

Extraction of the ^{12}C Longitudinal (\mathbf{R}_L) and (\mathbf{R}_T) Nuclear Electromagnetic Response Functions from *all* Electron Scattering Measurements on Carbon

1. Testing first principle nuclear theory predictions
2. Provide a platform for verification of electron and neutrino MC generators over the entire kinematic range of interest

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Presented by Zihao Lin

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Sacramento, California

Validation of MC generators using new extractions of ^{12}C electromagnetic longitudinal and transverse response functions

We extract ^{12}C Longitudinal (R_L) and (R_T) Nuclear Electromagnetic Response Functions from *all* Electron Scattering Measurements on Carbon for:

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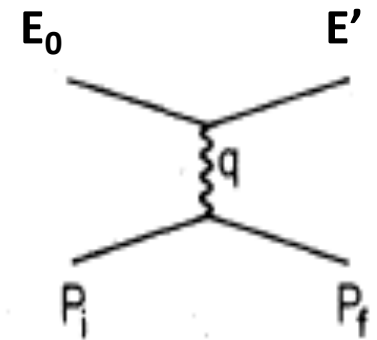
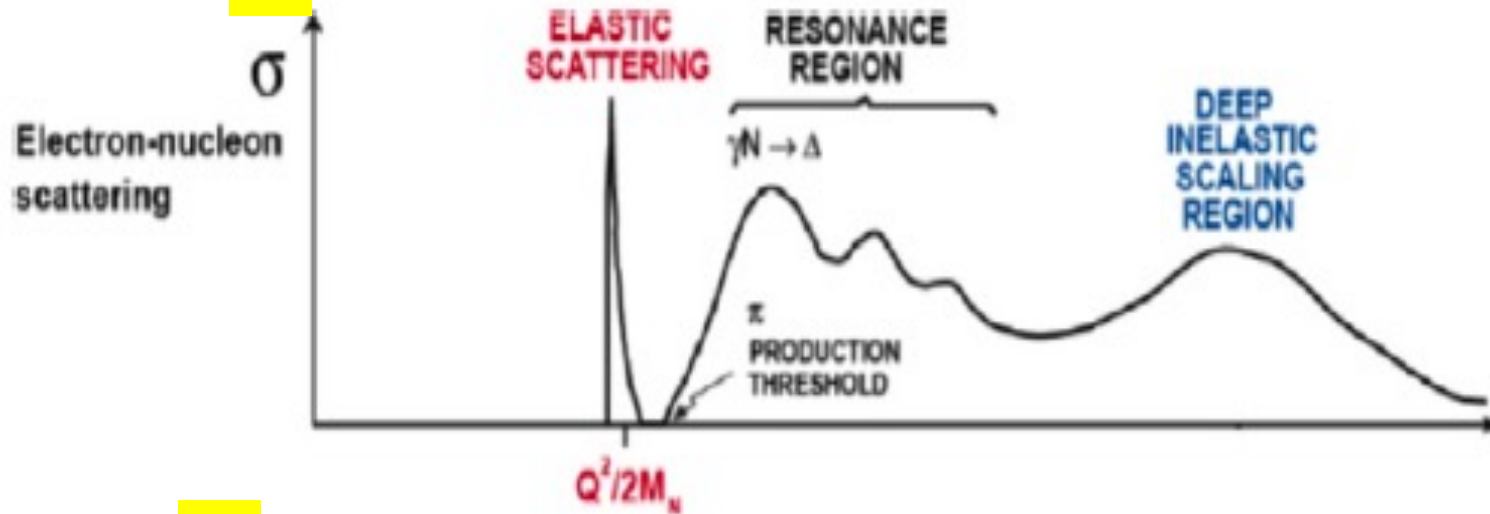
2nd Short-Baseline Experiment-Theory Workshop 2024, April 2-5, Santa Fe, NM

25 min + 5 min questions

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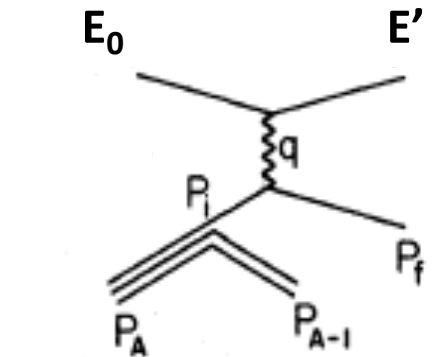
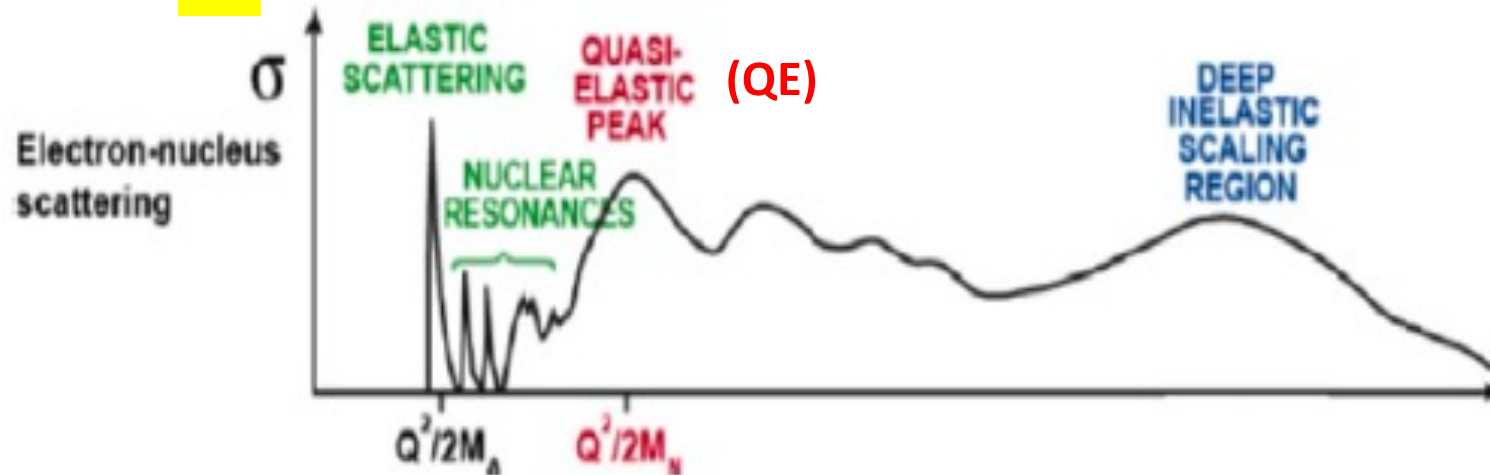
G.T. Garvey et al. / Physics Reports 580 (2015) 1–45

P, N



(i) (or ν)

^{12}C



(i) (or ν)

Nuclear corrections affect enhance the Transverse Response (R_T) and quench the Longitudinal Response (R_L) in QE scattering. There are three different formalisms to parametrize these two responses. (1) R_L and R_T (2) F_1 and F_L (3) σ_L and σ_T .

In this analysis we use R_L and R_T .

★

Nuclear Physics

This description is primarily used in the nuclear excitation and QE regions. The electron scattering differential cross section is written in terms of longitudinal ($\mathcal{R}_L(Q^2, \nu)$) and transverse ($\mathcal{R}_T(Q^2, \nu)$) nuclear response functions [24] energy transfer = ν (or ω)

$$\frac{d\sigma}{d\nu d\Omega} = \sigma_M [A \mathcal{R}_L(Q^2, \nu) + B \mathcal{R}_T(Q^2, \nu)] \quad (20)$$

where σ_M is the Mott cross section, $A = (Q^2/\mathbf{q}^2)^2$ and $B = \tan^2(\theta/2) + Q^2/2\mathbf{q}^2$.

Particle Physics

This description is primarily used in the inelastic continuum region. In the one-photon-exchange approximation, the spin-averaged cross section for inclusive electron-proton scattering can be expressed in terms of two structure functions as follows

$$\begin{aligned} \frac{d\sigma}{d\Omega dE'} &= \sigma_M [\mathcal{W}_2(W^2, Q^2) + 2 \tan^2(\theta/2) \mathcal{W}_1(W^2, Q^2)] \\ \sigma_M &= \frac{\alpha^2 \cos^2(\theta/2)}{[2E_0 \sin^2(\theta/2)]^2} = \frac{4\alpha^2 E'^2}{Q^4} \cos^2(\theta/2) \quad (10) \end{aligned}$$

$Q^2 = 4\text{-momentum transfer squared}$

$$Q^2 = (-q)^2 = 4E_0 E' \sin^2 \frac{\theta}{2},$$

$$\nu = E_0 - E'$$

$W^2 = \text{final state invariant mass squared}$

$$W^2 = M^2 + 2M\nu - Q^2$$

$\mathbf{q}^2 = 3\text{-momentum transfer squared}$

$$\mathbf{q}^2 = Q^2 + \nu^2$$

$E_x = \text{Excitation energy}$

$$E_x = \nu - \nu_{\text{elastic}}$$

$$x = Q^2/(2M\nu).$$

$$\mathcal{F}_1 = M\mathcal{W}_1 \text{ and } \mathcal{F}_2 = \nu\mathcal{W}_2.$$

$$\mathcal{F}_L(x, Q^2) = \mathcal{F}_2 \left(1 + \frac{4M^2 x^2}{Q^2} \right) - 2x\mathcal{F}_1,$$

$$\mathcal{R}_T(\mathbf{q}, \nu) = \frac{2\mathcal{F}_1(\mathbf{q}, \nu)}{M}$$

$$\mathcal{R}_L(\mathbf{q}, \nu) = \frac{\mathbf{q}^2}{Q^2} \frac{\mathcal{F}_L(\mathbf{q}, \nu)}{2Mx}$$

σ_L and σ_T .

A. Description in terms of longitudinal and transverse virtual photon cross sections

This description is often used in the resonance region. In the one-photon-exchange approximation, the spin-averaged cross section for inclusive electron-proton scattering can be expressed in terms of the photon helicity coupling as

$$\frac{d\sigma}{d\Omega dE'} = \Gamma [\sigma_T(W^2, Q^2) + \epsilon \sigma_L(W^2, Q^2)], \quad (6)$$

where σ_T (σ_L) is the cross section for photo-absorption of purely transverse (longitudinal) polarized photons,

$$\Gamma = \frac{\alpha E' (W^2 - M_N^2)}{(2\pi)^2 Q^2 M E_0 (1 - \epsilon)} \quad (7)$$

is the flux of virtual photons, $\alpha = 1/137$ is the fine structure constant, and

$$\epsilon = \left[1 + 2 \left(1 + \frac{\nu^2}{Q^2} \right) \tan^2 \frac{\theta}{2} \right]^{-1} \quad (8)$$

is the relative flux of longitudinal virtual photons (sometimes referred to as the virtual photon polarization). Since Γ and ϵ are purely kinematic factors, it is convenient to define the reduced cross section

$$\sigma_r = \frac{1}{\Gamma} \frac{d\sigma}{d\Omega dE'} = \sigma_T(W^2, Q^2) + \epsilon \sigma_L(W^2, Q^2). \quad (9)$$

All the hadronic structure information is therefore, contained in σ_T and σ_L , which are only dependent on W^2

$$K = \frac{Q^2(1-x)}{2Mx} = \frac{2M\nu - Q^2}{2M} \quad (13)$$

$$\mathcal{F}_1 = \frac{MK}{4\pi^2\alpha} \sigma_T \quad (14)$$

$$\mathcal{F}_2 = \frac{\nu K(\sigma_L + \sigma_T)}{4\pi^2\alpha(1 + \frac{Q^2}{4M^2x^2})} \quad (15)$$

$$\mathcal{F}_L(x, Q^2) = \mathcal{F}_2 \left(1 + \frac{4M^2x^2}{Q^2} \right) - 2x\mathcal{F}_1, \quad (16)$$

$$\mathcal{R}_T(\mathbf{q}, \nu) = \frac{2\mathcal{F}_1(\mathbf{q}, \nu)}{M} = \frac{K}{2\pi^2\alpha} \sigma_T \quad (21)$$

$$\mathcal{R}_L(\mathbf{q}, \nu) = \frac{q^2}{Q^2} \frac{\mathcal{F}_L(\mathbf{q}, \nu)}{2Mx} = \frac{q^2}{Q^2} \frac{K}{4\pi^2\alpha} \sigma_L \quad (22)$$

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- **We have initiated a program to extract RL and RT values on various nuclei using all available data.**
Here we report on our extraction for Carbon.
- We extract RL and RT in all regions, nuclear elastic, nuclear excitations, quasielastic, resonance region and inelastic scattering.
- We extract at 18 values of fixed Q^2 values $0 < Q^2 < 3.45 \text{ GeV}^2$ as a function of energy transfer ν .
- And at 18 values of fixed values of 3-momentum transfer $0.1 < \mathbf{q} < 2.78 \text{ GeV}$ as a function of ν
- We energy transfer ν , from $\nu=0$ to the end of the resonance region $W=2.0 \text{ GeV}$

For Carbon we use 16,000 electron scattering and photproduction cross sections

Goals:

- Test first-principle nuclear theories at fixed values of Q^2 and \mathbf{q} .
- Use extracted RL and RT values to validate MC generators
Since we covers all kinematic regions this is a much preferable way than comparison with a few cross section measurements in a limited sets of kinematic regions.
- **Where there is no data, we provide the values from our universal fit to all electron scattering data.**
The fit will be also be made available for validation of electron and neutrino MC generators

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First Step: Christy- Bodek Universal Fit (needed for this analysis))

- We update the Bosted-Christy fit to **all of the world's electron scattering data** on H, D and nuclear targets to include the **lowest values of energy transfer ν and q^2** (for ^{12}C we fit **~16,000 electron scattering and photoabsorption cross sections**).
- We fit for: **QE cross section (including Transverse Enhancement/MEC and longitudinal low q Quenching)**. **Resonance and pion production**, **DIS**, **nuclear excitations**, elastic scattering data. Since the cross sections span a large range of energies and scattering angles, we **extract both the longitudinal RL and transverse RT** contributions, and also get the Coulomb Sum Rule.
- We parameterize both the **Enhancement of the Transverse QE cross section** and the **Quenching of the Longitudinal QE** cross section. **We extract the most precise Coulomb Sum rule as a function of q and compare to theoretical calculations.**
- The fit can be used **in lieu of data to benchmark Monte Carlo predictions** (e.g. for e-H, e-D and e- ^{12}C and e- ^{16}O cross sections, and to is being used **compute radiative corrections for electron scattering** experiments..

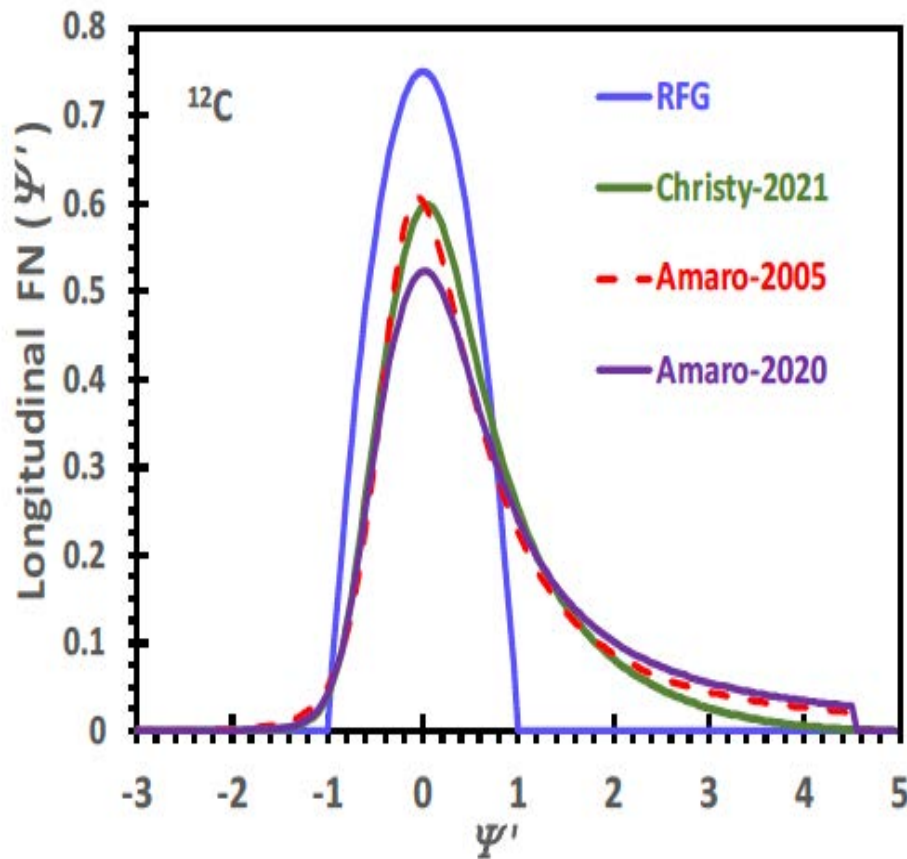
Modeling QE:

Use superscaling- Fit for the longitudinal scaling function parameters in the overall fit

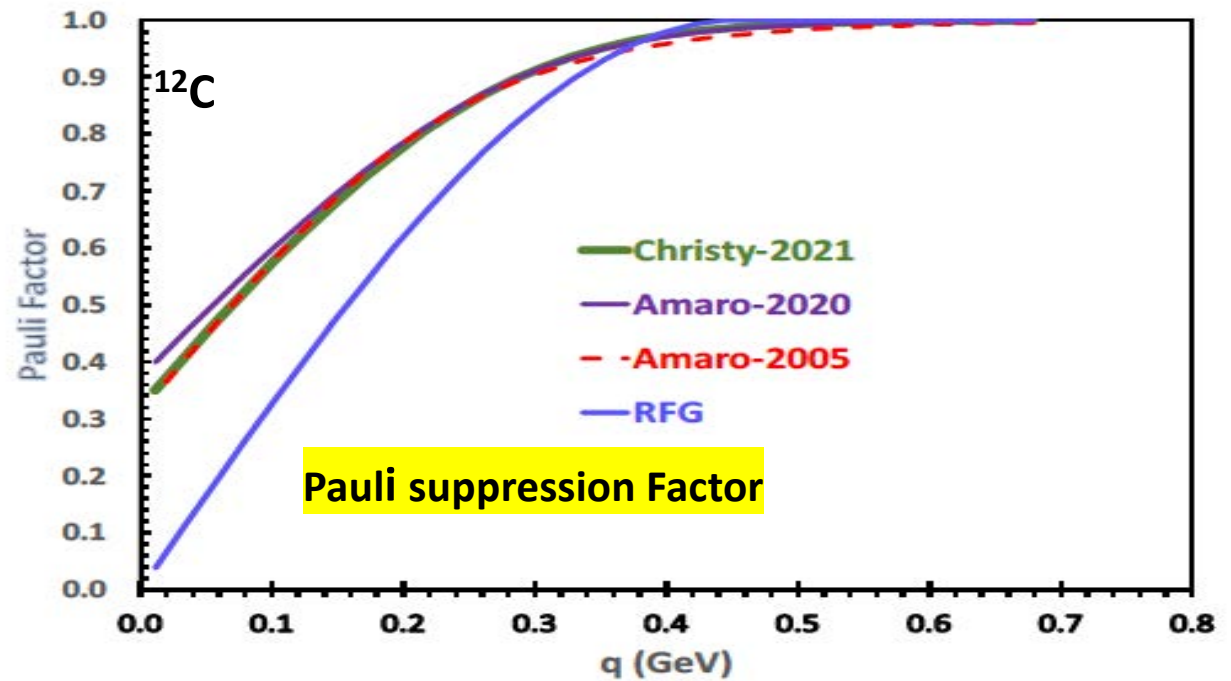
The ψ' superscaling variable is given by the following expression:

$$\psi' \equiv \frac{1}{\sqrt{\xi_F}} \frac{\lambda' - \tau'}{\sqrt{(1 + \lambda')\tau' + \kappa\sqrt{\tau'(1 + \tau')}}}, \quad (16)$$

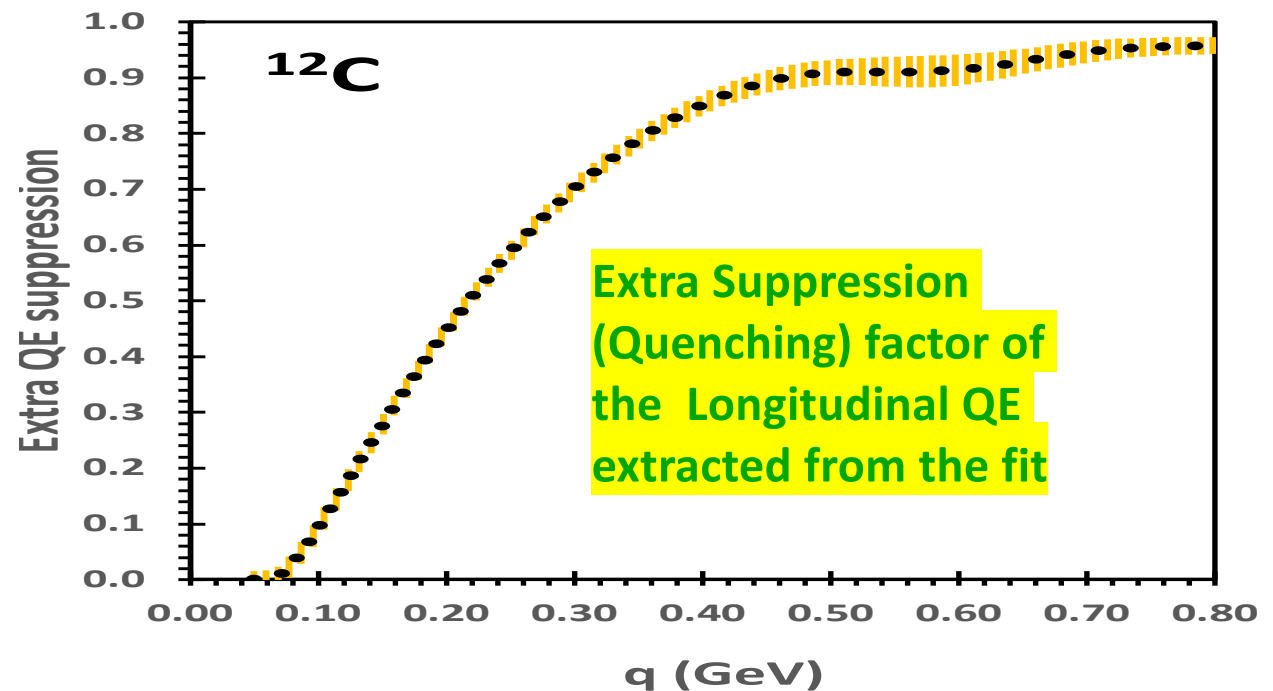
where $\xi_F \equiv [\sqrt{1 + \eta_F^2} - 1]$, $\eta_F \equiv K_F/M_n$, $\lambda \equiv \nu/2M_n$, $\kappa \equiv |\mathbf{q}|/2M_n$ and $\tau \equiv |Q^2|/4M_n^2 = \kappa^2 - \lambda^2$.



Pauli Suppression Factor. We use the Rosenfelder method.



We find that there is Quenching of the Longitudinal QE cross section (in addition to Pauli) extracted from our fit



★ Christy- Bodek Universal Fit: Carbon
In the Nuclear excitation region

Cross sections for
excitations less than 10
MeV multiplied by (1/6)

Nuclear excitation region

Ex < 50 MeV

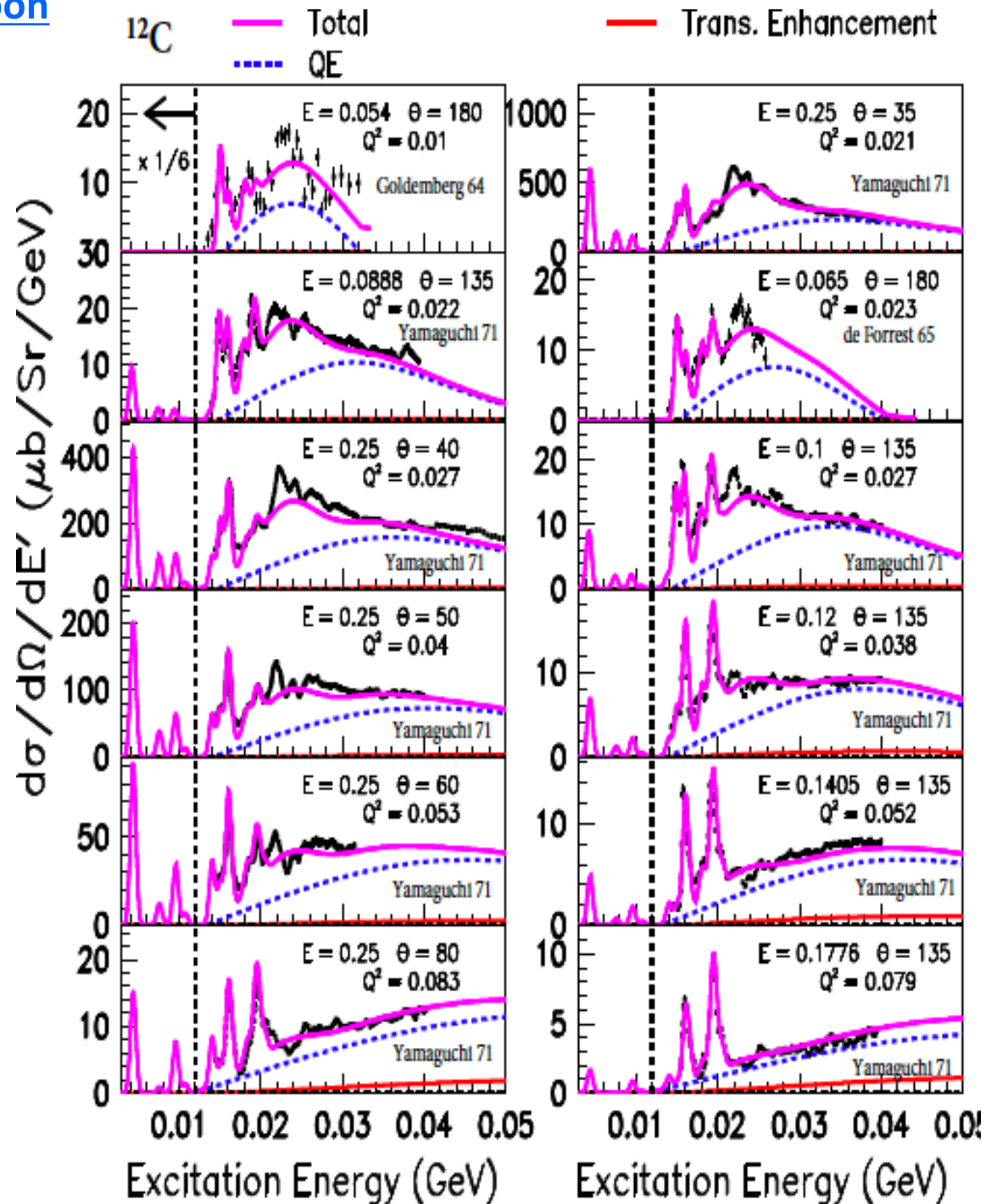
Comparison of our fit to
representative e-C12 data for
 $0.01 < q^2 < 0.08 \text{ GeV}^2$.

Shown: **Total including excitations**
: solid -----

Quasielastic (QE) contribution:
dashed-----

**Transverse Enhancement at large
angles accounts for Meson
Exchange Currents and
Enhancement of Transverse QE
response** dashed-----

Electron scattering cross sections



Quasielastic (QE) Region-I

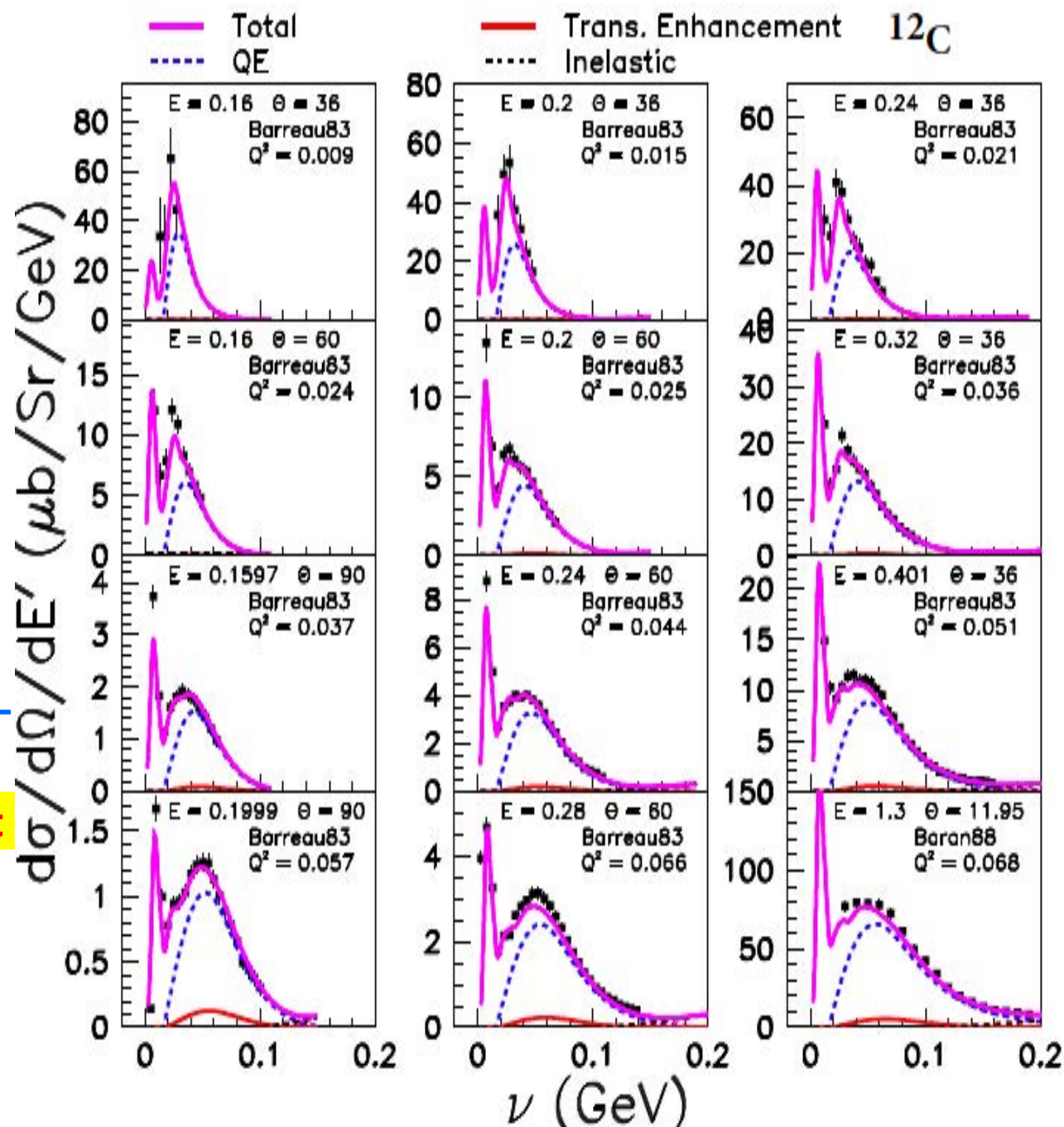
Comparison of our fit to representative e-C12 data

For $\nu < 0.2$ GeV and $0.01 < q^2 < 0.068$ GeV².

Shown: Total including excitations solid -----

Quasielastic (QE) contribution dashed -----

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE response dashed-----



Christy- Bodek Universal Fit

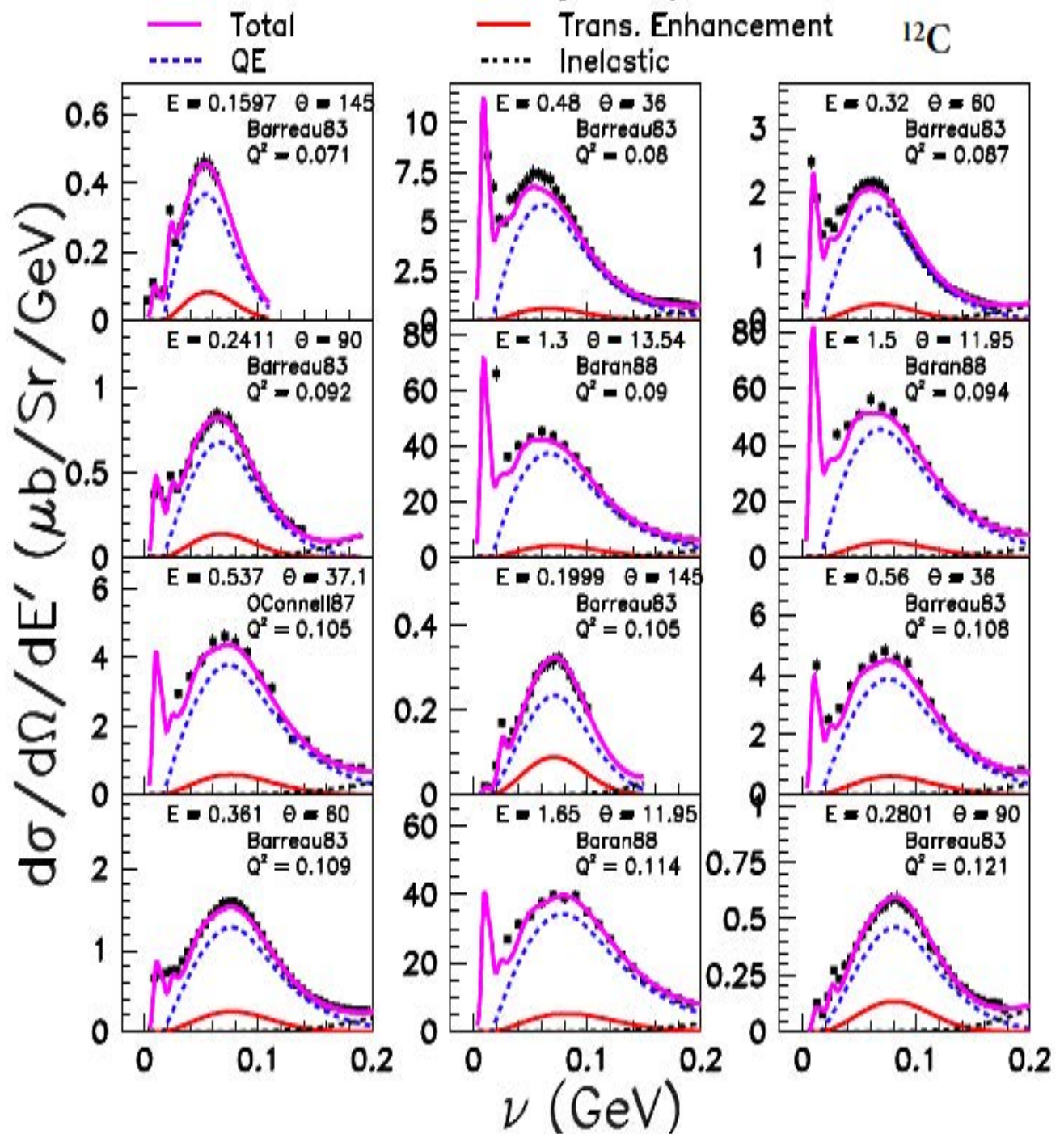
Quasielastic (QE) Region II

Comparison of our fit to representative e-C12 data for $< 0.2 \text{ GeV}$ and $0.071 < q^2 < 0.121 \text{ GeV}^2$.

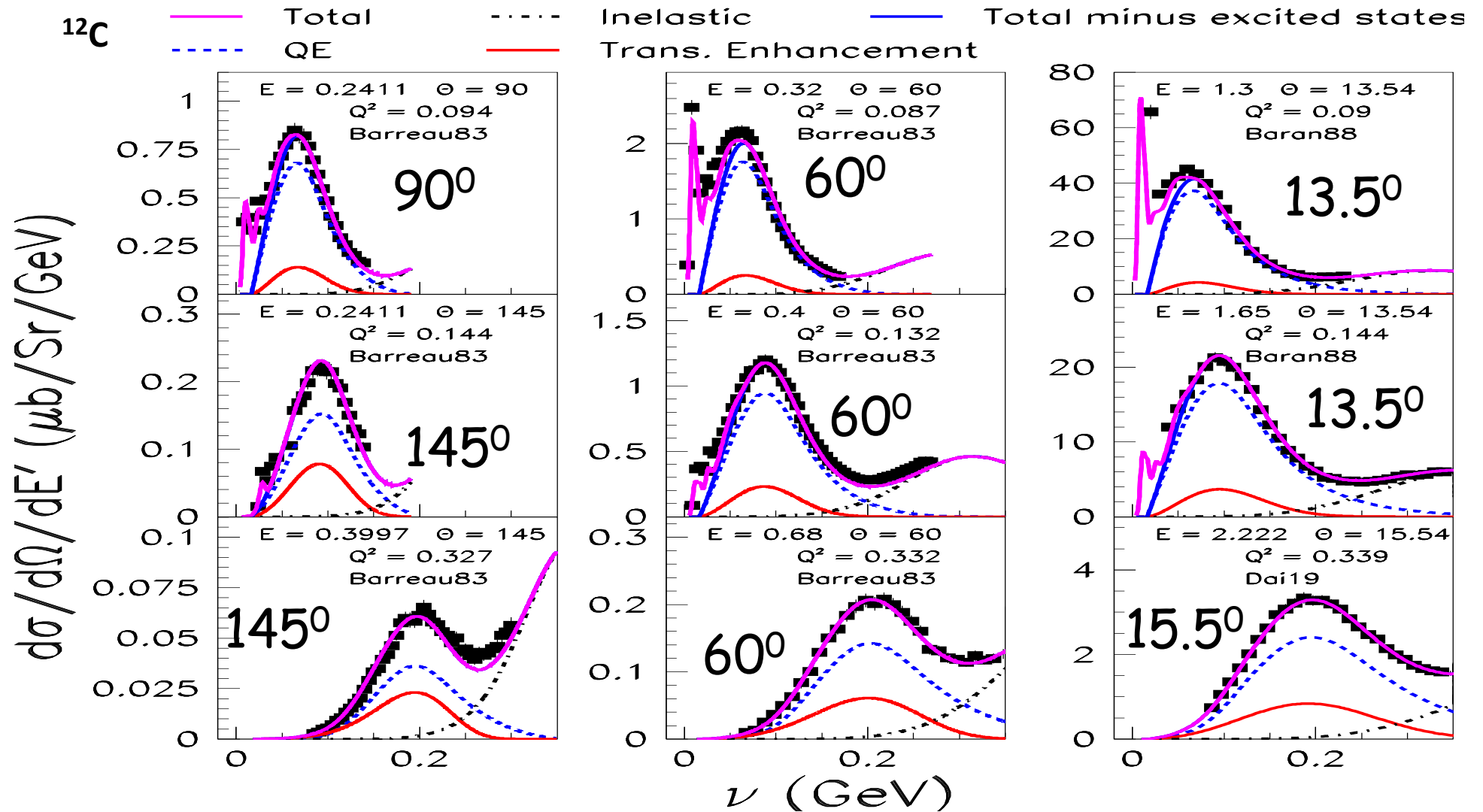
Shown: Total including excitations solid -----

Quasielastic (QE) contribution dashed -----

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE response dashed-----



Christy- Bodek Universal Fit - Fit functional forms for RL RT



The overall fit provides R_L and R_T at all values of q

Shown are **large and small angle cross sections at the same q** that provide the major contribution to the extraction of R_L and R_T at

$q^2=0.09, 0.15$ and 0.35 GeV^2 ($q=0.3, 0.38$ and 0.57 GeV)

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NEW: Individual R_L R_T extractions (Rosenbluth plots)

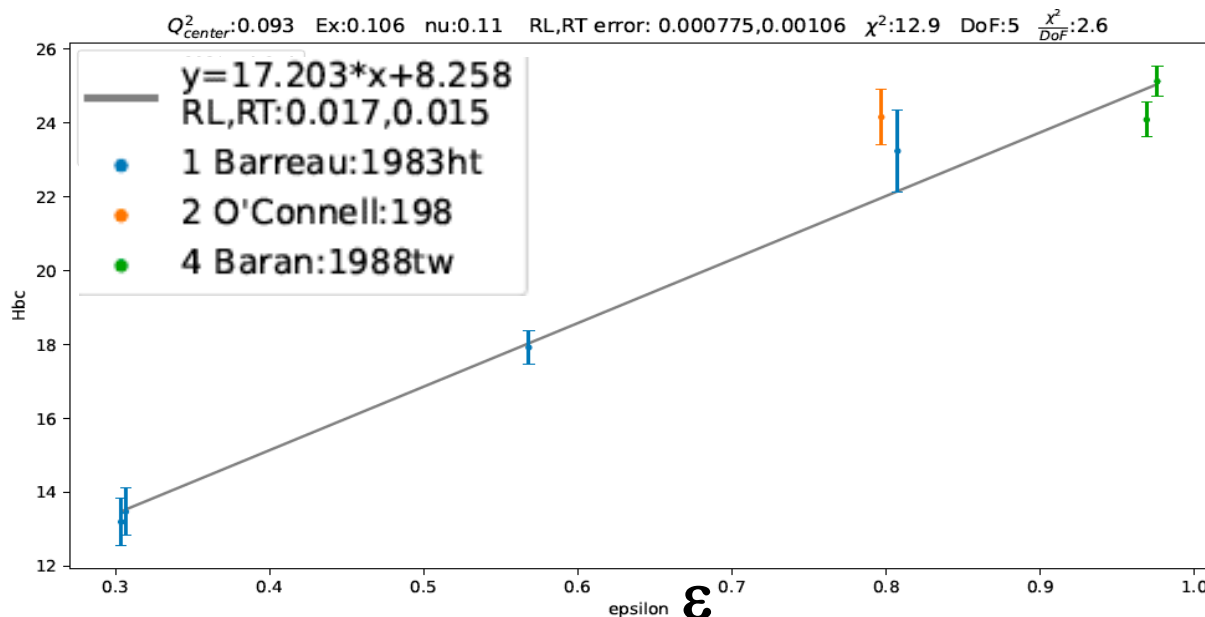
We apply Coulomb corrections in the analysis. We bin all cross sections in bins of q (we also do it for bins in Q^2) and apply bin centering corrections using the universal fit. Bin centers are at:

18 q values: 0.100, 0.148, 0.167, 0.205, 0.240, 0.300, 0.380, 0.475, 0.570,
0.649, 0.756, 0.991, 1.659, 1.921, 2.213, 2.500, 2.783, 3.500 GeV

18 Q^2 values: 0.00 (photoproduction), 0.010, 0.020, 0.026, 0.040, 0.056, 0.093, 0.120, 0.160, 0.265,
0.38, 0.50, 0.80, 1.25, 1.75, 2.25, 2.75, 3.25, 3.75 GeV²

2. Bin centering correction in q (or Q^2). For $\nu < 50$ MeV we bin-center the data in bins of Excitation energy E_x , and for $\nu > 50$ MeV we bin-center the data in bins of W^2 . We then perform Rosenbluth fits versus virtual photon polarization ϵ extract R_L and R_T . Later convert E_x and W^2 to ν .

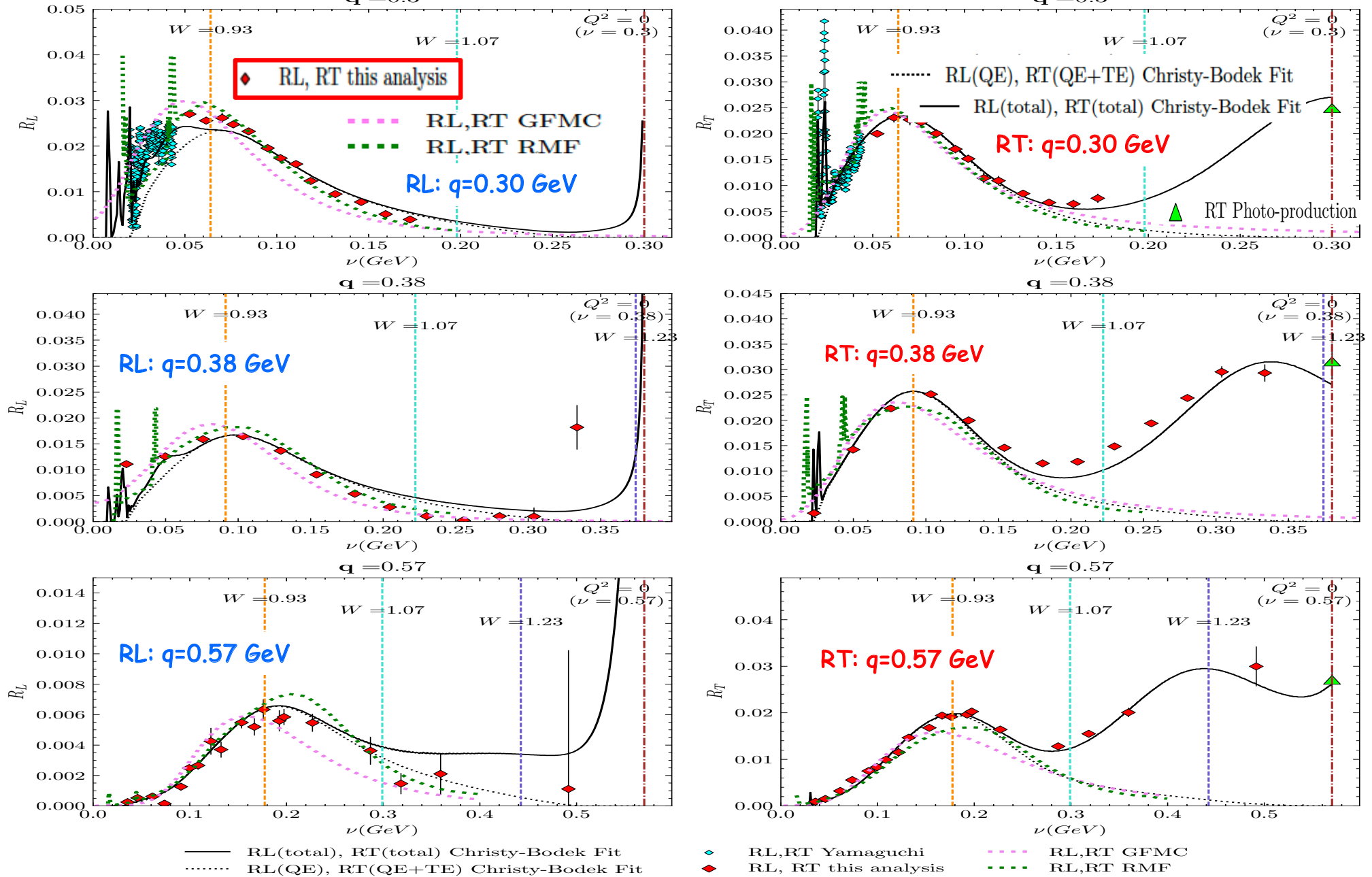
2. The universal fit includes all **16,000 cross sections**, The individual R_L R_T only use a **subset of the data** in which for the same values ν and Q^2 there is data spanning a significant range of angles. Therefore the Universal Fit provide R_L and R_T for a more extended kinematic range



ϵ = virtual
photon
polarization

$$\epsilon = \left[1 + 2 \left(1 + \frac{\nu^2}{Q^2} \right) \tan^2 \frac{\theta}{2} \right]^{-1}$$

*Comparison to nuclear theory for QE 1p1h process (available for $q = 0.3, 0.38, 0.57$ GeV) only



Shown are theory predictions for QE 1p1h process (including contribution from 1b and 2b currents).

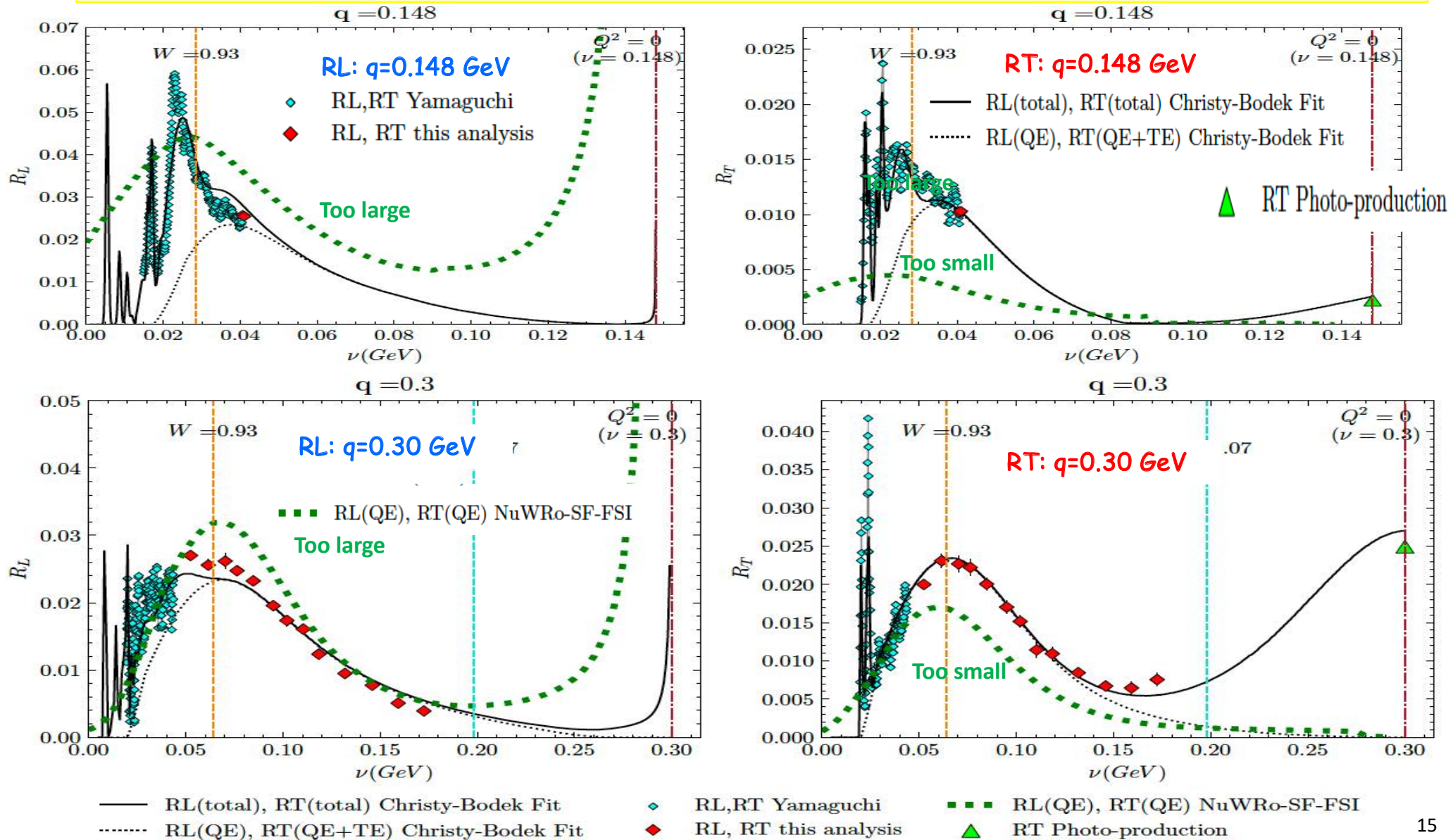
GFMC (Green's Function Monte Carlo – first principle)..... No nuclear states

ED-RMF (Energy Depending Relativistic mean field)..... Nuclear states grouped at 2 values of fixed

QE predictions reasonable but not perfect

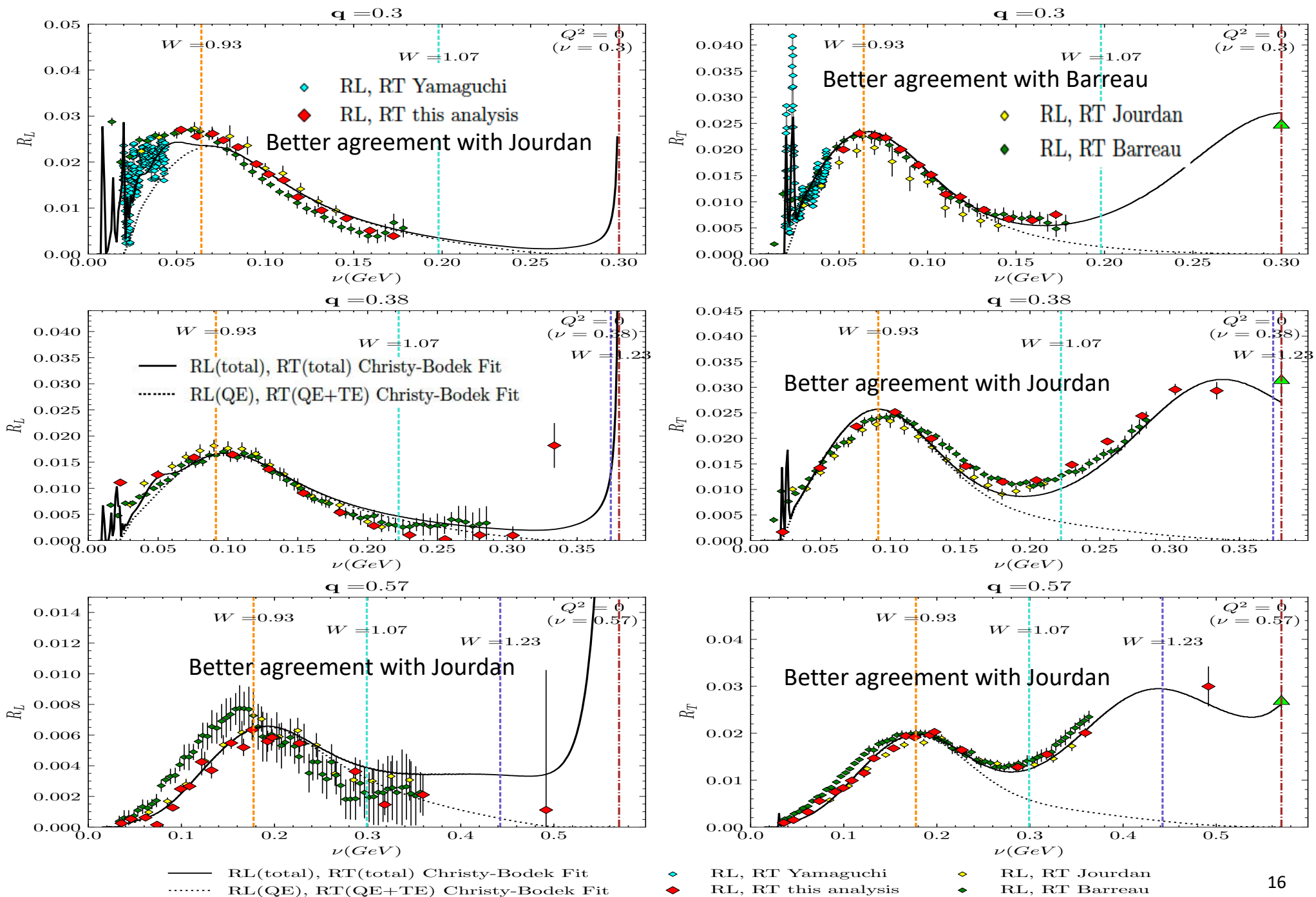
* Comparison to NuWRo Neutrino MC generator for QE (in electron scattering mode)

NuWRo; R_L is too high (needs longitudinal quenching), R_T is too low (needs Transverse Enhancement/MEC). No nuclear excitations are modeled in any NuWRo. Note: **NuWRo has Transverse Enhancement/MEC in neutrino but not in electron mode.** ---If MEC is implemented in electron mode, these data can be used to validate the model.



Comparisons with GENIE (in electron mode) will be available shortly

Comparison to previous R_L R_T extractions $q = 0.3, 0.38, 0.57$ GeV



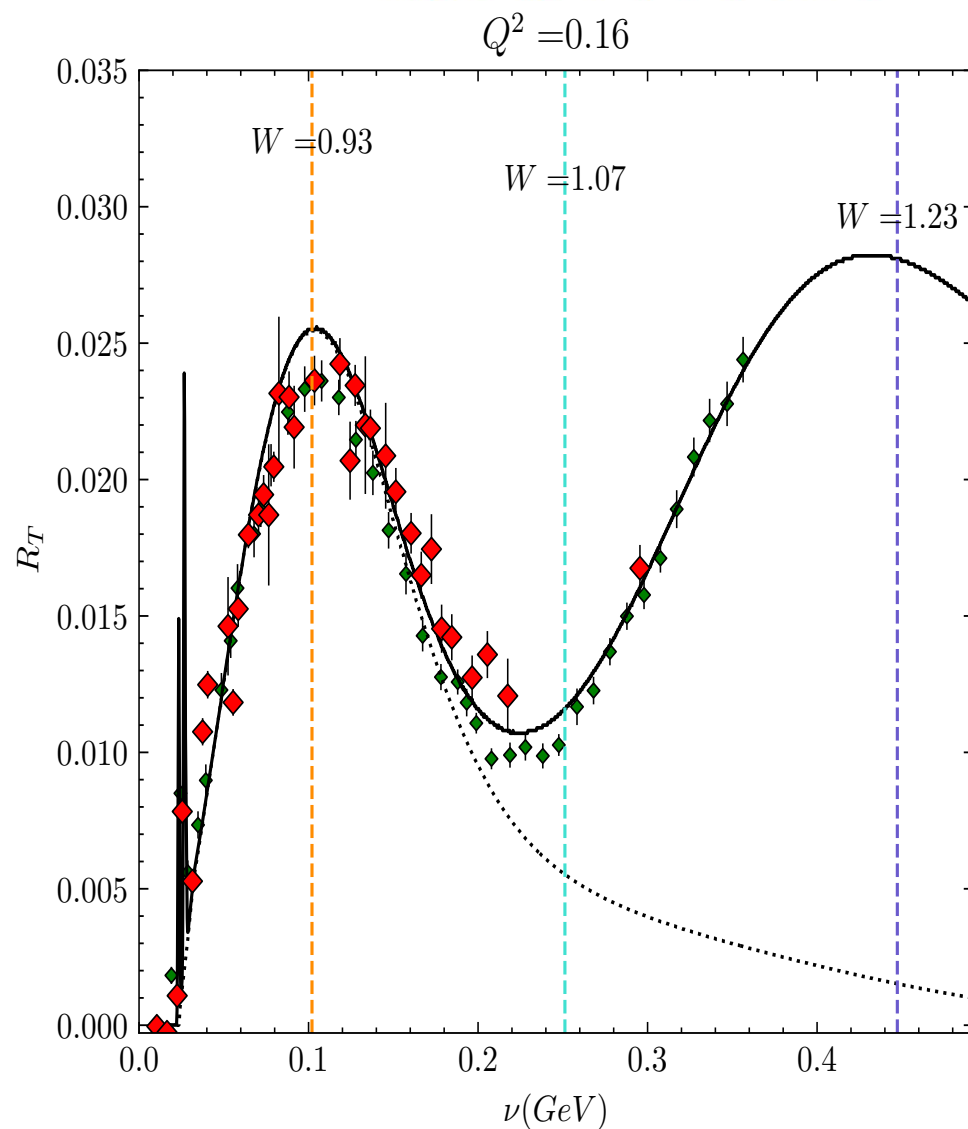
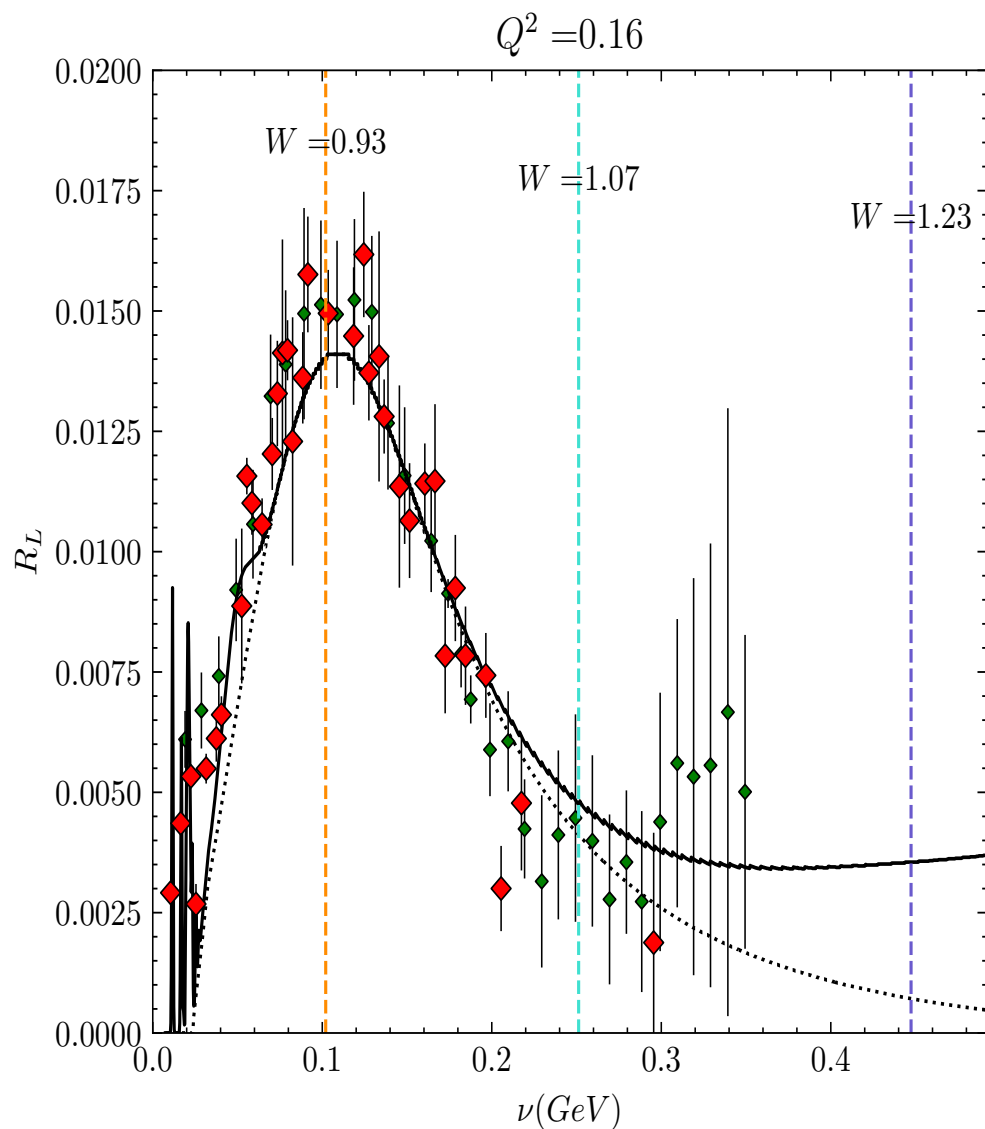
Comparison to previous R_L R_T extractions $Q^2=0.16 \text{ GeV}^2$

◆ R_L, R_T this analysis

◆ R_L, R_T Barreau

— $R_L(\text{total}), R_T(\text{total})$ Christy-Bodek Fit

⋯ $R_L(\text{QE}), R_T(\text{QE}+\text{TE})$ Christy-Bodek Fit



— $R_L(\text{total}), R_T(\text{total})$ Christy-Bodek Fit

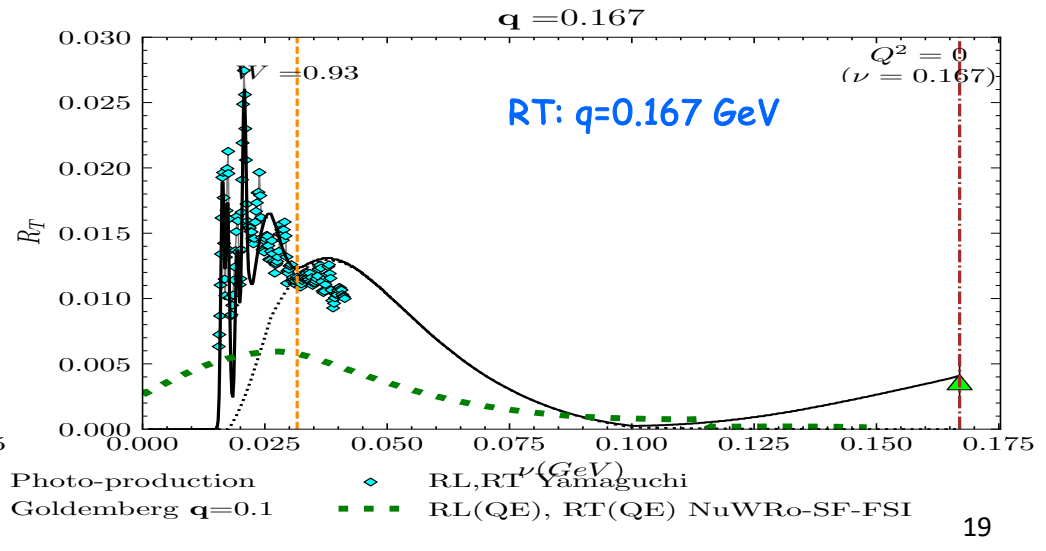
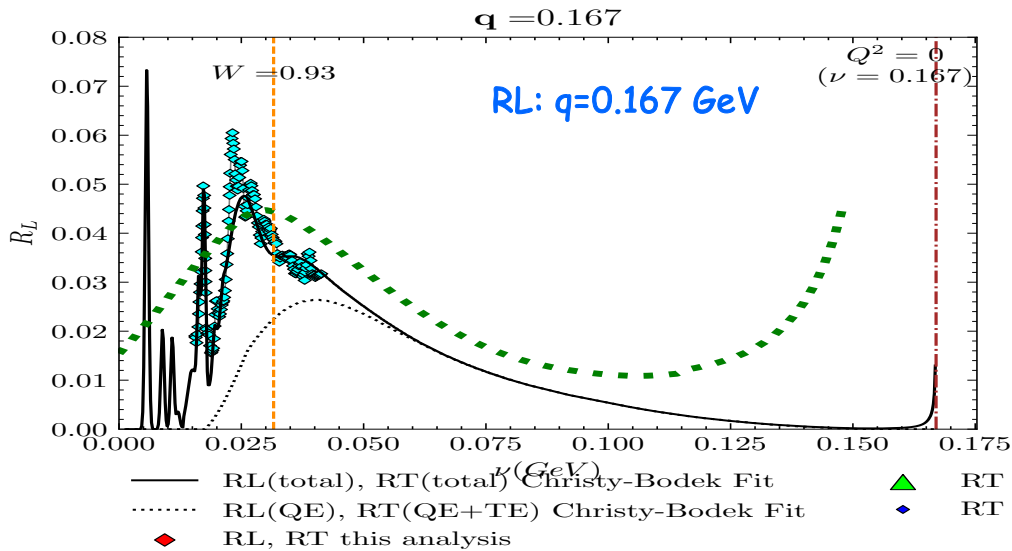
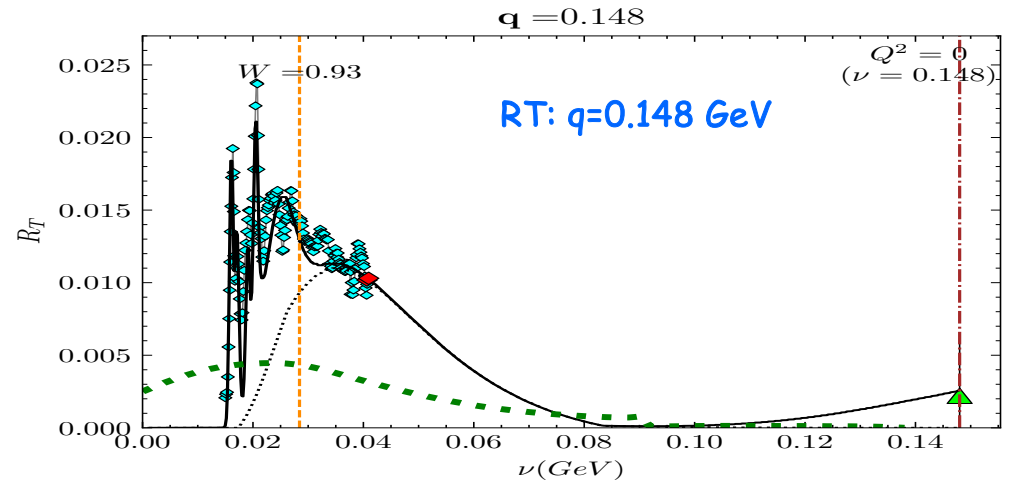
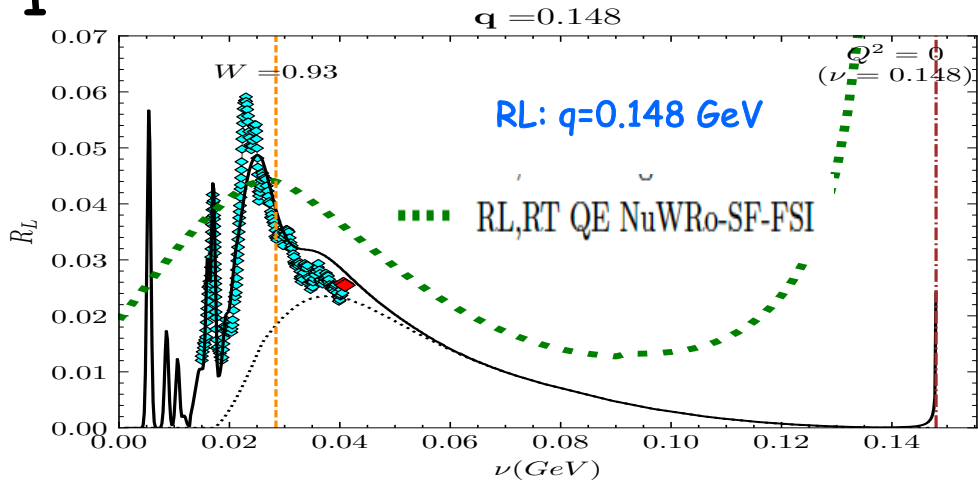
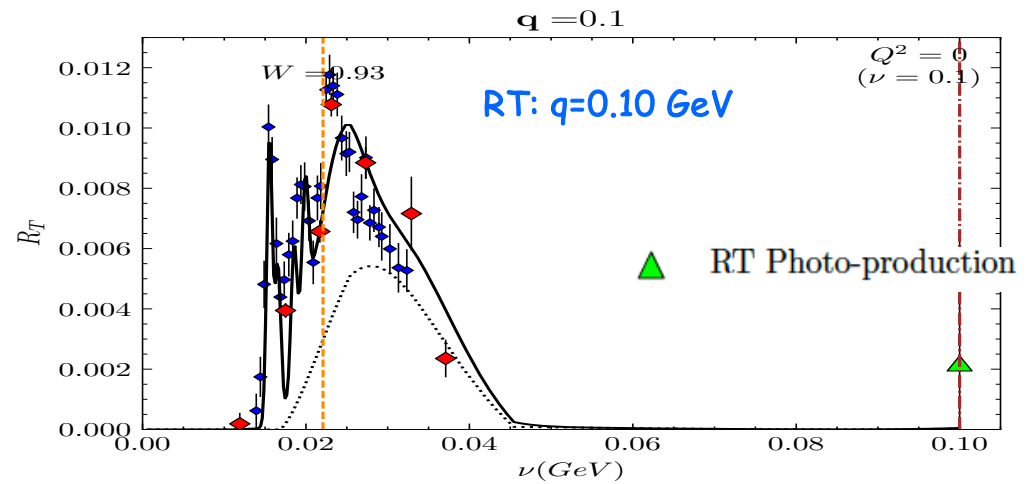
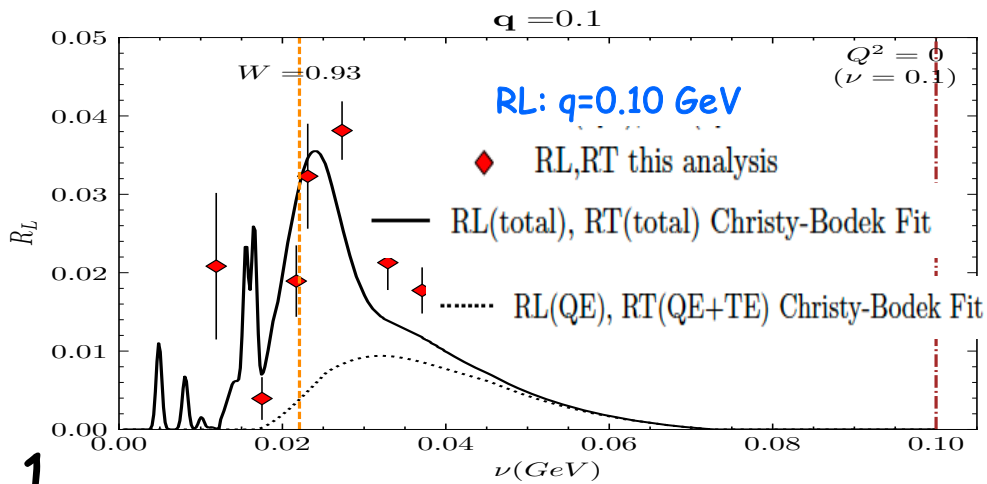
⋯ $R_L(\text{QE}), R_T(\text{QE}+\text{TE})$ Christy-Bodek Fit

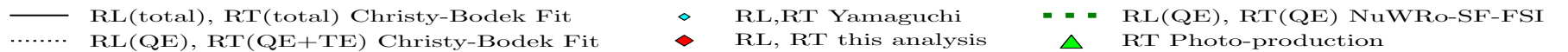
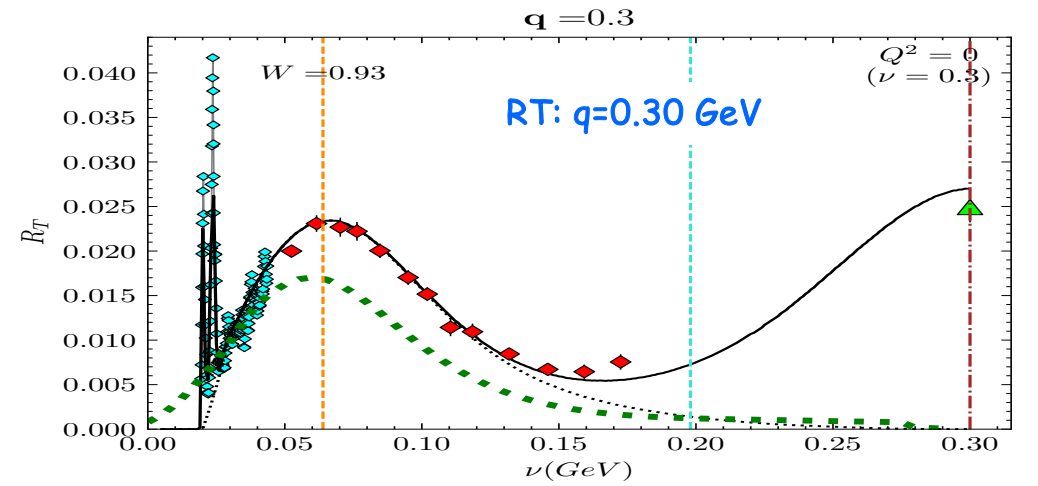
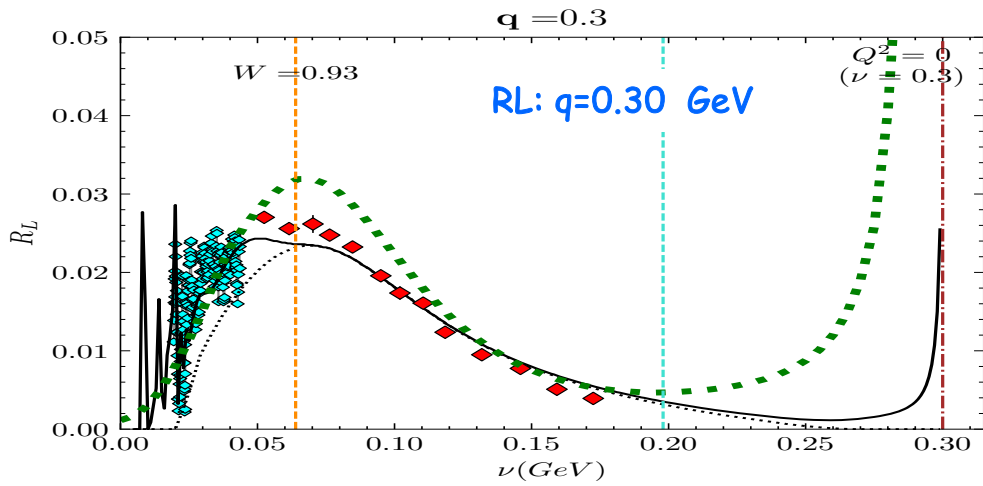
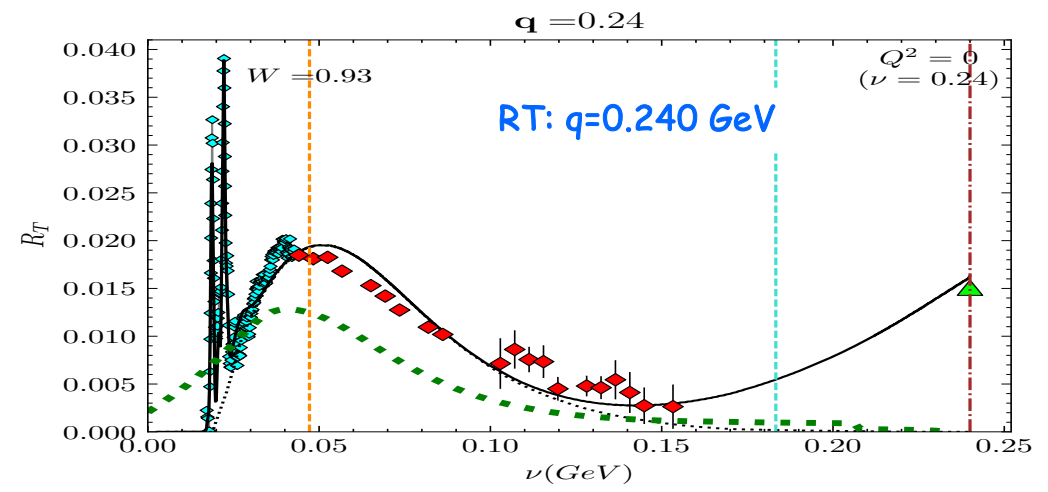
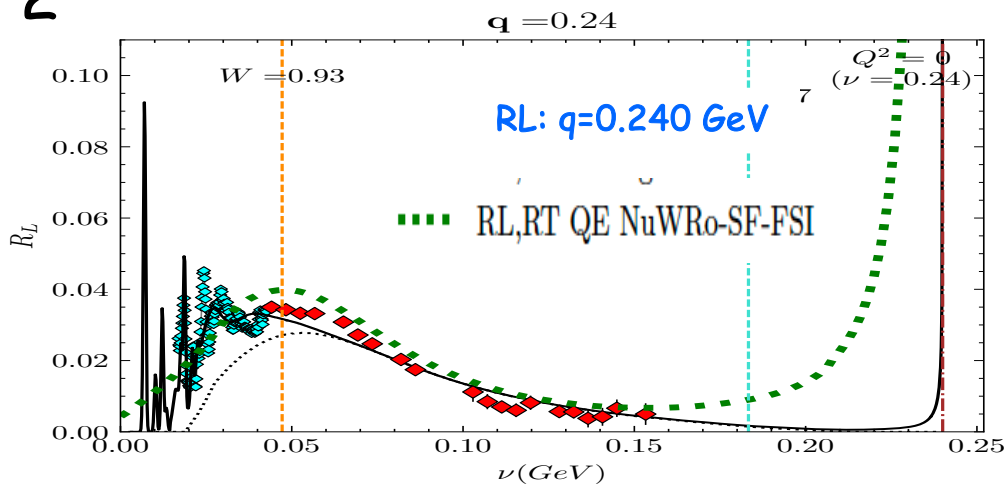
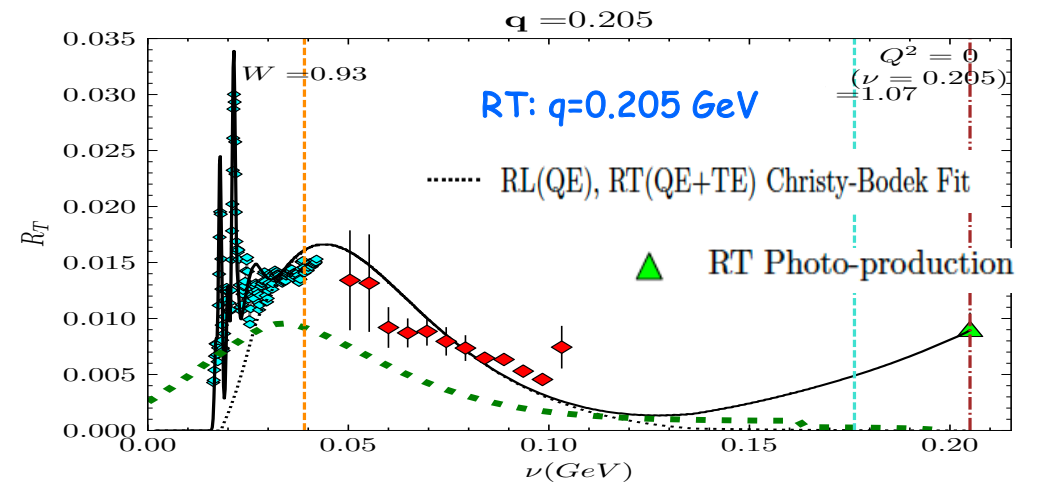
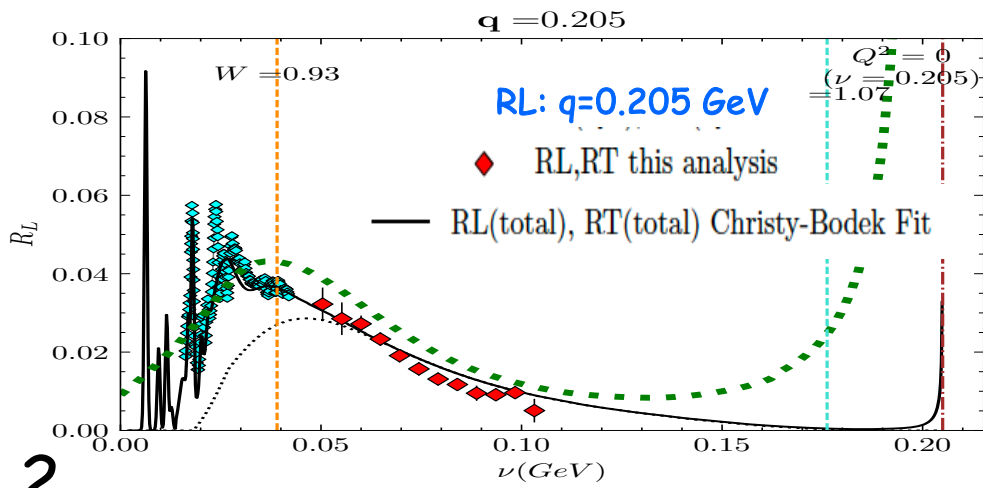
◆ R_L, R_T this analysis

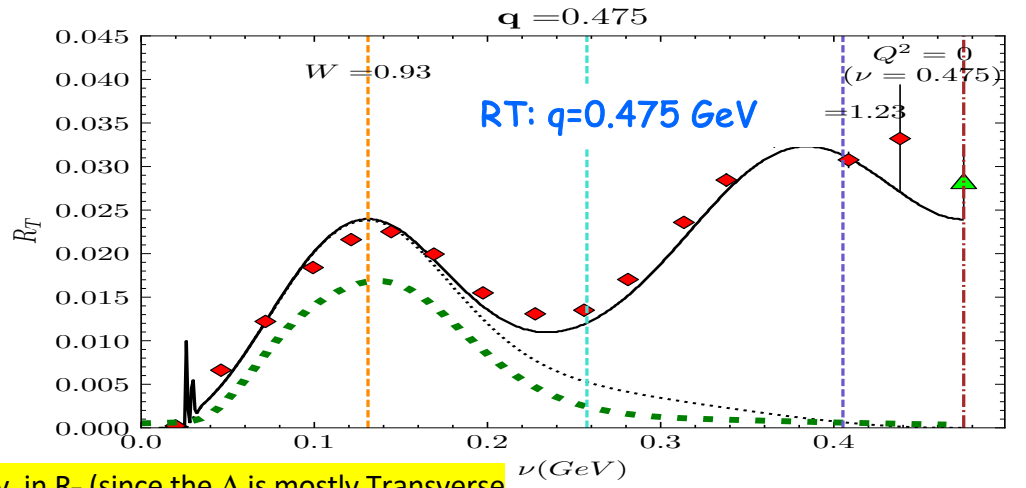
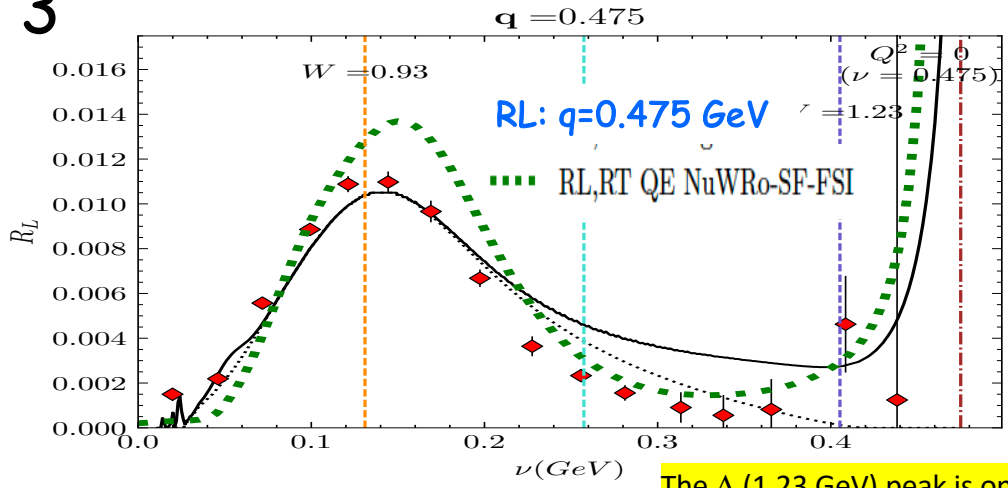
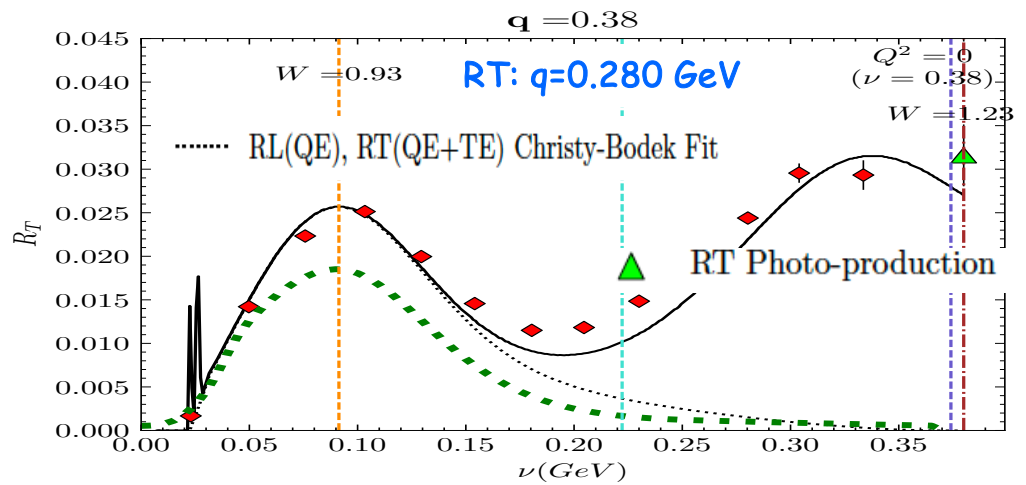
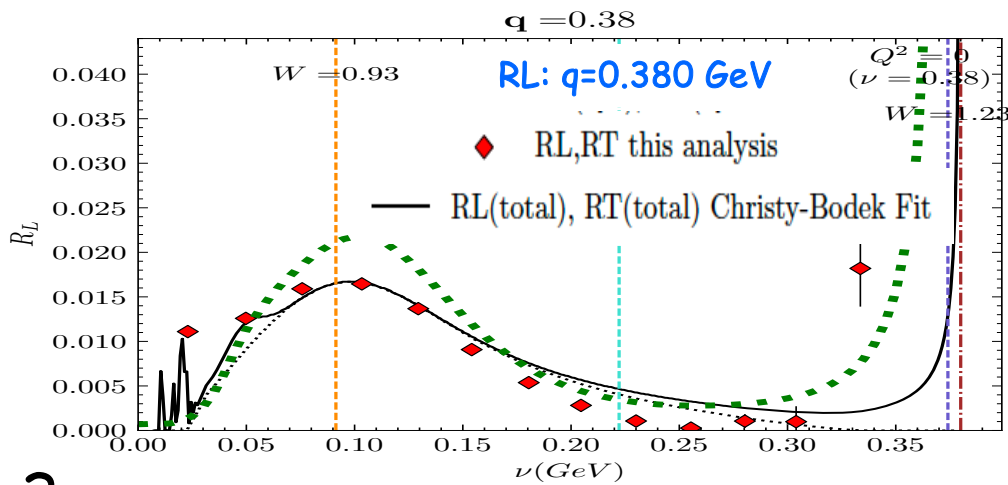
◆ R_L, R_T Barreau

Extraction of Individual Measurements of R_L and R_T

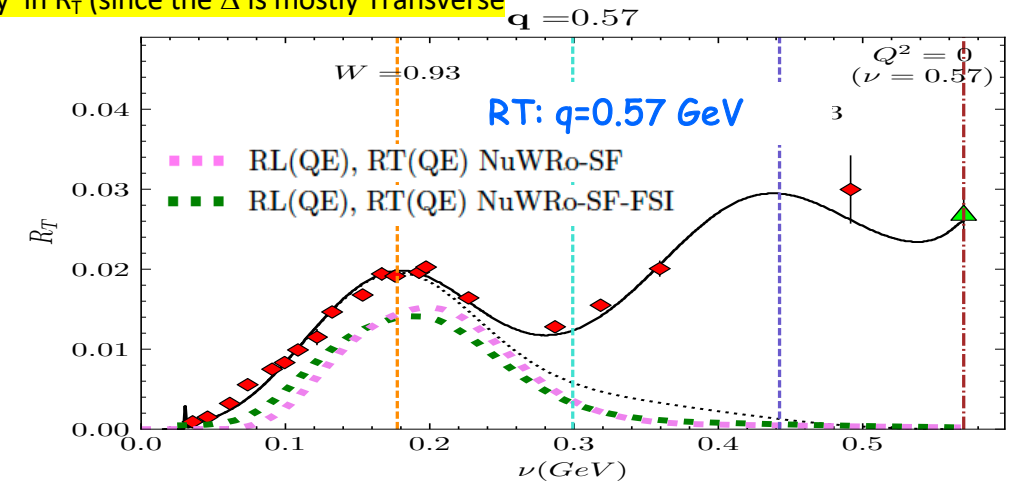
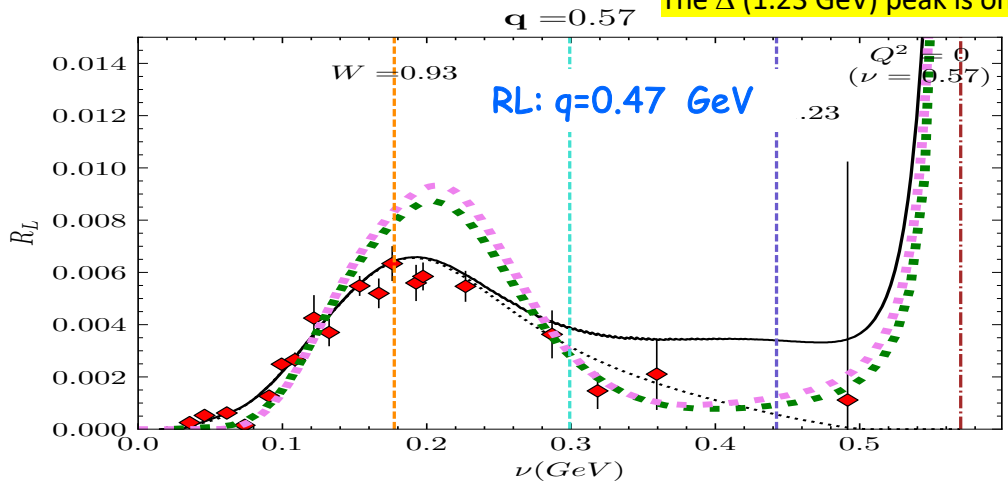
- The most effective way to validate **theoretical models** and neutrino and electron scattering MC generators is to compare to R_L and R_T measurements at fixed values of \mathbf{q} (or Q^2) versus ν . This way, the entire kinematic range at all scattering angles can be validated.
- The Christy-Bodek universal fit provides fits to R_L and R_T everywhere. However, it is also important to validate the universal fit everywhere.
- The Christy-Bodek universal fit includes all 16,000 cross sections measurements. In order to extract we need cross section measurements at the same \mathbf{q} (or Q^2) and ν , but at both at small and large.
- Therefore, only a subset of the cross sections can be used in individual R_L and R_T extractions since some cross sections are **only measured at small angles** and some are **only measured at large angles** (but at different \mathbf{q} (or Q^2) and ν). However, all cross sections are included in the Christy-Bodek universal fit .







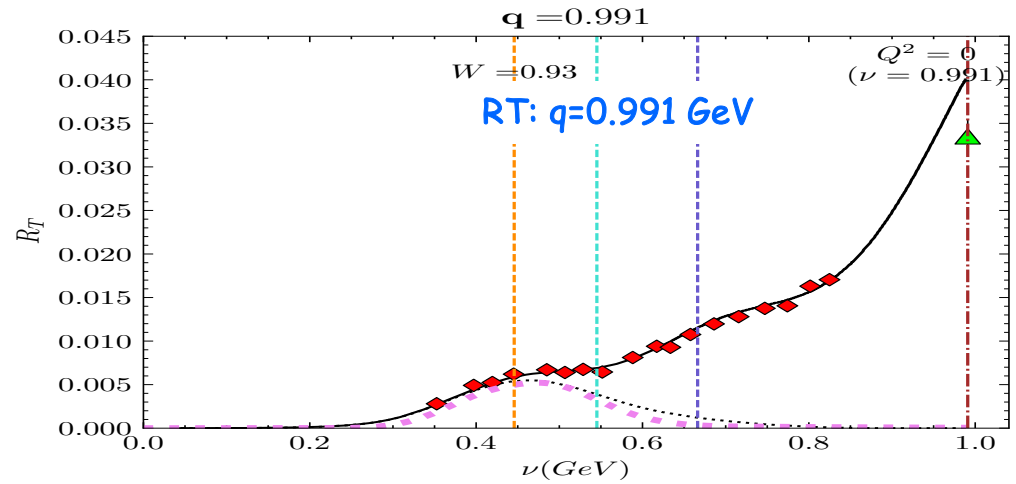
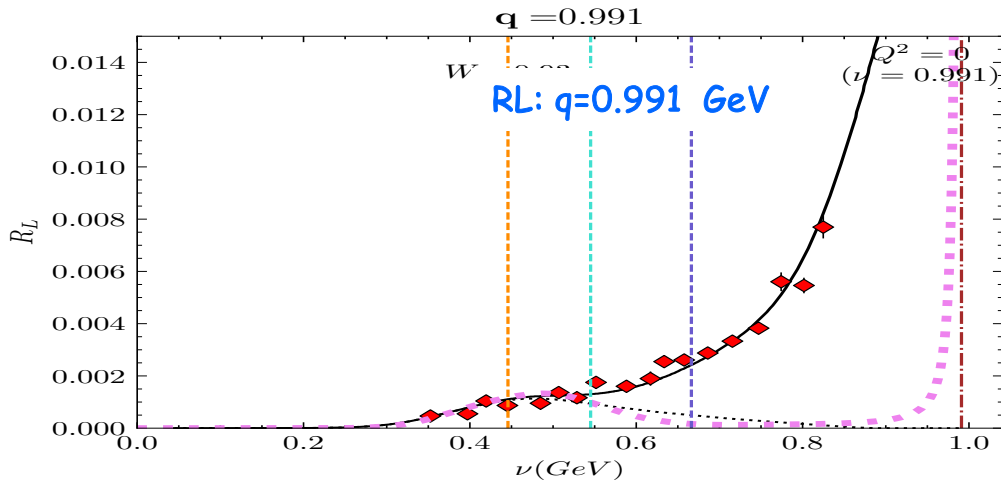
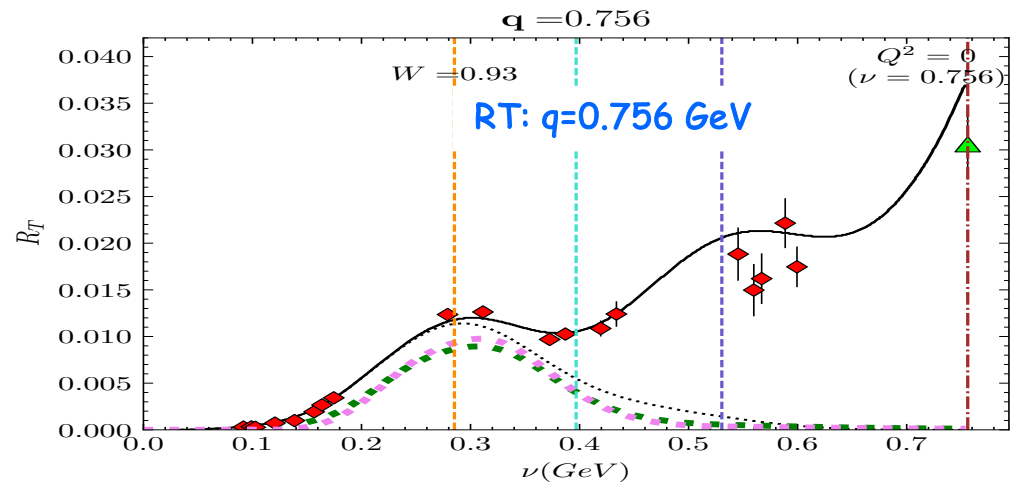
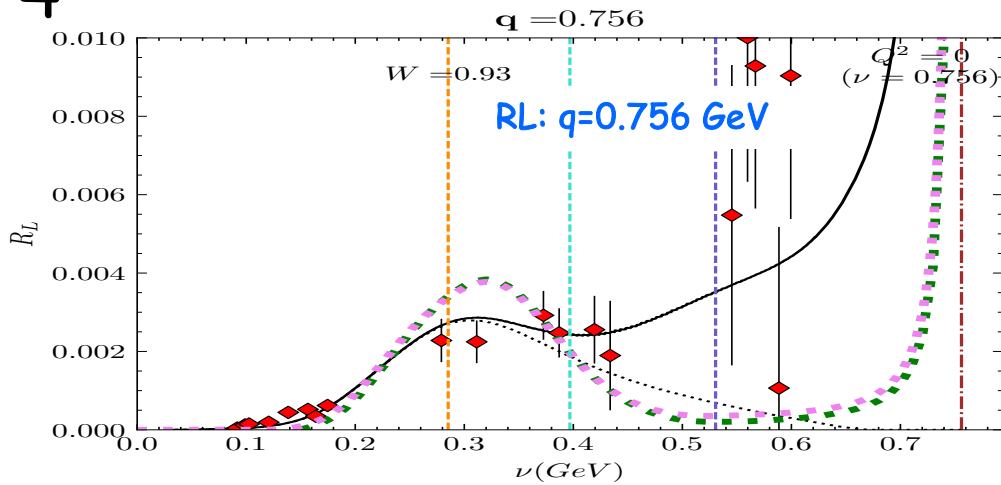
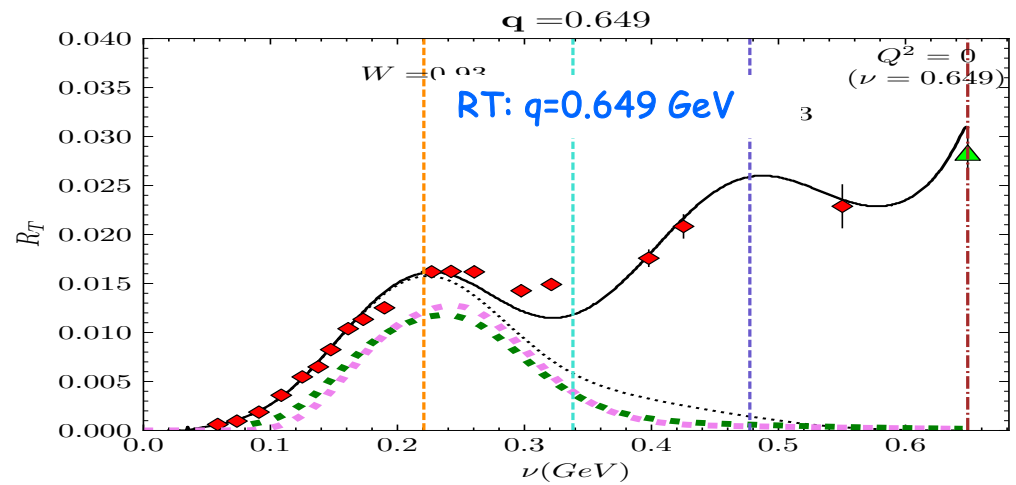
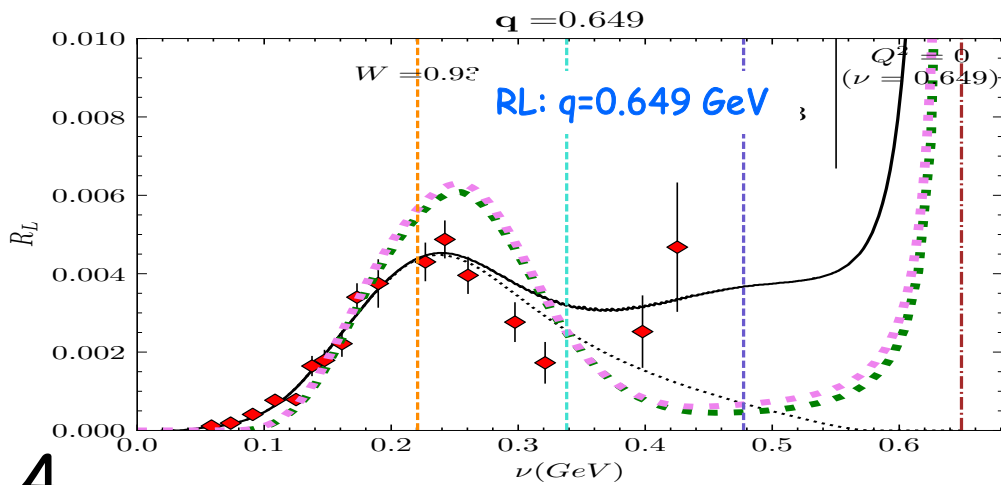
The Δ (1.23 GeV) peak is only in R_T (since the Δ is mostly Transverse)



— RL(total), RT(total) Christy-Bodek Fit
..... RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL, RT this analysis
..... RL(QE), RT(QE) NuWRo-SF-FSI

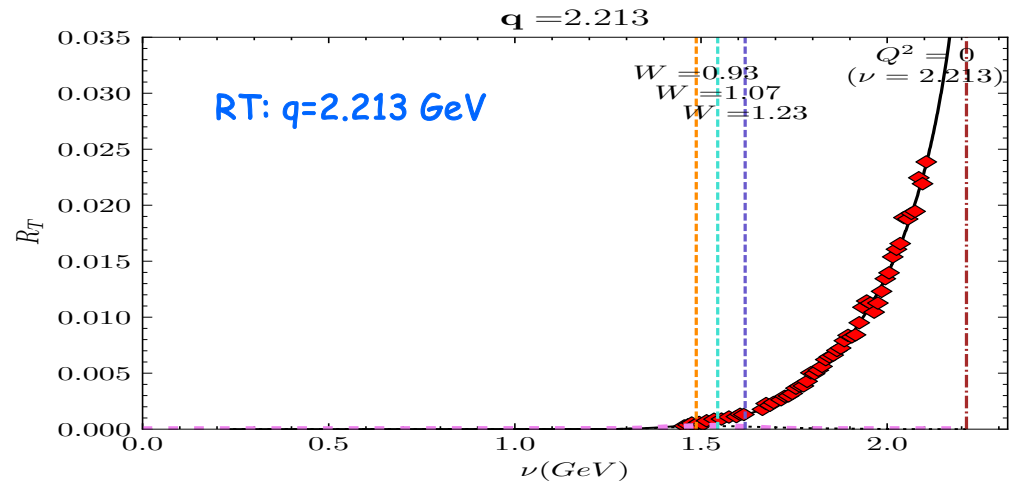
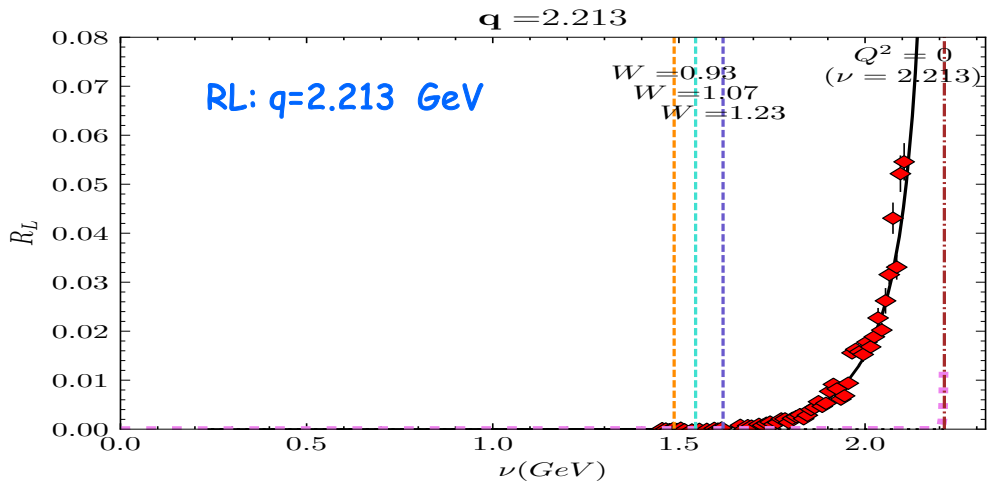
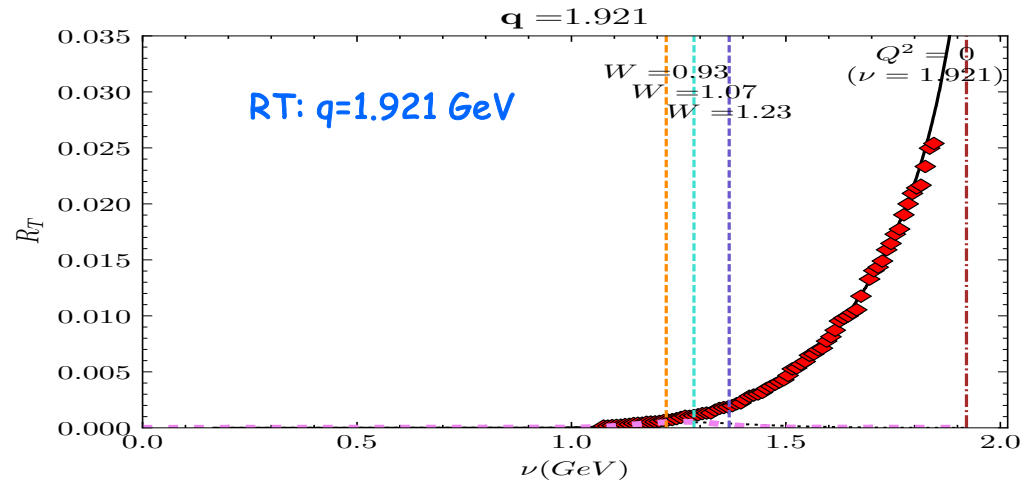
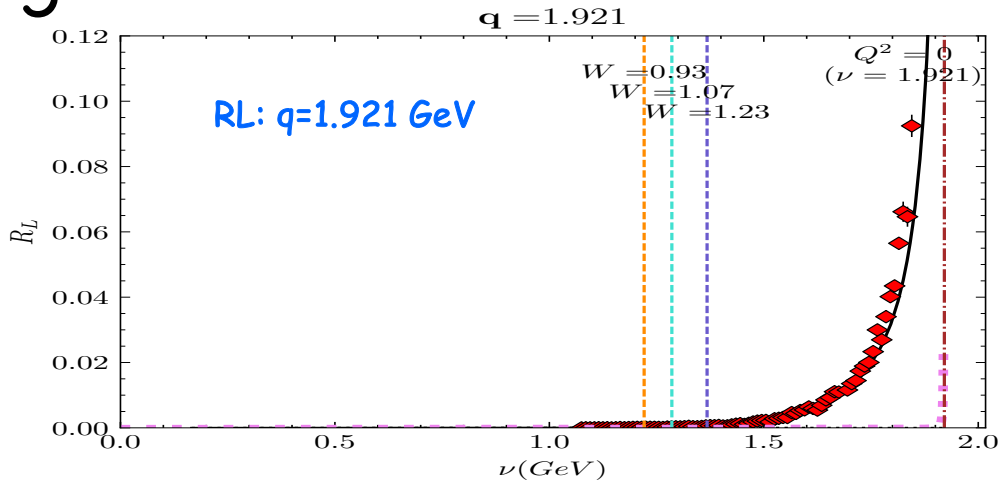
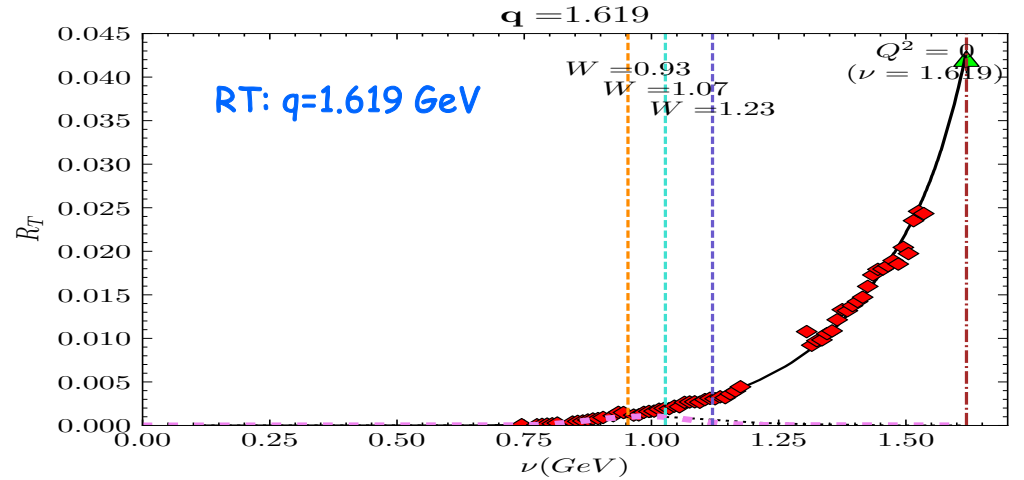
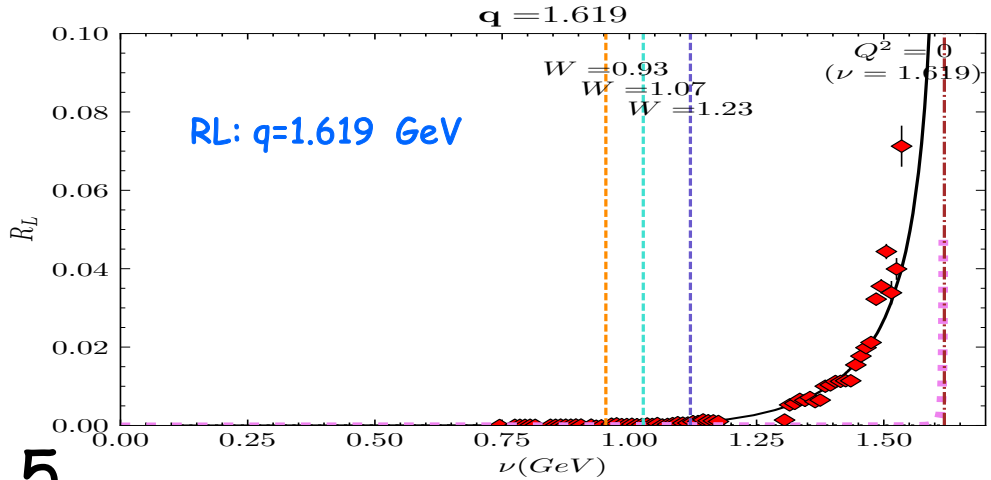
▲ RT Photo-production
..... RL(QE), RT(QE) NuWRo-SF



— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL, RT this analysis
 ■ RL(QE), RT(QE) NuWRo-SF-FSI

■ RL(QE), RT(QE) NuWRo-SF
 ▲ RT Photo-production

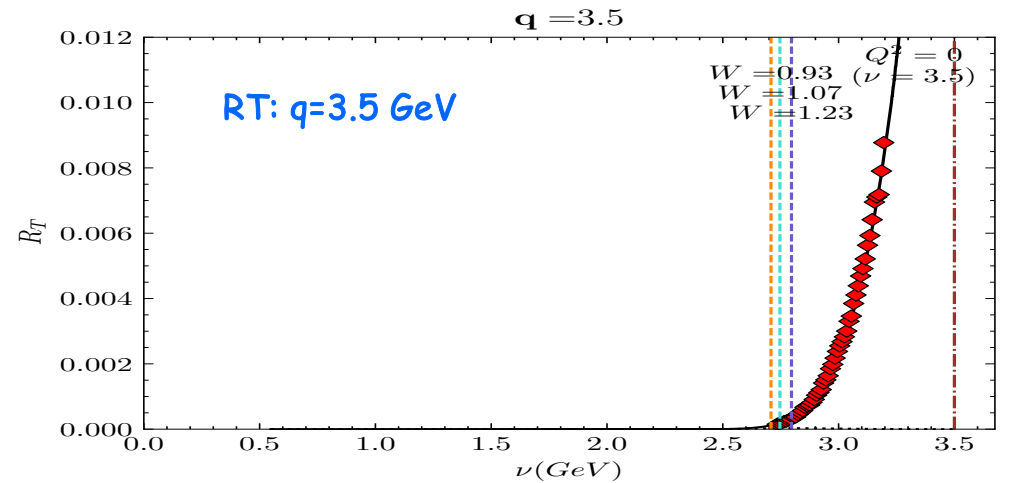
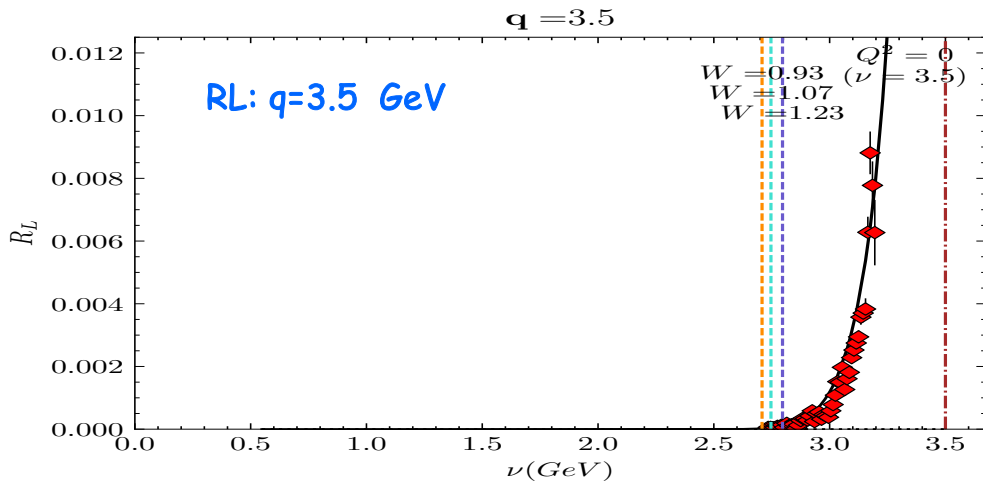
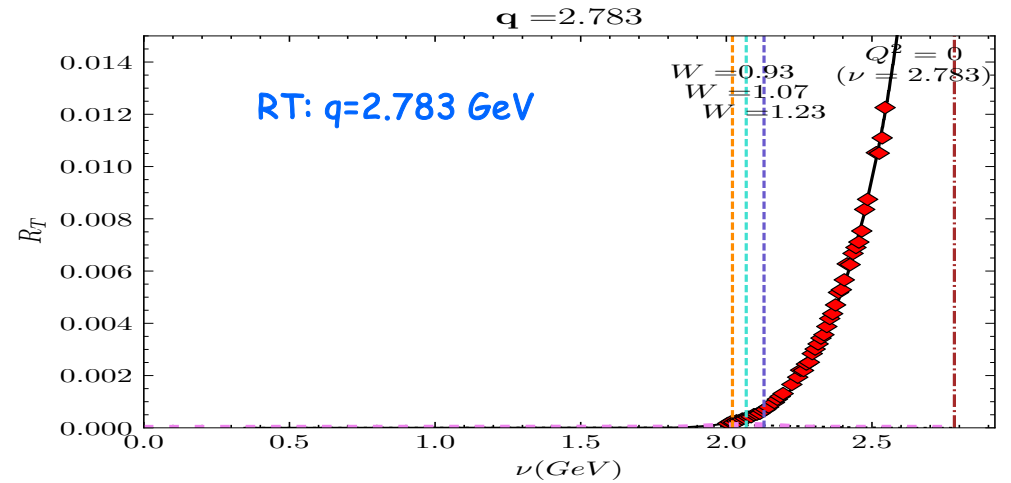
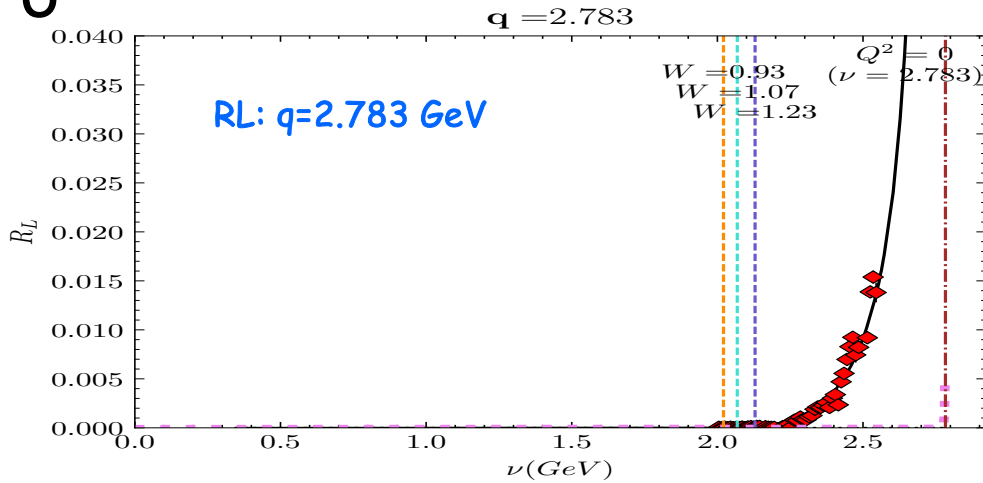
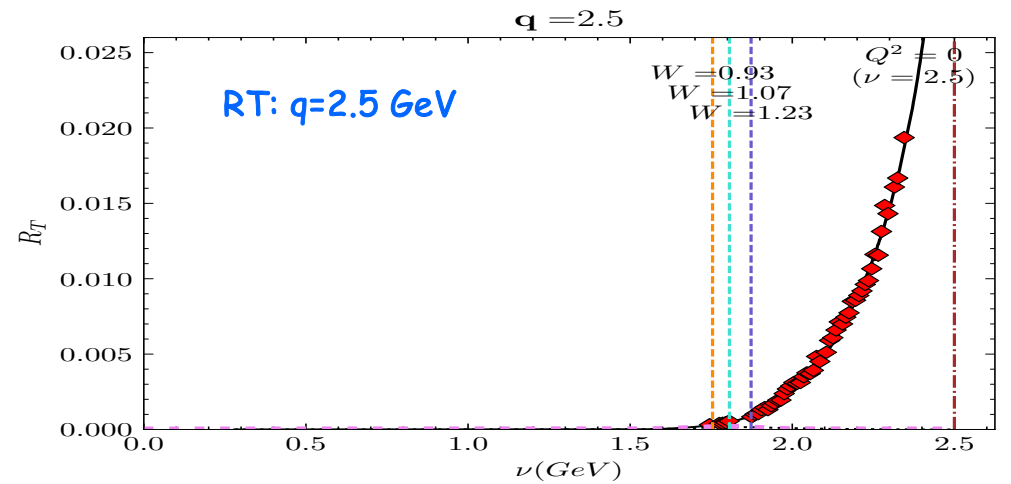
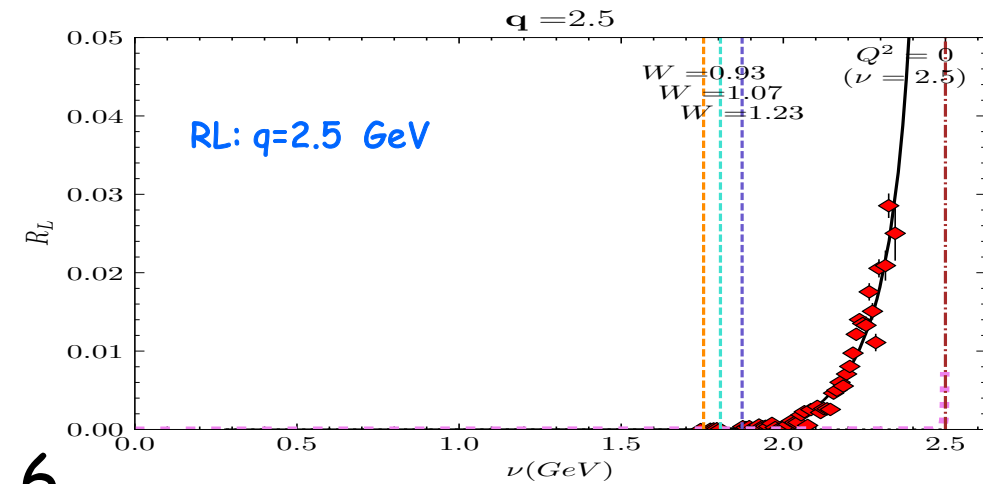


— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

♦ RL, RT this analysis
 - - - - - RL(QE), RT(QE) NuWRO-SF

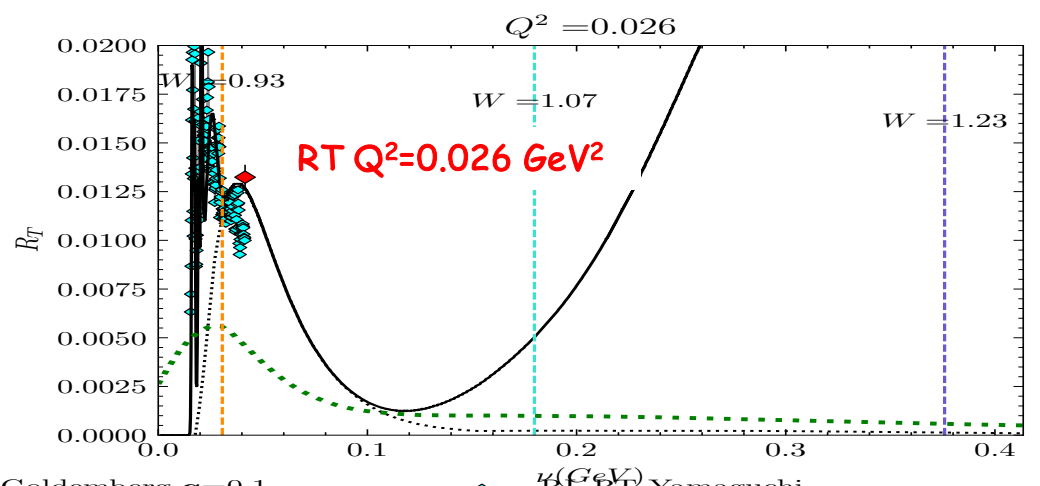
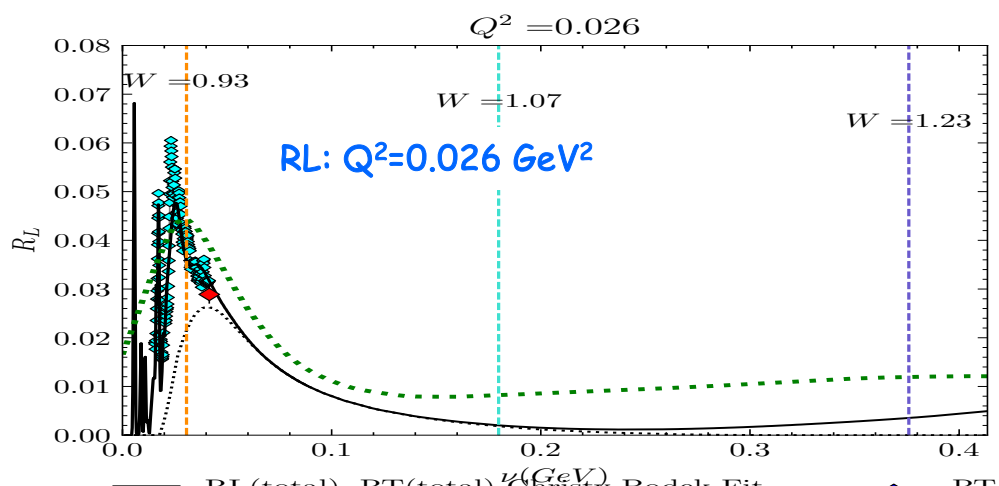
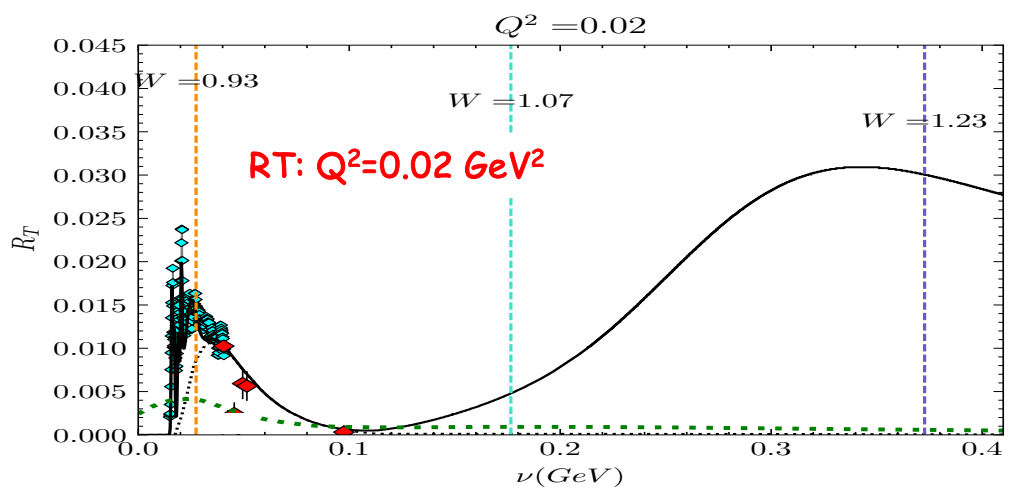
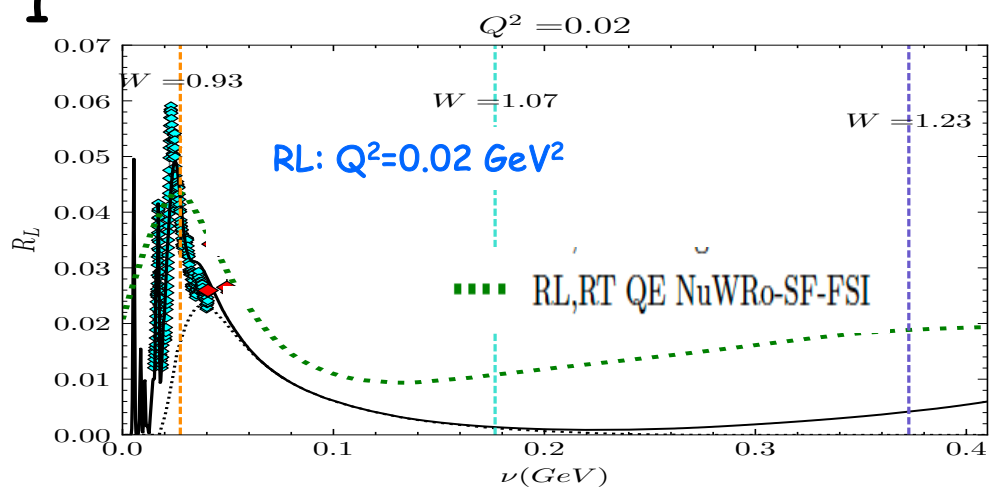
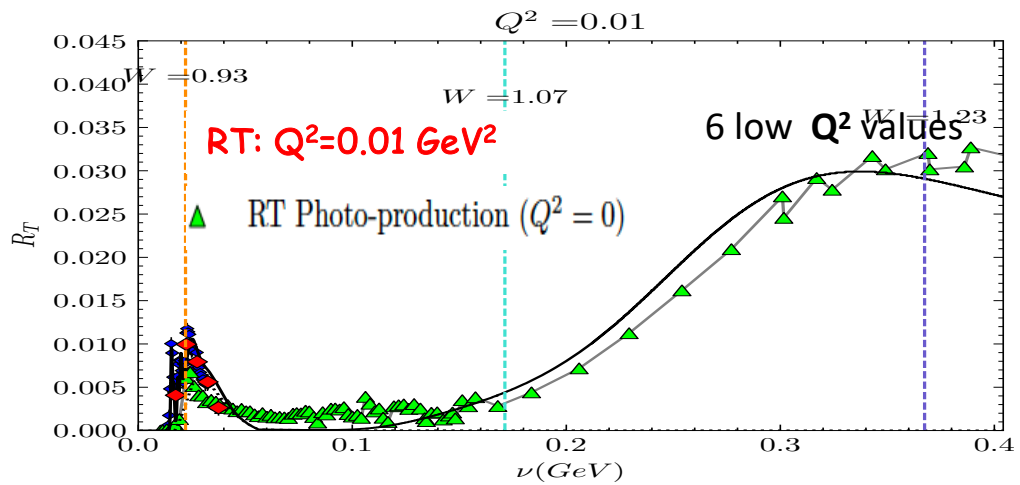
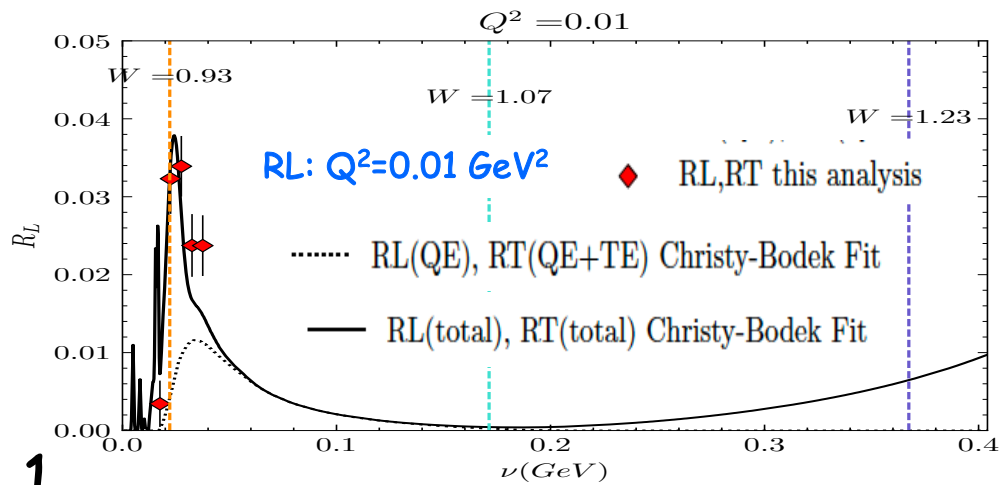
▲ RT Photo-production

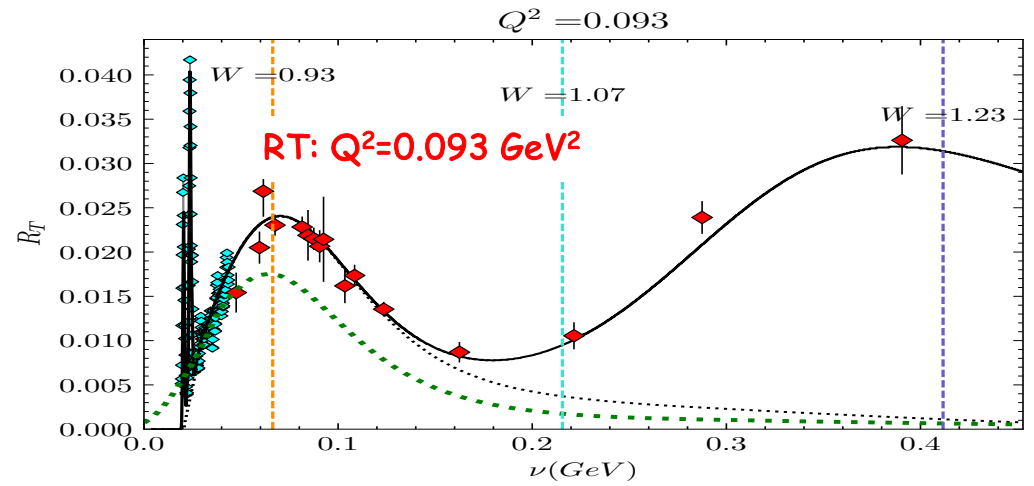
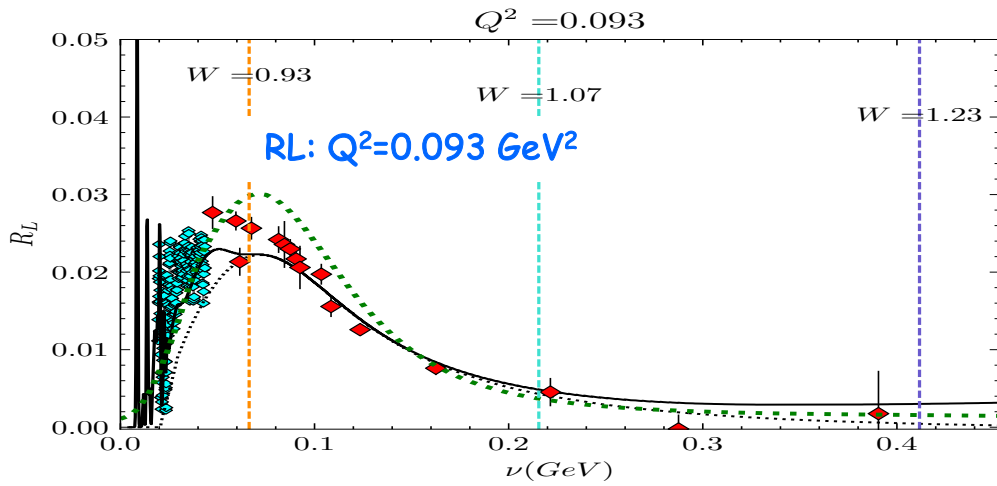
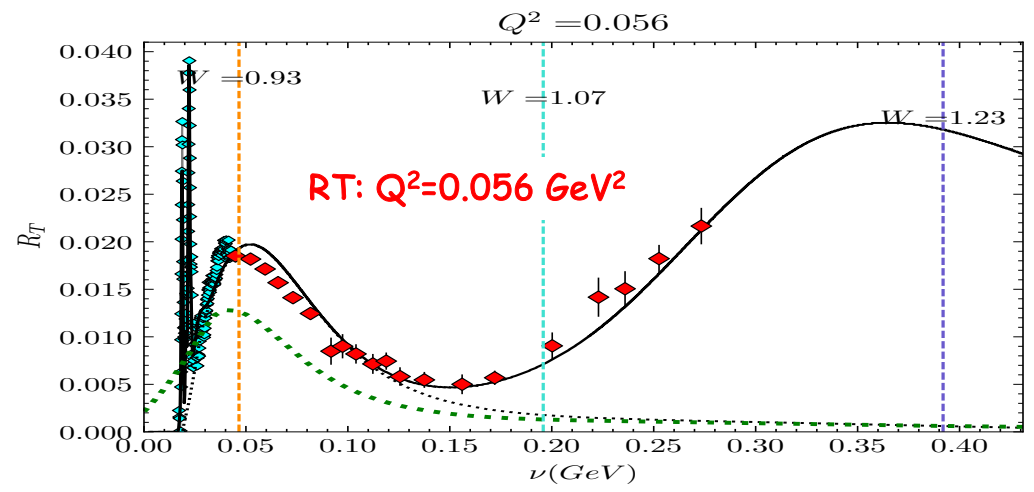
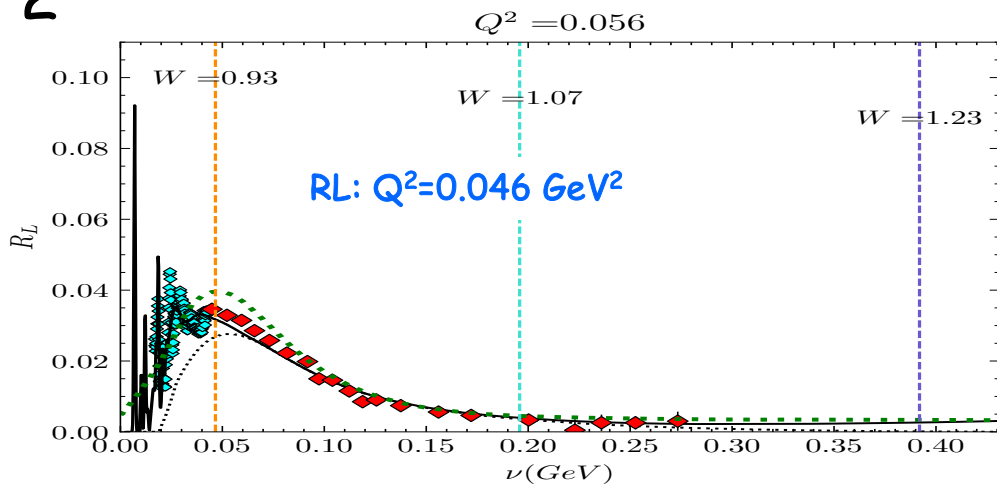
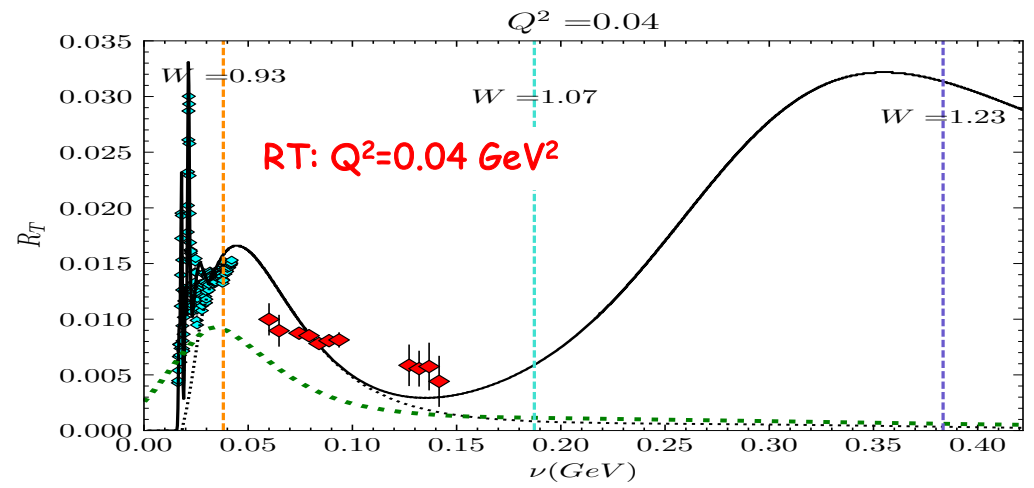
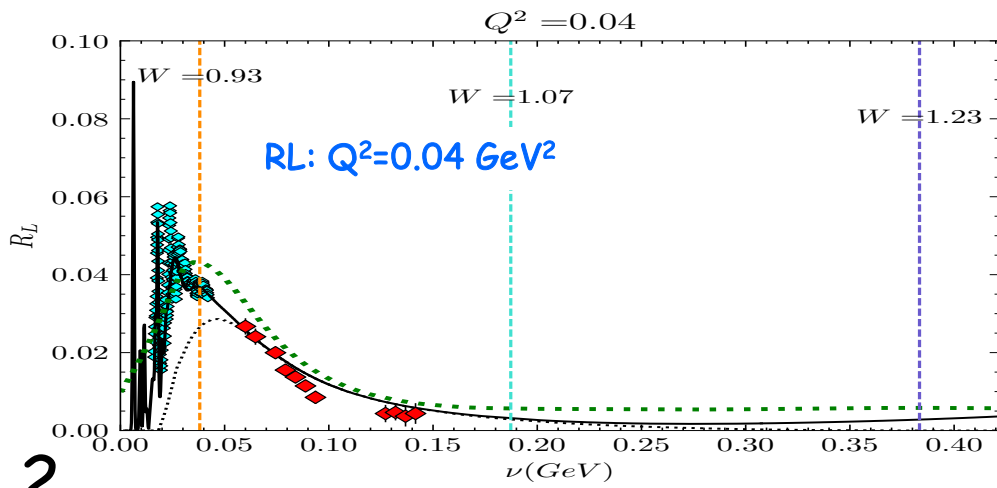
6



— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit
 ◆ RL, RT this analysis
 ■ RL(QE), RT(QE) NuWRo-SF

▲ RT Photo-production

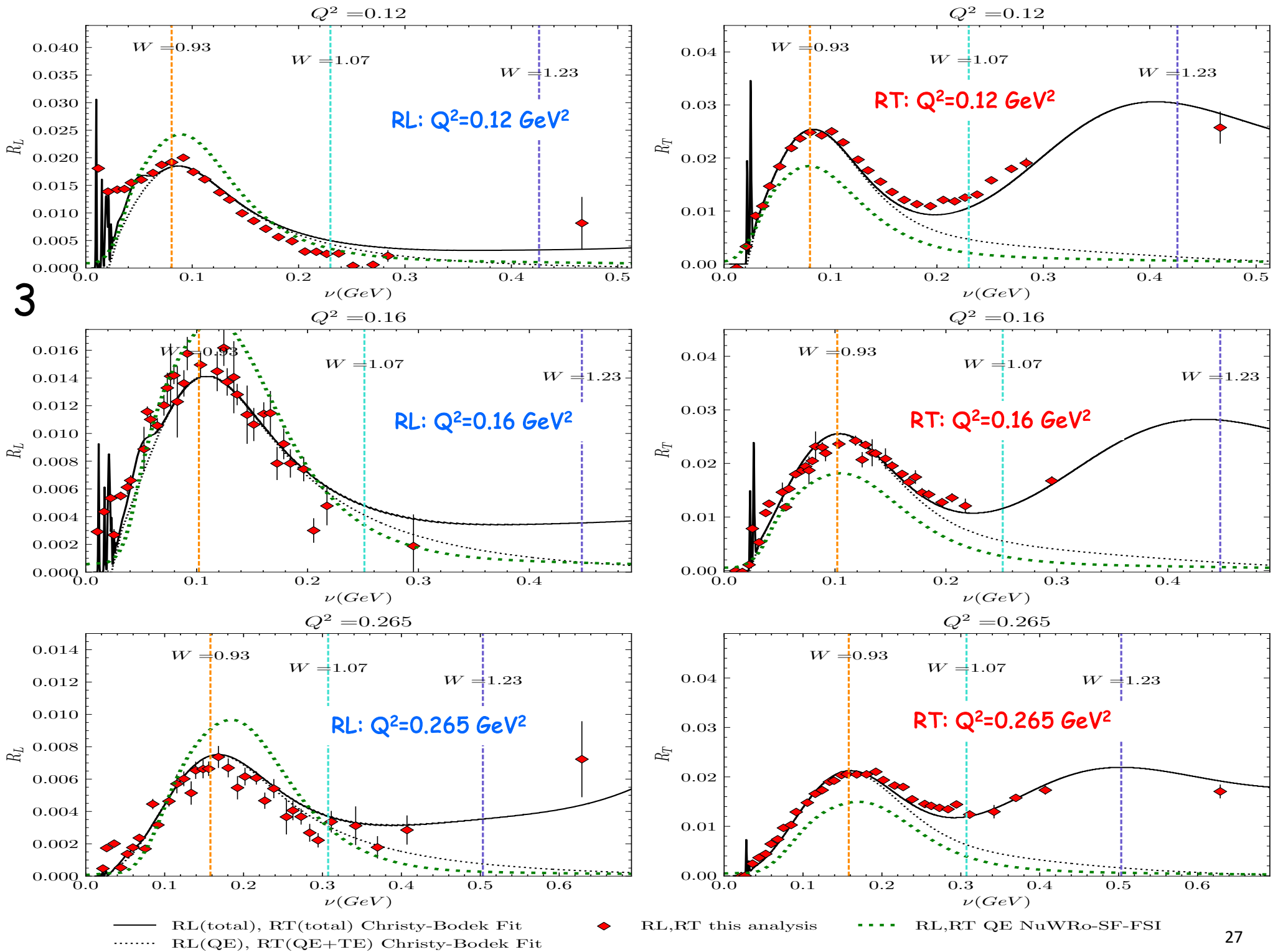


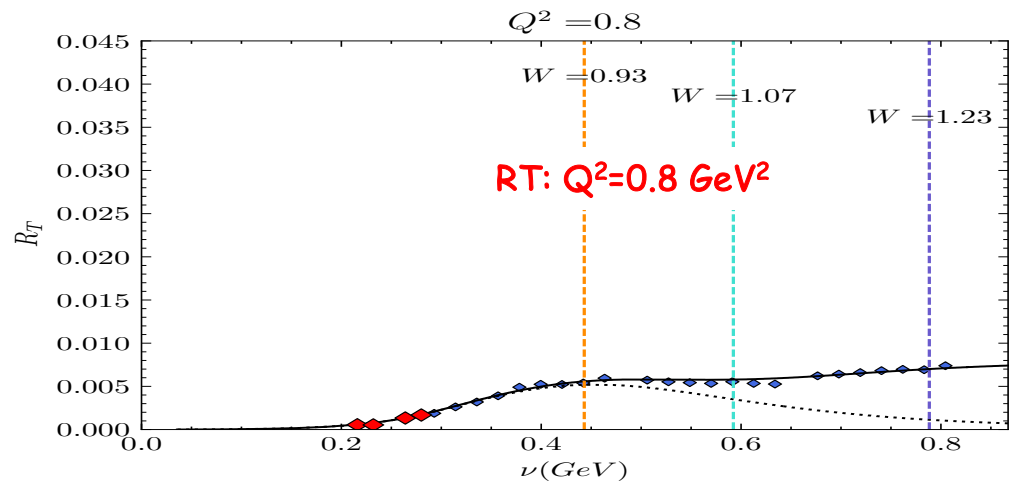
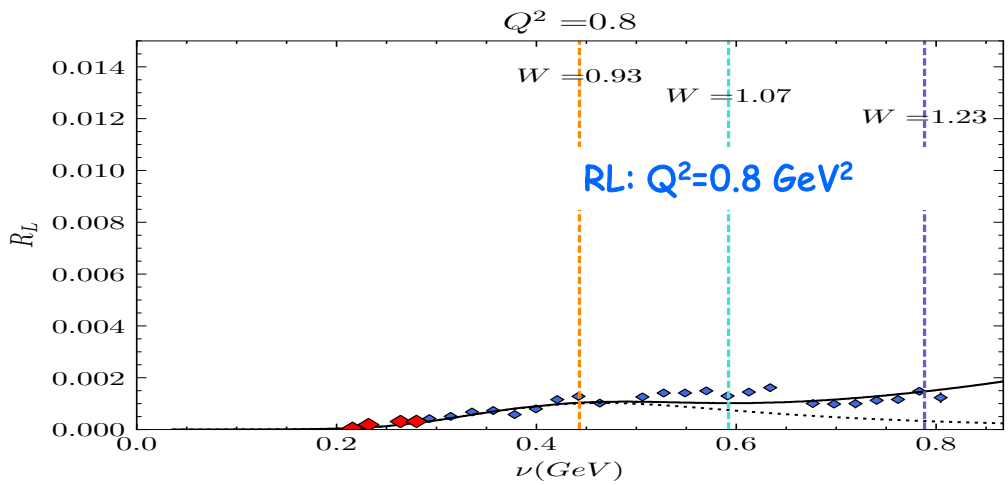
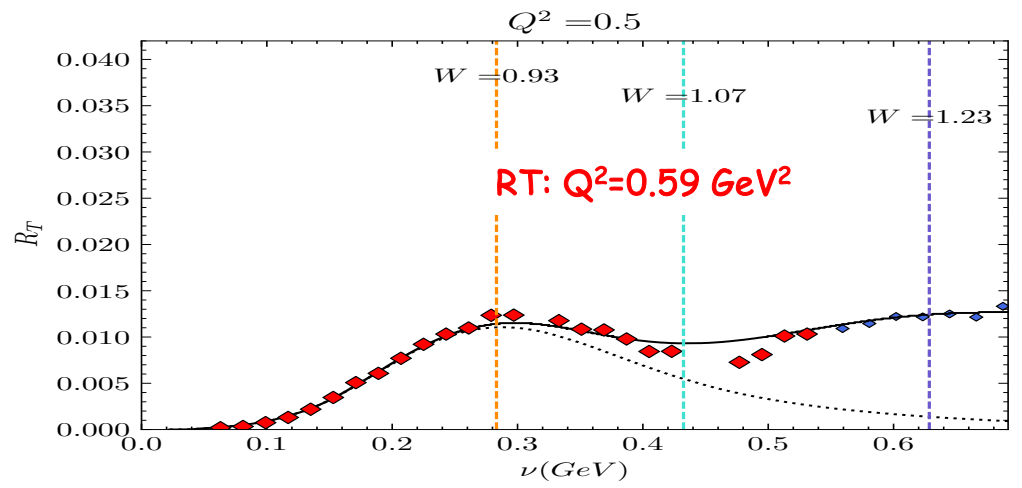
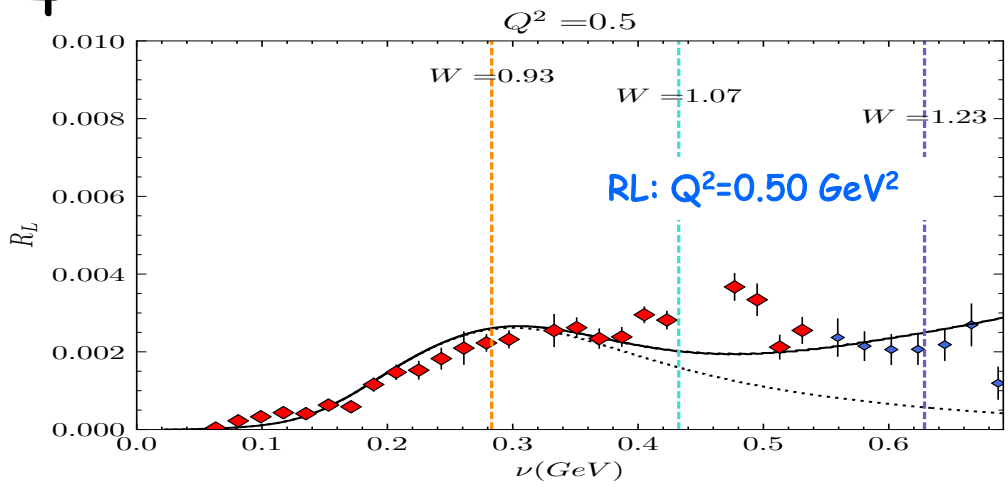
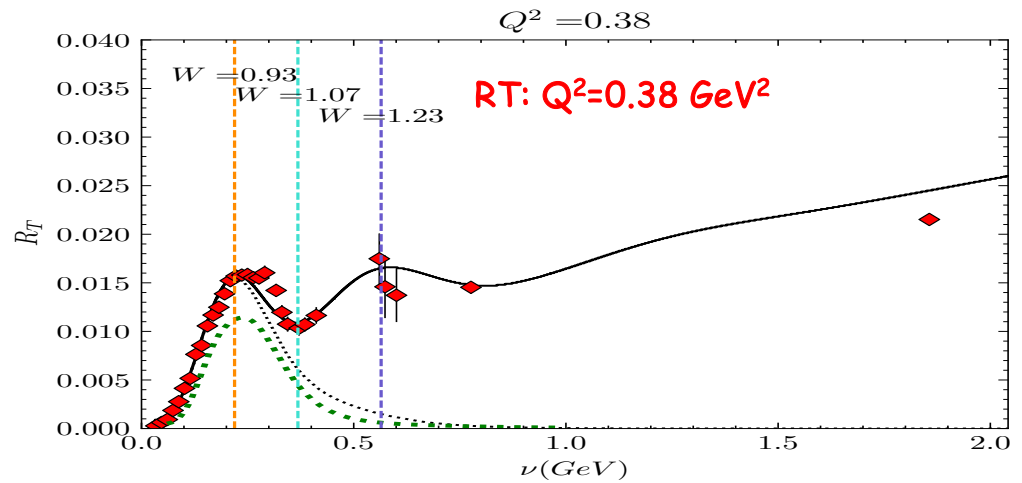
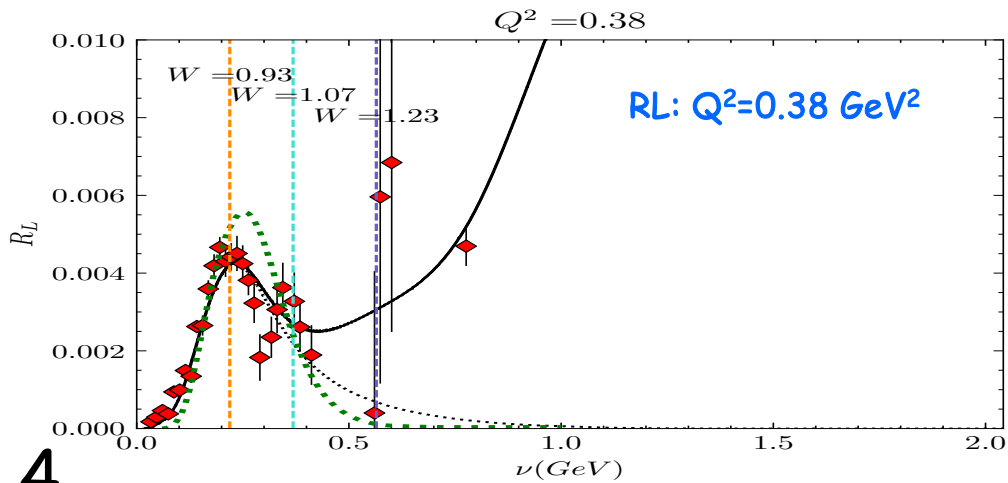


— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL,RT Yamaguchi
 ◆ RL,RT this analysis

..... RL,RT QE NuWro-SF-FSI



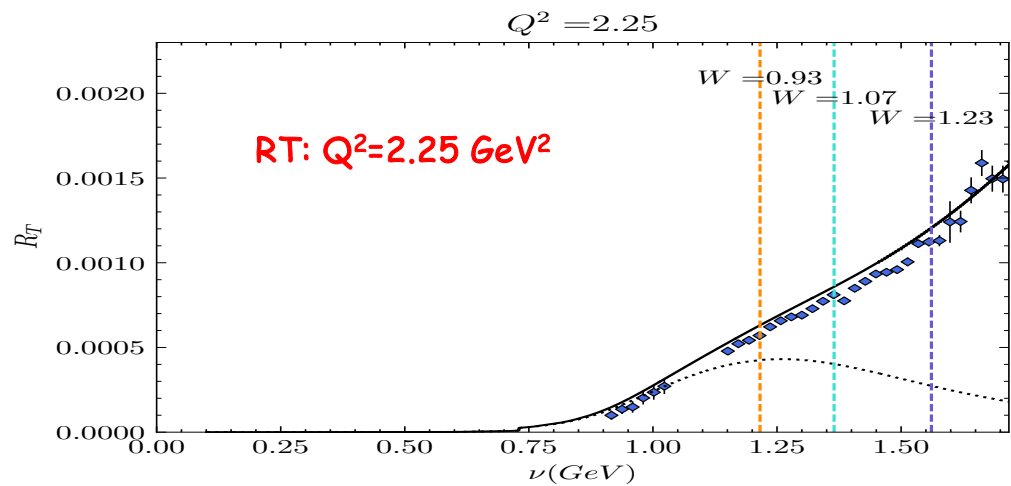
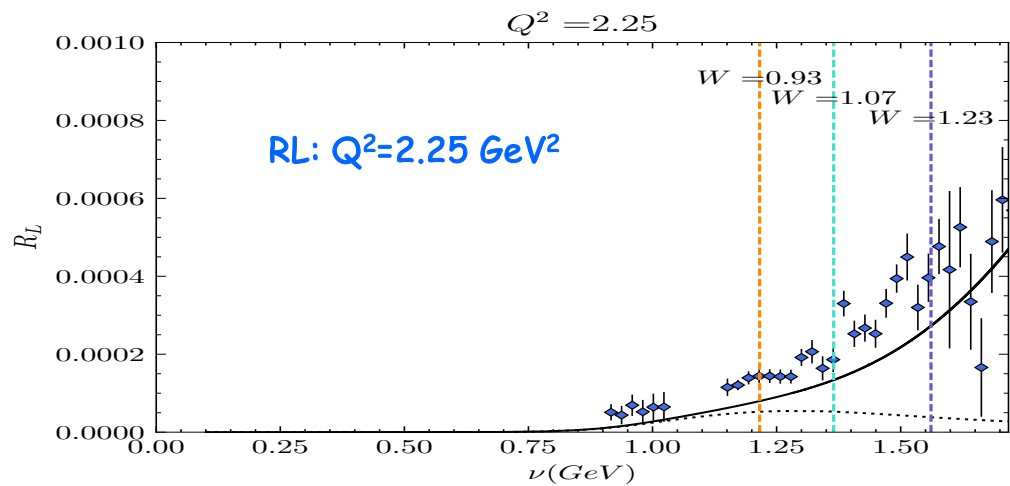
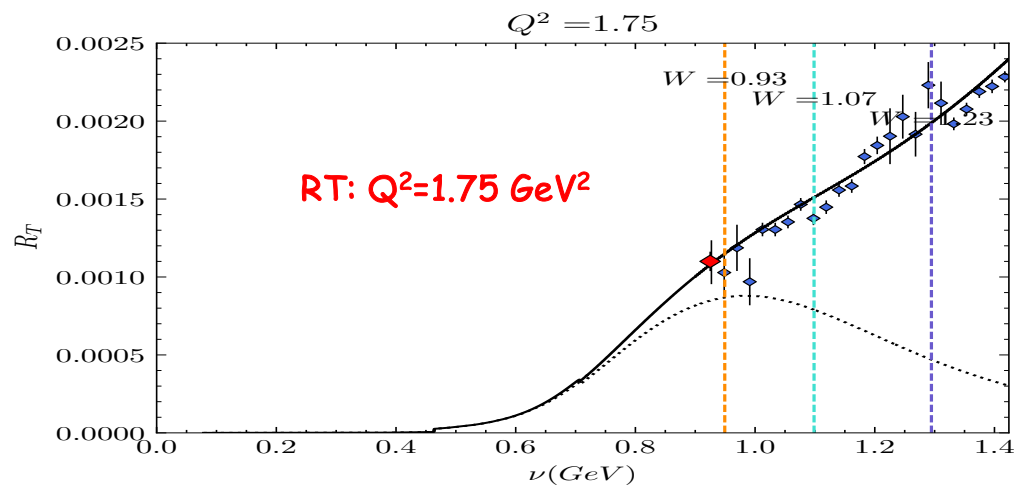
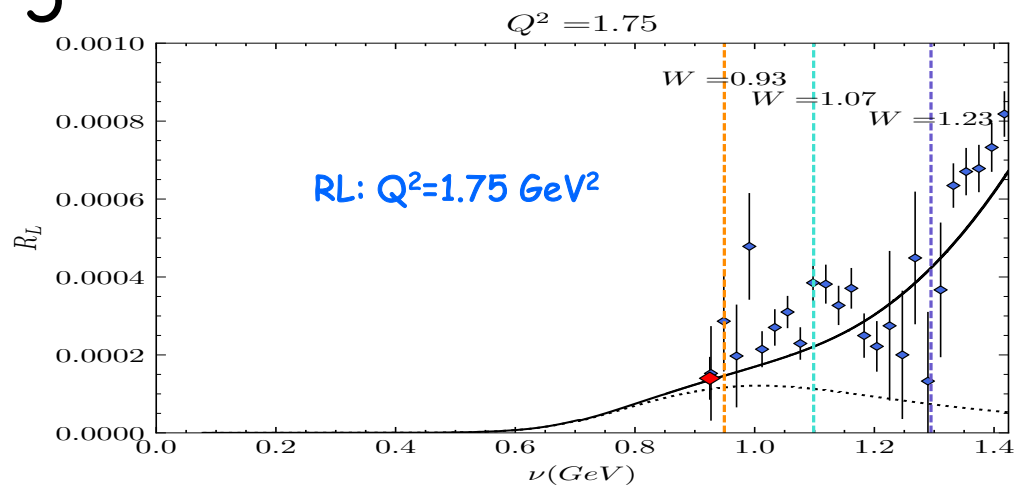
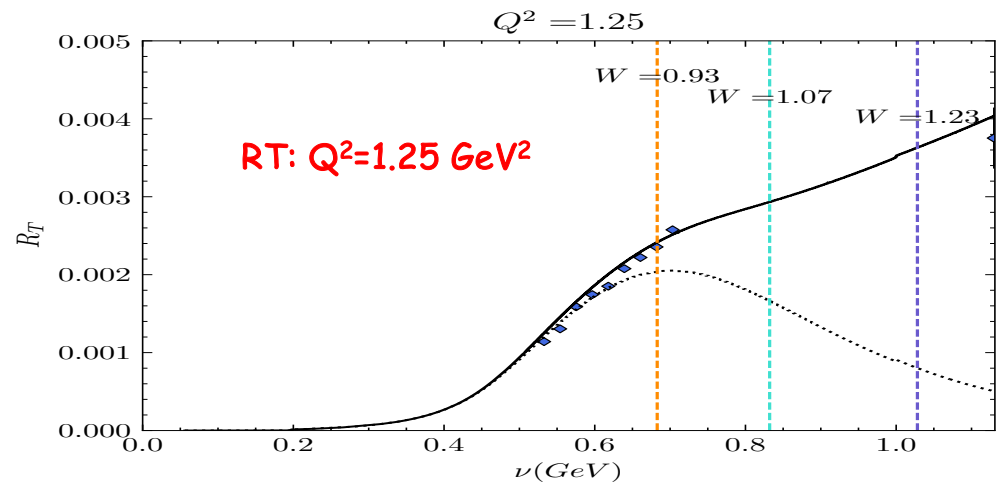
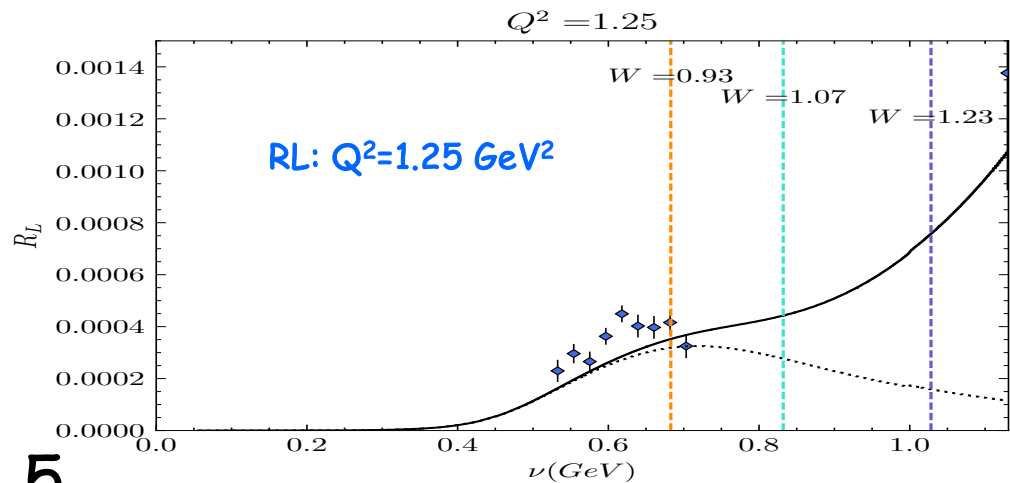


— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL,RT this analysis
 RL,RT QE NuWro-SF-FSI

◆ RL,RT JLab e04-001

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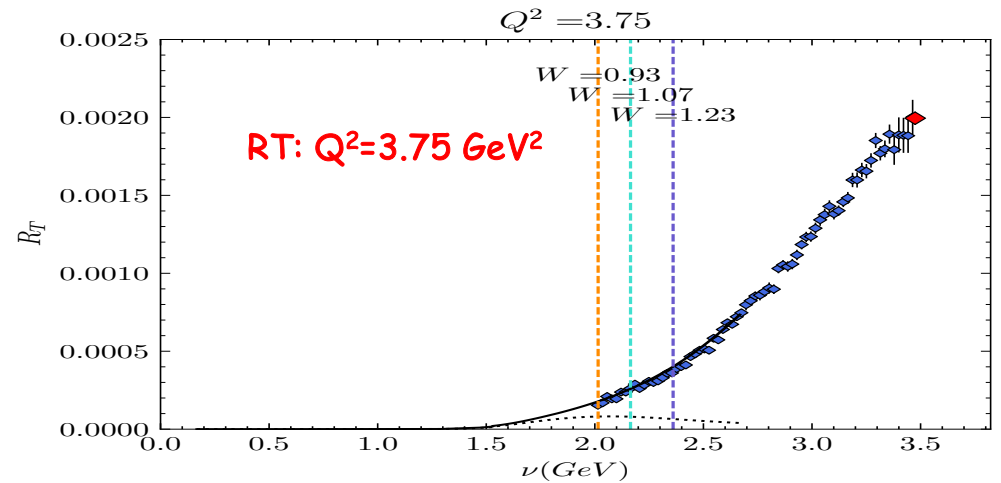
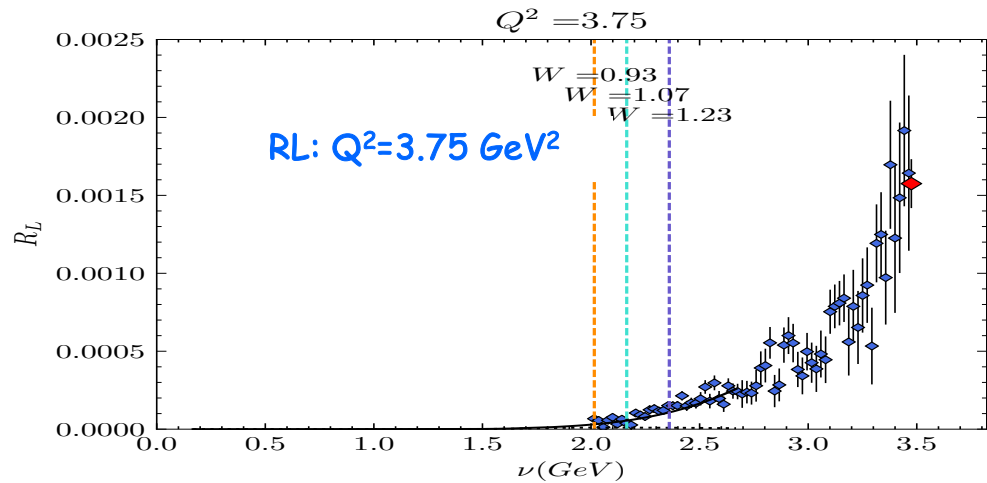
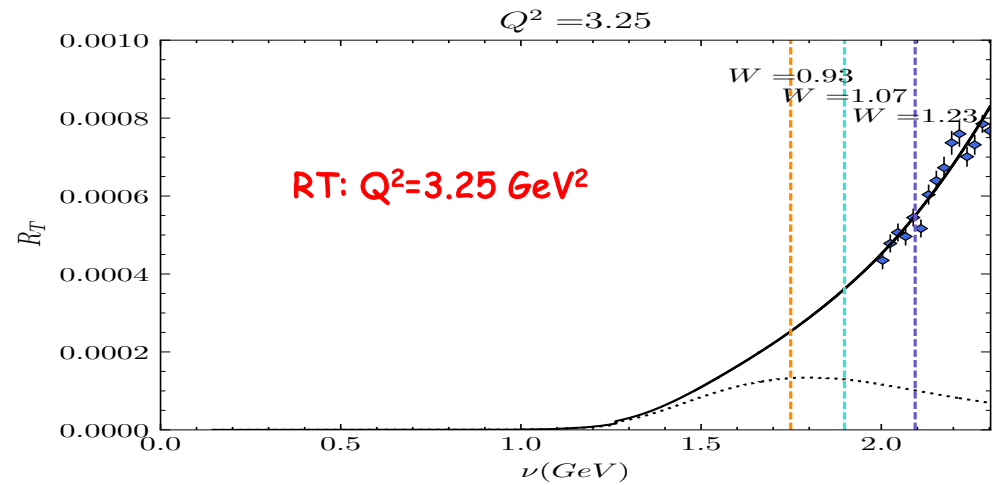
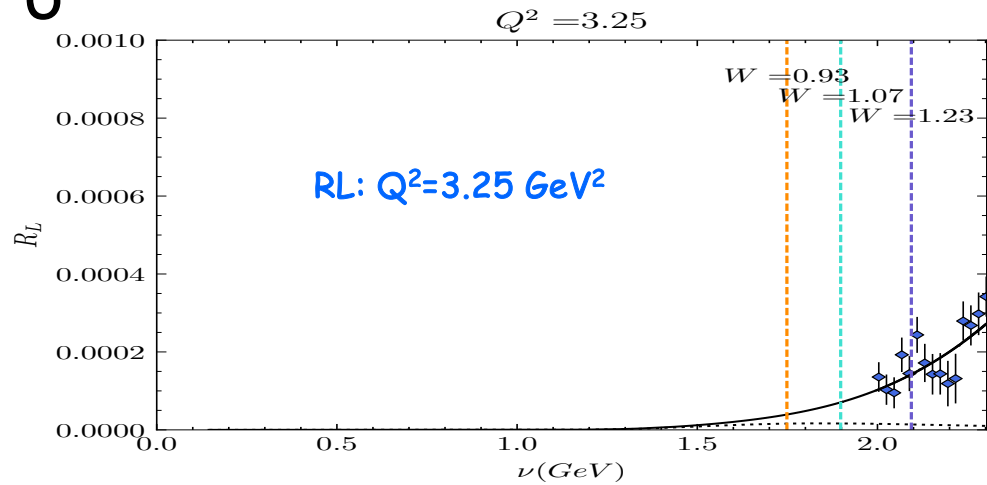
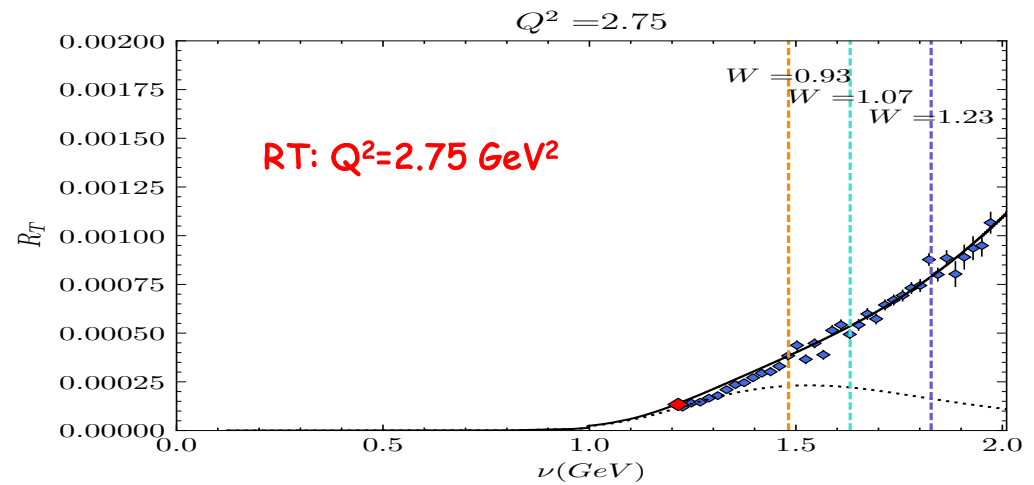
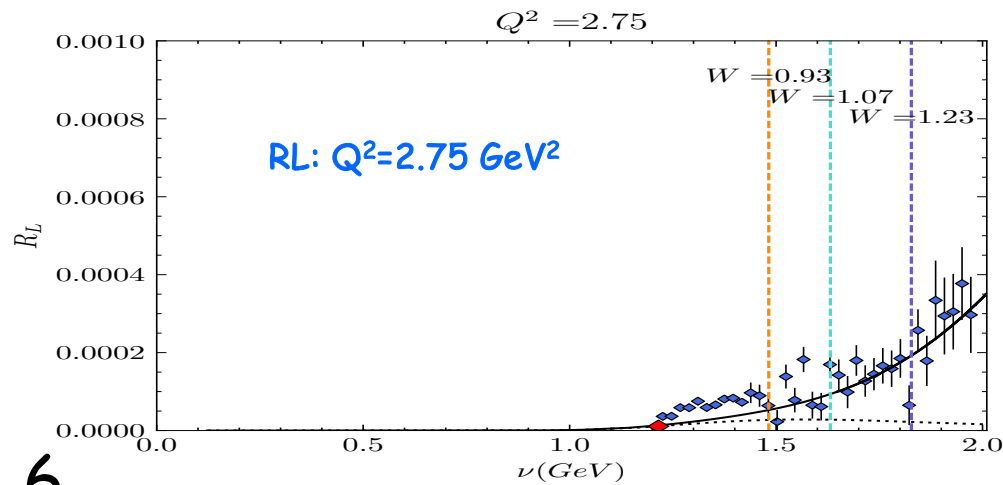


— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL,RT JLab e04-001

◆ RL,RT this analysis

6



— RL(total), RT(total) Christy-Bodek Fit
 RL(QE), RT(QE+TE) Christy-Bodek Fit

◆ RL,RT JLab e04-001 ◆ RL,RT this analysis

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Comments

- The extracted R_L and R_T are in good agreement with the Christy-Bodek Universal Fit. The fit can be used for validation of MC generators over a more extended range of q and ν .
- The Christy-Bodek fit includes **Longitudinal Quenching** at low q and **Transverse Enhancement** at intermediate q . It also includes nuclear excitations
- The Δ (1.23 GeV) peak is only seen in R_T (since the Δ is mostly Transverse).
- For fixed q the maximum value of ν is $\nu=q$ (where R_T can also be extracted from photoproduction data).
- Validation of Neutrino Generator NuWRo in electron model: NuWRo only models QE with a spectral function, and adds Final State Interaction (FSI)
- NuWRo (spectral function) requires FSI for better agreement with the data.
- However, even with FSI for $q < 0.3$ GeV NuWRo overestimates R_L (**requires Longitudinal Quenching**)
- NuWRo underestimates R_T (in neutrino mode it has Transverse Enhancement/MEC, but not in electron mode).
- Nuclear excitations are not included
- Comparisons with GENIE (in electron mode) will be available shortly.
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Conclusions

- The 18 R_L and R_T extractions cover a large kinematic range. The values are in excellent agreement with the Christy-Bodek Universal fit to all cross section values. The universal fit covers an even larger kinematic range.
- The R_L and R_T measurements as well as the universal fit provide a simple way to validate electron and neutrino MC generators over the entire kinematic range of interest.
- Good agreement in the QE region with nuclear theory for 3 values of q . Predictions for all other values of q not yet available.

In Supplemental Materials we will provide

Tables of the extracted values of R_L and R_T

- Tables of the Christy-Bodek Universal fit values for R_L and R_T for the 18 q and Q^2 values. The contributions of nuclear excitations, QE, transverse enhancement and inelastic scattering will be listed separately.
- Code for the Christy-Bodek Universal fit

Backup