Applications of algebraic effect theories

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Reasoning about programs

$$f_1 = \texttt{fun}\ x \mapsto x + x$$
$$f_2 = \texttt{fun}\ x \mapsto 2 * x$$

We know mathematical properties of operations $+$ and $*$ so we can argue that the programs are equal (whatever that means).

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This becomes harder with effectful operations, such as *print*.

$$print\_twice = \texttt{fun}\ s \mapsto print\ s;\ print\ s$$
$$print\_double = \texttt{fun}\ s \mapsto print\ s\ \hat{}\ s$$

“Implementing print”

We give meaning to operations with handlers. An example:

$$
\begin{aligned}
&\mathit{collect\_prints} \;=\; \texttt{handler}\,\{\\
&\quad \mid \mathit{print}(s;\ k) \mapsto\\
&\qquad\quad \texttt{do}\ (v, s') \leftarrow k\ ()\ \texttt{in}\ (v, s\,\hat{}\,s')\\
&\quad \mid \texttt{ret}\ x \mapsto (x, "")\\
&\}
\end{aligned}
$$

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and proceeds with the evaluation

$$\begin{aligned}&\texttt{do}\ (v, s') \leftarrow k\ ()\ \texttt{in}\\ &(v, \text{"}\mathit{test}\text{"}\ \hat{}\ s')\end{aligned}$$

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We want reasoning tools that can separate handlers from the rest of our code!

The theory of algebraic effects consists of operations and **equations** between them.

We can use equations such as

$$\mathcal{E}_{no\_sep} = print\ s_1;\ print\ s_2 \sim print\ s_1\hat{}\ s_2$$

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Can we achieve that without restricting ourselves to a global effect theory?

Idea

Include the desired equations in types.

$$\underline{C} = A!\Sigma/\mathcal{E}$$

Operations of type $\underline{C}$ either return a value of type A or call an operation from Σ in the effect theory $\mathcal{E}$.

Equations in $\mathcal{E}$ tell us what computations we deem equal, e.g. *print_double* and *print_twice* are equal at $\texttt{string} \to \texttt{unit}!\{\mathit{print}\}/\{\mathcal{E}_{no\_sep}\}$ but not at $\texttt{string} \to \texttt{unit}!\{\mathit{print}\}/\emptyset$

Assigning types to example handler

Without equations, *collect_prints* would have the type

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But before we wanted to use the theory

$$\mathcal{E}_{no\_sep} = print\ s_1;\ print\ s_2 \sim print\ s_1\,\hat{}\ s_2$$

so we could instead use the type

$$A!\{print\}/\{\mathcal{E}_{no\_sep}\} \Rightarrow (A * \mathtt{string})!\emptyset/\emptyset$$

Another example

$$\begin{aligned}
&\mathit{use\_newline} \;=\; \texttt{handler}\,\{\\
&\quad \mid \mathit{print}(s;\, k) \mapsto \mathit{print}\ s;\ \mathit{print}\ \text{"\textbackslash n"};\ k\,()\\
&\quad \mid \texttt{ret}\ x \mapsto x\\
&\}
\end{aligned}$$

$$\mathcal{E}_{no\_sep} = \mathit{print}\ s_1;\ \mathit{print}\ s_2 \sim \mathit{print}\ s_1\,\hat{}\ s_2$$

$$\mathcal{E}_{sep} = \mathit{print}\ s_1;\ \mathit{print}\ s_2 \sim \mathit{print}\ s_1\,\hat{}\ \text{"\textbackslash n"}\ \hat{}\ s_2$$

$$\mathit{use\_newline} : A!\{\mathit{print}\}/\{\mathcal{E}_{sep}\} \Rightarrow A!\{\mathit{print}\}/\{\mathcal{E}_{no\_sep}\}$$

How to type handlers?

All typing rules remain largely the same as when not using equations.

The only important change is

$$\frac{\Gamma, x : A \vdash c_r : \underline{D} \qquad \Gamma \vdash h : \Sigma \Rightarrow \underline{D} \text{ respects } \mathcal{E}}{\Gamma \vdash \texttt{handler}\,(\texttt{ret}\ x \mapsto c_r; h) : A!\Sigma/\mathcal{E} \Rightarrow \underline{D}}$$

How to type handlers?

For $\Gamma \vdash h : \Sigma \Rightarrow \underline{D}$ respects $\mathcal{E}$ we first check that the types match. We then check correctness in a logic, but we have a choice which logic to use.

The general idea is checking that our handler maps equivalent computations to equivalent computations.

The choice of logic also bears impact on the denotational semantics of our language.

Future work

- Implement the proposed equation system in Eff.
- Consider options for easier checks of the respects relation.
- How to define subtyping?
- Concrete applications (probabilistic programming).