About Controlling Fee Rates and P&L for Lightning Network Routing Nodes

Work in Progress: This peace of work should be interpreted as thoughts by the author only. It probably still contains minor and possibly even major errors at this stage.

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List of Contents

1. Summary

An approach for a more quantitative fee modelling for Lightning Network Routing Nodes is presented which considers inbound and outbound fee rates as well. A great basic assumption of the approach is, that a competitive fee setting is good for the network and that a node can incentivize inflows and outflows with their fee settings. If a node is using its fees as opportunity costs for the calculation of the fee limits, the fee settings influences also the rebalancing payments. The incentivization can only work, if the remote side has healthy fee settings too, at least in a certain range. If a node is working with high fee walls independent of the liquidity state of the channel and there are is no usage by the sender then there is basically the question whether the channel is keeping it open.

The basic dynamic of the proposed model is very simple, lower the outbound fees and raising the inbound fees if liquidity is local and the other way around if the liquidity is remote. But we are not modelling the fee rates directly, we model to components, called margin and spread, and derive

the fee rates from this components. Moreover introduce a five parameter model for the spread using exponential moving averages and for the margin a model having two parameters using mean reversion. The automatic determination of an optimal parameterization would be the subject of future research. However, it is theoretically already possible to use the model with manual parameterization, provided that the dynamics of the parameters are understood.

About the structure of the document

At first in Chapter 2 we show how we could introduce negative forward fees which are compatible with #bLIP18 using the inbound fees. We show, when these are arbitrage free, that means an attacker cannot gain money by sending through a node because of the negative fees. Together with the findings from chapter 3, we can conclude that the existence of a positive margin is an important condition for this.

In Chapter 3 we show how to model margin rates and spread rates, and in a second step we compose the fee rates using this information. This allows us also a better explain of the actual P&L, because we can view on forwards as liquidity trades buying liquidity on the incoming, selling liquidity on the outgoing channel and earning a margin. We show mathematically the decomposition for forwards fees and for the rebalancing costs in two views, one which separates the margin of the source channel and one which separates the margin of the target channel. This could allow node runners a more detailed explanation of its profit & losses on channel level.

In Chapter 4 and 5 we present an approach of modelling the spread rates and margin rates. The spread rates are adjusting using a kind of PID Controller, which increases the spread when the channel is depleted. Actually the spread rate controller uses an exponential moving for the integral, and an exponential decay for the integral. This allows to express the controller as a continuous function of the current depletion and five parameters. The margin rate instead is controlled by a mean reverting controller which converges smoothly to a given mean reversion level by given speed.

We assume that all parameters for the controllers are externally given. But for an automated node management we need also processes for the calibration of the parameters. Some rough ideas are represented for further automatization are presented in section 6. The overall goal is to build a model which maximize the returns of the node.

1.1. Comparison of the Dynamic Lndg Auto-Fees

We don't want to give a complete overview over the existing fee tools. But we can divide the existing algos in three main categories.

1. **static**: The fee rates are constant and until the node operator changes the configuration.
2. **semi-dynamic**: The fee rates are a function of the current state if a channel or all channels with one peer, e.g. a function of the current local balances. The same at different times implies the same fee rates. An example for this are proportional or other balance dependent configurations with charge-lnd.
3. **dynamic**: Dynamic algos are not only a function of the current channel state, they take also the historic evolvement into account. The main difference to the semi-dynamic is, that the same state to a different time can lead to different fee rates.

One example of dynamic fee algorithm are the Auto-Fees implemented in Lndg, which we'd like to compare with the dynamic of our proposed spread rate controller. We want to focus on the part where Lndg decreases the fees if the local liquidity goes above a given `ExcessLimit` (Code) and discuss two differences:

2. About Arbitrage Freeness with Negative Forward Fees

We'd like to introduce the whole theory in a world where negative forward where negative forward fees are possible, introducing them as inbound fee which overpays the outbound fees. In the review process of #6703, there was a discussion about allowing overall negative forward fees. At that point, it seemed better to prevent node runners from losing money by not allowing this feature.

We want to introduce a concept of arbitrage freeness here which prevents that an attacker can extract money from a node with just sending sats back and forth, because of bad relationships between inbound and outbound fees.

Let c be a channel policy, $r_{c,i}$, $r_{c,o}$ the inbound and outbound fee rates in decimals (e.g., 1000ppm = 0.001), $b_{c,i}$, $b_{c,o}$ the base fees in msat. All information are represented as a vector $c = (r_{c,o}, b_{c,o}, r_{c,i}, b_{c,oi})^T$ and $c \in C$ with the policy space $C = \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \times \mathbb{R} \times \mathbb{R}$.

The function

$$f_{c,o}(x) := (1 + r_{c,o}) \cdot x + b_{c,o}$$

calculates the amount to receive from an amount to send using the outbound fees of the channel c. The function

$$f_{c,i}(x) := (1 + r_{c,i}) \cdot x + b_{c,i}$$

calculates the amount to send from an amount to receive using the inbound fees of the channel c. This function is only valid with negative forward fees. At the moment the inbound fees are floored by the next outbound fee. i.e. the function is

$$\tilde{f}_{c,i}(x) := x + \max(r_{c,i} \cdot x + b_{c,i}; -k) = \max((1 + r_{c,i}) \cdot x + b_{c,i}; x - k) = \max(f_{c,i}(x), x - k)$$

with the previous outbound fee k.

With negative forward fees it can happen, that $f_{c,i}(x) < 0$ for small amounts of x if there is a negative inbound base fee $b_{c,i}$. Economically that would mean, that the routing node is able to pay an amount to the sender of the payment. We will assume that f becomes to

$$\bar{f}_{c,i}(x) = \max((1 + r_{c,i}) \cdot x + b_{c,i}, 0) = \max(f_{c,i}(x), 0)$$

for upcoming proofs. Moreover we want to assume, that there is no upper limit for the amount x, i.e. no constraints by channel capacity or max htlc amounts.

For each possible representation of f we can see that $f_{c,i}(x + \Delta) \geq f_{c,i}(x)$ for $x, \Delta \geq 0$, because $f_{c,i}(x + \Delta) = f_{c,i}(x) + \Delta(1 - r_{c,i})$ and the last summand is positive because we can assume $r_{c,i} \geq -1 = -100\%$.

2.1. Arbitrage Freeness

To introduce the concept of **arbitrage freeness** we want to analyze when the following attack doesn't lead to a profit for the attacker. We assume that the attacker controls all nodes of the channel's parties, hence they can send sats from one node to another without incurring additional hop costs. The attacker chooses an arbitrary sequence of channels with policies $c_1, \ldots, c_n$ with $n \geq 2$ and $c_1 = c_n$. They send an amount x_1 from c_1 to c_2, so they receive x_2 in c_2. They proceed by sending x_2 from c_2 to c_3, receiving x_3, etc. Our goal is to ensure that they don't receive more than they pay, i.e. $x_1 \geq x_n$.

We say a channel policy c is **weak arbitrage free** if an attacker cannot gain money with a self payment in the same channel, i.e it is not possible sending x_1 in the channel and receiving x_2 with $x_2 > x_1$.

Lemma: Under the given assumptions, a channel policy c is weak arbitrage free if and only if

$$f_{c,io}(x) := (f_{c,i} \circ f_{c,o})(x) \geq x$$

for all x. The condition is full filled if both conditions hold

- $r_{c,o} + r_{c,i} + r_{c,o} \cdot r_{c,i} \geq 0$
- $b_{c,o} + b_{c,i} + b_{c,o} \cdot r_{c,i} \geq 0$

Proof: We set $c = c_1 = c_2$, i.e. an attacker makes a circular payment in the same channel, sending x_1 in the channel and receiving x_2 with $x_1 = f_{c,i}(f_{c,o}(x_2)) = f_{c,io}(x_2)$. Hence $x_1 = f_{c,io}(x_2) \geq x_2$ is necessary and sufficient for each x_2.

Moreover we have

$$f_{c,io}(x) = x + (r_{c,o} + r_{c,i} + r_{c,o} \cdot r_{c,i}) \cdot x + b_{c,o} + b_{c,i} + b_{c,o} \cdot r_{c,i}$$

and if both inequalities hold, the value of the function is $\geq x$ for all x. And if we use $\tilde{f}_i$ or $\bar{f}_i$ instead of f_i the value of the function even only becomes higher. $\square$

At this point we cannot say whether an attacker can gain money if the channel policy c is used in combination with other policy. That's why we want to introduce the concept of **strong arbitrage freeness (SAF)** which is a property of a subset $\tilde{C} \subseteq C$. We say $\tilde{C}$ is SAF if for any sequence $c_1, \ldots, c_{n-1}$ with $c_i \in \tilde{C}$ and any amount x

$$((f_{c_1,i} \circ f_{c_2,o}) \circ \cdots \circ (f_{c_{n-2},i} \circ f_{c_{n-1},o}) \circ (f_{c_{n-1},i} \circ f_{c_1,o}))(x) \geq x$$

In other words: the attacker cannot gain money by sending an arbitrary amount along an arbitrary channel sequence.

If a set is SAF, then every subset is it too, because every policy sequence of elements of the subset is also a sequence of elements of the upper set. This leads us to the question if we can define a maximum set C_A which is SAF, i.e. there is no other set $\tilde{C}$ which is SAF and $C_A \subset \tilde{C}$

For the next lemma we define another composition $f_{c,oi} := f_{c,o} \circ f_{c,i}$

Lemma: $C_A := \{c \in C \mid \forall x \geq 0,\ f_{c,oi}(x) \geq x \ \wedge \ f_{c,io}(x) \geq x\}$ is the maximum set which is SAF.

Proof: The first part is taking an arbitrary set $\tilde{C}$ being SAF and showing that every channel policy $c \in \tilde{C}$ is also in C_A. As second part we show that the whole constructed set C_A is still SAF.

Part 1: First step is to realize that the zero channel policy $c_0 = (0, 0, 0, 0)^T$ can be used to construct a SAF by $C_0 = \{c_0\}$, because by definitions we have $f_{c_0,io}(x) = f_{c_0,oi}(x) = x$ and hence each composition leads to the identity which satisfies the SAF condition for each x.

Second if we have a SAF set $\tilde{C}$ which doesn't include c_0, then $\tilde{C} \cup \{c_0\}$ is also SAF. This can be shown by taking an arbitrary policy sequence out of $\tilde{C}$ and if we enrich this sequence with c_0, then the SAF definition still holds, because the identity function don't change the forward fees.

Third let $c \in \tilde{C}$ a policy of an arbitrary SAF set. By definition of SAF, c alone has to satisfy $f_{c,io}(x) \geq x$ for all x. Moreover $C_c := \{c_0, c\}$ is SAF by the rationale mentioned previously. But then we can follow from the SAF definition that we are also safe against an attacker sending in an amount in the channel with policy c_0 to the channel with policy c and back. Because of

$$((f_{c_0,i} \circ f_{c,o}) \circ (f_{c,i} \circ f_{c_0,o}))(x) = (f_{c,o} \circ f_{c,i})(x) = f_{c,oi}(x)$$

we have shown that $f_{c,oi}(x) \geq x$ for all x, and hence $c \in C_A$.

Part 2: Let $c_1, \ldots, c_n$ be a sequence with $c_i \in C_A$, we have to show that for all x

$$x_1 = ((f_{c_1,i} \circ f_{c_2,o}) \circ \cdots \circ (f_{c_{n-2},i} \circ f_{c_{n-1},o}) \circ (f_{c_{n-1},i} \circ f_{c_1,o}))(x) \geq x$$

hold.

Choosing an arbitrary amount $x_n \geq 0$ and using the fact that composition is associative, we have:

$$x_1 = (f_{c_1,i} \circ (f_{c_2,o} \circ f_{c_2,i}) \circ \cdots \circ (f_{c_{n-1},o} \circ f_{c_{n-1},i}) \circ f_{c_n,o})(x_n)$$

This leads, by our definitions, to:

$$x_1 = (f_{c_1,i} \circ (f_{c_2,oi} \circ \cdots \circ f_{c_{n-1},oi}) \circ f_{c_1,o})(x_n)$$

If we assume that $f_{c,oi}(x) \geq x$ for each channel c and every possible amount x, then the inequality also holds for any composition of such functions in sequence. As consequence there exists a $\Delta \geq 0$ such that:

$$(f_{c_2,oi} \circ \cdots \circ f_{c_{n-1},oi})(f_{c_1,o}(x_n)) = f_{c_1,o}(x_n) + \Delta$$

As already seen, for any $x, \Delta \geq 0$ we have $f_{c,i}(x + \Delta) \geq f_{c,i}(x)$ and hence:

$$x_1 = f_{c_1,i}(f_{c_1,o}(x_n) + \Delta) \geq f_{c_1,i}(f_{c_1,o}(x_n)) = f_{c_1,io}(x_n)$$

Because of C_A definition we have $f_{c,io}(x) \geq x$ for each channel c and any amount x, and hence $x_1 \geq f_{c_1,io}(x_n) \geq x_n$, proofing that C_A is SAF. □

Corollary: The set of channel policies which where the parameters satisfy the following restrictions is denoted by C_R:

1. $r_{c,o} + r_{c,i} + r_{c,o} \cdot r_{c,i} \geq 0$
2. $b_{c,o} + b_{c,i} + b_{c,o} \cdot r_{c,i} \geq 0$
3. $b_{c,o} + b_{c,i} + b_{c,i} \cdot r_{c,o} \geq 0$

C_R is SAF.

Proof: Independently whether we which function we use for f_i: $\tilde{f}_i$, $\bar{f}_i$ or f_i itself we can say that

$$f_{c,io}(x) \geq x + (r_{c,o} + r_{c,i} + r_{c,o} \cdot r_{c,i}) \cdot x + b_{c,o} + b_{c,i} + b_{c,o} \cdot r_{c,i}$$

$$f_{c,oi}(x) \geq x + (r_{c,o} + r_{c,i} + r_{c,o} \cdot r_{c,i}) \cdot x + b_{c,o} + b_{c,i} + b_{c,i} \cdot r_{c,o}$$

And with the given assumptions both right hand sides are ≥ 0, which implies the SAF.

Remark: We can also say that $C_R \subseteq C_A$, e.g. if we only work with policies without base fees and without flooring the fees at outbound fees.

3. Modelling Outbound and Inbound Fee Rates with Margin and Liquidity Spread

3.1. The Margin Spread Model

Assume that for a specific channel c there are two given variables: a non-negative margin rate m_c and a liquidity spread rate s_c, which can both be real numbers. We like to set the outbound and inbound fee rates with this known information.

If $m_c + s_c \geq 0$, we set

$$\begin{aligned} r_{c,o} &= \frac{1}{1-s_c} \cdot (m_A + s_c) \\ r_{c,i} &= -s_c \end{aligned}$$

A continuous prolongation of these functions for the case $m_c + s_c < 0$ is

$$\begin{aligned} r_{c,o} &= 0 \\ r_{c,i} &= m_c \end{aligned}$$

We can also consolidate the two blocks into one, by introducing $\tilde{s_c} := s_c$ if $s_c + m_c \geq 0$ and $\tilde{s_c} := -m_c$ if $s_c + m_c < 0$, and set

$$\begin{aligned} r_{c,o} &= \frac{1}{1-\tilde{s_c}} \cdot (\tilde{s_c} + m_c) \\ r_{c,i} &= -\tilde{s_c} \end{aligned}$$

This makes the following proofs easier because we only have to care about one case.

3.2. Rationale for Margin and Spread

3.2.1. Margin in Circular Case But why does it make sense to call the two variables margin and spread? Let's have a look at a circular payment with channel policy c first. Assuming there are no restrictions for the incoming fees, the net earnings of such a payment can be calculated by

$$E(x) = f_{c,oi}(x) - x = ((1 + r_{c,o}) \cdot (1 + r_{c,i}) - 1) \cdot x + B(x)$$

with a residual $B(x)$ driven by the base fees, which we want to ignore for now. With

$$(1 + r_{c,o}) \cdot (1 + r_{c,i}) - 1 = \left(1 + \frac{1}{1-\tilde{s_c}} \cdot (m_c + \tilde{s_c})\right) \cdot (1 - \tilde{s_c}) - 1 = m_c$$

Hence, a node runner earns a margin amount that scales by the margin rate m_c. Ignoring the $B(x)$, the spread has no influence on the earnings.

Moreover,

$$m_c = (1 + r_{c,o}) \cdot (1 + r_{c,i}) - 1 = r_{c,o} + r_{c,i} + r_{c,o} \cdot r_{c,i}$$

That's why we can replace the first inequality for checking $c \in C_R$ by the margin rate:

Corollary: Given a channel policy c, we have $c \in C_R$ if $m_c \geq 0$ and the second and third inequality are satisfied.

3.2.2. Margin in Two Party Case Now, look at the case where two channels with policies c_1 and c_2 are involved. At first, we have forwards from c_1 to c_2 and afterwards in the other direction. Also, assuming there are no restrictions of the incoming fees, the earnings are given by

$$E(x) = (f_{c_1,i} \circ f_{c_2,o} \circ f_{c_2,i} \circ f_{c_1,o})(x) - x = ((1 + r_{c_1,i}) \cdot (1 + r_{c_2,o}) \cdot (1 + r_{c_2,i}) \cdot (1 + r_{c_1,o}) - 1) \cdot x + B(x)$$

and after some algebra, we have

$$E(x) = ((1 + m_{c_1}) \cdot (1 + m_{c_2}) - 1) \cdot x + B(x)$$

What we see here is a compounding of the margin rates, and that the spread has no influence on the earnings if we ignore the term $B(x)$.

3.2.3. Role of the Spread As we have seen, the spread has no influence on the earnings, but this is only half the truth because the spread basically determines our price for the liquidity, either by rebalancing or by incoming forwards. A lower spread makes it harder to incentivize inbound liquidity, because the inbound fee rate is also lower in this case. Moreover, if one uses the outbound fee rates as opportunity costs for rebalancing, a lower spread implies a lower outbound fee rate and hence also a lower rebalancing budget.

On the other hand, a higher spread makes it easier to acquire inbound liquidity, but the outbound fee rates are also higher, which can restrict outgoing forwards in the channel.

We can conclude that the spread rate is more of a liquidity price, which influences the forwarding volume of a channel, and the forwarding volume influences the margin we can earn.

Also, the volatility of the spread is important. If the volatility is higher, i.e., one increases the spread fast if the liquidity is remote and decreases the spread fast if the liquidity is local, then it can happen that one buys liquidity at prices being not able to sell. This influences the total profits, because now one has a negative impairment effect driven by the spread rates. Per se, volatility isn't a bad thing, because it could also lead to higher volume on the node. The question is more about what the optimal ratio between volatility and total profits is.

3.3. Practical Setting

The model will help us later to understand what the origins of the P&L of a routing node are. And if we are able to model the margin and the liquidity, we can set the fee rates immediately. But in practice, we cannot set the fee rates to real numbers, and maybe we will not broadcast each changed ppm to the network. That's why we will introduce residuals $\epsilon_{c,o}$ and $\epsilon_{c,i}$, which are the results of rounding and other practical constraints. Moreover, in practice, the factor $\frac{1}{1-\tilde{s_c}} \sim 1$. We will set the factor to 1 and put the difference in $\epsilon_{c,o}$.

Summarizing:

$$\begin{aligned} r_{c,o} &= \tilde{s_c} + m_A + \epsilon_{c,o} \\ r_{c,i} &= -\tilde{s_c} + \epsilon_{c,i} \end{aligned}$$

3.4. Explain the P&L of a Routing Node

3.4.1 P&L Explain for Forwards We denote the fee a sender wants to pay us with f for a forward incoming in the channel A with Alice and outgoing in the channel B with Bob. $f(x) = x \cdot ((1 + r_{B,o}) \cdot (1 + r_{A,i}) - 1) + b_{B,o} \cdot (1 + r_{A,i}) + b_{A,i}$ is a function calculating the theoretical fee if there is no floor of at 0 by a given outgoing amount of x. Our goal is to build two views, one which separates the margin amount of channel A, and one which extracts the margin amount of channel B.

Forwards: Separation of the Source Margin Moreover, there is the identity

$$\begin{aligned} & x \cdot ((1 + r_{B,o}) \cdot (1 + r_{A,i}) - 1) = x \cdot (r_{B,o} + r_{A,i} + r_{B,o} \cdot r_{A,i}) \\ =& x \cdot r_{B,o} + (x + f) \cdot r_{A,i} + x \cdot r_{B,o} \cdot r_{A,i} - f \cdot r_{A,i}. \end{aligned}$$

If we also consider that

$$\begin{aligned} r_{A,i} &= -\tilde{s_A} + \epsilon_{A,i} = -(\tilde{s_A} + m_A + \epsilon_{A,o}) + m_A + (\epsilon_{A,i} + \epsilon_{A,o}) \\ &= -r_{A,o} + m_A + (\epsilon_{A,i} + \epsilon_{A,o}) \end{aligned}$$

we can decompose our total fee into seven parts

$$\begin{aligned} f &= \sum_i f_i \\ f_1 &= f - f(x)^+ \\ f_2 &= f(x)^+ - f(x) \\ f_3 &= b_o \cdot (1 + r_i) + b_i \\ f_4 &= (x + f) \cdot m_A \\ f_5 &= -(x + f) \cdot r_{A,o} \\ f_6 &= x \cdot r_{B,o} \\ f_7 &= x \cdot r_{B,o} r_{A,i} - f \cdot r_{A,i} + (x + f) \cdot (\epsilon_{A,i} + \epsilon_{A,o}). \end{aligned}$$

Let's explain the several components:

1. f_1 is an overpayment between the actual fee and the theoretical fee based on the formulas of bLIP18 considering a floor of the fee at 0, which is the current lnd implementation. Such an overpayment can happen if the payer is not aware of lower fees or cannot use the inbound discounts.
2. f_2 is an unexpected margin, which usually occurs when $(1+r_{B,o})\cdot(1+r_{A,i}) < 1$, i.e., the outbound fee is lower than the inbound discount. This is the case if we have a forward from a node which is more a sink to a node which is more a source. That's why this type of earning is not a usual one and should be separated from regular margins. Moreover, if we had negative forward fees, we could also aggregate f_1 and f_2 into one overpayment.
3. f_3 is the base margin, the net amount fees from the specified base fees.
4. f_4 is the margin amount using Alice's margin rate. It scales with the received incoming forwarding amount.
5. While all components up to f_4 had an earnings character, f_5 and f_6 have more of a character of purchasing costs and selling proceeds of a fee potential. We have received liquidity of an amount of $x + f$ in the channel of Alice. If we are able to sell it to another source, we can earn $(x + f) \cdot r_{A,o}$, and if we assume that the rate of $r_{A,o}$ is a market-conform rate, then f_5 represents the purchasing costs for the fee potential.
6. On the other hand, we received f_6 for selling the liquidity to Bob.
7. f_7 is a residual P&L. It consists of many second-order terms.

The concept of 'fee potential' was introduced by DerEwige. It is basically the product of the fee rate and its potential fees to earn. According to it, we can also define a spread potential for spread rates and a margin potential for margin rates.

Because of the purchasing and selling fee potentials, we have to think about routing more as liquidity trading. But f_5 and f_6 do not cause P&L directly. P&L is caused by the other components. There is only a P&L effect from the fee potential if the fee rates changed between purchasing and selling the potential.

Forwards: Separation of the Target Margin But we can also derive another decomposition that separates the margin of the target channel of Bob. With

$$
\begin{aligned}
r_{B,o} &= (\tilde{s_B} + \epsilon_{B,o}) + m_B \\
r_{A,i} &= -(\tilde{s_A} + \epsilon_{A,o}) + (\epsilon_{A,i} + \epsilon_{A,o})
\end{aligned}
$$

we decompose f into

$$
\begin{aligned}
f &= \sum_i f_i \\
f_1 &= f - f(x)^+ \\
f_2 &= f(x)^+ - f(x) \\
f_3 &= b_o \cdot (1 + r_i) + b_i \\
f_4 &= x \cdot m_B \\
f_5 &= -(x + f) \cdot (\tilde{s_A} + \epsilon_{A,o}) \\
f_6 &= x \cdot (\tilde{s_B} + \epsilon_{B,o}) \\
f_7 &= x \cdot r_{B,o} r_{A,i} - f \cdot r_{A,i} + (x + f) \cdot (\epsilon_{A,i} + \epsilon_{A,o}).
\end{aligned}
$$

f_1, f_2, f_3, and f_7 haven't changed. But now:

4. f_4 is the margin amount of Bob, which scales with the outgoing amount x.
5. f_5 and f_6: Now the liquidity trade isn't about the fee potential. It is only about the spread potential (and the epsilons). But forwarding can still be interpreted as a trade of liquidity.

Now we have two decompositions, one extracting the margin of the incoming channel and one extracting the margin of the outgoing channel. Then, any convex combination is a decomposition, e.g., choosing 50% of each.

3.4.2. P&L Explain for Rebalancing Costs In the context of rebalancing costs, we also want to decompose the rebalancing costs in two views.

Our rebalancing is from our source channel A with Alice to channel B with Bob, and it costs an amount of $c < 0$ in sats. Of course, in the context of negative forward fees in the future, we want to take into account the case of $c > 0$, e.g., if one rebalances from a sink to a source. Effectively, rebalancing for a routing node is also a liquidity trade, better described as a liquidity arbitrage trade. It involves selling liquidity in channel A and buying liquidity in channel B.

Rebalancing: Separation of the Benefit in the Source Margin

$$
\begin{aligned}
c &= \sum_i c_i \\
c_1 &= (x + c) \cdot r_{A,o} \\
c_2 &= -x \cdot r_{B,o} \\
c_3 &= m_B \cdot (x + f_B) - m_A \cdot (x + f_A) \\
c_4 &= c - c_1 - c_2 - c_3
\end{aligned}
$$

1. c_1 is the virtual amount received for selling the fee potential in the channel with Alice.
2. c_2 is the virtual amount paid for buying the fee potential in the channel with Bob.
3. c_3 is the advantage we have in the margin for forwarding from Alice to Bob compared to a forward from Bob to Alice. f_A and f_B are the hypothetical fees for these forwards to calculate the correct incoming amount in the two channels. Let's say A and B have nearly identical fee rates such that $c_1 + c_2 = 0$. It is only beneficial to rebalance in this case if we earn more margin from Alice to Bob than from Bob to Alice, i.e., if $c > c_3$.
4. c_4 is the difference of c_1 to c_3 to the actual paid fee c. It is basically our rebalancing margin, and rebalancing is only beneficial if $c_4 > 0$.

Rebalancing: Separation of the Benefit in the Target Margin The next step is building a decomposition that separates the margin benefit of the target channel.

$$\begin{aligned}
c &= \sum_i c_i \\
c_1 &= (x + c) \cdot (\tilde{s_A} + \epsilon_{A,o}) = (x + c) \cdot r_{A,o} - (x + c) \cdot m_A \\
c_2 &= -x \cdot (\tilde{s_B} + \epsilon_{B,o}) = -x \cdot r_{B,o} + x \cdot m_B \\
c_3 &= m_A \cdot (x + f_A) - m_B \cdot (x + f_B) \\
&= m_B \cdot (x + f_A) - m_A \cdot (x + f_B) + 2 \cdot (m_A \cdot (x + f_A) - m_B \cdot (x + f_B)) \\
c_4 &= c - c_1 - c_2 - c_3
\end{aligned}$$

We can interpret the components as in the previous decomposition, with the difference that now only the spread potentials (and the epsilons) are traded and not the whole fee potentials.

Adjustments of the Potentials We have seen that in the decomposition of the earnings and the costs, implicit buys and sells of liquidity to the price of the fee rates or spread rates are included. But these rates have to be the current rates at the time of the trade. However, the rates can change, e.g., between buying the liquidity with a rebalancing payment and selling it with a forward. If a node had to lower their fees in between, there is an additional negative P&L component, which is not included in the above-mentioned components. These additional value adjustments could be calculated with an additional process that revalues the fee potentials at defined time intervals.

1. For time T_{n-1}, we know the potential of the fees, spreads, or margins.
2. We determine the current potentials at time T_n by multiplying the current balance with the current fees, spreads, or margins.
3. We determine the net balance of bought liquidity and sold liquidity in between for fees, spreads, or margins.
4. Now we can compare the sum of 1. and 3. with 2. If the sum is lower, then we have a negative value adjustment, which is effectively a loss for the node. If it is higher, then there is an additional gain.

3.5. Margin Spread Model with #bLIP19 Inbound Fees

There is a different proposal for inbound fees with #bLIP19. The main idea is that a node communicates the inbound fee to the peer, and it increments its outbound fee.

> MUST ensure it's next `channel_update` message for the corresponding channel has its `fee_proportional_millionths` and `fee_base_msat` fields incremented by the `inbound_forwarding_fee_proportional_millionths` and `inbound_forwarding_fee_base_msat` fields, respectively. Fields which fall below zero MUST be set to zero.

The floor at zero would make it impossible to use the proposed margin spread model with such inbound fees because the inbound discounts would be limited by the outbound fees of the peer.

Hence, the proposed margin spread model could not fully be used with #bLIP19 inbound fees.

4. Modelling the Spread Rate with Exponential Moving Averages

4.1. Basics

The model for the spread rate is based on the idea of PID (Proportional-Integral- Derivative) controllers. PID controllers utilize a measured process variable, compare it to a target value, and calculate an error function, denoted as $e(t)$. They use a linear function to adjust the control variable based on the error, its integral over time, and its derivative, aiming to minimize the error in subsequent iterations.

We are not using classical integrals and derivatives. Instead, an exponential weighted moving average with a smoothing parameter (α_i) defines the implicit length of the error history. Additionally, an exponential decay with a parameter (α_d) is applied to the error delta as a derivative component. You can also specify a drift for adjustments that scale over time but are not influenced by the error.

For calculating the error, the remote balance is compared to a target value, typically derived from the average liquidity ratio of all channels. The difference between the observed remote balance and the target is mapped to an error (e) in the range [-0.5, 0.5] using linear interpolation. When the remote balance equals the target, e is set to 0.

4.2. Modelling the Dynamic of the Spread Rate

Let T be the current time, and T_0 represent the oldest observed historic timestamp.

Our approach is based on control theory, i.e., we have a system with a time- dependent input function, our error function $e(t)$. The system returns a time- dependent output $y(t)$, which is the marginal increase of the spread rate. Hence, we can model the spread rate by the following differential equation:

$$ds(t) = y(t)dt$$

This equation leads to:

$$s(T_n) = s(T_{n-1}) + \int_{T_{n-1}}^{T_n} y(t)dt$$

The system output $y(t)$ itself is a linear combination of several parts:

$$y(t) = K_p \cdot e(t) + K_i \cdot E_{\alpha_i}(t) + K_d \cdot D_{\alpha_d}(t)$$

The functions E_α and D_α are defined later. Our goal is that they have the following properties: while the first summand scales with the current value of the error function, the second summand should scale with an exponential moving average $E_{\alpha_i}(t)$ over a longer time horizon. α_i serves as a smoothing parameter for controlling the implicit length of the history. The third summand should scale with the recent changes in the error function. But we don't want to apply the whole changes immediately. Instead, we want to decay the changes smoothly with an exponential function $D_{\alpha_d}(t)$ parametrized by a smoothing parameter α_d for the implicit decay period.

Overall, the model has five parameters that need to be calibrated. For the next sections, we assume the parameters as externally given.

Our goal is now to find a recursive representation of the following integral, which allows us to update the controller incrementally:

$$\int_{T_{n-1}}^{T_n} y(t)dt = K_p \int_{T_{n-1}}^{T_n} e(t)dt + K_i \int_{T_{n-1}}^{T_n} E_{\alpha_i}(t)dt + K_d \int_{T_{n-1}}^{T_n} D_{\alpha_d}(t)dt$$

Now, let's delve into the different parts. We will assume that the error function $e(t)$ is piecewise linear over $t \in]T_{n-1}, T_n]$, i.e.,

$$e(t) = \beta_1 \cdot (t - T_{n-1}) + \beta_0,\ \beta_1 = \frac{e(T_n) - e(T_{n-1})}{T_n - T_{n-1}},\ \beta_0 = e(T_{n-1})$$

4.2.1. Proportional Part This part is relatively straightforward:

$$\begin{aligned} K_p \int_{T_{n-1}}^{T_n} e(t)dt &= K_p \cdot (T_n - T_{n-1}) \cdot \left(\frac{m}{2} \cdot (T_n - T_{n-1}) + \beta_0\right) \\ &= K_p \cdot (T_n - T_{n-1}) \cdot \frac{e(T_n) + e(T_{n-1})}{2} \end{aligned}$$

The outcome of the controller scales with the average error of the time period.

4.2.2. Exponential Decay of a Function Before we go into the concrete definition of our integral and derivative part, some general remarks about the decay of functions: given an integrable input function $x(t)$ (which will later be the error $e(t)$ or the derivative of it) and a parameter α, we define a weight function $W_{x,\alpha}(t)$ by

$$W_{x,\alpha}(t) := \int_{T_o}^{t} x(\tau) \cdot \alpha \cdot \exp\left(\alpha(\tau - t)\right) d\tau$$

and with $h_\alpha(t) = \alpha \exp(-\alpha t)$, we have

$$W_{x,\alpha}(t) := \int_{T_o}^{t} x(\tau) \cdot h_\alpha(t - \tau) d\tau$$

which is a convolution integral over the local area $[T_0; t]$. This fact is not relevant now but might be interesting for future analysis of the controller.

Now, given $T_{n-1} > T_0$, we want to express $W_{h,\alpha}(t)$ for $t > T_{n-1}$ recursively using $W_{h,\alpha}(T_{n-1})$. It becomes:

$$\begin{aligned} W_{x,\alpha}(t) &= \int_{T_{n-1}}^{t} x(\tau) \cdot \alpha \cdot \exp\left(\alpha(\tau - t)\right) d\tau + \exp(\alpha(T_{n-1} - t)) \int_{T_o}^{T_{n-1}} x(\tau) \cdot \alpha \cdot \exp\left(\alpha(\tau - T_{n-1})\right) d\tau \\ &= \int_{T_{n-1}}^{t} x(\tau) \cdot \alpha \cdot \exp\left(\alpha(\tau - t)\right) d\tau + \exp(\alpha(T_{n-1} - t)) \cdot W_{x,\alpha}(T_{n-1}) \end{aligned}$$

4.2.3. Integral Part

For calculating the integral part, we apply our piecewise linear-defined error function $e(t)$ to W_x as $x(t)$.

Our weight function will lead to an exponential weighted moving average, which we define by

$$E_\alpha(t) := W_{e,\alpha}(t)$$

and with the assumption for $e(t)$, we have to solve

$$E_\alpha(t) = \int_{T_{n-1}}^{t} \alpha \exp\left(\alpha(\tau - t)\right) \cdot (\beta_1 \cdot \tau + \beta_0) \cdot d\tau + \exp\left(\alpha(T_{n-1} - t)\right) \cdot E_\alpha(T_{n-1})$$

and the solution is

$$E_\alpha(t) = e(t) - \frac{\beta_1}{\alpha} + \exp\left(\alpha(T_{n-1} - t)\right) \cdot \left(E_\alpha(T_{n-1}) + \frac{\beta_1}{\alpha} - \beta_0\right)$$

Now we can set the integral part of the controller as:

$$\begin{aligned} &K_i \int_{T_{n-1}}^{T_n} E_\alpha(t)dt \\ =&K_i \left((T_n - T_{n-1}) \cdot \left(\frac{e(T_n) + e(T_{n-1})}{2} - \frac{\beta_1}{\alpha}\right) + \frac{1}{\alpha}(1 - \exp(\alpha(T_{n-1} - T_n))) \cdot \left(E_\alpha(T_{n-1}) + \frac{\beta_1}{\alpha} - \beta_0\right)\right) \end{aligned}$$

This allows us to update the controller recursively with only the knowledge of the value of $E_\alpha(T_{n-1})$.

4.2.4. Derivative Part

For the derivative part, we are using the partial derivative $\frac{\partial e}{\partial \tau}$ for $x(t)$. Hence, we have to solve

$$D_\alpha(t) := W_{\frac{\partial e}{\partial \tau},\alpha}(t) = \int_{T_{n-1}}^{t} \alpha \exp\left(\alpha(\tau - t)\right) \cdot \beta_1 \cdot d\tau + \exp\left(\alpha(T_{n-1} - t)\right) \cdot D_\alpha(T_{n-1})$$

and the solution is

$$D_\alpha(t) = \beta_1 + \exp\left(\alpha(T_{n-1} - t)\right) \cdot (D_\alpha(T_{n-1}) - \beta_1)$$

Now we can set the derivative part of the controller as:

$$\begin{aligned} &K_d \int_{T_{n-1}}^{T_n} D_\alpha(t)dt \\ =&K_d \left(e(T_n) - e(T_{n-1}) + \frac{1}{\alpha}(1 - \exp(\alpha(T_{n-1} - T_n))) \cdot (D_\alpha(T_{n-1}) - \beta_1)\right) \end{aligned}$$

This also allows us to update the controller recursively.

4.3. Behavior after a Simple Impulse

To show the dynamics of the spread controller, we want to investigate its long- term behavior after a simple impulse. At $T_0 = 0$, we set $s(T_0) = 0$ and $e(T_0) = e_1$, and for $t \geq T_1 = 1$, we observe $e(t) = e_1$. We want to determine an analytical function for the spread $s(T)$ for $T \geq T_1$ based on our derived formulas. For $t \in]0; 1]$, we have $\beta_1 = e_1 - e_0$ and $\beta_0 = e_0$; for $t > 1$, we have $\beta_1 = 0$ and $\beta_0 = e_0$. The other five model parameters are chosen arbitrarily.

First, we want to show how $E_\alpha(t)$ evolves if there is no change in the error from T_1. For $t > T_1$, we get:

$$E_\alpha(t) = e(t) - \frac{\beta_1}{\alpha} + \exp\left(\alpha(T_1 - t)\right) \cdot (E_\alpha(T_1) - e_1)$$

Because the second summand goes to zero for $t \to \infty$, we have $E_\alpha(t)$ converging to e_1, and hence the contribution of the integral part for time intervals with large values of T_{n-1} and T_n is nearly $K_i \cdot (T_n - T_{n-1}) \cdot e_1$.

If we apply the same thoughts to the derivative part, we see that $D_\alpha(t)$ converges to $\beta_1 = 0$, and hence the long-term contribution of this part also converges to 0. However, it is more interesting to show that the overall contribution of this part converges to $K_d \cdot (e_1 - e_0)$. We set $D_\alpha(T_0) = 0$ because there was no impulse before T_0. We get:

$$D_\alpha(T_1) = e_1 - e_0 - \beta_1 \cdot \exp(-\alpha) = \beta_1 \cdot (1 - \exp(-\alpha))$$

$$\int_{T_0}^{T_1} D_\alpha(t)dt = e_1 - e_0 - \frac{\beta_1}{\alpha} \cdot (1 - \exp(-\alpha))$$

and for arbitrary $T > T_1$:

$$\begin{aligned}\int_{T_1}^{T} D_\alpha(t)dt &= \frac{1}{\alpha} \cdot (1 - \exp(-\alpha(T - T_1))) \cdot D_\alpha(T_1)\\ &= \frac{1}{\alpha} \cdot (1 - \exp(-\alpha(T - 1))) \cdot \beta_1 \cdot (1 - \exp(-\alpha))\end{aligned}$$

Adding the two integrals leads to:

$$\begin{aligned}\int_{T_0}^{T} D_\alpha(t)dt &= e_1 - e_0 - \frac{\beta_1}{\alpha} \cdot (1 - \exp(-\alpha)) + \frac{1}{\alpha} \cdot (1 - \exp(-\alpha(T-1))) \cdot \beta_1 \cdot (1 - \exp(-\alpha))\\ &= e_1 - e_0 - \frac{\beta_1}{\alpha} \cdot (1 - \exp(-\alpha)) \cdot (-1 + (1 - \exp(-\alpha(T-1))))\\ &= e_1 - e_0 + \frac{\beta_1}{\alpha} \cdot (1 - \exp(-\alpha)) \cdot \exp(-\alpha(T-1))\end{aligned}$$

Because $\exp(-\alpha(T-1))$ converges to zero for large T, the integral converges to $e_1 - e_0$, and hence the overall contribution of the derivative part converges to $K_d \cdot (e_1 - e_0)$.

Moreover, the total spread rate adjustment converges to $K_\infty = e_1 \cdot (K_p + K_i)$, and because the bounded error function, the spread rate adjustments are bounded to values in $[-0.5 \cdot (K_p + K_i), 0.5 \cdot (K_p + K_i)]$.

4.4. Example for a Spread Rate Controller

We want to show how the spread rate controller evolves in the following scenario: The funds in our channel are completely local and haven't moved for a longer time. Hence $D_\alpha(t) = 0$ and $e(t) = E_\alpha(t) = 0.5$. Suddenly there is an impulse depleting the channel immediately. The error moves to a value of -0.5. For this scenario analysis, we use the following parameters:

$$K_p = 40,\ \alpha_i = 0.1,\ K_i = 80,\ \alpha_d = 0.9,\ K_d = 30$$

As a result, we get the following total adjustments of the spread rate. We interpret the time scale as days, but depending on the implementation, it could also be hours, weeks, or another time unit. The first 3 to 4 days, there is no significant increase in the spread rate. Up to 9 days, there is an increase of about 100 ppm. The next 100 ppm are raised up to day 12. By day 21, there is an increase of another 400 ppm. Hence, in average, it takes roughly 2 days for an increase of another 100 ppm, which is consistent with our observation that spread rate adjustments have to converge to $K_\infty = 0.5 \cdot (K_p + K_i) = 60$.

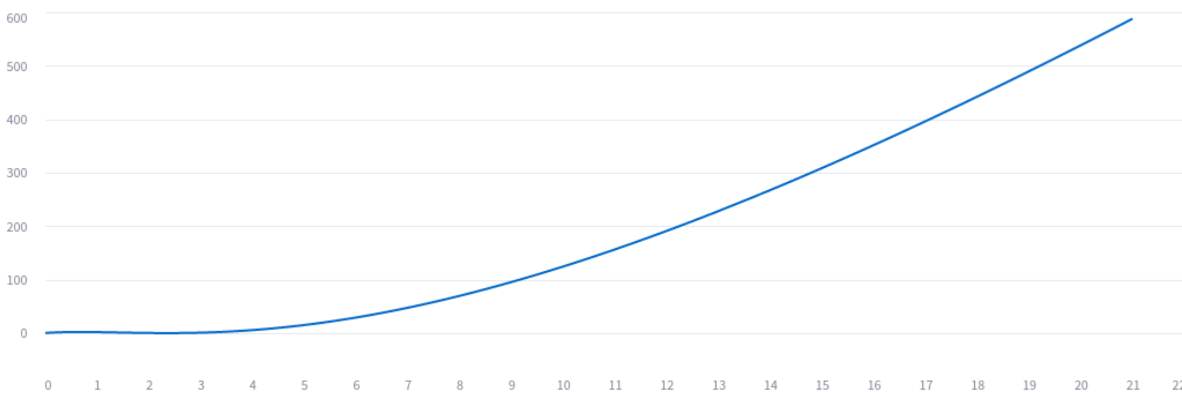
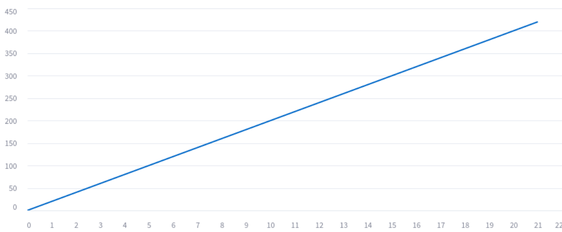


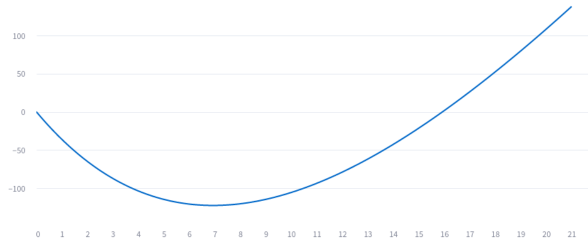
Figure 1: Aggregated Adjustments of the Spread Rate Controller

For a better understanding of the dynamics, we want to look at how the different parts of the controller evolve, i.e., we apply the parameters for one or two parts. The other parameters are set to zero. Obviously, the proportional part leads to a function with an increase of 20 ppm daily. The integral part decreases the spread rate at the beginning because our moving average $E_\alpha(t)$ is negative. As E rises, the contribution to the spread rate falls less and less until it begins to rise. This happens when E becomes positive. If we combine the proportional with the integral part, there is still a slight decrease in the spread rate at the beginning, with a minimum of around -30 ppm on day 3. But it makes no sense to decrease the fee rate after the channel has been depleted. Applying the derivative part fixes this. This part will contribute nearly 30 ppm to the controller on day 3. After day 3, there is almost no contribution anymore.

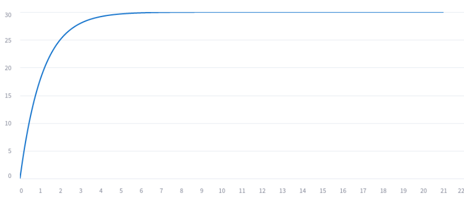
Proportional $K_p = 40$



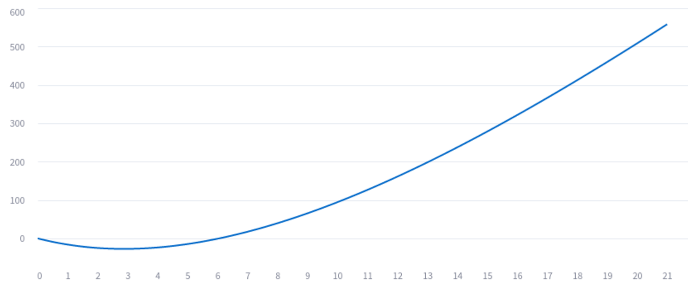
Integral $\alpha_i = 0.1,\ K_i = 80$



Derivative $\alpha_d = 0.9,\ K_d = 30$



Proportional & Integral $K_p = 40,\ \alpha_i = 0.1,\ K_i = 80$



4.4.1. Scaling the Example Interestingly, we can construct other functions for spread adjustments using the given parameters, where the curves have a similar shape. We want to have a function that is almost flat for around 15 days and converges to an adjustment rate of about 96 ppm per day.

The given parameters are flat for around 3 days. If we interpret one time unit as "per 5 days," then by dividing by 5, we get a new parameter set which leads to a flat curve until $t = 15$. The only parameter we keep the same is K_d because the 30 ppm only depends on the change and not on the time it occurs in.

$$K_p = 8,\ \alpha_i = 0.02,\ K_i = 16,\ \alpha_d = 0.18,\ K_d = 30$$

This controller converges to 12 ppm. If we scale the K values by a factor of 8, we get a controller converging to $K_\infty = 96$.

$$K_p = 64,\ \alpha_i = 0.02,\ K_i = 128,\ \alpha_d = 0.18,\ K_d = 240$$

This is the result of the controller. To be fair, there is a local maximum of the controller at around 15 ppm within the first 15 days. Before, it was around 2 ppm and practically irrelevant. With mathematical optimization techniques, it should be possible to make the curve much smoother.

4.5. Spread Rate Controller as a Linear Time-Invariant System

Due to the structure of the convolution integral, my research leads me to linear time-invariant systems (LTI). Our spread rate controller seems to be LTI because the output (the change of the controller) is linear depending on the input function (the error function), since the integrals are linear functions, and the derivative is linear as well. The output is also time-invariant, meaning if you apply the same error function to the system with some time delay, the change of the spread controller will return the same output with the exact time delay.

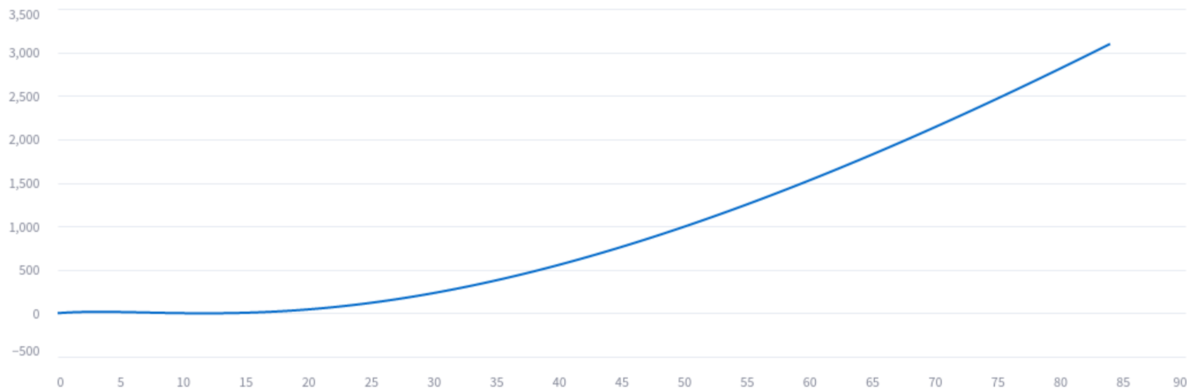


Figure 2: Scaled Spread Rate Controller

> The fundamental result in LTI system theory is that any LTI system can be characterized entirely by a single function called the system's impulse response.

This is also important for the calibration of the model. For example, if one calibrates the parameters in a way that the spread rate change is suitable for one given change of the error function, then one has also determined the spread rates for all other possible error functions.

4.6. Outlook: Scenario-Based Model Calibration of the Parameters

Calibration is the process of finding the optimal parameters for the model. One idea is to use a scenario-based calibration, i.e., the user or another algorithm suggests a dynamic for the controller in the depletion scenario we have investigated.

Only rough thoughts and ideas from here without deeper analysis

The dynamic itself could be described by three parameters: the adjustment rate K_∞ to which the controller converges, a time T_c up to which the spread adjustments are as small as possible, and an adjustment rate $K_{2\cdot T_c}$, which describes the marginal adjustment rate at time $2 \cdot T_c$. This redefinition makes the dynamic easier to understand, but we need to convert it into the actual five parameters of the controller. One way to find a parameter set with as small adjustments as possible at the beginning is to solve the following optimization problem with the given two constraints:

$$\text{argmin} \int_0^{T_c} \left(s(t) - s(0)\right)^2 dt$$

With the knowledge of the scaling properties of our model, it is possible to solve the problem with the parameters $K_\infty = 1$, $T_c = 1$, and $K_2 = \frac{K_{2\cdot T_c}}{K_\infty}$ first. Because of the parameters α, the objective function is not polynomial, and also the constraint fitting the K_2 is not polynomial.

Using a solver directly for the non-polynomial problem seems cumbersome. One alternative could be an alternating minimization:

1. Fix α_d and α_i.
2. Let K_p, K_i, and K_d be free. Now we have to minimize a quadratic function $f(x) = x^T Ax$ with two linear constraints. Such an optimization problem can be easily solved with a linear system of equations. Maybe A is not positive semi-definite, but it is possible to replace A with $\frac{1}{2}(A^T + A)$, which is p.s.d. by construction.

3. Determine the partial derivatives of 2. regarding α_d and α_i and, for example, use the gradient method to determine new alphas. Go to step 1 and repeat until some end condition is fulfilled.

A possible way to adjust the three parameters K_∞, T_c, and $K_{2\cdot T_c}$ is described in the next section.

5. Modelling the Margin Rate with a Mean Reverting Controller

The margin $m(t)$ is controlled by the following differential equation:

$$dm(t) = \alpha \cdot (K_m - m(t))dt$$

K_m is called the mean reversion level. If K_m equals $m(t)$, then $d_M(t) = 0$, and no further adjustments of the margin are needed. $\alpha > 0$ is called the mean reversion speed and determines how quickly $m(t)$ reverts to the mean reversion level K_m.

The solution of the differential equation for $t > T_{n-1}$ with a given initial value $m(T_{n-1})$ is:

$$m(t) = K_m \cdot (1 - \exp(\alpha(T_{n-1} - t))) + m(T_{n-1}) \cdot \exp(\alpha(T_{n-1} - t))$$

or equivalently:

$$m(t) = m(T_{n-1}) + (K_m - m(T_{n-1})) \cdot (1 - \exp(\alpha(T_{n-1} - t)))$$

A way to calibrate K_m is explained in the next section.

6. Overall Model Design and Calibration

There are likely many possibilities to build an overall model with the introduced building blocks. Here is one example the author of this paper will focus on in the next months, with the long-term goal of having a fully automated model that optimizes the risk-return ratio of the node. To accomplish this, we plan to implement various processes that each optimize different objective functions. The aim is that the interaction of these processes will lead to the desired outcome.

1. There is one margin controller for all channel parties and, for each channel party, a separate spread controller with a manual adjustment of all parameters. This is the current state of the prototype.

2. For each spread controller, we have to adjust K_∞ and $K_{2\cdot T_c}$. A simple approach would be setting the K's proportional to the margin, with fixed factors at the beginning. The factors should be chosen high enough so that changing T_c has a significant effect on the adjustment speed of the controllers.

3. For the margin controller, we can keep α constant at some preconfigured value, e.g., 1% to 3% per day. It is more important to calibrate the level K_m. We want to adjust the margin in a way that maximizes the profit of the node by considering the dependencies with flow and potential rebalancing margins. Necessary for this is the proposed P&L decomposition of each transaction. This will allow us to see the interaction between forwarding and rebalancing margins and the adjustments on the different potentials.

4. For setting T_c of each spread controller, it could be helpful to analyze the P&L at the channel level. If T_c was set too high, it is likely that the channel will be depleted for a longer time, maybe generating margins but without potential adjustments. If T_c was set too low, then it is likely to generate a lot of negative P&L with potential adjustments, as the spread controller is likely to be heavily overdriven due to the short T_c.

5. Each channel party needs a target. By default, the ratio of remote liquidity and capacity is used. However, for some parties, it may make sense to use different values, e.g., if the party lowers their fee rates very late.

6. Using the spread rate controllers and margin controllers for risk-averse calculation of the maximum rebalancing costs could be beneficial. For example, we could use lower cost limits for a target if the channel is almost full, compared to a depleted channel. Because if the channel is almost full, it is more likely that negative value adjustments could occur in the future. For source channels, it could make sense to lower the cost limits if they are almost full to anticipate future negative value adjustments. An advanced cost calculation could use a statistical simulation of the rates, e.g., with a Monte Carlo model.

7. It may be possible to automate the factors from step 2 as well, e.g., to fine- tune the volatility of the P&L to achieve a stable P&L over time.