2. When $\beta < 0$: $f(\beta) = \frac{1}{2}[\beta^2 - 2(\lambda+t)\beta]$ one possible minimizer is $\lambda+t$ if $\lambda+t<0$

then minimum is $f(\lambda+t) = -\frac{1}{2}(\lambda+t)^2$

When $\beta > 0$ $f(\beta) = \frac{1}{2}[\beta^2 + 2(\lambda-t)\beta]$ " " " " $t-\lambda$ if $t-\lambda>0$

" " " $f(t-\lambda) = -\frac{1}{2}(t-\lambda)^2$.

1° $t > 0$.

If $t-\lambda > 0$. then $t-\lambda$ is attainable for $\beta > 0$.

if $\beta = \lambda+t$ also attainable for $\beta < 0$.

then $\lambda t < 0$. thus $f(t-\lambda) < f(t+\lambda)$

Thus the minimizer is $t-\lambda$, consistent with $\text{sign}(t)[|t|-\lambda]_+$. (*)

If $\beta = \lambda+t > 0$ not attainable, then $f(\beta) \downarrow$ on $[-\infty, 0]$

thus $f(\beta) \geq 0$ on $[-\infty, 0]$. so the minimizer is still $t-\lambda$, consistent with (*).

If $t-\lambda < 0$. then $\lambda, t > 0$. thus both $t-\lambda < 0$ and $\lambda+t > 0$ are not attainable. Thus $f(\beta) \downarrow$ on $[-\infty, 0]$; $\uparrow$ on $[0, +\infty]$

Thus the minimizer is 0, still consistent with (*),

2° $t < 0$

If $t+\lambda < 0$, then $\lambda+t$ is attainable for $\beta < 0$.

If $\beta = t-\lambda$ is also attainable, then $\lambda t > 0$

thus $f(t+\lambda) < f(t-\lambda)$. Thus minimizer is $t-\lambda$, consistent with (*)

If $\beta = t-\lambda$ is not attainable, similarly $f(\beta) \geq 0$ on $[0, +\infty]$.

thus minimizer is $t-\lambda$. consistent with (*),

If $t+\lambda > 0$ then $\lambda > -t > 0$. thus $t-\lambda < 0$

Both $t-\lambda$ and $\lambda+t$ are not attainable.

Thus $f(\beta) \downarrow$ on $[-\infty, 0]$; $\uparrow$ on $[0, +\infty)$

Thus the minimizer is 0, consistent with (*).

From all the conditions listed above, we can see minimizer is indeed $\text{sign}(t)[|t|-\lambda]_+$