$$
\begin{array}{llll}
\text{values } v & ::= & x & \text{variable} \\
& | & () & \text{unit} \\
& | & n & \text{integer} \\
& | & \texttt{Left } v \mid \texttt{Right } v & \text{sum constructors} \\
& | & (v_1, v_2) & \text{pair} \\
& | & \texttt{fun } x \mapsto c & \text{function} \\
& | & \texttt{handler } (\texttt{ret } x \mapsto c_r; h) & \text{handler} \\
& | & (v : A) & \text{type annotation} \\
\\
\text{computations } c & ::= & \texttt{ret } v & \text{returned value} \\
& | & \texttt{match } v \texttt{ with } (x, y) \mapsto c & \text{product match} \\
& | & \texttt{match } v \texttt{ with Left } x \mapsto c_1 \mid \texttt{Right } x \mapsto c_2 & \text{sum match} \\
& | & v_1\, v_2 & \text{application} \\
& | & op(v; y.c) & \text{operation call} \\
& | & \texttt{let rec } f\ x : A \rightarrow \underline{C} = c_1 \texttt{ in } c_2 & \text{recursive function} \\
& | & \texttt{do } x \leftarrow c_1 \texttt{ in } c_2 & \text{sequencing} \\
& | & \texttt{with } v \texttt{ handle } c & \text{handling} \\
& | & (c : \underline{C}) & \text{type annotation} \\
\\
\text{operation clauses } h & ::= & \emptyset \mid h \cup \{op(x; k) \mapsto c_{op}\}
\end{array}
$$

Figure 1: *EEFF* Term Syntax

$$
\begin{array}{rcll}
\text{(value) type } A, B & ::= & \texttt{unit} & \text{unit type} \\
& | & \texttt{int} & \text{int type} \\
& | & \texttt{empty} & \text{empty type} \\
& | & A + B & \text{sum type} \\
& | & A \times B & \text{product type} \\
& | & A \to \underline{C} & \text{function type} \\
& | & \underline{C} \Rightarrow \underline{D} & \text{handler type} \\
\\
\text{computation type } \underline{C}, \underline{D} & ::= & A!\Sigma/\mathcal{E} \\
\\
\text{signature } \Sigma & ::= & \emptyset \mid \Sigma \cup \{op : A \to B\} \\
\\
\text{value context } \Gamma & ::= & \varepsilon \mid \Gamma, x : A \\
\\
\text{template context } \mathtt{Z} & ::= & \varepsilon \mid \mathtt{Z}, z : A \to * \\
\\
\text{template } T & ::= & z\,v & \text{applied variable} \\
& | & \texttt{match } v \texttt{ with } (x, y) \mapsto T & \text{product match} \\
& | & \texttt{match } v \texttt{ with Left } x \mapsto T_1 \mid \texttt{Right } x \mapsto T_2 & \text{sum match} \\
& | & op(v;\, y.\,T) & \text{operation call} \\
\\
\text{(effect) theory } \mathcal{E} & ::= & \emptyset \mid \mathcal{E} \cup \{\Gamma; \mathtt{Z} \vdash T_1 \sim T_2\}
\end{array}
$$

Figure 2: *EEFF* Type Syntax

$$\frac{\Gamma \vdash v \Leftarrow A}{\Gamma \vdash (v:A) \Rightarrow A} \qquad \frac{\Gamma \vdash v \Rightarrow A}{\Gamma \vdash v \Leftarrow A} \qquad \frac{(x:A) \in \Gamma}{\Gamma \vdash x \Rightarrow A} \qquad \frac{}{\Gamma \vdash () \Rightarrow \text{unit}}$$

$$\frac{}{\Gamma \vdash n \Rightarrow \text{int}} \qquad \frac{\Gamma \vdash v \Leftarrow A}{\Gamma \vdash \text{Left } v \Leftarrow A + B} \qquad \frac{\Gamma \vdash v \Leftarrow B}{\Gamma \vdash \text{Right } v \Leftarrow A + B}$$

$$\frac{\Gamma \vdash v_1 \Leftarrow A \qquad \Gamma \vdash v_2 \Leftarrow B}{\Gamma \vdash (v_1, v_2) \Leftarrow A \times B} \qquad \frac{\Gamma \vdash v_1 \Rightarrow A \qquad \Gamma \vdash v_2 \Rightarrow B}{\Gamma \vdash (v_1, v_2) \Rightarrow A \times B}$$

$$\frac{\Gamma, x:A \vdash c \Leftarrow \underline{C}}{\Gamma \vdash \text{fun } x \mapsto c \Leftarrow A \to \underline{C}} \qquad \frac{\Gamma, x:A \vdash c_r \Leftarrow \underline{D} \qquad \Gamma \vdash h : \Sigma \Rightarrow \underline{D} \text{ respects } \mathcal{E}}{\Gamma \vdash \text{handler } (\text{ret } x \mapsto c_r; h) \Leftarrow A!\Sigma/\mathcal{E} \Rightarrow \underline{D}}$$

$$\frac{}{\Gamma \vdash \emptyset \Leftarrow \emptyset \Rightarrow \underline{D}}$$

$$\frac{\Gamma \vdash h \Leftarrow \Sigma \Rightarrow \underline{D} \qquad \Gamma, x:A_{op}, k:B_{op} \to \underline{D} \vdash c_{op} \Leftarrow \underline{D} \qquad op \notin \Sigma}{\Gamma \vdash h \cup \{op(x;\ k) \mapsto c_{op}\} \Leftarrow (\Sigma \cup \{op:A_{op} \to B_{op}\}) \Rightarrow \underline{D}}$$

$$\frac{\Gamma \vdash c \Leftarrow \underline{C}}{\Gamma \vdash (c:\underline{C}) \Rightarrow \underline{C}} \qquad \frac{\Gamma \vdash c \Rightarrow \underline{C} \qquad \color{red}{\text{effect subtyping?}}}{\Gamma \vdash c \Leftarrow \underline{C}}$$

$$\frac{\Gamma \vdash v \Rightarrow A + B \qquad \Gamma, x:A \vdash c_l \Leftarrow \underline{C} \qquad \Gamma, x:B \vdash c_r \Leftarrow \underline{C}}{\Gamma \vdash \text{match } v \text{ with Left } x \mapsto c_l \mid \text{Right } x \mapsto c_r \Leftarrow \underline{C}}$$

$$\frac{\Gamma \vdash v \Rightarrow A \times B \qquad \Gamma, x:A, y:B \vdash c \Rightarrow \underline{C}}{\Gamma \vdash \text{match } v \text{ with } (x, y) \mapsto c \Rightarrow \underline{C}} \qquad \frac{\Gamma \vdash v_1 \Rightarrow A \to \underline{C} \qquad \Gamma \vdash v_2 \Leftarrow A}{\Gamma \vdash v_1\ v_2 \Rightarrow \underline{C}}$$

$$\frac{\Gamma, f:A \to \underline{C}, x:A \vdash c_1 \Leftarrow \underline{C} \qquad \Gamma, f:A \to \underline{C} \vdash c_2 \Rightarrow \underline{D}}{\Gamma \vdash \text{let rec } f\ x : A \to \underline{C} = c_1 \text{ in } c_2 \Rightarrow \underline{D}} \qquad \frac{\Gamma \vdash v \Rightarrow A}{\Gamma \vdash \text{ret } v \Rightarrow A!\emptyset/\emptyset}$$

$$\frac{(op:A_{op} \to B_{op}) \in \Sigma \qquad \Gamma \vdash v \Leftarrow A_{op} \qquad \Gamma, y:B_{op} \vdash c \Leftarrow A!\Sigma/\mathcal{E}}{\Gamma \vdash op(v;\ y.c) \Leftarrow A!\Sigma/\mathcal{E}}$$

$$\frac{\Gamma \vdash c_1 \Rightarrow A!\Sigma/\mathcal{E} \qquad \Gamma, x:A \vdash c_2 \Leftarrow B!\Sigma/\mathcal{E} \qquad \color{red}{\text{effect subtyping?}}}{\Gamma \vdash \text{do } x \leftarrow c_1 \text{ in } c_2 \Leftarrow B!\Sigma/\mathcal{E}}$$

$$\frac{\Gamma \vdash v \Rightarrow \underline{C} \Rightarrow \underline{D} \qquad \Gamma \vdash c \Leftarrow \underline{C}}{\Gamma \vdash \text{with } v \text{ handle } c \Rightarrow \underline{D}}$$

Figure 3: *EEFF* Type System

3

$$
\begin{array}{llll}
\text{values } v & ::= & x & \text{variable} \\
& | & () & \text{unit} \\
& | & n & \text{integer} \\
& | & \texttt{Left } v \mid \texttt{Right } v & \text{sum constructors} \\
& | & (v_1, v_2) & \text{pair} \\
& | & \texttt{fun } x \mapsto c & \text{function} \\
& | & \texttt{handler}\,(\texttt{ret } x \mapsto c_r; h) & \text{handler} \\
\\
\text{computations } c & ::= & \texttt{ret } v & \text{returned value} \\
& | & \texttt{match } v \texttt{ with } (x, y) \mapsto c & \text{product match} \\
& | & \texttt{match } v \texttt{ with Left } x \mapsto c_1 \mid \texttt{Right } x \mapsto c_2 & \text{sum match} \\
& | & v_1\ v_2 & \text{application} \\
& | & op(v;\ y.c) & \text{operation call} \\
& | & \texttt{let rec } f\ x = c_1 \texttt{ in } c_2 & \text{recursive function} \\
& | & \texttt{do } x \leftarrow c_1 \texttt{ in } c_2 & \text{sequencing} \\
& | & \texttt{with } v \texttt{ handle } c & \text{handling} \\
\end{array}
$$

$$
\text{operation clauses } h \quad ::= \quad \emptyset \mid h \cup \{ op(x;\ k) \mapsto c_{op} \}
$$

Figure 4: *EEFF* Term Syntax

$$\frac{(x\!:\!A) \in \Gamma}{\Gamma \vdash x\!:\!A} \qquad \frac{}{\Gamma \vdash ()\!:\!\mathtt{unit}} \qquad \frac{}{\Gamma \vdash n\!:\!\mathtt{int}} \qquad \frac{\Gamma \vdash v\!:\!A}{\Gamma \vdash \mathtt{Left}\ v\!:\!A + B}$$

$$\frac{\Gamma \vdash v\!:\!B}{\Gamma \vdash \mathtt{Right}\ v\!:\!A + B} \qquad \frac{\Gamma \vdash v_1\!:\!A \qquad \Gamma \vdash v_2\!:\!B}{\Gamma \vdash (v_1, v_2)\!:\!A \times B} \qquad \frac{\Gamma \vdash v_1\!:\!A \qquad \Gamma \vdash v_2\!:\!B}{\Gamma \vdash (v_1, v_2)\!:\!A \times B}$$

$$\frac{\Gamma, x\!:\!A \vdash c\!:\!\underline{C}}{\Gamma \vdash \mathtt{fun}\ x \mapsto c\!:\!A \to \underline{C}} \qquad \frac{\Gamma, x\!:\!A \vdash c_r\!:\!\underline{D} \qquad \Gamma \vdash h\!:\!\Sigma \Rightarrow \underline{D}\ \text{respects}\ \mathcal{E}}{\Gamma \vdash \mathtt{handler}\ (\mathtt{ret}\ x \mapsto c_r; h)\!:\!A!\Sigma/\mathcal{E} \Rightarrow \underline{D}}$$

$$\frac{}{\Gamma \vdash \emptyset\!:\!\emptyset \Rightarrow \underline{D}} \qquad \frac{\Gamma \vdash h\!:\!\Sigma \Rightarrow \underline{D} \qquad \Gamma, x\!:\!A_{op}, k\!:\!B_{op} \to \underline{D} \vdash c_{op}\!:\!\underline{D} \qquad op \notin \Sigma}{\Gamma \vdash h \cup \{op(x;\ k) \mapsto c_{op}\}\!:\!(\Sigma \cup \{op\!:\!A_{op} \to B_{op}\}) \Rightarrow \underline{D}}$$

$$\frac{\Gamma \vdash v\!:\!A + B \qquad \Gamma, x\!:\!A \vdash c_l\!:\!\underline{C} \qquad \Gamma, x\!:\!B \vdash c_r\!:\!\underline{C}}{\Gamma \vdash \mathtt{match}\ v\ \mathtt{with}\ \mathtt{Left}\ x \mapsto c_l \mid \mathtt{Right}\ x \mapsto c_r\!:\!\underline{C}}$$

$$\frac{\Gamma \vdash v\!:\!A \times B \qquad \Gamma, x\!:\!A, y\!:\!B \vdash c\!:\!\underline{C}}{\Gamma \vdash \mathtt{match}\ v\ \mathtt{with}\ (x, y) \mapsto c\!:\!\underline{C}} \qquad \frac{\Gamma \vdash v_1\!:\!A \to \underline{C} \qquad \Gamma \vdash v_2\!:\!A}{\Gamma \vdash v_1\ v_2\!:\!\underline{C}}$$

$$\frac{\Gamma, f\!:\!A \to \underline{C}, x\!:\!A \vdash c_1\!:\!\underline{C} \qquad \Gamma, f\!:\!A \to \underline{C} \vdash c_2\!:\!\underline{D}}{\Gamma \vdash \mathtt{let}\ \mathtt{rec}\ f\ x = c_1\ \mathtt{in}\ c_2\!:\!\underline{D}} \qquad \frac{\Gamma \vdash v\!:\!A}{\Gamma \vdash \mathtt{ret}\ v\!:\!A!\emptyset/\emptyset}$$

$$\frac{(op\!:\!A_{op} \to B_{op}) \in \Sigma \qquad \Gamma \vdash v\!:\!A_{op} \qquad \Gamma, y\!:\!B_{op} \vdash c\!:\!A!\Sigma/\mathcal{E}}{\Gamma \vdash op(v;\ y.c)\!:\!A!\Sigma/\mathcal{E}}$$

$$\frac{\Gamma \vdash c_1\!:\!A!\Sigma/\mathcal{E} \qquad \Gamma, x\!:\!A \vdash c_2\!:\!B!\Sigma/\mathcal{E}}{\Gamma \vdash \mathtt{do}\ x \leftarrow c_1\ \mathtt{in}\ c_2\!:\!B!\Sigma/\mathcal{E}} \qquad \frac{\Gamma \vdash v\!:\!\underline{C} \Rightarrow \underline{D} \qquad \Gamma \vdash c\!:\!\underline{C}}{\Gamma \vdash \mathtt{with}\ v\ \mathtt{handle}\ c\!:\!\underline{D}}$$

Figure 5: *EEFF* Type System

$$\frac{}{\texttt{match }(v_1, v_2)\texttt{ with }(x, y) \mapsto c \; \rightsquigarrow \; c[x \mapsto v_1, y \mapsto v_2]}$$

$$\frac{}{\texttt{match Left } v \texttt{ with Left } x \mapsto c_l \mid \texttt{Right } x \mapsto c_r \; \rightsquigarrow \; c_l[x \mapsto v]}$$

$$\frac{}{\texttt{match Right } v \texttt{ with Left } x \mapsto c_l \mid \texttt{Right } x \mapsto c_r \; \rightsquigarrow \; c_r[x \mapsto v]}$$

$$\frac{}{(\texttt{fun } x \mapsto c)\, v \; \rightsquigarrow \; c[x \mapsto v]}$$

$$\frac{}{\texttt{let rec } f \; x = A \rightarrow \underline{C} \texttt{ in } c_1 c_2 \; \rightsquigarrow \; c_2[f \mapsto (\texttt{fun } x \mapsto \texttt{let rec } f \; x = A \rightarrow \underline{C} \texttt{ in } c_1 c_1)]}$$

$$\frac{c_1 \rightsquigarrow c_1'}{\texttt{do } x \leftarrow c_1 \texttt{ in } c_2 \; \rightsquigarrow \; \texttt{do } x \leftarrow c_1' \texttt{ in } c_2} \qquad \frac{}{\texttt{do } x \leftarrow \texttt{ret } v \texttt{ in } c \; \rightsquigarrow \; c[x \mapsto v]}$$

$$\frac{}{\texttt{do } x \leftarrow op(v;\, y.c_1) \texttt{ in } c_2 \; \rightsquigarrow \; op(v;\, y.\texttt{do } x \leftarrow c_1 \texttt{ in } c_2)}$$

$$\frac{c \rightsquigarrow c'}{\texttt{with } v \texttt{ handle } c \; \rightsquigarrow \; \texttt{with } v \texttt{ handle } c'}$$

$$\frac{}{\texttt{with }(\texttt{handler }(\texttt{ret } x \mapsto c_r;\, h))\texttt{ handle }(\texttt{ret } v) \; \rightsquigarrow \; c_r[x \mapsto v]}$$

$$\frac{H = (\texttt{handler }(\texttt{ret } x \mapsto c_r;\, h)) \qquad (op(x;\, k) \mapsto c_{op}) \in h}{\texttt{with } H \texttt{ handle }(op(v;\, y.c)) \; \rightsquigarrow \; c_{op}[x \mapsto v, k \mapsto (\texttt{fun } y \mapsto \texttt{with } H \texttt{ handle } c)]}$$

Figure 6: *EEFF* Operational Semantics