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4 DEVELOPMENT OF SCALABLE APPROACHES TO NEUTRINO MASS 5 MEASUREMENT WITH THE PROJECT 8 EXPERIMENT

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²⁰ Abstract

²¹ Neutrinos are fundamental particles in the standard model and play an important role
²² in the current understanding of the universe, however, the masses of the neutrinos, one
²³ of the most fundamental parameters for any particles, is currently unknown. This fact
²⁴ represents a gaping hole in our current knowledge of the universe that may provide clues
²⁵ to the energy scale of possible physics beyond the standard model. This dissertation
²⁶ summarizes research and development as a member of the Project 8 collaboration towards
²⁷ an experiment to measure the neutrino mass to a sensitivity below $50 \text{ meV}/c^2$, which
²⁸ is an order of magnitude below the most sensitive direct measurements of the neutrino
²⁹ mass to date. Project 8 will perform this measurement using Cyclotron Radiation
³⁰ Emission Spectroscopy (CRES) to measure the beta-decay endpoint spectrum of atomic
³¹ tritium. I present an analysis of the signal reconstruction performance of an antenna
³² array system designed to perform large-scale CRES measurements. Next, I discuss an
³³ approach to calibrating an antenna array CRES experiment using a unique probe antenna
³⁴ designed to mimic radiation from CRES events. Finally, I present design studies for a
³⁵ resonant cavity that could be used to perform a CRES experiment with atomic tritium
³⁶ at multi-cubic-meter scales.

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781 **Chapter 1** |
782 **Introduction**

783 **1.1 Summary**

784 Neutrinos are one of the fundamental particles that comprise the standard model of
785 particle physics and account for a significant fraction of the matter in the universe.
786 Neutrinos are the most abundant fermions in the universe, but due to their weak
787 interactions neutrinos seldom interact with other particles. Regardless, neutrinos play a
788 unique role in the evolution of the early-universe, therefore, a detailed understanding of
789 the properties of the neutrino is key to understanding the universe at the cosmological
790 scale as well as the smallest particle physics regime.

791 It was uncertain that neutrinos had nonzero mass until vacuum neutrino flavor
792 oscillations were observed in the late 90's and early 00's. A simple relativistic argument
793 as to why oscillations are evidence for neutrino masses is that oscillations imply neutrinos
794 experience time, which means that they do not propagate at the speed of light, therefore
795 the masses of the neutrinos must be non-zero. Current neutrino oscillation data supports
796 that neutrino flavor states are actually a superposition of three separate neutrino states
797 with well-defined masses. Measurements of neutrino oscillations that have taken place
798 over the past couple of decades have measured the differences between neutrino mass
799 eigenstates with increasing precision. However, oscillation measurements cannot tell
800 us the mass scale of the neutrinos, which is required in order to measure the absolute
801 neutrino masses.

802 The neutrino mass scale remains an unknown quantity in the standard model of
803 particle physics. The value of the neutrino mass influences the evolution of the early
804 universe and is likely relevant to the energy-scale of new physics responsible for the factor
805 of 10^{-6} difference between the neutrino and electron masses. A model-independent way
806 to measure the neutrino mass is to measure the tritium beta-decay spectrum near its
807 endpoint. Energy conservation requires that the neutrino mass carry away some kinetic

808 energy from the beta-decay electron in the form of its mass, which causes a distortion in
809 the shape of the tritium beta-decay spectrum near the endpoint. The isotope tritium has
810 many advantages for this measurement, and has been used by the KATRIN collaboration
811 to perform the most sensitive direct neutrino mass measurement to date.

812 KATRIN represents the state-of-the-art in the current generation of neutrino mass
813 direct measurement experiments with a projected neutrino mass sensitivity of $m_\nu < 200$ meV.
814 This sensitivity does not fully exhaust the allowed parameter space of neutrino
815 masses under the normal and inverted neutrino mass ordering scenarios, which motivates
816 the development of a next generation of neutrino mass measurement experiments.

817 The Project 8 collaboration is developing a next-generation neutrino mass experiment
818 with a goal neutrino mass sensitivity of $m_\nu < 40$ meV. This sensitivity is sufficient to
819 exhaust the range of neutrino masses allowed under the inverted mass ordering regime.
820 Project 8 intends to achieve its sensitivity goal utilizing two technologies that are novel
821 to the space of direct neutrino mass measurements — atomic tritium and cyclotron
822 radiation emission spectroscopy (CRES). Atomic tritium is required in order to avoid
823 systematic broadening the tritium beta-decay spectrum caused by the final state of the
824 $^3\text{He}^+ \text{-T}$ molecule, and the CRES technique enables a differential measurement of the
825 tritium spectrum that is background-free and able to be directly integrated with the
826 atomic tritium source.

827 The Project 8 collaboration is currently engaged in a research and development
828 program intended to simultaneously develop the atomic tritium and CRES technologies
829 so that they can be combined in a next-generation experiment. This past year (2022)
830 Project 8 has used the CRES technique to measure the molecular tritium beta-decay
831 spectrum and place an upper limit on the neutrino mass: $m_\beta \leq 152$ eV. This measurement,
832 while not competitive scientifically, represents the first proof-of-principle that the CRES
833 technique can be used to measure the neutrino mass.

834 The future goals of the Project 8 collaboration are to develop the technologies
835 and techniques necessary to scale-up the volume in which CRES measurements can
836 be performed. Project 8's first neutrino mass measurement with CRES utilized a
837 measurement volume on the cubic-centimeter scale, however, sensitivity calculations
838 estimate that an experiment sensitive to neutrino masses of 40 meV will require several
839 tens of cubic-meters of experiment volume filled with atomic tritium. Developing a new
840 approach to performing CRES measurements that can be successfully scaled to these
841 volumes is a necessary step towards Project 8's neutrino mass measurement goal, and is
842 the primary topic of my dissertation research.

843 A parallel development is the technology necessary to produce, cool, trap, and
844 recirculate a supply of atomic tritium that is compatible with CRES measurements. The
845 atomic tritium system is equally important as the large-volume CRES measurement
846 technology, but will not be discussed at depth here.

847 The Project 8 collaboration has identified two scalable approaches to neutrino mass
848 measurement using the CRES technique. One approach is to use an array of antennas
849 that surrounds a volume of trapped atomic tritium that can perform CRES measurements
850 by collection the cyclotron radiation emitted by beta-decay electrons into free-space. The
851 other approach uses a resonant cavity filled with atomic tritium to perform CRES by
852 measuring the excitation of resonant cavity modes caused by the motion of electrons
853 trapped inside the cavity volume.

854 The cavity and antenna approaches to CRES have been studied in detail over the past
855 five years, and, while both approaches offer a physically viable path towards a 40 meV
856 neutrino mass measurement the collaboration has elected to pursue the cavity approach
857 for the foreseeable future. The major advantage of the cavity approach is a significant
858 reduction in the cost and complexity of the experiment design and data analysis, which
859 provides a lower risk path to Project 8’s scientific goals.

860 In this dissertation I summarize my most impactful contributions to the research and
861 development of antenna array and cavity CRES. In short these contributions are

- 862 • the development and analysis of signal reconstruction algorithms for antenna array
863 CRES, which provide key inputs to sensitivity analyses of antenna array CRES
864 experiments.
- 865 • The development of a specialized antenna, designed to synthesize fake CRES
866 radiation, which enables bench-top testing and validation of the antenna array
867 CRES technique.
- 868 • The development of an open-cavity design for CRES measurement, whose mode
869 structure can be tuned using perturbations that modify the impedance of the cavity
870 walls. The development of this cavity concept was one of many developments that
871 eventually lead to the adoption of cavities as the CRES technology of choice for
872 the future of Project 8.

873 1.2 Outline

874 The outline of this dissertation is as follows. In Chapter 2 I provide an introduction to
875 the basic physics of neutrinos and beta-decay, which provides context for a discussion of
876 various methods to measure the neutrino absolute mass scale.

877 Chapter 3 is an overview of the CRES technique and the Project 8 collaboration.
878 I highlight the Project 8 Phase II experiment, which was the first measurement of
879 the tritium beta-decay spectrum with CRES, and I discuss the planned research and
880 development for an antenna array CRES experiment in Phase III of the Project 8
881 collaboration’s experiment plan. I end Chapter 3 with a discussion of the pilot-scale and
882 Phase IV experiments, that will combine a scalable CRES measurement technology with
883 atomic tritium and measure the neutrino mass with 40 meV sensitivity.

884 Chapter 4 discusses the first of my contributions mentioned above, which is the
885 development of signal reconstruction techniques for antenna array CRES and an antenna
886 array demonstrator experiment called the FSCD. I discuss the key tools that Project 8
887 uses to simulate antenna array CRES before introducing signal reconstruction algorithms
888 that can be used to detect CRES signals using the array. I end Chapter 4 with a paper
889 that summarizes a detailed analysis and comparison of the signal detection performance
890 of each algorithm.

891 Chapter 5 describes my contributions to the development of antennas and an antenna
892 measurement system for Project 8, which is the second major contribution of this
893 dissertation. I begin with a general overview of basic principle of antennas and antenna
894 measurements, before including a paper that describes the development of unique antenna
895 designed to mimic the cyclotron radiation emitted by electrons in free-space when trapped
896 in a magnetic field. I call this antenna the synthetic cyclotron radiation antenna (SYNCA)
897 and its main purpose is to serve as a fake electron for laboratory validation measurements
898 of Project 8’s antenna array CRES simulations. Chapter 5 ends with an overview of
899 laboratory measurements of a prototype antenna array using the SYNCA, which were
900 compared with simulations to provide upper bounds on reconstruction errors caused by
901 imperfections in real-life measurements.

902 Chapter 6 discusses the cavity approach to CRES, which was adopted as the preferred
903 CRES technology for Phase IV late into my dissertation work. The chapter stars by
904 discussing resonant cavities in general before introducing the operating principles of the
905 cavity approach to CRES. I end the chapter by discussing a study of and open-cavity
906 design that could be used for CRES measurements and integrated with atomic tritium

907 and an electron gun calibration source for the pilot-scale and Phase IV experiments.

908 Finally, in Chapter 7 I conclude by briefly discussing the future directions of the
909 Project 8 collaboration as we continue towards a direct measurement of the neutrino
910 mass.

₉₁₁ **Chapter 2 |**

₉₁₂ **Neutrinos and Neutrino Masses**

₉₁₃ **2.1 Introduction**

₉₁₄ In this chapter I provide a cursory overview of background information relevant to
₉₁₅ neutrinos and neutrino mass measurements.

₉₁₆ In Section 2.2 I provide background information on the history of neutrinos and beta-
₉₁₇ decay. In Section 2.3 I describe the discovery of neutrino oscillations, which demonstrated
₉₁₈ unambiguously that neutrinos have non-zero masses. In Section 2.4 I discuss the current
₉₁₉ state of the theoretical understanding of neutrino masses in the standard model. Lastly,
₉₂₀ in Section 2.5 I discuss a few methods for measuring the absolute scale of the neutrino
₉₂₁ mass.

₉₂₂ **2.2 Neutrinos and Beta-decay**

₉₂₃ Late in the 19th century the phenomena of radioactivity was first observed in experiments
₉₂₄ performed by Henri Becquerel with uranium, and further studied using thorium and
₉₂₅ radium by Marie and Pierre Curie [4, 5]. Early work in radioactivity classified different
₉₂₆ forms of radiation based on it's ability to penetrate different materials. Rutherford was
₉₂₇ the first to separate radioactive emissions into two types, alpha and beta radiation [6].
₉₂₈ Alpha rays were easily stopped by a piece of paper or thin foil of metal, whereas beta
₉₂₉ radiation could penetrate metal several millimeters thick. Later a third form of radiation
₉₃₀ was identified by Villard [7], which was still more penetrating, later termed gamma
₉₃₁ radiation by Rutherford.

₉₃₂ When these forms of radioactivity were first discovered it was unclear what physically
₉₃₃ constituted an alpha, beta, or gamma particle. Experiments with radioactivity in magnetic
₉₃₄ fields were eventually able to identify the charge composition of the different forms of

radiation. In particular, experiments by Becquerel identified [8] that beta radiation had an identical charge-to-mass ratio to the electron. This was strongly suggestive that beta particles were indeed electrons.

Studies of beta radiation lead to the discovery that radioactivity resulted in the transmutation of elements [9] caused by the decay of a heavier nucleus to a lighter species. One feature of beta radiation, which we will refer to now as beta-decay, that differentiated it from alpha and gamma radiation is that the electrons produced by beta-decay have a continuous spectrum of kinetic energies, whereas, alpha and gamma particles are emitted with discrete energies. This feature of beta-decay was first observed by Chadwick in 1914 [10], and was extremely puzzling at the time, since the continuous spectrum apparently violates energy conservation [11].

Famously, in 1930 Pauli proposed the existence of a new neutral particle, which he termed the "neutron", that was also produced during beta-decay to resolve the missing energy problem posed by the beta-decay spectrum [12]. Because this particle carried no charge, it was hypothesized that it had simply not been observed in any previous experiments. This "neutron", which was initially estimated to have a mass no larger than that of an electron, was eventually renamed the "neutrino" by Fermi [13] after the discovery of the neutron by Chadwick in 1932 [14]. Later, in 1933, Fermi developed a quantum mechanical theory for beta-decay in which an electron and neutrino are produced by the decay of a neutron to a proton inside the radioactive nucleus [15].

Little more than a speculation when first introduced, indirect evidence for the existence of neutrinos was obtained in 1938 by the simultaneous observation of the electron and recoiling nucleus in cloud chambers by Crane and Halpern [16]. However, it wasn't until the Cowan-Reines experiment [17] in 1956 that direct evidence for the existence of neutrinos was observed through the observation of inverse beta-decays caused by neutrinos from a nuclear reactor interacting with protons contained in water molecules. The difficulty in detecting neutrinos is caused by their weak interactions with other particles. Later experiments revealed the existence of different types or flavors of neutrinos based on the nature of the leptons produced in neutrino charged-current interactions [18], but the existence of a neutrino mass remained an open question that would take more than 40 years to resolve.

966 2.3 Neutrino Oscillations

967 One of the first clues that neutrino flavor transitions or neutrino oscillations were occurring
 968 was the solar neutrino problem. The solar neutrino problem is a discrepancy between
 969 the measured and predicted flux of ν_e from the sum. The solar neutrino problem was
 970 famously observed by Ray Davis Jr. and collaborators in the 1960's [19] at the Homestake
 971 mine in South Dakota. In the early 2000's, the SNO experiment was able to resolve the
 972 solar neutrino problem by identifying neutrino oscillations as the cause of the observed
 973 deficit [20]. Furthermore, measurements of the atmospheric flux of neutrinos by the
 974 Super-Kamiokande experiment and others revealed that fewer muon-type neutrinos
 975 survived passage through the earth than expected providing strong evidence for neutrino
 976 oscillations for both flavors [21].

977 Neutrino oscillations occur because the weakly-interacting neutrino eigenstates are
 978 distinct from the mass eigenstates [22]. The neutrino mass eigenstates represent physical
 979 particles in that they are solutions to the free-particle Hamiltonian, whereas, the neutrino
 980 weak eigenstates correspond to the neutrino states that interact via the weak charged-
 981 current interaction. The neutrino weak eigenstates are a linear superposition of the
 982 neutrino mass eigenstates

$$983 \quad \nu_\ell = \sum_i U_{\ell i} \nu_i, \quad (2.1)$$

983 where $\ell = e, \mu, \tau$ and $i = 1, 2, 3$. The matrix elements $U_{\ell i}$ are the elements of the
 984 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix that describes the mixing between
 985 the neutrino flavor and mass states.

986 A standard parameterization [23] of the PMNS matrix is

$$987 \quad U_{PMNS} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \\ 988 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.2) \\ 989 \times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

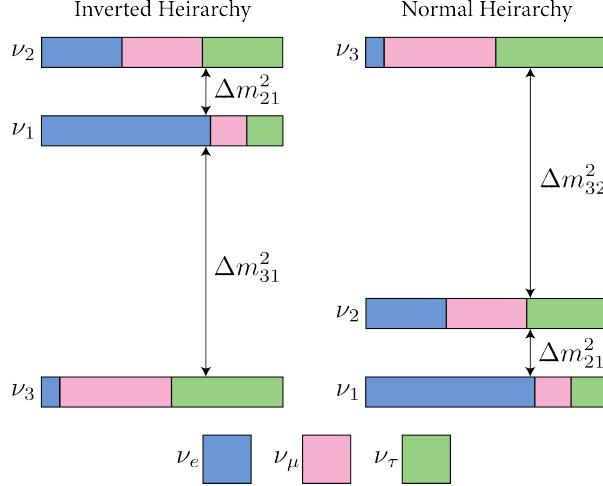


Figure 2.1. A diagram of two different neutrino mass ordering scenarios. In the inverted hierarchy (inverted mass ordering) the lightest neutrino mass is m_3 , whereas, in the normal hierarchy (normal mass ordering) m_1 is the lightest neutrino. What cannot be measured by neutrino oscillations is the neutrino absolute mass scale, which is essentially the mass of the lightest neutrino mass eigenstate.

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. The parameters α_1 and α_2 are only included in the PNMS matrix if neutrinos are Majorana particles, something which represents a current area of research in neutrino physics. The phase δ quantifies the degree of CP-violation in the neutrino sector. Including the Majorana phases the PMNS matrix contains six independent parameters. Neutrino oscillation probabilities also depend on the squared mass differences between neutrino mass eigenstates

$$\Delta m_{ij}^2 = m_i^2 - m_j^2, \quad (2.3)$$

where $ij = 12, 32, 31$ respectively. Because $\Delta m_{32}^2 = \Delta m_{31}^2 - \Delta m_{21}^2$, this adds an additional two parameters that must be constrained by neutrino oscillations.

A large experimental effort over the past couple decades has greatly contained the majority of parameters in the PMNS matrix, many to relative uncertainties of only a few percent. However, certain ambiguities remain, which is the origin of the current uncertainty in the ordering of the neutrino masses (see Figure 2.1). The neutrino masses can be arranged by their relative masses. Current neutrino oscillation data supports that $m_2 > m_1$, however, the sign of Δm_{32}^2 is still unknown. Therefore, two mass-ordering scenarios are allowed, one where neutrino masses are arranged $m_3 > m_2 > m_1$, which is called the normal mass ordering (NMO), or alternatively neutrino masses may be ordered $m_2 > m_1 > m_3$, which is called the inverted mass ordering (IMO). Next-

1004 generation neutrino oscillation experiments such as JUNO [24], Hyper-Kamiokande [25],
 1005 and DUNE [26] are poised to resolve this ambiguity in the coming years.

1006 Neutrino oscillation probabilities are sensitive to the neutrino masses via the squared
 1007 mass differences. Therefore, oscillation probabilities are unaffected by the absolute scale
 1008 of the neutrino mass. However, oscillations can be used to obtain a lower bound on the
 1009 neutrino masses by setting the mass of the lightest neutrino mass state to zero. This
 1010 results in different lower limits depending on the ordering of the neutrino mass states.
 1011 Current best-fit values [23] with 1σ -uncertainties for the squared mass differences are

$$\Delta m_{21}^2 = (7.42^{+0.21}_{-0.20}) \times 10^{-5} \text{ eV}^2, \quad (2.4)$$

$$\Delta m_{31}^2 = (2.5176^{+0.026}_{-0.028}) \times 10^{-3} \text{ eV}^2 \text{ (NMO)}, \quad (2.5)$$

1012 for the normal mass ordering, and for the inverted ordering the limit is

$$\Delta m_{32}^2 = (-2.498^{+0.028}_{-0.028}) \times 10^{-3} \text{ eV}^2 \text{ (IMO).} \quad (2.6)$$

1013 The parameter Δm_{21}^2 is the same in the NMO and the IMO. Allowing the lightest neutrino
 1014 mass in each ordering scenario (m_{least}) to take on a range of values one can visualize the
 1015 relative masses of the neutrinos as a function of m_{least} (see Figure 2.2). The absolute
 1016 neutrino mass scale is effectively the value of this m_{least} parameter.

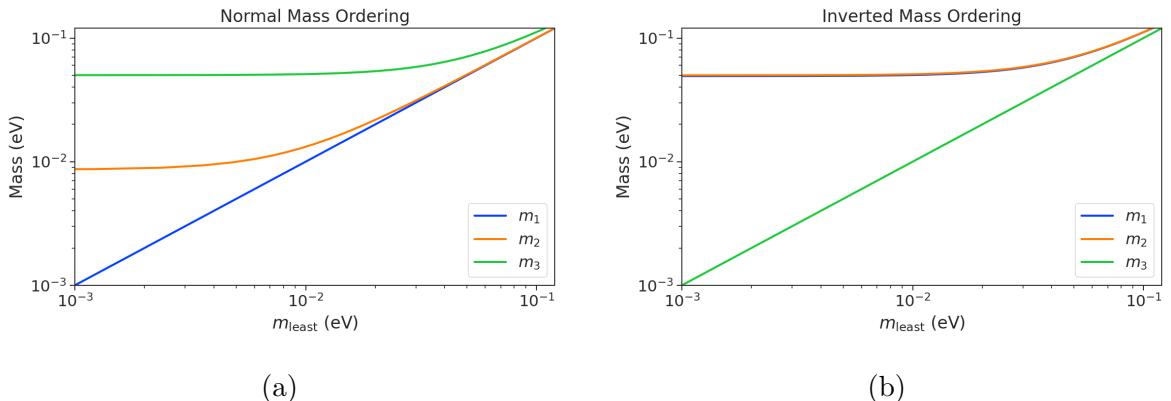


Figure 2.2. The masses of the neutrinos as a function of the lightest neutrino mass in both the normal (a) and inverted (b) mass ordering regimes.

2.4 Neutrino Masses in the Standard Model

In this section, I briefly summarize the current theoretical understanding of neutrino masses in the standard model [27–29]. Neutrinos are spin 1/2 particles, which are described using the Dirac equation.

$$(i\hbar\gamma^\mu\partial_\mu - mc)\psi(x) = 0, \quad (2.7)$$

where the field that describes the particle is denoted as $\psi(x)$. In the standard model fermions acquire mass through the Yukawa interaction, which add to the standard model Lagrangian terms of the form

$$\mathcal{L}_{\text{Yukawa}} = -Y_{ij}^\ell \bar{L}_{Li} \phi E_{Rj} + \text{h.c.}, \quad (2.8)$$

where Y_{ij}^ℓ is an element of the 3×3 Yukawa coupling matrix for leptons, L_{Li} is the left-handed lepton doublet for generation i , ϕ is the Higgs doublet, and E_{Rj} is the right-handed lepton field for generation j . Neutrinos are represented only as left-handed neutrinos and right-handed antineutrinos in the standard model, which is consistent with experimental observations. Since there are no right-handed neutrino singlet fields, there are no Yukawa interaction terms, thus neutrinos in the standard model are strictly massless. Therefore, non-zero neutrino mass is evidence for physics beyond the standard model.

For the charged leptons, the Yukawa interaction leads to masses of the form

$$m_{ij}^\ell = Y_{ij}^\ell \frac{v}{\sqrt{2}}, \quad (2.9)$$

where v is the Higgs vacuum expectation value. The observation of massive neutrinos motivates the extension of the standard model to explain the origin of neutrino masses, which can be approached in different ways, but all approaches add additional degrees of freedom to the standard model.

One approach is to introduce to the standard model a right-handed neutrino field that allows one to include Yukawa terms of the form

$$\mathcal{L}_{\nu \text{Yukawa}} = -Y_{ij}^\ell \bar{L}_{Li} \phi \nu_{Rj} + \text{h.c.} \quad (2.10)$$

where ν_{Rj} is the right-handed neutrino singlet. Because experimental evidence strongly

1040 predicts only three active neutrinos, these additional neutrinos are sterile and do not in-
1041 teract via the strong, weak, or electromagnetic interactions. After spontaneous symmetry
1042 breaking, the Yukawa interaction leads to mass terms given by

$$\mathcal{L}_D = -M_{Dij}\bar{\nu}_{Ri}\nu_{Lj} + \text{h.c.}, \quad (2.11)$$

1043 which is called a Dirac mass term. One of the issues with constructing neutrino masses
1044 in this way is that the required Yukawa couplings are at least a factor of 10^6 smaller than
1045 that of an electron, which begs the question: why are the Yukawa couplings so small for
1046 the neutrinos?

1047 An alternative approach is to allow the neutrinos to have a Majorana mass, which is
1048 possible because neutrinos are electrically neutral particles. The Majorana mass terms
1049 for neutrinos have the form

$$\mathcal{L}_M = -\frac{1}{2}(M_{Rij}\bar{\nu}_{Ri}\nu_{Rj}^c M_{Lij}\bar{\nu}_{Li}\nu_{Lj}^c) + \text{h.c.}, \quad (2.12)$$

1050 where M_{Rij} and M_{Lij} are right-handed and left-handed Majorana mass matrices. A
1051 consequence of neutrinos being Majorana particles is lepton number violation, which
1052 predicts the occurrence of neutrino-less double beta-decay at a rate proportional to the
1053 neutrino mass.

1054 In the most general case neutrinos have both Dirac and Majorana mass terms, which
1055 allows one to generate neutrino masses with Yukawa couplings similar to the rest of the
1056 standard model. Considering a single generation of neutrinos for demonstration, the
1057 combined neutrino mass Lagrangian can be written as

$$\mathcal{L}_{D+M} = -m_D\bar{\nu}_R\nu_L - \frac{1}{2}(m_L\bar{\nu}_L\nu_L^c + m_R\bar{\nu}_R\nu_R^c) + \text{h.c.}, \quad (2.13)$$

1058 or equivalently,

$$\mathcal{L}_{D+M} = -\frac{1}{2} \begin{bmatrix} \bar{\nu}_L & \bar{\nu}_R^c \end{bmatrix} \begin{bmatrix} m_L & m_D \\ m_D & m_R \end{bmatrix} \begin{bmatrix} \nu_L^c \\ \nu_R \end{bmatrix} + \text{h.c..} \quad (2.14)$$

1059 An example mass generation mechanism with this approach is the Type-I see-saw
1060 mechanism [30], in which we take $m_L = 0$ and $m_R \gg m_D$. By diagonalizing Equation
1061 2.14 one obtains the mass eigenvalues that represent the physical masses of the neutrinos.
1062 The light neutrino mass eigenstate, which represents the observed neutrino mass, has a
1063 mass given by

$$m_1 \approx \frac{m_D^2}{m_R}, \quad (2.15)$$

1064 and the heavy neutrino mass eigenstate, which represents the unobserved sterile neutrino,
1065 has a mass

$$m_2 \approx m_R. \quad (2.16)$$

1066 For m_D similar to the other quark or lepton masses, one obtains physical neutrino masses
1067 consistent with observations from sterile neutrino masses of $m_R \approx O(10^{15})$ GeV. This
1068 mass scale is well beyond the capabilities of modern particle accelerators to probe.

1069 2.5 Neutrino Absolute Mass Scale

1070 The neutrino absolute mass scale or simply "neutrino mass" cannot be probed with
1071 neutrino oscillations, since oscillation probabilities are determined by the squared mass
1072 differences between neutrino mass eigenstates, therefore, alternative techniques are needed
1073 to perform an effective measurement of the neutrino mass.

1074 2.5.1 Limits from Cosmology

1075 The Λ CDM model summarizes the current cosmological understanding of the universe [23].
1076 Λ CDM predicts that the universe originated from a single expansion event colloquially
1077 called the "Big Bang". During the Big Bang, the universe originated as a hot spacetime
1078 singularity, which abruptly experienced rapid expansion in a process known as inflation.
1079 After expansion the inflationary field eventually decayed into a population of quarks,
1080 gluons, leptons, and photons, which were kept in thermal equilibrium by the high-
1081 temperatures of the early universe.

1082 As the universe continued to expand it's density and temperature decreased until
1083 the formation of neutral atoms, primarily hydrogen, was possible. At which point the
1084 population of photons produced during the Big Bang decoupled from the primordial
1085 universe and began to freely propagate. A direct prediction of the Λ CDM model is that
1086 this population of photons is still present, but with a significantly reduced temperature
1087 due to the subsequent expansion of the universe. This is consistent with the observation of
1088 the CMB (cosmic microwave background), which is a population of microwave radiation
1089 with a blackbody temperature of 2.7 K. The CMB is extremely uniform in all directions
1090 with slight anisotropies that can be analyzed to study the evolution of the early universe.
1091 A series of experiments have measured the CMB with increasing levels of precision, which
1092 has lead to a significant increase in our current understanding of cosmology.

1093 In addition to the CMB, inflation predicts the existence of a $C\nu B$ (cosmic neutrino

background) [31], which are the remnant neutrinos produced during the Big Bang. Since neutrinos only interact via the weak force, they decouple from the Big Bang plasma at an earlier time than the CMB photons. The temperature at which the C ν B decouples depends on the neutrino rest mass. Neutrinos play a unique role in the Λ CDM model, due to the fact that neutrinos act as radiation early in the universe but as matter in the late universe. This leads to specific signatures that impact the expected anisotropies of the CMB as well as the distribution of matter in the universe [32]. By combining measurements of the CMB with measurements of the large-scale structure (LSS) of the universe one can constrain the neutrino mass scale by fitting these datasets with the Λ CDM model. This analysis results in some of the most stringent constraints on the neutrino mass. Recent analyses [23] have been able to constrain the neutrino mass scale to

$$\Sigma_{m_\nu} \equiv \sum_i m_i < 0.11 \text{ eV}, \quad (2.17)$$

where m_i are the neutrino mass eigenstates.

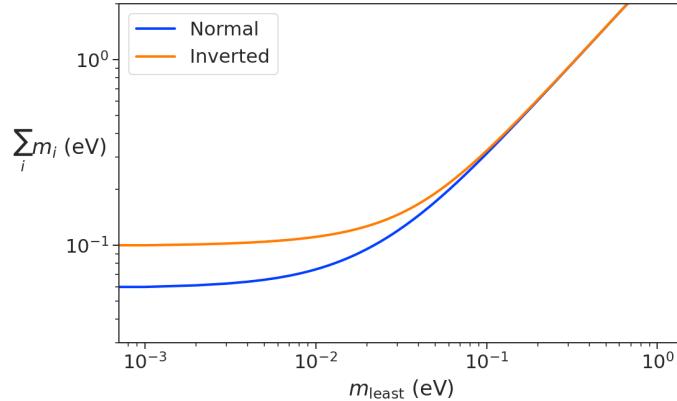


Figure 2.3. The neutrino mass observable measured by cosmology as a function of the lightest neutrino mass eigenstate.

The observable Σ_{m_ν} constrains the neutrino mass by setting the mass of the lightest neutrino mass eigenstate (m_{least}) (see Figure 2.3). In the normal mass ordering Σ_{m_ν} can be rewritten in the form

$$\Sigma_{m_\nu} = m_{\text{least}} + \sqrt{\Delta m_{21}^2 + m_{\text{least}}^2} + \sqrt{\Delta m_{32}^2 + m_{\text{least}}^2}, \quad (2.18)$$

where it is clear that a measurement of Σ_{m_ν} effectively sets the neutrino mass scale

1111 through m_{least} . The analogous formula for the inverted mass ordering is

$$\Sigma_{m_\nu} = m_{\text{least}} + \sqrt{-\Delta m_{32}^2 + m_{\text{least}}^2} + \sqrt{-\Delta m_{31}^2 + m_{\text{least}}^2}. \quad (2.19)$$

1112 Upcoming experiments [33] are planned to refine measurements of the CMB, LSS,
 1113 and other cosmological observables. With this additional data it is possible that in the
 1114 near future cosmological measurements will be able to positively constrain the neutrino
 1115 absolute mass scale. However, the strength of these limits strictly depend on the accuracy
 1116 of the Λ CDM model, which highlights the need for direct experimental measurements of
 1117 the neutrino mass to confirm the predictions of cosmology and to fix the neutrino mass
 1118 parameter in future cosmological analyses.

1119 2.5.2 Limits from Neutrinoless Double Beta-decay Searches

1120 If neutrinos are Majorana fermions, then the neutrino is equivalent to its own antiparticle
 1121 and lepton conservation is not an exact law of nature [34]. Limits on the rate of
 1122 neutrinoless double beta-decay ($0\nu\beta\beta$), are some of the most powerful current tests of
 1123 lepton number conservation [23]. If $0\nu\beta\beta$ were observed, it would direct evidence that
 1124 neutrinos are Majorana fermions and provide a method for measuring the neutrino mass
 1125 scale.

1126 Standard double beta-decay occurs when two neutrons in an unstable nucleus spon-
 1127 taneously decay into two protons, which results in the production of two electrons and
 two neutrinos (see Figure 2.4). Whereas, during $0\nu\beta\beta$ the two neutrinos self-annihilate

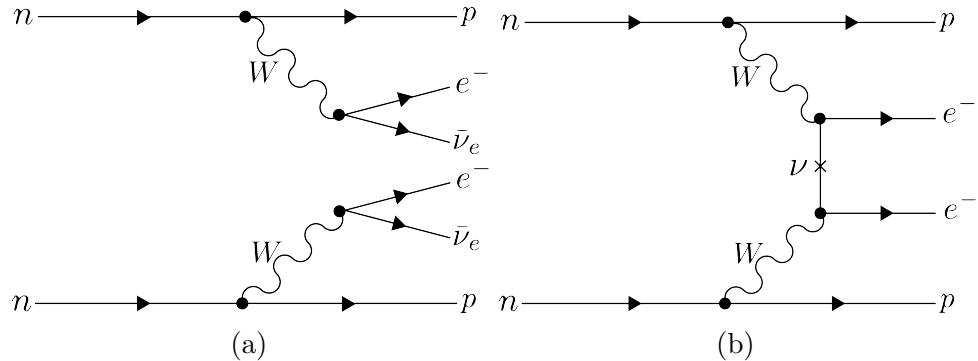


Figure 2.4. Feynman diagrams for double beta-decay (a) and $0\nu\beta\beta$ (b).

1128

1129 producing only two electrons, which violates lepton number by two.

1130 Assuming that the exchange of two Majorana neutrinos is the dominant channel for
 1131 $0\nu\beta\beta$, then a measurement of the $0\nu\beta\beta$ half-life for a particular isotope can be used to

1132 set the neutrino absolute mass scale [35]. The half-life is written in terms of the effective
 1133 neutrino mass for $0\nu\beta\beta$ ($m_{\beta\beta}$) using the equation

$$T_{1/2}^{0\nu} = \frac{1}{G|\mathcal{M}|^2 m_{\beta\beta}^2}, \quad (2.20)$$

1134 where G is the phase-space factor for the decay and \mathcal{M} is the relevant nuclear matrix
 1135 element. $m_{\beta\beta}$ is given by an incoherent sum of the neutrino mass eigenstates weighted
 1136 by the PMNS mixing matrix parameters,

$$m_{\beta\beta} = \left| \sum_i U_{ei}^2 m_i \right|. \quad (2.21)$$

1137 The information provided from $0\nu\beta\beta$ on the neutrino mass scale can be visualized by
 1138 expressing the value of $m_{\beta\beta}$ in terms of m_{least} and two relative Majorana phases [1]. The
 1139 allowed regions for $m_{\beta\beta}$ as a function of m_{least} are shown in Figure 2.5 as the regions
 1140 bounded by the black curves overlayed with the discovery probabilities of future $0\nu\beta\beta$
 decay experiments based on current neutrino data.

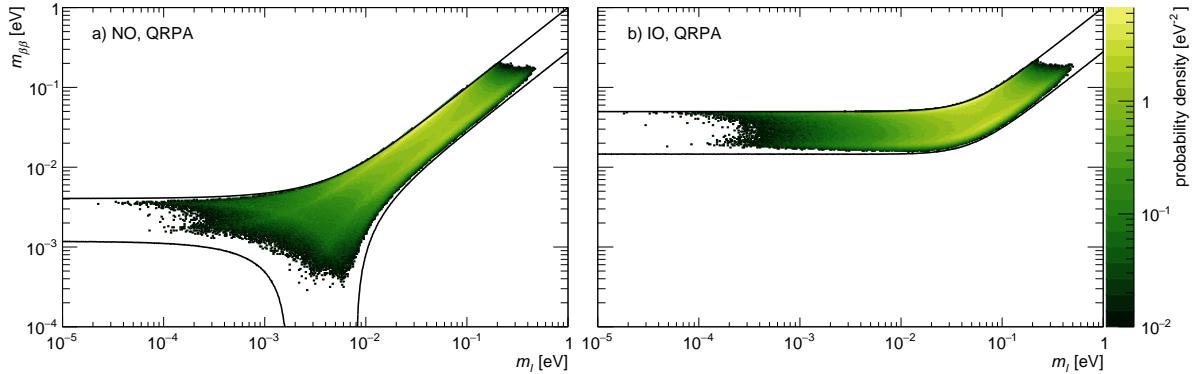


Figure 2.5. The discovery probabilities for the future generation of $0\nu\beta\beta$ experiments as a function of $m_{\beta\beta}$ and m_{least} . Figure from [1].

1141
 1142 Because of the possibility of cancellation due to the unknown Majorana phases included
 1143 in the sum specified by Equation 2.21, the neutrino mass information gained from $0\nu\beta\beta$
 1144 is necessarily imperfect. Additionally, theoretical uncertainties in the calculation of the
 1145 nuclear matrix elements complicates the calculation of $m_{\beta\beta}$ from a measurement of $0\nu\beta\beta$
 1146 half-life. Similar to cosmology, there is a high degree of complementarity between direct
 1147 measurements of the neutrino mass and $0\nu\beta\beta$. In particular, a measurement of m_{least} to
 1148 less than 0.1 eV sensitivity provides significant information for $0\nu\beta\beta$ searches based on
 1149 the discovery probabilities displayed in Figure 2.5.

2.5.3 Limits from Beta-decay

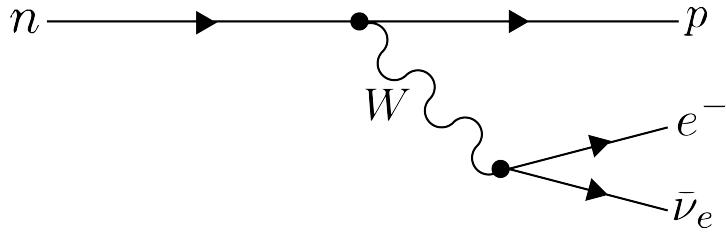


Figure 2.6. A Feynman diagram of beta decay

Certain processes involving neutrinos, in particular beta-decay (see Figure 2.6), have initial states with well-defined total energies and final states that can be measured with high accuracy and precision. Beta-decay involves the decay of an unstable isotope where a neutron spontaneously converts to a proton and emits an electron and anti-neutrino ("neutrino" for brevity) to conserve charge and lepton number [4]. Therefore, by applying the principles of energy and momentum conservation, a measurement of the kinematics of the final state can be used to constrain the neutrino mass [36].

Using beta-decay to measure the neutrino mass can be tied back to Fermi's original 1934 theory of nuclear beta-decay [15] (see Figure 2.7). Because the constraints on the

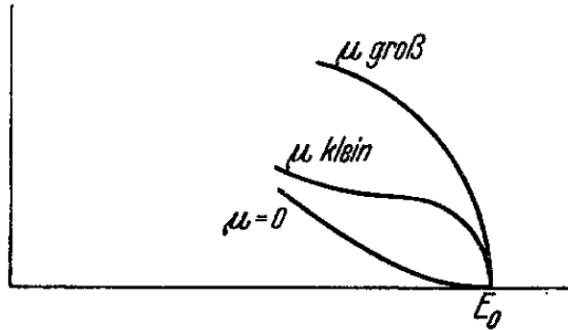


Figure 2.7. A figure from Fermi's 1934 paper on a theory of beta-decay depicting the kinetic energy spectrum of the emitted electron. The effect of the neutrino mass, written as μ , is to distort the shape of the spectrum near the endpoint from the zero-mass spectrum.

neutrino mass from beta-decay depend only on the final state measurement capabilities and the principles of energy and momentum conservation, neutrino mass measurements with beta-decay are called direct measurements. A direct measurement like beta-decay contrasts with other neutrino mass measurements approaches that are model-dependent such as cosmology and $0\nu\beta\beta$, which provide complementary ways to study the physics of massive neutrinos.

1166 The isotope of choice for direct neutrino mass measurements with beta-decay has
 1167 been tritium (3H_2) for many decades, because it conveniently fulfills many experimental
 1168 requirements. Of upmost importance is a decay with a low Q-value, which is the available
 1169 kinetic energy based on the mass difference between the initial and final states. The
 1170 effect of a massive neutrino on the shape of the spectrum is magnified for low Q-values
 1171 and tritium has an unusually low Q-value of 18.6 keV.

1172 Additionally, tritium beta-decay is super-allowed, which results in a relatively short
 1173 half-life of 12.3 years. Therefore, high source activity can be obtained with a relatively
 1174 small source mass. High-activity is desirable because of the low-activity near the tritium
 1175 spectrum endpoint. For tritium beta-decays, only a factor of 3×10^{-13} of the decays
 1176 occur in the last 1 eV of the spectrum. Isotopes with Q-values lower than tritium are
 1177 known [36], but this is outweighed by exceedingly long half-lives leading to unobtainable
 1178 source masses.

1179 The endpoint measurement approach involves quantifying the effect of the neutrino's
 1180 mass on shape of the electron's kinetic energy spectrum near the endpoint. The shape of
 1181 the kinetic energy spectrum (see Figure 2.8) is given by

$$\frac{d\Gamma}{dE} = \frac{G_F^2 |V_{ud}|^2}{2\pi^3} (G_V^2 + 3G_A^2) F(Z, \beta) \beta (E + m_e)^2 (E_0 - E) \\ \times \sum_{i=1,2,3} |U_{ei}|^2 [(E_0 - E)^2 - m_i^2]^{1/2} \Theta(E_0 - E - m_i), \quad (2.22)$$

1182 where G_F is the Fermi coupling constant, V_{ud} is an element of the CKM matrix, E
 1183 is the kinetic energy of the electron, β is the velocity of the electron divided by the
 1184 speed of light, E_0 is the endpoint energy assuming zero neutrino mass, $F(Z, \beta)$ is the
 1185 Fermi function, and $\Theta(E_0 - E - m_i)$ is the Heaviside function, which enforces energy
 1186 conservation. One can see that the decay spectrum is actually a combination of three
 1187 spectra with different endpoints based on the values of the neutrino mass eigenstates, m_i .
 1188 This produces "kinks" in the spectrum shape due to overlapping spectra with different
 1189 endpoint values, but such an effect would be nearly impossible to resolve given the finite
 1190 energy resolution of a real experiment.

1191 The neutrino mass scale variable measured by beta-decay is given by

$$m_\beta^2 = \sum_i |U_{ei}|^2 m_i^2, \quad (2.23)$$

1192 where m_β is the electron-weighted neutrino mass or simply "neutrino mass" for brevity.

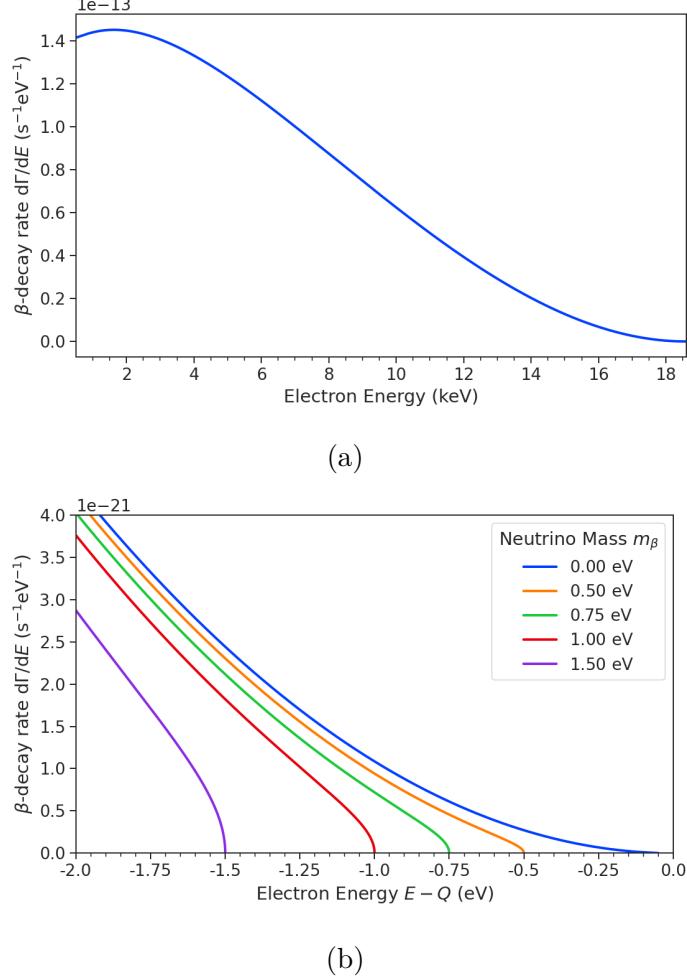


Figure 2.8. The tritium beta-decay spectrum. The effect of a massive neutrino on the spectrum is to change its shape near the endpoint by an amount proportional to the size of the neutrino mass. A sufficiently high-statistic and high-resolution measurement of the spectrum endpoint would be able to measure the neutrino mass.

1193 m_β corresponds to a particular weighted sum of the neutrino masses, which is distinct
 1194 from effective neutrino masses such as $m_{\beta\beta}$ [36]. Assuming unitarity, the neutrino mass
 1195 can be expressed in terms of the PMNS matrix elements, squared mass differences, and
 1196 the lightest neutrino mass eigenstate. For the normal mass ordering the equation is

$$m_\beta^2 = m_{\text{least}}^2 + |U_{e2}|^2 \Delta m_{21}^2 + |U_{e3}|^2 \Delta m_{31}^2, \quad (2.24)$$

1197 and for the inverted ordering the equation is

$$m_\beta^2 = m_{\text{least}}^2 + |U_{e1}|^2 (-\Delta m_{32}^2 - \Delta m_{21}^2) + |U_{e2}|^2 (-\Delta m_{32}^2). \quad (2.25)$$

1198 Therefore, a measurement of the neutrino mass in combination with neutrino mixing
1199 parameters is effectively a measurement of m_{least} .

1200 Since the neutrino mass is small (< 1 eV), it's effect on the spectrum is limited to
1201 the endpoint region. The affect of a non-zero neutrino mass on the endpoint spectrum is
1202 plotted for the reader in Figure 2.8. Resolving the small changes in the spectrum shape
1203 requires an experimental technique with high statistics, excellent energy resolution, and
1204 low background activity.

1205 **Chapter 3** | 1206 **Direct Measurement of the Neutrino Mass** 1207 **with Project 8**

1208 **3.1 Introduction**

1209 A promising technique for direct measurements of the neutrino mass beyond the projected
1210 200 meV limit of the KATRIN experiment [37] is tritium beta-decay spectroscopy with
1211 an atomic tritium source [38]. Atomic tritium, combined with a large-volume, high-
1212 resolution energy measurement technique, is capable of measuring the neutrino mass
1213 with sensitivity below the 50 meV, which exhausts the range of neutrino masses allowed
1214 under the inverted hierarchy.

1215 Cyclotron Radiation Emission Spectroscopy (CRES) is a high-resolution energy
1216 measurement technique compatible with atomic tritium production and storage that can
1217 enable the next-generation of neutrino mass direct measurement experiments [39]. The
1218 Project 8 collaboration is currently engaged in a program of research and development
1219 (R&D) aimed at developing the technology necessary for a 40 meV sensitivity measurement
1220 of the neutrino mass using CRES and atomic tritium [40].

1221 In Section 3.2 I provide an introduction to the basics of the CRES technique as well as
1222 the goals of the Project 8 experiment. Additionally, I sketch out the phased experiment
1223 development plan being implemented by Project 8 to build towards a next-generation
1224 neutrino mass experiment.

1225 In Section 3.3 I give an overview of Phase II of the Project 8 experiment [41,42], which
1226 completed early in 2023. Although the bulk of the work presented in this dissertation is
1227 relevant to designs of future Project 8 experiments, a description of the work in Phase II
1228 provides useful context.

1229 In Section 3.4 I introduce a CRES measurement concept based on antenna arrays [43],
1230 which could be the basis for the ultimate Project 8 neutrino mass experiment. A

1231 significant portion of the R&D efforts of Project 8 in Phase III were directed towards
1232 simulating and modeling this experimental concept in order to understand the achievable
1233 sensitivity to the neutrino mass.

1234 Lastly, in Section 3.5 I introduce conceptual designs of pilot-scale experiments and
1235 Phase IV that combine atomic CRES with a large-volume CRES detection technique.
1236 This includes a design concept for an antenna array based experiment, but also a design
1237 for a resonant cavity based experiment. Resonant cavities are discussed in more depth in
1238 Chapter 6 and have become the default choice for the Phase IV experiment.

1239 **3.2 Project 8 and Cyclotron Radiation Emission Spec-** 1240 **troscopy**

1241 **3.2.1 Cyclotron Radiation Emission Spectroscopy — CRES**

1242 Time and frequency are two of the most precisely measured quantities in physics. Atomic
1243 clocks, which operate by measuring the frequencies of various atomic transitions, have
1244 been used to measure time with astounding relative uncertainties of 10^{-18} seconds [44].
1245 The extreme precision possible with frequency measurements is often summarized using
1246 the a quote from the Physicist Arthur Schawlow who said advise his students to "Never
1247 measure anything but frequency!" [45].

1248 Neutrino mass measurements using tritium beta-decay require the measurement of
1249 perturbations to the 18600 eV tritium endpoint with a precision as small as 0.1 eV,
1250 therefore, a spectroscopic technique with extremely high resolution is required. Frequency
1251 measurements are capable of such high-resolutions for the intuitive reason that they are
1252 essentially digital counting measurements, which average the number of oscillations of a
1253 physical system over time. By observing a rapidly oscillating system over a sufficient
1254 length of time one can obtain essentially arbitrary precision on a frequency limited only
1255 by the measurement time and signal-to-noise ratio (SNR) of the system.

1256 A method is required for translating an electron kinetic energy measurement into a
1257 frequency measurement. A straightforward way to accomplish this is to place a gaseous
1258 supply of tritium into a magnetic field, therefore, when a beta-decay occurs the resulting
1259 electron will immediately begin to orbit around a magnetic field line at the cyclotron
1260 frequency, proportional to its kinetic energy (see Figure 3.1). The acceleration caused
1261 by the orbit leads to the emission of cyclotron radiation that can be detected using an
1262 array of antennas or resonant cavity. The starting frequency of the radiation gives the

1263 electron's initial kinetic energy, which is used to build the beta-decay spectrum and
 1264 measure the neutrino mass. The name for this measurement technique is Cyclotron
 1265 Radiation Emission Spectroscopy or CRES [39].

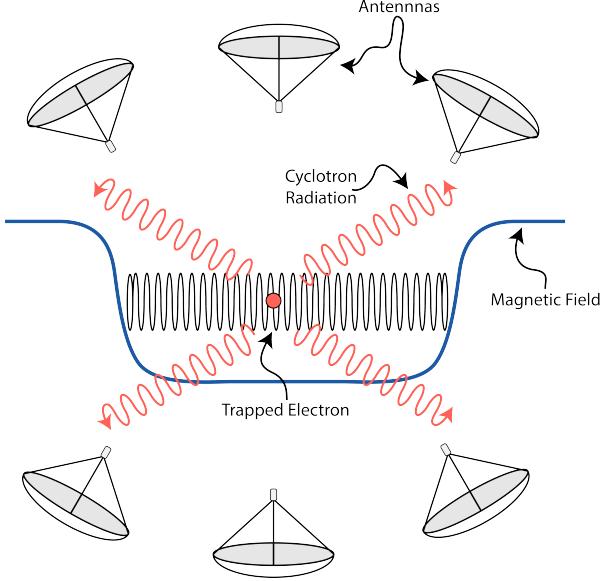


Figure 3.1. A cartoon illustration of the CRES technique. An electron is contained in a magnetic trap, which is a local minimum in the magnetic field, so that its cyclotron radiation can be detected by an array of antennas. Detecting the cyclotron radiation allows us to measure its cyclotron frequency and determine its kinetic energy.

1266 In the non-relativistic case, the cyclotron frequency is simply a function of the
 1267 charge-to-mass ratio of the particle, however, the relativistic correction to the cyclotron
 1268 frequency

$$f_c = \frac{qB}{2\pi m_e \gamma} = \frac{1}{2\pi} \frac{qB}{m_e + E_{\text{kin}}/c^2}, \quad (3.1)$$

1269 introduces a dependence of the kinetic energy (E_{kin}) to the inverse of the cyclotron
 1270 frequency (f_c). Electrons with kinetic energies of 18.6 keV are in the weakly relativistic
 1271 regime with $\beta = \frac{v}{c} = 0.263$ and $\gamma = 1.036$.

1272 The frequency resolution of a CRES measurement can be estimated by differentiating
 1273 Equation 3.1,

$$\frac{df_c}{dE_{\text{kin}}} = \frac{1}{2\pi} \frac{-qBc^2}{(m_e c^2 + E_{\text{kin}})^2}, \quad (3.2)$$

1274 from which we obtain the relationship between fractional differences in energy and
 1275 frequency,

$$\frac{df_c}{f_c} = \frac{1 - \gamma}{\gamma} \frac{dE_{\text{kin}}}{E_{\text{kin}}}. \quad (3.3)$$

1276 Therefore, an energy precision of 1 eV for an 18.6 keV electron requires a frequency
 1277 precision of approximately 2 ppm.

1278 The minimum observation time required to achieve this resolution can be estimated
 1279 using the uncertainty principle as formulated by Gabor [46]. Electrons from tritium
 1280 beta-decay experience random collisions with the background gas particles, which limits
 1281 the uninterrupted radiation lifetime. The time between collision events, referred to as
 1282 "track length", is an exponentially distributed variable. Differences in the track lengths
 1283 of a population of mono-energetic electrons leads to an uncertainty or broadening in the
 1284 distribution of measured frequencies, which is proportional to the mean track length, τ_λ .
 1285 The resulting frequency distribution has a Lorentzian profile, whose width is given by
 1286 the Gabor limit,

$$\tau_\lambda \Delta f_c = \frac{1}{2\pi} \implies \Delta f_c = \frac{1}{2\pi\tau_\lambda}. \quad (3.4)$$

1287 The cyclotron frequency for a 18.6-keV electron in a 1 T field is approximately
 1288 27 GHz, consequently, the minimum observation time for a 2 ppm frequency resolution
 1289 is approximately 3 μ sec. The Gabor limit is not the true lower bound on the frequency
 1290 resolution for a CRES signal, since it derives from the Fourier representation of a fixed
 1291 length time-series using a basis of infinite duration sinusoids. If one takes the approach of
 1292 fitting the CRES signal in the time-domain, then the lower limit on frequency precision
 1293 is given by the Cramér-Rao lower bound (CRLB) [47], which depends on the track length
 1294 and SNR. In general, the CRLB allows for better precision on the cyclotron frequency.

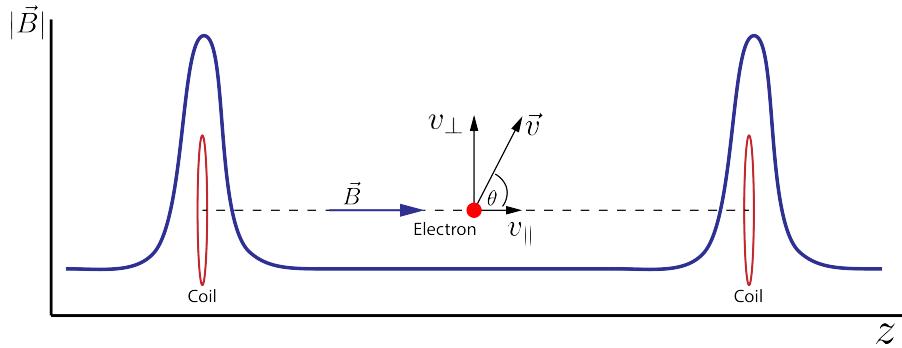


Figure 3.2. An illustration of an electron in a bathtub magnetic trap generated by two well-separated coils.

1295 Ensuring that an electron remains under observation long enough so that it's frequency
 1296 can be precisely measured requires a magnetic trap. A magnetic trap is a local minimum
 1297 in a background magnetic field generated an appropriate configuration of electromagnetic
 1298 coils. Since magnetic fields can do no work, there is no danger of the magnetic trap

1299 affecting the kinetic energy electron after it is emitted from the beta-decay. One common
 1300 approach to creating a magnetic trap is the "bathtub" trap configuration, which can be
 1301 produced using two magnetic pinch coils aligned on a central axis that are separated by
 1302 a distance that is large compared to the coil radius (see Figure 3.2). This configuration
 1303 produces a trap with a uniform bottom and relatively steep walls, which is ideal for
 1304 CRES measurements.

1305 Electrons produced in the trap oscillate back and forth between the trap walls at
 1306 a frequency that depends upon the pitch angle, unless they are produced with pitch
 1307 angles too small to be contained in the trap. Pitch angle is defined as the angle between
 1308 the component of the electron's velocity perpendicular to the magnetic field and the
 1309 component parallel to the magnetic field

$$\tan \theta = \frac{v_{\perp}}{v_{\parallel}}. \quad (3.5)$$

1310 The axial motion of the electron leads to variation in the cyclotron frequency caused
 1311 by the changing value of the magnetic field. This leads to frequency modulation that
 1312 generate sidebands in the cyclotron radiation spectrum. Resolving these sideband
 1313 frequency components is necessary for a complete reconstruction of the CRES signal in
 1314 the experiment.

1315 Electrons trapped in a cylindrically symmetric trap have three primary components of
 motion (see Figure 3.3). The dominant component, typically with the highest frequency,

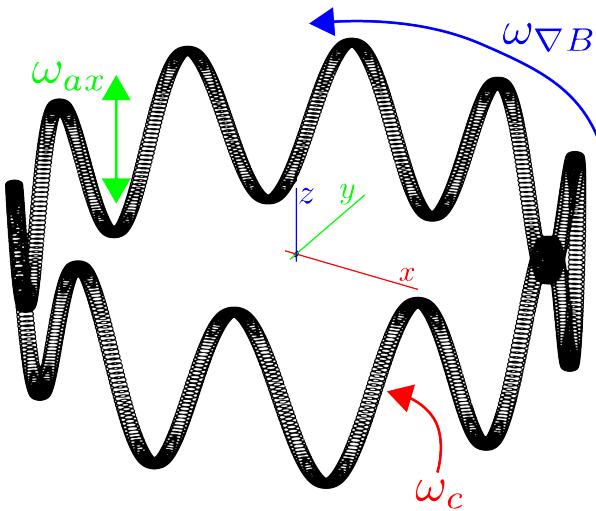


Figure 3.3. A plot of the main components of an electron's trajectory in a cylindrically symmetric trap.

1316
 1317 is the electron's cyclotron orbit, which encodes information on the electron's kinetic

1318 energy. Axial motion from the electron's pitch angle leads to frequency modulation,
 1319 and a shift in the average magnetic field experienced by an electron. This leads to a
 1320 correlation between the kinetic energy of the electron and the pitch angle depending on
 1321 the particular shape of the magnetic trap, which can negatively impact energy resolution.
 1322 To reduce this correlation one must engineer the trap to have a flat bottom with very
 1323 steep walls, which is more easily achieved with a small aspect ratio bathtub trap. Radial
 1324 gradients in the trap leads to a third component of motion called grad-B drift [48]. The
 1325 equation for the drift velocity is

$$\mathbf{v}_{\nabla B} = \frac{m_e v_{\perp}^2}{2qB} \frac{\mathbf{B} \times \nabla B}{B^2}. \quad (3.6)$$

1326 The total power of the radiation emitted by an electron in a free-space environment
 1327 is given by the Larmor equation [49]

$$P(\gamma, \theta_p) = \frac{1}{4\pi\epsilon_0} \frac{2}{3} \frac{q^2 \omega_c^2}{c} (\gamma^2 - 1) \sin^2 \theta_p, \quad (3.7)$$

1328 where ω_c is the cyclotron frequency multiplied by 2π and θ_p is the pitch angle to distinguish
 1329 it from the spherical angle coordinate. A single electron with a 90° pitch angle and
 1330 18.6 keV of kinetic energy in a 1 T magnetic field emits a total radiation power of 1.2 fW,
 1331 furthermore, one is typically only able to receive a fraction of this total power with an
 1332 antenna or other detection system. Therefore, RF systems in CRES experiments must be
 1333 operated at cryogenic temperatures to limit the noise power such that adequate SNR can
 1334 be achieved for signal detection and reconstruction. Alternatively, longer tracks enable
 1335 detection of weaker signals due to the increase in the total signal energy available for the
 1336 detection algorithm.

1337 3.2.2 The Project 8 Collaboration

1338 The Project 8 collaboration¹ is a group of institutions in the United States and Germany
 1339 aiming to measure the neutrino mass by developing a novel spectrometer technology
 1340 based on CRES. In the ultimate Project 8 experiment, the CRES technique will be used
 1341 to measure the beta-decay spectrum using a large source of atomic tritium sufficient to
 1342 achieve the required statistics in the last $O(10)$ eV of the decay spectrum. Project 8 is
 1343 targeting a neutrino mass sensitivity below 50 meV [50], which exhausts the range of
 1344 possible neutrino masses under the inverted hierarchy and is a factor of four less than

¹<https://www.project8.org/>

1345 sensitivity projections for the ongoing KATRIN experiment.

1346 Project 8's proposed experiment requires the development of two novel technologies:
1347 the production and trapping of a source of atomic tritium on cubic-meter scales and
1348 technology to enable CRES measurements of individual electrons in the same volume.

1349 **Atomic Tritium**

1350 Previous measurements of the tritium beta-decay spectrum for neutrino mass measure-
1351 ments have relied on sources of molecular tritium for their measurements [37, 51, 52] due
1352 to the technical challenges associated with the production and storage of atomic tritium.

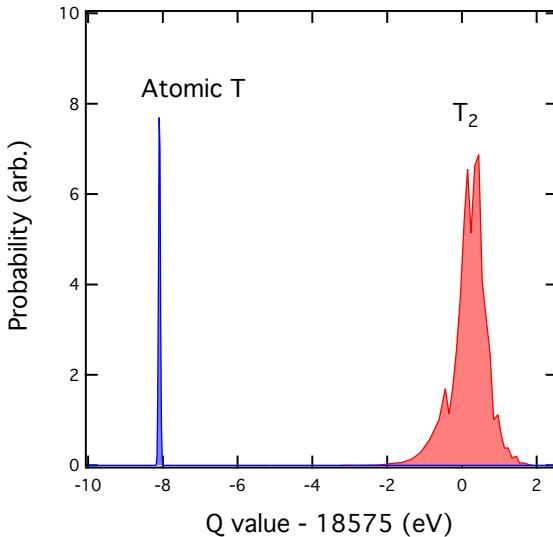


Figure 3.4. A plot of the final state distributions of atomic and molecular tritium. The final state distribution provides the primary contribution to the width of the molecular spectrum whereas thermal doppler broadening is responsible for the width of the atomic spectrum.

1353 One must supply sufficient energy to the tritium molecules to break the molecular
1354 bond and create atomic tritium. Common approaches include the use of hot coaxial
1355 filament atom crackers as well as plasma sources. Both involve heating the tritium atoms
1356 to temperatures of > 2500 K, which must then be cooled to temperatures on the order
1357 of a few mK so that the tritium atoms can be trapped. Cooling the atoms requires the
1358 construction of a large tritium infrastructure and cooling system that can supply a source
1359 of cold atoms to the trap.

1360 Once cold tritium atoms are produced they cannot make contact with any surfaces
1361 to avoid recombination of the atoms to molecules. Therefore, a magnetic trap is required
1362 to store the atoms for a sufficient length of time that they have a chance to decay before

1363 escaping the trap. Trapping the atoms requires the construction of a large and complex
1364 magnet system that must be cooled to cryogenic temperatures.

1365 The significant experimental complexity caused by atomic tritium makes a molecular
1366 source the obvious choice from practical considerations. However, the drawback of
1367 molecular tritium for neutrino mass measurement is the irreducible broadening in the
1368 electron's kinetic energy due to the final state spectrum of molecular tritium (see Figure
1369 3.4). The broadening of the final state spectra has a RMS amplitude of 436 meV [53, 54]
1370 caused by variation in the final vibrational state of the daughter molecule.

1371 For atomic tritium the primary sources of broadening in the final state spectrum are
1372 magnetic hyperfine splittings (magnitude of $O(10^{-5})$ eV) and thermal Doppler broadening
1373 caused by the motion of the trapped atom. Atomic tritium at a temperature of 1 mK
1374 has a broadening which is dominated by thermal Doppler broadening, providing about
1375 1 meV RMS of broadening to the electron's kinetic energy.

1376 The larger energy broadening with molecular tritium leads to an irreducible statistical
1377 uncertainty that limits the achievable sensitivity to approximately 100 meV at 90%
1378 confidence. For previous direct measurements of the neutrino mass this uncertainty is an
1379 insignificant contribution to the overall uncertainty budget, however, for experiments
1380 like Project 8 atomic tritium is a key component to the success of the experiment.

1381 CRES for Neutrino Mass Measurement

1382 Several features of the CRES technique make it an attractive choice for a next generation
1383 neutrino mass measurement experiment. Because CRES is a remote-sensing technique,
1384 it is possible to observe the kinetic energy of the electron without altering its trajectory
1385 or directly interacting with the particle, therefore, in a CRES experiment the source
1386 gas volume can be the same as the CRES spectrometer volume. Tritium gas is also
1387 transparent to cyclotron radiation, which means that the kinetic energies of electrons can
1388 be measured using a cavity or antenna array, located directly outside the atom trapping
1389 volume.

1390 Because source and spectrometer can be colocated, CRES experiments have an
1391 advantageous scaling law relative to the current state-of-the-art beta-decay spectroscopy
1392 experiment, KATRIN. KATRIN utilizes the magnetic adiabatic collimation with an
1393 electrostatic filter (MAC-E filter) technique to measure the beta-decay spectrum of
1394 molecular tritium. In this approach, a source of molecular tritium is located outside the
1395 spectrometer. When a beta-decay occurs the electron is guided out of the tritium source
1396 using a magnetic field and is transported through the MAC-E filter before it is detected

1397 on the other side of the filter using a charge sensor. The measurement statistics of the
1398 MAC-E filter are limited by the transverse area of the tritium source and filter due to the
1399 need to travel through the experiment without scattering. This scaling is less favorable
1400 than the volumetric scaling of CRES due to the ability to colocate source and detector.

1401 Another promising aspect of the CRES technique is the inherently high precision
1402 of frequency based measurements. The endpoint of the molecular tritium beta-decay
1403 spectrum is approximately 18.6 keV, which dwarfs the neutrino mass scale of $< 1 \text{ eV}/c^2$
1404 by at least a factor of 10^5 . Measuring the effect of such a small mass on a high energy
1405 electron requires excellent energy resolution. Since frequency measurements are essentially
1406 counting measurements they are intrinsically quite accurate due to the ability to measure
1407 the cyclotron frequency by effectively averaging over millions of cyclotron orbits. Using
1408 off-the-shelf RF components its is possible to achieve part-per-million accuracy on the
1409 kinetic energy with the CRES technique.

1410 CRES is also nearly immune to typical sources of backgrounds that can plague other
1411 experiments. Since CRES operates via a non-destructive measurement of the electron's
1412 cyclotron frequency, sources of background electrons are effectively filtered out by limiting
1413 the frequency bandwidth of the measurement. The fiducial volume of the experiment is
1414 free from any surfaces that could introduce stray electrons, and electrons from sources
1415 outside the fiducial volume can be prevented from entering the experiment.

1416 Neutrino Mass Sensitivity Goals

1417 Project 8's ultimate goal is to combine CRES with atomic tritium to measure the neutrino
1418 mass with 40 meV sensitivity at the 90% confidence level (see Figure 3.5). This sensitivity
1419 is sufficient to fully exhaust the range of allowable neutrino masses under the inverted
1420 neutrino mass ordering regime and is approximately an order of magnitude less than the
1421 projected final sensitivity of the KATRIN experiment. Excluding the full neutrino mass
1422 parameter space would require a sensitivity an order of magnitude lower than what is
1423 proposed by Project 8, which would require an experiment whose size and complexity
1424 are currently well beyond proposals for the next-generation of neutrino mass direct
1425 measurement experiments.

1426 **3.2.3 The Project 8 Phased Development Plan**

1427 Reaching 40 meV sensitivity requires the simultaneous development and eventually
1428 combination of CRES and atomic tritium. These technologies require a significant up-front

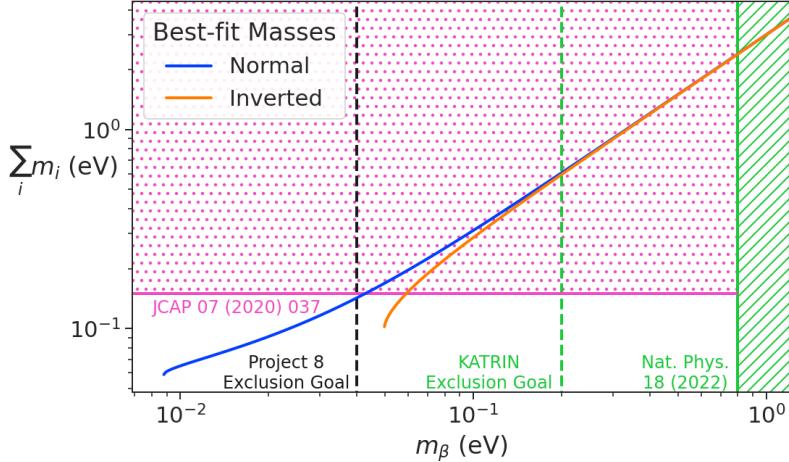


Figure 3.5. Neutrino mass exclusion plot including limits from cosmological measurements and the KATRIN experiment. Allowed ranges for neutrino masses under the normal and inverted hierarchies are shown as the blue and orange lines respectively. The black dashed line shows Project 8's goal neutrino mass sensitivity for the Phase IV experiment.

1429 R&D investment to build-out the required capabilities for a 40 meV CRES experiment.
 1430 Therefore, Project 8 is following a phased experiment plan in which incremental progress
 1431 can be made towards the ultimate goal of a 40 meV neutrino mass measurement with
 1432 CRES.

1433 Phase I and II: Proof of Principle and First Tritium Measurements

1434 The earlier phases of the Project 8 experiment, Phase I and II, were focused on demon-
 1435 stration and development of the CRES technique itself as well as a proof-of-principle
 1436 measurement of the neutrino mass using the CRES technique.

1437 In Phase I, Project 8 performed a proof-of-principle measurement of the ^{83m}Kr
 1438 spectrum using CRES, which marked the first ever kinetic energy spectrum measurement
 1439 with CRES. The experiment included all the components of a basic CRES experiment.
 1440 An electron source consisting of a gas of ^{83m}Kr was supplied to a waveguide gas cell
 1441 constructed out of a segment of WR-42 waveguide and sealed with Kapton windows at
 1442 the top and bottom. A magnetic trapping region was created in the waveguide cell using
 1443 a single electromagnetic coil wrapped around the waveguide which provided a trapping
 1444 volume on the order of a few cubic-millimeters. Detection of the cyclotron radiation was
 1445 performed by connecting the waveguide cell to an additional segment of waveguide that
 1446 transmitted the radiation to a cryogenic amplifier.

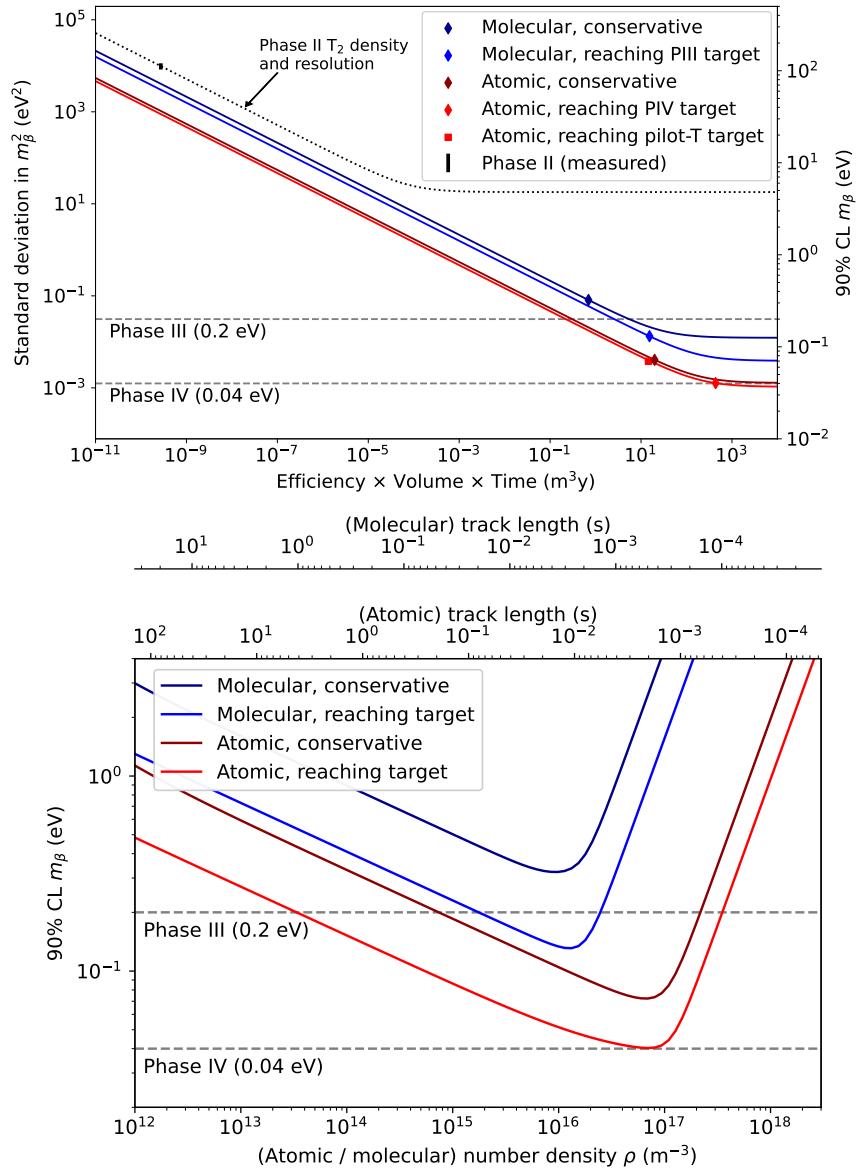


Figure 3.6. Sensitivity calculations for a cavity based CRES experiment that demonstrate the neutrino mass measurement goals of the Project 8 collaboration throughout the phased development plan. The blue curves indicate molecular tritium sources and the red curves indicate atomic tritium sources. In the current plan, Phase III contains two tritium experiments. The first is the Low-frequency Apparatus (LFA), which is a molecular tritium experiment, and the second is the atomic tritium pilot-scale experiment that officially ends Phase III. The sensitivity of these experiments is primarily a function of statistics, however, there is a critical density beyond which CRES electrons do not have enough time to radiate between collisions for a high-resolution frequency measurement leading to worse sensitivity.

¹⁴⁴⁷ Success in Phase I was achieved with the 2014 publication of the measured ^{83m}Kr
¹⁴⁴⁸ conversion spectrum [55], which contains a mono-energetic 17.8-keV as well as several
¹⁴⁴⁹ other conversion lines at higher energies. Publication of this result marked the official
¹⁴⁵⁰ end of Phase I and the start of Phase II, in which Project 8 shifted its focus to the
¹⁴⁵¹ demonstration of the first tritium beta-decay spectrum using CRES. For more information
¹⁴⁵² on Phase II please see Section 3.3.

¹⁴⁵³ **Phase III: Research and Development and a Pilot-scale Experiment**

¹⁴⁵⁴ After completing Phase II, Project 8 has shifted focus towards R&D aimed at the
¹⁴⁵⁵ construction of an experiment that demonstrates all the technologies required for a
¹⁴⁵⁶ 40 meV measurement of the neutrino mass. The culmination of Phase III is a pilot-scale
¹⁴⁵⁷ experiment that successfully retires all technological and engineering risks associated
¹⁴⁵⁸ with the Phase IV experiment, while also being a scientifically interesting experiment in
¹⁴⁵⁹ its own right. Sensitivity estimates of the pilot-scale experiment predict a neutrino mass
¹⁴⁶⁰ sensitivity on par with the projected sensitivity of the KATRIN experiment.

¹⁴⁶¹ Phase III R&D is divided into two main efforts — atomic tritium and CRES detection
¹⁴⁶² techniques. Atomic tritium development in Phase III must retire all risks associated
¹⁴⁶³ with the atomic tritium system. This includes the production of tritium atoms, atomic
¹⁴⁶⁴ cooling and recirculation systems, purity and isotope concentration monitoring, and
¹⁴⁶⁵ atom trapping. Currently, Project 8 is operating small scale atom cracking demonstrator
¹⁴⁶⁶ systems to show that atom production at the estimated rates needed for Phase IV is
¹⁴⁶⁷ achievable. Future efforts will continue the current developments on atom production
¹⁴⁶⁸ and expand to include demonstrations of atomic cooling with an evaporative beam line
¹⁴⁶⁹ as well as atom trapping using Halbach magnet arrays.

¹⁴⁷⁰ The need for new CRES detection techniques is driven by the drastic increase in scale
¹⁴⁷¹ from Phase II to the pilot-scale experiments. The physical volume used for CRES in
¹⁴⁷² Phase II was on the order of a few cubic-centimeters, and achieving Project 8's sensitivity
¹⁴⁷³ target of 40 meV requires an experiment volume on the multi-cubic meter scale. Therefore,
¹⁴⁷⁴ the waveguide gas cell CRES detection technique used in Phase II is not a feasible option
¹⁴⁷⁵ for the future of Project 8 due to its inability to scale to the required size.

¹⁴⁷⁶ Two alternative CRES detection techniques have been proposed for the pilot-scale
¹⁴⁷⁷ experiment — antenna arrays and resonant cavities (see Section 3.4 and Chapter 6).
¹⁴⁷⁸ Both approaches have relative advantages and disadvantages, however, the improved
¹⁴⁷⁹ understanding of the antenna array and cavity approaches to CRES in the recent years
¹⁴⁸⁰ has led to cavities being the preferred technology for the pilot-scale experiment and

¹⁴⁸¹ Phase IV due to the estimated reduced cost and complexity of this approach. Since
¹⁴⁸² a large degree of the work presented in this dissertation is focused specifically on the
¹⁴⁸³ development of the antenna array CRES technique as well as the design of demonstrator
¹⁴⁸⁴ experiments, we described the proposed R&D plan for antenna array Section 3.4. A
¹⁴⁸⁵ description of the cavity approach to CRES can be found in Chapter 6.

¹⁴⁸⁶ Cavity CRES R&D consists of a series of demonstrator experiments intended to
¹⁴⁸⁷ demonstrate cavity CRES at a variety of scales and magnetic fields. Radioactive sources
¹⁴⁸⁸ gases include ^{83m}Kr and molecular tritium, as well as electrons produced by an electron-
¹⁴⁸⁹ gun, which is a key calibration tool for future CRES experiments. The near-term cavity
¹⁴⁹⁰ effort in Project 8 is the cavity CRES apparatus (CCA), which is a small-scale cavity
¹⁴⁹¹ experiment operating near 26 GHz. The CCA will perform the first CRES measurements
¹⁴⁹² using a small cavity, and will pave the way towards larger scale cavity experiments in
¹⁴⁹³ preparation for the eventual pilot-scale tritium experiment.

¹⁴⁹⁴ The pilot-scale experiment is the first experiment, which will combine atomic tritium
¹⁴⁹⁵ and large-volume CRES detection in the same experiment. It will directly demonstrate
¹⁴⁹⁶ all the technologies required for Phase IV such that no technical risks remain for scaling
¹⁴⁹⁷ the experiment to required scale. A robust approach to scaling the pilot-scale experiment
¹⁴⁹⁸ is to simply build multiple copies of it for the Phase IV experiment.

¹⁴⁹⁹ **Phase IV: Project 8's Ultimate Neutrino Mass Experiment**

¹⁵⁰⁰ The design of Phase IV should be a direct extension of the pilot-scale CRES experiment
¹⁵⁰¹ that marks the official end of Phase III (see Section 3.5). The Phase IV experiment
¹⁵⁰² represents the final experiment in the Project 8 neutrino mass measurement experiment
¹⁵⁰³ plan and will have sensitivity to neutrino masses of 40 meV.

¹⁵⁰⁴ **3.3 Phase II: First Tritium Beta Decay Spectrum and ¹⁵⁰⁵ Neutrino Mass Measurement with CRES**

¹⁵⁰⁶ In Phase II, Project 8 demonstrated the first ever measurement of the tritium beta-decay
¹⁵⁰⁷ spectrum endpoint using the CRES technique, which lead to the first neutrino mass
¹⁵⁰⁸ measurement by the Project 8 collaboration. This milestone was made possible by many
¹⁵⁰⁹ improvements in the CRES technique and in the understanding of CRES systematics,
¹⁵¹⁰ which takes an important first step towards larger scale measurements of the tritium
¹⁵¹¹ beta-decay spectrum with CRES. In this section, I briefly describe some important

elements of the Phase II experiment, with the goal of contextualizing the research and development efforts for Phases III and IV of Project 8. For more complete descriptions of the work that lead to Project 8’s Phase II results please refer to the relevant publications by the collaboration [41, 42].

3.3.1 The Phase II CRES Apparatus

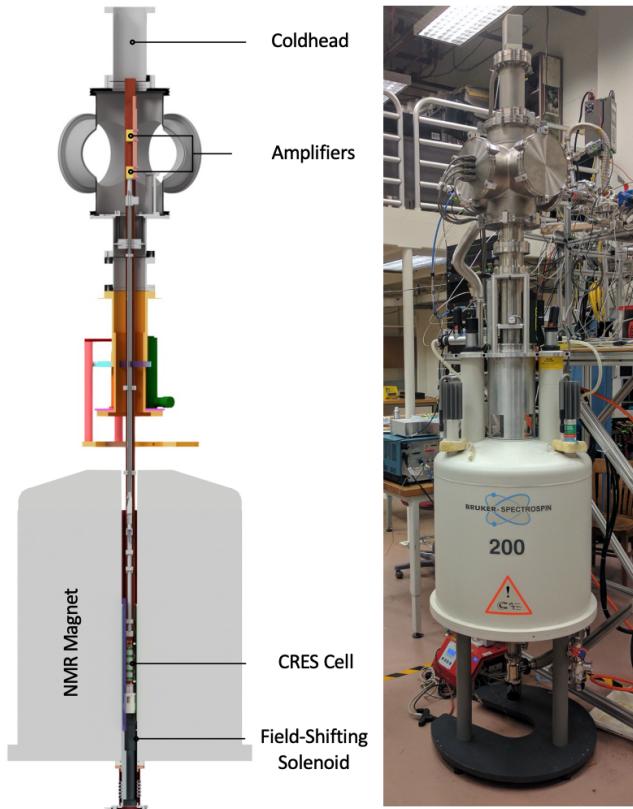


Figure 3.7. The Phase II CRES apparatus used to perform the first measurement of the tritium beta-decay spectrum using CRES.

Magnet and Cryogenics

The magnetic field for the Phase II experiment is provided by a nuclear magnetic resonance (NMR) spectroscopy magnet with a central bore diameter of 52 mm (see Figure 3.7). The magnet produces a background magnetic field with an average value of 0.959 T with a 10 ppm variation across the bore diameter achieved using several shim coils built into the magnet. Using an external NMR field probe, the variation of

1523 the magnetic field along the vertical axis of the magnet bore was measured to obtain
1524 an accurate model of the magnetic field so that the CRES cell could be positioned for
1525 optimal magnetic field uniformity.

1526 An external solenoid magnet was installed inside the magnet bore to provide the
1527 ability to shift the magnitude of the background magnetic field by a few mT. The solenoid
1528 has inside diameter of 46 mm and a length of 350 mm, which terminates in a vacuum
1529 flange that allows it to be inserted into the NMR magnet bore from the bottom. By
1530 shifting the value of the magnetic field by a few mT, the cyclotron frequencies of electrons
1531 produced by the 17.8 keV ^{83m}Kr internal-conversion line [56] can be shifted by frequencies
1532 of ± 100 MHz. This allows one to study the frequency dependent behavior of several
1533 CRES systematics such as detection efficiency that directly affect the measured shape of
1534 the tritium spectrum.

1535 The inside of the magnet bore diameter was pumped down to a vacuum of less than
1536 10 μtorr using a turbomolecular pump, which allows for cryogenic cooling of the CRES
1537 cell and RF system. Cooling power was supplied to the Phase II apparatus using a
1538 cryopump with its coldhead mounted above the primary magnet and CRES cell. This
1539 arrangement allowed for sufficient cooling power to be delivered to the amplifiers to cool
1540 them to a temperature of ≈ 40 K, while keeping the amplifiers far enough from the
1541 magnet so as not to be damaged by the large field strength. Thermal contact between
1542 the coldhead, amplifiers, RF system, and CRES cell is achieved using a copper bar that
1543 runs the full length of the apparatus. To prevent freeze-out of ^{83m}Kr on the walls of the
1544 CRES cell a separate heater was installed to keep the CRES cell near a temperature of
1545 85 K during the operation of the experiment.

1546 CRES Cell

1547 Located in the most uniform region of the magnetic field is the CRES cell, which is
1548 the region of the apparatus where radioactive decays of ^{83m}Kr and T_2 produce electrons
1549 that can be trapped and measured using CRES (see Figure 3.8). The CRES cell is
1550 manufactured from a segment of cylindrical waveguide designed to operate at K-band
1551 frequencies near 26 GHz. The diameter of the waveguide determines which resonant
1552 modes of the waveguide will couple to the electron and transmit its radiation to the
1553 amplifiers. For Phase II a waveguide diameter of 1 cm was selected, which allows electrons
1554 to couple to the TE_{11} and TM_{01} cylindrical waveguide modes. To reduce complexity in
1555 modeling and analyzing the CRES data, it is ideal to select a diameter that prevents
1556 electrons from coupling to higher-order waveguide modes beyond the fundamental TE

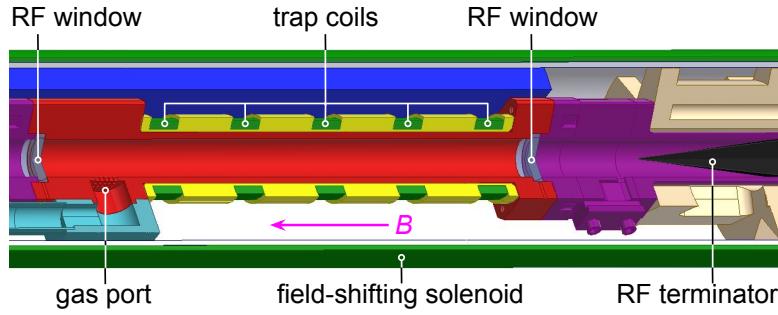


Figure 3.8. Diagram of the CRES cell portion of the Phase II apparatus.

1557 and TM modes.

1558 Around the exterior of the cylindrical waveguide are several magnetic coils used to
 1559 produce magnetic traps inside the CRES cell volume. Without a magnetic trap electrons
 1560 produced from decays inside the CRES cell quickly impact the cell wall, which prevents
 1561 a measurement of their cyclotron frequency using CRES. Each coil along the length of
 1562 the waveguide produces a separate trap that is approximately harmonic in shape. By
 1563 independently controlling the currents provided to each coil the traps can be configured
 1564 to have equal values of the magnetic field at the trap bottom despite a non-uniform field
 1565 from the NMR magnet.

1566 Two primary magnetic trap configurations were used during the Phase II experiment.
 1567 The first was a shallow trap configuration used primarily for it's high energy resolution to
 1568 study systematics using ^{83m}Kr decays, and the second was a deeper trap that could trap a
 1569 higher percentage of pitch angles. The trade-off with this trap is that the higher trapping
 1570 efficiency comes at the cost of lower energy resolution due to the greater variation in pitch
 1571 angle. The deep trap was the trap used to measure the tritium beta-decay spectrum in
 1572 Phase II.

1573 The source gases were delivered into the CRES cell through a gas port located near the
 1574 top end of the cylindrical waveguide. To prevent the gases from escaping the cell, vacuum
 1575 tight RF transparent windows are needed to contain the tritium and krypton source
 1576 gas across a 1 atm pressure differential, while still transmitting the cyclotron radiation
 1577 without distortion. The crystalline material, CaF_2 , which has a thermal expansion
 1578 coefficient similar to that of copper, was used for this purpose in the CRES cell. Two
 1579 windows, each 2.4 mm thick, were used to seal off the ends of the CRES cell. The
 1580 thickness of 2.4 mm corresponds to half of a cyclotron wavelength when one accounts for
 1581 the permittivity of CaF_2 .

1582 **RF System**

1583 The RF system in the Phase II apparatus propagates the cyclotron radiation from the
 1584 CRES cell to the receiver chain. The receiver chain performs the down-conversion and
 1585 digitization required to obtain signals that can be analyzed to determine the cyclotron
 frequencies of electrons in the CRES cell (see Figure 3.9).

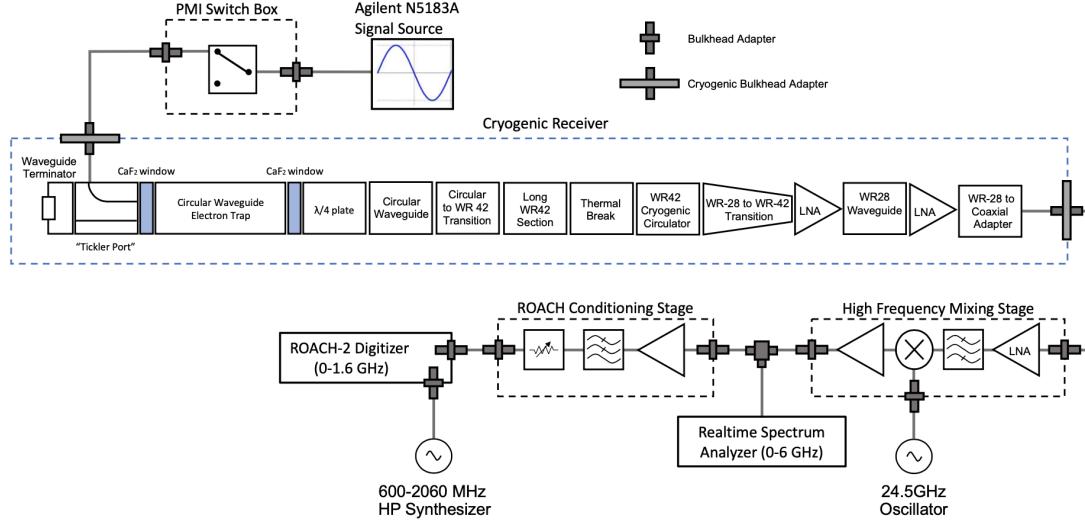


Figure 3.9. RF system diagram for the Phase II apparatus.

1586
 1587 Below the CRES cell, at the bottom of the Phase II apparatus, is a tickler port and
 1588 waveguide terminator. The tickler port is used to inject signals into the CRES cell and
 1589 RF system for testing and calibration purposes. The waveguide terminator is designed to
 1590 absorb cyclotron radiation emitted by electrons that transmits out of the bottom of the
 1591 CRES cell. This lowers the total power received from electrons in the CRES cell, since all
 1592 the energy radiated downwards is absorbed into the terminator. Earlier iterations of the
 1593 Phase II apparatus used an RF short in this location that reflected this power up towards
 1594 the amplifiers, however, interference between the upward traveling and reflected radiation
 1595 led to a disappearance in the signal carrier that made reconstruction impossible.

1596 Radiation traveling upward passes through the CaF_2 window passes through a $\lambda/4$
 1597 plate, which transforms the circularly polarized cyclotron radiation into linear polarization.
 1598 The linearly polarized fields next travel through a segment of circular waveguide that
 1599 transitions into a long segment of WR-42 waveguide that carries the fields out of the
 1600 high magnetic field region. A thermal break segment is included, which consists of a a
 1601 segment of gold-plated stainless steel WR-42 waveguide, to help thermally isolate the
 1602 relatively warm CRES cell from the colder amplifiers. The radiation then passes through

1603 a cryogenic circular, which prevents signals reflected from the amplifiers from interfering
1604 with the CRES cell before a WR-42 to WR-28 transition connects the waveguide to the
1605 first of the cryogenic amplifiers. The radiation passes through two cryogenic amplifiers
1606 before being coupled to a coaxial termination at the top of the Phase II apparatus.

1607 The coaxial cable transfers the cyclotron radiation signals to a high-frequency mixing
1608 stage that performs an analog frequency down-conversion using a 24.5 GHz LO. Two forms
1609 of digitization can be used at this stage to readout the CRES data. One is a real-time
1610 spectrum analyzer that digitizes the CRES signal data in time-domain and computes the
1611 frequency spectrum in real-time, which allows for direct visualization of CRES signal
1612 spectrograms as the experiment is running. The real-time spectrum analyzer is most
1613 useful for taking small amount of streamed data for debugging and analysis of the system.
1614 The other method, which was used to collect the majority of the CRES data in Phase II,
1615 is a ROACH-2 FPGA and digitizer system. The ROACH system consists of a fast ADC
1616 that samples the CRES signal data at 3.2 GSps. Internal digital down-conversion stages
1617 implemented in the FPGA perform a mixing operation that reduces the bandwidth of the
1618 CRES signals to 100 MHz. The FPGA implements a 8192 sample FFT and packetizes
1619 time and frequency domain records in parallel. The packetized data is then transferred
1620 from the ROACH to be analyzed by the data-processing pipeline.

1621 **3.3.2 CRES Track and Event Reconstruction**

1622 **Time-Frequency Spectrogram**

1623 The online data-processing software uses a real-time triggering algorithm that identifies
1624 interesting data that could contain CRES signals. Triggered data are collected into files
1625 that are transferred to a server for offline processing and analysis. The data files contain
1626 a continuous series of time-domain samples, broken into a set of records, which are 4096
1627 samples long. The time-series is made up of 8-bit IQ samples acquired at 100 MHz.

1628 Each time-series record is accompanied by an associated frequency spectrum consisting
1629 of 4096 frequency bins approximately 24.4 kHz wide, which is represented as a power
1630 spectral density. The individual frequency spectra can be organized temporally to create
1631 a time-frequency spectrogram that represents the evolution of the cyclotron frequency
1632 spectrum over the course of the CRES event (see Figure 3.10). The time-frequency
1633 spectrogram is represented as a two-dimensional image where the color of each pixel is
1634 proportional to the power spectral density. Each vertical slice of pixels in the image
1635 represents a frequency spectrum, therefore, each horizontal bin represents the data

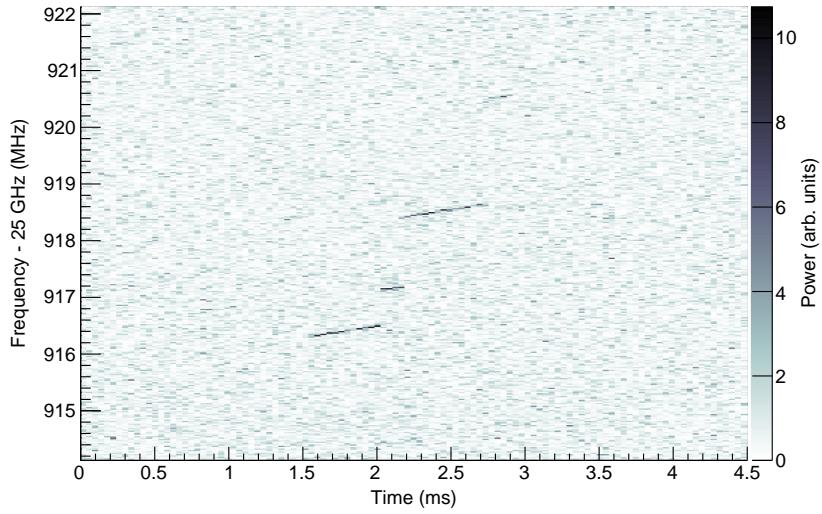


Figure 3.10. The time-frequency spectrogram of a tritium CRES event in the Phase II apparatus.

1636 obtained over a duration of $4096 \times 0.01 \text{ MHz}^{-1} = 40.96 \mu\text{sec}$.

1637 CRES Event Data Features

1638 Phenomenologically, a CRES signal appears as a sinusoidal signal whose frequency slow
 1639 increases over time in what is called a frequency "chirp". Axial motion of the electron in
 1640 the trap leads to the formation of frequency sidebands that surround the more powerful
 1641 carrier frequency. The critical piece of information that must be extracted from the track
 1642 and event reconstruction procedure is the carrier frequency, since it is this frequency that
 1643 gives the cyclotron frequency and thus the kinetic energy. Axial motion from non- 90°
 1644 pitch angles changes the average magnetic field experienced by an electron, which leads to
 1645 different cyclotron frequencies being measured for electrons with the same kinetic energy.
 1646 However, because of the low-SNR in Phase II sidebands were unable to be observed,
 1647 so no attempt to directly correct for this effect was attempted in the Phase II analysis.
 1648 The effect of different pitch angles is to broaden the peak of a monoenergetic electron
 1649 line, which can be quantified by measuring the instrumental resolution of the Phase II
 1650 apparatus.

1651 In the time-frequency spectrogram representation, the chirping carrier frequency
 1652 appears as a linear track of high-power frequency bins (see Figure 3.10). The vertical
 1653 slope of the tracks is caused by the emission of energy from the electron in the form of
 1654 cyclotron radiation, therefore, the size of the slope parameter is directly proportional

1655 to the Larmour power. The continuous track is periodically interrupted by random
1656 jumps to higher frequency and lower energy caused by random inelastic collisions with
1657 background gas molecules. The length of a track is an exponentially distributed variable
1658 whose mean value is inversely proportional to the gas density. The size of the frequency
1659 discontinuities is directly proportional to the energies of the rotational and vibrational
1660 states of background gas molecules.

1661 A CRES event refers to the collection of tracks produced by a trapped electron until
1662 it inevitably scatters into a pitch angle that can no longer be trapped. The goal of track
1663 and event reconstruction is to identify the set of tracks in a time-frequency spectrogram
1664 that represents a segment of data acquired in the Phase II apparatus. These tracks must
1665 be clustered into events, from which one can determine the first track produced by the
1666 electron and thus estimate it's starting cyclotron frequency and kinetic energy.

1667 Track Reconstruction

1668 The first step in CRES event reconstruction is the identification of tracks in the time-
1669 frequency spectrogram, which is essentially an image processing task. Track finding
1670 starts by normalizing the power spectral density based on the average noise power. Next
1671 a power threshold is applied to the normalized spectrogram where only bins that have a
1672 SNR ratio greater than five are selected to build tracks. In this case SNR is defined as the
1673 ratio between the normalized, unitless power of a bin divided by the average normalized
1674 power across the full frequency spectrum.

1675 The sparse spectrogram produced by this power cut consists only of a sparse collection
1676 of high-power frequency bins that could be part of a CRES signal track (see Figure
1677 3.11). In this form is it much easier to identify tracks "by eye", however, for the Phase II
1678 analysis Project 8 developed its own custom-made track finding algorithm, called the
1679 sequential track finder (STF).

1680 The STF algorithm processes the sparse spectrogram in sequential fashion, processing
1681 each time-slice one-by-one until the end of the spectrogram is reached. Tracks are found
1682 by searching for points in the sparse spectrogram that appear to fall on a straight line.
1683 Multiple configurable parameters are built into the STF algorithm that allow the user to
1684 tune the criteria for adding a point to an existing track or creating a new track. These
1685 include parameters such as maximum time and frequency differences between subsequent
1686 points in a track as well as minimum SNR values for the start and endpoints of the track.
1687 Additionally, tracks are required to have a minimum length and slope to be considered
1688 potential CRES tracks rather than random noise fluctuations.

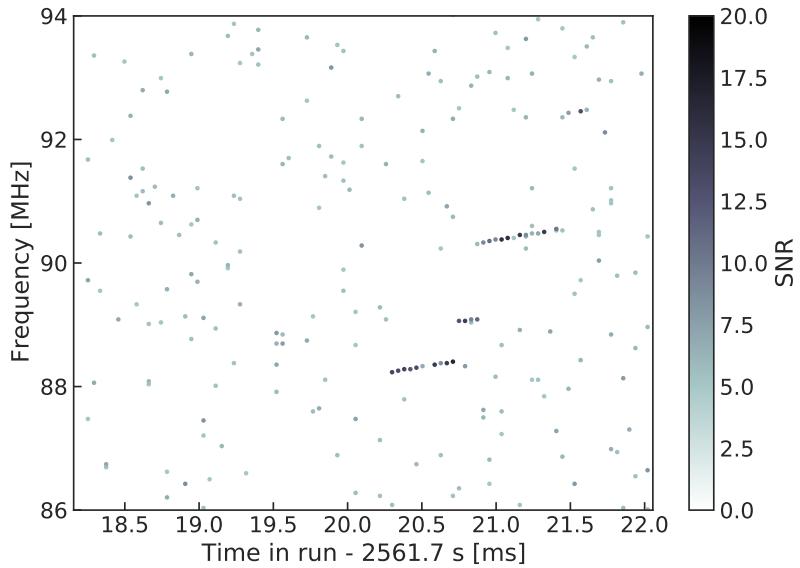


Figure 3.11. The sparse spectrogram obtained by placing a power cut on the raw spectrogram shown in Figure 3.10.

The resulting output of the STF is a collection of track objects that consist of the track point objects and their properties. The final step is to calculate track-level properties and apply cuts to reject false tracks found by the STF. This involves the fitting of a line to the collection of track points as well as the total and average power of the track obtained by computing the sum and mean of the points powers. The starting frequency of the track is determined by calculating the time coordinate that intersects with the linear fit. A cut is performed to remove all tracks that do not have a specified average power over their duration, which helps to remove the majority of noise fluctuations that have passed all previous cuts up to this point.

1698 Event Reconstruction

After track reconstruction comes event reconstruction where the identified tracks are grouped into events that correspond to the trajectory of a single electron in the trap. This procedure attempts to match tracks head to tail by checking if the start and end times of a pair of tracks fall within a certain tolerance. This tolerance is a configurable parameter that can be tuned to an optimal value using Monte Carlo simulations of events in the Phase II apparatus.

After the event building procedure has completed there remains a small likelihood that false tracks have made it through to the event reconstruction stage. Typically, cuts

at the track level are able to remove 95% of the false tracks identified by the STF, which leads to a significant number of false tracks at the event building stage. However, the additional event-level information makes it possible to reject events that contain these false tracks with a high degree of confidence.

Two event level features are associated with events caused by real electrons — the duration of the first track as well as the number of tracks in the event. Real electrons tend to have event structures with longer first tracks and a higher number of total tracks. Based on the values of these two criteria, a minimum threshold on the average power in the first track was configured to reject false events. The average power in the first track was chosen due to the critical nature of the starting frequency of the first track in an event to the krypton and tritium spectrum analyses.

3.3.3 Results from Phase II

The main result from Phase II was the measurement of the tritium beta-decay spectrum using CRES, which lead to the first neutrino mass limit with CRES. However, Phase II also included a significant ^{83m}Kr measurement campaign to understand important systematics relevant to the tritium spectrum measurement, but also to understanding the fundamentals of the CRES technique itself. This required high-resolution measurements of the ^{83m}Kr internal-conversion spectrum [56], which is an interesting science result in its own right.

The results from Phase II represents a significant effort from the entire Project 8 collaboration over several years. Because the focus of my contributions to Project 8 is directed towards the research and development efforts for the Phase III experiments, the goal in this section is not to provide a detailed description of the analyses that lead to the Phase II results. Rather, I will provide brief descriptions of a few plots representative of the main results from Phase II.

Measurements with Krypton

Measurements with krypton were a key calibration tool for Phase II of the experiment and will continue to be useful in Phase III. In the context of Project 8 krypton measurements refers to CRES measurements of the internal-conversion spectrum of the metastable state of krypton-83, ^{83m}Kr , produced by electron capture decays of ^{83}Rb . A supply of ^{83}Rb was built into the Phase II apparatus gas system that supplied the CRES cell with ^{83m}Kr via emanation.

1739 The ^{83m}Kr internal-conversion spectrum consists of several lines based on the orbital
 1740 of the electron ejected during the decay. The conversion lines useful to Project 8 are
 1741 those that emit electrons with kinetic energies that fall inside the detectable frequency
 1742 bandwidth of the Phase II apparatus. These are the K; L2 and L3; M2 and M3; and N2
 1743 and N3 lines with kinetic energies of 17.8 keV, \approx 30.4 keV, \approx 31.9 keV, and \approx 32.1 keV,
 1744 respectively. The different energies of the lines allow a onw to test the linearity of the
 1745 relationship between kinetic energy and frequency across the range of frequencies covered
 1746 by the continuous tritium spectrum.

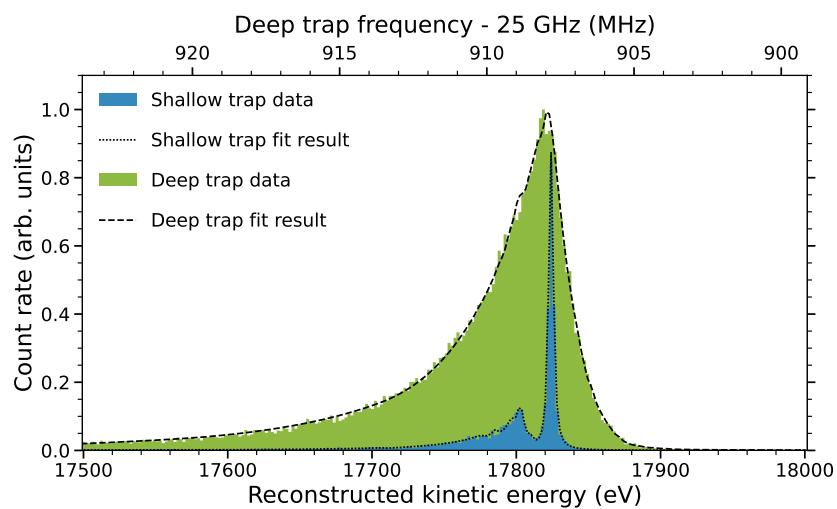


Figure 3.12. Fits to the measured 17.8-keV ^{83m}Kr conversion line using the deep and shallow trap configurations.

1747 Numerous detector related effects relevant to the tritium analysis can be characterized
 1748 by measuring the shape of the krypton spectrum. Specific examples include variations
 1749 in the magnetic field as a function of the radial position of the electron, variation in
 1750 the magnetic field caused by the trap shape, variation in the average magnetic field for
 1751 electrons with different pitch angles, and the effect of missing tracks due to scattering.
 1752 These spectrum shape measurements focused on the 17.8-keV krypton line and utilized
 1753 different trap geometries based on the particular goal of the dataset (see Figure 3.12).

1754 Krypton measurements with a shallow trap allow for high energy resolution, since
 1755 variation in frequency due to pitch angle differences is sharply reduced in the shallow
 1756 trap configuration. With this trap the main 17.8-keV peak of the conversion spectrum is
 1757 clearly visible along with additional satellite peaks at lower energy, which correspond to
 1758 the shakeup/shakeoff spectrum of the decay. The high accuracy of the fit demonstrates a
 1759 high degree of understanding of the CRES systematics.

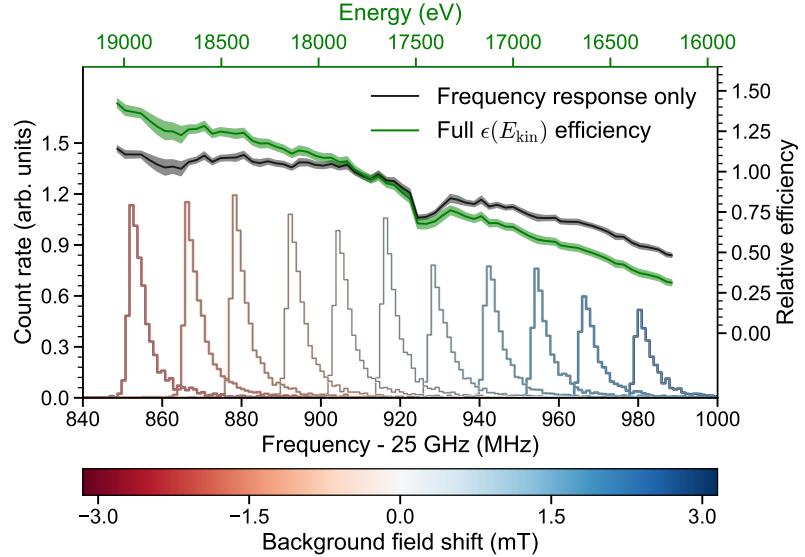


Figure 3.13. Measurements of the 17.8-keV ^{83m}Kr line using the deep trap configuration for different values of the magnetic field from the field shifting solenoid.

The broadening of the krypton spectrum seen for the deeper track is due to the large range of electron pitch angles that can be trapped. Furthermore, with a deeper trap there is a larger parameter space of electron that could be produced with pitch angles that are trappable but not visible in the time-frequency spectrogram. These electrons live in the trap and can scatter multiple times before randomly scattering to a visible pitch angle. This leads to one or more missing tracks earlier in the event, which leads to a misreconstruction of the true starting frequency. By measuring the krypton spectrum shape in the same trap used to detect tritium events, the effect this has on the spectrum shape can be characterized to mitigate its impact on the tritium measurements.

Changes in the Krypton spectrum shape as a function of CRES frequency were used to study the detection efficiency of the Phase II apparatus. Variations in the detection efficiency as a function of frequency directly influences the measured shape of the continuous tritium spectrum, which can lead to errors in the neutrino mass estimate if not modeled appropriately. Using the field shifting solenoid the cyclotron frequency of the krypton 17.83 keV line was shifted across the full frequency range of the tritium spectrum data (see Figure 3.13). Variations in the deep trap krypton spectrum shape can be used to infer the detection efficiency as a function of frequency and correct for this affect in the tritium measurements.

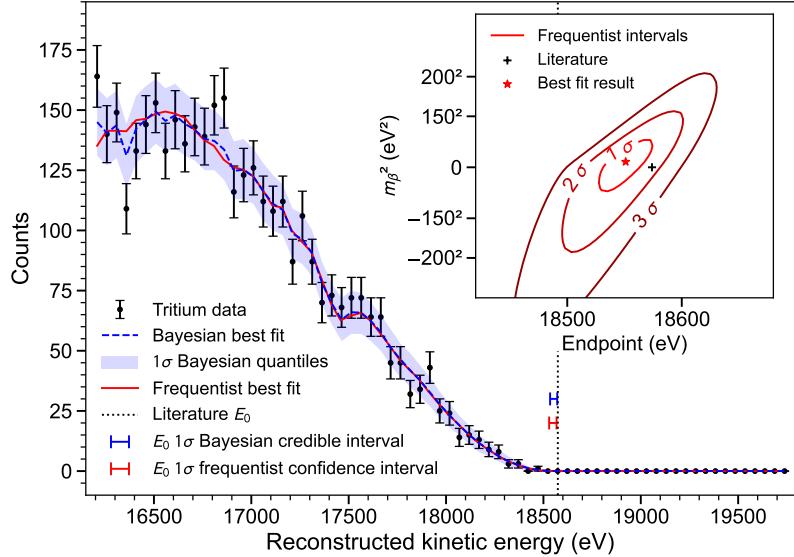


Figure 3.14. The measured tritium spectrum from Phase II with Bayesian and frequentist fits.

1778 Tritium Spectrum and Neutrino Mass Results

1779 The tritium measurement campaign resulted in the collection of 82 days of detector
 1780 live time during which 3770 total tritium events were detected. The track and event
 1781 reconstruction analysis extracted the starting frequencies of these tritium events, which
 1782 were used to build a frequency spectrum of tritium beta-decays. The resulting frequency
 1783 spectrum was then converted to an energy spectrum using the information gleaned from
 1784 the krypton measurement campaign to obtain the tritium beta-decay spectrum (see
 1785 Figure 3.14).

1786 CRES is inherently a very low background technique with the dominant source of
 1787 noise being random RF fluctuations. Monte Carlo simulations backed validated using
 1788 measurements of the RF noise background were used to set track and event cuts to
 1789 guarantee that zero false events would occur over the duration of the experiment with
 1790 90% confidence. Notably, the measured spectrum has zero events beyond the tritium
 1791 spectrum endpoint, which allows us to constrain the background rate in the Phase II
 1792 apparatus to less than 3×10^{-10} counts/ev/s. Achieving a low background is critical for
 1793 future neutrino mass experiments that seek to measure the neutrino mass with less than
 1794 100 meV sensitivity.

1795 Bayesian and frequentist based fits to the measured tritium spectrum, incorporating
 1796 information gained about CRES systematics from the krypton measurements, were
 1797 performed to extract upper limits on the tritium beta-decay spectrum endpoint as well as

1798 the neutrino mass. The estimated spectrum endpoints are 18553^{+18}_{-19} eV for the Bayesian
1799 analysis and 18548^{+19}_{-19} eV for the frequentist analysis. The quoted uncertainties are
1800 $1-\sigma$, and both results are within $2-\sigma$ of the literature endpoint value of 15574 eV. The
1801 estimated neutrino mass for both results is consistent with $m_\beta^2 = 0$. The 90% confidence
1802 upper limits for the Bayesian analysis is $m_\beta < 155$ eV/c² and $m_\beta < 152$ eV/c for the
1803 frequentist analysis.

1804 Though the neutrino mass results from Phase II are not competitive with KATRIN
1805 the experiment was a promising first step towards the development of more precise
1806 neutrino mass measurements using CRES. The low-background and high-resolution
1807 achievable with krypton measurements are promising features of the technique that were
1808 demonstrated with the Phase II apparatus. As new technologies are developed to enable
1809 CRES measurements in larger volume, many of the lessons learned from Phase II will
1810 continue to influence the operation and design of future experiments.

1811 **3.4 Phase III R&D: Antenna Array CRES**

1812 The goal of Phase III in the Project 8 experimental program is to develop the technologies
1813 and expertise required to build an experiment that uses CRES to measure the neutrino
1814 mass with a target sensitivity of 40 meV. One of the key technologies is a method for
1815 performing high resolution CRES measurements in a large volume, which allows one to
1816 observe a sufficient quantity of tritium to measure the low-activity endpoint region of
1817 the tritium spectrum.

1818 **3.4.1 The Basic Approach**

1819 One possible approach, suggested in the original CRES publication [39], is to use many
1820 antennas to surround a volume of tritium gas in a magnetic field (see Figure 3.15). When
1821 a decay occurs the electron will emit cyclotron radiation that can be collected by the array
1822 and used to perform CRES. Each antenna in the array collects only a small fraction of
1823 the electron's signal power, which is less than 1 fW for a 18.6 keV kinetic energy electron
1824 in a 1 T magnetic field. Scaling to large volumes with the antenna array approach is
1825 accomplished by increasing the number of antennas in the array, which increases the
1826 volume under observation proportionally.

1827 Several features of the antenna array approach make it an attractive candidate technol-
1828 ogy for a large volume experiment. One example is the accurate position reconstruction

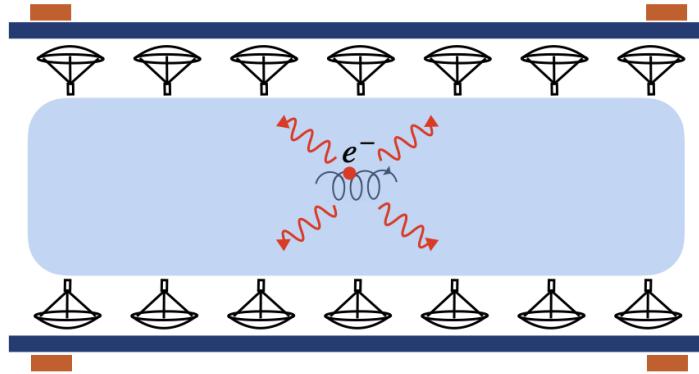


Figure 3.15. A cartoon illustration of the basics of the antenna array CRES technique.

possible with a multichannel antenna array. Using techniques like digital beamforming, it is possible to estimate the radial and azimuthal positions of the electron in the magnetic trap with a precision significantly less than the size of the cyclotron wavelength. This capability allows one to perform event-by-event estimations of the magnetic fields experienced by an electron, which helps achieve high energy resolution with the CRES technique.

The easy availability of position information with the antennas array approach is potentially a unique advantage that provides significant flexibility in the magnetic field uniformity requirements compared to other proposed approaches to large volume CRES (see Chapter 6). Spatial discrimination using digital beamforming leads to pileup reduction, which helps to reduce the potential of background events caused by missing tracks or by incorrectly clustering a group of tracks into an event. Limits on the background rate for a neutrino mass measurement with 40 meV sensitivity are stringent and the total activity of the tritium source is gigantic relative to the activity near the endpoint. Thus, pileup discrimination could be an important tool for a large scale CRES experiment.

Another beneficial quality of antenna arrays is that the volume of the experiment can be scaled independent of frequency by simply adding more antennas to the array (see Figure 3.19). Resonant cavities, the proposed alternative large volume CRES technology, are ideally operated in magnetic fields that cause electrons to move with cyclotron frequencies near the fundamental cavity resonance, to avoid complex coupling of the electron to multiple cavity modes simultaneously. This leads to a coupling between the cavity volume and the magnetic field magnitude, which forces one to lower the magnetic field in order to increase the experiment scale. Whereas, for antenna arrays, in principle there is no physical limitation on the size of the antenna array that can be used at a

particular magnetic field. However, this approach to scaling an antenna array experiment leads to rapidly increasing cost and complexity due to the large number of antennas, amplifiers, and data streams which require substantial computer processing power to effectively utilize.

3.4.2 The FSCD: Free-space CRES Demonstrator

The complexity of the antenna array CRES technique requires the construction of a small scale demonstration experiment to develop an understanding of technique itself and relevant systematics. Without a demonstrator experiment it is not possible to sufficiently retire the technical risks associated with the full-scale experiment. Therefore, Phase III of the Project 8 experimental program is primarily focused on the development and operation of demonstrator experiments to inform the design of the Phase IV experiment.

The Phase III demonstrator experiment for antenna array CRES is called the Free-space CRES Demonstrator or FSCD. The FSCD is also a capable neutrino mass measurement experiment in its own right, with a target neutrino mass sensitivity of a few eV using a molecular tritium source.

Magnetic Field

The background magnetic field for the FSCD is provided by a hospital-grade MRI magnet (see Figure 3.16). The magnet produces a magnetic field of approximately 0.958 T, which corresponds to a tritium spectrum endpoint frequency of approximately 25.86 GHz. The magnet is installed in the Project 8 laboratory located at the University of Washington, Seattle, and is shimmed to produce a uniform magnetic field with variations on the ppm-level. Measurements of the magnetic field non-uniformities are performed using a NMR probe and rotational gantry to capture measurements of the magnetic field around an elliptical surface in the center of the MRI magnet. During the operation of the FSCD an array of Hall or NMR magnetometers would be used to periodically measure the magnetic field to monitor its time stability.

Inside the field of the MRI magnet additional electromagnets would be installed that provide the capability to shift the value of the background magnetic field and produce a magnetic trap. Shifting the background magnetic field by a few μ T lets one control the cyclotron frequencies of electrons with a fixed kinetic energy, which is key to an effective calibration of the FSCD. The preferred calibration method for the FSCD is a mono-energetic electron gun that can inject electrons into the magnetic trap with a



Figure 3.16. An image of the MRI magnet installed in the Project 8 laboratory at the University of Washington, Seattle.

known kinetic energy. In combination with the field shifting magnet, one can vary the cyclotron frequencies of the electrons to measure the response of the antenna array as a function of the radiation frequency and electron position. This procedure characterizes the response of the antenna array and provides further information on magnetic field uniformity, which is important to achieving good energy resolution.

The design of the magnetic trap is absolutely critical to the success of a CRES experiment. The ideal shape is the perfect magnetic box, which has a flat bottom and step function walls. Any variation in the average magnetic field experienced by an electron leads to changes in the cyclotron frequency that can make determining the true starting kinetic energy more difficult. This includes changes in the magnetic field caused by the walls of the magnetic trap as well as radial magnetic field variations.

The ideal box trap is completely uniform and has infinitely steep walls that cause no change in the electron's cyclotron frequency as it is reflected from the trap wall, however, such a trap cannot be made from any combination of magnetic coils since it violates Maxwell's equations. One of the goals of magnetic trap design is to identify the configuration of coils that produces a trap that approximates the perfect box trap as closely as possible.

1903 **Antenna Array**

1904 The canonical antenna array design for CRES is a uniform cylindrical array of antennas
1905 that surrounds the magnetic trap volume. Since the FSCD is a demonstrator experiment,

1906 the antenna array design is the simplest form of the uniform cylindrical array, which is a single circular ring of antennas with a diameter of 20 cm (see Figure 3.17). Along this

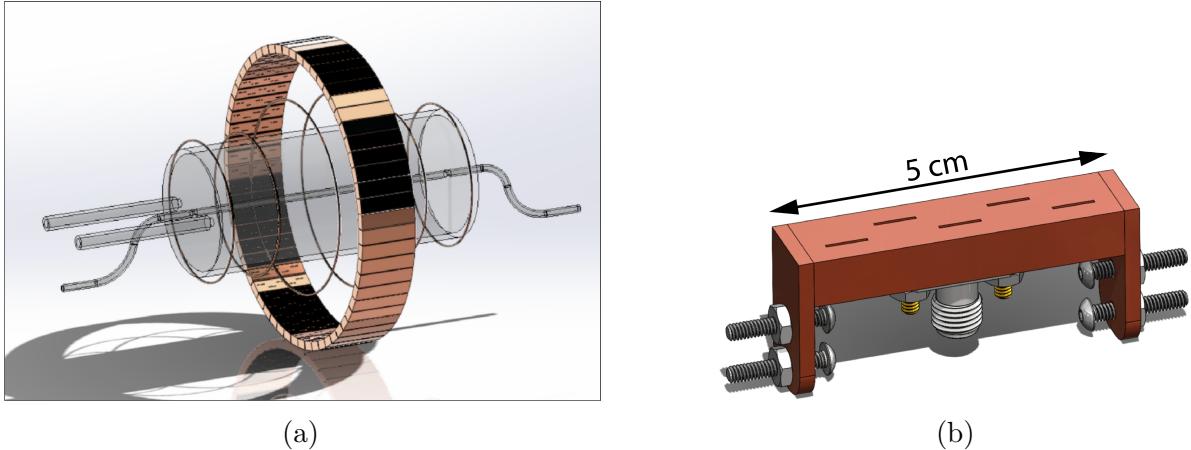


Figure 3.17. (a) A model of the FSCD antenna array, magnetic trap, and tritium containment vessel design.(b) A more detailed model of a prototype design for the 5-slot waveguide antenna design.

1907
1908 circle are sixty slotted waveguide antennas that fully populate the available space around
1909 the array circumference. In order to maximize the power collected from each electron
1910 it is optimal to cover as large a fraction of the solid angle around the magnetic trap as
1911 possible.

1912 The distance between antennas around the circumference of the array is proportional
1913 to the wavelength of the cyclotron radiation. Therefore, maximizing the solid angle
1914 coverage of the array, while minimizing channel count to keep the hardware and data
1915 acquisition costs manageable, biases one towards smaller array diameters. Antenna
1916 near-field effects limit the minimum diameter of the array for a given antenna design,
1917 since the radiation from electrons that are too close to the array cannot be detected due
1918 to destructive interference.

1919 Slotted waveguide antennas are used in the FSCD antenna array due to their high
1920 efficiency and low loss, which comes from the lack of dielectric materials in the antenna
1921 structure. Coupling to the waveguide is performed with a coaxial cable connected at the
1922 center of the antenna. One of the drawbacks of waveguide antennas is the large amount
1923 of space required to fit them inside the limited MRI magnet volume. Alternative antenna
1924 designs, constructed from microstrip printed circuit boards require significantly less space
1925 at the cost of slightly higher energy losses in the antenna structure.

1926 The FSCD antenna design is a 5 cm long segment of WR-34 waveguide with 5 vertical
1927 slots cut into the side. The distance between slots along the length of the waveguide is

1928 a half wavelength for optimal power combination between the individual antenna slots.
1929 Each slot is offset from the center of the antenna face a small distance in order to most
1930 effectively couple the slot to waveguide modes inside the antenna.

1931 The passive power combination achieved by placing 5 slots in a single waveguide is a
1932 compromise intended to reduce the cost and complexity of the antenna array system.
1933 Each additional channel in the array requires it's own cryogenic amplifier and also increase
1934 the required computer power to process the raw data collected by digitizing each channel.
1935 Passive summation, achieved by combining antennas into arrays axially, reduces the
1936 array channel count at the cost of losses from imperfect passive combination.

1937 Interference and re-radiation eventually limit the achievable the axial extent of passive
1938 power combination. The 5-slot designed developed for the FSCD is optimized to minimize
1939 the impact of these losses while achieving the maximum amount of axial coverage with a
1940 single ring of antennas. Scaling beyond the volume covered by a single ring of antennas is
1941 achieved by stacking additional rings of antennas together to cover a larger trap volume.
1942 A likely scenario for the FSCD experiment involves a staged experiment approach, where
1943 first a series of measurements is performed using only a single ring of antennas followed by
1944 experiments that add additional rings to the FSCD. The goal would be to first understand
1945 the principles of antenna array CRES using the simplest possible experiment, before
1946 attempting to scale the technique by expanding the antenna array size.

1947 Tritium Source

1948 While the primary purpose of the FSCD is as a technology demonstrator, it is impossible to
1949 retire all risks with the Phase IV experiment without an intermediate scale measurement
1950 of the neutrino mass. Therefore, the FSCD has the scientific goal of measuring the
1951 neutrino mass with a rough sensitivity goal in the range of a few eV. This level of precision
1952 is achievable using a molecular tritium source with a volume of approximately 1 L at a
1953 density comparable to potential Phase IV scenarios.

1954 Unlike previous CRES experiments, where the tritium source could be colocated
1955 with the receiving antenna inside a waveguide transmission line, the tritium source
1956 in the FSCD is thermally isolated from the antenna array to avoid freeze-out of the
1957 tritium molecules. The tiny radiation power emitted by electrons requires a system noise
1958 temperature of ≈ 10 K or less, in order to detect events at a high enough efficiency to
1959 reach the neutrino mass sensitivity goals of the experiment. Achieving a system noise of
1960 10 K requires that the antenna array and amplifiers operate at liquid helium temperatures
1961 of ≈ 4 K, which significantly lowers the vapor pressure of molecular tritium. By keeping

1962 the molecular tritium isolated in an RF-transparent vessel the tritium gas can be kept
1963 at a relatively warmer temperature in the range of 30 K to avoid the accumulation of
1964 tritium on the experiment surfaces.

1965 **Data Acquisition and Reconstruction**

1966 A fundamental change in the data acquisition system for the FSCD is the shift from
1967 single to multichannel reconstruction. This transition results in a significant increase in
1968 the data-generation rate, which is linearly related to the number of independent channels
1969 in the array. The larger data volume coincides with an increased demand for computer
1970 processing power based on the need for more precise signal reconstruction algorithms
1971 driven by the FSCD and Phase IV sensitivity goals. Therefore, the data acquisition
1972 system for the FSCD is likely to represent a significantly larger fraction of the experiment
1973 cost and complexity than in Phase II.

1974 Each antenna is connected to a cryogenic amplifier and down-converted from the
1975 26 GHz CRES frequency using an IQ-mixer to reduce the size of the analysis window.
1976 Using an LO with a frequency of approximately 25.80 GHz the antenna array signals can
1977 be digitized at a rate of 200 MHz, which is sufficient bandwidth to resolve the complete
1978 sideband spectrum produced by axial oscillations of electrons in the FSCD magnetic
1979 trap.

1980 Direct storage of the raw FSCD antenna array data is undesirable, since the estimated
1981 amount of raw data generated is $O(1)$ exabyte per year. The storage of such a large
1982 dataset is infeasible for a demonstrator experiment like the FSCD, since it would represent
1983 a disproportionate fraction of the total experiment budget in Phase III and Phase IV.
1984 Therefore, a goal of the FSCD experiment is the development of real-time reconstruction
1985 methods that could reduce the raw data volume by detecting and reconstructing CRES
1986 events in real-time. Ultimately, a real-time CRES reconstruction pipeline is desired, which
1987 takes raw voltages samples from the antenna array and converts them into measured
1988 starting kinetic energy values for electrons.

1989 The feasibility of a real-time reconstruction pipeline rests on the development of
1990 computationally efficient algorithms that can be implemented without the need for
1991 enormous computing resources. One challenge with the antenna array approach is that
1992 the small radiation power of a single electron is distributed among all channels in the
1993 array, such that reconstruction using only the information in a single channel is not
1994 possible. Therefore, the simply performing the initial step in reconstruction — signal
1995 detection — requires orders of magnitude more computational power than previous CRES

1996 experiments. This operation will then be followed by other, potentially more expensive,
1997 reconstruction steps that are required in order to determine the kinetic energy of the
1998 electron.

1999 **3.5 Pilot-scale Experiments**

2000 **3.5.1 Choice of Frequency**

2001 The optimal CRES frequency for Project 8 is that which allows us reach our target
2002 sensitivity of 40 meV, while minimizing the cost and complexity of the overall experiment.
2003 The magnitude of the background magnetic field determines the cyclotron frequency,
2004 which affects the entirety of the CRES detection system design, therefore, specifying the
2005 operating frequency of the CRES experiments is one of the first steps towards developing
2006 a full design.

2007 **Scaling Laws**

2008 The Phase I and II experiments utilized a background magnetic field of 0.959 T provided
2009 by an NMR magnet. This magnetic field was selected primarily for convenience, however,
2010 the cyclotron frequencies for electrons near the tritium endpoint in a 0.959 T field ranges
2011 from 25 to 26 GHz, which is within the standard RF Ka-band. Therefore, microwave
2012 electronics specialized for these frequencies are easily obtainable for relatively low cost.
2013 The operating frequency for the large-scale experiments must be selected in a more
2014 rigorous manner due to the increased scale and complexity of the systems as well as the
2015 requirements of the 40 meV neutrino mass science goal.

2016 There is a bias towards lower frequencies in a large-volume experiment, due to the
2017 direct relationship between wavelength and the physical size of the compatible RF
2018 components like antennas and cavities. With a longer wavelength more volume can
2019 be surrounded by an array with fewer antennas, which reduces hardware and data-
2020 processing costs. Additionally, the size of a cavity experiment is directly proportional to
2021 the wavelength, since this sets the physical dimensions of the cavity. It is also simpler to
2022 engineer a magnet that provides a uniform magnetic field across several cubic-meters of
2023 space at lower magnetic fields, which provides advantages in terms of cost-reduction.

2024 A concern with lower magnetic fields and frequencies is the scaling of the Larmour
2025 power equation, which is proportional to the square of the frequency. Naively, one would
2026 predict that the SNR would decrease with lower fields, however, two additional scaling

laws that affect the noise power also come into play. Noise power is directly proportional to the required bandwidth, which decreases linearly with the magnetic field. Furthermore, at lower frequencies it is possible to purchase amplifiers with lower noise temperatures until approximately 300 MHz at which point this relationship tends to flatten. Therefore, it is expected that the SNR remains approximately constant as the frequency decreases.

The SNR directly impacts the overall efficiency of the experiment through its effects on signal detection and energy resolution. Thus, the expectation that SNR remains the same at lower frequencies clearly biases large-scale experiments in this direction. One drawback of lower magnetic fields is the increased influence of external magnetic fields on the experiment. This includes magnetic fields from the building materials as well as variations in the earth's magnetic field. To deal with these affects a suitable magnetic field correction system will need to be devised, which includes constant monitoring of external fields.

Atomic Tritium Considerations

The pilot-scale experiments will be the first Project 8 experiments to combine CRES with atomic tritium, therefore, the optimal frequency should take into account the affect of the background magnetic field on the atom trap. The primary influence of the background

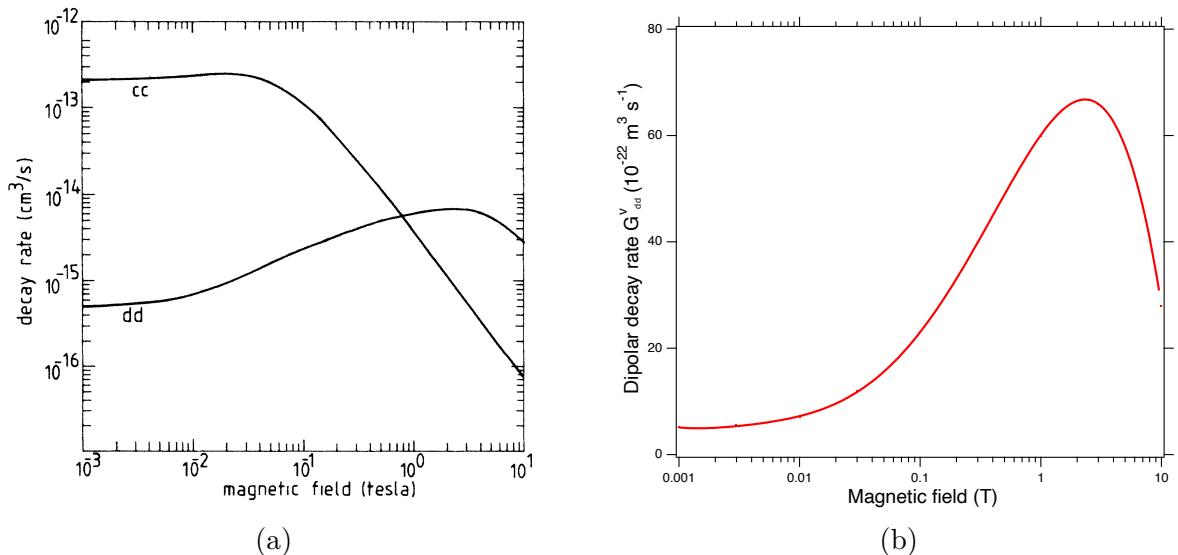


Figure 3.18. (a) A plot of the decay rate for the two-body dipolar spin exchange interaction for cc and dd state. (b) A plot of the decay rate of the dipolar spin exchange interaction for d+d states as a function of magnetic field magnitude. Lowering the magnetic field is key for reducing the losses from this interaction.

2043

2044 field magnitude is through the rate of dipolar spin-flips caused by a spin exchange
2045 interaction between trapped atoms [57].

2046 Atomic tritium is a simple quantum system with a hyperfine structure given by the
2047 addition of the nuclear and atomic spins. The addition of two spins leads to a hyperfine
2048 structure with four states in the (m_s, m_I) basis [58]. The states with atomic spins directed
2049 anti-parallel to the magnetic field have $m_s = -1/2$ and are labeled as the a and b states.
2050 The a and b states are colloquially known as high-field seeking states, since their energy is
2051 minimized when in regions of higher magnetic field. This leads to losses in the magnetic
2052 trap as these atoms are drawn to higher fields away from the trap center. Alternatively,
2053 the c and d states, with atomic spin $m_s = +1/2$, minimize their energy in low magnetic
2054 fields because of the parallel alignment between spin and the magnetic field. Therefore,
2055 these low-field seeking states tend to stay trapped significantly longer than the high-field
2056 seeking states.

2057 It would be advantageous to prepare tritium atoms in purely c and d states before
2058 trapping, however, even in this case losses still occur due to dipolar interactions between
2059 pairs of c and d states leading to flipped atomic spins and subsequent losses from high-
2060 field seeking atoms. The rate of these interactions depends on the magnitude of the
2061 background magnetic field and is maximal for dd interactions around 1 T (see Figure
2062 3.18). The rate of losses from these interactions at 1 T requires atomic tritium production
2063 at a rate two orders of magnitude larger than at 0.1 T, thus, requirements on the whole
2064 atomic tritium system are significantly relaxed at lower magnetic fields, which provides
2065 powerful argument for moving to lower frequencies with the pilot-scale experiments and
2066 Phase IV.

2067 **3.5.2 Pilot-scale Experiment Concepts**

2068 While the pilot-scale experiments are still in the early stages, enough is known to sketch
2069 the general features of these experiments at the conceptual level.

2070 **Pilot-scale Antenna Array CRES Experiment Concept**

2071 A conceptual design for an antenna-based CRES experiment is shown in Figure 3.19.
2072 A large solenoid magnet provides a uniform background magnetic field less than 0.1 T
2073 in magnitude. Inside this region is the atom trapping magnet that generates a high
2074 magnetic field at the walls, which decays exponentially towards the central region. Known
2075 magnet designs that produce suitable atom trapping fields include Ioffe-Prichard traps,

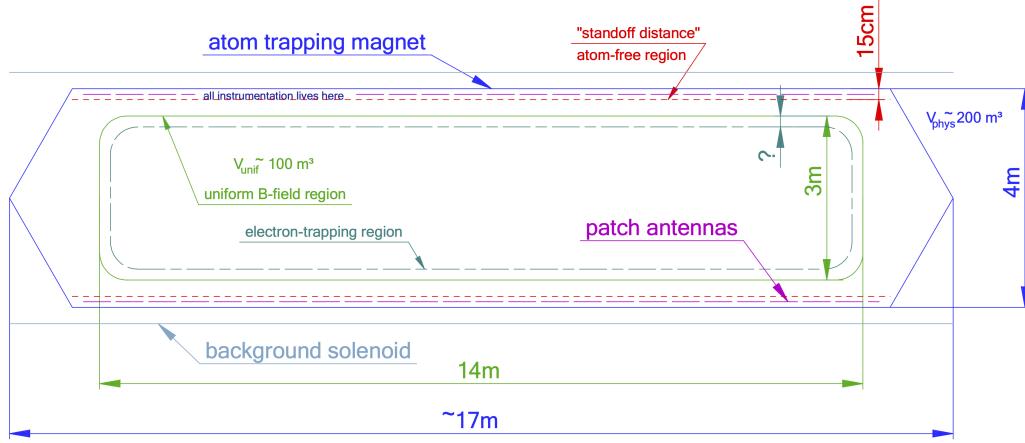


Figure 3.19. A conceptual sketch of a large-volume antenna array based CRES experiment to measure the neutrino mass.

which use conducting coils, as well as a Halbach array made from permanent magnets. Either magnet choice produces a region of high magnetic fields, which excludes atoms and allows for the placement of antennas inside the experiment.

Inside this region an array of microstrip patch antennas is inserted to collect the cyclotron radiation without providing a surface for atomic tritium recombination. Due to the lower frequency of cyclotron radiation antennas of a larger size can be used, which lowers the total number of antennas required to observe the experiment volume. Because of this scaling, the lower frequency experiment uses a similar number of antennas compared to a much smaller demonstrator experiment with a 1 T magnetic field.

The atomic tritium beamline that supplies fresh tritium atoms to the experiment is not shown in the figure. The general configuration would matches the one shown for the pilot-scale cavity experiment (see Figure 3.20).

2088 Pilot-scale Cavity CRES Experiment Concept

The pilot-scale cavity experiment includes both an atomic tritium system and cavity CRES system. The atomic system consists of a thermal atom cracker located at the start of an evaporatively cooled atomic beamline. The atomic tritium system provides a supply of tritium atoms to the trap with temperatures on the order of a few mK. Atoms at this temperature can be trapped magneto-gravitationally, which is the reason for the vertical orientation of the cavity. At these low magnetic fields the trapping requirements for electrons and atoms differ enough such that it is advantageous to decouple the trapping potentials to avoid radioactive heating of the tritium atoms from excess trapped

2097 electrons. Electron trapping is provided by a set of magnetic pinch coils at the top and
2098 bottom of the cavity and a multi-pole Ioffe or Halbach magnet serves to contain the
2099 atoms.

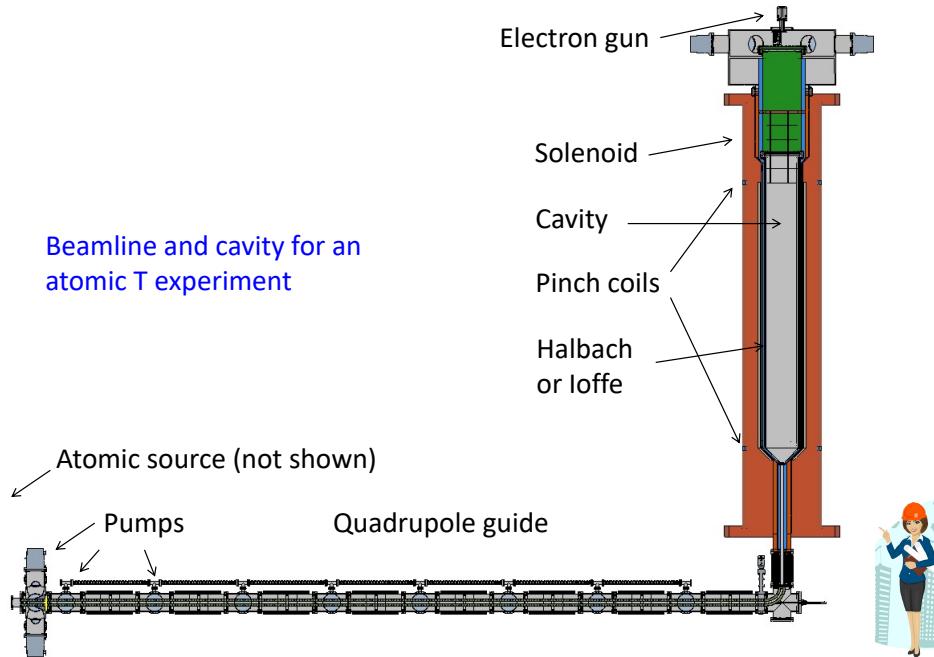


Figure 3.20. A conceptual sketch of a pilot-scale cavity CRES experiment with an atomic tritium beamline.

2100 The cavity design for the pilot-scale experiment consists of a large cylindrical cavity
2101 with a TE011 resonance of 325 MHz. Such a cavity is truly enormous, with a diameter
2102 of approximately 1.2 m and a height of 11 m. When an electron is produced inside
2103 the cavity with a cyclotron frequency that matches the TE011 resonant frequency it's
2104 cyclotron orbit couples the electron to the TE011, which drives a resonance in the cavity.
2105 These resonant fields can be read-out using an appropriate cavity coupling mechanism
2106 located at the center of the cavity. For more information on the cavity approach to
2107 CRES see Chapter 6.

2108 The bottom of the cavity has a cone termination to match the contour of the atom
2109 trapping magnet. This shape still allows for TE011 resonances with high internal Qs,
2110 which are required for good SNR in the cavity experiment. A small opening in the bottom
2111 of the cone serves as an entry point for the tritium atoms. To allow for calibration of
2112 the magnetic field inhomogeneities with an electron gun, the top of the cavity is left
2113 nearly completely open. Normally, this would drastically lower the Q-factor of the TE011
2114 mode, but a specially configured coaxial partition is inserted at the top. This termination

2115 scheme is designed to act as a perfect short for the TE011 mode since the circular shape
2116 of the partition matches the electric field boundary conditions for the TE011 mode.
2117 Simulations with HFSS have confirmed that this design results in a high quality TE011
2118 resonance despite the nearly completely open end.

2119 **3.6 Phase IV**

2120 The baseline CRES technology being pursued by the Project 8 collaboration are resonant
2121 cavities, which, due to their geometric properties, simple CRES signal structure, and low
2122 channel count, appear to be the better option for Phase IV. The current knowledge of the
2123 antenna array CRES approach reveals no technical obstacles that would preclude it as a
2124 baseline technology for Phase IV though it would most certainly be significantly more
2125 expensive. Therefore, antenna arrays represent a fallback approach if resonant cavities
2126 prove infeasible.

2127 The sensitivity of the pilot-scale atomic tritium experiment is estimated to be on
2128 the order of 0.1 eV, which means that increasing the sensitivity to reach the Phase IV
2129 goal will require an even larger experiment. Because of the direct coupling between the
2130 RF characteristics of a cavity and its geometry, the baseline plan is to build multiple
2131 copies of the pilot-scale experiment (see Figure 3.21) to obtain the required amount of
2132 volume rather than increase the size of the cavity beyond the pilot-scale. The built-in
2133 redundancy of this approach is useful in the sense that the experiment has no single
2134 point of failure, additionally, building several copies of the a pilot-scale experiment will
2135 minimize new engineering and design effort.

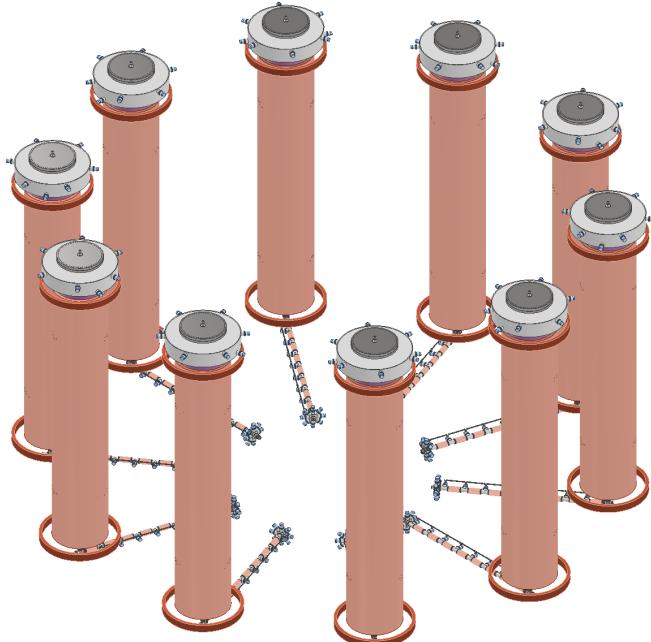


Figure 3.21. An illustration of a possible arrangement of ten pilot-scale cavity experiments for Phase IV. The experiments are arranged in a circle with an approximate diameter of 50 meters. Each atomic beamline connected to the bottom of each cavity is approximately 10 m in length. The cavities themselves are designed to operate at 325 MHz and are approximately 11 m tall. The circular arrangement of cavities has some advantages when it comes to cancellation of fringe fields from neighboring magnets, which is important due to the small magnetic field magnitudes consistent with these CRES frequencies. The advantage of ten independent atomic sources and cavities is that there is no single point of failure for the experiment. If an experiment goes down for repairs the other nine may continue running. Figure courtesy of Michael Huehn at UW-Seattle.

2136 **Chapter 4** |

2137 **Signal Reconstruction Techniques for An-**

2138 **tenna Array CRES and the FSCD**

2139 **4.1 Introduction**

2140 The transition from a waveguide CRES experiment to an antenna array CRES experi-
2141 ment introduces new challenges related to data acquisition, signal detection, and signal
2142 reconstruction caused by the multi-channel nature of the data. The development of signal
2143 reconstruction algorithms is crucial to the design of antenna array based experiments like
2144 the FSCD, because these algorithms directly influence the detection efficiency and energy
2145 resolution of the CRES experiment. In this Chapter I summarize my contributions to
2146 the development and analysis of signal reconstruction and detection algorithms for the
2147 FSCD experiment.

2148 In Section 4.2 I discuss the primary tool for this work, which is the Locust simulations
2149 package developed by the Project 8 experiment. Locust is used to simulate CRES events
2150 in the detector. Locust uses Kassiopeia to calculate particle trajectory solutions for
2151 electrons in the magnetic trap. The trajectories are then used to calculate the response
2152 of the antenna array to the cyclotron radiation produced by the electron, which results
2153 in signals that can be used to analyze the performance of different signal reconstruction
2154 algorithms. More recently, Project 8 has developed CREsana, which is a new simulations
2155 package that takes a more analytical approach to CRES signal simulations for antenna
2156 arrays. Although CREsana signals were not used for the signal reconstruction algorithm
2157 development detailed here, we introduce the software as it plays a role in the antenna
2158 array measurements presented in Section 5.5.

2159 In Section 4.3 I discuss the signal reconstruction and detection approaches analyzed
2160 for the FSCD experiment. In general there are two steps to signal reconstruction —
2161 detection and parameter estimation. With signal detection one is primarily concerned

2162 only with distinguishing between data that contains a signal versus data that contains only
2163 noise, whereas, with parameter estimation one is interested in extracting the kinematic
2164 parameters of the electron encoded in the cyclotron radiation signal shape. Due to
2165 the low signal power of electrons near the spectrum endpoint in the FSCD experiment,
2166 signal detection is a non-trivial problem. This is magnified by the need to maximize the
2167 detection efficiency of the experiment in order to achieve the neutrino mass sensitivity
2168 goals. My contributions to signal reconstruction analysis for the FSCD are focused on
2169 this signal detection component of reconstruction.

2170 After the discussion of various signal detection approaches, in Section 4.4 I present a
2171 more detailed analysis of the detection performance of three algorithms, which could be
2172 used to signal detection in the FSCD. This section was originally prepared for publication
2173 in JINST as a separate paper. The algorithms include a digital beamforming algorithm,
2174 a matched filter algorithm, and a neural network algorithm, which I analyze in terms of
2175 classification accuracy and estimated computational cost.

2176 **4.2 FSCD Simulations**

2177 Antenna array CRES and the FSCD requires a combination of different capabilities
2178 not often found in a single simulation tool. First of all, accurate calculations of the
2179 magneto-static fields produced by current-carrying coils are required in order to accurately
2180 model the magnetic trap and background magnets. The resulting magnetic fields must
2181 then be used to calculate the exact relativistic trajectory of electrons, which is required
2182 in order to calculate the electro-magnetic (EM) fields produced by the acceleration of
2183 the electron. Finally, the simulation has to model the interaction of the antenna and
2184 RF receiver chain with these EM-fields in order to produce the simulated voltage signals
2185 produced by the antenna array during the CRES event. At the time when Project 8 was
2186 developing this simulation capability, no single available simulation tool was known to
2187 adequately perform this suite of calculations, which prompted the development of custom
2188 simulation framework to simulate the FSCD. This simulation framework includes custom
2189 simulation tools developed by Project 8 as well as other open-source and proprietary
2190 software developed by third-parties.

2191 **4.2.1 Kassiopeia**

2192 Kassiopeia¹ is a particle tracking and static EM-field solver developed by the KATRIN
2193 collaboration for simulations of their spectrometer based on magnetic adiabatic collimation
2194 with an electrostatic filter [59]. Due to the measurement technique employed by the
2195 KATRIN collaboration, Kassiopeia is not designed to solve for the EM-fields produced by
2196 electrons in magnetic fields. However, it does provide efficient solvers for static electric
2197 and magnetic fields and charged particle trajectory solvers. Because of this, Project 8
2198 has incorporated parts of Kassiopeia into its own simulation framework.

2199 **Magnetostatic Field Solutions**

2200 The solutions to the electric and magnetic fields generated by a static configuration of
2201 charges and currents is given by Maxwell's equations in the limit where the time-dependent
2202 terms go to zero. In their static form Maxwell's equations [48] are

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (4.1)$$

$$\nabla \times \mathbf{E} = 0 \quad (4.2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (4.3)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \quad (4.4)$$

2203 where we can see that the electric and magnetic fields are now completely decoupled
2204 from each other. The solution for the magnetic field in this boundary value problem is
2205 given by the Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int dr' \frac{r'^3 \mathbf{J}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r}' - \mathbf{r}|^3}, \quad (4.5)$$

2206 which Kassiopeia uses a variety of numeric integration techniques to solve for a user
2207 defined current distribution.

2208 **Kassiopeia Simulation of the FSCD Magnetic Trap**

2209 The trap developed for the FSCD experiment utilizes six current carrying coils, which
2210 surround a cylindrical tritium containment vessel (see Figure 4.1). Some critical aspects
2211 of the trap design include the total trapping volume, the maximum trap depth, the

¹<https://github.com/KATRIN-Experiment/Kassiopeia>

steepness of the trap walls, as well as the radial and azimuthal uniformity of the magnetic fields.

The volume of the FSCD trap is a cylindrically shaped region with a radius of 5 cm and a length of 15 cm resulting in a roughly 1 L total trap volume. The trap volume is an important design feature, because it sets the volume of the experiment that is potentially usable for CRES measurements. Trapping a larger volume allows one to observe a larger number of tritium atoms, which increases the statistical power and sensitivity of the neutrino mass measurement. Due to the cost of constructing magnets with large and uniform magnetic fields it is important that the trap use as much of the available volume as possible to limit the overall cost of the experiment.

Coil	Radius (mm)	Z Pos. (mm)	Current (A×Turns)
1	50.0	-92.3	750.0
2	50.1	-56.9	-220.3
3	68.5	-19.5	-250.0
4	68.5	19.5	-250.0
5	50.1	56.9	-220.3
6	50.0	92.3	750.0

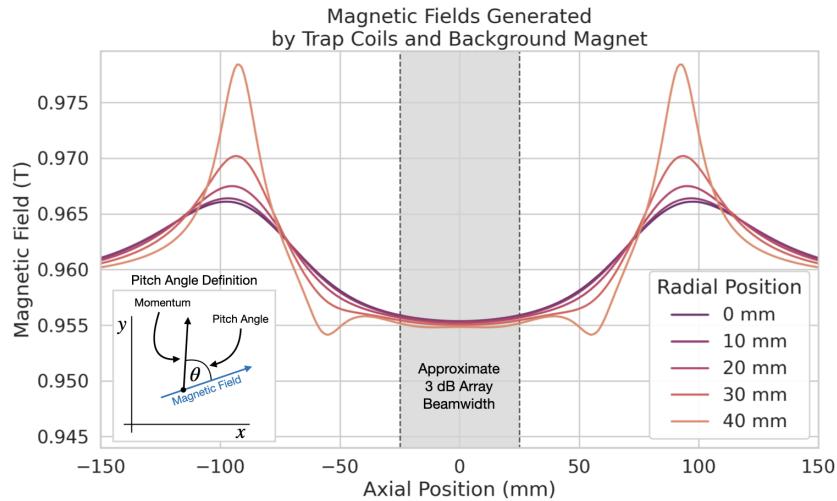
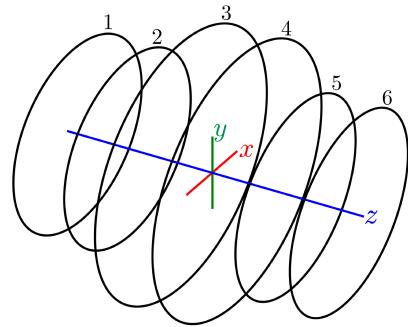


Figure 4.1. The geometry and parameters of the coils used to simulate the FSCD magnetic trap in Kassiopéia. Some axial profiles of the magnetic trap at different radial positions are shown to demonstrate the shape of the magnetic field and trap depth as a function of position. Calculation of the magnetic field profiles was graciously done by René Reimann.

The depth of the FSCD trap is approximately 10 mT when measured along the central axis, which is sufficient to trap electrons with pitch angles as small as 84° . The trap depth factors into the efficiency of the experiment by directly controlling the range

of electron pitch angles that can be trapped. If a higher fraction of pitch angles are trapped then, in principle, more decay events can be observed. However, the signals from electrons with small pitch angles are typically significantly harder to detect than larger pitch angles when using an antenna array, which increases the likelihood of not detecting the first track of the CRES event and harms the energy resolution of the experiment.

The steepness of the trap walls as well as any non-uniformities in the magnetic field contribute to the total energy resolution of the CRES measurement by causing uncertainty in the relationship between an electron's kinetic energy and it's cyclotron frequency. When an electron is trapped, it oscillates back and forth along the trap z-axis (see Figure 4.1) unless it is produced with a pitch angle of exactly 90° [60]. As the electron is reflected from the trap walls it experiences a change in the total magnetic field, which causes a modulation in the cyclotron frequency. This change in magnetic field from the trap introduces a correlation between the pitch angle and kinetic energy parameters of the electron that can reduce energy resolution. In order to mitigate this effect it is important to make the trap walls as steep as possible.

Particle Trajectory Solutions

The magnetic fields solved by direct integration of the electron's current density can be used by Kassiopeia to solve for the trajectory of electrons based on user specified initial conditions. Various distributions are available within Kassiopeia that can be sampled in order to replicate realistic event statistics, including uniform, Gaussian, and Lorentzian among others. In general, an electron has six kinematic parameters that define its trajectory, which are the three-dimensional coordinates of the initial position and the three components of the electron's momentum vector. However, when simulating CRES events it is more common to parameterize the electron's trajectory in terms of it's initial position, the kinetic energy, the pitch angle, and the initial direction of the component of the electron's momentum perpendicular to the magnetic field. This parameterization is completely equivalent to specify each component of the electrons initial position and momentum vectors.

From the initial parameters of the electron and the magnetic field, Kassiopeia solves for the trajectory of the electron. The direct approach proceeds by solving the motion of the electron using the Lorentz force equation, which takes the form of a set of differential equations

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}}{\gamma m} \quad (4.6)$$

$$\frac{d\mathbf{p}}{dt} = e(\mathbf{E} + \frac{\mathbf{p} \times \mathbf{B}}{\gamma m}), \quad (4.7)$$

where \mathbf{r} is the position of the electron, \mathbf{p} is the electron's momentum, e is the charge of the electron, m is the electron's mass, and γ is the relativistic Lorentz term. To account for kinetic energy losses from radiation Kassiopeia includes an additional term in the momentum differential equation, which calculates the change in the electron's momentum induced by synchrotron radiation. Kassiopeia solves this pair of differential equations using numerical integration, however, the exact trajectory can be computationally intensive to solve. If the adiabatic approximation can be applied, then Kassiopeia can make use of a simpler set of equations that can be more readily solved numerically.

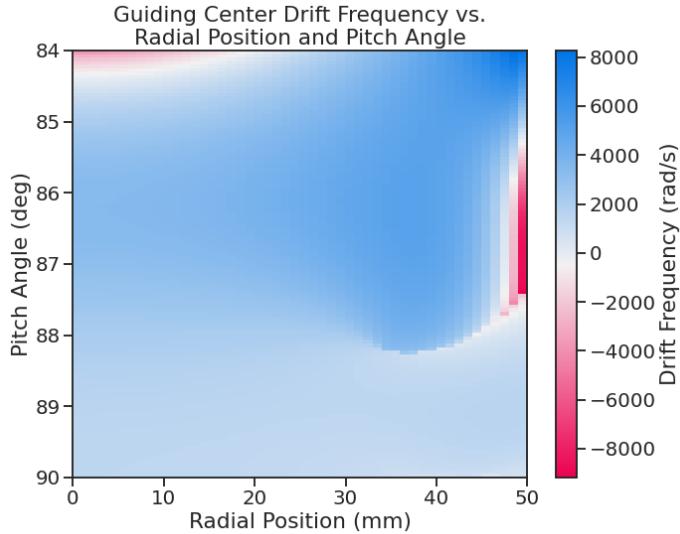


Figure 4.2. A map of the average ∇B -drift frequency for electrons trapped in the prototype FSCD trap shown in Figure 4.1. Negative drift frequencies indicate electrons that are drifting opposite to the standard direction, which means that they are close to escaping the magnetic trap.

Even though Kassiopeia is not directly capable of simulating the cyclotron radiation, it is still an invaluable CRES simulation tool, due to the accurate trajectory solutions for electrons in magnetic traps. With Kassiopeia it is possible to test the efficiency of a particular trap design and analyze features of the electron trajectories that are important to the position, track, and event reconstruction algorithms (see Section 4.3). One example of this for the FSCD is the analysis of the average ∇B -drift frequency as a function of the electrons radial position and pitch angle in the magnetic trap (see Figure 4.2). Radial gradients in the trap cause the guiding center of the electron to drift around the center of

the magnetic trap with an average frequency on the order of 10^3 rad/s. This frequency, while slow compared to the length of a typical CRES time-slice, is large enough to cause a significant loss in efficiency of certain signal reconstruction algorithms. Therefore, it is important to model the drift of the electron in the reconstruction algorithm in order to mitigate the effects of this motion on the reconstruction.

4.2.2 Locust

The Locust² software package [61] is the primary simulation tool developed and used by the Project 8 collaboration for CRES experiments. Locust simulates the responses of antennas and receiver electronics chain to rapidly time-varying electric fields using a flexible approach that allows one to choose from a variety of electric field sources and antennas. Similarly, one can simulate the receiver chain using a series of modular generators that include standard signal processing operations such as down-mixing and fast Fourier transforms (FFT). Since the primary focus of this chapter is the application of Locust to analyses of the FSCD, we shall describe only the most relevant aspects of the software rather than provide a comprehensive description.

Cyclotron Radiation Field Solutions

Simulating CRES events in the FSCD requires that we calculate the electric fields produced by the acceleration of the electron. In the general case, this can be a complicated question to answer, due to back-reaction forces on the electron from its own electric fields that occur when the electron is surrounded by conductive material such as a waveguide or cavity. However, in the case of the FSCD it is possible to ignore such effects and approximate the electron as radiating into a free-space environment.

The equations that describe the electromagnetic fields from a relativistic moving point particle are the Liénard-Wiechert field equations [62, 63], which are obtained by differentiating the Liénard-Wiechert potentials. In their full form the Liénard-Wiechert field equations are

$$\mathbf{E} = e \left[\frac{\hat{n} - \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta} \cdot \hat{n})^3 |\mathbf{R}|^2} \right]_{t_r} + \frac{e}{c} \left[\frac{\hat{n} \times [(\hat{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}]}{(1 - \boldsymbol{\beta} \cdot \hat{n})^3 |\mathbf{R}|} \right]_{t_r} \quad (4.8)$$

$$\mathbf{B} = [\hat{n} \times \mathbf{E}]_{t_r}, \quad (4.9)$$

²https://github.com/project8/locust_mc/tree/master

2299 where e is the charge of the particle, \hat{n} is the unit vector pointing from the particle to the
 2300 position where the fields are calculated, β and $\dot{\beta}$ are the velocity and acceleration of the
 2301 particle divided by the speed of light (c), \mathbf{R} is the distance from the particle to the field
 2302 calculation position, and γ is the relativistic Lorentz term. The subscript t_r indicates
 2303 that the equations must be evaluated at the retarded time so that the time-delay from
 2304 the travel time of the electromagnetic radiation is correctly accounted for.

2305 The only required input to calculate the electric field at the position of an FSCD
 2306 antenna is the velocity and acceleration of the electron, which can be obtained from
 2307 Kassiopeia simulations. Therefore, when simulating a CRES event Locust first runs a
 2308 Kassiopeia simulation of the electron and calculates the electric field incident on the
 2309 antenna. The only difficulty with this approach is the determination of the retarded time.
 2310 The retarded time corresponds to the time that a photon, which has just arrived at an
 2311 antenna at the space-time position (t, \mathbf{r}) , was actually emitted by the electron at the
 2312 space-time position of $(t_r, \mathbf{r}_e(t_r))$. Defined in this way, finding the retarded time requires
 2313 solving

$$c(t - t_r) = |\mathbf{r} - \mathbf{r}_e(t_r)|, \quad (4.10)$$

2314 where the distance traveled by the photon between the measurement and retarded times
 2315 is equal to the distance between the antenna and the electron at the retarded time.
 2316 Locust solves Equation 4.10 using a built-in root finding algorithm to find the retarded
 2317 time, and thus the electric field produced by the electron at the position of each antenna
 2318 in the FSCD array.

2319 Antenna Response Modeling

2320 With the electric field it is possible, in principle, to calculate the resulting voltages pro-
 2321 duced in the antenna. However, direct simulation of the antenna itself is computationally
 2322 expensive since it would require the modeling of complex interactions of the electron's
 2323 electric fields with charge carriers in the conductive elements of the antenna. Direct
 2324 simulation of the antenna in Locust can be avoided by modeling the antenna response
 2325 using the antenna factor, or antenna transfer function, approach. The antenna factor
 2326 defines the voltage produced in the antenna terminal for an incident electric field [64],

$$A_F = \frac{V}{|\mathbf{E}|}, \quad (4.11)$$

where V is the voltage and $|\mathbf{E}|$ is the magnitude of the incident electric field. To obtain the antenna factor for the antennas developed for the FSCD Project 8 employs Ansys HFSS. HFSS is a commercially available finite element method electromagnetic solver widely used throughout the antenna engineering industry [65]. HFSS is capable of calculating the antenna factor and gain patterns for complex antenna designs and outputting the resulting quantities in the form of a text file that can be used as an input to the Locust simulation.

The antenna factor defines the steady-state response of the antenna to electromagnetic plane waves and is a function of the frequency of the radiation. Therefore, in order to apply the transfer function for the calculation of the antenna voltage response in the time domain, Locust models the antenna as a linear time-invariant system [66]. In this formalism the response of the system to the driving force is given by

$$y[n] = h * x = \sum_k h[k]x[n - k], \quad (4.12)$$

where $y[n]$ is the discretely sampled response, x is the driving force stimulus, and h is the finite impulse response (FIR) filter. When applied to the FSCD array, this formalism calculates the voltage time-series produced in each antenna by convolving the electric field time-series with the antenna FIR filter, which is obtained by performing a inverse Fourier transform on the transfer function from HFSS.

Radio-frequency Receiver and Signal Processing

After obtaining the voltage time-series by computing the electron trajectory and antenna response, Locust simulates the signal processing associated with the radio-frequency receiver chain. The standard receiver chain used in Locust simulations of the FSCD attempts to mimic the operations that would actually occur in hardware (see Figure 4.3).

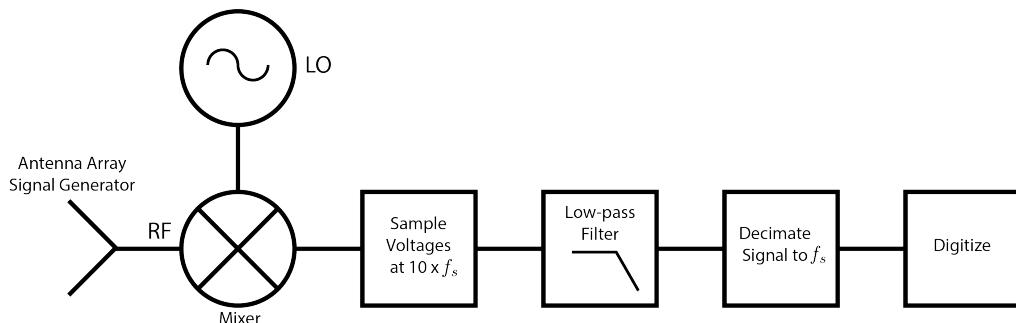


Figure 4.3. The receiver chain used by Locust when simulating CRES events in the FSCD.

2349 Frequency down-conversion is used in the FSCD to reduce the digitization bandwidth
 2350 required to read-out CRES data. According to the Nyquist sampling theorem [67], the
 2351 minimal sampling rate that guarantees no information loss for a signal with a bandwidth
 2352 Δf is given by

$$f_{\text{Nyq}} = 2\Delta f. \quad (4.13)$$

2353 The total bandwidth of CRES signal frequencies from tritium beta-decay ranges from 0
 2354 to 26 GHz in a 0.95 T magnetic field, therefore, direct digitization of CRES signals from
 2355 the FSCD would require sampling frequencies greater than 50 GHz, which is infeasible for
 2356 a real experiment. However, for the purposes of neutrino mass measurement we are only
 2357 interested in measuring the shape of the spectrum in the last 100 eV, which corresponds
 2358 to a frequency bandwidth of 5 MHz. Down-conversion is a technique for reducing the
 2359 base frequencies of signals in a bandwidth given by $[f_{\text{LO}}, f_{\text{LO}} + \Delta f]$ to the bandwidth
 2360 $[0, \Delta f]$, by performing the following multiplication

$$x(t) \rightarrow x(t)e^{-2\pi f_{\text{LO}} t}. \quad (4.14)$$

2361 In down-conversion the signal ($x(t)$) is multiplied by a sinusoidal signal with frequency
 2362 f_{LO} to reduce the absolute frequencies of the signals in the bandwidth. In the FSCD this
 2363 allows us to detect events in the last 100 eV of the tritium spectrum while sampling the
 2364 data far below 50 GHz. The standard bandwidth used in the FSCD is 200 MHz, which
 2365 allows for higher frequency resolution than the minimum sampling frequency for 100 eV
 2366 of energy bandwidth.

2367 Trying to directly simulate down-conversion with a frequency multiplication in Locust
 2368 would require the sampling of the electric fields at each antenna in the FSCD array with
 2369 a period of ≈ 20 ps, which is extremely slow computationally. To avoid this Locust
 2370 performs the down-conversion by intentionally under-sampling the electric fields with
 2371 a frequency of 2 GHz. Sampling below the Nyquist limit causes the higher frequency
 2372 components of the CRES signal to alias, however, Locust can remove these aliased
 2373 frequency peaks using a combination of low-pass filtering and decimation to recreate
 2374 frequency down-conversion. After filtering and decimation, Locust simulates digitization
 2375 by an 8-bit digitizer at a sampling frequency of 200 MHz to recreate the conditions of
 2376 the FSCD. The voltage offset and the digitizer range must be configured by the user
 2377 based on the characteristics of the simulation.

2378 **Data**

2379 The output of Locust simulations for the FSCD primarily consists of two data files. The
2380 first is the electron trajectory information calculated by Kassiopiea, which is output in
2381 the form of a `.root` file [68]. This file contains important kinematic information about
2382 the electron such as it's position and pitch angle as a function of time. The other file is
2383 produced by Locust and it contains the digitized signals acquired from each antenna in
2384 the FSCD array. The Locust output files conform to the Monarch specification developed
2385 by Project 8, which is based on the commonly used HDF5 file format, and matches the
2386 format of the files produced by the Project 8 data acquisition software. This makes it
2387 possible to use the same data analysis code to analyze both simulated and real data.

2388 **4.2.3 CRESana**

2389 Locust is the primary simulation tool used by Project 8 in the development and simulation
2390 of the FSCD. However, simulations of CRES events in larger antenna arrays (≥ 100
2391 antennas) using Locust can take several hours to complete, which is prohibitively long
2392 when one is performing a sensitivity analysis for a large scale antenna experiment. One
2393 of the reasons for Locust's slow operation is that the electric fields from the electron
2394 must be solved numerically for each time-step for each of the antennas in the array.
2395 These numerical solutions allow Locust to accurately simulate the electric fields from
2396 arbitrarily complicated electron trajectories at the cost of more computations and slower
2397 simulations. Therefore, an additional simulation tool that sacrifices some accuracy for
2398 computational efficiency would be extremely useful simulations and sensitivity analyses
2399 of larger antenna array experiments.

2400 To fill this need, Project has developed a new simulations package called CRESana³,
2401 specifically designed to perform analytical simulations of antenna array based CRES
2402 experiments. CRESana is not as flexible as Locust, but it provides a significant increase
2403 in simulation speed. It does this by using well-justified analytical approximations of the
2404 electrons motion in the magnetic field and the resulting electric fields from the electron's
2405 acceleration. The electric fields and signals generated by CRESana are consistent with
2406 theoretical calculations of the electron's radiation, and are test for accuracy using
2407 well-known test-case simulations and consistency checks.

³<https://github.com/MCFlowMace/CRESana>

2408 4.3 Signal Detection and Reconstruction Techniques for 2409 Antenna Array CRES

2410 Antenna Array CRES Signal Reconstruction

2411 A robust set of FSCD simulation tools are vital to the development of the analysis
2412 algorithms necessary for antenna array CRES to succeed. In order to perform CRES
2413 measurements using an antenna array, one must develop an algorithm that uses the
2414 multi-channel time-series obtained by digitizing the array to estimate the starting kinetic
2415 energies of electrons produced in the magnetic trap. This procedure consists of a multi-
2416 stage process of detecting a CRES signal then estimating the parameters of the electron
2417 that produced and is often referred to as simply CRES signal reconstruction.

2418 Compared with the signal reconstruction approaches of the Phase I and II CRES
2419 experiments, antenna array CRES requires a significantly different approach to signal
2420 reconstruction. In Phase I and II, CRES was performed using a waveguide gas cell that
2421 could be directly connected to a waveguide transmission line. The transmission line
2422 efficiently transmits the cyclotron radiation along it's length to an antenna at either end
2423 of the waveguide. However, with an antenna array the electron is essentially radiating
2424 into free-space, therefore, the cyclotron radiation power collected by the array is directly
2425 proportional to the solid angle surrounding the electron that is covered with antennas.
2426 Because it is not practical to fully surround the magnetic trap with antennas, some of the
2427 cyclotron radiation power that would have been collected by the waveguide escapes into
2428 free-space. Furthermore, the power that is collected by the antenna array is split between
2429 every channel in the antenna array, which significantly lowers the signal-to-noise ratio
2430 (SNR) of CRES signals in a single antenna channel compared to a waveguide apparatus.
2431 Therefore, a suite of completely new signal reconstruction techniques are needed in order
2432 to perform CRES in the FSCD.

2433 Changes to the approach to CRES signal reconstruction are also motivated by the
2434 more ambitious scientific goals of the FSCD experiment. A measurement of the tritium
2435 beta-decay spectrum that is sensitive to neutrino masses as small as 40 meV requires that
2436 we measure the kinetic energies of individual electrons with a total energy broadening
2437 of 115 meV [69]. This resolution includes all sources of uncertainty in the electron's
2438 kinetic energy such as magnetic field inhomogeneities. This level of energy resolution is
2439 compatible only with an event-by-event signal reconstruction approach where the kinetic
2440 energies, pitch angles, and other parameters of the CRES events are estimated before

2441 constructing the beta-decay spectrum.

2442 The event-by-event approach is distinct from the analysis done for the Phase I and
2443 Phase II experiments where only the starting cyclotron frequency of the event was
2444 estimated by analyzing the tracks formed by the carrier frequency in the time-frequency
2445 spectrogram. These frequencies were then combined into a frequency spectrogram, which
2446 was converted to the beta-decay energy spectrum using an ensemble approach that
2447 averaged over all other event parameters. The ensemble approach to signal reconstruction
2448 results in poor energy resolution because other kinematic parameters such as pitch angle
2449 change the cyclotron carrier frequency due to changes in the average magnetic field
2450 experience by the electron, and it is therefore incompatible with the future goals of the
2451 Project 8 collaboration.

2452 Components of Reconstruction: Signal Detection and Parameter Estimation

2453 CRES signal reconstruction can be viewed as a two-step procedure consisting of signal
2454 detection followed by parameter estimation. In the former, one is concerned with
2455 identifying CRES signals in the data regardless of the signal parameters, whereas, in the
2456 latter one operates under the assumption that a signal is present and then estimates its
2457 parameters.

2458 More formally, signal detection is essentially a binary hypothesis test between the
2459 signal and noise data classes and parameter estimation describes a procedure of fitting a
2460 model to the observed data. While both of these processes are required for a complete
2461 reconstruction (see Figure 4.4), the focus of my work and this chapter is on the signal
2462 detection aspect of antenna array CRES signal reconstruction.

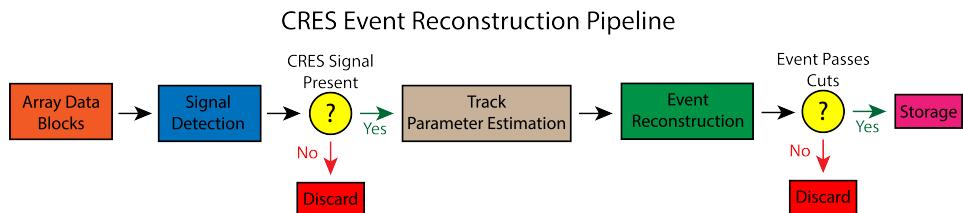


Figure 4.4. A high-level diagram depicting the process of CRES event reconstruction. The first step consists of identifying the presence of a signal in the data. This step is necessary to avoid the danger of performing a reconstruction of a false event, which would constitute a background contribution to the tritium spectrum measured by CRES.

2463 **Detection Theory**

2464 The problem of signal detection can be posed as a statistical hypothesis test [70]. For
2465 CRES signals, which are essentially vectors with added white Gaussian noise (WGN),
2466 one needs to choose between two hypotheses

$$\mathcal{H}_0 : \mathbf{y} = \boldsymbol{\nu} \quad (4.15)$$

$$\mathcal{H}_1 : \mathbf{y} = \mathbf{x} + \boldsymbol{\nu}, \quad (4.16)$$

2467 where \mathbf{y} is the CRES data vector, $\boldsymbol{\nu}$ is a sample of WGN, and \mathbf{x} represents the CRES
2468 signal. The hypothesis that the data contains only noise is labeled \mathcal{H}_0 and the hypothesis
2469 that the data contains a signal is labeled \mathcal{H}_1 .

2470 For illustrative purposes one can examine the case where one the first sample of
2471 data is used to distinguish between \mathcal{H}_0 and \mathcal{H}_1 . The value of the first data sample is
2472 distributed according to two gaussian distributions corresponding to \mathcal{H}_0 and \mathcal{H}_1 (see
2473 Figure 4.5). By setting a decision threshold on the value of this sample, one can choose
2474 the correct hypothesis with a probability given by the areas underneath the probability
2475 distribution curves. A true positive corresponds to correctly identifying that the data
2476 contains signal, whereas, a true negative means that one has correctly identified the data
2477 as noise. The rate at which the detector performs a true positive classification is given
2478 by the green region underneath $p(\mathbf{y}[0]; \mathcal{H}_0)$, and the rate at which the detector performs
2479 a true negative classification is given by the orange region underneath $p(\mathbf{y}[0]; \mathcal{H}_1)$. Two
2480 types of misclassifications are possible. Either we declare noise data as signal, which is
2481 call a false positive, or we declare signal data as noise, which is a false negative. Note
2482 that it is only possible to trade off these two types of errors by tuning the detection
2483 threshold. One cannot simultaneously reduce the rate of false positives without also
2484 increasing the rate of false negatives.

2485 The approach taken with CRES signals is to fix the rate of false positives by setting
2486 a minimum value for a detection threshold. The rate of false positives that is acceptable
2487 at the detection stage depends upon the rate of background events compatible with the
2488 sensitivity goals of the experiment. The ultimate goal of a neutrino mass measurement
2489 with 40 meV sensitivity in general has strict requirements on the number of background
2490 events, which requires a relatively high detection threshold to achieve. Consequently,
2491 the ideal signal detection algorithm is the one that achieves the maximum rate of true
2492 positives for a fixed rate of false positives, so that the detection efficiency of the experiment
2493 is maximized and potential sources of background are kept to a minimum.

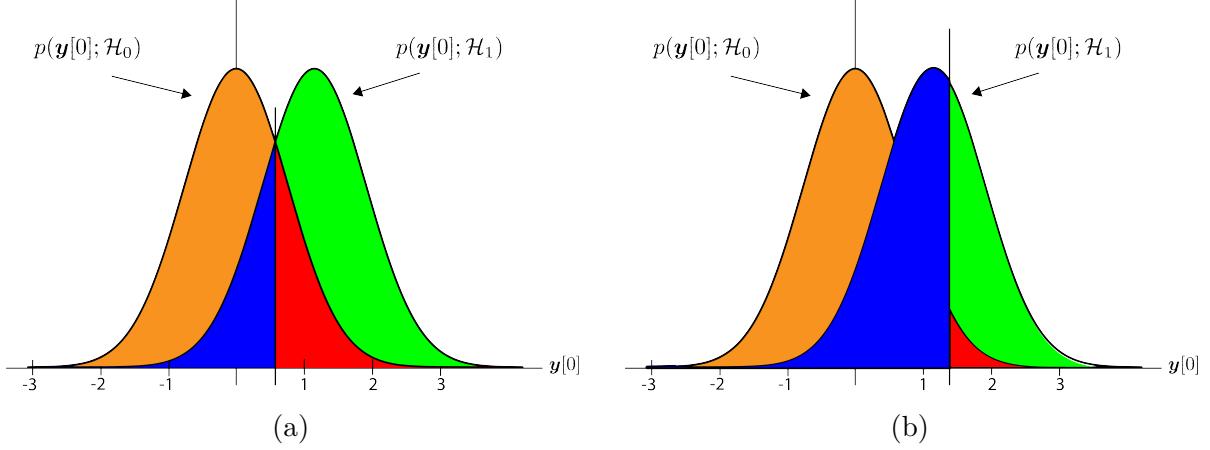


Figure 4.5. An illustration of two PDFs associated with a binary hypothesis test. The decision threshold is represented by the vertical line that partitions both distributions. The orange and red areas correspond to the true negative and false positive probabilities and the blue and green areas correspond to the false negative and true positive probabilities respectively. To decide between the two hypotheses we perform the likelihood ratio test specified by the Neyman-Pearson theorem. This approach achieves the highest true positive probability for a given false positive probability.

According to the Neyman-Pearson theorem [71], the statistical hypothesis test that maximizes the probability of detection for a fixed rate of false positives is the likelihood ratio test, which is formed by computing the ratio of the signal likelihood to the noise likelihood,

$$L(x) = \frac{P(\mathbf{y}; \mathcal{H}_1)}{P(\mathbf{y}; \mathcal{H}_0)} > \gamma. \quad (4.17)$$

Here, the likelihood of the hypotheses \mathcal{H}_0 and \mathcal{H}_1 are described by the probability distributions $P(\mathbf{y}; \mathcal{H}_0)$ and $P(\mathbf{y}; \mathcal{H}_1)$ respectively, and γ is the threshold for deciding \mathcal{H}_1 . The decision threshold is determined by integrating $P(\mathbf{y}; \mathcal{H}_0)$ such that

$$P_{\text{FP}} = \int_{\gamma}^{\infty} P(\tilde{\mathbf{y}}; \mathcal{H}_0) d\tilde{\mathbf{y}} = \alpha, \quad (4.18)$$

where α is the desired false positive detection rate given by the red colored areas shown in Figure 4.5. The true positive detection rate is given by the similar integral

$$P_{\text{TP}} = \int_{\gamma}^{\infty} P(\tilde{\mathbf{y}}; \mathcal{H}_1) d\tilde{\mathbf{y}}, \quad (4.19)$$

which corresponds to the green areas in Figure 4.5.

Changing the decision threshold allows one to trade-off between P_{TP} and P_{FP} as

appropriate for the given situation. It is common to summarize the relationship between P_{TP} and P_{FP} using the receiver operating characteristic (ROC) curve, which is obtained by evaluating the true positive and false positive probabilities as a function of the decision threshold value (see Figure 4.6). The ROC curve provides a convenient way to compare

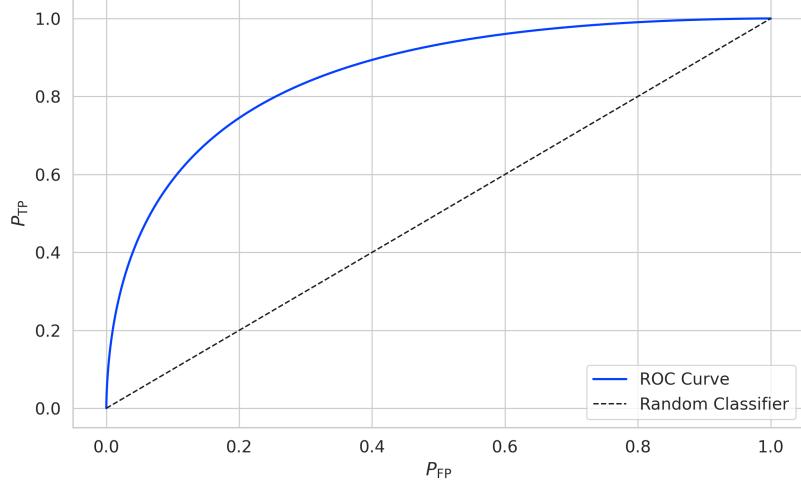


Figure 4.6. An example ROC curve formed by computing the P_{FP} and the P_{TP} for a given likelihood ratio test. As the decision threshold is increased P_{FP} decreases at the expense of a lower P_{TP} . The black dashed line indicates the lower bound ROC curve obtained by randomly deciding between \mathcal{H}_0 and \mathcal{H}_1 .

the performance of different signal detection algorithms. In general, a classifier with a higher the P_{TP} as a function of P_{FP} is desirable, which corresponds to a larger area underneath the respective ROC curve. A perfect classifier has an area underneath the curve of 1.0, however, such a classifier is almost never achievable in practice.

4.3.1 Digital Beamforming

Introduction to Beamforming

Beamforming refers to a suite of antenna array signal processing techniques that are designed to enhance the radiation or gain of the array in certain directions and suppress it in other direction [64]. Beamforming is of interest to Project 8 as a first level of signal reconstruction for the FSCD and other antenna array CRES experiments, which operates at the signal detection stage of reconstruction.

Beamforming is accomplished by performing a phased summation of the signals received by the antenna array. The beamforming phases are chosen such that the signals

2522 emitted by the array will constructively interfere at the point of interest (see Figure
 2523 4.7). As a consequence of the principle of reciprocity [72], when the array is operating in
 2524 receive mode, the signals emitted from a source at the same point will constructively
 interfere when summed. The origin of the phase delays in beamforming is the path-

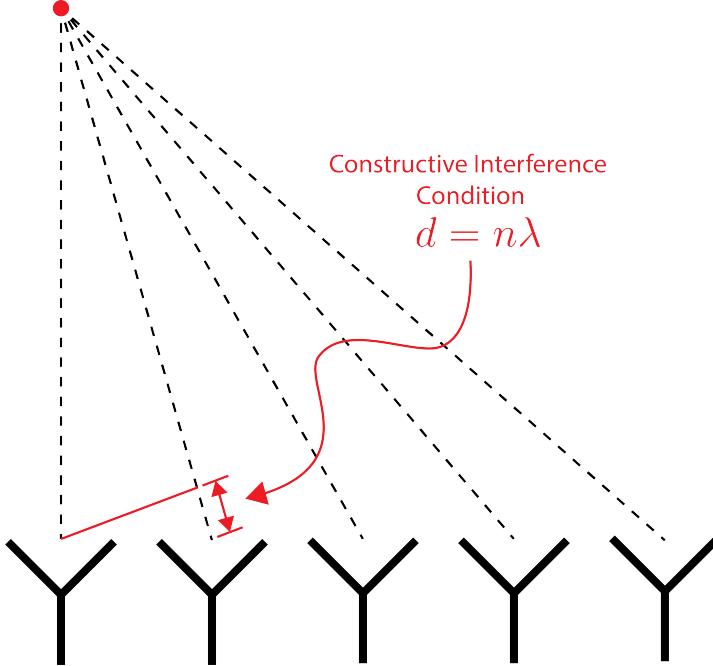


Figure 4.7. An illustration of the constructive interference condition which is the operating principle of digital beamforming using a uniform linear array as an example.

2525
 2526 length difference to the beamforming point between different antennas in the array. The
 2527 relationship between the phase delay and the path-length difference is given by the
 2528 familiar equation

$$\phi = \frac{2\pi d}{\lambda}, \quad (4.20)$$

2529 where ϕ is the phase delay, d is the path-length difference, and λ is the wavelength of
 2530 the radiation. In practice, one chooses the values of d by specifying the beamforming
 2531 positions of interest and then calculates the beamforming phases using Equation 4.20,
 2532 which is guaranteed to follow the constructive interference condition shown in Figure 4.7.

2533 Beamforming can be neatly expressed mathematically using the vector equation

$$y[n] = \Phi^T[n] \mathbf{x}[n], \quad (4.21)$$

2534 where $\mathbf{x}[n]$ is the array snapshot vector, $\Phi[n]$ is a vector of beamforming shifts, and
 2535 $y[n]$ is the resulting summed signal. The beamforming shifts consist of a set of complex

2536 numbers that contain the beamforming phase shift and an amplitude weighting factor,

$$\Phi[n] = [A_0[n]e^{-2\pi i \phi_0[n]}, A_1[n]e^{-2\pi i \phi_1[n]}, \dots, A_{N-1}[n]e^{-2\pi i \phi_{N-1}[n]}], \quad (4.22)$$

2537 where the set of magnitudes $A_i[n]$ are amplitude weighting factors and $\phi_i[n]$ are the phase
2538 shifts from the path-length differences. The index i is used to denote the antenna channel
2539 number. The amplitude weighting factor is the relative magnitude of the signal received
2540 by a particular antenna to the other antennas in the array, such that the antennas that
2541 receive signals with higher amplitude, due to being closer to the source, have more
2542 weight in the beamforming summation. The input and outputs signals beamforming
2543 are naturally expected to be functions of time as indicated by the index $[n]$, however, it
2544 is also possible to use time dependent beamforming phases that shift the beamforming
2545 position of the array over time.

2546 Digital beamforming is the type of beamforming algorithm of interest to Project 8 for
2547 CRES. Specifically, digital beamforming means that the beamforming phases are applied
2548 to the array signals in software rather than employing fixed beamforming phase shifts in
2549 the receiver chain hardware. The advantage of digital beamforming is that for a given
2550 series of array snapshots one can specify a large number of beamforming positions and
2551 effectively search for electrons by performing the beamforming summation associated
2552 with each point and applying a signal detection algorithm to identify the presence of a
2553 CRES signal.

2554 One of the most attractive features of digital beamforming is the spatial filtering
2555 effect, which is a direct consequence of the constructive interference condition used to
2556 define the beamforming phases. Spatial filtering allows for signals from multiple electrons
2557 at different positions in the trap to be effectively separated, because the constructive
2558 interference condition will force the signals from electrons at positions different from the
2559 beamforming position to cancel. This helps to reduce signal pile-up that could become
2560 an issue for large scale CRES experiments using a dense tritium source.

2561 The digital beamforming positions can be specified with arbitrary densities limited
2562 only by the available computational resources. This provides a very straight-forward way
2563 to estimate the position of the electron in the trap by using a dense grid of beamforming
2564 positions and maximizing the output power of the beamforming summation over this
2565 grid. This natural approach to position reconstruction is attractive due the requirements
2566 of an event-by-event signal reconstruction, which needs an accurate estimation of the
2567 exact magnetic field experienced by the electron in order to correctly estimate it's kinetic

2568 energy. Combined with an accurate map of the magnetic field inhomogeneities of the
 2569 trap obtained from calibrations, beamforming allows one to apply this magnetic field
 2570 correction with a spatial resolution that is a fraction of the cyclotron wavelength.

2571 **Laboratory Beamforming Demonstrations**

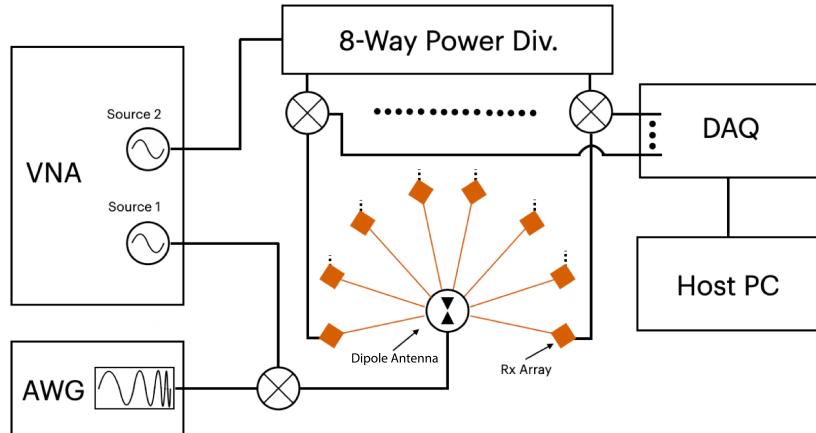


Figure 4.8. System level diagram of the laboratory setup used for beamforming demonstrations at Penn State. For more information on this system see Chapter 5. Signals near 26 GHz are fed to a dipole antenna using an arbitrary waveform generator (AWG) and vector network analyzer (VNA), which drive a mixer. The dipole radiation is collected by an array of antennas connected to the digitizer data acquisition (DAQ) system.

2572 As part of the development of antenna array CRES for the FSCD, an antenna
 2573 measurement setup was constructed at Penn State to serve as a testbed for antenna
 2574 prototypes and to perform laboratory validations of array simulations. This system
 2575 is discussed in more detail in Chapter 5. Early versions of the antenna measurement
 2576 system (see Figure 4.8 and Figure 4.9) were used to perform beamforming reconstruction
 2577 studies of a simple probe antenna to better understand the principles of beamforming
 2578 and confirm the estimated beamforming performance of Locust.

2579 Signals from an arbitrary waveform generator were up-converted to 26 GHz using a
 2580 mixer and a high-frequency source from a vector network analyzer and fed to the dipole
 2581 antenna through a balun. The radiation from the dipole antenna was received by an
 2582 array of horn antennas. The signals from the horn antennas were then down-converted
 2583 to baseband using a collection of mixers and an 8-way power divider. The signals were
 2584 then digitized and saved to a host computer for analysis.

2585 The data collected using the dipole and horn antenna array is reconstructed using the
 2586 beamforming reconstruction approach specified in Section 4.3.1. A two-dimensional grid

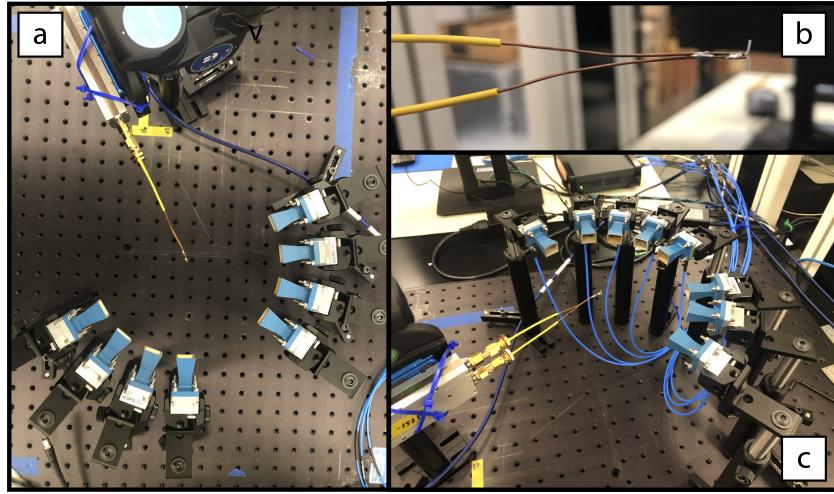


Figure 4.9. Photographs of the beamforming demonstration setup. In (a) I show a top-down view of the dipole antenna and the array of eight horn antennas. Manual repositioning of the horn antennas allows one to synthesize a full-circular antenna array. The dipole antenna is mounted on a camera tripod mount that allows for manual position tuning. (b) is a close up image of the dipole, which is manufactured from two segments of semi-rigid coaxial cable. (c) is another image of the dipole and array.

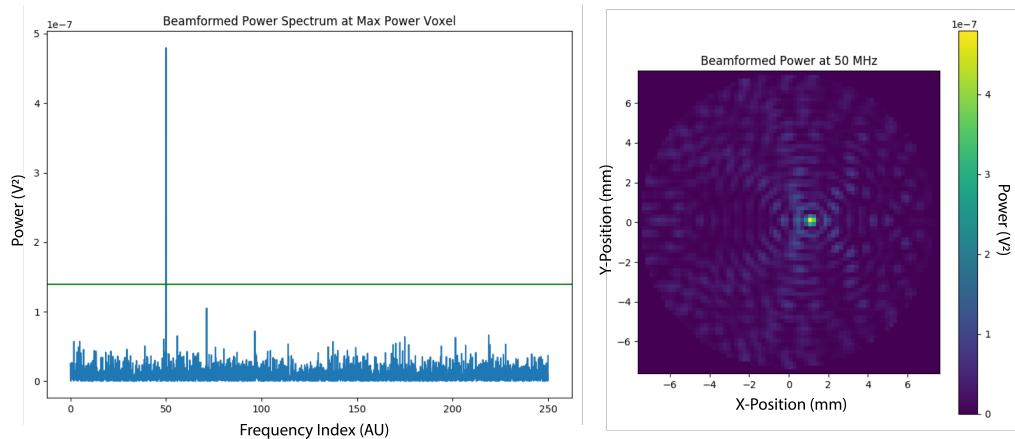


Figure 4.10. An example of digital beamforming reconstruction of a dipole antenna using a synthetic array of horn antennas. The beamforming image on the right is constructed by computing the time-averaged power of the summed signals for a two-dimensional grid of beamforming positions. In the image one can see a clear maximum that corresponds to the position of the dipole antenna. On the left I show the frequency spectrum of the time-series at the maximum power pixel. White gaussian noise is added to the signal to mimic a more realistic signal-to-noise-ratio. The signal emitted by the dipole is clearly visible as the high power peak in the frequency spectrum.

of xy-positions is defined and the beamforming phase shifts for each of these positions is calculated. The phased summation can be visualized by plotting the time-averaged power for each of the summations as a pixel in the resulting beamforming image (see Figure 4.10). White Gaussian noise (WGN) can be added to the data at this stage to simulate more realistic signal-to-noise ratios (SNR) if desired. The beamforming peak maxima is expected to have a Bessel function shape due to the circular symmetry of the array, and by analyzing the size of the beamforming maxima one can confirm that the beamforming reconstruction measurement has similar position resolution as expected from Locust simulations. Additionally, signal detection rates can be estimated from the data by comparing the magnitude of the beamforming signal peak in the frequency spectra to simulation.

2598 FSCD Beamforming Simulations

Using Locust simulations of the FSCD one can perform beamforming reconstruction studies using the simulated CRES signal data. As we mentioned in the previous section, the beamforming procedure begins by specifying a set of beamforming positions and corresponding beamforming shifts. The beamforming positions form a grid that covers the region of interest in the field of view of the antenna array. There are effectively an infinite number of ways to specify the grid positions, however, uniform square grids are the most commonly used due to their simplicity. In the FSCD experiment the number and pattern of the grid positions would be optimized to cover the most important regions of the trap volume to maximize detection efficiency while minimizing superfluous calculations.

The beamforming grids used for signal reconstruction with the FSCD consist of a set of points that cover a region of the two-dimensional plane formed by the perimeter of the antenna array. The axial dimension is left out of the beamforming grid because the electrons are assumed to occupy only an average axial position, which corresponds to the center of the magnetic trap. This is because it is impossible to resolve the axial position of the electron as a function of time due to the rapid axial oscillation frequencies of trapped electrons relative to the FSCD time-slice duration.

After beamforming, a summed time-series is obtained for each beamforming position that can be evaluated for the presence of a signal using a detection algorithm. A beamforming image is a visualization method that is equivalent to arranging the beamforming grid points according to their physical locations to form a three-dimensional matrix where the first two dimensions encode the XY-position of the beamforming point and the third dimension contains the summed time-series. The image is formed by taking the

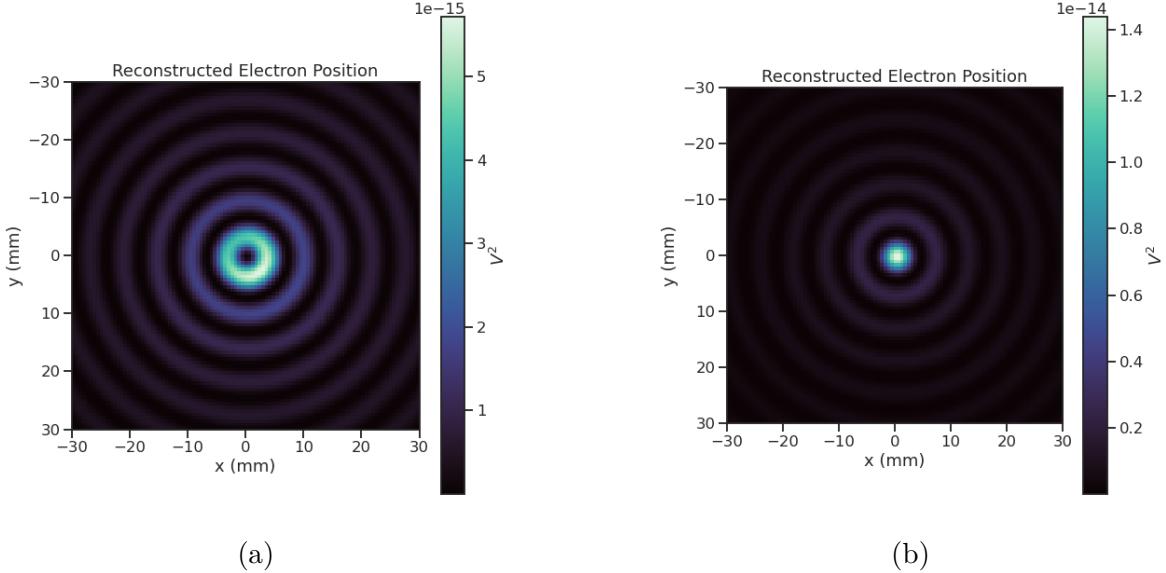


Figure 4.11. Beamforming images visualizing the reconstruction of an electron without (a) and with (b) the cyclotron phase correction. The images were generated using data from Locust simulations. The cyclotron phase refers to a phase offset equal to the relative azimuthal position of an antenna in the array. This phase offset is caused by the circular electron orbit and must be corrected for during reconstruction.

time-averaged power (see Figure 4.11). Beamforming images are purely for the purposes of visualization and are not particularly useful for signal detection or reconstruction.

If the beamforming phases consist only of the spatial phase component from Equation 4.20, then the resulting beamforming image contains a relatively high-power ring-shaped region that is centered on the position of the electron (see Figure 4.11a). The origin of this shape is an additional phase offset particular to a cyclotron radiation source. Essentially, the circular motion that produces the cyclotron radiation introduces a relative phase offset to the electric fields that is equal to the azimuthal position of the field measurement point. For example, if we have two antennas, one located at an azimuthal position of 0° and another located at an azimuthal position of 90° , then the CRES signals received by these antennas will be out of phase by 90° , which is the difference in their azimuthal positions. This phase offset can be corrected by adding an additional term to the beamforming phase equation that is equal to the azimuthal position of the antenna relative to the electron,

$$\phi_i[n] = \frac{2\pi d_i[n]}{\lambda} + \Delta\varphi_i[n], \quad (4.23)$$

where $\Delta\varphi_i$ is difference between the azimuthal position of the electron and the i -th antenna channel. Using the updated beamforming phases in the summation changes the

ring feature into a Bessel function peak whose maximum corresponds to the position of the electron. Including this cyclotron phase correction significantly improves the signal detection and reconstruction capabilities of beamforming by more than doubling the summed signal power and shrinking the beamforming maxima feature size.

The beamforming image examples in Figure 4.11 were produced using an electron located on the central axis of the magnetic trap, which do not experience ∇B -drift. However, for electrons produced at non-zero radial position the beamforming phases must be made time-dependent in order to track the position of the electron's guiding center over time. Without this correction the ∇B -drift causes the electron to move between beamforming positions, which effectively spreads the cyclotron radiation power over a wider area in the beamforming image (see Figure 4.12). This effect significantly

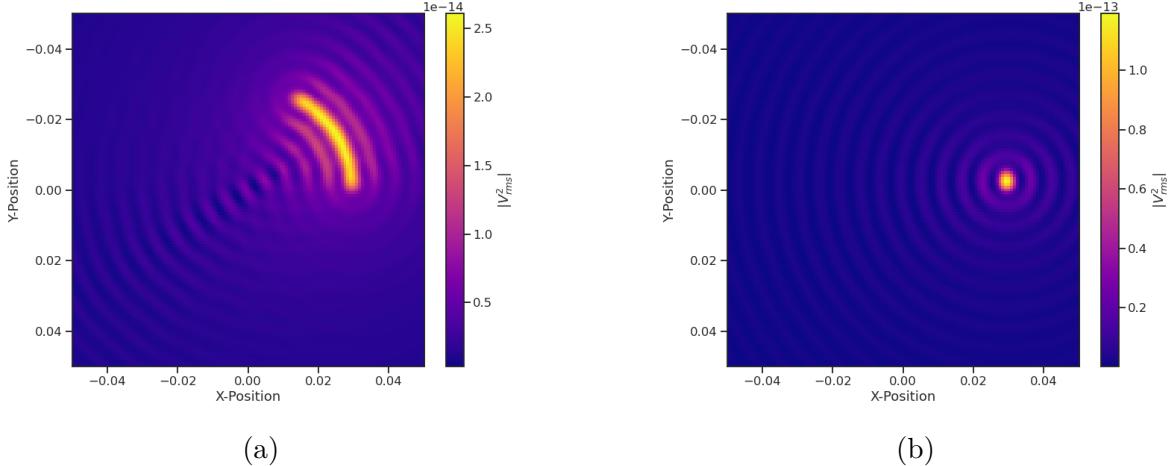


Figure 4.12. Beamforming images visualizing the reconstruction of an electron located off the central axis of the FSCD trap. In (a) we performing beamforming without the ∇B -drift correction, and in (b) we include the ∇B -drift correction.

reduces the power of the beamforming maxima and increases the size of the beamforming features, simultaneously harming detection efficiency and position reconstruction.

The ∇B -drift correction simply adds a circular time-dependence to the beamforming positions as a function of time,

$$r[n] = r_0 \quad (4.24)$$

$$\varphi[n] = \varphi_0 + \omega_{\nabla B} t[n], \quad (4.25)$$

where $\omega_{\nabla B}$ is the drift frequency and $t[n]$ is the time vector. In the ideal case the ∇B -drift frequencies from Figure 4.2 for the correct pitch angle and radial position would be used,

2654 however, it is not possible to know the electron's pitch angle a priori. In principle, one
 2655 could perform multiple beamforming summations for a given beamforming position using
 2656 different drift frequencies and choose the one that maximizes the summed power, but
 2657 this approach leads to a huge computational burden that would be impractical for a
 2658 real FSCD experiment. A compromise is to use an average value of $\omega_{\nabla B}$ obtained by
 2659 averaging over the drift frequencies for electrons of different pitch angle at a particular
 2660 radius. This approach keeps the computational cost of time-dependent beamforming to a
 2661 minimum while still providing a significant increase in the detection efficiency of digital
 2662 beamforming.

2663 **Signal Detection with Beamforming and a Power Threshold**

2664 Up to this point we have neglected any specific discussion of how digital beamforming is
 2665 used for signal detection and reconstruction. This is because, strictly speaking, digital
 2666 beamforming consists only of the phased summation of the array signals and cannot
 2667 be used alone for signal detection. The example beamforming images shown in Figure
 2668 4.11 and Figure 4.12 were produced using simulated data that contained no noise, which
 2669 significantly degrades the utility of analyzing the beamforming images for signal detection
 2670 and reconstruction.

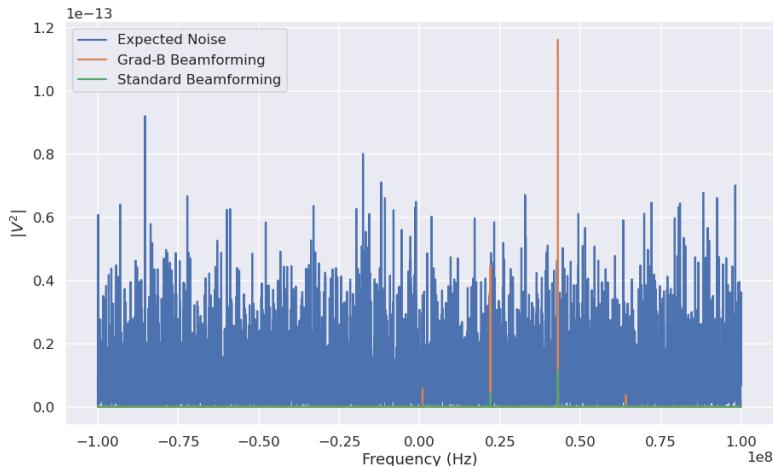


Figure 4.13. A plot of a typical frequency spectrum obtained by applying a Fourier transform to the time-series obtained from beamforming. The frequency spectra are plotted without noise on top of an example of a typical noise spectrum to visualize a realistic signal-to-noise ratio. In the example we see that without beamforming it would not be possible to detect anything since the signal amplitudes would be reduced by a factor of sixty relative to the noise. Additionally, we see that the ∇B -drift correction is needed to detect this electron since it comes from a simulation of an electron with a significant off-axis position.

2671 Digital beamforming as a detection algorithm is understood to mean digital beam-
 2672 forming plus a detection threshold placed on the amplitude of the frequency spectrum
 2673 obtained by applying a fast Fourier transform (FFT) to the summed time-series (see
 2674 Figure 4.13). This approach is most similar to the time-frequency spectrogram analysis
 2675 employed in previous CRES experiments, however, in principle any signal detection
 2676 algorithm could be used after the beamforming procedure. In Section 4.4 I analyze the
 2677 signal detection performance of the power threshold approach in detail.

2678 From the example frequency spectra in Figure 4.13 it is clear that without a re-
 2679 construction technique that coherently combines the signals from the full antenna our
 2680 ability to detect CRES signals will be drastically reduced. Because the CRES signals are
 2681 in-phase at the correct beamforming position the summed power increases as a function
 2682 of N^2 compared to a single antenna channel, where N is the number of antennas. It
 2683 is true that the noise power is also increased by beamforming, but, because the noise
 2684 is incoherent, its power only increases linearly. Consequently, the signal-to-noise ratio
 2685 (SNR) of the CRES signal increases linearly with the number of antennas, which greatly
 2686 improves detection efficiency compared to using only the information in a single antenna.

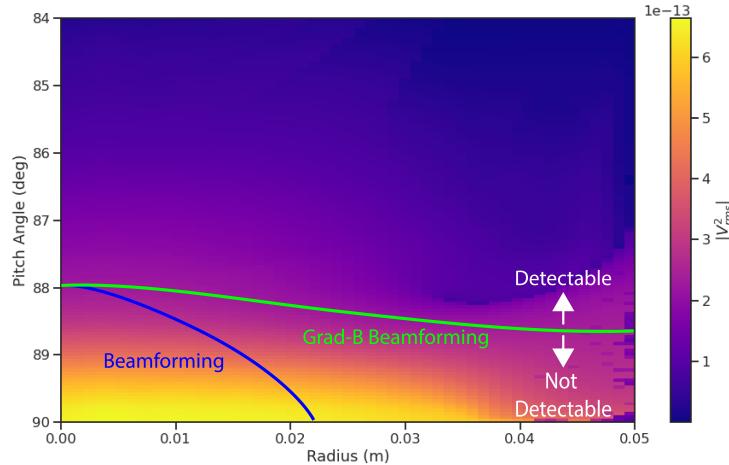


Figure 4.14. A plot of the total signal power received by the FSCD array from trapped electrons with different radial positions and pitch angles generated using Locust simulations. The lines on the plot indicate a 10 dB detection threshold above the mean value of the noise in the frequency spectrum. With static beamforming electrons with radial positions larger than about two centimeters are undetectable due to the change in the electron's position over time causing losses from beamforming phase mismatch. This is corrected by including ∇B -drift frequencies in the beamforming phases. Both beamforming techniques fail to detect electrons below $\approx 88.0^\circ$, since these signal are composed of several relatively weak sidebands that are comparable to the noise.

2687 The power threshold detection algorithm searches for high-power frequency bins that
 2688 should correspond to a frequency component of the CRES signal. In order to prevent
 2689 random noise fluctuations from being mistaken as CRES signals the power threshold
 2690 must be set high enough so that it is unlikely that random noise could be responsible. A
 2691 consequence of this is that many electrons that can be trapped will go undetected because
 2692 the modulation caused by axial oscillations leads to the cyclotron carrier power to falling
 2693 below the decision threshold. The time-dependent beamforming used to correct for the
 2694 ∇B -drift increases the volume of the magnetic trap where electrons can be detected,
 2695 but it is ineffective at increasing the range of detectable pitch angles (see Figure 4.14).
 2696 Fundamentally, this is because the power threshold only uses a fraction of the signal
 2697 power to detect electrons and ignores the power present in the frequency sidebands. In
 2698 the subsequent sections I examine two other signal detection algorithms that seek to
 2699 improve the detection efficiency of the FSCD by utilizing the more of the signal shape to
 2700 compute the detection test statistics.

2701 **4.3.2 Matched Filtering**

2702 **Introduction to Matched Filtering**

2703 The problem of CRES signal detection is the problem of detecting a signal buried in
 2704 WGN, which has been examined at great depth in the signal processing literature [70].
 2705 For a fully known signal in WGN the optimal detector is the matched filter, which means
 2706 that it achieves the highest true positive rate for a fixed rate of false positives. The
 2707 matched filter test statistic is calculated by taking the inner product of the data with
 2708 the matched filter template

$$\mathcal{T} = \left| \sum_n h^\dagger[n] y[n] \right|, \quad (4.26)$$

2709 where $h[n]$ is the matched filter template and $y[n]$ is the data. The matched filter test
 2710 statistic defines a binary hypothesis test in which the data vector is assumed to be an
 2711 instance of two possible data classes. By setting a decision threshold on the value of \mathcal{T} ,
 2712 one can classify a given data vector as belonging to two distinct hypotheses. Under the
 2713 first hypothesis the data is composed of pure WGN, and under the second hypothesis the
 2714 data is composed of the known signal with additive WGN. The matched filter template

2715 is obtained by rescaling the known signal in the following way

$$h[n] = \frac{x[n]}{\sqrt{\tau \sum_n x^\dagger[n]x[n]}}, \quad (4.27)$$

2716 where τ is the variance of the WGN and $x[n]$ is the known signal. Strictly speaking,
2717 Equation 4.27 is only true for noise with a diagonal covariance matrix, however, in the
2718 context of the FSCD we are justified in assuming this to be true. Defining the matched
2719 filter templates in this way guarantees that the expectation value of \mathcal{T} is equal to one
2720 when the data contains only noise, which is the standard matched filter normalization in
2721 the signal processing literature.

2722 Although matched filters are canonically formulated in terms of a perfectly known
2723 signal, it is still possible to apply the matched filter technique given imperfect information
2724 about the signal provided that the signal is deterministic. From our discussion of CRES
2725 simulation tools for the FSCD (see Section 4.2) we know that the shape of CRES signals
2726 are completely determined by the initial parameters of the electron. The random collisions
2727 with background gas molecules which cause the formation of signal tracks are the only
2728 stochastic component of the CRES event after the initial beta-decay, therefore, it is
2729 possible to develop a matched filter for the detection of CRES signal tracks which are fully
2730 determined by the parameters of the electron after the initial beta-decay or subsequent
2731 collision events.

2732 The matched filter test statistic for CRES signals is a modified version of Equation
2733 4.26

$$\mathcal{T} = \max_{\mathbf{h}, m} |\mathbf{h} * \mathbf{y}| = \max_{\mathbf{h}, m} \left| \sum_k h^\dagger[k]x[m - k] \right|, \quad (4.28)$$

2734 where the matched filter inner product has been replaced with a convolution operation
2735 and a maximization over the template and convolution delay (m). Replacing the inner
2736 product with a convolution accounts for the fact that the start time of the CRES signal is
2737 now an unknown parameter, in addition, we now perform a maximization of the matched
2738 filter convolution over a number of different templates. Because the shape of the signal is
2739 unknown we are forced to guess a number of different signal shapes to create a template
2740 bank with which we can identify unknown signals by performing an exhaustive search.

2741 The template bank approach to matched filtering, while quite powerful, can quickly
2742 become computationally intractable. This is especially true in the case of the FSCD
2743 because of the large amount of raw data produced by the array that must be analyzed.
2744 Specifically, the time-domain convolution specified by Equation 4.28 is particularly

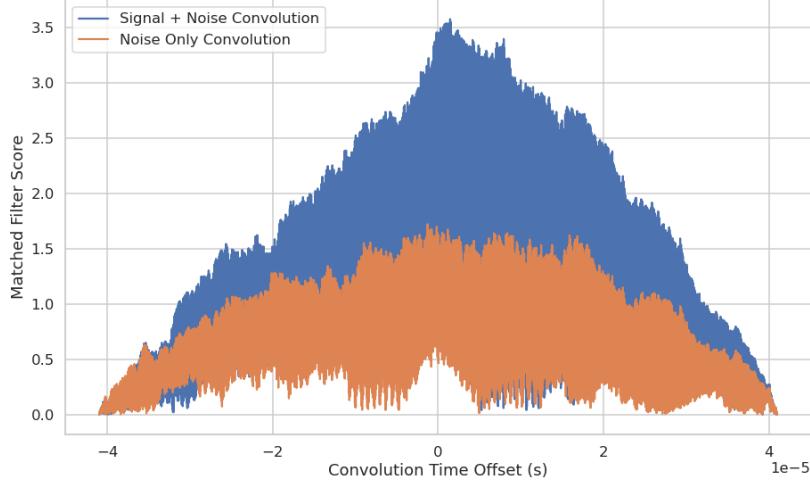


Figure 4.15. Example of a convolution of a CRES signal template with a segment of noisy data. A simulated CRES signal was simulated using Locust and normalized to create a matched filter template. When this template is convolved with noisy data the contains the matching signal the convolution output increases dramatically compared to data with only noise. The decreasing convolution output as the time offset of the convolution increases is caused by zero-padding of the data and template.

2745 computationally intensive and is a major barrier towards the implementation of a
 2746 matched filter for signal detection in an experiment like the FSCD. This can be avoided
 2747 by using the convolution theorem to replace the time-domain convolution with an inner
 2748 product in the frequency domain.

2749 The convolution theorem states that

$$\mathbf{f} * \mathbf{g} = \mathcal{F}^{-1}(\mathbf{F} \cdot \mathbf{G}) \quad (4.29)$$

2750 where \mathbf{f} and \mathbf{g} are discretely sampled time-series, \mathbf{F} and \mathbf{G} are the respective discrete
 2751 Fourier transforms, and \mathcal{F}^{-1} is the inverse discrete Fourier transform operator. The
 2752 convolution theorem allows us to perform the matched filter convolution by first com-
 2753 puting the Fourier transform of the template and data, then performing a point-wise
 2754 multiplication of the two frequency series, and finally performing the inverse Fourier
 2755 transform to obtain the convolution output. Because discrete Fourier transforms can be
 2756 performed extremely efficiently, the convolution theorem is almost always used in lieu of
 2757 directly computing the convolution.

2758 One thing to note here is that the convolution theorem for discrete sequences shown
 2759 here, is technically valid only for circular convolutions, which is not directly specified
 2760 in Equation 4.28. However, because typical CRES track lengths are much longer than

2761 the Fourier analysis window and also that the frequency chirp rates are small compared
2762 to the time-slice duration, it is relatively safe to use circular convolutions to evaluate
2763 matched filter scores for CRES signals, which allows us to apply the convolution theorem
2764 to compute matched filter scores using the frequency representation of the data and
2765 matched filter template.

2766 **Matched Filter Analysis of the FSCD**

2767 The optimality provided by the matched filter makes it a useful algorithm for analysis
2768 of CRES experiment designs for sensitivity analyses, since it indicates the best possible
2769 detection efficiency achievable by an experiment configuration. The standard approach to
2770 performing these studies involves generating a large number of simulated electron signals
2771 that span the kinematic parameter space of electrons in the magnetic trap. In general,
2772 electrons have six kinematic parameters along with an additional start time parameter.

2773 In order to limit the number of simulations required to evaluate the detection efficiency
2774 the standard approach is to fix the starting axial position, starting azimuthal position,
2775 starting direction of the perpendicular component of the electron's momentum, and event
2776 start time to reduce the parameter space to starting radial position, starting kinetic
2777 energy, and starting pitch angle. The fixed variables are true nuisance parameters that do
2778 not affect the detection efficiency estimates for the FSCD design, because they manifest
2779 as phases which are marginalized during the calculation of the matched filter score.

2780 Across radial position, kinetic energy, and pitch angle one defines a regular grid of
2781 parameters and uses Locust to simulate the corresponding signals (see Figure 4.16). This
2782 grid of simulated signals can be used to estimate the likelihood of detecting signals,
2783 because the matched filter score specifies the shape of the PDF that defines the detection
2784 probability and the size of the template bank influences the likelihood of a good match
2785 between a template and a random signal.

2786 The matched filter approach can also be used to estimate the achievable energy
2787 resolution of the experiment by using a dense grid of templates generated with parameters
2788 close to the unknown signal (see figure 4.17). Because matched filter templates with
2789 similar parameters have signal shapes that are also similar, templates with incorrect
2790 parameters can have nearly identical matched filter scores as the correct template. Since
2791 only one sample of noise is included in a sample of real data, one cannot guarantee that
2792 the best matching template corresponds to the ground truth parameters of the signal.
2793 This introduces uncertainty into the signal parameter estimation that manifests as an
2794 energy broadening. Dense grids of matched filter templates allows one to quantify this

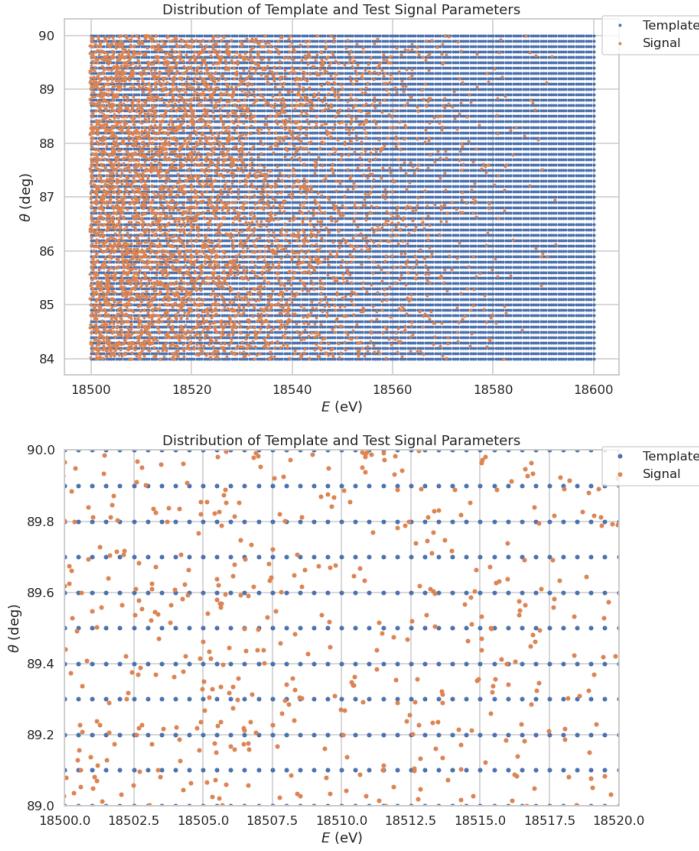


Figure 4.16. An example two-dimensional parameter distribution of a matched filter template bank and random test signals. θ refers to the pitch angle of the electron and E is the kinetic energy. The template bank forms a regular grid of in pitch angle and energy, whereas, the test signals are uniformly distributed in pitch angle and follow the tritium beta-decay kinetic energy distribution. This is why there are fewer test signals at higher energies. The need for high match across the full parameter space prevents one from reducing the density of templates in this low activity region. A zoomed in version of the template bank illustrates the relative density of templates and signals needed for match $> 90\%$.

broadening by analyzing the parameter space of templates with matched filter scores close to the ground truth. This approach is analogous to maximum likelihood estimation and is one key component of a complete sensitivity analysis for an antenna array CRES experiment.

A key parameter for describing the performance of a matched filter template bank at signal detection is match, which we define as the average ratio of the highest matched filter score for a random signal to the matched filter score for a perfectly matching

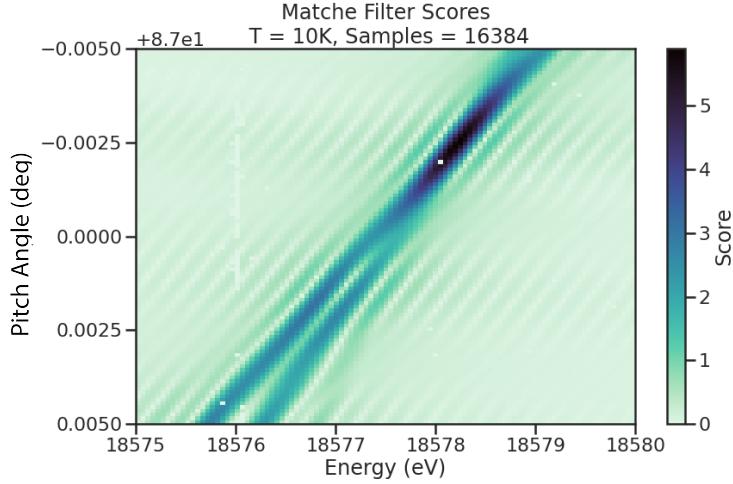


Figure 4.17. The matched filter scores of a dense grid of templates in pitch angle energy space. Dense template grids allow one to estimate the kinetic energy of the electron by identifying the best matching template. The uncertainty on this value is proportional to the space of templates that also match the test signal well. In the worst case matched filter templates can be completely degenerate where templates with different parameters match a signal with equal likelihood.

2802 template. In equation form this is

$$\text{Match} \equiv \Gamma = \frac{\mathcal{T}_{\text{best}}}{\mathcal{T}_{\text{ideal}}}, \quad (4.30)$$

2803 where $\mathcal{T}_{\text{best}}$ is the matched filter score of the best fitting template in the bank and $\mathcal{T}_{\text{ideal}}$ is
 2804 the hypothetical matched filter score one would measure if the signal perfectly matched
 2805 the template. Generally, one desires an average match as close to one as possible, however,
 2806 the average match value is an exponential function of the number of templates in the
 2807 template bank (see Figure 4.18). This behavior is observed for dense matched filter grids
 2808 like the one in Figure 4.17. A dense grid was used to calculate the average value of match
 2809 for different template bank sizes shown in Figure 4.18.

2810 The exponential relationship between match and template bank size is also evident
 2811 for template banks that cover a wide range of parameters, such as the template bank
 2812 visualized in Figure 4.16. Since no prior knowledge of the signal parameters is available,
 2813 one has no choice but to use a template bank that covers a large range of parameters for
 2814 signal detection. Achieving a high average match in this scenario can easily overwhelm
 2815 the available computational resources, so in practice only a limited number of templates
 2816 could be used at the detection stage. Therefore, accurately modeling the effects of match
 2817 is key to correct sensitivity calculations.

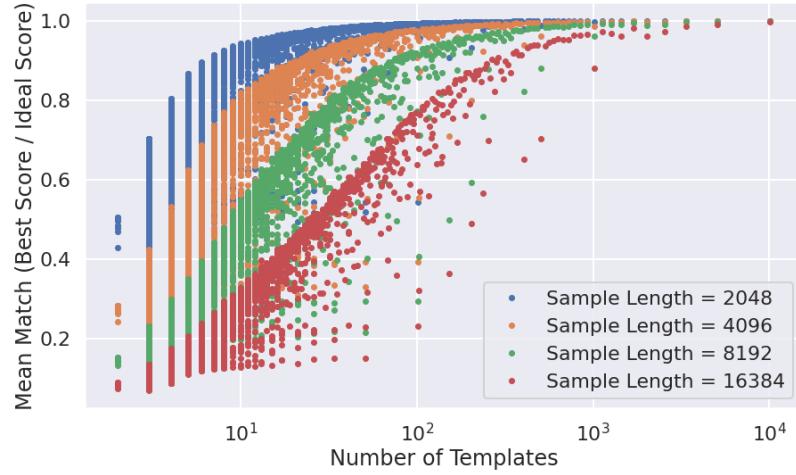


Figure 4.18. The mean match of the dense template grid shown in Figure 4.17 for different numbers of templates. Grids of different sizes were obtained by decimating a dense grid of templates and the average match for each grid was computed using the same set of randomly distributed test signals. Plotting the mean match against the size of the grid allows one to visualize the exponential relationship between match and template bank size. The noise in each curve is caused by sampling effects from the decimation algorithm. In general, longer templates are harder to than shorter templates.

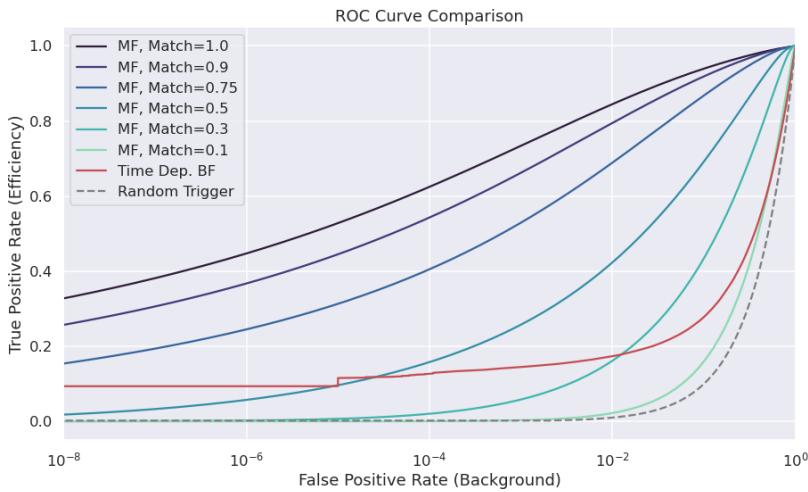


Figure 4.19. Matched filter template bank ROC curves as a function of mean match. One can see that for low match a matched filter is on average worse than the more straight forward beamforming detection approach.

2818 The effect of match on the detection efficiency of the matched filter template bank can
2819 be summarized using the ROC curve (see Figure 4.19). A single ROC curve is obtained
2820 by averaging over the PDFs that describe the detection probabilities of each individual
2821 template. The matched filter score for a template follows a Rician distribution with a
2822 mean value equal to the matched filter score multiplied by the match ratio between the
2823 template and signal. Therefore, the distribution that describes the average matched filter
2824 score when there is a signal in the data is obtained by averaging over the distributions
2825 for every template, whose expectation values are multiplied by the average match ratio.

2826 The distribution of the matched filter score when there is no signal in the data follows
2827 a Rayleigh distribution. Therefore, a trials penalty, which is the statistical penalty one
2828 pays for randomly checking many templates in order to avoid a random match between
2829 noise and a template, is included by computing the joint distribution of N_{template} Rayleigh
2830 distributions, where N_{template} is the size of the template bank. For more information on
2831 the calculation of matched filter template bank ROC curves please refer to Section 4.4.

2832 An alternative way to visualize the detection performance for each algorithm is to
2833 specify a minimum acceptable false positive rate at the trigger level. This is equivalent
2834 to specifying a minimum threshold on the value of the matched filter score or the size of
2835 a frequency peak for a beamforming power threshold trigger. One can then draw regions
2836 of detectable signals as a function of the electron's pitch angle and radial position (see
2837 Figure 4.20). A kinetic energy shift is equivalent to an overall frequency shift of the
2838 signal and should have no effect on the detection probability assuming sufficient density
2839 of matched filter templates in the energy dimension. A electron is declared "detectable"
2840 for the regions in Figure 4.20 if the signal has at least 50% probability of falling above the
2841 decision threshold of the respective classifier. One can see that the parameter space of
2842 detectable signals is greatly expanded beyond the beamforming power threshold trigger
2843 with a matched filter (MF) or deep neural network (DNN) (see Section 4.3.3). Plots such
2844 as Figure 4.20 are useful for visualization, but, since the handling of detection likelihood
2845 is not sufficiently rigorous, the detection probability boundaries are not well-suited to
2846 sensitivity estimates.

2847 **Optimized Matched Filtering Implementation for the FSCD**

2848 The biggest practical obstacle to the implementation of a matched filter template bank
2849 detection approach is oftentimes the computational cost associated with exhaustively
2850 calculating the matched filter scores of the template bank, and the FSCD is no exception
2851 in this regard. At a basic level computing a matched filter score requires the convolution

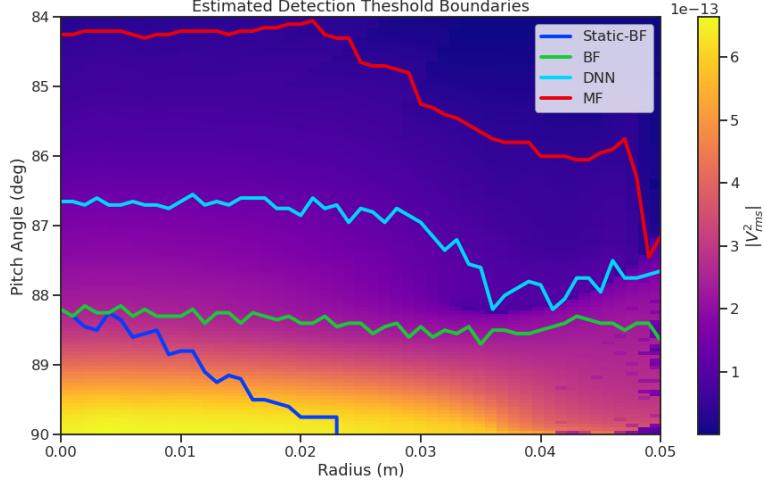


Figure 4.20. Boundaries of detectable electrons in pitch angle kinetic energy space for a series of different signal detection algorithms. A detectable signal is defined as a signal that is above a consistent decision with at least 50% probability. This non-rigorous treatment of detection probability is primarily useful for the visualization the relative increases in detection performance provided by the different algorithms. The static beamforming (Static-BF) algorithm is the digital beamforming algorithm introduced above without the ∇B -drift correction. The DNN algorithm refers to a convolutional neural network classifier trained to detect CRES signals (see Section 4.3.3).

of two vectors, which can be performed very efficiently by computers if the convolution theorem and fast Fourier transforms (FFT) are utilized. Furthermore, one can consider applying digital beamforming as a pre-processing step to reduce the dimensionality of the data before the matched filter is applied. In order to understand the relative gain in computational efficiency offered by these optimizations we analyze the total number of floating-point operations (FLOP) of several matched filter implementations in big O notation that utilize different combinations of optimizations.

A direct implementation of a matched filter as specified by Equation 4.28 involves the convolution of N_{ch} signals of length N_s with template signals of length N_t . As a uniform metric we shall compare the FLOP of the various matched filter implementations on a per-template basis, since each implementation scales linearly with the number of templates. The direct convolution approach to matched filtering costs

$$O(N_{\text{ch}}) \times O(N_s \times N_t) \quad (4.31)$$

FLOP per-template, whose cost is dominated by the $O(M \times N)$ convolution operation. The computational cost of the direct matched filter approach can be significantly

2866 reduced by exploiting the convolution theorem and FFT algorithms. If we restrict
 2867 ourselves to signals and templates that contain equal numbers of samples then the
 2868 convolution can be calculated by Fourier transforming both vectors, performing the
 2869 point-wise multiplication, and then performing the inverse Fourier transform to obtain
 2870 the convolution result. The FFT algorithm is able to compute the Fourier transform
 2871 utilizing only $O(N \log N)$ operations compared to $O(N^2)$ for a naive Fourier transform
 2872 implementation. This optimization results in a computational cost per-template of

$$O(N_{\text{ch}}) \times O(N_s \log N_s) \quad (4.32)$$

2873 A typical signal vector in the FSCD contains $O(10^4)$ samples in which case the FFT
 2874 reduces the computational cost of the matched filter by a factor of $O(10^3)$. This large
 2875 reduction in computational cost implies that a direct implementation of a matched filter
 2876 is completely infeasible in the FSCD due to resource constraints.

2877 Rather than relying solely on the matched filter it is tempting to consider using
 2878 digital beamforming as an initial step in the signal reconstruction for the purposes of
 2879 data reduction. The primary motivation is to reduce the dimensionality of the data by
 2880 a factor of N_{ch} by combining the array outputs coherently into a single channel. One
 2881 can view the beamforming operation as a partial matched filter, in the sense that the
 2882 matched filter convolution contains the beamforming phased summation along with a
 2883 prediction of the signal shape. By separating beamforming from the signal shape one
 2884 hopes to reduce the overall computational cost by effectively shrinking the number of
 2885 templates and reducing the number of operations required to check each one.

2886 The nature of this optimization requires that we account for the number of templates
 2887 used for pure matched filtering versus the hybrid approach. To first order, the total
 2888 number of templates at the trigger stage is a product of the number of guesses for each
 2889 of the electron's parameters

$$N_T = N_E \times N_\theta \times N_r \times N_\varphi, \quad (4.33)$$

2890 where N_E is the number of kinetic energies, N_θ is the number of pitch angles, N_r is the
 2891 number of starting radial positions, and N_φ is the number of starting azimuthal positions.
 2892 The starting axial position and cyclotron motion phase are not necessary to include in
 2893 the template bank since these parameters manifest themselves as the starting phase of
 2894 the signal, which is effectively marginalized when using a FFT to compute the matched
 2895 filter convolution. Therefore, the total number of operations required by a matched filter

2896 to detect a signal in a segment of array data is on the order of

$$O(N_T) \times O(N_{ch}) \times O(N_s \log N_s) \quad (4.34)$$

2897 With the hybrid approach we attempt to remove the spatial parameters from the
 2898 template bank by using beamforming to combine the array signals into a single channel.
 2899 Beamforming explicitly assumes a starting position, which allows us to only use matched
 2900 filter templates that span the two-dimensional space of kinetic energy and pitch angle.
 2901 The total computational cost of the hybrid method is directly proportional to the number
 2902 of beamforming positions. For the time-dependent beamforming defined in Section 4.3.1,
 2903 the number of beamforming positions is given by

$$N_{BF} = N_r \times N_\varphi \times N_{\omega_{\nabla B}}, \quad (4.35)$$

2904 where N_r and N_φ are the same spatial parameters encountered in the pure matched
 2905 filter template bank and $N_{\omega_{\nabla B}}$ is the number of ∇B -drift frequency assumptions. If a
 2906 unique drift frequency is used for each pitch angle then the hybrid approach is effectively
 2907 equivalent to a pure matched filter in the number of operations. The key efficiency gain
 2908 of the hybrid approach is to exploit the relatively small differences in $\omega_{\nabla B}$ for electrons
 2909 of different pitch angles by using only a small number of average drift frequencies.

2910 The total number of operations for the hybrid approach can be expressed as a sum of
 2911 the operations required by the beamforming and matched filtering steps,

$$O(N_{BF}) \times O(N_{ch} N_s) + O(N_{BF}) \times O(N_E N_\theta) \times O(N_s \log N_s). \quad (4.36)$$

2912 The first product in the sum is the number of operations required by beamforming,
 2913 which is simply the number of beamforming points times the computational cost of the
 2914 beamforming matrix multiplication, and the second product is the computational cost
 2915 of matched filtering the summed signal generated by each beamforming position. To
 2916 compare this to pure matched filtering we take the ratio of Equations 4.34 and 4.36 to
 2917 obtain

$$\Gamma_{BFMF} = \frac{O(N_{\omega_{\nabla B}})}{O(N_E N_\theta) \times O(\log N_s)} + \frac{O(N_{\omega_{\nabla B}})}{O(N_{ch})}. \quad (4.37)$$

2918 This expression can be simplified by observing that $O(N_E N_\theta) \times O(\log N_s) \gg O(N_{ch})$,

2919 which means that the ratio of computational cost for the two methods can be reduced to

$$\Gamma_{\text{BFMF}} \approx \frac{O(N_{\omega_{\nabla B}})}{O(N_{\text{ch}})}. \quad (4.38)$$

2920 If we limit ourselves to a number of estimated drift frequencies of $O(1)$ then we see that
2921 the estimated computational cost reduction of the hybrid approach is of $O(N_{\text{ch}})$. This is
2922 quite a large reduction considering that the FSCD antenna array contains sixty antennas
2923 in the baseline design.

2924 The main drawback of the hybrid approach is that the limited number of allowed
2925 drift frequency guesses can lead to detection efficiency loss due to phase mismatch. The
2926 degree of phase error from an incorrect drift frequency is proportional to the length of
2927 the array data vector used by the signal detection algorithm. For signals with lengths
2928 equal to the baseline FSCD Fourier analysis window of 8192 samples, typical phase errors
2929 from using an average versus the exact ∇B -drift frequency are on the order of a few
2930 percent in terms of the signal energy. This has a relatively small impact on the overall
2931 detection efficiency, however, future experiments with antenna array CRES will want to
2932 balance optimizations such as these during the design phase to keep experiment costs to
2933 a minimum while still achieving scientific goals.

2934 Kinetic Energy and Pitch Angle Degeneracy

2935 More accurate modeling of a matched filter requires that we consider the effects of
2936 mismatched signals and template, since this more accurately reflects the real-world usage
2937 of a matched filter where many incorrect templates are convolved with the data until the
2938 matching template is found. One way to study this is to use the grid of simulated signals
2939 to compute the matched filter scores between mismatched signals and templates and
2940 evaluate the matched filter scores under this scenario. What one finds when performing
2941 this analysis is that templates for kinetic energies and pitch angles that do not match
2942 the underlying signal can have matched filter scores that are indistinguishable from the
2943 matched filter score of the correct template (see Figure 4.21 and Figure 4.21).

2944 This degeneracy in matched filter score is the result of correlations between the kinetic
2945 energy of the electron and the pitch angle caused by changes in the average magnetic field
2946 experienced by an electron for different pitch angles. While in principle helpful for the
2947 purposes of signal detection these correlations are unacceptable since they greatly reduce
2948 the energy resolution of the experiment by causing electrons with specific kinetic energy
2949 to templates across a wide range of energies. It is important to emphasize that this

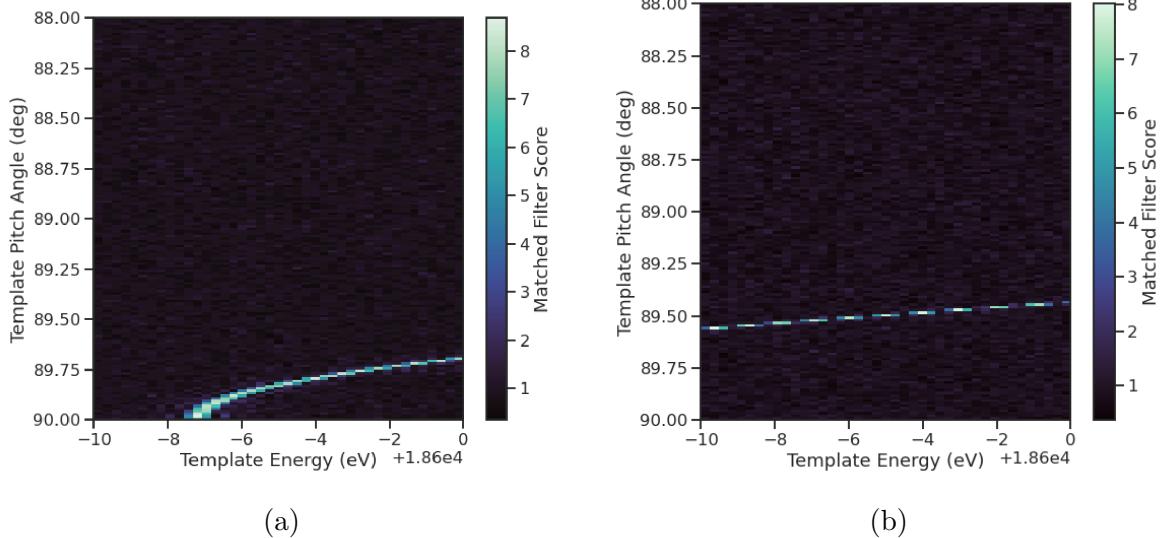


Figure 4.21. Two example illustrations of the correlation between kinetic energy and pitch angle imparted by the shape of the FSCD magnetic trap. The correlations manifest themselves as degeneracies in the matched filter score where multiple matched filter templates have the same matched filter for a particular signal. These degeneracies are a sign that the magnetic trap must be redesigned in order to break the correlation between pitch angle and kinetic energy.

2950 degeneracy cannot be fixed by implementing a different signal reconstruction algorithm.
 2951 As revealed by the matched filter scores the shapes of the signals for different parameters
 2952 are identical. Resolving this degeneracy between pitch angle and energy requires the
 2953 design of a new magnetic trap with steeper walls so that the average magnetic field
 2954 experienced by an electron is less dependent on pitch angle.

2955 **4.3.3 Machine Learning**

2956 Machine learning is a vast and rapidly developing field of research [73]. In this Section
 2957 we shall attempt to provided a brief introduction to some of the concepts and techniques
 2958 of machine learning that were applied to CRES signal detection rather than attempt a
 2959 comprehensive overview.

2960 **Introduction to Machine Learning**

2961 Digitization of the FSCD antenna array generates large amounts of data that must be
 2962 rapidly processed to enable real-time signal detection and reconstruction. While digital
 2963 beamforming combined with a power threshold is relatively computationally inexpensive,
 2964 it is relatively ineffective at detecting CRES signal with small pitch angles, since it relies

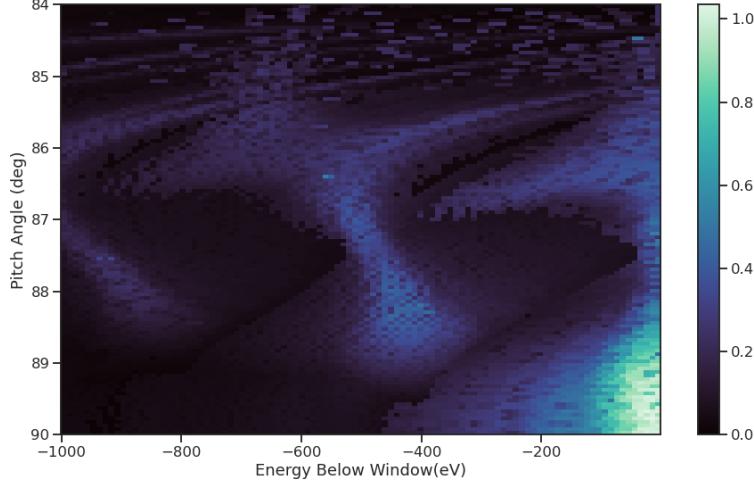


Figure 4.22. A visualization of the correlation between energy and pitch angle in the FSCD magnetic trap. The image is formed by computing the match of the best template from a grid consisting of pitch angles from 84 to 90 degrees in steps of 0.05 degrees, kinetic energies from 17574 to 18574 eV, located at 2 cm from the central axis, and simulated for a length of three FSCD time-slices. The signals used to compute the best matching template consisted of a grid from 84 to 90 degrees in steps of 0.05 degrees, kinetic energies from 18550 to 18575 eV in steps of 0.25 eV, located 2 cm from the central axis, and simulated for three FSCD time-slices. The colored regions of the plot show how well signals with lower energy can match those of higher energy for the FSCD magnetic trap, which is proportional to the achievable energy resolution of the FSCD design.

on a visible frequency peak above the noise. On the other hand, a matched filter is able to detect signals with a significantly larger range of parameters, however, the exhaustive search of matched filter templates can be computationally expensive. Machine learning based triggering algorithms have been used successfully in many different high-energy physics experiments [74] and recent developments have shown success in the detection of gravitational wave signals using machine learning techniques [75, 76] in place of the more traditional matched filtering method. This motivates the exploration of machine learning as a potential CRES signal detection algorithm.

There are several different approaches to machine learning, but the one most important to our discussion here is the supervised learning approach. In supervised machine learning one uses a differentiable model or function that is designed to map the input data to the appropriate label [73]. The data is represented as a multidimensional matrix of floating point values such as an image or a time-series, and the label is generally a class name such as signal or noise for classification problems or a continuous value like kinetic energy in the case of regression problems.

2980 In supervised learning the model is trained to map from the data to the correct label
2981 by evaluating the output of the model using a training dataset consisting of a set of
2982 paired data and labels. To evaluate the difference between the model output and the
2983 correct label a loss function is used to quantify the error between the model prediction
2984 and the ground truth. For example, a common loss function in regression problems is the
2985 squared error loss function, which quantifies error using the squared difference between
2986 the model output and label.

2987 Using the outputs of the loss function the next step in supervised learning is to
2988 compute the gradient of error with respect to the model parameters in a process called
2989 backpropagation. Using the model parameter gradients the last step in the supervised
2990 learning process is to perform an update of the parameter values in order to minimize
2991 the error in the model predictions across the whole dataset. This loop is performed many
2992 times while randomly shuffling the dataset until the error converges to a minimum value
2993 at which point the training procedure has finished. It is standard practice to monitor
2994 the training procedure by evaluating the performance of the model using a separate
2995 validation dataset that matches the statistical distribution of the training data and to
2996 check the performance of the model after training using yet another dataset called the
2997 test dataset. These practices help to guard against overtraining which is a concern for
2998 models with many parameters.

2999 Convolutional Neural Networks

3000 A popular class of machine learning models are neural networks. A neural network is
3001 essentially a function composed of a series of linear operations called layers which take a
3002 piece of data typically represented as a matrix, multiplies the elements of the data by a
3003 weight, and then sums these products to produce an output matrix. Neural networks
3004 composed of purely linear operations are unable to model complex non-linear behavior,
3005 therefore, non-linear activation functions are applied to the outputs of each of the layers
3006 to increase the ability of the neural network to model complex relationships between the
3007 data.

3008 Neural networks are typically composed of at least three layers, but with the present
3009 capabilities of computer hardware they more often contain many more than this. The
3010 first layer in a neural network is called the input layer, because it takes the data objects
3011 as input, and the last layer in a neural network is known as the output layer. The
3012 output layer is trained by machine learning to map the data to a desired output using
3013 the supervised learning procedure described in Section 4.3.3. In between the input and

3014 the output layer are typically several hidden layers that receive inputs from and transmit
3015 outputs to other layers in the neural network model. The term deep neural network
3016 (DNN) refers to those neural networks that have at least one hidden layer, which have
3017 proven to be extremely powerful tools for pattern recognition and function approximation.

3018 An important type of DNN are convolutional neural networks (CNN) that typically
3019 contain several layers which perform a convolution of the input with a set of filters. These
3020 convolution operations are typically accompanied by layers that attempt to down-sample
3021 the data along with the standard neural network activation functions. A standard CNN
3022 is composed of several convolutional layers at the beginning of the network and ends
3023 with a series of fully-connected neural network layers at the output. Intuitively, one
3024 can imagine that the convolutional layers are extracting features from the data that
3025 fully-connected layers use to perform the classification or regression task.

3026 **Deep Filtering for Signal Detection in the FSCD**

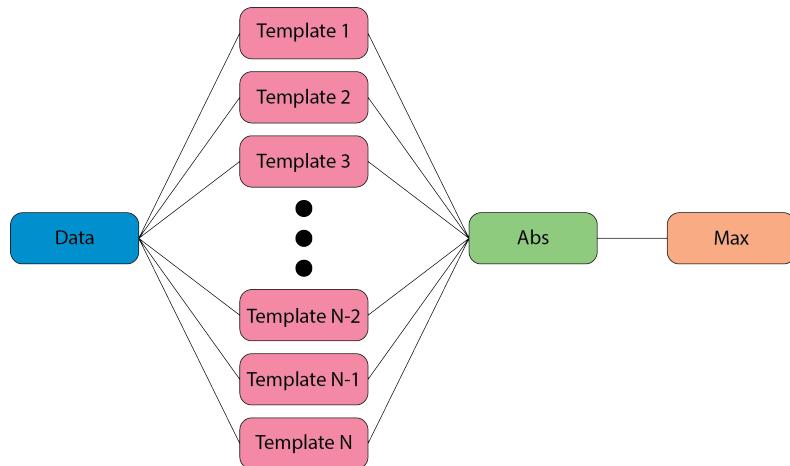


Figure 4.23. A representation of a matched filter template bank as a convolutional neural network. The network has a single layer composed of the templates, which act as convolutional filters. The activation of the neural network is an absolute value followed by a max operator.

3027 CNNs have been extremely influential in the field of computer vision, particularly tasks
3028 such as image segmentation and classification, but have also been applied in numerous
3029 experimental physics contexts. Given the particular challenge posed by signal detection
3030 and reconstruction in the FSCD we are interested in exploring the potential of machine
3031 learning as an effective algorithm for real-time signal detection, since this application
3032 requires both high efficiency and fast evaluation.

3033 In the machine learning paradigm signal detection is equivalent to a binary classifi-

3034 cation problem between the signal and noise data classes, and my investigation focuses
3035 specifically on the application of CNNs to signal detection in the FSCD, which is moti-
3036 vated by relatively recent demonstrations of CNNs achieving classification accuracies for
3037 gravitational wave time-series signals comparable to a matched filter template bank. In
3038 this framework it is possible to interpret the matched filter as a type of CNN composed
3039 of a single convolutional layer with the templates making up the layer filters (see Figure
3040 4.23). Since this neural network has no hidden layers, it is not a DNN like we have
3041 been discussing so far, but we can attempt to construct a proper CNN that attempts to
3042 reproduce the classification performance of the matched filter network.

3043 The name deep filtering refers to this scheme of replacing a matched filter template
3044 bank with a DNN. The reason why one might want to do this is that it may be possible to
3045 exploit redundancies and correlations between templates that may allow one to perform
3046 signal detection with similar accuracy but with fewer computations, which is important
3047 for real-time detection scenarios like the FSCD experiment. In Section 4.4 we perform a
3048 detailed comparison of the signal detection performance of a CNN to beamforming and a
3049 matched filter template bank.

3050 Deep filtering is conceptually a simple technique. Similar to a matched filter template
3051 bank a large number of simulated CREs signals are generated and used to train a model
3052 to distinguish between signal and noise data (see Figure 4.24). In order to reduce the
3053 dimensionality of the input FSCD data a digital beamforming summation is applied
3054 to the raw time-series data generated by Locust to compress the 60-channel data to a
3055 single time-series. CREs signal have a sparse frequency representation and experiments
3056 training CNN's on time-series and frequency series data found that models trained on
3057 frequency spectrum data performed significantly better, therefore, an FFT is applied to
3058 the summed time-series before being normalized and fed to the classification model.

3059 The data used to train the model consists of an equal proportion of signal and noise
3060 frequency spectra. Unique samples of WGN are generated and added to the signals during
3061 training time to avoid having to pre-generate and store large samples of noise data. The
3062 binary cross-entropy loss function combined with the ADAM optimizer proved effective
3063 at training the models to classify CREs data. A simple hyperparameter optimization
3064 was performed by manually tuning model, loss function, and optimizer parameters. The
3065 model and training loops were implemented in python using the PyTorch deep learning
3066 framework. Standard machine learning best practices were followed when training the
3067 models, such as overtraining monitoring using a validation dataset. Models were trained
3068 until the training loss and accuracy converged and then evaluated using a separate test

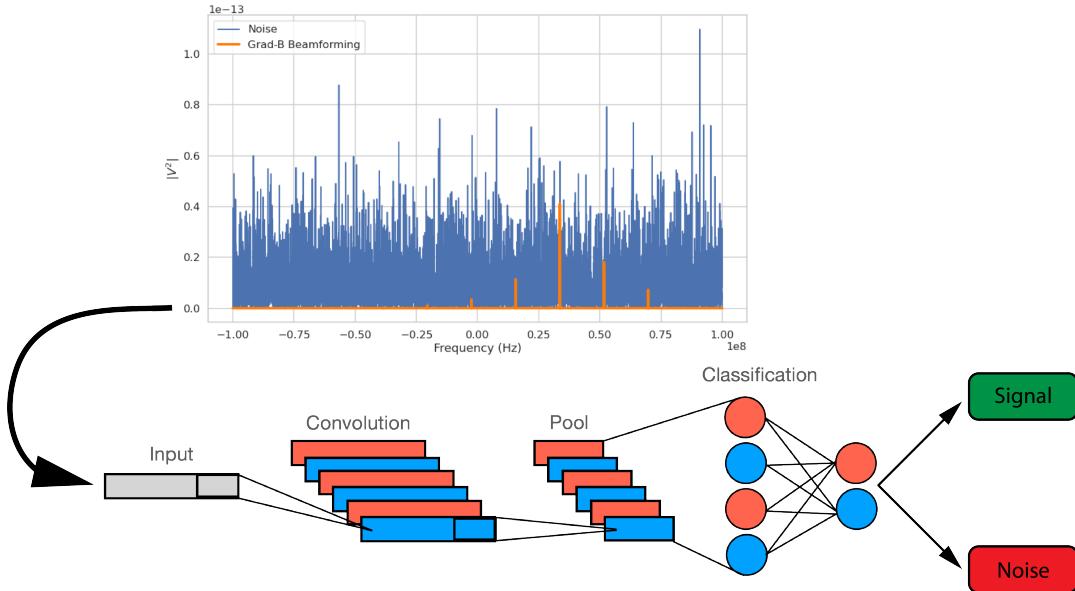


Figure 4.24. A graphical depiction of CRES signal detection using a CNN. A noisy segment of data is converted to a frequency series using digital beamforming and a FFT. The complex-valued frequency series is input into a trained CNN model that classifies the data as signal or noise using a decision threshold on the CNN output.

3069 data set.

3070 The classification results of the test dataset are used to quantify the relationship
 3071 between the true positive rate and the false positive rate for the model. The true positive
 3072 rate is analogous to detection efficiency and the false positive rate is a potential source of
 3073 background in the detector. One can limit the rate of false positives using a sufficiently
 3074 high threshold on the model output at the cost of a lower detection efficiency (see Figure
 3075 4.25 and Figure 4.26). As expected, the performance of the model at signal classification
 3076 is negatively effected the noise power, which is quantified by the noise temperature.

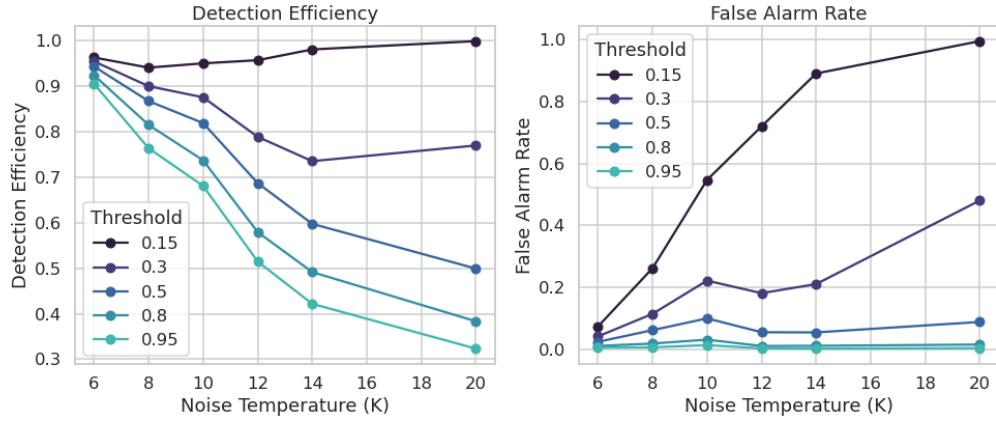


Figure 4.25. The detection efficiency and false alarm rate (false positive rate) as a function of the decision threshold for different values of the noise temperature. The model is trained to output a value close to one for data that contains a signal and outputs a value near zero when the data contains only noise. One sees that a lower decision threshold will have a high detection efficiency at the cost of a high rate of false alarms.

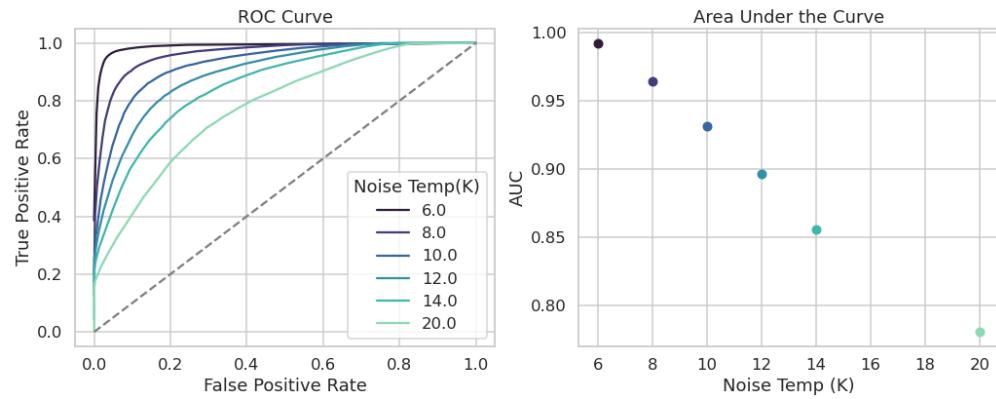


Figure 4.26. ROC curves for a CNN model classifying CRES signals. One can see that the area under the curve, which is a figure of merit that describes the performance of the classifier, is roughly linearly dependent with the noise temperature.

3077 **4.4 Analysis of Signal Detection Algorithms for the An-** 3078 **tenna Array Demonstrator**

3079 This section contains an early version of the manuscript for the triggering paper prepared
3080 for publication in JINST. In it I present a relatively detailed analysis of the signal
3081 detection performance of the three signal detection approaches discussed so far using a
3082 population of simulated CRES signals generated with Locust. The focus of the paper is
3083 on the performance of the signal detection algorithms for pitch angles below 88.5° where
3084 the beamforming power threshold begins to fail.

3085 **4.4.1 Introduction**

3086 Cyclotron Radiation Emission Spectroscopy (CRES) is a technique for measuring the
3087 kinetic energies of charged particles by observing the frequency of the cyclotron radiation
3088 that is emitted as they travel through a magnetic field [39]. The Project 8 Collaboration
3089 is developing the CRES technique as a next-generation approach to tritium beta-decay
3090 endpoint spectroscopy for neutrino mass measurement. Recently, Project 8 has used
3091 CRES to perform the first ever tritium beta-decay energy spectrum and neutrino mass
3092 measurement [41, 42].

3093 Previous CRES measurements have utilized relatively small volumes of gas that are
3094 directly integrated with a waveguide transmission line, which transmits the cyclotron
3095 radiation emitted by the trapped electrons to a cryogenic amplifier. While this technology
3096 has had demonstrable success, it is not a feasible option for scaling up to significantly
3097 larger measurement volumes. In particular, the goal of the Project 8 Collaboration
3098 is to use CRES combined with atomic tritium to measure the neutrino mass with a
3099 40 meV sensitivity. Achieving this sensitivity goal will require a multi-cubic-meter scale
3100 measurement volume in order to obtain the required event statistics in the tritium
3101 beta-spectrum endpoint region; hence, there is a need for new techniques to enable large
3102 volume CRES measurements for future experiments.

3103 One approach is to surround a large volume with an array of antennas that together
3104 collect the cyclotron radiation emitted by trapped electrons [40, 77]. A promising
3105 array design is an inward-facing uniform cylindrical array that surrounds the tritium
3106 containment volume. Increasing the size of the antenna array, by adding additional
3107 rings of antennas along vertical axis, allows one to grow the experimental volume until a
3108 sufficient amount of tritium gas can be observed by the array. A challenging aspect of

3109 this approach is that the total radiated power emitted by an electron near the tritium
3110 spectrum endpoint is on the order of 1 fW or less, which is then distributed between
3111 all the antennas in the array. Consequently, detecting the presence of a CRES signal
3112 and determining the electron's kinetic energy requires reconstructing the entire antenna
3113 array output over the course of the CRES event, posing a significant data acquisition
3114 and signal reconstruction challenge.

3115 Project 8 has developed a triggering system to enable real-time identification of CRES
3116 events using an antenna array [78]. Previous measurements with the CRES technique
3117 have utilized a threshold on the frequency spectrum formed from a segment of CRES
3118 time-series data. This algorithm relies on the detection of a frequency peak above the
3119 thermal noise background, which limits the kinematic parameter space of detectable
3120 electrons. Due to the limitations of this power threshold, Project 8 has been investigating
3121 alternative signal identification approaches, including both matched filtering and machine
3122 learning based classifiers, to improve the detection efficiency of the experiment. In
3123 order to evaluate the relative gains in detection efficiency that come from utilizing
3124 these alternative algorithms, we develop analytical models for the power threshold and
3125 matched filter signal classifier performance applicable to an antenna array based CRES
3126 detector. In addition, we implement and test a basic convolutional neural network (CNN)
3127 as a first step towards the development of neural-network based classifiers for CRES
3128 measurements. These results allow us to compare the estimated detection efficiencies of
3129 each of these methods, which we weigh against the associated computational costs for
3130 real-time applications.

3131 The outline of this paper is as follows. In Section 4.4.2 we give an overview of a
3132 prototypical antenna array CRES experiment, and describe the major steps involved
3133 in the proposed approach to real-time signal identification. In Section 4.4.3 we develop
3134 models for the power threshold and matched filter algorithms, and introduce the machine
3135 learning approach and CNN architecture. In Section 4.4.4 we describe our process for
3136 generating simulated CRES signal data and the details of training the CNN. Finally,
3137 in Section 4.4.5 we perform a comparison of the signal classification accuracy of the
3138 three approaches and discuss the relevant trade-offs in terms of detection efficiency and
3139 computational cost.

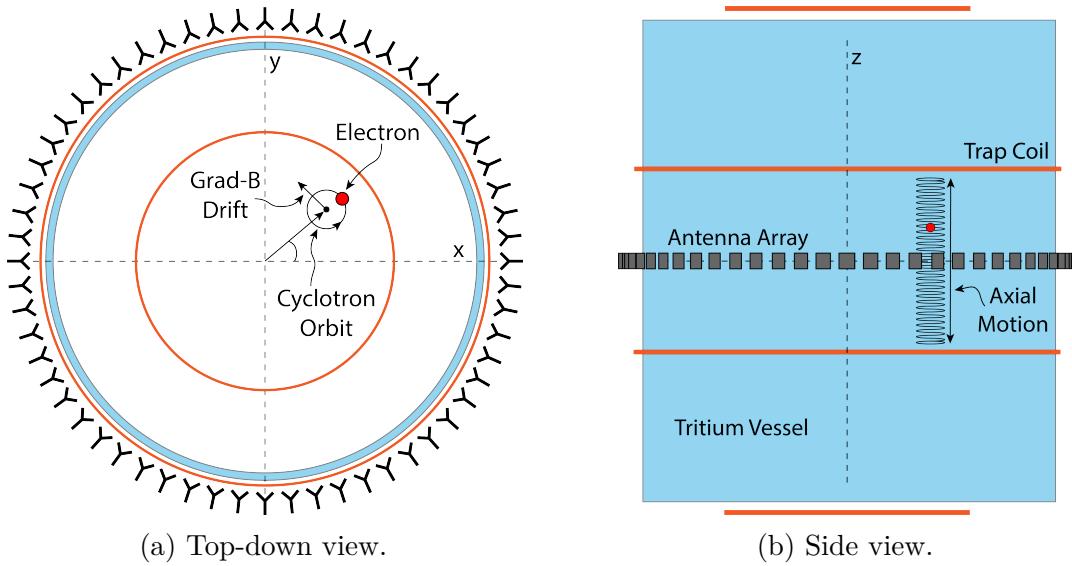


Figure 4.27. An illustration of the conceptual design for an antenna array CRES tritium beta-decay spectrum measurement. The antenna array geometry consists of a 20 cm interior diameter with 60 independent antenna channels arranged evenly around the circumference. The nominal antenna design is sensitive to radiation in the frequency range of 25-26 GHz, which corresponds to the cyclotron frequency of electrons emitted near the tritium beta-spectrum endpoint in a 1 T magnetic field. The array is located at the center of the magnetic trap produced by a set of current-carrying coils. The nominal magnetic trap design is capable of trapping electrons up to 5 cm away from the central axis of the array and traps electrons within an approximately 6 cm long axial region centered on the antenna array.

3140 4.4.2 Signal Detection with Antenna Array CRES

3141 4.4.2.1 Antenna Array and DAQ System

3142 In order to explore the potential of antenna array CRES for neutrino mass measurement,
 3143 the Project 8 Collaboration has developed a conceptual design for a prototype antenna
 3144 array CRES experiment [40, 77], called the Free-space CRES Demonstrator or FSCD,
 3145 which could be used as a demonstration of the antenna array measurement technique
 3146 (see Figure 4.27). The FSCD utilizes a single ring of antennas, which is the simplest
 3147 form of a uniform cylindrical array configuration, to surround a radio-frequency (RF)
 3148 transparent tritium gas vessel. A prototype version of this antenna array has been built
 3149 and tested by the Project 8 collaboration to validate simulations of the array radiation
 3150 pattern and beamforming algorithms [43]. In the FSCD the antenna array is positioned
 3151 at the center of the magnetic trap formed by a set of electro-magnetic coils that are
 3152 designed to produce a magnetic trap with flat central region and steep walls both radially
 3153 and axially.

3154 When a beta-decay electron is trapped its motion consists of three primary components.
 3155 The component with the highest frequency is the cyclotron orbit whose frequency is
 3156 determined by the size of the background magnetic field. The FSCD design assumes
 3157 a background magnetic field value of approximately 0.96 T, which results in cyclotron
 3158 frequencies for electrons with kinetic energies near the tritium beta-spectrum endpoint
 3159 from 25 to 26 GHz. The component with the next highest frequency is the axial oscillation
 3160 experienced by electrons with pitch angles of less than 90° [60]. The flat region of the
 3161 FSCD magnetic trap extends approximately 3 cm above and below the antenna array
 3162 plane causing electrons to move back and forth as they are reflected from the trap walls.
 3163 Typical oscillation frequencies are on the order of 10's of MHz, which results in an
 3164 oscillation period that is $O(10^3)$ smaller than the duration of a typical CRES event.
 3165 Therefore, when reconstructing CRES events we treat the electron as occupying only an
 3166 average axial position at the center of the magnetic trap, since we are not able to resolve
 3167 the axial position as a function of time. The component of motion with the smallest
 3168 frequency is ∇B -drift caused by radial field gradients in the trap, producing an orbit of
 3169 the electron around the central axis of the trap with a frequency on the order of a few
 3170 kHz, dependent on the pitch angle and the radial position of the electron.

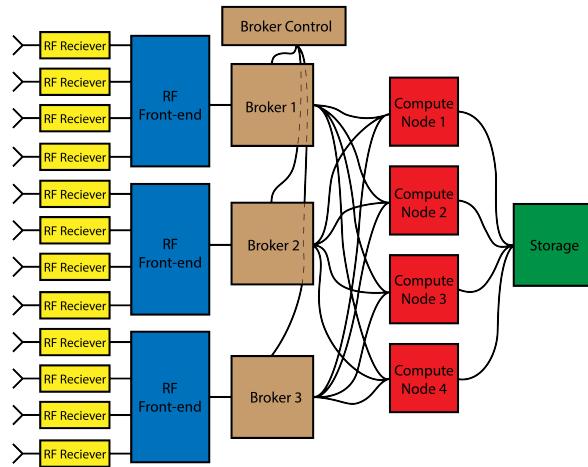


Figure 4.28. A high-level diagram of the DAQ system archtecture envisioned for the FSCD.

3171 The data acquisition (DAQ) system digitizes the signals from the antenna array and
 3172 combines thee data streams into a time-ordered matrix of array snapshots that can be
 3173 used by the reconstruction algorithms. The FSCD DAQ system design [78] is divided into
 3174 three layers 4.28. The first layer is the RF front-end, which includes the antenna array,
 3175 the RF receiver boards, and the digitization electronics. The receiver board contains an
 3176 amplifier, RF mixer, and bandpass filter to enable down-conversion, and is followed by

3177 the digitization electronics that sample the CRES signals at 200 MHz. In order to achieve
 3178 an adequate signal-to-noise ratio to detect CRES events, the DAQ system for the antenna
 3179 array demonstrator must have a total system noise temperature of ≈ 10 K, which we
 3180 can achieve by using low-noise amplifiers and operating at cryogenic temperatures. After
 3181 digitization, the array data must be reorganized from individual data streams sorted
 3182 by channel into array snapshots sorted by time. In order to solve this data transfer
 3183 and networking problem the second layer of the DAQ system consists of a set of broker
 3184 computer nodes that reorganize the array data into time-ordered chunks. This approach
 3185 allows us to accommodate different data transfer requirements by scaling the number of
 3186 broker nodes in this layer accordingly. Next, the broker layer distributes these chunks
 3187 of array data to the final layer of the DAQ system, which consists of a set of identical
 3188 reconstruction nodes that perform the calculations required for CRES reconstruction.
 3189 Similar to the broker layer, the number of reconstruction nodes can be increased or
 3190 decreased depending on the amount of computer power required for real-time CRES
 3191 reconstruction.

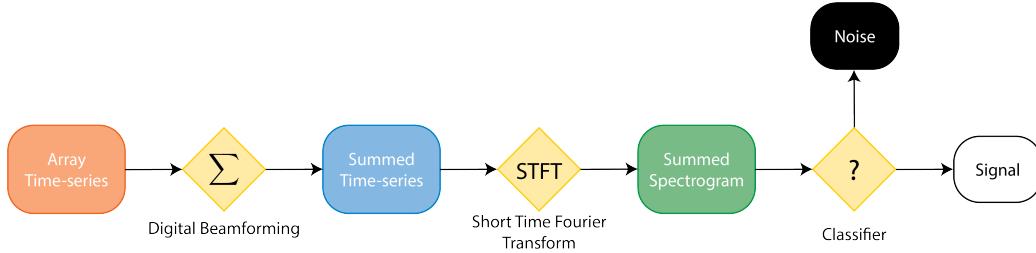


Figure 4.29. A block diagram illustration of the real-time triggering algorithm proposed for antenna array CRES reconstruction.

3192 The design of the FSCD DAQ system is intended to enable a significant portion of
 3193 the CRES event reconstruction to occur in real-time. The motivation for this comes from
 3194 the fact that the FSCD antenna array generates approximately 1 exabyte of raw data
 3195 per year of operation. Therefore, in order to reduce the data-storage requirements, it is
 3196 ideal to perform at least some of the CRES event reconstruction in real-time so that it
 3197 is possible to save a reduced form of the data for offline analysis. The first step of the
 3198 real-time reconstruction would be a real-time signal detection algorithm, which is the
 3199 focus of this paper. Our approach consists of three main operations performed on the
 3200 time-series data blocks including digital beamforming, a short time Fourier transform
 3201 (STFT), and a binary classification algorithm to distinguish between signal and noise
 3202 data (see Figure 4.29).

3203 4.4.2.2 Real-time Signal Detection

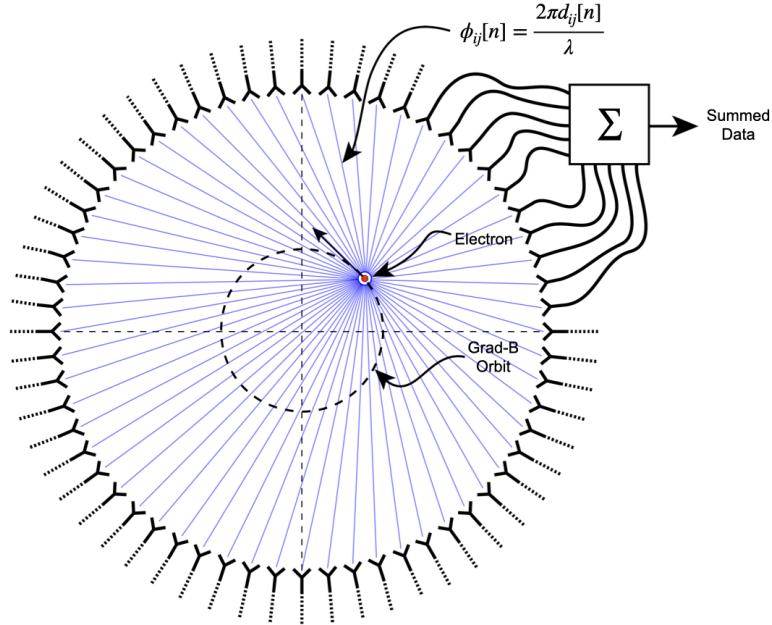


Figure 4.30. An illustration of the digital beamforming procedure. The blue lines indicate the various distances from the beamforming position to the antennas. In the situation depicted the actual position of the electron matches the beamforming position, so we should expect constructive interference when the phase shifted signals are summed. To prevent the electron's ∇B -motion from moving the electron off of the beamforming position, the beamforming phase include a time-dependence to follow the trajectory of the electron in the magnetic trap.

3204 The first step in the real-time detection algorithm is digital beamforming, which is
 3205 a phased summation of the signals received by individual antennas in the array (see
 3206 Figure 5.21). The phase shifts correspond to the path length differences between a spatial
 3207 position and each individual antenna such that, when there is an electron located at
 3208 the beamforming position, all the signals received by the array constructively interfere.
 3209 Since we do not know ahead of time where an electron will be produced in the detector,
 3210 we define a grid of beamforming positions that cover the entire region where electrons
 3211 can be trapped and perform a phased summation for each of these points for every
 3212 time-step in the array data block. As we saw in Section 4.4.2.1, the axial oscillation
 3213 of the electrons prevents us from resolving it's position along the Z-axis of the trap,
 3214 therefore our beamforming grid need only cover the possible positions of the electron in
 3215 the two-dimensional plane defined by the antenna array.

3216 The equation defining digital beamforming can be expressed as

$$\mathbf{y}[n] = \Phi^T[n]\mathbf{x}[n], \quad (4.39)$$

3217 where $\mathbf{x}[n]$ is array snapshot vector at the sampled time n , $\Phi[n]$ is the matrix of
 3218 beamforming phase shifts, and $\mathbf{y}[n]$ is summed output vector that contains the voltages
 3219 for each of the summed channels that correspond to a particular beamforming position.
 3220 The elements of the beamforming phase shift matrix can be expressed as a weighted
 3221 complex exponential

$$\Phi_{ij}[n] = A_{ij}[n] \exp(2\pi i \phi_{ij}[n]), \quad (4.40)$$

3222 where the indices i and j label the beamforming and antenna positions respectively. The
 3223 weight A_{ij} accounts for the relative power increase for antennas that are closer to the
 3224 position of the electron, and ϕ_{ij} is the total beamforming phase shift for the j -th antenna
 3225 at the i -th beamforming position.

3226 The beamforming phase shift is a sum of two terms

$$\phi_{ij}[n] = \frac{2\pi d_{ij}[n]}{\lambda} + \theta_{ij}[n], \quad (4.41)$$

3227 where the first term is the phase shift originating from the path length difference ($d_{ij}[n]$)
 3228 between the beamforming and antenna positions, which are represented by the vectors
 3229 (r_j, θ_j) and $(r_i, \theta_i[n])$, and the second term is the angular separation ($\theta_{ij}[n]$) of the two
 3230 positions. The angular separation enters into the beamforming phase due to an effect
 3231 caused by the circular orbit of the electron that produces radiation whose phase is linearly
 3232 dependent on the relative azimuthal position of the antenna [79, 80]. The time-dependence
 3233 of the beamforming phases is intended to correct for the effects of ∇B -drift, which cause
 3234 the guiding centers of electrons to orbit the center of the magnetic trap. By including a
 3235 linear time-dependence in the azimuthal beamforming position,

$$\theta_i[n] = \omega_{\nabla B} t[n] + \theta_{i,0}, \quad (4.42)$$

3236 where $\omega_{\nabla B}$ is the azimuthal grad-B drift frequency, $t[n]$ is the time vector and, $\theta_{i,0}$ is the
 3237 starting azimuthal position, we can configure the beamforming phases to effectively track
 3238 the XY-position of the guiding center over the event duration. Predicting accurate values
 3239 of $\omega_{\nabla B}$ for a specific trap and set of kinematic parameters will be done by simulations,
 3240 which are performed using the Kassiopeia software package [59] by Project 8.

3241 After digital beamforming, we apply a short-time Fourier transform (STFT) to the

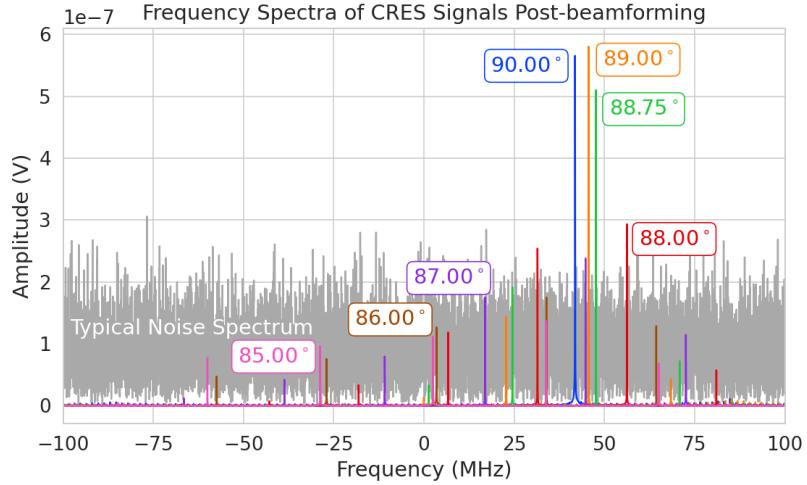


Figure 4.31. Frequency spectra of simulated CRES signals post-beamforming. The signal of a 90° electron consists of a single frequency component that is easy to detect with a power threshold on the frequency spectrum. This power threshold is still effective for signals with relatively large pitch angles such as 89.0° and 88.75° , which are composed of a main carrier and a few small sidebands. Signals with smaller pitch angles, below about 88.5° , tend to be dominated by sidebands such that no single frequency component can be reliably distinguished from the noise with a power threshold.

3242 summed time-series to obtain the frequency spectrum representation of the signals (see
 3243 Figure 4.31). From the detection perspective, the frequency representation of the CRES
 3244 data is advantageous compared to the time domain, because the frequency spectra of
 3245 CRES signals are well-approximated by a frequency and amplitude modulated sinusoidal
 3246 whose carrier frequency increases as a linear chirp. The modulation is caused by the axial
 3247 oscillation of the electron in the magnetic trap and produce frequency spectra that are
 3248 well-described by a small number of frequency components. The linear chirp is caused
 3249 by the energy loss due to cyclotron radiation, which results in a relatively slow increase
 3250 in the frequency components of the CRES signal over time. During the standard Fourier
 3251 analysis window for the FSCD of $40.96 \mu\text{sec}$, we expect a typical CRES signal to increase
 3252 in frequency by approximately 15 kHz, which is smaller than the frequency bin width
 3253 given the 200 MHz sample rate. Therefore when considering a single frequency spectrum
 3254 it is justifiable to neglect the effects of the linear frequency chirp.

3255 In the cases where the electron's pitch angle is $\gtrsim 88.5^\circ$, the majority of the signal
 3256 power is contained in a single frequency component, with the remaining signal power
 3257 contained in a small number of sidebands proportional to the electron's axial modulation
 3258 (see Figure 4.31). In these cases detection is relatively straight-forward by implementing

3259 a power threshold on the STFT, since the amplitude of the main signal peak is distinct
 3260 from the thermal noise spectrum. However, as the pitch angle of the electron is decreased
 3261 below 88.5° , the modulation index of the signal increases causing the maximum amplitude
 3262 of the frequency spectrum to be comparable to typical noise fluctuations. At this point,
 3263 the power threshold trigger is no longer able to distinguish between signal and noise
 3264 leading to a reduction in detection efficiency. The neutrino mass sensitivity of the FSCD
 3265 is directly linked to the overall detection efficiency. And, because the distribution of
 3266 electron pitch angles is effectively uniformly distributed across the range of pitch angles
 3267 that can be trapped, the overall detection efficiency is directly influenced by the range of
 3268 pitch angles that have detectable signals. Therefore, utilizing a signal detection algorithm
 3269 that can more effectively identify signals with pitch angles less than 88.5° will improve
 3270 both detection efficiency and ultimately the neutrino mass sensitivity of the FSCD and
 3271 other CRES experiments.

3272 Modeling the detection performance of alternative signal detection algorithms for
 3273 the FSCD requires that we pose the signal detection problem in a consistent manner.
 3274 The approach we take is to perform a binary hypothesis test on the frequency spectra
 3275 generated by the STFT. Mathematically, this is expressed as,

$$\mathcal{H}_0 : y[n] = \nu[n] \quad (4.43)$$

$$\mathcal{H}_1 : y[n] = x[n] + \nu[n]. \quad (4.44)$$

3276 Where under hypothesis \mathcal{H}_0 , the vector representing the frequency spectrum ($y[n]$) is
 3277 composed of pure white Gaussian noise (WGN) represented by $\nu[n]$, and under hypothesis
 3278 \mathcal{H}_1 the frequency spectrum is composed of a CRES signal ($x[n]$) with added WGN. The
 3279 dominant source of noise in a FSCD-like experiment is expected to be thermal Nyquist-
 3280 Johnson noise, which is well approximated by a WGN distribution. In order to decide
 3281 between these two hypotheses we follow the standard Neyman-Pearson approach by
 3282 performing a log-likelihood ratio test between the probability distributions of the signal
 3283 classifier output under \mathcal{H}_1 and \mathcal{H}_0 [70]. The output of the log-likelihood ratio test is
 3284 called the test statistic, which is used to assign the data as belonging to the noise (\mathcal{H}_0)
 3285 or signal (\mathcal{H}_1) classes by setting a decision threshold on the value of the test statistic.

3286 In practice, we select the decision threshold by finding the value of the test statistic
 3287 that guarantees an acceptable rate of false positives and then attempt to maximize
 3288 the signal detection probability under that fixed false positive rate. Because the signal
 3289 classifier will be used to evaluate the spectra of $O(10^2)$ beamforming positions every

3290 40.96 μ sec, we will require the signal classifiers to operate with decision thresholds that
 3291 provide false positive rates significantly smaller than 1%. This reduces the burden placed
 3292 on later stages of the CRES reconstruction chain to reject these false positives and
 3293 decreases the overall likelihood of reconstructing a false event. Below, we calculate the
 3294 probability distributions that allow us characterize how different detection algorithms
 3295 will perform for CRES signals in an FSCD experiment.

3296 **4.4.3 Signal Detection Algorithms**

3297 **4.4.3.1 Power Threshold**

3298 The power threshold detection algorithm uses the maximum amplitude of the frequency
 3299 spectra as the detection test statistic. To model the performance of this approach,
 3300 consider first the case where the signal is pure WGN. For a single bin in the frequency
 3301 spectrum, the probability that the amplitude falls below a specific threshold value is
 3302 given by the Rayleigh cumulative distribution function (CDF),

$$\text{Ray}(x; \tau) = 1 - \exp(-|x|^2/\tau), \quad (4.45)$$

3303 where the complex amplitude of the frequency bin is x , and τ is the WGN variance.
 3304 Because the noise samples for each frequency bin are independent and identically dis-
 3305 tributed (IID), the probability that every bin in the frequency spectrum falls below the
 3306 threshold is the joint CDF formed by the product of each individual frequency bin CDF,

$$F_0(x; \tau, N_{\text{bin}}) = \text{Ray}(x; \tau)^{N_{\text{bin}}}. \quad (4.46)$$

3307 The PDF for the power threshold classifier can then be obtained by differentiating the
 3308 CDF.

3309 The probability distribution for the power threshold classifier under \mathcal{H}_1 is formed in
 3310 a similar way, but the frequency bins that contain signal must be treated separately. For
 3311 a frequency bin that contains both signal and noise we can describe the probability that
 3312 the amplitude of the bin will fall below our threshold using the Rician CDF,

$$\text{Rice}(x; \tau, \nu) = 1 - Q_1 \left(\frac{|\nu|}{\sqrt{2\tau}}, \frac{|x|}{\sqrt{2\tau}} \right), \quad (4.47)$$

3313 where the parameter $|\nu|$ defines the noise-free amplitude of the signal and Q_1 is the
 3314 Marcum Q-function. This time the CDF that describes the probability that the entire

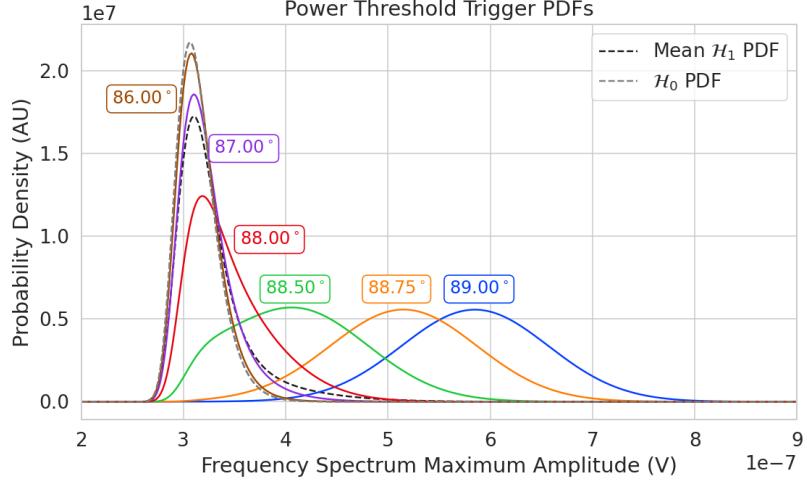


Figure 4.32. PDFs of the power threshold test statistic for CRES signals with various pitch angles as well as the PDF for the noise-only signal case. The average PDF computed for pitch angles ranging from 85.5 to 88.5° is also shown. As the pitch angle is decreased the signal PDF converges towards the noise PDF which indicates that the power threshold trigger is unable to distinguish between small pitch angle signals and noise.

3315 spectrum falls below the decision threshold is the product of both signal and noise CDFs,

$$F_1(x; \tau, \nu, N_{\text{bin}}, N_s) = \text{Ray}(x; \tau)^{N_{\text{bin}} - N_s} \prod_{k=0}^{N_s} \text{Rice}(x; \tau, \nu_k). \quad (4.48)$$

3316 The first half of Equation 4.48 is the contribution from the bins in the frequency spectrum
 3317 that contain only noise, and the second half is the product of the Rician CDFs for the
 3318 frequency bins that contain signal peaks with a noise-free amplitude of $|\nu_k|$. In Figure
 3319 we show plots of example PDFs under \mathcal{H}_1 and \mathcal{H}_0 .

3320 **4.4.3.2 Matched Filtering**

3321 The shape of a CRES signal is completely determined by the initial conditions of the
 3322 electron as it is emitted from beta-decay, which implies that it is possible to apply
 3323 matched filtering as a signal detection algorithm. With a matched filter one uses the
 3324 shape of the known signal, which is called a template, to filter the incoming data by
 3325 computing the convolution between the signal and the data [70]. For cases where the
 3326 signal is buried in WGN, the matched filter is the optimal detector in that it achieves
 3327 the maximum probability of a true detection for a fixed false positive rate. Since CRES
 3328 signals have an unknown shape but are deterministic, we can apply a matched filter by

3329 using simulations to generate a large number of signal templates called a template bank,
 3330 which spans the parameter space of possible signals. Then at detection time, we use the
 3331 template bank to identify signals by performing the matched filter convolution for each
 3332 template in an exhaustive search.

3333 As we saw from the frequency spectra in Figure 4.31, CRES signals are highly periodic
 3334 in nature. In such cases, it is advantageous to utilize the convolution theorem to replace
 3335 the matched filter convolution with an inner product in the frequency-domain. With the
 3336 convolution theorem, the matched filter test statistic that describes the detection of a
 3337 signal buried in WGN using a matched filter template bank is given by

$$\mathcal{T} = \max_{\mathbf{h}} \left| \sum_{n=0}^{N_{\text{bin}}} h^\dagger[n] y[n] \right|, \quad (4.49)$$

3338 where $h^\dagger[n]$ is the complex conjugate of the signal template. For the case when our
 3339 template bank consists of only a single template it is possible to derive an exact analytical
 3340 form for the PDF describing the matched filter test statistic. First, we derive PDF under
 3341 the signal hypothesis, where the equation describing the matched filter test statistic, also
 3342 known as the matched filter score, becomes

$$\mathcal{T} = \left| \sum_{n=0}^{N_{\text{bin}}} h^\dagger[n] y[n] \right|. \quad (4.50)$$

3343 Each noisy frequency bin represented by $y[n]$ is the sum between value of the signal
 3344 at that bin and complex WGN, which means that $y[n]$ is itself Gaussian distributed.
 3345 Therefore, the value of the inner product between the template and the data is also a
 3346 complex Gaussian variable; and, since the matched filter score is the magnitude of this
 3347 inner product, it must follow a Rician distribution.

3348 We can derive the equation for the Rician PDF by expressing the matched filter
 3349 template \mathbf{h} in terms of the corresponding simulated signal, which we write as \mathbf{x}_h to
 3350 distinguish from the signal in the data. Using the standard normalization and assuming
 3351 uncorrelated WGN, the matched filter templates can be written as

$$\mathbf{h} = \frac{\mathbf{x}_h}{\sqrt{\tau |\mathbf{x}_h|^2}} \quad (4.51)$$

3352 where τ is the noise variance. Inserting this into Equation 4.49 and expressing the data

3353 as a sum between a signal and a WGN vector yields,

$$\mathcal{T} = \frac{1}{\sqrt{\tau|\mathbf{x}_h|^2}} \left| \sum_{n=1}^{N_{\text{bin}}} x_h[n] (x[n] + \nu[n]) \right|. \quad (4.52)$$

3354 Next, we transform the expression by isolating the randomly distributed components
 3355 giving

$$\mathcal{T} = \frac{\left| \sum_{n=1}^{N_{\text{bin}}} x_h[n] x[n] \right|}{\sqrt{\tau|\mathbf{x}_h|^2}} + \frac{1}{\sqrt{\tau|\mathbf{x}_h|^2}} \left| \sum_{n=1}^{N_{\text{bin}}} x_h[n] \nu[n] \right|. \quad (4.53)$$

3356 The first term of 4.53 can be simplified by using the Cauchy-Schawrz inequality to express
 3357 the magnitude of the inner product in terms of the magnitudes of the signal and template
 3358 as well as an orthogonality constant which we call "match" (Γ). Using this we obtain,

$$\mathcal{T} = |\mathbf{h}| |\mathbf{x}| \Gamma + \frac{1}{\sqrt{\tau|\mathbf{x}_h|^2}} \left| \sum_{n=1}^{N_{\text{bin}}} x_h[n] \nu[n] \right|. \quad (4.54)$$

3359 The second term is a sum of Gaussian distributed variables, which we should expect also
 3360 follows a Gaussian distribution. Each of the samples $\nu[n]$ is described by

$$\nu[n] \sim \mathcal{N}(0, \tau), \quad (4.55)$$

3361 where $\mathcal{N}(0, \tau)$ is a complex Gaussian distribution with zero mean and variance τ . There-
 3362 fore,

$$\frac{x_h[n]}{\sqrt{\tau|\mathbf{x}_h|^2}} \nu[n] \sim \mathcal{N}\left(0, \frac{x_h[n]^2}{|\mathbf{x}_h|^2}\right), \quad (4.56)$$

$$\sum_{n=1}^{N_{\text{bin}}} \frac{x_h[n]}{\sqrt{\tau|\mathbf{x}_h|^2}} \nu[n] \sim \mathcal{N}\left(0, \frac{\sum_{n=1}^{N_{\text{bin}}} x_h[n]^2}{|\mathbf{x}_h|^2}\right) = \mathcal{N}(0, 1), \quad (4.57)$$

$$|\mathbf{h}| |\mathbf{x}| \Gamma + \sum_{n=1}^{N_{\text{bin}}} \frac{x_h[n]}{\sqrt{\tau|\mathbf{x}_h|^2}} \nu[n] \sim \mathcal{N}(|\mathbf{h}| |\mathbf{x}| \Gamma, 1). \quad (4.58)$$

3363 We see that \mathcal{T} is magnitude of a complex variable with mean $|\mathbf{h}| |\mathbf{x}| \Gamma$ and variance one. In
 3364 order to simply the expression a bit further, we define the quantity $\mathcal{T}_{\text{ideal}} = |\mathbf{h}| |\mathbf{x}| \Gamma$, which
 3365 we call the ideal matched filter score, because it represents the value of the matched
 3366 filter inner product that we would expect if no noise was present in the signal. We can
 3367 write the matched filter test statistic as the magnitude of a two-dimensional vector in

3368 the complex plane

$$\mathcal{T} = |(\mathcal{T}_{\text{ideal}} + n_r, n_i)|, \quad (4.59)$$

3369 where n_r and n_i are the real and imaginary components of the noise each with variance
3370 $1/2$, which is modeled by a Rician distribution with shape factor $\mathcal{T}_{\text{ideal}}$. Therefore, the
3371 probability distribution of the matched filter test statistic is given by,

$$P_1(x; \mathcal{T}_{\text{ideal}}) = 2x \exp(- (x^2 + \mathcal{T}_{\text{ideal}}^2)) I_0(2x\mathcal{T}_{\text{ideal}}), \quad (4.60)$$

3372 where I_0 is the zeroth-order modified Bessel function.

3373 The shape of the matched filter score distribution is controlled by the parameter
3374 $\mathcal{T}_{\text{ideal}}$, which is effectively the value of the matched filter score if the data contained no
3375 noise. Without noise, the data vector reduces to the signal, \mathbf{x} , in which case Equation
3376 4.50 becomes the magnitude of an inner product between two vectors. We can write
3377 the magnitude of an inner product in terms of the lengths of the individual vectors and
3378 a constant that describes the degree of orthogonality between them. Applying this to
3379 Equation 4.50, we obtain

$$\mathcal{T}_{\text{ideal}} = |\mathbf{h}^\dagger \cdot \mathbf{x}| = |\mathbf{h}| |\mathbf{x}| \Gamma \quad (4.61)$$

3380 where Γ describes the orthogonality between \mathbf{h} and \mathbf{x} . From the point of view of matched
3381 filtering, we can interpret Γ as describing how well the template matches the underlying
3382 signal in the data.

3383 The matched filter score PDF under the noise hypothesis can be readily obtained
3384 from Equation 4.60 by setting the value of $\mathcal{T}_{\text{ideal}}$ to zero, since the data contains no signal
3385 in the noise case. Doing this, we obtain the Rayleigh distribution that describes the
3386 matched filter score under \mathcal{H}_0 ,

$$P_0(x) = 2x \exp(-x^2). \quad (4.62)$$

3387 Equations 4.60 and 4.62 describe the behavior of the matched filter test statistic
3388 under \mathcal{H}_0 and \mathcal{H}_1 for a single template. However, defining a PDF that describes the
3389 matched filter test statistic in the case of multiple templates is in general a mathematically
3390 intractable problem, since there is no guarantee of orthogonality between matched filter
3391 templates. This leads to correlations between the matched filter scores of different
3392 templates because only one sample of noise is used to compute the matched filter scores
3393 of the template bank. In order to proceed, we need to make the simplifying assumption
3394 that we can treat the matched filter scores as IID variables, which allows to ignore

3395 correlations between templates. The overall effect of this will be an underestimate of the
 3396 performance of the matched filter, since we are under counting the number of templates
 3397 that could contribute a detectable score.

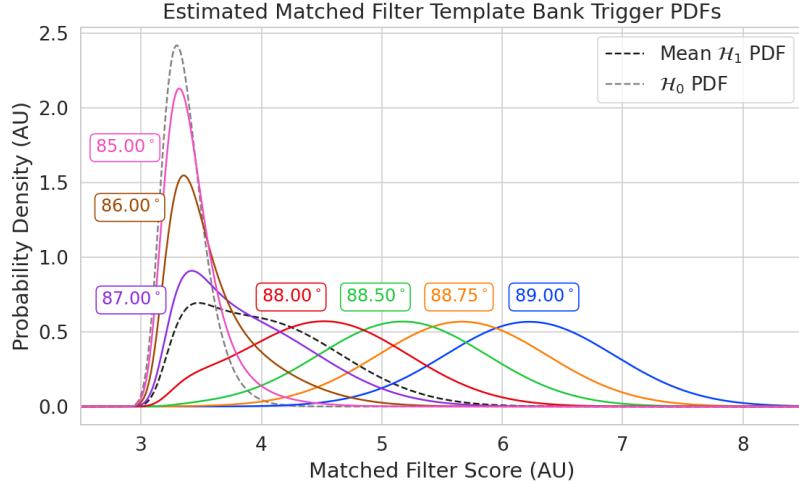


Figure 4.33. Plots of the estimated PDFs for the matched filter template bank test statistic for CRES signals with various pitch angles as well as the estimated PDF for the noise only signal case. We assume an estimated number of templates of 10^5 and perfect match between signal and template i.e. $\Gamma_{\text{best}} = 1$. The mean PDF includes signals ranging from $85.5 - 88.5^\circ$ in pitch angle. There is a much larger distinction between the signal PDFs at small pitch angle compared to the power threshold indicating a higher detection efficiency for these signals.

3398 For \mathcal{H}_0 we model the probability that the matched filter score falls below our threshold
 3399 using the CDF obtained by integrating Equation 4.62. Because we are assuming that
 3400 the matched filter scores using different templates are independent, the probability that
 3401 the matched filter score for all templates falls below a threshold value is the joint CDF
 3402 formed by multiplying the CDF for each template. Under \mathcal{H}_0 this is

$$F_0(x) = \left(1 - e^{-x^2}\right)^{N_t}, \quad (4.63)$$

3403 where x is the matched filter score threshold and N_t is the number of templates. We
 3404 should expect that the distribution describing the matched filter template bank maximum
 3405 score depends on N_t , because with more templates there is a greater chance of a random
 3406 match between the template and data.

3407 For \mathcal{H}_1 , we start by denoting the CDF of the best matching template as $F_{\text{best}}(x; \mathcal{T}_{\text{best}})$,
 3408 and treat the matched filter scores for all other templates as negligible ($\mathcal{T}_{\text{ideal}} \approx 0$). Then
 3409 we form the joint CDF by combining the distributions for all templates used during

3410 detection. Since we are exhaustively checking the matched filter scores, the number of
3411 templates checked will be a randomly distributed variable that ranges from zero to the
3412 total number of available templates. If we assume that signals are uniformly distributed
3413 across the parameter space spanned by the template bank then on average we check
3414 $(N_t - 1)/2 \approx N_t/2$ templates for each inference. Therefore, the estimated CDF under \mathcal{H}_1
3415 is

$$F_1(x; \mathcal{T}_{\text{best}}) = F_{\text{best}}(x; \mathcal{T}_{\text{best}}) \left(1 - e^{-x^2}\right)^{N_t/2}. \quad (4.64)$$

3416 In Figure 4.33 we show plots of the estimated matched filter template bank classifier
3417 PDFs under both \mathcal{H}_0 and \mathcal{H}_1 .

3418 4.4.3.3 Machine Learning

3419 In this paper we focus on Convolutional Neural Networks (CNN) as an example of
3420 a machine learning based signal classifier. CNNs are constructed using a series of
3421 convolutional layers, each composed of a set of filters that are convolved with the input
3422 data. The individual convolutional filters can be viewed as matched filter templates that
3423 are learned from a set of simulated data rather than being directly generated. This opens
3424 the possibility of finding a more efficient representation of the matched filter templates
3425 during the training process that can potentially reduce computational cost at inference
3426 time while still offering good classification performance.

3427 The machine learning approach is distinct from both the power threshold and matched
3428 filtering in that we do not attempt to manually engineer a test statistic that is computed
3429 from the data for classification. Instead, we attempt calculate the test statistic by
3430 constructing a differentiable function that maps the complex frequency series generated
3431 by the STFT to a binary classification as either signal or noise. The test statistic for the
3432 machine learning classifier can be expressed as

$$\mathcal{T} = G(\mathbf{y}; \boldsymbol{\Omega}) \quad (4.65)$$

3433 where \mathbf{y} is the noisy data vector and $G(\mathbf{y}; \boldsymbol{\Omega})$ is the machine learning model parameterized
3434 by the weights $\boldsymbol{\Omega}$. By using supervised learning on a labeled set of training signals, we
3435 can modify the function parameters to learn the mapping from the data to the likelihood
3436 of \mathbf{y} belonging to either \mathcal{H}_1 or \mathcal{H}_0 .

3437 The CNN architecture used for this work is summarized by Table 4.1. No strategic
3438 hyper-parameter optimization approach was implemented beyond the manual testing
3439 of different CNN architecture variations, so this particular model is best viewed as a

Table 4.1. A summary of the CNN model layers and parameters. The output of each 1D-Convolution and Fully Connected layer is passed through a LeakyReLU activation function and re-normalized using batch normalization before being passed to the next layer in the model. The output of the final Fully Connected layer in the model is left without activation so that the model outputs can be directly passed to the Binary Cross-entropy loss function used during training.

Layer	Type	Input Channels	Output Channels	Parameters
1	1D-Convolution	2	15	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 1$)
2	Maximum Pooling	15	15	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 4$)
3	1D-Convolution	15	20	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 1$)
4	Maximum Pooling	20	20	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 4$)
5	1D-Convolution	20	25	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 1$)
6	Maximum Pooling	25	25	($N_{\text{kernel}} = 4$, $N_{\text{stride}} = 4$)
7	Fully Connected	3200	512	NA
8	Fully Connected	512	64	NA
9	Fully Connected	64	2	NA

³⁴⁴⁰ proof-of-concept rather than a rigorously optimized design. Numerous model variations
³⁴⁴¹ were tested, some with significantly more layers and convolutions filters per layer, as
³⁴⁴² well as others that were even smaller than the architecture in Table 4.1. Ultimately, the
³⁴⁴³ model architecture choice was driven by the motivation to find the minimal model whose
³⁴⁴⁴ classification performance was still comparable to the larger CNN’s tested, because of
³⁴⁴⁵ the importance of minimizing computational cost in real-time applications. It is possible
³⁴⁴⁶ that more sophisticated machine learning models could improve upon the classification
³⁴⁴⁷ results achieved here, but we leave this investigation for future work.

³⁴⁴⁸ 4.4.4 Methods

³⁴⁴⁹ 4.4.4.1 Data Generation

³⁴⁵⁰ To test the triggering performance of the classifiers, simulated CRES signals were
³⁴⁵¹ generated using the Locust simulations package [61, 79] developed by the Project 8
³⁴⁵² collaboration. Locust uses the separately developed Kassiopeia package to calculate the
³⁴⁵³ magnetic fields produced by a user defined set of current carrying coils along with any
³⁴⁵⁴ specified background magnetic fields, resulting in a magnetic trap. Next, Kassiopeia
³⁴⁵⁵ calculates the trajectory of an electron in this magnetic field starting from a set of user
³⁴⁵⁶ specified initial conditions. The Locust software then uses the electron trajectories from
³⁴⁵⁷ Kassiopeia to calculate the resulting electromagnetic fields using the Liénard-Wiechert
³⁴⁵⁸ equations, and determine the voltages generated in the antenna array with the antenna

3459 transfer function. Locust then simulates the down-conversion, filtering, and digitization
3460 steps resulting in the simulated CRES signals for an electron.

3461 The shape of the received CRES signal is determined by the initial kinematic param-
3462 eters, including the starting position of the electron, the starting kinetic energy of the
3463 electron, and the pitch angle. For the studies performed here we constrain ourselves to a
3464 single initial electron position located at $(x, y, z) = (5, 0, 0)$ mm, and using this starting
3465 position we generate two datasets by varying the initial kinetic energy and the starting
3466 pitch angle. The first dataset consists of a two-dimensional square grid of kinetic energy
3467 and pitch angle spanning an energy range from 18575-18580 eV with a spacing of 0.1 eV,
3468 and pitch angles from 85.5-88.5° with a spacing of 0.001°, resulting in 153051 signals with
3469 a unique energy-pitch angle combination. This dataset is intended to represent a matched
3470 filter template bank. The second dataset was generated by randomly sampling uniform
3471 probability distributions covering the same parameter space to produce approximately
3472 50000 signals randomly parameterized in energy and pitch angle. This dataset provides
3473 the training and test data for the machine learning approach, and acts as a representative
3474 sample of signals to evaluate the performance of the matched filter template bank.

3475 Each signal was simulated for a duration of 40.96 μ s, which is equivalent to 8192
3476 samples at the FSCD digitization rate, and begins at time $t = 0$ s for all simulations.
3477 This duration represents a single frequency spectrum generated by the STFT. The output
3478 of the Locust simulation is a matrix of array snapshots with size given by the number of
3479 channels times the event length ($N_{\text{ch}} \times N_{\text{sample}}$), which we pre-process using the digital
3480 beamforming summation and STFT described in Section 4.4.2.2. The ∇B -drift correction
3481 uses the exact value of $\omega_{\nabla B}$, obtained from the Kassiopeia simulation of that electron.
3482 In practice, an average value for $\omega_{\nabla B}$ could be used, because there is limited variation in
3483 drift frequency across this parameter space.

3484 4.4.4.2 Template Number and Match Estimation

3485 The estimated PDF for the matched filter template bank depends on the score of the
3486 best matching template or equivalently the match of the best template (Γ_{best}) as well
3487 as the number of templates. One expects that with a higher number of templates the
3488 average value of Γ_{best} will increase, however, there is a point of diminishing returns at
3489 which more templates will not significantly increase match, but will still increase the
3490 likelihood of false positives. Therefore, it is desirable to use the minimum number of
3491 templates that provide an acceptable mean value of Γ_{best} .

3492 To quantify the relationship between match and template number, we calculated

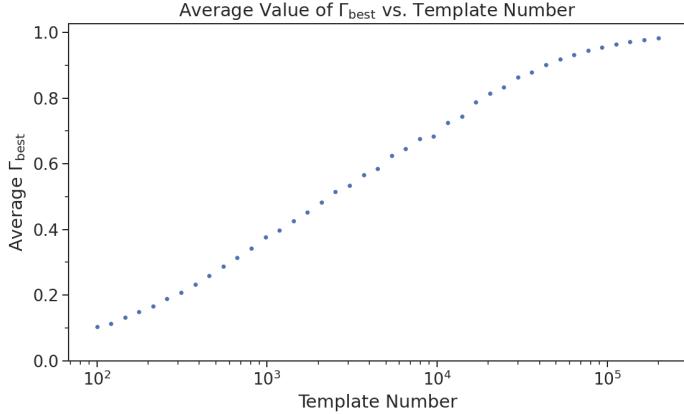


Figure 4.34. The mean match of the matched filter template bank to a test set of randomly parameterized signals as a function of the number or density of templates. The parameter space includes pitch angles from $85.5 - 88.5^\circ$ and energies from 18575 – 18580 eV.

the mean match of the random dataset to a selection of templates obtained from the regularly spaced dataset. The results are shown in Figure 4.34, where we find that the average value of Γ_{best} is an exponential function of the number of templates. From this plot we select the desired value of mean match at which we would like to evaluate the matched filter PDF and can infer the required number of templates.

4.4.4.3 CNN Training and Data Augmentation

To prepare the data for training the model, we split the random dataset in half to create distinct training and test datasets. Additionally, a randomly selected 20% of the training data is isolated for use as a validation set during the training loop. The size of the training, validation, and test datasets are then tripled by appending two additional copies of the data to increase the sample size of the dataset after data augmentation. The data is loaded with no noise, which is added to each data batch during the training phase by generating a new noise sample from a complex WGN distribution. In order to ensure an even split between signal and noise data we append to the noise-free signals an equal number of empty signals composed of all zeros. Therefore, as the data is randomly shuffled during training, on average an equal number of empty signals will be included with the training signals. After adding the sample of WGN to the data batch, the empty signals represent the noise-only data that the model must distinguish from signal data.

As the training signals are loaded we apply a unique random phase shift as the first form of data augmentation. Since the data is generated using the same initial axial position and cyclotron orbit phase, the randomization is an attempt to prevent

3514 overtraining on these features. During each training epoch the data is randomly shuffled
 3515 and split into batches of 2500 signals. Each batch of signals is then circularly shifted
 3516 by a random number of frequency bins to simulate a kinetic energy shift from -75 to
 3517 20 eV to simulate a training dataset with a larger energy range. Next, a sample of
 3518 complex WGN, consistent with the expected 10 K Nyquist-Johnson noise expected for
 3519 the FSCD, is generated and added to the signal, which prevents overtraining on noise
 3520 features. As a final step, the data is renormalized by the standard deviation of the noise
 3521 so that the range of values in the data is close to $[-1, 1]$, which helps ensure well-behaved
 3522 back-propagation.

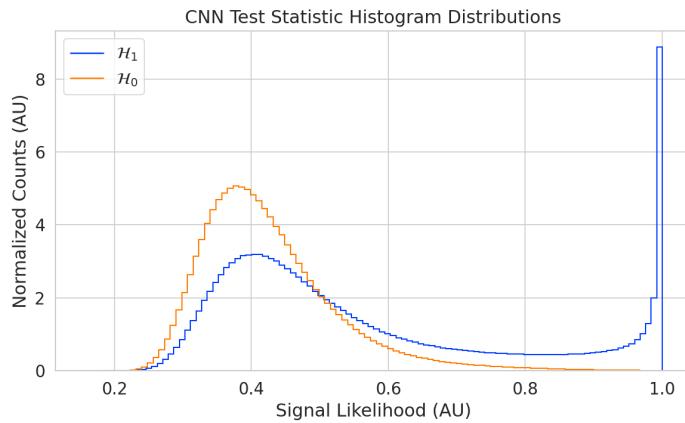


Figure 4.35. Histograms of the trained CNN model output from the test dataset. The blue histogram shows the model outputs for signal data. The oddly shaped peak near the end is the result of the softmax function mapping the long tail of the raw output distribution to the range $[0, 1]$.

3523 The Binary Cross-entropy loss function is used to compute the loss for each batch of
 3524 data and the model weights are updated using the ADAM optimizer with a learning rate
 3525 of 5×10^{-3} . After each training epoch, the loss and classification accuracy of the validation
 3526 dataset are computed to monitor for overtraining. It was noticed that the relatively high
 3527 noise power and the fact that a new sample of noise was used for each batch together
 3528 provided a strong form of regularization, since no evidence of over-training was observed
 3529 even after several thousand epochs. Typically, the loss and classification accuracy of
 3530 the model converged after a few hundred training epochs, but the training loop was
 3531 extended to 3000 epochs to attempt to achieve the best possible performance. The
 3532 training procedure generally took about 24 hrs using a single NVIDIA V100 GPU [81].

3533 After training the model, we use it to classify the test dataset and generate
 3534 histograms of the model outputs for both classes of data. The data augmentation

procedure for the evaluation of the test data mirrors the training procedure without the validation split. Since a random circular shift and a new sample of WGN is added to each batch, the testing evaluation loop is run for 100 epochs to get a representative sample of noise and circular shifts. The model outputs for each batch are passed through a softmax activation and then combined into histograms, which we show in Figure 4.35.

4.4.5 Results and Discussion

4.4.5.1 Trigger Classification Performance

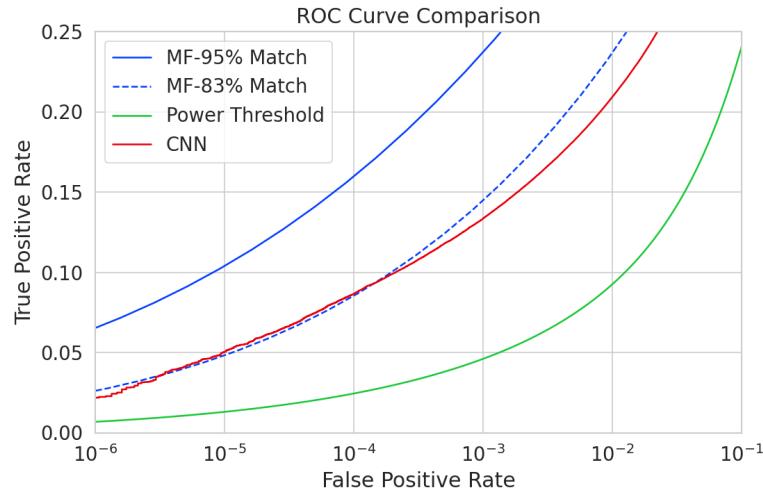


Figure 4.36. ROC curves describing the detection efficiency or true positive rates for the three signal classification algorithms examined in this paper.

Using the matched filter and power threshold CDFs, along with the classification results from the CNN, we compare detection performance by computing receiver operating characteristic (ROC) curves. Specifically, we compare the detection performance averaged over the full signal parameter space in order to get a measure of the overall detection efficiency achieved by each algorithm. For the power threshold and matched filter algorithms, we obtain the mean ROC curve by taking the average over all signals in the regularly spaced dataset. In the case of the matched filter, we examine two cases using different numbers of templates, which have different values of mean match. The ROC curve describing the CNN is obtained by forming a histogram of the network outputs for each class of signal and from this computing the estimated CDFs and ROC curve. In Figure 4.36, we show the ROC curves obtained for each of the detection algorithms, visualized in terms of true positive rate and false positive rate.

3554 The true positive rate of a signal classifier is equivalent to its detection efficiency, and
3555 we see that for the population of signals with pitch angles $< 88.5^\circ$ the power threshold
3556 has a consistently lower detection efficiency than the CNN and the matched filter. This
3557 result could have been predicted from the visualization of signal spectra in Figure 4.31,
3558 where we see that there is no way to distinguish between a noise peak and a signal peak
3559 with high confidence at small pitch angles. The CNN offers a significant and consistent
3560 increase in detection efficiency over the power threshold approach, with the relative
3561 improvement in detection efficiency increasing as the false positive rate decreases. If
3562 we compare the CNN to the matched filter, we see that the performance of the tested
3563 network is roughly equivalent to a matched filter detector with an average match of
3564 about 83%, which uses approximately 20000 matched filter templates. The overall best
3565 detection efficiency is achieved by the matched filter classifier if a large enough template
3566 bank is used. We show in the plot the ROC curve for a matched filter template bank
3567 with 95% average match, which is achieved with approximately 100000 templates. Since
3568 the matched filter is known to be statistically optimal for detecting a known signal in
3569 WGN, it is somewhat expected that this algorithm has the highest detection efficiency.

3570 A potentially impactful difference between the matched filter and CNN algorithms is
3571 that the CNN relies upon convolutions as its fundamental calculation mechanism, whereas
3572 our implementation of a matched filter utilizes an inner product. Since convolution is
3573 a translation invariant operation, the detection performance of CNN can be extended
3574 to a wider range of CRES event kinetic energies with less cost than the matched filter,
3575 a feature that we exploited during the CNN training by including circular translations
3576 of the CRES frequency spectra in the training loop. Increasing the range of kinetic
3577 energies detectable by a matched filter requires a proportional increase in the number of
3578 templates, which directly translates into increased computational and hardware costs.
3579 From a practical perspective, the detection algorithm is always limited by the available
3580 computational hardware, so estimating the relative costs is a key factor in determining
3581 their feasibility. Below we perform a more detailed analysis of the relative costs of each
3582 of the detection algorithms.

3583 4.4.5.2 Computational Cost and Hardware Requirements

3584 In the process of investigating triggering approaches for an antenna array CRES experi-
3585 ment, we have uncovered a strong tension between detection efficiency and computational
3586 resources. To relate the computational cost estimates to actual costs, we compare the
3587 theoretical amount of computer hardware required to implement the signal classifiers

for real-time detection in an FSCD experiment. To do this we shall utilize order of magnitude estimates of the theoretical peak performance values for currently available Graphics Processing Units (GPUs) as a metric. This approach will underestimate the amount of required hardware, since it is unlikely that any CRES detection algorithm could reach the theoretical peak performance of the hardware.

Of the three detection algorithms tested, the power threshold classifier is the least expensive. It requires that we check whether the amplitude of each frequency bin in the STFT is below or above our decision threshold. The STFT combined with digital beamforming produces $N_{\text{bin}}N_b$ frequency bins that must be checked every N_{bin}/f_s seconds. This requires approximately $O(10^{10})$ FLOPS to check in real-time. Current generations of GPUs have peak theoretical performances in the range of $O(10^{13}) - O(10^{14})$ FLOPS [82], dependent on the required floating-point precision of the computation. Therefore, the entire computational needs of a real-time triggering system using a power threshold classifier, including digital beamforming and generation of the STFT, could be met by a single high-end GPU or a small number of less powerful GPUs. Since triggering is only one step of the full real-time signal reconstruction approach, limiting the computational cost of this stage is ideal. However, we have seen that the power threshold classifier does not provide sufficient detection efficiency across the entire range of possible signals, which is the primary motivation for exploring more complicated triggering solutions.

As discussed, the computational cost of the matched filter approach requires counting the number of templates that must be checked for each frequency spectra produced by the STFT. Computing the matched filter scores requires $O(N_b N_t N_{\text{bin}})$ operations, since for each of the N_b beamforming positions we must multiply N_t templates with a data vector that has length N_{bin} . The time within which we must perform this calculation is equal to N_{bin}/f_s to keep up with the data generation rate. To cover the 5 eV kinetic energy range spanned by the template bank, we saw that 10^4 to 10^5 templates are required in order to match or exceed the detection efficiency of the CNN. If the number of templates scales linearly with then kinetic energy range of interest as expected, then we would require 10^5 to 10^6 matched filter templates with this more realistic range of energies. Considering this, the estimated computational cost of the matched filter is between $O(10^{15})$ to $O(10^{16})$ FLOPS, which is $O(10^2)$ to $O(10^3)$ high-end GPUs.

Lastly, we have the CNN classifier. To estimate the computational cost we simply sum the number of convolutions and matrix multiplications specified by the network architecture shown in Table 4.1. Each convolutional layer consists of $N_{\text{in}}N_{\text{out}}N_{\text{kernel}}L_{\text{input}}$ floating-point operations, where N_{in} is the number of input channels, N_{out} is the number

3623 of output channels, N_{kernel} is the size of the convolutional kernel, and L_{input} is the length
3624 of the input vector, and the fully connected layers each contribute $N_{\text{in}}N_{\text{out}}$ operations.
3625 Summing all the neural network layers we estimate that the CNN would require $O(10^6)$
3626 floating point operations for each frequency spectra; therefore, the total computation
3627 cost of the CNN trigger is this cost times the number of beamforming positions per the
3628 data acquisition time, which is $O(10^{13})$ FLOPS or $O(10^0)$ GPUs.

3629 Compared with the matched filter approach the CNN requires $O(100)$ to $O(1000)$
3630 fewer GPUs to implement, dependent on the exact number of templates used in the
3631 template bank. The 100 eV kinetic energy range is motivated by the application of these
3632 detection algorithms to an FSCD-like neutrino mass measurement experiment. However,
3633 if a significantly larger range of kinetic energies is required, a CNN may be the preferred
3634 detection approach despite the lower average detection efficiency due to computational
3635 cost considerations. The low estimated computational cost of the CNN is directly related
3636 to the small network size.

3637 Additional experiments with larger CNNs, generated by increasing the depth and
3638 width of the neural network, and we observed that these changes provided minimal
3639 ($\lesssim 1\%$) improvement in the classification accuracy of the model. A potential reason
3640 for this could be the sparse nature of the signals in the frequency domain and the low
3641 SNR which makes for a challenging dataset to learn from. Future work could investigate
3642 modifications to the neural network architecture such as sparse convolutions, which may
3643 improve the classification accuracy of the model or further reduce the computational
3644 costs of this approach. Alternatively, more complicated CNN architectures such as a
3645 ResNet [83] or VGG model [84] may provide improved classification performance over a
3646 basic CNN. An additional promising area of investigation are recurrent neural networks,
3647 which may be able to exploit the time-ordered features of the STFT for more accurate
3648 signal detection if the electron signals last for multiple Fourier transform windows.

3649 Our estimate of the computational cost of the matched filter is somewhat naive if
3650 we notice that the majority of the values that make up a CRES frequency spectra are
3651 zero (see Figure 4.31). Therefore, the majority of operations in the matched filter inner
3652 product are unnecessary, and we could instead evaluate the matched filter inner product
3653 using only the $\lesssim 10$ frequency peaks that make up CRES signal. This optimization
3654 reduces the number of operations required to check each template by a factor of $O(100)$
3655 to $O(1000)$, which brings the estimated computational cost of the matched filter in
3656 line with the CNN. Although this level of sparsity results in a multiplication with very
3657 low arithmetic complexity, the resulting sparse matched filter algorithm is still likely

3658 to be constrained by memory access speed rather than compute speed. Ultimately, the
3659 comparison of the relative computational and hardware costs between the matched filter
3660 and CNN will depend on the efficiency of the software implementation and hardware
3661 support for neural network and sparse matrix calculations.

3662 **4.4.6 Conclusion**

3663 Increasing the detection efficiency and overall event rate of the CRES technique represents
3664 a key developmental path towards new scientific results and broader applications of the
3665 CRES technique. It is what motivates both the antenna array detection approach and
3666 the development of real-time signal reconstruction algorithms. We have demonstrated
3667 that significant gains in the detection efficiency of the CRES technique are achievable
3668 by utilizing triggering algorithms that account for the specific shape of CRES signals in
3669 the detector. These algorithms emphasize the need for accurate and fast methods for
3670 CRES simulation, since they directly contribute to the success of matched filter methods
3671 by providing a way to generate expected signal templates and also serve as a source of
3672 training data for machine learning approaches.

3673 The improvements in detection efficiency offered by these alternative approaches to
3674 triggering are crucial to the success of efforts to develop scalable technologies for CRES
3675 measurement, since they provide a significant increase in the detectable parameter space
3676 of CRES events, which allows for a better utilization of the larger detection volume.
3677 While we have focused on the real-time detection of CRES signals from antenna arrays,
3678 these same signal classifiers could be used in CRES experiments utilizing a different
3679 detector technologies, since the same principles of signal detection will apply. For example,
3680 previous CRES measurements by the Project 8 collaboration that utilized a waveguide
3681 gas cell, could have improved their detection efficiency by employing a matched filter
3682 or neural network classifier to identify trapped electrons with pitch angles that are too
3683 small to be detected by the power threshold approach. Furthermore, alternative CRES
3684 detector technologies such as resonant cavities [40] could also see similar improvements
3685 in detection efficiency, which is of crucial importance to future efforts by the Project 8
3686 collaboration to utilize CRES to measure the neutrino mass.

Chapter 5

Antenna and Antenna Measurement System Development for the Project 8 Experiment

5.1 Introduction

The FSCD and antenna array CRES represent an innovative approach to beta-decay spectroscopy. While much can be learned from simulations about the systematics of CRES with antenna arrays, laboratory measurements and demonstrations provide critical inputs to sensitivity and simulation models as well as provide a means for calibration and commissioning of the experiment. Therefore, a robust program of antenna and antenna measurement hardware development is important to the success of the FSCD and the development of antenna array CRES more broadly.

In this chapter we summarize the development of an antenna measurement system at Penn State to implement and test the techniques of antenna array CRES on the bench-top, in order to support the efforts of the Project 8 collaboration. In Section 5.2 we provide an introduction to some fundamental parameters and concepts related to antenna measurements as well as an overview of the Penn State antenna measurement system hardware. In Section 5.3 we include the manuscript of a paper [80] which details the design and characterization of a specialized antenna developed to mimic the electric fields emitted by an electron in a CRES experiment. This antenna, called the Synthetic Cyclotron Antenna (SYNCA), is intended as a calibration tool for antenna arrays developed for CRES measurements. Lastly, in Section 5.5 we summarize a set of prototype FSCD antenna array measurements with the SYNCA [43], which we use to validate the simulated performance of the antenna array and estimate systematic errors associated with the antenna array.

3712 5.2 Antenna Measurements for CRES experiments

3713 5.2.1 Antenna Parameters

3714 Antenna characterization measurements are intended to validate simulations of the
3715 antenna array performance, which ultimately informs the neutrino mass sensitivity of
3716 the experiment. In this section, I shall summarize a few fundamental concepts relating
3717 to antennas and antenna measurement, before introducing how Project 8 uses antenna
3718 measurement for the development of antenna array CRES.

3719 5.2.1.1 Radiation Patterns

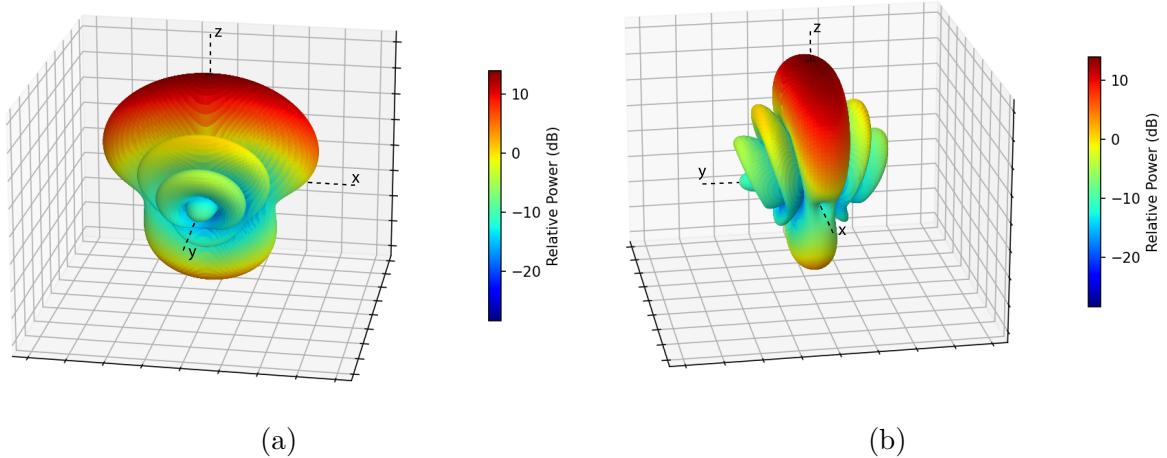


Figure 5.1. An example radiation pattern generated using HFSS simulations. The color and radial distance of the surface from the origin indicate the relative magnitude of radiation power emitted by the antenna in that direction. The primary goal of most antenna measurements is typically to measure the antenna pattern, which is used to derive many useful antenna performance parameters.

3720 Antennas are conductive structures designed to carry alternating electric currents
3721 in order to transmit energy in the form of electro-magnetic (EM) waves [64]. Perhaps
3722 the most fundamental way to characterize an antenna, is to map out the radiated power
3723 density as a function of position, which is called the radiation pattern (see Figure 5.1).
3724 We find the radiation power density by calculating the time-averaged Poynting vector for
3725 all positions surrounding the antenna, which in equation form is

$$\mathbf{W}(x, y, z) = \langle \mathbf{E}(x, y, z, t) \times \mathbf{H}^*(x, y, z, t) \rangle_t, \quad (5.1)$$

3726 where $\mathbf{E}(x, y, z, t)$ and $\mathbf{H}(x, y, z, t)$ are the time-dependent electric and magnetic fields
 3727 produced by the antenna [48]. The radiation power density has units of W/m^2 and is
 3728 more typically called the energy flux density in physics applications, since it is a measure
 3729 of the amount of energy passing through a unit area over time.

3730 Because the radiation power density is a measure of power per unit area, its value
 3731 in a particular direction will depend on the distance from the antenna at which we are
 3732 measuring. This is undesirable for practical applications A related quantity, which is
 3733 distance independent, is the energy flux per unit solid angle or radiation intensity, which
 3734 is computed directly from the radition power density by multiplying by the squared
 3735 distance from the antenna. Specifically,

$$U = r^2 W(x, y, z), \quad (5.2)$$

3736 where r is the distance from the antenna to the field measurement point. The radiation
 3737 intensity is typically defined in regions where the Poynting vector consists only of a radial
 3738 component where it is safe to treat as a scalar quantity.

3739 5.2.1.2 Directivity and Gain

3740 Since the radiation intensity is a measure of average power per unit solid angle, it is
 3741 independent of distance and more useful as feature for antenna measurement. However,
 3742 most antenna measurements are performed in terms of the directly related directivity
 3743 and gain quantities. Directivity is defined as the ratio between the radiation intensity at
 3744 particular point on the radiation pattern to the average radiation intensity computed
 3745 over all solid angles [64]. The equation that relates the radiation intensity to directivity
 3746 is

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}}, \quad (5.3)$$

3747 where U_0 is the average radiation intensity over all solid angles, which simply the total
 3748 radiated power (P_{rad}) divided by 4π . Closely related to directivity is concept of gain,
 3749 which accounts for energy losses that occur inside then antenna when attempting to
 3750 transmit or receive a signal. The antenna gain is given by

$$G = \frac{4\pi U}{P_{\text{in}}}, \quad (5.4)$$

3751 where P_{in} is the total power delivered to the antenna. Gain can be thought of as the ratio
 3752 of the antenna's radiation intensity to that of a hypothetical isotropic, lossless radiator.

3753 The maximum values of gain and directivity exhibited by the main lobe of the antenna
 3754 pattern as well as the ratio between the gain of the main lobe and any side-lobes are
 3755 important figures of merit used to evaluate antenna designs.

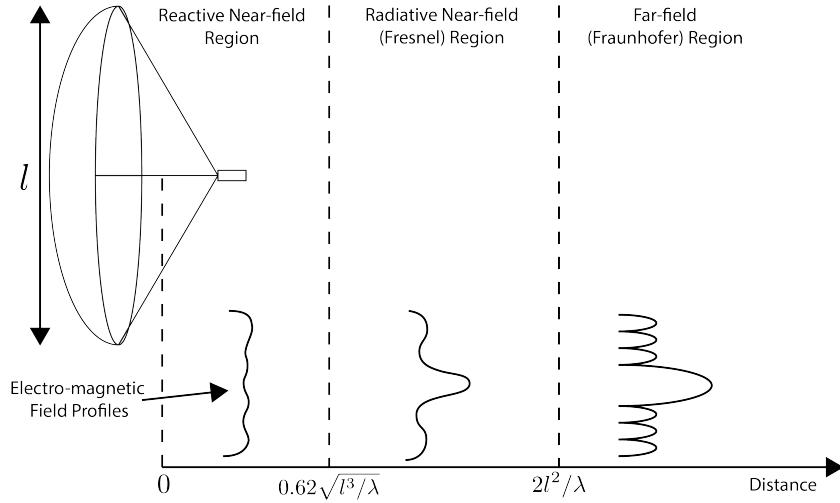


Figure 5.2. An illustration of the three field regions important for the analysis of an antenna system. Very close to the antenna the electric fields are primarily reactive so there is no radiation. If a receiving antenna were placed in this region most of the energy would be reflected back to the transmitter. Outside of the reactive near-field is the radiative near field. At these distances the antenna does radiate, but the radiation pattern is not well-defined since it changes based on the distance of the receiving antenna. It is only in the far-field region where the radiation pattern becomes constant as a function of distance, which is where the majority of antenna engineering is assumed to take place. The antenna arrays developed by Project 8 for CRES measurements operate in the radiative near-field due to the importance of limiting power loss from free-space propagation, which complicates the design of the antenna system.

3756 5.2.1.3 Far-field and Near-field

3757 Radiation patterns are only well-defined in regions where the shape of the radiation
 3758 pattern is independent of distance. The region where this approximation is valid is called
 3759 the "far-field", and in this region we can treat the EM fields from the antenna as spherical
 3760 plane waves. A rule of thumb for antennas is that the far-field approximation is valid
 3761 when the condition

$$R > \frac{2l^2}{\lambda} \quad (5.5)$$

3762 is met. In this expression, R is the distance from the antenna, l is the largest characteristic
 3763 dimension of the antenna, and λ is the wavelength of the radiation (see Figure 5.2).

3764 The region very close to the antenna is called the reactive near-field, because in this
 3765 region the reactive component of the EM field is dominant. Unlike radiative electric

3766 fields, the reactive electric and magnetic fields are out of phase from each other by
 3767 90° , since they are the result of electrostatic and magnetostatic effects coming from the
 3768 self-capacitance and self-inductance of the antenna. The reactive fields are unable to
 3769 transfer energy a significant distance from the antenna and are thus completely negligible
 3770 for most antenna applications. The limit of the reactive near-field for an electrically-large
 3771 antenna is typically taken to be

$$R < 0.62\sqrt{l^3/\lambda}. \quad (5.6)$$

3772 The unique application of antennas by Project 8 is somewhat limited by reactive near-
 3773 field effects in the form of a maximum radial position for electrons inside the uniform
 3774 cylindrical antenna array. If electrons are too close to the edge of the array than reactive
 3775 near-field effects leads to a large reduction in the received power and consequently
 3776 detection efficiency. This leads to a significant volume inside of the antenna array that
 3777 is unsuitable for CRES lowering the volumetric efficiency of the antenna array CRES
 3778 technique relative to a cavity experiment.

3779 In between the reactive near-field and the far-field is the radiative near-field region.
 3780 In this region the fields are primarily radiative, however we are still too close to the
 3781 antenna for the spherical plane wave approximation to apply. Therefore, interference
 3782 effects between EM waves emitted from different points on the antenna occur causing the
 3783 shape of the radiation pattern to change as a function of distance from the antenna. If we
 3784 evaluate the far-field distance limit for the FSCD one finds an estimated far-field distance
 3785 of 43 cm, which is a factor of four larger than the radius of the antenna array designed for
 3786 the experiment. Consequently, we expect near-field effects to influence the performance
 3787 of the antenna array highlighting the importance of calibration and characterization
 3788 measurements.

3789 5.2.1.4 Polarization

3790 The polarization of an EM wave defines the spatial orientation of the electric field
 3791 oscillations in the plane perpendicular to the direction of the propagation, and is defined
 3792 in terms of orthogonal polarization components. In our application, one analyzes the
 3793 properties of radiation propagating along the radial (\hat{r}) direction away from the antenna,
 3794 which implies that the electric fields can be described as a linear combination of orthogonal
 3795 polarization components

$$\mathbf{E}_{\text{tot}} = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}, \quad (5.7)$$

3796 in Cartesian coordinates, or

$$\mathbf{E}_{\text{tot}} = E_\theta \hat{\theta} + E_\phi \hat{\phi}, \quad (5.8)$$

3797 in spherical coordinates.

3798 In general, one defines partial radiation patterns, directivities, and gains so that the
3799 performance of the antenna for the desired polarization can be analyzed. The radiation
3800 pattern defined in terms of partial patterns is

$$U_{\text{tot}} = U_\phi + U_\theta, \quad (5.9)$$

3801 where U_ϕ and U_θ are the radiation intensities in a particular direction for the respective
3802 polarization components. Similarly, a quantity such as gain can be written in terms of
3803 partial gains,

$$G_{\text{tot}} = G_\phi + G_\theta = \frac{2\pi U_\phi}{P_{\text{in}}} + \frac{2\pi U_\theta}{P_{\text{in}}}. \quad (5.10)$$

3804 If we view an electron performing a circular orbit in the XY-plane from the side, that
3805 is, along the X or Y axes, then we would observe the electron to be performing a linear
3806 oscillation perpendicular to the viewing axis. From this intuitive picture, we can predict
3807 that the primary polarization of electric fields from CRES events to be linearly polarized
3808 in the $\hat{\phi}$ direction when viewed with an antenna positioned in the XY-plane.

3809 5.2.1.5 Antenna Factor and Effective Aperture

3810 A useful way to characterize the performance of an antenna is to measure the electric
3811 field magnitude required to produce a signal with an amplitude of one volt in the antenna
3812 terminals. This ratio between the magnitude of the incoming electric field and the
3813 magnitude of the signal produced by the antenna is called the antenna factor, which is
3814 written as

$$A_F = \frac{|\mathbf{E}_{\text{in}}|}{V_{\text{ant}}}, \quad (5.11)$$

3815 where A_F is the antenna factor, E_{in} is the incoming electric field, and V_{ant} is the magnitude
3816 of the voltage produced by the antenna.

3817 The antenna factor can be expressed in terms of the antenna's gain through a related
3818 quantity called the effective aperture. The effective aperture defines for a given incident
3819 radiation power density (W/m^2) the power that is received by the antenna. Therefore,

3820 the effective aperture gives the equivalent area of the antenna,

$$A_{\text{eff}} = \frac{P_{\text{rec}}}{P_{\text{in}}} = \frac{\lambda^2}{4\pi} G, \quad (5.12)$$

3821 where the received power is P_r and the total incoming power is P_{in} .

3822 If we express the incident power in terms of the magnitude of the Poynting vector,
3823 then

$$|\mathbf{S}_{\text{in}}| = |\mathbf{E}_{\text{in}}|^2 / \eta_0, \quad (5.13)$$

3824 where η_0 is the impedance of free-space, which relates the magnitudes of the electric and
3825 magnetic fields in a vacuum, and is defined by

$$\eta_0 = \frac{|\mathbf{E}|}{|\mathbf{H}|} = \sqrt{\frac{\epsilon_0}{\mu_0}}. \quad (5.14)$$

3826 The total received power by the antenna can therefore be expressed as

$$P_{\text{rec}} = |\mathbf{S}_{\text{in}}| A_{\text{eff}} = |\mathbf{S}_{\text{in}}| \frac{\lambda^2}{4\pi} G = \frac{|\mathbf{E}_{\text{in}}|^2 \lambda^2 G}{4\pi \eta_0}. \quad (5.15)$$

3827 To relate this to the antenna factor recall that we can relate the voltage produced by
3828 the antenna to the received power with

$$P_{\text{rec}} = \frac{V_{\text{ant}}^2}{Z} = \frac{|\mathbf{E}_{\text{in}}|^2}{A_F^2 Z}, \quad (5.16)$$

3829 where Z is the system impedance. Setting Equations 5.15 and 5.16 equal to each other,
3830 we obtain the following expression for antenna factor in terms of gain

$$A_F = \sqrt{\frac{4\pi\eta_0}{ZG\lambda^2}} = \frac{9.73}{\lambda\sqrt{G}}. \quad (5.17)$$

3831 The second expression in Equation 5.17 is obtained by evaluating the constant terms
3832 assuming a system impedance of 50Ω .

3833 We have gone through the effort of expressing the antenna factor in terms of gain
3834 to highlight that the majority of antenna parameters that we care to measure for a
3835 CRES experiment can be obtained from the radiation or gain pattern of the antenna.
3836 The antenna factor is a particularly important parameter for CRES measurements
3837 due to its relevance to antenna array simulations with the Locust software [61, 79].
3838 Specifically, Locust simulates the trajectory of an electron in a magnetic trap by running

3839 the Kassiopeia software package [59] and then uses the Liénard-Wiechert equations [62,63]
3840 to calculate the electric fields that are incident on the antenna.

3841 To compute the response of the antenna to the electric field, Locust relies upon
3842 linear time-invariant system theory, which computes the response of the antenna (i.e. the
3843 voltage time series generated by the antenna) using a convolution between the electric field
3844 time-series and the antenna impulse response. This approach is necessary for correctly
3845 modeling the antenna response to the electric field due to the broadband and non-
3846 stationary nature of the electric fields from CRES events. Since antenna measurements
3847 take place under steady-state conditions, parameters such as the radiation pattern, gain,
3848 and antenna factor are defined in the frequency domain. However, by performing an
3849 inverse Fourier transform on the antenna factor we can obtain the antenna impulse
3850 response, which allows us to simulate CRES events in the antenna array demonstrator
3851 experiment.

3852 **5.2.2 Antenna Measurement Fundamentals**

3853 **5.2.2.1 Friis Transmission Equation**

3854 The antenna factor, sometimes called the antenna transfer function, is used to model
3855 how the antenna will respond to electric fields emitted from a CRES event. Therefore,
3856 being able to measure the antenna transfer function of the antenna array is a key step
3857 in the commissioning and calibration phases of an antenna array CRES experiment. A
3858 common approach to antenna characterization is to perform a two antenna transmit-
3859 receive measurement where an antenna with a known gain is used to characterize the
unknown gain of the antenna under test (see Figure 5.3).

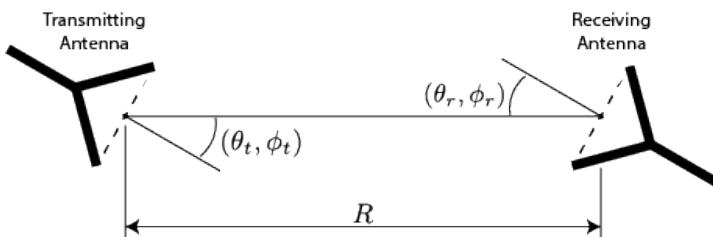


Figure 5.3. An illustration of the Friis measurement technique commonly used for antenna characterization measurements.

3860
3861 To analyze this two antenna setup we seek to calculate the amount of power from
3862 the transmitting antenna that we will detect with the receiving antenna. Using our
3863 understanding of antenna gain, we can calculate the power density transmitted by an

3864 antenna in a direction (θ_t, ϕ_t) at frequency f and distance R , which is given by

$$w_t = \frac{P_t}{4\pi R^2} G_t(\theta_t, \phi_t, f). \quad (5.18)$$

3865 Here, P_t is the total power delivered to the transmitting antenna and $G_t(\theta_t, \phi_t, f)$ is
3866 the value of the transmitting antenna gain. The power density is the power per unit
3867 area, so to calculate the total power delivered to the receiving antenna we multiply the
3868 transmitted power density by the effective area of the receiving antenna,

$$P_r = w_t A_{eff,r} = P_t \frac{G_t(\theta_t, \phi_t, f) G_r(\theta_r, \phi_r, f) c^2}{(4\pi R f)^2}, \quad (5.19)$$

3869 where $G_r(\theta_r, \phi_r, f)$ is the gain of the receiving antenna. Equation 5.19 is called the Friis
3870 transmission equation [85], which is of fundamental importance for antenna measurements,
3871 since it allows one to measure the gain of an unknown antenna by measuring the power
3872 received from an antenna with a known gain pattern. Alternatively, if no antenna with a
3873 known gain pattern is available, two identical antennas with unknown gain patterns can
3874 be used.

3875 5.2.2.2 S-Parameters and Network Analyzers

3876 Instead of directly measuring the power received by the antenna under test, it is more
3877 common to measure the ratio of the received power to the transmitted power,

$$\frac{P_r}{P_t} = \frac{G_t(\theta_t, \phi_t, f) G_r(\theta_r, \phi_r, f) c^2}{(4\pi R f)^2}. \quad (5.20)$$

3878 This power ratio can be easily measured using a vector network analyzer (VNA), which
3879 automates a significant fraction of the measurement process. Network analyzers are
3880 used to measure the scattering or S-parameters of a multi-port RF device [86], which
3881 describes how waves are scattered between the device ports. The antenna measurements
3882 we have been considering can be modeled as a two-port microwave device that we can
3883 characterize by measuring how incident voltage waves are transmitted or reflected (see
3884 Figure 5.4). We can write the scattered waves (V_1^- and V_2^-) in terms of the incident (V_1^+
3885 and V_2^+) waves using the scattering matrix

$$\begin{pmatrix} V_1^- \\ V_2^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \end{pmatrix}, \quad (5.21)$$

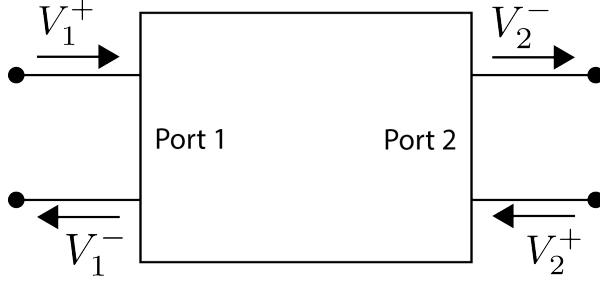


Figure 5.4. Illustration of a two-port S-parameter measurement setup. S-parameters characterize how incoming waves of voltage or power scatter off of the RF device under test. This allows you to measure important properties of the device. In particular, we can use this framework to model a two antenna radiation pattern measurement, which we can then automate using a VNA.

3886 where the elements of the matrix are the device S-parameters. It is assumed that,
 3887 when exciting the device from a particular port, that all other ports in the network are
 3888 terminated at the system impedance. This ensures that the incident waves from other
 3889 ports in the network are zero. Therefore, the S-parameters are the ratios between the
 3890 scattered and incident waves,

$$S_{ij} = \frac{V_i^-}{V_j^+}. \quad (5.22)$$

3891 Alternatively, S-parameters can be defined as the ratio of the scattered and incident
 3892 power, which is proportional to the ratio of the squared voltage waves. Returning to
 3893 our antenna measurement setup, we see that measuring the ratio of the received to the
 3894 transmitted power is equivalent to measuring the ratio of power being scattered from port
 3895 1 to port 2 in a RF network. Therefore, measuring an antenna's gain can be accomplished
 3896 quite easily, by using a VNA to perform a two port S_{21} measurement.

3897 5.2.2.3 Antenna Array Commissioning and Calibration Measurements

3898 Up to this point we have been discussing calibration and commissioning measurements
 3899 as they apply to a single antenna. While these measurements play an important role
 3900 in validating the radiation patterns of the individual array elements, the ultimate goal
 3901 is to use a phased array of these antennas. Therefore, we must also consider antenna
 3902 measurement techniques that apply to the whole array system.

3903 By measuring the gain of each individual array element we can predict the features of
 3904 the signals received during a CRES event using the antenna factor (see Section 5.2.1.5).
 3905 However, unpredictable changes to the antenna performance can be introduced by the
 3906 incorporation of the antennas into the circular array geometry, therefore, we employ both

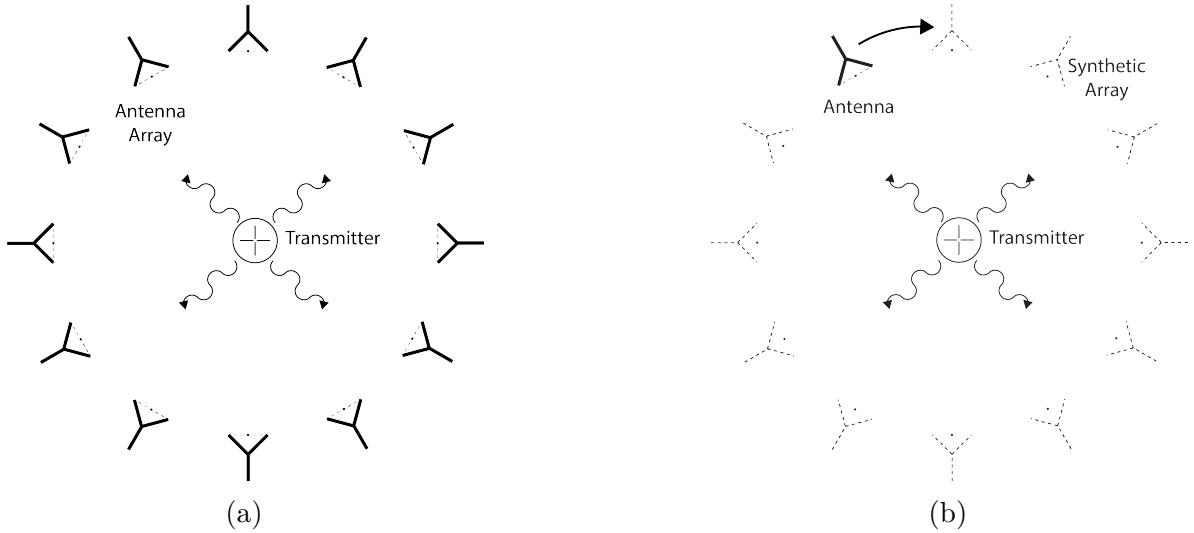


Figure 5.5. Two measurement approaches to characterizing an antenna array for CRES measurements. The full-array approach (a) requires a complete antenna array with all the associated hardware. The synthetic array approach (b) utilizes a single antenna and a set of rotation/translation stages to reposition the transmitter or the receiving antenna to synthesize the signals that would be received by the full-array. This approach reduces the cost and complexity of array measurements. A down-side of the synthetic array approach is that multi-channel effects such as reflections cannot be measured. Utilizing both the full-array and the synthetic array is a powerful way to quantify the impact of errors from the multi-channel array.

3907 individual antenna and full-array measurements in the commissioning of the FSCD to
 3908 account for these effects.

3909 There are two main approaches to array measurements that could be used for
 3910 characterization and calibration (see Figure 5.5). One approach is to construct the
 3911 complete array and use an omni-directional transmitting antenna to measure the power
 3912 received by each channel in the antenna array. In Section 5.3 we describe the development
 3913 of an omni-directional transmitter that also mimics the radiation phase characteristics of a
 3914 CRES event, which is useful because the entire array can be tested without repositioning.
 3915 Alternatively, a full antenna array can be synthesized by repeatedly moving and measuring
 3916 a single array element. This approach is ideal for identifying if different channels in the
 3917 antenna array are affecting each other through multi-path interference by comparing the
 3918 measurement results of the synthetic array to the real array.

5.2.3 The Penn State Antenna Measurement System

The development of antenna array based CRES requires the capability to test and calibrate different antenna array designs to validate the performance of the as-built antenna array before and during the experiment. With these aims in mind we developed an antenna measurement system at Penn State specifically designed to mimic the characteristics of the antenna experiment designed for demonstration of the antenna array CRES technique by the Project 8 collaboration.

The Penn State antenna measurement system utilizes a two antenna measurement configuration with a stationary reference antenna and a test antenna mounted on a set of motorized translation and rotation stages (see Figure 5.6). The antenna measurement system can be operated in two distinct modes, one focused on the characterization of the radiation patterns of prototype antennas and the other focused on the validation of data-acquisition (DAQ) and CRES signal reconstruction techniques to bridge the gap between real measurements and simulation. In both measurement configurations it is critical to isolate the antennas from the environment so that multi-path reflections do not negatively influence the measurement results. For this reason we surround the measurement volume with microwave absorber foam (AEMI AEC-1.5) specifically designed to attenuate microwave radiation near the 26 GHz measurement range of the system.

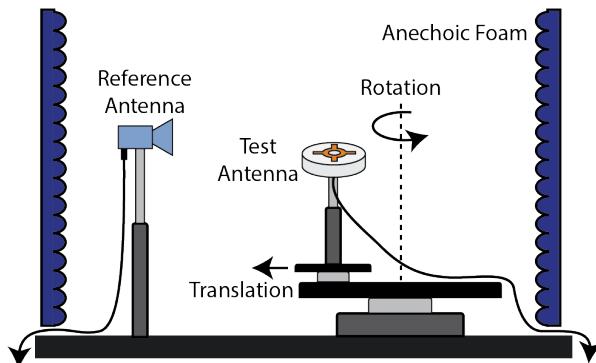


Figure 5.6. Illustration of the antenna measurement system developed for the Project 8 Collaboration. The reference and test antennas can be connected to different data acquisition configurations depending on the measurement goals. The reference antenna is typically a standard horn antenna and the test antenna is mounted on a set of translation stages for positioning. Automated translation stages allow for relatively painless data-taking enabling synthetic antenna array measurements using only a single receiving antenna. Anechoic foam designed to mitigate RF reflections surrounds the setup.

In the first measurement configuration the reference antenna is typically a well-

3939 characterized horn antenna as pictured, since horn antennas have well-known and stable
 3940 radiation patterns making them ideal as standard references. For characterization
 3941 measurements, the test antenna represents the antenna-under-test whose pattern we wish
 3942 to characterize. Mounting the test antenna on motorized rotation and translation stages
 3943 allows us to automate the procedure significantly speeding up the radiation pattern
 3944 measurement process.

3945 In the second measurement configuration one is interested in recreating the conditions
 3946 of an antenna array CRES experiment as it concerns the antenna array and DAQ system.
 3947 In this case, the reference antenna is a prototype FSCD antenna, which will be used to
 3948 construct the antenna array in the FSCD experiment, and the test antenna is a specially
 3949 designed synthetic cyclotron antenna (SYNCA) as picture in Figure 5.6. The SYNCA is
 3950 designed such that the radiation pattern mimics that of a CRES electron so that the
 3951 signals received by the prototype CRES array antenna mimic what is expected for a real
 3952 CRES experiment.

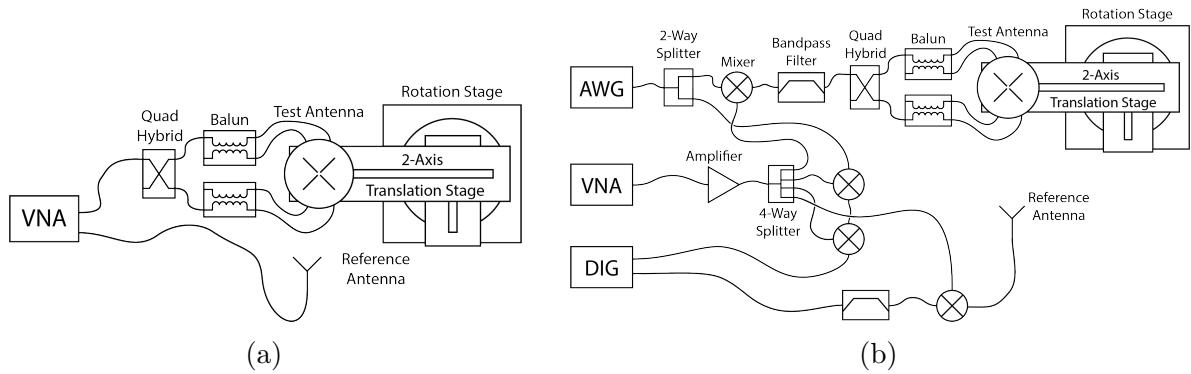


Figure 5.7. Diagrams of two measurement system configurations. Configuration (a) utilizes a VNA and is more suited to antenna characterization. Configuration (b) utilizes an AWG and VNA as a signal generation system and digitizer to collect measurement data, which is more suited to simulating CRES measurements. The transmission chain utilizes a quadrature hybrid and a pair of baluns to drive the cross-dipole variant test antenna developed for synthetic CRES measurements.

3953 In Figure 5.7 we show two high-level system diagrams of the Penn State antenna
 3954 measurement system that depict the important system components and the connections
 3955 between them. The two configurations of the measurement system utilize different
 3956 hardware. For characterization and radiation pattern measurements, one prefers the
 3957 configuration shown in Figure 5.7a. In this case a vector network analyzer (VNA) is
 3958 used as both the transmission source and data acquisition system and it is relatively
 3959 easy to calibrate over a wide range of frequencies. Whereas, if one is more interested

3960 in recreating what would take place in the FSCD experiment then the configuration
3961 shown in Figure 5.7b is preferable, since this system effectively mimics the receiver chain
3962 envisioned for the FSCD experiment.

3963 The characterization configuration utilizes a network analyzer (Keysight N5222A)
3964 with two independent sources and four measurement ports as the primary measurement
3965 tool. A standard reference antenna is connected to one measurement port, and the test
3966 antenna is connected to a separate port. The typical reference antenna used for these
3967 studies is a Pasternack PF9851 horn antenna . In the measurement shown, the test
3968 antenna represents a SYNCA antenna, which requires a transmission chain consisting of
3969 quadrature hybrid coupler (Marki QH-0226) connected to two baluns (Marki BAL-0026)
3970 to generate feed signals with the appropriate phases. The VNA measures the radiation
3971 pattern by performing a transmission S-parameter measurement, which can be used with
3972 the knowledge of the reference antenna's radiation pattern to determine the radiation
3973 pattern of the test antenna (see Section 5.2.1).

3974 The second configuration is more complicated and incorporates more hardware
3975 components in order to more closely mimic the DAQ system envisioned for the FSCD
3976 experiment. The basic approach is to produce CRES-like radiation and use an antenna
3977 combined with a realistic RF receiver chain to acquire the signals. On the transmit side,
3978 an arbitrary waveform generator (AWG, RIGOL DG5252) is used to generate a waveform
3979 that mimics a CRES signal at a baseband frequency up to 250 MHz. This frequency is
3980 then up-converted to the CRES signal frequency band of 25.8 to 26.0 GHz using a mixer
3981 (Marki MM1-0832L) and a bandpass filter (K&L Microwave 3C62-25900/T200-K/K) to
3982 reject unwanted mixing components outside out of the 200 MHz CRES signal band. The
3983 local oscillator signal for mixing is provided by one of the VNA sources configured to run
3984 in a continuous wave setting. On the receive side, a prototype antenna is used to detect
3985 the radiation emitted by the test antenna, which is down-converted and filtered using
3986 the same mixer and bandpass filter as the transmission chain. Lastly, data acquisition is
3987 performed using a 14-bit ADC sampling at 500 MSa/s (CAEN DT530) to digitize the
3988 down-converted signals.

3989 In order to distribute the LO to all mixers a 4-way power splitter (MiniCircuits
3990 ZC4PD-18263-S+) along with an amplifier (Marki APM-6848) is used to drive the four
3991 mixers used in the measurement system. A limitation of using the VNA as an LO source
3992 is that there is no control of the LO phase when a measurement is triggered by the
3993 control script, which leads to a random phase offset between acquisitions. This makes it
3994 impossible to perform synthetic array measurements, which require strict control over

3995 the starting phase of the transmitted signal. In order to monitor the random phase of the
3996 LO, a 2-way power splitter (MiniCircuits Z99SC-62-S+) is used to split the signal from
3997 the AWG between the transmission path and a LO monitoring path. The LO monitoring
3998 path consists of an up-conversion and down conversion using two mixers connected by a
3999 coaxial cable, and monitors the relative phase of the LO using a channel on the digitizer
4000 to sample this path. A phase shift in the LO will lead to a proportional phase shift in
4001 the mixed signal, which is measured and removed from the received signals.

4002 The test antenna is mounted on a set of motorized stages, which are identical for
4003 both measurement configurations. A rotational stage (ThorLabs PRMTZ8) is used as
4004 the base layer with additional translation stages mounted on top of this. The rotational
4005 stage is ideal for measuring a complete azimuthal scan of the test antenna's radiation
4006 pattern as well as for moving a SYNCA antenna in circular motion to recreate the
4007 symmetry of the FSCD antenna array. On top of the rotational stage we mount two
4008 linear translation stages (ThorLabs MTS50-Z8 and MTS25-Z8) in a cross-wise manner
4009 so that the test antenna can be moved along two perpendicular axes. Using the linear
4010 stages in combination with the rotational stage allows one to fine-tune the positioning of
4011 the test antenna so that it can be perfectly aligned with the central axis of the array.
4012 A LabView script was developed to automate the measurement of a full 360° radiation
4013 pattern and control the measurement electronics. Data from these acquisitions is stored
4014 on university provided cloud storage.

4015 **5.3 Development of a Synthetic Cyclotron Antenna (SYNCA) 4016 for Antenna Array Calibration**

4017 This section is the manuscript of the publication [80] detailing the development of a
4018 Synthetic Cyclotron Antenna (SYNCA) for antenna array characterization measurements
4019 by the Project 8 collaboration.

4020 **5.3.1 Introduction**

4021 Neutrinos are the most abundant standard model fermions in our universe, but due to
4022 weak interaction cross-sections with other particles, neutrinos are particularly difficult
4023 to study. Consequently, many fundamental properties of neutrinos are still unknown
4024 including the absolute scale of the neutrino mass [23]. Direct, kinematic measurements of
4025 the neutrino mass are particularly valuable due to their model independent nature [36].

4026 To date the most sensitive direct neutrino mass measurements have been performed by
 4027 the KATRIN collaboration [87], which measures the molecular tritium β -decay spectrum
 4028 to infer the neutrino mass. Current data from neutrino oscillation measurements [23]
 4029 allow for neutrino masses significantly smaller than the design sensitivity of the KATRIN
 4030 experiment; therefore, there is a need for new technologies for performing direct neutrino
 4031 mass measurements to probe lower neutrino masses.

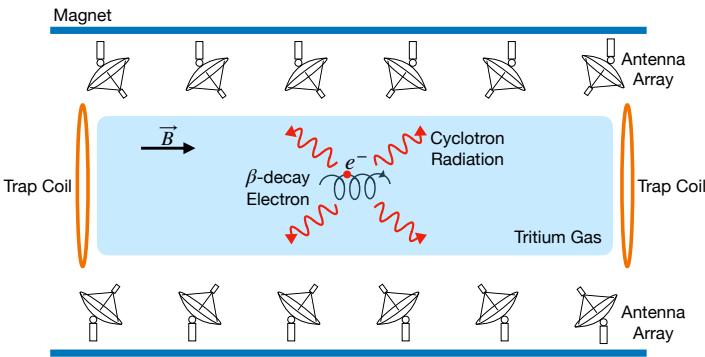


Figure 5.8. A sketch of an antenna array large-volume CRES experiment. Electrons from β -decays are confined in a magnetic field using a set of trap coils. The cyclotron radiation produced by the motion of the trapped electrons can be detected by a surrounding antenna array to determine the electron energies. Measuring the energies of many electrons produces a β -decay spectrum.

4032 The Project 8 collaboration is developing new methods for neutrino mass measurement
 4033 based on Cyclotron Radiation Emission Spectroscopy (CRES) [55, 88–90], with the goal
 4034 of measuring the absolute scale of the neutrino mass with a 40 meV/c² sensitivity [?, 36].
 4035 This sensitivity goal will require the development of two separate technical capabilities.
 4036 First is the development of an atomic tritium source, which avoids significant spectral
 4037 broadening due to molecular final states [54]. Second is the technology for performing
 4038 CRES in a multi-cubic-meter experimental volume with high combined detection and
 4039 reconstruction efficiency, which is required in order to obtain sufficient event statistics
 4040 near the tritium spectrum endpoint.

4041 One approach for a large-volume CRES experiment is to use an array of antennas,
 4042 which surrounds a volume of tritium gas, to detect the cyclotron radiation produced
 4043 by the β -decay electrons when they are trapped in a background magnetic field using a
 4044 set of magnetic trapping coils (see Figure 5.8). Project 8 has developed a conceptual
 4045 experiment design to study the feasibility of this approach. The design consists of a
 4046 single circular array of antennas with a radius of 10 cm and 60 independent channels
 4047 positioned around the center of the magnetic trap. The motivation behind this antenna

array design is to first develop an understanding of the antenna array approach to CRES
 with a small scale experiment before attempting to scale the technique to large volumes
 by using multiple antenna rings to construct the full cylindrical array. The development
 of the antenna array approach to CRES has largely proceeded through simulations using
 the Locust software package [79, 91], which is used to model the fields emitted by CRES
 events and predict the signals received by the surrounding antenna array. To validate
 these simulations, a dedicated test stand is being constructed to perform characterization
 measurements of the prototype antenna array developed by Project 8 (see Figure 5.9)
 and benchmark signal reconstruction methods using a specially designed transmitting
 calibration probe antenna.

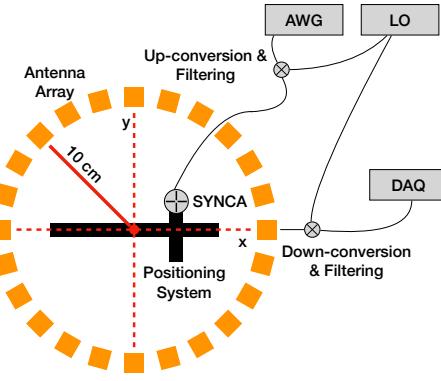


Figure 5.9. A schematic of the antenna array test stand. The circular antenna array has a radius of 10 cm with 60 independent channels (limited number shown for clarity). The test stand includes an arbitrary waveform generator (AWG), local oscillator (LO), and data acquisition (DAQ) hardware. Finally, a specialized Synthetic Cyclotron Antenna (SYNCA) is used to inject signals to test the antenna array.

We call this probe antenna the Synthetic Cyclotron Antenna or SYNCA. The SYNCA
 is a novel antenna design that mimics the cyclotron radiation generated by individual
 charged particles trapped in a magnetic field, which will be used in the antenna test
 stand to perform characterization measurements, simulation validation, and reconstruc-
 tion benchmarking. This paper provides an overview of the design, construction, and
 characterization measurements of the SYNCA performed in preparation for its usage as
 a transmitting calibration probe.

In Section 5.3.2 we provide a description of the cyclotron radiation field characteristics
 that we recreate with the SYNCA. In Section 5.3.3 we give an overview of the simulations
 performed to develop an antenna design that mimics the characteristics of cyclotron
 radiation. In Section 5.3.4 we outline characterization measurements to validate that
 the fields generated by the SYNCA match simulation, and finally in Section 5.3.5 we

4070 demonstrate an application of the SYNCA to test phased array reconstruction techniques
 4071 on the bench-top.

4072 5.3.2 Cyclotron Radiation Phenomenology

4073 To understand the cyclotron radiation phenomenology that the SYNCA should mimic,
 4074 we consider a charged particle moving at relativistic speed in the presence of an external
 4075 magnetic field (see Figure 5.10). In the special case we shall examine, the entirety of
 4076 the electron's momentum is directed perpendicular to the magnetic field; therefore, the
 4077 trajectory of the electron is confined to the cyclotron orbit plane. Because the momentum
 4078 vector is oriented perpendicular to the magnetic field, electrons with these trajectories
 4079 are said to have pitch angles of 90°.

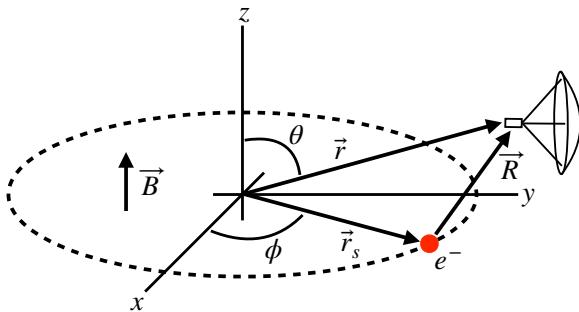


Figure 5.10. An electron (red dot) performing cyclotron motion in the x-y plane. The resulting cyclotron radiation is observed by an antenna located at the field point of interest.

4080 The cyclotron radiation fields generated by this circular trajectory are those which
 4081 we aim to reproduce with the SYNCA. We can describe the electromagnetic (EM) fields
 4082 using the Liénard-Wiechert equations [48, 79], which in non-covariant form express the
 4083 electric field as

$$\vec{E} = e \left[\frac{\hat{n} - \vec{\beta}}{\gamma^2 (1 - \vec{\beta} \cdot \hat{n})^3 |\vec{R}|^2} \right]_{t_r} + \frac{e}{c} \left[\frac{\hat{n} \times [(\hat{n} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \vec{\beta} \cdot \hat{n})^3 |\vec{R}|} \right]_{t_r}, \quad (5.23)$$

4084 where e is the particle's charge, $\hat{n} = (\vec{r} - \vec{r}_s)/|\vec{r} - \vec{r}_s|$ is the unit vector pointing from the
 4085 electron to the field measurement point, $\vec{\beta} = \vec{r}_s/c$ is the velocity of the particle divided
 4086 by the speed of light, and γ is the relativistic Lorentz factor. The equation is meant to
 4087 be evaluated at the retarded time as indicated by $t_r = t - |\vec{R}|/c$, which accounts for the
 4088 time delay due to the finite speed of light between the point where the field was emitted
 4089 and the point where the field is detected.

4090 We would like to simplify Equation 5.23 it at all possible. As a first step we analyze
 4091 the relative magnitudes of the electric field polarization components. Consider an electron
 4092 following a circular cyclotron orbit in a uniform magnetic field whose guiding center
 4093 is positioned at the origin of the coordinate system. The equation of motion can be
 4094 expressed as

$$\vec{r}_s = (r_c \cos \omega_c t_r) \hat{x} + (r_c \sin \omega_c t_r) \hat{y}. \quad (5.24)$$

4095 For single antenna located along the y -axis at position $\vec{r} = r_a \hat{y}$ we are interested in the
 4096 incident electric fields from the electron. The electric field is given by Equation 5.23,
 4097 which we evaluate in the regime where $r_a \gg r_c$. This limit can be justified by comparing
 4098 the radius of the cyclotron orbit for an electron with the tritium beta-spectrum endpoint
 4099 energy of 18.6 keV in a 1 T magnetic field to the typical ($r_a \simeq 100$ mm) radial position
 4100 of the receiving antenna. We find that the cyclotron orbit has a radius of 0.46 mm which
 4101 is approximately a factor of 200 smaller than the typical antenna radial position. In this
 4102 regime we can make the approximation $\vec{R} \simeq r_a \hat{y}$ and the expression for the electric field
 4103 at the antenna's position becomes

$$\vec{E} = \frac{e}{\gamma^2 r_a^2} \frac{\hat{x}(\frac{r_c \omega_c}{c} \sin \omega_c t_r) + \hat{y}(1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r)}{(1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r)^3} - \frac{e}{cr_a} \frac{\hat{x}(\frac{r_c^2 \omega_c^3}{c^2} - \frac{r_c \omega_c^2}{c} \cos \omega_c t_r)}{(1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r)^3}. \quad (5.25)$$

4104 Since the receiving antenna is part of a circular array of antennas, it is useful to rewrite
 4105 Equation 5.25 in terms of the azimuthal ($\hat{\phi}$) and radial (\hat{r}) polarizations. Making use of
 4106 the fact that for an antenna located at $R = r_a \hat{y}$ that $\hat{\phi} = -\hat{x}$ and $\hat{r} = \hat{y}$ we find

$$\vec{E} = \hat{\phi} E_\phi + \hat{r} E_r \quad (5.26)$$

$$E_\phi = \frac{e}{(1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r)^3} \left[-\frac{\frac{r_c \omega_c}{c} \sin \omega_c t_r}{\gamma^2 r_a^2} + \frac{\omega_c \left(\frac{r_c^2 \omega_c^2}{c^2} - \frac{r_c \omega_c}{c} \cos \omega_c t_r \right)}{cr_a} \right] \quad (5.27)$$

$$E_r = \frac{e(1 - \frac{r_c \omega_c}{c} \sin \omega_c t_r)}{\gamma^2 r_a^2 (1 - \frac{r_c \omega_c}{c} \cos \omega_c t_r)^3}. \quad (5.28)$$

4107 For the purposes of designing a synthetic cyclotron radiation antenna we are interested
 4108 in the dominant electric field polarization emitted by the electron. The antenna is being
 4109 designed to mimic the cyclotron radiation produced by electrons with kinetic energies of
 4110 approximately 18.6 keV in a 1 T magnetic field [54]. Since the relativistic beta factor for
 4111 an electron with this kinetic energy is $|\vec{\beta}| \simeq \frac{1}{4}$, the approximations $\gamma \simeq 1$ and $\frac{r_c \omega_c}{c} \simeq \frac{1}{4}$ are
 4112 justified. Inserting these expressions into the equations for the electric field components

4113 above simplifies the comparison of the magnitudes of the two components. Additionally,
 4114 we compare the time-averaged magnitudes to evaluate the root mean squared electric
 4115 field ratio. The time-averaged ratio of the radial and azimuthally polarized electric fields
 4116 with the above simplifications is given by

$$\frac{\langle |E_r| \rangle}{\langle |E_\phi| \rangle} = \frac{8 - \sqrt{2}}{\left| 1 - \frac{r_a}{r_c} \frac{1-2\sqrt{2}}{8} \right|} \simeq \frac{r_c}{r_a} \frac{8(8 - \sqrt{2})}{2\sqrt{2} - 1} = 0.13, \quad (5.29)$$

4117 where we have made use of the fact that for these magnetic fields and kinetic energies
 4118 the cyclotron radius is much smaller than the radius of the antenna array.

4119 From Equation 5.29 we see that the time-averaged azimuthal polarization is larger than
 4120 the radial polarization by about a factor of 8, which makes it the dominant contribution
 4121 to the electric fields at the position of the antenna. We must also consider the directivity
 4122 of the receiving antenna which can have a gain that is disproportionately large for a
 4123 specific polarization component. Because the E_ϕ component is dominant, the receiving
 4124 antenna array is designed with an azimuthal polarization, which negates the voltages
 4125 induced in the antenna from the radially polarized fields. Therefore, we conclude that
 4126 for the purpose of designing the SYNCA antenna it is acceptable to approximate the
 4127 electric fields from Equation 5.23 as purely azimuthally or ϕ -polarized. The simplified
 4128 expression for the electric field received by an antenna becomes

$$\vec{E} = E_\phi \hat{\phi} = \frac{e \frac{r_c \omega_c}{c}}{4r_a r_c} \left[\frac{\frac{r_c \omega_c}{c} - \cos \omega_c t - \frac{4r_c}{r_a} \sin \omega_c t}{(1 - \frac{r_c \omega_c}{c} \cos \omega_c t)^3} \right]_{t_r} \hat{\phi}, \quad (5.30)$$

4129 where the radius of the cyclotron orbit is called r_c , the cyclotron frequency is called ω_c ,
 4130 and the radial position of the receiving antenna is called r_a . Equation 5.30 has been
 4131 evaluated in the non-relativistic limit where $\gamma \simeq 1$, which is justified by the fact that
 4132 $|\vec{\beta}| \simeq \frac{c}{4}$ for an electron with an 18.6 keV kinetic energy in a 1 T magnetic field.

4133 This rather complicated expression can be simplified using Fourier analysis. Assuming
 4134 a background magnetic field of 1 T and a kinetic energy of 18.6 keV we calculate
 4135 numerically the electric field using Equation 5.30 and apply a discrete Fourier Transform
 4136 to visualize the frequency spectrum (see Figure 5.11).

4137 We observe that the azimuthally polarized electric field is periodic with a base cyclotron
 4138 frequency of 25.898 GHz corresponding to the highest power frequency component in
 4139 Figure 5.11. The frequency spectrum reveals that the signal is composed of a constant
 4140 term with zero frequency and a series of harmonics separated by 25.898 GHz. Therefore,

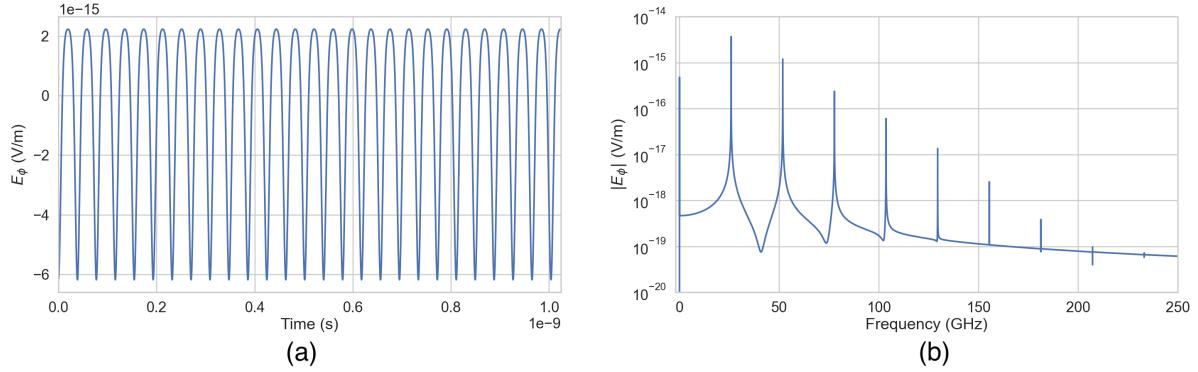


Figure 5.11. A plot of the numeric solution to Equation 5.31. The time-domain representation of the signal (a) is composed of a zero frequency term and a series of harmonics separated by the main cyclotron frequency as shown in the plot of the frequency spectrum (b). We can see that the relative amplitude of the harmonics beyond $k = 7$ are smaller than the main carrier by a factor of about 10^{-5} and are completely negligible.

we can represent the azimuthal electric fields from the electron as a linear combination of pure sinusoids with frequencies given by $\omega_k = k\omega_c$ ($k \in 0, 1, 2, \dots$) and amplitudes extracted from the Fourier representation. Using this representation we can transform the equation for the azimuthally polarized electric fields in Equation 5.30 into

$$E_\phi = \frac{e^{\frac{r_c \omega_c}{c}}}{4r_a r_c} \sum_{k=0}^7 A_k e^{i\omega_k t_r}, \quad (5.31)$$

where we have truncated the sum over harmonics at the 7th order for completeness. The amplitudes A_k are dimensionless complex numbers, which encode the relative powers of the harmonics as well as the starting overall phase of the cyclotron radiation. Because magnitude of the relative amplitudes exponentially decreases for higher harmonics, it is usually sufficient to consider only the terms up to $k = 4$ where the relative amplitude of the harmonics has decreased from the main carrier by a factor of approximately 100. However, for completeness we include harmonics up to 7th order in Equation 5.31. The range of frequencies to which the receiving antenna array in the antenna test stand is sensitive is defined by the antenna's transfer function. The receptive bandwidth for the antennas used in the test stand is a range of frequencies with a bandwidth on the order of a few GHz centered around the main cyclotron carrier frequency of 25.898 GHz. Therefore, the higher order harmonics as well as the zero frequency term can be ignored when considering only the signals that will be received by the antenna array.

Considering only the 1st order harmonic term from Equation 5.31, which represents the portion of the electric field that will be detected by the array, and evaluating this at

⁴¹⁶⁰ the retarded time we obtain the following for the ϕ -polarized electric fields

$$E_\phi \propto \cos \left(\omega_c \left(t - |\vec{R}|/c \right) - \Delta \right), \quad (5.32)$$

⁴¹⁶¹ where the arbitrary phase Δ is defined by $A_k = |A_k|e^{i\Delta}$. We are interested in the
⁴¹⁶² characteristics of the amplitude of the electric field as a function of the radial distance
⁴¹⁶³ component ($|\vec{R}|$) of the retarded time. In particular, the maximum of E_ϕ occurs when
⁴¹⁶⁴ the argument of the cosine function is equal $n\pi$ where $n \in \{0, \pm 2, \pm 4, \dots\}$; however, the
⁴¹⁶⁵ solutions where n is negative can be discarded since they represent unphysical negative
⁴¹⁶⁶ overall phases. Applying this condition to Equation 5.32 gives a condition on the radial
⁴¹⁶⁷ position of the maximum of E_ϕ

$$\omega_c(t - |\vec{R}|/c) - \Delta = n\pi, \quad (5.33a)$$

$$|\vec{R}| = \frac{c}{\omega_c} ((\omega_c t - \Delta) - n\pi), \quad (5.33b)$$

⁴¹⁶⁸ which is a function of time in the frame of the moving electron (t). Equation 5.33 can
⁴¹⁶⁹ be further simplified by noticing that the azimuthal position of the electron ($\phi_e(t)$) as a
⁴¹⁷⁰ function of time is defined by $\phi_e(t) = \omega_c t - \Delta$ which reduces Equation 5.33 to

$$|\vec{R}| = \frac{c}{\omega_c} (\phi_e(t) - n\pi). \quad (5.34)$$

⁴¹⁷¹ Equation 5.34 represents an archimedian spiral which is formed when plotting the
⁴¹⁷² amplitude of E_ϕ in the x-y plane. The solution where $n = 0$ represents the leading edge
⁴¹⁷³ of the radiation spiral which propagates outward from the electron at the speed of light.
⁴¹⁷⁴ The additional solutions for $n > 0$ represent the persistent spiral at radii inside the
⁴¹⁷⁵ leading edge of the radiated fields that have not yet been detected by the receiver at the
⁴¹⁷⁶ current time. In Figure 5.12a we show the expected spiral pattern for the maxima of the
⁴¹⁷⁷ cyclotron radiation.

⁴¹⁷⁸ In particular, we note that for the circular array geometry of the test stand, depicted
⁴¹⁷⁹ as the series of circles in Figure 5.12a, each antenna receives a linearly polarized wave
⁴¹⁸⁰ with a phase offset that corresponds to the azimuthal angle for that antenna element.
⁴¹⁸¹ Therefore, as we show in Figure 5.12b, when the relative phase of the received signal is
⁴¹⁸² plotted as a function of the receiving antenna's azimuthal position the result is also an
⁴¹⁸³ Archimedean spiral.

⁴¹⁸⁴ Based on these analytical calculations we can characterize the magnitude, polarization,
⁴¹⁸⁵ and phase of the signals received by the antenna array using three criteria. These criteria

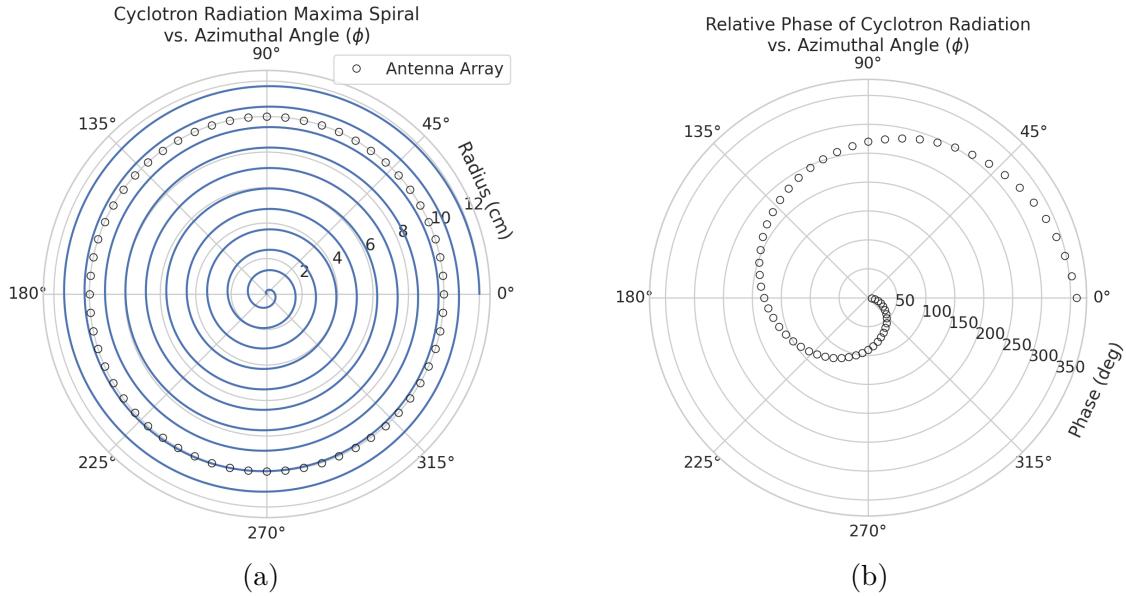


Figure 5.12. The amplitude maxima of the cyclotron radiation form an Archimedean spiral as the radiation propagates outward from the cyclotron orbit center (a). A circular antenna array located at a fixed radius from the orbit center will receive electric fields with equal magnitude in each of its channels, but the phase of the electric field incident on each array channel will be linearly out of phase from its neighbor antennas by an amount equal to the angular separation of the two channels (b).

4186 are the basis of comparison for the radiation produced by the SYNCA and cyclotron
 4187 radiation emitted by electrons and will be used to evaluate the performance of antenna
 4188 designs. The criteria are:

- 4189 1. Electric fields that are ϕ -polarized near $\theta = 90^\circ$
- 4190 2. Uniform time-averaged electric field magnitudes around the circumference of a
 4191 circle centered on the antenna
- 4192 3. Electric fields whose phase is equal to the azimuthal angle at the point of measure-
 4193 ment plus a constant

4194 The Locust simulation package [91] can be used to directly simulate the EM fields
 4195 generated by electrons performing cyclotron motion to validate the analytical calculations.
 4196 Locust simulates the EM fields by first calculating the trajectory of the electrons in
 4197 the magnetic trap using the Kassiopeia software package [92]. The trajectory can then
 4198 be used to solve for the EM fields using the Liénard-Wiechert equations directly with
 4199 no approximations. The resulting electric field solutions drive a receiving antenna by

convolving the time-domain fields with the finite-impulse response filter of the antenna
 or they can be examined directly to study the field characteristics that the SYNCA must
 reproduce. In the next section we compare the radiation field patterns for electrons
 simulated with Locust to patterns from a SYNCA antenna design.

5.3.3 SYNCA Simulations and Design

One potential SYNCA design is the crossed-dipole antenna [93]. A crossed-dipole antenna
 consists of two dipole antennas, one of which is rotated 90° with respect to the other,
 which are fed with signals that are out of phase from the opposite dipole by 90° (see
 Figure 5.13). This arrangement causes the signals fed to each arm of the dipole to be

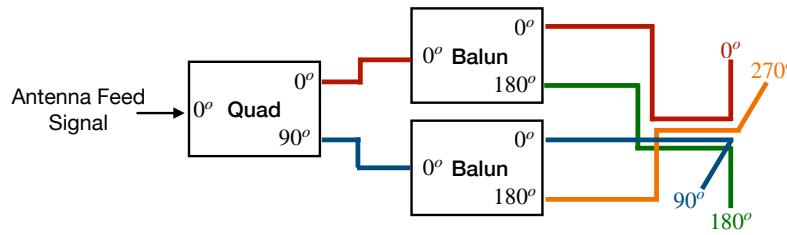


Figure 5.13. An idealized crossed-dipole antenna consists of two electric dipole antennas oriented perpendicular to each other and is fed with four signals with a quadrature phase relationship. An example antenna feed circuit is shown which is composed of a chained combination of a quadrature hybrid-coupler (Quad) and two baluns.

out of phase from each of the neighboring arms by 90°, which mirrors the spatial phase
 relationship of cyclotron radiation fields.

A potential drawback of this design is that standard crossed-dipole antennas do not
 radiate uniform electric fields near the $\theta = \pi/2$ plane. Typical crossed-dipole antennas
 use dipole arm lengths equal to $\lambda/4$ or larger [93], where λ is the wavelength at the
 desired operating frequency. Such large arm lengths cause the electric field magnitude
 to vary significantly around the circumference of the antenna. However, making the
 antenna electrically small by shrinking the arm length can improve the antenna pattern
 uniformity.

In general, the criterion for an electrically small antenna is that the largest dimension
 of the antenna (D) obey $D \lesssim \lambda/10$ [64]. In our application, we are attempting to mimic
 the cyclotron radiation emitted by electrons produced from tritium β -decay with energies
 near the spectrum endpoint. For a background magnetic field of 1 T, the corresponding

4222 cyclotron frequency of tritium endpoint electrons is approximately 26 GHz. Therefore, the
4223 electrically small condition would require that the largest dimension of the crossed-dipole
4224 antenna be smaller than 1.2 mm.

4225 A crossed-dipole antenna with an overall size of 1.2 mm is challenging to fabricate due
4226 to the small dimensions of the dipole arms that, in practice, are fragile and unsuitable
4227 for use as a calibration probe. To mitigate some of the challenges with the fabrication
4228 of such a small antenna, a variant crossed-dipole antenna design using printed circuit
4229 board (PCB) technology (see Figure 5.14) was developed in partnership with an antenna
prototyping company, Field Theory Consulting ¹.

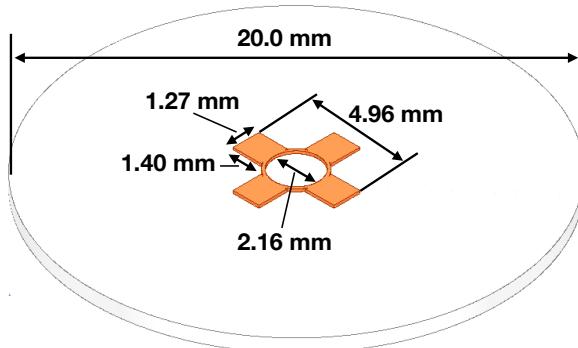


Figure 5.14. A model of the PCB crossed-dipole antenna with dimensions. The design has an inside diameter of 2.16 mm for the central circular trace, which is 0.13 mm wide. The dipole arms each have a width of 1.27 mm and protrude beyond the circular trace by 1.40 mm, which gives an overall width of 4.96 mm for the length of the antenna PCB trace from end-to-end. The overall size of the antenna is 20.0 mm the majority of which is the PCB dielectric material. This design was observed in simulation to maintain the field characteristics of the idealized crossed-dipole while being simpler to fabricate due to the increased size of the antenna.

4230
4231 The PCB crossed-dipole design uses four rectangular pads to represent the dipole arms,
4232 which are connected by a thin circular trace. The circular trace both adds mechanical
4233 stability to the antenna and improves the azimuthal uniformity of the electric fields
4234 compared to a more standard crossed-dipole geometry. Furthermore, the circular trace
4235 allows for a greater separation between dipole arms than standard crossed-dipoles, which
4236 is required to accommodate the coaxial connections to each pad. The pads each contain
4237 a through-hole solder joint to connect coaxial transmission lines using hand soldering.
4238 The antenna PCB has no ground plane on the bottom layer as this was observed in
4239 simulation to significantly distort the radiation pattern in the plane of the PCB. The
4240 only ground planes present in the model are the outer conductors of the four coaxial

¹<https://fieldtheoryinc.com/>

4241 transmission lines which feed the antenna. These are left unterminated on the bottom of
 4242 the PCB dielectric material.

4243 The antenna design development utilized a combination of Locust electron simula-
 4244 tions and antenna simulations using ANSYS HFSS [65], a commercial finite-element
 4245 electromagnetic simulation software. Two antenna designs were simulated: an idealized
 4246 electrically small crossed-dipole antenna with an arm length of 0.40 mm and an arm
 4247 separation of 0.05 mm, as well as a PCB crossed-dipole antenna with the dimensions
 4248 shown in Figure 5.14. Plotting the magnitude of the electric fields generated by the
 4249 antennas across a 10 cm square located in the same plane as the respective antennas
 4250 reveals the expected cyclotron spiral pattern (see Figure 5.15) which closely matches
 4251 the prediction for simulated electrons. The spiral pattern demonstrates that the electric
 4252 fields have the appropriate phases to mimic cyclotron radiation, which fulfills SYNCA
 criterion 3 identified in Section 5.3.2.

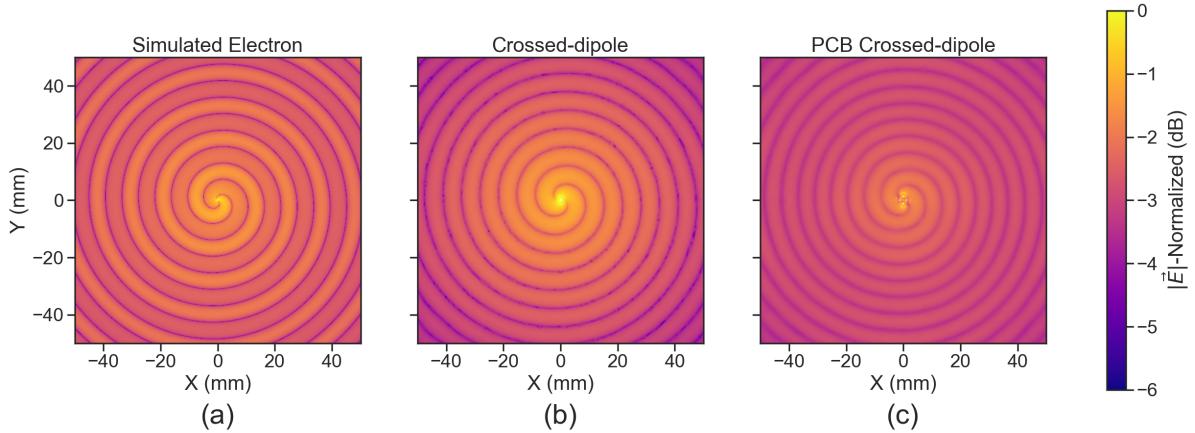


Figure 5.15. A comparison of the electric field magnitudes, normalized by the maximum value of the electric field in each simulation, plotted on a 10 cm square to visualize the Archimedean spirals formed by the electron (a), the crossed-dipole antenna (b), and a PCB crossed-dipole antenna (c). The matching patterns indicate that the electric fields have similar phase characteristics. These images were generated using Locust simulations for the electron and ANSYS HFSS for both antennas.

4253
 4254 As we can see from Figure 5.16, the crossed-dipole antenna, which uses an idealized
 4255 geometry, exhibits good agreement with simulation. The antenna has a maximum
 4256 deviation from a simulated electron of approximately 0.5 dB in the total electric field, 1
 4257 dB for the ϕ -polarized electric field and 1 dB for the θ -polarized electric field.

4258 In comparison, the pattern of the PCB crossed-dipole antenna, because the simulation
 4259 incorporates the geometry of the coax transmission lines, exhibits some distortion from
 4260 the idealized cross-dipole simulations. The vertically oriented ground planes of the coax

4261 lines introduce more θ -polarized electric fields than are observed for simulated electrons
 4262 near $\theta = 90^\circ$. The significant θ -polarized field minimum is still present but shifted
 to approximately $\theta = 65^\circ$. The θ -polarized field deviations of the PCB crossed-dipole

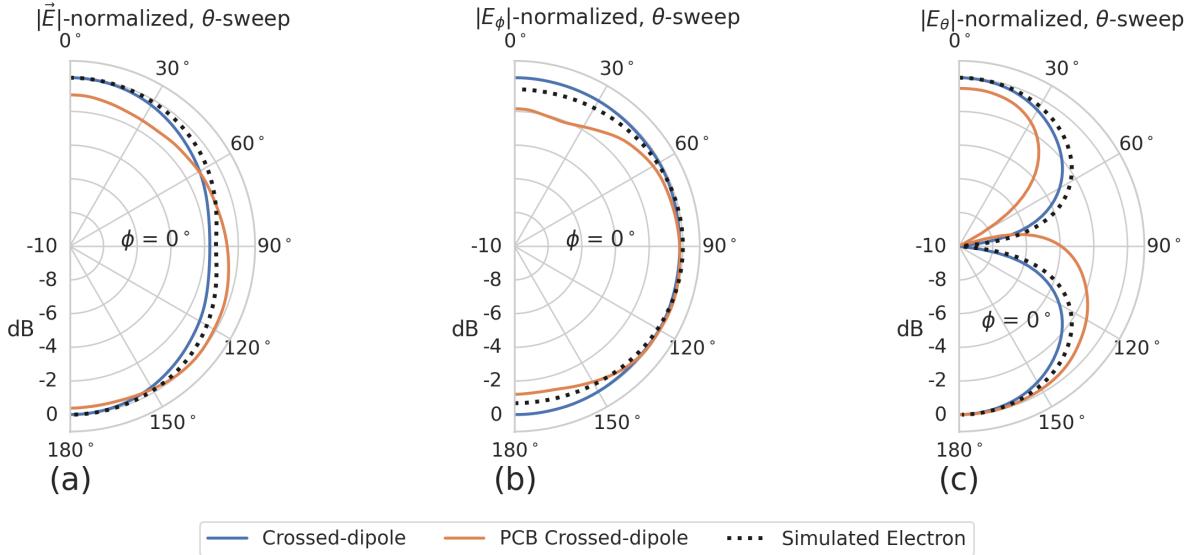


Figure 5.16. A comparison of the normalized electric field magnitudes for the ideal crossed-dipole, PCB crossed-dipole, and a simulated electron as a function of the polar angle (θ). (a) Shows the total electric field, (b) shows the ϕ -polarized electric field component, and (c) shows the θ -polarized electric field component. These images were generated using Locust simulations for the electron and ANSYS HFSS for both antennas.

4263
 4264 antenna should not greatly impact the performance of the antenna because the receiving
 4265 antenna array is primarily ϕ -polarized. Therefore deviations in the θ -polarized fields
 4266 will be suppressed due to the polarization mismatch. More importantly, the ϕ -polarized
 4267 electric field pattern generated by the PCB crossed-dipole closely matches simulated
 4268 electrons across the polar angle range of $50^\circ < \theta < 150^\circ$. In this region the PCB crossed-
 4269 dipole differs by less than 0.5 dB from simulated electrons. This range greatly exceeds
 4270 the beamwidth of the receiving antenna array which is designed to be most sensitive
 4271 to fields produced near $\theta = 90^\circ$. Therefore, we conclude that the PCB crossed-dipole
 4272 antenna generates a ϕ -polarized radiation pattern that fulfills SYNCA criterion 1 from
 4273 Section 5.3.2.

4274 The final SYNCA criterion is related to the uniformity of the electric fields when
 4275 measured azimuthally around the antenna. As we saw for real electrons in Section 5.3.2
 4276 it is expected that the magnitude of the electric field be completely uniform as a function
 4277 of the azimuthal angle due to the symmetry of the cyclotron orbit. In Figure 5.17 we plot
 4278 the total electric field as a function of azimuthal angle for an electron, the crossed-dipole

antenna, and the PCB crossed-dipole antenna. The crossed-dipole antenna exhibits

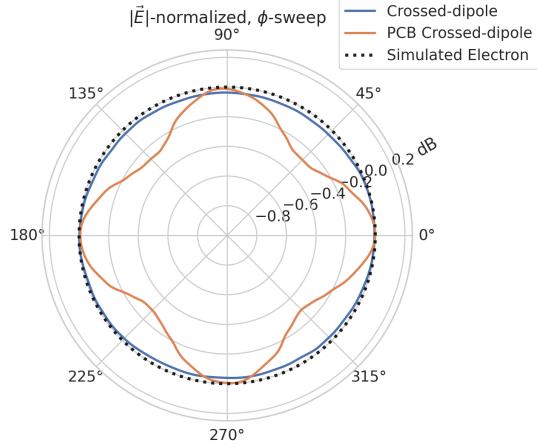


Figure 5.17. A comparison of the normalized electric field magnitudes for the crossed-dipole, PCB crossed-dipole, and a simulated electron as a function of the azimuthal angle (ϕ) evaluated at $\theta = 90^\circ$. This image was generated using Locust simulations for the electron and ANSYS HFSS for both antennas.

perfect uniformity around the azimuthal angle, whereas the PCB crossed-dipole has a small periodic deviation with a maximum difference of 0.3 dB caused by the coaxial transmission lines below the PCB. Such a small deviation from uniformity is acceptable since it is smaller than the expected variation in uniformity caused by imperfections in the antenna fabrication process, which modifies the antenna shape in an uncontrolled manner by introducing solder blobs with a typical size of a few tenths of a millimeter on the dipole arms (see Figure 5.18). Additionally, the SYNCA will be separately calibrated to account for azimuthal differences in the electric field magnitude. Therefore we see from the simulated performance of the PCB crossed-dipole antenna that this antenna design meets all three of the SYNCA criteria.

5.3.4 Characterization of the SYNCA

Two SYNCAs were manufactured using the PCB crossed-dipole design (see Figure 5.18). The antenna PCB (Matrix Circuit Board Materials, MEGTRON 6) is connected to four 2.92 mm coaxial connectors (Fairview Microwave, SC5843) using semi-rigid coax (Fairview Microwave, FMBC002), which also physically support the antenna PCB. The antenna PCB consists only of two layers which correspond to the copper antenna trace and the PCB dielectric. Each coax line is connected to the associated dipole arm using through-hole soldering and phase matched to ensure that the electrical length of each of the transmission lines is identical at the operating frequency. The antenna PCB is

4299 further reinforced using custom cut polystyrene foam blocks, which have an electrical
 4300 permittivity nearly identical to air. A custom 3D printed mount is included at the base
 4301 of the antenna to support the coax connectors and to provide a sturdy mounting base.

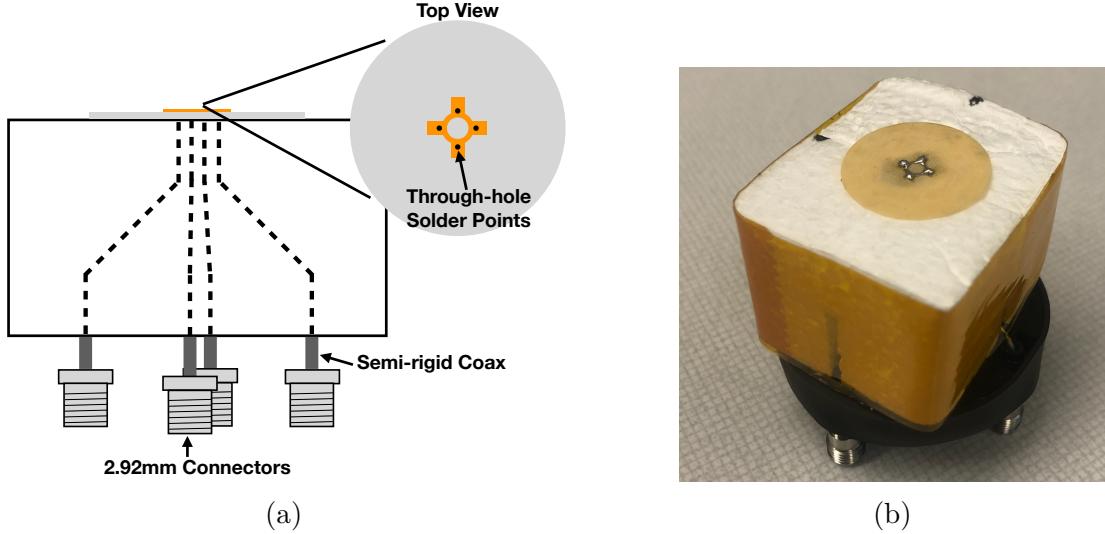


Figure 5.18. (a) A cartoon schematic which highlights the routing of the semi-rigid coax transmission lines. (b) A photograph of a SYNCA constructed using the modified crossed-dipole PCB antenna design. Visible in the photograph of the SYNCA are four blobs of solder which are an artifact of the SYNCA's hand-soldered construction. These solder blobs are the most significant deviation from the SYNCA design shown in Figure 5.14 and are responsible for a significant fraction of the irregularities seen in the antenna pattern.

4302 Characterization measurements were performed using a Vector Network Analyzer
 4303 (VNA) to measure the electric field magnitude and phase radiated by the SYNCA to
 4304 verify the radiation pattern (see Figure 5.19). The VNA is connected to the SYNCA
 4305 at one port through a hybrid-coupler whose outputs are connected to two baluns to
 4306 generate the signals with the appropriate phases to feed the SYNCA (see Figure 5.13).
 4307 The other port of the VNA is connected to a single reference horn antenna that serves
 4308 as a field probe. To position the SYNCA, a combination of translation and rotation
 4309 stages are used to characterize the antenna's fields across the entire radiation pattern
 4310 circumference. This measurement scheme is equivalent to measuring the fields generated
 4311 by the SYNCA using a full circular array of probe antennas.

4312 The antenna measurement space is surrounded by RF anti-reflective foam to isolate
 4313 the measurements from the lab environment (see Figure 5.19b) and remaining reflections
 4314 are removed using the VNA's time-gating feature. The SYNCA is affixed to the stages
 4315 by a custom RF transparent mount made of polystyrene foam. The coaxial cables deliver

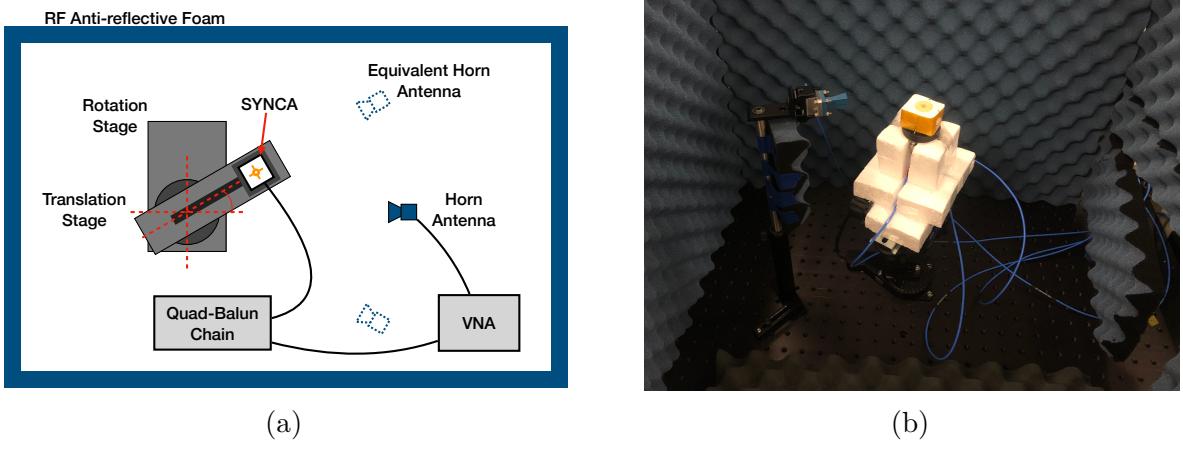


Figure 5.19. A schematic of the VNA characterization measurements (a). This setup allows for antenna gain and phase measurements across a full 360° of azimuthal angles using a motorized rotation stage and control of the radial position of the SYNCA using a translation stage. A photo of the setup in the lab is shown in (b).

4316 the antenna feed signals generated by the VNA to the SYNCA while still allowing
 4317 unrestricted rotation. The horn antenna probe is nominally positioned in the plane
 4318 formed by the antenna PCB ($\theta = 90^\circ$ or $z = 0$ mm) at a distance of 10 cm from the
 4319 SYNCA, to match the expected position of the antenna array relative to the SYNCA in
 4320 the antenna array test stand. The horn antenna can be manually raised or lowered to
 4321 different relative vertical positions to characterize the radiation pattern at different polar
 4322 angles.

4323 Several 360° scans were performed with probe vertical offsets of -10.0 mm, -5.0 mm,
 4324 0.0 mm, 5.0 mm, and 10.0 mm relative to the antenna PCB plane. These probe offsets
 4325 cover a 2 cm wide vertical region centered on the SYNCA PCB, approximately equal to
 4326 ± 6 degrees of polar angle. The measurements show that the SYNCA is generating fields
 4327 with nearly isotropic magnitude across the probed region. The standard deviation of the
 4328 electric field magnitude measured around the antenna circumference is approximately
 4329 2.9 dB for a typical rotational scan. The presence of a significant pattern null is noted
 4330 near 45° (see Figure 5.20), which we attribute to small imperfections in the antenna
 4331 PCB that could be introduced from the hand soldered terminations connecting the coax
 4332 cables to the antenna. There is no significant difference in the radiation pattern when
 4333 measured across the 2 cm vertical range. The measured relative phases closely follow
 4334 the expectation for an electron, being linear with the measurement rotation angle and
 4335 forming the expected spiral pattern. Other than the small phase imperfections there is

4336 a slight sinusoidal bias to the phase data, which we determined is the result of a small
 4337 ($\lesssim 1$ mm) offset of the antenna's phase center from the rotation axis of the automated
 4338 stages.

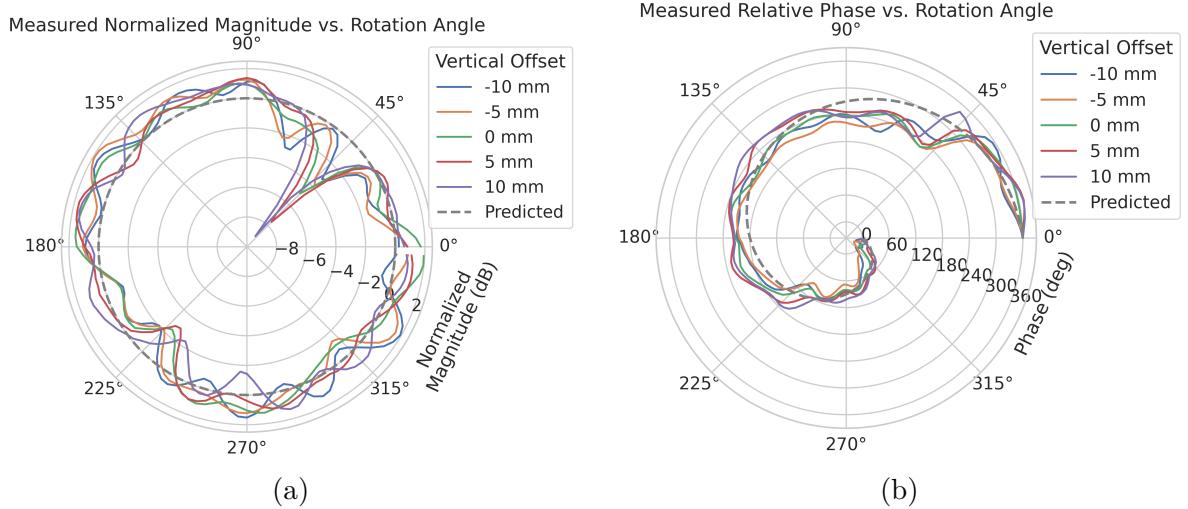


Figure 5.20. Linear interpolations of the measured electric field magnitude (a) and phase (b). The data was acquired using a VNA at 120 points spaced by 3 degrees from 0 to 357 degrees of azimuthal angle. The different color lines indicate the vertical offset of the horn antenna relative to the SYNCA PCB and the dashed line shows the expected shape from electron simulations. No significant difference in the antenna pattern is observed for the measured vertical offsets.

4339 The characterization measurements confirm the simulated performance of the SYNCA.
 4340 As expected the fields generated by the antenna are nearly isotropic in magnitude, ϕ -
 4341 polarized, and are linearly out of phase around the circumference of the antenna as
 4342 predicted for cyclotron radiation in Section 5.3.2. Small imperfections in the magnitude
 4343 and phase of the antenna are expected, particularly at the antenna's high operating
 4344 frequency of 26 GHz where small geometric changes can have significant impacts on
 4345 electrical properties. However, calibration through careful characterization measurements
 4346 can be used to remove the majority of these pattern imperfections, including the relatively
 4347 large pattern null near 45°, which will allow for the usage of the SYNCA as a test source
 4348 for free-space CRES experiments utilizing antenna arrays. In the next section we use the
 4349 VNA measurements obtained here as a calibration for signal reconstruction using digital
 4350 beamforming.

4351 5.3.5 Beamforming Measurements with the SYNCA

4352 Digital beamforming is a standard technique for signal reconstruction using a phased
 4353 array [94]. The SYNCA, since it exhibits the same cyclotron phases as a trapped electron,
 4354 can be used to perform simulated CRES digital beamforming reconstruction experiments
 4355 on the bench-top without the need for the magnet, cryogenics, and vacuum systems
 4356 required by a full CRES experiment. The fields received by the individual elements
 4357 of the antenna array will have phases dependent on the spatial position of the source
 4358 relative to the antennas. Therefore, a simple summation of the received signals will fail
 4359 to reconstruct the signal due to destructive interference between the individual channels
 4360 in the array. However, applying a phase shift associated with the source's spatial position
 4361 removes phase differences and results in a constructive summation of the channel signals
 4362 (see Figure 5.21). We can summarize the digital beamforming operation succinctly using
 4363 the following equation

$$y[t_n] = \sum_{m=0}^{N-1} x_m[t_n] A_m e^{i\phi_m}, \quad (5.35)$$

4364 where $y[t_n]$ represents the summed array signal at time t_n , $x_m[t_n]$ is the signal received
 4365 by channel m at time t_n , ϕ_m is the phase shift applied to the signal received at channel
 4366 m , and A_m is an amplitude weighting factor that accounts for the different signal power
 4367 received by individual channels. By changing the digital beamforming phases, the point
 4368 of constructive interference can be scanned across the sensitive region of the array to

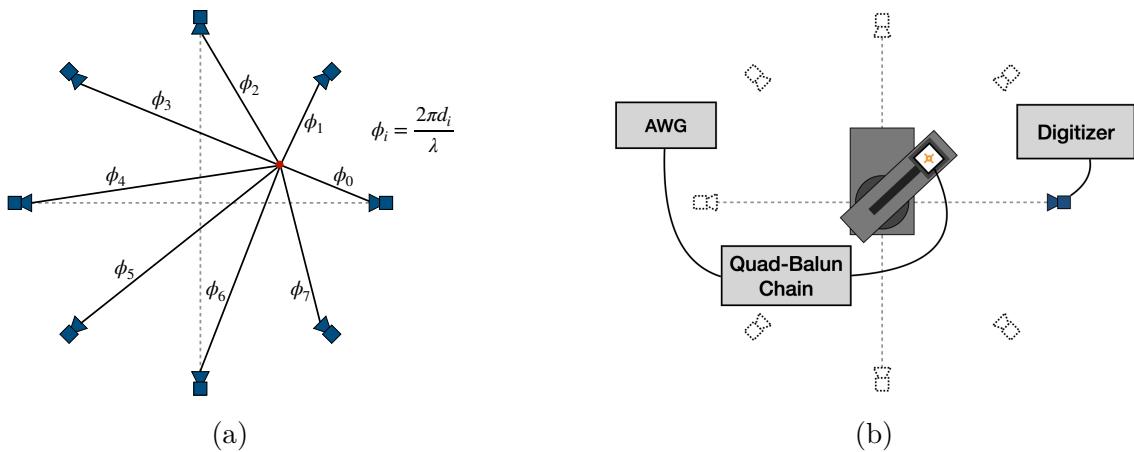


Figure 5.21. (a) A depiction of the relative phase differences for signals received by a circular antenna array from an isotropic source. The phases correspond to a unique spatial position.
 (b) A schematic of the setup used to perform digital beamforming.

4369 search for the location of a radiating source, which is identified as the point of maximum
 4370 summed signal power above a specified threshold. The digital beamforming phases consist
 4371 of two components,

$$\phi_m = 2\pi d_m/\lambda + \theta_m, \quad (5.36)$$

4372 where d_m is the distance from the m -th array element to the source, and θ_m is the
 4373 relative angle between the source position and the m -th antenna. The first component is
 4374 the standard digital beamforming phase that corresponds to the spatial position of the
 4375 source, and the second component is the cyclotron phase that corresponds to the relative
 4376 azimuthal phase offset.

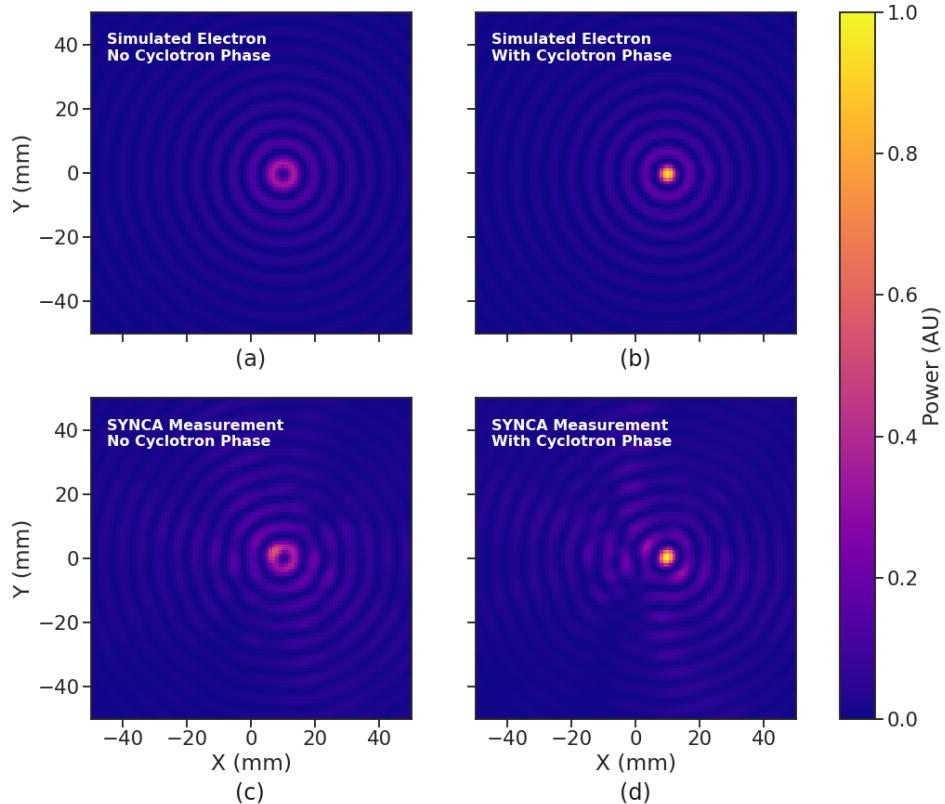


Figure 5.22. Digital beamforming maps generated using a simulated 60 channel array and electron simulated using the Locust package. (a) and (b) show the beamforming maps for simulated electrons without the cyclotron spiral phases and with the cyclotron spiral phases respectively. (c) and (d) show the beamforming maps produced from SYNCA measurements. We observe good agreement between simulated electrons and the SYNCA measurements.

4377 With a small modification to the hardware used to characterize the SYNCA (see
 4378 Figure 5.19), we can perform a digital beamforming reconstruction of a synthetic CRES

4379 event. By replacing the VNA with an arbitrary waveform generator (AWG), the SYNCA
4380 can be used to generate cyclotron radiation with an arbitrary signal structure, which
4381 can then be detected by digitizing the signals received by the horn antenna. Rotational
4382 symmetry allows us to use the rotational stage of the positioning system to rotate the
4383 SYNCA to recreate the signals that would have been received by a complete circular
4384 array of antennas.

4385 Using this setup, signals from a 60 channel circular array of equally spaced horn
4386 antennas were generated with the SYNCA positioned 10 mm off the central array axis,
4387 reconstructed using digital beamforming, and compared to Locust simulation (see Figure
4388 5.22). When the cyclotron spiral phases are not used, which is equivalent to setting θ_m
4389 in Equation 5.36 to zero, the SYNCA's position is reconstructed as a relatively faint ring
4390 as predicted by simulation. However, when the appropriate cyclotron phases are used
4391 during the beamforming procedure, both the simulated electron and the SYNCA appear
4392 as a single peak of high relative power corresponding to the source position. Therefore,
4393 we observe good agreement between the simulated and SYNCA reconstructions. While it
4394 may seem that for the case with no cyclotron phase corrections the ring reconstructs the
4395 position of the electron as effectively as beamforming with the cyclotron phase corrections,
4396 it is important to note that the simulations and measurements were generated without a
4397 realistic level of thermal noise. The larger maxima region and lower signal power, which
4398 occurs without the cyclotron phase corrections, significantly reduce the probability of
4399 detecting an electron in a realistic noise background.

4400 To bound the beamforming capabilities of the synthetic array of horn antennas, we
4401 performed a series of beamforming reconstructions where the SYNCA was progressively
4402 moved off the central axis of the array (see Figure 5.23). To extract an estimate of the
4403 position of the SYNCA using the digital beamforming image we apply a 2-dimensional
4404 (2D) Gaussian fit to the image data and extract the estimated centroid value. We find
4405 that the synthetic horn antenna array reconstructs the position of the SYNCA with a
4406 1σ -error of 0.3 mm with no apparent trend across the 30 mm measurement range. This
4407 reconstruction error is an order of magnitude larger than mean fit position uncertainty
4408 of 0.02 mm indicating that systematic effects related to the SYNCA positioning system
4409 could be contributing additional uncertainty to the measurements. Note that the current
4410 mean reconstruction error of 0.3 mm is a factor of 20 smaller than the full width at half
4411 maximum of the digital beamforming peak (6 mm), which could be interpreted as a naive
4412 estimate of the position reconstruction performance of this technique. Because these
4413 measurements are intended as a proof-of-principle demonstration, we do not investigate

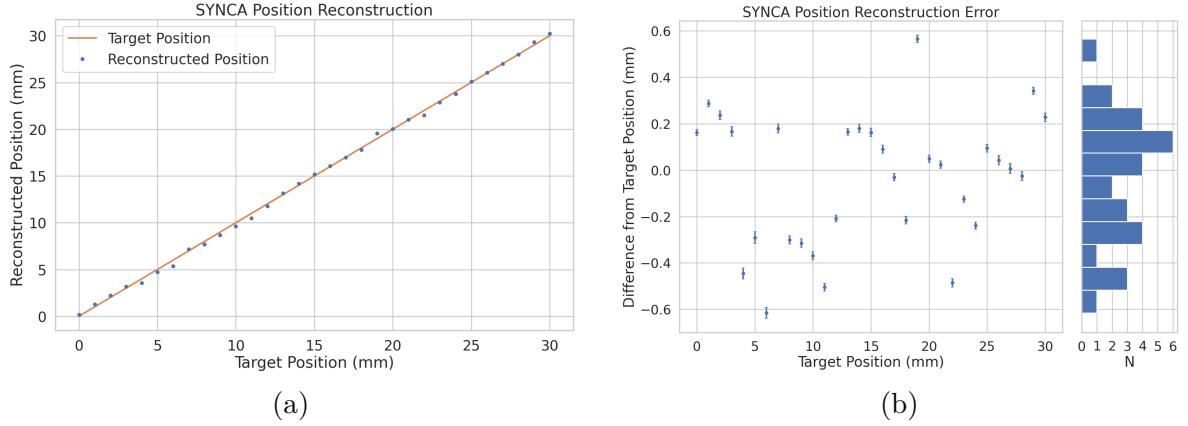


Figure 5.23. A plot of the SYNCA’s reconstructed position using the synthesized horn-antenna array and digital beamforming. (a) Shows the reconstructed position of the SYNCA compared with the target position indicated by the positioning system readout. (b) Shows the reconstruction error, which is the difference between the target and reconstructed positions. The error bars in (b) are the uncertainty in the mean position of the 2D Gaussian used to fit the digital beamforming reconstruction peak obtained from the fit covariance matrix. The mean fit position uncertainty of 0.02 mm is an order of magnitude smaller than the typical reconstruction error of 0.3 mm obtained by calculating the standard deviation of the difference between the reconstructed and target position.

4414 potential sources of systematic errors further; however, we expect that a similar and
 4415 more thorough investigation will be performed using the Project 8 antenna array test
 4416 stand, where typical reconstruction errors can be used to estimate the energy resolution
 4417 limits of antenna array designs.

4418 5.3.6 Conclusions

4419 In this paper we have introduced the SYNCA, which is a novel antenna design that
 4420 emits radiation that mimics the unique properties of the cyclotron radiation generated by
 4421 charged particles moving in a magnetic field. The characterization measurements of the
 4422 SYNCA validated the simulated performance of the PCB crossed-dipole antenna design.
 4423 Additionally, the SYNCA was used to estimate the position reconstruction capabilities
 4424 of a synthesized array of horn antennas and experimentally reproduced the simulated
 4425 digital beamforming reconstruction of electrons.

4426 While the SYNCA performs well, there exist discrepancies in the phase and magnitude
 4427 of the radiation pattern compared to the simulated SYNCA design that are related to
 4428 the small geometric differences in the soldered connections. Future design iterations that
 4429 replace the soldered connections with a fully surface mount design could improve the

4430 radiation pattern at the cost of some complexity and expense. Furthermore, improving
4431 the design of the antenna PCB and mounting system would allow the antenna to be
4432 inserted into a cryogenic and vacuum environment where in-situ antenna measurement
4433 calibrations could be performed.

4434 The discrepancies in the radiation pattern and phases exhibited by the as-built
4435 SYNCA should not greatly impact its performance as a calibration probe. Both magni-
4436 tude and phase variations can be accounted by applying the SYNCA characterization
4437 measurements as a calibration to the data collected by the antenna array test stand. The
4438 separate calibration of the SYNCA radiation does not impact the primary goals for the
4439 antenna array test stand which are array calibration and signal reconstruction algorithm
4440 performance characterization, because it can be performed with standard reference horn
4441 antennas with well understood characteristics.

4442 The SYNCA antenna technology advances the CRES technique by providing a
4443 mechanism to characterize free-space antenna arrays for CRES measurements without
4444 the need for a magnet and cryogenics system, which would be required for calibration
4445 using electron sources. Both the Project 8 collaboration as well as future collaborations
4446 which are developing antenna array based CRES experiments can make use of SYNCA
4447 antennas as an important component of their calibration and commissioning phases.

4448 5.4 SYNCA Development Discussion

4449 A crossed-dipole antenna (see Figure 5.24) was identified early on as a candidate SYNCA
4450 design. The crossed-dipole is a circularly polarized antenna, consequently, the electric
4451 fields measured in the plane of the dipole antenna exhibit the same relative phase offsets
4452 as a 90° electron in a magnetic trap. This is explained in greater detail in Section 5.3.
4453 These phase offsets were measured with the first rudimentary crossed-dipole prototype
4454 manufactured from coaxial cables with the insulation and shield stripped away.

4455 Because the SYNCA is ultimately a calibration tool, it is desireable that the antenna
4456 have a well-characterized and robust antenna pattern. Therefore, manufacturing a
4457 SYNCA using the stripped wire method shown in Figure 5.24 is infeasible. Studies of
4458 crossed-dipole antennas manufactured out of printed circuit boards were performed using
4459 HFSS to identify an antenna design that imitated an electron, while being more robust
4460 and simpler to manufacture (see Figure 5.25).

4461 Identifying a design that was robust, manufacturable, and most importantly matched
4462 the electric fields of a trapped electron proved to be a non-trivial task. The primary

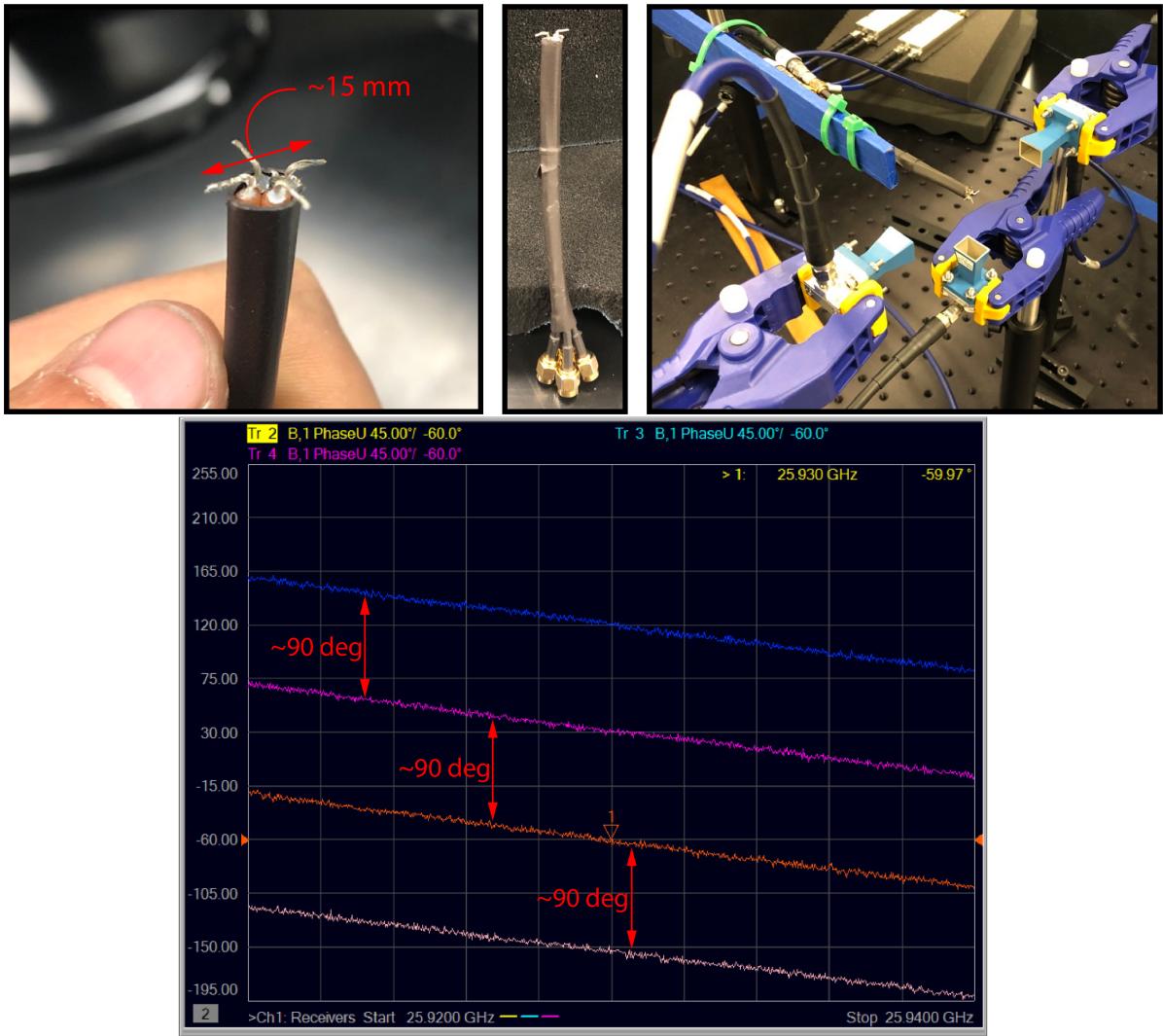


Figure 5.24. Images of an early prototype crossed-dipole antenna manufactured by hand and the first measurement setup. The antenna was constructed by hand using four stripped coaxial cables. The antenna was connected to one port of the VNA, and the remaining three ports on the VNA were connected to horn antenna arranged with 90 deg offsets around the crossed-dipole. The measured unwrapped S-parameter phases exhibit the desired relative phase behavior for a SYNCA. These early measurements were the first laboratory proof-of-principle for the crossed-dipole SYNCA.

factor driving the difficulty was the high operating frequency of the antenna (26 GHz) combined with the requirement that the antenna be electrically-small. An antenna that is electrically-small at 26 GHz has a largest dimension on the order of 1 mm, which poses significant manufacturability challenges given the limited available budget for SYCNA fabrication.

One of the key limitations with the small size requirements is the diameter of the

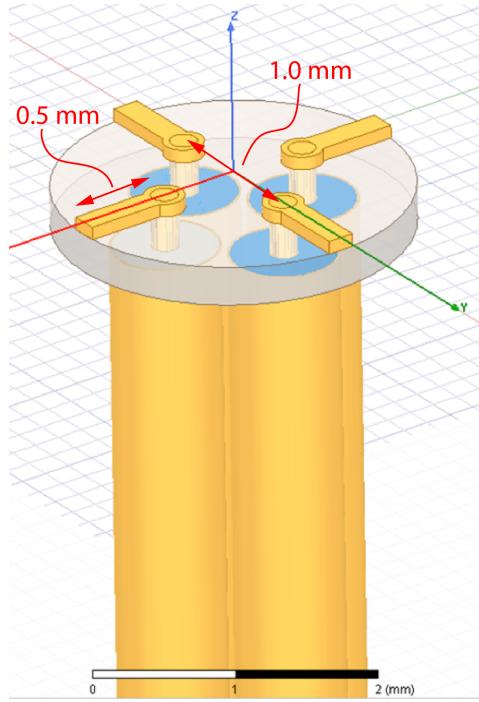


Figure 5.25. An early iteration of a crossed-dipole SYNCA antenna simulated in HFSS. The antenna is electrically small at 26 GHz, which requires dipole arms on the order of 1 mm long. This design is limited by the minimum achievable distance between the dipole arms caused by the available diameters of coaxial cables. The assumed termination scheme for the coaxial cables to the antenna is hand-soldering, which introduces random variation in the antenna pattern from the inevitable blobs of solder left on the surface of the PCB.

4469 coaxial cables needed to feed the crossed-dipole antenna. The smallest commonly available
 4470 rigid coaxial cables available on the market have diameters of approximately 0.5 mm,
 4471 which limited the spacing between dipole arms to a minimum of about 1 mm. The
 4472 crossed-dipole antenna performs better as a SYNCA if the dipole arm separation is
 4473 significantly less than the operating wavelength. Therefore, the high operating frequency
 4474 ultimately limited how well the SYNCA could mimic an electron. If the desired cyclotron
 4475 frequency was lowered by an order of magnitude to approximately 3 GHz a significantly
 4476 higher quality SYNCA could be manufactured at lower cost.

4477 The decision to use coaxial transmission lines terminated on the antenna PCB with a
 4478 hand-soldered connection was driven primarily to limit the costs of SYNCA development
 4479 and contributed to the observable variations in the SYNCA's gain and phase patterns.
 4480 A second iteration of the SYNCA design that minimized hand-soldering by using surface-
 4481 mount components could significantly reduce variations in the antenna pattern. The
 4482 major drawback in the development of a surface-mount SYNCA is the cost, and given the

4483 transition to a cavity based design for Phase IV, such a design was never investigated.

4484 5.5 FSCD Antenna Array Measurements with the SYNCA

4485 5.5.1 Introduction

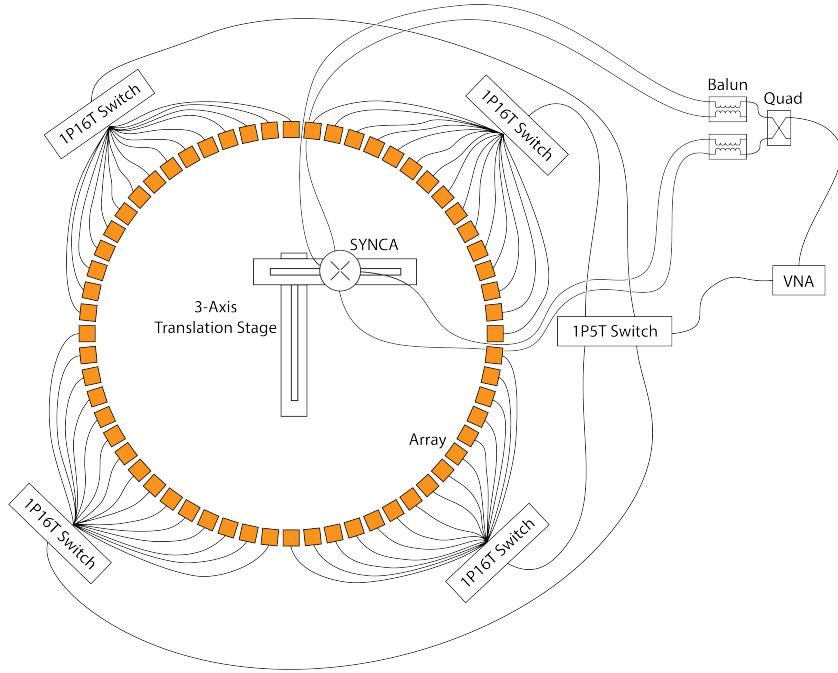


Figure 5.26. A diagram of the array measurement system used to test the prototype FSCD antenna array. A VNA is used as the primary measurement tool, which is connected to the array through a series of switches. The other port of the VNA connects to the SYNCA through the quad-balun chain used to provide the SYNCA feed signals. During measurements the SYNCA is positioned inside the center of the antenna array and translated to different radial and axial positions using a 3-axis manual translation stage setup.

4486 Using the SYNCA we can perform full-array measurements of prototype versions
4487 of the FSCD antenna array to test its performance with a realistic cyclotron radiation
4488 source (see Figure 5.26). The goal is to check how the measured power received by
4489 the array compares to FSCD simulations as a function of the radial and axial position
4490 of the SYNCA. These measurements are intended to validate the antenna research
4491 and development by Project 8, which has been driven primarily by simulations with
4492 Locust [61] and CRESana (see Section 4.2.3), and identify any discrepancies with these
4493 simulations tools. This knowledge will provide confidence in the simulations necessary
4494 for the analysis of the sensitivity of larger antenna array based CRES experiment designs

4495 to the neutrino mass.

4496 As shown in Section 5.3, the SYNCA does have some radiation pattern imperfections
4497 that complicate the comparison between measurement and simulation data. One way to
4498 disentangle some of the effects of these imperfections is to perform an additional set of
4499 measurements using a synthetic antenna array setup along with the SYNCA antenna.
4500 Since the synthetic array setup uses only a single array antenna, the data should be
4501 free of errors associated with individual antenna differences and multi-path interference,
4502 which are two error sources being tested with the full-array setup. By comparing the
4503 synthetic array data to the FSCD array data and to simulation data one can evaluate the
4504 significance of these effects relative to the errors introduced by SYNCA imperfections.

4505 **5.5.2 Measurement Setups**

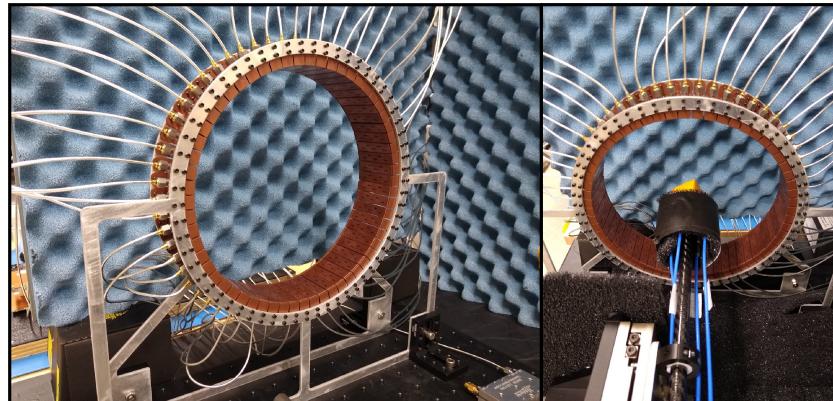
4506 **5.5.2.1 FSCD Array Setup**

4507 The antenna design that composes the array is the 5-slot waveguide antenna developed
4508 for the FSCD experiment (see Figure 5.27a). The antenna is 5 cm long and is constructed
4509 out of WR-34 waveguide with a 2.92 mm coax connector located at the center of the
4510 antenna. Copper flanges located on both ends of the antenna are used to mount the
4511 antenna in the array support structure. The antennas are supported by two circular steel
4512 brackets that can be bolted to both ends of the waveguide to construct the circular array
4513 (see Figure 5.27b). The antenna array consists of sixty identical waveguide antennas
4514 with a radius of 10 cm. The array is mounted perpendicular to an optical breadboard
4515 surface using a pair of the steel brackets, which provide sufficient space for the coaxial
4516 cable connections and allows for easy positioning of the SYNCA antenna. The SYNCA is
4517 mounted on the end of a carbon fiber rod attached to a set of manual translation stages,
4518 which are used to move the SYNCA antenna to different positions inside the array (see
4519 Figure 5.27c). The stages allow for independent motion in three different axes and can
4520 position the SYNCA at radial distances up to 5 cm from the center.

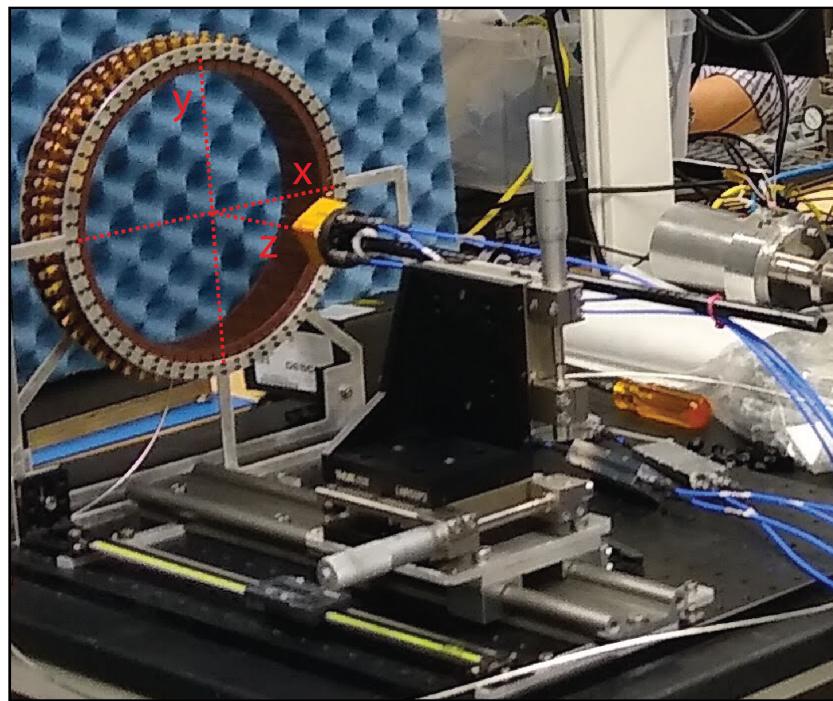
4521 Data acquisition is accomplished using a two-port VNA in combination with a series
4522 of microwave switches that allow the VNA to connect to each channel in the array . The
4523 first port of the VNA is connected to the quad-balun chain used to feed the SYNCA (see
4524 Section 5.3), and the second port of the VNA connects to a 1P5T microwave switch. The
4525 1P5T switch is connected to four separate 1P16T switch boards that connect directly
4526 to the array. The data acquisition is controlled by a python script running on a lab
4527 computer, which is connected to the VNA and an Arduino board programmed to control



(a)



(b)



(c)

Figure 5.27. Photos of the prototype FSCD antenna (a), the FSCD array and SYNCA (b), and the translation stages and coordinate system used to position the SYNCA (c).

4528 the microwave switches. The script uses the switches to iteratively connect each of the
4529 antennas in the array to the VNA. The VNA is configured to load a specific calibration
4530 file for each antenna channel and performs the measurements of all available S-parameters.
4531 The separate calibration files is an attempt to remove phase and magnitude errors caused
4532 by different propagation through the RF switches. Array measurements were performed
4533 for the set of SYNCA positions consisting of radial (x-axis) positions from 0 to 50 mm in
4534 5 mm steps and axial (z-axis) positions from 0 to 50 mm in 5 mm steps resulting in 121
4535 array measurements. At each SYNCA position we measured the two-port S-parameter
4536 matrix using a linear frequency sweep from 25.1 to 26.1 GHz with 101 discrete frequencies.

4537 5.5.2.2 Synthetic Array Setup

4538 A photograph of the setup used to perform the synthetic array measurements is shown
4539 in Figure 5.28. One important difference between this setup and the FSCD array setup
4540 is that the synthetic array measurements were performed with a waveform generator and
4541 digitizer instead of a VNA. The electronics configuration is identical to the diagram in
4542 Figure 5.7b. Despite the differences, one is still able to compare the measured phases of
4543 the synthetic array and the relative magnitude of the power, since the digitized signal
4544 power is directly proportional to S21.

4545 The arbitrary waveform generator in the setup is configured to produce a 64 MHz
4546 sine wave signal that is up-converted to 25.864 GHz using a mixer and the VNA source.
4547 This signal is passed through a bandpass filter and fed to the SYNCA quad-balun chain.
4548 A single FSCD antenna is positioned 10 cm from the SYNCA and aligned vertically so
4549 that the center of the 5-slot waveguide is in the plane of the SYNCA PCB (see Figure
4550 5.28). This position corresponds to $z = 0$ in Figure 5.27c. The SYNCA is rotated
4551 in three degree steps to synthesize an antenna array with 120 channels. This channel
4552 count is more than could physically fit in a 10 cm radius array, but there is no cost to
4553 over-sampling. Additionally, over-sampling allows for a check of the smoothness of the
4554 antenna array radiation pattern. The signals from the FSCD antenna are down-converted
4555 using the second mixer connected to the VNA source before being digitized at 250 MHz
4556 and saved to disk. Several synthetic array measurement scans were performed by using
4557 the linear translation stage to change the radial position of the SYNCA. In total eight
4558 scans were taken from 0 to 35 mm using a radial position step size of 5 mm.

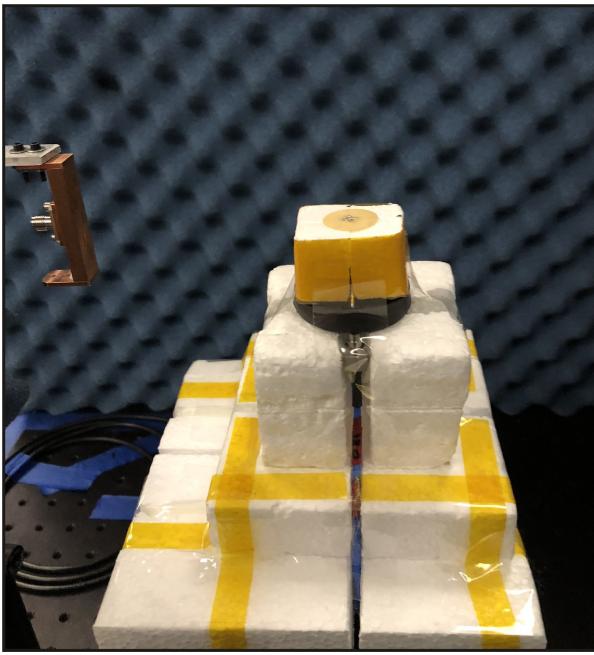


Figure 5.28. A photo of the FSCD antenna and the SYNCA in the synthetic array measurement setup at Penn State.

4559 5.5.3 Simulations, Analysis, and Results

4560 The Locust and CRESana simulation packages utilize the antenna transfer functions
4561 to calculate the power that would be received by each antenna from a CRES electron.
4562 The equivalent quantity in the measurement setup is the S21 matrix element, which
4563 indicates the ratio of the power received by an antenna in the array to the amount of
4564 power delivered to the SYNCA. Therefore, the analysis focuses on comparing the relative
4565 magnitudes and phase of the S21 parameters measured by the VNA as a function of
4566 the array channel and the SYNCA position. Additionally, we apply a beamforming
4567 reconstruction to the S21 data to evaluate how the summed power and beamforming
4568 images change as a function of the position of the SYNCA.

4569 5.5.3.1 Simulations

4570 Simulations for the FSCD array measurements were performed using CRESana, which
4571 performs analytical calculations of the EM-fields produced by an electron at the position
4572 of the antennas. At each sampled time CRESana computes the electric field vector at the
4573 antenna positions, which is projected onto the antenna polarization axis to obtain the
4574 co-polar electric field. The magnitude of the co-polar electric field is then multiplied by
4575 a flat antenna transfer function to calculate the corresponding voltage signal. CRESana

4576 simulations exploit the flat transfer functions of the FSCD antennas, which allows the
 4577 electric field to be multiplied by the antenna transfer function rather than performing
 4578 the full FIR calculation. These calculations produce a voltage time-series for each of the
 4579 antennas in the array that can be compared to the laboratory measurements.

4580 CRESana was configured to simulate a 90° electron in a constant background magnetic
 4581 field of ≈ 0.958 T with a kinetic energy of 18.6 keV. These parameters were chosen
 4582 in order to mimic a CRES event near the tritium beta-decay spectrum endpoint in
 4583 the FSCD experiment. The constant background magnetic field guarantees that the
 4584 guiding center of the electron is stationary across the duration of the simulation which is
 4585 consistent with the SYNCA in the laboratory measurements. Simulations were performed
 4586 with the electron's guiding center at radial positions from 0 to 45 mm in steps of 1 mm
 4587 and axial positions from 0 to 30 mm in steps of 1 mm. The simulations generated time
 4588 series consisting of 8192 samples at 200 MHz for the sixty channel FSCD antenna array
 4589 geometry.

4590 5.5.3.2 Phase Analysis

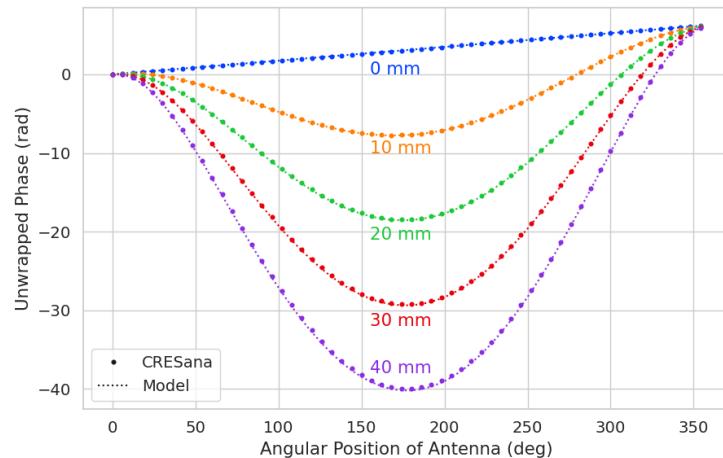


Figure 5.29. The unwrapped phases of signals received by the FSCD antenna array from an electron with a 90° pitch angle located in the plane of the antenna array. The data points indicated the phases extracted from simulation and the dashed lines show the model predictions.

4591 Correct modeling of the signal phases is fundamental to reconstruction for both
 4592 beamforming and matched filter approaches. The beamforming reconstruction relies on
 4593 a signal phase model developed from Locust simulations, which allows one to predict the
 4594 relative signal phases for a specific magnetic trap and electron position. The equation

4595 for the model is

$$\phi_{ij}(t) = \frac{2\pi d_{ij}(t)}{\lambda} + \theta_{ij}(t), \quad (5.37)$$

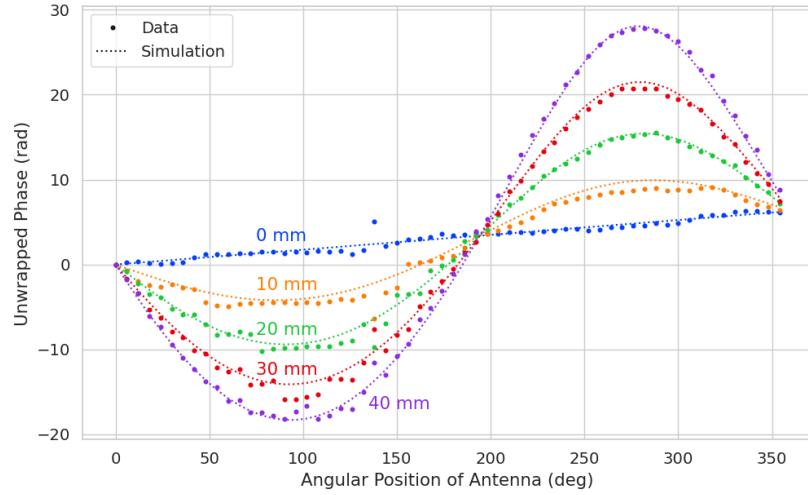
4596 where $d_{ij}(t)$ is distance between the assumed electron position and the antenna position,
4597 and $\theta_{ij}(t)$ is the angular separation between the electron and antenna positions. For
4598 details on the components of the phase model see Section 5.3.2. In Figure 5.29 we
4599 compare the phases predicted by Equation 5.37 to phases extracted from CREsana
4600 simulations of an electron located in the plane of the antenna array at a series of radial
4601 positions. One observes excellent agreement between the model and simulation.

4602 The measured signal phases from the FSCD array and synthetic array are shown
4603 in Figures 5.30a and 5.30b compared to the signal phase model. The axial position of
4604 the SYNCA in both plots is $z = 0$ mm, such that the plane of the PCB is aligned with
4605 the center of the FSCD antenna. The data shown in Figure 5.30a corresponds to the
4606 S-parameters measured at 25.80 GHz which is the frequency closest to the one used in
4607 the synthetic array setup. The different slope and sinusoidal phases exhibited by Figure
4608 5.30a and 5.30b reflects differences in the coordinate system for each setup. In general,
4609 we see that the phase model predicts the large scale features of the phases quite well,
4610 but there are some small scale deviations or errors from the phase model that do not
4611 appear to be present in simulation.

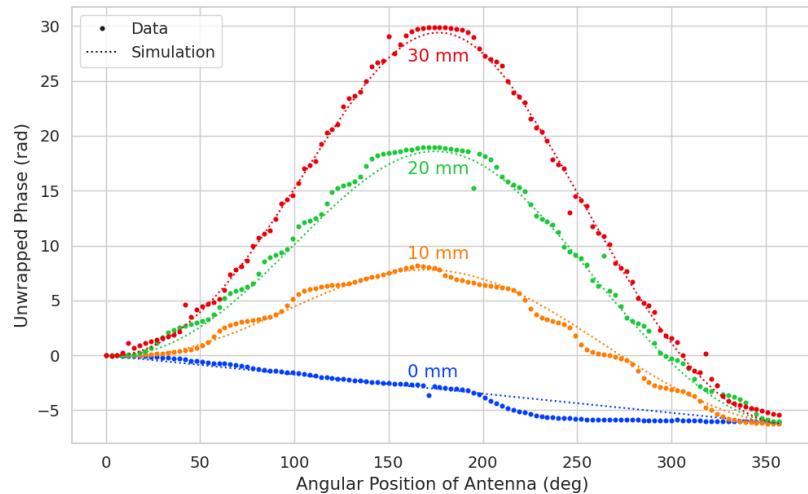
4612 A comparison of the phase errors, which are the difference between measurement and
4613 model is shown in Figure 5.31. The FSCD array data is referred to as the JUGAAD
4614 data in the plot legend, which is an alternative name for the FSCD array setup.

4615 The phase error at $R = 0$ in Figure 5.31 forms a smooth curve, with the exception of
4616 an outlier data point caused by a bug in the data acquisition script. One can attribute
4617 the observed phase error at this position to imperfections in the antenna pattern of the
4618 SYNCA. As the SYNCA is moved away from $R = 0$ mm one observes that the phase
4619 error exhibits oscillations whose frequency increases as a function of the radial position
4620 of the SYNCA. These oscillations have the appearance of a diffraction pattern, which
4621 is particularly clear for the radii ≥ 15 mm, due to the bilateral symmetry of the phase
4622 error peaks around 180° .

4623 One can observe a higher average variance in the phase errors measured for the FSCD
4624 array compared to the synthetic array. This is best seen by comparing the curves at
4625 $R \leq 15$ mm where the smooth synthetic array curves are distinct from the relatively
4626 noisy FSCD array errors. The extra noise in the FSCD array is most likely caused by
4627 differences in the radiation patterns of the antennas that make up the array as well as
4628 differences in the transmission lines through the switch network that introduce additional



(a)



(b)

Figure 5.30. Plots of the measured unwrapped phases from the FSCD array (a) and the synthetic array (b) compared to the model predictions for a series of radial positions. The different phases of the sinusoidal phase oscillations in the two plots reflects differences in the coordinate systems of the measurements.

phase errors into the measurement. Since the synthetic array measurements use only a single antenna, these extra error terms are not present, which explains the relatively smoother phase error curves. Despite the extra phase errors in the FSCD array, it is still possible to observe a similar phase error oscillation effect as the SYNCA is moved away from $R = 0$ mm.

The diffraction pattern exhibited by the phase error oscillations is more easily observed

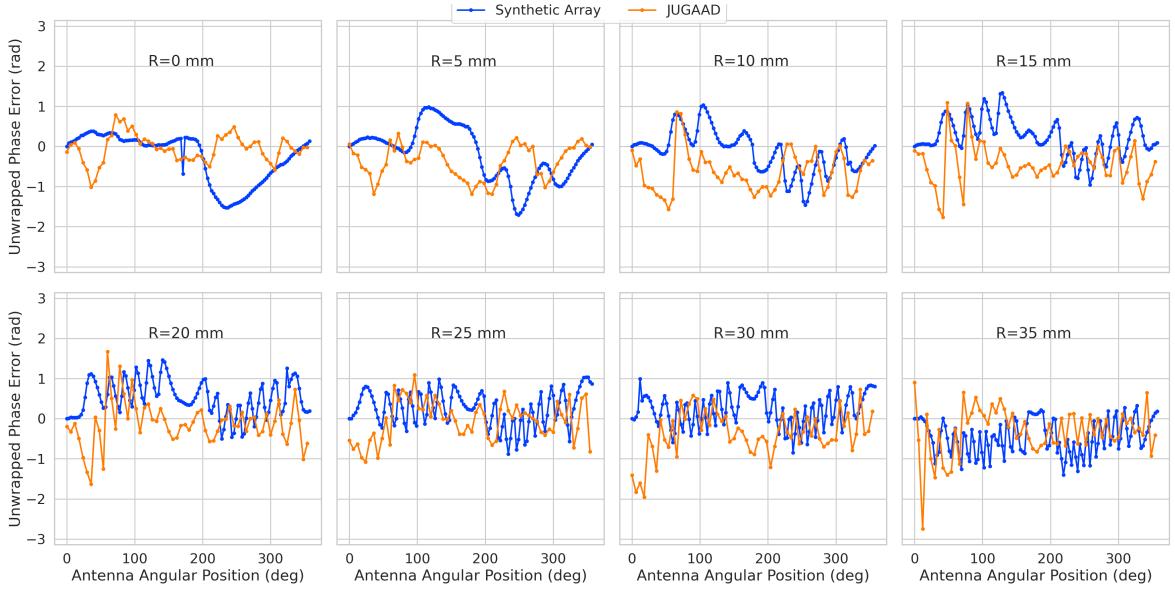
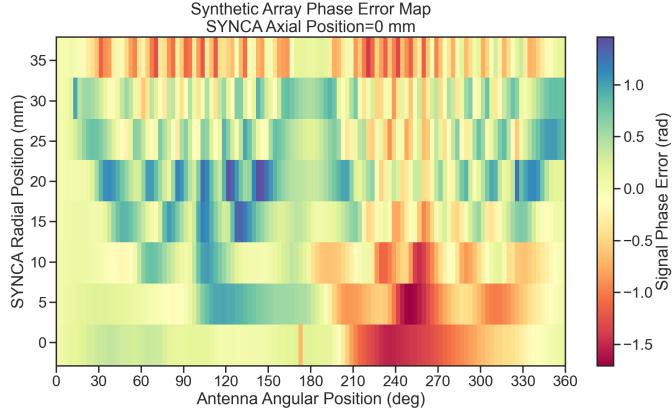


Figure 5.31. The phase errors between the measurement and model for the synthetic array (blue) and the FSCD array (orange) for a series of radial positions. The label JUGAAD refers to an alternative name for the FSCD array setup. As the SYNCA is translated off-axis phase errors with progressively higher oscillation frequency enter into the measurements.

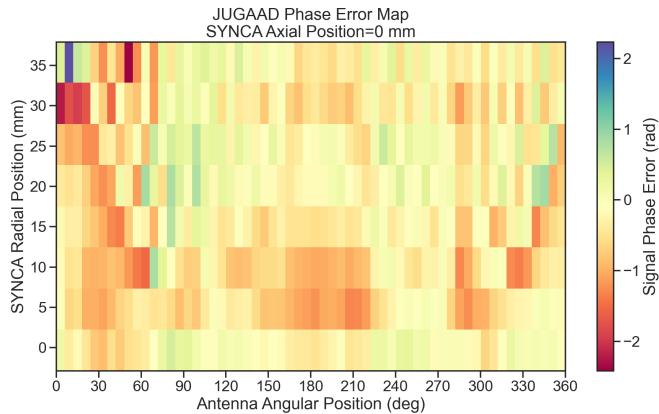
4635 by plotting the phase errors in a two-dimensional map, which is done in Figures 5.32a and
 4636 5.32b. For the synthetic array data ones observes a relatively clear diffraction pattern
 4637 that emerges as the SYNCA is moved radially. The bilateral symmetry of the diffraction
 4638 patterns is due to the bilateral symmetry of the circular synthetic array around the
 4639 translation axis of the SYNCA. A similar pattern is also visible in the FSCD array data,
 4640 although, it is obscured by the additional phase error that results from the multi-channel
 4641 array.

4642 The physical origin of the phase error diffraction pattern is attributed to interference
 4643 effects arising from path-length differences between the individual slots in the FSCD
 4644 antenna and the SYNCA transmitter. Since we are operating in the radiative near-field of
 4645 the FSCD antenna, the path length differences between the slots introduces a significant
 4646 change in the summation of the signals that occurs inside the waveguide, which causes
 4647 the radiation pattern of the antenna to change as a function of distance. Therefore, when
 4648 the SYNCA is positioned off-axis the different path-lengths from the SYNCA to each
 4649 antenna results in different radiation patterns leading to the observed diffraction pattern.

4650 This near-field effect is not present in simulations, because in order to simplify the
 4651 calculations we assume that the far-field approximation can be applied to the FSCD
 4652 antennas. This means that the radiation pattern and antenna transfer functions are



(a)



(b)

Figure 5.32. Two dimensional plots of the phase errors for the synthetic array (a) and the FSCD (JUGAAD) array (b). In both plots we observe evidence of a similar diffraction pattern with bilateral symmetry, but the FSCD array measurements have an additional phase error contribution from the different antennas and paths through the switch network.

independent of the distance between the transmitter and the receiving antenna. In principle, we can account for these near-field effects with a more detailed simulation of the FSCD antennas either in CRESana or Locust, which would result in an additional term in the beamforming phase model. However, this would increase the computational intensity of the simulation software. In the next section we briefly discuss the impact of these near-field effects on the measured magnitude of the signals.

4659 5.5.3.3 Magnitude Analysis

4660 Exactly as for the signal phase, one can use simulations to construct a model that
4661 describes the magnitude of the signals received by each channel in the antenna array.
4662 By examining the results of simulations or by analyzing the Liénard-Wiechert equation
4663 one can show that radiation pattern from a 90° pitch angle electron in a magnetic field
4664 is omni-directional. Therefore the relative magnitudes of the signals received by each
4665 channel will be determined by the free-space power loss, which is proportional to the
4666 inverse distance between the assumed electron position and the antenna.

4667 A consequence of this is that the signals produced in the array for electrons off the
4668 central axis will have larger amplitudes for the antennas closer to the electron compared
4669 to those which are further away. The amplitudes of the signals received by the array
from an electron located at a series of radial positions are shown in Figure 5.33.

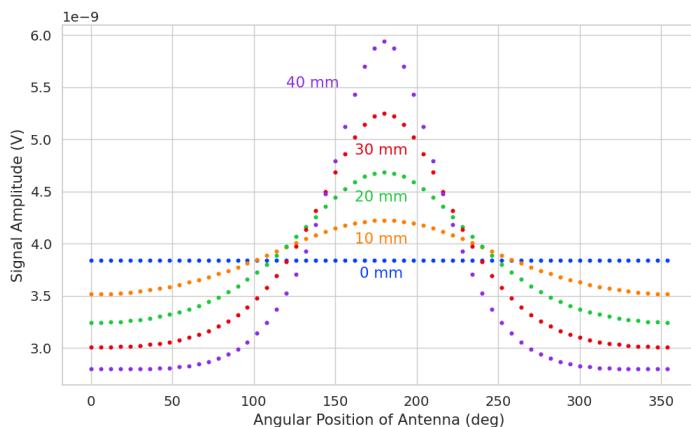


Figure 5.33. The amplitude of the signals from CRESana for the FSCD array from a 90° electron. As the electron is moved from $R = 0$ the signals begin to have unequal amplitudes depending on the distance from the electron to the antenna.

4670
4671 One expects to see a similar trend in the signal magnitudes in both the FSCD and
4672 synthetic arrays. The normalized signal magnitudes extracted from the full and synthetic
4673 array setups for a series of radial SYNCA positions are shown in Figure 5.34. The data
4674 corresponds to a SYNCA axial position of $z = 0$ mm and at a frequency 25.86 GHz. One
4675 complication is that the radiation pattern of the SYNCA is not perfectly omni-directional,
4676 which causes the measured magnitudes at $R = 0$ mm to diverge from the perfectly flat
4677 behavior exhibited by electrons.

4678 As the SYNCA is moved off-axis one observes a similar increase in the number of
4679 magnitude peaks in the synthetic array data that one would expect from a diffraction

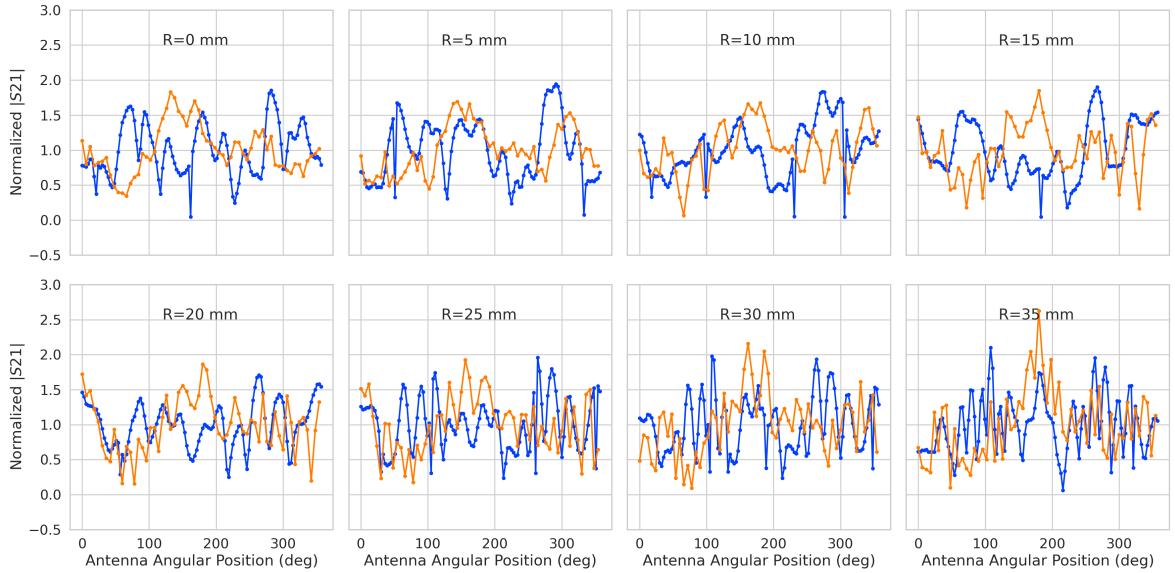
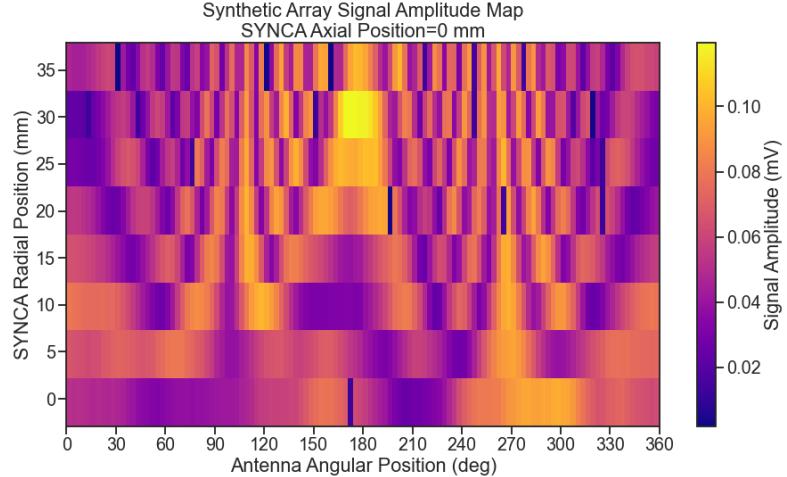


Figure 5.34. The normalized magnitudes of the S21 parameters measured in the FSCD (orange) and synthetic array (blue) setups. The dominant observed behavior as a function of radius is the increase in the number of magnitude peaks, which was noted in the phase error curves. There does not appear to be a strong change in the relative amplitude of a group of antennas as predicted by CRESana.

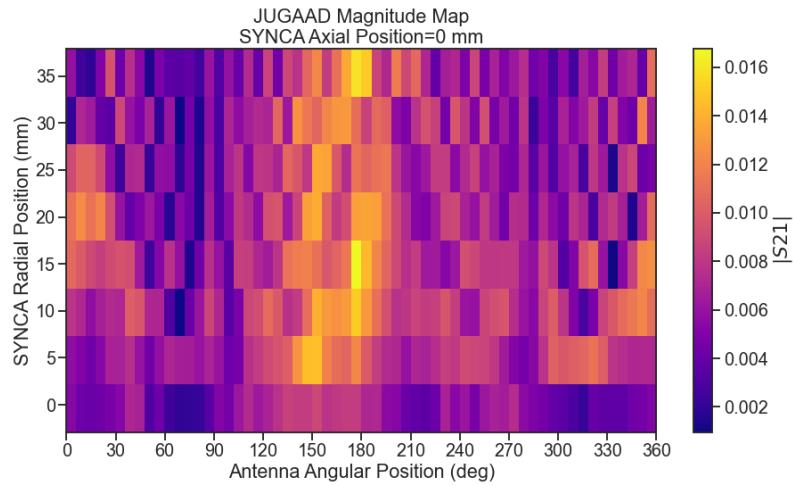
pattern, although this trend is not as stark compared to the phase data. Noticeably, there does not appear to be a set of channels with disproportionately larger amplitude at large R , which would be expected based on the trends from CRESana.

Comparing the magnitudes of the synthetic array to the FSCD array in Figure 5.34 we see that there is a similar amount of variability in the magnitudes at $R = 0$ mm, although there is potentially more small scale error in the magnitude curve caused by channel differences in the FSCD array. We observe a similar trend in the number of magnitude error peaks in the FSCD array data to the synthetic array data, which mirrors the diffraction effect observed in the phase data. The diffraction effect can be visualized more clearly by plotting a similar two-dimensional map of the magnitudes (see Figure 5.35).

The fact that one observes a similar diffraction pattern in the signal magnitudes as a function the SYNCA position reinforces the conclusions from the phase analysis that near-field effects are having a significant impact on the radiation pattern of the FSCD array. These near-field effects lead to changes in the magnitude and phase of the radiation pattern of the FSCD antenna as a function of distance. If left uncorrected these errors reduce detection efficiency by causing power loss in the beamforming or matched



(a)



(b) The two-dimensional maps showing the diffractive pattern exhibited by the FSCD and synthetic array signal magnitudes.

Figure 5.35.

4697 filter reconstruction due to phase mismatch. We explore the impact of these phase and
4698 magnitude errors on beamforming in the next section.

4699 5.5.3.4 Beamforming Characterization

4700 Errors in the signal magnitudes and phases lead to errors in signal reconstruction. For
4701 example, a matched filter reconstruction requires accurate knowledge of the signals in
4702 each channel to achieve optimal performance. Uncorrected errors leads to mismatches
4703 between the template and signal, which reduces detection efficiency and introduces

uncertainty in the parameter estimation. In this section, we analyze the beamformed signal amplitude as a function of the position of the SYNCA to quantify the impact of the phase and magnitude errors on signal reconstruction. Because of the imperfections in the SYNCA source, it is inappropriate to directly compare the beamformed signal amplitude of the FSCD array or synthetic array. Such a comparison would not allow one to disentangle losses that occur because of the antenna array from those that occur because of the source. Therefore, we focus on comparing the beamforming of the FSCD array to the synthetic array.

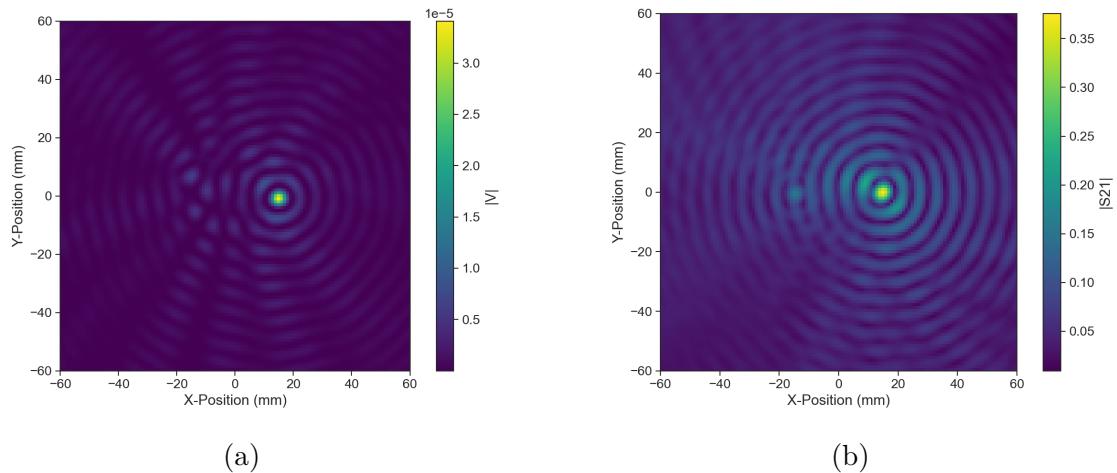


Figure 5.36. Beamforming images from the synthetic array (a) and FSCD array (b) setups with the SYNCA positioned 15 mm off the central axis. In both images we see a clear maxima that corresponds to the true SYNCA position. However, in the FSCD array there is an additional faint peak located at the opposite position of the beamforming maximum. This additional peak is the mirror of the true peak and is the result of reflections between antennas in the FSCD array.

The first method of comparison is to analyze the images generated by applying the beamforming reconstruction specified in Section 4.3.1 to the FSCD and synthetic array data (see Figure 5.36). The beamforming grid consisting of a square 121×121 grid spanning a range of -60-mm to 60 mm in the x and y dimensions. The beamforming images formed from the synthetic array produces a three-dimensional matrix where each grid position contains a summed time series. A single beamforming image is formed from this data matrix by taking the mean over the time dimension. In the case of the FSCD array, the VNA generates frequency domain data such that each grid position contains a summed frequency series produced by the VNA sweep. For this data a single image is formed by averaging in the frequency domain.

There is a clear difference between the synthetic and FSCD array beamforming images,

4723 which is the additional faint beamforming maxima located directly opposite the maxima
 4724 corresponding to the SYNCA position. The images in Figure 5.36 were generated with
 4725 data collected at a SYNCA radial position of 15 mm, which agrees well with the observed
 4726 beamforming maximum in both images. We observe that the faint beamforming peak is
 4727 located directly opposite of the true beamforming maximum similar to a mirror image.
 4728 Therefore, the origin of this additional feature appears to be reflections between the two
 4729 sides of the circular antenna array that are not present for the synthetic array since only
 4730 a single physical antenna is used.

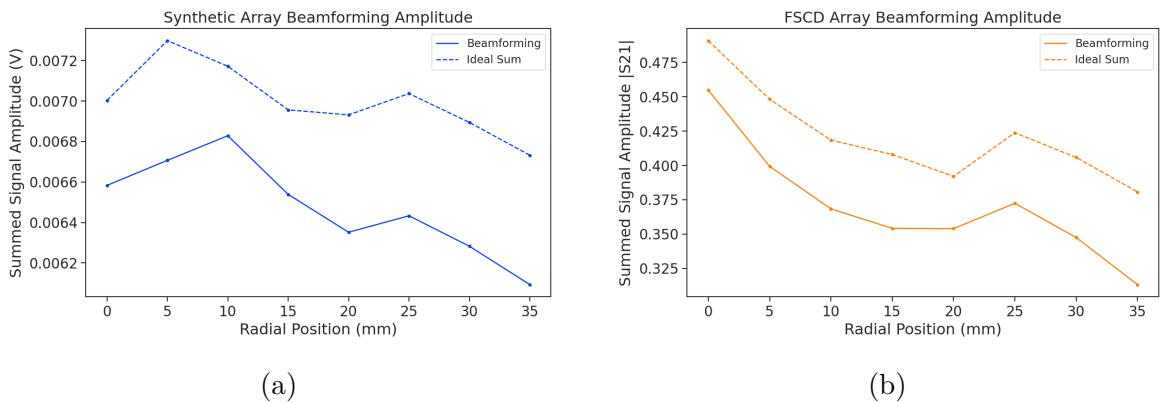


Figure 5.37. A comparison of the maximum signal amplitude obtained by beamforming to the signal amplitude obtained with an ideal summation as a function of the radial position of the SYNCA. The amplitudes for the synthetic array are shown in (a) and the FSCD array are shown in (b). In both setups we observe that the signal amplitudes obtained from beamforming are smaller than the signal amplitude that could be attained with the ideal summation without phase mismatch.

4731 From the beamforming images we extract the maximum amplitude, which we plot
 4732 as a function of the radial position of the SYNCA (see Figure 5.37). The phase errors
 4733 we observed in the FSCD and synthetic arrays leads to power loss at the beamforming
 4734 stage due to phase mismatches between the signals at different channels. This power
 4735 loss can be quantified by comparing the signal amplitude obtained from beamforming to
 4736 the amplitude which would be obtained from an ideal summation. We perform the ideal
 4737 summation by phase shifting each array channel to the same phase and then summing.
 4738 The comparison between the beamforming and ideal sums is shown in Figure 5.37, where
 4739 we observe that both the synthetic and FSCD arrays experience power losses from the
 4740 beamforming summation.

4741 The beamforming power loss can be quantified using the ratio of the beamforming to
 4742 ideal signal amplitudes. Computing this ratio as a function of SYNCA radial position

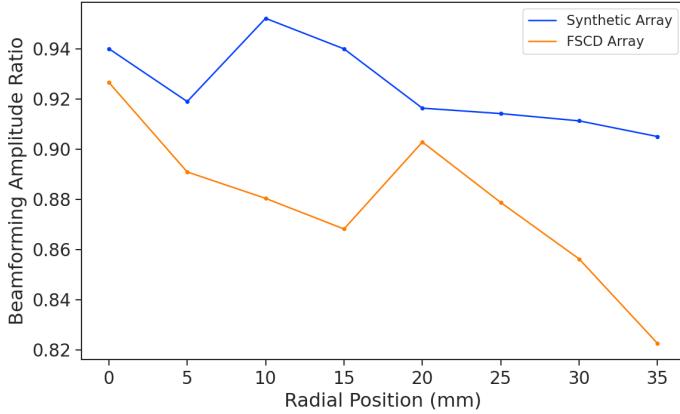


Figure 5.38. The ratio of the beamforming signal amplitude to the ideal signal amplitude for the FSCD and synthetic arrays. We see that the FSCD array has a larger power loss from phase error compare to the synthetic array which indicates that calibration errors associated with the multiple channels as well as reflections are impacting the signal reconstruction.

radius for the FSCD and synthetic arrays we find that the FSCD array has a uniformly smaller beamforming amplitude ratio, which means that the FSCD array has a larger beamforming power loss (see Figure 5.38). The primary contributions to the beamforming power loss in the synthetic array are phase errors from the SYNCA and phase errors from the FSCD antenna near-field. Both of these phase errors contribute to beamforming losses in the FSCD array, but there are clearly additional phase errors in the FSCD array measurements contributing to the smaller ratio. Two potential error sources include phase differences in the different antenna channels that could not be corrected by calibration as well as reflections between antennas in the array. The total effect of these additional phase errors is to reduce the beamforming amplitude ratio by about 5% from the beamforming ratio of the synthetic array. Therefore, we estimate that if no effort is made to correct these phase errors in an FSCD-like experiment, then we expect approximately a 10% total signal amplitude loss from a beamforming signal reconstruction.

5.5.4 Conclusions

The estimated power loss of a beamforming reconstruction obtained from this analysis provides valuable inputs to sensitivity calculations of a FSCD-like antenna array experiment to measure the neutrino mass, since it helps to bound systematic uncertainties from the antenna array and reconstruction pipeline. This power loss lowers the estimated detection efficiency of the experiment since some of the signal power is lost due to improper combining between channels and also increases the uncertainty in the electron's

⁴⁷⁶³ kinetic energy by contributing to errors in the estimation of the electron's cyclotron
⁴⁷⁶⁴ frequency.

⁴⁷⁶⁵ If these reconstruction losses prove unacceptable there are steps that can be taken
⁴⁷⁶⁶ to mitigate their effects. Some examples include the development of a more accurate
⁴⁷⁶⁷ antenna simulation approach that can reproduce the observed near-field interference
⁴⁷⁶⁸ patterns of the FSCD antennas and the implementation of a calibration approach that
⁴⁷⁶⁹ allows for the relative phase delays of the array to be measured without changing or
⁴⁷⁷⁰ disconnecting the antenna array configuration.

Chapter 6

Development of Resonant Cavities for Large Volume CRES Measurements

6.1 Introduction

The cavity approach was originally an alternative CRES measurement technology under consideration by the Project 8 collaboration for the Phase IV experiment. After pursuing an antenna array based CRES demonstrator design for several years, the increasing costs and complexity of the antenna arrays led to a reconsideration of the baseline technology for the ultimate CRES experiment planned by Project 8. Currently, a cavity based CRES experiment is the preferred technology choice for future experiments by the Project 8 collaboration including the Phase IV experiment.

In this chapter I provide a brief summary of resonant cavities and sketch out the key features of a cavity based CRES experiment. In Section 6.2 I provide a brief introduction to cylindrical resonant cavities and the solutions for the electromagnetic fields in the cavity volume.

In Section 6.3 I describe the main components of a cavity based CRES experiment, including the background and trap magnets, cavity geometry and design, and cavity coupling considerations. I also discuss some relevant trade-offs between an antenna array and cavity CRES experiment, and highlight some reasons for the transition of Project 8 to the development of a cavity based experiment.

Finally, in Sections 6.4 and 6.5, I present the design and development of an open mode-filtered cavity that could be used in a cavity based CRES experiment with atomic tritium. The results of the cavity simulations are confirmed by laboratory measurements of a proof-of-principle prototype that demonstrates key features of the design.

4795 6.2 Cylindrical Resonant Cavities

4796 Resonant cavities are sealed conductive containers, which allows us to describe the
4797 electromagnetic (EM) fields contained in the cavity volume as a superposition of resonant
4798 modes [86]. The field shapes of the resonant modes are determined by Maxwell's equations
4799 and the boundary conditions enforced by the cavity geometry. Of interest to Project 8
4800 for CRES measurements are cylindrical cavities due to their ease of construction and
4801 integration with atom and electron trapping magnets.

4802 6.2.1 General Field Solutions

4803 Consider a long segment of conducting material with a cylindrical cross-section (see
4804 Figure 6.1). A geometry such as this can be used as a waveguide transmission line to
4805 transfer EM energy from point to point, or, if conducting shorts are inserted on both
4806 ends of the cylinder, the waveguide becomes a resonant cavity.

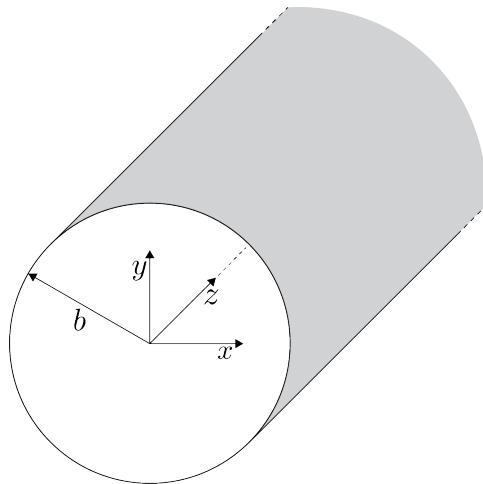


Figure 6.1. Geometry of a cylindrical waveguide with radius b .

4807 The fields allowed inside a cylindrical cavity are determined by the boundary conditions
4808 of the cylindrical geometry. The general approach to solving the fields begins by assuming
4809 solutions to Maxwell's equations of the form

$$\mathbf{E}(x, y, z) = (\mathbf{e}(x, y) + \hat{z}e_z(x, y))e^{-i\beta z}, \quad (6.1)$$

$$\mathbf{H}(x, y, z) = (\mathbf{h}(x, y) + \hat{z}h_z(x, y))e^{-i\beta z}. \quad (6.2)$$

4810 The solutions assume a harmonic time dependence of the form $e^{i\omega t}$ and propagation

4811 along the positive z-axis. The functions $\mathbf{e}(x, y)$ and $\mathbf{h}(x, y)$ represent the transverse
 4812 (\hat{x}, \hat{y}) components of the electric and magnetic fields respectively, and $e_z(x, y)$, $h_z(x, y)$
 4813 represent the longitudinal components. The version of Maxwell's equations in the case
 4814 where there are no source terms can be written as a pair of coupled differential equations,

$$\nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}, \quad (6.3)$$

$$\nabla \times \mathbf{H} = i\omega\epsilon\mathbf{E}, \quad (6.4)$$

4815 where ϵ and μ are the permittivity and permeability of the material inside the waveguide
 4816 or cavity. Using the field solutions from Equations 6.1 and 6.2 one can solve for the
 4817 transverse components of the fields in terms of the longitudinal fields. Because we
 4818 are interested in cylindrical cavities it is advantageous to write the field solutions in
 4819 cylindrical coordinates. After performing this transformation the set of four equations
 4820 for the transverse field components are,

$$H_\rho = \frac{i}{k_c^2} \left(\frac{\omega\epsilon}{\rho} \frac{\partial E_z}{\partial\phi} - \beta \frac{\partial H_z}{\partial\rho} \right), \quad (6.5)$$

$$H_\phi = \frac{-i}{k_c^2} \left(\omega\epsilon \frac{\partial E_z}{\partial\rho} + \frac{\beta}{\rho} \frac{\partial H_z}{\partial\phi} \right), \quad (6.6)$$

$$E_\rho = \frac{-i}{k_c^2} \left(\beta \frac{\partial E_z}{\partial\rho} + \frac{\omega\mu}{\rho} \frac{\partial H_z}{\partial\phi} \right), \quad (6.7)$$

$$E_\phi = \frac{i}{k_c^2} \left(\frac{-\beta}{\rho} \frac{\partial E_z}{\partial\phi} + \omega\mu \frac{\partial H_z}{\partial\rho} \right), \quad (6.8)$$

4821 where k_c is the cutoff wavenumber defined by $k_c^2 = k^2 - \beta^2$ with $k = \omega\sqrt{\mu\epsilon}$ being the
 4822 wavenumber of the EM radiation.

4823 This set of equations can be used to solve for a variety of different modes that can be
 4824 obtained by setting conditions on E_z and H_z . For cylindrical cavities two types of modes
 4825 are allowed, which correspond to solutions where $E_z = 0$ and $H_z = 0$ respectively.

4826 6.2.2 TE and TM Modes

4827 The TE family of modes corresponds to the case where $E_z = 0$. This implies that H_z is
 4828 a solution to the Helmholtz wave equation

$$(\nabla^2 + k^2)H_z = 0. \quad (6.9)$$

4829 For solutions of the form $H_z(\rho, \phi, z) = h_z(\rho, \phi)e^{-i\beta z}$, Equation 6.9 can be solved using
 4830 the standard technique of separation of variables. Rather than reproduce the derivation
 4831 here we shall simply quote the solutions for the transverse fields [86], which are

$$H_\rho = \frac{-i\beta}{k_{c_{nm}}} (A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.10)$$

$$H_\phi = \frac{-i\beta n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.11)$$

$$E_\rho = \frac{-i\omega\mu n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.12)$$

$$E_\phi = \frac{i\omega\mu}{k_{c_{nm}}} (A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}. \quad (6.13)$$

4832 One can observe that the solutions have a periodic dependence on ϕ , and radial profiles
 4833 given by the Bessel functions of the first kind. The integer indices n and m arise from
 4834 continuity conditions on the EM fields in the azimuthal and radial directions. For the
 4835 TE modes $n \geq 0$ and $m \geq 1$. $k_{c_{nm}}$ is the cutoff wavenumber for the TE_{nm} mode given by

$$k_{c_{nm}} = \frac{p'_{nm}}{b}, \quad (6.14)$$

4836 where b is the radius of the cavity or waveguide and p'_{nm} is the m -th root of the derivative
 4837 of the n -th order Bessel function (see Table 6.1).

Table 6.1. A table of the values of p'_{nm} .

n	p'_{n1}	p'_{n2}	p'_{n3}
0	3.832	7.016	10.174
1	1.841	5.331	8.536
2	3.054	6.706	9.970

4838 The TM mode family corresponds to the case where $H_z = 0$, and $(\nabla^2 + k^2)E_z = 0$.
 4839 Again, we assume solutions of the form $E_z(\rho, \phi, z) = e_z(\rho, \phi)e^{-i\beta z}$, for which the general
 4840 form of the solutions is the same as for the TE modes. However, the different boundary
 4841 conditions for the TM modes results in particular solutions with a different from, which
 4842 we shall quote here without derivation. The transverse fields of the TM modes are given
 4843 by

$$H_\rho = \frac{-i\omega\epsilon n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.15)$$

$$H_\phi = \frac{-i\omega\epsilon}{k_{c_{nm}}} (A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z} \quad (6.16)$$

$$E_\rho = \frac{-i\beta}{k_{c_{nm}}} (A \sin n\phi + B \cos n\phi) J'_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.17)$$

$$E_\phi = \frac{-i\beta n}{k_{c_{nm}}^2 \rho} (A \cos n\phi - B \sin n\phi) J_n(k_{c_{nm}}\rho) e^{-i\beta_{nm}z}, \quad (6.18)$$

which one may notice are the same solutions as the TE modes with H and E flipped.
 The cutoff wavenumber for the TM modes is given by, $k_{c_{nm}} = p_{nm}/b$, where the values of p_{nm} correspond to the m -th zero of the n -th order Bessel function (see Table 6.2).

Table 6.2. A table of the values of p_{nm} .

n	p_{n1}	p_{n2}	p_{n3}
0	2.405	5.520	8.654
1	3.832	7.016	10.174
2	5.135	8.417	11.620

6.2.3 Resonant Frequencies of a Cylindrical Cavity

A cylindrical cavity is constructed by taking a section of cylindrical waveguide and shorting both ends with conductive material. This means that the electric fields inside a cylindrical cavity are exactly those we derived in Section 6.2.2 with the additional condition that the electric fields must go to zero at $z = 0$ and $z = L$ (see Figure 6.2).

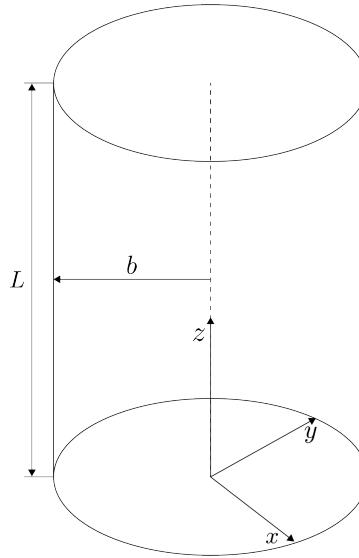


Figure 6.2. The geometry of a cylindrical cavity with length L and radius b .

4852 The transverse electric field solutions for a cylindrical waveguide are of the form

$$\mathbf{E}(\rho, \phi, z) = \mathbf{e}(\rho, \phi) (A_+ e^{-i\beta_{nm}z} + A_- e^{i\beta_{nm}z}), \quad (6.19)$$

4853 where A_+ and A_- are arbitrary amplitudes of forward and backward propagating waves.

4854 In order to enforce that \mathbf{E} is zero at both ends of the cavity we require that

$$\beta_{nm}L = 2\pi\ell, \quad (6.20)$$

4855 where $\ell = 0, 1, 2, 3, \dots$. Using this constraint on the propagation constant we can solve

4856 for the resonant frequencies of the TE_{nml} and the TM_{nml} modes in a cylindrical cavity.

4857 For the TE modes the resonant frequencies are

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{p'_{nm}}{b}\right)^2 + \left(\frac{\ell\pi}{L}\right)^2}, \quad (6.21)$$

4858 and the frequencies of the TM modes are

$$f_{nml} = \frac{c}{2\pi\sqrt{\mu_r\epsilon_r}} \sqrt{\left(\frac{p_{nm}}{b}\right)^2 + \left(\frac{\ell\pi}{L}\right)^2}. \quad (6.22)$$

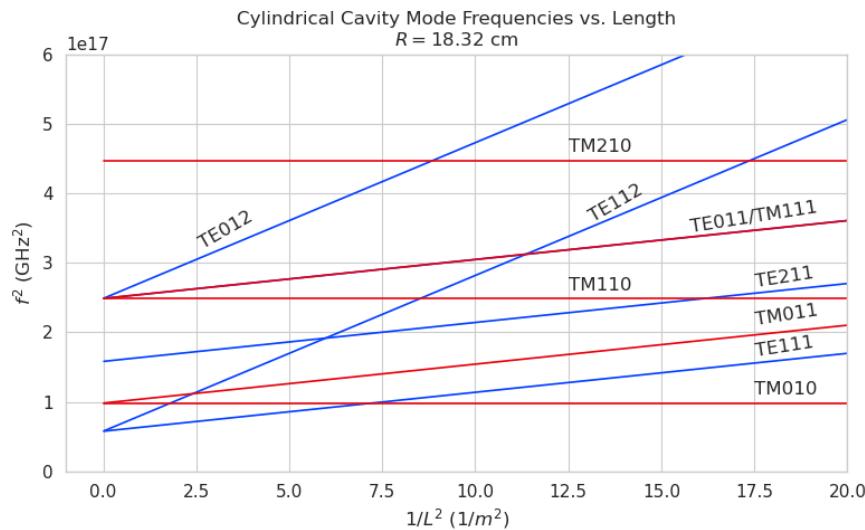


Figure 6.3. Relation of mode frequency to cavity length for a cylindrical cavity with a radius of 18.32 cm.

4859 6.2.4 Cavity Q-factors

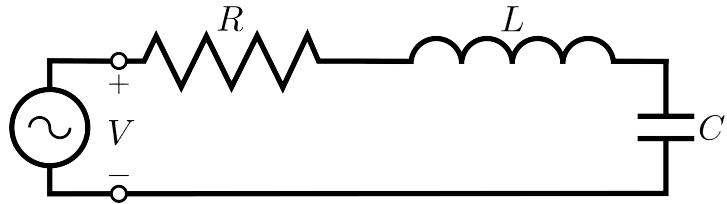


Figure 6.4. A series RLC circuit.

4860 The resonant behavior of cylindrical cavities can be modeled as a series RLC circuit
 4861 (see figure 6.4). The input impedance of the circuit can be obtained by applying
 4862 Kirchhoff's laws to calculate the impedance of the equivalent circuit. For a series RLC
 4863 circuit the input impedance is

$$Z_{\text{in}} = \left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C \right). \quad (6.23)$$

4864 The resistance in the circuit represents all sources of loss in the cavity, which is primarily
 4865 caused by the finite conductivity of the cavity walls. The inductor and capacitor represent
 4866 the energy stored in the cavity in the form of electric and magnetic fields. If the circuit
 4867 is being driven by an external power source we can write the input power in terms of the
 4868 circuit input impedance and the source voltage

$$P_{\text{in}} = \frac{1}{2} Z_{\text{in}} |I|^2 = \frac{1}{2} |I|^2 \left(\frac{1}{R} + \frac{1}{i\omega L} + i\omega C \right). \quad (6.24)$$

4869 The resistor introduces a loss into the system with a power given by

$$P_{\text{loss}} = \frac{1}{2} |I|^2 R, \quad (6.25)$$

4870 and the capacitor and inductor store energies given by

$$W_e = \frac{1}{4} \frac{|I|^2}{\omega^2 C}, \quad (6.26)$$

$$W_m = \frac{1}{4} |I|^2 L, \quad (6.27)$$

4871 respectively. Using these expressions we can write the input power and input impedance

4872 expressions in terms of the lost power and stored energy

$$P_{\text{in}} = P_{\text{loss}} + 2i\omega(W_m - W_e), \quad (6.28)$$

$$Z_{\text{in}} = \frac{P_{\text{loss}} + 2i\omega(W_m - W_e)}{\frac{1}{2}|I|^2}. \quad (6.29)$$

4873 The condition for resonance in the RLC circuit is that the stored magnetic energy
 4874 is equal to the stored electric energy ($W_e = W_m$). When this occurs $Z_{\text{in}} = R$, which is a
 4875 purely real impedance, and $P_{\text{in}} = P_{\text{loss}}$. The resonant frequency of the circuit can be
 4876 determined from the condition $W_e = W_m$ from which one finds that

$$\omega_0 = \frac{1}{\sqrt{LC}}. \quad (6.30)$$

4877 An important performance parameter for any resonant system is the Q-factor, which
 4878 quantifies the quality of the resonator as the ratio of the stored energy multiplied by the
 4879 resonant frequency to the average energy lost per second. For the series RLC circuit, the
 4880 Q-factor is given by the expression

$$Q_0 = \omega \frac{W_e + W_m}{P_{\text{loss}}} = \frac{1}{\omega_0 RC}, \quad (6.31)$$

4881 from which one observes that as the resistance of the RLC circuit is decreased the quality
 4882 factor of the resonator increases. From the perspective of cylindrical cavities this implies
 4883 that as one decreases the resistance of the cavity walls it is expected that the Q-factor of
 4884 the cavity should increase, which is indeed the case. In certain applications where a high
 4885 Q is desireable it is possible to manufacture a cavity out of superconducting materials in
 4886 order to minimize the power losses of the system.

4887 The Q-factor of the resonator also determines with bandwidth (BW) of the system.
 4888 A cavity with a high Q-factor will resonant with a smaller range of frequencies than a
 4889 cavity with a low Q-factor. To see this we can examine the behavior of the RLC circuit
 4890 when driven by frequencies near the resonance. For a frequency $\omega = \omega_0 + \Delta\omega$, where
 4891 $\Delta\omega = \omega - \omega_0 \ll \omega_0$, we can write the input impedance as

$$Z_{\text{in}} = R + i\omega L \left(\frac{\omega^2 - \omega_0^2}{\omega^2} \right), \quad (6.32)$$

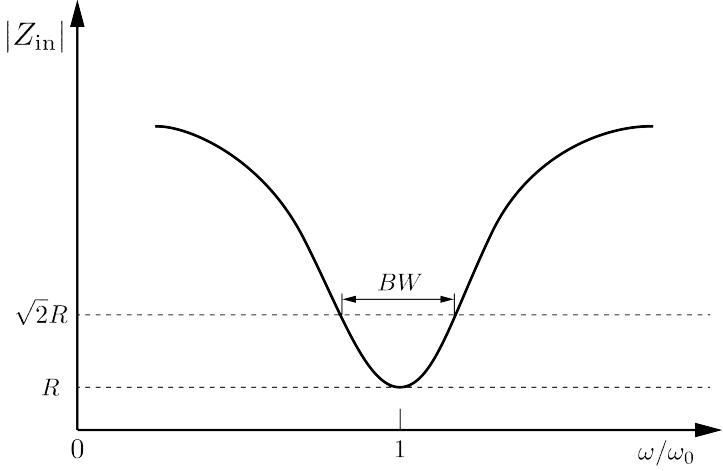


Figure 6.5. Illustration of the behavior of the input impedance of the series RLC circuit as a function of the driving frequency. The BW is proportion to the width of the resonance, which is inversely proportional to Q.

and by expanding $(\omega^2 - \omega_0^2)/\omega^2$ to first order in $\Delta\omega$, we obtain

$$Z_{\text{in}} \approx R + i \frac{2RQ_0\Delta\omega}{\omega_0}. \quad (6.33)$$

Therefore, the magnitude of the input impedance near the resonance is given by

$$|Z_{\text{in}}| = R \sqrt{1 + 4Q_0^2 \frac{\Delta\omega^2}{\omega^2}}, \quad (6.34)$$

from which we observe that for the series RLC circuit the input impedance is minimized at the resonant frequency, which corresponds to the maximum input power (see Figure 6.5). The half-power BW is the range of frequencies over which the input power drops to half the input power on resonance. This occurs when $|Z_{\text{in}}| = \sqrt{2}R$, which corresponds to $\Delta\omega/\omega = \text{BW}/2$. Using Equation 6.34 one can find that

$$2R^2 = R^2(1 + Q_0^2\text{BW}^2), \quad (6.35)$$

which implies

$$\text{BW} = \frac{1}{Q_0} \quad (6.36)$$

It is important to emphasize that the Q-factor defined here, Q_0 , is technically the unloaded Q. It reflects the quality of the cavity or resonant circuit without the influence of any external circuitry. In practice, however, a cavity is invariably coupled to an

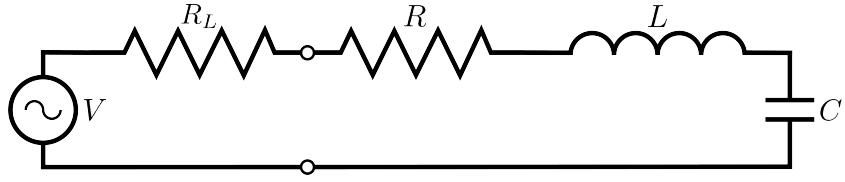


Figure 6.6. A series RLC circuit coupled to an external circuit with input impedance R_L .

4903 external circuit to drive a cavity resonance or to measure the energy of a resonant mode.
 4904 Coupling a cavity to an external circuit changes the Q by loading the equivalent cavity
 4905 RLC circuit (see Figure 6.6). The Q-factor of the cavity when it is loaded by an external
 4906 circuit is called the loaded Q, which is the quantity that one actually measures when
 4907 exciting a resonance in the cavity. Using the series RLC circuit model one can see that
 4908 the load resistor in Figure 6.6 will add in series with the resistor in the circuit for a total
 4909 equivalent resistance of $R + R_L$. Therefore, the loaded Q is given by

$$Q_L = \frac{1}{\omega_0(R + R_L)C}, \quad (6.37)$$

4910 from which one observes that the loaded Q is always less than the intrinsic Q of the
 4911 cavity.

4912 The amount of coupling that is desireable depends on the specific application of
 4913 the resonator. If one wants a resonator that is particular frequency selective than it
 4914 makes sense to limit the amount of coupling to the cavity to maintain a small BW,
 4915 alternatively, if a larger BW is need one can increase the cavity coupling by tuning the
 4916 input impedance of the external circuit. The critical point, where maximum power is
 4917 transferred between the cavity and the external circuit, occurs when the input impedance
 4918 of the cavity matches the input impedance of the external transmission line. For the
 4919 series RLC circuit on resonance, this matching condition corresponds to

$$Z_0 = Z_{in} = R, \quad (6.38)$$

4920 where Z_0 is the impedance of the transmission line. The loaded Q at this critical point
 4921 is, therefore,

$$Q_L = \frac{1}{2\omega_0 Z_0 C} = \frac{Q_0}{2}. \quad (6.39)$$

4922 One can described the degree of coupling between the cavity and an external circuit by

4923 defining a coupling factor, g , such that,

$$g = \frac{Q_0}{Q_L} - 1. \quad (6.40)$$

4924 When $g = 1$ then $Q_L = Q_0/2$, and the cavity is said to be critically coupled as we
4925 described. If $Q_L < Q_0/2$, then the cavity is undercoupled to the transmission line,
4926 corresponding to $g < 1$. Alternatively, if $Q_L > Q_0/2$, then $g > 1$, and the cavity is
4927 overcoupled to the transmission line. Various specialized circuits can be used to tune the
4928 input impedance of the external circuit as seen by the cavity to achieve a wide range of
4929 different coupling factors based on the desired application of the cavity.

4930 6.3 The Cavity Approach to CRES

4931 6.3.1 A Sketch of a Molecular Tritium Cavity CRES Experiment

4932 Resonant cavities can be used to perform CRES measurements, and they represent the
4933 current preferred technology by the Project 8 collaboration. The basic approach to a
4934 neutrino mass measurement using a resonant cavity and molecular tritium beta-decay
source is illustrated by Figure 6.7.

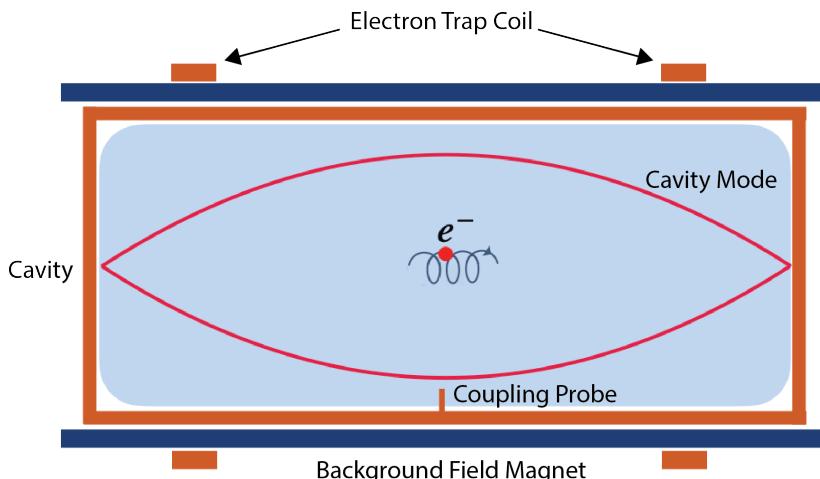


Figure 6.7. A cartoon depiction of a cavity CRES experiment. A metallic cavity filled with tritium gas is inserted into a uniform background magnetic field to perform CRES measurements. Electrons from beta-decays inside the cavity can be trapped and used to excite a resonant mode(s). By coupling to the cavity mode with a suitable probe one can measure the cyclotron frequency of the electron and perform CRES.

4935

4936 At the core of the experiment is a large resonant cavity filled with tritium gas. The
4937 filled cavity is then placed in a uniform magnetic field provided by a primary magnet
4938 that provides the background magnetic field. The value of the background magnetic field
4939 sets the range of cyclotron frequencies for electrons emitted near the tritium spectrum
4940 endpoint. When a beta-decay electron is produced in the cavity it is trapped using a set
4941 of magnetic pinch coils that keep electrons inside the cavity volume.

4942 Electrons trapped inside the cavity do not radiate in the same way as electrons
4943 in free-space. Effectively, the same boundary conditions that were used to derive the
4944 resonant modes of a cylindrical cavity in Section 6.2 apply to the radiation of the electron
4945 as well. The coupling of an electron performing cyclotron motion in a cavity has been
4946 studied in detail for measurements of the electron’s magnetic moment [95–97] If an
4947 electron is emitted with a kinetic energy that corresponds to a cyclotron frequency that
4948 matches a resonant frequency of the cavity, then energy radiated by the electron excites
4949 a corresponding resonance in the cavity. The strength of the electron’s coupling to the
4950 cavity is given to first order by the dot product between the electrons trajectory and
4951 the electric field vector of the resonant mode. Additional effects, such as the Purcell
4952 enhancement [98], alter the emitted power from the free-space Larmor equation [49]. If an
4953 electron is moving with a cyclotron frequency that is far from any resonant modes in the
4954 cavity, then radiation from the electron is suppressed. One can interpret this somewhat
4955 surprising effect as the metallic walls of the cavity reflecting the radiated energy back to
4956 the electron.

4957 Detecting an electron in the cavity is accomplished by coupling the cavity to an
4958 external transmission line that leads to an amplifier and RF receiver chain [99]. The
4959 coupling of the cavity resonance to the amplifier occurs through a coupling probe or
4960 aperture designed to read-out the excitation of the mode(s) excited by the electron. For
4961 CRES measurements, the placement of a wire antenna coupling probe inside the cavity
4962 volume leads to unacceptable losses of tritium atoms due to recombination to molecular
4963 tritium on the antenna surface, therefore, apertures are the preferred coupling method
4964 for cavity CRES experiments.

4965 One of the attractive features of the CRES technique for neutrino mass measurement
4966 is the gain in statistics that comes from the differential nature of the tritium spectrum
4967 measurement. Initially, this seems incompatible with cavities, due to the narrow reso-
4968 nances of cavity modes giving relatively small bandwidth. However, by intentionally
4969 over-coupling to a single cavity mode one can achieve bandwidths of a few 10’s of MHz
4970 (see Section 6.2), which is sufficient for a measurement of the tritium spectrum endpoint

4971 region.

4972 **6.3.2 Magnetic Field, Cavity Geometry, and Resonant Modes**

4973 **Magnetic Field and Volume Scaling**

4974 For a CRES experiment, cylindrical cavities are a natural choice since they match
4975 the geometry of standard solenoid magnets, which are needed in order to produce the
4976 background magnetic field for CRES measurements. Furthermore, the cylindrical shape is
4977 compatible with a Halbach array, which is the leading choice of atom trapping technology
4978 for future atomic tritium experiments by the Project 8 collaboration. Cylindrical
4979 cavities also benefit from well-established machining practices that are able to achieve
4980 high geometric precision at large lengths scales. More exotic cavity designs are under-
4981 consideration and there are on-going efforts to investigate the potential advantages these
4982 may have over the standard cylindrical geometry.

4983 As we saw in Section 6.2, the physical dimensions of the cavity are directly coupled
4984 to the resonant frequencies of the cavity. This dependency links the size of the cavity to
4985 the magnitude of the background magnetic field, because the magnetic field determines
4986 the cyclotron frequencies of trapped electrons. Specifically, as the size of the cavity is
4987 increased to accommodate larger volumes of tritium gas, the frequencies of the resonant
4988 modes decrease proportionally. This requires that the magnetic field also decrease in
4989 order to maintain coupling between electrons and the desired cavity mode.

4990 The required cavity size is ultimately determined by the required statistics in the
4991 tritium spectrum endpoint region. Because the gas density must be kept below a certain
4992 level to ensure that electrons have sufficient time to radiate before scattering, larger
4993 volumes become the only way to achieve higher event statistics. To achieve the sensitivity
4994 goals of Phase III and IV cavity volumes on the order of several cubic-meters are required,
4995 which pushes one towards frequencies in the range of 100's of MHz.

4996 **Single-mode Cavity CRES**

4997 It is tempting to consider maintaining a high magnetic field, while still increasing the size
4998 of the cavity, in order to increase the radiated power from trapped electrons for better
4999 SNR. However, if one were to maintain the same magnetic field while increasing the
5000 size of the cavity, the electrons would begin to couple to higher order modes with more
5001 complicated transverse geometries. The danger with this approach is that a complicated
5002 mode structure could introduce systematic errors into the CRES signals. Example

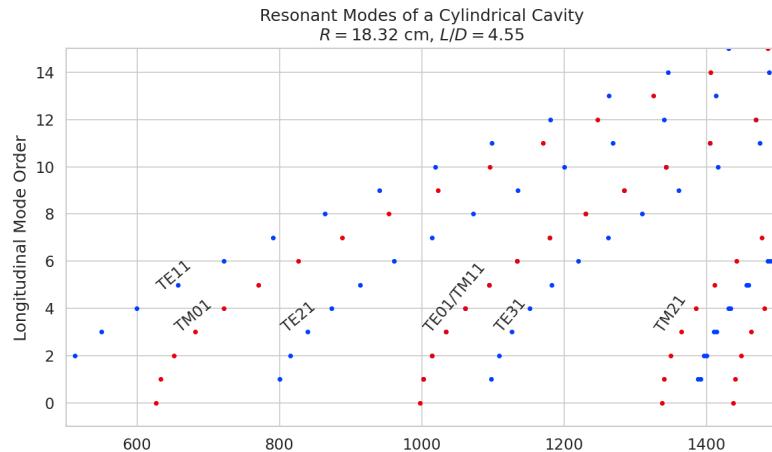
5003 systematics include unpredicted mode hybridization or changes in the mode shapes from
5004 imperfections in the cavity construction, which would prevent reconstruction of the
5005 electron's starting kinetic energies with adequate resolution. For this reason, it is ideal
5006 to operate with magnetic fields that give cyclotron frequencies near the fundamental
5007 frequency of the cavity, where the mode structure is relatively simple (see Figure 6.8).
5008 In this frequency region it is possible to perform CRES by coupling to only a single
5009 resonant mode, however, it is currently an open question if a single mode measurement
5010 will provide enough information about an individual electron's position to reconstruct
5011 the full event. Regardless, developing a solid understanding of the CRES phenomenology
5012 when an electron is coupling to a single mode will be a necessary step towards a future
5013 multi-mode cavity experiment.

5014 Considerations for Resonant Mode Selection

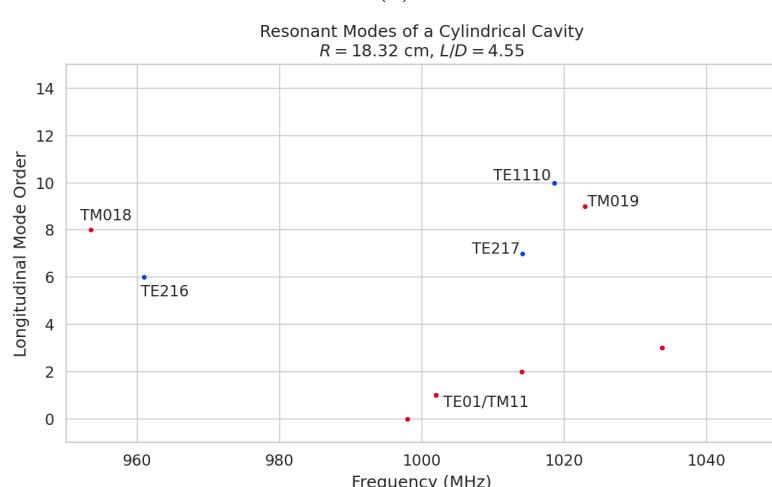
5015 A single-mode cavity experiment begs the question, which resonant mode is best for
5016 CRES measurements? There is an immediate bias towards low order TE_{nm} and TM_{nm}
5017 modes due to the multi-mode considerations discussed above. Additionally, there is a
5018 preference towards modes with longitudinal index $\ell = 1$ with a single antinode along the
5019 vertical axis of the cylindrical cavity. The reason for this is that there is a phase change
5020 in the electric fields between antinodes that leads to modulation effects that destroy the
5021 carrier frequency signal information.

5022 A second consideration for mode selection is the volumetric efficiency of the mode.
5023 Volumetric efficiency can be thought of as an integral over the volume of the cavity
5024 weighted by the relative amplitude of the mode. From the perspective of simply maximiz-
5025 ing the volume useable for CRES measurements this integral would be as close to unity
5026 as possible. However, there is a requirement to reconstruct the position of the electrons
5027 inside the cavity volume so that the local magnetic fields can be used to convert the
5028 measured cyclotron frequency to a kinetic energy. With a single mode this necessarily
5029 requires a variable transverse mode amplitude, which lowers the volumetric efficiency, so
5030 that position of the electron in the cavity can be estimated from the average amplitude
5031 of the CRES signal. Longitudinal indices of $\ell = 1$ have an advantage in volumetric
5032 efficiency over higher order ℓ modes, since there are only two longitudinal nodes, one at
5033 each end of the cavity. Therefore, the average coupling strength of trapped electrons as
5034 they oscillate axially is higher for $\ell = 1$ modes.

5035 The longitudinal variation in the mode strength is ultimately critical for achieving the
5036 energy resolution required for neutrino mass measurements. Correcting for the change in



(a)



(b)

Figure 6.8. Examples of the resonant mode frequencies of a cylindrical cavity. This cavity has a radius of 18.32 cm and a length to diameter ratio of 4.55.

5037 the average magnetic fields experienced by electrons with different pitch angles requires
 5038 that information on the axial motion of the electron be encoded into the CRE signal.
 5039 The longitudinal variation in the mode amplitude leads to amplitude modulation of the
 5040 CRE signal with a frequency proportional to the electron's pitch angle.

5041 An additional factor for mode selection is the intrinsic or unloaded Q of the mode. In
 5042 terms of SNR it is advantageous to use a mode with a very high Q_0 , which is then highly
 5043 overcoupled to achieve the necessary bandwidth to cover the tritium endpoint spectrum.
 5044 This scheme leads to a decoupling of the physical cavity temperature from the effective
 5045 noise temperature after the amplifier, which allows us to achieve adequate SNR without

5046 the requirement of cooling the entire cavity to single Kelvin temperatures.

5047 An example of a resonant mode that exhibits these traits is the TE₀₁₁ mode. At present
5048 the TE₀₁₁ mode is the preferred resonance for a single-mode cavity CRES experiment
5049 by the Project 8 collaboration. TE₀₁₁ is a low order mode located in a region relatively
5050 far from other cavity modes. Furthermore, the separation of the TE₀₁₁ mode can be
5051 improved by various mode-filtering techniques discussed in Section 6.4.2 below. TE₀₁₁
5052 consists of a single longitudinal antinode that can provide pitch angle information in the
5053 form of amplitude modulation, and has an electric field with a radial profile given by the
5054 J'_0 Bessel function allowing for radial position estimation. Lastly, the TE₀₁₁ mode has a
5055 relatively high intrinsic Q compared to nearby modes, which helps with SNR. Unloaded
5056 Q's greater than 80000 are achievable for a 1 GHz TE₀₁₁ resonance using a copper walled
5057 cavity.

5058 **6.3.3 Trade-offs Between the Antenna and Cavity Approaches**

5059 The choice between cavities and antennas for large-scale CRES measurements is not
5060 without trade-offs. Both the antenna array and cavity approaches are relatively immature
5061 techniques, at present there are no known obstacles that would prevent either approach
5062 from being used for a large scale neutrino mass experiment. The preference for cavities
5063 is largely driven by important practical considerations that could make a cavity based
5064 experiment significantly cheaper than an antenna experiment of similar size and scope.
5065 However, the switch to cavities also introduces new challenges less relevant to the
5066 antenna array, which must be solved in order for Project 8 to achieve its neutrino mass
5067 measurement goals.

5068 One of the major relative drawbacks of the antenna array approach is the size and
5069 complexity of the data-acquisition system. A large-scale antenna array experiment
5070 requires $O(100)$ antennas independently digitized at rates of $O(10)$ to $O(100)$ MHz. Since
5071 there is insufficient information in a single antenna channel to detect or reconstruct the
5072 CRES signal, the entire array output must be processed during the signal reconstruction.
5073 Because data storage becomes an issue with these data volumes, there is a real-time
5074 signal reconstruction requirement that allows one to detect CRES signals buried in the
5075 thermal noise. As we discuss in Section 4.4, the computational cost of these real-time
5076 detection algorithms are potentially quite large for even a small scale antenna array
5077 experiment. However, the operating principle of a cavity experiment allows the CRES
5078 signal to be detected using only a single read-out channel digitized at rates of $O(10)$ MHz,
5079 which reduces the cost of the data acquisition system by many orders of magnitude.

5080 From an engineering perspective, the simple geometry and thin-walls of a cylindrical
5081 cavity are simpler to interface with the cryogenic and magnetic subsystems needed for a
5082 CRES experiment. Whereas, the antenna array requires careful design and engineering
5083 to accommodate the antenna array and receiver electronics in proximity to the trapping
5084 magnets. Additionally, due to near-field interference effects, the antenna array is unable
5085 to reconstruct CRES events within the reactive near-field distance of the antennas.
5086 Because atom trapping requirements require magnetic fields which correspond to cyclotron
5087 frequencies for endpoint electrons less than 1 GHz, the required stand-off distance leads to
5088 a significant loss in useable experiment volume, necessitating larger and more expensive
5089 magnets.

5090 Another advantage to the cavity approach is the relatively compact sideband structure,
5091 which is a result of the low modulation index for cavity CRES signals. The axial motion
5092 in an antenna array experiment leads to frequency modulation and sidebands. The shape
5093 of the sideband structure is determined by the modulation index, $h = \frac{\Delta f}{f_a}$, where Δf
5094 is the size of the frequency deviation and f_a is the axial frequency. The large electron
5095 traps required for a cubic-meter-scale experiment leads to high modulation indices, which
5096 causes the signal spectrum to be made up of numerous low power sidebands that make
5097 reconstruction and detection challenging. This behavior was observed in simulations
5098 of the FSCD in which carrier power decreased with pitch angle due to the increase in
5099 modulation index (see Figure 4.31). For cavities, however, the modulation index remains
5100 near $h = 1$ even for very long magnetic traps due to the high phase velocity in cavities
5101 relative to the axial velocity of the electron. This results in an almost ideal spectrum
5102 shape that has a strong carrier frequency with a few sidebands whose relative amplitudes
5103 encode pitch angle information.

5104 A downside of the cavity approach is the apparent difficulty of estimating the position
5105 of the electron using only the coupling of the electron to a single mode. The amplitude of
5106 the TE₀₁₁ mode is completely independent of the azimuthal coordinate, therefore, position
5107 reconstruction using the TE₀₁₁ mode is only able to estimate the radial position of the
5108 electron. This position degeneracy may lead to magnetic field uniformity requirements
5109 that are too challenging to meet due to mechanical uncertainties in cavity and magnet
5110 construction, as well as uncertainties caused by nuisance external magnetic fields such
5111 as the Earth's field and magnetic fields from building materials. A multi-mode cavity
5112 experiment may provide a way to extract more precise information on the position of
5113 the electron by analyzing the coupling of the electron to several modes that overlap in
5114 different ways.

6.4 Single-mode Resonant Cavity Design and Simulations

The single-mode cylindrical cavities envisioned for the Phase III and IV experiments must be carefully engineered in order to measure the neutrino mass with the desired sensitivity. In this section I summarize some simulation studies performed to analyze early design concepts for a single-mode cavity. The primary tool for these investigations was Ansys HFSS, which was also used for the development of the SYNCA antenna described in Section 5.3.

6.4.1 Open Cylindrical Cavities with Coaxial Terminations

Design Concept

A basic cavity design question relevant to Project 8's ultimate goal of an atomic tritium CRES experiment is how to build a cavity that can be efficiently filled with atomic tritium. To keep the rate of atom loss from recombination on surfaces it is ideal if the ends of the cylindrical cavity are as open as possible so that tritium atoms can flow inside unimpeded. Additionally, one of the primary calibration techniques planned for future CRES experiments involves CRES measurements using electrons injected from an electron gun source, which also requires an opening at the cavity end. Cylindrical cavities with open ends can be manufactured, however, the intrinsic Q-factors of these cavities are orders of magnitude less than their sealed counterparts, which reduces the signal-to-noise ratio when that cavity is used for CRES measurement.

Cylindrical cavities with mostly open ends that also exhibit Q values for the $TE_{01\ell}$ modes similar to sealed cavities can be built by using coaxial endcaps to terminate the cavity. Cavities of this type have been manufactured for specialized applications related to the measurements of the dielectric constants of liquefied gasses (see Figure 6.9) [2, 3]. This cavity design leaves the ends of the cavity wide open, but retains high Q-values for the $TE_{01\ell}$ modes due to the coaxial endcap, which are designed to perfectly reflect the electric fields of $TE_{01\ell}$ modes. Coupling to the $TE_{01\ell}$ mode is achieved via an aperture located at the center of the cavity wall.

A cavity similar to Figure 6.9 is a candidate design for the future CRES experiments by Project 8, since it appears to elegantly solve many practical issues that arise when combining cavity CRES and atomic tritium. The coaxial endcaps leave significant regions of the cavity ends completely open, which allows for the entrance of atomic tritium as well as the pumping away of molecular tritium that has recombined on the cavity walls.

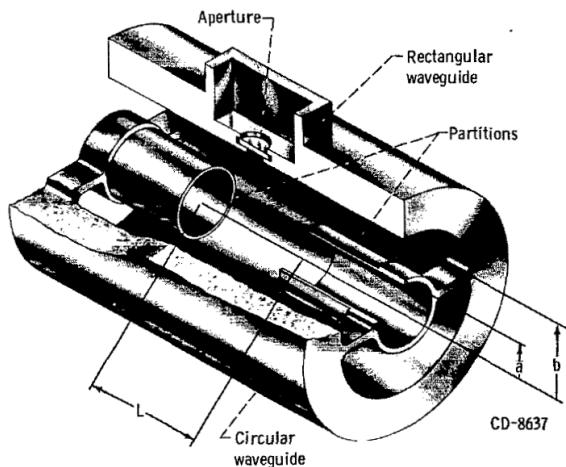


Figure 6.9. An image of an open cavity with coaxial terminations used for dielectric constant measurements. Figure from [2].

These open ends are achieved while preserving the high Q-values of the $\text{TE}_{01\ell}$ modes, which is important for extracting as much signal power from the electron as possible. In subsequent sections we shall analyze this cavity design in more detail, primarily by using HFSS simulations to analyze the resonant mode structure of this cavity geometry.

Coaxial Terminator Constraints

The reason that coaxial endcaps can be used to achieve high Q-values for the $\text{TE}_{01\ell}$ modes is that the electric fields for these modes are purely azimuthally polarized (see Equations 6.12 and 6.13). Therefore, the boundary conditions that require the electric field to go to zero at the cavity ends can be supplied using a coaxial partition of the correct radius (see Figure 6.10). Because the cylindrical shape enforced by the partition does not match the boundary conditions of other cavity modes, these terminations also significantly suppress the Q-factors of non- $\text{TE}_{01\ell}$ modes, which is potentially beneficial for a single-mode cavity CRES experiment.

The correct radius of the cylindrical partition is derived by setting up the boundary value problem in Figure 6.10, and analyzing the reflection and transmission coefficients for waves incident on the coaxial terminators. The basic problem is to identify the radius a where the reflection coefficient for the $\text{TE}_{01\ell}$ modes becomes equal to 1. One can show that if the coaxial partitions are made sufficiently long relative to the wavelength of the TE_{01} modes than perfect reflection can be achieved. This derivation is quite lengthy and complex and is presented in full in [3]. Here, we shall simply explain the resulting

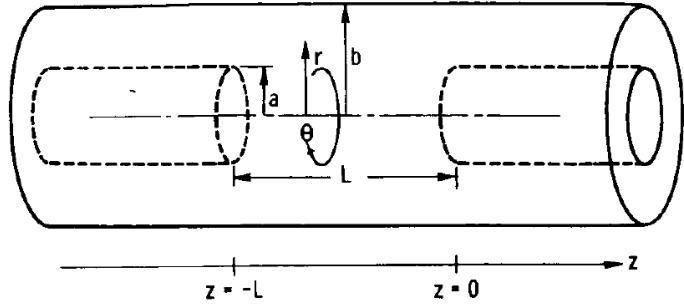


Figure 6.10. The simplified geometry of an open cavity with coaxial terminations. Figure from [3].

5167 conditions on the partition radius for perfect reflection.

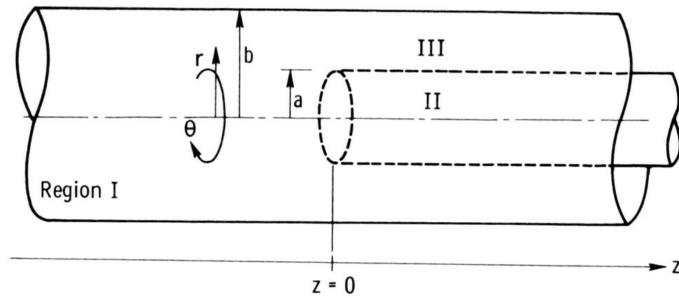


Figure 6.11. Electric field regions for the open cavity boundary value problem. Figure from [3].

5168 The open cavity boundary value problem is solved by expressing the forms of the
 5169 electric fields in the different regions of the cavity and requiring that the electric fields are
 5170 continuous. There are effectively three distinct regions in the open cavity corresponding
 5171 to the central cavity volume, the inner coaxial volume, and the outer coaxial volume (see
 5172 Figure 6.11).

5173 In Region I, the boundary conditions are those of a cylindrical waveguide, and we
 5174 require that E_ϕ for the TE_{0m} modes go to zero at the cavity wall ($r = b$). This requires
 5175 that $J'_{0m}(k_{c0m} b) = 0$. We aim to solve for the radius a in the specific situation where the
 5176 TE_{01} mode can propagate but all other TE_{0m} modes are below the cutoff frequency for
 5177 the circular waveguide. This is equivalent to requiring

$$3.832 < k_{c0m} b < 7.016, \quad (6.41)$$

5178 where the numbers 3.832 and 7.016 correspond to the first and second zeros of the Bessel
 5179 function (see Table 6.1).

5180 In Region II the boundary conditions are those of a cylindrical waveguide, but with
 5181 a smaller radius. The condition that $E_\phi = 0$ at the cylindrical partition radius is that
 5182 $J'_{0m}(k_{c0m}a) = 0$. To ensure perfect reflection, we want all modes in Region 1 of the cavity
 5183 to be below the cutoff frequency of the circular waveguide formed by the inner volume of
 5184 the coaxial terminator. Therefore, we consider the solutions where

$$k_{c0m}a < 3.832. \quad (6.42)$$

5185 Finally, in Region III the boundary condition are those of a coaxial waveguide. We
 5186 need to guarantee that $E_\phi = 0$ at both $r = b$ and $r = a$, which involves finding the
 5187 eigenvalues of the following equation

$$J'_0(k_{c0m}a)Y'_0(k_{c0m}b) - J'_0(k_{c0m}b)Y'_0(k_{c0m}a) = 0, \quad (6.43)$$

5188 where Y'_0 the zeroth-order derivatives of the Bessel function of the second kind. The
 5189 solutions to this equation depend on the value of the ratio b/a . The approximate solution
 5190 is given by

$$\delta_n a \simeq \frac{n\pi}{b/a - 1}, \quad (6.44)$$

5191 where δ_n are eigenvalues of Equation 6.43. Similar to Region II, we are interested in
 5192 solutions for which the TE₀₁ modes of Region I are below the cutoff of Region III.
 5193 Therefore, we require that

$$k_{c0m} < \delta_1. \quad (6.45)$$

5194 In general, one has some freedom in specifying the value of b/a . A value typically used
 5195 in practice is $b/a = 2.082$, which corresponds to positioning the radius of the cylindrical
 5196 partition at the maxima of the TE₀₁ electrical fields.

5197 Using the constraints from the three field regions one can develop a coaxial terminator
 5198 that acts as a virtual perfectly conducting surface for the TE₀₁ modes. The only required
 5199 inputs are the desired frequency of the TE₀₁₁ mode and a choice for the value of b/a .

5200 6.4.2 Mode Filtering

5201 The general case of an electron coupling to a resonant cavity is complicated. This is
 5202 because cavities contain an infinite number of resonant modes, which for higher order
 5203 modes, have couplings to the electron with a complex spatial dependence. The danger is
 5204 that improper modeling of the electron's coupling to the cavity can lead to systematic

5205 errors in the CRES measurements that prevent a high-resolution measurement of the
5206 electron's kinetic energy. This in part drives the preference for a single-mode cavity
5207 experiment that uses only the electron's coupling to the TE₀₁₁ mode to perform CRES,
5208 assuming that sufficient information on the electron's position can be obtained with a
5209 single mode.

5210 The TE₀₁₁ mode is in a region where there are relatively few other modes to which
5211 the electron could couple(see Figure 6.8). However, one can see that the frequency of
5212 the TE₀₁₁ is perfectly degenerate with the TM₁₁₁ mode, which means that electrons will
5213 inevitably couple to both modes if they have the correct cyclotron frequency.

5214 The magnitude of the impact of the electron coupling to both TE₀₁₁ and TM₁₁₁ is
5215 currently unknown. To first order an electron coupling to more both modes will lose more
5216 energy overtime, which can be measured by observing the frequency chirp rate of the
5217 signal. This effect may be small enough to be negligible or simple enough to model that
5218 the cavity can be treated as an effective single-mode cavity. Alternatively, the one could
5219 consider devising a coupling scheme that is sensitive to both the TE₀₁₁ and the TM₁₁₁
5220 modes. By measuring the coupling of the electron to both modes more information on
5221 the position of the electron could be obtained, which could improve the position and
5222 energy resolution of the CRES measurements.

5223 A different approach is the mode filtering approach, which seeks to obtain a single
5224 TE₀₁₁ mode cavity using perturbations to the cavity walls that selectively impede the
5225 TM modes, while leaving the TE modes mostly unperturbed. The type of perturbations
5226 required can be determined by visualizing the surface currents induced in the cavity
5227 walls by each type of mode (see Figure 6.12). By definition, all TM have electric fields
5228 directed along the vertical axis of the cylindrical cavity, which means that perturbations
5229 that impede currents in this direction will modify TM resonances. On the other hand,
5230 the TE₀₁ modes induce azimuthal currents in the cavity walls, therefore, it is possible to
5231 break the degeneracy between TE₀₁ and TM₁₁ using a cavity perturbation that impedes
5232 axial currents, but does not affect the flow of azimuthal currents.

5233 Figure 6.12 shows two cavity design concepts that achieve this selective current
5234 perturbation. The resistive approach inserts a series of thin dielectric rings into the walls
5235 of the cavity that introduces a resistive and capacitive impedance to the longitudinal
5236 currents, while leaving azimuthal current paths intact. Cavities of this type with high
5237 TE₀₁ Q's have also been constructed by tightly wrapping a thin, dielectric coated wire
5238 around a mold to form the cavity wall. An alternative method is to introduce an inductive
5239 impedance by cutting grooves or a thread pattern on the inside wall of the cavity. For

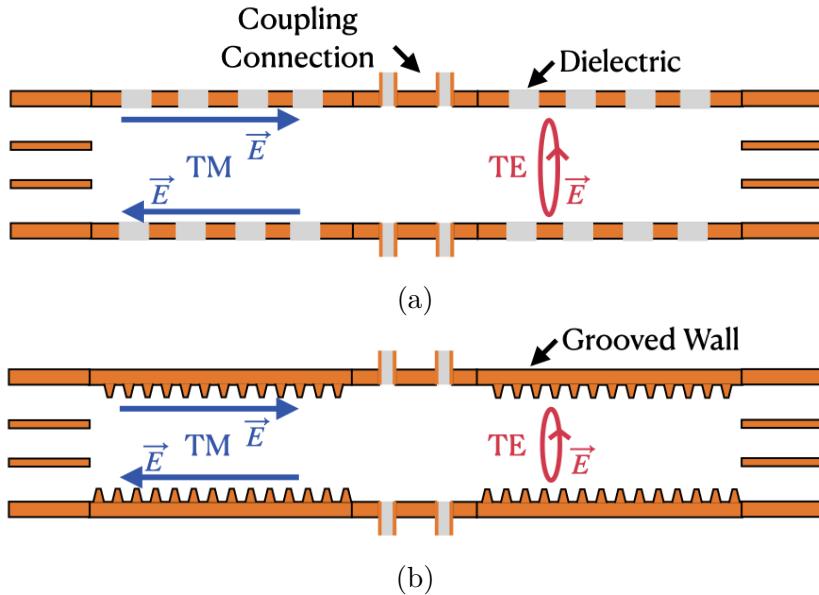


Figure 6.12. Two mode filtering concepts to break the degeneracy of TE_{01} and TM_{11} modes. The resistive approach uses dielectric materials to impede currents that travel vertically along the cavity while leaving azimuthal currents unperturbed. An alternative approach is to impede the currents using grooves cut into the cavity wall, which achieve the same effect with an inductive impedance.

5240 reasons of manufacturability and compatibility with tritium the grooved cavity approach
 5241 is the preferred method for mode-filtered cavity construction by Project 8.

5242 **6.4.3 Simulations of Open, Mode-filtered Cavities**

5243 A candidate design for a single TE_{011} mode CRES experiment is a cavity that utilizes
 5244 the coaxial terminations combined with a mode-filtering wall. The first step towards
 5245 validating that a cavity that combines these two design features will operate as expected
 5246 is a thorough simulation effort for which finite element method (FEM) simulation software
 5247 is invaluable. The primary tool for electromagnetic FEM calculations inside Project 8 is
 5248 Ansys HFSS, which has a robust and well-established eigenmode solver that can identify
 5249 the resonant frequencies and associated Q-factors for given structure.

5250 Four variations of a cavity design with a ~ 1 GHz TE_{011} resonance were implemented
 5251 in HFSS (see Figure 6.13). The four designs include a standard cylindrical cavity, an
 5252 open cavity with smooth walls, an open cavity with resistive walls, and an open cavity
 5253 with grooved walls. The relevant design parameters are summarized in Table 6.3. All
 5254 cavities were simulated using copper walls and filled with a vacuum dielectric. The
 5255 identities of the resonant modes found by HFSS were validated by visual inspection of

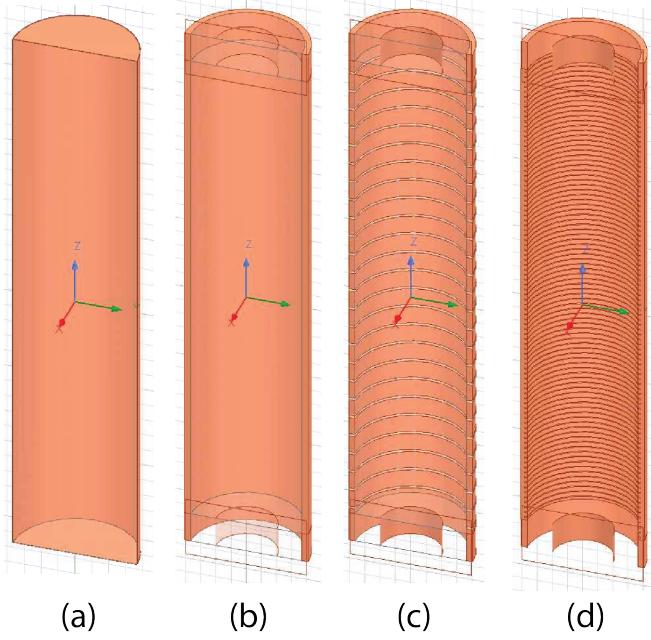


Figure 6.13. Four cavity design variations. (a) is a standard sealed cylindrical cavity, (b) is an open cavity with smooth walls, (c) is an open cavity with resistive walls, and (d) is an open cavity with grooved walls. The main cavity and coaxial terminator parameter are identical for all four cavities.

5256 the electric and magnetic field patterns and by comparison to analytical calculations of
 5257 the mode frequencies.

Table 6.3. A table of cavity design parameters used for HFSS simulations.

Name	Qty.	Unit	Description
D_{cav}	326.4	mm	Cavity diameter
L_{cav}	1668.0	mm	Cavity length
D_{term}	200.2	mm	Inner diameter of coaxial terminator
L_{term}	100.0	mm	Terminator length
l_{die}	8.3	mm	Dielectric spacer thickness
Δl_{die}	66.7	mm	Distance between dielectric spacers
l_{groove}	3.0	mm	Groove height
d_{groove}	9.0	mm	Groove depth
Δl_{groove}	18.3	mm	Distance between grooves

5258 The results of the HFSS simulations validate our predictions of the resonant behavior
 5259 of an open, mode-filtered cavity developed in the preceding sections (see Figure 6.14) One
 5260 can see that for a standard cavity the TE_{01} and the TM_{11} are degenerate in frequency
 5261 with relatively high Q-factors. The open-ended cavity preserves the high Q-factors of
 5262 the TE_{01} modes, while the other modes, since their boundary conditions do not match

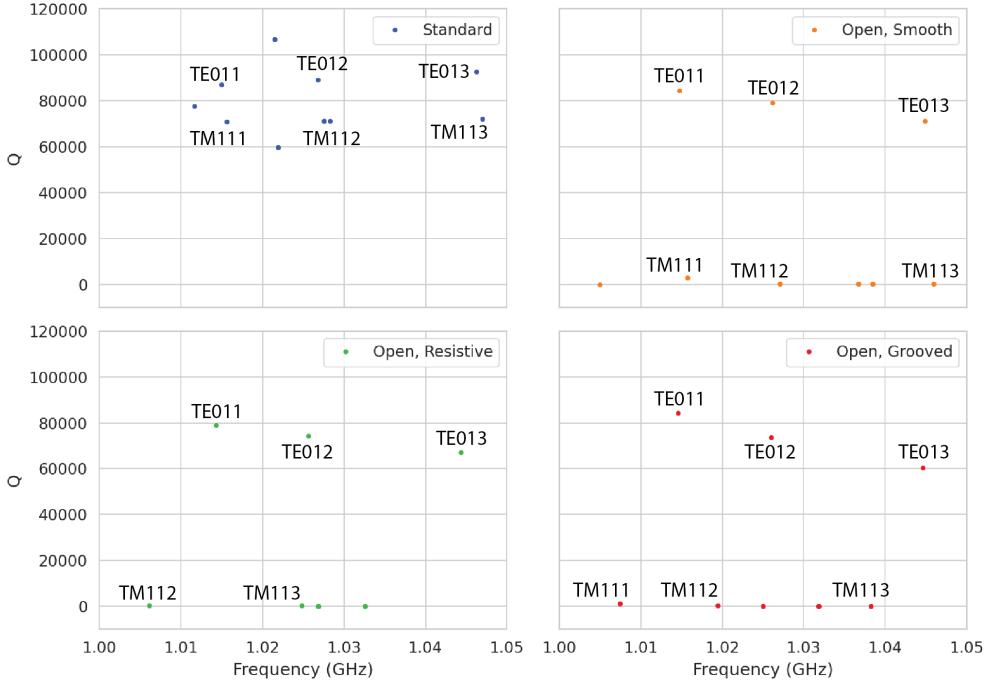


Figure 6.14. The frequencies and Q-factors of the resonant modes identified by HFSS for the cavity variations shown in Figure 6.13. The fully-sealed cavity with smooth walls has several high-Q modes near the TE_{011} resonance. Introducing the open-termination preserves the Q-factors of the $TE_{01\ell}$ modes and suppresses the Q-factors of the modes whose boundary conditions do not match the cylindrical partition. Both the resistive and grooved wall perturbations shift the resonant frequencies of the TM modes away from the TE_{011} mode. By properly tuning the geometry of the grooves or the resistive spacers several MHz of frequency separation can be achieved.

the coaxial geometry, have their Q-factors suppressed. One can see that the effect of the resistive and inductive mode-filtering schemes is to effectively shift the resonant frequencies of the TM_{11} modes below those of the associated TE_{01} modes, which breaks the degeneracy. Optimization of the dielectric spacer or groove parameters can ensure that the TE_{011} mode is isolated from other modes by $O(10)$ MHz, which provides sufficient bandwidth for a measurement of the tritium spectrum endpoint.

Further optimization of the cavity design requires a more detailed cavity simulation that includes the cavity coupling mechanism as well as other geometry modifications required for integration into the magnetic and tritium gas subsystems. Perhaps more important is the development of the capability to simulate the interaction of electrons with the cavity so that simulated CRES signals can be generated using cavities designed for CRES measurements. Simulated CRES signals can then be used to estimate the neutrino mass sensitivity of the experiment, which allows for the optimization of the cavity

5276 design towards the configuration that provides the best measurement of the neutrino
5277 mass.

5278 **6.5 Single-mode Resonant Cavity Measurements**

5279 Measurement test stands play an important role in the research and development process
5280 that cannot be replaced by simulations. For example, constructing a prototype CRES
5281 cavity forces one to consider important practical issues such as manufacturability and
5282 machine tolerances that may require modifications to the design. Furthermore, by
5283 comparing laboratory measurements of a real cavity to simulations, one can quantify
5284 the impact of imperfections and real-life measurement systematics, which allows for
5285 more accurate sensitivity estimates of the experiment. Lastly, the development of these
5286 prototypes helps to build the necessary experience and expertise within the collaboration
5287 required for more complicated experiments to succeed.

5288 In this spirit a prototype cavity was constructed to demonstrate the open, mode-
5289 filtered cavity concept explored in the previous sections. The primary goal of the
5290 measurements was to validate that an open, mode-filtered cavity suppressed the TM_{11}
5291 modes as predicted by HFSS simulations.

5292 **6.5.1 Cavities and Setup**

5293 Two rudimentary, cavities were constructed using segments of copper pipe available from
5294 McMaster-Carr (see Figure 6.15). The design consists of copper pipes of two diameters.
5295 The larger diameter pipe forms the main cavity wall and the smaller diameter pipe is
5296 used to create a coaxial termination. The diameter of the outer pipe was chosen to
5297 produce a TE_{011} resonance of approximately 6 GHz, while the diameter of the smaller
5298 pipe was selected based on the open termination criteria introduced in Section 6.4.1. The
5299 approximate diameters and lengths of the copper pipe are summarized in Table 6.4.

5300 Coupling to the cavity was achieved using a hand-formable segment of coaxial cable
5301 stripped at one end to form a loop antenna. This was inserted into a small hole located
5302 at the center of the main cavity wall. The coaxial terminators were supported inside the
5303 main cavity by carving a spacer from polystyrene foam (styrofoam) so that they could
5304 be easily inserted into the cavity and repositioned. The dielectric constant of styrofoam
5305 is quite close to air at microwave frequencies so this is expected to have minimal impact
5306 on the resonant properties of the cavity.

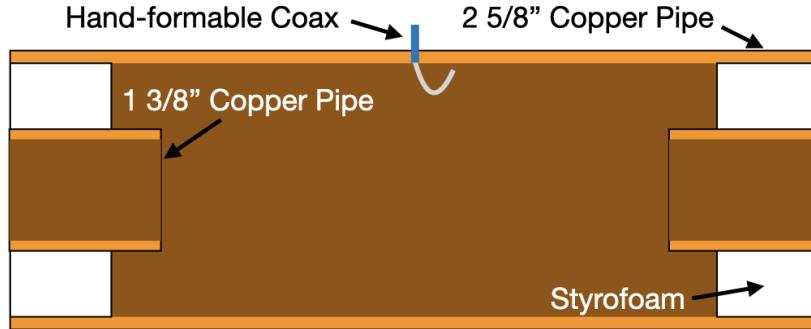


Figure 6.15. A cartoon depicting the design of the open-ended cavity prototype designed to operate at approximately 6 GHz. The main cavity wall was composed of a single copper pipe. A mode-filtered version of this cavity was constructed by

Table 6.4. A table of parameters describing the cavity prototypes. Certain values such as the cavity length and the distance between dielectric spacers are approximate due to variation in the machining of the copper. In particular, the filtered cavity was constructed from conducting copper segments that varied in size from 1.50" to 1.85".

Name	Qty.	Unit	Description
D_{cav}	2.625	in	Cavity diameter
L_{cav}	≈ 13	in	Cavity length
D_{term}	1.375	in	Inner diameter of coaxial terminator
L_{term}	1.575	in	Terminator length
l_{die}	0.75	in	Dielectric spacer thickness
Δl_{die}	≈ 1.50 to 1.85	in	Distance between dielectric spacers

5307 The actual length of the cavity is given by the distance between the inner edges of the
 5308 coaxial terminations. The length of the outer section of pipe that forms the main wall of
 5309 the cavity is approximately 16" in length which leads to a cavity length of $\approx 13"$ when
 5310 both terminators are inserted in the cavity. Because the terminators were not rigidly
 5311 mounted this distance is only approximate, however, the uncertain length of the cavity
 5312 will not prevent us from validating the open cavity design.

5313 Along with the smooth-walled open cavity a resistively mode-filtered cavity was
 5314 constructed by creating dielectric spacers out of segments of clear PVC pipe (see Figure
 5315 6.16). The spacers were machined such that the conductive segments of the cavity would
 5316 be separated by 0.75" when the cavity was fully assembled. Due to variations in the
 5317 lengths of the copper segments that make up the cavity wall the distance between spacers
 5318 has significant variation with average value of about 1.7". Eight total spacers were used
 5319 to build the cavity, which when assembled was approximately 16" in total length similar
 5320 to the non-filtered cavity.

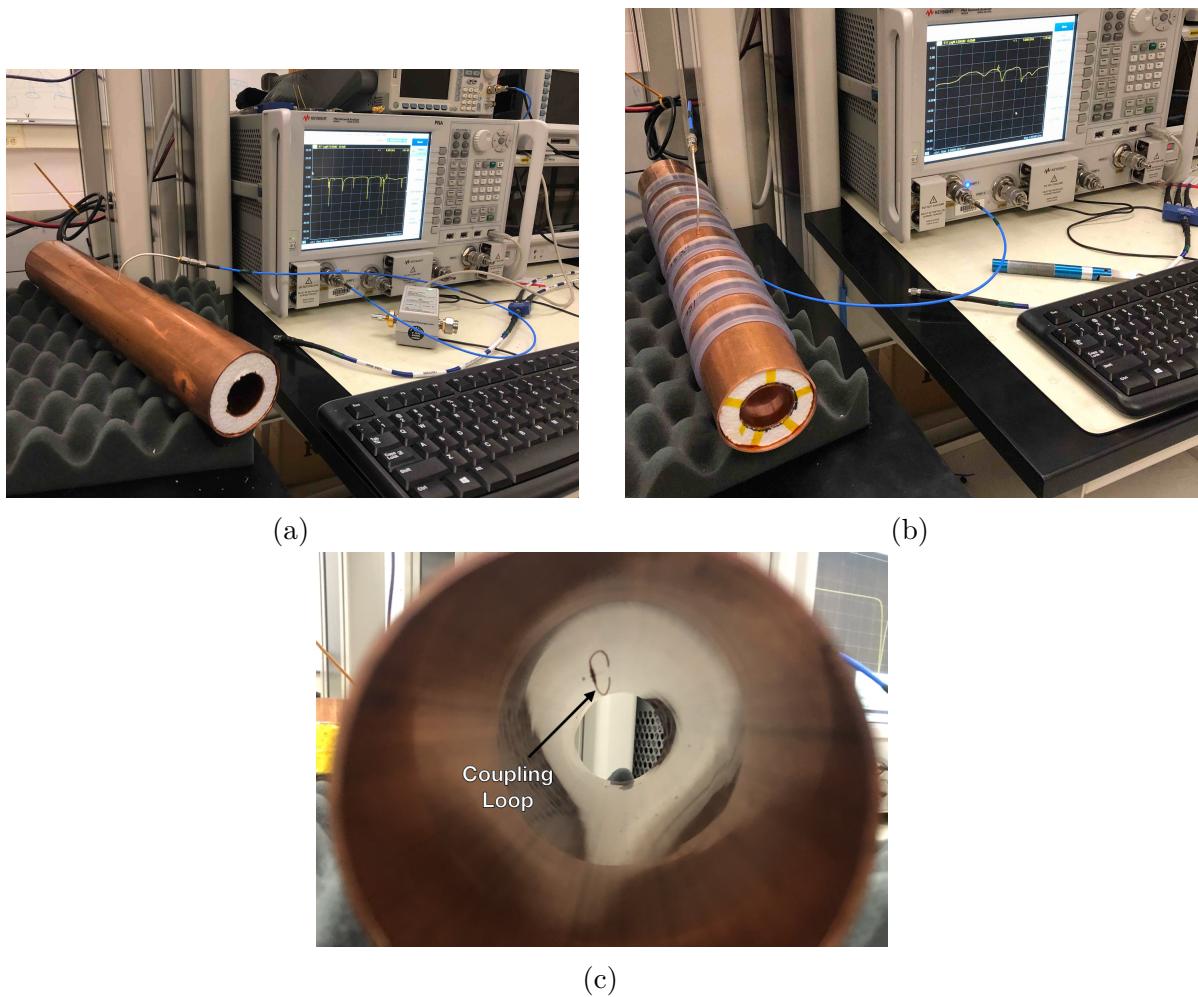


Figure 6.16. Images depicting the measurement of the filtered and non-filtered open cavities using the VNA. The coupling loop in the figure is shown in the TE orientation.

5321 Measurements of both cavities were performed using a VNA connected to the cavity
 5322 coupling probe (see Figure 6.16). By measuring the return loss over a range of frequencies
 5323 one can measure the frequencies and relative Q-factors of the resonant modes in the
 5324 cavity. Due to the opposite polarity of the electric fields for the TE and TM modes,
 5325 the loop coupling probe must be rotated 90° to change the polarity of the loop antenna.
 5326 When the antenna is oriented such that the loop opening faces the ends of the cavity, it
 5327 couples primarily to the TE modes which have magnetic fields directed along the long
 5328 axis of the cavity (see Figure 6.16). If the coupling loop is turned by 90° from where
 5329 it is shown in the image then it will couple to the TM modes which have azimuthally
 5330 directed magnetic fields. In this way both the TE and TM resonances can be measured
 5331 independently.

6.5.2 Results and Discussion

The primary analysis for the prototype cavities involved a simple visualization of the return loss as measured by the VNA and a comparison between the filtered and non-filtered variations. Since the resonances measured by the VNA are not labeled, there is an uncertainty about the true identities of the modes measured by the VNA. To resolve this I performed a simulation of the simplest possible cavity that could be created from the prototype components, which is a fully open cavity created by removing the coaxial inserts. The fully-open cavity with the as-built dimensions was simulated in HFSS to get estimates on the positions of the TE₀₁₁ and TM₁₁₁ modes (see Figure 6.17).

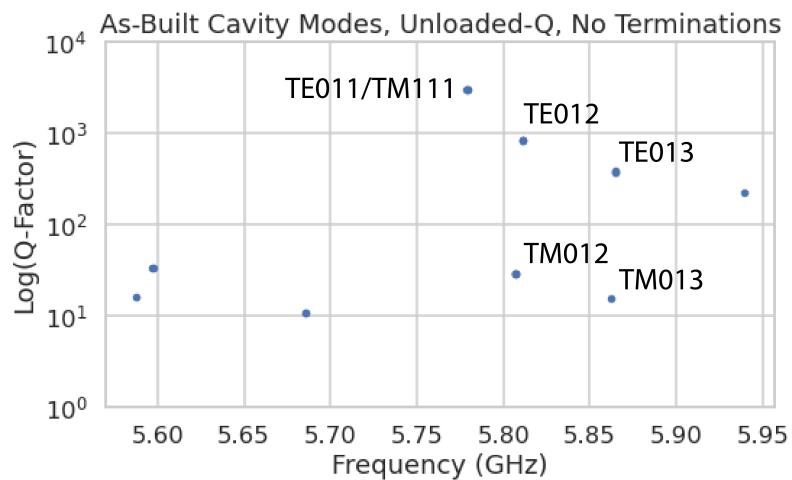


Figure 6.17. HFSS simulation results for a the as-built cavity with the coaxial terminators removed. The TE₀₁₁/TM₁₁₁ frequency is approximately 5.78 GHz.

Simulation of the fully open cavity shows that the TE₀₁₁/TM₁₁₁ modes have a frequency of approximately 5.78 GHz in the fully open cavity. If the frequency of this mode is compared to the measurements of the filtered and non-filtered cavities with the terminators removed one can easily identify the TE₀₁₁ mode at approximately 5.75 GHz (see Figure 6.18).

Both variations of the non-filtered cavities one sees that the TE₀₁₁ mode is degenerate in frequency with what appears to be a doublet of TM modes located at the TM₁₁₁ frequency position. This doublet is actually the TM₁₁₁ mode, which has two polarizations with opposite polarizations. Because the pipe used to construct the cavity is not perfectly round, the frequency degeneracy between the two polarizations is broken resulting in the doublet peaks.

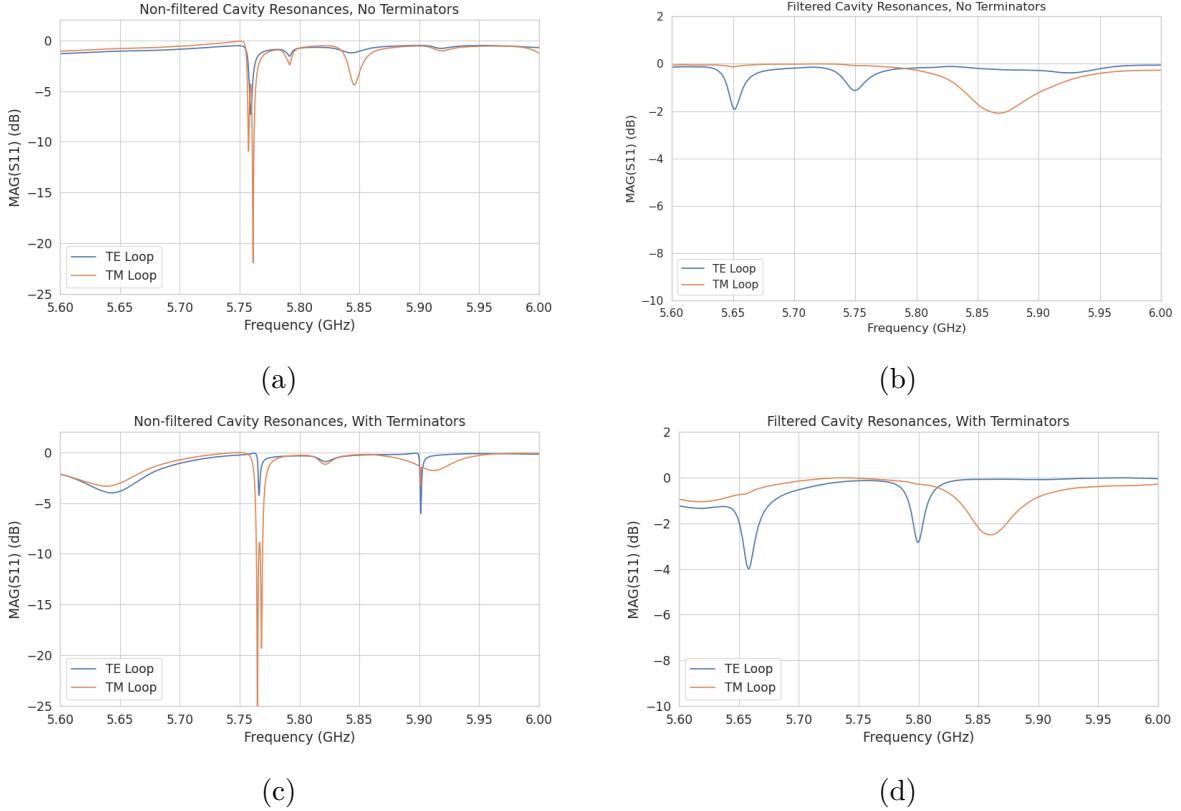


Figure 6.18. Measurements of the filtered and non-filtered prototype cavities acquired with the VNA.

5352 The S-parameter plot for the filtered cavity without terminators has an isolated TE
 5353 resonance at 5.65 GHz, associated with the TE_{011} mode. The frequency of this mode
 5354 is lower than the non-filtered cavity due to a difference in the overall lengths of the
 5355 cavities. An obvious difference between the filtered and non-filtered cavities is that
 5356 there is no TM_{111} doublet at the TE_{011} frequency. This is what one would expect if
 5357 the mode-filtering was suppressing the TM modes. There appears to be a noticeable
 5358 difference in the Q of the TE_{011} resonance between non-filtered and filtered variations as
 5359 indicated by the increased resonance depth for the filtered cavity. Overall, the Q-factors
 5360 of the filtered cavity appear significantly smaller than the non-filtered cavity due to the
 5361 increase in resonance width. This is likely caused by the relatively large widths of the
 5362 dielectric spacers, which are partially impeding the TE modes.

5363 One can see from these cavity measurements that, in principle, resistive mode-filtering
 5364 can be used to separate the TE_{011} resonance from the degenerate TM_{111} modes in
 5365 combination with the open cavity endcaps. This finding agrees with the expectations
 5366 from HFSS, which should provide confidence that the eigenmode solver is correctly

5367 modeling the behavior of the cavity. Although I did not perform a similar study using
5368 a cavity with grooved walls it is expected that the resonant mode structure would be
5369 similar to the cavity studied here.

5370 While this prototype cavity is a good first step, several deficiencies prevent this setup
5371 from providing more than qualitative information to the design of cavities for CRES. This
5372 includes the rudimentary approach to cavity coupling using a stripped coax antenna and
5373 the inability to map the field density in the cavity volume. Improvements in these areas
5374 are required so that measurements from a real cavity can provide useful information to
5375 cavity CRES simulations that will ultimately inform neutrino mass sensitivity estimates.

5376 Future work with prototype cavities must include an improved cavity coupling scheme,
5377 which is robust and compatible with atomic tritium. Since the cavity will ultimately
5378 be filled with atomic tritium, a coupling antenna cannot be used due to the losses of
5379 atomic tritium caused by recombination on the antenna surfaces. Possible non-invasive
5380 coupling schemes include aperture coupling, where the cavity is coupled to an external
5381 waveguide structure through an aperture, or a split-ring coupling approach, where the
5382 center segment of the cylindrical cavity wall is replaced an isolated conductive ring with
5383 a small vertical slit. The aperture coupling approach is a standard coupling scheme [86]
5384 used in a wide range of applications, but at low frequencies the size of the external
5385 waveguide conflicts with design of the atom trapping magnet and cryogenics system.
5386 The split-ring approach could potentially be coupled to a small coaxial transmission line
5387 which is more compatible with the rest of the experiment design. A challenge is achieving
5388 adequate coupling through impedance tuning, which is a focus of current research.

5389 The robustness of the coupling mechanism is relevant due to the difficulty in modeling
5390 its effect on the cavity modes. Small changes in geometry can have a large influence on
5391 the coupling and hence the performance of the cavity, therefore, correctly modeling the
5392 cavity coupling is critical for accurate CRES simulations. Coupling schemes that rely
5393 on connections to coaxial lines are potentially at a disadvantage in this regard due to
5394 the affect of soldering imperfections or unintended bends in the coax on the coupling.
5395 Future work will identify a coupling scheme for the cavity compatible with the neutrino
5396 mass goals of Project 8.

5397 Imperfections in the geometry of a real cavity will necessarily distort the resonant
5398 modes away from simulation predictions. This will change the coupling of an electron
5399 to the cavity and thus change the expected signal structure. Ultimately, this effect will
5400 limit the achievable energy resolution of the experiment unless the differences between
5401 simulation and a real cavity can be sufficiently characterized and calibrated. One possible

5402 approach to this is to utilize a "bead puller" system [100] to strategically perturb the
5403 cavity by moving a conductive bead through the cavity volume. The small perturbation
5404 caused by the bead affects the phase of the cavity resonances proportional to the total
5405 magnitude of the electric field at that position, so by moving the bead through the
5406 cavity volume the total electric field can be mapped and compared to simulation. This
5407 information can provide bounds on the relative perturbations to the cavity mode structure
5408 from real-life imperfections compared to the idealized cavity in HFSS.

5409 **Chapter 7 |**

5410 **Conclusion and Future Prospects**

5411 In this dissertation we have discussed research and development efforts towards the
5412 development of a scalable CRES measurement technology that can be used to build a
5413 CRES experiment at cubic-meter scales with sensitivity to neutrino masses of 40 meV.
5414 The primary contributions of my dissertation are the development and analysis of signal
5415 reconstruction algorithms for an antenna array based CRES experiment [101], which leads
5416 to estimates of the neutrino mass sensitivity; the development of a synthetic cyclotron
5417 radiation antenna (SYNCA) [80], which allowed for laboratory validation of antenna
5418 array CRES simulation models [43]; and the development of an open-ended cavity design
5419 compatible with atomic tritium for a cavity based CRES experiment. A measurable
5420 impact of this work is the transition of the Project 8 collaboration's experimental plan
5421 from an antenna array based approach to a cavity based approach, where my work played
5422 a key role in demonstrating the significantly higher cost and complexity of the antenna
5423 array experiment.

5424 The transition from antenna arrays to cavities requires a new set of demonstrator
5425 experiments to make incremental progress towards a 40 meV measurement of the neutrino
5426 mass. At the time of writing, the near-term plan of Project 8 is to design and construct a
5427 small-scale cavity CRES experiment utilizing the 1 T magnet installed in the UW-Seattle.
5428 This cavity is designed to have a TE011 resonance with a frequency of about 26 GHz with
5429 a length-to-diameter ratio that mimics the larger cavities intended for the pilot-scale and
5430 Phase IV experiments. The goal of this experiment is to demonstrate cavity CRES as
5431 well as validate models of CRES systematics using electrons from ^{83m}Kr and an electron
5432 gun. Though the primary goal is demonstration, near-term physics measurements are
5433 available in the form of high-resolution measurements of the ^{83m}Kr conversion spectrum
5434 of interest to the KATRIN collaboration.

5435 Furthermore, Project 8 is currently constructing a low-frequency CRES setup located
5436 at Yale University to better understand the principles of cavity based CRES at lower

5437 magnetic fields. The Low, UHF Cavity Krypton Experiment at Yale (LUCKEY) is
5438 a 1.5 GHz cavity CRES experiment the will use conversion electrons from ^{83m}Kr to
5439 perform CRES measurements at the lowest frequencies ever attempted with the technique.
5440 LUCKEY will validate frequency scaling models developed by Project 8 and will pave
5441 the way for the future Low-Frequency Apparatus (LFA), which will be a larger, 1 GHz
5442 cavity CRES experiment that includes a molecular tritium source. The target for the
5443 LFA is a measurement of the neutrino mass with a sensitivity of approximately 0.2 eV,
5444 which will build towards the atomic pilot-scale CRES experiment.

5445 In parallel to the development of cavity CRES is the development of the atomic
5446 tritium source. Recent demonstrations of the production of atomic hydrogen are excellent
5447 steps towards the atomic tritium production needed for the pilot-scale experiment. One
5448 area of future study includes the development of a more detailed understanding of the
5449 efficiency of atomic hydrogen production. Near-term plans include the development of a
5450 magnetic, evaporatively cooled beamline, as well as the prototyping of a Halbach array
5451 atoms trap. Nearly all of the components of the atomic tritium system will require
5452 demonstration before the complete system can be built. The long-term goal of the
5453 atomic tritium work is to construct a full atomic tritium prototype that demonstrates
5454 the production, cooling, trapping, and recycling of tritium at the rates needed for the
5455 pilot-scale experiment.

5456 More broadly, the long-term goal of the Project 8 collaboration is to fully develop
5457 both the atomic tritium and cavity CRES technologies so that both can be combined in
5458 a pilot-scale CRES experiment. It is envisioned that this process will take approximately
5459 10 years for both atomic tritium and cavity CRES. After these developments comes
5460 the pilot-scale experiment which will be the first CRES experiment that simultaneously
5461 demonstrates all the required technologies for Phase IV. Scaling to Phase IV with cavity
5462 CRES will require the construction of multiple copies (approximately 10) of the pilot-scale
5463 experiment to obtain sufficient statistics for 40 meV sensitivity.

5464 Development of the CRES experimental technique by Project 8 has led to new
5465 experiments utilizing the CRES technique for basic physics research, such as the ^6He -
5466 CRES collaboration [102], and has also found applications as a new approach to x-ray
5467 spectroscopy [103]. Recently, a new experimental effort called CRESDA has begun in
5468 the UK to develop new quantum technologies applied to CRES measurements for the
5469 neutrino mass [104]. This flourishing of new experimental efforts based on the CRES
5470 technique is likely to continue as Project 8 continues to develop the technique towards
5471 its neutrino mass measurement goal.

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Education

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- Doctor of Philosophy, Physics, The Pennsylvania State University, University Park, Pennsylvania, USA, 2023
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Selected Publications

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- Astari Esfahani, A. et al. (2023) "Antenna Arrays for CRES-based Neutrino Mass Measurement", *Phys. Rev. C*, In preparation.
- Astari Esfahani, A. et al. (2023) "Real-time Signal Detection for Cyclotron Radiation Emission Spectroscopy Measurements using Antenna Arrays", *Journal of Instrumentation*, In preparation.
- Astari Esfahani, A. et al. (2023) "Tritium Beta Spectrum and Neutrino Mass Limit from cyclotron Radiation Emission Spectroscopy", *Phys. Rev. Lett.*, Accepted.
- Astari Esfahani, A. et al. (2022) "SYNCA: A Synthetic Cyclotron Antenna for the Project 8 Collaboration", *Journal of Instrumentation*, **18**(01).

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Selected Presentations

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- *New Developments in the CRES Technique for Neutrino Mass Measurement*, Invited Talk, Fall 2022 Meeting of the APS Division of Nuclear Physics, New Orleans, Louisiana, USA, 2022
- *Signal Detection Algorithms for Phase III of the Project 8 Experiment*, Contributed Talk, APS April Meeting 2022, New York, New York, 2022
- *Synthetic Electron Antenna for Calibrating the Project 8 Neutrino Mass Experiment*, Contributed Talk, Fall 2021 Meeting of the APS Division of Nuclear Physics, Virtual, 2021

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