MATHEMATICS

Close packing of small spheres around a large one

Asked 3 years, 6 months ago Active 3 years, 3 months ago Viewed 720 times

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It is well known that, given a sphere, the maximum number of identical spheres that we can pack around it is exactly 12, corresponding to a face centered cubic or hexagonal close packed lattice.

My question is: given a sphere of radius R, how many spheres of radius $r < R$ can we closely pack around it?

With disks, the problem is rather easy to solve. Indeed, with reference to the picture at the bottom, we can see that we must have

$$\theta = \frac{2\pi}{n} = 2\arctan\left(\frac{r}{\sqrt{R^2+2Rr}}\right)$$

from which

$$n = \left\lfloor \frac{\pi}{\arctan\left(\frac{r}{\sqrt{R^2+2Rr}}\right)} \right\rfloor$$

The last expression gives the correct result for $R = r$, namely $n = 6$ (hexagonal lattice). Moreover, when $R \gg r$, we get

$$n \simeq \left\lfloor \frac{\pi R}{r} \right\rfloor$$

which is completely reasonable.

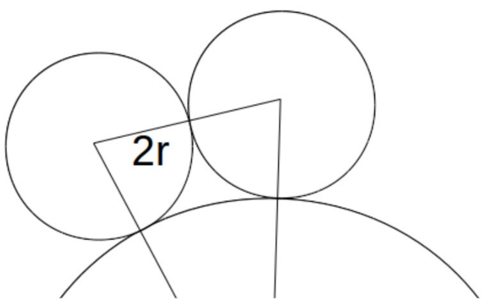
How can I tackle the same problem in the 3D case (spheres)?

It is clear that for $R \gg r$ we must get

$$n \simeq \left\lfloor \frac{4\pi R^2}{\pi r^2} \right\rfloor$$

and also that we must have $n(R = r) = 12$.

Any hint/suggestion is appreciated.



Long story short, you can't. Even the kissing number of 12, though tentatively known for centuries, was notoriously difficult to prove. – Ivan Neretin Jun 29 '17 at 10:14

@valerio: While the two packings you list are the only *lattice packings of space*, twelve balls of radius R touching a central ball of radius R have "continuous flexibility", in the lax sense that *individual balls can be moved while keeping others fixed*. A natural approach for finding a lower bound is to surround a ball of radius $r < R$ with "as many r-balls as possible, leaving one gap", then to repeat the process (with as many pairwise-tangent triples as possible) until no more balls can be added. There is certainly no simple formula, and rigorous bounds are difficult. – Andrew D. Hwang Jun 29 '17 at 11:02

Actually, in the limit $R \gg r$ you shouldn't expect to get $n \simeq \lfloor \frac{4\pi R^2}{\pi r^2} \rfloor$, but rather $n \simeq \lfloor \frac{\pi\sqrt{3}}{6} \frac{4\pi R^2}{\pi r^2} \rfloor$, with $\frac{\pi\sqrt{3}}{6} \approx 0.9069$. This is because the problem becomes a circle-packing problem. – m3tro Oct 11 '18 at 16:41

1 Answer

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My question was how many spheres within the large ball given that the spheres have a radius of smallest known atom and the number of dimensions of packing is equal to the radius of smallest known atom

answered Aug 26 '17 at 14:13

user474951

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Do you think that this answers the qquestion ? – Claude Leibovici Aug 26 '17 at 14:31

It might help if we start the packing from center to circumference we might be able to propagate the answer beyond circumference .maybe? ! – user474951 Aug 26 '17 at 15:01

It is one and the same question if we start with one sphere of smallest radius ever and stack around it spheres of similar radii to fill up a bigger ball ,you are stacking spheres around a ball on a larger scale.what really matters is radius of sphere and number of dimensions in space in which you are doing the stacking.The number of dimensions depend on the radius of the center sphere,which can be as tiny as we are able to measure.The power to which that number is multiplied can guide us to the number of dimensions in space we can do the stacking in – user474951 Aug 26 '17 at 15:19

the smallest value for R is r .If R <r then the sphere to be stacked would overlap the center sphere and go beyond its circumference. – user474951 Aug 26 '17 at 15:31