

values $v$	$::=$	$x$ $()$ $n$ $\text{Left } v \mid \text{Right } v$ $(v_1, v_2)$ $\text{fun } x \mapsto c$ $\text{handler } (\text{ret } x \mapsto c_r; h)$	variable unit integer sum constructors pair function handler
computations $c$	$::=$	$\text{ret } v$ $\text{match } v \text{ with } (x, y) \mapsto c$ $\text{match } v \text{ with Left } x \mapsto c_1 \mid \text{Right } x \mapsto c_2$ $v_1 \ v_2$ $op(v; y.c)$ $\text{let rec } f \ x : A \rightarrow \underline{C} = c_1 \text{ in } c_2$ $\text{do } x \leftarrow c_1 \text{ in } c_2$ $\text{with } v \text{ handle } c$	returned value product match sum match application operation call recursive function sequencing handling
operation clauses $h$	$::=$	$\emptyset \mid h \cup \{op(x; k) \mapsto c_{op}\}$	

Figure 1: *EEFF* Term Syntax

$$\begin{array}{c}
\frac{}{\text{match } (v_1, v_2) \text{ with } (x, y) \mapsto c \rightsquigarrow c[x \mapsto v_1, y \mapsto v_2]} \\
\\
\frac{}{\text{match Left } v \text{ with Left } x \mapsto c_l \mid \text{Right } x \mapsto c_r \rightsquigarrow c_l[x \mapsto v]} \\
\\
\frac{}{\text{match Right } v \text{ with Left } x \mapsto c_l \mid \text{Right } x \mapsto c_r \rightsquigarrow c_r[x \mapsto v]} \\
\\
\frac{}{(\text{fun } x \mapsto c) v \rightsquigarrow c[x \mapsto v]} \\
\\
\frac{}{\text{let rec } f \text{ } x : A \rightarrow \underline{C} = c_1 \text{ in } c_2 \rightsquigarrow c_2[f \mapsto (\text{fun } x \mapsto \text{let rec } f \text{ } x : A \rightarrow \underline{C} = c_1 \text{ in } c_1)]} \\
\\
\frac{c_1 \rightsquigarrow c'_1}{\text{do } x \leftarrow c_1 \text{ in } c_2 \rightsquigarrow \text{do } x \leftarrow c'_1 \text{ in } c_2} \qquad \frac{}{\text{do } x \leftarrow \text{ret } v \text{ in } c \rightsquigarrow c[x \mapsto v]} \\
\\
\frac{}{\text{do } x \leftarrow \text{op}(v; y.c_1) \text{ in } c_2 \rightsquigarrow \text{op}(v; y.\text{do } x \leftarrow c_1 \text{ in } c_2)} \\
\\
\frac{c \rightsquigarrow c'}{\text{with } v \text{ handle } c \rightsquigarrow \text{with } v \text{ handle } c'} \\
\\
\frac{}{\text{with } (\text{handler } (\text{ret } x \mapsto c_r; h)) \text{ handle } (\text{ret } v) \rightsquigarrow c_r[x \mapsto v]} \\
\\
\frac{H = (\text{handler } (\text{ret } x \mapsto c_r; h)) \quad (\text{op}(x; k) \mapsto c_{op}) \in h}{\text{with } H \text{ handle } (\text{op}(v; y.c)) \rightsquigarrow c_{op}[x \mapsto v, k \mapsto (\text{fun } y \mapsto \text{with } H \text{ handle } c)]}
\end{array}$$

Figure 2: *EEFF* Operational Semantics

(value) type $A, B$	$::=$	$\text{unit}$ $ $ $\text{int}$ $ $ $\text{empty}$ $ $ $A + B$ $ $ $A \times B$ $ $ $A \rightarrow \underline{C}$ $ $ $\underline{C} \Rightarrow \underline{D}$	unit type int type empty type sum type product type function type handler type
computation type $\underline{C}, \underline{D}$	$::=$	$A! \Sigma / \mathcal{E}$	
signature $\Sigma$	$::=$	$\emptyset \mid \Sigma \cup \{op : A \rightarrow B\}$	
value context $\Gamma$	$::=$	$\varepsilon \mid \Gamma, x : A$	
template context $Z$	$::=$	$\varepsilon \mid Z, z : A \rightarrow *$	
template $T$	$::=$	$z \ v$ $ $ $\text{match } v \text{ with } (x, y) \mapsto T$ $ $ $\text{match } v \text{ with Left } x \mapsto T_1 \mid \text{Right } x \mapsto T_2$ $ $ $op(v; y.T)$	applied variable product match sum match operation call
(effect) theory $\mathcal{E}$	$::=$	$\emptyset \mid \mathcal{E} \cup \{\Gamma; Z \vdash T_1 \sim T_2\}$	

Figure 3: *EEFF* Type Syntax

$$\begin{array}{c}
\frac{\Gamma \vdash v \Leftarrow A}{\Gamma \vdash (v:A) \Rightarrow A} \quad \frac{\Gamma \vdash v \Rightarrow A' \quad A = A'}{\Gamma \vdash v \Leftarrow A} \quad \frac{(x:A) \in \Gamma}{\Gamma \vdash x \Rightarrow A} \\
\\
\frac{}{\Gamma \vdash () \Rightarrow \text{unit}} \quad \frac{}{\Gamma \vdash n \Rightarrow \text{int}} \quad \frac{\Gamma \vdash v \Leftarrow A}{\Gamma \vdash \text{Left } v \Leftarrow A + B} \\
\\
\frac{\Gamma \vdash v \Leftarrow B}{\Gamma \vdash \text{Right } v \Leftarrow A + B} \quad \frac{\Gamma \vdash v_1 \Leftarrow A \quad \Gamma \vdash v_2 \Leftarrow B}{\Gamma \vdash (v_1, v_2) \Leftarrow A \times B} \\
\\
\frac{\Gamma \vdash v_1 \Rightarrow A \quad \Gamma \vdash v_2 \Rightarrow B}{\Gamma \vdash (v_1, v_2) \Rightarrow A \times B} \quad \frac{\Gamma, x:A \vdash c \Leftarrow \underline{C}}{\Gamma \vdash \text{fun } x \mapsto c \Leftarrow A \rightarrow \underline{C}} \\
\\
\frac{\Gamma, x:A \vdash c_r \Leftarrow \underline{D} \quad \Gamma \vdash h:\Sigma \Rightarrow \underline{D} \text{ respects } \mathcal{E}}{\Gamma \vdash \text{handler } (\text{ret } x \mapsto c_r; h) \Leftarrow A! \Sigma / \mathcal{E} \Rightarrow \underline{D}} \quad \frac{}{\Gamma \vdash \emptyset \Leftarrow \emptyset \Rightarrow \underline{D}} \\
\\
\frac{\Gamma \vdash h \Leftarrow \Sigma \Rightarrow \underline{D} \quad \Gamma, x:A_{op}, k:B_{op} \rightarrow \underline{D} \vdash c_{op} \Leftarrow \underline{D} \quad op \notin \Sigma}{\Gamma \vdash h \cup \{op(x; k) \mapsto c_{op}\} \Leftarrow (\Sigma \cup \{op:A_{op} \rightarrow B_{op}\}) \Rightarrow \underline{D}} \\
\\
\frac{\Gamma \vdash c \Leftarrow \underline{C}}{\Gamma \vdash (c:\underline{C}) \Rightarrow \underline{C}} \quad \frac{\Gamma \vdash c \Rightarrow \underline{C'} \quad \underline{C} = \underline{C'} \quad \text{effect subtyping?}}{\Gamma \vdash c \Leftarrow \underline{C}} \\
\\
\frac{\Gamma \vdash v \Rightarrow A + B \quad \Gamma, x:A \vdash c_l \Leftarrow \underline{C} \quad \Gamma, x:B \vdash c_r \Leftarrow \underline{C}}{\Gamma \vdash \text{match } v \text{ with Left } x \mapsto c_l \mid \text{Right } x \mapsto c_r \Leftarrow \underline{C}} \\
\\
\frac{\Gamma \vdash v \Rightarrow A \times B \quad \Gamma, x:A, y:B \vdash c \Rightarrow \underline{C}}{\Gamma \vdash \text{match } v \text{ with } (x, y) \mapsto c \Rightarrow \underline{C}} \quad \frac{\Gamma \vdash v_1 \Rightarrow A \rightarrow \underline{C} \quad \Gamma \vdash v_2 \Leftarrow A}{\Gamma \vdash v_1 \ v_2 \Rightarrow \underline{C}} \\
\\
\frac{\Gamma, f:A \rightarrow \underline{C}, x:A \vdash c_1 \Leftarrow \underline{C} \quad \Gamma, f:A \rightarrow \underline{C} \vdash c_2 \Rightarrow \underline{D}}{\Gamma \vdash \text{let rec } f \ x : A \rightarrow \underline{C} = c_1 \text{ in } c_2 \Rightarrow \underline{D}} \quad \frac{\Gamma \vdash v \Rightarrow A}{\Gamma \vdash \text{ret } v \Rightarrow A! \emptyset / \emptyset} \\
\\
\frac{(op:A_{op} \rightarrow B_{op}) \in \Sigma \quad \Gamma \vdash v \Leftarrow A_{op} \quad \Gamma, y:B_{op} \vdash c \Leftarrow A! \Sigma / \mathcal{E}}{\Gamma \vdash op(v; y.c) \Leftarrow A! \Sigma / \mathcal{E}} \\
\\
\frac{\Gamma \vdash c_1 \Rightarrow A! \Sigma / \mathcal{E} \quad \Gamma, x:A \vdash c_2 \Leftarrow B! \Sigma / \mathcal{E} \quad \text{effect subtyping?}}{\Gamma \vdash \text{do } x \leftarrow c_1 \text{ in } c_2 \Leftarrow B! \Sigma / \mathcal{E}} \\
\\
\frac{\Gamma \vdash v \Rightarrow \underline{C} \Rightarrow \underline{D} \quad \Gamma \vdash c \Leftarrow \underline{C}}{\Gamma \vdash \text{with } v \text{ handle } c \Rightarrow \underline{D}}
\end{array}$$

Figure 4: *EEFF* Type System