Annu. Rev. Fluid Mech. 1992. 24 : 21–46
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MODELING OF TWO-PHASE SLUG FLOW

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KEY WORDS: gas-liquid flow, intermittency, bubbly flow, separated flow, bubble entrainment

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INTRODUCTION

When gas and liquid flow in a pipe, over a range of flow rates, a flow pattern results in which sequences of long bubbles, almost filling the pipe cross section, are successively followed by liquid slugs that may contain small bubbles (see Figure 1). This flow pattern, usually called "slug flow", is encountered in numerous practical situations, for example:

- production of hydrocarbons in wells and their transportation in pipelines;
- production of steam and water in geothermal power plants;
- boiling and condensation in liquid-vapor systems of thermal power plants;
- emergency core cooling of nuclear reactors;
- heat and mass transfer between gas and liquid in chemical reactors.

Slug flow belongs to a class of intermittent flows that has very distinctive features. Even if the inlet conditions are stationary, the flow as seen by an observer, is an unsteady phenomenon, dispersed flow appearing alternately with separated flow. These two states follow in a random-like manner, inducing pressure and velocity fluctuations. In vertical flow, the large bubbles—typically longer than, say, one pipe diameter—rise with a round shaped front followed by a cylindrical main body surrounded by an annular liquid film. In the literature, these long bubbles are frequently referred to as Taylor bubbles or Dumitrescu bubbles. In the film, gravity forces the

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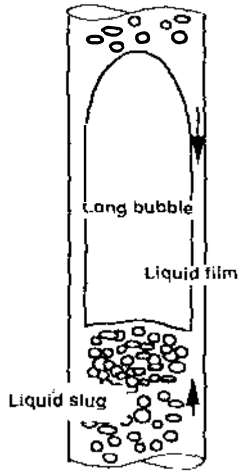

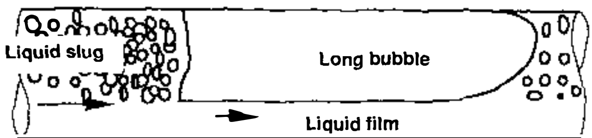

Figure 1 Slug flow in a vertical pipe (*top*), in a horizontal pipe (*bottom*).

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liquid to fall so that it impinges the liquid slug, causing a flow separation at the bubble tail. Under certain conditions small bubbles are entrained in the slug; they may either coalesce at the front of the following bubble or flow in the next falling film. When the pipe is other than vertical, the symmetry of the long bubbles is lost. Even for small deviation angles, the transverse component of gravity causes the interface structure of long bubbles to evolve from an annular to a stratified flow pattern. For the same reason, the small bubbles are more or less stratified in the liquid slugs, under the opposing effects of buoyancy and turbulence dispersion.

The range of gas and liquid flow rates for which slug flow occurs is presented in flow pattern maps that display the transitions to other flow patterns (e.g. Taitel & Dukler 1976, Taitel et al 1980, Barnea 1987). Some authors have claimed to discern the "plug flow" pattern in which elongated bubbles alternate with nonbubbly slugs at very low gas velocity (Barnea et al 1980). Ruder & Hanratty (1990) distinguished plug flow from slug flow by the shape of the bubble tail: similar to the fronts or staircase-shaped for plug flow; agitated and characterized by a hydraulic jump for slug flow. We shall not distinguish between these two types in this paper, since both flow patterns involve similar physical modeling.

The study of the transitions between slug flow and either dispersed flow or separated flow provides a suitable framework for understanding the mechanisms of slug formation. For near horizontal flow, when the gas flow rate is increased, the stratified flow may undergo a flow bifurcation leading to slug formation. The Kelvin-Helmholtz instability, occurring when gas inertia is high enough to overcome the stabilizing effect of gravity, was recognized early on to be the origin of the onset of slugging. The K-H inviscid theory has been qualitatively interpreted in the most widely used model of Taitel & Dukler (1976): However this model fails at high pressure as well as for highly viscous liquids. As the classical inviscid theory underpredicts the gas velocity at which the transition occurs, new approaches have been followed within the framework of linear stability theory, restricted to long waves and including viscous effects (Ferschneider et al 1985, Lin & Hanratty 1986). Even though the theory satisfactorily predicts the onset of slugging for atmospheric pressure and sufficiently weak gas velocity, the criterion must be interpreted as a sufficient condition of instability leading possibly to two-dimensional waves but not necessarily to slugs (Andritsos & Hanratty 1987). That the onset of vertical slug flow results from an instability of bubbly flow is not currently accepted. However there exists strong experimental evidence (Matuszkiewicz et al 1987), as well as theoretical justification from linear stability theory (Bouré 1988), that the void fraction waves—that is waves of volume fraction of gas—become unstable when the gas flow rate is increased. The understanding of the onset of slugging is still fragmentary and the obvious limits of the linear stability theory suggest that nonlinear analyses must be developed.

The complexity of slug flow arises from its particular structure, which is neither periodic in space nor in time. This suggests that one should formulate balance equations, without ignoring the chaotic nature of the flow—a basic problem reviewed in the second section. Because the gas is mainly contained in the long bubbles, special attention must be paid to understanding their motion (reviewed in the third section). The distribution of phases between the long bubbles and the liquid slugs is the third fundamental subject to be discussed. Even if the mechanism of slug formation is more or less understood, there is no evidence that a predictable model could give the characteristic scales of slug flow, such as the frequency or the probability distribution of the lengths of the long bubbles and liquid slugs. This problem will be reviewed in the last section.

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The following notation is used: q denotes the flow pattern, S separated flow, D dispersed flow; k denotes the phases, G gas and L liquid; w and i refer to wall and interface respectively.

SLUG FLOW EQUATIONS AND THE CLOSURE PROBLEM

From the physical point of view the most distinctive feature of slug flow is its intermittency. Thus, any attempt to model the flow by a classical time-averaging procedure regardless of the flow structure would be highly restrictive. Slug flow modeling requires a more detailed analysis, taking into account the intermittent behavior. The need to discriminate between the flow patterns was recognized early on in order to express closure laws adapted to each. In slug flow the space-time occurrence of long bubbles and liquid slugs requires closure models used for both separated and dispersed flow. An additional distinctive closure problem arises from thc intermittency, which is extensively treated in this review.

Two approaches have been explored over the last 25 years. Initially, semimechanistic models based upon physical observations were developed intuitively. By reducing intermittency to periodicity, the actual very complex flow structure was simplified to an equivalent cell consisting of a long bubble and a liquid slug. The long bubble was approximated by a cylindrical capsule, while the liquid slug was considered as either a single-phase or a homogeneous bubbly flow. The pioneering works on slug flow modeling are seen as those of Griffith & Wallis (1961) and Nicklin et al (1962), who were the first to recognize the importance of the long bubble motion. However, the equivalent unit-cell concept was introduced for the first time by Wallis (1969) and used in a simplified model predicting the pressure gradient arising from three contributions: the liquid slug, the ends of the long bubble, and its main body. The "unit-cell model" was developed by Dukler & Hubbard (1975) for horizontal flow and thereafter by Fernandcs et al (1983) for vertical slug flow. The balance equations are written in a frame of reference moving with the cell so the flow appears steady: They express the requirement that mass and momentum be conserved across the moving boundary separating the liquid slug from the long bubble. This approach has been reviewed recently by Taitel & Barnea (1990) but it remains conceptually limited.

In the last decade, a formalism has been developed for the physical properties to be statistically averaged over each flow pattern, namely the separated and the dispersed flow regions. The method leads to the prediction of the dependent variables within each flow pattern and of the characteristics of the intermittency from a "statistical cellular model"

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based on conditioned averaging of conservation equations. The first attempts were by Ferschneider (1982) and Fabre et al (1983) for horizontal slug flow, then Liné (1983) for vertical slug flow; the most recent refinements are due to Fabre et al (1989) specifically for transient flow. Conceptually, this approach is similar to the conditioned averaging applied to intermittent turbulence in single-phase flows by Libby (1975) and Dopazo (1977). The method is based on the introduction of a characteristic function of intermittency χ (x, t): equal to 1 if some flow conditions are realized at (x, t), and 0 otherwise. It is arbitrarily assigned to 1 for separated flow and expressed as the sum of Heaviside functions, each being linked to the Lagrangian coordinates of the bubble front and back. Provided that the cell velocity V is a continuous function having the actual discrete value at each front and back location, it is easily demonstrated that

$$\frac{\partial \chi}{\partial x} = \bar{\omega}' - \bar{\omega}'' \quad \text{and} \quad \frac{\partial \chi}{\partial t} = (\bar{\omega}' - \bar{\omega}'')V \Rightarrow \frac{\partial \chi}{\partial t} + V\frac{\partial \chi}{\partial x} = 0, \tag{1}$$

where $\bar{\omega}'$ (or $\bar{\omega}''$) is the sum of Dirac delta functions linked to the front (or the back) of the long bubbles. Equation (1) indicates that the cellular structure propagates as a kinematic wave (Whitham 1974). On weighting the cross-section averaged balance equations of each phase by the intermittency function one obtains the instantaneous balance equations, which separate into two sets of equations for conditioned quantities in each flow pattern. These equations contain additional terms that formally result from the spatial and temporal derivatives of the intermittency function and physically account for the mass and momentum fluxes across the moving boundaries between the liquid slugs and the long bubbles.

The equations are then written in a statistical average form as in the procedure adopted by Dopazo (1977) for the time-averaged equations of statistically stationary turbulent flow. They involve the statistical probability of the occurrence of separated flow defined by $\beta^{S} = \bar{\chi}$ as well as the conditioned statistical average G_{k}^{S} of the physical quantity G defined by $G_{k}^{S} = \overline{\chi G_{kS}}/\beta^{S}$. Similar definitions are used for the dispersed flow. In fact, the averaging operator denoted by an overbar must satisfy certain conditions to be applied to the sum of the Dirac delta functions. On averaging Equation (1) Fabre et al (1989) pointed out that β^{S} may undergo spatial variations even in steady flow because its derivatives are expressed as

$$\frac{\partial \beta^{S}}{\partial t} = \frac{V'}{\lambda'} - \frac{V''}{\lambda''} \quad \text{and} \quad \frac{\partial \beta^{S}}{\partial x} = \frac{1}{\lambda''} - \frac{1}{\lambda'}, \tag{2}$$

where λ' and λ'' are the mean length scales of the cells, V' and V'' their mean velocities determined at the front and the back of the long bubbles,

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and consequently ν' and ν'' defined by $\nu = V/\lambda$ are the frequencies. Interestingly enough in steady flow, ν' and ν'' are equal—unlike the length scales and velocities. This difference has been observed in vertical flow by Zai-Sha Mao & Dukler (1989) from the measurements of both V' and V'': As expected the bubble front velocity is greater due to the gas expansion.

The *flow pattern equations* take the general form illustrated below by the continuity equation

$$\frac{\partial}{\partial t}(\beta^q\rho_k^q\alpha_k^q)+\frac{\partial}{\partial x}(\beta^q\rho_k^q\alpha_k^q U_k^q) = \frac{\phi''_{kq}}{\lambda''} - \frac{\phi'_{kq}}{\lambda'}, \tag{3}$$

where α_k^q is the mean phase fraction of phase k in flow pattern q, ρ_k^q the mean density, and U_k^q the mean velocity. This formulation introduces on the right-hand side the mean fluxes $\phi_{kq} = \rho_{kq}\alpha_{kq}(V-U_{kq})$ expressing the rate at which mass is exchanged between the long bubbles and the liquid slugs. In fully-developed stationary flow the mean fluxes ϕ''_{kq}, ϕ'_{kq} entering and leaving the slugs are equal yielding

$$\phi_k = \rho_k\alpha_k V - m_k = \rho_{kS}\alpha_{kS}(V-U_{kS}) = \rho_{kD}\alpha_{kD}(V-U_{kD}), \tag{4}$$

where m_k is the inlet mass flux and α_k is the global phase fraction defined by $(\beta^S\rho_k^S\alpha_k^S+\beta^D\rho_k^D\alpha_k^D)/\rho_k$. Strictly, the mean physical quantities averaged at the ends, e.g. α_{kq} and U_{kq}, are not the same as their average over the flow pattern, α_k^q and U_k^q—however, the issue is still an open one. Relations similar to (4) have been used in the unit-cell model (see for example Fernandes et al 1983) but the above problem does not appear since the two kinds of averages are not distinguished.

The equation for the conservation of momentum takes the same form as (3) except that it contains unclosed extra terms arising from the nonlinearities of the basic equations (Fabre et al 1989). In fully-developed stationary flow it simplifies to

$$\alpha_k^q\frac{\partial P_k^q}{\partial x} - T_k^{wq} - T_k^{iq} + \Pi_k^{iq} + \rho_k^q\alpha_k^q g\sin\theta$$

$$= \frac{1}{\beta^q\lambda}\left[\phi_k(U'_{kq} - U''_{kq}) - (\alpha'_{kq}P'_{kq} - \alpha''_{kq}P''_{kq})\right]. \tag{5}$$

The left-hand side is a classical momentum balance averaged over the flow pattern q in which P is the pressure, T the contribution of shear stresses at the wall and the interface, and Π the contribution of interfacial pressure. The peculiar feature of (5) involves the terms on the right-hand side—due to the lack of experimental information, however, they have been ignored. Relations (4) and (5) may be used, together with the appropriate closure assumptions, for the determination of phase fractions and velocities in each flow pattern.

The *global equations* for mass and momentum are obtained by summing the flow pattern equations weighted by the probability of occurrence of each. Thus the terms related to the exchanges at the front and the back formally disappear. In the particular case of fully-developed steady flow the momentum equation reduces to

$$\frac{dP}{dx} = [\beta^S(T_L^{wS} + T_G^{wS}) + \beta^D(T_L^{wD} + T_G^{wD})] - (\rho_L\alpha_L + \rho_G\alpha_G)g \sin\theta. \tag{6}$$

Equation (6) is used in order to determine the pressure gradient, which is of primary importance from the practical point of view. An interesting feature arises from the sign of the first term of the right-hand side (which represents the mean friction at the wall): In vertically upward flow it may be positive (e.g. Koeck 1980, Souhar 1982, Fréchou 1986), contrary to intuitive expectations but without any violation of the Second Law of Thermodynamics.

The statistical cellular model offers the right framework for future modeling. Nevertheless, in the present state of knowledge, it requires such drastic assumptions, namely in the exchange terms between the flow regions, that it leads to similar equations found in the unit-cell model, whose formulation is at least clear cut. The resulting equations have to be closed by a set of physical laws that concern the velocity V of long bubbles and the distribution of phases between the separated and dispersed flow regions. These specific problems are discussed in the following paragraphs.

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MOTION OF LONG BUBBLES

In slug flow the gas is mainly conveyed by long bubbles; successful modeling rests on an understanding of their motion. To give a rough idea of its importance, the void fraction in liquid slugs is rarely greater than 25% although its value in long bubbles may be as high as 90%. The velocity of these long bubbles depends on the gravitational acceleration, the diameter and inclination of the pipe, the volumetric fluxes of both phases, and the fluid properties, namely viscosity, density, and surface tension. Moreover, some internal parameters might be of importance in the interaction mechanism between successive bubbles: The bubble and slug lengths, and the void fraction in liquid slugs are expected to play such a role. However, very little has been reported on this subject and the problem is practically ignored. As a consequence, the prediction of long bubble velocity is greatly simplified since it only depends on input flow rates.

Following Nicklin et al (1962) it is always possible to represent the velocity of these long bubbles as the sum of two terms:

$$V = C_0 U_m + V_\infty, \tag{7}$$

where V_∞ is the bubble velocity in a fluid at rest at infinity and U_m the cross-sectional average of the velocity of the fluid. Such a decomposition does not mean that V is a linear function of U_m since C_0 is a dimensionless coefficient that may depend on U_m through the other dimensionless parameters of the problem, namely the Reynolds, Weber, and Froude numbers. Nevertheless, for most practical purposes the velocity of long bubbles may be interpreted from the above relation as that resulting from the combination of transport by the mean flow and the driving force.

Considerable work has been done to understand the motion of long bubbles in tubes. As a first step we shall summarize the theoretical and experimental work on the drift velocity of long bubbles in a stagnant liquid. The influence of gravity and fluid properties will be considered for both vertical and horizontal cases. As a second step we shall discuss the influence of the liquid motion by analyzing the rise velocity in a flowing liquid.

Motion of a Single Bubble in a Stagnant Liquid

As long as the pressure in the gas is assumed constant, the drift velocity V_∞ of a long bubble moving in a pipe of diameter D in a liquid at rest at infinity depends on the viscosity μ_L and the density ρ_L of the liquid, the density ρ_G of the gas (or the difference of densities $\Delta\rho = \rho_L - \rho_G$), and the surface tension σ. This dependency may be expressed through the general relationship (Zukoski 1966):

$$V_\infty = C_\infty(N_f, \mathrm{Eo}, \theta)\sqrt{g'D}, \tag{8}$$

where the gravitational acceleration g has been replaced for convenience by $g' = g\Delta\rho/\rho_L$, θ is the pipe inclination, N_f the dimensionless inverse viscosity defined by $D^{3/2}g'^{1/2}\rho_L/\mu_L$, and Eo the Eötvös number, which is the inverse of the dimensionless surface tension equal to $\rho_L g' D^2/\sigma$.

VERTICAL MOTION Zukoski (1966) observed that long bubbles rising in vertical tubes were stable to external perturbations. Their front has the shape of a prolate spheroid determined by the flow conditions upstream of the bubbles and is independent of their length (Nicklin et al 1962). The film region is, however, length dependent since it results from a balance between friction and gravity as well as from the curvature at the bubble front. The shape at the rear depends on whether or not the viscous force is negligible: When negligible, the bubble has a flat back indicating that

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flow separation occurs; in contrast, when viscous effects are significant, the bubble back is shaped like an oblate spheroid following a standing wavelet on the film (Goldsmith & Mason 1962). Thus two asymptotic cases are of special interest: the inertia-controlled regime and the viscous regime. We note in passing that the influence of surface tension may be disregarded when the Eötvös number is high enough, say for Eo > 100 (Wallis 1969).

For the *inertia-controlled regime* the theoretical solution may be found by assuming an inviscid fluid. The condition is realized if N_f is greater than 300, i.e. when the pipe diameter is greater than 50 $(\mu_L^2 \rho_L^{-2} g'^{-1})^{1/3}$. It may be expected that most of the practical cases of slug flow will fall within the frame of the inviscid fluid assumption: For example the condition is fulfilled when $D > 2.5$ mm for a gas bubble rising in water. Dumitrescu (1943) was the first to give the solution by retaining three terms in the series expansion of the potential flow around a prescribed spherical front, leading to the value of 0.351 for C_∞. The agreement of this result with the experiments of White & Beardmore (1962), Nicklin et al (1962), and Zukoski (1966), as well as with the numerical simulations of Zai-Sha Mao & Dukler (1990), is quite impressive. Reference is frequently made to the later solution obtained independently by Davies & Taylor (1950), although it gave the less accurate result of 0.328—this was because they retained only the first term of the series expansion to satisfy the constant gas pressure condition on the arbitrary assumption of a spherical interface at $D/4$. The paper of Collins (1967) gives a detailed discussion of the different theoretical treatments including that of Layzer (1955): Although the constant pressure condition cannot be satisfied for a prescribed spherical front all along the surface, the result of Dumitrescu was confirmed provided the volume of the bubble was greater than $(0.4D)^3$; the ratio of the radius of curvature at the vertex to the pipe radius is then equal to 0.71.

The same analysis can be extended to the case in which surface tension is not negligible, since it only appears in the boundary conditions at the free surface of the bubble. The secondary influence of the Eötvös number in Equation (8) has been analyzed theoretically for high Reynolds number by Tung & Parlange (1976) and more recently by Bendiksen (1985). Tung & Parlange used the maximum velocity principle of Garabedian (1957) for selecting the physical solution, while Bendiksen followed the method of Dumitrescu without imposing an a priori spherical shape. Both found that surface tension monotonically reduces the rise velocity—in convincing agreement with their own experiments as well as with the experiments of Zukoski (1966) (Figure 2). The simplified result of Bendiksen is given here (after the correction of a typographical error in the author's paper):

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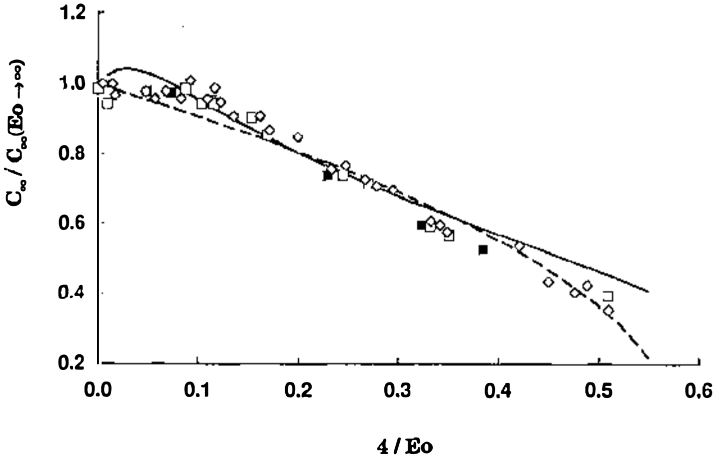

Figure 2 Influence of Eötvös number on C_∞. Theories of Bendiksen (1985) (*solid line*) and Tung & Parlange (1976) (*dashed line*). Experiments of Bendiksen (1985) (*solid squares*), Tung & Parlange (1976) (*diamonds*), and Zukoski (1966) (*open squares*).

$$C_\infty(N_f \to \infty, \mathrm{Eo}, 90°)$$
$$= 0.344 \frac{1-0.96\,e^{-0.0165\,\mathrm{Eo}}}{(1-0.52\,e^{-0.0165\,\mathrm{Eo}})^{3/2}} \sqrt{1+\frac{20}{\mathrm{Eo}}\left(1-\frac{6.8}{\mathrm{Eo}}\right)}. \quad (9)$$

The range of validity of the above relation is sufficient for most practical cases. We note in passing the special case of two-dimensional bubbles in a rectangular channel for which an infinite number of solutions satisfying both Laplace's equation and the boundary conditions at the bubble surface were found by Garabedian (1957) by using four terms of the series. Couët & Strumolo (1987) confirmed the possibility of multiple solutions using a numerical method that allowed the constant pressure condition to be satisfied at N points of the free surface. They selected the maximum velocity out of the infinite set of discrete values although no theoretical justification was given. Such a problem of choice arises in purely numerical methods in which the bubble velocity is unknown. Zai-Sha Mao & Dukler (1990) selected the solution for which $dK/ds = 0$ at the vertex, where K is the mean curvature and s the curvilinear distance from the axis. Their results are in excellent agreement with the accepted values for both plane and axisymmetrical bubbles.

The *viscous regime* is encountered when N_f is less than 2 (Wallis 1969): It requires that the pipe diameter be less than 1.6 $(\mu_L^2 \rho_L^{-2} g'^{-1})^{1/3}$, which may arise with highly viscous liquids. The dimensional analysis leads to C_∞ being expressed as

$$C_\infty(N_f, \text{Eo} \to \infty, 90^\circ) = 0.01 N_f, \tag{10}$$

where the coefficient was found by White & Beardmore (1962) and Wallis (1969) from their experiments. Wallis (1969, p. 290) proposed a correlation—taking into account inertia, viscosity, and surface tension effects—that may be useful for practical situations of slug flow.

The case of very small tubes is beyond the scope of this review. However, it should be mentioned that the bubbles can be motionless. This phenomenon, which has been analyzed theoretically by Bretherton (1961), occurs when the gravity force is negligible compared to the surface tension force, that is for $\text{Eo} < 3.37$.

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HORIZONTAL MOTION The motion of long bubbles in horizontal pipes has been less studied theoretically for the obvious reason that the symmetry with respect to the axis is lost. In contrast to the previous situation, the wall is wetted by the gas so that, in the fully-developed region far downstream from the front, the interface is plane. The motion of long bubbles in still liquid has for a long time been a matter of controversy [as pointed out by Weber (1981), some investigators believed that they should be stationary while others did not]. This problem has been discussed theoretically by Benjamin (1968) for the inertia-controlled regime by considering the case of a horizontal cavity filled with liquid and open at one end. As the tube is emptying, a bubble front propagates towards the closed end. The determination of the bubble velocity for a rectangular channel may be considered as a classical inviscid flow exercise for students; for pipe flow it leads to the result $C_\infty = 0.542$, in agreement with the experimental values of Zukoski (1966) corresponding to the highest Eötvös numbers. A drift velocity greater in horizontal than in vertical situations is not, intuitively, what one would expect.

The influence of surface tension deserves special attention. As in vertical flow, the drift velocity decreases as the surface tension increases. However, Zukoski (1966) suggested, from his experiments in the range $100 < \text{Eo} < 4000$, that in a horizontal pipe the velocity might not approach a limiting value, in contrast to vertical flow where the asymptotic value is reached for $\text{Eo} > 100$. This contradiction with the theory of Benjamin does not seem reasonable: According to Spedding & Nguyen (1978) there is no experimental evidence of an increase in bubble velocity beyond $\text{Eo} = 10{,}000$. However, the asymptotic behavior of the bubble velocity remains obscure at very high Eötvös number—further experiments are needed. If the theory of Benjamin is accepted, the experimental results of Zukoski are well correlated by (Weber 1981):

$$C_\infty(N_f \to \infty, \text{Eo}, 0^\circ) = 0.54 - 1.76\,\text{Eo}^{-0.56}. \tag{11}$$

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INFLUENCE OF PIPE INCLINATION Experimental studies of bubble motion with a stationary liquid have been carried out by Zukoski (1966), Spedding & Nguyen (1978), and Weber et al (1986) for pipe inclinations ranging from 0 to 90°. The effect of inclination is complex because of the change in bubble geometry. Below 30° the tube is wetted by the gas, the contact angle of the bubble at the wall being acute; beyond 40° this angle is obtuse. At high Eötvös numbers the velocity is a maximum for an inclination of about 35°, roughly corresponding to contact at right angles with the wall. A general theory for an inclined pipe is lacking at present. If the empirical correlation of Bendiksen (1984), improved by Weber et al (1986), may be safely used for the inertia-controlled regime, the coupled influence of inclination, viscosity, and surface tension are not really understood. Following Couët & Strumolo (1987) it is necessary as a first step to reduce the complexity of the problem to the study of plane flow.

Bubble Motion in a Moving Liquid

When a liquid flows through a tube, the motion of the long bubbles results from the complex influence of both buoyancy and mean motion of the liquid. Let us focus at first upon the case of vertical flow where two different situations occur. In upward flow, experiments carried out with single bubbles (Griffith & Wallis 1961, Nicklin et al 1962, Bendiksen 1984) show that the front is smooth, spherical, and symmetrical with respect to the pipe axis. In downward flow the structure of the free surface is more complex. Griffith & Wallis (1961) and Nicklin et al (1962) report that the bubble migrates with an asymmetrical shape. Moreover, above some critical liquid flow rate, the motion is observed to be very unsteady, the bubble being distorted alternately on one side of the tube and then the other. Martin (1976) observed the same trends for rising and descending bubbles, except for the case of small pipe diameters where surface tension tends to reduce eccentricity as well as unstable behavior.

A theoretical analysis of upflow has been made by Collins et al (1978) and has been extended by Bendiksen (1985) in order to take into account surface tension effects. Before discussing the method of Collins et al let us briefly recall the earlier experimental results of Nicklin et al (1962). Decomposing the velocity according to Equation (7) they plotted C_0 versus the mean velocity of the liquid U_m: C_0 increases from 0.9 for negative values to a maximum of 1.8 near $U_m = 0$ and then decreases towards an asymptotic value of 1.2, which is reached when the Reynolds number $\mathrm{Re} = \rho_L U_m D/\mu_L$ is greater than 8000. As the value of C_0 at high Reynolds number is close to the ratio of the maximum to the mean velocity, they concluded that "the bubble velocity is very nearly the sum of the velocity on the centerline above the bubble plus the characteristic velocity in still liquid". While crude, this explanation predicts the rise velocity with sufficient accuracy for most purposes. In particular, any physical mechanism having an effect on the rise velocity—such as the flow regime or the liquid rheology for laminar flow—does so through its effect on the liquid centerline velocity.

INFLUENCE OF MEAN LIQUID MOTION IN VERTICAL PIPE FLOW The theoretical influence of upstream liquid flow has been studied under the restrictive assumption of an inviscid fluid. As previously mentioned, the velocity distribution must have an important effect on the bubble motion: Thus the rise velocity is expected to depend on the liquid viscosity through the liquid Reynolds number. It might seem surprising to discuss the effects of viscosity within the framework of inviscid theory. However, viscosity acts essentially to develop the liquid velocity profile far ahead of the bubble—but it has no influence near the front if inertia still dominates. The condition is satisfied provided $N_f > 300$: This occurs at Reynolds numbers for which the upstream liquid flow can be either laminar or turbulent. For inviscid axisymmetric flow, Stokes's stream function satisfies a Poisson equation, which is solved by applying the boundary conditions at the bubble surface. Collins et al (1978) used this approach to obtain an approximate solution for both laminar and turbulent flow with prescribed upstream vorticity: $C_0 = 2.27$ for laminar flow [in agreement with the result of purely viscous flow (Taylor 1961)] and $1.2 < C_0 < 1.4$ for turbulent flow dcpending on thc Reynolds numbcr. Note that thc bubble rise is not equal to the centerline velocity plus the velocity in still liquid, although it is close to it. Bendiksen (1985) used the same assumption of rotational flow of an inviscid fluid to include the effect of surface tension. The results may be summarized as follows:

$$C_0 = 2.29\left[1 - \frac{20}{\mathrm{Eo}}\left(1 - e^{0.0125\,\mathrm{Eo}}\right)\right] \tag{12}$$

for laminar flow, and

$$C_0 = \frac{\log \mathrm{Re} + 0.309}{\log \mathrm{Re} - 0.743}\left[1 - \frac{2}{\mathrm{Eo}}\left(3 - e^{-0.025\,\mathrm{Eo}} \log \mathrm{Re}\right)\right] \tag{13}$$

for turbulent flow.

These results confirm the experiments of Nicklin et al (1962), who showed that the bubbles rise faster in laminar than in turbulent flow. The bubble velocity is sensitive to surface tension since for both flow regimes, C_0 decreases when Eo decreases. Note that viscosity has two different effects: On the one hand increasing μ_L decreases N_f and thus the bubble

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velocity through V_∞; on the other hand it decreases Re so that C_0 may increase significantly if a change in flow regime occurs.

INFLUENCE OF PIPE INCLINATION The experimental work of Bendiksen (1984) provides an extensive data set for pipe inclinations ranging from $-30°$ to $90°$. The author concluded that the bubble velocity is well predicted if the coefficient C_0 is expressed as a function of Re, $\mathrm{Fr} = U_m/(g'D)^{1/2}$, Eo, and θ. We note here the two main conclusions for positive θ: For $\mathrm{Fr} \leq 3.5$ the magnitude of C_0 ranges from 1.00 to 1.20; for $\mathrm{Fr} \geq 3.5$ the values of C_0 approach 1.19–1.20.

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Motion of a Train of Long Bubbles in Slug Flow

There is no evidence that a train of bubbles behaves like a single bubble: Its motion may be influenced by the dynamic interactions caused by the wake and the small bubbles in the liquid slugs. To our knowledge no theoretical study has been undertaken to illuminate the possible interaction between two consecutive cylindrical bubbles. The problem must thus be examined on the basis of experimental results.

Various methods have been used for determining the average velocity of long bubbles in slug flow. They involve different measurement techniques as well as specific data processing to discriminate between the large bubbles and smaller ones. The results are generally given in the classical form of Equation (7). However, contrary to the proposal of Nicklin et al (1962), most authors have taken V_∞ as the intercept on the ordinate of a linear relation between V and U_m [see Equation (7)]. Note that U_m represents the mixture velocity in the case of bubbly slugs.

UPWARD MOTION Surprisingly, models for predicting the motion of a single bubble apply satisfactorily to the more complex situation of slug flow. For turbulent flow in the range Re = 20,000–100,000, Fernandes (1981) reported $C_0 = 1.29$ and $C_\infty = 0.35$, whereas in a similar facility and for almost the same Reynolds numbers, Fréchou (1986) found 1.1–1.2 for C_0 and 0.35 for C_∞.

When the Reynolds number of the mixture decreases below some critical value, laminar flow is expected and the bubble velocity should increase. This particular behavior has been pointed out by Grace & Clift (1979) and later by Fréchou (1986). When the Reynolds number is greater than 2000 the flow is turbulent and C_0 decreases when Re increases; below Re = 500, C_0 approaches asymptotically the value of 2.3 corresponding to laminar flow. The numerical experiments of Zai-Sha Mao & Dukler (1991) are very promising because they predict a rise velocity in remarkable agreement with the experiments, provided the flow regime is prescribed (see

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Figure 3). The Re values at which the transition occurs are lower than expected. It is in fact likely that the occurrence of turbulence in the liquid slugs is also dependent on the turbulence in the falling film as well as on the length of the slugs. Specific experiments have to be performed to clarify this point.

DOWNWARD MOTION Griffith & Wallis (1961) were probably the first to report the unstable motion of cylindrical bubbles in downward liquid flow. In contrast to upward flow where bubbles remain centered, in downward flow they are eccentrically located, with surface tension acting to restore the symmetry. Thus it is to be expected that the higher the surface tension the higher C_0. In specific experiments Martin (1976) reported values of 0.93, 0.90, and 0.86, considerably smaller than for upward flow, corresponding to estimated Eötvös numbers of 90, 1400, and 2670. There is no doubt that downward slug flow is an important issue and raises several questions that are linked mainly to the flow stability: In particular the bubble motion at very high Eötvös number is poorly understood.

NEARLY HORIZONTAL MOTION Some authors claim that the drift velocity V_∞ is zero in horizontal flow [see for example Gregory & Scott (1969) and Dukler & Hubbard (1975)]. As previously mentioned, this contradicts the experimental results of Zukoski (1966) and Bendiksen (1984) as well as the theory of Benjamin (1968). The data obtained for a pipe of 4.5 cm inside diameter and 50 m length by Ferré (1979) are helpful in clarifying this point. He found two different critical Froude numbers: for Fr < 2, $C_0 = 1.1$ and $C_\infty = 0.45$; if $2 > \mathrm{Fr} < 8$, $C_0 = 1.30$ and $C_\infty = 0$; and if

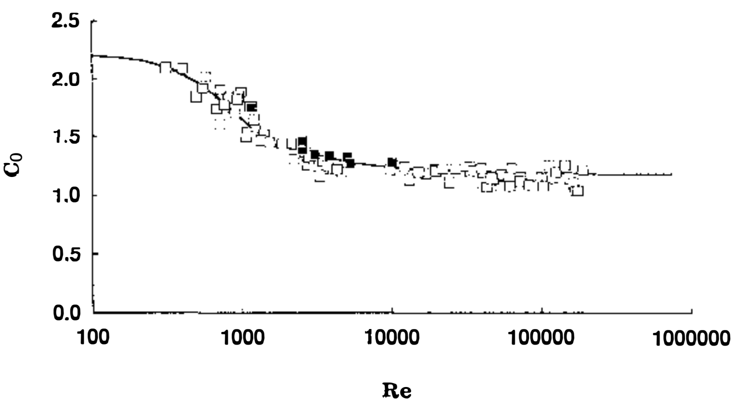

Figure 3 Influence of mixture Reynolds number on C_0. Correlation of Fréchou (1986) (*solid line*). Experiments of Fréchou (1986) (*open squares*); numerical experiments of Zai-Sha Mao & Dukler (1991) (*solid squares*).

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Fr > 8, $C_0 = 1.02$ and $C_\infty = 3$. The experimental results of Théron (1989) confirm the first transition. This was also noticed by Bendiksen (1984) in analyzing the motion of single bubbles in a pipe of 2.5 cm diameter—though at the higher Froude number of 3.5—suggesting the secondary influence of surface tension. Following his conclusion the centered motion of the bubble is restored as soon as the inertia effect is sufficient to overcome gravity: At small enough Froude number the stagnation point should be located at the wall as assumed in Benjamin's theory. When the eccentricity of the bubble decreases under the combined effects of inertia and surface tension, the stagnation point ought to be located somewhere on the bubble front. A theoretical study of the possible solutions starting with 2-D flow is highly desirable to throw light on this problem.

MOTION IN THE ABSENCE OF GRAVITY In the absence of a body force the bubble motion is controlled only by the mean flow so that the relevant parameters for C_0 are the Reynolds number and the capillary number $\mathrm{Ca} = U_m \mu_L / \sigma$. The above conditions are encountered in capillary tubes where the conditions $\mathrm{Eo} \ll 1$ and $N_f \ll 1$ are satisfied. However they also correspond to small Reynolds numbers leading to creeping flow. The problem has been solved theoretically by Bretherton (1961) and numerically by Reinelt (1987) for small capillary numbers (a case beyond the scope of the present review). For Ca > 1 the experiments of Taylor (1961) show that $C_0 \rightarrow 2.27$ for viscous flow, in complete agreement with the theory of Collins et al (1978). For obvious reasons the case of turbulent flow with negligible gravity cannot be achieved in capillary tubes. In recent years the development of space programs has offered the possibility of micro-gravity experiments. Some results have been obtained during parabolic flights by Colin (1990) confirming the value of $C_0 = 1.2$ predicted theoretically for turbulent flow.

DISTRIBUTION OF PHASES

The prediction of the mean void fractions both in the long bubbles and the liquid slugs involves (*a*) the distinct momentum interactions in separated flow and bubbly flow and (*b*) the phase exchanges between the long bubbles and the liquid slugs.

Void Fraction in Long Bubbles

The void fraction in long bubbles results from the momentum balance applied to the liquid film (5): The film equilibrium involves the gravity force, wall and interfacial shear stresses, and inertia. In most models, the flow of the liquid film is regarded as fully-developed so that inertia may be neglected. Thus, solving the momentum balance requires only expressions for the wall and interface friction factors. The wall friction factor has generally been satisfactorily calculated from classical single-phase relations [e.g. Dukler & Bergelin (1952), Fulford (1964), Liné & Masbernat (1985), and Orell & Rembrand (1986) for vertical flow; Dukler & Hubbard (1975) and Ferschneider (1982) for horizontal flow]. Nevertheless, the film Reynolds number at which the transition between laminar and turbulent regime occurs remains unclear. Interfacial friction modeling is still considered an open problem for both annular and stratified flow patterns. In vertical flow, hopefully, this force is of minor importance and has been frequently neglected (Fernandes et al 1983, Orell & Rembrand 1986). This approximation is supported by the fact that the pressure is almost constant in the long bubble (Laird & Chisholm 1956, Akagawa et al 1971). For the calculation of the interfacial friction factor Wallis (1970) suggested a widely used correlation corresponding to a smooth interface.

Because of the fully-developed flow assumption, the method underestimates the mean film thickness. This weakness is particularly important for bubbles a few diameters long. Lusseyran (1990) observed that, even for long bubbles, the equilibrium thickness was seldom reached. In addition to this problem, we should point out that the prediction of the pressure gradient may be greatly affected by inertia in the front curvature region as indicated by Barnea (1990).

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Gas Entrainment and Phase Fractions in Liquid Slugs

The experimental observations show that the void fraction in a liquid slug may be very sensitive to the size and the slope of the pipe as well as the fluid properties (e.g. Gregory et al 1978, Ferschneider 1982, Andreussi & Bendiksen 1989). The presence of small bubbles in the liquid slugs results from coupled phenomena involving their production by fragmentation of the bubble tail, their entrainment out of the bubble wake, and their drift relative to the mean flow; the major difficulty is to isolate and understand each of these effects.

Apart from pure correlations for horizontal flow (Gregory et al 1978, Garcia 1980, Ferschneider 1982), the most interesting approach explored consists of determining the entrainment rate ϕ_G at the tail from a mass balance between the bubble production rate ϕ_{Ge} and the loss rate ϕ_{Gb} due to gas entering the long bubbles [Fernandes et al (1983), Liné & Masbernat (1985) for vertical flow; Andreussi & Bendiksen (1989) for horizontal and inclined flow]. The void fraction is then obtained provided the drift of the small bubbles is known. These similar models will be compared after noting that:

1. The model of Andreussi & Bendiksen is in close agreement with the experiments of several authors (e.g. Ferschneider 1982).
2. The model of Fernandes seems to have only limited applicability to viscous liquids since it overpredicts the void fraction (Fréchou 1986).

BUBBLE PRODUCTION Very few experiments have been carried out on the mechanism of slug aeration. Jepson (1987) studied a stationary slug by forming, in a horizontal pipe, a hydraulic jump that bridges the pipe cross section. He showed that part of the air which is entrained across the jump is returned to the pseudo-long bubble. From these experimental observations, it seems that the mechanism of aeration corresponds to swarms of bubbles formed in the mixing region and cyclically detached from the mixing vortex, the frequency of these pulses increasing with the velocity of the liquid. Since the source of small gas bubbles is located at the rear of the long bubbles, a strong evolution of void fraction within the liquid slug is observed (Ferschneider 1982). The physics of the fragmentation of the long bubble tail has not been understood clearly. The experimental data show that there exists a liquid film velocity, relative to the tail, below which no entrainment occurs (e.g. Gregory et al 1978, Ferschneider 1982, Andreussi & Bendiksen 1989). This feature has been experimentally confirmed by Nydal & Andreussi (1991), who analyzed the aeration process of an advancing front of water in a horizontal pipe. Also important is the influence of the fluid properties: It may be verified from the available data that, at a fixed Fr, the greater the Eötvös number, the higher the void fraction, indicating that surface tension might play an appreciable role in determining whether or not the interface can avoid breakup.

In vertical flow, Fernandes et al (1983) assumed that the production rate of bubbles ϕ_{Ge} corresponds to the gas flux over the domain of cylindrical bubbles in which the gas has a negative velocity with respect to the bubbles themselves: ϕ_{Ge} can be easily derived as a function of the friction velocity at the film interface and the mean velocity $\phi_{\mathrm{G}}/(\rho_{\mathrm{G}}\alpha_{\mathrm{G}}^{\mathrm{S}})$ from the velocity-defect law. The model involves the interfacial shear stress. However, it does not take into account the influence of surface tension. It is also limited to vertical flow because of the assumed symmetry. In a different approach, validated in nearly horizontal flow, Andreussi & Bendiksen (1989) assumed that ϕ_{Ge} is proportional to the liquid flux across the tail, provided it exceeds a threshold value corresponding to the conditions where vortex flow occurs at the tail. The corresponding critical velocity is given by an empirical correlation in which surface tension is surprisingly absent. It involves a dimensional constant as well, which underlines the limitations of existing knowledge on the onset of entrainment. Several research projects are currently under way to remedy this situation.

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BUBBLE LOSS For vertical flow, Fernandes et al (1983) suggested that the flux of gas ϕ_{Gb} that flows back into the long bubble must depend on both turbulence and void fraction in the bubble wake. The turbulence intensity was estimated from the theory of free turbulence, whereas the void fraction was calculated assuming no bubble drift. In fact neither the turbulence level nor the void fraction were measured in the wake—so the model remains purely intuitive. In horizontal flow Andreussi & Bendiksen (1989) estimated ϕ_{Gb} to be proportional to the void fraction in the liquid slug α_G^D and to the rise velocity U_{G0} of the small bubbles given by Equation (14).

SMALL BUBBLE DRIFT IN LIQUID SLUGS The drift of small bubbles caused by a force balance involving buoyancy, surface tension, viscous, and inertial forces is beyond the scope of this review. Although there is considerable interest in understanding their motion, particularly for vertical slug flow, no study specifically directed at this problem has been carried out. The very first studies on the drag force exerted on a bubble by an external uniform flow were reported by Levich (1962); more recent reviews have been published by Harper (1972) and Clift et al (1978). The relationship for the rise velocity given by Harmathy (1960) deserves to be quoted since it is frequently used in practical models:

$$U_{G0} = 1.53(\sigma g'/\rho_L)^{1/4}. \tag{14}$$

The bubble drift is also influenced by the wall proximity. The drift flux model of Zuber & Findlay (1964)—used for vertical bubbly flow—takes the drift into account through an equation similar to (14), and the wall effect, through the velocity and void distributions. The experiments of Serizawa et al (1975) on the turbulence in a dispersed flow confirm that the local slip velocity varies little over the pipe cross section and can be estimated by the rise velocity of a bubble in an infinite medium.

Because Equation (14) is size-independent there is no need to predict the bubble diameter d_B. For a gas bubble in water (Clift et al 1978, p. 172) the rise velocity is almost constant in the 1–20 mm range with a minimum for a 7 mm bubble. Thus for air-water experiments the bubble size has a minor influence. However for small enough Morton numbers, i.e. if $\mathrm{Mo} = g'\mu_L^4\rho_L^{-1}\sigma^{-3} < 10^{-2}$ (corresponding to very viscous liquids), there is no range of diameters for which the rise velocity is independent of size (e.g. Comolet 1979). There is therefore a need to predict the bubble size. Most models involving the bubble size follow the breakup theory of Hinze (1955) proposed initially for the emulsification of a liquid in a turbulent flow of another liquid. He assumed that dynamic pressure fluctuations due to eddy motion determine the size of the largest drops. Thus, provided the

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Kolmogorov energy distribution law is valid, the critical diameter above which breakup occurs is given by:

$$d_{\text{crit}} = 0.725(\sigma/\rho_L)^{3/5}\varepsilon^{-2/5}, \tag{15}$$

where ε is the turbulent energy dissipation per unit mass. Sevik & Park (1973) theoretically derived a coefficient with a value of 1.15 for bubble breakup and confirmed its validity by experiments. In addition to the fact that ε is not well-approximated by the single-phase-flow relationship (e.g. Taitel et al 1980), the Hinze model does not predict satisfactorily the maximum bubble size as shown by the bubbly flow experiments carried out by Colin (1990) with and without gravity.

From our experience the aforementioned models developed for bubbly flow are not applicable to bubble drift in liquid slugs. Their weaknesses arise essentially from the fact that the flow develops from the highly aerated region of the wake, in which the bubbles are almost captive, to a region of smaller void, where bubbly flow models might apply [e.g. Ferschneider (1982) for horizontal flow, Barnea & Shemer (1989) for vertical flow]. Moreover in a large diameter pipe, void fractions greater than 25% have been observed. This leads to the formation of a swarm of bubbles in which bubbles interact with one another. Our present knowledge of this problem appears very limited. Moreover the mechanisms affecting bubble size are open to question since the bubble diameter depends essentially on the turbulence in the wake of the long bubble where these swarms are formed.

CHARACTERISTIC LENGTH SCALE

In most slug flow models neither the characteristic length scale λ of the cells nor their frequency ν are needed to calculate the void fraction and the pressure gradient. However these parameters are of fundamental importance in the design of industrial facilities, e.g. for hydrocarbon transportation.

The mean length of liquid slugs has been experimentally observed to be about 12–30 pipe diameters in horizontal flow (Nicholson et al 1978, Dukler & Hubbard 1975, Ferré 1979); other papers report the mean frequency, which is related to the mean length since $\nu = V/\lambda$ (e.g. Gregory & Scott 1969, Vermeulen & Ryan 1971, Dukler & Hubbard 1975, Heywood & Richardson 1979, and Knervold et al 1984). In vertical flow the mean length of the liquid slugs is almost in the range of 8–25 diameters (Moissis & Griffith 1962, Fitremann 1977, Fernandes 1981, Fréchou 1986). Unlike the cell velocity V, the bubble and slug length distributions are widely dispersed about their average. Typical values of the ratio between the standard deviation and the average, as reported by Ferré (1979), are within 30–70% for the liquid slug lengths. In vertical slug flow where the development length is shorter the standard deviation of slug length is about 20%.

Heuristic Analysis

The flow structure, i.e. the alternation of long bubbles and liquid slugs characterized by the intermittency function χ, may be understood as a kinematic wave propagating at the velocity V, as shown by Equation (1). The characteristics of the wavelength scale or frequency depend on the phenomena at the section where the slugs are formed ("entrance mechanism") and their evolution by coalescence ("overtaking mechanism") or fragmentation of long bubbles as well as by gas expansion. The fragmentation of long bubbles is, in fact, seldom observed and the gas expansion may be easily taken into account. Thus we shall focus upon the entrance mechanism and bubble coalescence, which have provided the starting point for two conceptually different kinds of models. Indeed, the models based upon the entrance mechanism implicitly postulate that the wave is not dispersive, its structure depending on the way it is formed, whereas models based upon bubble coalescence suppose that a fully-developed solution independent of entry conditions exists.

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ENTRANCE MECHANISM Assuming that the long bubbles do not coalesce Taitel & Dukler (1977) solved the transient separated horizontal flow adopting a two-fluid model and calculated the time scale necessary for a perturbation to span the pipe. Although the model involves a drastic assumption to keep the equations hyperbolic, their results are in close agreement with the experimental data for small pipes. However for larger diameters and for hydrocarbon fluids the model does not predict the cell frequency with reasonable accuracy (see the comparison with experimental data by Bayini 1983). Tronconi (1990) reported that the slug frequency in a horizontal slug flow is one half of that of the unstable waves—responsible for the generation of the slugs at the inlet region of the pipe—whose frequency is derived from finite amplitude wave theory.

BUBBLE OVERTAKING For vertical flow, Taitel et al (1980) suggested that the minimum stable slug length is related to the distance needed to re-establish the fully-developed turbulent velocity distribution in the liquid slug after it is disturbed by the falling film entering at its front. In their analytical model, Dukler et al (1985) solved the boundary layer equations for quasi-parallel flow to calculate the developing length. The model gives a possible explanation of the experimental results of Fernandes (1981) but it fails to predict the correct trends in the case of air-oil flow (Fréchou 1986). Maron et al (1982) proposed a slug length model both for horizontal

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and vertical slug flows based on the concept of periodic relaxation and recovery of the wall boundary layer at the front of the slug. Barnea & Brauner (1985), neglecting the viscous effect of the solid wall, proposed a minimum stable length in horizontal slug flow of 32 pipe diameters.

It is worth noting that, even starting from different points of view, these various approaches arrive at a correct estimate of the slug lengths. However, they do not tell us whether the characteristic length scales are controlled by the entrance effect or by overtaking mechanisms.

Statistical Analysis

From field data in horizontal and nearly horizontal pipes, the slug-length distribution is shown to be sensitive to the pipe diameter: For given fluids and flow rates, both the mean value and the standard deviation of slug lengths increase with the pipe diameter. The slug-length distribution also changes from a very peaked distribution at the inlet, to a more dispersed distribution downstream. The first attempt at expressing the probability distribution of the liquid slug lengths is due to Brill et al (1981) who fitted their field data by a log-normal distribution. However, the theoretical arguments or stochastic process that might justify this distribution are missing. More recently, Bernicot & Drouffe (1989) proposed a probabilistic approach to model the mechanism of slug formation at the pipe entrance. The probability for a slug to be formed was assumed uniform over some inlet region whose length depends on the location at which the previous slug was formed. The probabilistic model of slug formation was completed by a mechanism of length evolution. The results are quite encouraging since the distribution far from the entrance is close (though not identical) to the log-normal distribution of Brill et al. Moreover, the standard deviation of the logarithm of slug lengths tends asymptotically towards 0.5, which is close to the observed value 0.556.

Fractal statistics were used by Sæther et al (1990) to analyze the stochastic fluctuations of slug lengths in nearly horizontal flow. They concluded that slug flow is persistent, or in other words that there is a correlation between past and future, contrary to the requirement for a random process. The persistence is characterized by the Hurst's exponent (equal to 0.5 for random process), which was found to increase linearly with the mixture velocity from 0.53 to 0.76. The method seems promising since it allows the prediction of length series, provided both the average and standard deviation as well as the Hurst exponent are known.

Despite the deterministic nature of the local instantaneous equations, slug flow is unpredictable for most flow conditions. The theory of dynamical systems provides the framework for characterizing their chaotic behavior. During the last few years, considerable attention has been paid in various fields, to understanding the onset of deterministic chaos and to characterizing the chaotic attractors. Only recently has a first attempt been made by Lusseyran (1990) to analyze the chaotic behavior of upward slug flow. Starting from a time series of the wall shear stress, he constructed the trajectory in an N-dimensional phase space by the method of time delays. The fractal dimension of the resulting attractor was found to be 2.28 for the single case that was analyzed. Although a generalization to other flow conditions must be avoided, there is possibly an indication that slug flow could be controlled by a small number of degrees of freedom.

CONCLUSION

The physical modeling of gas-liquid slug flow, which was raised 30 years ago in engineering systems, has evolved from more or less empirical correlations to a more rigorous formulation leading to the statistical cellular model. Such a model, which derives from a statistical average of a set of nonlinear equations, involves several closure problems, some of which remain incompletely solved.

1. The long bubble motion is satisfactorily understood from an inviscid analysis, except for vertical downflow where unstable motion has been observed. In addition some other aspects, which have not yet been investigated, deserve special attention, in particular: the influence of slug length on bubble motion, which plays a central role in the coalescence of long bubbles; and the added mass force exerted on the long bubble, which must be known for transient flow modeling.
2. The problem of gas exchange between the long bubbles and the liquid slugs is a complex phenomenon whose treatment is at present based upon correlations of gas entrainment; this approach has to be improved. The fragmentation of the long bubble tail into small bubbles needs to be analyzed theoretically in order to bring out the influence of fluid properties and film dynamics. The gas flux also depends on the size of the small bubbles; this problem has not yet been investigated in slug flow.
3. The basic idea consisting of decomposing the flow into a separated flow region and a bubbly flow region works reasonably well, even though each flow is assumed to be fully developed. It does represent, however, a serious limitation that needs to be overcome by adopting a more sophisticated formulation including flow development.
4. Last but not least, the physics of slug formation and long bubble coalescence requires much effort in the years to come. This is the key

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issue of slug flow, which provokes fundamental and highly current questions about its chaotic nature.

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