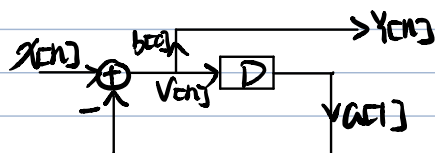Assignment: EE599 Homework 1.
Name: Zifan Wang
ID: 9525587296

Problem 2. Using filters in Python

1. a) First order (L=1) AR filter

Difference Equation:

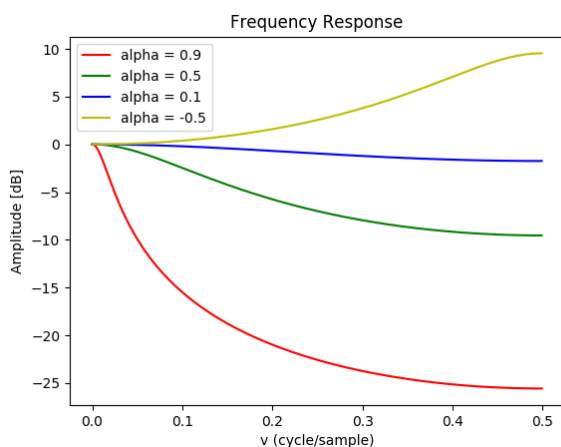$$y[n] = b[0] \cdot x[n] + a[1] \cdot y[n-1]$$

Z-transform:

$$y[n] - a[1] \cdot y[n-1] = b[0] \cdot x[n]$$

$$Y[z] - a[1] \cdot Y[z] \cdot z^{-k\gg1} = b[0] \, z^{-k\gg0}$$

$$(1 - a[1] \cdot z^{-1}) \cdot Y[z] = b[0]$$

$$Y[z] = \frac{b[0]}{1 - a[1] \cdot z^{-1}}, \quad b[0] = (1-\alpha), \quad a[1] = \alpha$$

b).

c). $\alpha = 0.9$:

$$y[0] = 1 - 0.9 = 0.1 \longrightarrow 0.1 \times 20\% = 0.02 \longrightarrow n = 15$$

$\alpha = 0.5$:

$$y[0] = 1 - 0.5 = 0.5 \longrightarrow 0.5 \times 20\% = 0.1 \longrightarrow n = 2.5$$

$\alpha = 0.1$:

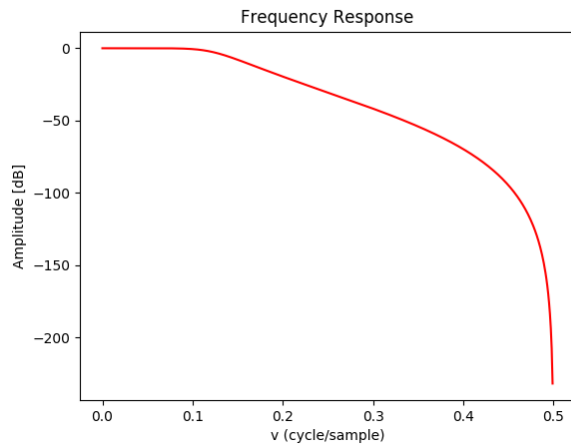$$y[0] = 1 - 0.1 = 0.9 \longrightarrow 0.9 \times 20\% = 0.18 \longrightarrow n = 1$$

1. Design filter.

a).

± |HW1-Problem2 ?:3 ✗| → python3 AR_Filter.pyNumerator: [0.01020948 0.04083792 0.06125688 0.04083792 0.01020948]
Denominator: [1.   -1.96842779 1.73586071 -0.72447083 0.1203896]

$$H[z] = \frac{0.01021 + 0.0408\,z^{-1} + 0.0613\,z^{-2} + 0.0408\,z^{-3} + 0.01021\,z^{-4}}{1 - 1.968\,z^{-1} + 1.736\,z^{-2} - 0.72452\,z^{-3} + 0.1204\,z^{-4}}$$

$\therefore b[0] = 0.01021, \quad b[1] = 0.0408, \quad b[2] = 0.0613 + \quad b[4] = 0.0408, \quad b[5] = 0.01021$

$a[1] = -1.968, \quad a[2] = 1.736, \quad a[3] = -0.72455, \quad a[4] = 0.1204$

b).