CMSC 25025

HW 2.

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1. First, fix μ and optimize over λ_is. $\|\hat{x}_i - V_k\lambda_i\|$

Now we want to minimizer $\|x_i - \mu - V_k\lambda_i\|^2_{(*)}$, $\forall i = 1, \dots, n$.

let $x_i - \mu$ be $\hat{x}_i$. let $R(V_k) = \{y : y = V_k z \quad z \in R^n\}$

and $R^{\perp}(V_k) = \{ y : y^T x = 0 \quad \forall x \in R(V_k)\}$

Then $\hat{x}_i = \hat{x}_{i1} + \hat{x}_{i2}$ where $\hat{x}_{i1} \in R(V_k)$. $\hat{x}_{i2} \in R^{\perp}(V_k)$

Then $(*) = \|V_k\lambda_i - \hat{x}_{i1} - \hat{x}_{i2}\|^2 = \underbrace{\|V_k\lambda_i - \hat{x}_{i1}\|^2}_{(*_2)} + \|\hat{x}_{i2}\|^2$

Then to minimize $*$ $\Leftrightarrow$ minimize $(*_2)$

~~Since~~ $\cancel{*_2 = \quad *_2 = \hat{x}_i - V_k\lambda_i}$

minimize $\|V_k\lambda_i - \hat{x}_{i1}\|^2 \Leftrightarrow V_k\lambda_i = \hat{x}_{i1}$ $(*_3)$ since V_k is a full rank matrix and the equations $(*_3)$ can be solved

Thus $V_k^T V_k \lambda_i = V_k^T \hat{x}_{i1}$

Thus $\lambda_i = V_k^T \hat{x}_{i1} = V_k^T \hat{x}_{i1} + V_k^T \hat{x}_{i2} = V_k^T \hat{x}_i$

so $\hat{\lambda}_{i}(\mu) = V_k^T(x_i - \mu)$

Plug this into $\sum_{i=1}^{n} \|x_i - \mu - V_k\lambda_i\|^2 = \sum_{i=1}^{n} \|(I - V_kV_k^T)(x_i - \mu)\|^2 = f(\mu)$

$Df(\mu) = 2\sum_{i=1}^{n}(I - V_kV_k^T)^2(\mu - x_i) = 2n(I - V_kV_k^T)^2(\mu - \bar{x})$

$D^2f(\mu) \geq 0$

When $\mu = \bar{x}$ $Df(\mu) = 0$ certainly a minimum

Other minimizer of μ belong to the set $\{\mu : (I - V_kV_k^T)^2(\mu - \bar{x}) = 0\}$