values v	$::=$	x	variable
	$\mid$	$()$	unit
	$\mid$	n	integer
	$\mid$	$\texttt{Left}\ v \mid \texttt{Right}\ v$	sum constructors
	$\mid$	(v_1, v_2)	pair
	$\mid$	$\texttt{fun}\ x \mapsto c$	function
	$\mid$	$\texttt{handler}\,(\texttt{ret}\ x \mapsto c_r\,;h)$	handler
	$\mid$	$(v : A)$	type annotation
computations c	$::=$	$\texttt{ret}\ v$	returned value
	$\mid$	$\texttt{match}\ v\ \texttt{with}\ (x, y) \mapsto c$	product match
	$\mid$	$\texttt{match}\ v\ \texttt{with}\ \texttt{Left}\ x \mapsto c_1 \mid \texttt{Right}\ x \mapsto c_2$	sum match
	$\mid$	$v_1\ v_2$	application
	$\mid$	$op(v;\ y.c)$	operation call
	$\mid$	$\texttt{let rec}\ f\ x\ : A \to \underline{C} = c_1\ \texttt{in}\ c_2$	recursive function
	$\mid$	$\texttt{do}\ x \leftarrow c_1\ \texttt{in}\ c_2$	sequencing
	$\mid$	$\texttt{with}\ v\ \texttt{handle}\ c$	handling
	$\mid$	$(c : \underline{C})$	type annotation
operation clauses h	$::=$	$\emptyset \mid h \cup \{op(x;\ k) \mapsto c_{op}\}$	

Figure 1: *EEFF* Term Syntax

In the case of closed handlers, how does the subtyping change???
What happens to the equations in the case of open handlers???

$$
\begin{array}{rcll}
\text{(value) type } A, B & ::= & \mathtt{unit} & \text{unit type} \\
 & \mid & \mathtt{int} & \text{int type} \\
 & \mid & \mathtt{empty} & \text{empty type} \\
 & \mid & A + B & \text{sum type} \\
 & \mid & A \times B & \text{product type} \\
 & \mid & A \to \underline{C} & \text{function type} \\
 & \mid & \underline{C} \Rightarrow \underline{D} & \text{handler type} \\
 & & & \\
\text{computation type } \underline{C}, \underline{D} & ::= & A!\Sigma/\mathcal{E} & \\
 & & & \\
\text{signature } \Sigma & ::= & \emptyset \mid \Sigma \cup \{op : A \to B\} & \\
 & & & \\
\text{value context } \Gamma & ::= & \varepsilon \mid \Gamma, x : A & \\
 & & & \\
\text{template context } \mathtt{Z} & ::= & \varepsilon \mid \mathtt{Z}, z : A \to * & \\
 & & & \\
\text{template } T & ::= & z\ v & \text{applied variable} \\
 & \mid & \mathtt{match}\ v\ \mathtt{with}\ (x, y) \mapsto T & \text{product match} \\
 & \mid & \mathtt{match}\ v\ \mathtt{with}\ \mathtt{Left}\ x \mapsto T_1 \mid \mathtt{Right}\ x \mapsto T_2 & \text{sum match} \\
 & \mid & op(v;\ y.\, T) & \text{operation call} \\
 & & & \\
\text{(effect) theory } \mathcal{E} & ::= & \emptyset \mid \mathcal{E} \cup \{\Gamma; \mathtt{Z} \vdash T_1 \sim T_2\} &
\end{array}
$$

Figure 2: *EEFF* Type Syntax

$$\frac{\Gamma \vdash v \Leftarrow A}{\Gamma \vdash (v : A) \Rightarrow A} \qquad \frac{\Gamma \vdash v \Rightarrow A}{\Gamma \vdash v \Leftarrow A} \qquad \frac{(x : A) \in \Gamma}{\Gamma \vdash x \Rightarrow A} \qquad \frac{}{\Gamma \vdash () \Rightarrow \texttt{unit}}$$

$$\frac{}{\Gamma \vdash n \Rightarrow \texttt{int}} \qquad \frac{\Gamma \vdash v \Leftarrow A}{\Gamma \vdash \texttt{Left}\ v \Leftarrow A + B} \qquad \frac{\Gamma \vdash v \Leftarrow B}{\Gamma \vdash \texttt{Right}\ v \Leftarrow A + B}$$

$$\frac{\Gamma \vdash v_1 \Leftarrow A \qquad \Gamma \vdash v_2 \Leftarrow B}{\Gamma \vdash (v_1, v_2) \Leftarrow A \times B} \qquad \frac{\Gamma \vdash v_1 \Rightarrow A \qquad \Gamma \vdash v_2 \Rightarrow B}{\Gamma \vdash (v_1, v_2) \Rightarrow A \times B}$$

$$\frac{\Gamma, x : A \vdash c \Leftarrow \underline{C}}{\Gamma \vdash \texttt{fun}\ x \mapsto c \Leftarrow A \to \underline{C}} \qquad \frac{\Gamma, x : A \vdash c_r \Leftarrow \underline{D} \qquad \Gamma \vdash h : \Sigma \Rightarrow \underline{D} \text{ respects } \mathcal{E}}{\Gamma \vdash \texttt{handler}\ (\texttt{ret}\ x \mapsto c_r ; h) \Leftarrow A!\Sigma/\mathcal{E} \Rightarrow \underline{D}}$$

$$\frac{}{\Gamma \vdash \emptyset \Leftarrow \emptyset \Rightarrow \underline{D}}$$

$$\frac{\Gamma \vdash h \Leftarrow \Sigma \Rightarrow \underline{D} \qquad \Gamma, x : A_{op}, k : B_{op} \to \underline{D} \vdash c_{op} \Leftarrow \underline{D} \qquad op \notin \Sigma}{\Gamma \vdash h \cup \{op(x;\ k) \mapsto c_{op}\} \Leftarrow (\Sigma \cup \{op : A_{op} \to B_{op}\}) \Rightarrow \underline{D}}$$

$$\frac{\Gamma \vdash c \Leftarrow \underline{C}}{\Gamma \vdash (c : \underline{C}) \Rightarrow \underline{C}} \qquad \frac{\Gamma \vdash c \Rightarrow \underline{C} \qquad \text{effect subtyping?}}{\Gamma \vdash c \Leftarrow \underline{C}}$$

$$\frac{\Gamma \vdash v \Rightarrow A + B \qquad \Gamma, x : A \vdash c_l \Leftarrow \underline{C} \qquad \Gamma, x : B \vdash c_r \Leftarrow \underline{C}}{\Gamma \vdash \texttt{match}\ v\ \texttt{with Left}\ x \mapsto c_l \mid \texttt{Right}\ x \mapsto c_r \Leftarrow \underline{C}}$$

$$\frac{\Gamma \vdash v \Rightarrow A \times B \qquad \Gamma, x : A, y : B \vdash c \Rightarrow \underline{C}}{\Gamma \vdash \texttt{match}\ v\ \texttt{with}\ (x, y) \mapsto c \Rightarrow \underline{C}} \qquad \frac{\Gamma \vdash v_1 \Rightarrow A \to \underline{C} \qquad \Gamma \vdash v_2 \Leftarrow A}{\Gamma \vdash v_1\ v_2 \Rightarrow \underline{C}}$$

$$\frac{\Gamma, f : A \to \underline{C}, x : A \vdash c_1 \Leftarrow \underline{C} \qquad \Gamma, f : A \to \underline{C} \vdash c_2 \Rightarrow \underline{D}}{\Gamma \vdash \texttt{let rec}\ f\ x\ : A \to \underline{C} = c_1\ \texttt{in}\ c_2 \Rightarrow \underline{D}} \qquad \frac{\Gamma \vdash v \Rightarrow A}{\Gamma \vdash \texttt{ret}\ v \Rightarrow A!\emptyset/\emptyset}$$

$$\frac{(op : A_{op} \to B_{op}) \in \Sigma \qquad \Gamma \vdash v \Leftarrow A_{op} \qquad \Gamma, y : B_{op} \vdash c \Leftarrow A!\Sigma/\mathcal{E}}{\Gamma \vdash op(v;\ y.c) \Leftarrow A!\Sigma/\mathcal{E}}$$

$$\frac{\Gamma \vdash c_1 \Rightarrow A!\Sigma/\mathcal{E} \qquad \Gamma, x : A \vdash c_2 \Leftarrow B!\Sigma/\mathcal{E} \qquad \text{effect subtyping?}}{\Gamma \vdash \texttt{do}\ x \leftarrow c_1\ \texttt{in}\ c_2 \Leftarrow B!\Sigma/\mathcal{E}}$$

$$\frac{\Gamma \vdash v \Rightarrow \underline{C} \Rightarrow \underline{D} \qquad \Gamma \vdash c \Leftarrow \underline{C}}{\Gamma \vdash \texttt{with}\ v\ \texttt{handle}\ c \Rightarrow \underline{D}}$$

Figure 3: *EEFF* Type System

$$
\begin{array}{rcll}
\text{values } v & ::= & x & \text{variable} \\
& | & () & \text{unit} \\
& | & n & \text{integer} \\
& | & \texttt{Left}\ v \ \mid\ \texttt{Right}\ v & \text{sum constructors} \\
& | & (v_1, v_2) & \text{pair} \\
& | & \texttt{fun}\ x \mapsto c & \text{function} \\
& | & \texttt{handler}\,(\texttt{ret}\ x \mapsto c_r\,; h) & \text{handler} \\
\\
\text{computations } c & ::= & \texttt{ret}\ v & \text{returned value} \\
& | & \texttt{match}\ v\ \texttt{with}\ (x, y) \mapsto c & \text{product match} \\
& | & \texttt{match}\ v\ \texttt{with}\ \texttt{Left}\ x \mapsto c_1 \mid \texttt{Right}\ x \mapsto c_2 & \text{sum match} \\
& | & v_1\ v_2 & \text{application} \\
& | & op(v;\ y.c) & \text{operation call} \\
& | & \texttt{let rec}\ f\ \ x = c_1\ \texttt{in}\ c_2 & \text{recursive function} \\
& | & \texttt{do}\ x \leftarrow c_1\ \texttt{in}\ c_2 & \text{sequencing} \\
& | & \texttt{with}\ v\ \texttt{handle}\ c & \text{handling} \\
\\
\text{operation clauses } h & ::= & \emptyset \mid h \cup \{op(x;\ k) \mapsto c_{op}\} &
\end{array}
$$

Figure 4: *EEFF* Term Syntax

$$\frac{(x : A) \in \Gamma}{\Gamma \vdash x : A} \qquad \frac{}{\Gamma \vdash () : \texttt{unit}} \qquad \frac{}{\Gamma \vdash n : \texttt{int}} \qquad \frac{\Gamma \vdash v : A}{\Gamma \vdash \texttt{Left}\ v : A + B}$$

$$\frac{\Gamma \vdash v : B}{\Gamma \vdash \texttt{Right}\ v : A + B} \qquad \frac{\Gamma \vdash v_1 : A \qquad \Gamma \vdash v_2 : B}{\Gamma \vdash (v_1, v_2) : A \times B} \qquad \frac{\Gamma \vdash v_1 : A \qquad \Gamma \vdash v_2 : B}{\Gamma \vdash (v_1, v_2) : A \times B}$$

$$\frac{\Gamma, x : A \vdash c : \underline{C}}{\Gamma \vdash \texttt{fun}\ x \mapsto c : A \to \underline{C}} \qquad \frac{\Gamma, x : A \vdash c_r : \underline{D} \qquad \Gamma \vdash h : \Sigma \Rightarrow \underline{D}\ \textsf{respects}\ \mathcal{E}}{\Gamma \vdash \texttt{handler}\ (\texttt{ret}\ x \mapsto c_r ; h) : A!\Sigma/\mathcal{E} \Rightarrow \underline{D}}$$

$$\frac{}{\Gamma \vdash \emptyset : \emptyset \Rightarrow \underline{D}} \qquad \frac{\Gamma \vdash h : \Sigma \Rightarrow \underline{D} \qquad \Gamma, x : A_{op}, k : B_{op} \to \underline{D} \vdash c_{op} : \underline{D} \qquad op \notin \Sigma}{\Gamma \vdash h \cup \{op(x;\ k) \mapsto c_{op}\} : (\Sigma \cup \{op : A_{op} \to B_{op}\}) \Rightarrow \underline{D}}$$

$$\frac{\Gamma \vdash v : A + B \qquad \Gamma, x : A \vdash c_l : \underline{C} \qquad \Gamma, x : B \vdash c_r : \underline{C}}{\Gamma \vdash \texttt{match}\ v\ \texttt{with Left}\ x \mapsto c_l \mid \texttt{Right}\ x \mapsto c_r : \underline{C}}$$

$$\frac{\Gamma \vdash v : A \times B \qquad \Gamma, x : A, y : B \vdash c : \underline{C}}{\Gamma \vdash \texttt{match}\ v\ \texttt{with}\ (x, y) \mapsto c : \underline{C}} \qquad \frac{\Gamma \vdash v_1 : A \to \underline{C} \qquad \Gamma \vdash v_2 : A}{\Gamma \vdash v_1\ v_2 : \underline{C}}$$

$$\frac{\Gamma, f : A \to \underline{C}, x : A \vdash c_1 : \underline{C} \qquad \Gamma, f : A \to \underline{C} \vdash c_2 : \underline{D}}{\Gamma \vdash \texttt{let rec}\ f\ x = c_1\ \texttt{in}\ c_2 : \underline{D}} \qquad \frac{\Gamma \vdash v : A}{\Gamma \vdash \texttt{ret}\ v : A!\emptyset/\emptyset}$$

$$\frac{(op : A_{op} \to B_{op}) \in \Sigma \qquad \Gamma \vdash v : A_{op} \qquad \Gamma, y : B_{op} \vdash c : A!\Sigma/\mathcal{E}}{\Gamma \vdash op(v;\ y.c) : A!\Sigma/\mathcal{E}}$$

$$\frac{\Gamma \vdash c_1 : A!\Sigma/\mathcal{E} \qquad \Gamma, x : A \vdash c_2 : B!\Sigma/\mathcal{E}}{\Gamma \vdash \texttt{do}\ x \leftarrow c_1\ \texttt{in}\ c_2 : B!\Sigma/\mathcal{E}} \qquad \frac{\Gamma \vdash v : \underline{C} \Rightarrow \underline{D} \qquad \Gamma \vdash c : \underline{C}}{\Gamma \vdash \texttt{with}\ v\ \texttt{handle}\ c : \underline{D}}$$

Figure 5: *EEFF* Type System

$$\frac{}{\texttt{match}\ (v_1, v_2)\ \texttt{with}\ (x, y) \mapsto c\ \rightsquigarrow\ c[x \mapsto v_1, y \mapsto v_2]}$$

$$\frac{}{\texttt{match Left}\ v\ \texttt{with Left}\ x \mapsto c_l \mid \texttt{Right}\ x \mapsto c_r\ \rightsquigarrow\ c_l[x \mapsto v]}$$

$$\frac{}{\texttt{match Right}\ v\ \texttt{with Left}\ x \mapsto c_l \mid \texttt{Right}\ x \mapsto c_r\ \rightsquigarrow\ c_r[x \mapsto v]}$$

$$\frac{}{(\texttt{fun}\ x \mapsto c)\ v\ \rightsquigarrow\ c[x \mapsto v]}$$

$$\frac{}{\texttt{let rec}\ f\ x = A \rightarrow \underline{C}\ \texttt{in}\ c_1 c_2\ \rightsquigarrow\ c_2[f \mapsto (\texttt{fun}\ x \mapsto \texttt{let rec}\ f\ x = A \rightarrow \underline{C}\ \texttt{in}\ c_1 c_1)]}$$

$$\frac{c_1\ \rightsquigarrow\ c_1'}{\texttt{do}\ x \leftarrow c_1\ \texttt{in}\ c_2\ \rightsquigarrow\ \texttt{do}\ x \leftarrow c_1'\ \texttt{in}\ c_2} \qquad \frac{}{\texttt{do}\ x \leftarrow \texttt{ret}\ v\ \texttt{in}\ c\ \rightsquigarrow\ c[x \mapsto v]}$$

$$\frac{}{\texttt{do}\ x \leftarrow op(v;\ y.c_1)\ \texttt{in}\ c_2\ \rightsquigarrow\ op(v;\ y.\texttt{do}\ x \leftarrow c_1\ \texttt{in}\ c_2)}$$

$$\frac{c\ \rightsquigarrow\ c'}{\texttt{with}\ v\ \texttt{handle}\ c\ \rightsquigarrow\ \texttt{with}\ v\ \texttt{handle}\ c'}$$

$$\frac{}{\texttt{with}\ (\texttt{handler}\ (\texttt{ret}\ x \mapsto c_r; h))\ \texttt{handle}\ (\texttt{ret}\ v)\ \rightsquigarrow\ c_r[x \mapsto v]}$$

$$\frac{H = (\texttt{handler}\ (\texttt{ret}\ x \mapsto c_r; h)) \qquad (op(x;\ k) \mapsto c_{op}) \in h}{\texttt{with}\ H\ \texttt{handle}\ (op(v;\ y.c))\ \rightsquigarrow\ c_{op}[x \mapsto v, k \mapsto (\texttt{fun}\ y \mapsto \texttt{with}\ H\ \texttt{handle}\ c)]}$$

Figure 6: *EEFF* Operational Semantics