observed neutrino mass, has a
1086 mass given by

$$m_1 \approx \frac{m_D^2}{m_R}, \quad (2.15)$$

1087 and the heavy neutrino mass eigenstate, which represents the unobserved sterile neutrino,
1088 has a mass

$$m_2 \approx m_R. \quad (2.16)$$

1089 For m_D similar to the other quark or lepton masses, one obtains physical neutrino masses
1090 consistent with observations from sterile neutrino masses of $m_R \approx O(10^{15})$ GeV. This
1091 mass scale is well beyond the capabilities of modern particle accelerators to probe.

1092 2.5 Neutrino Absolute Mass Scale

1093 The neutrino absolute mass scale or simply "neutrino mass" cannot be probed with
1094 neutrino oscillations, since oscillation probabilities are determined by the squared mass
1095 differences between neutrino mass eigenstates, therefore, alternative techniques are needed
1096 to perform an effective measurement of the neutrino mass.

1097 2.5.1 Limits from Cosmology

1098 The Λ CDM model summarizes the current cosmological understanding of the universe [24].
1099 Λ CDM predicts that the universe originated from a single expansion event colloquially
1100 called the "Big Bang". During the Big Bang, the universe originated as a hot spacetime
1101 singularity, which abruptly experienced rapid expansion in a process known as inflation.
1102 After expansion the inflationary field eventually decayed into a population of quarks,
1103 gluons, leptons, and photons, which were kept in thermal equilibrium by the high-
1104 temperatures of the early universe.

1105 As the universe continued to expand it's density and temperature decreased until
1106 the formation of neutral atoms, primarily hydrogen, was possible. At which point the
1107 population of photons produced during the Big Bang decoupled from the primordial
1108 universe and began to freely propagate. A direct prediction of the Λ CDM model is that
1109 this population of photons is still present, but with a significantly reduced temperature
1110 due to the subsequent expansion of the universe. This is consistent with the observation of
1111 the CMB (cosmic microwave background), which is a population of microwave radiation
1112 with a blackbody temperature of 2.7 K. The CMB is extremely uniform in all directions
1113 with slight anisotropies that can be analyzed to study the evolution of the early universe.
1114 A series of experiments have measured the CMB with increasing levels of precision, which
1115 has lead to a significant increase in our current understanding of cosmology.

1116 In addition to the CMB, inflation predicts the existence of a $C\nu B$ (cosmic neutrino

background) [32], which are the remnant neutrinos produced during the Big Bang. Since neutrinos only interact via the weak force, they decouple from the Big Bang plasma at an earlier time than the CMB photons. The temperature at which the C ν B decouples depends on the neutrino rest mass. Neutrinos play a unique role in the Λ CDM model, due to the fact that neutrinos act as radiation early in the universe but as matter in the late universe. This leads to specific signatures that impact the expected anisotropies of the CMB as well as the distribution of matter in the universe [33]. By combining measurements of the CMB with measurements of the large-scale structure (LSS) of the universe one can constrain the neutrino mass scale by fitting these datasets with the Λ CDM model. This analysis results in some of the most stringent constraints on the neutrino mass. Recent analyses [24] have been able to constrain the neutrino mass scale to

$$\Sigma_{m_\nu} \equiv \sum_i m_i < 0.11 \text{ eV}, \quad (2.17)$$

where m_i are the neutrino mass eigenstates.

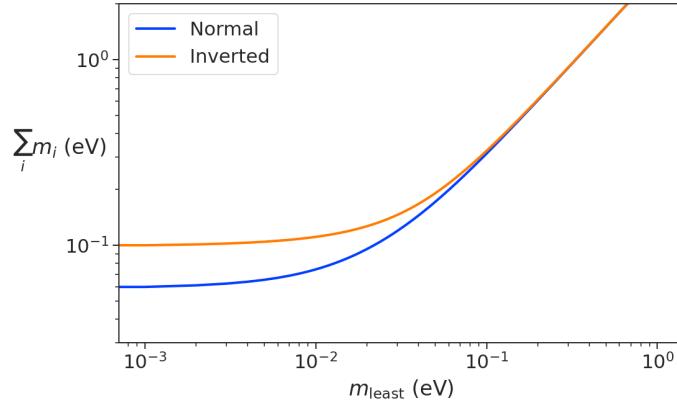


Figure 2.3. The neutrino mass observable measured by cosmology as a function of the lightest neutrino mass eigenstate.

The observable Σ_{m_ν} constrains the neutrino mass by setting the mass of the lightest neutrino mass eigenstate (m_{least}) (see Figure 2.3). In the normal mass ordering Σ_{m_ν} can be rewritten in the form

$$\Sigma_{m_\nu} = m_{\text{least}} + \sqrt{\Delta m_{21}^2 + m_{\text{least}}^2} + \sqrt{\Delta m_{32}^2 + m_{\text{least}}^2}, \quad (2.18)$$

where it is clear that a measurement of Σ_{m_ν} effectively sets the neutrino mass scale

1134 through m_{least} . The analogous formula for the inverted mass ordering is

$$\Sigma_{m_\nu} = m_{\text{least}} + \sqrt{-\Delta m_{32}^2 + m_{\text{least}}^2} + \sqrt{-\Delta m_{31}^2 + m_{\text{least}}^2}. \quad (2.19)$$

1135 Upcoming experiments [34] are planned to refine measurements of the CMB, LSS,
1136 and other cosmological observables. With this additional data it is possible that in the
1137 near future cosmological measurements will be able to positively constrain the neutrino
1138 absolute mass scale. However, the strength of these limits strictly depend on the accuracy
1139 of the Λ CDM model, which highlights the need for direct experimental measurements of
1140 the neutrino mass to confirm the predictions of cosmology and to fix the neutrino mass
1141 parameter in future cosmological analyses.

1142 2.5.2 Limits from Neutrinoless Double Beta-decay Searches

1143 If neutrinos are Majorana fermions, then the neutrino is equivalent to its own antiparticle
1144 and lepton conservation is not an exact law of nature [35]. Limits on the rate of
1145 neutrinoless double beta-decay ($0\nu\beta\beta$), are some of the most powerful current tests of
1146 lepton number conservation [24]. If $0\nu\beta\beta$ were observed, it would be direct evidence that
1147 neutrinos are Majorana fermions and provide a method for measuring the neutrino mass
1148 scale.

1149 Standard double beta-decay occurs when two neutrons in an unstable nucleus spon-
1150 taneously decay into two protons, which results in the production of two electrons and
 two neutrinos (see Figure 2.4). Whereas, during $0\nu\beta\beta$ the two neutrinos self-annihilate

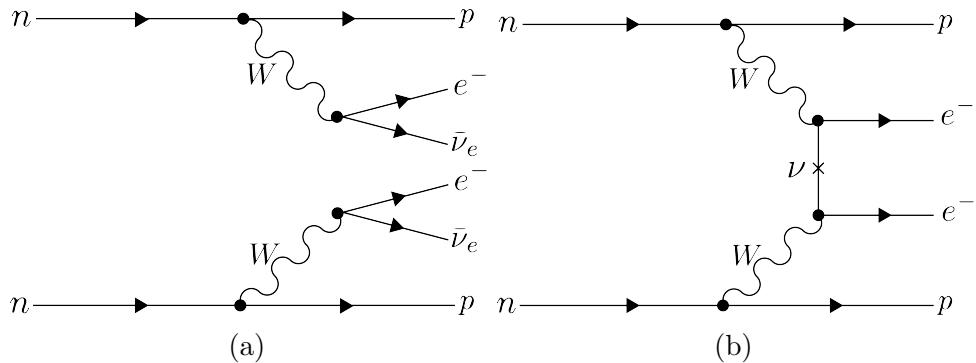


Figure 2.4. Feynman diagrams for double beta-decay (a) and $0\nu\beta\beta$ (b).

1151
1152 producing only two electrons, which violates lepton number by two.

1153 Assuming that the exchange of two Majorana neutrinos is the dominant channel for
1154 $0\nu\beta\beta$, then a measurement of the $0\nu\beta\beta$ half-life for a particular isotope can be used to

1155 set the neutrino absolute mass scale [36]. The half-life is written in terms of the effective
 1156 neutrino mass for $0\nu\beta\beta$ ($m_{\beta\beta}$) using the equation

$$T_{1/2}^{0\nu} = \frac{1}{G|\mathcal{M}|^2 m_{\beta\beta}^2}, \quad (2.20)$$

1157 where G is the phase-space factor for the decay and \mathcal{M} is the relevant nuclear matrix
 1158 element. $m_{\beta\beta}$ is given by an incoherent sum of the neutrino mass eigenstates weighted
 1159 by the PMNS mixing matrix parameters,

$$m_{\beta\beta} = \left| \sum_i U_{ei}^2 m_i \right|. \quad (2.21)$$

1160 The information provided from $0\nu\beta\beta$ on the neutrino mass scale can be visualized by
 1161 expressing the value of $m_{\beta\beta}$ in terms of m_{least} and two relative Majorana phases [2]. The
 1162 allowed regions for $m_{\beta\beta}$ as a function of m_{least} are shown in Figure 2.5 as the regions
 1163 bounded by the black curves overlayed with the discovery probabilities of future $0\nu\beta\beta$
 decay experiments based on current neutrino data.

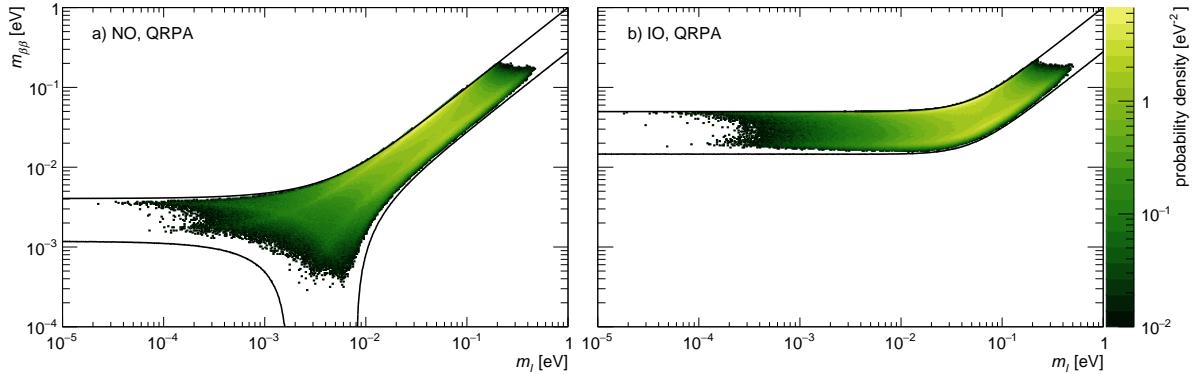


Figure 2.5. The discovery probabilities for the future generation of $0\nu\beta\beta$ experiments as a function of $m_{\beta\beta}$ and m_{least} . Figure from [2].

1164
 1165 Because of the possibility of cancellation due to the unknown Majorana phases included
 1166 in the sum specified by Equation 2.21, the neutrino mass information gained from $0\nu\beta\beta$
 1167 is necessarily imperfect. Additionally, theoretical uncertainties in the calculation of the
 1168 nuclear matrix elements complicates the calculation of $m_{\beta\beta}$ from a measurement of $0\nu\beta\beta$
 1169 half-life. Similar to cosmology, there is a high degree of complementarity between direct
 1170 measurements of the neutrino mass and $0\nu\beta\beta$. In particular, a measurement of m_{least} to
 1171 less than 0.1 eV sensitivity provides significant information for $0\nu\beta\beta$ searches based on
 1172 the discovery probabilities displayed in Figure 2.5.

2.5.3 Limits from Beta-decay

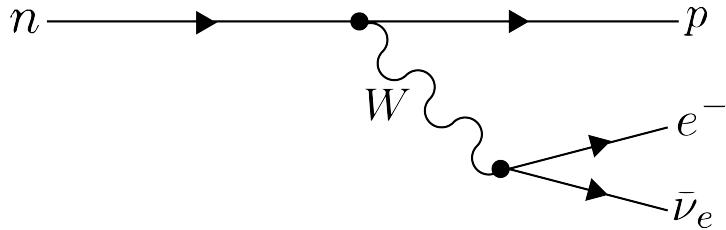


Figure 2.6. A Feynman diagram of beta decay

Certain processes involving neutrinos, in particular beta-decay (see Figure 2.6), have initial states with well-defined total energies and final states that can be measured with high accuracy and precision. Beta-decay involves the decay of an unstable isotope where a neutron spontaneously converts to a proton and emits an electron and anti-neutrino ("neutrino" for brevity) to conserve charge and lepton number [5]. Therefore, by applying the principles of energy and momentum conservation, a measurement of the kinematics of the final state can be used to constrain the neutrino mass [37].

Using beta-decay to measure the neutrino mass can be tied back to Fermi's original 1934 theory of nuclear beta-decay [16] (see Figure 2.7). Because the constraints on the

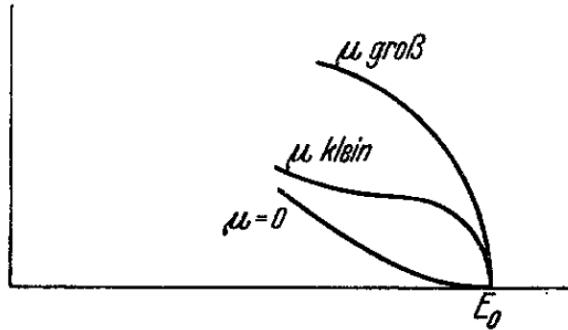


Figure 2.7. A figure from Fermi's 1934 paper on a theory of beta-decay depicting the kinetic energy spectrum of the emitted electron. The effect of the neutrino mass, written as μ , is to distort the shape of the spectrum near the endpoint from the zero-mass spectrum.

neutrino mass from beta-decay depend only on the final state measurement capabilities and the principles of energy and momentum conservation, neutrino mass measurements with beta-decay are called direct measurements. A direct measurement like beta-decay contrasts with other neutrino mass measurements approaches that are model-dependent such as cosmology and $0\nu\beta\beta$, which provide complementary ways to study the physics of massive neutrinos.

1189 The isotope of choice for direct neutrino mass measurements with beta-decay has
 1190 been tritium (3H_2) for many decades, because it conveniently fulfills many experimental
 1191 requirements. Of upmost importance is a decay with a low Q-value, which is the available
 1192 kinetic energy based on the mass difference between the initial and final states. The
 1193 effect of a massive neutrino on the shape of the spectrum is magnified for low Q-values
 1194 and tritium has an unusually low Q-value of 18.6 keV.

1195 Additionally, tritium beta-decay is super-allowed, which results in a relatively short
 1196 half-life of 12.3 years. Therefore, high source activity can be obtained with a relatively
 1197 small source mass. High-activity is desirable because of the low-activity near the tritium
 1198 spectrum endpoint. For tritium beta-decays, only a factor of 3×10^{-13} of the decays
 1199 occur in the last 1 eV of the spectrum. Isotopes with Q-values lower than tritium are
 1200 known [37], but this is outweighed by exceedingly long half-lives leading to unobtainable
 1201 source masses.

1202 The endpoint measurement approach involves quantifying the effect of the neutrino's
 1203 mass on shape of the electron's kinetic energy spectrum near the endpoint. The shape of
 1204 the kinetic energy spectrum (see Figure 2.8) is given by

$$\frac{d\Gamma}{dE} = \frac{G_F^2 |V_{ud}|^2}{2\pi^3} (G_V^2 + 3G_A^2) F(Z, \beta) \beta (E + m_e)^2 (E_0 - E) \\ \times \sum_{i=1,2,3} |U_{ei}|^2 [(E_0 - E)^2 - m_i^2]^{1/2} \Theta(E_0 - E - m_i), \quad (2.22)$$

1205 where G_F is the Fermi coupling constant, V_{ud} is an element of the CKM matrix, E
 1206 is the kinetic energy of the electron, β is the velocity of the electron divided by the
 1207 speed of light, E_0 is the endpoint energy assuming zero neutrino mass, $F(Z, \beta)$ is the
 1208 Fermi function, and $\Theta(E_0 - E - m_i)$ is the Heaviside function, which enforces energy
 1209 conservation. One can see that the decay spectrum is actually a combination of three
 1210 spectra with different endpoints based on the values of the neutrino mass eigenstates, m_i .
 1211 This produces "kinks" in the spectrum shape due to overlapping spectra with different
 1212 endpoint values, but such an effect would be nearly impossible to resolve given the finite
 1213 energy resolution of a real experiment.

1214 The neutrino mass scale variable measured by beta-decay is given by

$$m_\beta^2 = \sum_i |U_{ei}|^2 m_i^2, \quad (2.23)$$

1215 where m_β is the electron-weighted neutrino mass or simply "neutrino mass" for brevity.

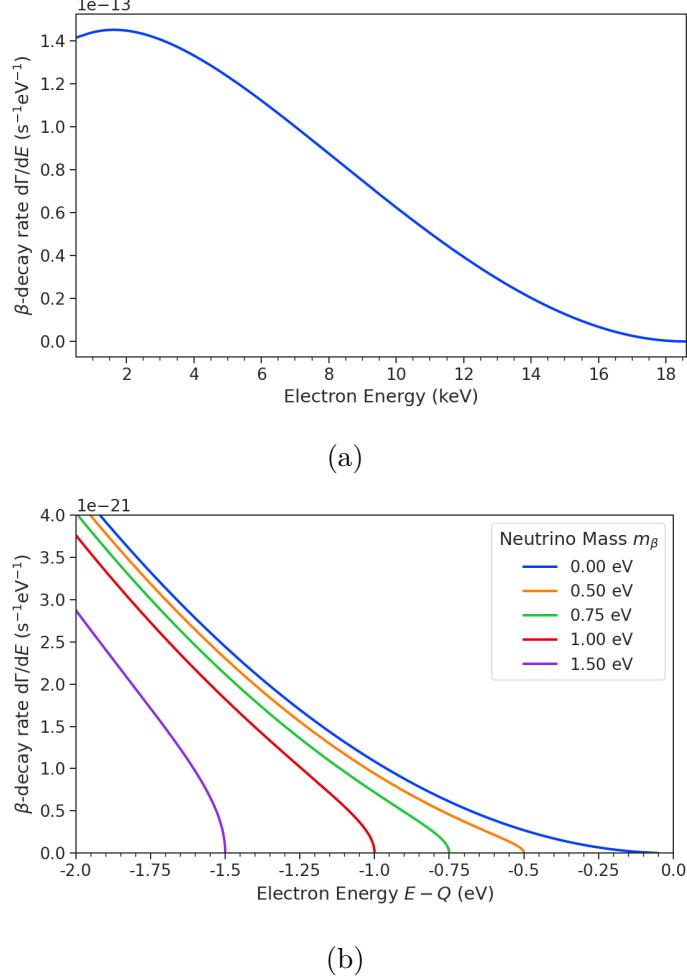


Figure 2.8. The tritium beta-decay spectrum. The effect of a massive neutrino on the spectrum is to change its shape near the endpoint by an amount proportional to the size of the neutrino mass. A sufficiently high-statistic and high-resolution measurement of the spectrum endpoint would be able to measure the neutrino mass.

The quantity m_β corresponds to a particular weighted sum of the neutrino masses, which is distinct from effective neutrino masses such as $m_{\beta\beta}$ [37]. Assuming unitarity, the neutrino mass can be expressed in terms of the PMNS matrix elements, squared mass differences, and the lightest neutrino mass eigenstate. For the normal mass ordering the equation is

$$m_\beta^2 = m_{\text{least}}^2 + |U_{e2}|^2 \Delta m_{21}^2 + |U_{e3}|^2 \Delta m_{31}^2, \quad (2.24)$$

and for the inverted ordering the equation is

$$m_\beta^2 = m_{\text{least}}^2 + |U_{e1}|^2 (-\Delta m_{32}^2 - \Delta m_{21}^2) + |U_{e2}|^2 (-\Delta m_{32}^2). \quad (2.25)$$

1222 Therefore, a measurement of the neutrino mass in combination with neutrino mixing
1223 parameters is effectively a measurement of m_{least} .

1224 Since the neutrino mass is small (< 1 eV), its effect on the spectrum is limited to the
1225 endpoint region. The affect of a non-zero neutrino mass on the endpoint spectrum is
1226 plotted for the reader in Figure 2.8. Resolving the small changes in the spectrum shape
1227 requires an experimental technique with high statistics, excellent energy resolution, and
1228 low background activity.

1229 **Chapter 3 |**

1230 **Direct Measurement of the Neutrino Mass**

1231 **with Project 8**

1232 **3.1 Introduction**

1233 A promising technique for direct measurements of the neutrino mass beyond the projected
1234 200 meV limit of the KATRIN experiment [38] is tritium beta-decay spectroscopy with
1235 an atomic tritium source [39]. Atomic tritium, combined with a large-volume, high-
1236 resolution energy measurement technique, is capable of measuring the neutrino mass
1237 with sensitivity below the 50 meV, which exhausts the range of neutrino masses allowed
1238 under the inverted hierarchy.

1239 Cyclotron Radiation Emission Spectroscopy (CRES) is a high-resolution energy
1240 measurement technique compatible with atomic tritium production and storage that can
1241 enable the next-generation of neutrino mass direct measurement experiments [40]. The
1242 Project 8 collaboration is currently engaged in a program of research and development
1243 (R&D) aimed at developing the technology necessary for a measurement of the neutrino
1244 mass using CRES and atomic tritium with a sensitivity of 40 meV [41].

1245 In Section 3.2 I provide an introduction to the basics of the CRES technique as well as
1246 the goals of the Project 8 experiment. Additionally, I sketch out the phased experiment
1247 development plan being implemented by Project 8 to build towards a next-generation
1248 neutrino mass experiment.

1249 In Section 3.3 I give an overview of Phase II of the Project 8 experiment [42,43], which
1250 completed early in 2023. Although the bulk of the work presented in this dissertation is
1251 relevant to designs of future Project 8 experiments, a description of the work in Phase II
1252 provides useful context.

1253 In Section 3.4 I introduce a CRES measurement concept based on antenna arrays [44],
1254 which could be the basis for the ultimate Project 8 neutrino mass experiment. A

1255 significant portion of the R&D efforts of Project 8 in Phase III were directed towards
1256 simulating and modeling this experimental concept in order to understand the achievable
1257 sensitivity to the neutrino mass.

1258 Lastly, in Section 3.5 I introduce conceptual designs of pilot-scale experiments and
1259 Phase IV that combine atomic CRES with a large-volume CRES detection technique.
1260 This includes a design concept for an antenna array based experiment, but also a design
1261 for a resonant cavity based experiment. Resonant cavities are discussed in more depth in
1262 Chapter 6 and have become the default choice for the Phase IV experiment.

1263 **3.2 Project 8 and Cyclotron Radiation Emission Spectroscopy**

1264

1265 **3.2.1 Cyclotron Radiation Emission Spectroscopy — CRES**

1266 Time and frequency are two of the most precisely measured quantities in physics. Atomic
1267 clocks, which operate by measuring the frequencies of various atomic transitions, have
1268 been used to measure time with astounding relative uncertainties of 10^{-18} seconds [45].
1269 The extreme precision possible with frequency measurements is often summarized using
1270 the a quote from the Physicist Arthur Schawlow who said advise his students to "Never
1271 measure anything but frequency!" [46].

1272 Neutrino mass measurements using tritium beta-decay require the measurement
1273 of perturbations to the 18.6-keV tritium endpoint with a precision as small as 0.1 eV.
1274 Therefore, a spectroscopic technique with extremely high resolution is required. Frequency
1275 measurements are capable of such high-resolutions for the intuitive reason that they are
1276 essentially digital counting measurements, which average the number of oscillations of a
1277 physical system over time. By observing a rapidly oscillating system over a sufficient
1278 length of time one can obtain essentially arbitrary precision on a frequency limited only
1279 by the measurement time and signal-to-noise ratio (SNR) of the system.

1280 A method is required for translating an electron kinetic energy measurement into a
1281 frequency measurement. A straightforward way to accomplish this is to place a gaseous
1282 supply of tritium into a magnetic field, therefore, when a beta-decay occurs the resulting
1283 electron will immediately begin to orbit around a magnetic field line at the cyclotron
1284 frequency, proportional to its kinetic energy (see Figure 3.1). The acceleration caused
1285 by the orbit leads to the emission of cyclotron radiation that can be detected using an
1286 array of antennas or resonant cavity. The starting frequency of the radiation gives the

1287 electron's initial kinetic energy, which is used to build the beta-decay spectrum and
 1288 measure the neutrino mass. The name for this measurement technique is Cyclotron
 1289 Radiation Emission Spectroscopy or CRES [40].

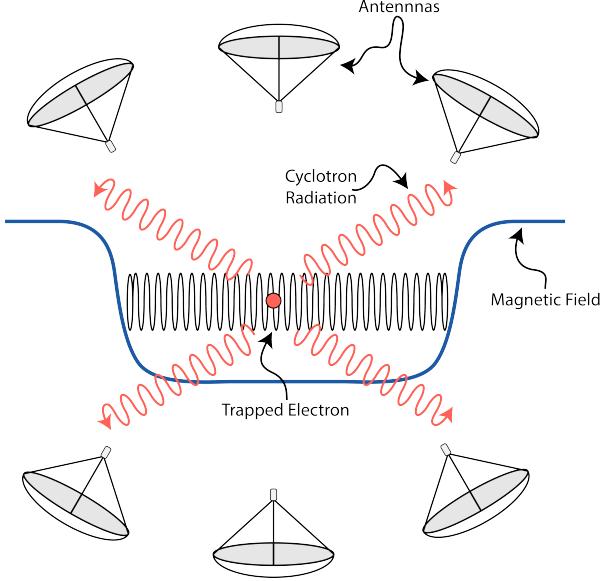


Figure 3.1. A cartoon illustration of the CRES technique. An electron is contained in a magnetic trap, which is a local minimum in the magnetic field, so that its cyclotron radiation can be detected by an array of antennas. Detecting the cyclotron radiation allows one to measure its cyclotron frequency and determine its kinetic energy.

1290 In the non-relativistic case, the cyclotron frequency is simply a function of the
 1291 charge-to-mass ratio of the particle, however, the relativistic correction to the cyclotron
 1292 frequency

$$f_c = \frac{qB}{2\pi m_e \gamma} = \frac{1}{2\pi} \frac{qB}{m_e + E_{\text{kin}}/c^2}, \quad (3.1)$$

1293 introduces a dependence of the kinetic energy (E_{kin}) to the inverse of the cyclotron
 1294 frequency (f_c). Electrons with kinetic energies of 18.6 keV are in the weakly relativistic
 1295 regime with $\beta = \frac{v}{c} = 0.263$ and $\gamma = 1.036$.

1296 The frequency resolution of a CRES measurement can be estimated by differentiating
 1297 Equation 3.1,

$$\frac{df_c}{dE_{\text{kin}}} = \frac{1}{2\pi} \frac{-qBc^2}{(m_e c^2 + E_{\text{kin}})^2}, \quad (3.2)$$

1298 from which one obtains the relationship between fractional differences in energy and
 1299 frequency,

$$\frac{df_c}{f_c} = \frac{1 - \gamma}{\gamma} \frac{dE_{\text{kin}}}{E_{\text{kin}}}. \quad (3.3)$$

1300 Therefore, an energy precision of 1 eV for an 18.6 keV electron can be achieved with a
 1301 frequency precision of approximately 2 ppm.

1302 The minimum observation time required to achieve this resolution can be estimated
 1303 using the uncertainty principle as formulated by Gabor [47]. Electrons from tritium
 1304 beta-decay experience random collisions with the background gas particles, which limits
 1305 the uninterrupted radiation lifetime. The time between collision events, referred to as
 1306 "track length", is an exponentially distributed variable. Differences in the track lengths
 1307 of a population of mono-energetic electrons leads to an uncertainty or broadening in the
 1308 distribution of measured frequencies, which is proportional to the mean track length, τ_λ .
 1309 The resulting frequency distribution has a Lorentzian profile, whose width is given by
 1310 the Gabor limit,

$$\tau_\lambda \Delta f_c = \frac{1}{2\pi} \implies \Delta f_c = \frac{1}{2\pi\tau_\lambda}. \quad (3.4)$$

1311 The cyclotron frequency for a 18.6-keV electron in a 1 T field is approximately
 1312 27 GHz, consequently, the minimum observation time for a frequency resolution of 2 ppm
 1313 is approximately 3 μ sec. The Gabor limit is not the true lower bound on the frequency
 1314 resolution for a CRES signal, since it derives from the Fourier representation of a fixed
 1315 length time-series using a basis of infinite duration sinusoids. If one takes the approach of
 1316 fitting the CRES signal in the time-domain, then the lower limit on frequency precision
 1317 is given by the Cramér-Rao lower bound (CRLB) [48], which depends on the track length
 1318 and SNR. The CRLB is the minimum variance achievable by an unbiased estimator
 1319 for an unknown but deterministic parameter. In general, the CRLB allows for better
 1320 precision on the cyclotron frequency.

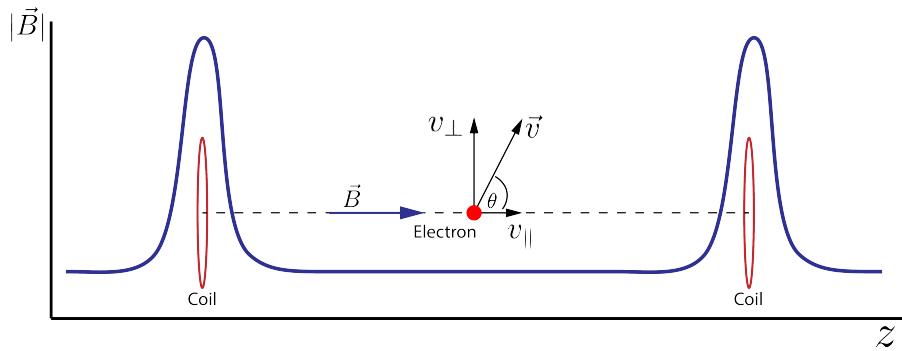


Figure 3.2. An illustration of an electron in a bathtub magnetic trap generated by two well-separated coils.

1321 Ensuring that an electron remains under observation long enough so that its frequency
 1322 can be precisely measured can be achieved using a magnetic trap. A magnetic trap is a

local minimum in a background magnetic field generated an appropriate configuration of electromagnetic coils. Since magnetic fields can do no work, there is no danger of the magnetic trap affecting the kinetic energy of the electron after it is emitted from the beta-decay. One common approach to creating a magnetic trap is the "bathtub" trap configuration, which can be produced using two magnetic pinch coils aligned on a central axis that are separated by a distance that is large compared to the coil radius (see Figure 3.2). This configuration produces a trap with a uniform bottom and relatively steep walls, which is ideal for CRES measurements.

The electron's pitch angle is a useful parameter for describing its motion in the magnetic trap. Pitch angle is defined in terms of the ratio between the component of the electron's velocity perpendicular to the magnetic field and the component parallel to the magnetic field

$$\tan \theta_p = \frac{v_{\perp}}{v_{\parallel}}. \quad (3.5)$$

Electrons with pitch angles less than 90° oscillate back and forth in the magnetic trap, which leads to variations in the cyclotron frequency caused by the changing value of the magnetic field along the electron's path. This leads to frequency modulation that produces sidebands in the cyclotron radiation spectrum. Resolving these sideband frequency components is necessary for a complete reconstruction of the CRES signal in the experiment.

Electrons trapped in a cylindrically symmetric trap have three primary components of motion (see Figure 3.3). The dominant component, typically with the highest frequency,

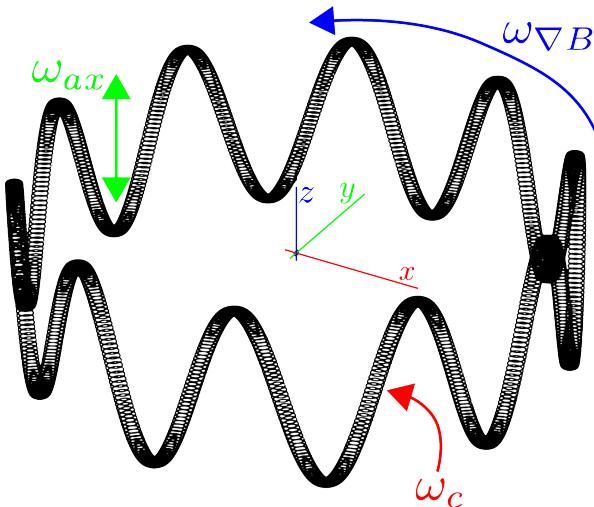


Figure 3.3. A plot of the main components of an electron's trajectory in a cylindrically symmetric trap.

1342

1343 is the electron's cyclotron orbit, which encodes information on the electron's kinetic
 1344 energy. Axial motion from the electron's pitch angle leads to frequency modulation,
 1345 and a shift in the average magnetic field experienced by an electron. This leads to a
 1346 correlation between the kinetic energy of the electron and the pitch angle depending on
 1347 the particular shape of the magnetic trap, which can negatively impact energy resolution.
 1348 Generally, more variation in the magnetic field along the electron's trajectory leads to
 1349 a worse energy resolution. The magnetic trap can be engineered to have a flat bottom
 1350 with very steep walls to mitigate this effect. A bathtub trap design, where the distance
 1351 between the coils is much greater than the coil radius, is the trap that best approximates
 1352 this ideal design. Radial gradients in the trap leads to a third component of motion
 1353 called grad-B drift [49]. The equation for the drift velocity is

$$\mathbf{v}_{\nabla B} = \frac{m_e v_{\perp}^2}{2qB} \frac{\mathbf{B} \times \nabla B}{B^2}. \quad (3.6)$$

1354 The total power of the radiation emitted by an electron in a free-space environment
 1355 is given by the Larmor equation [50]

$$P(\gamma, \theta_p) = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{q^2 \omega_c^2}{c} (\gamma^2 - 1) \sin^2 \theta_p, \quad (3.7)$$

1356 where ω_c is the cyclotron frequency multiplied by 2π and θ_p is the pitch angle to distinguish
 1357 it from the spherical angle coordinate. A single electron with a 90° pitch angle and
 1358 18.6 keV of kinetic energy in a 1 T magnetic field emits a total radiation power of 1.2 fW.
 1359 Furthermore, one is typically only able to receive a fraction of this total power with an
 1360 antenna or other detection system. Therefore, RF (radio-frequency) systems in CRES
 1361 experiments must be operated at cryogenic temperatures to limit the noise power such
 1362 that adequate SNR can be achieved for signal detection and reconstruction. Alternatively,
 1363 longer tracks enable detection of weaker signals due to the increase in the total signal
 1364 energy available for the detection algorithm.

1365 3.2.2 Project 8

1366 The Project 8 collaboration¹ is a group of institutions in the United States and Germany
 1367 building an experiment to measure the neutrino mass by developing a novel spectrometer
 1368 technology based on CRES. In the ultimate Project 8 experiment, the CRES technique
 1369 will be used to measure the beta-decay spectrum using a large source of atomic tritium

¹<https://www.project8.org/>

1370 sufficient to achieve the required statistics in the last $O(10)$ eV of the decay spectrum.
1371 Project 8 is targeting a neutrino mass sensitivity below 50 meV [51], which exhausts the
1372 range of possible neutrino masses under the inverted hierarchy and is a factor of four less
1373 than sensitivity projections for the ongoing KATRIN experiment.

1374 Project 8's proposed experiment requires the development of two novel technologies:
1375 the production and trapping of a source of atomic tritium on cubic-meter scales and
1376 technology to enable CRES measurements of individual electrons in the same volume.

1377 Atomic Tritium

1378 Previous measurements of the tritium beta-decay spectrum for neutrino mass measure-
1379 ments have relied on sources of molecular tritium for their measurements [38, 52, 53] due
1380 to the technical challenges associated with the production and storage of atomic tritium.

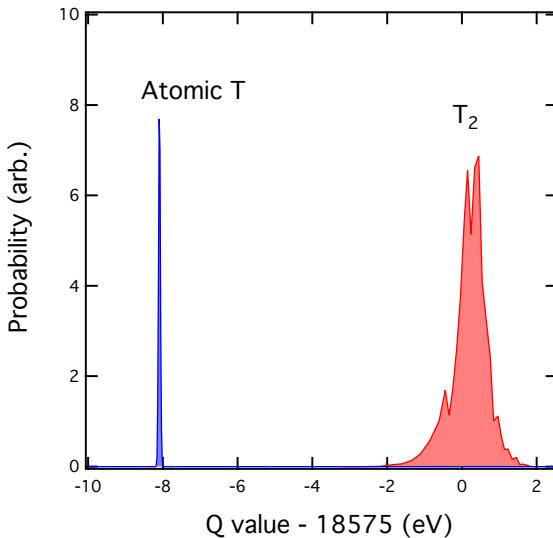


Figure 3.4. A plot of the final state distributions of atomic and molecular tritium. The final state distribution provides the primary contribution to the width of the molecular spectrum whereas thermal doppler broadening is responsible for the width of the atomic spectrum.

1381 One must supply sufficient energy to the tritium molecules to break the molecular
1382 bond and create atomic tritium. Common approaches include the use of hot coaxial
1383 filament atom crackers as well as plasma sources. Both involve heating the tritium atoms
1384 to temperatures of > 2500 K, which must then be cooled to temperatures on the order
1385 of a few mK so that the tritium atoms can be trapped. Cooling the atoms requires the
1386 construction of a large tritium infrastructure and cooling system that can supply a source
1387 of cold atoms to the trap.

1388 Once cold tritium atoms are produced they cannot make contact with any surfaces
1389 to avoid recombination of the atoms to molecules. Therefore, a magnetic trap is required
1390 to store the atoms for a sufficient length of time that they have a chance to decay before
1391 escaping the trap. Trapping the atoms requires the construction of a large and complex
1392 magnet system that must be cooled to cryogenic temperatures.

1393 The significant experimental complexity caused by atomic tritium makes a molecular
1394 source the obvious choice from practical considerations. However, the drawback of
1395 molecular tritium for neutrino mass measurement is the irreducible broadening in the
1396 electron's kinetic energy due to the final state spectrum of molecular tritium (see Figure
1397 3.4). The broadening of the final state spectra has a RMS amplitude of 436 meV [54, 55]
1398 caused by variation in the final vibrational state of the daughter molecule.

1399 For atomic tritium, the primary sources of broadening in the final state spectrum are
1400 magnetic hyperfine splittings (magnitude of $O(10^{-5})$ eV) and thermal Doppler broadening
1401 caused by the motion of the trapped atom. Atomic tritium at a temperature of 1 mK
1402 has a broadening which is dominated by thermal Doppler broadening, providing about
1403 1 meV RMS of broadening to the electron's kinetic energy.

1404 The larger energy broadening with molecular tritium leads to an irreducible statistical
1405 uncertainty that limits the achievable sensitivity to approximately 100 meV at 90%
1406 confidence. For previous direct measurements of the neutrino mass, this uncertainty is
1407 an insignificant contribution to the overall uncertainty budget. However, for experiments
1408 like Project 8 atomic tritium is a key component to the success of the experiment.

1409 **CRES for Neutrino Mass Measurement**

1410 Several features of the CRES technique make it an attractive choice for a next generation
1411 neutrino mass measurement experiment. Because CRES is a remote-sensing technique,
1412 it is possible to observe the kinetic energy of the electron without altering its trajectory
1413 or directly interacting with the particle, therefore, in a CRES experiment the source
1414 gas volume can be the same as the CRES spectrometer volume. Tritium gas is also
1415 transparent to cyclotron radiation, which means that the kinetic energies of electrons can
1416 be measured using a cavity or antenna array, located directly outside the atom trapping
1417 volume.

1418 Because source and spectrometer can be colocated, CRES experiments have an
1419 advantageous scaling law relative to the current state-of-the-art beta-decay spectroscopy
1420 experiment, KATRIN. KATRIN utilizes the magnetic adiabatic collimation with an
1421 electrostatic filter (MAC-E filter) technique to measure the beta-decay spectrum of

1422 molecular tritium. In this approach, a source of molecular tritium is located outside the
1423 spectrometer. When a beta-decay occurs the electron is guided out of the tritium source
1424 using a magnetic field and is transported through the MAC-E filter before it is detected
1425 on the other side of the filter using a charge sensor. The measurement statistics of the
1426 MAC-E filter are limited by the transverse area of the tritium source and filter due to the
1427 need to travel through the experiment without scattering. This scaling is less favorable
1428 than the volumetric scaling of CRES due to the ability to colocate source and detector.

1429 Another promising aspect of the CRES technique is the inherently high precision
1430 of frequency based measurements. The endpoint of the molecular tritium beta-decay
1431 spectrum is approximately 18.6 keV, which dwarfs the neutrino mass scale of $< 1 \text{ eV}/c^2$
1432 by at least a factor of 10^5 . Measuring the effect of such a small mass on a high energy
1433 electron requires excellent energy resolution. Since frequency measurements are essentially
1434 counting measurements they are intrinsically quite accurate due to the ability to measure
1435 the cyclotron frequency by effectively averaging over millions of cyclotron orbits. It
1436 is possible to achieve part-per-million accuracy on the kinetic energy with the CRES
1437 technique using the off-the-shelf RF components.

1438 CRES is also nearly immune to typical sources of backgrounds that can plague other
1439 experiments. Since CRES operates via a non-destructive measurement of the electron's
1440 cyclotron frequency, sources of background electrons are effectively filtered out by limiting
1441 the frequency bandwidth of the measurement. The fiducial volume of the experiment is
1442 free from any surfaces that could introduce stray electrons, and electrons from sources
1443 outside the fiducial volume can be prevented from entering the experiment.

1444 Neutrino Mass Sensitivity Goals

1445 Project 8's ultimate goal is to combine CRES with atomic tritium to measure the neutrino
1446 mass with 40 meV sensitivity at the 90% confidence level (see Figure 3.5). This sensitivity
1447 is sufficient to fully exhaust the range of allowable neutrino masses under the inverted
1448 neutrino mass ordering regime and is approximately an order of magnitude less than the
1449 projected final sensitivity of the KATRIN experiment. Excluding the full neutrino mass
1450 parameter space would require a sensitivity an order of magnitude lower than what is
1451 proposed by Project 8, which would require an experiment whose size and complexity
1452 are currently well beyond proposals for the next-generation of neutrino mass direct
1453 measurement experiments.

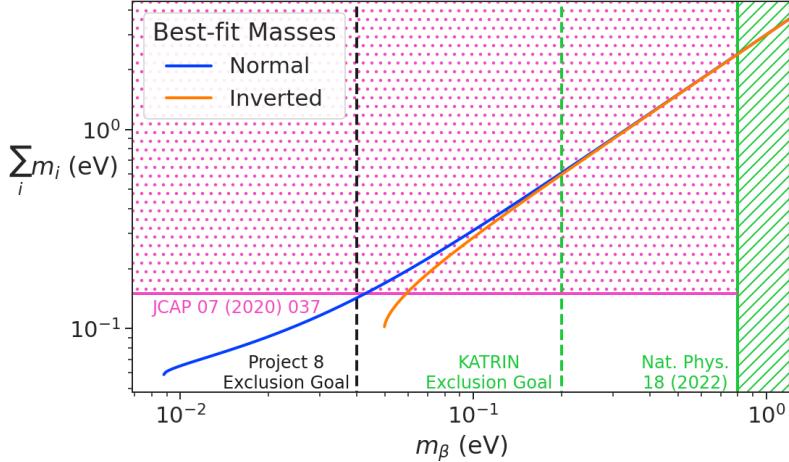


Figure 3.5. Neutrino mass exclusion plot including limits from cosmological measurements and the KATRIN experiment. Allowed ranges for neutrino masses under the normal and inverted hierarchies are shown as the blue and orange lines respectively. The black dashed line shows Project 8’s goal neutrino mass sensitivity for the Phase IV experiment.

1454 3.2.3 The Project 8 Phased Development Plan

1455 Reaching 40 meV sensitivity requires the simultaneous development and eventually
 1456 combination of CRES and atomic tritium. These technologies require a significant up-front
 1457 R&D investment to build-out the required capabilities for a 40 meV CRES experiment.
 1458 Therefore, Project 8 is following a phased experiment plan in which incremental progress
 1459 can be made towards the ultimate goal of a 40 meV neutrino mass measurement with
 1460 CRES.

1461 Project 8’s experiment plan is divided into four phases. The first phase, called
 1462 Phase I, consisted of a demonstration of the CRES technique and a measurement of the
 1463 internal conversion spectrum of ^{83m}Kr . Phase II was the first measurement of the tritium
 1464 beta-decay spectrum and neutrino mass measurement with CRES. Currently, Project 8
 1465 is engaged in Phase III, which is research and development towards a scalable CRES
 1466 measurement technique and atomic tritium source for the final Project 8 experiment in
 1467 Phase IV. Phase IV is the ultimate experiment by Project 8 that will combine CRES
 1468 with atomic tritium to measure the neutrino mass with a sensitivity of 40 meV.

1469 Phase I and II: Proof of Principle and First Tritium Measurements

1470 The earlier phases of the Project 8 experiment, Phase I and II, were focused on demon-
 1471 stration and development of the CRES technique itself as well as a proof-of-principle

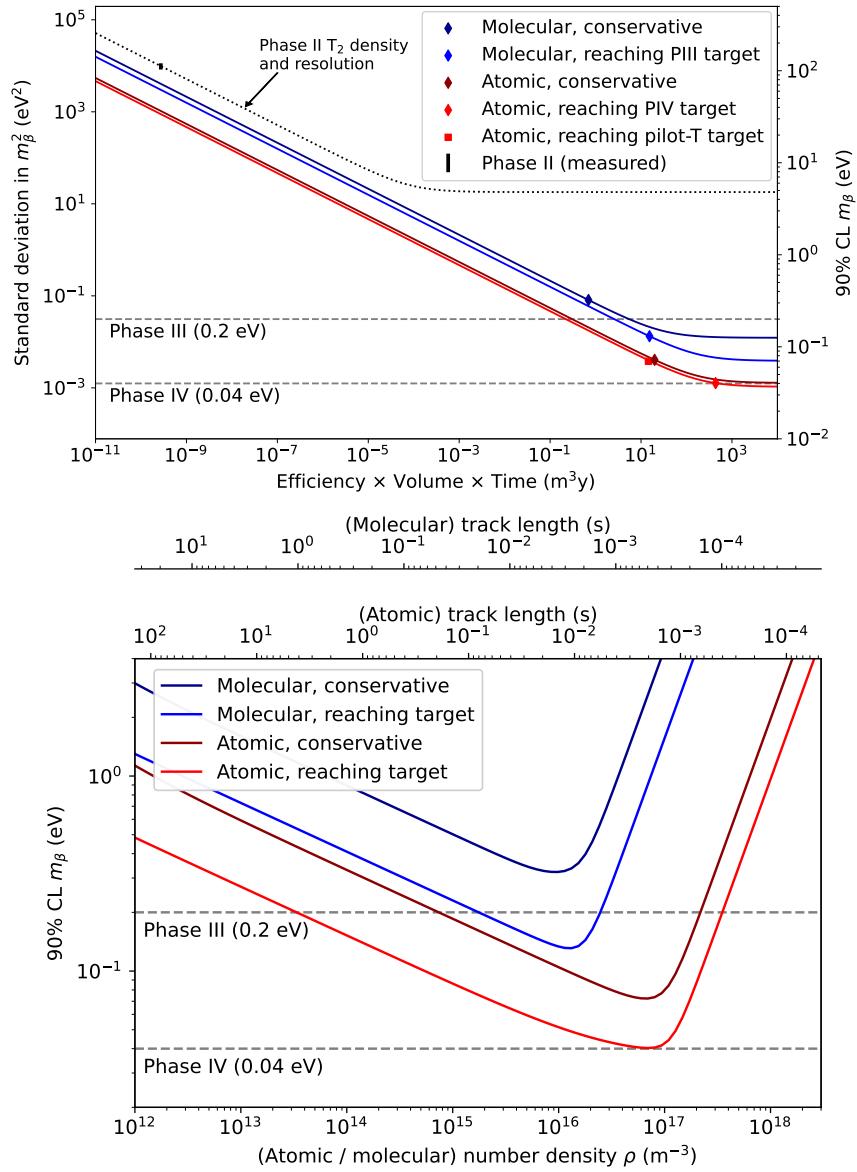


Figure 3.6. Sensitivity calculations for a cavity based CRES experiment that demonstrate the neutrino mass measurement goals of Project 8 throughout the phased development plan. The blue curves indicate molecular tritium sources and the red curves indicate atomic tritium sources. In the current plan, Phase III contains two tritium experiments. The first is the Low-frequency Apparatus (LFA), which is a molecular tritium experiment, and the second is the atomic tritium pilot-scale experiment that officially ends Phase III. The sensitivity of these experiments is primarily a function of statistics, however, there is a critical density beyond which CRES electrons do not have enough time to radiate between collisions for a high-resolution frequency measurement leading to worse sensitivity.

¹⁴⁷² measurement of the neutrino mass using the CRES technique.

¹⁴⁷³ In Phase I, Project 8 performed a proof-of-principle measurement of the ^{83m}Kr
¹⁴⁷⁴ spectrum using CRES, which marked the first ever kinetic energy spectrum measurement
¹⁴⁷⁵ with CRES. The experiment included all the components of a basic CRES experiment.
¹⁴⁷⁶ An electron source consisting of a gas of ^{83m}Kr was supplied to a waveguide gas cell
¹⁴⁷⁷ constructed out of a segment of WR-42 waveguide and sealed with Kapton windows at
¹⁴⁷⁸ the top and bottom. A magnetic trapping region was created in the waveguide cell using
¹⁴⁷⁹ a single electromagnetic coil wrapped around the waveguide which provided a trapping
¹⁴⁸⁰ volume on the order of a few cubic-millimeters. Detection of the cyclotron radiation was
¹⁴⁸¹ performed by connecting the waveguide cell to an additional segment of waveguide that
¹⁴⁸² transmitted the radiation to a cryogenic amplifier.

¹⁴⁸³ Success in Phase I was achieved with the 2014 publication of the measured ^{83m}Kr
¹⁴⁸⁴ conversion spectrum [56], which contains a mono-energetic 17.8-keV line as well as several
¹⁴⁸⁵ other conversion lines at higher energies. Publication of this result marked the official
¹⁴⁸⁶ end of Phase I and the start of Phase II, in which Project 8 shifted its focus to the
¹⁴⁸⁷ demonstration of the first tritium beta-decay spectrum using CRES. Phase II is described
¹⁴⁸⁸ below in Section 3.3.

¹⁴⁸⁹ **Phase III: Research and Development and a Pilot-scale Experiment**

¹⁴⁹⁰ After completing Phase II, Project 8 has shifted focus towards R&D aimed at the
¹⁴⁹¹ construction of an experiment that demonstrates all the technologies required for a
¹⁴⁹² 40 meV measurement of the neutrino mass. The culmination of Phase III is a pilot-scale
¹⁴⁹³ experiment that successfully retires all technological and engineering risks associated
¹⁴⁹⁴ with the Phase IV experiment, while also being a scientifically interesting experiment in
¹⁴⁹⁵ its own right. Sensitivity estimates of the pilot-scale experiment predict a neutrino mass
¹⁴⁹⁶ sensitivity on par with the projected sensitivity of the KATRIN experiment.

¹⁴⁹⁷ Phase III R&D is divided into two main efforts — atomic tritium and CRES detection
¹⁴⁹⁸ techniques. Atomic tritium development in Phase III must retire all risks associated
¹⁴⁹⁹ with the atomic tritium system. This includes the production of tritium atoms, atomic
¹⁵⁰⁰ cooling and recirculation systems, purity and isotope concentration monitoring, and
¹⁵⁰¹ atom trapping. Currently, Project 8 is operating small scale atom cracking demonstrator
¹⁵⁰² systems to show that atom production at the estimated rates needed for Phase IV is
¹⁵⁰³ achievable. Future efforts will continue the current developments on atom production
¹⁵⁰⁴ and expand to include demonstrations of atomic cooling with an evaporative beam line
¹⁵⁰⁵ as well as atom trapping using Halbach magnet arrays.

1506 The need for new CRES detection techniques is driven by the drastic increase in scale
1507 from Phase II to the pilot-scale experiments. The physical volume used for CRES in
1508 Phase II was on the order of a few cubic-centimeters, and achieving Project 8’s sensitivity
1509 target of 40 meV requires an experiment volume on the multi-cubic meter scale. Therefore,
1510 the waveguide gas cell CRES detection technique used in Phase II is not a feasible option
1511 for the future of Project 8 due to its inability to scale to the required size.

1512 Two alternative CRES detection techniques have been proposed for the pilot-scale
1513 experiment — antenna arrays and resonant cavities (see Section 3.4 and Chapter 6).
1514 Both approaches have relative advantages and disadvantages, however, the improved
1515 understanding of the antenna array and cavity approaches to CRES in the recent years
1516 has led to cavities being the preferred technology for the pilot-scale experiment and
1517 Phase IV due to the estimated reduced cost and complexity of this approach. Since
1518 a large degree of the work presented in this dissertation is focused specifically on the
1519 development of the antenna array CRES technique as well as the design of demonstrator
1520 experiments, I describe the proposed R&D plan for antenna array CRES in Section 3.4.
1521 A description of the cavity approach to CRES can be found in Chapter 6.

1522 Cavity CRES R&D consists of a series of demonstrator experiments intended to
1523 demonstrate cavity CRES at a variety of scales and magnetic fields. Radioactive sources
1524 gases include ^{83m}Kr and molecular tritium, as well as electrons produced by an electron-
1525 gun, which is a key calibration tool for future CRES experiments. The near-term cavity
1526 effort in Project 8 is the cavity CRES apparatus (CCA), which is a small-scale cavity
1527 experiment operating near 26 GHz. The CCA will perform the first CRES measurements
1528 using a small cavity, and will pave the way towards larger scale cavity experiments in
1529 preparation for the eventual pilot-scale tritium experiment.

1530 The pilot-scale experiment will be the first experiment to combine atomic tritium and
1531 large-volume CRES detection. It will directly demonstrate all the technologies required
1532 for Phase IV such that no technical risks remain for scaling the experiment to required
1533 scale. A robust approach to scaling the pilot-scale experiment is to simply build multiple
1534 copies of it for the Phase IV experiment.

1535 **Phase IV: Project 8’s Ultimate Neutrino Mass Experiment**

1536 The design of Phase IV should be a direct extension of the pilot-scale CRES experiment
1537 that marks the official end of Phase III (see Section 3.5). The Phase IV experiment
1538 represents the final experiment in the Project 8 neutrino mass measurement experiment
1539 plan and will have sensitivity to neutrino masses of 40 meV.

3.3 Phase II: First Tritium Beta Decay Spectrum and Neutrino Mass Measurement with CRES

In Phase II, Project 8 demonstrated the first ever measurement of the tritium beta-decay spectrum endpoint using the CRES technique, which lead to the first neutrino mass measurement by Project 8. This milestone was made possible by many improvements in the CRES technique and in the understanding of CRES systematics, which takes an important first step towards larger scale measurements of the tritium beta-decay spectrum with CRES. In this section, I briefly describe some important elements of the Phase II experiment, with the goal of contextualizing the research and development efforts for Phases III and IV of Project 8. For more complete descriptions of the work that lead to Project 8's Phase II results please refer to the relevant publications by the collaboration [42, 43].

3.3.1 The Phase II CRES Apparatus

Magnet and Cryogenics

The magnetic field for the Phase II experiment is provided by a nuclear magnetic resonance (NMR) spectroscopy magnet with a central bore diameter of 52 mm (see Figure 3.7). The magnet produces a background magnetic field with an average value of 0.959 T with a 10 ppm variation across the bore diameter achieved using several shim coils built into the magnet. Using an external NMR field probe, the variation of the magnetic field along the vertical axis of the magnet bore was measured to obtain an accurate model of the magnetic field so that the CRES cell could be positioned for optimal magnetic field uniformity.

An external solenoid magnet was installed inside the magnet bore to provide the ability to shift the magnitude of the background magnetic field by a few mT. The solenoid has inside diameter of 46 mm and a length of 350 mm, which terminates in a vacuum flange that allows it to be inserted into the NMR magnet bore from the bottom. By shifting the value of the magnetic field by a few mT, the cyclotron frequencies of electrons produced by the 17.8 keV ^{83m}Kr internal-conversion line [57] can be shifted by frequencies of ± 100 MHz. This allows one to study the frequency dependent behavior of several CRES systematics, such as detection efficiency, that directly affect the measured shape of the tritium spectrum.

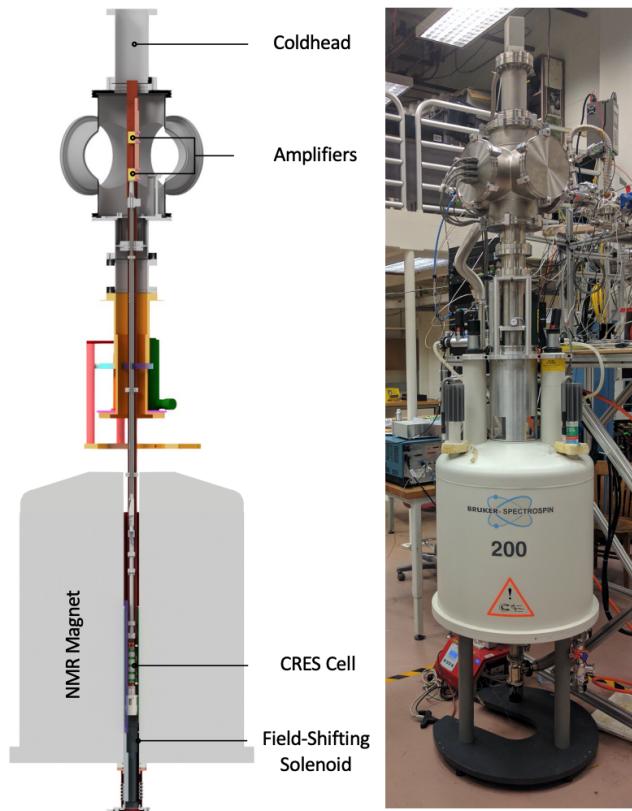
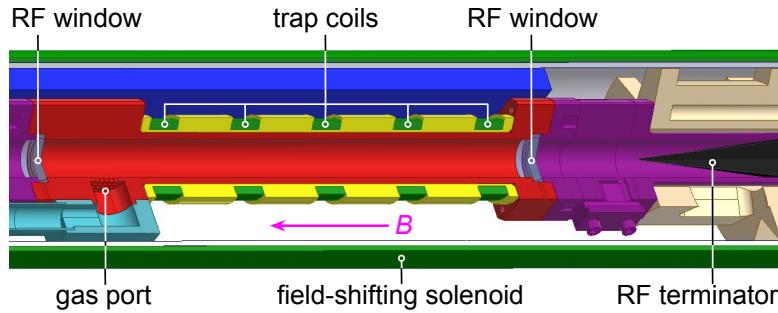


Figure 3.7. The Phase II CRES apparatus used to perform the first measurement of the tritium beta-decay spectrum using CRES.

1571 The inside of the magnet bore diameter was pumped down to a vacuum of less than
 1572 10 μ torr using a turbomolecular pump, which allows for cryogenic cooling of the CRES
 1573 cell and RF system. Cooling power was supplied to the Phase II apparatus using a
 1574 cryopump with its coldhead mounted above the primary magnet and CRES cell. This
 1575 arrangement allowed for sufficient cooling power to be delivered to the amplifiers to cool
 1576 them to a temperature of ≈ 40 K, while keeping the amplifiers far enough from the
 1577 magnet so as not to be damaged by the large field strength. Thermal contact between
 1578 the coldhead, amplifiers, RF system, and CRES cell is achieved using a copper bar that
 1579 runs the full length of the apparatus. To prevent freeze-out of ^{83m}Kr on the walls of the
 1580 CRES cell a separate heater was installed to keep the CRES cell near a temperature of
 1581 85 K during the operation of the experiment.

1582 **CRES Cell**

1583 Located in the most uniform region of the magnetic field is the CRES cell, which is
1584 the region of the apparatus where radioactive decays of ^{83m}Kr and T_2 produce electrons
that can be trapped and measured using CRES (see Figure 3.8). The CRES cell is



1585 **Figure 3.8.** Diagram of the CRES cell portion of the Phase II apparatus.

1586 manufactured from a segment of cylindrical waveguide designed to operate at K-band
1587 frequencies near 26 GHz. The diameter of the waveguide determines which resonant
1588 modes of the waveguide will couple to the electron and transmit its radiation to the
1589 amplifiers. For Phase II a waveguide diameter of 1 cm was selected, which allows electrons
1590 to couple to the TE_{11} and TM_{01} cylindrical waveguide modes. To reduce complexity in
1591 modeling and analyzing the CRES data, it is ideal to select a diameter that prevents
1592 electrons from coupling to higher-order waveguide modes beyond the fundamental TE
1593 and TM modes.

1594 Around the exterior of the cylindrical waveguide are several magnetic coils used to
1595 produce magnetic traps inside the CRES cell volume. Without a magnetic trap electrons
1596 produced from decays inside the CRES cell quickly impact the cell wall, which prevents
1597 a measurement of their cyclotron frequency using CRES. Each coil along the length of
1598 the waveguide produces a separate trap that is approximately harmonic in shape. By
1599 independently controlling the currents provided to each coil, the traps can be configured
1600 to have equal values of the magnetic field at the trap bottom despite a non-uniform field
1601 from the NMR magnet.

1602 Two primary magnetic trap configurations were used during the Phase II experiment.
1603 The first was a shallow trap configuration used primarily for its high energy resolution to
1604 study systematics using ^{83m}Kr decays, and the second was a deeper trap that could trap a
1605 higher percentage of pitch angles. The trade-off with this trap is that the higher trapping
1606 efficiency comes at the cost of lower energy resolution due to the greater variation in
1607 pitch angle (see Section 3.2.1). The deep trap was the trap used to measure the tritium

1608 beta-decay spectrum in Phase II.

1609 The source gases were delivered into the CRES cell through a gas port located near the
1610 top end of the cylindrical waveguide. To prevent the gases from escaping the cell, vacuum
1611 tight RF transparent windows are needed to contain the tritium and krypton source
1612 gas across a 1 atm pressure differential, while still transmitting the cyclotron radiation
1613 without distortion. The crystalline material, CaF₂, which has a thermal expansion
1614 coefficient similar to that of copper, was used for this purpose in the CRES cell. Two
1615 windows, each 2.4 mm thick, were used to seal off the ends of the CRES cell. The
1616 thickness of 2.4 mm corresponds to half of a cyclotron wavelength when one accounts for
1617 the permittivity of CaF₂.

1618 RF System

1619 The RF system in the Phase II apparatus propagates the cyclotron radiation from the
1620 CRES cell to the receiver chain. The receiver chain performs the down-conversion and
1621 digitization required to obtain signals that can be analyzed to determine the cyclotron
frequencies of electrons in the CRES cell (see Figure 3.9).

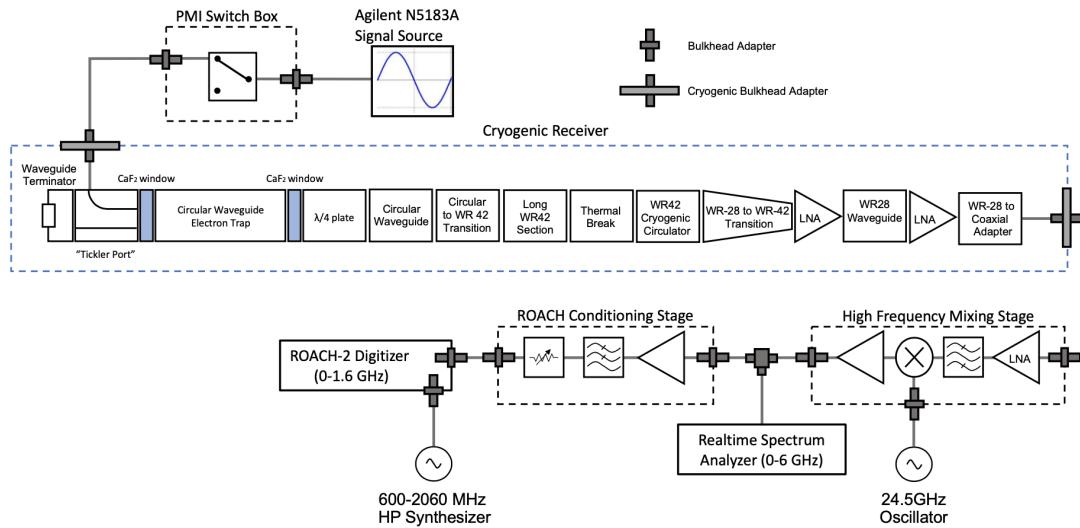


Figure 3.9. RF system diagram for the Phase II apparatus.

1622
1623 Below the CRES cell, at the bottom of the Phase II apparatus, is a tickler port and
1624 waveguide terminator. The tickler port is used to inject signals into the CRES cell and
1625 RF system for testing and calibration purposes. The waveguide terminator is designed to
1626 absorb cyclotron radiation emitted by electrons that transmits out of the bottom of the
1627 CRES cell. This lowers the total power received from electrons in the CRES cell, since all

1628 the energy radiated downwards is absorbed into the terminator. Earlier iterations of the
1629 Phase II apparatus used an RF short in this location that reflected this power up towards
1630 the amplifiers, however, interference between the upward traveling and reflected radiation
1631 led to a disappearance in the signal carrier that made reconstruction impossible.

1632 Radiation traveling upward passes through the CaF₂ window passes and a λ/4 plate,
1633 which transforms the circularly polarized cyclotron radiation into linear polarization.
1634 The linearly polarized fields next travel through a segment of circular waveguide that
1635 transitions into a long segment of WR-42 waveguide that carries the fields out of the
1636 high magnetic field region. A thermal break segment is included, which consists of a
1637 segment of gold-plated stainless steel WR-42 waveguide, to help thermally isolate the
1638 relatively warm CRES cell from the colder amplifiers. The radiation then passes through
1639 a cryogenic circulator, which prevents signals reflected from the amplifiers from interfering
1640 with the CRES cell before a WR-42 to WR-28 transition connects the waveguide to the
1641 first of the cyrogenic amplifiers. The radiation passes through two cyrogenic amplifiers
1642 before being coupled to a coaxial termination at the top of the Phase II apparatus.

1643 The coaxial cable transfers the cyclotron radiation signals to a high-frequency mixing
1644 stage that performs an analog frequency down-conversion using a 24.5 GHz LO. Two forms
1645 of digitization can be used at this stage to readout the CRES data. One is a real-time
1646 spectrum analyzer that digitizes the CRES signal data in time-domain and computes the
1647 frequency spectrum in real-time, which allows for direct visualization of CRES signal
1648 spectrograms as the experiment is running. The real-time spectrum analyzer is most
1649 useful for taking small amount of streamed data for debugging and analysis of the system.
1650 The other method, which was used to collect the majority of the CRES data in Phase II,
1651 is a ROACH-2 FPGA and digitizer system. The ROACH system consists of a fast ADC
1652 that samples the CRES signal data at 3.2 GSps. Internal digital down-conversion stages
1653 implemented in the FPGA perform a mixing operation that reduces the bandwidth of the
1654 CRES signals to 100 MHz. The FPGA implements a 4096 sample FFT and packetizes
1655 time and frequency domain records in parallel. The packetized data is then transferred
1656 from the ROACH to be analyzed by the data-processing pipeline.

1657 **3.3.2 CRES Track and Event Reconstruction**

1658 **Time-Frequency Spectrogram**

1659 The online data-processing software uses a real-time triggering algorithm that identifies
1660 interesting data that could contain CRES signals. Triggered data are collected into files

1661 that are transferred to a server for offline processing and analysis. The data files contain
1662 a continuous series of time-domain samples, broken into a set of records, which are 4096
1663 samples long. The time-series is made up of 8-bit IQ samples acquired at 100 MHz.

1664 Each time-series record is accompanied by an associated frequency spectrum consisting
1665 of 4096 frequency bins approximately 24.4 kHz wide, which is represented as a power
1666 spectral density. The individual frequency spectra can be organized temporally to create
1667 a time-frequency spectrogram that represents the evolution of the cyclotron frequency
spectrum over the course of the CRES event (see Figure 3.10). The time-frequency

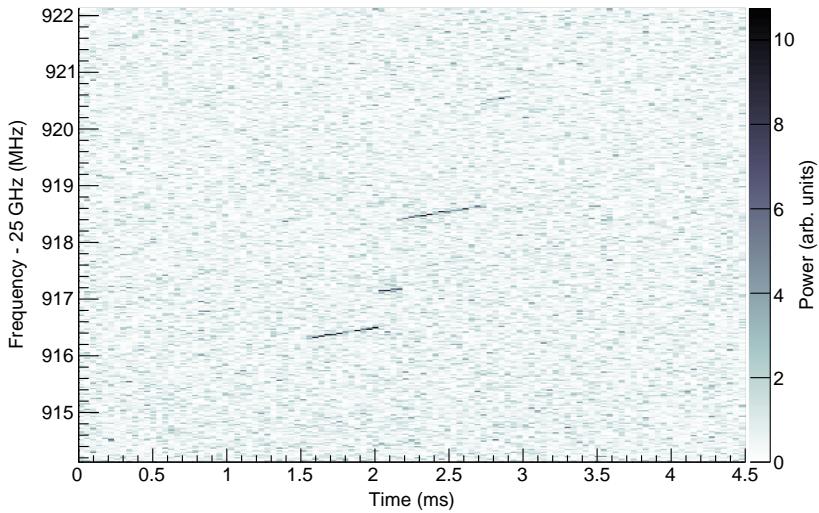


Figure 3.10. The time-frequency spectrogram of a tritium CRES event in the Phase II apparatus.

1668
1669 spectrogram is represented as a two-dimensional image where the color of each pixel is
1670 proportional to the power spectral density. Each vertical slice of pixels in the image
1671 represents a frequency spectrum, therefore, each horizontal bin represents the data
1672 obtained over a duration of $4096 \times 0.01 \text{ MHz}^{-1} = 40.96 \mu\text{sec}$.

1673 CRES Event Data Features

1674 Phenomenologically, a CRES signal appears as a sinusoidal signal whose frequency slowly
1675 increases over time in what is called a frequency "chirp". Axial motion of the electron in
1676 the trap leads to the formation of frequency sidebands that surround the more powerful
1677 carrier frequency. The critical piece of information that must be extracted from the track
1678 and event reconstruction procedure is the carrier frequency, since it is this frequency that
1679 gives the cyclotron frequency and thus the kinetic energy. Axial motion from non-90°

1680 pitch angles changes the average magnetic field experienced by an electron, because
1681 the electron effectively samples the magnetic field along its trajectory. The change in
1682 the average magnetic field with pitch angle leads to different cyclotron frequencies that
1683 correspond to the same kinetic energy. However, because of the low-SNR in Phase
1684 II, sidebands were unable to be observed, so no attempt to correct for this effect was
1685 attempted. The effect of different pitch angles is to broaden the peak of a monoenergetic
1686 electron line, which can be quantified by measuring the instrumental resolution of the
1687 Phase II apparatus.

1688 In the time-frequency spectrogram representation, the chirping carrier frequency
1689 appears as a linear track of high-power frequency bins (see Figure 3.10). The vertical
1690 slope of the tracks is caused by the emission of energy from the electron in the form of
1691 cyclotron radiation, therefore, the size of the slope parameter is directly proportional
1692 to the Larmour power. The continuous track is periodically interrupted by random
1693 jumps to higher frequency (lower energy) caused by random inelastic collisions with
1694 background gas molecules. The length of a track is an exponentially distributed variable
1695 whose mean value is inversely proportional to the gas density. The size of the frequency
1696 discontinuities is directly proportional to the energies of the rotational and vibrational
1697 states of background gas molecules.

1698 A CRES event refers to the collection of tracks produced by a trapped electron until
1699 it inevitably scatters into a pitch angle that can no longer be trapped. The goal of track
1700 and event reconstruction is to identify the set of tracks in a time-frequency spectrogram
1701 that represents a segment of data acquired in the Phase II apparatus. These tracks must
1702 be clustered into events, from which one can determine the first track produced by the
1703 electron and thus estimate its starting cyclotron frequency and kinetic energy.

1704 **Track Reconstruction**

1705 The first step in CRES event reconstruction is the identification of tracks in the time-
1706 frequency spectrogram, which is essentially an image processing task. Track finding
1707 starts by normalizing the power spectral density based on the average noise power. Next
1708 a power threshold is applied to the normalized spectrogram where only bins that have a
1709 SNR ratio greater than five are selected to build tracks. In this case SNR is defined as the
1710 ratio between the normalized, unitless power of a bin divided by the average normalized
1711 power across the full frequency spectrum.

1712 The sparse spectrogram produced by this power cut consists only of a sparse collection
1713 of high-power frequency bins that could be part of a CRES signal track (see Figure

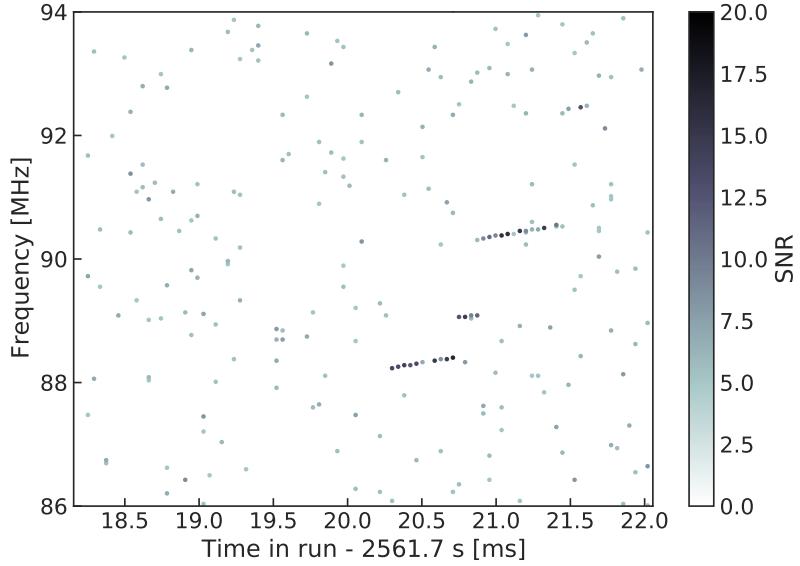


Figure 3.11. The sparse spectrogram obtained by placing a power cut on the raw spectrogram shown in Figure 3.10.

1714 3.11). In this form is it much easier to identify tracks "by eye", however, for the Phase II
 1715 analysis Project 8 developed its own custom-made track finding algorithm, called the
 1716 sequential track finder (STF).

1717 The STF algorithm processes the sparse spectrogram in sequential fashion, processing
 1718 each time-slice one-by-one until the end of the spectrogram is reached. Tracks are found
 1719 by searching for points in the sparse spectrogram that appear to fall on a straight line.
 1720 Multiple configurable parameters are built into the STF algorithm that allow the user to
 1721 tune the criteria for adding a point to an existing track or creating a new track. These
 1722 include parameters such as maximum time and frequency differences between subsequent
 1723 points in a track as well as minimum SNR values for the start and endpoints of the track.
 1724 Additionally, tracks are required to have a minimum length and slope to be considered
 1725 potential CRES tracks rather than random noise fluctuations.

1726 The resulting output of the STF is a collection of track objects that consist of the track
 1727 point objects and their properties. The final step is to calculate track-level properties and
 1728 apply cuts to reject false tracks found by the STF. This involves the fitting of a line to
 1729 the collection of track points as well as the total and average power of the track obtained
 1730 by computing the sum and mean of the points powers. The starting frequency of the
 1731 track is determined by calculating the time coordinate that intersects with the linear fit.
 1732 A cut is then performed to remove all tracks that do not have a specified average power
 1733 over their duration, which helps to remove the majority of noise fluctuations that have

¹⁷³⁴ passed all previous cuts up to this point.

¹⁷³⁵ Event Reconstruction

¹⁷³⁶ After track reconstruction comes event reconstruction, where the identified tracks are
¹⁷³⁷ grouped into events that correspond to the trajectory of a single electron in the trap.
¹⁷³⁸ This procedure attempts to match tracks head to tail by checking if the start and end
¹⁷³⁹ times of a pair of tracks falls within a certain tolerance. This tolerance is a configurable
¹⁷⁴⁰ parameter that can be tuned to an optimal value using Monte Carlo simulations of events
¹⁷⁴¹ in the Phase II apparatus.

¹⁷⁴² After the event building procedure has completed, there remains a small likelihood
¹⁷⁴³ that false tracks have made it through to the event reconstruction stage. Typically, cuts
¹⁷⁴⁴ at the track level are able to remove 95% of the false tracks identified by the STF, which
¹⁷⁴⁵ leads to a significant number of false tracks at the event building stage. However, the
¹⁷⁴⁶ additional event-level information makes it possible to reject events that contain these
¹⁷⁴⁷ false tracks with a high degree of confidence.

¹⁷⁴⁸ Two event level features are associated with events caused by real electrons — the
¹⁷⁴⁹ duration of the first track as well as the number of tracks in the event. Real electrons
¹⁷⁵⁰ tend to have event structures with longer first tracks and a higher number of total tracks.
¹⁷⁵¹ Based on the values of these two criteria, a minimum threshold on the average power in
¹⁷⁵² the first track was configured to reject false events. The average power in the first track
¹⁷⁵³ was chosen due to the critical nature of the starting frequency of the first track in an
¹⁷⁵⁴ event to the krypton and tritium spectrum analyses.

¹⁷⁵⁵ 3.3.3 Results from Phase II

¹⁷⁵⁶ The main result from Phase II was the measurement of the tritium beta-decay spectrum
¹⁷⁵⁷ using CRES, which lead to the first neutrino mass limit with CRES. However, Phase
¹⁷⁵⁸ II also included a significant ^{83m}Kr measurement campaign to understand important
¹⁷⁵⁹ systematics relevant to the tritium spectrum measurement and the fundamentals of
¹⁷⁶⁰ the CRES technique itself. This required high-resolution measurements of the ^{83m}Kr
¹⁷⁶¹ internal-conversion spectrum [57], which is an interesting science result in its own right.

¹⁷⁶² The results from Phase II represents a significant effort from entire Project 8 over
¹⁷⁶³ several years. Because the focus of my contributions to Project 8 is directed towards the
¹⁷⁶⁴ research and development efforts for the Phase III experiments, the goal in this section
¹⁷⁶⁵ is not to provide a detailed description of the analyses that lead to the Phase II results.

1766 Rather, I will provide brief descriptions of a few plots representative of the main results
1767 from Phase II as reported in [42, 43].

1768 Measurements with Krypton

1769 Measurements with krypton were a key calibration tool for Phase II of the experiment
1770 and will continue to be useful in Phase III. Krypton measurements refers to CRES
1771 measurements of the internal-conversion spectrum of the metastable state of krypton-83,
1772 ^{83m}Kr , produced by electron capture decays of ^{83}Rb . A supply of ^{83}Rb was built into the
1773 Phase II apparatus gas system that supplied the CRES cell with ^{83m}Kr via emanation.

1774 The ^{83m}Kr internal-conversion spectrum consists of several lines based on the orbital
1775 of the electron ejected during the decay. The conversion lines useful to Project 8 are
1776 those that emit electrons with kinetic energies that fall inside the detectable frequency
1777 bandwidth of the Phase II apparatus. These are the K; L2 and L3; M2 and M3; and N2
1778 and N3 lines; with kinetic energies of 17.8 keV, \approx 30.4 keV, \approx 31.9 keV, and \approx 32.1 keV,
1779 respectively. The different energies of the lines allows one to test the linearity of the
1780 relationship between kinetic energy and frequency across the range of frequencies covered
1781 by the continuous tritium spectrum.

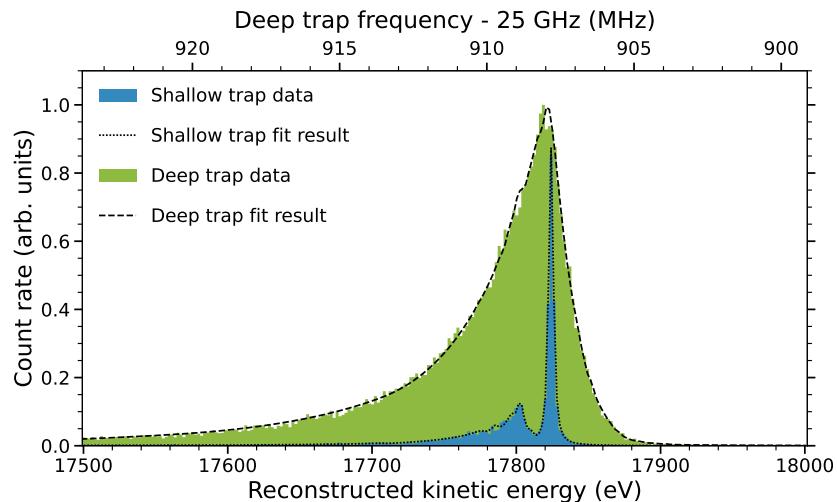


Figure 3.12. Fits to the measured 17.8-keV ^{83m}Kr conversion line using the deep and shallow trap configurations.

1782 Numerous detector-related effects relevant to the tritium analysis can be characterized
1783 by measuring the shape of the krypton spectrum. Specific examples include variations
1784 in the magnetic field as a function of the radial position of the electron, variation in
1785 the magnetic field caused by the trap shape, variation in the average magnetic field for

1786 electrons with different pitch angles, and the effect of missing tracks due to scattering.
 1787 These spectrum shape measurements focused on the 17.8-keV krypton line and utilized
 1788 different trap geometries based on the particular goal of the dataset (see Figure 3.12).

1789 Krypton measurements with a shallow trap allow for high energy resolution, since
 1790 variation in frequency due to pitch angle differences is sharply reduced in the shallow
 1791 trap configuration. With this trap the main 17.8-keV peak of the conversion spectrum is
 1792 clearly visible along with additional satellite peaks at lower energy, which correspond to
 1793 the shakeup/shakeoff spectrum of the decay. The high accuracy of the fit demonstrates a
 1794 high degree of understanding of the CRES systematics.

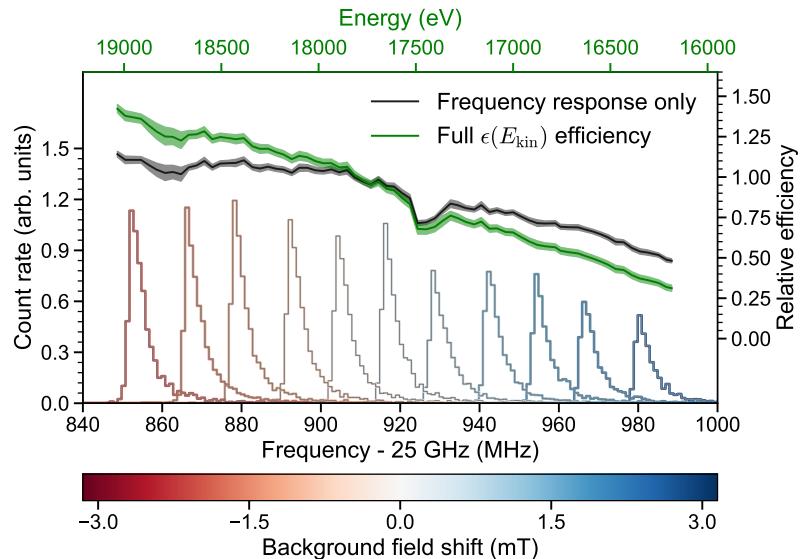


Figure 3.13. Measurements of the 17.8-keV ^{83m}Kr line using the deep trap configuration for different values of the magnetic field from the field shifting solenoid.

1795 The broadening of the krypton spectrum seen for the deeper track is due to the large
 1796 range of electron pitch angles that can be trapped. Furthermore, with a deeper trap
 1797 there is a larger parameter space of electron that could be produced with pitch angles
 1798 that are trappable but not visible in the time-frequency spectrogram. These electrons
 1799 remain in the trap and can scatter multiple times before randomly scattering to a visible
 1800 pitch angle. This leads to one or more missing tracks earlier in the event, which leads to
 1801 a misreconstruction of the true starting frequency. By measuring the krypton spectrum
 1802 shape in the same trap used to detect tritium events, the effect this has on the spectrum
 1803 shape can be characterized to mitigate its impact on the tritium measurements.

1804 Changes in the Krypton spectrum shape as a function of CRES frequency were

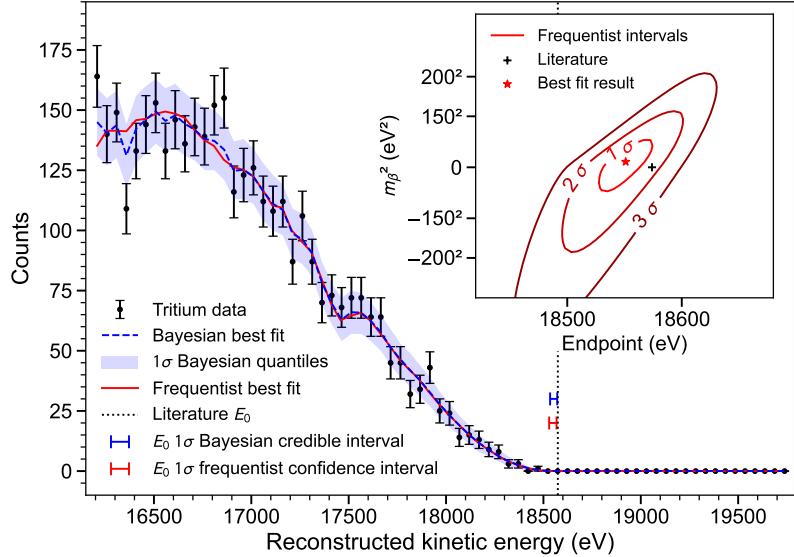


Figure 3.14. The measured tritium spectrum from Phase II with Bayesian and frequentist fits.

used to study the detection efficiency of the Phase II apparatus. Variations in the detection efficiency as a function of frequency directly influences the measured shape of the continuous tritium spectrum, which can lead to errors in the neutrino mass estimate if not modeled appropriately. Using the field-shifting solenoid (FSS) the cyclotron frequency of the krypton 17.83 keV line was shifted across the full frequency range of the tritium spectrum data (see Figure 3.13). The FSS is a wound copper solenoid magnet that surrounds the CRES cell. Controlling the current through this magnet allows one change the value of the background magnetic field and the frequency of the krypton conversion lines. Variations in the deep trap krypton spectrum shape can be used to infer the detection efficiency as a function of frequency and correct for this affect in the tritium measurements.

1816 Tritium Spectrum and Neutrino Mass Results

1817 The tritium measurement campaign resulted in the collection of 82 days of detector
 1818 live time during which 3770 total tritium events were detected. The track and event
 1819 reconstruction analysis extracted the starting frequencies of these tritium events, which
 1820 were used to build a frequency spectrum of tritium beta-decays. The resulting frequency
 1821 spectrum was then converted to an energy spectrum using the information gleaned from
 1822 the krypton measurement campaign to obtain the tritium beta-decay spectrum (see
 1823 Figure 3.14).

1824 CRES is inherently a very low background technique with the dominant source of noise
1825 being random RF fluctuations. Monte Carlo simulations, validated using measurements
1826 of the RF noise background, were used to set track and event cuts to guarantee that
1827 zero false events would occur over the duration of the experiment with 90% confidence.
1828 Notably, the measured spectrum has zero events beyond the tritium spectrum endpoint,
1829 which allows one to constrain the background rate in the Phase II apparatus to less than
1830 3×10^{-10} counts/ev/s. Achieving a low background is critical for future neutrino mass
1831 experiments that seek to measure the neutrino mass with less than 100 meV sensitivity.

1832 Bayesian and frequentist based fits to the measured tritium spectrum, incorporating
1833 information gained about CRES systematics from the krypton measurements, were
1834 performed to extract upper limits on the tritium beta-decay spectrum endpoint as well as
1835 the neutrino mass. The estimated spectrum endpoints are 18553^{+18}_{-19} eV for the Bayesian
1836 analysis and 18548^{+19}_{-19} eV for the frequentist analysis. The quoted uncertainties are
1837 $1-\sigma$, and both results are within $2-\sigma$ of the literature endpoint value of 18574 eV. The
1838 estimated neutrino mass for both results is consistent with $m_\beta^2 = 0$. The 90% confidence
1839 upper limits for the Bayesian analysis is $m_\beta < 155$ eV/c² and $m_\beta < 152$ eV/c for the
1840 frequentist analysis.

1841 Though the neutrino mass results from Phase II are not competitive with KATRIN,
1842 the experiment was a promising first step towards the development of more precise
1843 neutrino mass measurements using CRES. The low-background and high-resolution
1844 achievable with krypton measurements are promising features of the technique that were
1845 demonstrated with the Phase II apparatus. As new technologies are developed to enable
1846 CRES measurements in larger volume, many of the lessons learned from Phase II will
1847 continue to influence the operation and design of future experiments.

1848 **3.4 Phase III R&D: Antenna Array CRES**

1849 The goal of Phase III in the Project 8 experimental program is to develop the technologies
1850 and expertise required to build an experiment that uses CRES to measure the neutrino
1851 mass with a target sensitivity of 40 meV. One of the key technologies is a method for
1852 performing high resolution CRES measurements in a large volume, which allows one to
1853 observe a sufficient quantity of tritium to measure the low-activity endpoint region of
1854 the tritium spectrum.

1855 3.4.1 The Basic Approach

1856 One possible approach, suggested in the original CRES publication [40], is to use many
1857 antennas to surround a volume of tritium gas in a magnetic field (see Figure 3.15). When
1858 a decay occurs the electron will emit cyclotron radiation that can be collected by the array
and used to perform CRES. Each antenna in the array collects only a small fraction of

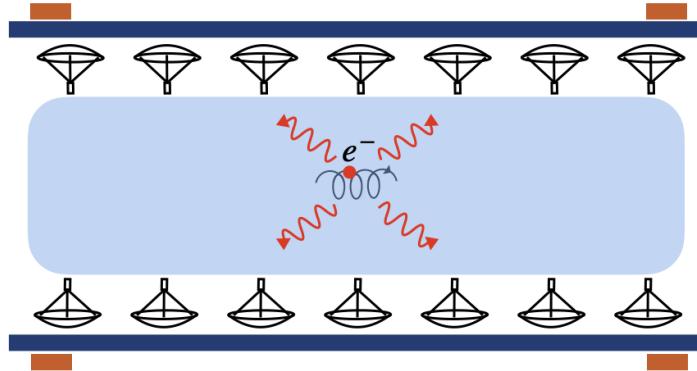


Figure 3.15. A cartoon illustration of the basics of the antenna array CRES technique.

1859
1860 the electron's signal power, which is less than 1 fW for a 18.6 keV kinetic energy electron
1861 in a 1 T magnetic field. Scaling to large volumes with the antenna array approach is
1862 accomplished by increasing the number of antennas in the array, which increases the
1863 volume available for CRES measurements.

1864 Several features of the antenna array approach make it an attractive candidate technol-
1865 ogy for a large volume experiment. One example is the accurate position reconstruc-
1866 tion possible with a multichannel antenna array. Using techniques like digital beamforming,
1867 it is possible to estimate the radial and azimuthal positions of the electron in the mag-
1868 netic trap with a precision significantly less than the size of the cyclotron wavelength.
1869 This capability allows one to perform event-by-event estimations of the magnetic fields
1870 experienced by an electron, which helps achieve high energy resolution with the CRES
1871 technique.

1872 The easy availability of position information with the antennas array approach
1873 is potentially a unique advantage that provides significant flexibility in the magnetic
1874 field uniformity requirements compared to other proposed approaches to large volume
1875 CRES (see Chapter 6). Spatial discrimination using digital beamforming leads to pileup
1876 reduction, which helps to reduce the potential of background events caused by missing
1877 tracks or by incorrectly clustering a group of tracks into an event. Limits on the
1878 background rate for a neutrino mass measurement with 40 meV sensitivity are stringent

1879 and the total activity of the tritium source is gigantic relative to the activity near the
1880 endpoint. Thus, pileup discrimination could be an important tool for a large scale CRES
1881 experiment.

1882 Another beneficial quality of antenna arrays is that the volume of the experiment can
1883 be scaled independent of frequency by simply adding more antennas to the array (see
1884 Figure 3.19). Resonant cavities, the proposed alternative large volume CRES technology,
1885 are ideally operated in magnetic fields that cause electrons to move with cyclotron
1886 frequencies near the fundamental cavity resonance, to avoid complex coupling of the
1887 electron to multiple cavity modes simultaneously. This leads to a coupling between the
1888 cavity volume and the magnetic field magnitude, which forces one to lower the magnetic
1889 field in order to increase the experiment scale. Whereas, for antenna arrays, in principle
1890 there is no physical limitation on the size of the antenna array that can be used at a
1891 particular magnetic field. However, this approach to scaling an antenna array experiment
1892 leads to rapidly increasing cost and complexity due to the large number of antennas,
1893 amplifiers, and data streams, which require substantial computer processing power to
1894 effectively utilize. The link between array size and computational cost will be explored
1895 in Section 4.3.

1896 **3.4.2 The FSCD: Free-space CRES Demonstrator**

1897 The complexity of the antenna array CRES technique requires the construction of a
1898 small scale demonstration experiment to develop an understanding of technique itself and
1899 relevant systematics. Without a demonstrator experiment it is not possible to sufficiently
1900 retire the technical risks associated with the full-scale experiment. Therefore, Phase
1901 III of the Project 8 experimental program is primarily focused on the development and
1902 operation of demonstrator experiments to inform the design of the Phase IV experiment.

1903 The Phase III demonstrator experiment for antenna array CRES is called the Free-
1904 space CRES Demonstrator or FSCD. The FSCD is also a capable neutrino mass mea-
1905 surement experiment in its own right, with a target neutrino mass sensitivity of a few
1906 eV using a molecular tritium source. The higher-costs associated with antenna-based
1907 CRES, which were identified over the course of the development of the FSCD, have lead
1908 to the adoption of resonant cavities as the technology of choice for Phase III. Therefore,
1909 all future development of the FSCD and antenna-based CRES in Project 8 has been
1910 suspended.

1911 **Magnetic Field**

1912 The background magnetic field for the FSCD is provided by a hospital-grade MRI magnet
1913 (see Figure 3.16). The magnet produces a magnetic field of approximately 0.958 T, which corresponds to a tritium spectrum endpoint frequency of approximately 25.86 GHz. The



Figure 3.16. An image of the MRI magnet installed in the Project 8 laboratory at the University of Washington, Seattle.

1914
1915 magnet is installed in the Project 8 laboratory located at the University of Washington,
1916 Seattle, and is shimmed to produce a uniform magnetic field with variations on the
1917 ppm-level. Measurements of the magnetic field non-uniformities are performed using a
1918 NMR probe and rotational gantry to capture measurements of the magnetic field around
1919 an elliptical surface in the center of the MRI magnet. During the operation of the FSCD
1920 an array of Hall or NMR magnetometers would be used to periodically measure the
1921 magnetic field to monitor its time stability.

1922 Inside the field of the MRI additional electromagnets would be installed that provide
1923 the capability to shift the value of the background magnetic field and produce a magnetic
1924 trap. Shifting the background magnetic field by a few μ T lets one control the cyclotron
1925 frequencies of electrons with a fixed kinetic energy, which is key to an effective calibration
1926 of the FSCD. The preferred calibration method for the FSCD is a mono-energetic electron
1927 gun that can inject electrons into the magnetic trap with a known kinetic energy. In
1928 combination with the field shifting magnet, one can vary the cyclotron frequencies of the
1929 electrons to measure the response of the antenna array as a function of the radiation
1930 frequency and electron position. This procedure characterizes the response of the antenna

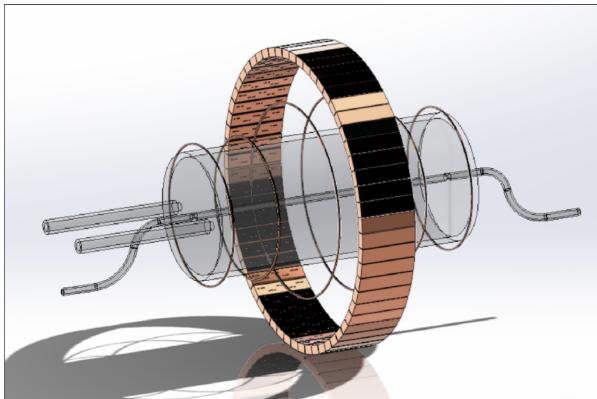
array and provides further information on magnetic field uniformity, which is important to achieving good energy resolution.

The design of the magnetic trap is absolutely critical to the success of a CRES experiment. The ideal shape is the perfect magnetic box, which has a flat bottom and step function walls. Any variation in the average magnetic field experienced by an electron leads to changes in the cyclotron frequency that can make determining the true starting kinetic energy more difficult. This includes changes in the magnetic field caused by the walls of the magnetic trap as well as radial magnetic field variations.

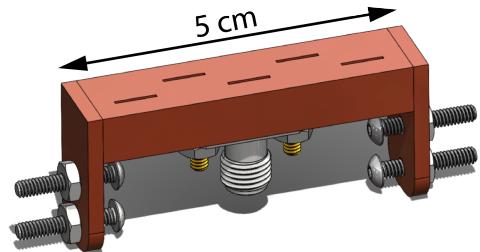
The ideal box trap is completely uniform and has infinitely steep walls that cause no change in the electron's cyclotron frequency as it is reflected from the trap wall. However, such a trap cannot be made from any combination of magnetic coils since it violates Maxwell's equations. One of the goals of magnetic trap design is to identify the configuration of coils that produces a trap that approximates the perfect box trap as closely as possible.

Antenna Array

The canonical antenna array design for CRES is a uniform cylindrical array of antennas that surrounds the magnetic trap volume. Since the FSCD is a demonstrator experiment, the antenna array design is the simplest form of the uniform cylindrical array, which is a single circular ring of antennas with a diameter of 20 cm (see Figure 3.17). Along



(a)



(b)

Figure 3.17. (a) A model of the FSCD antenna array, magnetic trap, and tritium containment vessel design.(b) A more detailed model of a prototype design for the 5-slot waveguide antenna design.

1949

1950 this circle are sixty slotted waveguide antennas that fully populate the available space

1951 around the array circumference. It is optimal to cover as large a fraction of the solid
1952 angle around the magnetic trap as possible in order to maximize the power collected
1953 from each electron .

1954 The distance between antennas around the circumference of the array is proportional
1955 to the wavelength of the cyclotron radiation. Therefore, maximizing the solid angle
1956 coverage of the array, while minimizing channel count to keep the hardware and data
1957 acquisition costs manageable, biases one towards smaller array diameters. Antenna
1958 near-field effects limit the minimum diameter of the array for a given antenna design,
1959 since the radiation from electrons that are too close to the array cannot be detected due
1960 to destructive interference.

1961 Slotted waveguide antennas are used in the FSCD antenna array due to their high
1962 efficiency and low loss, which comes from the lack of dielectric materials in the antenna
1963 structure. Coupling to the waveguide is performed with a coaxial cable connected at the
1964 center of the antenna. One of the drawbacks of waveguide antennas is the large amount
1965 of space required to fit them inside the limited MRI magnet volume. Alternative antenna
1966 designs, constructed from microstrip printed circuit boards require significantly less space
1967 at the cost of slightly higher energy losses in the antenna structure.

1968 The FSCD antenna design is a 5 cm long segment of WR-34 waveguide with 5 vertical
1969 slots cut into the side. The distance between slots along the length of the waveguide is
1970 a half wavelength for optimal power combination between the individual antenna slots.
1971 Each slot is offset from the center of the antenna face a small distance in order to most
1972 effectively couple the slot to waveguide modes inside the antenna.

1973 The passive power combination achieved by placing 5 slots in a single waveguide is a
1974 compromise intended to reduce the cost and complexity of the antenna array system.
1975 Each additional channel in the array requires its own cryogenic amplifier and also increase
1976 the required computer power to process the raw data collected by digitizing each channel.
1977 Passive summation, achieved by combining antennas into arrays axially, reduces the
1978 array channel count at the cost of losses from imperfect passive combination.

1979 Interference and re-radiation eventually limit the axial extent of passive power combi-
1980 nation. The 5-slot designed developed for the FSCD is optimized to minimize the impact
1981 of these losses while achieving the maximum amount of axial coverage with a single ring
1982 of antennas. Scaling beyond the volume covered by a single ring of antennas is achieved
1983 by stacking additional rings of antennas together to cover a larger trap volume. A likely
1984 scenario for the FSCD experiment involves a staged experiment approach, where first
1985 a series of measurements is performed using only a single ring of antennas followed by

1986 experiments that add additional rings to the FSCD. The goal would be to first understand
1987 the principles of antenna array CRES using the simplest possible experiment, before
1988 attempting to scale the technique by expanding the antenna array size.

1989 **Tritium Source**

1990 While the primary purpose of the FSCD is as a technology demonstrator, it is impossible to
1991 retire all risks with the Phase IV experiment without an intermediate scale measurement
1992 of the neutrino mass. Therefore, the FSCD has the scientific goal of measuring the
1993 neutrino mass with a rough sensitivity goal in the range of a few eV. This level of precision
1994 is achievable using a molecular tritium source with a volume of approximately 1 L at a
1995 density comparable to potential Phase IV scenarios.

1996 Unlike previous CRES experiments, where the tritium source could be colocated
1997 with the receiving antenna inside a waveguide transmission line, the tritium source
1998 in the FSCD is thermally isolated from the antenna array to avoid freeze-out of the
1999 tritium molecules. The tiny radiation power emitted by electrons requires a system noise
2000 temperature of ≈ 10 K or less, in order to detect events at a high enough efficiency to
2001 reach the neutrino mass sensitivity goals of the experiment. Achieving a system noise of
2002 10 K requires that the antenna array and amplifiers operate at liquid helium temperatures
2003 of ≈ 4 K, which significantly lowers the vapor pressure of molecular tritium. By keeping
2004 the molecular tritium isolated in an RF-transparent vessel the tritium gas can be kept
2005 at a relatively warmer temperature in the range of 30 K to avoid the accumulation of
2006 tritium on the experiment surfaces.

2007 **Data Acquisition and Reconstruction**

2008 A fundamental change in the data acquisition system for the FSCD is the shift from
2009 single to multichannel reconstruction. This transition results in a significant increase in
2010 the data-generation rate, which is linearly related to the number of independent channels
2011 in the array. The larger data volume coincides with an increased demand for computer
2012 processing power based on the need for more precise signal reconstruction algorithms
2013 driven by the FSCD and Phase IV sensitivity goals. Therefore, the data acquisition
2014 system for the FSCD is likely to represent a significantly larger fraction of the experiment
2015 cost and complexity than in Phase II.

2016 Each antenna is connected to a cryogenic amplifier and down-converted from the
2017 26 GHz CRES frequency using an IQ-mixer to reduce the size of the analysis window.
2018 Using an LO with a frequency of approximately 25.80 GHz the antenna array signals can

2019 be digitized at a rate of 200 MHz, which is sufficient bandwidth to resolve the complete
2020 sideband spectrum produced by axial oscillations of electrons in the FSCD magnetic
2021 trap.

2022 Direct storage of the raw FSCD antenna array data is undesirable, since the estimated
2023 amount of raw data generated is $O(1)$ exabyte per year. The storage of such a large
2024 dataset is infeasible for a demonstrator experiment like the FSCD, since it would represent
2025 a disproportionate fraction of the total experiment budget in Phase III and Phase IV.
2026 Therefore, a goal of the FSCD experiment is the development of real-time reconstruction
2027 methods that could reduce the raw data volume by detecting and reconstructing CRES
2028 events in real-time. Ultimately, a real-time CRES reconstruction pipeline is desired, which
2029 takes raw voltages samples from the antenna array and converts them into measured
2030 starting kinetic energy values for electrons.

2031 The feasibility of a real-time reconstruction pipeline rests on the development of
2032 computationally efficient algorithms that can be implemented without the need for
2033 enormous computing resources. One challenge with the antenna array approach is that
2034 the small radiation power of a single electron is distributed among all channels in the array,
2035 such that reconstruction using only the information in a single channel is not possible.
2036 Therefore, simply performing the initial step in reconstruction — signal detection —
2037 requires orders of magnitude more computational power than previous CRES experiments.
2038 This operation will then be followed by other, potentially more expensive, reconstruction
2039 steps that are required in order to determine the kinetic energy of the electron.

2040 **3.5 Pilot-scale Experiments**

2041 The Project 8 pilot-scale experiment represents the experiment that retires all technical
2042 and engineering risks with Project 8’s neutrino mass measurement approach, by combining
2043 all the required components of Phase IV in a multi-cubic-meter experiment. The larger
2044 scope and complexity of the pilot-scale experiment requires a careful choice of magnetic
2045 field and cyclotron frequency since this directly affects the design of nearly all parts of
2046 the experiment. Currently, designs for the pilot-scale experiment are in the conceptual
2047 stage, but a goal of Phase III is to translate these design concepts into detailed technical
2048 designs and specifications.

2049 **3.5.1 Choice of Frequency**

2050 The optimal CRES frequency for Project 8 is that which reaches the target sensitivity of
2051 40 meV, while minimizing the cost and complexity of the overall experiment. The CRES
2052 frequency is directly linked to the magnetic field, which is coupled to nearly all aspects
2053 of the experiment design, therefore, an optimization of CRES frequency is effectively an
2054 optimization of the sensitivity of the overall experiment.

2055 **Frequency Scaling Laws**

2056 The Phase I and II experiments utilized a background magnetic field of 0.959 T provided
2057 by an NMR magnet. Since this magnet was already available, the 0.959 T background
2058 field was selected for convenience. However, one additional reason to use this background
2059 field is that the cyclotron frequencies for electrons near the tritium endpoint in a 0.959 T
2060 field are approximately 26 GHz, which is within the standard RF Ka-band. Therefore,
2061 microwave electronics specialized for these frequencies are obtainable for relatively low
2062 cost. The operating frequency for the large-scale experiments must be selected in a more
2063 rigorous manner due to the increased scale and complexity of the systems as well as the
2064 requirements of the 40 meV neutrino mass science goal.

2065 There is a bias towards lower frequencies in a large-volume experiment, due to the
2066 direct relationship between wavelength and the physical size of the compatible RF
2067 components like antennas and cavities. With a longer wavelength more volume can
2068 be surrounded by an array with fewer antennas, which reduces hardware and data-
2069 processing costs. Additionally, the size of a cavity experiment is directly proportional to
2070 the wavelength, since this sets the physical dimensions of the cavity. It is also simpler to
2071 engineer a magnet that provides a uniform magnetic field across several cubic-meters of
2072 space at lower magnetic fields, which provides advantages in terms of cost-reduction.

2073 A concern with lower magnetic fields and frequencies is the power scaling as described
2074 by the Larmour equation, in which power is proportional to the square of the frequency.
2075 Naively, one would predict that the SNR would decrease with lower fields, however, two
2076 additional scaling laws that affect the noise power also come into play. Noise power
2077 is directly proportional to the required bandwidth, which decreases linearly with the
2078 magnetic field. Furthermore, at lower frequencies it is possible to purchase amplifiers with
2079 lower noise temperatures until approximately 300 MHz, at which point this relationship
2080 tends to flatten. Therefore, it is expected that the SNR remains approximately constant
2081 as the frequency decreases.

2082 The facts that large-volume experiments are simpler to achieve at lower frequencies
 2083 and SNR is expected to be approximately the same together motivate the usage of lower
 2084 magnetic fields in the large-scale experiments. This is simply because a low-frequency
 2085 experiment is less costly than a high-frequency experiment and there is little to no penalty
 2086 in SNR or detection efficiency at these fields.

2087 One drawback of lower magnetic fields is the increased influence of external magnetic
 2088 fields on the experiment. This includes magnetic fields from the building materials as well
 2089 as variations in the earth's magnetic field. A suitable magnetic field correction system
 2090 will need to be devised to deal with these effects, which includes constant monitoring of
 2091 external fields.

2092 Atomic Tritium Considerations

2093 The pilot-scale experiments will be the first Project 8 experiments to combine CRES with
 2094 atomic tritium, therefore, the optimal frequency should take into account the affect of the
 background magnetic field on the atom trap. The primary influence of the background

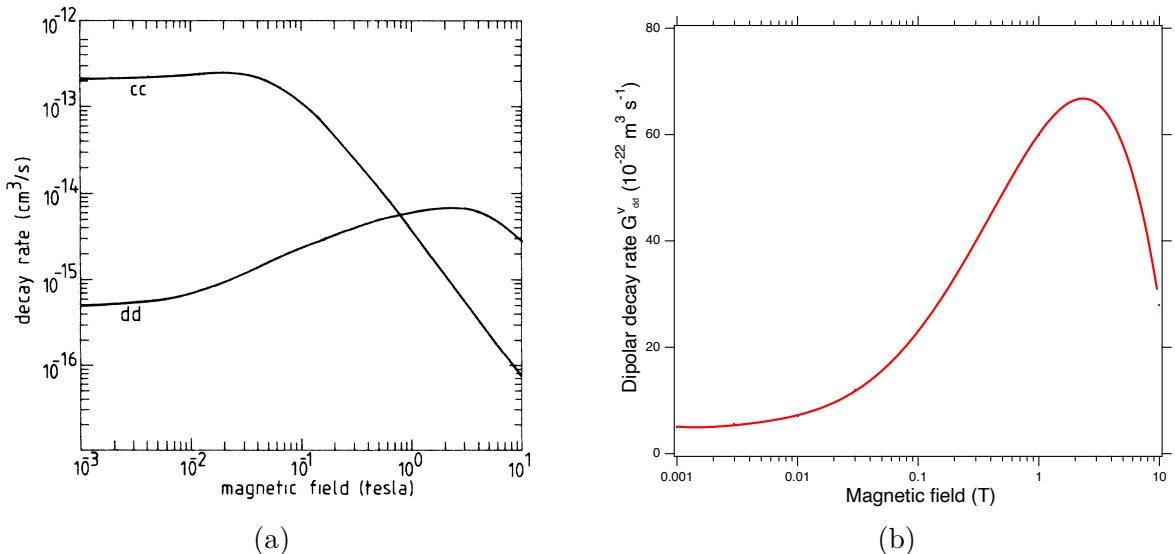


Figure 3.18. (a) A plot of the decay rate for the two-body dipolar spin exchange interaction for cc and dd state. (b) A plot of the decay rate of the dipolar spin exchange interaction for d+d states as a function of magnetic field magnitude. Lowering the magnetic field is key for reducing the losses from this interaction.

2095 field magnitude is through the rate of dipolar spin-flips caused by a spin exchange
 2096 interaction between trapped atoms [58].

2098 Atomic tritium is a simple quantum system with a hyperfine structure given by the
 2099 addition of the nuclear and atomic spins. The addition of two spins leads to a hyperfine

structure with four states in the (m_s, m_I) basis [59]. The states with atomic spins directed anti-parallel to the magnetic field have $m_s = -1/2$ and are labeled as the a and b states. The a and b states are colloquially known as high-field seeking states, since their energy is minimized when in regions of higher magnetic field. This leads to losses in the magnetic trap as these atoms are drawn to higher fields away from the trap center. Alternatively, the c and d states, with atomic spin $m_s = +1/2$, minimize their energy in low magnetic fields because of the parallel alignment between spin and the magnetic field. Therefore, these low-field seeking states tend to stay trapped significantly longer than the high-field seeking states.

It would be advantageous to prepare tritium atoms in purely c and d states before trapping, however, even in this case losses still occur due to dipolar interactions between pairs of c and d states leading to flipped atomic spins and subsequent losses from high-field seeking atoms. The rate of these interactions depends on the magnitude of the background magnetic field and is maximal for dd interactions around 1 T (see Figure 3.18). The rate of losses from these interactions at 1 T requires atomic tritium production at a rate two orders of magnitude larger than at 0.1 T, thus, requirements on the whole atomic tritium system are significantly relaxed at lower magnetic fields, which provides powerful argument for moving to lower frequencies with the pilot-scale experiments and Phase IV.

3.5.2 Pilot-scale Experiment Concepts

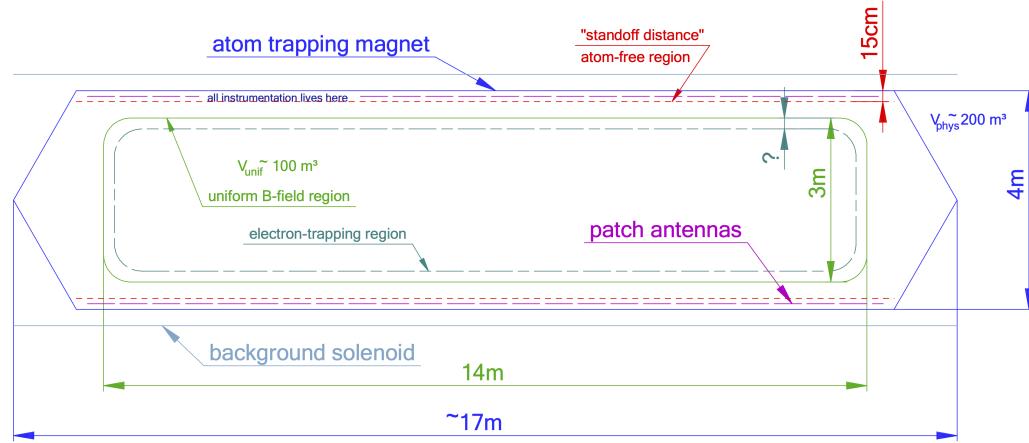


Figure 3.19. A conceptual sketch of a large-volume antenna array based CRES experiment to measure the neutrino mass.

While the pilot-scale experiments are still in the early stages, enough is known to

2121 sketch the general features of these experiments at the conceptual level. Development of
2122 the antenna-based experiment has been suspended in favor of the cavity-based experiment.

2123 **Pilot-scale Antenna Array CRES Experiment Concept**

2124 A conceptual design for an antenna-based CRES experiment is shown in Figure 3.19. A
2125 large solenoid magnet provides a uniform background magnetic field less than 0.1 T in
2126 magnitude. Inside this region is the atom trapping magnet that generates a high magnetic
2127 field at the walls, which decays exponentially towards the central region. Known magnet
2128 designs that produce suitable atom trapping fields include Ioffe-Pritchard traps [60],
2129 which use conducting coils, as well as a Halbach array [61] made from permanent magnets.
2130 Either magnet choice produces a region of high magnetic fields, which excludes atoms
2131 and allows for the placement of antennas inside the experiment.

2132 Inside this region an array of microstrip patch antennas is inserted to collect the
2133 cyclotron radiation without providing a surface for atomic tritium recombination. Due
2134 to the lower frequency of cyclotron radiation antennas of a larger size can be used,
2135 which lowers the total number of antennas required to observe the experiment volume.
2136 Because of this scaling, the lower frequency experiment uses a similar number of antennas
2137 compared to a much smaller demonstrator experiment with a 1 T magnetic field.

2138 The atomic tritium beamline that supplies fresh tritium atoms to the experiment is
2139 not shown in the figure. The general configuration would matches the one shown for the
2140 pilot-scale cavity experiment (see Figure 3.20).

2141 **Pilot-scale Cavity CRES Experiment Concept**

2142 The pilot-scale cavity experiment includes both an atomic tritium system and cavity
2143 CRES system. The atomic system consists of a thermal atom cracker located at the
2144 start of an evaporatively cooled atomic beamline. The atomic tritium system provides a
2145 supply of tritium atoms to the trap with temperatures on the order of a few mK. Atoms
2146 at this temperature can be trapped magneto-gravitationally, which is the reason for the
2147 vertical orientation of the cavity. At these low magnetic fields the trapping requirements
2148 for electrons and atoms differ enough such that it is advantageous to decouple the the
2149 trapping potentials to avoid radioactive heating of the tritium atoms from excess trapped
2150 electrons. Electron trapping is provided by a set of magnetic pinch coils at the top and
2151 bottom of the cavity and a multi-pole Ioffe or Halbach magnet serves to contain the
2152 atoms.

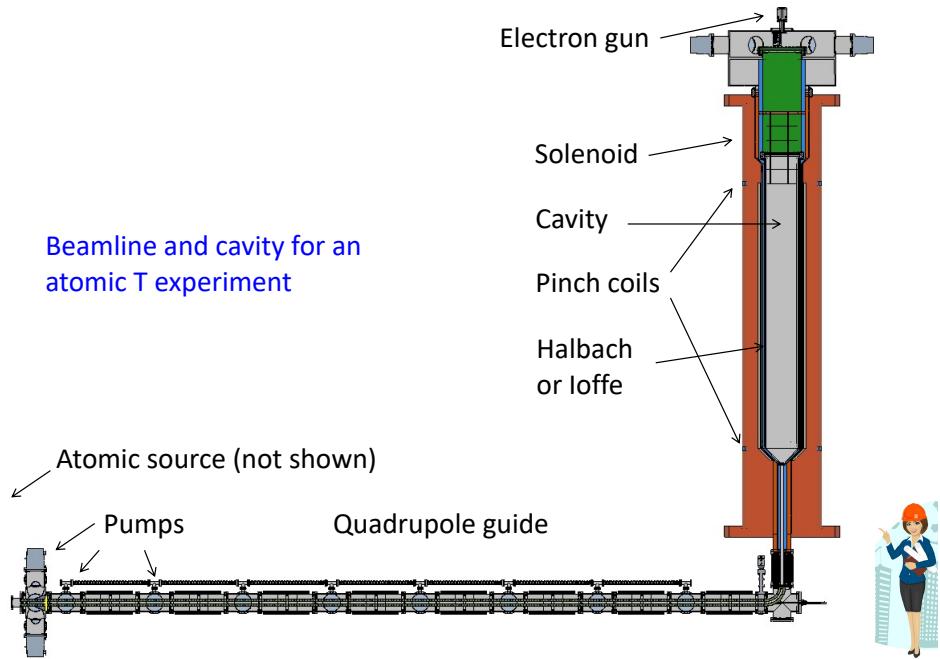


Figure 3.20. A conceptual sketch of a pilot-scale cavity CRES experiment with an atomic tritium beamline.

2153 The cavity design for the pilot-scale experiment consists of a large cylindrical cavity
 2154 with a TE011 resonance of 325 MHz. Such a cavity is truly enormous, with a diameter of
 2155 approximately 1.2 m and a height of 11 m. When an electron is produced inside the cavity
 2156 with a cyclotron frequency that matches the TE011 resonant frequency, its cyclotron
 2157 orbit couples the electron to the TE011 mode, which drives a resonance in the cavity.
 2158 These resonant fields can be read-out using an appropriate cavity coupling mechanism
 2159 located at the center of the cavity. For more information on the cavity approach to
 2160 CRES see Chapter 6.

2161 The bottom of the cavity has a cone termination to match the contour of the atom
 2162 trapping magnet. This shape still allows for TE011 resonances with high internal Qs,
 2163 which are required for good SNR in the cavity experiment. A small opening in the bottom
 2164 of the cone serves as an entry point for the tritium atoms. To allow for calibration of
 2165 the magnetic field inhomogeneities with an electron gun, the top of the cavity is left
 2166 nearly completely open. Normally, this would drastically lower the Q-factor of the TE011
 2167 mode, but a specially configured coaxial partition is inserted at the top. This termination
 2168 scheme is designed to act as a perfect short for the TE011 mode since the circular shape
 2169 of the partition matches the electric field boundary conditions for the TE011 mode.
 2170 Simulations with HFSS have confirmed that this design results in a high quality TE011

²¹⁷¹ resonance despite the nearly completely open end.

²¹⁷² 3.6 Phase IV

²¹⁷³ The baseline CRES technology being pursued by Project 8 are resonant cavities, which,
²¹⁷⁴ due to their geometric properties, simple CRES signal structure, and low channel
²¹⁷⁵ count, appear to be the better option for Phase IV. The current knowledge of the
²¹⁷⁶ antenna array CRES approach reveals no technical obstacles that would preclude it
²¹⁷⁷ as a baseline technology for Phase IV, though it would certainly be significantly more
²¹⁷⁸ expensive. Therefore, antenna arrays represent a fallback approach if resonant cavities
²¹⁷⁹ prove infeasible.

²¹⁸⁰ The sensitivity of the pilot-scale atomic tritium experiment is estimated to be on
²¹⁸¹ the order of 0.1 eV, which means that increasing the sensitivity to reach the Phase IV
²¹⁸² goal will require an even larger experiment. Because of the direct coupling between the
²¹⁸³ RF characteristics of a cavity and its geometry, the baseline plan is to build multiple
²¹⁸⁴ copies of the pilot-scale experiment (see Figure 3.21) to obtain the required amount of
²¹⁸⁵ volume rather than increase the size of the cavity beyond the pilot-scale. The built-in
²¹⁸⁶ redundancy of this approach is useful in the sense that the experiment has no single
²¹⁸⁷ point of failure, additionally, building several copies of the a pilot-scale experiment will
²¹⁸⁸ minimize new engineering and design effort.

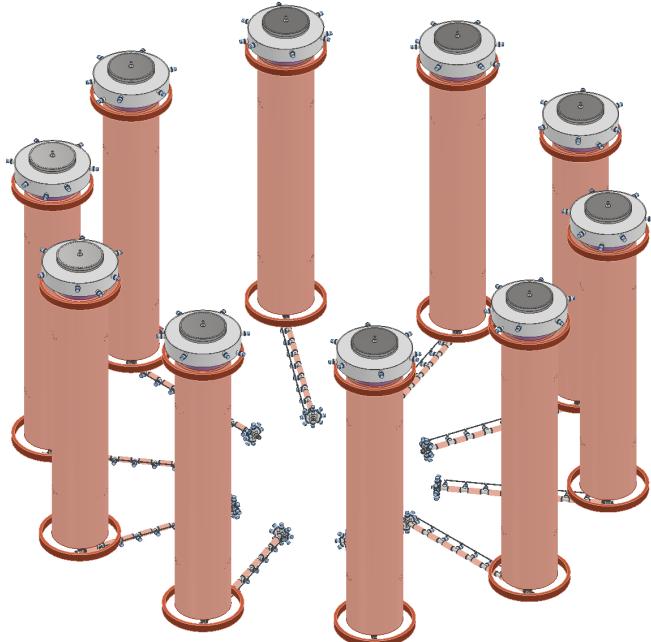


Figure 3.21. An illustration of a possible arrangement of ten pilot-scale cavity experiments for Phase IV. The experiments are arranged in a circle with an approximate diameter of 50 meters. Each atomic beamline connected to the bottom of each cavity is approximately 10 m in length. The cavities themselves are designed to operate at 325 MHz and are approximately 11 m tall. The circular arrangement of cavities has some advantages when it comes to cancellation of fringe fields from neighboring magnets, which is important due to the small magnetic field magnitudes consistent with these CRES frequencies. The advantage of ten independent atomic sources and cavities is that there is no single point of failure for the experiment. If an experiment goes down for repairs the other nine may continue running. Figure courtesy of Michael Huehn at UW-Seattle.

Chapter 4

Signal Reconstruction Techniques for Antenna Array CRES and the FSCD

4.1 Introduction

An antenna array CRES (Cyclotron Radiation Emission Spectroscopy) experiment introduces new challenges related to data acquisition, signal detection, and signal reconstruction caused by the multi-channel nature of the data. The development of signal reconstruction algorithms [62] is crucial for the design of antenna array based experiments like the FSCD (Free Space CRES Demonstrator, described in Section 3.4.2), because these algorithms directly influence the detection efficiency and energy resolution of the CRES experiment. In this Chapter I summarize my contributions to the development and analysis of signal reconstruction and detection algorithms for the FSCD experiment.

In Section 4.2 I discuss the primary tool for this work, which is the Locust simulations package developed by the Project 8 experiment. Locust is used to simulate CRES events in the detector, which begins with calling a second software package — Kassiopeia — to calculate particle trajectory solutions for electrons in the magnetic trap. The trajectories are subsequently used to calculate the response of the antenna array to the cyclotron radiation produced by the electron, which results in signals that can be used to analyze the performance of different signal reconstruction algorithms. More recently, Project 8 has developed CREsana, which is a new simulations package that takes a more analytical approach to CRES signal simulations for antenna arrays. Although CREsana signals were not used for the signal reconstruction algorithm development detailed here, I introduce the software as it is the simulation software used to model the antenna array measurements presented in Section 5.4.

In Section 4.3 I discuss the signal reconstruction and detection approaches analyzed for the FSCD experiment. In general there are two steps to signal reconstruction — detection

and parameter estimation. With signal detection one is concerned with distinguishing between data that contains a signal versus data that contains only noise, whereas, with parameter estimation one extracts the kinematic parameters of the electron encoded in the cyclotron radiation signal shape. Due to the low signal power of electrons near the spectrum endpoint in the FSCD experiment, signal detection is a non-trivial problem. This is magnified by the need to maximize the detection efficiency of the experiment in order to achieve the neutrino mass sensitivity goals. My contributions to signal reconstruction analyses for the FSCD are focused on the signal detection component of reconstruction.

After discussing various signal detection approaches, in Section 4.4 I present a detailed analysis of the detection performance of three algorithms, which could be used to signal detection in the FSCD. This section was prepared for publication in JINST as a separate paper. The algorithms include a digital beamforming algorithm, a matched filter algorithm, and a neural network algorithm, which I analyze in terms of classification accuracy and estimated computational cost.

4.2 FSCD Simulations

Antenna array CRES and the FSCD require a combination of different capabilities not often found in a single simulation tool. In particular, accurate calculations of the magneto-static fields produced by current-carrying coils are needed to accurately model the magnetic trap and background magnets. The resulting magnetic fields must then be used to calculate the exact relativistic trajectory of electrons. The electron trajectories are required to calculate the electro-magnetic (EM) fields produced by the acceleration of the electron. Finally, the simulation must model the interaction of the antenna and RF (radio-frequency) receiver chain with the EM-fields in order to yield the simulated voltage signals from the antenna array. No available simulation tools adequately perform these combined functions, therefore, Project 8 developed a custom simulation framework to simulate the FSCD and CRES. This simulation framework includes custom simulation tools developed by Project 8, as well as open-source and proprietary software developed by third-parties.

2244 4.2.1 Kassiopeia

2245 Kassiopeia¹ is a particle tracking and static EM-field solver developed by the KATRIN
2246 collaboration for simulations of their spectrometer based on the MAC-E (magnetic
2247 adiabatic collimation with electrostatic)filter technique [63]. Unfortunately, Kassiopeia is
2248 not designed to solve for the EM-fields radiated by electrons in magnetic fields. However,
2249 it does provide efficient solvers for static electric and magnetic fields and charged particle
2250 trajectory solvers. Because of this, Project 8 has incorporated parts of Kassiopeia into
2251 the Locust simulation framework.

2252 Magnetostatic Field Solutions

2253 The solutions to the electric and magnetic fields generated by a static configuration of
2254 charges and currents is given by Maxwell's equations in the limit where the time-dependent
2255 terms go to zero. In their static form Maxwell's equations [49] are

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (4.1)$$

$$\nabla \times \mathbf{E} = 0 \quad (4.2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (4.3)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \quad (4.4)$$

2256 where it can be seen that the electric and magnetic fields are completely decoupled from
2257 one another. The solution for the magnetic field in this boundary value problem is given
2258 by the Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int dr' \frac{r'^3 \mathbf{J}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r}' - \mathbf{r}|^3}, \quad (4.5)$$

2259 which Kassiopeia can use a variety of numeric integration techniques to solve for a
2260 particular current distribution.

2261 Kassiopeia Simulation of the FSCD Magnetic Trap

2262 The trap developed for the FSCD experiment utilizes six current carrying coils, which
2263 surround a cylindrical tritium containment vessel (see Figure 4.1). Some critical aspects
2264 of the trap design include the total trapping volume, the maximum trap depth, the

¹<https://github.com/KATRIN-Experiment/Kassiopeia>

steepness of the trap walls, as well as the radial and azimuthal uniformity of the magnetic fields.

The volume of the FSCD trap is a cylindrically shaped region with a radius of 5 cm and a length of 15 cm resulting in a roughly 1 L total trap volume. The trap volume is an important design feature, because it sets the volume of the experiment that is potentially usable for CRES measurements. Trapping a larger volume allows one to observe a larger number of tritium atoms, which increases the statistical power and sensitivity of the neutrino mass measurement. Due to the cost of constructing magnets with large and uniform magnetic fields it is important that the trap use as much of the available volume as possible to limit the overall cost of the experiment.

Coil	Radius (mm)	Z Pos. (mm)	Current (Amp. \times Turns)
1	50.0	-92.3	750.0
2	50.1	-56.9	-220.3
3	68.5	-19.5	-250.0
4	68.5	19.5	-250.0
5	50.1	56.9	-220.3
6	50.0	92.3	750.0

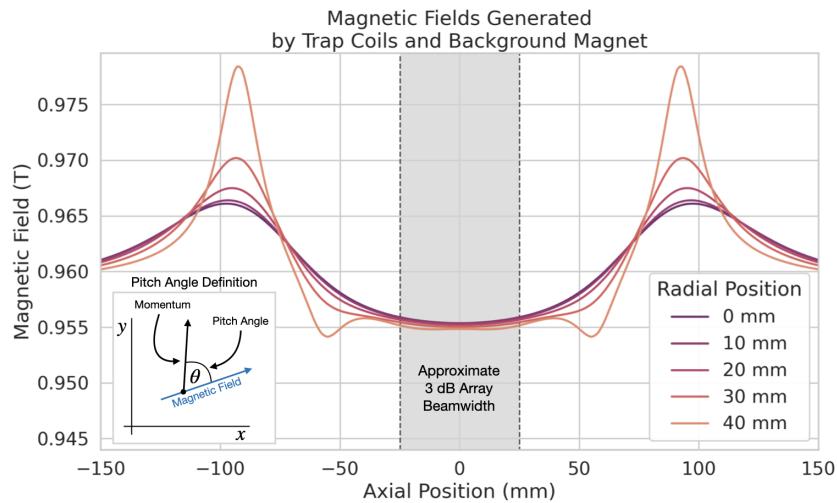
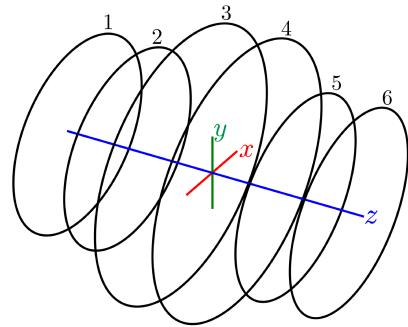


Figure 4.1. The geometry and parameters of the coils used to simulate the FSCD magnetic trap in Kassiopeia. Some axial profiles of the magnetic trap at different radial positions are shown to demonstrate the shape of the magnetic field and trap depth as a function of position. Calculation of the magnetic field profiles was graciously done by René Reimann.

The depth of the FSCD trap is approximately 10 mT when measured along the central axis, which is sufficient to trap electrons with pitch angles as small as 84° . The trap depth influences the efficiency of the experiment by directly controlling the range of

2278 electron pitch angles that can be trapped. If a higher fraction of pitch angles are trapped,
2279 in principle, more decay events can be observed. However, the signals from electrons
2280 with small pitch angles are significantly harder to detect in the FSCD than large pitch
2281 angles, which increases the likelihood of not detecting the first track of the CRES event
2282 and harms the energy resolution of the experiment.

2283 The steepness of the trap walls as well as non-uniformities in the magnetic field
2284 contribute to the total energy resolution of the CRES measurement by causing uncertainty
2285 in the relationship between an electron's kinetic energy and its cyclotron frequency. When
2286 an electron is trapped, it oscillates back and forth along the trap z-axis (see Figure 4.1)
2287 unless it has a pitch angle of exactly 90° [64]. As the electron is reflected from the trap
2288 walls it experiences a change in the total magnetic field, which causes a modulation in the
2289 cyclotron frequency. This change in magnetic field from the trap introduces a correlation
2290 between the pitch angle and kinetic energy parameters of the electron that can reduce
2291 energy resolution. In order to mitigate this effect it is important to make the trap walls
2292 as steep as possible.

2293 Particle Trajectory Solutions

2294 The magnetic fields solved by direct integration of the coil current densities are used to
2295 calculate the trajectories of electrons based on user specified initial conditions. Various
2296 statistical distributions are available, which can be sampled to replicate realistic event
2297 statistics. These include uniform, Gaussian, and Lorentzian distributions among others.
2298 In general, an electron has six kinematic parameters that define its trajectory, which are
2299 the three-dimensional coordinates of the initial position and the three components of the
2300 electron's momentum vector. However, when simulating CRES events it is common to
2301 parameterize the electron's trajectory in terms of the initial position, kinetic energy, pitch
2302 angle, and initial direction of the component of the electron's momentum perpendicular
2303 to the magnetic field. This parameterization is completely equivalent to specifying the
2304 starting position and momentum vectors.

2305 From the initial parameters of the electron and the magnetic field, Kassiopeia solves
2306 for the trajectory of the electron. The direct approach proceeds by solving the motion of
2307 the electron using the Lorentz force equation, which takes the form of a set of differential
2308 equations

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}}{\gamma m} \quad (4.6)$$

$$\frac{d\mathbf{p}}{dt} = e(\mathbf{E} + \frac{\mathbf{p} \times \mathbf{B}}{\gamma m}), \quad (4.7)$$

where \mathbf{r} is the position of the electron, \mathbf{p} is the electron's momentum, e is the charge of the electron, m is the electron's mass, and γ is the relativistic Lorentz term. Kassiopeia solves this pair of differential equations using numerical integration, however, the exact trajectory can be computationally intensive to solve. If the adiabatic approximation can be applied, then Kassiopeia can make use of a simpler set of equations that can be more readily solved numerically.

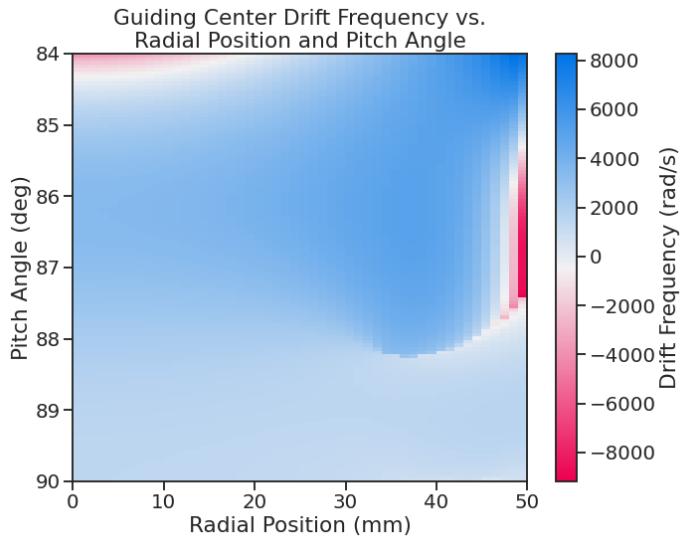


Figure 4.2. A map of the average ∇B -drift frequency for electrons trapped in the prototype FSCD trap shown in Figure 4.1. Negative drift frequencies indicate electrons that are drifting opposite to the standard direction, which means that they are close to escaping the magnetic trap.

Though Kassiopeia is not directly capable of simulating the cyclotron radiation, it is an invaluable CRES simulation tool. With Kassiopeia it is possible to test the efficiency of a particular trap design, and analyze features of the electron trajectories that are important to the position, track, and event reconstruction (see Section 4.3). An example is the analysis of the average ∇B -drift frequency as a function of the electrons radial position and pitch angle in the FSCD trap (see Figure 4.2). Radial gradients in the trap cause the guiding center of the electron to drift around the center of the magnetic trap with an average frequency on the order of 10^3 rad/s. This frequency, while slow compared to the length of a typical CRES time-slice, is large enough to cause a significant loss in efficiency of certain signal reconstruction algorithms. Therefore, it is important to model

2325 the drift of the electron in the reconstruction algorithm in order to mitigate the effects
2326 of this motion on the reconstruction.

2327 4.2.2 Locust

2328 The Locust² software package [65] is the primary simulation tool developed and used
2329 by the Project 8 collaboration for CRES experiments. Locust simulates the responses
2330 of antennas and receiver electronics chain to rapidly time-varying electric fields using
2331 a flexible approach that allows one to choose from a variety of electric field sources
2332 and antennas. Similarly, one can simulate the receiver chain using a series of modular
2333 generators that include standard signal processing operations such as down-mixing and
2334 fast Fourier transforms (FFT). Since the primary focus of this chapter is the application
2335 of Locust to analyses of the FSCD, I shall describe only the most relevant aspects of the
2336 software rather than provide a comprehensive description.

2337 Cyclotron Radiation Field Solutions

2338 Simulating CRES events in the FSCD requires one to calculate the electric fields produced
2339 by the acceleration of the electron. In the general case, this can be a complicated
2340 computation, due to back-reaction forces on the electron. However, in the case of the
2341 FSCD it is possible to ignore such effects and approximate the electron as radiating into
2342 a free-space environment.

2343 The equations that describe the EM fields from a relativistic moving point particle
2344 are the Liénard-Wiechert equations [66, 67], which are obtained by differentiating the
2345 Liénard-Wiechert potentials. In their full form, the Liénard-Wiechert field equations are

$$\mathbf{E} = e \left[\frac{\hat{n} - \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta} \cdot \hat{n})^3 |\mathbf{R}|^2} \right]_{t_r} + \frac{e}{c} \left[\frac{\hat{n} \times [(\hat{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{n})^3 |\mathbf{R}|} \right]_{t_r} \quad (4.8)$$

$$\mathbf{B} = [\hat{n} \times \mathbf{E}]_{t_r}, \quad (4.9)$$

2346 where e is the charge of the particle, \hat{n} is the unit vector pointing from the particle to
2347 the position where the fields are calculated, $\boldsymbol{\beta}$ and $\dot{\boldsymbol{\beta}}$ are the velocity and acceleration
2348 of the particle divided by the speed of light (c), \mathbf{R} is the distance from the particle to
2349 the field calculation position, and γ is the relativistic Lorentz term. The subscript t_r
2350 indicates that the equations are evaluated at the retarded time so that the time-delay
2351 from the travel time of the electromagnetic radiation is taken into account.

²https://github.com/project8/locust_mc/tree/master

2352 The only required input to calculate the electric field at the position of an FSCD
 2353 antenna is the velocity and acceleration of the electron, which can be obtained from
 2354 Kassiopeia simulations. Therefore, when simulating a CRES event Locust first runs
 2355 a Kassiopeia simulation of the electron and subsequently calculates the electric field
 2356 incident on the antenna. This requires one to calculate the retarded time. The retarded
 2357 time corresponds to the time that a photon, which has just arrived at an antenna at
 2358 the space-time position (t, \mathbf{r}) , was actually emitted by the electron at the space-time
 2359 position of $(t_r, \mathbf{r}_e(t_r))$. To calculate the retarded time one solves

$$c(t - t_r) = |\mathbf{r} - \mathbf{r}_e(t_r)|, \quad (4.10)$$

2360 where the distance traveled by the photon between the measurement and retarded times
 2361 is equal to the distance between the antenna and the electron at the retarded time.
 2362 Locust solves Equation 4.10 using root finding algorithm to calculate the retarded time,
 2363 which yields the electric field emitted by the electron, at the position of each antenna in
 2364 the FSCD array.

2365 Antenna Response Modeling

2366 The electric field solutions are used to calculate the resulting voltages produced in the
 2367 antenna. However, direct simulation of the antenna itself is computationally expensive,
 2368 since it requires modeling the complex interactions of the electron's electric fields with
 2369 charge carriers in the antenna. Direct simulation of the antenna in Locust is avoided by
 2370 modeling the antenna response using the antenna factor, or antenna transfer function.
 2371 The antenna factor defines the voltage produced in the antenna terminal for an incident
 2372 electric field [68],

$$A_F = \frac{V}{|\mathbf{E}|}, \quad (4.11)$$

2373 where V is the voltage and $|\mathbf{E}|$ is the magnitude of the incident electric field. To obtain the
 2374 antenna factor for the antennas developed for the FSCD Project 8 employs Ansys HFSS.
 2375 HFSS is a commercially available finite element method electromagnetic solver widely
 2376 used throughout the antenna engineering industry [69]. HFSS is capable of calculating
 2377 the antenna factor and gain patterns for complex antenna designs and outputting the
 2378 resulting quantities in the form of a text file that can be used as a configuration input to
 2379 Locust.

2380 The antenna factor defines the steady-state response of the antenna to electromagnetic

2381 plane waves in the frequency-domain. Since the antenna response is calculated in the
 2382 time-domain Locust models the antenna as a linear time-invariant system [70]. In this
 2383 formalism the response of the system to the driving force is given by

$$y[n] = h * x = \sum_k h[k]x[n - k], \quad (4.12)$$

2384 where $y[n]$ is the discretely sampled response, x is the driving force stimulus, and h is
 2385 the finite impulse response (FIR) filter. When applied to the FSCD array, this formalism
 2386 calculates the voltage time-series produced in each antenna by convolving the electric
 2387 field time-series with the antenna FIR filter, which is obtained by performing an inverse
 2388 Fourier transform on the transfer function from HFSS.

2389 Radio-frequency Receiver and Signal Processing

2390 After obtaining the voltage time-series by computing the electron trajectory and antenna
 2391 response, Locust simulates the signal processing performed by the RF receiver chain.
 2392 The simulated Locust receiver chain includes all operations that would be performed by
 2393 the RF hardware (see Figure 4.3).

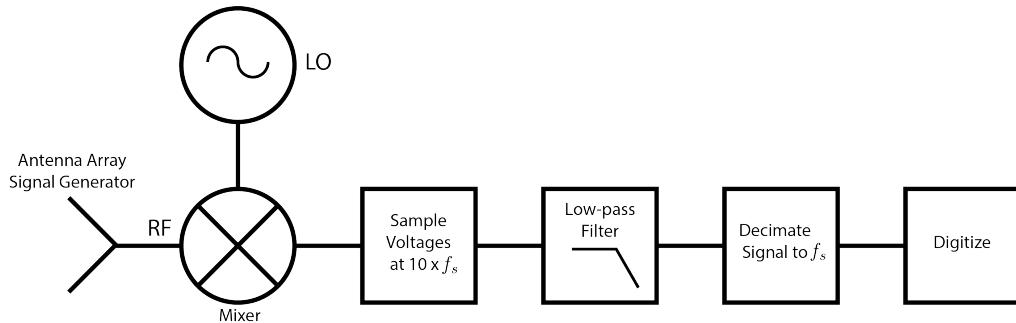


Figure 4.3. The receiver chain used by Locust when simulating CRES events in the FSCD.

2394 Frequency down-conversion reduces the digitization bandwidth required to read-out
 2395 CRES data. According to the Nyquist sampling theorem [71], the minimal sampling rate
 2396 that guarantees no information loss for a signal with a bandwidth Δf is given by

$$f_{\text{Nyq}} = 2\Delta f. \quad (4.13)$$

2397 The total bandwidth for CRES events ranges from 0 to 26 GHz in a 0.95 T magnetic field,
 2398 therefore, direct digitization of CRES signals from the FSCD would require sampling
 2399 frequencies greater than 50 GHz, which is infeasible for a real experiment. However, one

2400 need only measure the shape of the spectrum in the last 100 eV, which corresponds to a
2401 frequency bandwidth of 5 MHz, to effectively measure the neutrino mass.

2402 Down-conversion is a technique for reducing the base frequencies of signals in a
2403 bandwidth given by $[f_{\text{LO}}, f_{\text{LO}} + \Delta f]$ to the bandwidth $[0, \Delta f]$, by performing the following
2404 multiplication

$$x(t) \rightarrow x(t)e^{-2\pi f_{\text{LO}} t}. \quad (4.14)$$

2405 The signal, $x(t)$, is multiplied by a sinusoidal signal with frequency f_{LO} to reduce the
2406 absolute frequencies of the signals in the bandwidth. In the FSCD, this allows one to
2407 detect events in the last 100 eV of the tritium spectrum, while sampling the data far
2408 below 50 GHz. The standard bandwidth used in the FSCD is 200 MHz, which allows for
2409 higher frequency resolution than the minimum sampling frequency for 100 eV of energy
2410 bandwidth.

2411 Directly simulating down-conversion with a frequency multiplication in Locust requires
2412 sampling the electric fields at each antenna in the FSCD array with a period of ≈ 20 ps,
2413 which is extremely slow computationally. To avoid this, Locust performs the down-
2414 conversion by intentionally under-sampling the electric fields with a frequency of 2 GHz.
2415 Sampling below the Nyquist limit causes the higher frequency components of the CRES
2416 signal to alias, however, Locust can remove these aliased frequency peaks using a
2417 combination of low-pass filtering and decimation to recreate frequency down-conversion.
2418 After filtering and decimation, Locust simulates digitization by an 8-bit digitizer at a
2419 sampling frequency of 200 MHz to recreate the conditions of the FSCD. The voltage
2420 offset and digitizer range must be configured by the user based on the characteristics of
2421 the simulation.

2422 Data

2423 The output of Locust simulations for the FSCD primarily consists of two data files. The
2424 first is the electron trajectory information calculated by Kassiopiea, which is output in
2425 the form of a `.root` file [72]. This file contains important kinematic information about
2426 the electron such as its position and pitch angle as a function of time. The other file
2427 is produced by Locust and contains the digitized signals acquired from each antenna
2428 in the array. The Locust output files conform to the Monarch specification³ developed
2429 by Project 8, which is based on the commonly used HDF5 file format, and matches the
2430 format of the files produced by the Project 8 data acquisition software. This makes it

³<https://github.com/project8/monarch>

2431 possible to use the same data analysis code to analyze both simulated and real data.

2432 4.2.3 CRESana

2433 Locust is the primary simulation tool used by Project 8 in the development and simulation
2434 of the FSCD. However, simulations of CRES events in larger antenna arrays (≥ 100
2435 antennas) can take several hours to complete, which is prohibitively long when one is
2436 performing a sensitivity analysis and optimization. One reason for Locust's slow operation
2437 is that the electric fields from the electron must be solved numerically for each time-step
2438 for all antennas in the array. These numerical solutions allow Locust to accurately
2439 simulate the electric fields from arbitrarily complicated electron trajectories at the cost
2440 of more computations and slower simulations. Therefore, an additional simulation tool
2441 that sacrifices the accuracy of numerical approaches for computational efficiency is a
2442 useful tool for studying large antenna array experiments.

2443 Recently, Project 8 has developed a new simulations package called CRESana⁴, specif-
2444 ically designed to perform analytical simulations of antenna-based CRES experiments.
2445 CRESana provides a significant increase in simulation speed by using well-justified an-
2446 alytical approximations of the electrons motion and electric fields in a magnetic trap.
2447 The electric fields and signals generated by CRESana are consistent with theoretical
2448 calculations of the electron's radiation, and are tested for accuracy using well-known
2449 test-case simulations and consistency checks.

2450 4.3 Signal Detection and Reconstruction Techniques for 2451 Antenna Array CRES

2452 Antenna Array CRES Signal Reconstruction

2453 Antenna array CRES requires one to use the multichannel time-series obtained by
2454 digitizing the array to estimate the starting kinetic energies of electrons produced in
2455 the magnetic trap using CRES signal reconstruction algorithm. This procedure consists
2456 of a multi-stage process of detecting a CRES signal followed by an estimation of the
2457 electron's parameters.

2458 Antenna array CRES requires a significantly different approach to signal reconstruction
2459 than previous Project 8 experiments. In Phases I and II, CRES was performed using a

4<https://github.com/MCflowMace/CRESana>

2460 waveguide gas cell directly integrated into a waveguide transmission line. The transmission
2461 line efficiently propagates the cyclotron radiation along its length to an antenna at the
2462 ends of the waveguide. However, with an antenna array the electron is radiating into
2463 free-space, therefore, the cyclotron radiation power collected by the array is directly
2464 proportional to the solid angle surrounding the electron that is covered with antennas.
2465 Because it is not practical to fully surround the magnetic trap with antennas, some of the
2466 cyclotron radiation power that would have been collected by the waveguide escapes into
2467 free-space. Furthermore, the power that is collected by the antenna array is split between
2468 every channel in the antenna array, which significantly lowers the signal-to-noise ratio
2469 (SNR) of CRES signals in a single antenna channel compared to a waveguide apparatus.
2470 Therefore, a suite of completely new signal reconstruction techniques are needed in order
2471 to perform CRES in the FSCD.

2472 Changes to the approach to CRES signal reconstruction are also motivated by the
2473 scientific goals of Project 8. A measurement of the tritium beta-decay spectrum that is
2474 sensitive to neutrino masses as small as 40 meV requires that we measure the kinetic
2475 energies of individual electrons with a total energy broadening of 115 meV [73]. This
2476 resolution includes all sources of uncertainty in the electron's kinetic energy such as
2477 magnetic field inhomogeneities. This precise energy resolution is only achieved by an
2478 event-by-event signal reconstruction approach where the kinetic energies, pitch angles,
2479 and other parameters of the CRES events are estimated for individual electrons before
2480 constructing the beta-decay spectrum.

2481 The event-by-event approach is distinct from the analysis done for the Phase I
2482 and Phase II experiments, where the starting cyclotron frequency of the event was
2483 measured by analyzing the tracks formed by the electron's carrier in a time-frequency
2484 spectrogram. These frequencies were then combined into a frequency beta-spectrum,
2485 which was converted to the beta-decay energy spectrum using an ensemble approach that
2486 averaged over all other event parameters. The ensemble approach to signal reconstruction
2487 results in poor energy resolution because other kinematic parameters such as pitch angle
2488 change the cyclotron carrier frequency due to changes in the average magnetic field
2489 experience by the electron.

2490 Components of Reconstruction: Signal Detection and Parameter Estimation

2491 CRES signal reconstruction is a two-step procedure consisting of signal detection followed
2492 by parameter estimation. In the former, one is concerned with identifying CRES signals
2493 in the data regardless of the signal parameters, whereas, in the latter one operates under

2494 the assumption that a signal is present and then estimates it's parameters.

2495 More formally, signal detection can be posed as a binary hypothesis test between
2496 the signal and noise data classes, and parameter estimation is a process of fitting a
2497 signal model to the observed data. While both of these are required for a complete
2498 reconstruction (see Figure 4.4), the focus of my work and this chapter is on the signal
2499 detection aspect of antenna array CRES signal reconstruction.

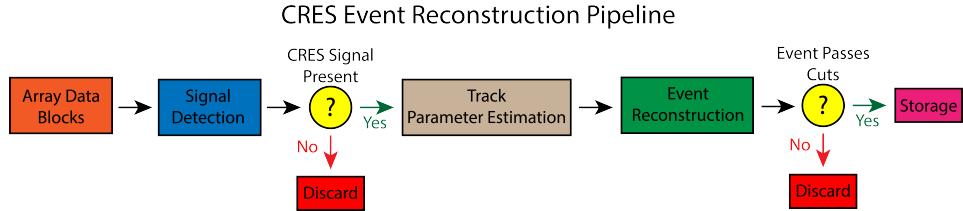


Figure 4.4. A high-level diagram depicting the process of CRES event reconstruction. The first step consists of identifying the presence of a signal in the data. This step is necessary to avoid the danger of performing a reconstruction of a false event, which would constitute a background contribution to the tritium spectrum measured by CRES.

2500 Detection Theory

2501 Signal detection is the process of deciding whether noisy data contains signal or noise,
2502 which can be posed as a statistical hypothesis test [74]. For CRES signals, which are
2503 represented by signal vectors with added white Gaussian noise (WGN), one needs to
2504 choose between

$$\mathcal{H}_0 : \mathbf{y} = \boldsymbol{\nu} \quad (4.15)$$

$$\mathcal{H}_1 : \mathbf{y} = \mathbf{x} + \boldsymbol{\nu}, \quad (4.16)$$

2505 where \mathbf{y} is the CRES data vector, $\boldsymbol{\nu}$ is a sample of WGN, and \mathbf{x} represents the CRES
2506 signal. The hypothesis that the data contains only noise is labeled \mathcal{H}_0 and the hypothesis
2507 that the data contains a signal is labeled \mathcal{H}_1 .

2508 For illustrative purposes, it is useful to study the case where only the first sample of
2509 data is used to distinguish between \mathcal{H}_0 and \mathcal{H}_1 . The value of the first data sample is
2510 distributed according to two possible Gaussian distributions(see Figure 4.5). By setting a
2511 decision threshold on the value of this sample, one can choose the correct hypothesis with
2512 a probability given by the area underneath the probability distribution curves. A true
2513 positive corresponds to correctly identifying that the data contains signal, whereas, a true
2514 negative means that one has correctly identified the data as noise. The rate at which the

2515 detector performs a true positive classification is given by the green region underneath
 2516 $p(\mathbf{y}[0]; \mathcal{H}_0)$, and the rate at which the detector performs a true negative classification is
 given by the orange region underneath $p(\mathbf{y}[0]; \mathcal{H}_1)$. Two types of misclassifications are

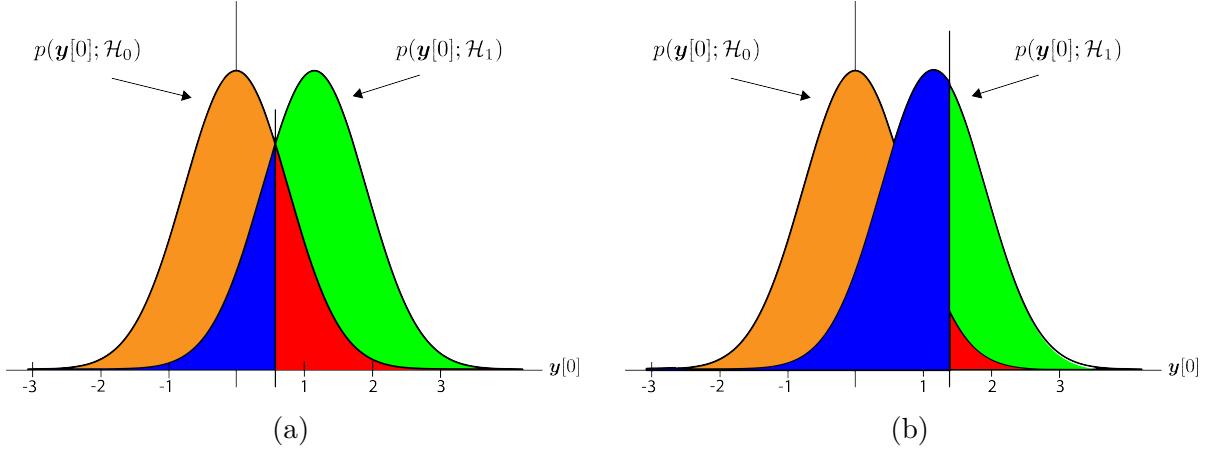


Figure 4.5. An illustration of two PDFs associated with a binary hypothesis test. The decision threshold is represented by the vertical line that partitions both distributions. The orange and red areas correspond to the true negative and false positive probabilities and the blue and green areas correspond to the false negative and true positive probabilities respectively. To decide between the two hypotheses the likelihood ratio test specified by the Neyman-Pearson theorem is applied. This approach achieves the highest true positive probability for a given false positive probability.

2517
 2518 possible. Either one declares noise data as signal, which is called a false positive, or one
 2519 declares signal data as noise, which is a false negative. Note that it is only possible to
 2520 trade off these two types of errors by tuning the detection threshold. One cannot reduce
 2521 the rate of false positives without also increasing the rate of false negatives.

2522 The approach taken with CRES signals is to fix the rate of false positives by setting
 2523 a minimum decision threshold value. The rate of false positives that is acceptable at the
 2524 detection stage depends upon the total rate of background events compatible with the
 2525 sensitivity goals of the experiment. The ultimate goal of a neutrino mass measurement
 2526 with 40 meV sensitivity in general has strict requirements on the number of background
 2527 events, which requires a relatively high detection threshold to achieve. Consequently,
 2528 the ideal signal detection algorithm is the one that achieves the maximum rate of true
 2529 positives for a fixed rate of false positives, so that the detection efficiency of the experiment
 2530 is maximized and potential sources of background are kept to a minimum.

2531 According to the Neyman-Pearson theorem [75], the statistical hypothesis test that
 2532 maximizes the probability of detection for a fixed rate of false positives is the likelihood
 2533 ratio test, which is formed by computing the ratio of the signal likelihood to the noise

2534 likelihood,

$$L(x) = \frac{P(\mathbf{y}; \mathcal{H}_1)}{P(\mathbf{y}; \mathcal{H}_0)} > \gamma. \quad (4.17)$$

2535 Here, the likelihood of the hypotheses \mathcal{H}_0 and \mathcal{H}_1 are described by the probability
2536 distributions $P(\mathbf{y}; \mathcal{H}_0)$ and $P(\mathbf{y}; \mathcal{H}_1)$ respectively, and γ is the threshold for deciding \mathcal{H}_1 .

2537 The decision threshold is determined by integrating $P(\mathbf{y}; \mathcal{H}_0)$ such that

$$P_{FP} = \int_{\gamma}^{\infty} P(\tilde{\mathbf{y}}; \mathcal{H}_0) d\tilde{\mathbf{y}} = \alpha, \quad (4.18)$$

2538 where α is the desired false positive detection rate given by the red colored areas shown
2539 in Figure 4.5. The true positive detection rate is given by the similar integral

$$P_{TP} = \int_{\gamma}^{\infty} P(\tilde{\mathbf{y}}; \mathcal{H}_1) d\tilde{\mathbf{y}}, \quad (4.19)$$

2540 which corresponds to the green areas in Figure 4.5.

2541 Changing the decision threshold allows one to trade-off between P_{TP} and P_{FP} as
2542 appropriate for the given situation. It is standard to summarize the relationship between
2543 P_{TP} and P_{FP} using the receiver operating characteristic (ROC) curve, which is obtained
2544 by evaluating the true positive and false positive probabilities as a function of the decision
threshold value (see Figure 4.6). The ROC curve provides a convenient way to compare

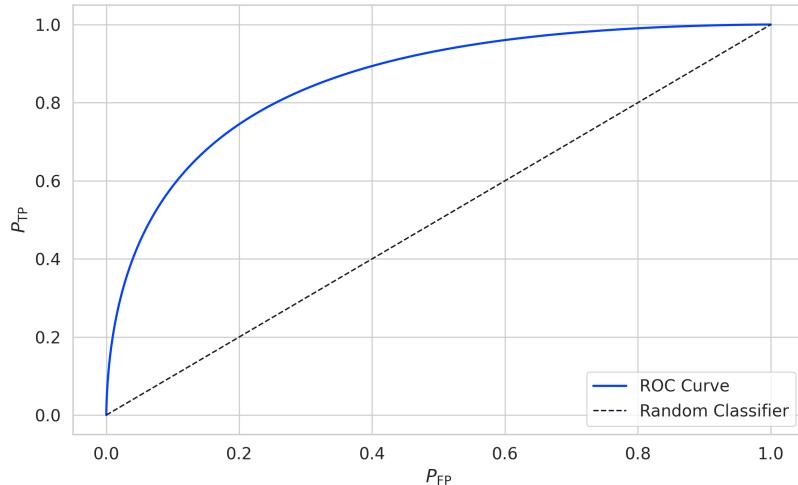


Figure 4.6. An example ROC curve formed by computing the P_{FP} and the P_{TP} for a given likelihood ratio test. As the decision threshold is increased P_{FP} decreases at the expense of a lower P_{TP} . The black dashed line indicates the lower bound ROC curve obtained by randomly deciding between \mathcal{H}_0 and \mathcal{H}_1 .

2545

2546 the performance of different signal detection algorithms. In general, a classifier with
2547 a higher the P_{TP} as a function of P_{FP} is desirable, which corresponds to a larger area
2548 underneath the respective ROC curve. A perfect classifier has an area underneath the
2549 curve of 1.0, however, such a classifier is never achieved in practice.

2550 4.3.1 Digital Beamforming

2551 Introduction to Beamforming

2552 Beamforming is an antenna array signal processing technique designed to enhance the
2553 radiation of the array in a particular direction and suppress it in other directions [68].
2554 Beamforming is of interest to Project 8 as a first level of signal reconstruction for the
2555 FSCD and other antenna array CRES experiments, which operates at the signal detection
2556 stage of reconstruction.

2557 Beamforming is performed using a phased summation of the signals received by the
2558 antenna array. The beamforming phases are selected such that the signals emitted by
2559 the array will constructively interfere at the point of interest (see Figure 4.7). As a
2560 consequence of the principle of reciprocity [76], when the array is operating in receive
2561 mode, the signals emitted from a source at the same point will constructively interfere
2562 when summed. The origin of the phase delays in beamforming is the path-length difference
2563 to the beamforming point between different antennas in the array. The relationship
2564 between the phase delay and the path-length difference is given by the familiar equation

$$\phi = \frac{2\pi d}{\lambda}, \quad (4.20)$$

2565 where ϕ is the phase delay, d is the path-length difference, and λ is the wavelength of
2566 the radiation. In practice, one chooses the values of d by specifying the beamforming
2567 positions of interest and then calculates the beamforming phases using Equation 4.20,
2568 which is guaranteed to follow the constructive interference condition shown in Figure 4.7.

2569 Beamforming can be neatly expressed mathematically using the vector equation

$$y[n] = \Phi^T[n] \mathbf{x}[n], \quad (4.21)$$

2570 where $\mathbf{x}[n]$ is the array snapshot vector, $\Phi[n]$ is a vector of beamforming shifts, and
2571 $y[n]$ is the resulting summed signal. The beamforming shifts consist of a set of complex

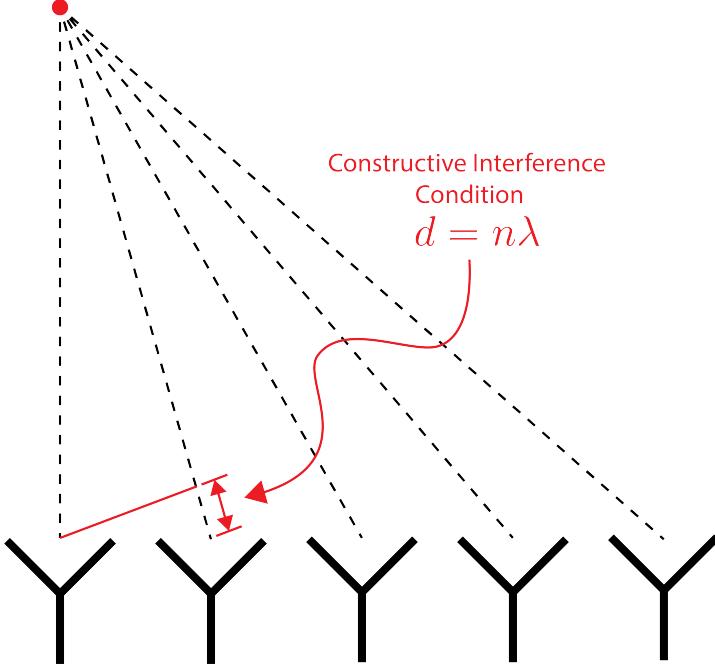


Figure 4.7. An illustration of the constructive interference condition which is the operating principle of digital beamforming using a uniform linear array as an example.

2572 numbers that contain the beamforming phase shift and an amplitude weighting factor,

$$\Phi[n] = [A_0[n]e^{-2\pi i \phi_0[n]}, A_1[n]e^{-2\pi i \phi_1[n]}, \dots, A_{N-1}[n]e^{-2\pi i \phi_{N-1}[n]}], \quad (4.22)$$

2573 where the set of magnitudes $A_i[n]$ are amplitude weighting factors and $\phi_i[n]$ are the
 2574 phase shifts from the path-length differences. The index i is used to denote the antenna
 2575 channel number. The amplitude weighting factor is the relative magnitude of the signal
 2576 received by a particular antenna in the array. This factor properly accounts for antennas
 2577 that are closer to the radiating source. In general, the beamforming phases can also be
 2578 functions of time to track the motion of a non-stationary source.

2579 Digital beamforming specifically is the type of beamforming algorithm of interest to
 2580 Project 8 for CRES. With digital beamforming, the phase shifts are applied to the array
 2581 signals in software rather than employing fixed beamforming phase shifts in the receiver
 2582 chain hardware. The advantage of digital beamforming is that for any given series of
 2583 array data one can specify an arbitrarily large number of beamforming positions and
 2584 search for electrons using a flexible and easily configurable beamforming grid.

2585 Digital beamforming can be viewed as the spatial filtering, which is a direct conse-
 2586 quence of the constructive interference condition used to define the beamforming phases.

2587 Digital beamforming causes signals from multiple electrons at different positions in the
2588 trap to be separated, because the interference condition will cause the signals from
2589 electrons at other position to cancel out. This spatial filtering effect reduces pile-up that
2590 could become an issue for large scale CRES experiments using a dense tritium source.

2591 Beamforming positions can be specified with arbitrary densities limited only by
2592 the available computational resources. This provides a very straight-forward way to
2593 estimate the position of the electron in the trap by using a dense grid of beamforming
2594 positions and maximizing the output power of the beamforming summation over this
2595 grid. This approach to position reconstruction is attractive due the requirements of an
2596 event-by-event signal reconstruction, which needs an accurate estimation of the exact
2597 magnetic field experienced by the electron in order to correctly estimate its kinetic
2598 energy. Combined with an accurate map of the magnetic field inhomogeneities of the
2599 trap obtained from calibrations, beamforming allows one to apply this magnetic field
2600 correction with a spatial resolution that is a fraction of the cyclotron wavelength.

2601 **Laboratory Beamforming Demonstrations**

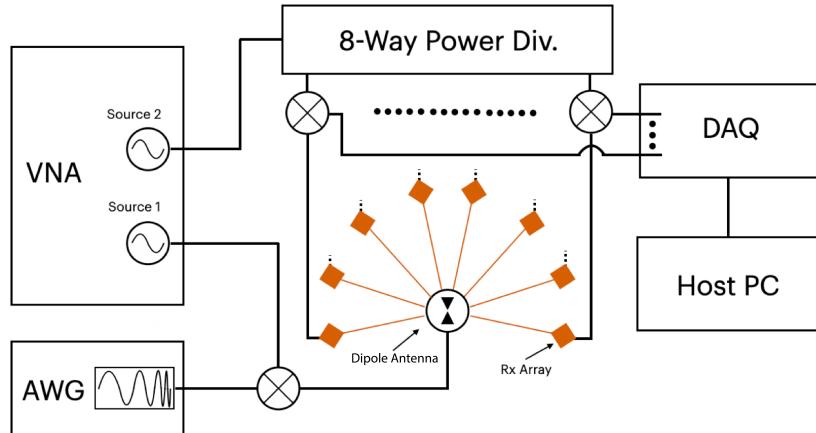


Figure 4.8. A system level diagram of the laboratory setup used for beamforming demonstrations at Penn State. For more information on this system see Chapter 5. Signals near 26 GHz are fed to a dipole antenna using an arbitrary waveform generator (AWG) and vector network analyzer (VNA), which drive a mixer. The dipole radiation is collected by an array of antennas connected to the digitizer data acquisition (DAQ) system.

2602 An antenna measurement setup was constructed at Penn State to serve as a testbed
2603 for antenna prototypes and to perform laboratory validations of array simulations for
2604 the FSCD. This system is discussed in more detail in Chapter 5. Early versions of

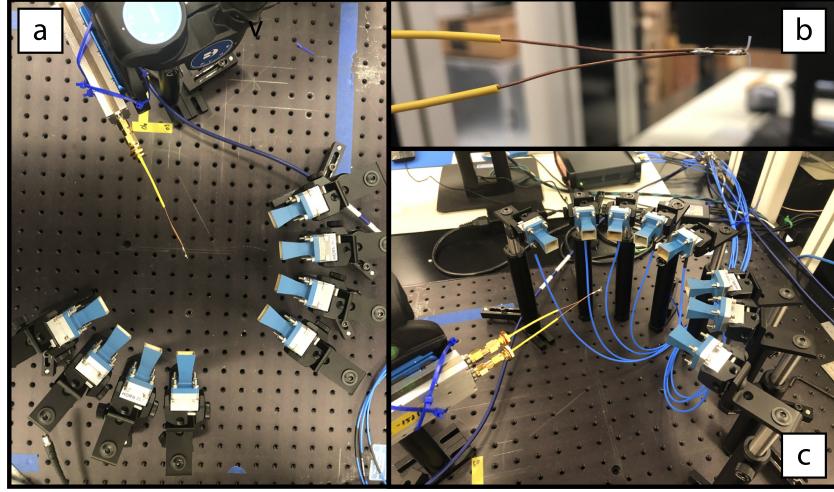


Figure 4.9. Photographs of the beamforming demonstration setup. In (a) I show a top-down view of the dipole antenna and the array of eight horn antennas. Manual repositioning of the horn antennas allows one to synthesize a full-circular antenna array. The dipole antenna is mounted on a camera tripod mount that allows for manual position tuning. (b) is a close up image of the dipole, which is manufactured from two segments of semi-rigid coaxial cable. (c) is another image of the dipole and array.

2605 the antenna measurement system (see Figure 4.8 and Figure 4.9) were used to perform
 2606 beamforming reconstruction studies of a simple probe antenna.

2607 Signals from an arbitrary waveform generator were up-converted to 26 GHz using a
 2608 mixer and a high-frequency source from a vector network analyzer and fed to a dipole
 2609 antenna through a balun. The radiation from the dipole antenna was received by an
 2610 array of horn antennas. The signals from the horn antennas were down-converted to
 2611 baseband using a collection of mixers and an 8-way power divider. The signals were then
 2612 digitized and saved to a host computer for analysis.

2613 The data collected using the dipole and horn antenna array is reconstructed using the
 2614 beamforming reconstruction approach specified in Section 4.3.1. A two-dimensional grid
 2615 of xy-positions is defined and the beamforming phase shifts for each of these positions
 2616 is calculated. The phased summation can be visualized by plotting the time-averaged
 2617 power for each of the summations as a pixel in the resulting beamforming image (see
 2618 Figure 4.10). White Gaussian noise (WGN) can be added to the data at this stage
 2619 to simulate more realistic SNR if desired. The beamforming peak maxima is expected
 2620 to have a Bessel function shape due to the circular symmetry of the array, and by
 2621 analyzing the size of the beamforming maxima one can confirm that the beamforming
 2622 reconstruction measurement has similar position resolution as expected from Locust
 2623 simulations. Additionally, signal detection rates can be estimated from the data by

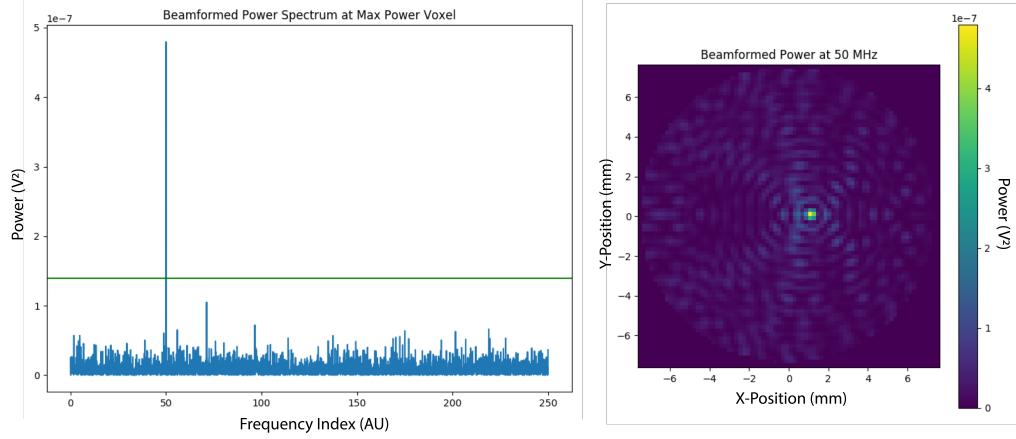


Figure 4.10. An example of digital beamforming reconstruction of a dipole antenna using a synthetic array of horn antennas. The beamforming image on the right is constructed by computing the time-averaged power of the summed signals for a two-dimensional grid of beamforming positions. In the image, one can see a clear maximum that corresponds to the position of the dipole antenna. On the left I show the frequency spectrum of the time-series at the maximum power pixel. White Gaussian noise is added to the signal to mimic a more realistic signal-to-noise-ratio. The signal emitted by the dipole is clearly visible as the high power peak in the frequency spectrum.

2624 comparing the magnitude of the beamforming signal peak in the frequency spectra to
 2625 simulation.

2626 FSCD Beamforming Simulations

2627 Locust simulations of the FSCD are used to generate simulated CRES signal data to
 2628 perform beamforming reconstruction studies. As mentioned in the previous section,
 2629 the beamforming procedure beings by specifying a set of beamforming positions and
 2630 corresponding beamforming shifts. The beamforming positions form a grid that covers
 2631 the region of interest. There are effectively an infinite number of ways to specify the
 2632 grid positions, however, uniform square grids are the most commonly used due to their
 2633 simplicity. In the actual experiment the number and pattern of beamforming positions
 2634 would be optimized to cover the most important regions of the trap volume, which
 2635 maximizes detection efficiency and minimizes superfluous calculations.

2636 The beamforming grids used for signal reconstruction with the FSCD consist of a set
 2637 of points that cover the two-dimensional plane formed by the perimeter of the antenna
 2638 array. The axial dimension is left out because electrons are treated as if they occupy only
 2639 their average axial position, which corresponds to the center of the magnetic trap. This
 2640 treatment is valid since it is impossible to resolve the axial position of the electron as a

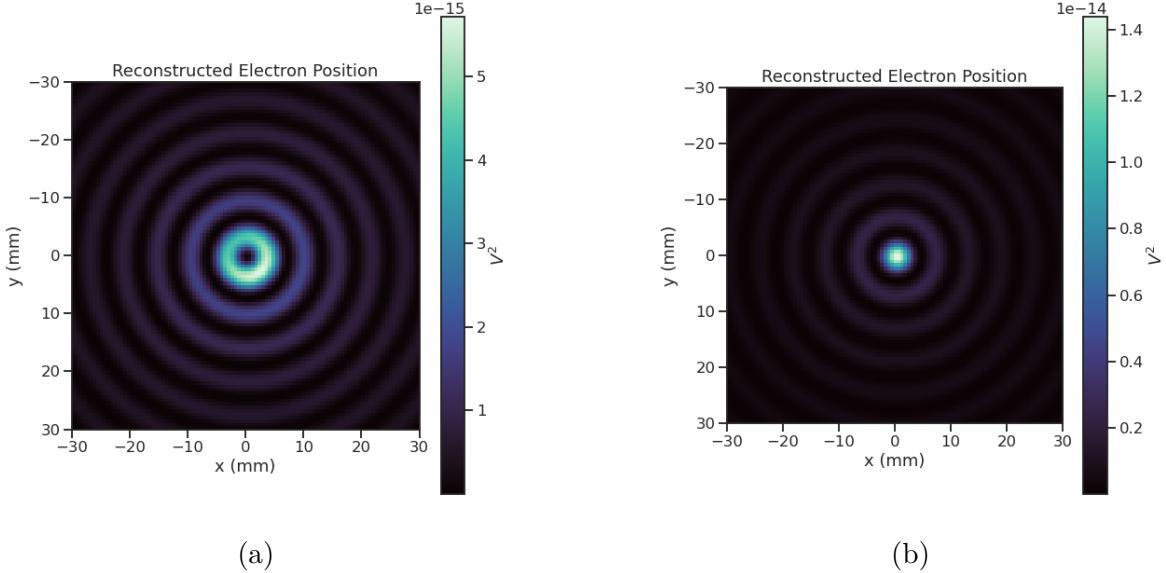


Figure 4.11. Beamforming images visualizing the reconstruction of an electron without (a) and with (b) the cyclotron phase correction. The images were generated using data from Locust simulations. The cyclotron phase refers to a phase offset equal to the relative azimuthal position of an antenna in the array. This phase offset is caused by the circular electron orbit and must be corrected for during reconstruction.

function of time due to the rapid oscillation frequencies of trapped electrons.

After beamforming, a summed time-series is obtained for each beamforming position that can be checked for a signal using a detection algorithm. A beamforming image is a visualization method that is equivalent to arranging the beamforming grid points according to their physical locations. Each pixel in the image corresponds to a summed time-series obtained for a digital beamforming position, and the image is obtained taking the time-averaged power at every pixel(see Figure 4.11).

If only the spatial beamforming phase component from Equation 4.20 is used, then the resulting image contains a ring-shaped feature centered on the position of the electron (see Figure 4.11a). The origin of this shape is an additional phase offset particular to a cyclotron radiation source. The circular cyclotron orbit introduces a relative phase offset to the electric fields equal to the azimuthal position of the field measurement point [77, 78]. Therefore, two antennas, one located at an azimuthal position of 0° and another located at an azimuthal position of 90° , will receive CRES signals out of phase by 90° , which is the difference in their azimuthal positions. This phase offset can be corrected by adding an additional term to the beamforming phase equation that is equal

2657 to the azimuthal position of the antenna relative to the electron,

$$\phi_i[n] = \frac{2\pi d_i[n]}{\lambda} + \Delta\varphi_i[n], \quad (4.23)$$

2658 where $\Delta\varphi_i$ is difference between the azimuthal position of the electron and the i -th
 2659 antenna channel. Using the updated beamforming phases changes the ring feature into
 2660 the expected Bessel peak whose maximum corresponds to the position of the electron.
 2661 Including this cyclotron phase correction significantly improves the signal detection and
 2662 reconstruction capabilities of beamforming by more than doubling the summed signal
 2663 power and shrinking the beamforming maxima feature size.

2664 The beamforming image examples in Figure 4.11 were produced using an electron
 2665 located on the central axis of the magnetic trap, which do not experience ∇B -drifts.
 2666 However, electrons produced at non-zero radial position the beamforming phases must
 2667 be made time-dependent to track the position of the electron's guiding center over
 2668 time. Without this correction the ∇B -drift causes the electron to move away from the
 2669 beamforming position, which effectively spreads the cyclotron radiation power over a
 wider area in the beamforming image (see Figure 4.12). This effect significantly reduces

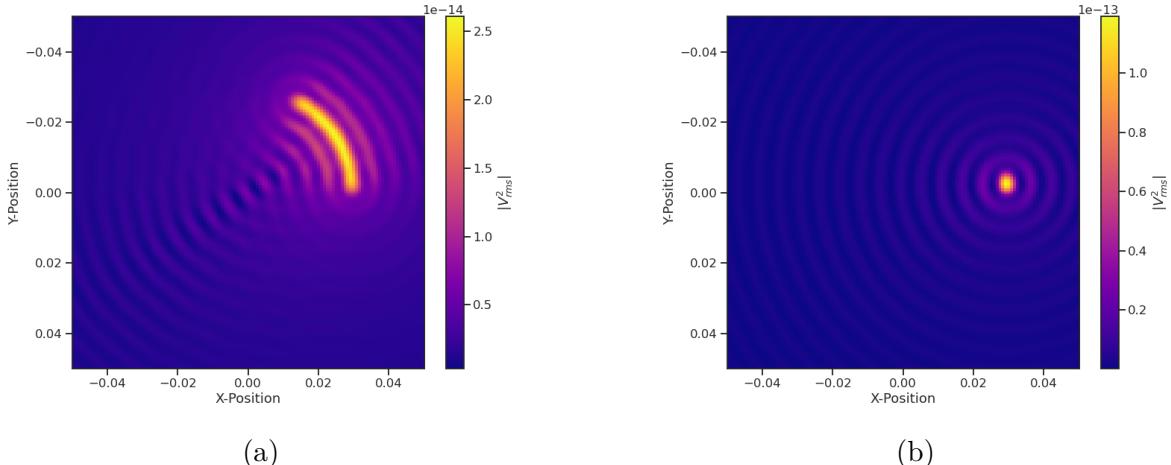


Figure 4.12. Beamforming images visualizing the reconstruction of an electron located off the central axis of the FSCD trap. In (a) beamforming is being performed without the ∇B -drift correction, and in (b) it is included.

2670
 2671 the power of the beamforming maxima and increases the size of the beamforming features,
 2672 simultaneously harming detection efficiency and position reconstruction.

2673 The ∇B -drift correction simply adds a circular time-dependence to the beamforming

2674 positions as a function of time,

$$r[n] = r_0 \quad (4.24)$$

$$\varphi[n] = \varphi_0 + \omega_{\nabla B} t[n], \quad (4.25)$$

2675 where $\omega_{\nabla B}$ is the drift frequency and $t[n]$ is the time vector. In the ideal case the ∇B -drift
2676 frequencies from Figure 4.2 for the correct pitch angle and radial position would be used,
2677 however, it is not possible to know the electron's pitch angle a priori. In principle, one
2678 could perform multiple beamforming summations for a given beamforming position using
2679 different drift frequencies and choose the one that maximizes the summed power, but
2680 this approach leads to a huge computational burden that would be impractical for a
2681 real FSCD experiment. A compromise is to use an average value of $\omega_{\nabla B}$ obtained by
2682 averaging over the drift frequencies for electrons of different pitch angle at a particular
2683 radius. This approach keeps the computational cost of time-dependent beamforming to a
2684 minimum while still providing a significant increase in the detection efficiency of digital
2685 beamforming.

2686 **Signal Detection with Beamforming and a Power Threshold**

2687 Up to this point I have neglected a specific discussion of how digital beamforming is used
2688 for signal detection and reconstruction. Because, strictly speaking, digital beamforming
2689 consists only of the phased summation of the array signals and cannot be used alone for
2690 signal detection. The example beamforming images shown in Figure 4.11 and Figure 4.12
2691 were produced using simulated data that contained no noise, which significantly degrades
2692 the utility of analyzing the beamforming images for signal detection and reconstruction.

2693 In Project 8, digital beamforming as a detection algorithm is understood to mean
2694 digital beamforming plus a power or amplitude threshold placed on the frequency
2695 spectrum obtained by applying a fast Fourier transform (FFT) to the summed time-series
2696 (see Figure 4.13). This approach is similar to the time-frequency spectrogram analysis
2697 employed in Phase I and II. However, it is possible to use any signal detection algorithm
2698 after beamforming. In Section 4.4 I analyze the signal detection performance of the
2699 power threshold approach in detail.

2700 Without a reconstruction technique that coherently combines the signals from the
2701 full antenna, the ability to detect CRES signals is drastically reduced (see Figure 4.13).
2702 Because the CRES signals are in-phase at the correct beamforming position, the summed
2703 power increases as a function of N^2 compared to a single antenna channel, where N is

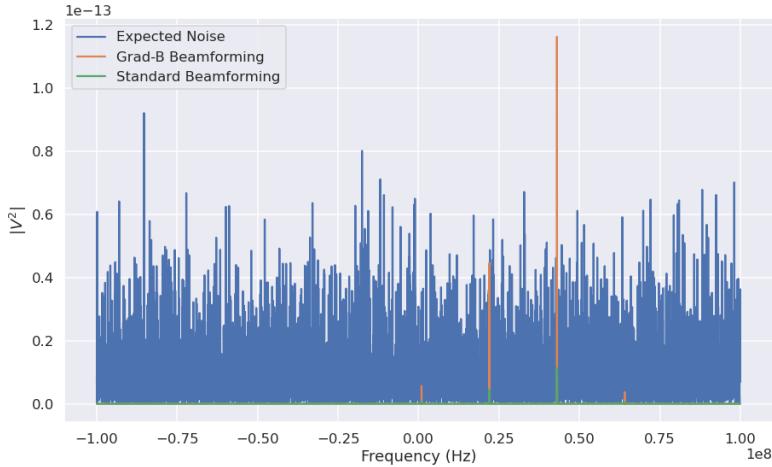


Figure 4.13. A plot of a typical frequency spectrum obtained by applying a Fourier transform to the time-series obtained from beamforming. The frequency spectra are plotted without noise on top of an example of a typical noise spectrum to visualize a realistic signal-to-noise ratio. In the example, without beamforming it would not be possible to detect anything since the signal amplitudes would be reduced by a factor of sixty relative to the noise. Additionally, it is clear the ∇B -drift correction is needed to detect this electron in the presence of noise.

2704 the number of antennas. It is true that the noise power is also increased by beamforming,
 2705 but, because the noise is incoherent, its power only increases linearly. Consequently, the
 2706 SNR of the CRES signal increases linearly with the number of antennas, which greatly
 2707 improves detection efficiency compared to using only the information in a single antenna.

2708 The power threshold detection algorithm searches for high-power frequency bins that
 2709 should correspond to a frequency component of the CRES signal. In order to prevent
 2710 random noise fluctuations from being mistaken as CRES signals the power threshold
 2711 must be set high enough so that it is unlikely that random noise could be responsible. A
 2712 consequence of this is that many electrons that can be trapped will go undetected because
 2713 the modulation caused by axial oscillations leads to the cyclotron carrier power to falling
 2714 below the decision threshold. The time-dependent beamforming used to correct for the
 2715 ∇B -drift increases the volume of the magnetic trap where electrons can be detected,
 2716 but it is ineffective at increasing the range of detectable pitch angles (see Figure 4.14).
 2717 Fundamentally, this is because the power threshold only uses a fraction of the signal
 2718 power to detect electrons and ignores the power present in the frequency sidebands. In
 2719 the subsequent sections I examine two other signal detection algorithms that seek to
 2720 improve the detection efficiency of the FSCD by utilizing the more of the signal shape to
 2721 compute the detection test statistics.

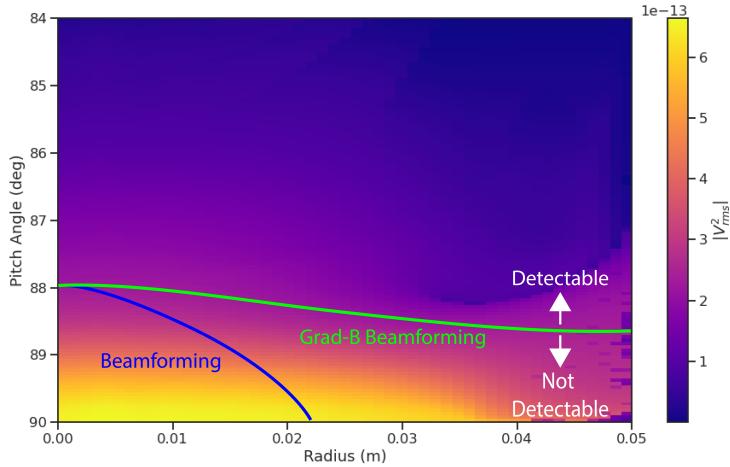


Figure 4.14. A plot of the total signal power received by the FSCD array from trapped electrons with different radial positions and pitch angles generated using Locust simulations. The lines on the plot indicate a 10 dB detection threshold above the mean value of the noise in the frequency spectrum. With static beamforming electrons with radial positions larger than about two centimeters are undetectable due to the change in the electron's position over time causing losses from beamforming phase mismatch. This is corrected by including ∇B -drift frequencies in the beamforming phases. Both beamforming techniques fail to detect electrons below $\approx 88.0^\circ$, since these signal are composed of several relatively weak sidebands that are comparable to the noise.

2722 4.3.2 Matched Filtering

2723 Introduction to Matched Filtering

2724 The problem of CRES signal detection is the problem of detecting a signal buried in
 2725 WGN, which has been examined at great depth in the signal processing literature [74].
 2726 For a fully known signal in WGN the optimal detector is the matched filter, which means
 2727 that it achieves the highest true positive rate for a fixed rate of false positives.

2728 The matched filter test statistic is calculated by taking the inner product of the data
 2729 with the matched filter template

$$\mathcal{T} = \left| \sum_n h^\dagger[n] y[n] \right|, \quad (4.26)$$

2730 where $h[n]$ is the matched filter template and $y[n]$ is the data. The matched filter test
 2731 statistic defines a binary hypothesis test in which the data vector is assumed to be an
 2732 instance of two possible data classes. By setting a decision threshold on the value of \mathcal{T} ,
 2733 one can classify a given data vector as belonging to two distinct hypotheses. Under the

2734 first hypothesis the data is composed of pure WGN, and under the second hypothesis
 2735 the data is composed of the known signal with additive WGN.

2736 The matched filter template is obtained by rescaling the known signal in the following
 2737 way

$$h[n] = \frac{x[n]}{\sqrt{\tau \sum_n x^\dagger[n]x[n]}}, \quad (4.27)$$

2738 where τ is the variance of the WGN and $x[n]$ is the known signal. Strictly speaking,
 2739 Equation 4.27 is only true for noise with a diagonal covariance matrix, which is assumed
 2740 to be true for the FSCD. Defining the matched filter templates in this way guarantees
 2741 that the expectation value of \mathcal{T} is equal to one when the data contains only noise, which
 2742 is the standard matched filter normalization.

2743 Although matched filters are canonically formulated in terms of a perfectly known
 2744 signal, it is possible to apply the matched filter technique with imperfect information
 2745 provided the signal is deterministic. From the discussion of CRES simulation tools (see
 2746 Section 4.2) it was shown that the shape of CRES signals are completely determined
 2747 by the initial parameters of the electron. The random collisions with background gas
 2748 molecules, which cause the formation of signal tracks, are the only stochastic component
 2749 of the CRES event after the initial beta-decay. Therefore, a matched filter can be used
 2750 for the detection of deterministic CRES signal tracks between scattering events.

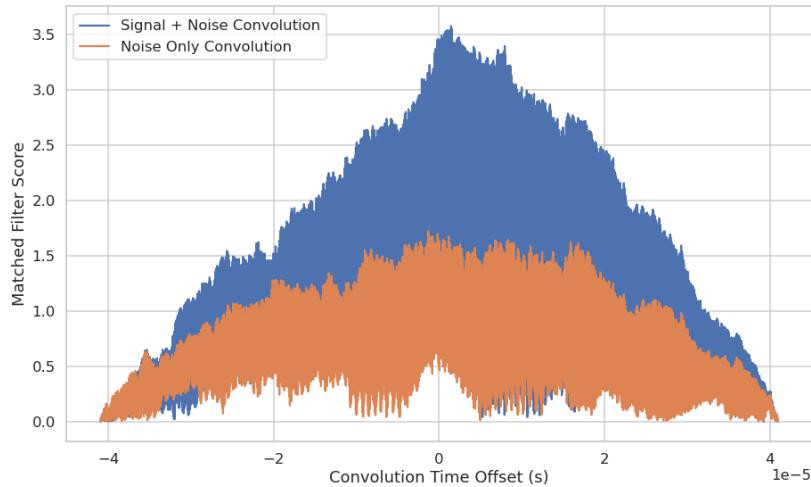


Figure 4.15. Example of a convolution of a CRES signal template with a segment of noisy data. A simulated CRES signal was simulated using Locust and normalized to create a matched filter template. When this template is convolved with noisy data the contains the matching signal the convolution output increases dramatically compared to data with only noise. The decreasing convolution output as the time offset of the convolution increases is caused by zero-padding of the data and template.

2751 The matched filter test statistic for CRES signals is a modified version of Equation
2752 4.26

$$\mathcal{T} = \max_{h,m} |\mathbf{h} * \mathbf{y}| = \max_{h,m} \left| \sum_k h^\dagger[k] x[m-k] \right|, \quad (4.28)$$

2753 where the matched filter inner product has been replaced with a convolution operation
2754 and a maximization over the template and convolution delay (m). Replacing the inner
2755 product with a convolution accounts for the fact that the start time of the CRES signal is
2756 now an unknown parameter. In addition, a maximization of the matched filter convolution
2757 is performed over a number of different templates. Because the shape of the signal is
2758 unknown, a range of different signal shapes, called a template bank, must be checked
2759 using an exhaustive search.

2760 Matched Filtering in the Frequency Domain

2761 The template bank approach, while powerful, can become computationally intractable.
2762 Specifically, the time-domain convolution specified by Equation 4.28 is particularly
2763 computationally intensive and is a major barrier towards the implementation of a
2764 matched filter for signal detection in an experiment like the FSCD. This can be avoided
2765 by using the convolution theorem to replace the time-domain convolution with an inner
2766 product in the frequency domain.

2767 The convolution theorem states that

$$\mathbf{f} * \mathbf{g} = \mathcal{F}^{-1}(\mathbf{F} \cdot \mathbf{G}) \quad (4.29)$$

2768 where \mathbf{f} and \mathbf{g} are discretely sampled time-series, \mathbf{F} and \mathbf{G} are the respective discrete
2769 Fourier transforms, and \mathcal{F}^{-1} is the inverse discrete Fourier transform operator. The
2770 convolution theorem allows us to perform the matched filter convolution by first com-
2771 puting the Fourier transform of the template and data, then performing a point-wise
2772 multiplication of the two frequency series, and finally performing the inverse Fourier
2773 transform to obtain the convolution output. Because discrete Fourier transforms can be
2774 performed extremely efficiently, the convolution theorem is almost always used in lieu of
2775 directly computing the convolution.

2776 One thing to note here is that the convolution theorem for discrete sequences shown
2777 here, is technically valid only for circular convolutions, which is not directly specified
2778 in Equation 4.28. However, because typical CRES track lengths are much longer than
2779 the Fourier analysis window and the frequency chirp rates are small compared to the

2780 time-slice duration, it is safe to use circular convolutions to evaluate matched filter scores
2781 for CRES signals, which allows one to apply the convolution theorem to compute matched
2782 filter scores for the FSCD.

2783 **Matched Filter Analysis of the FSCD**

2784 Since the matched filter is the optimal signal detection approach, it provides the ultimate
2785 upper bounds on signal detection. This makes it a useful algorithm for assessing the
2786 upper bounds on neutrino mass sensitivity for the FSCD, since it indicates the best
2787 possible detection efficiency achievable for that experiment configuration. The standard
2788 approach to performing these studies involves generating numerous simulated electron
2789 signals that span the kinematic parameter space of electrons.

2790 To limit the number of simulations required to evaluate the detection efficiency,
2791 the standard approach is to fix the starting axial position, starting azimuthal position,
2792 starting direction of the perpendicular component of the electron’s momentum, and event
2793 start time. This reduces the dimensionality of the simulated parameter space to three
2794 parameters — the starting radial position, starting kinetic energy, and starting pitch
2795 angle. The fixed variables are nuisance parameters, which do not affect the detection
2796 efficiency estimates for the FSCD design, because they simply introduce overall phase
2797 offsets that can be marginalized during the calculation of the matched filter score. Across
2798 radial position, kinetic energy, and pitch angle one defines a regular grid of parameters
2799 and uses Locust to simulate the corresponding signals (see Figure 4.16). This grid of
2800 simulated signals is used to estimate detection efficiency by calculating the detection
2801 probability of a randomly parameterized signal using the grid as a set of matched filter
2802 templates (see Section 4.4).

2803 The matched filter approach can also be used to estimate the achievable energy
2804 resolution of the experiment by using a dense grid of templates generated with parameters
2805 close to the unknown signal (see figure 4.17). Because matched filter templates with similar
2806 parameters have closely matching signal shapes, templates with incorrect parameters can
2807 have nearly identical matched filter scores as the correct template. Since only one sample
2808 of noise is included in a sample of real data, one cannot guarantee that the template
2809 with the maximum score corresponds to the ground truth parameters of the signal. This
2810 introduces uncertainty into the signal parameter estimation that manifests as an energy
2811 broadening. Dense grids of matched filter templates allow one to quantify this broadening
2812 by analyzing the parameter space of templates with matched filter scores close to the
2813 ground truth. This approach is analogous to maximum likelihood estimation and is one

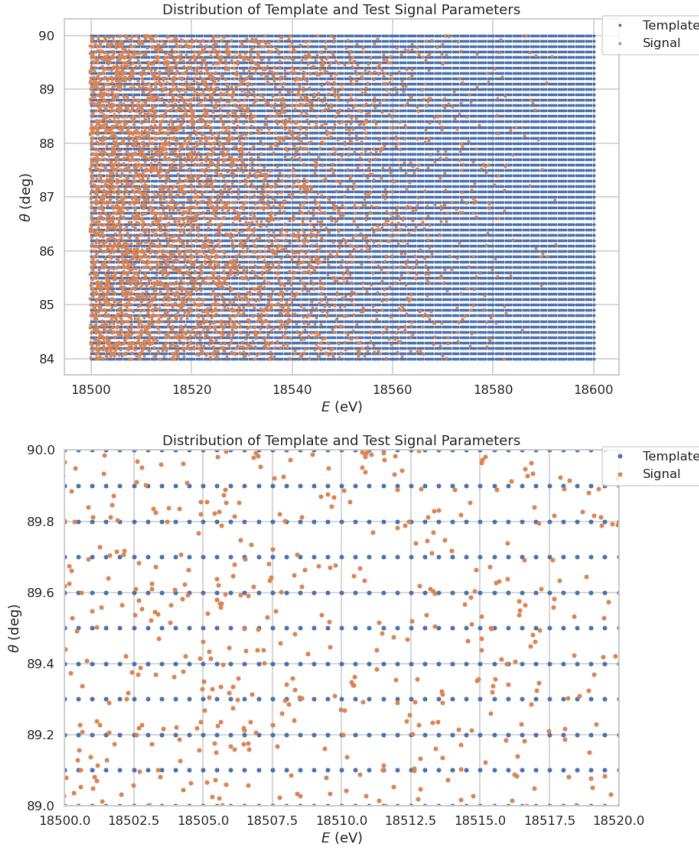


Figure 4.16. An example two-dimensional parameter distribution of a matched filter template bank and random test signals. θ refers to the pitch angle of the electron and E is the kinetic energy. The template bank forms a regular grid of in pitch angle and energy, whereas, the test signals are uniformly distributed in pitch angle and follow the tritium beta-decay kinetic energy distribution. This is why there are fewer test signals at higher energies. The need for high match across the full parameter space prevents one from reducing the density of templates in this low activity region. A zoomed in version of the template bank illustrates the relative density of templates and signals needed for match $> 90\%$.

2814 key component of a complete sensitivity analysis for an antenna array CRES experiment.

2815 A figure of merit that summarizes the performance of a matched filter template
 2816 bank at signal detection is "mean match", which is defined as the average ratio of the
 2817 highest matched filter score for a random signal to the matched filter score for a perfectly
 2818 matching template. In equation form the match ratio for a single template is given by

$$\text{Match} \equiv \Gamma = \frac{\mathcal{T}_{\text{best}}}{\mathcal{T}_{\text{ideal}}}, \quad (4.30)$$

2819 where $\mathcal{T}_{\text{best}}$ is the matched filter score of the best fitting template in the bank and $\mathcal{T}_{\text{ideal}}$

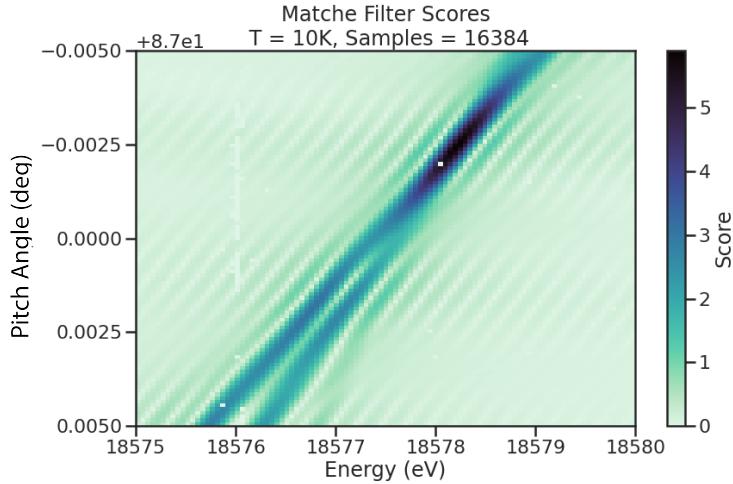


Figure 4.17. The matched filter scores of a dense grid of templates in pitch angle energy space. Dense template grids allow one to estimate the kinetic energy of the electron by identifying the best matching template. The uncertainty on this value is proportional to the space of templates that also match the test signal well. In the worst case matched filter templates can be completely degenerate where templates with different parameters match a signal with equal likelihood.

is the hypothetical score one would measure if the signal perfectly matched the template. The mean match is the average value of match for a typical signal inside the parameter range covered by the matched filter template bank. Generally, one desires a mean match as close to unity as possible, which is typically an exponential function of the number of templates in the template bank (see Figure 4.18).

The exponential relationship between match and template bank size manifests for dense and sparse template grids. Sparse template grids are used for signal detection when no prior information on the signal is available, whereas, dense templates grids are more useful for parameter estimation. The mean match value directly influences the detection efficiency of the template bank, but due to the exponential scaling, achieving a high average match at the detection stage can easily overwhelm the available computational resources.

The effect of match on the detection efficiency of the matched filter template bank can be summarized using the ROC curve (see Figure 4.19). The average performance of the template bank can be described by a single ROC curve obtained by averaging over the PDFs that describe the detection probabilities of each template in the bank.

The distribution that describes the matched filter score under the signal hypothesis is a Rician distribution, which has a mean value equal to the matched filter score multiplied by the match ratio (see Section 4.4). Alternatively, the distribution of the matched

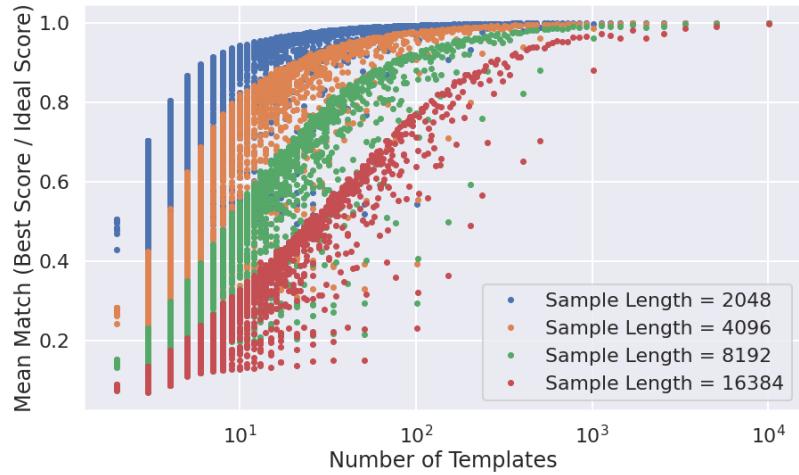


Figure 4.18. The mean match of the dense template grid shown in Figure 4.17 for different numbers of templates. Grids of different sizes were obtained by decimating a dense grid of templates and the average match for each grid was computed using the same set of randomly distributed test signals. Plotting the mean match against the size of the grid allows one to visualize the exponential relationship between match and template bank size. The noise in each curve is caused by sampling effects from the decimation algorithm. In general, longer templates are harder to match than shorter templates.

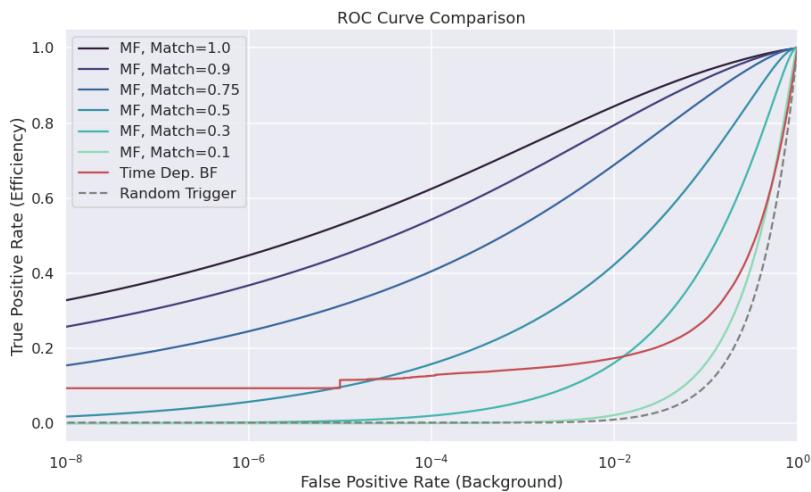


Figure 4.19. Matched filter template bank ROC curves as a function of mean match. One can see that for low match a matched filter is on average worse than the more straight forward beamforming detection approach.

filter score when there is no signal in the data follows a Rayleigh distribution, which is equivalent to a Rician distribution with zero mean. The matched filter score for each template in the template bank is described by a separate Rician distribution. Therefore, one way to model detection probability for a given signal is to average across all matched filter distributions in the template bank to obtain a single distribution that describes the statistical behavior of the matched filter score.

A different way to visualize the detection performance for each algorithm is to specify a minimum acceptable false positive rate at the trigger level. This is equivalent to specifying a minimum threshold on the value of the matched filter score or the size of a frequency peak for a beamforming power threshold trigger. One can then draw regions of detectable signals as a function of the electron's pitch angle and radial position (see Figure 4.20). A kinetic energy shift is equivalent to an overall frequency shift of the

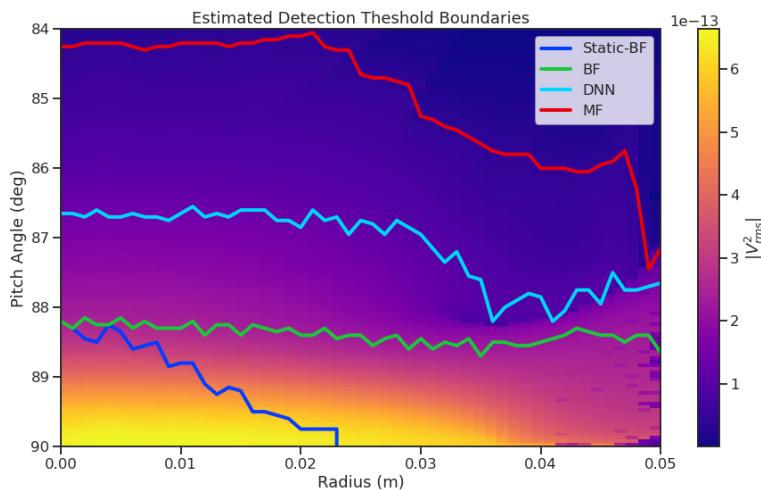


Figure 4.20. Boundaries of detectable electrons in pitch angle kinetic energy space for a series of different signal detection algorithms. A detectable signal is defined as a signal that is above a consistent decision with at least 50% probability. This non-rigorous treatment of detection probability is primarily useful for the visualization the relative increases in detection performance provided by the different algorithms. The static beamforming (Static-BF) algorithm is the digital beamforming algorithm introduced above without the ∇B -drift correction. The DNN algorithm refers to a convolutional neural network classifier trained to detect CRES signals (see Section 4.3.3).

signal and should have no effect on the detection probability assuming sufficient density of matched filter templates in the energy dimension. A electron is declared "detectable" for the regions in Figure 4.20 if the signal has at least 50% probability of falling above the decision threshold of the respective classifier. One can see that the parameter space of detectable signals is greatly expanded beyond the beamforming power threshold trigger

2856 with a matched filter (MF) or deep neural network (DNN) (see Section 4.3.3). Plots such
2857 as Figure 4.20 are useful for visualization, but, since the handling of detection likelihood
2858 is not sufficiently rigorous, the detection probability boundaries are not well-suited to
2859 sensitivity estimates.

2860 Optimized Matched Filtering Implementation for the FSCD

2861 The biggest practical obstacle to the implementation of a matched filter template bank is
2862 the computational cost associated with exhaustively calculating the matched filter scores,
2863 therefore, one must employ several optimizations in a practical setting.

2864 Computing a matched filter score requires the convolution of two vectors, which can
2865 be performed very efficiently by computers if the convolution theorem and fast Fourier
2866 transforms (FFT) are utilized. Furthermore, one can apply digital beamforming as a
2867 pre-processing step to reduce the dimensionality of the data before the matched filter.
2868 In order to understand the relative gain in computational efficiency offered by these
2869 optimizations I analyze the total number of floating-point operations (FLOP) of several
2870 matched filter implementations in big O notation that utilize different combinations of
2871 optimizations.

2872 A direct implementation of a matched filter as specified by Equation 4.28 involves
2873 the convolution of N_{ch} signals of length N_s with template signals of length N_t . The
2874 FLOPs of the various matched filter implementations on a per-template basis will be
2875 used as a consistent metric, since each implementation scales linearly with the number of
2876 templates. The direct convolution approach to matched filtering costs

$$O(N_{\text{ch}}) \times O(N_s \times N_t) \quad (4.31)$$

2877 FLOP per-template, whose cost is dominated by the $O(M \times N)$ convolution operation.

2878 The computational cost of the direct matched filter approach can be significantly
2879 reduced by exploiting the convolution theorem and FFT algorithms. By restricting oneself
2880 to signals and templates that contain equal numbers of samples, the convolution can be
2881 calculated by Fourier transforming both vectors, performing the point-wise multiplication,
2882 and taking the inverse Fourier transform to obtain the convolution result. The FFT
2883 algorithm is able to compute the Fourier transform utilizing only $O(N \log N)$ operations.
2884 This optimization results in a computational cost per-template of

$$O(N_{\text{ch}}) \times O(N_s \log N_s) \quad (4.32)$$

2885 A typical signal vector in the FSCD contains $O(10^4)$ samples in which case the FFT
 2886 reduces the computational cost of the matched filter by a factor of $O(10^3)$. In practice,
 2887 due to the large reduction in computational cost with a frequency-domain matched filter,
 2888 direct implementations of the matched filter using a time-domain convolution are almost
 2889 never attempted in practice. Particularly, a time-domain matched filter is completely
 2890 computationally infeasible for the the FSCD due to resource constraints.

2891 Rather than relying solely on the matched filter it is tempting to consider using
 2892 digital beamforming as an initial step in the signal reconstruction for the purposes of
 2893 data reduction. The primary motivation is to reduce the dimensionality of the data by
 2894 a factor of N_{ch} by combining the array outputs coherently into a single channel. One
 2895 can view the beamforming operation as a partial matched filter, in the sense that the
 2896 matched filter convolution contains the beamforming phased summation along with a
 2897 prediction of the signal shape. By separating beamforming from the signal shape one
 2898 hopes to reduce the overall computational cost by effectively shrinking the number of
 2899 templates and reducing the number of operations required to check each one.

2900 The nature of this optimization requires that one account for the number of templates
 2901 used for pure matched filtering versus the hybrid approach. To first order, the total
 2902 number of templates at the trigger stage is a product of the number of guesses for each
 2903 of the electron's parameters

$$N_T = N_E \times N_\theta \times N_r \times N_\varphi, \quad (4.33)$$

2904 where N_E is the number of kinetic energies, N_θ is the number of pitch angles, N_r is the
 2905 number of starting radial positions, and N_φ is the number of starting azimuthal positions.
 2906 The starting axial position and cyclotron motion phase are not necessary to include in
 2907 the template bank, since these parameters manifest themselves as the starting phase of
 2908 the signal, which is effectively marginalized when using a FFT to compute the matched
 2909 filter convolution. Therefore, the total number of operations required by a matched filter
 2910 to detect a signal in a segment of array data is on the order of

$$O(N_T) \times O(N_{\text{ch}}) \times O(N_s \log N_s) \quad (4.34)$$

2911 With the hybrid approach one removes spatial parameters from the template bank
 2912 by using beamforming to combine the array signals into a single channel. Beamforming
 2913 explicitly assumes a starting position, which allows one to use matched filter templates
 2914 that span the two-dimensional space of kinetic energy and pitch angle. The total

computational cost of the hybrid method is directly proportional to the number of beamforming positions. For the time-dependent beamforming defined in Section 4.3.1, the number of beamforming positions is given by

$$N_{\text{BF}} = N_r \times N_\varphi \times N_{\omega_{\nabla B}}, \quad (4.35)$$

where N_r and N_φ are the same spatial parameters encountered in the pure matched filter template bank and $N_{\omega_{\nabla B}}$ is the number of ∇B -drift frequency assumptions. If a unique drift frequency is used for each pitch angle then the hybrid approach is effectively equivalent to a pure matched filter in the number of operations. The key efficiency gain of the hybrid approach is to exploit the relatively small differences in $\omega_{\nabla B}$ for electrons of different pitch angles by using only a few average drift frequencies.

The total number of operations for the hybrid approach can be expressed as a sum of the operations required by the beamforming and matched filtering steps,

$$O(N_{\text{BF}}) \times O(N_{\text{ch}}N_s) + O(N_{\text{BF}}) \times O(N_E N_\theta) \times O(N_s \log N_s). \quad (4.36)$$

The first product in the sum is the number of operations required by beamforming, which is simply the number of beamforming points times the computational cost of the beamforming matrix multiplication, and the second product is the computational cost of matched filtering the summed signal generated by each beamforming position. To compare this to pure matched filtering, one takes the ratio of Equations 4.34 and 4.36 to obtain

$$\Gamma_{\text{BFMF}} = \frac{O(N_{\omega_{\nabla B}})}{O(N_E N_\theta) \times O(\log N_s)} + \frac{O(N_{\omega_{\nabla B}})}{O(N_{\text{ch}})}. \quad (4.37)$$

This expression can be simplified by observing that $O(N_E N_\theta) \times O(\log N_s) \gg O(N_{\text{ch}})$, which means that the ratio of computational cost for the two methods can be reduced to

$$\Gamma_{\text{BFMF}} \approx \frac{O(N_{\omega_{\nabla B}})}{O(N_{\text{ch}})}. \quad (4.38)$$

Limiting oneself to a number of estimated drift frequencies of $O(1)$, then it can be seen that the estimated computational cost reduction of the hybrid approach is of $O(N_{\text{ch}})$. This is a large reduction considering that the FSCD antenna array contains sixty antennas in the baseline design.

The main drawback of the hybrid approach is that the limited number of allowed drift frequency guesses can lead to detection efficiency loss due to phase mismatch. The

2940 degree of phase error from an incorrect drift frequency is proportional to the length of
 2941 the array data vector used by the signal detection algorithm. For signals with lengths
 2942 equal to the baseline FSCD Fourier analysis window of 8192 samples, typical phase errors
 2943 from using an average versus the exact ∇B -drift frequency are on the order of a few
 2944 percent in terms of the signal energy. This has a relatively small impact on the overall
 2945 detection efficiency, however, future experiments with antenna array CRES will want to
 2946 balance optimizations such as these during the design phase to keep experiment costs to
 2947 a minimum while still achieving scientific goals.

2948 **Kinetic Energy and Pitch Angle Degeneracy**

2949 Accurate modeling of a matched filter requires one to consider the effects of mismatched
 2950 signals and template, since this more accurately reflects the real-world usage of a matched
 2951 filter. One way to study this is to use a signal grid to compute the matched filter scores
 2952 between mismatched signals and templates and evaluate the matched filter scores under
 2953 this scenario. What one finds when performing this analysis is that templates for signals
 2954 with incorrect parameters can have matched filter scores that are indistinguishable from
 the matched filter score of the correct template (see Figure 4.21 and Figure 4.21).

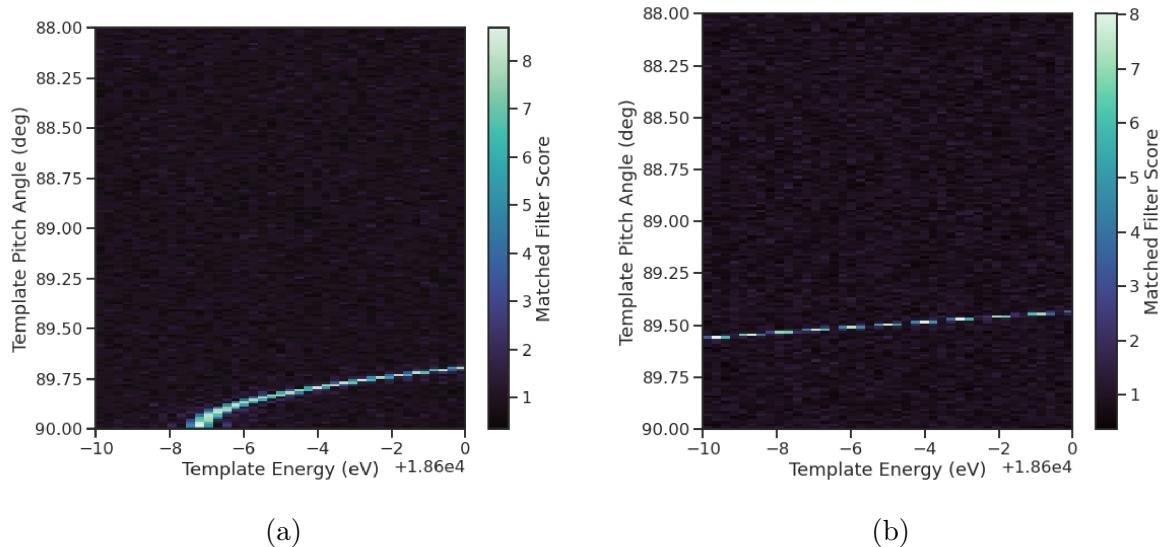


Figure 4.21. Two example illustrations of the correlation between kinetic energy and pitch angle imparted by the shape of the FSCD magnetic trap. The correlations manifest themselves as degeneracies in the matched filter score where multiple matched filter templates have the same matched filter for a particular signal. These degeneracies are a sign that the magnetic trap must be redesigned in order to break the correlation between pitch angle and kinetic energy.

2955

2956 This degeneracy in matched filter score is the result of correlations between the kinetic
 2957 energy and pitch angle of the electron caused by the magnetic trap. These correlations
 2958 are unacceptable since they greatly reduce the energy resolution of the experiment by
 2959 causing electrons with specific kinetic energy to match templates across a wide range of
 energies.

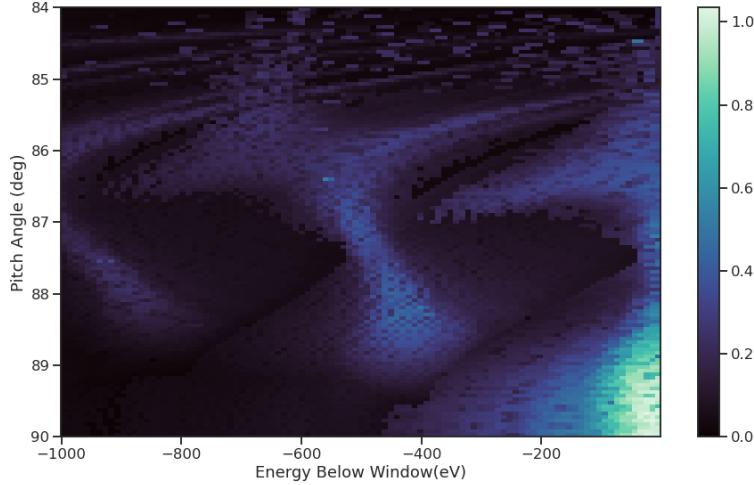


Figure 4.22. A visualization of the correlation between energy and pitch angle in the FSCD magnetic trap. The image is formed by computing the match of the best template from a grid consisting of pitch angles from 84 to 90 degrees in steps of 0.05 degrees, kinetic energies from 17574 to 18574 eV, located at 2 cm from the central axis, and simulated for a length of three FSCD time-slices. The signals used to compute the best matching template consisted of a grid from 84 to 90 degrees in steps of 0.05 degrees, kinetic energies from 18550 to 18575 eV in steps of 0.25 eV, located 2 cm from the central axis, and simulated for three FSCD time-slices. The colored regions of the plot show how well signals with lower energy can match those of higher energy for the FSCD magnetic trap, which is proportional to the achievable energy resolution of the FSCD design.

2960
 2961 This degeneracy cannot be fixed by implementing a different signal reconstruction
 2962 algorithm. As revealed by the matched filter scores the shapes of the signals for different
 2963 parameters are identical. Resolving this degeneracy between pitch angle and energy
 2964 requires the design of a new magnetic trap with steeper walls so that the average magnetic
 2965 field experienced by an electron is less dependent on pitch angle.

2966 4.3.3 Machine Learning

2967 Machine learning is a broad field of research [79] that has been particularly transformative
 2968 in the recent past. In this Section I provide a brief introduction to some concepts and

2969 techniques of machine learning that were applied to CRES signal detection in my
2970 dissertation.

2971 **Introduction to Machine Learning**

2972 Digitization of the FSCD antenna array generates large amounts of data that must
2973 be rapidly processed for real-time signal detection and reconstruction. While digital
2974 beamforming combined with a power threshold is relatively computationally inexpensive,
2975 it is ineffective at detecting CRES signal with small pitch angles, since it relies on a
2976 visible frequency peak above the noise. On the other hand, a matched filter is able to
2977 detect signals with a significantly larger range of parameters, however, the exhaustive
2978 search of matched filter templates can be computationally expensive. Machine learning
2979 based triggering algorithms have been used successfully in many high-energy physics
2980 experiments [80], and recently have shown success in the detection of gravitational wave
2981 signals [81, 82] in place of more traditional matched filtering methods. The success of
2982 machine learning in these domains motivates the exploration of machine learning as a
2983 potential CRES signal detection algorithm.

2984 Various approaches to machine learning are possible, but the one most important
2985 to the discussion here is the supervised learning approach. In supervised learning, one
2986 uses a differentiable model or function that is designed to map the input data to the
2987 appropriate label [79]. The data is represented as a multidimensional matrix of floating
2988 point values such as an image or a time-series, and the label is typically a class name such
2989 as signal or noise for classification problems, or a continuous value like kinetic energy for
2990 regression problems.

2991 In supervised learning the model is trained to map from the data to the correct label
2992 by evaluating the output of the model using a training dataset consisting of a set of
2993 paired data and labels. To evaluate the difference between the model output and the
2994 correct label a loss function is used to quantify the error between the model prediction
2995 and the ground truth. For example, a common loss function in regression problems is the
2996 squared error loss function, which quantifies error using the squared difference between
2997 the model output and label.

2998 Using the outputs of the loss function the next step in supervised learning is to
2999 compute the gradient of error with respect to the model parameters in a process called
3000 backpropagation. The gradients are used to update the model parameter values in
3001 order to minimize errors in the model predictions across the whole dataset. This loop is
3002 performed many times while randomly shuffling the dataset until the error converges to a

3003 minimum value at which point the training procedure has finished. It is standard practice
3004 to monitor the training procedure by evaluating the performance of the model using a
3005 separate validation dataset that matches the statistical distribution of the training data
3006 and to check the performance of the model after training using yet another dataset called
3007 the test dataset. These practices help to guard against overtraining which is a concern
3008 for models with many parameters.

3009 **Convolutional Neural Networks**

3010 A popular class of machine learning models are neural networks. A neural network is
3011 a function composed of a series of linear operations called layers, which take a piece of
3012 data typically represented as a matrix, multiply the elements of the data by a weight,
3013 and then sums these products to produce an output matrix. Neural networks composed
3014 of purely linear operations are unable to model complex non-linear behavior. Therefore,
3015 non-linear activation functions are applied to the outputs of each of the layers to increase
3016 the ability of the neural network to model complex relationships between the data.

3017 Neural networks are typically composed of at least three layers, but with the present
3018 capabilities of computer hardware they typically contain much more than this. The first
3019 layer in a neural network is called the input layer, because it takes the data objects
3020 as input, and the last layer in a neural network is known as the output layer. The
3021 output layer is trained by machine learning to map the data to an output label using
3022 the supervised learning procedure described in Section 4.3.3. Between the input and the
3023 output layers are typically several hidden layers that receive inputs from and transmit
3024 outputs to other layers in the neural network model. The term deep neural network
3025 (DNN) refers to those neural networks that have at least one hidden layer, which have
3026 proven to be extremely powerful tools for pattern recognition and function approximation.

3027 An important type of DNN are convolutional neural networks (CNN) that typically
3028 contain several layers which perform a convolution of the input with a set of filters. These
3029 convolution operations are typically accompanied by layers that attempt to down-sample
3030 the data along with the standard neural network activation functions. A standard CNN
3031 is composed of several convolutional layers at the beginning of the network and ends
3032 with a series of fully-connected neural network layers at the output. Intuitively, one
3033 can imagine that the convolutional layers are extracting features from the data that
3034 fully-connected layers use to perform the classification or regression task.

3035 **Deep Filtering for Signal Detection in the FSCD**

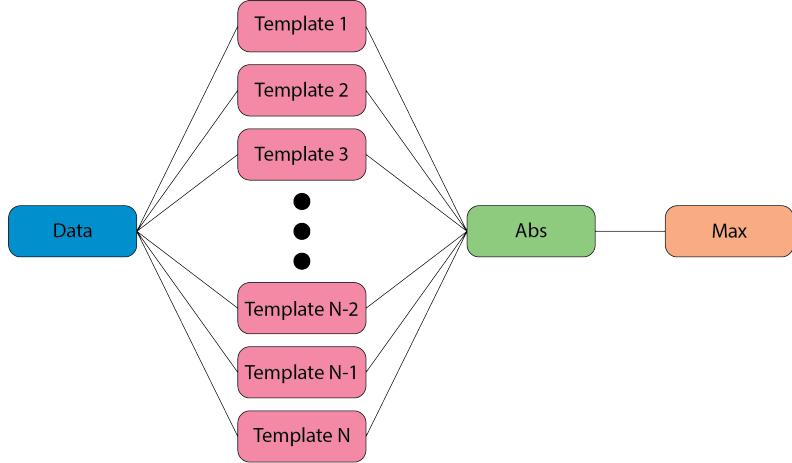


Figure 4.23. A representation of a matched filter template bank as a convolutional neural network. The network has a single layer composed of the templates, which act as convolutional filters. The activation of the neural network is an absolute value followed by a max operator.

3036 CNNs have been extremely influential in the field of computer vision, particularly tasks
 3037 such as image segmentation and classification, but have also been applied in numerous
 3038 experimental physics contexts. Given the particular challenge posed by signal detection
 3039 and reconstruction in the FSCD, CNNs are an interesting choice for real-time signal
 3040 detection, since this application requires both high efficiency and fast evaluation.

3041 In the machine learning paradigm, signal detection is a binary classification problem
 3042 between the signal and noise data classes. My investigation focuses specifically on the
 3043 application of CNNs to signal detection in the FSCD, which is motivated by relatively
 3044 recent demonstrations of CNNs achieving classification accuracies for gravitational wave
 3045 time-series signals comparable to a matched filter template bank. In this framework
 3046 it is possible to interpret the matched filter as a type of CNN composed of a single
 3047 convolutional layer with the templates making up the layer filters (see Figure 4.23).
 3048 Since this neural network has no hidden layers, it is not a DNN, but one can attempt to
 3049 construct a proper CNN that attempts to reproduce the classification performance of the
 3050 matched filter network, which can be referred to as "deep filtering".

3051 The reason why deep filtering can be effective is that it may be possible to exploit
 3052 redundancies and correlations between templates, which allows one to perform signal
 3053 detection with similar accuracy but with fewer computations. This is relevant to real-time
 3054 detection scenarios like the FSCD experiment. In Section 4.4 I perform a detailed
 3055 comparison of the signal detection performance of a CNN to beamforming and a matched

3056 filter template bank.

3057 Deep filtering is conceptually a simple technique. Similar to a matched filter template
3058 bank, many simulated CRES signals are generated and used to train a model to distinguish
3059 between signal and noise data (see Figure 4.24). To reduce the dimensionality of the
3060 input FSCD data, a digital beamforming summation is applied to the raw time-series
3061 data generated by Locust to compress the 60-channel data to a single time-series. CRES
3062 signals have a sparse frequency representation and experiments training CNN's on time-
3063 series and frequency-series data found that models trained on frequency spectrum data
3064 performed significantly better. Therefore, an FFT is applied to the summed time-series
3065 before being normalized and fed to the classification model.

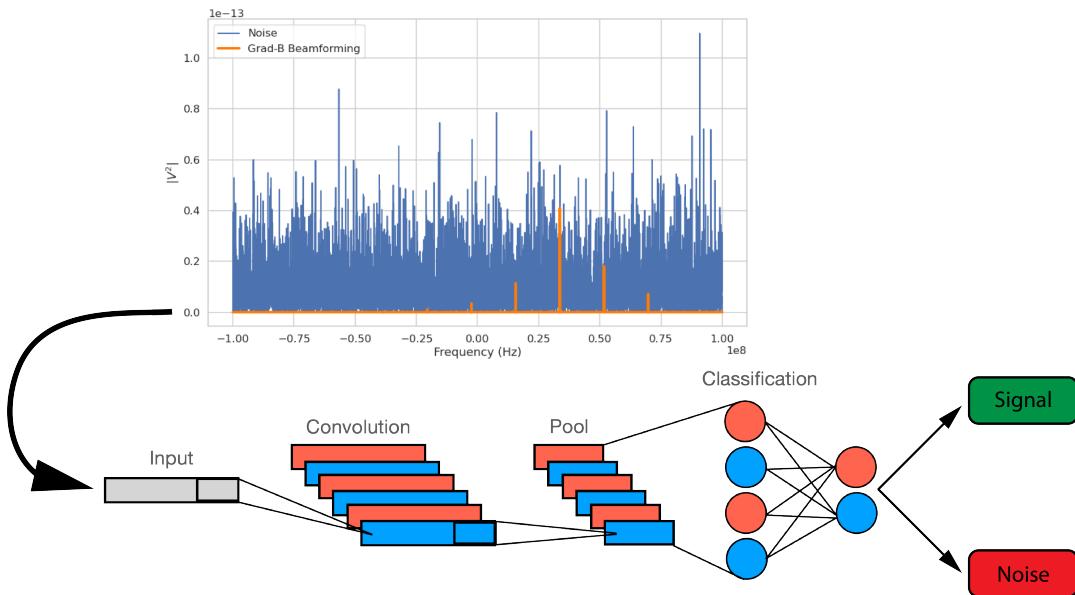


Figure 4.24. A graphical depiction of CRES signal detection using a CNN. A noisy segment of data is converted to a frequency series using digital beamforming and a FFT. The complex-valued frequency series is input into a trained CNN model that classifies the data as signal or noise using a decision threshold on the CNN output.

3066 The data used to train the model consists of an equal proportion of signal and noise
3067 frequency spectra. Unique samples of WGN are generated and added to the signals during
3068 training time to avoid having to pre-generate and store large samples of noise data. The
3069 binary cross-entropy loss function combined with the ADAM optimizer proved effective
3070 at training the models to classify CRES data. A simple hyperparameter optimization
3071 was performed by manually tuning model, loss function, and optimizer parameters. The
3072 model and training loops was implemented in python using the PyTorch deep learning
3073 framework. Standard machine learning practices were followed when training the models,

such as overtraining monitoring using a validation dataset. Models were trained until the training loss and accuracy converged and then evaluated using a separate test data set.

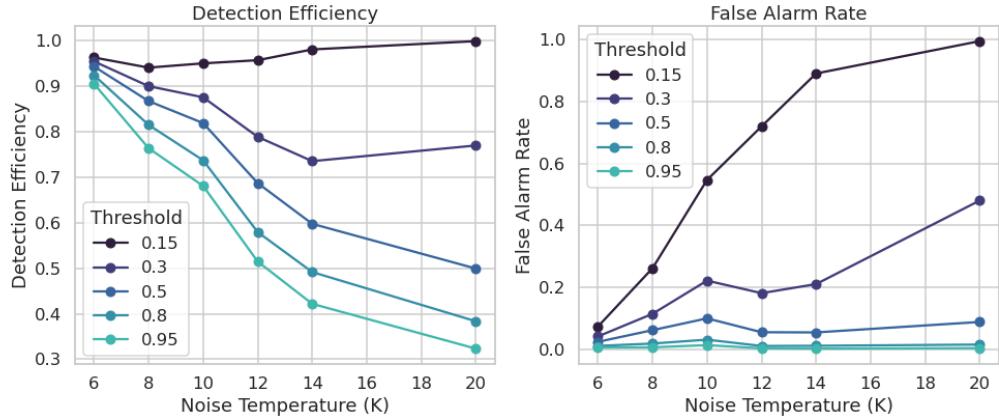


Figure 4.25. The detection efficiency and false alarm rate (false positive rate) as a function of the decision threshold for different values of the noise temperature. The model is trained to output a value close to one for data that contains a signal and outputs a value near zero when the data contains only noise. One sees that a lower decision threshold will have a high detection efficiency at the cost of a high rate of false alarms.

The classification results of the test dataset are used to quantify the relationship between the true positive rate and the false positive rate for the model. The true positive rate is analogous to detection efficiency and the false positive rate is a potential source of background in the detector. One can limit the rate of false positives using a sufficiently high threshold on the model output at the cost of a lower detection efficiency (see Figure 4.25 and Figure 4.26). As expected, the performance of the model at signal classification is negatively effected by the noise power, which is quantified by the noise temperature.

4.4 Analysis of Signal Detection Algorithms for the FSCD

This section consists of a modified manuscript for an antenna-based CRES signal detection paper prepared for publication in JINST. The contents of this paper were still undergoing collaboration review at the time of writing. In it I present a detailed analysis of the signal detection performance of the three signal detection approaches discussed so far using a population of simulated CRES signals generated with Locust. The focus of the paper is on the performance of the signal detection algorithms for pitch angles below 88.5° where the beamforming power threshold is least effective.

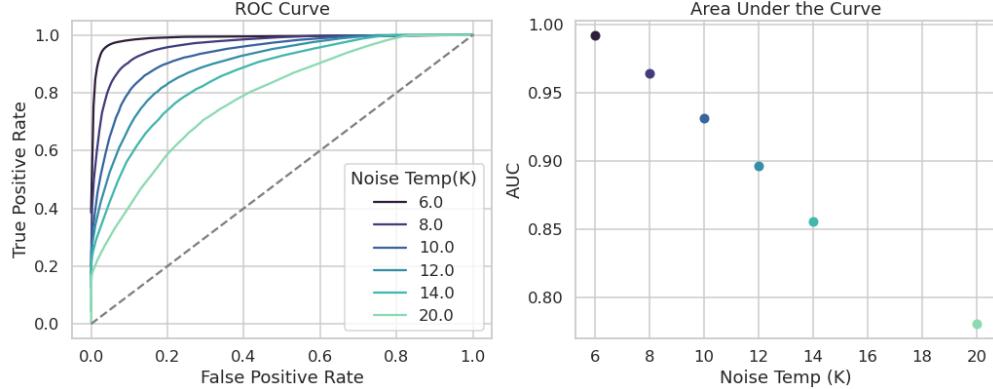


Figure 4.26. ROC curves for a CNN model classifying CRES signals. One can see that the area under the curve, which is a figure of merit that describes the performance of the classifier, is roughly linearly dependent with the noise temperature.

3091 4.4.1 Introduction

3092 One approach to large volume CRES is to surround a large volume with an array of
 3093 antennas that together collect a portion of the cyclotron radiation emitted by trapped
 3094 electrons [41, 83]. A promising design is an inward-facing uniform cylindrical array that
 3095 surrounds the tritium containment volume. Increasing the size of the antenna array,
 3096 by adding additional rings of antennas along the vertical axis, allows one to grow the
 3097 experiment volume until a sufficient amount of tritium gas can be observed by the array.
 3098 A challenging aspect of this approach is that the total radiated power emitted by an
 3099 electron near the tritium spectrum endpoint is on the order of 1 fW or less in a 1 T
 3100 magnetic field, which is then distributed among all antennas in the array. Because the
 3101 CRES signal information is spread across the antenna array, detecting the presence of
 3102 a CRES signal and determining the electron's kinetic energy requires reconstructing
 3103 the entire array output over the course of the CRES event, posing a significant data
 3104 acquisition and signal reconstruction challenge.

3105 Previous measurements with the CRES technique (see Section 3.3) have utilized
 3106 a threshold on the frequency spectrum formed from a segment of CRES time-series
 3107 data. This algorithm relies on the detection of a frequency peak above the thermal
 3108 noise background, which limits the kinematic parameter space of detectable electrons
 3109 (see Section 4.4.2.2). Although a power threshold based classification was adequate for
 3110 smaller detectors, improvements in detection efficiency are needed for better sensitivity
 3111 to the neutrino mass. Better detection efficiency is possible by taking advantage of the
 3112 deterministic CRES signal structure with a matched filter or machine learning based

3113 classifier [84]. In order to evaluate the relative gains in detection efficiency that come
3114 from utilizing these algorithms for antennas, analytical models that describe the detection
3115 performance a power threshold and matched filter classifier are developed. In addition,
3116 a basic convolutional neural network (CNN) is implemented and tested as a first step
3117 towards the development of neural-network based classifiers for antenna array based
3118 CRES measurements. These results allow for a comparison between the estimated
3119 detection efficiencies of each of these methods, which are weighed against the associated
3120 computational costs for real-time applications.

3121 The outline of the remainder of this chapter is as follows. Section 4.4.2 is an overview
3122 of a prototypical antenna array CRES experiment, and describes the approach to real-time
3123 signal identification. Section 4.4.3 develops models for the power threshold and matched
3124 filter algorithms and introduces the machine learning approach and CNN architecture.
3125 Section 4.4.4 describes the process for generating simulated CRES signal data and the
3126 details of training the CNN. Finally, Section 4.4.5 compares the signal classification
3127 accuracy for the three approaches and discusses the relevant trade-offs in terms of
3128 detection efficiency and computational cost.

3129 **4.4.2 Signal Detection with Antenna Array CRES**

3130 **4.4.2.1 Antenna Array and Data Rate Estimates**

3131 In order to explore the potential of antenna array CRES for neutrino mass measurement,
3132 the Project 8 Collaboration has developed a conceptual design for a prototype antenna
3133 array CRES experiment [41,83], called the Free-space CRES Demonstrator or FSCD (see
3134 Figure 4.27). The FSCD utilizes a single ring of antennas, which is the simplest form of a
3135 uniform cylindrical array configuration, to surround a radio-frequency (RF) transparent
3136 tritium gas vessel. A prototype version of this antenna array has been built and tested
3137 by the Project 8 collaboration [44] to validate simulations of the array radiation pattern
3138 and beamforming algorithms [85]. In the FSCD the antenna array is positioned at the
3139 center of the magnetic trap formed by a set of electromagnetic coils, which create a local
3140 minimum in the magnetic field with flat central region and steep walls in the radial and
3141 axial directions.

3142 When an electron is trapped its motion consists of three primary components. The
3143 component with the highest frequency is the cyclotron orbit whose frequency is determined
3144 by the size of the background magnetic field. The FSCD design assumes a background
3145 magnetic field value of approximately 0.96 T, which results in cyclotron frequencies

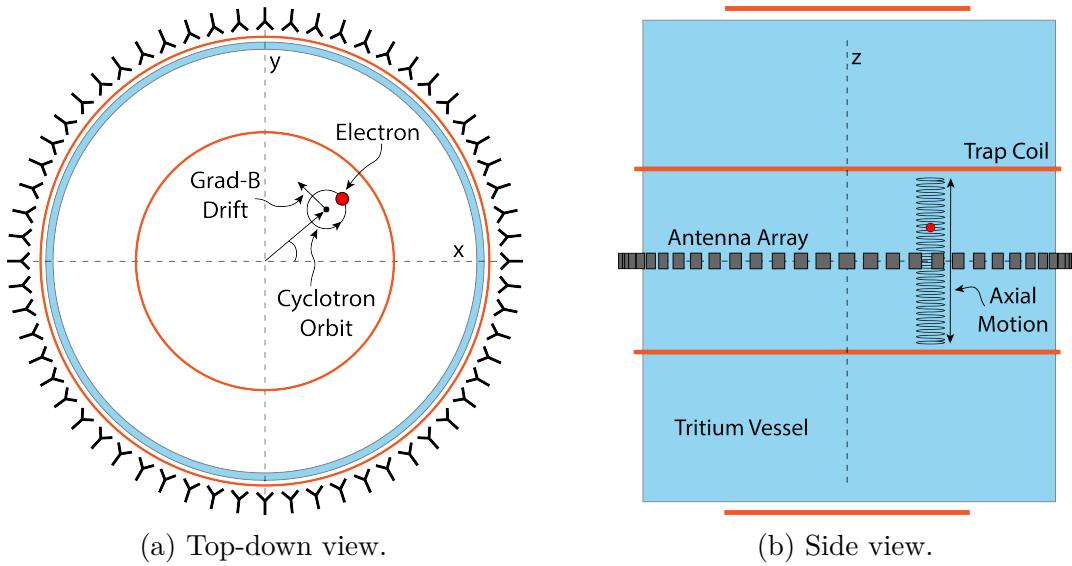


Figure 4.27. An illustration of the conceptual design for an antenna array CRES tritium beta-decay spectrum measurement. The antenna array geometry consists of a 20 cm interior diameter with 60 independent antenna channels arranged evenly around the circumference. The nominal antenna design is sensitive to radiation in the frequency range of 25-26 GHz, which corresponds to the cyclotron frequency of electrons emitted near the tritium beta-spectrum endpoint in a 0.96 T magnetic field. The array is located at the center of the magnetic trap produced by a set of current-carrying coils. The nominal magnetic trap design is capable of trapping electrons up to 5 cm away from the central axis of the array and traps electrons within an approximately 6 cm long axial region centered on the antenna array.

for electrons with kinetic energies near the tritium beta-spectrum endpoint of 26 GHz.
 The component with the next highest frequency is the axial oscillation experienced by electrons with pitch angles⁵ of less than 90° [64]. The flat region of the FSCD magnetic trap extends approximately 3 cm above and below the antenna array plane causing electrons to move back and forth as they are reflected from the trap walls. Typical oscillation frequencies are on the order of 10's of MHz, which results in an oscillation period that is $O(10^3)$ smaller than the duration of a typical CRES event. Therefore, the axial extent of the electron's motion is generally ignored for the purposes of reconstruction, since the electron can be treated as if it is located in the average axial position at the bottom of the magnetic trap. The component of motion with the smallest frequency is the ∇B -drift caused by radial field gradients in the trap, producing an orbit of the electron around the central axis of the trap with a frequency on the order of a few kHz, dependent on the pitch angle and the radial position of the electron.

⁵Pitch angle is defined as the angle of the particle's total momentum with respect to the local magnetic field.

3159 Each component of motion influences the shape of the cyclotron radiation signals
 3160 received by the antenna array, therefore, the data acquisition (DAQ) system must be
 3161 properly designed in order to resolve the effects of the cyclotron motion, pitch angle, and
 3162 ∇ -B drift on the signal shape. Frequency down-conversion allows for intentional under-
 3163 sampling of the CRES signals at a nominal bandwidth of 200 MHz. The bandwidth is
 3164 required to be large enough to contain all sidebands produced by pitch angle modulation,
 3165 but must be limited to reduce the Nyquist-Johnson noise power for adequate signal-to-
 3166 noise ratio. The estimated noise temperature for the FSCD is ≈ 10 K, achievable with
 3167 low-noise HEMT amplifiers and cryogenic temperatures.

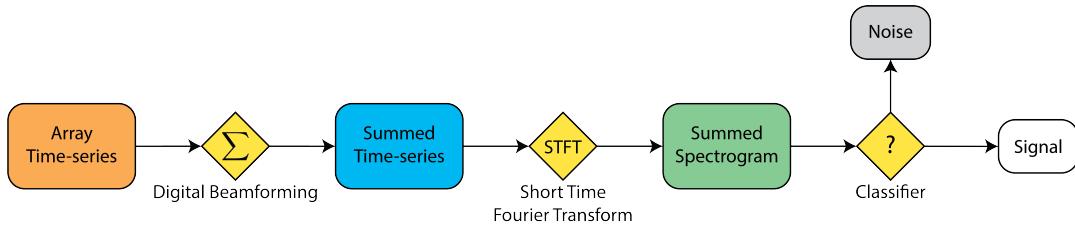


Figure 4.28. A block diagram illustration of the real-time triggering algorithm proposed for antenna array CRES reconstruction.

3168 A design goal for the FSCD DAQ system is to enable a significant portion of the
 3169 CRES event reconstruction to occur in real-time. The estimated data volume generated
 3170 by the FSCD is 1 exabyte of raw data per year of operation, with the nominal array size
 3171 of 60 antennas sampled at 200 MHz, which would be too expensive to store for offline
 3172 processing. Therefore, it is ideal to perform some CRES event reconstruction in real-time
 3173 so that it is possible to save a reduced form of the data for offline analysis. The first step
 3174 of the real-time reconstruction would be a real-time signal detection algorithm, which is
 3175 the focus of this paper. The basic approach consists of three operations performed on the
 3176 time-series data blocks including digital beamforming, a short time Fourier transform
 3177 (STFT), and a binary classification algorithm to distinguish between signal and noise
 3178 data (see Figure 4.28).

3179 4.4.2.2 Real-time Signal Detection

3180 The first step in the real-time detection algorithm is digital beamforming, which is a
 3181 phased summation of the signals received by the array (see Figure 4.29). The phase shifts
 3182 correspond to the path length differences between a spatial position and the antennas
 3183 such that, when there is an electron located at the beamforming position, all the signals
 3184 received by the array constructively interfere. Since one does not know a priori where an

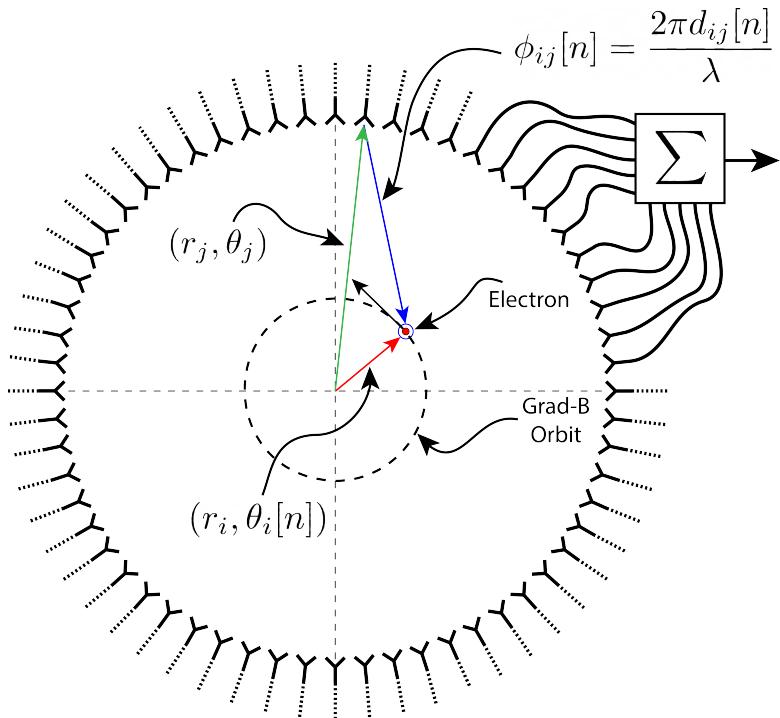


Figure 4.29. An illustration of the digital beamforming procedure. The blue lines indicate the distances from the beamforming position to each antenna. In the situation depicted the actual position of the electron matches the beamforming position, therefore, one expects constructive interference when the phase shifted signals are summed. To prevent the electron's ∇B -motion from moving the electron off of the beamforming position, the beamforming phases include time-dependence to follow the trajectory of the electron in the magnetic trap.

3185 electron will be produced in the detector, a grid of beamforming positions is designed to
 3186 cover the entire azimuthal plane where electrons can be trapped. A beamforming phased
 3187 summation is performed for all points in the grid at each time-step. As shown in Section
 3188 4.4.2.1, the axial oscillation of the electrons prevents one from resolving its position along
 3189 the z-axis, therefore, the beamforming grid need only cover the possible positions of the
 3190 electron in the two-dimensional plane defined by the antenna array.

3191 Digital beamforming can be expressed as

$$\mathbf{y}[n] = \Phi^T[n]\mathbf{x}[n], \quad (4.39)$$

3192 where $\mathbf{x}[n]$ is the array snapshot vector at the sampled time n , $\Phi[n]$ is the matrix of
 3193 beamforming phase shifts, and $\mathbf{y}[n]$ is the summed output vector that contains the
 3194 voltages for each of the summed channels corresponding to a particular beamforming
 3195 position. The elements of the beamforming phase shift matrix can be expressed as a

3196 weighted complex exponential

$$\Phi_{ij}[n] = A_{ij}[n] \exp(2\pi i \phi_{ij}[n]), \quad (4.40)$$

3197 where the indices i and j label the beamforming and antenna positions respectively. The
3198 weight A_{ij} accounts for the relative power increase for antennas that are closer to the
3199 position of the electron, and ϕ_{ij} is the total beamforming phase shift for the j -th antenna
3200 at the i -th beamforming position.

3201 The beamforming phase shift is a sum of two terms described by Equation 4.23, which
3202 in the notation of Figure 4.29 is

$$\phi_{ij}[n] = \frac{2\pi d_{ij}[n]}{\lambda} + \theta_{ij}[n]. \quad (4.41)$$

3203 The ∇B -drift in the trap is accounted for using time-dependent beamforming phases as
3204 in Equation 4.25. In the coordinate system of Figure 4.29 the time-dependent azimuthal
3205 position of the electron is

$$\theta_i[n] = \omega_{\nabla B} t[n] + \theta_{i,0}. \quad (4.42)$$

3206 Predicting accurate values of $\omega_{\nabla B}$ for a specific trap and set of kinematic parameters can
3207 be done with simulations, which are performed using the Locust software package [65]
3208 developed by Project 8.

3209 After digital beamforming, a short-time Fourier transform (STFT) is applied to the
3210 summed time-series to obtain the signal frequency spectrum (see Figure 4.30). From the
3211 detection perspective, the frequency representation of the CRES data is advantageous
3212 compared to the time domain, due to the sparseness of CRES signals in the frequency
3213 domain. The frequency spectra of CRES signals are well-approximated by a frequency and
3214 amplitude modulated sinusoidal whose carrier frequency increases as a linear chirp [64].
3215 The modulation is caused by the axial oscillation of the electron in the magnetic trap,
3216 and the linear chirp is caused by the energy loss due to cyclotron radiation, which results
3217 in a relatively slow increase in the frequency components of the CRES signal over time.
3218 A typical CRES signal increases in frequency by approximately 15 kHz during the
3219 standard Fourier analysis window of 40.96 μ sec, which is smaller than the frequency
3220 bin width for a 200 MHz sample rate. Therefore, when considering a single frequency
3221 spectrum it is justifiable to neglect the effects of the linear frequency chirp.

3222 The majority of the CRES signal power for electrons in the FSCD trap is contained in
3223 a single frequency component when the electron has a pitch angle $\gtrsim 88.5^\circ$. The remain-

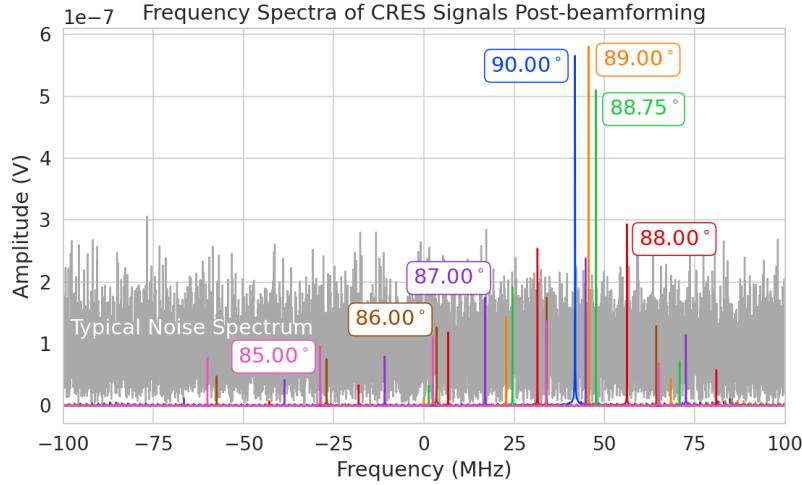


Figure 4.30. Frequency spectra of simulated CRES events in the FSCD magnetic trap after beamforming. The signal of a 90° electron consists of a single frequency component that is clearly detectable using a power threshold on the frequency spectrum. This power threshold remains effective for signals with relatively large pitch angles such as 89.0° and 88.75° , which are composed of a main carrier and a few small sidebands. Signals with smaller pitch angles, below about 88.5° , are dominated by sidebands such that no single frequency component can be reliably distinguished from the noise with a power threshold.

ing signal power is distributed between a small number of sidebands with amplitudes proportional to the electron's axial modulation (see Figure 4.30). Signal detection for these pitch angles is straightforward using a simple power threshold on the STFT, since the amplitude of the main signal peak is well above the thermal noise spectrum. However, as the pitch angle of the electron is decreased below 88.5° , the maximum amplitude of the frequency spectrum becomes comparable to typical noise fluctuations. At this point, the power threshold trigger is no longer able to distinguish between signal and noise leading to a reduction in detection efficiency, which is directly linked to the neutrino mass sensitivity of the FSCD. Because the distribution of electron pitch angles is effectively uniform, utilizing a signal detection algorithm that can improve efficiency for pitch angles less than 88.5° will lead to improvements in the neutrino mass sensitivity of the FSCD.

4.4.3 Signal Detection Algorithms

Modeling detection performance requires one to pose the signal detection problem in a consistent manner. The approach studied here uses the frequency spectra obtained from a STFT applied to the beamformed time-series from the FSCD to perform a binary hypothesis test. The basic mathematics of this hypothesis test approach was described

3240 above (Section 4.3), where the noise and signal hypotheses represent the following samples
 3241 of data,

$$\mathcal{H}_0 : y[n] = \nu[n] \quad (4.43)$$

$$\mathcal{H}_1 : y[n] = x[n] + \nu[n]. \quad (4.44)$$

3242 Under hypothesis \mathcal{H}_0 the vector representing the frequency spectrum ($y[n]$) is composed of
 3243 complex white Gaussian noise (cWGN, $\nu[n]$) with total variance τ , and under hypothesis
 3244 \mathcal{H}_1 the frequency spectrum is composed of a CRES signal ($x[n]$) with added cWGN.
 3245 The dominant noise source for the FSCD is expected to be thermal Nyquist-Johnson
 3246 noise, which is well approximated by a cWGN distribution. The hypothesis test is
 3247 performed by calculating the ratio between the log-likelihood probability distributions
 3248 for the classifier under \mathcal{H}_1 and \mathcal{H}_0 , which is the standard Neyman-Pearson approach
 3249 to hypothesis testing [74]. The output of the log-likelihood ratio test is called the test
 3250 statistic, which is used to assign the data as belonging to the noise or signal classes using
 3251 a decision threshold.

3252 In practice, the decision threshold is selected by finding the value of the test statistic
 3253 that guarantees a tolerable rate of false positives. Given this false positive rate (FPR),
 3254 one attempts to find a classifier that maximizes the true positive rate (TPR), which is
 3255 the probability of correctly identifying if a piece of data contains signal or noise. Because
 3256 FSCD signal classifiers will be used to evaluate the spectra of $O(10^2)$ beamforming
 3257 positions every $40.96 \mu\text{sec}$, there is a requirement that the signal classifiers with FPR
 3258 significantly smaller than 1% to reduce the burden placed on later stages of the CRES
 3259 reconstruction chain.

3260 4.4.3.1 Power Threshold

3261 The power threshold detection algorithm uses the maximum amplitude of the frequency
 3262 spectrum as the detection test statistic. Consider the \mathcal{H}_0 hypothesis where the signal is
 3263 pure cWGN. The performance of the power threshold can be modeled by first analyzing
 3264 a single bin in the frequency spectrum. The probability that the amplitude of a single
 3265 frequency bin falls below the decision threshold is given by the Rayleigh cumulative
 3266 distribution function (CDF),

$$\text{Ray}(x; \tau) = 1 - \exp(-|x|^2/\tau), \quad (4.45)$$

3267 where the complex value of the frequency bin is x , and τ is the cWGN variance. Because
 3268 the noise samples are independent and identically distributed (IID), the probability that
 3269 all bins in the frequency spectrum fall below the threshold is the joint CDF formed by
 3270 the product of each individual frequency bin CDF,

$$F_0(x; \tau, N_{\text{bin}}) = \text{Ray}(x; \tau)^{N_{\text{bin}}}. \quad (4.46)$$

3271 Finally, the PDF for the power threshold classifier can be obtained by differentiating
 3272 Equation 4.46.

3273 The noise variance of a beamformed frequency spectrum can be obtained directly
 3274 from the estimated noise power in a single antenna channel. The Nyquist-Johnson noise
 3275 power is given by $k_B T \Delta f$, where k_B is Boltzmann's constant, T is the system noise
 3276 temperature, and Δf is the sample rate. The beamformed noise variance is increased
 3277 by a factor of N_{ch} , where N_{ch} is the number of antennas, caused by the summation of
 3278 incoherent noise samples, however, the noise variance per frequency bin is decreased by a
 3279 factor equal to the number of samples in the STFT (N_{FFT}). The final expression for the
 3280 noise variance of the beamformed frequency spectrum is given by

$$\tau = k_B T \Delta f N_{\text{ch}} R / N_{\text{FFT}}, \quad (4.47)$$

3281 where the system impedance (R) has been used to convert from power to voltage-squared.

3282 The probability distribution for the power threshold classifier under \mathcal{H}_1 is calculated
 3283 in a similar way, but the frequency bins that contain signal must be treated separately.
 3284 The probability that the amplitude of a frequency bin containing both signal and noise
 3285 bin falls below the decision threshold is described by a Rician CDF,

$$\text{Rice}(x; \tau, \alpha) = 1 - \int_x^\infty d|\tilde{x}| \frac{2|\tilde{x}|}{\tau} \exp\left(-\frac{|\tilde{x}|^2 + |\alpha|^2}{\tau}\right) \mathcal{I}_0\left(\frac{2|\tilde{x}||\alpha|}{\tau}\right), \quad (4.48)$$

3286 where the parameter $|\alpha|$ defines the noise-free amplitude of the signal. The CDF that
 3287 describes the probability that the entire spectrum falls below the decision threshold is
 3288 the product of both signal and noise CDFs,

$$F_1(x; \tau, \alpha, N_{\text{bin}}, N_s) = \text{Ray}(x; \tau)^{N_{\text{bin}} - N_s} \prod_{k=0}^{N_s} \text{Rice}(x; \tau, \alpha_k). \quad (4.49)$$

3289 The first half of Equation 4.49 is the contribution from the bins in the frequency spectrum

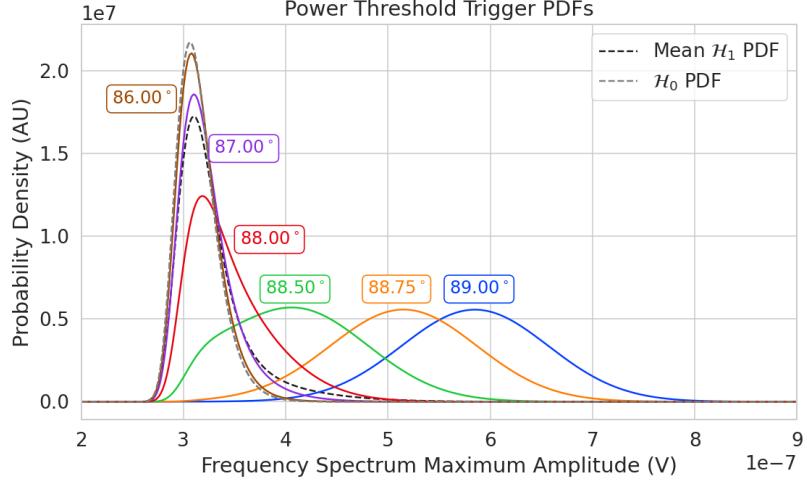


Figure 4.31. PDFs of the power threshold test statistic for CRES signals with various pitch angles as well as the PDF for the noise-only signal case. The average PDF computed for pitch angles ranging from 85.5 to 88.5° is also shown. As the pitch angle is decreased the signal PDF converges towards the noise PDF which indicates that the power threshold trigger is unable to distinguish between small pitch angle signals and noise.

3290 that contain only noise, and the second half is the product of the Rician CDFs for the
 3291 frequency bins that contain signal peaks with a noise-free amplitude of $|\alpha_k|$. Figure 4.31
 3292 shows plots of example PDFs under \mathcal{H}_1 and \mathcal{H}_0 .

3293 4.4.3.2 Matched Filtering

3294 The shape of a CRES signal in-between random scattering events with the background
 3295 gas is completely determined by the initial conditions of the electron, which implies that
 3296 it is possible to apply matched filtering as a signal detection algorithm. A matched filter
 3297 uses the shape of the known signal, which is called a template, to filter the incoming
 3298 data by computing the convolution between the signal and the data [74]. The matched
 3299 filter is the optimal detector, which means it achieves the maximum TPR for a particular
 3300 FPR, under the assumption that the signal is perfectly known and the noise is Gaussian
 3301 distributed. Since CRES signals have an unknown shape but are deterministic, the
 3302 matched filter can be applied by using simulations to generate a large number of signal
 3303 templates, called a "template bank", which spans the parameter space of possible signals.
 3304 Then at detection time, the template bank is used to identify signals by performing the
 3305 matched filter convolution for each template in an exhaustive search.

3306 CRES signals are highly periodic in nature. In such cases, it is advantageous to utilize
 3307 the convolution theorem to replace the matched filter convolution with an inner product

3308 in the frequency-domain. Utilizing the matched filter formalism described above (Section
 3309 4.3.2) the matched filter test statistic can be written as

$$\mathcal{T} = \max_h \left| \sum_{n=0}^{N_{\text{bin}}} h^\dagger[n] y[n] \right|, \quad (4.50)$$

3310 where $h^\dagger[n]$ is the complex conjugate of the signal template.

3311 The approach to deriving PDFs that describe the matched filter template bank will
 3312 be to first derive PDFs for \mathcal{H}_0 and \mathcal{H}_1 in the case of a single template and use these
 3313 solutions to create PDFs that describe the multi-template case. In the case when the
 3314 template bank consists of only a single template it is possible to derive an exact analytical
 3315 form for the PDF. Consider the \mathcal{H}_1 case, where the equation describing the matched
 3316 filter test statistic, also known as the matched filter score, becomes

$$\mathcal{T} = \left| \sum_{n=0}^{N_{\text{bin}}} h^\dagger[n] y[n] \right|. \quad (4.51)$$

3317 Each noisy frequency bin is a sum of signal and cWGN, which means $y[n]$ is also a
 3318 Gaussian distributed variable. Therefore, the value of the inner product between the
 3319 template and the data is also a complex Gaussian variable; and, since the matched filter
 3320 score is the magnitude of this inner product, it must follow a Rician distribution.

3321 The distribution that describes the matched filter score under \mathcal{H}_1 can be derived
 3322 starting with the matched filter template equation. The matched filter template \mathbf{h} is a
 3323 simulated signal (\mathbf{x}_h) with a normalization factor

$$\mathbf{h} = \frac{\mathbf{x}_h}{\sqrt{\tau |\mathbf{x}_h|^2}}, \quad (4.52)$$

3324 where τ is the noise variance. Inserting this into Equation 4.50 and expressing the data
 3325 as a sum between a signal and a WGN vector yields,

$$\mathcal{T} = \frac{1}{\sqrt{\tau |\mathbf{x}_h|^2}} \left| \sum_{n=1}^{N_{\text{bin}}} x_h^\dagger[n] x[n] + \sum_{n=1}^{N_{\text{bin}}} x_h^\dagger[n] \nu[n] \right|. \quad (4.53)$$

3326 The first term is a scalar product between the signal and template vectors and the
 3327 second term is a complex Gaussian distributed variable with variance one. For the
 3328 purposes of identifying the statistical distribution, it is useful to rewrite the summation

³³²⁹ describing an inner product

$$\sum_{n=1}^{N_{\text{bin}}} x_h^\dagger[n] x[n] = \mathbf{x}_h \cdot \mathbf{x} = |\mathbf{x}_h \cdot \mathbf{x}| e^{i\vartheta} \leq |\mathbf{x}_h| |\mathbf{x}| e^{i\vartheta}, \quad (4.54)$$

³³³⁰ the last step utilizes the Cauchy-Schawrz inequality, where equality is guaranteed when
³³³¹ $\mathbf{x} = \mathbf{x}_h$. Instead of the inequality it is useful to define a quantity called "match" such that

$$|\mathbf{x}_h \cdot \mathbf{x}| e^{i\vartheta} = |\mathbf{x}_h| |\mathbf{x}| \Gamma e^{i\vartheta}, \quad (4.55)$$

³³³² where the match factor $\Gamma \in [0, 1]$. The match factor quantifies how well the template
³³³³ matches the signal.

³³³⁴ The fact that the second term is a random complex Gaussian variable with unity
³³³⁵ variance can be seen by noting that each of the noise samples are drawn from the complex
³³³⁶ Gaussian distribution, $\mathcal{N}(0, \tau)$. Therefore,

$$\frac{x_h^\dagger[n]}{\sqrt{\tau |\mathbf{x}_h|^2}} \nu[n] \sim \mathcal{N}\left(0, \frac{x_h^\dagger[n] x_h[n]}{|\mathbf{x}_h|^2}\right), \quad (4.56)$$

$$n = \sum_{n=1}^{N_{\text{bin}}} \frac{x_h[n]}{\sqrt{\tau |\mathbf{x}_h|^2}} \nu[n] \sim \mathcal{N}\left(0, \frac{\sum_{n=1}^{N_{\text{bin}}} x_h^\dagger[n] x_h[n]}{|\mathbf{x}_h|^2}\right) = \mathcal{N}(0, 1). \quad (4.57)$$

³³³⁷ Equation 4.53 can now be simplified

$$\mathcal{T} = ||\mathbf{h}||\mathbf{x}| \Gamma e^{i\vartheta} + n |, \quad (4.58)$$

³³³⁸ where Equation 4.52 has been used to redefine the inner product term. The quantity
³³³⁹ $||\mathbf{h}||\mathbf{x}| \Gamma$ is a real number, which is the matched filter score that one would expect if the
³³⁴⁰ data contained no noise. The final simplification is to define $\mathcal{T}_{\text{ideal}} = ||\mathbf{h}||\mathbf{x}| \Gamma$, from which
³³⁴¹ one obtains

$$\mathcal{T} = |\mathcal{T}_{\text{ideal}} e^{i\vartheta} + n|. \quad (4.59)$$

³³⁴² From Equation 4.59 on can see that \mathcal{T} is simply the magnitude of a complex number
³³⁴³ with added cWGN of variance 1, which follows the Rician distribution, therefore the
³³⁴⁴ distribution that describes the matched filter score for a single template under \mathcal{H}_1 is

$$P_1(x; \mathcal{T}_{\text{ideal}}) = 2x \exp(- (x^2 + \mathcal{T}_{\text{ideal}}^2)) I_0(2x\mathcal{T}_{\text{ideal}}). \quad (4.60)$$

³³⁴⁵ The shape of the matched filter score distribution is controlled by the parameter $\mathcal{T}_{\text{ideal}}$,

3346 which is effectively the value of the matched filter score if the data contained no noise.
 3347 Without noise, the data vector reduces to the signal, \mathbf{x} , in which case Equation 4.51
 3348 becomes the magnitude of an inner product between two vectors. The magnitude of an
 3349 inner product can be expressed in terms of the magnitudes of the vectors and a constant
 3350 that describes the degree of orthogonality between them. Applying this to Equation 4.51,
 3351 one obtains

$$\mathcal{T}_{\text{ideal}} = |\mathbf{h}^\dagger \cdot \mathbf{x}| = |\mathbf{h}| |\mathbf{x}| \Gamma \quad (4.61)$$

3352 where Γ describes the orthogonality between \mathbf{h} and \mathbf{x} . Γ effectively quantifies how well
 3353 the template matches the unknown signal in the data.

3354 The matched filter score PDF under \mathcal{H}_0 is readily obtained from Equation 4.60 by
 3355 setting the value of $\mathcal{T}_{\text{ideal}}$ to zero, since the data contains no signal in the noise case.
 3356 Doing this, one obtains a Rayleigh distribution,

$$P_0(x) = 2x \exp(-x^2). \quad (4.62)$$

3357 Equations 4.60 and 4.62 describe the behavior of the matched filter test statistic
 3358 under \mathcal{H}_0 and \mathcal{H}_1 for a single template. However, defining a PDF that describes the
 3359 matched filter test statistic in the case of multiple templates is in general a mathematically
 3360 intractable problem, since there is no guarantee of orthogonality between matched filter
 3361 templates. This leads to correlations between the matched filter scores of different
 3362 templates, because only one sample of noise is used to compute the matched filter scores
 3363 of the template bank. In order to proceed, it is assumed that the matched filter scores for
 3364 all templates are IID variables, which allows one to ignore correlations between templates.
 3365 The overall effect of this will be an underestimate of the performance of the matched
 3366 filter by over-estimating the required number of templates and, therefore, the magnitude
 3367 of the statistical trials penalty.

3368 The probability that the matched filter score falls below the decision threshold under
 3369 \mathcal{H}_0 is again given by the CDF. Because of the assumption that matched filter scores from
 3370 different templates are independent, the probability that the matched filter score for all
 3371 templates falls below the threshold value is simply the joint CDF, which is

$$F_0(x) = \left(1 - e^{-x^2}\right)^{N_t}, \quad (4.63)$$

3372 where x is the matched filter score threshold and N_t is the number of templates. One
 3373 should expect that the distribution describing the maximum score of the matched filter

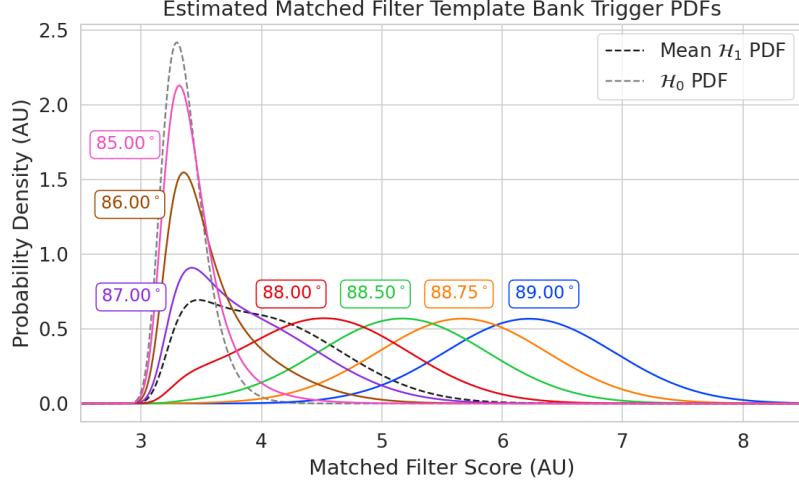


Figure 4.32. Plots of PDFs that describe the matched filter template bank test statistic for CRES signals with various pitch angles, as well as the estimated PDF for the noise only case. 10^5 matched filter templates are used and perfect match between signal and template i.e. $\Gamma_{\text{best}} = 1$ is assumed. The mean PDF includes signals ranging from $85.5 - 88.5^\circ$ in pitch angle. There is a larger distinction between the signal PDFs at small pitch angles compared to the power threshold, which indicates a higher detection efficiency for these signals.

3374 template bank depends on N_t , because with more templates there is a greater chance of
 3375 a random match between the template and data.

3376 The CDF that describes \mathcal{H}_1 is derived by starting with the CDF of the best matching
 3377 template, $F_{\text{best}}(x; \mathcal{T}_{\text{best}})$. Because of the orthogonality assumption, the matched filter
 3378 scores for all other templates are negligible ($\mathcal{T}_{\text{ideal}} \approx 0$). The joint CDF that describes
 3379 the total template bank is obtained by combining the distributions for all templates used
 3380 during detection. Therefore, the estimated CDF under \mathcal{H}_1 is

$$F_1(x; \mathcal{T}_{\text{best}}) = F_{\text{best}}(x; \mathcal{T}_{\text{best}}) \left(1 - e^{-x^2}\right)^{N_t}. \quad (4.64)$$

3381 Figure 4.32 shows plots of the matched filter template bank PDFs under \mathcal{H}_0 and \mathcal{H}_1 .

3382 4.4.3.3 Machine Learning

3383 The focus in this paper is on the potential of Convolutional Neural Networks (CNN)
 3384 as a machine learning based signal classifier at the trigger level. CNNs are constructed
 3385 using a series of convolutional layers, each composed of a set of filters that are convolved
 3386 with the input data. The individual convolutional filters can be viewed heuristically
 3387 as matched filter templates [86] that are learned from a set of simulated data rather

than being directly generated. This opens the possibility of finding a more efficient representation of the matched filter templates during the training process that can potentially reduce computational cost at inference time while retaining good classification performance.

The machine learning approach is distinct from the power threshold and matched filtering in that there is no attempt to manually engineer a test statistic that can be computed from the input data. Instead, a test statistic is calculated by constructing a differentiable function that maps the complex frequency series to a binary classification as signal or noise. The differentiable function is trained using supervised learning to correctly perform this mapping. The test statistic for the machine learning classifier is expressed mathematically as

$$\mathcal{T} = G(\mathbf{y}; \boldsymbol{\Omega}) \quad (4.65)$$

where \mathbf{y} is the noisy data vector and $G(\mathbf{y}; \boldsymbol{\Omega})$ is the machine learning model parameterized by the weights $\boldsymbol{\Omega}$.

Table 4.1. A summary of the CNN model layers and parameters. The output of each 1D-Convolution and Fully Connected layer is passed through a LeakyReLU activation function and re-normalized using batch normalization before being passed to the next layer in the model. The output of the final Fully Connected layer in the model is left without activation so that the model outputs can be directly passed to the Binary Cross-entropy loss function used during training. The first layer in the network has two input channels for the real and imaginary components of the spectrum.

Layer	Type	Input Channels	Output Channels	Parameters
1	1D-Convolution	2	15	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 1$)
2	Maximum Pooling	15	15	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 4$)
3	1D-Convolution	15	20	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 1$)
4	Maximum Pooling	20	20	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 4$)
5	1D-Convolution	20	25	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 1$)
6	Maximum Pooling	25	25	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 4$)
7	Fully Connected	3200	512	NA
8	Fully Connected	512	64	NA
9	Fully Connected	64	2	NA

The CNN architecture used for this work is summarized by Table 4.1. No strategic hyper-parameter optimization approach was implemented beyond the manual testing of different CNN architecture variations, so this particular model is best viewed as a proof-of-concept rather than a rigorously optimized design. Numerous model variations were tested, some with significantly more layers and convolutions filters per layer, as well as others that were even smaller than the architecture in Table 4.1. Ultimately, the

3407 model architecture choice was driven by the motivation to find the minimal model whose
3408 classification performance was still comparable to the larger CNN's tested, because of
3409 the importance of minimizing computational cost in real-time applications. It is possible
3410 that more sophisticated machine learning models could improve upon the classification
3411 results achieved here, but this investigation is left for future work.

3412 4.4.4 Methods

3413 4.4.4.1 Data Generation

3414 Simulated CRES signals were generated using the Locust simulations package [65, 77].
3415 Locust uses the separately developed Kassiopeia package [63] to calculate the magnetic
3416 fields produced by a user defined set of current carrying coils along with any specified
3417 background magnetic fields, resulting in a magnetic trap. Next, Kassiopeia calculates the
3418 trajectory of an electron in this magnetic field starting from a set of user specified initial
3419 conditions. The Locust software then uses the electron trajectories from Kassiopeia
3420 to calculate the resulting electromagnetic fields using the Liénard-Wiechert equations,
3421 and determines the voltages generated in the antenna array with the antenna transfer
3422 function. Locust then simulates the down-conversion, filtering, and digitization steps
3423 resulting in the simulated CRES signals for an electron.

3424 The shape of the received CRES signal is determined by the initial kinematic param-
3425 eters, including the starting position of the electron, the starting kinetic energy of the
3426 electron, and the pitch angle. The studies performed here are constrained to a single
3427 initial electron position located at $(x, y, z) = (5, 0, 0)$ mm. Two datasets are generated
3428 using this starting position by varying the initial kinetic energy and pitch angle. The
3429 first dataset consists of a two-dimensional square grid spanning an energy range from
3430 18575-18580 eV with a spacing of 0.1 eV, and pitch angles from 85.5-88.5° with a spacing
3431 of 0.001°, resulting in 153051 signals with a unique energy-pitch angle combination. This
3432 dataset is intended to represent a matched filter template bank. The upper range of pitch
3433 angles is limited because of the greater relative detection efficiency of the matched filter
3434 and neural network classifiers in this pitch angle range. The second dataset was generated
3435 by randomly sampling uniform probability distributions covering the same parameter
3436 space to produce approximately 50000 signals randomly parameterized in energy and
3437 pitch angle. This dataset provides the training and test data for the machine learning
3438 approach, and acts as a representative sample of signals to evaluate the performance of
3439 the matched filter template bank.

3440 Each signal was simulated for a duration of $40.96 \mu\text{s}$ or 8192 samples starting at
 3441 time $t = 0$ s for all simulations. This duration represents a single frequency spectrum
 3442 generated by the STFT. The FSCD antenna array has sixty channels, and the output of
 3443 the Locust simulations are a matrix of array snapshots with a size given by the number
 3444 of channels times the event length ($N_{\text{ch}} \times N_{\text{sample}}$). The raw data from Locust is first
 3445 summed using digital beamforming and converted to frequency spectra using a Fourier
 3446 transform. The beamforming procedure uses the exact position and ∇B -drift correction
 3447 to simplify the comparison between trigger algorithms. Many beamforming positions
 3448 would be used in practice and potentially several estimates of a typical $\omega_{\nabla B}$ depending
 3449 on the variation of the ∇B -drift frequency with pitch angle.

3450 **4.4.4.2 Template Number and Match Estimation**

3451 The estimated PDF for the matched filter template bank on the number of templates and
 3452 the mean match (Γ_{best}). A given signal with random parameters will have a template in
 3453 the filter bank that gives the highest matched filter score, therefore, the mean match
 3454 ratio is obtained by averaging over the best matching templates for a representative
 3455 population of test signals. Γ_{best} is a figure of merit that characterizes the performance of
 3456 a template bank at signal detection. One expects that with more templates the value
 3457 of Γ_{best} will increase, however, there is a point of diminishing returns at which more
 3458 templates will not significantly increase match, but will still increase the likelihood of
 3459 false positives. Therefore, it is desirable to use the minimum number of templates that
 provide an acceptable mean value of Γ_{best} .

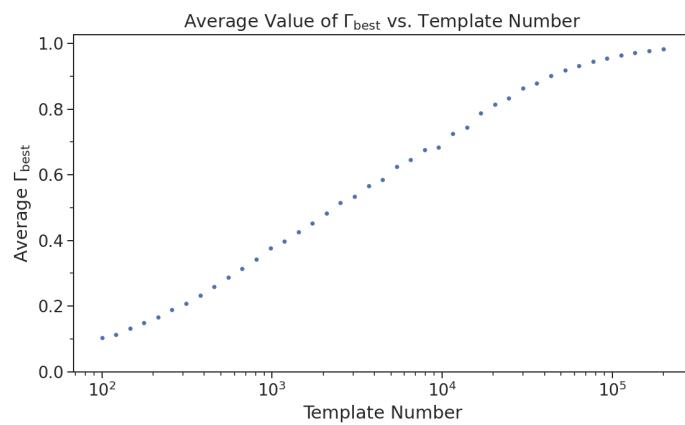


Figure 4.33. The mean match of the matched filter template bank to a test set of randomly parameterized signals as a function of the number or density of templates. The parameter space includes pitch angles from $85.5 - 88.5^\circ$ and energies from 18575 – 18580 eV.

3460 To quantify the relationship between match and template number, the mean match
3461 of the random dataset to a selection of templates from the regularly spaced dataset was
3462 calculated using a subset of the mean match values from Figure 4.18 with a sample
3463 length of 8192. Again, one sees that the average value of Γ_{best} is an exponential function
3464 of the number of templates (see Figure 4.33). Using this plot one can infer the required
3465 number of templates for the desired value of mean match.

3466 **4.4.4.3 CNN Training and Data Augmentation**

3467 The random dataset is split in half to create distinct training and test datasets for
3468 training the model. A randomly selected 20% of the training data is isolated for use as
3469 a validation set during the training loop. The size of the training, validation, and test
3470 datasets are tripled by appending two additional copies of the data to increase the sample
3471 size of the dataset after data augmentation. A different sample of noise is added to the
3472 simulation data during the training loop, which prevents the model from overtraining on
3473 noise features. The training and test datasets contain an equal split between signal and
3474 noise data, which are randomly shuffled after each training epoch.

3475 The Locust simulation data was augmented to make the datasets more representative
3476 of actual experiment data. As the signals are loaded for training a unique random phase
3477 shift is applied. Since the simulations are generated using the same initial axial position
3478 and cyclotron orbit phase, the randomization is an attempt to prevent overtraining on
3479 these features. During each training epoch the data is randomly shuffled and split into
3480 batches of 2500 signals. Each batch of signals is then circularly shifted by a random
3481 number of frequency bins to simulate a kinetic energy shift from -75 to 20 eV, which
3482 imitates a dataset with a larger energy range. Next, a sample of cWGN, consistent
3483 with 10 K Nyquist-Johnson noise, is generated and added to the signal, which prevents
3484 overtraining on noise features. As a final step, the data is renormalized by the standard
3485 deviation of the noise so that the range of values in the data is close to $[-1, 1]$, which
3486 ensures well-behaved back-propagation.

3487 The Binary Cross-entropy loss function is used to compute the loss for each batch of
3488 data, and the model weights are updated using the ADAM optimizer with a learning
3489 rate of 5×10^{-3} . After each training epoch, the loss and classification accuracy of the
3490 validation dataset are computed to monitor for overtraining. It was noticed that because
3491 of the relatively high noise power and the fact that a new sample of noise was used for
3492 each batch, it was nearly impossible to over-train the model. Typically, the loss and

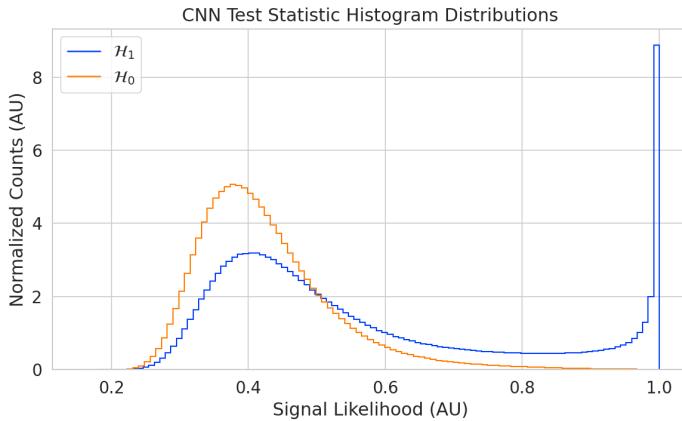


Figure 4.34. Histograms of the trained CNN model output from the test dataset. The blue histogram shows the model outputs for signal data. The oddly shaped peak near the end is the result of the softmax function mapping the long tail of the raw output distribution to the range $[0, 1]$.

classification accuracy of the model converged after a few hundred training epochs, but the training loop was extended to 3000 epochs to attempt to achieve the best possible performance. The training procedure generally took about 24 hrs using a single NVIDIA V100 GPU [87].

After training the model, it was used to classify the test dataset and generate histograms of the model outputs for both classes of data. The data augmentation procedure for the evaluation of the test data mirrors the training procedure without the validation split. Since a random circular shift and a new sample of WGN is added to each batch, the testing evaluation loop is run for 100 epochs to get a representative sample of noise and circular shifts. The model outputs are passed through a softmax activation and then combined into histograms (see Figure 4.34).

4.4.5 Results and Discussion

4.4.5.1 Trigger Classification Performance

The detection performance of the signal classifiers can be compared by computing the receiver operating characteristic (ROC) curves (see Figure 4.35). A single ROC curve is obtained for the matched filter and power threshold classifiers by averaging over analytical ROC curves obtained from the distributions in Section 4.4.3. Two ROC curves are calculated for the matched filter with different numbers of templates and mean match. The ROC curve describing the CNN is obtained numerically from the histograms of the

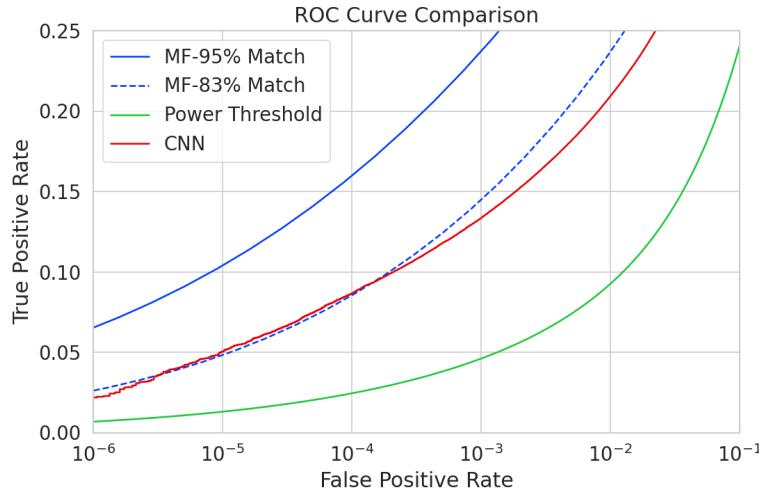


Figure 4.35. ROC curves describing the detection efficiency or true positive rates for the three signal classification algorithms examined in this paper. The matched filter (MF) and Power Threshold curves are computed analytically using the distribution functions introduced in Section 4.4.3, and the CNN curve is computed numerically using the classification results on the test dataset. The percent match indicated in the legend refers to the mean match of the classifier.

3512 model outputs for each signal class.

3513 The TPR of a signal classifier is equivalent to its detection efficiency, and one sees
 3514 that for the population of signals with pitch angles $< 88.5^\circ$ the power threshold has
 3515 a consistently lower detection efficiency than the CNN and the matched filter. This
 3516 result might have been predicted from the visualization of signal spectra in Figure 4.30,
 3517 where it can be seen that a noise peak and a signal peak cannot be distinguished with
 3518 high-confidence at small pitch angles. The CNN offers a significant and consistent increase
 3519 in detection efficiency over the power threshold approach, with the relative improvement
 3520 in detection efficiency increasing as the false positive rate decreases.

3521 If one compares the CNN to the matched filter, it can be seen that the performance of
 3522 the tested network is roughly equivalent to a matched filter detector with a mean match
 3523 of about 83%, which uses approximately 2×10^4 matched filter templates. The overall
 3524 best detection efficiency is achieved by the matched filter classifier if a large enough
 3525 template bank is used. The plot displays the ROC curve for a matched filter template
 3526 bank with 95% mean match, which is achieved with approximately 10^5 templates. Since
 3527 the matched filter is known to be statistically optimal for detecting a known signal in
 3528 WGN, it is unsurprising that this algorithm has the highest detection efficiency.

3529 An important difference between the matched filter and CNN algorithms is that the

3530 CNN relies upon convolutions as its fundamental calculation mechanism, whereas our
3531 implementation of a matched filter utilizes an inner product. Since convolution is a
3532 translation invariant operation, the detection performance of CNN can be extended to
3533 a wider range of CRES event kinetic energies with less cost than the matched filter, a
3534 feature that is exploited during the CNN training by including circular translations of
3535 the CRES frequency spectra in the training loop. Increasing the range of detectable
3536 kinetic energies with a matched filter requires a proportional increase in the number of
3537 templates, which directly translates into increased computational and hardware costs.
3538 From a practical perspective, the detection algorithm is always limited by the available
3539 computational hardware, so estimating the relative costs is a key factor in determining
3540 their feasibility. A more detailed analysis of the relative costs of each of the detection
3541 algorithms is performed below.

3542 4.4.5.2 Computational Cost and Hardware Requirements

3543 The trade-off between better detection efficiency and computational cost is common
3544 to many signal detection problems and the FSCD is no exception. Computational
3545 costs can be related to actual hardware costs by calculating the theoretical amount of
3546 computer hardware required to implement the signal classifiers for real-time detection.
3547 The approach taken here utilizes order of magnitude estimates of the theoretical peak
3548 performance values for currently available Graphics Processing Units (GPUs) as a metric.
3549 This approach underestimates the amount of required hardware, since it is unlikely that
3550 any CRES detection algorithm could reach the theoretical peak performance of the
3551 hardware.

3552 Since the signal detection algorithms are designed to work using beamformed frequency
3553 spectra, the computational cost of beamforming combined with a fast Fourier transform
3554 (FFT) is constant for all classifiers. The beamforming grid is assumed to contain N_{bf}
3555 beamforming positions, each of which will produce a frequency spectrum containing N_{bin}
3556 after the FFT.

3557 Considering the power threshold classifier, this results in $N_{\text{bin}}N_{\text{b}}$ frequency bins
3558 that must be checked every N_{bin}/f_s seconds. The 20 cm diameter FSCD array requires
3559 $N_{\text{bf}} \approx O(10^2)$ for sufficient coverage and has a sampling frequency $f_s = 200$ MHz with a
3560 Fourier analysis window of $N_{\text{bin}} = 8192$ samples. Therefore the power threshold requires
3561 approximately $O(10^{10})$ FLOPS to check in real-time with these parameters

3562 Current generations of GPUs have peak theoretical performances in the range of
3563 $O(10^{13}) - O(10^{14})$ FLOPS [88], dependent on the required floating-point precision of

3564 the computation. Therefore, the entire computational needs of a real-time triggering
3565 system using a power threshold classifier, including digital beamforming and generation
3566 of the STFT, could be met by a single high-end GPU or a small number of less powerful
3567 GPUs. Since triggering is only one step of the full real-time signal reconstruction
3568 approach, limiting the computational cost of this stage is ideal. However, the power
3569 threshold classifier does not provided sufficient detection efficiency across the entire
3570 range of possible signals, which is the primary motivation for exploring more complicated
3571 triggering solutions.

3572 As discussed, the computational cost of the matched filter approach requires counting
3573 the number of templates that must be checked for each frequency spectra produced by
3574 the STFT. Computing the matched filter scores requires $O(N_{\text{bf}}N_t N_{\text{bin}})$ operations, since
3575 for each of the beamforming positions one must multiply N_t templates with a data vector
3576 that has length N_{bin} . The computation must be performed in a time less-than or equal
3577 to N_{bin}/f_s to keep up with the data generation rate. A 5 eV range of kinetic energies
3578 required 10^4 to 10^5 templates in order for the matched filter to exceed the performance
3579 of the CNN. The number of templates is expected to scale linearly with the total kinetic
3580 energy range of interest, therefore, 10^5 to 10^6 matched filter templates would be expected
3581 for the nominal 50 eV analysis window of the FSCD. Considering this, the estimated
3582 computational cost of the matched filter is between $O(10^{15})$ to $O(10^{16})$ FLOPS, which is
3583 $O(10^2)$ to $O(10^3)$ high-end GPUs.

3584 The computational cost of the CNN can be estimated by simply summing the compu-
3585 tational costs of the convolutions and matrix multiplications specified by the network
3586 architecture shown in Table 4.1. Each convolutional layer consists of $N_{\text{in}}N_{\text{out}}N_{\text{kernel}}L_{\text{input}}$
3587 floating-point operations, where N_{in} is the number of input channels, N_{out} is the number
3588 of output channels, N_{kernel} is the size of the convolutional kernel, and L_{input} is the length
3589 of the input vector, and the fully connected layers each contribute $N_{\text{in}}N_{\text{out}}$ operations.
3590 Summing all the neural network layers it is estimated that the CNN requires $O(10^6)$
3591 floating point operations to evaluate each frequency spectra; therefore, the total com-
3592 putational cost of the CNN trigger is value multiplied by the number of beamforming
3593 positions per the data acquisition time, which is $O(10^{13})$ FLOPS or $O(10^0)$ GPUs.

3594 Compared with the matched filter approach the CNN requires $O(100)$ to $O(1000)$
3595 fewer GPUs to implement, dependent on the exact number of templates used in the
3596 template bank. The 50 eV kinetic energy range is motivated by the application of these
3597 detection algorithms to an FSCD-like neutrino mass measurement experiment. However,
3598 if a significantly larger range of kinetic energies is required, a CNN may be the preferred

3599 detection approach despite the lower mean detection efficiency due to computational cost
3600 considerations.

3601 Additional experiments with larger CNNs, generated by increasing the depth and
3602 width of the neural network, were performed. It was observed that these changes
3603 provided minimal ($\lesssim 1\%$) improvement in the classification accuracy of the model. A
3604 potential reason for this could be the sparse nature of the signals in the frequency
3605 domain and the low SNR, which makes for a challenging dataset to learn from. Future
3606 work might investigate modifications to the neural network architecture such as sparse
3607 convolutions, which may improve the classification accuracy of the model or further
3608 reduce the computational costs of this approach. Alternatively, more complicated CNN
3609 architectures such as a ResNet [89] or VGG model [90] may provide improved classification
3610 performance over a basic CNN. An additional promising area of investigation are recurrent
3611 neural networks, which may be able to exploit the time-ordered features of the STFT for
3612 more accurate signal detection if the electron signals last for multiple Fourier transform
3613 windows.

3614 The estimate of the computational costs of the matched filter is somewhat naive if one
3615 notices that the majority of the values that make up a CRES frequency spectrum are zero
3616 (see Figure 4.30). Therefore, the majority of operations in the matched filter inner product
3617 are unnecessary, and one could instead evaluate the matched filter inner product using
3618 only the $\lesssim 10$ frequency peaks that make up the CRES signal. This optimization reduces
3619 the number of operations required to check each template by a factor of $O(100)$ to $O(1000)$,
3620 which brings the estimated computational cost of the matched filter in line with the
3621 CNN. Although this level of sparsity results in a multiplication with very low arithmetic
3622 complexity, the resulting sparse matched filter algorithm is still likely to be constrained
3623 by memory access speed rather than compute speed. Ultimately, the comparison of
3624 the relative computational and hardware costs between the matched filter and CNN
3625 will depend on the efficiency of the software implementation and hardware support for
3626 neural network and sparse matrix calculations, which will need to be determined using
3627 real-world benchmarks.

3628 **4.4.6 Conclusion**

3629 Increasing the detection efficiency and overall event rate of the CRES technique represents
3630 a key developmental path towards new scientific results and broader applications of the
3631 CRES technique. It is what motivates both the antenna array detection approach and
3632 the development of real-time signal reconstruction algorithms. The work presented here

3633 demonstrates that significant gains in the detection efficiency of the CRES technique
3634 are achievable by utilizing triggering algorithms that account for the specific shape of
3635 CRES signals in the detector. These algorithms emphasize the need for accurate and fast
3636 methods for CRES simulation, since they directly contribute to the success of matched
3637 filter methods by providing a way to generate expected signal templates and also serve
3638 as a source of training data for machine learning approaches.

3639 The down-side of these more advanced approaches to signal detection is the increase
3640 in computational resources required to implement them. However, it was shown that a
3641 CNN of minimal size was able to significantly improve detection performance above the
3642 baseline power threshold trigger algorithm with a theoretical computational cost of only
3643 $O(1)$ high-end GPU. This algorithm improves on detection performance while requiring
3644 at least a factor $O(10^2)$ less in computer relative to a matched filter template bank,
3645 which would be the classical approach to signal detection in Gaussian noise. Future work
3646 obtaining real-life benchmarks of the CNN and matched filter algorithms are required to
3647 support these conclusions, but this study has indicated that a real-time signal detection
3648 algorithm for an antenna array CRES experiment is computationally feasible without
3649 extraordinary compute power.

3650 While this work has focused on the real-time detection of CRES signals from antenna
3651 arrays, these same signal classifiers could be used in CRES experiments utilizing different
3652 detector technologies, since the same principles of signal detection will apply. For example,
3653 previous CRES measurements by the Project 8 collaboration that utilized a waveguide
3654 gas cell, could have improved their detection efficiency by employing a matched filter
3655 or neural network classifier to identify trapped electrons with pitch angles that are too
3656 small to be detected by the power threshold approach. Furthermore, alternative CRES
3657 detector technologies such as resonant cavities [41] could also see similar improvements
3658 in detection efficiency, which is of crucial importance to future efforts by the Project 8
3659 collaboration to utilize CRES to measure the neutrino mass.

3660 **Chapter 5** |

3661 **Antenna and Antenna Measurement Sys-**

3662 **tem Development for the Project 8 Ex-**

3663 **periment**

3664 **5.1 Introduction**

3665 The FSCD (free space CRES demonstrator) and antenna array CRES (cyclotron radiation
3666 emission spectroscopy) represent an innovative approach to beta-decay spectroscopy.
3667 While much can be learned from simulations about the systematics of CRES with antenna
3668 arrays, laboratory measurements and demonstrations provide critical inputs to sensitivity
3669 and simulation models, and provide a means for calibration and commissioning of the
3670 experiment. Therefore, a robust program of antenna and antenna measurement hardware
3671 development is key to the success of the FSCD and the development of antenna array
3672 CRES more broadly.

3673 In this chapter I summarize the development of an antenna measurement system at
3674 Penn State to implement and test the techniques of antenna array CRES on the bench-top.
3675 In Section 5.2 I provide an introduction to some fundamental parameters and concepts
3676 related to antenna measurements as well as an overview of the Penn State antenna
3677 measurement system hardware. In Section 5.3 I include the manuscript of a paper [78]
3678 which details the design and characterization of a specialized antenna developed to mimic
3679 the electric fields emitted by an electron in a CRES experiment. This antenna, called
3680 the Synthetic Cyclotron Antenna (SYNCA), is intended as a calibration tool for antenna
3681 arrays developed for CRES measurements. Lastly, in Section 5.4 I summarize a set of
3682 prototype FSCD antenna array measurements with the SYNCA [44], which I use to
3683 validate the simulated performance of the antenna array and estimate systematic errors
3684 associated with the antenna array.

3685 5.2 Antenna Measurements for CRES experiments

3686 5.2.1 Antenna Parameters

3687 Antenna characterization measurements are intended to validate simulations of the
3688 antenna array performance, which ultimately informs the neutrino mass sensitivity of
3689 the experiment. In this section, I shall summarize a few fundamental concepts relating
3690 to antennas and antenna measurement, before introducing how Project 8 uses antenna
3691 measurements for the development of antenna array CRES.

3692 5.2.1.1 Radiation Patterns

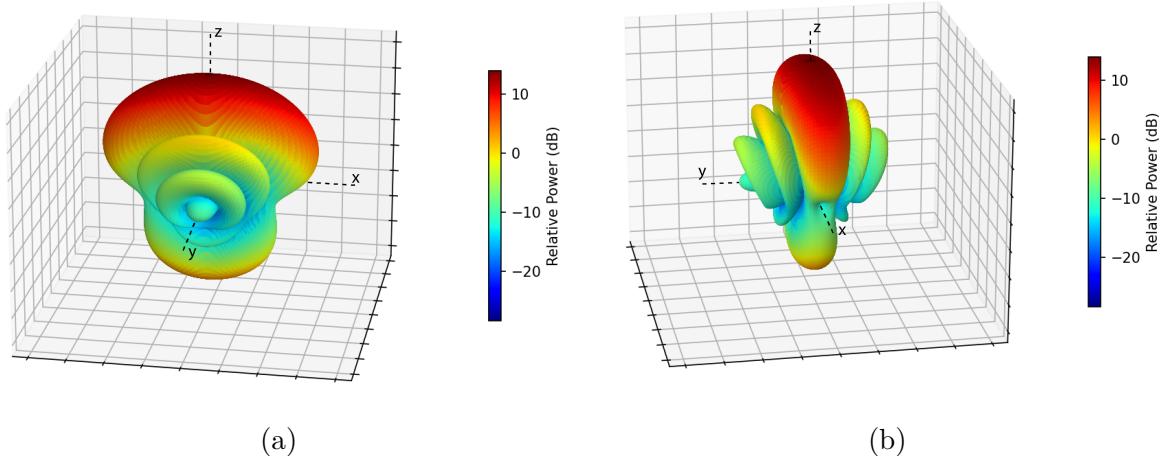


Figure 5.1. An example radiation pattern generated using HFSS simulations. The color and radial distance of the surface from the origin indicate the relative magnitude of radiation power emitted by the antenna in that direction. The primary goal of most antenna measurements is typically to measure the antenna pattern, which is used to derive many useful antenna performance parameters.

3693 Antennas are conductive structures designed to carry alternating electric currents
3694 to transmit energy in the form of EM (electro-magnetic) waves [68]. Perhaps the most
3695 fundamental way to characterize an antenna, is to map out the radiated power density
3696 as a function of position, which is called the radiation pattern (see Figure 5.1). The
3697 radiation power density is obtained by calculating the time-averaged Poynting vector for
3698 all positions surrounding the antenna, which in equation form is

$$\mathbf{W}(x, y, z) = \langle \mathbf{E}(x, y, z, t) \times \mathbf{H}^*(x, y, z, t) \rangle_t, \quad (5.1)$$

3699 where $\mathbf{E}(x, y, z, t)$ and $\mathbf{H}(x, y, z, t)$ are the time-dependent electric and magnetic fields
 3700 produced by the antenna [49]. The radiation power density has units of W/m^2 and is
 3701 more typically called the energy flux density in physics applications, since it is a measure
 3702 of the amount of energy passing through a unit area over time.

3703 Because the radiation power density is a measure of power per unit area, its value
 3704 in a particular direction will depend on the distance from the antenna at which one is
 3705 measuring. This is undesirable for practical applications. A related quantity, which is
 3706 distance independent, is the energy flux per unit solid angle or radiation intensity, which
 3707 is computed directly from the radiation power density by multiplying by the squared
 3708 distance from the antenna. Specifically,

$$U = r^2 W(x, y, z), \quad (5.2)$$

3709 where r is the distance from the antenna to the field measurement point. The radiation
 3710 intensity is typically defined in regions where the Poynting vector consists only of a radial
 3711 component where it is safe to treat as a scalar quantity.

3712 5.2.1.2 Directivity and Gain

3713 Since the radiation intensity is a measure of average power per unit solid angle, it is
 3714 independent of distance and more useful as feature for antenna measurement. The
 3715 radiation intensity is directly related to antenna directivity and gain, which are common
 3716 antenna engineering figures-of-merit. Directivity is defined as the ratio between the
 3717 radiation intensity at particular point on the radiation pattern to the average radiation
 3718 intensity computed over all solid angles [68]. The equation that relates the radiation
 3719 intensity to directivity is

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}}, \quad (5.3)$$

3720 where U_0 is the average radiation intensity over all solid angles, which simply the total
 3721 radiated power (P_{rad}) divided by 4π . Closely related to directivity is antenna gain, which
 3722 accounts for energy losses that occur inside then antenna when attempting to transmit
 3723 or receive a signal. The antenna gain is given by

$$G = \frac{4\pi U}{P_{\text{in}}}, \quad (5.4)$$

3724 where P_{in} is the total power delivered to the antenna. Gain can be thought of as the ratio
 3725 of the antenna's radiation intensity to that of a hypothetical isotropic, lossless radiator.

3726 The maximum values of gain and directivity exhibited by the main lobe of the antenna
 3727 pattern as well as the ratio between the gain of the main lobe and any side-lobes are
 3728 important figures-of-merit to evaluate antenna design performance.

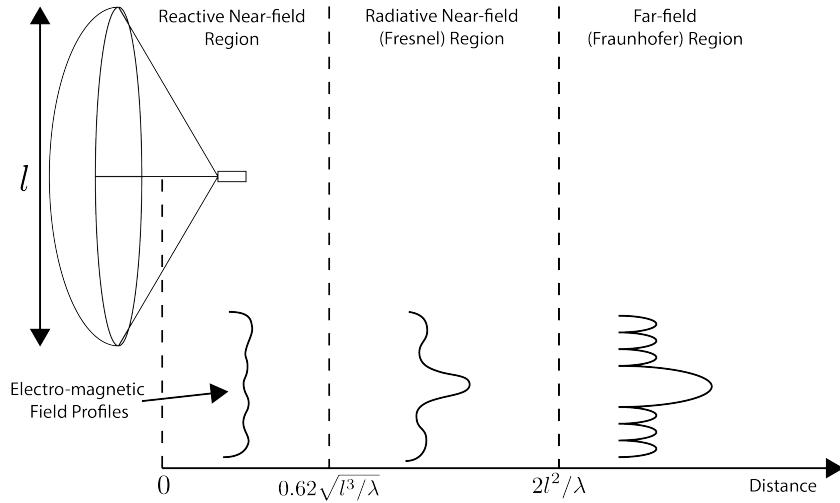


Figure 5.2. An illustration of the three field regions important for the analysis of an antenna system. Very close to the antenna the electric fields are primarily reactive so there is no radiation. If a receiving antenna were placed in this region most of the energy would be reflected back to the transmitter. Outside of the reactive near-field is the radiative near field. At these distances the antenna does radiate, but the radiation pattern is not well-defined since it changes based on the distance of the receiving antenna. It is only in the far-field region where the radiation pattern becomes constant as a function of distance, which is where the majority of antenna engineering is assumed to take place. The antenna arrays developed by Project 8 for CRES measurements operate in the radiative near-field due to the importance of limiting power loss from free-space propagation, which complicates the design of the antenna system.

3729 5.2.1.3 Far-field and Near-field

3730 Radiation patterns are well-defined only in regions where the shape of the radiation
 3731 pattern is independent of distance. The region where this approximation is valid is called
 3732 the "far-field", and in this region the EM fields from the antenna can be approximated as
 3733 spherical plane waves. A rule of thumb for antennas is that the far-field approximation
 3734 applies when the condition

$$R > \frac{2l^2}{\lambda} \quad (5.5)$$

3735 is true. In this expression, R is the distance from the antenna, l is the largest characteristic
 3736 dimension of the antenna, and λ is the wavelength of the radiation (see Figure 5.2).

3737 The region very close to the antenna is called the reactive near-field, because in this
 3738 region the reactive component of the EM field is dominant. Unlike radiative electric

3739 fields, the reactive electric and magnetic fields are out of phase from each other by 90° ,
3740 since they are caused by electrostatic and magnetostatic effects from the self-capacitance
3741 and self-inductance of the antenna. The reactive fields are unable to transfer energy a
3742 significant distance from the antenna and are thus completely negligible for most antenna
3743 applications. The limit of the reactive near-field for an electrically-large antenna is
3744 typically taken to be

$$R < 0.62\sqrt{l^3/\lambda}. \quad (5.6)$$

3745 The unique application of antennas by Project 8 is limited by reactive near-field effects,
3746 since it defines an absolute minimum distance for detectable electrons inside the uniform
3747 cylindrical antenna array. If electrons are too close to the edge of the array than reactive
3748 near-field effects leads to a large reduction in the received power and detection efficiency.
3749 This leads to a significant volume inside the antenna array that is unsuitable for CRES
3750 lowering the volumetric efficiency of the antenna array CRES technique.

3751 Between the reactive near-field and the far-field is the radiative near-field region. In
3752 this region the fields are primarily radiative, however, it is too close to the antenna for
3753 the spherical plane wave approximation to apply. Therefore, interference effects between
3754 EM waves emitted from different points on the antenna occur causing the shape of the
3755 radiation pattern to change as a function of distance from the antenna. Evaluating the
3756 far-field distance limit for the FSCD antennas one finds an estimated far-field distance
3757 of 43 cm, which is a factor of four larger than the radius of the antenna array designed
3758 for the experiment. Consequently, it is expected that near-field effects will influence
3759 the performance of the antenna array highlighting the importance of calibration and
3760 characterization measurements to mitigate these effects.

3761 5.2.1.4 Polarization

3762 The polarization of an EM wave defines the spatial orientation of the electric field
3763 oscillations. Conventionally, polarization vectors \mathbf{a} defined in the plane perpendicular
3764 to the direction of propagation for the EM wave. For radiation moving in the radial (\hat{r})
3765 direction the electric field can be decomposed into the orthogonal basis

$$\mathbf{E}_{\text{tot}} = E_\theta \hat{\theta} + E_\phi \hat{\phi}, \quad (5.7)$$

3766 assuming a spherical coordinate system.

3767 In general, one defines partial radiation patterns, directivities, and gains so that the
3768 performance of the antenna can be analyzed for the desired polarization. The radiation

³⁷⁶⁹ pattern defined in terms of partial patterns is

$$U_{\text{tot}} = U_\phi + U_\theta, \quad (5.8)$$

³⁷⁷⁰ where U_ϕ and U_θ are the radiation intensities in a particular direction for the respective
³⁷⁷¹ polarization components. Similarly, a quantity such as gain can be written in terms of
³⁷⁷² partial gains,

$$G_{\text{tot}} = G_\phi + G_\theta = \frac{2\pi U_\phi}{P_{\text{in}}} + \frac{2\pi U_\theta}{P_{\text{in}}}. \quad (5.9)$$

³⁷⁷³ An electron performing a circular orbit in the XY-plane from the side, viewed along
³⁷⁷⁴ the X or Y axes, would be seen as performing a linear oscillation perpendicular to the
³⁷⁷⁵ viewing axis. From this picture, one would predict that the primary polarization of
³⁷⁷⁶ electric fields from CRES events is linearly polarization in the $\hat{\phi}$ direction in the XY-plane.

³⁷⁷⁷ 5.2.1.5 Antenna Factor and Effective Aperture

³⁷⁷⁸ A useful way to characterize the performance of an antenna is to measure the electric
³⁷⁷⁹ field magnitude required to produce a signal with an amplitude of one volt in the antenna
³⁷⁸⁰ terminals. This ratio between the magnitude of the incoming electric field and the
³⁷⁸¹ magnitude of the signal produced by the antenna is called the antenna factor, which is
³⁷⁸² written as

$$A_F = \frac{|\mathbf{E}_{\text{in}}|}{V_{\text{ant}}}, \quad (5.10)$$

³⁷⁸³ where A_F is the antenna factor, E_{in} is the incoming electric field, and V_{ant} is the magnitude
³⁷⁸⁴ of the voltage produced by the antenna.

³⁷⁸⁵ The antenna factor can be expressed in terms of the antenna's gain through a related
³⁷⁸⁶ quantity called the effective aperture. The effective aperture defines for a given incident
³⁷⁸⁷ radiation power density (W/m^2) the power that is received by the antenna. Therefore,
³⁷⁸⁸ the effective aperture gives the equivalent area of the antenna,

$$A_{\text{eff}} = \frac{P_{\text{rec}}}{P_{\text{in}}} = \frac{\lambda^2}{4\pi} G, \quad (5.11)$$

³⁷⁸⁹ where the received power is P_r and the total incoming power is P_{in} .

³⁷⁹⁰ The magnitude of the Poynting vector can be written as

$$|\mathbf{S}_{\text{in}}| = |\mathbf{E}_{\text{in}}|^2 / \eta_0, \quad (5.12)$$

3791 where η_0 is the impedance of free-space, which relates the magnitudes of the electric and
3792 magnetic fields in a vacuum, and is defined by

$$\eta_0 = \frac{|\mathbf{E}|}{|\mathbf{H}|} = \sqrt{\frac{\epsilon_0}{\mu_0}}. \quad (5.13)$$

3793 Therefore, the total received power by the antenna is

$$P_{\text{rec}} = |\mathbf{S}_{\text{in}}| A_{\text{eff}} = |\mathbf{S}_{\text{in}}| \frac{\lambda^2}{4\pi} G = \frac{|\mathbf{E}_{\text{in}}|^2 \lambda^2 G}{4\pi \eta_0}. \quad (5.14)$$

3794 To relate this to the antenna factor recall that the voltage produced by the antenna
3795 is related to the received power by

$$P_{\text{rec}} = \frac{V_{\text{ant}}^2}{Z} = \frac{|\mathbf{E}_{\text{in}}|^2}{A_{\text{F}}^2 Z}, \quad (5.15)$$

3796 where Z is the system impedance. Setting Equations 5.14 and 5.15 equal to each other,
3797 one obtains the following expression for antenna factor in terms of gain

$$A_{\text{F}} = \sqrt{\frac{4\pi\eta_0}{ZG\lambda^2}} = \frac{9.73}{\lambda\sqrt{G}}. \quad (5.16)$$

3798 The second expression in Equation 5.16 is obtained by evaluating the constant terms
3799 assuming a system impedance of 50Ω .

3800 This exercise highlights that the majority of antenna parameters that one cares
3801 to measure about an antenna can be obtained from the radiation or gain pattern of
3802 the antenna. The antenna factor is a particularly important parameter for CRES
3803 measurements due to it's relevance to antenna array simulations with the Locust software
3804 [65, 77].

3805 To compute the response of the antenna to the electric field, Locust relies upon
3806 linear time-invariant system theory, which computes the response of the antenna (i.e. the
3807 voltage time series generated by the antenna) using a convolution between the electric field
3808 time-series and the antenna impulse response. This approach is necessary for correctly
3809 modeling the antenna response to the electric field due to the broadband and non-
3810 stationary nature of the electric fields from CRES events. Since antenna measurements
3811 take place under steady-state conditions, parameters such as the radiation pattern, gain,
3812 and antenna factor are defined in the frequency domain. However, by performing an
3813 inverse Fourier transform on the antenna factor one obtains the antenna impulse response,

3814 which is used to calculate CRES signal voltages in Locust.

3815 5.2.2 Antenna Measurement Fundamentals

3816 5.2.2.1 Friis Transmission Equation

3817 The antenna factor or antenna transfer function is used to model how the antenna
3818 responds to electric fields emitted from a CRES event. Therefore, directly measuring the
3819 antenna transfer functions of the array is a key step in the commissioning and calibrating
3820 the FSCD experiment. A common approach to antenna characterization is to perform a
3821 two antenna transmit-receive measurement where an antenna with a known gain is used
to characterize the unknown gain of the antenna under test (see Figure 5.3).

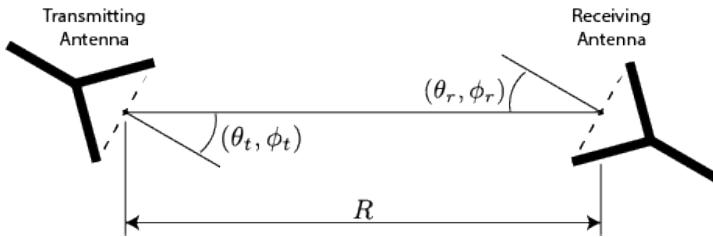


Figure 5.3. An illustration of the Friis measurement technique commonly used for antenna characterization measurements.

3822
3823 Analyzing this two antenna setup involves calculating the power received from the
3824 transmitting antenna. The received power density is expressed as a function of the
3825 antenna gain in a direction (θ_t, ϕ_t) at frequency f and distance R

$$w_t = \frac{P_t}{4\pi R^2} G_t(\theta_t, \phi_t, f), \quad (5.17)$$

3826 where the subscript t denotes the transmitting antenna, and P_t is the total power delivered
3827 to the transmitting antenna. The power density is power per unit area, so the total
3828 power delivered to the receiving antenna is the transmitted power density multiplied by
3829 the effective area of the receiving antenna

$$P_r = w_t A_{\text{eff},r} = P_t \frac{G_t(\theta_t, \phi_t, f) G_r(\theta_r, \phi_r, f) c^2}{(4\pi R f)^2}, \quad (5.18)$$

3830 where $G_r(\theta_r, \phi_r, f)$ is the gain of the receiving antenna. Equation 5.18 is called the Friis
3831 transmission equation [91], which is of fundamental importance for antenna measurements,
3832 since it allows one to measure the gain of an unknown antenna by measuring the power

3833 received from an antenna with a known gain pattern. Alternatively, if an antenna with a
 3834 known gain pattern is unavailable, two identical antennas with unknown gain patterns
 3835 can be used.

3836 **5.2.2.2 S-Parameters and Network Analyzers**

3837 It is more common to measure the ratio of the received power to the transmitted power
 3838 instead of the absolute received power

$$\frac{P_r}{P_t} = \frac{G_t(\theta_t, \phi_t, f) G_r(\theta_r, \phi_r, f) c^2}{(4\pi R f)^2}. \quad (5.19)$$

3839 This power ratio can be easily measured using a vector network analyzer (VNA), which
 3840 automates a significant fraction of the measurement process. Network analyzers are used
 3841 to measure the scattering or S-parameters of a multi-port RF device [92], which describes
 3842 how waves are scattered between the device ports. Friis antenna measurements can be
 3843 modeled as a two-port microwave device that is characterized by measuring how incident
 voltage waves are transmitted or reflected (see Figure 5.4). The scattered waves (V_1^-

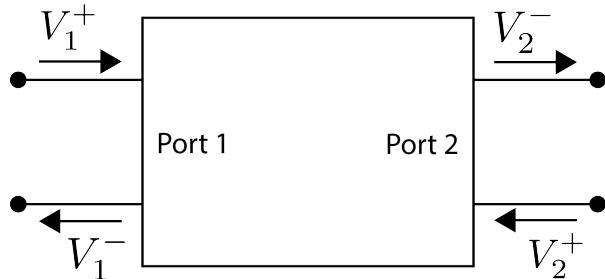


Figure 5.4. Illustration of a two-port S-parameter measurement setup. S-parameters characterize how incoming waves of voltage or power scatter off of the RF device under test. This allows you to measure important properties of the device. In particular, this framework can be used to model a two antenna radiation pattern measurement, which can be automated using a VNA.

3844
 3845 and V_2^-) can be written in terms of the incident (V_1^+ and V_2^+) waves using the scattering
 3846 matrix

$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}, \quad (5.20)$$

3847 where the elements of the matrix are the device S-parameters. It is assumed that,
 3848 when exciting the device from a particular port, that all other ports in the network are
 3849 terminated at the system impedance. This ensures that the incident waves from other
 3850 ports in the network are zero. Therefore, the S-parameters are the ratios between the

3851 scattered and incident waves,

$$S_{ij} = \frac{V_i^-}{V_j^+}. \quad (5.21)$$

3852 Alternatively, S-parameters can be defined as the ratio of the scattered and incident
3853 power, which is proportional to the ratio of the squared voltage waves.

3854 Returning to the antenna measurement setup, it is clear that measuring the ratio of
3855 the received to the transmitted power is equivalent to measuring the ratio of power being
3856 scattered from port 1 to port 2 in a RF (radio-frequency) network. Therefore, measuring
3857 an antenna's gain can be accomplished quite easily using a VNA to perform a two port
3858 S_{21} measurement.

3859 **5.2.2.3 Antenna Array Commissioning and Calibration Measurements**

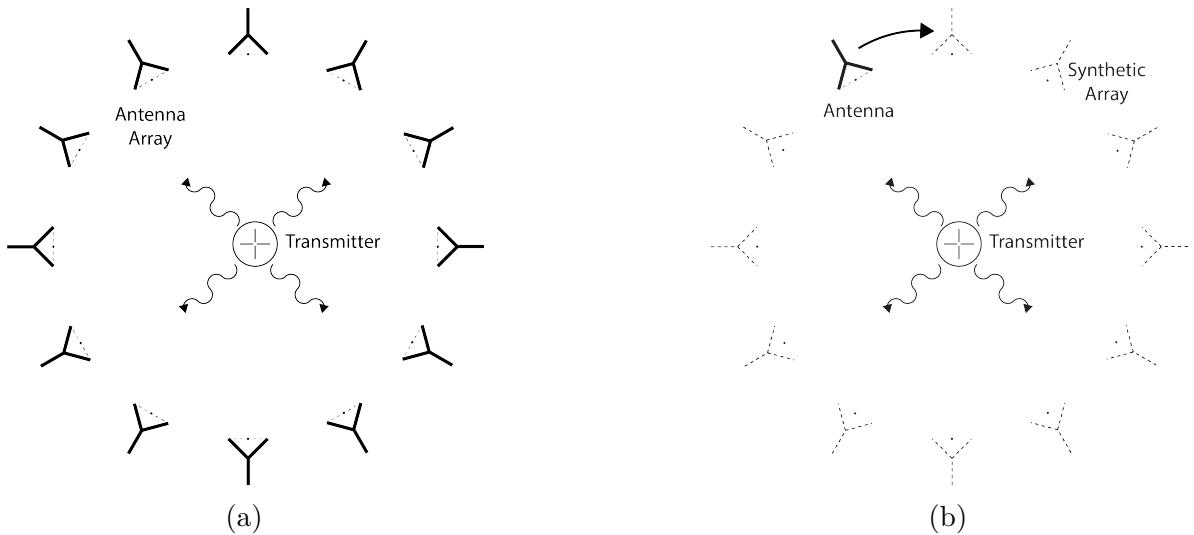


Figure 5.5. Two measurement approaches to characterizing an antenna array for CRES measurements. The full-array approach (a) requires a complete antenna array with all the associated hardware. The synthetic array approach (b) utilizes a single antenna and a set of rotation/translation stages to reposition the transmitter or the receiving antenna to synthesize the signals that would be received by the full-array. This approach reduces the cost and complexity of array measurements. A down-side of the synthetic array approach is that multi-channel effects such as reflections cannot be measured. Utilizing both the full-array and the synthetic array is a powerful way to quantify the impact of errors from the multi-channel array.

3860 Measuring the gain of each individual array element allows to predict the features of
3861 the signals received during a CRES event (see Section 5.2.1.5). However, unpredictable
3862 changes to the antenna performance can be introduced by the incorporation of the

3863 antennas into the circular array geometry, therefore, both individual antenna and full-
3864 array characterization measurements are performed as part of the commissioning of the
3865 FSCD.

3866 There are two main approaches to array measurements that could be used for
3867 characterization and calibration (see Figure 5.5). One approach is to construct the
3868 complete array and use a omni-directional transmitting antenna to measure the power
3869 received by each channel in the antenna array. In Section 5.3 I describe the development
3870 of an omni-directional transmitter that also mimics the radiation phase characteristics of a
3871 CRES event, which is useful because the entire array can be tested without repositioning.
3872 Alternatively, a full antenna array can be synthesized by repeatedly moving and measuring
3873 a single array element. This approach is ideal for identifying if different channels in the
3874 antenna array are affecting each other through multi-path interference by comparing the
3875 measurement results of the synthetic array to the real array.

3876 **5.2.3 The Penn State Antenna Measurement System**

3877 The development of antenna array based CRES requires the capability to test and
3878 calibrate different antenna array designs to validate the performance of the as-built
3879 antenna array before and during the experiment. With these aims in mind an antenna
3880 measurement system was developed at Penn State specifically designed to mimic the
3881 characteristics of the FSCD experiment.

3882 The Penn State antenna measurement system utilizes a two antenna measurement
3883 configuration with a stationary reference antenna and a test antenna mounted on a set
3884 of motorized translation and rotation stages (see Figure 5.6). The antenna measurement
3885 system can be operated in two distinct modes, one focused on the characterization of
3886 the radiation patterns of prototype antennas, and the other focused on the validation of
3887 data-acquisition (DAQ) and CRES signal reconstruction techniques to bridge the gap
3888 between real measurements and simulation. In both measurement configurations, it is
3889 critical to isolate the antennas from the environment so that multi-path reflections do not
3890 negatively influence the measurement results. For this reason the measurement volume
3891 is surrounded with microwave absorber foam (AEMI AEC-1.5) specifically designed to
3892 attenuate microwave radiation near the 26 GHz measurement range of the system.

3893 In the first measurement configuration, the reference antenna is a well-characterized
3894 horn antenna as pictured, since horn antennas have well-known and stable radiation
3895 patterns making them ideal as standard references. For characterization measurements,
3896 the test antenna represents the antenna-under-test whose pattern is being characterized.

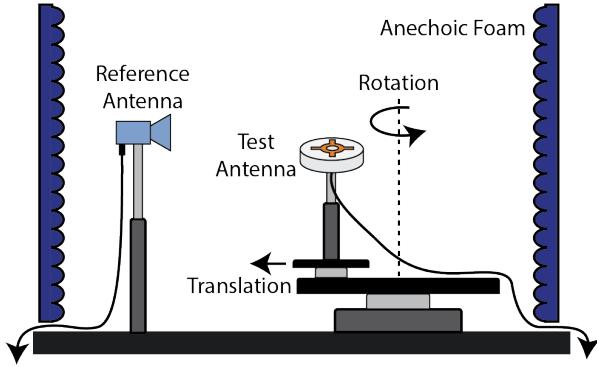


Figure 5.6. Illustration of the antenna measurement system developed for the Project 8 Collaboration. The reference and test antennas can be connected to different data acquisition configurations depending on the measurement goals. The reference antenna is typically a standard horn antenna and the test antenna is mounted on a set of translation stages for positioning. Automated translation stages allows for relatively painless data-taking enabling synthetic antenna array measurements using only a single receiving antenna. Anechoic form designed to mitigate RF reflections surrounds the setup.

3897 Mounting the test antenna on motorized rotation and translation stages allows for
 3898 automation, which significantly speeds up the radiation pattern measurement process.

3899 The second measurement configuration mimics the conditions of the FSCD as it
 3900 concerns the antenna array and DAQ system. In this configuration, the reference antenna
 3901 is a prototype FSCD antenna, and the test antenna is a specially designed synthetic
 3902 cyclotron antenna (SYNCA) as picture in Figure 5.6. The SYNCA is designed such that
 3903 the radiation pattern mimics that of a CRES electron so that the signals received by the
 3904 prototype CRES array antenna mimic what is expected for a real CRES experiment.

3905 Figure 5.7 shows two high-level system diagrams of the Penn State antenna measure-
 3906 ment system that depict the important system components and the connections between
 3907 them. The two configurations of the measurement system utilize different hardware. For
 3908 characterization and radiation pattern measurements, the configuration shown in Figure
 3909 5.7a is used. In this case a vector network analyzer (VNA) is used as the transmission
 3910 source and data acquisition system, which is easy to calibrate over a wide range of
 3911 frequencies. The configuration in 5.7b is used to mimic the FSCD experiment, since this
 3912 system includes a more realistic receiver chain.

3913 The characterization configuration utilizes a network analyzer (Keysight N5222A)
 3914 with two independent sources and four measurement ports as the primary measurement
 3915 tool. A standard reference antenna is connected to one measurement port, and the test
 3916 antenna is connected to a second port. The typical reference antenna used for these
 3917 studies is a Pasternack PF9851 horn antenna. In the measurement shown, the test

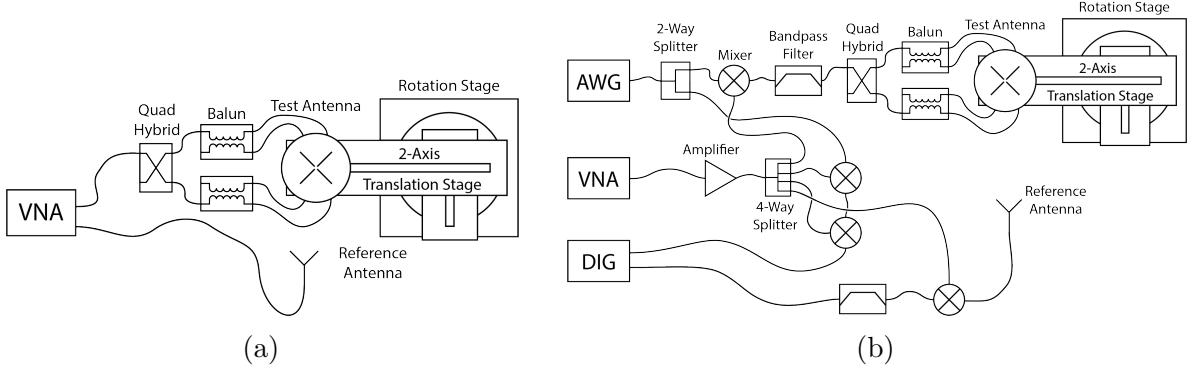


Figure 5.7. Diagrams of two measurement system configurations. Configuration (a) utilizes a VNA and is more suited to antenna characterization. Configuration (b) utilizes an AWG (arbitrary waveform generator) and VNA as a signal generation system and digitizer to collect measurement data, which is more suited to simulating CRES measurements. The transmission chain utilizes a quadrature hybrid and a pair of baluns to drive the cross-dipole variant test antenna developed for synthetic CRES measurements.

3918 antenna represents a SYNCA antenna, which requires a transmission chain consisting of
 3919 quadrature hybrid coupler (Marki QH-0226) connected to two baluns (Marki BAL-0026)
 3920 to generate feed signals with the appropriate phases. The VNA measures the radiation
 3921 pattern by performing a transmission S-parameter measurement, which can be used with
 3922 the knowledge of the reference antenna's radiation pattern to determine the radiation
 3923 pattern of the test antenna (see Section 5.2.1).

3924 The second configuration incorporates more hardware components to mimic the DAQ
 3925 system envisioned for the FSCD experiment. The basic approach is to produce CRES-like
 3926 radiation and use an antenna combined with a realistic RF receiver chain to acquire the
 3927 signals. On the transmit side, an arbitrary waveform generator (AWG, RIGOL DG5252)
 3928 is used to generate a waveform that mimics a CRES signal at a baseband frequency up
 3929 to 250 MHz. This frequency is then up-converted to the CRES signal frequency band
 3930 of 25.8 to 26.0 GHz using a mixer (Marki MM1-0832L) and a bandpass filter (K&L
 3931 Microwave 3C62-25900/T200-K/K) to reject unwanted mixing components outside out
 3932 of the 200 MHz CRES signal band. The local oscillator signal for mixing is provided by
 3933 one of the VNA sources configured to run in a continuous wave setting. On the receive
 3934 side, a prototype antenna is used to detect the radiation emitted by the test antenna,
 3935 which is down-converted and filtered using the same mixer and bandpass filter as the
 3936 transmission chain. Lastly, data acquisition is performed using a 14-bit ADC sampling
 3937 at 500 MSa/s (CAEN DT530) to digitize the down-converted signals.

3938 In order to distribute the LO to all mixers a 4-way power splitter (MiniCircuits

3939 ZC4PD-18263-S+) along with an amplifier (Marki APM-6848) is used to drive the four
3940 mixers used in the measurement system. A limitation of using the VNA as an LO source
3941 is that there is no control of the LO phase when a measurement is triggered by the
3942 control script, which leads to a random phase offset between acquisitions. This makes it
3943 impossible to perform synthetic array measurements, which require strict control over
3944 the starting phase of the transmitted signal. In order to monitor the random phase of the
3945 LO, a 2-way power splitter (MiniCircuits Z99SC-62-S+) is used to split the signal from
3946 the AWG between the transmission path and a LO monitoring path. The LO monitoring
3947 path consists of an up-conversion and down conversion using two mixers connected by a
3948 coaxial cable, and monitors the relative phase of the LO using a channel on the digitizer
3949 to sample this path. A phase shift in the LO will lead to a proportional phase shift in
3950 the mixed signal, which is measured and removed from the received signals.

3951 The test antenna is mounted on a set of motorized stages, which are identical for
3952 both measurement configurations. A rotational stage (ThorLabs PRMTZ8) is used as
3953 the base layer with additional translation stages mounted on top. The rotational stage is
3954 ideal for measuring a complete azimuthal scan of the test antenna's radiation pattern
3955 as well as for moving a SYNCA antenna in circular motion to recreate the symmetry
3956 of the FSCD antenna array. On top of the rotational stage, are mounted two linear
3957 translation stages (ThorLabs MTS50-Z8 and MTS25-Z8) in a cross-wise manner so that
3958 the test antenna can be moved along two perpendicular axes. Using the linear stages in
3959 combination with the rotational stage allows one to fine-tune the positioning of the test
3960 antenna so that it can be perfectly aligned with the central axis of the array. A LabView
3961 script was developed to automate the measurement of a full 360° radiation pattern and
3962 control the measurement electronics. Data from these acquisitions is stored on university
3963 provided cloud storage.

3964 **5.3 Development of a Synthetic Cyclotron Antenna (SYNCA)** 3965 **for Antenna Array Calibration**

3966 This section is the manuscript of the publication [78] detailing the development of a
3967 Synthetic Cyclotron Antenna (SYNCA) for antenna array characterization measurements
3968 by the Project 8 collaboration.

3969 5.3.1 Introduction

3970 Neutrinos are the most abundant standard model fermions in our universe, but due to
3971 weak interaction cross-sections with other particles, neutrinos are particularly difficult
3972 to study. Consequently, many fundamental properties of neutrinos are still unknown
3973 including the absolute scale of the neutrino mass [24]. Direct, kinematic measurements of
3974 the neutrino mass are particularly valuable due to their model independent nature [37].
3975 To date the most sensitive direct neutrino mass measurements have been performed by
3976 the KATRIN collaboration [93], which measures the molecular tritium β -decay spectrum
3977 to infer the neutrino mass. Current data from neutrino oscillation measurements [24]
3978 allow for neutrino masses significantly smaller than the design sensitivity of the KATRIN
3979 experiment; therefore, there is a need for new technologies for performing direct neutrino
3980 mass measurements to probe lower neutrino masses.

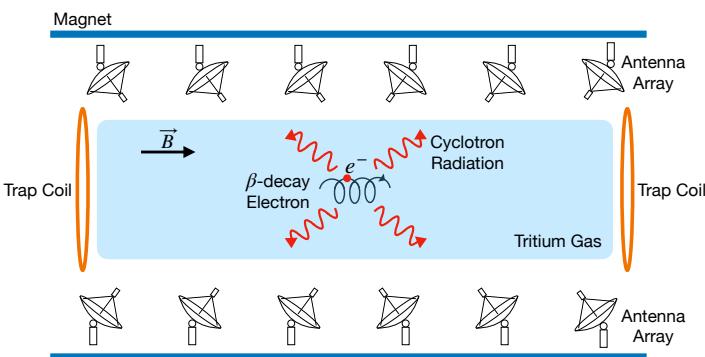


Figure 5.8. A sketch of an antenna array large-volume CRES experiment. Electrons from β -decays are confined in a magnetic field using a set of trap coils. The cyclotron radiation produced by the motion of the trapped electrons can be detected by a surrounding antenna array to determine the electron energies. Measuring the energies of many electrons produces a β -decay spectrum.

3981 The Project 8 collaboration is developing new methods for neutrino mass measurement
3982 based on Cyclotron Radiation Emission Spectroscopy (CRES) [56, 94–96], with the goal
3983 of measuring the absolute scale of the neutrino mass with a 40 meV/c² sensitivity [37, 73].
3984 This sensitivity goal will require the development of two separate technical capabilities.
3985 First is the development of an atomic tritium source, which avoids significant spectral
3986 broadening due to molecular final states [55]. Second is the technology for performing
3987 CRES in a multi-cubic-meter experimental volume with high combined detection and
3988 reconstruction efficiency, which is required in order to obtain sufficient event statistics
3989 near the tritium spectrum endpoint.

One approach for a large-volume CRES experiment is to use an array of antennas, which surrounds a volume of tritium gas, to detect the cyclotron radiation produced by the β -decay electrons when they are trapped in a background magnetic field using a set of magnetic trapping coils (see Figure 5.8). Project 8 has developed a conceptual experiment design to study the feasibility of this approach. The design consists of a single circular array of antennas with a radius of 10 cm and 60 independent channels positioned around the center of the magnetic trap. The motivation behind this antenna array design is to first develop an understanding of the antenna array approach to CRES with a small scale experiment before attempting to scale the technique to large volumes by using multiple antenna rings to construct the full cylindrical array. The development of the antenna array approach to CRES has largely proceeded through simulations using the Locust software package [77, 97], which is used to model the fields emitted by CRES events and predict the signals received by the surrounding antenna array. To validate these simulations, a dedicated test stand is being constructed to perform characterization measurements of the prototype antenna array developed by Project 8 (see Figure 5.9) and benchmark signal reconstruction methods using a specially designed transmitting calibration probe antenna.

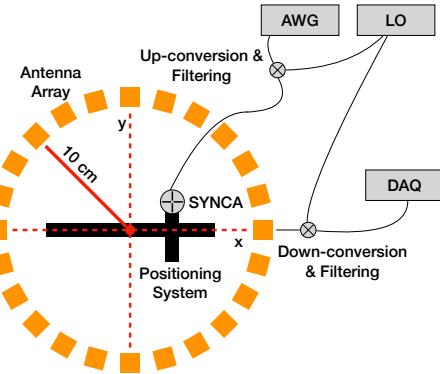


Figure 5.9. A schematic of the antenna array test stand. The circular antenna array has a radius of 10 cm with 60 independent channels (limited number shown for clarity). The test stand includes an arbitrary waveform generator (AWG), local oscillator (LO), and data acquisition (DAQ) hardware. Finally, a specialized Synthetic Cyclotron Antenna (SYNCA) is used to inject signals to test the antenna array.

We call this probe antenna the Synthetic Cyclotron Antenna or SYNCA. The SYNCA is a novel antenna design that mimics the cyclotron radiation generated by individual charged particles trapped in a magnetic field, which will be used in the antenna test stand to perform characterization measurements, simulation validation, and reconstruction benchmarking. This paper provides an overview of the design, construction, and

characterization measurements of the SYNCA performed in preparation for its usage as a transmitting calibration probe.

In Section 5.3.2 we provide a description of the cyclotron radiation field characteristics that we recreate with the SYNCA. In Section 5.3.3 we give an overview of the simulations performed to develop an antenna design that mimics the characteristics of cyclotron radiation. In Section 5.3.4 we outline characterization measurements to validate that the fields generated by the SYNCA match simulation, and finally in Section 5.3.5 we demonstrate an application of the SYNCA to test phased array reconstruction techniques on the bench-top.

5.3.2 Cyclotron Radiation Phenomenology

To understand the cyclotron radiation phenomenology that the SYNCA should mimic, we consider a charged particle moving at relativistic speed in the presence of an external magnetic field (see Figure 5.10). In the special case we shall examine, the entirety of the electron's momentum is directed perpendicular to the magnetic field; therefore, the trajectory of the electron is confined to the cyclotron orbit plane. Because the momentum vector is oriented perpendicular to the magnetic field, electrons with these trajectories are said to have pitch angles of 90°.

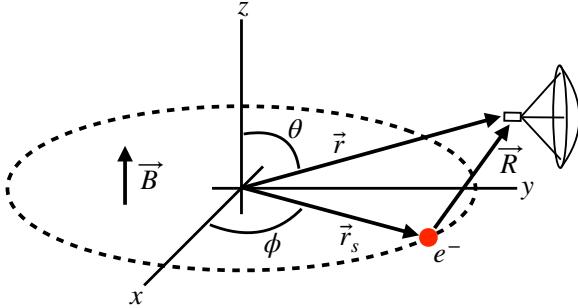


Figure 5.10. An electron (red dot) performing cyclotron motion in the x-y plane. The resulting cyclotron radiation is observed by an antenna located at the field point of interest.

The cyclotron radiation fields generated by this circular trajectory are those which we aim to reproduce with the SYNCA. We can describe the electromagnetic (EM) fields using the Liénard-Wiechert equations [49, 77], which in non-covariant form express the electric field as

$$\vec{E} = e \left[\frac{\hat{n} - \vec{\beta}}{\gamma^2(1 - \vec{\beta} \cdot \hat{n})^3 |\vec{R}|^2} \right]_{tr} + \frac{e}{c} \left[\frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \vec{\beta} \cdot \hat{n})^3 |\vec{R}|} \right]_{tr}, \quad (5.22)$$

4033 where e is the particle's charge, $\hat{n} = (\vec{r} - \vec{r}_s)/|\vec{r} - \vec{r}_s|$ is the unit vector pointing from the
 4034 electron to the field measurement point, $\vec{\beta} = \dot{\vec{r}}_s/c$ is the velocity of the particle divided
 4035 by the speed of light, and γ is the relativistic Lorentz factor. The equation is meant to
 4036 be evaluated at the retarded time as indicated by $t_r = t - |\vec{R}|/c$, which accounts for the
 4037 time delay due to the finite speed of light between the point where the field was emitted
 4038 and the point where the field is detected.

4039 We would like to simplify Equation 5.22 it at all possible. As a first step we analyze
 4040 the relative magnitudes of the electric field polarization components. Consider an electron
 4041 following a circular cyclotron orbit in a uniform magnetic field whose guiding center
 4042 is positioned at the origin of the coordinate system. The equation of motion can be
 4043 expressed as

$$\vec{r}_s = (r_c \cos \omega_c t_r) \hat{x} + (r_c \sin \omega_c t_r) \hat{y}. \quad (5.23)$$

4044 For single antenna located along the y -axis at position $\vec{r} = r_a \hat{y}$ we are interested in the
 4045 incident electric fields from the electron. The electric field is given by Equation 5.22,
 4046 which we evaluate in the regime where $r_a \gg r_c$. This limit can be justified by comparing
 4047 the radius of the cyclotron orbit for an electron with the tritium beta-spectrum endpoint
 4048 energy of 18.6 keV in a 1 T magnetic field to the typical ($r_a \simeq 100$ mm) radial position
 4049 of the receiving antenna. We find that the cyclotron orbit has a radius of 0.46 mm which
 4050 is approximately a factor of 200 smaller than the typical antenna radial position. In this
 4051 regime we can make the approximation $\vec{R} \simeq r_a \hat{y}$ and the expression for the electric field
 4052 at the antenna's position becomes

$$\vec{E} = \frac{e}{\gamma^2 r_a^2} \frac{\hat{x} \left(\frac{r_c \omega_c}{c} \sin \omega_c t_r \right) + \hat{y} \left(1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r \right)}{(1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r)^3} - \frac{e}{cr_a} \frac{\hat{x} \left(\frac{r_c^2 \omega_c^3}{c^2} - \frac{r_c \omega_c^2}{c} \cos \omega_c t_r \right)}{(1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r)^3}. \quad (5.24)$$

4053 Since the receiving antenna is part of a circular array of antennas, it is useful to rewrite
 4054 Equation 5.24 in terms of the azimuthal ($\hat{\phi}$) and radial (\hat{r}) polarizations. Making use of
 4055 the fact that for an antenna located at $R = r_a \hat{y}$ that $\hat{\phi} = -\hat{x}$ and $\hat{r} = \hat{y}$ we find

$$\vec{E} = \hat{\phi} E_\phi + \hat{r} E_r \quad (5.25)$$

$$E_\phi = \frac{e}{(1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r)^3} \left[-\frac{\frac{r_c \omega_c}{c} \sin \omega_c t_r}{\gamma^2 r_a^2} + \frac{\omega_c \left(\frac{r_c^2 \omega_c^2}{c^2} - \frac{r_c \omega_c}{c} \cos \omega_c t_r \right)}{cr_a} \right] \quad (5.26)$$

$$E_r = \frac{e \left(1 - \frac{r_c \omega_c}{c} \sin \omega_c t_r \right)}{\gamma^2 r_a^2 (1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r)^3}. \quad (5.27)$$

For the purposes of designing a synthetic cyclotron radiation antenna we are interested in the dominant electric field polarization emitted by the electron. The antenna is being designed to mimic the cyclotron radiation produced by electrons with kinetic energies of approximately 18.6 keV in a 1 T magnetic field [55]. Since the relativistic beta factor for an electron with this kinetic energy is $|\vec{\beta}| \simeq \frac{1}{4}$, the approximations $\gamma \simeq 1$ and $\frac{r_c \omega_c}{c} \simeq \frac{1}{4}$ are justified. Inserting these expressions into the equations for the electric field components above simplifies the comparison of the magnitudes of the two components. Additionally, we compare the time-averaged magnitudes to evaluate the root mean squared electric field ratio. The time-averaged ratio of the radial and azimuthally polarized electric fields with the above simplifications is given by

$$\frac{\langle |E_r| \rangle}{\langle |E_\phi| \rangle} = \frac{8 - \sqrt{2}}{\left| 1 - \frac{r_a}{r_c} \frac{1-2\sqrt{2}}{8} \right|} \simeq \frac{r_c}{r_a} \frac{8(8 - \sqrt{2})}{2\sqrt{2} - 1} = 0.13, \quad (5.28)$$

where we have made use of the fact that for these magnetic fields and kinetic energies the cyclotron radius is much smaller than the radius of the antenna array.

From Equation 5.28 we see that the time-averaged azimuthal polarization is larger than the radial polarization by about a factor of 8, which makes it the dominant contribution to the electric fields at the position of the antenna. We must also consider the directivity of the receiving antenna which can have a gain that is disproportionately large for a specific polarization component. Because the E_ϕ component is dominant, the receiving antenna array is designed with an azimuthal polarization, which negates the voltages induced in the antenna from the radially polarized fields. Therefore, we conclude that for the purpose of designing the SYNCA antenna it is acceptable to approximate the electric fields from Equation 5.22 as purely azimuthally or ϕ -polarized. The simplified expression for the electric field received by an antenna becomes

$$\vec{E} = E_\phi \hat{\phi} = \frac{e^{\frac{r_c \omega_c}{c}}}{4r_a r_c} \left[\frac{\frac{r_c \omega_c}{c} - \cos \omega_c t - \frac{4r_c}{r_a} \sin \omega_c t}{(1 - \frac{r_c \omega_c}{c} \cos \omega_c t)^3} \right]_{t_r} \hat{\phi}, \quad (5.29)$$

where the radius of the cyclotron orbit is called r_c , the cyclotron frequency is called ω_c , and the radial position of the receiving antenna is called r_a . Equation 5.29 has been evaluated in the non-relativistic limit where $\gamma \simeq 1$, which is justified by the fact that $|\vec{\beta}| \simeq \frac{c}{4}$ for an electron with an 18.6 keV kinetic energy in a 1 T magnetic field.

This rather complicated expression can be simplified using Fourier analysis. Assuming a background magnetic field of 1 T and a kinetic energy of 18.6 keV we calculate

4084 numerically the electric field using Equation 5.29 and apply a discrete Fourier Transform
to visualize the frequency spectrum (see Figure 5.11).

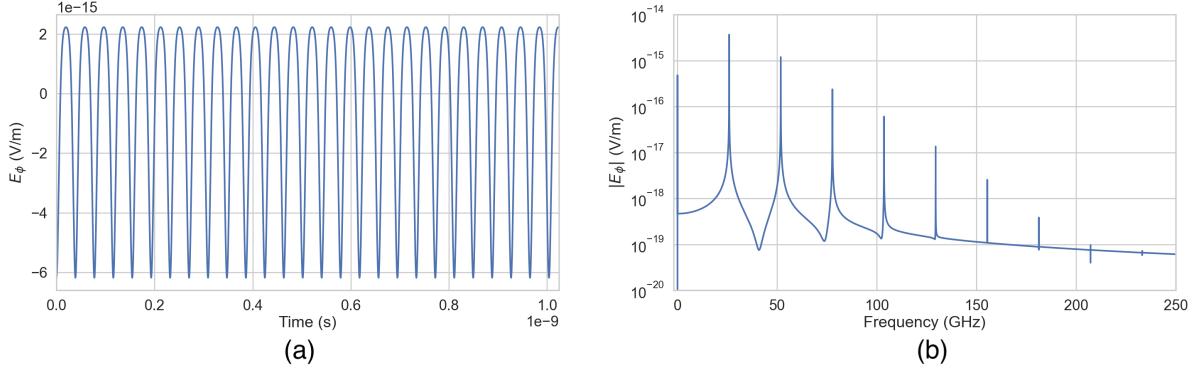


Figure 5.11. A plot of the numeric solution to Equation 5.30. The time-domain representation of the signal (a) is composed of a zero frequency term and a series of harmonics separated by the main cyclotron frequency as shown in the plot of the frequency spectrum (b). We can see that the relative amplitude of the harmonics beyond $k = 7$ are smaller than the main carrier by a factor of about 10^{-5} and are completely negligible.

4085
4086 We observe that the azimuthally polarized electric field is periodic with a base cyclotron
4087 frequency of 25.898 GHz corresponding to the highest power frequency component in
4088 Figure 5.11. The frequency spectrum reveals that the signal is composed of a constant
4089 term with zero frequency and a series of harmonics separated by 25.898 GHz. Therefore,
4090 we can represent the azimuthal electric fields from the electron as a linear combination
4091 of pure sinusoids with frequencies given by $\omega_k = k\omega_c$ ($k \in 0, 1, 2, \dots$) and amplitudes
4092 extracted from the Fourier representation. Using this representation we can transform
4093 the equation for the azimuthally polarized electric fields in Equation 5.29 into

$$E_\phi = \frac{e^{\frac{r_c \omega_c}{c}}}{4r_a r_c} \sum_{k=0}^7 A_k e^{i\omega_k t_r}, \quad (5.30)$$

4094 where we have truncated the sum over harmonics at the 7th order for completeness. The
4095 amplitudes A_k are dimensionless complex numbers, which encode the relative powers of
4096 the harmonics as well as the starting overall phase of the cyclotron radiation. Because
4097 magnitude of the relative amplitudes exponentially decreases for higher harmonics, it is
4098 usually sufficient to consider only the terms up to $k = 4$ where the relative amplitude
4099 of the harmonics has decreased from the main carrier by a factor of approximately 100.
4100 However, for completeness we include harmonics up to 7th order in Equation 5.30. The
4101 range of frequencies to which the receiving antenna array in the antenna test stand is
4102 sensitive is defined by the antenna's transfer function. The receptive bandwidth for

4103 the antennas used in the test stand is a range of frequencies with a bandwidth on the
 4104 order of a few GHz centered around the main cyclotron carrier frequency of 25.898 GHz.
 4105 Therefore, the higher order harmonics as well as the zero frequency term can be ignored
 4106 when considering only the signals that will be received by the antenna array.

4107 Considering only the 1st order harmonic term from Equation 5.30, which represents
 4108 the portion of the electric field that will be detected by the array, and evaluating this at
 4109 the retarded time we obtain the following for the ϕ -polarized electric fields

$$E_\phi \propto \cos \left(\omega_c \left(t - |\vec{R}|/c \right) - \Delta \right), \quad (5.31)$$

4110 where the arbitrary phase Δ is defined by $A_k = |A_k|e^{i\Delta}$. We are interested in the
 4111 characteristics of the amplitude of the electric field as a function of the radial distance
 4112 component ($|\vec{R}|$) of the retarded time. In particular, the maximum of E_ϕ occurs when
 4113 the argument of the cosine function is equal $n\pi$ where $n \in \{0, \pm 2, \pm 4, \dots\}$; however, the
 4114 solutions where n is negative can be discarded since they represent unphysical negative
 4115 overall phases. Applying this condition to Equation 5.31 gives a condition on the radial
 4116 position of the maximum of E_ϕ

$$\omega_c(t - |\vec{R}|/c) - \Delta = n\pi, \quad (5.32a)$$

$$|\vec{R}| = \frac{c}{\omega_c} ((\omega_c t - \Delta) - n\pi), \quad (5.32b)$$

4117 which is a function of time in the frame of the moving electron (t). Equation 5.32 can
 4118 be further simplified by noticing that the azimuthal position of the electron ($\phi_e(t)$) as a
 4119 function of time is defined by $\phi_e(t) = \omega_c t - \Delta$ which reduces Equation 5.32 to

$$|\vec{R}| = \frac{c}{\omega_c} (\phi_e(t) - n\pi). \quad (5.33)$$

4120 Equation 5.33 represents an archimedian spiral which is formed when plotting the
 4121 amplitude of E_ϕ in the x-y plane. The solution where $n = 0$ represents the leading edge
 4122 of the radiation spiral which propagates outward from the electron at the speed of light.
 4123 The additional solutions for $n > 0$ represent the persistent spiral at radii inside the
 4124 leading edge of the radiated fields that have not yet been detected by the receiver at the
 4125 current time. In Figure 5.12a we show the expected spiral pattern for the maxima of the
 4126 cyclotron radiation.

4127 In particular, we note that for the circular array geometry of the test stand, depicted
 4128 as the series of circles in Figure 5.12a, each antenna receives a linearly polarized wave

4129 with a phase offset that corresponds to the azimuthal angle for that antenna element.
 4130 Therefore, as we show in Figure 5.12b, when the relative phase of the received signal is
 4131 plotted as a function of the receiving antenna's azimuthal position the result is also an
 4132 Archimedean spiral.

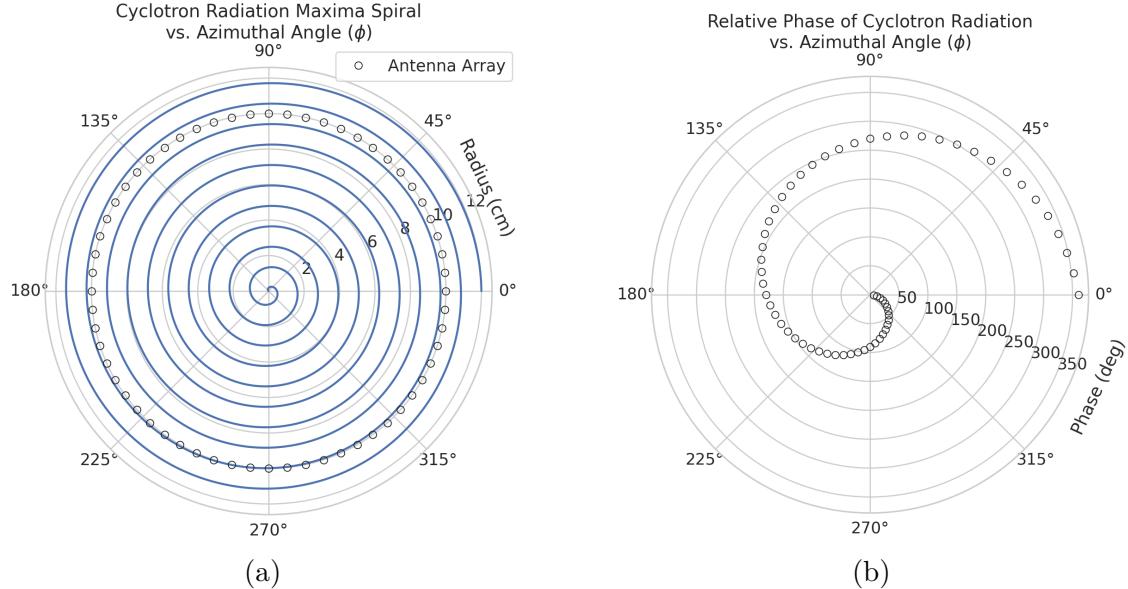


Figure 5.12. The amplitude maxima of the cyclotron radiation form an Archimedean spiral as the radiation propagates outward from the cyclotron orbit center (a). A circular antenna array located at a fixed radius from the orbit center will receive electric fields with equal magnitude in each of its channels, but the phase of the electric field incident on each array channel will be linearly out of phase from its neighbor antennas by an amount equal to the angular separation of the two channels (b).

4133 Based on these analytical calculations we can characterize the magnitude, polarization,
 4134 and phase of the signals received by the antenna array using three criteria. These criteria
 4135 are the basis of comparison for the radiation produced by the SYNCA and cyclotron
 4136 radiation emitted by electrons and will be used to evaluate the performance of antenna
 4137 designs. The criteria are:

- 4138 1. Electric fields that are ϕ -polarized near $\theta = 90^\circ$
- 4139 2. Uniform time-averaged electric field magnitudes around the circumference of a
4140 circle centered on the antenna
- 4141 3. Electric fields whose phase is equal to the azimuthal angle at the point of measure-
4142 ment plus a constant

4143 The Locust simulation package [97] can be used to directly simulate the EM fields
4144 generated by electrons performing cyclotron motion to validate the analytical calculations.
4145 Locust simulates the EM fields by first calculating the trajectory of the electrons in
4146 the magnetic trap using the Kassiopeia software package [98]. The trajectory can then
4147 be used to solve for the EM fields using the Liénard-Wiechert equations directly with
4148 no approximations. The resulting electric field solutions drive a receiving antenna by
4149 convolving the time-domain fields with the finite-impulse response filter of the antenna
4150 or they can be examined directly to study the field characteristics that the SYNCA must
4151 reproduce. In the next section we compare the radiation field patterns for electrons
4152 simulated with Locust to patterns from a SYNCA antenna design.

4153 **5.3.3 SYNCA Simulations and Design**

4154 **Early SYNCA Design Development**

4155 A crossed-dipole antenna [99] (see Figure 5.13) was identified early on as a candidate
4156 SYNCA design. The crossed-dipole is a circularly polarized antenna, consequently, the
4157 electric fields measured in the plane of the dipole antenna exhibit the same relative phase
4158 offsets as a 90° electron in a magnetic trap. These phase offsets were measured with the
4159 first rudimentary crossed-dipole prototype manufactured from coaxial cables with the
4160 insulation and shield stripped away.

4161 Because the SYNCA is ultimately a calibration tool, it is desireable that the antenna
4162 have a well-characterized and robust antenna pattern. Therefore, manufacturing a
4163 SYNCA using the stripped wire method shown in Figure 5.13 is infeasible. Studies of
4164 crossed-dipole antennas manufactured out of printed circuit boards were performed using
4165 HFSS to identify an antenna design that imitated an electron, while being more robust
4166 and simpler to manufacture (see Figure 5.14).

4167 Identifying a design that was robust, manufacturable, and matched the electric fields
4168 of a trapped electron proved to be a non-trivial task. The primary factor driving the
4169 difficulty was the high operating frequency of the antenna (26 GHz) combined with
4170 the requirement that the antenna be electrically-small. An antenna that is electrically-
4171 small at 26 GHz has a largest dimension on the order of 1 mm, which poses significant
4172 manufacturability challenges given the limited available budget for SYCNA fabrication.

4173 One of the key limitations with the small size requirements is the diameter of the
4174 coaxial cables needed to feed the crossed-dipole antenna. The smallest commonly available
4175 rigid coaxial cables available on the market have diameters of approximately 0.5 mm,

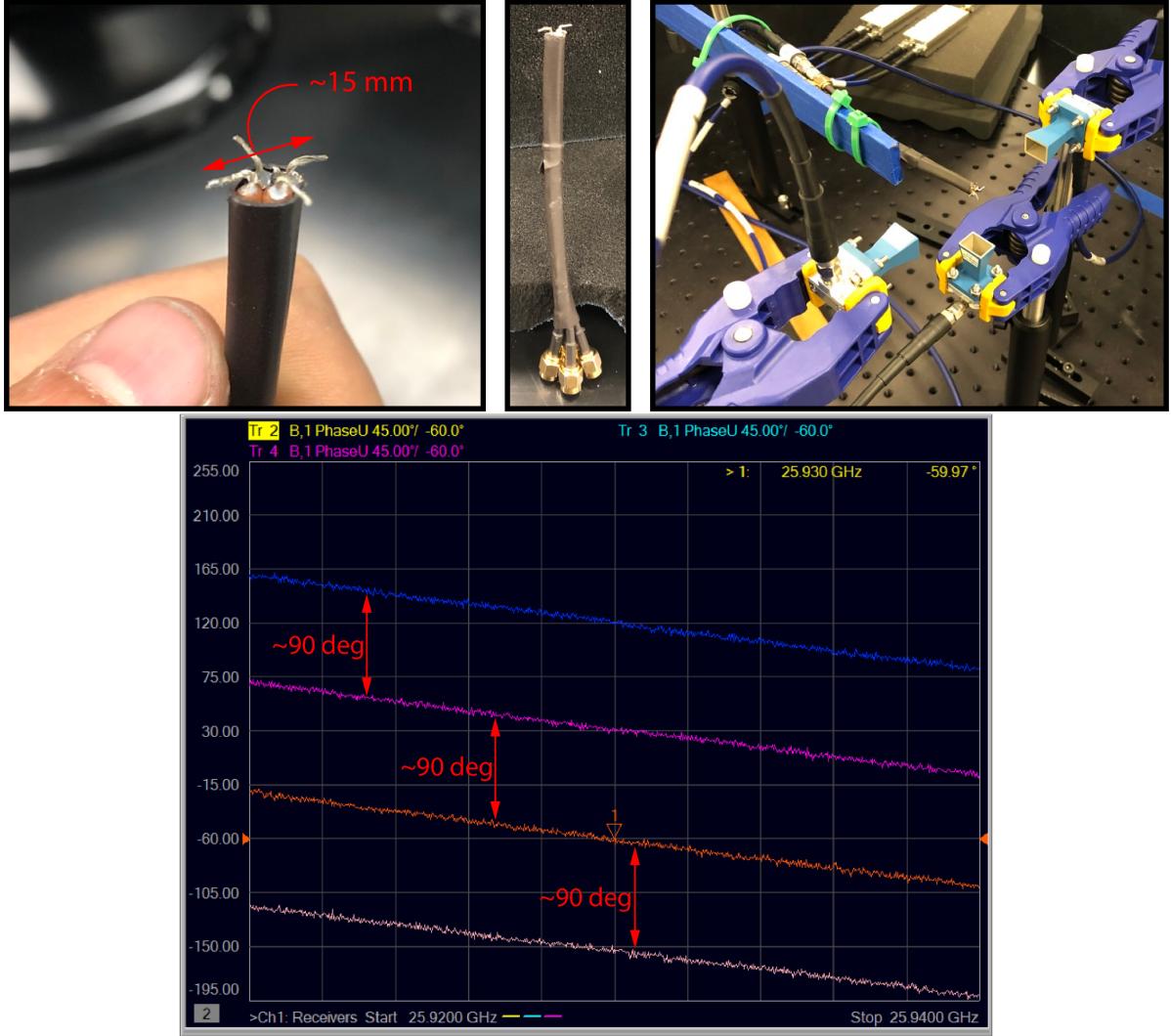


Figure 5.13. Images of an early prototype crossed-dipole antenna manufactured by hand and the first measurement setup. The antenna was constructed by hand using four stripped coaxial cables. The antenna was connected to one port of the VNA, and the remaining three ports on the VNA were connected to horn antenna arranged with 90 deg offsets around the crossed-dipole. The measured unwrapped S-parameter phases exhibit the desired relative phase behavior for a SYNCA. These early measurements were the first laboratory proof-of-principle for the crossed-dipole SYNCA.

4176 which limited the spacing between dipole arms to a minimum of about 1 mm. The
 4177 crossed-dipole antenna performs better as a SYNCA if the dipole arm separation is
 4178 significantly less than the operating wavelength. Therefore, the high operating frequency
 4179 ultimately limited how well the SYNCA could mimic an electron. If the desired cyclotron
 4180 frequency was lowered by an order of magnitude to approximately 3 GHz a significantly
 4181 higher quality SYNCA could be manufactured at lower cost.

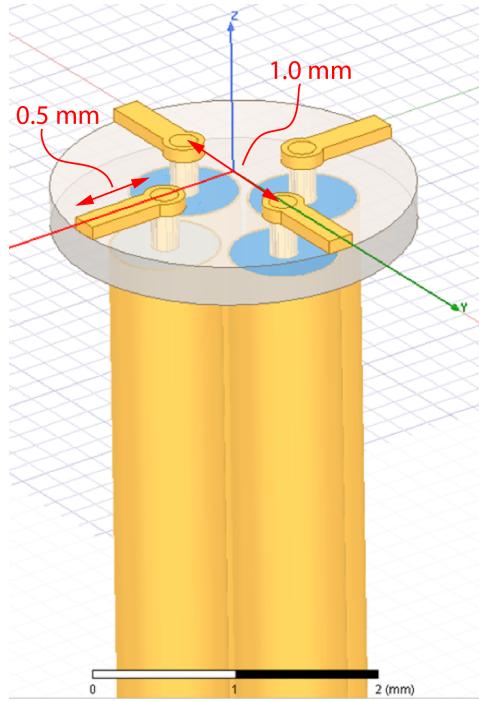


Figure 5.14. An early iteration of a crossed-dipole SYNCA antenna simulated in HFSS. The antenna is electrically small at 26 GHz, which requires dipole arms on the order of 1 mm long. This design is limited by the minimum achievable distance between the dipole arms caused by the available diameters of coaxial cables. The assumed termination scheme for the coaxial cables to the antenna is hand-soldering, which introduces random variation in the antenna pattern from the inevitable blobs of solder left on the surface of the PCB.

4182 The decision to use coaxial transmission lines terminated on the antenna PCB with a
 4183 hand-soldered connection was driven primarily to limit the costs of SYNCA development
 4184 and contributed to the observable variations in the SYNCA's gain and phase patterns.
 4185 A second iteration of the SYNCA design that minimized hand-soldering by using surface-
 4186 mount components could significantly reduce variations in the antenna pattern. The
 4187 major drawback in the development of a surface-mount SYNCA is the cost, and given the
 4188 transition to a cavity based design for Phase IV, such a design was never investigated.

4189 **Finalized SYNCA Simulations and Design**

4190 One of the main drawbacks of a standard crossed-dipole design is that they do not radiate
 4191 uniform electric fields near the $\theta = \pi/2$ plane. Typical crossed-dipole antennas use dipole
 4192 arm lengths equal to $\lambda/4$ or larger [99], where λ is the wavelength at the desired operating
 4193 frequency. Such large arm lengths cause the electric field magnitude to vary significantly
 4194 around the circumference of the antenna. However, making the antenna electrically small

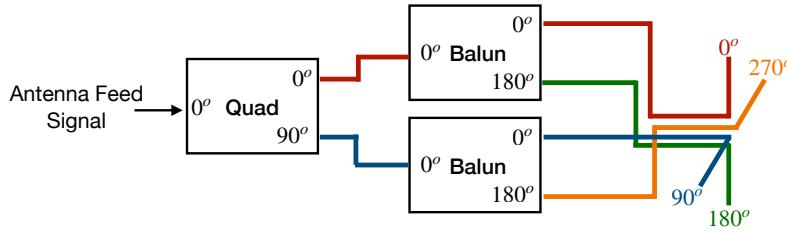


Figure 5.15. An idealized crossed-dipole antenna consists of two electric dipole antennas oriented perpendicular to each other and is fed with four signals with a quadrature phase relationship. An example antenna feed circuit is shown which is composed of a chained combination of a quadrature hybrid-coupler (Quad) and two baluns.

4195 by shrinking the arm length can improve the antenna pattern uniformity.

4196 In general, the criterion for an electrically small antenna is that the largest dimension
 4197 of the antenna (D) obey $D \lesssim \lambda/10$ [68]. In our application, we are attempting to mimic
 4198 the cyclotron radiation emitted by electrons produced from tritium β -decay with energies
 4199 near the spectrum endpoint. For a background magnetic field of 1 T, the corresponding
 4200 cyclotron frequency of tritium endpoint electrons is approximately 26 GHz. Therefore, the
 4201 electrically small condition would require that the largest dimension of the crossed-dipole
 4202 antenna be smaller than 1.2 mm.

4203 A crossed-dipole antenna with an overall size of 1.2 mm is challenging to fabricate due
 4204 to the small dimensions of the dipole arms that, in practice, are fragile and unsuitable
 4205 for use as a calibration probe. To mitigate some of the challenges with the fabrication
 4206 of such a small antenna, a variant crossed-dipole antenna design using printed circuit
 4207 board (PCB) technology (see Figure 5.16) was developed in partnership with an antenna
 4208 prototyping company, Field Theory Consulting ¹.

4209 The PCB crossed-dipole design uses four rectangular pads to represent the dipole arms,
 4210 which are connected by a thin circular trace. The circular trace both adds mechanical
 4211 stability to the antenna and improves the azimuthal uniformity of the electric fields
 4212 compared to a more standard crossed-dipole geometry. Furthermore, the circular trace
 4213 allows for a greater separation between dipole arms than standard crossed-dipoles, which
 4214 is required to accommodate the coaxial connections to each pad. The pads each contain
 4215 a through-hole solder joint to connect coaxial transmission lines using hand soldering.
 4216 The antenna PCB has no ground plane on the bottom layer as this was observed in

¹<https://fieldtheoryinc.com/>

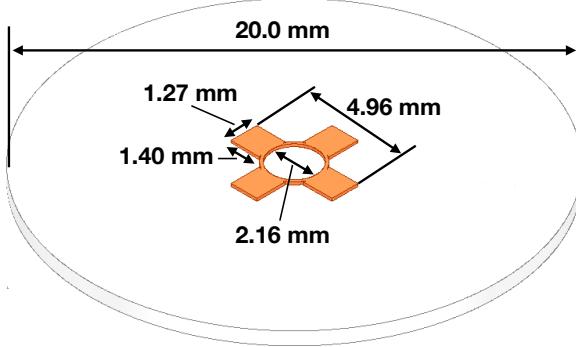


Figure 5.16. A model of the PCB crossed-dipole antenna with dimensions. The design has an inside diameter of 2.16 mm for the central circular trace, which is 0.13 mm wide. The dipole arms each have a width of 1.27 mm and protrude beyond the circular trace by 1.40 mm, which gives an overall width of 4.96 mm for the length of the antenna PCB trace from end-to-end. The overall size of the antenna is 20.0 mm the majority of which is the PCB dielectric material. This design was observed in simulation to maintain the field characteristics of the idealized crossed-dipole while being simpler to fabricate due to the increased size of the antenna.

4217 simulation to significantly distort the radiation pattern in the plane of the PCB. The
 4218 only ground planes present in the model are the outer conductors of the four coaxial
 4219 transmission lines which feed the antenna. These are left unterminated on the bottom of
 4220 the PCB dielectric material.

4221 The antenna design development utilized a combination of Locust electron simula-
 4222 tions and antenna simulations using ANSYS HFSS [69], a commercial finite-element
 4223 electromagnetic simulation software. Two antenna designs were simulated: an idealized
 4224 electrically small crossed-dipole antenna with an arm length of 0.40 mm and an arm
 4225 separation of 0.05 mm, as well as a PCB crossed-dipole antenna with the dimensions
 4226 shown in Figure 5.16. Plotting the magnitude of the electric fields generated by the
 4227 antennas across a 10 cm square located in the same plane as the respective antennas
 4228 reveals the expected cyclotron spiral pattern (see Figure 5.17) which closely matches
 4229 the prediction for simulated electrons. The spiral pattern demonstrates that the electric
 4230 fields have the appropriate phases to mimic cyclotron radiation, which fulfills SYNCA
 4231 criterion 3 identified in Section 5.3.2.

4232 As we can see from Figure 5.18, the crossed-dipole antenna, which uses an idealized
 4233 geometry, exhibits good agreement with simulation. The antenna has a maximum
 4234 deviation from a simulated electron of approximately 0.5 dB in the total electric field, 1
 4235 dB for the ϕ -polarized electric field and 1 dB for the θ -polarized electric field.

4236 In comparison, the pattern of the PCB crossed-dipole antenna, because the simulation
 4237 incorporates the geometry of the coax transmission lines, exhibits some distortion from

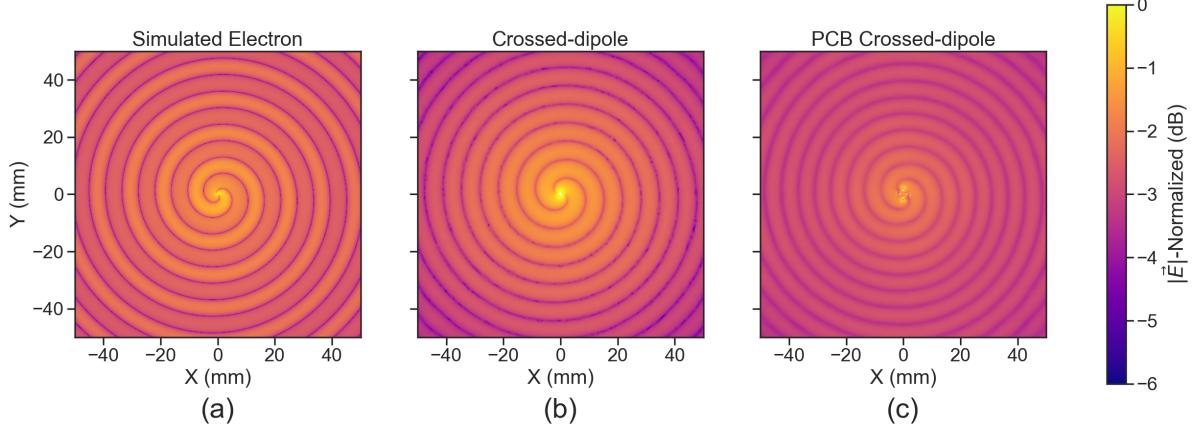


Figure 5.17. A comparison of the electric field magnitudes, normalized by the maximum value of the electric field in each simulation, plotted on a 10 cm square to visualize the Archimedean spirals formed by the electron (a), the crossed-dipole antenna (b), and a PCB crossed-dipole antenna (c). The matching patterns indicate that the electric fields have similar phase characteristics. These images were generated using Locust simulations for the electron and ANSYS HFSS for both antennas.

the idealized cross-dipole simulations. The vertically oriented ground planes of the coax lines introduce more θ -polarized electric fields than are observed for simulated electrons near $\theta = 90^\circ$. The significant θ -polarized field minimum is still present but shifted to approximately $\theta = 65^\circ$. The θ -polarized field deviations of the PCB crossed-dipole antenna should not greatly impact the performance of the antenna because the receiving antenna array is primarily ϕ -polarized. Therefore deviations in the θ -polarized fields will be suppressed due to the polarization mismatch. More importantly, the ϕ -polarized electric field pattern generated by the PCB crossed-dipole closely matches simulated electrons across the polar angle range of $50^\circ < \theta < 150^\circ$. In this region the PCB crossed-dipole differs by less than 0.5 dB from simulated electrons. This range greatly exceeds the beamwidth of the receiving antenna array which is designed to be most sensitive to fields produced near $\theta = 90^\circ$. Therefore, we conclude that the PCB crossed-dipole antenna generates a ϕ -polarized radiation pattern that fulfills SYNCA criterion 1 from Section 5.3.2.

The final SYNCA criterion is related to the uniformity of the electric fields when measured azimuthally around the antenna. As we saw for real electrons in Section 5.3.2 it is expected that the magnitude of the electric field be completely uniform as a function of the azimuthal angle due to the symmetry of the cyclotron orbit. In Figure 5.19 we plot the total electric field as a function of azimuthal angle for an electron, the crossed-dipole antenna, and the PCB crossed-dipole antenna. The crossed-dipole antenna exhibits

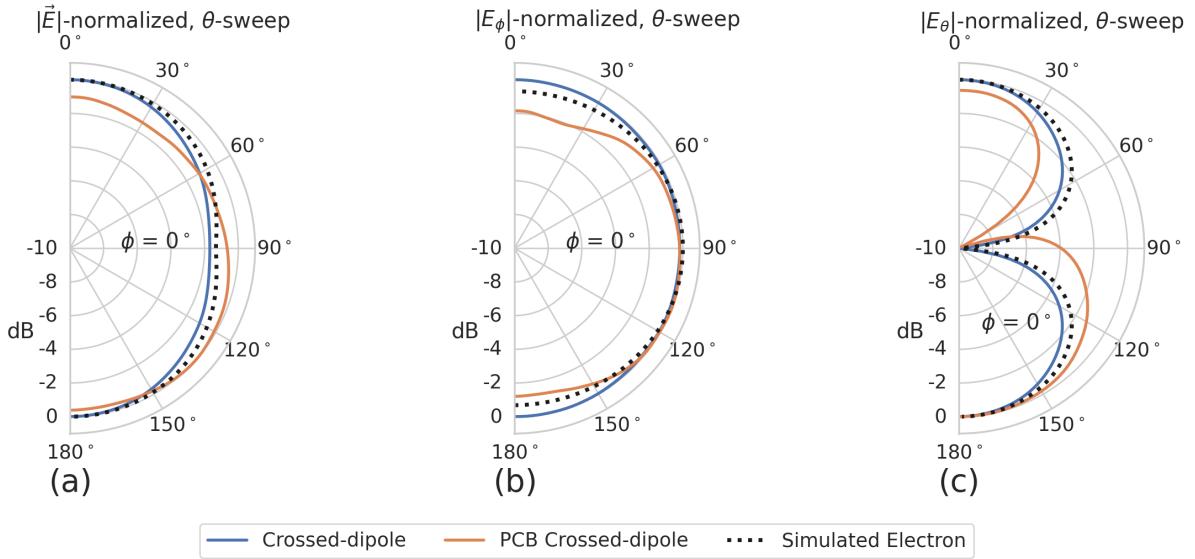


Figure 5.18. A comparison of the normalized electric field magnitudes for the ideal crossed-dipole, PCB crossed-dipole, and a simulated electron as a function of the polar angle (θ). (a) Shows the total electric field, (b) shows the ϕ -polarized electric field component, and (c) shows the θ -polarized electric field component. These images were generated using Locust simulations for the electron and ANSYS HFSS for both antennas.

perfect uniformity around the azimuthal angle, whereas the PCB crossed-dipole has a small periodic deviation with a maximum difference of 0.3 dB caused by the coaxial transmission lines below the PCB. Such a small deviation from uniformity is acceptable since it is smaller than the expected variation in uniformity caused by imperfections in the antenna fabrication process, which modifies the antenna shape in an uncontrolled manner by introducing solder blobs with a typical size of a few tenths of a millimeter on the dipole arms (see Figure 5.20). Additionally, the SYNCA will be separately calibrated to account for azimuthal differences in the electric field magnitude. Therefore we see from the simulated performance of the PCB crossed-dipole antenna that this antenna design meets all three of the SYNCA criteria.

5.3.4 Characterization of the SYNCA

Two SYNCAs were manufactured using the PCB crossed-dipole design (see Figure 5.20). The antenna PCB (Matrix Circuit Board Materials, MEGTRON 6) is connected to four 2.92 mm coaxial connectors (Fairview Microwave, SC5843) using semi-rigid coax (Fairview Microwave, FMBC002), which also physically support the antenna PCB. The antenna PCB consists only of two layers which correspond to the copper antenna trace

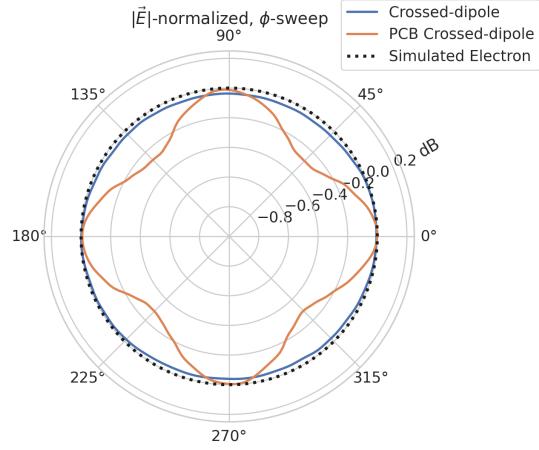


Figure 5.19. A comparison of the normalized electric field magnitudes for the crossed-dipole, PCB crossed-dipole, and a simulated electron as a function of the azimuthal angle (ϕ) evaluated at $\theta = 90^\circ$. This image was generated using Locust simulations for the electron and ANSYS HFSS for both antennas.

and the PCB dielectric. Each coax line is connected to the associated dipole arm using through-hole soldering and phase matched to ensure that the electrical length of each of the transmission lines is identical at the operating frequency. The antenna PCB is further reinforced using custom cut polystyrene foam blocks, which have an electrical permittivity nearly identical to air. A custom 3D printed mount is included at the base of the antenna to support the coax connectors and to provide a sturdy mounting base.

Characterization measurements were performed using a Vector Network Analyzer (VNA) to measure the electric field magnitude and phase radiated by the SYNCA to verify the radiation pattern (see Figure 5.21). The VNA is connected to the SYNCA at one port through a hybrid-coupler whose outputs are connected to two baluns to generate the signals with the appropriate phases to feed the SYNCA (see Figure 5.15). The other port of the VNA is connected to a single reference horn antenna that serves as a field probe. To position the SYNCA, a combination of translation and rotation stages are used to characterize the antenna's fields across the entire radiation pattern circumference. This measurement scheme is equivalent to measuring the fields generated by the SYNCA using a full circular array of probe antennas.

The antenna measurement space is surrounded by RF anti-reflective foam to isolate the measurements from the lab environment (see Figure 5.21b) and remaining reflections are removed using the VNA's time-gating feature. The SYNCA is affixed to the stages by a custom RF transparent mount made of polystyrene foam. The coaxial cables deliver the antenna feed signals generated by the VNA to the SYNCA while still allowing

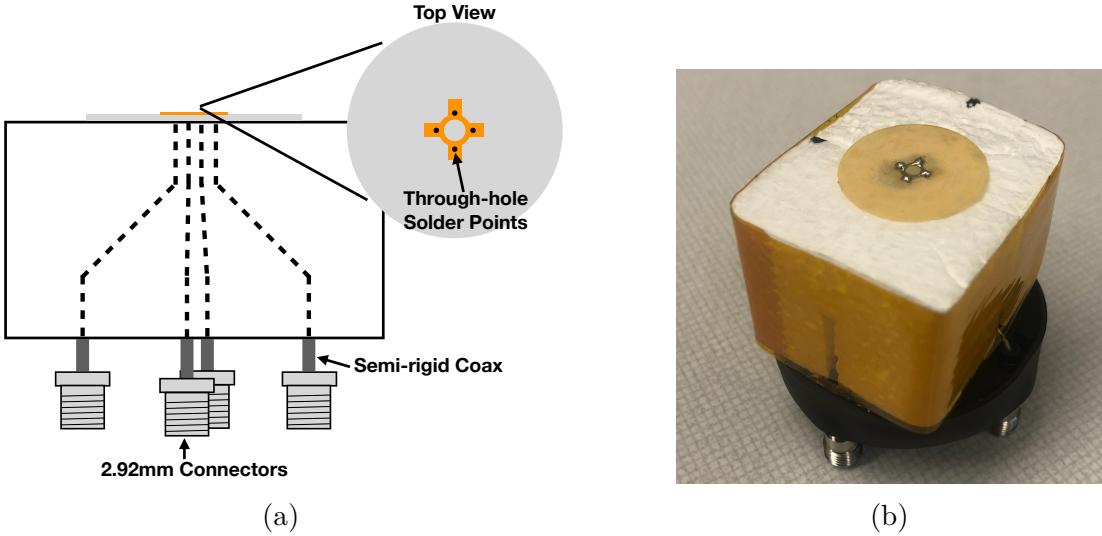


Figure 5.20. (a) A cartoon schematic which highlights the routing of the semi-rigid coax transmission lines. (b) A photograph of a SYNCA constructed using the modified crossed-dipole PCB antenna design. Visible in the photograph of the SYNCA are four blobs of solder which are an artifact of the SYNCA’s hand-soldered construction. These solder blobs are the most significant deviation from the SYNCA design shown in Figure 5.16 and are responsible for a significant fraction of the irregularities seen in the antenna pattern.

unrestricted rotation. The horn antenna probe is nominally positioned in the plane formed by the antenna PCB ($\theta = 90^\circ$ or $z = 0$ mm) at a distance of 10 cm from the SYNCA, to match the expected position of the antenna array relative to the SYNCA in the antenna array test stand. The horn antenna can be manually raised or lowered to different relative vertical positions to characterize the radiation pattern at different polar angles.

Several 360° scans were performed with probe vertical offsets of -10.0 mm, -5.0 mm, 0.0 mm, 5.0 mm, and 10.0 mm relative to the antenna PCB plane. These probe offsets cover a 2 cm wide vertical region centered on the SYNCA PCB, approximately equal to ± 6 degrees of polar angle. The measurements show that the SYNCA is generating fields with nearly isotropic magnitude across the probed region. The standard deviation of the electric field magnitude measured around the antenna circumference is approximately 2.9 dB for a typical rotational scan. The presence of a significant pattern null is noted near 45° (see Figure 5.22), which we attribute to small imperfections in the antenna PCB that could be introduced from the hand soldered terminations connecting the coax cables to the antenna. There is no significant difference in the radiation pattern when measured across the 2 cm vertical range. The measured relative phases closely follow



Figure 5.21. A schematic of the VNA characterization measurements (a). This setup allows for antenna gain and phase measurements across a full 360° of azimuthal angles using a motorized rotation stage and control of the radial position of the SYNCA using a translation stage. A photo of the setup in the lab is shown in (b).

4312 the expectation for an electron, being linear with the measurement rotation angle and
 4313 forming the expected spiral pattern. Other than the small phase imperfections there is
 4314 a slight sinusoidal bias to the phase data, which we determined is the result of a small
 4315 ($\lesssim 1$ mm) offset of the antenna's phase center from the rotation axis of the automated
 4316 stages.

4317 The characterization measurements confirm the simulated performance of the SYNCA.
 4318 As expected the fields generated by the antenna are nearly isotropic in magnitude, ϕ -
 4319 polarized, and are linearly out of phase around the circumference of the antenna as
 4320 predicted for cyclotron radiation in Section 5.3.2. Small imperfections in the magnitude
 4321 and phase of the antenna are expected, particularly at the antenna's high operating
 4322 frequency of 26 GHz where small geometric changes can have significant impacts on
 4323 electrical properties. However, calibration through careful characterization measurements
 4324 can be used to remove the majority of these pattern imperfections, including the relatively
 4325 large pattern null near 45° , which will allow for the usage of the SYNCA as a test source
 4326 for free-space CRES experiments utilizing antenna arrays. In the next section we use the
 4327 VNA measurements obtained here as a calibration for signal reconstruction using digital
 4328 beamforming.

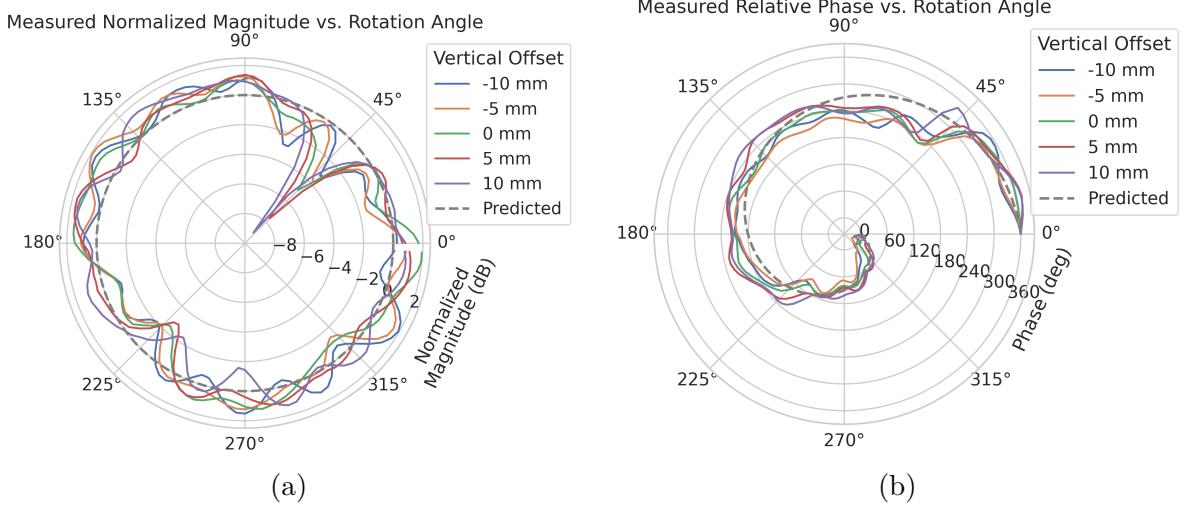


Figure 5.22. Linear interpolations of the measured electric field magnitude (a) and phase (b). The data was acquired using a VNA at 120 points spaced by 3 degrees from 0 to 357 degrees of azimuthal angle. The different color lines indicate the vertical offset of the horn antenna relative to the SYNCA PCB and the dashed line shows the expected shape from electron simulations. No significant difference in the antenna pattern is observed for the measured vertical offsets.

4329 5.3.5 Beamforming Measurements with the SYNCA

4330 Digital beamforming is a standard technique for signal reconstruction using a phased
 4331 array [100]. The SYNCA, since it exhibits the same cyclotron phases as a trapped
 4332 electron, can be used to perform simulated CRES digital beamforming reconstruction
 4333 experiments on the bench-top without the need for the magnet, cryogenics, and vacuum
 4334 systems required by a full CRES experiment. The fields received by the individual
 4335 elements of the antenna array will have phases dependent on the spatial position of the
 4336 source relative to the antennas. Therefore, a simple summation of the received signals
 4337 will fail to reconstruct the signal due to destructive interference between the individual
 4338 channels in the array. However, applying a phase shift associated with the source's
 4339 spatial position removes phase differences and results in a constructive summation of the
 4340 channel signals (see Figure 5.23). We can summarize the digital beamforming operation
 4341 succinctly using the following equation

$$y[t_n] = \sum_{m=0}^{N-1} x_m[t_n] A_m e^{i\phi_m}, \quad (5.34)$$

4342 where $y[t_n]$ represents the summed array signal at time t_n , $x_m[t_n]$ is the signal received
 4343 by channel m at time t_n , ϕ_m is the phase shift applied to the signal received at channel
 4344 m , and A_m is an amplitude weighting factor that accounts for the different signal power
 4345 received by individual channels. By changing the digital beamforming phases, the point
 4346 of constructive interference can be scanned across the sensitive region of the array to
 4347 search for the location of a radiating source, which is identified as the point of maximum
 4348 summed signal power above a specified threshold. The digital beamforming phases consist
 4349 of two components,

$$\phi_m = 2\pi d_m / \lambda + \theta_m, \quad (5.35)$$

4350 where d_m is the distance from the m -th array element to the source, and θ_m is the
 4351 relative angle between the source position and the m -th antenna. The first component is
 4352 the standard digital beamforming phase that corresponds to the spatial position of the
 4353 source, and the second component is the cyclotron phase that corresponds to the relative
 4354 azimuthal phase offset.

4355 With a small modification to the hardware used to characterize the SYNCA (see
 4356 Figure 5.21), we can perform a digital beamforming reconstruction of a synthetic CRES
 4357 event. By replacing the VNA with an arbitrary waveform generator (AWG), the SYNCA
 4358 can be used to generate cyclotron radiation with an arbitrary signal structure, which
 4359 can then be detected by digitizing the signals received by the horn antenna. Rotational
 4360 symmetry allows us to use the rotational stage of the positioning system to rotate the
 4361 SYNCA to recreate the signals that would have been received by a complete circular

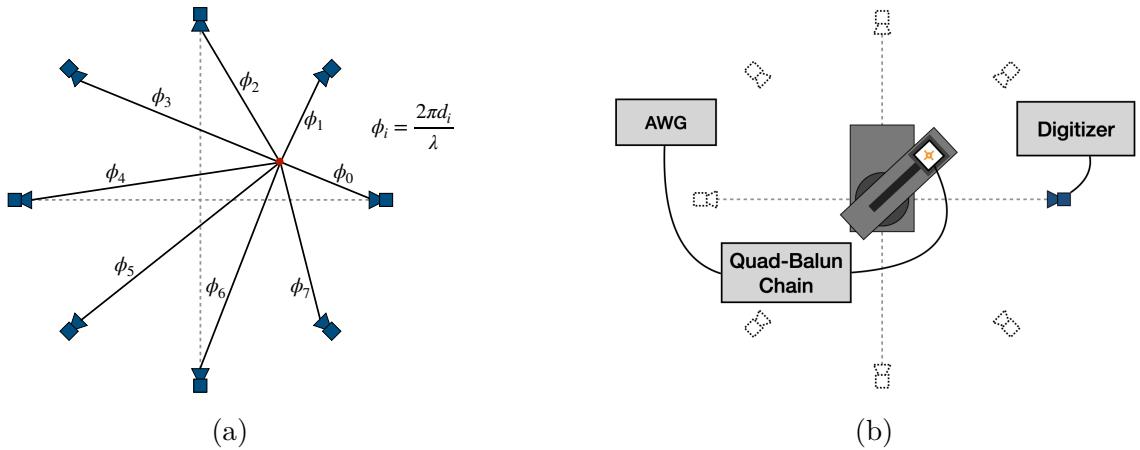


Figure 5.23. (a) A depiction of the relative phase differences for signals received by a circular antenna array from an isotropic source. The phases correspond to a unique spatial position. (b) A schematic of the setup used to perform digital beamforming.

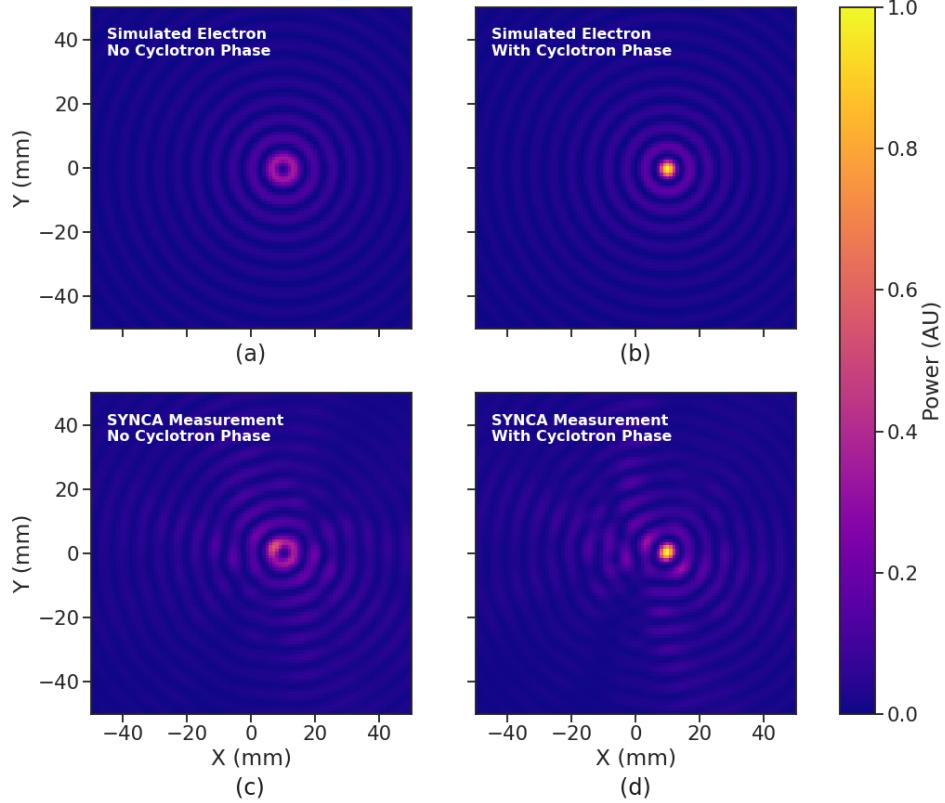


Figure 5.24. Digital beamforming maps generated using a simulated 60 channel array and electron simulated using the Locust package. (a) and (b) show the beamforming maps for simulated electrons without the cyclotron spiral phases and with the cyclotron spiral phases respectively. (c) and (d) show the beamforming maps produced from SYNCA measurements. We observe good agreement between simulated electrons and the SYNCA measurements.

4362 array of antennas.

4363 Using this setup, signals from a 60 channel circular array of equally spaced horn
 4364 antennas were generated with the SYNCA positioned 10 mm off the central array axis,
 4365 reconstructed using digital beamforming, and compared to Locust simulation (see Figure
 4366 5.24). When the cyclotron spiral phases are not used, which is equivalent to setting θ_m
 4367 in Equation 5.35 to zero, the SYNCA's position is reconstructed as a relatively faint ring
 4368 as predicted by simulation. However, when the appropriate cyclotron phases are used
 4369 during the beamforming procedure, both the simulated electron and the SYNCA appear
 4370 as a single peak of high relative power corresponding to the source position. Therefore,
 4371 we observe good agreement between the simulated and SYNCA reconstructions. While it
 4372 may seem that for the case with no cyclotron phase corrections the ring reconstructs the
 4373 position of the electron as effectively as beamforming with the cyclotron phase corrections,

4374 it is important to note that the simulations and measurements were generated without a
 4375 realistic level of thermal noise. The larger maxima region and lower signal power, which
 4376 occurs without the cyclotron phase corrections, significantly reduce the probability of
 detecting an electron in a realistic noise background.

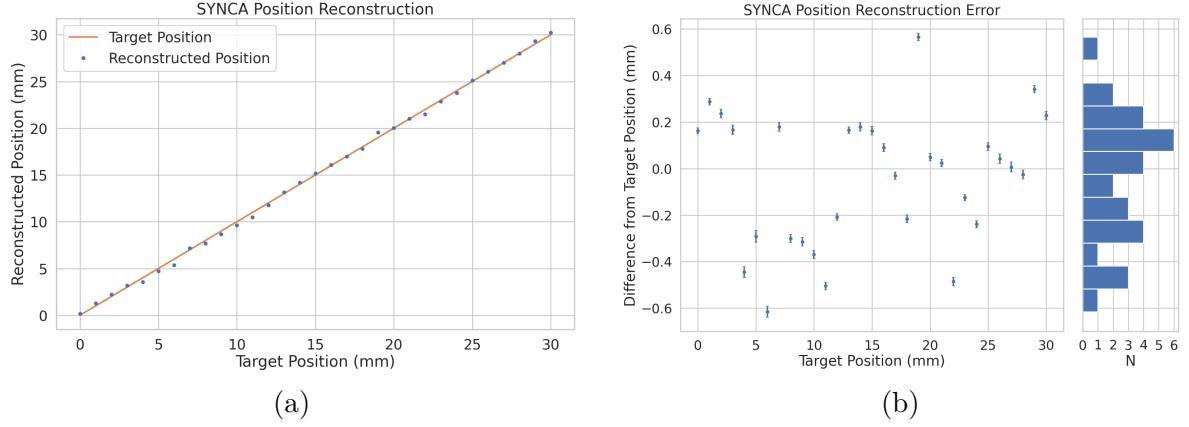


Figure 5.25. A plot of the SYNCA’s reconstructed position using the synthesized horn-antenna array and digital beamforming. (a) Shows the reconstructed position of the SYNCA compared with the target position indicated by the positioning system readout. (b) Shows the reconstruction error, which is the difference between the target and reconstructed positions. The error bars in (b) are the uncertainty in the mean position of the 2D Gaussian used to fit the digital beamforming reconstruction peak obtained from the fit covariance matrix. The mean fit position uncertainty of 0.02 mm is an order of magnitude smaller than the typical reconstruction error of 0.3 mm obtained by calculating the standard deviation of the difference between the reconstructed and target position.

4377
 4378 To bound the beamforming capabilities of the synthetic array of horn antennas, we
 4379 performed a series of beamforming reconstructions where the SYNCA was progressively
 4380 moved off the central axis of the array (see Figure 5.25). To extract an estimate of the
 4381 position of the SYNCA using the digital beamforming image we apply a 2-dimensional
 4382 (2D) Gaussian fit to the image data and extract the estimated centroid value. We find
 4383 that the synthetic horn antenna array reconstructs the position of the SYNCA with a
 4384 1σ -error of 0.3 mm with no apparent trend across the 30 mm measurement range. This
 4385 reconstruction error is an order of magnitude larger than mean fit position uncertainty
 4386 of 0.02 mm indicating that systematic effects related to the SYNCA positioning system
 4387 could be contributing additional uncertainty to the measurements. Note that the current
 4388 mean reconstruction error of 0.3 mm is a factor of 20 smaller than the full width at half
 4389 maximum of the digital beamforming peak (6 mm), which could be interpreted as a naive
 4390 estimate of the position reconstruction performance of this technique. Because these
 4391 measurements are intended as a proof-of-principle demonstration, we do not investigate

4392 potential sources of systematic errors further; however, we expect that a similar and
4393 more thorough investigation will be performed using the Project 8 antenna array test
4394 stand, where typical reconstruction errors can be used to estimate the energy resolution
4395 limits of antenna array designs.

4396 5.3.6 Conclusions

4397 In this paper we have introduced the SYNCA, which is a novel antenna design that
4398 emits radiation that mimics the unique properties of the cyclotron radiation generated by
4399 charged particles moving in a magnetic field. The characterization measurements of the
4400 SYNCA validated the simulated performance of the PCB crossed-dipole antenna design.
4401 Additionally, the SYNCA was used to estimate the position reconstruction capabilities
4402 of a synthesized array of horn antennas and experimentally reproduced the simulated
4403 digital beamforming reconstruction of electrons.

4404 While the SYNCA performs well, there exist discrepancies in the phase and magnitude
4405 of the radiation pattern compared to the simulated SYNCA design that are related to
4406 the small geometric differences in the soldered connections. Future design iterations that
4407 replace the soldered connections with a fully surface mount design could improve the
4408 radiation pattern at the cost of some complexity and expense. Furthermore, improving
4409 the design of the antenna PCB and mounting system would allow the antenna to be
4410 inserted into a cryogenic and vacuum environment where in-situ antenna measurement
4411 calibrations could be performed.

4412 The discrepancies in the radiation pattern and phases exhibited by the as-built
4413 SYNCA should not greatly impact its performance as a calibration probe. Both magni-
4414 tude and phase variations can be accounted by applying the SYNCA characterization
4415 measurements as a calibration to the data collected by the antenna array test stand. The
4416 separate calibration of the SYNCA radiation does not impact the primary goals for the
4417 antenna array test stand which are array calibration and signal reconstruction algorithm
4418 performance characterization, because it can be performed with standard reference horn
4419 antennas with well understood characteristics.

4420 The SYNCA antenna technology advances the CRES technique by providing a
4421 mechanism to characterize free-space antenna arrays for CRES measurements without
4422 the need for a magnet and cryogenics system, which would be required for calibration
4423 using electron sources. Both the Project 8 collaboration as well as future collaborations
4424 which are developing antenna array based CRES experiments can make use of SYNCA
4425 antennas as an important component of their calibration and commissioning phases.

4426 **5.4 FSCD Antenna Array Measurements with the SYNCA**

4427 **5.4.1 Introduction**

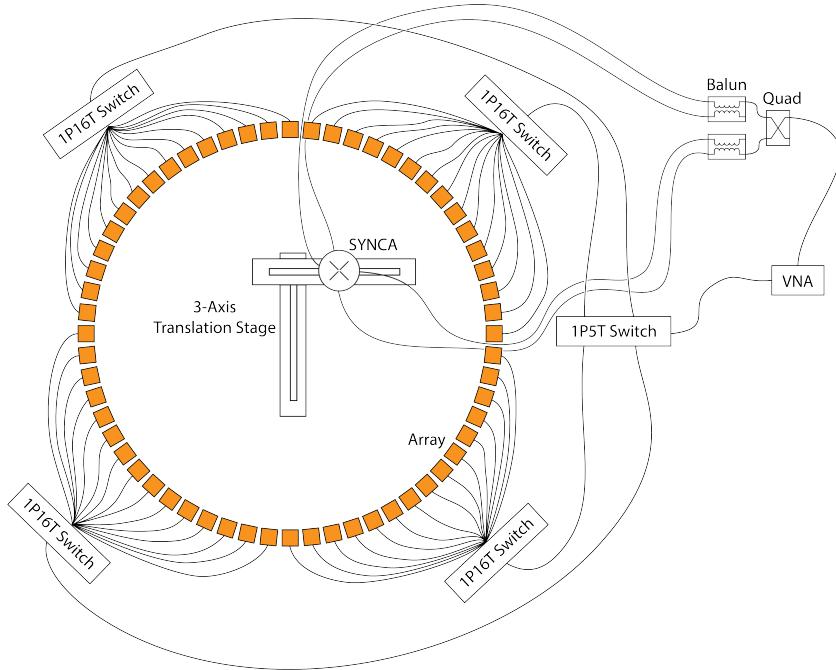


Figure 5.26. A diagram of the array measurement system used to test the prototype FSCD antenna array. A VNA is used as the primary measurement tool, which is connected to the array through a series of switches. The other port of the VNA connects to the SYNCA through the quad-balun chain used to provide the SYNCA feed signals. During measurements the SYNCA is positioned inside the center of the antenna array and translated to different radial and axial positions using a 3-axis manual translation stage setup.

4428 Using the SYNCA it is possible perform full-array measurements of prototype versions
4429 of the FSCD antenna array with a realistic cyclotron radiation source (see Figure 5.26).
4430 The goal is to compare the measured power received to FSCD simulations as a function
4431 of the radial and axial position of the SYNCA source. These measurements are intended
4432 to validate the antenna research and development by Project 8, which has been driven
4433 primarily by simulations with Locust [65] and CRESana (see Section 4.2.3), and identify
4434 any discrepancies with these simulations tools. This knowledge will provide confidence
4435 in the simulations necessary for the analysis of the sensitivity of larger antenna array
4436 based CRES experiment designs to the neutrino mass.

4437 As shown in Section 5.3, the SYNCA has some radiation pattern imperfections
4438 that complicate the comparison between measurement and simulation data. One way

4439 to disentangle the effects of these imperfections is to perform an additional set of
4440 measurements using a synthetic antenna array setup along with the SYNCA antenna.
4441 Since the synthetic array setup uses only a single array antenna, the data should be
4442 free of errors associated with individual antenna differences and multi-path interference,
4443 which are two error sources being tested with the full-array setup. By comparing the
4444 synthetic array data to the FSCD array data and to simulation data one can evaluate the
4445 significance of these effects relative to the errors introduced by SYNCA imperfections.

4446 **5.4.2 Measurement Setups**

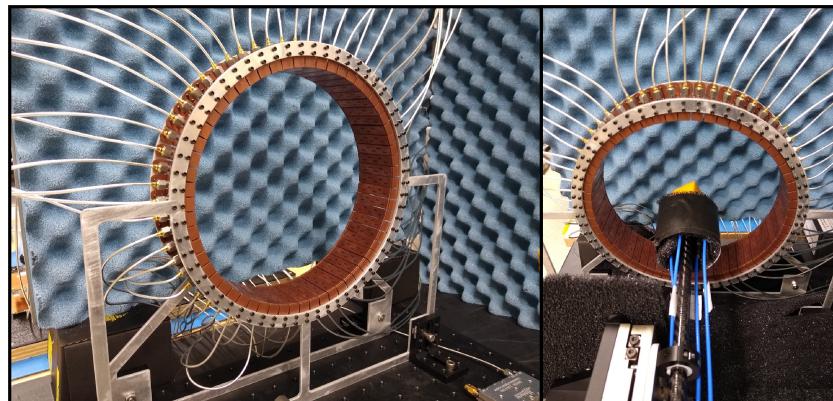
4447 **5.4.2.1 FSCD Array Setup**

4448 The antenna design that composes the array is the 5-slot waveguide antenna developed
4449 for the FSCD experiment (see Figure 5.27a). The antenna is 5 cm long and is constructed
4450 out of WR-34 waveguide with a 2.92 mm coax connector located at the center of the
4451 antenna. Copper flanges located on both ends of the antenna are used to mount the
4452 antenna in the array support structure. The antennas are supported by two circular steel
4453 brackets that can be bolted to both ends of the waveguide to construct the circular array
4454 (see Figure 5.27b). The antenna array consists of sixty identical waveguide antennas
4455 with a radius of 10 cm. The array is mounted perpendicular to an optical breadboard
4456 surface using a pair of the steel brackets, which provide sufficient space for the coaxial
4457 cable connections and allows for easy positioning of the SYNCA antenna. The SYNCA is
4458 mounted on the end of a carbon fiber rod attached to a set of manual translation stages,
4459 which are used to move the SYNCA antenna to different positions inside the array (see
4460 Figure 5.27c). The stages allow for independent motion in three different axes and can
4461 position the SYNCA at radial distances up to 5 cm from the center.

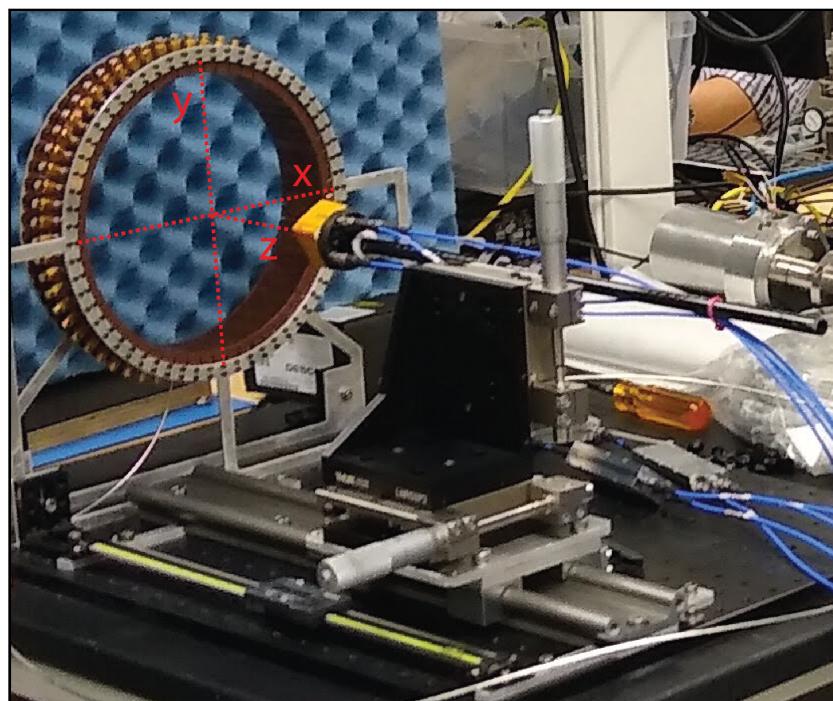
4462 Data acquisition is accomplished using a two-port VNA in combination with a series
4463 of microwave switches that allow the VNA to connect to each channel in the array . The
4464 first port of the VNA is connected to the quad-balun chain used to feed the SYNCA (see
4465 Section 5.3), and the second port of the VNA connects to a 1P5T microwave switch. The
4466 1P5T switch is connected to four separate 1P16T switch boards that connect directly
4467 to the array. The data acquisition is controlled by a python script running on a lab
4468 computer, which is connected to the VNA and an Arduino board programmed to control
4469 the microwave switches. The script uses the switches to iteratively connect each of the
4470 antennas in the array to the VNA. The VNA is configured to load a specific calibration
4471 file for each antenna channel and performs the measurements of all available S-parameters.



(a)



(b)



(c)

Figure 5.27. Photos of the prototype FSCD antenna (a), the FSCD array and SYNCA (b), and the translation stages and coordinate system used to position the SYNCA (c).

4472 The separate calibration files is an attempt to remove phase and magnitude errors caused
4473 by different propagation through the RF switches. Array measurements were performed
4474 for the set of SYNCA positions consisting of radial (x-axis) positions from 0 to 50 mm
4475 in 5 mm steps and axial (z-axis) positions from 0 to 50 mm in 5 mm steps resulting in
4476 121 array measurements. At each SYNCA position the two-port S-parameter matrix
4477 is measured using a linear frequency sweep from 25.1 to 26.1 GHz with 101 discrete
4478 frequencies.

4479 **5.4.2.2 Synthetic Array Setup**

4480 A photograph of the setup used to perform the synthetic array measurements is shown
4481 in Figure 5.28. A difference between this setup and the FSCD array setup is that the
4482 synthetic array measurements were performed with a waveform generator and digitizer
4483 instead of a VNA. The electronics configuration is identical to the diagram in Figure
4484 5.7b. Despite the differences, one is still able to compare the measured phases of the
4485 synthetic array and the relative magnitude of the power, since the digitized signal power
4486 is directly proportional to S21.

4487 The arbitrary waveform generator in the setup is configured to produce a 64 MHz
4488 sine wave signal that is up-converted to 25.864 GHz using a mixer and the VNA source.
4489 This signal is passed through a bandpass filter and fed to the SYNCA quad-balun chain.
4490 A single FSCD antenna is positioned 10 cm from the SYNCA and aligned vertically so
4491 that the center of the 5-slot waveguide is in the plane of the SYNCA PCB (see Figure
4492 5.28). This position corresponds to $z = 0$ in Figure 5.27c. The SYNCA is rotated
4493 in three degree steps to synthesize an antenna array with 120 channels. This channel
4494 count is more than could physically fit in a 10 cm radius array, but there is no cost to
4495 over-sampling. The signals from the FSCD antenna are down-converted using the second
4496 mixer connected to the VNA source before being digitized at 250 MHz and saved to
4497 disk. Several synthetic array measurement scans were performed by using the linear
4498 translation stage to change the radial position of the SYNCA. In total eight scans were
4499 taken from 0 to 35 mm using a radial position step size of 5 mm.

4500 **5.4.3 Simulations, Analysis, and Results**

4501 The Locust and CRESana simulation packages utilize the antenna transfer functions
4502 to calculate the power that would be received by each antenna from a CRES electron.
4503 The equivalent quantity in the measurement setup is the S21 matrix element, which

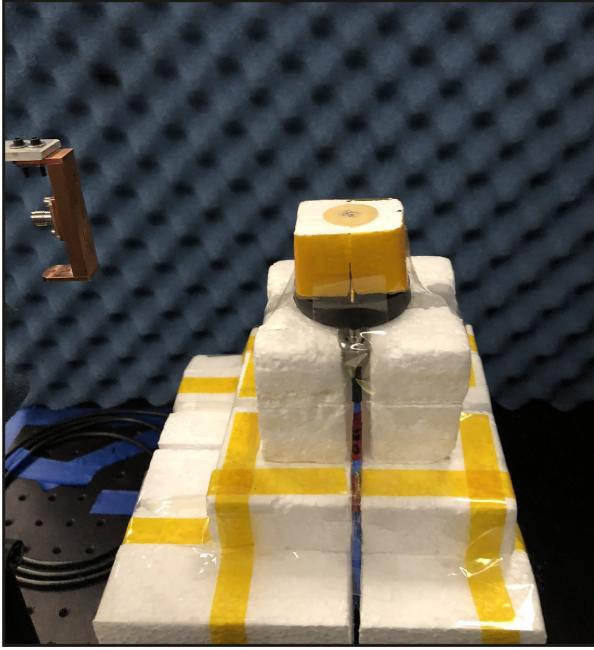


Figure 5.28. A photo of the FSCD antenna and the SYNCA in the synthetic array measurement setup at Penn State.

4504 indicates the ratio of the power received by an antenna in the array to the amount of
4505 power delivered to the SYNCA. Therefore, the analysis focuses on comparing the relative
4506 magnitudes and phase of the S21 parameters measured by the VNA as a function of the
4507 array channel and the SYNCA position. Additionally, a beamforming reconstruction
4508 using the S21 data is done to evaluate how the summed power and beamforming images
4509 change as a function of the position of the SYNCA.

4510 **5.4.3.1 Simulations**

4511 Simulations for the FSCD array measurements were performed using CREsana, which
4512 performs analytical calculations of the EM-fields produced by an electron at the position
4513 of the antennas. At each sampled time CREsana computes the electric field vector at the
4514 antenna positions, which is projected onto the antenna polarization axis to obtain the
4515 co-polar electric field. The magnitude of the co-polar electric field is then multiplied by
4516 a flat antenna transfer function to calculate the corresponding voltage signal. CREsana
4517 simulations exploit the flat transfer functions of the FSCD antennas, which allows the
4518 electric field to be multiplied by the antenna transfer function rather than performing
4519 the full FIR calculation. These calculations produce a voltage time-series for each of the
4520 antennas in the array that can be compared to the laboratory measurements.

CRESana was configured to simulate a 90° electron in a constant background magnetic field of ≈ 0.958 T with a kinetic energy of 18.6 keV. These parameters were chosen in order to mimic a CRES event near the tritium beta-decay spectrum endpoint in the FSCD experiment. The constant background magnetic field guarantees that the guiding center of the electron is stationary across the duration of the simulation which is consistent with the SYNCA in the laboratory measurements. Simulations were performed with the electron's guiding center at radial positions from 0 to 45 mm in steps of 1 mm and axial positions from 0 to 30 mm in steps of 1 mm. The simulations generated time series consisting of 8192 samples at 200 MHz for the sixty channel FSCD antenna array geometry.

5.4.3.2 Phase Analysis

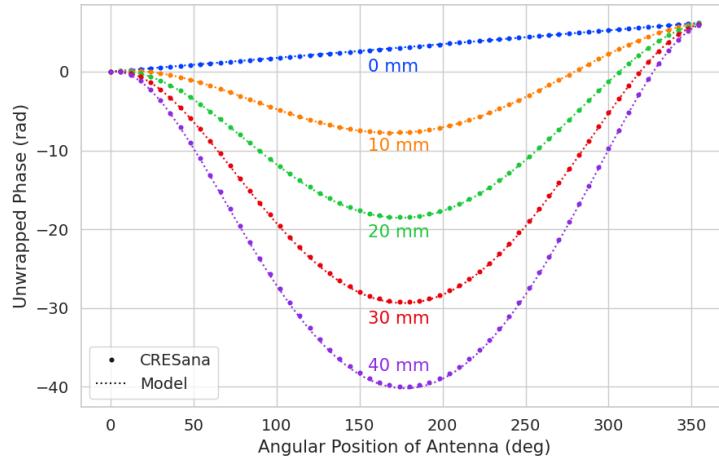


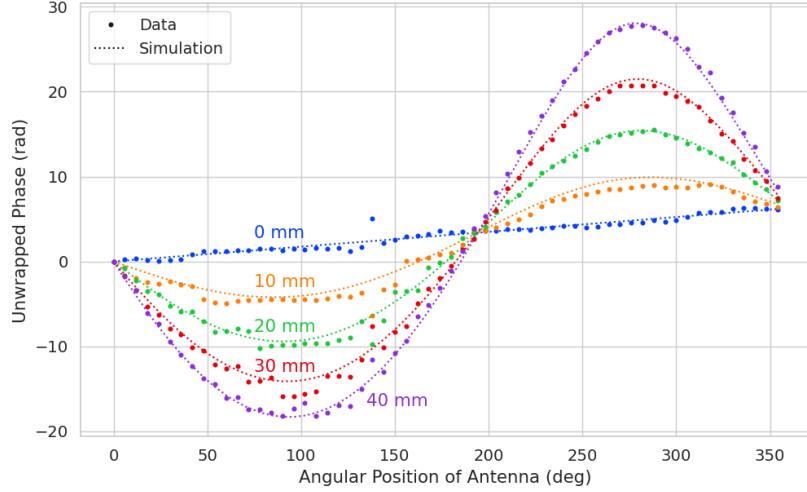
Figure 5.29. The unwrapped phases of signals received by the FSCD antenna array from an electron with a 90° pitch angle located in the plane of the antenna array. The data points indicated the phases extracted from simulation and the dashed lines show the model predictions.

Correct modeling of the signal phases is fundamental to reconstruction for both beamforming and matched filter approaches. The beamforming reconstruction relies on a signal phase model developed from Locust simulations, which allows one to predict the relative signal phases for a specific magnetic trap and electron position. The equation for the model is

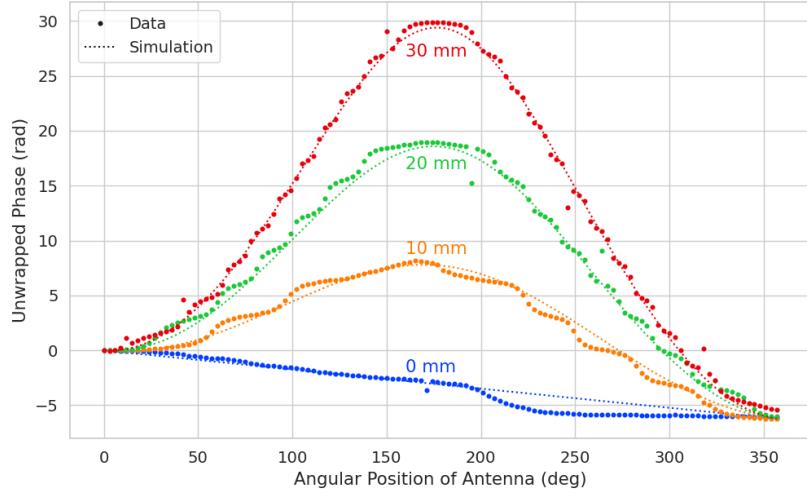
$$\phi_{ij}(t) = \frac{2\pi d_{ij}(t)}{\lambda} + \theta_{ij}(t), \quad (5.36)$$

where $d_{ij}(t)$ is distance between the assumed electron position and the antenna position, and $\theta_{ij}(t)$ is the angular separation between the electron and antenna positions. For

4539 details on the components of the phase model see Section 5.3.2. In Figure 5.29 I compare
 4540 the phases predicted by Equation 5.36 to phases extracted from CRESana simulations of
 4541 an electron located in the plane of the antenna array at a series of radial positions. One
 4542 observes excellent agreement between the model and simulation.



(a)



(b)

Figure 5.30. Plots of the measured unwrapped phases from the FSCD array (a) and the synthetic array (b) compared to the model predictions for a series of radial positions. The different phases of the sinusoidal phase oscillations in the two plots reflects differences in the coordinate systems of the measurements.

4543 The measured signal phases from the FSCD array and synthetic array are shown
 4544 in Figures 5.30a and 5.30b compared to the signal phase model. The axial position of

4545 the SYNCA in both plots is $z = 0$ mm, such that the plane of the PCB is aligned with
 4546 the center of the FSCD antenna. The data shown in Figure 5.30a corresponds to the
 4547 S-parameters measured at 25.80 GHz which is the frequency closest to the one used in
 4548 the synthetic array setup. The different slope and sinusoidal phases exhibited by Figure
 4549 5.30a and 5.30b reflects differences in the coordinate system for each setup. In general,
 4550 the phase model predicts the large scale features of the phases well, but there are some
 4551 small scale deviations or errors from the phase model that do not appear to be present
 4552 in simulation.

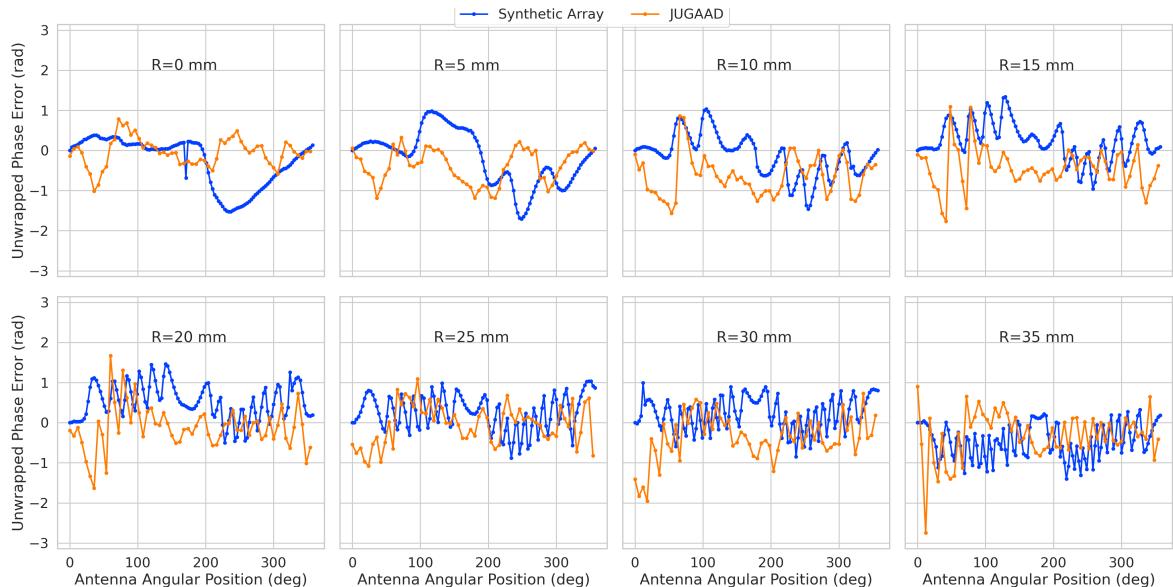


Figure 5.31. The phase errors between the measurement and model for the synthetic array (blue) and the FSCD array (orange) for a series of radial positions. The label JUGAAD refers to an alternative name for the FSCD array setup. As the SYNCA is translated off-axis phase errors with progressively higher oscillation frequency enter into the measurements.

4553 A comparison of the phase errors, which are the difference between measurement and
 4554 model is shown in Figure 5.31. The FSCD array data is referred to as the JUGAAD
 4555 data in the plot legend, which is an alternative name for the FSCD array setup.

4556 The phase error at $R = 0$ in Figure 5.31 forms a smooth curve, with the exception of
 4557 an outlier data point caused by a bug in the data acquisition script. One can attribute
 4558 the observed phase error at this position to imperfections in the antenna pattern of the
 4559 SYNCA. As the SYNCA is moved away from $R = 0$ mm one observes that the phase
 4560 error exhibits oscillations whose frequency increases as a function of the radial position
 4561 of the SYNCA. These oscillations have the appearance of a diffraction pattern, which
 4562 is particularly clear for the radii ≥ 15 mm, due to the bilateral symmetry of the phase

4563 error peaks around 180° .

4564 One can observe a higher average variance in the phase errors measured for the FSCD
4565 array compared to the synthetic array. This is best seen by comparing the curves at
4566 $R \leq 15$ mm where the smooth synthetic array curves are distinct from the relatively
4567 noisy FSCD array errors. The extra noise in the FSCD array is most likely caused by
4568 differences in the radiation patterns of the antennas that make up the array as well as
4569 differences in the transmission lines through the switch network that introduce additional
4570 phase errors into the measurement. Since the synthetic array measurements use only
4571 a single antenna, these extra error terms are not present, which explains the relatively
4572 smoother phase error curves. Despite the extra phase errors in the FSCD array, it is still
4573 possible to observe a similar phase error oscillation effect as the SYNCA is moved away
4574 from $R = 0$ mm.

4575 The diffraction pattern exhibited by the phase error oscillations is more easily observed
4576 by plotting the phase errors in a two-dimensional map, which is done in Figures 5.32a and
4577 5.32b. For the synthetic array data ones observes a relatively clear diffraction pattern
4578 that emerges as the SYNCA is moved radially. The bilateral symmetry of the diffraction
4579 patterns is due to the bilateral symmetry of the circular synthetic array around the
4580 translation axis of the SYNCA. A similar pattern is also visible in the FSCD array data,
4581 although, it is obscured by the additional phase error that results from the multi-channel
4582 array.

4583 The physical origin of the phase error diffraction pattern is attributed to interference
4584 effects arising from path-length differences between the individual slots in the FSCD
4585 antenna and the SYNCA transmitter. Since measurements are being performed in the
4586 radiative near-field of the FSCD antenna, the path length differences between the slots
4587 introduces a significant change in the summation of the signals that occurs inside the
4588 waveguide, which causes the radiation pattern of the antenna to change as a function of
4589 distance. Therefore, when the SYNCA is positioned off-axis the different path-lengths
4590 from the SYNCA to each antenna results in different radiation patterns leading to the
4591 observed diffraction pattern.

4592 This near-field effect is not present in simulations, because in order to simplify the
4593 calculations it is assumed that the far-field approximation can be applied to the FSCD
4594 antennas. This means that the radiation pattern and antenna transfer functions are
4595 independent of the distance between the transmitter and the receiving antenna. In
4596 principle, the near-field effects can be accounted for with a more detailed simulation of
4597 the FSCD antennas either in CRESana or Locust, which would result in an additional

4598 term in the beamforming phase model. However, this would increase the computational
4599 intensity of the simulation software. In the next section I briefly discuss the impact of
4600 these near-field effects on the measured magnitude of the signals.

4601 **5.4.3.3 Magnitude Analysis**

4602 Exactly as for the signal phase, one can use simulations to construct a model that
4603 describes the magnitude of the signals received by each channel in the antenna array.
4604 By examining the results of simulations or by analyzing the Liénard-Wiechert equation
4605 one can show that radiation pattern from a 90° pitch angle electron in a magnetic field
4606 is omni-directional. Therefore the relative magnitudes of the signals received by each
4607 channel will be determined by the free-space power loss, which is proportional to the
4608 inverse distance between the assumed electron position and the antenna.

4609 A consequence of this is that the signals produced in the array for electrons off the
4610 central axis will have larger amplitudes for the antennas closer to the electron compared
4611 to those which are further away. The amplitudes of the signals received by the array
4612 from an electron located at a series of radial positions are shown in Figure 5.33.

4613 One expects to see a similar trend in the signal magnitudes in both the FSCD and
4614 synthetic arrays. The normalized signal magnitudes extracted from the full and synthetic
4615 array setups for a series of radial SYNCA positions are shown in Figure 5.34. The data
4616 corresponds to a SYNCA axial position of $z = 0$ mm and at a frequency 25.86 GHz. One
4617 complication is that the radiation pattern of the SYNCA is not perfectly omni-directional,
4618 which causes the measured magnitudes at $R = 0$ mm to diverge from the perfectly flat
4619 behavior exhibited by electrons.

4620 As the SYNCA is moved off-axis one observes a similar increase in the number of
4621 magnitude peaks in the synthetic array data that one would expect from a diffraction
4622 pattern, although this trend is not as stark compared to the phase data. Noticeably,
4623 there does not appear to be a set of channels with disproportionately larger amplitude at
4624 large R , which would be expected based on the trends from CREsana.

4625 Comparing the magnitudes of the synthetic array to the FSCD array in Figure 5.34,
4626 one observes a similar amount of variability in the magnitudes at $R = 0$ mm, although
4627 there is potentially more small scale error in the magnitude curve caused by channel
4628 differences in the FSCD array. A similar trend is seen in the number of magnitude error
4629 peaks in the FSCD array data to the synthetic array data, which mirrors the diffraction
4630 effect observed in the phase data. The diffraction effect can be visualized more clearly
4631 by plotting a similar two-dimensional map of the magnitudes (see Figure 5.35).

4632 The fact that one observes a similar diffraction pattern in the signal magnitudes
4633 as a function the SYNCA position reinforces the conclusions from the phase analysis
4634 that near-field effects are having a significant impact on the radiation pattern of the
4635 FSCD array. These near-field effects lead to changes in the magnitude and phase of the
4636 radiation pattern of the FSCD antenna as a function of distance. If left uncorrected these
4637 errors reduce detection efficiency by causing power loss in the beamforming or matched
4638 filter reconstruction due to phase mismatch. I explore the impact of these phase and
4639 magnitude errors on beamforming in the next section.

4640 **5.4.3.4 Beamforming Characterization**

4641 Errors in the signal magnitudes and phases lead to errors in signal reconstruction. For
4642 example, a matched filter reconstruction requires accurate knowledge of the signals in
4643 each channel to achieve optimal performance. Uncorrected errors leads to mismatches
4644 between the template and signal, which reduces detection efficiency and introduces
4645 uncertainty in the parameter estimation. In this section, I analyze the beamformed signal
4646 amplitude as a function of the position of the SYNCA to quantify the impact of the
4647 phase and magnitude errors on signal reconstruction. Because of the imperfections in
4648 the SYNCA source, it is inappropriate to directly compare the measured beamformed
4649 signal amplitudes of the FSCD array or synthetic array to simulations. Because such a
4650 comparison, would not allow one to disentangle losses that occur because of the antenna
4651 array from those that occur because of the source. Therefore, I focus on comparing
4652 the measured beamforming results of the FSCD array to the synthetic array, since the
4653 SYNCA imperfections are common to both setups.

4654 The first method of comparison is to analyze the images generated by applying the
4655 beamforming reconstruction specified in Section 4.3.1 to the FSCD and synthetic array
4656 data (see Figure 5.36). The beamforming grid consisting of a square 121×121 grid
4657 spanning a range of -60-mm to 60 mm in the x and y dimensions. The beamforming
4658 images formed from the synthetic array produces a three-dimensional matrix where each
4659 grid position contains a summed time series. A single beamforming image is formed from
4660 this data matrix by taking the mean over the time dimension. In the case of the FSCD
4661 array, the VNA generates frequency domain data such that each grid position contains a
4662 summed frequency series produced by the VNA sweep. For this data a single image is
4663 formed by averaging in the frequency domain.

4664 There is a clear difference between the synthetic and FSCD array beamforming images,
4665 which is the additional faint beamforming maxima located directly opposite the maxima

4666 corresponding to the SYNCA position. The images in Figure 5.36 were generated with
4667 data collected at a SYNCA radial position of 15 mm, which agrees well with the observed
4668 beamforming maximum in both images. The faint beamforming peak is located directly
4669 opposite of the true beamforming maximum similar to a mirror image. Therefore, the
4670 origin of this additional feature appears to be reflections between the two sides of the
4671 circular antenna array that are not present for the synthetic array since only a single
4672 physical antenna is used.

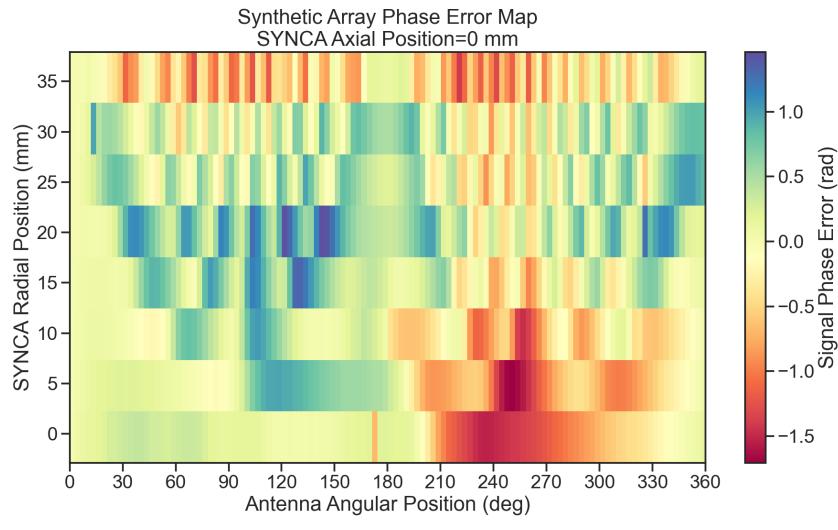
4673 From the beamforming images the maximum amplitude is extracted, which can be
4674 plotted as a function of the radial position of the SYNCA (see Figure 5.37). The phase
4675 errors observed in the FSCD and synthetic arrays leads to power loss at the beamforming
4676 stage due to phase mismatches between the signals at different channels. This power loss
4677 can be quantified by comparing the signal amplitude obtained from beamforming to the
4678 amplitude which would be obtained from an ideal summation. The ideal summation is
4679 performed by phase shifting each array channel to an identical phase and then summing.
4680 The comparison between the beamforming and ideal sums is shown in Figure 5.37,
4681 where it is seen that the synthetic and FSCD arrays experience power losses from the
4682 beamforming summation.

4683 The beamforming power loss can be quantified using the ratio of the beamforming to
4684 ideal signal amplitudes. Computing this ratio as a function of SYNCA radial position
4685 radius for the FSCD and synthetic arrays, it is found that the FSCD array has a uniformly
4686 smaller beamforming amplitude ratio, which means that the FSCD array has a larger
4687 beamforming power loss (see Figure 5.38). The primary contributions to the beamforming
4688 power loss in the synthetic array are phase errors from the SYNCA and phase errors
4689 from the FSCD antenna near-field. Both of these phase errors contribute to beamforming
4690 losses in the FSCD array, but there are clearly additional phase errors in the FSCD array
4691 measurements contributing to the smaller ratio. Two potential error sources include phase
4692 differences in the different antenna channels that could not be corrected by calibration as
4693 well as reflections between antennas in the array. The total effect of these additional phase
4694 errors is to reduce the beamforming amplitude ratio by about 5% from the beamforming
4695 ratio of the synthetic array. Therefore, it is estimated that if no effort is made to correct
4696 these phase errors in an FSCD-like experiment, then one would expect approximately a
4697 10% total signal amplitude loss from a beamforming signal reconstruction.

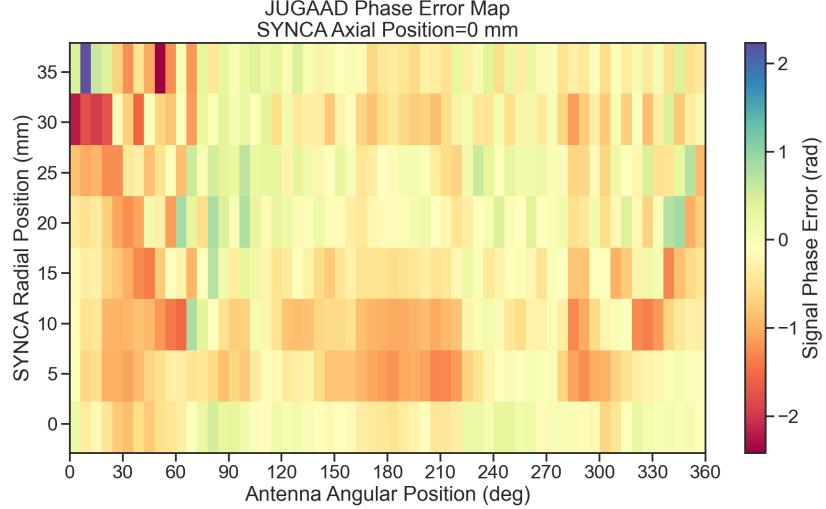
4698 5.4.4 Conclusions

4699 The estimated power loss of a beamforming reconstruction obtained from this analysis
4700 provides valuable inputs to sensitivity calculations of a FSCD-like antenna array exper-
4701 iment to measure the neutrino mass, since it helps to bound systematic uncertainties
4702 from the antenna array and reconstruction pipeline. This power loss lowers the estimated
4703 detection efficiency of the experiment since some of the signal power is lost due to
4704 improper combining between channels and also increases the uncertainty in the electron's
4705 kinetic energy by contributing to errors in the estimation of the electron's cyclotron
4706 frequency.

4707 If these reconstruction losses prove unacceptable there are steps that can be taken
4708 to mitigate their effects. Some examples include the development of a more accurate
4709 antenna simulation approach that can reproduce the observed near-field interference
4710 patterns of the FSCD antennas and the implementation of a calibration approach that
4711 allows for the relative phase delays of the array to be measured without changing or
4712 disconnecting the antenna array configuration.



(a)



(b)

Figure 5.32. Two dimensional plots of the phase errors for the synthetic array (a) and the FSCD (JUGAAD) array (b). In both plots there is evidence of a similar diffraction pattern with bilateral symmetry, but the FSCD array measurements have an additional phase error contribution from the different antennas and paths through the switch network.

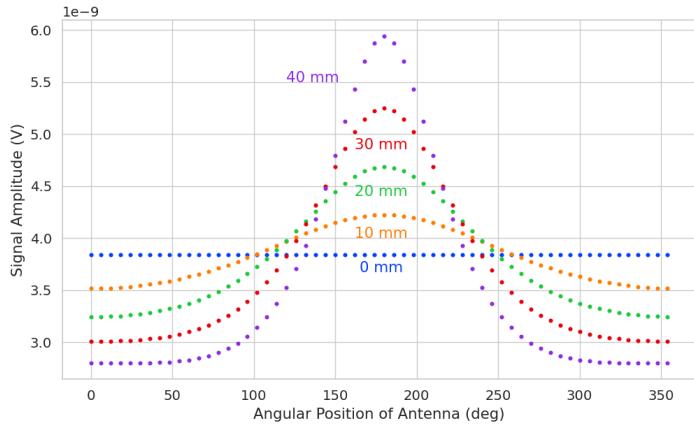


Figure 5.33. The amplitude of the signals from CREsana for the FSCD array from a 90° electron. As the electron is moved from $R = 0$ the signals begin to have unequal amplitudes depending on the distance from the electron to the antenna.

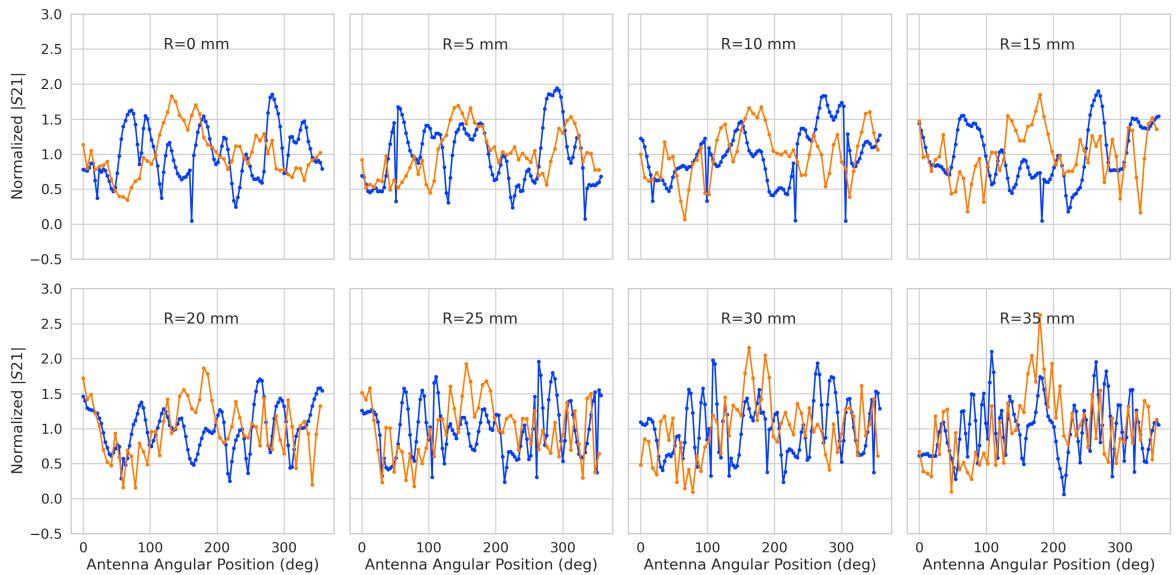
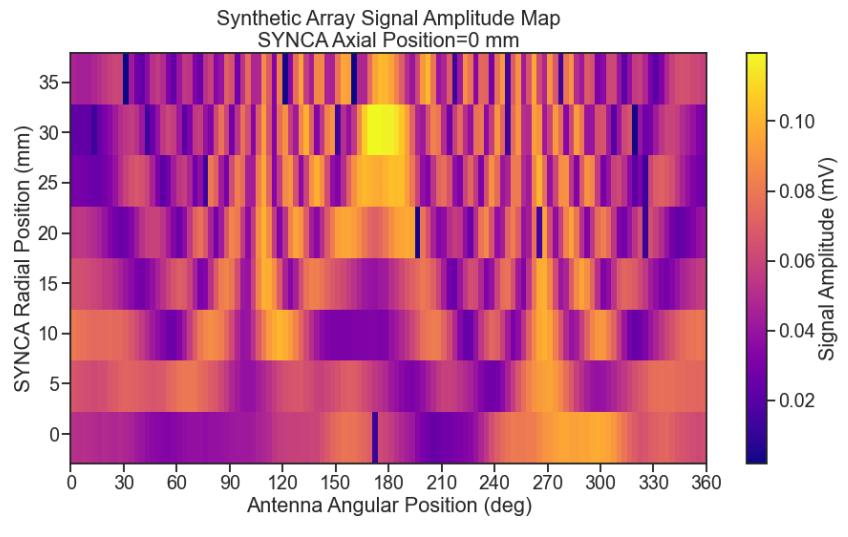
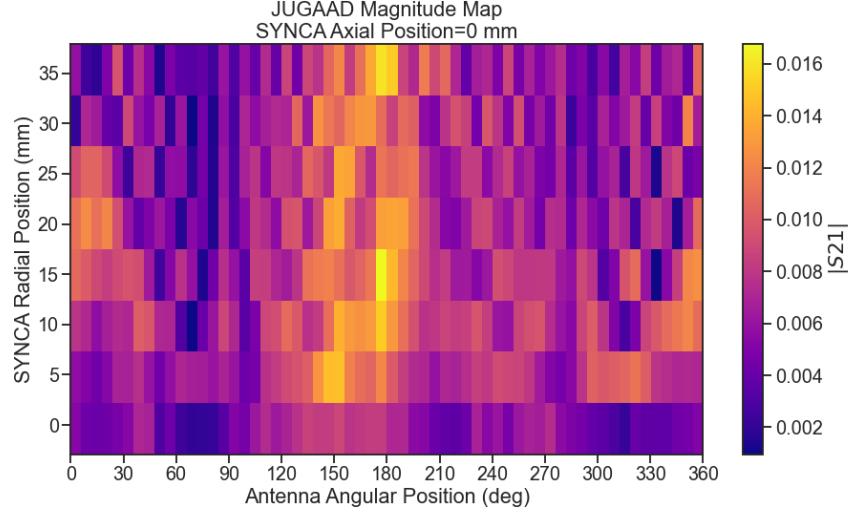


Figure 5.34. The normalized magnitudes of the S21 parameters measured in the FSCD (orange) and synthetic array (blue) setups. The dominant observed behavior as a function of radius is the increase in the number of magnitude peaks, which was noted in the phase error curves. There does not appear to be a strong change in the relative amplitude of a group of antennas as predicted by CREsana.



(a)



(b) The two-dimensional maps showing the diffractive pattern exhibited by the FSCD and synthetic array signal magnitudes.

Figure 5.35.

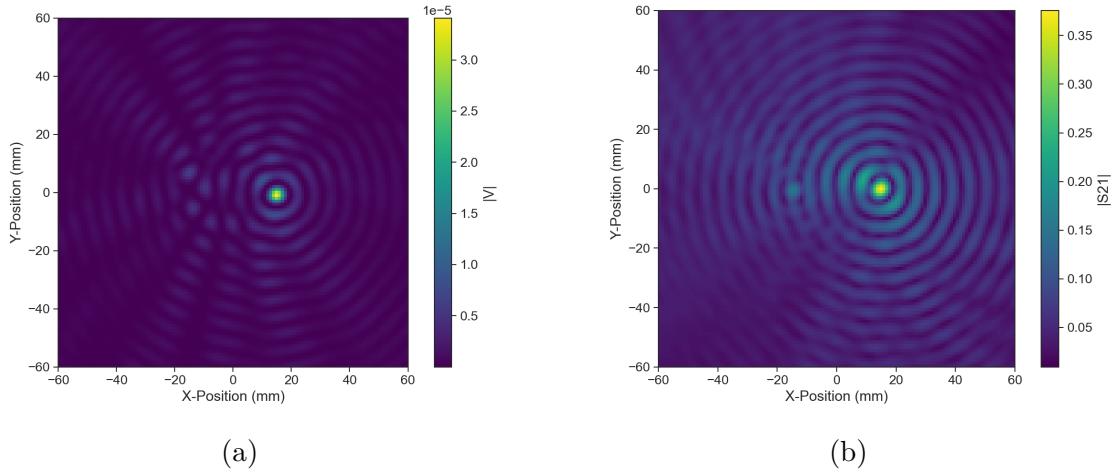


Figure 5.36. Beamforming images from the synthetic array (a) and FSCD array (b) setups with the SYNCA positioned 15 mm off the central axis. In both images, there is a clear maxima that corresponds to the true SYNCA position. However, in the FSCD array there is an additional faint peak located at the opposite position of the beamforming maximum. This additional peak is the mirror of the true peak and is the result of reflections between antennas in the FSCD array.

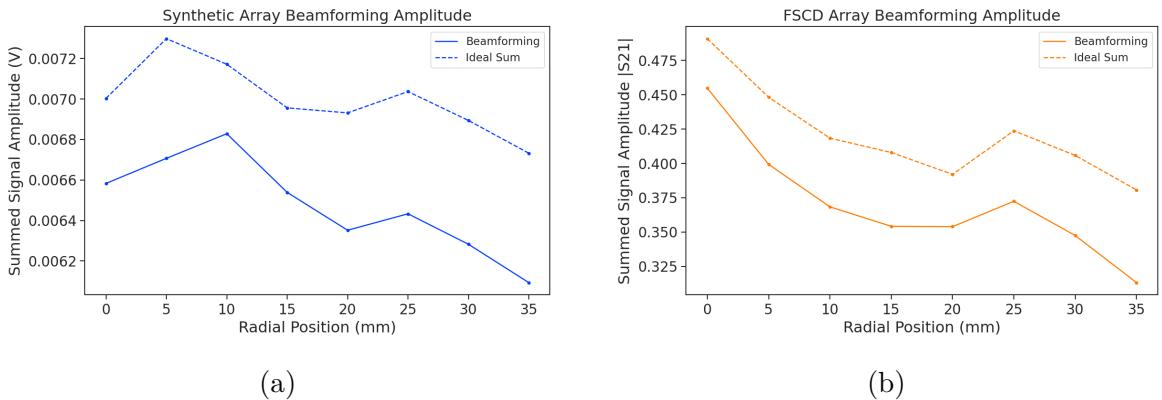


Figure 5.37. A comparison of the maximum signal amplitude obtained by beamforming to the signal amplitude obtained with an ideal summation as a function of the radial position of the SYNCA. The amplitudes for the synthetic array are shown in (a) and the FSCD array are shown in (b). In both setups, the signal amplitudes obtained from beamforming are smaller than the signal amplitude that could be attained with the ideal summation without phase mismatch.

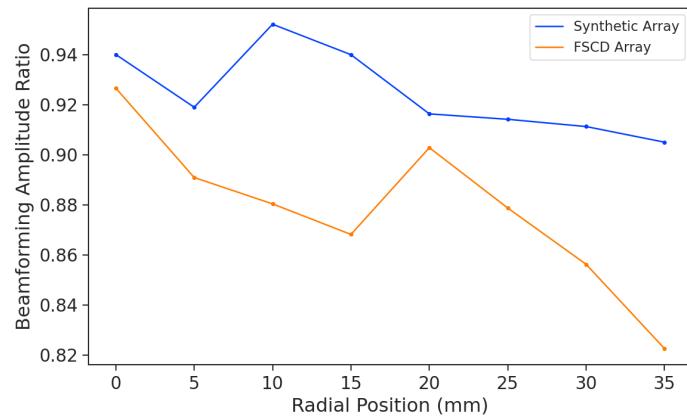


Figure 5.38. The ratio of the beamforming signal amplitude to the ideal signal amplitude for the FSCD and synthetic arrays. The FSCD array has a larger power loss from phase error compared to the synthetic array which indicates that calibration errors associated with the multiple channels as well as reflections are impacting the signal reconstruction.

Chapter 6

Development of Resonant Cavities for Large Volume CRES Measurements

6.1 Introduction

The cavity approach was originally an alternative CRES measurement technology under consideration by the Project 8 collaboration for the Phase IV experiment. After pursuing an antenna array based CRES demonstrator design for several years, the increasing costs and complexity of the antenna arrays led to a reconsideration of the baseline technology for the ultimate CRES experiment planned by Project 8. Currently, a cavity based CRES experiment is the preferred technology choice for future experiments by the Project 8 collaboration including the Phase IV experiment.

In this chapter I provide a brief summary of resonant cavities and sketch out the key features of a cavity based CRES experiment. In Section 6.2 I provide a brief introduction to cylindrical resonant cavities and the solutions for the electromagnetic fields in the cavity volume.

In Section 6.3 I describe the main components of a cavity based CRES experiment, including the background and trap magnets, cavity geometry and design, and cavity coupling considerations. I also discuss some relevant trade-offs between an antenna array and cavity CRES experiment, and highlight some reasons for the transition of Project 8 to the development of a cavity based experiment.

Finally, in Sections 6.4 and 6.5, I present the design and development of an open mode-filtered cavity that could be used in a cavity based CRES experiment with atomic tritium. The results of the cavity simulations are confirmed by laboratory measurements of a proof-of-principle prototype that demonstrates key features of the design.

6.2 Cylindrical Resonant Cavities

Resonant cavities are sealed conductive containers, which allows one to describe the electromagnetic (EM) fields contained in the cavity volume as a superposition of resonant modes [92]. The field shapes of the resonant modes are determined by Maxwell's equations and the boundary conditions enforced by the cavity geometry. Of interest to Project 8 for CRES measurements are cylindrical cavities due to their ease of construction and integration with atom and electron trapping magnets.

6.2.1 General Field Solutions

Consider a long segment of conducting material with a cylindrical cross-section (see Figure 6.1). A geometry such as this can be used as a waveguide transmission line to transfer EM energy from point to point, or, if conducting shorts are inserted on both ends of the cylinder, the waveguide becomes a resonant cavity.

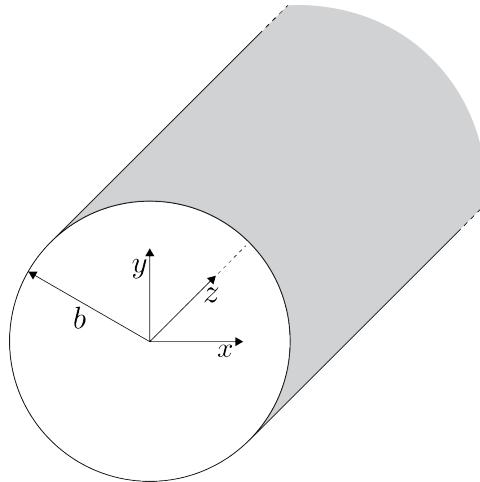


Figure 6.1. Geometry of a cylindrical waveguide with radius b .

The fields allowed inside a cylindrical cavity are determined by the boundary conditions of the cylindrical geometry. The general approach to solving the fields begins by assuming solutions to Maxwell's equations of the form

$$\mathbf{E}(x, y, z) = (\mathbf{e}(x, y) + \hat{z}e_z(x, y))e^{-i\beta z}, \quad (6.1)$$

$$\mathbf{H}(x, y, z) = (\mathbf{h}(x, y) + \hat{z}h_z(x, y))e^{-i\beta z}. \quad (6.2)$$

The solutions assume a harmonic time dependence of the form $e^{i\omega t}$ and propagation

4753 along the positive z-axis. The functions $\mathbf{e}(x, y)$ and $\mathbf{h}(x, y)$ represent the transverse
 4754 (\hat{x}, \hat{y}) components of the electric and magnetic fields respectively, and $e_z(x, y)$, $h_z(x, y)$
 4755 represent the longitudinal components. The version of Maxwell's equations in the case
 4756 where there are no source terms can be written as a pair of coupled differential equations,

$$\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}, \quad (6.3)$$

$$\nabla \times \mathbf{H} = i\omega\epsilon\mathbf{E}, \quad (6.4)$$

4757 where ϵ and μ are the permittivity and permeability of the material inside the waveguide
 4758 or cavity. Using the field solutions from Equations 6.1 and 6.2 one can solve for the
 4759 transverse components of the fields in terms of the longitudinal fields. Because cylindrical
 4760 cavities are of interest, it is advantageous to write the field solutions in cylindrical
 4761 coordinates. After performing this transformation, the set of four equations for the
 4762 transverse field components are

$$H_\rho = \frac{i}{k_c^2} \left(\frac{\omega\epsilon}{\rho} \frac{\partial E_z}{\partial\phi} - \beta \frac{\partial H_z}{\partial\rho} \right), \quad (6.5)$$

$$H_\phi = \frac{-i}{k_c^2} \left(\omega\epsilon \frac{\partial E_z}{\partial\rho} + \frac{\beta}{\rho} \frac{\partial H_z}{\partial\phi} \right), \quad (6.6)$$

$$E_\rho = \frac{-i}{k_c^2} \left(\beta \frac{\partial E_z}{\partial\rho} + \frac{\omega\mu}{\rho} \frac{\partial H_z}{\partial\phi} \right), \quad (6.7)$$

$$E_\phi = \frac{i}{k_c^2} \left(\frac{-\beta}{\rho} \frac{\partial E_z}{\partial\phi} + \omega\mu \frac{\partial H_z}{\partial\rho} \right), \quad (6.8)$$

4763 where k_c is the cutoff wavenumber defined by $k_c^2 = k^2 - \beta^2$ with $k = \omega\sqrt{\mu\epsilon}$ being the
 4764 wavenumber of the EM radiation.

4765 This set of equations can be used to solve for a variety of different modes, which can
 4766 be obtained by setting conditions on E_z and H_z . For cylindrical cavities two types of
 4767 modes are allowed, which correspond to solutions where $E_z = 0$ and $H_z = 0$ respectively.

4768 6.2.2 TE and TM Modes

4769 The TE family of modes corresponds to the case where $E_z = 0$. This implies that H_z is
 4770 a solution to the Helmholtz wave equation

$$(\nabla^2 + k^2)H_z = 0. \quad (6.9)$$

4771 For solutions of the form $H_z(\rho, \phi, z) = h_z(\rho, \phi)e^{-i\beta z}$, Equation 6.9 can be solved using
 4772 the standard technique of separation of variables. Rather than reproduce the derivation
 4773 here I shall simply quote the solutions for the transverse fields [92], which are

$$H_\rho = \frac{-i\beta}{k_{c_{nm}}} (A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.10)$$

$$H_\phi = \frac{-i\beta n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.11)$$

$$E_\rho = \frac{-i\omega\mu n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.12)$$

$$E_\phi = \frac{i\omega\mu}{k_{c_{nm}}} (A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}. \quad (6.13)$$

4774 One observes that the solutions have a periodic dependence on ϕ , and radial profiles
 4775 given by the Bessel functions of the first kind. The integer indices n and m arise from
 4776 continuity conditions on the EM fields in the azimuthal and radial directions. For the
 4777 TE modes, the indices range from $n \geq 0$ and $m \geq 1$. $k_{c_{nm}}$ is the cutoff wavenumber for
 4778 the TE_{nm} mode given by

$$k_{c_{nm}} = \frac{p'_{nm}}{b}, \quad (6.14)$$

4779 where b is the radius of the cavity or waveguide and p'_{nm} is the m -th root of the derivative
 4780 of the n -th order Bessel function (see Table 6.1).

Table 6.1. A table of the values of p'_{nm} .

n	p'_{n1}	p'_{n2}	p'_{n3}
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.970

4781 The TM mode family corresponds to the case where $H_z = 0$, and $(\nabla^2 + k^2)E_z = 0$.
 4782 Again, solutions are assumed of the form $E_z(\rho, \phi, z) = e_z(\rho, \phi)e^{-i\beta z}$, for which the general
 4783 form of the solutions is the same as for the TE modes. However, the different boundary
 4784 conditions for the TM modes results in particular solutions with a different form, which I
 4785 shall quote here without derivation. The transverse fields of the TM modes are given by

$$H_\rho = \frac{-i\omega\epsilon n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.15)$$

$$H_\phi = \frac{-i\omega\epsilon}{k_{c_{nm}}} (A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z} \quad (6.16)$$

$$E_\rho = \frac{-i\beta}{k_{c_{nm}}} (A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.17)$$

$$E_\phi = \frac{-i\beta n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.18)$$

⁴⁷⁸⁶ which one may notice are the same solutions as the TE modes with H and E flipped.

⁴⁷⁸⁷ The cutoff wavenumber for the TM modes is given by, $k_{c_{nm}} = p_{nm}/b$, where the values of

⁴⁷⁸⁸ p_{nm} correspond to the m -th zero of the n -th order Bessel function (see Table 6.2).

Table 6.2. A table of the values of p_{nm} .

n	p_{n1}	p_{n2}	p_{n3}
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

⁴⁷⁸⁹ 6.2.3 Resonant Frequencies of a Cylindrical Cavity

⁴⁷⁹⁰ A cylindrical cavity is constructed by taking a section of cylindrical waveguide and
⁴⁷⁹¹ shorting both ends with conductive material. This means that the electric fields inside a
⁴⁷⁹² cylindrical cavity are exactly those derived in Section 6.2.2 with the additional condition
 that the electric fields must go to zero at $z = 0$ and $z = L$ (see Figure 6.2).

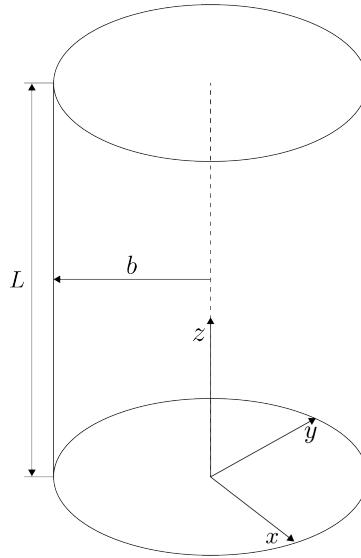


Figure 6.2. The geometry of a cylindrical cavity with length L and radius b .

⁴⁷⁹³

4794

The transverse electric field solutions for a cylindrical waveguide are of the form

$$\mathbf{E}(\rho, \phi, z) = \mathbf{e}(\rho, \phi) (A_+ e^{-i\beta_{nm}z} + A_- e^{i\beta_{nm}z}), \quad (6.19)$$

4795 where A_+ and A_- are arbitrary amplitudes of forward and backward propagating waves.4796 In order to enforce that \mathbf{E} is zero at both ends of the cavity it is required that

$$\beta_{nm}L = 2\pi\ell, \quad (6.20)$$

4797 where $\ell = 0, 1, 2, 3, \dots$. Using this constraint on the propagation constant one can solve
4798 for the resonant frequencies of the TE_{nml} and the TM_{nml} modes in a cylindrical cavity.
4799 For the TE modes the resonant frequencies are

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{p'_{nm}}{b}\right)^2 + \left(\frac{\ell\pi}{L}\right)^2}, \quad (6.21)$$

4800 and the frequencies of the TM modes are

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{p_{nm}}{b}\right)^2 + \left(\frac{\ell\pi}{L}\right)^2}. \quad (6.22)$$

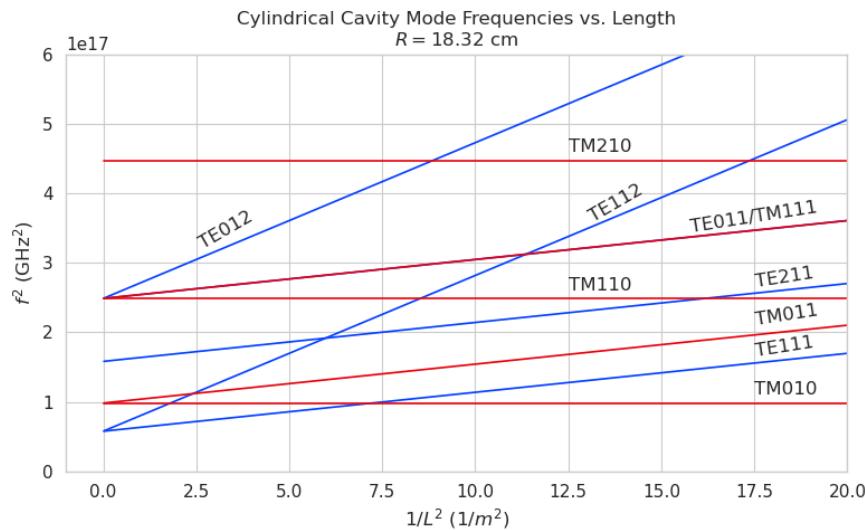


Figure 6.3. Relation of mode frequency to cavity length for a cylindrical cavity with a radius of 18.32 cm.

4801 6.2.4 Cavity Q-factors

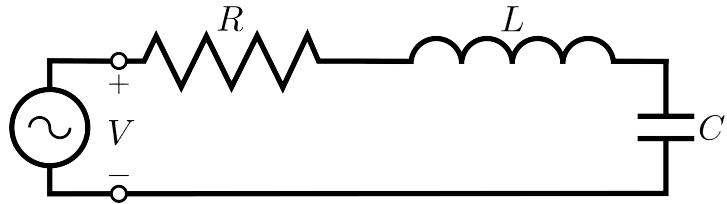


Figure 6.4. A series RLC circuit.

4802 The resonant behavior of cylindrical cavities can be modeled as a series RLC circuit
 4803 (see figure 6.4). The input impedance of the circuit can be obtained by applying
 4804 Kirchhoff's laws to calculate the impedance of the equivalent circuit. For a series RLC
 4805 circuit the input impedance is

$$Z_{\text{in}} = \left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C \right). \quad (6.23)$$

4806 The resistance in the circuit represents all sources of loss in the cavity, which is primarily
 4807 caused by the finite conductivity of the cavity walls. The inductor and capacitor represent
 4808 the energy stored in the cavity in the form of electric and magnetic fields. If the circuit
 4809 is being driven by an external power source the input power can be written in terms of
 4810 the circuit input impedance and the source voltage

$$P_{\text{in}} = \frac{1}{2} Z_{\text{in}} |I|^2 = \frac{1}{2} |I|^2 \left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C \right). \quad (6.24)$$

4811 The resistor introduces a loss into the system with a power given by

$$P_{\text{loss}} = \frac{1}{2} |I|^2 R, \quad (6.25)$$

4812 and the capacitor and inductor store energies given by

$$W_e = \frac{1}{4} \frac{|I|^2}{\omega^2 C}, \quad (6.26)$$

$$W_m = \frac{1}{4} |I|^2 L, \quad (6.27)$$

4813 respectively. Using these expressions the input power and input impedance can be written

4814 in terms of the lost power and stored energy

$$P_{\text{in}} = P_{\text{loss}} + 2i\omega(W_m - W_e), \quad (6.28)$$

$$Z_{\text{in}} = \frac{P_{\text{loss}} + 2i\omega(W_m - W_e)}{\frac{1}{2}|I|^2}. \quad (6.29)$$

4815 The condition for resonance in the RLC circuit is that the stored magnetic energy
4816 is equal to the stored electric energy ($W_e = W_m$). When this occurs $Z_{\text{in}} = R$, which is a
4817 purely real impedance, and $P_{\text{in}} = P_{\text{loss}}$. The resonant frequency of the circuit can be
4818 determined from the condition $W_e = W_m$ from which one finds that

$$\omega_0 = \frac{1}{\sqrt{LC}}. \quad (6.30)$$

4819 An important performance parameter for any resonant system is the Q-factor, which
4820 quantifies the quality of the resonator as the ratio of the stored energy multiplied by the
4821 resonant frequency to the average energy lost per second. For the series RLC circuit, the
4822 Q-factor is given by the expression

$$Q_0 = \omega \frac{W_e + W_m}{P_{\text{loss}}} = \frac{1}{\omega_0 RC}, \quad (6.31)$$

4823 from which one observes that as the resistance of the RLC circuit is decreased the quality
4824 factor of the resonator increases. From the perspective of cylindrical cavities this implies
4825 that as one decreases the resistance of the cavity walls it is expected that the Q-factor of
4826 the cavity should increase, which is indeed the case. In certain applications where a high
4827 Q is desireable it is possible to manufacture a cavity out of superconducting materials in
4828 order to minimize the power losses of the system.

4829 The Q-factor of the resonator also determines with bandwidth (BW) of the system. A
4830 cavity with a high Q-factor will resonant with a smaller range of frequencies than a cavity
4831 with a low Q-factor. To see this examine the behavior of the RLC circuit when driven by
4832 frequencies near the resonance. For a frequency $\omega = \omega_0 + \Delta\omega$, where $\Delta\omega = \omega - \omega_0 \ll \omega_0$,
4833 the input impedance can be written as

$$Z_{\text{in}} = R + i\omega L \left(\frac{\omega^2 - \omega_0^2}{\omega^2} \right), \quad (6.32)$$

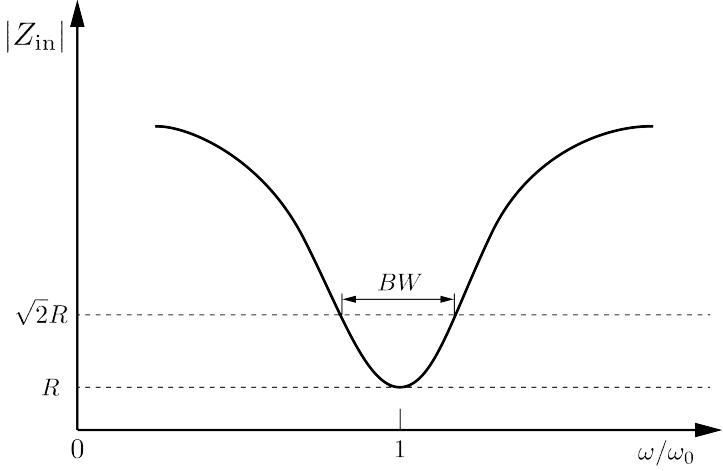


Figure 6.5. Illustration of the behavior of the input impedance of the series RLC circuit as a function of the driving frequency. The BW is proportion to the width of the resonance, which is inversely proportional to Q.

4834 and by expanding $(\omega^2 - \omega_0^2)/\omega^2$ to first order in $\Delta\omega$, one obtains

$$Z_{\text{in}} \approx R + i \frac{2RQ_0\Delta\omega}{\omega_0}. \quad (6.33)$$

4835 Therefore, the magnitude of the input impedance near the resonance is given by

$$|Z_{\text{in}}| = R \sqrt{1 + 4Q_0^2 \frac{\Delta\omega^2}{\omega^2}}, \quad (6.34)$$

4836 from which it is seen that for the series RLC circuit the input impedance is minimized
 4837 at the resonant frequency, which corresponds to the maximum input power (see Figure
 4838 6.5). The half-power BW is the range of frequencies over which the input power drops to
 4839 half the input power on resonance. This occurs when $|Z_{\text{in}}| = \sqrt{2}R$, which corresponds to
 4840 $\Delta\omega/\omega = \text{BW}/2$. Using Equation 6.34 one can find that

$$2R^2 = R^2(1 + Q_0^2\text{BW}^2), \quad (6.35)$$

4841 which implies

$$\text{BW} = \frac{1}{Q_0} \quad (6.36)$$

4842 It is important to emphasize that the Q-factor defined here, Q_0 , is technically the
 4843 unloaded Q. It reflects the quality of the cavity or resonant circuit without the influence
 4844 of any external circuitry. In practice, however, a cavity is invariably coupled to an

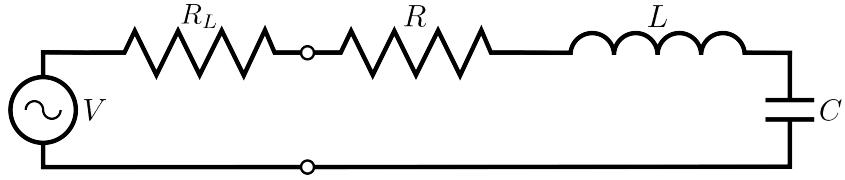


Figure 6.6. A series RLC circuit coupled to an external circuit with input impedance R_L .

4845 external circuit to drive a cavity resonance or to measure the energy of a resonant mode.
 4846 Coupling a cavity to an external circuit changes the Q by loading the equivalent cavity
 4847 RLC circuit (see Figure 6.6). The Q-factor of the cavity when it is loaded by an external
 4848 circuit is called the loaded Q, which is the quantity that one actually measures when
 4849 exciting a resonance in the cavity. Using the series RLC circuit model one can see that
 4850 the load resistor in Figure 6.6 will add in series with the resistor in the circuit for a total
 4851 equivalent resistance of $R + R_L$. Therefore, the loaded Q is given by

$$Q_L = \frac{1}{\omega_0(R + R_L)C}, \quad (6.37)$$

4852 from which one observes that the loaded Q is always less than the intrinsic Q of the
 4853 cavity.

4854 The amount of coupling that is desireable depends on the specific application of
 4855 the resonator. If one wants a resonator that is particular frequency selective then it
 4856 makes sense to limit the amount of coupling to the cavity to maintain a small BW,
 4857 alternatively, if a larger BW is need one can increase the cavity coupling by tuning the
 4858 input impedance of the external circuit. The critical point, where maximum power is
 4859 transferred between the cavity and the external circuit, occurs when the input impedance
 4860 of the cavity matches the input impedance of the external transmission line. For the
 4861 series RLC circuit on resonance, this matching condition corresponds to

$$Z_0 = Z_{in} = R, \quad (6.38)$$

4862 where Z_0 is the impedance of the transmission line. The loaded Q at this critical point
 4863 is, therefore,

$$Q_L = \frac{1}{2\omega_0 Z_0 C} = \frac{Q_0}{2}. \quad (6.39)$$

4864 One can described the degree of coupling between the cavity and an external circuit by

4865 defining a coupling factor, g , such that,

$$g = \frac{Q_0}{Q_L} - 1. \quad (6.40)$$

4866 When $g = 1$ then $Q_L = Q_0/2$, and the cavity is said to be critically coupled. If
4867 $Q_L < Q_0/2$, then the cavity is undercoupled to the transmission line, corresponding to
4868 $g < 1$. Alternatively, if $Q_L > Q_0/2$, then $g > 1$, and the cavity is overcoupled to the
4869 transmission line. Various specialized circuits can be used to tune the input impedance
4870 of the external circuit as seen by the cavity to achieve a wide range of different coupling
4871 factors based on the desired application of the cavity.

4872 6.3 The Cavity Approach to CRES

4873 6.3.1 A Sketch of a Molecular Tritium Cavity CRES Experiment

4874 Resonant cavities can be used to perform CRES measurements, and they represent the
4875 current preferred technology by the Project 8 collaboration. The basic approach to a
4876 neutrino mass measurement using a resonant cavity and molecular tritium beta-decay
source is illustrated by Figure 6.7.

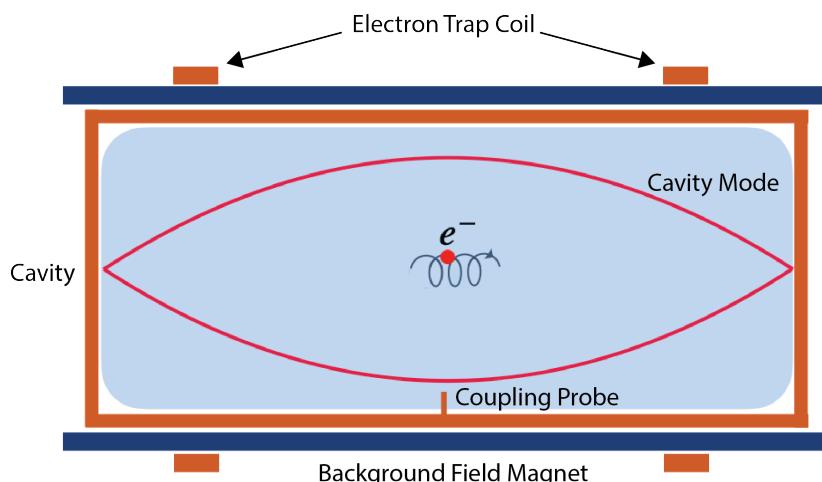


Figure 6.7. A cartoon depiction of a cavity CRES experiment. A metallic cavity filled with tritium gas is inserted into a uniform background magnetic field to perform CRES measurements. Electrons from beta-decays inside the cavity can be trapped and used to excite a resonant mode(s). By coupling to the cavity mode with a suitable probe one can measure the cyclotron frequency of the electron and perform CRES.

4877

4878 At the core of the experiment is a large resonant cavity filled with tritium gas. The
4879 filled cavity is then placed in a uniform magnetic field provided by a primary magnet,
4880 which provides the background magnetic field. The value of the background magnetic field
4881 sets the range of cyclotron frequencies for electrons emitted near the tritium spectrum
4882 endpoint. When a beta-decay electron is produced in the cavity it is trapped using a set
4883 of magnetic pinch coils that keep electrons inside the cavity volume.

4884 Electrons trapped inside the cavity do not radiate in the same way as electrons
4885 in free-space. Effectively, the same boundary conditions that were used to derive the
4886 resonant modes of a cylindrical cavity in Section 6.2 apply to the radiation of the electron
4887 as well. The coupling of an electron performing cyclotron motion in a cavity has been
4888 studied in detail for measurements of the electron’s magnetic moment [101–103]. If an
4889 electron is emitted with a kinetic energy that corresponds to a cyclotron frequency that
4890 matches a resonant frequency of the cavity, then energy radiated by the electron excites
4891 a corresponding resonance in the cavity. The strength of the electron’s coupling to the
4892 cavity is given to first order by the dot product between the electrons trajectory and
4893 the electric field vector of the resonant mode. Additional effects, such as the Purcell
4894 enhancement [104], alter the emitted power from the free-space Larmor equation [50]. If
4895 an electron is moving with a cyclotron frequency that is far from any resonant modes
4896 in the cavity, then radiation from the electron is suppressed. One can interpret this
4897 somewhat surprising effect as the metallic walls of the cavity reflecting the radiated
4898 energy back to the electron.

4899 Detecting an electron in the cavity is accomplished by coupling the cavity to an
4900 external transmission line that leads to an amplifier and RF receiver chain [105]. The
4901 coupling of the cavity resonance to the amplifier occurs through a coupling probe or
4902 aperture designed to read-out the excitation of the mode(s) excited by the electron. For
4903 CRES measurements, the placement of a wire antenna coupling probe inside the cavity
4904 volume leads to unacceptable losses of tritium atoms due to recombination to molecular
4905 tritium on the antenna surface, therefore, apertures are the preferred coupling method
4906 for cavity CRES experiments.

4907 One of the attractive features of the CRES technique for neutrino mass measurement
4908 is the gain in statistics that comes from the differential nature of the tritium spectrum
4909 measurement. Initially, this seems incompatible with cavities, due to the narrow reso-
4910 nances of cavity modes giving relatively small bandwidth. However, by intentionally
4911 over-coupling to a single cavity mode one can achieve bandwidths of a few 10’s of MHz
4912 (see Section 6.2), which is sufficient for a measurement of the tritium spectrum endpoint

4913 region.

4914 **6.3.2 Magnetic Field, Cavity Geometry, and Resonant Modes**

4915 **Magnetic Field and Volume Scaling**

4916 For a CRES experiment, cylindrical cavities are a natural choice since they match
4917 the geometry of standard solenoid magnets, which are needed in order to produce the
4918 background magnetic field for CRES measurements. Furthermore, the cylindrical shape is
4919 compatible with a Halbach array, which is the leading choice of atom trapping technology
4920 for future atomic tritium experiments by the Project 8 collaboration. Cylindrical cavities
4921 also benefit from well-established machining practices that are able to achieve high
4922 geometric precision at large lengths. More exotic cavity designs are under-consideration
4923 and there are ongoing efforts to investigate the potential advantages these may have over
4924 the standard cylindrical geometry.

4925 As shown in Section 6.2, the physical dimensions of the cavity are directly coupled to
4926 the resonant frequencies of the cavity. This dependency links the size of the cavity to
4927 the magnitude of the background magnetic field, because the magnetic field determines
4928 the cyclotron frequencies of trapped electrons. Specifically, as the size of the cavity is
4929 increased to accommodate larger volumes of tritium gas, the frequencies of the resonant
4930 modes decrease proportionally. This requires that the magnetic field also decrease in
4931 order to maintain coupling between electrons and the desired cavity mode.

4932 The required cavity size is ultimately determined by the required statistics in the
4933 tritium spectrum endpoint region. Because the gas density must be kept below a certain
4934 level to ensure that electrons have sufficient time to radiate before scattering, larger
4935 volumes become the only way to achieve higher event statistics. To achieve the sensitivity
4936 goals of Phase III and IV cavity volumes on the order of several cubic-meters are required,
4937 which pushes one towards frequencies in the range of 100's of MHz.

4938 **Single-mode Cavity CRES**

4939 It is tempting to consider maintaining a high magnetic field, while still increasing the size
4940 of the cavity, in order to increase the radiated power from trapped electrons for better
4941 SNR. However, if one were to maintain the same magnetic field while increasing the
4942 size of the cavity, the electrons would begin to couple to higher order modes with more
4943 complicated transverse geometries. The danger with this approach is that a complicated
4944 mode structure could introduce systematic errors into the CRES signals. Example

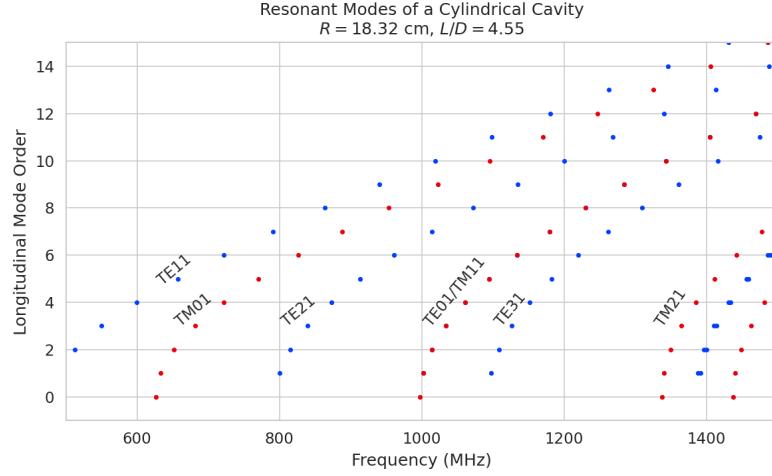
4945 systematics include unpredicted mode hybridization or changes in the mode shapes from
4946 imperfections in the cavity construction, which would prevent reconstruction of the
4947 electron's starting kinetic energies with adequate resolution. For this reason, it is ideal
4948 to operate with magnetic fields that give cyclotron frequencies near the fundamental
4949 frequency of the cavity, where the mode structure is relatively simple (see Figure 6.8).
4950 In this frequency region it is possible to perform CRES by coupling to only a single
4951 resonant mode; however, it is currently an open question if a single mode measurement
4952 will provide enough information about an individual electron's position to reconstruct
4953 the full event. Regardless, developing a solid understanding of the CRES phenomenology
4954 when an electron is coupling to a single mode will be a necessary step towards a future
4955 multi-mode cavity experiment.

4956 Considerations for Resonant Mode Selection

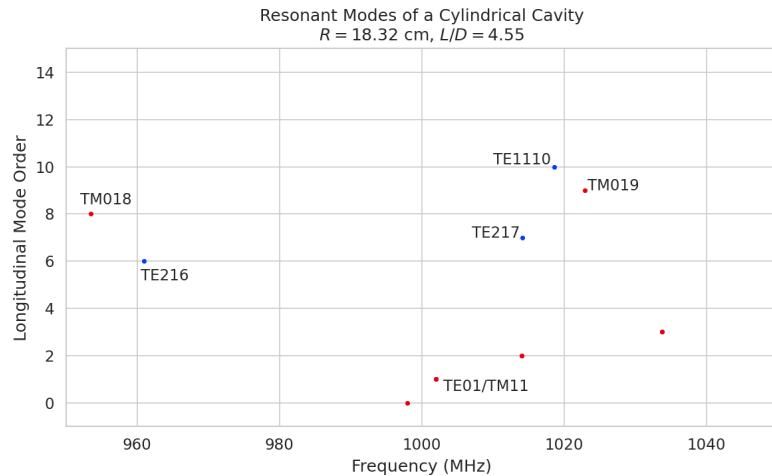
4957 A single-mode cavity experiment begs the question, which resonant mode is best for
4958 CRES measurements? There is an immediate bias towards low order TE_{nm} and TM_{nm}
4959 modes due to the multi-mode considerations discussed above. Additionally, there is a
4960 preference towards modes with longitudinal index $\ell = 1$ with a single antinode along the
4961 vertical axis of the cylindrical cavity. The reason for this is that there is a phase change
4962 in the electric fields between antinodes that leads to modulation effects that destroy the
4963 carrier frequency signal information.

4964 A second consideration for mode selection is the volumetric efficiency of the mode.
4965 Volumetric efficiency can be thought of as an integral over the volume of the cavity
4966 weighted by the relative amplitude of the mode. From the perspective of simply maximiz-
4967 ing the volume useable for CRES measurements, this integral would be as close to unity
4968 as possible. However, there is a requirement to reconstruct the position of the electrons
4969 inside the cavity volume so that the local magnetic fields can be used to convert the
4970 measured cyclotron frequency to a kinetic energy. With a single mode this necessarily
4971 requires a variable transverse mode amplitude, which lowers the volumetric efficiency, so
4972 that position of the electron in the cavity can be estimated from the average amplitude
4973 of the CRES signal. Longitudinal indices of $\ell = 1$ have an advantage in volumetric
4974 efficiency over higher order ℓ modes, since there are only two longitudinal nodes, one at
4975 each end of the cavity. Therefore, the average coupling strength of trapped electrons as
4976 they oscillate axially is higher for $\ell = 1$ modes.

4977 An additional factor for mode selection is the intrinsic or unloaded Q of the mode.
4978 In terms of SNR it is advantageous to use a mode with a very high Q_0 , which is then



(a)



(b)

Figure 6.8. Examples of the resonant mode frequencies of a cylindrical cavity. This cavity has a radius of 18.32 cm and a length to diameter ratio of 4.55. Several families of resonant modes are relevant in the ≈ 800 MHz bandwidth of (a); however, after zooming in to a ~ 80 MHz bandwidth centered on TE011 one sees that only a handful of resonant modes have frequencies close to TE011. Since the bandwidth required for a cavity CRES experiment is $O(10)$'s MHz, a significant number of resonant modes can be safely ignored since their resonant frequencies are far from the CRES bandwidth.

4979 highly overcoupled to achieve the necessary bandwidth to cover the tritium endpoint
4980 spectrum. A high intrinsic Q (Q_0) combined with a small loaded Q (Q_L) requires a
4981 large cavity coupling factor (g , see Equation 6.40). Because of the impedance mismatch
4982 associated with large coupling factors, the noise power contributed by the physical cavity
4983 temperature is reduced by a factor proportional to $1/g$, which allows one to achieve

4984 adequate SNR without the requirement of cooling the entire cavity to single Kelvin
4985 temperatures.

4986 An example of a resonant mode that exhibits these traits is the TE₀₁₁ mode. At present
4987 the TE₀₁₁ mode is the preferred resonance for a single-mode cavity CRES experiment
4988 by the Project 8 collaboration. TE₀₁₁ is a low order mode located in a region relatively
4989 far from other cavity modes. Furthermore, the separation of the TE₀₁₁ mode can be
4990 improved by various mode-filtering techniques discussed in Section 6.4.2 below. TE₀₁₁
4991 consists of a single longitudinal antinode that can provide pitch angle information in the
4992 form of amplitude modulation, and has an electric field with a radial profile given by the
4993 J'_0 Bessel function allowing for radial position estimation. Lastly, the TE₀₁₁ mode has a
4994 relatively high intrinsic Q compared to nearby modes, which helps with SNR. Unloaded
4995 Q's greater than 80000 are achievable for a 1 GHz TE₀₁₁ resonance using a copper walled
4996 cavity.

4997 6.3.3 Trade-offs Between the Antenna and Cavity Approaches

4998 The choice between cavities and antennas for large-scale CRES measurements is not
4999 without trade-offs. Both the antenna array and cavity approaches are relatively immature
5000 techniques, and at present there are no known obstacles that would prevent either
5001 approach from being used for a large scale neutrino mass experiment. The preference for
5002 cavities is largely driven by important practical considerations that could make a cavity
5003 based experiment significantly cheaper than an antenna experiment of similar size and
5004 scope. However, the switch to cavities also introduces new challenges less relevant to the
5005 antenna array, which must be solved in order for Project 8 to achieve its neutrino mass
5006 measurement goals.

5007 One of the major relative drawbacks of the antenna array approach is the size and
5008 complexity of the data-acquisition system. A large-scale antenna array experiment
5009 requires $O(100)$ antennas independently digitized at rates of $O(10)$ to $O(100)$ MHz. Since
5010 there is insufficient information in a single antenna channel to detect or reconstruct the
5011 CRES signal, the entire array output must be processed during the signal reconstruction.
5012 Because data storage becomes an issue with these data volumes, there is a real-time
5013 signal reconstruction requirement that allows one to detect CRES signals buried in the
5014 thermal noise. As discussed in Section 4.4, the computational cost of these real-time
5015 detection algorithms are potentially quite large for even a small scale antenna array
5016 experiment. However, the operating principle of a cavity experiment allows the CRES
5017 signal to be detected using only a single read-out channel digitized at rates of $O(10)$ MHz,

5018 which reduces the cost of the data acquisition system by many orders of magnitude.

5019 From an engineering perspective, the simple geometry and thin-walls of a cylindrical
5020 cavity are simpler to interface with the cryogenic and magnetic subsystems needed
5021 for a CRES experiment. Conversely, the antenna array requires careful design and
5022 engineering to accommodate the antenna array and receiver electronics in proximity to
5023 the trapping magnets. Additionally, due to near-field interference effects, the antenna
5024 array is unable to reconstruct CRES events within the reactive near-field distance of the
5025 antennas. Because atom trapping requirements require magnetic fields which correspond
5026 to cyclotron frequencies for endpoint electrons less than 1 GHz, the required stand-off
5027 distance leads to a significant loss in useable experiment volume, necessitating larger and
5028 more expensive magnets.

5029 Another advantage to the cavity approach is the relatively compact sideband structure,
5030 which is a result of the low modulation index for cavity CRES signals. The axial motion
5031 in an antenna array experiment leads to frequency modulation and sidebands. The shape
5032 of the sideband structure is determined by the modulation index, $h = \frac{\Delta f}{f_a}$, where Δf
5033 is the size of the frequency deviation and f_a is the axial frequency. The large electron
5034 traps required for a cubic-meter-scale experiment leads to high modulation indices, which
5035 causes the signal spectrum to be made up of numerous low power sidebands that make
5036 reconstruction and detection challenging. This behavior was observed in simulations
5037 of the FSCD in which carrier power decreased with pitch angle due to the increase in
5038 modulation index (see Figure 4.30). For cavities, however, the modulation index remains
5039 near $h = 1$ even for very long magnetic traps due to the high phase velocity in cavities
5040 relative to the axial velocity of the electron. This results in an almost ideal spectrum
5041 shape that has a strong carrier frequency with a few sidebands whose relative amplitudes
5042 encode pitch angle information.

5043 A downside of the cavity approach is the apparent difficulty of estimating the position
5044 of the electron using only the coupling of the electron to a single mode. The amplitude of
5045 the TE₀₁₁ mode is completely independent of the azimuthal coordinate, and thus position
5046 reconstruction using the TE₀₁₁ mode is only able to estimate the radial position of the
5047 electron. This position degeneracy may lead to magnetic field uniformity requirements
5048 that are too challenging to meet due to mechanical uncertainties in cavity and magnet
5049 construction, as well as uncertainties caused by nuisance external magnetic fields such
5050 as the Earth's field and magnetic fields from building materials. A multi-mode cavity
5051 experiment may provide a way to extract more precise information on the position of
5052 the electron by analyzing the coupling of the electron to several modes that overlap in

5053 different ways.

5054 **6.4 Single-mode Resonant Cavity Design and Simulations**

5055 The single-mode cylindrical cavities envisioned for the Phase III and IV experiments must
5056 be carefully engineered in order to measure the neutrino mass with the desired sensitivity.
5057 In this section I summarize some simulation studies performed to analyze early design
5058 concepts for a single-mode cavity. The primary tool for these investigations was Ansys
5059 HFSS, which was also used for the development of the SYNCA antenna described in
5060 Section 5.3.

5061 **6.4.1 Open Cylindrical Cavities with Coaxial Terminations**

5062 **Design Concept**

5063 A basic cavity design question relevant to Project 8's ultimate goal of an atomic tritium
5064 CRES experiment is how to build a cavity that can be efficiently filled with atomic
5065 tritium. To keep the rate of atom loss from recombination on surfaces low, it is ideal if
5066 the ends of the cylindrical cavity are as open as possible so that tritium atoms can flow
5067 inside unimpeded. Additionally, one of the primary calibration techniques planned for
5068 future CRES experiments involves CRES measurements using electrons injected from
5069 an electron gun source, which also requires an opening at the cavity end. Cylindrical
5070 cavities with open ends can be manufactured, however, the intrinsic Q-factors of these
5071 cavities are orders of magnitude less than their sealed counterparts, which reduces the
5072 signal-to-noise ratio when that cavity is used for CRES measurement.

5073 Cylindrical cavities with mostly open ends that also exhibit Q values for the $TE_{01\ell}$
5074 modes similar to sealed cavities can be built by using coaxial endcaps to terminate the
5075 cavity. Cavities of this type have been manufactured for specialized applications related
5076 to the measurements of the dielectric constants of liquefied gasses (see Figure 6.9) [3, 4].
5077 This cavity design leaves the ends of the cavity wide open, but retains high Q-values for
5078 the $TE_{01\ell}$ modes due to the coaxial endcap, which are designed to perfectly reflect the
5079 electric fields of $TE_{01\ell}$ modes. Coupling to the $TE_{01\ell}$ mode is achieved via an aperture
5080 located at the center of the cavity wall.

5081 A cavity similar to Figure 6.9 is a candidate design for the future CRES experiments
5082 by Project 8, since it appears to elegantly solve many practical issues that arise when
5083 combining cavity CRES and atomic tritium. The coaxial endcaps leave significant regions

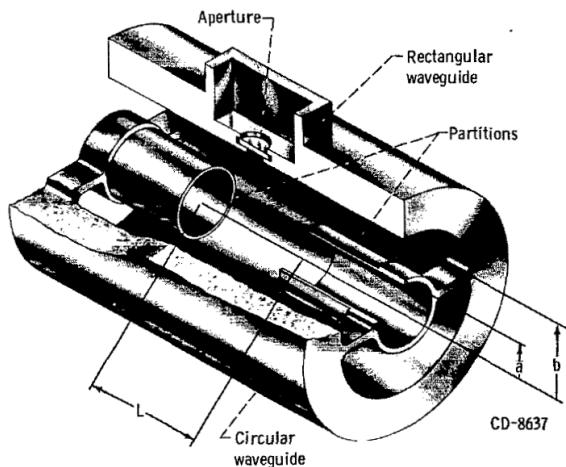


Figure 6.9. An image of an open cavity with coaxial terminations used for dielectric constant measurements. Figure from [3].

of the cavity ends completely open, which allows for the entrance of atomic tritium as well as the pumping away of molecular tritium that has recombined on the cavity walls. These open ends are achieved while preserving the high Q-values of the $TE_{01\ell}$ modes, which is important for extracting as much signal power from the electron as possible. In subsequent sections this cavity design will be analyzed in more detail, primarily by using HFSS simulations to analyze the resonant mode structure of this cavity geometry.

Coaxial Terminator Constraints

The reason that coaxial endcaps can be used to achieve high Q-values for the $TE_{01\ell}$ modes is that the electric fields for these modes are purely azimuthally polarized (see Equations 6.12 and 6.13). Therefore, the boundary conditions that require the electric field to go to zero at the cavity ends can be supplied using a coaxial partition of the correct radius (see Figure 6.10). Because the cylindrical shape enforced by the partition does not match the boundary conditions of other cavity modes, these terminations also significantly suppress the Q-factors of non- $TE_{01\ell}$ modes, which is potentially beneficial for a single-mode cavity CRES experiment.

The correct radius of the cylindrical partition is derived by setting up the boundary value problem in Figure 6.10, and analyzing the reflection and transmission coefficients for waves incident on the coaxial terminators. The basic problem is to identify the radius a where the reflection coefficient for the $TE_{01\ell}$ modes becomes equal to 1. One can show that if the coaxial partitions are made sufficiently long relative to the wavelength of the

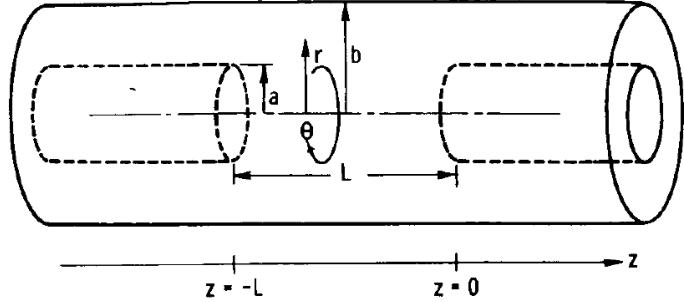


Figure 6.10. The simplified geometry of an open cavity with coaxial terminations. Figure from [4].

5104 TE₀₁ modes then perfect reflection can be achieved. This derivation is quite lengthy
 5105 and complex and is presented in full in [4]. Here, I shall simply explain the resulting
 5106 conditions on the partition radius for perfect reflection.

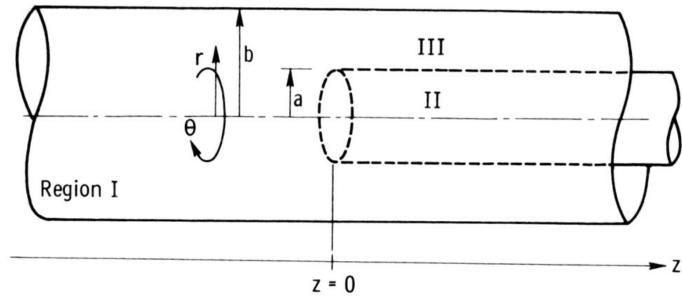


Figure 6.11. Electric field regions for the open cavity boundary value problem. Figure from [4].

5107 The open cavity boundary value problem is solved by expressing the forms of the
 5108 electric fields in the different regions of the cavity and requiring that the electric fields are
 5109 continuous. There are effectively three distinct regions in the open cavity corresponding
 5110 to the central cavity volume, the inner coaxial volume, and the outer coaxial volume (see
 5111 Figure 6.11).

5112 In Region I, the boundary conditions are those of a cylindrical waveguide, and it
 5113 is required that E_ϕ for the TE_{0m} modes go to zero at the cavity wall ($r = b$). This
 5114 necessitates $J'_{0m}(k_{c0m}b) = 0$. A solution for the radius a is desired such that the TE₀₁
 5115 mode propagates, but other TE_{0m} modes are below the cutoff frequency for the circular
 5116 waveguide. This is equivalent to requiring

$$3.832 < k_{c0m}b < 7.016, \quad (6.41)$$

5117 where the numbers 3.832 and 7.016 correspond to the first and second zeros of the Bessel
 5118 function (see Table 6.1).

5119 In Region II the boundary conditions are those of a cylindrical waveguide, but with
 5120 a smaller radius. The condition that $E_\phi = 0$ at the cylindrical partition radius is that
 5121 $J'_{0m}(k_{c0m}a) = 0$. To ensure perfect reflection, all modes in Region 1 of the cavity must be
 5122 below the cutoff frequency of the circular waveguide formed by the inner volume of the
 5123 coaxial terminator. Therefore, solutions where the condition

$$k_{c0m}a < 3.832, \quad (6.42)$$

5124 is true are required.

5125 Finally, in Region III the boundary condition are those of a coaxial waveguide. One
 5126 needs to guarantee that $E_\phi = 0$ at both $r = b$ and $r = a$, which involves finding the
 5127 eigenvalues of the following equation

$$J'_0(k_{c0m}a)Y'_0(k_{c0m}b) - J'_0(k_{c0m}b)Y'_0(k_{c0m}a) = 0, \quad (6.43)$$

5128 where Y'_0 the zeroth-order derivatives of the Bessel function of the second kind. The
 5129 solutions to this equation depend on the value of the ratio b/a . The approximate solution
 5130 is given by

$$\delta_n a \simeq \frac{n\pi}{b/a - 1}, \quad (6.44)$$

5131 where δ_n are eigenvalues of Equation 6.43. Similar to Region II, solutions for which
 5132 the TE₀₁ modes of Region I are below the cutoff frequency of Region III are needed.
 5133 Therefore, it is required that

$$k_{c0m} < \delta_1. \quad (6.45)$$

5134 In general, one has some freedom in specifying the value of b/a . A value typically used
 5135 in practice is $b/a = 2.082$, which corresponds to positioning the radius of the cylindrical
 5136 partition at the maxima of the TE₀₁ electrical fields.

5137 Using the constraints from the three field regions one can develop a coaxial terminator
 5138 that acts as a virtual perfectly conducting surface for the TE₀₁ modes. The only required
 5139 inputs are the desired frequency of the TE₀₁₁ mode and a choice for the value of b/a .

5140 6.4.2 Mode Filtering

5141 The general case of an electron coupling to a resonant cavity is complicated. This is
5142 because cavities contain an infinite number of resonant modes, which for higher order
5143 modes, have couplings to the electron with a complex spatial dependence. The danger is
5144 that improper modeling of the electron's coupling to the cavity can lead to systematic
5145 errors in the CRES measurements that prevent a high-resolution measurement of the
5146 electron's kinetic energy. This in part drives the preference for a single-mode cavity
5147 experiment that uses only the electron's coupling to the TE₀₁₁ mode to perform CRES,
5148 assuming that sufficient information on the electron's position can be obtained with a
5149 single mode.

5150 The TE₀₁₁ mode is in a region where there are relatively few other modes to which
5151 the electron could couple(see Figure 6.8). However, one can see that the frequency of
5152 the TE₀₁₁ is perfectly degenerate with the TM₁₁₁ mode, which means that electrons will
5153 inevitably couple to both modes if they have the correct cyclotron frequency.

5154 The magnitude of the impact of the electron coupling to both TE₀₁₁ and TM₁₁₁ is
5155 currently unknown. To first order an electron coupling to both modes will lose more
5156 energy over time, which can be measured by observing the frequency chirp rate of the
5157 signal. This effect may be small enough to be negligible or simple enough to model, so
5158 that the cavity can be treated as an effective single-mode cavity. Alternatively, one could
5159 consider devising a coupling scheme that is sensitive to both the TE₀₁₁ and the TM₁₁₁
5160 modes. By measuring the coupling of the electron to both modes more information on
5161 the position of the electron could be obtained, which could improve the position and
5162 energy resolution of the CRES measurements.

5163 A different approach is the mode filtering approach, which seeks to obtain a single
5164 TE₀₁₁ mode cavity using perturbations to the cavity walls that selectively suppress the
5165 TM modes, while leaving the TE modes mostly unperturbed. The type of perturbations
5166 required can be determined by visualizing the surface currents induced in the cavity
5167 walls by each type of mode (see Figure 6.12). By definition, all TM have electric fields
5168 directed along the vertical axis of the cylindrical cavity, which means that perturbations
5169 that impede currents in this direction will modify TM resonances. On the other hand,
5170 the TE₀₁ modes induce azimuthal currents in the cavity walls; therefore, it is possible
5171 to break the degeneracy between TE₀₁ and TM₁₁ by using a cavity perturbation that
5172 suppresses axial currents, but does not affect the flow of azimuthal currents.

5173 Figure 6.12 shows two cavity design concepts that achieve this selective current
5174 perturbation. The resistive approach inserts a series of thin dielectric rings into the walls

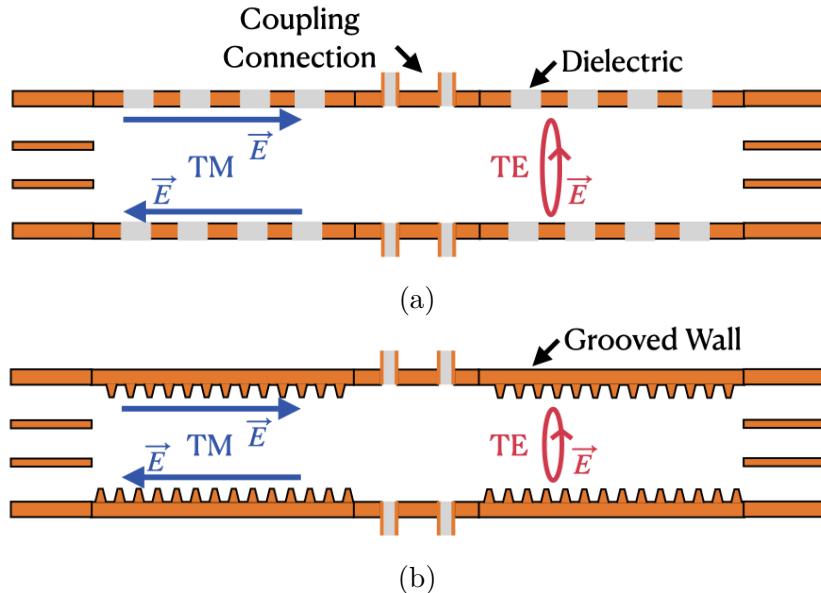


Figure 6.12. Two mode filtering concepts to break the degeneracy of TE_{01} and TM_{11} modes. The resistive approach uses dielectric materials to impede currents that travel vertically along the cavity while leaving azimuthal currents unperturbed. An alternative approach is to impede the currents using grooves cut into the cavity wall, which achieve the same effect with an inductive impedance.

of the cavity that introduces a resistive and capacitive impedance to the longitudinal currents, while leaving azimuthal current paths intact. Cavities of this type with high TE_{01} Q's have also been constructed by tightly wrapping a thin, dielectric coated wire around a mold to form the cavity wall. An alternative method is to introduce an inductive impedance by cutting grooves or a thread pattern on the inside wall of the cavity. For reasons of manufacturability and compatibility with tritium the grooved cavity approach is the preferred method for mode-filtered cavity construction by Project 8.

6.4.3 Simulations of Open, Mode-filtered Cavities

A candidate design for a single TE_{011} mode CRES experiment is a cavity that utilizes the coaxial terminations combined with a mode-filtering wall. The first step towards validating that a cavity that combines these two design features will operate as expected is a thorough simulation effort for which finite element method (FEM) simulation software is invaluable. The primary tool for electromagnetic FEM calculations inside Project 8 is Ansys HFSS, which has a robust and well-established eigenmode solver that can identify the resonant frequencies and associated Q-factors for given structure.

Four variations of a cavity design with a ~ 1 GHz TE_{011} resonance were implemented

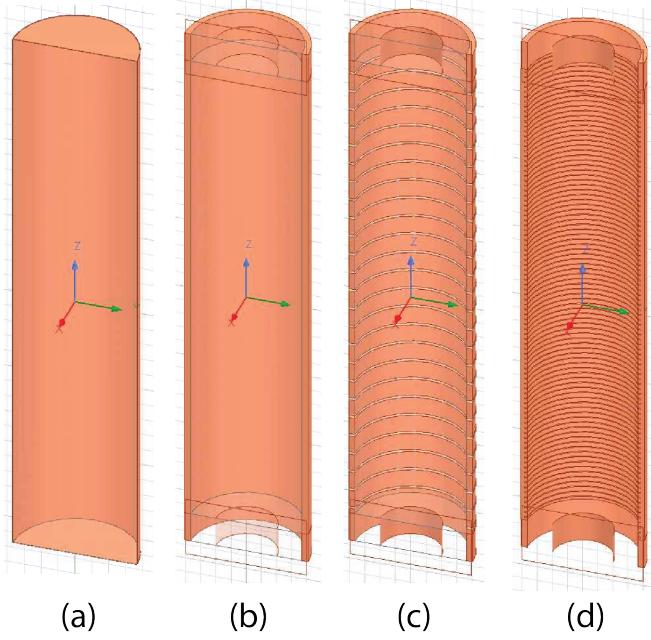


Figure 6.13. Four cavity design variations. (a) is a standard sealed cylindrical cavity, (b) is an open cavity with smooth walls, (c) is an open cavity with walls made of alternating conductor and dielectric, and (d) is an open cavity with grooved walls. The main cavity and coaxial terminator parameter are identical for all four cavities.

in HFSS (see Figure 6.13). The four designs include a standard cylindrical cavity, an open cavity with smooth walls, an open cavity with walls made of alternating conductor and dielectric, and an open cavity with grooved walls. The relevant design parameters are summarized in Table 6.3. All cavities were simulated using copper walls and filled with a vacuum dielectric. The identities of the resonant modes found by HFSS were validated by visual inspection of the electric and magnetic field patterns and by comparison to analytical calculations of the mode frequencies.

Table 6.3. A table of cavity design parameters used for HFSS simulations.

Name	Qty.	Unit	Description
D_{cav}	326.4	mm	Cavity diameter
L_{cav}	1668.0	mm	Cavity length
D_{term}	200.2	mm	Inner diameter of coaxial terminator
L_{term}	100.0	mm	Terminator length
l_{die}	8.3	mm	Dielectric spacer thickness
Δl_{die}	66.7	mm	Distance between dielectric spacers
l_{groove}	3.0	mm	Groove height
d_{groove}	9.0	mm	Groove depth
Δl_{groove}	18.3	mm	Distance between grooves

5198 The results of the HFSS simulations validate our predictions of the resonant behavior
of an open, mode-filtered cavity developed in the preceding sections (see Figure 6.14) One

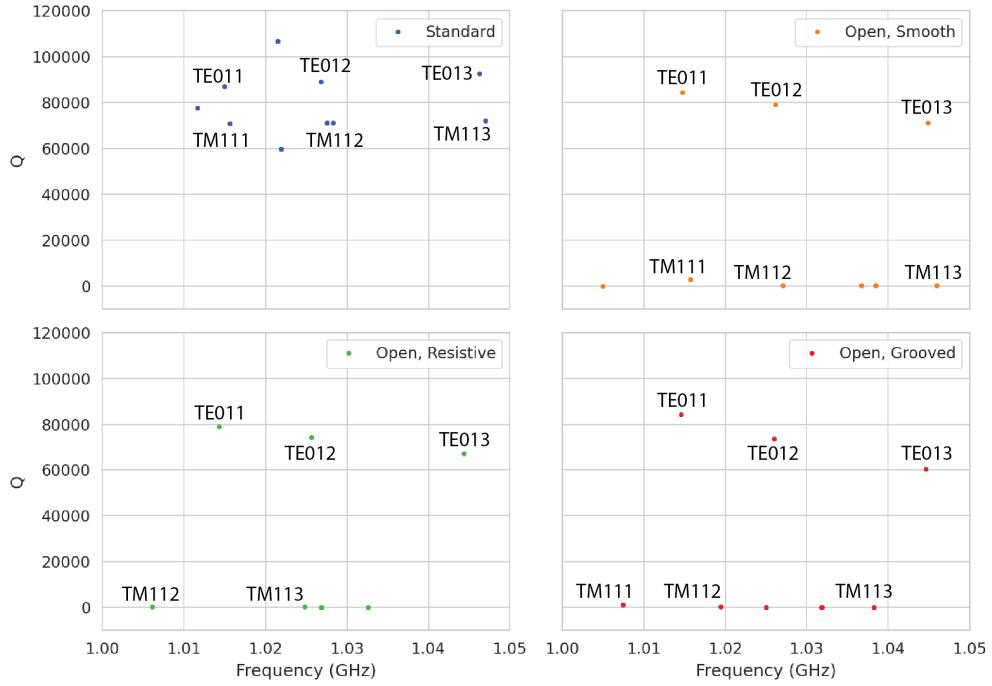


Figure 6.14. The frequencies and Q-factors of the resonant modes identified by HFSS for the cavity variations shown in Figure 6.13. The fully-sealed cavity with smooth walls has several high-Q modes near the TE_{011} resonance. Introducing the open-termination preserves the Q-factors of the $TE_{01\ell}$ modes and suppresses the Q-factors of the modes whose boundary conditions do not match the cylindrical partition. Both the resistive and grooved wall perturbations shift the resonant frequencies of the TM modes away from the TE_{011} mode. By properly tuning the geometry of the grooves or the resistive spacers several MHz of frequency separation can be achieved.

5199
5200 can see that for a standard cavity the TE_{01} and the TM_{11} are degenerate in frequency
5201 with relatively high Q-factors. The open-ended cavity preserves the high Q-factors of
5202 the TE_{01} modes, while the other modes, since their boundary conditions do not match
5203 the coaxial geometry, have their Q-factors suppressed. One can see that the effect of
5204 the resistive and inductive mode-filtering schemes is to effectively shift the resonant
5205 frequencies of the TM_{11} modes below those of the associated TE_{01} modes, which breaks
5206 the degeneracy. Optimization of the dielectric spacer or groove parameters can ensure
5207 that the TE_{011} mode is isolated from other modes by $O(10)$ MHz, which provides sufficient
5208 bandwidth for a measurement of the tritium spectrum endpoint.

5209 Further optimization of the cavity design requires a more detailed cavity simulation
5210 that includes the cavity coupling mechanism as well as other geometry modifications

5211 required for integration into the magnetic and tritium gas subsystems. Perhaps more
5212 important is the development of the capability to simulate the interaction of electrons
5213 with the cavity so that simulated CRES signals can be generated using cavities designed
5214 for CRES measurements. Simulated CRES signals can then be used to estimate the
5215 neutrino mass sensitivity of the experiment, which allows for the optimization of the cavity
5216 design towards the configuration that provides the best measurement of the neutrino
5217 mass.

5218 **6.5 Single-mode Resonant Cavity Measurements**

5219 Measurement test stands play an important role in the research and development process
5220 that cannot be replaced by simulations. For example, constructing a prototype CRES
5221 cavity forces one to consider important practical issues such as manufacturability and
5222 machine tolerances that may require modifications to the design. Furthermore, by
5223 comparing laboratory measurements of a real cavity to simulations, one can quantify
5224 the impact of imperfections and real-life measurement systematics, which allows for
5225 more accurate sensitivity estimates of the experiment. Lastly, the development of these
5226 prototypes helps to build the necessary experience and expertise within the collaboration
5227 required for more complicated experiments to succeed.

5228 With these objectives in mind a prototype cavity was constructed to demonstrate the
5229 open, mode-filtered cavity concept explored in the previous sections. The primary goal
5230 of the measurements was to validate that an open, mode-filtered cavity suppressed the
5231 TM₁₁ modes as predicted by HFSS simulations.

5232 **6.5.1 Cavities and Setup**

5233 Two rudimentary cavities were constructed using segments of copper pipe available from
5234 McMaster-Carr (see Figure 6.15). The design consists of copper pipes of two diameters.
5235 The larger diameter pipe forms the main cavity wall and the smaller diameter pipe is
5236 used to create a coaxial termination. The diameter of the outer pipe was chosen to
5237 produce a TE₀₁₁ resonance of approximately 6 GHz, while the diameter of the smaller
5238 pipe was selected based on the open termination criteria introduced in Section 6.4.1. The
5239 approximate diameters and lengths of the copper pipe are summarized in Table 6.4.

5240 Coupling to the cavity was achieved using a hand-formable segment of coaxial cable
5241 stripped at one end to form a loop antenna. This was inserted into a small hole located

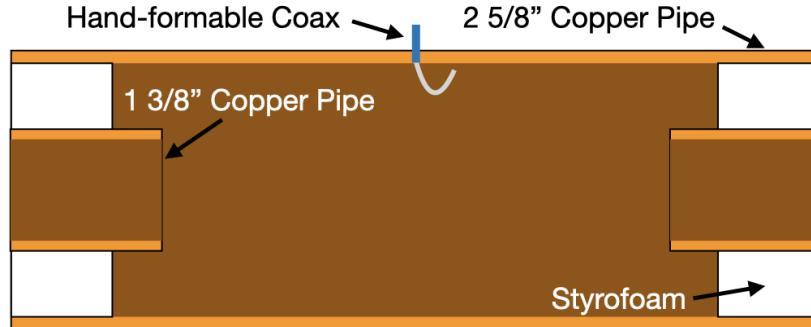


Figure 6.15. A cartoon depicting the design of the open-ended cavity prototype designed to operate at approximately 6 GHz. The main cavity wall was composed of a single copper pipe. A mode-filtered version of this cavity was constructed by

5242 at the center of the main cavity wall. The coaxial terminators were supported inside the
 5243 main cavity by carving a spacer from polystyrene foam (styrofoam) so that they could
 5244 be easily inserted into the cavity and repositioned. The dielectric constant of styrofoam
 5245 is quite close to air at microwave frequencies so this is expected to have minimal impact
 5246 on the resonant properties of the cavity.

Table 6.4. A table of parameters describing the cavity prototypes. Certain values such as the cavity length and the distance between dielectric spacers are approximate due to variation in the machining of the copper. In particular, the filtered cavity was constructed from conducting copper segments that varied in size from 1.50" to 1.85".

Name	Qty.	Unit	Description
D_{cav}	2.625	in	Cavity diameter
L_{cav}	≈ 13	in	Cavity length
D_{term}	1.375	in	Inner diameter of coaxial terminator
L_{term}	1.575	in	Terminator length
l_{die}	0.75	in	Dielectric spacer thickness
Δl_{die}	≈ 1.50 to 1.85	in	Distance between dielectric spacers

5247 The effective length of the cavity is given by the distance between the inner edges of
 5248 the coaxial terminations. The length of the outer section of pipe that forms the main wall
 5249 of the cavity is approximately 16" in length which leads to a cavity length of $\approx 13"$ when
 5250 both terminators are inserted in the cavity. Because the terminators were not rigidly
 5251 mounted this distance is only approximate, but the uncertain length of the cavity will
 5252 not prevent us from validating the open cavity design.

5253 Along with the smooth-walled open cavity, a resistively mode-filtered cavity was
 5254 constructed by creating dielectric spacers out of segments of clear PVC pipe (see Figure
 5255 6.16). The spacers were machined such that the conductive segments of the cavity would

5256 be separated by 0.75" when the cavity was fully assembled. Due to variations in the
 5257 lengths of the copper segments that make up the cavity wall the distance between spacers
 5258 has significant variation with average value of about 1.7". Eight total spacers were used
 5259 to build the cavity, which when assembled was approximately 16" in total length similar
 to the non-filtered cavity.

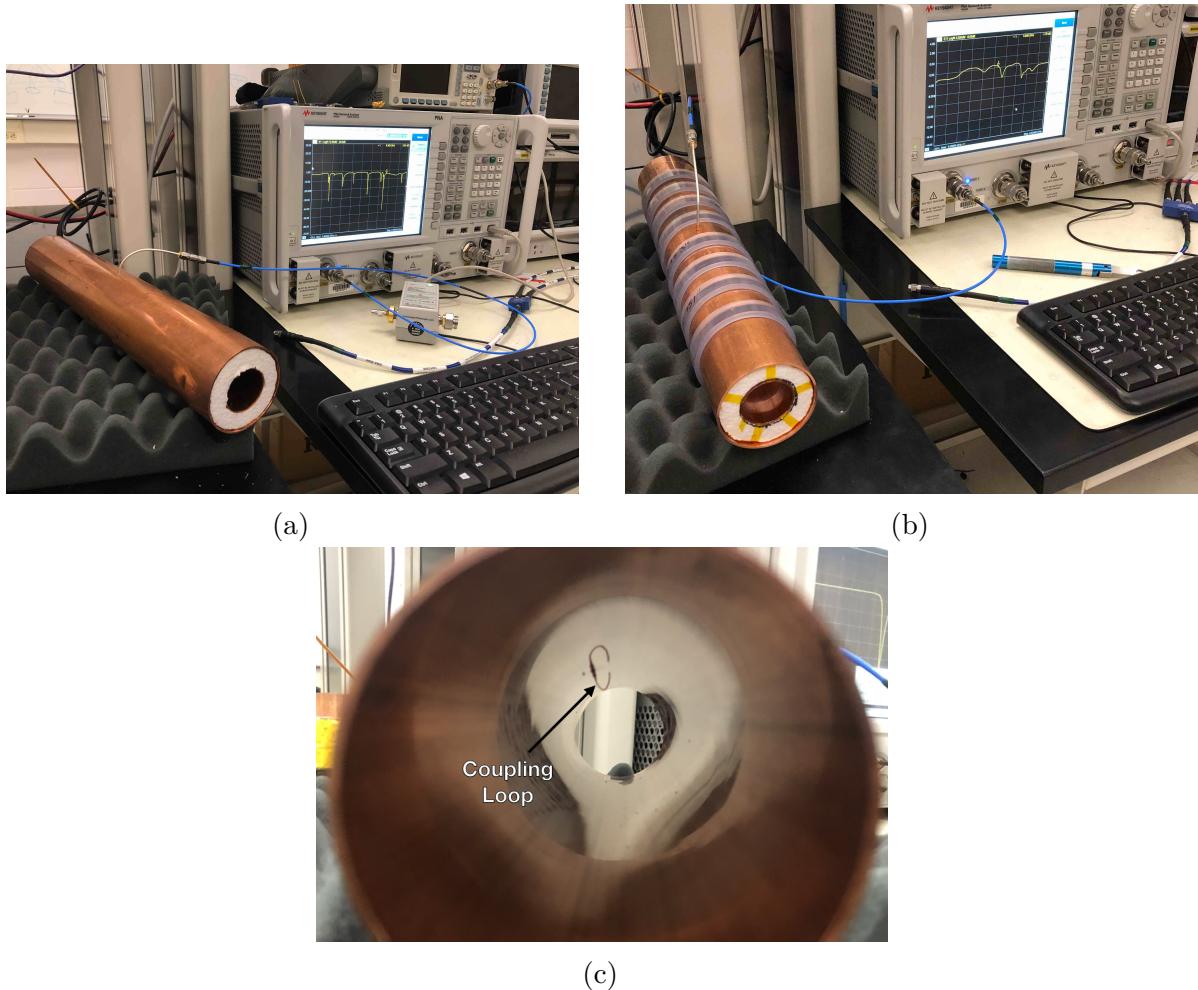


Figure 6.16. Images depicting the measurement of the filtered and non-filtered open cavities using the VNA. The coupling loop in the figure is shown in the TE orientation.

5260 Measurements of both cavities were performed using a VNA connected to the cavity
 5261 coupling probe (see Figure 6.16). By measuring the return loss over a range of frequencies
 5262 one can measure the frequencies and relative Q-factors of the resonant modes in the
 5263 cavity. Due to the opposite polarity of the electric fields for the TE and TM modes,
 5264 the loop coupling probe must be rotated 90° to change the polarity of the loop antenna.
 5265 When the antenna is oriented such that the loop opening faces the ends of the cavity, it

5267 couples primarily to the TE modes which have magnetic fields directed along the long
 5268 axis of the cavity (see Figure 6.16). If the coupling loop is turned by 90° from where
 5269 it is shown in the image then it will couple to the TM modes which have azimuthally
 5270 directed magnetic fields. In this way both the TE and TM resonances can be measured
 5271 independently.

5272 **6.5.2 Results and Discussion**

5273 The primary analysis for the prototype cavities involved a simple visualization of the return loss as measured by the VNA and a comparison between the filtered and non-
 5274 filtered variations. Since the resonances measured by the VNA are not labeled, there is an uncertainty about the true identities of the modes measured by the VNA. To resolve
 5275 this I performed a simulation of the simplest possible cavity that could be created from the prototype components, which is a fully open cavity created by removing the coaxial
 5276 inserts. The fully-open cavity with the as-built dimensions was simulated in HFSS to get
 5277 estimates on the positions of the TE_{011} and TM_{111} modes (see Figure 6.17).

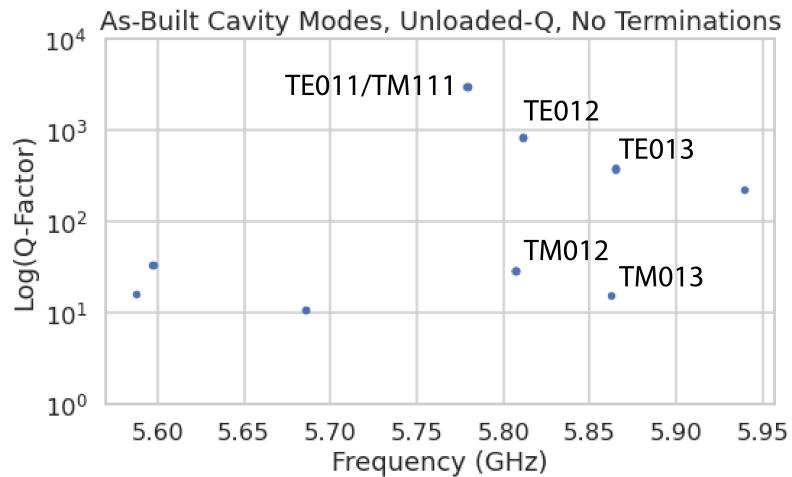


Figure 6.17. HFSS simulation results for a the as-built cavity with the coaxial terminators removed. The $\text{TE}_{011}/\text{TM}_{111}$ frequency is approximately 5.78 GHz.

5281 Simulation of the fully open cavity shows that the $\text{TE}_{011}/\text{TM}_{111}$ modes have a
 5282 frequency of approximately 5.78 GHz in the fully open cavity. If the frequency of this
 5283 mode is compared to the measurements of the filtered and non-filtered cavities with the
 5284 terminators removed one can easily identify the TE_{011} mode at approximately 5.75 GHz
 5285 (see Figure 6.18).

5286 In both variations of the non-filtered cavities one sees that the TE_{011} mode is degen-
 5287 erate with what appears to be a doublet of TM modes located at the TM_{111} frequency
 5288 position. This doublet is actually the TM_{111} mode, which has two polarizations with
 5289 opposite polarizations. Because any real pipe used to construct a cavity will not be
 5290 perfectly round due to manufacturing tolerances, it is likely that the frequency degeneracy
 between the two polarizations is broken, which results in the doublet peaks.

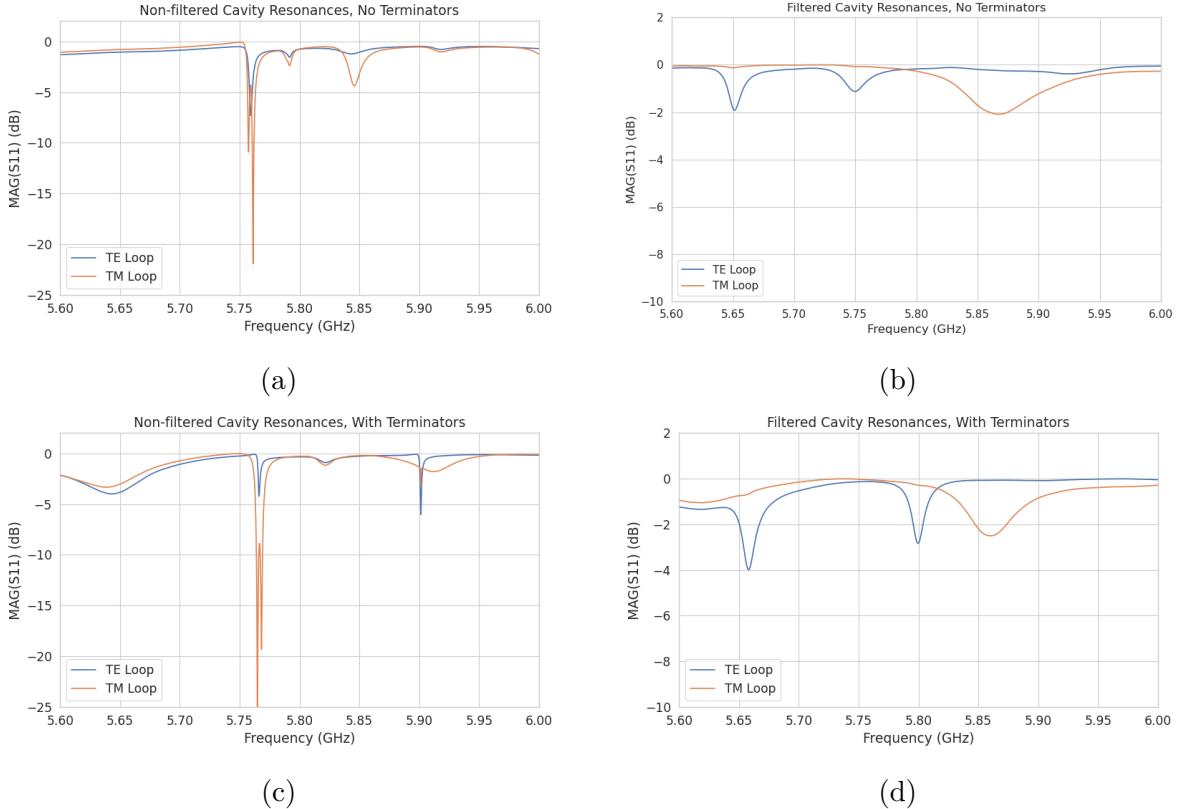


Figure 6.18. Measurements of the filtered and non-filtered prototype cavities acquired with the VNA.

5291
 5292 The S-parameter plot for the filtered cavity without terminators has an isolated TE
 5293 resonance at 5.65 GHz, associated with the TE_{011} mode. The frequency of this mode
 5294 is lower than the non-filtered cavity due to a difference in the overall lengths of the
 5295 cavities. An obvious difference between the filtered and non-filtered cavities is that
 5296 there is no TM_{111} doublet at the TE_{011} frequency. This is what one would expect if
 5297 the mode-filtering was suppressing the TM modes. There appears to be a noticeable
 5298 difference in the Q of the TE_{011} resonance between non-filtered and filtered variations as
 5299 indicated by the increased resonance depth for the filtered cavity. Overall, the Q-factors
 5300 of the filtered cavity appear significantly smaller than the non-filtered cavity due to the

5301 increase in resonance width. This is likely caused by the relatively large widths of the
5302 dielectric spacers, which are partially impeding the TE modes.

5303 One can see from these cavity measurements that, in principle, resistive mode-filtering
5304 can be used to separate the TE_{011} resonance from the degenerate TM_{111} modes in
5305 combination with the open cavity endcaps. This finding agrees with the expectations
5306 from HFSS, which should provide confidence that the eigenmode solver is correctly
5307 modeling the behavior of the cavity. Although I did not perform a similar study using
5308 a cavity with grooved walls, it is expected that the resonant mode structure would be
5309 similar to the cavity studied here.

5310 While this prototype cavity is a good first step, several deficiencies prevent this setup
5311 from providing more than qualitative information to the design of cavities for CRES. This
5312 includes the rudimentary approach to cavity coupling using a stripped coax antenna and
5313 the inability to map the field density in the cavity volume. Improvements in these areas
5314 are required so that measurements from a real cavity can provide useful information to
5315 cavity CRES simulations that will ultimately inform neutrino mass sensitivity estimates.

5316 Future work with prototype cavities must include an improved cavity coupling scheme,
5317 which is robust and compatible with atomic tritium. Since the cavity will ultimately
5318 be filled with atomic tritium, a coupling antenna cannot be used due to the losses of
5319 atomic tritium caused by recombination on the antenna surfaces. Possible non-invasive
5320 coupling schemes include aperture coupling, where the cavity is coupled to an external
5321 waveguide structure through an aperture, or a split-ring coupling approach, where the
5322 center segment of the cylindrical cavity wall is replaced an isolated conductive ring with
5323 a small vertical slit. The aperture coupling approach is a standard coupling scheme [92]
5324 used in a wide range of applications, but at low frequencies the size of the external
5325 waveguide conflicts with design of the atom trapping magnet and cryogenics system.
5326 The split-ring approach could potentially be coupled to a small coaxial transmission line
5327 which is more compatible with the rest of the experiment design. A challenge with this
5328 coupling scheme is achieving a large enough coupling factor by correctly engineering
5329 the geometry of the split-ring and the connection to the coax transmission line. The
5330 design and optimization of this coupling scheme is an area of current research in the
5331 collaboration.

5332 The robustness of the coupling mechanism is relevant due to the difficulty in modeling
5333 its effect on the cavity modes. Small changes in geometry can have a large influence on
5334 the coupling and hence the performance of the cavity, therefore, correctly modeling the
5335 cavity coupling is critical for accurate CRES simulations. Coupling schemes that rely

5336 on connections to coaxial lines are potentially at a disadvantage in this regard due to
5337 the affect of soldering imperfections or unintended bends in the coax on the coupling.
5338 Future work will identify a coupling scheme for the cavity compatible with the neutrino
5339 mass goals of Project 8.

5340 Imperfections in the geometry of a real cavity will necessarily distort the resonant
5341 modes away from simulation predictions. This will change the coupling of an electron
5342 to the cavity and thus change the expected signal structure. Ultimately, this effect will
5343 limit the achievable energy resolution of the experiment unless the differences between
5344 simulation and a real cavity can be sufficiently characterized and calibrated. One possible
5345 approach to this is to utilize a "bead puller" system [106] to strategically perturb the
5346 cavity by moving a conductive bead through the cavity volume. The small perturbation
5347 caused by the bead affects the phase of the cavity resonances proportional to the total
5348 magnitude of the electric field at that position, so by moving the bead through the
5349 cavity volume the total electric field can be mapped and compared to simulation. This
5350 information can provide bounds on the relative perturbations to the cavity mode structure
5351 from real-life imperfections compared to the idealized cavity in HFSS.

5352 **Chapter 7 |**

5353 **Conclusion and Future Prospects**

5354 In this dissertation I have discussed research and development efforts towards the
5355 development of a scalable CRES measurement technology that can be used to build a
5356 CRES experiment at cubic-meter scales with sensitivity to neutrino masses of 40 meV.
5357 The primary contributions of my dissertation are the development and analysis of signal
5358 reconstruction algorithms for an antenna-based CRES experiment [62], which vital to
5359 estimating the neutrino mass sensitivity of the experiment; the development of a synthetic
5360 cyclotron radiation antenna (SYNCA) [78], which allowed for laboratory validation of
5361 antenna array CRES simulation models [44]; and the development of an open-ended cavity
5362 design compatible with atomic tritium for a cavity based CRES experiment. A measurable
5363 impact of this work is the transition of the Project 8 collaboration's experimental plan
5364 from an antenna array based approach to a cavity based approach, after my work played
5365 a key role in demonstrating the significantly higher cost and complexity of the antenna
5366 array experiment.

5367 The transition from antenna arrays to cavities requires a new set of demonstrator
5368 experiments to make incremental progress towards a 40 meV measurement of the neutrino
5369 mass. At the time of writing, the near-term plan of Project 8 is to design and construct a
5370 small-scale cavity CRES experiment utilizing the 1 T magnet installed in the UW-Seattle.
5371 This cavity is designed to have a TE011 resonance with a frequency of about 26 GHz with
5372 a length-to-diameter ratio that mimics the larger cavities intended for the pilot-scale and
5373 Phase IV experiments. The goal of this experiment is to demonstrate cavity CRES as
5374 well as validate models of CRES systematics using electrons from ^{83m}Kr and an electron
5375 gun. Though the primary goal is demonstration, near-term physics measurements are
5376 available in the form of high-resolution measurements of the ^{83m}Kr conversion spectrum
5377 of interest to the KATRIN collaboration.

5378 Furthermore, Project 8 is currently constructing a low-frequency CRES setup located
5379 at Yale University to better understand the principles of cavity based CRES at lower

5380 magnetic fields. The Low, UHF Cavity Krypton Experiment at Yale (LUCKEY) is
5381 a 1.5 GHz cavity CRES experiment the will use conversion electrons from ^{83m}Kr to
5382 perform CRES measurements at the lowest frequencies ever attempted with the technique.
5383 LUCKEY will validate frequency scaling models developed by Project 8 and will pave
5384 the way for the future Low-Frequency Apparatus (LFA), which will be a larger, 1 GHz
5385 cavity CRES experiment that includes a molecular tritium source. The target for the
5386 LFA is a measurement of the neutrino mass with a sensitivity of approximately 0.2 eV,
5387 which will build towards the atomic pilot-scale CRES experiment.

5388 In parallel to the development of cavity CRES is the development of the atomic
5389 tritium source. Recent demonstrations of the production of atomic hydrogen are excellent
5390 steps towards the atomic tritium production needed for the pilot-scale experiment. One
5391 area of future study includes the development of a more detailed understanding of the
5392 efficiency of atomic hydrogen production. Near-term plans include the development
5393 of a magnetic, evaporatively cooled beamline, as well as the prototyping of a Halbach
5394 array atom trap. Nearly all the components of the atomic tritium system will require
5395 demonstration before the complete system can be built. The long-term goal of the
5396 atomic tritium work is to construct a full atomic tritium prototype that demonstrates
5397 the production, cooling, trapping, and recycling of tritium at the rates needed for the
5398 pilot-scale experiment.

5399 More broadly, the long-term goal of the Project 8 collaboration is to fully develop
5400 both the atomic tritium and cavity CRES technologies so that both can be combined in
5401 a pilot-scale CRES experiment. It is envisioned that this process will take approximately
5402 10 years for both atomic tritium and cavity CRES. After these developments comes
5403 the pilot-scale experiment which will be the first CRES experiment that simultaneously
5404 demonstrates all the required technologies for Phase IV. Scaling to Phase IV with cavity
5405 CRES will require the construction of multiple copies (approximately 10) of the pilot-scale
5406 experiment to obtain sufficient statistics for 40 meV sensitivity.

5407 Development of the CRES experimental technique by Project 8 has led to new
5408 experiments utilizing the CRES technique for basic physics research, such as the ^6He -
5409 CRES collaboration [107], and has also found applications as a new approach to x-ray
5410 spectroscopy [108]. Recently, a new experimental effort called CRESDA has begun in
5411 the UK to develop new quantum technologies applied to CRES measurements for the
5412 neutrino mass [109]. This flourishing of new experimental efforts based on the CRES
5413 technique is likely to continue as Project 8 continues to develop the technique towards
5414 its neutrino mass measurement goal.

5415 The work described in this dissertation represents important R&D steps towards
5416 a next-generation neutrino mass measurement with CRES. A direct neutrino mass
5417 measurement, which sets both an upper and lower bound on the neutrino mass will
5418 be important to several physics subfields. For example, fixing the neutrino mass scale
5419 with an independent measurement will enable cosmological fits to use the neutrino mass
5420 as a fixed parameter, which will allow current data to provide more information on
5421 other parameters in cosmological models. Additionally, a precise measurement of the
5422 neutrino mass scale will be important for constraining the energy-scales of possible new
5423 physics responsible for the smallness of the neutrino mass relative to other standard
5424 model particles.

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Education

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- Doctor of Philosophy, Physics, The Pennsylvania State University, University Park, Pennsylvania, USA, 2023
- Bachelor of Science, Physics, The University of Minnesota, Minneapolis, Minnesota, USA, 2017

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Selected Publications

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- Astari Esfahani, A. et al. (2023) "Antenna Arrays for CRES-based Neutrino Mass Measurement", *Phys. Rev. C*, In preparation.
- Astari Esfahani, A. et al. (2023) "Real-time Signal Detection for Cyclotron Radiation Emission Spectroscopy Measurements using Antenna Arrays", *Journal of Instrumentation*, In preparation.
- Astari Esfahani, A. et al. (2023) "Tritium Beta Spectrum and Neutrino Mass Limit from cyclotron Radiation Emission Spectroscopy", *Phys. Rev. Lett.*, Accepted.
- Astari Esfahani, A. et al. (2022) "SYNCA: A Synthetic Cyclotron Antenna for the Project 8 Collaboration", *Journal of Instrumentation*, **18**(01).

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Selected Presentations

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- *New Developments in the CRES Technique for Neutrino Mass Measurement*, Invited Talk, Fall 2022 Meeting of the APS Division of Nuclear Physics, New Orleans, Louisiana, USA, 2022
- *Signal Detection Algorithms for Phase III of the Project 8 Experiment*, Contributed Talk, APS April Meeting 2022, New York, New York, USA, 2022
- *Synthetic Electron Antenna for Calibrating the Project 8 Neutrino Mass Experiment*, Contributed Talk, Fall 2021 Meeting of the APS Division of Nuclear Physics, Virtual, 2021

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