Extraction of the ^{12}C Longitudinal (R_L) and (R_T) Nuclear Electromagnetic Response Functions from *all* Electron Scattering Measurements on Carbon

1. Testing first principle nuclear theory predictions
2. Provide a platform for verification of electron and neutrino MC generators over the entire kinematic range of interest

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Presented by Zihao Lin

American Physical Society April Meeting 2024

In-Person & Virtual APS April Meeting: April 3 - 6, 2024, in Sacramento, California

10 min talk + 2 min questions

Validation of MC generators using new extractions of ^{12}C electromagnetic longitudinal and transverse response functions

We extract ^{12}**C** Longitudinal (**R_L**) and **(R_T)** Nuclear Electromagnetic Response Functions from *all* Electron Scattering Measurements on Carbon for:

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Presented by A. Bodek

2nd Short-Baseline Experiment-Theory Workshop 2024, April 2-5, Santa Fe

25 min + min questions

Nuclear Physics

This description is primarily used in the nuclear excitation and QE regions. The electron scattering differential cross section is written in terms of longitudinal ($\mathcal{R}_L(Q^2,\nu)$) and transverse ($\mathcal{R}_T(Q^2,\nu)$) nuclear response functions [24]

$$\frac{d\sigma}{d\nu d\Omega} = \sigma_M[A\mathcal{R}_L(Q^2,\nu) + B\mathcal{R}_T(Q^2,\nu)] \quad (20)$$

where σ_M is the Mott cross section, $A = (Q^2/\mathbf{q}^2)^2$ and $B = \tan^2(\theta/2) + Q^2/2\mathbf{q}^2$.

Particle Physics

This description is primarily used in the inelastic continuum region. In the one-photon-exchange approximation, the spin-averaged cross section for inclusive electron-proton scattering can be expressed in terms of two structure functions as follows

$$\frac{d\sigma}{d\Omega dE'} = \sigma_M[\mathcal{W}_2(W^2,Q^2) + 2\tan^2(\theta/2)\mathcal{W}_1(W^2,Q^2)]$$

$$\sigma_M = \frac{\alpha^2\cos^2(\theta/2)}{[2E_0 sin^2(\theta/2)]^2} = \frac{4\alpha^2 E'^2}{Q^4}\cos^2(\theta/2) \quad (10)$$

Q^2 = 4-momentum transfer squared

$$Q^2 = (-q)^2 = 4E_0E'\sin^2\frac{\theta}{2},$$

$$\nu = E_0 - E'.$$

W^2 =final state invariant mass squared

$$W^2 = M^2 + 2M\nu - Q^2$$

$\mathbf{q}^2$ = 3-momentum transfer squared

$$\mathbf{q}^2 = Q^2 + \nu^2$$

E_x = Excitation energy

$$E_x = \nu - \nu_{elastic}$$

$$x = Q^2/(2M\nu).$$

$$\mathcal{F}_1 = M\mathcal{W}_1 \text{ and } \mathcal{F}_2 = \nu\mathcal{W}_2.$$

$$\mathcal{F}_L(x,Q^2) = \mathcal{F}_2\left(1 + \frac{4M^2x^2}{Q^2}\right) - 2x\mathcal{F}_1,$$

$$\mathcal{R}_T(\mathbf{q},\nu) = \frac{2\mathcal{F}_1(\mathbf{q},\nu)}{M}$$

$$\mathcal{R}_L(\mathbf{q},\nu) = \frac{\mathbf{q}^2}{Q^2}\frac{\mathcal{F}_L(\mathbf{q},\nu)}{2Mx}$$

A. Description in terms of longitudinal and transverse virtual photon cross sections

This description is often used in the resonance region. In the one-photon-exchange approximation, the spin-averaged cross section for inclusive electron-proton scattering can be expressed in terms of the photon helicity coupling as

$$\frac{d\sigma}{d\Omega dE'} = \Gamma \left[\sigma_T(W^2, Q^2) + \epsilon\sigma_L(W^2, Q^2)\right], \qquad (6)$$

where σ_T (σ_L) is the cross section for photo-absorption of purely transverse (longitudinal) polarized photons,

$$\Gamma = \frac{\alpha E'(W^2 - M_N^2)}{(2\pi)^2 Q^2 M E_0 (1-\epsilon)} \qquad (7)$$

is the flux of virtual photons, $\alpha = 1/137$ is the fine structure constant, and

$$\epsilon = \left[1 + 2(1 + \frac{\nu^2}{Q^2})\tan^2\frac{\theta}{2}\right]^{-1} \qquad (8)$$

is the relative flux of longitudinal virtual photons (sometimes referred to as the virtual photon polarization). Since Γ and ϵ are purely kinematic factors, it is convenient to define the reduced cross section

$$\sigma_r = \frac{1}{\Gamma}\frac{d\sigma}{d\Omega dE'} = \sigma_T(W^2, Q^2) + \epsilon\sigma_L(W^2, Q^2). \qquad (9)$$

All the hadronic structure information is therefore, contained in σ_T and σ_L, which are only dependent on W^2

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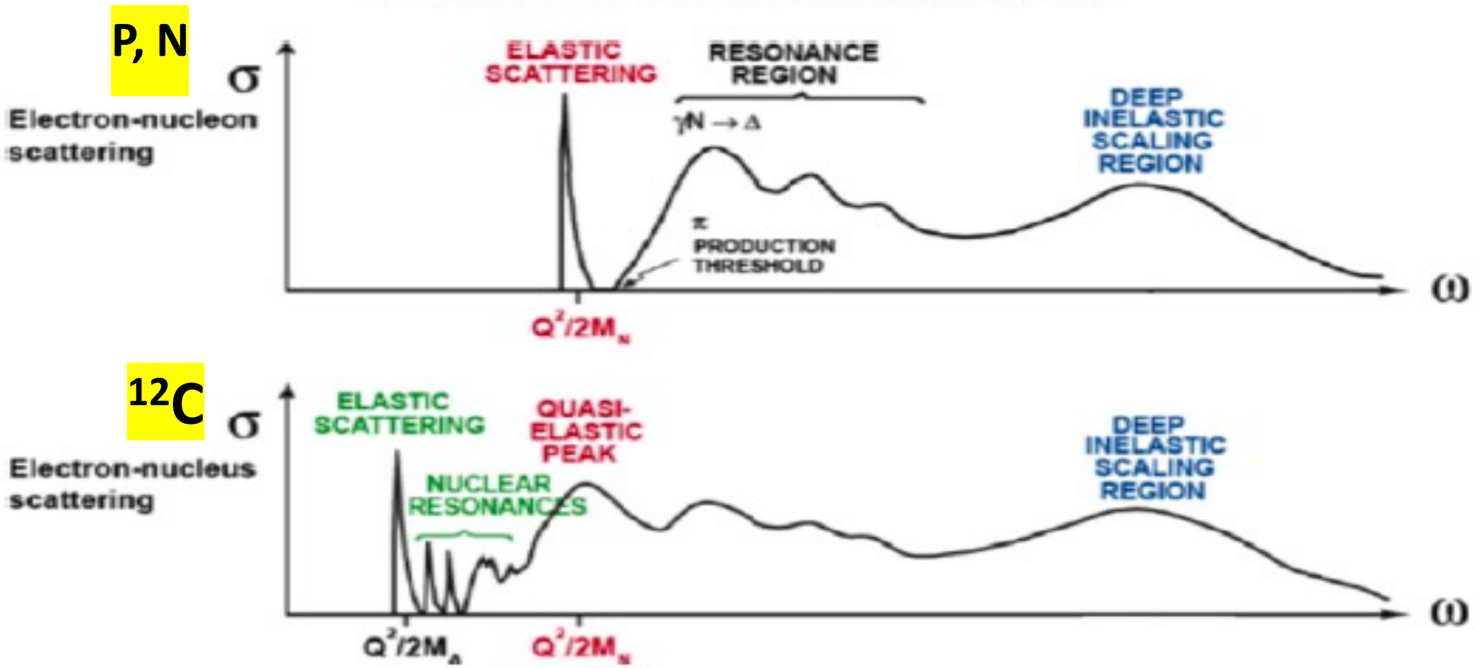


We have initiated a program to extract RL and RT values on various nuclei using all available data. Here we report on our extraction for Carbon. We extract extract RL and RT in all regions, nuclear elastic, nuclear excitations, quasielastic, resonance region and inelastic scattering. We cover the kinematic range $0 < Q^2 < 3.45$ GeV^2, and energy transfer ν, from ν=0 to the end of the resonance region W=2.0 GeV. We extract at fixed Q^2 values as a function of ν and also at fixed values of $0.1 < \mathbf{q} < 2.78$ GeV and $0 < \nu < (\nu = \mathbf{q}$, i.e. Q^2=0). For Carbon we use 16,000 electron scattering and photoproduction cross sections

Goals:

1. Test first-principle nuclear theories at fixed values of Q^2 and **q**.
2. Comparing to extracted RL and RT values to MC generators covers all kinematic regions is much preferable to validating with cross section data in limited sets of kinematic regions.
3. **Where there is no data, we provide the values from our universal fit to all electron scattering data.**
4. This fit will be made available for validation of electron and neutrino MC generators

First Step: Christy- Bodek Universal Fit (needed for this analysis)

- Fit to all of the world's electron scattering data on H, D and nuclear targets to include the lowest values of energy transfer ν and q^2 (for carbon we fit about 16,000 e-^{12}C and photoprodution cross section We fit the QE cross section (including Transverse Enhancement/MEC, +longitudinal low q suppression) resonance and pion production, DIS, nuclear excitations, elastic scattering data.
  Note: Nuclear excitations are significant at low ν and contribute up to 30% to the longitudinal Inelastic Coulomb Sum Rule (CSR)

- Since the cross sections span a large range of energies and scattering angles, we extract fits to both the longitudinal R_L and transverse R_T contributions.

- We parameterize both the Enhancement of the Transverse QE cross section and the Suppression of the Longitudinal QE cross section. We also extracted the most precise Coulomb Sum rule as a function of q and compare to theoretical calculations.

- The fit can be used in lieu of data to benchmark Monte Carlo predictions (e.g. for e-H, e-D and e-^{12}C and e-^{1}O cross sections, and to is being used compute radiative corrections for electron scattering experiments.

Bakcup: 1 Christy- Bodek Universal Fit Nuclear excitations as example

Cross sections for excitations less than 10 MeV multiplied by (1/6)

Nuclear excitation region Ex < 50 MeV

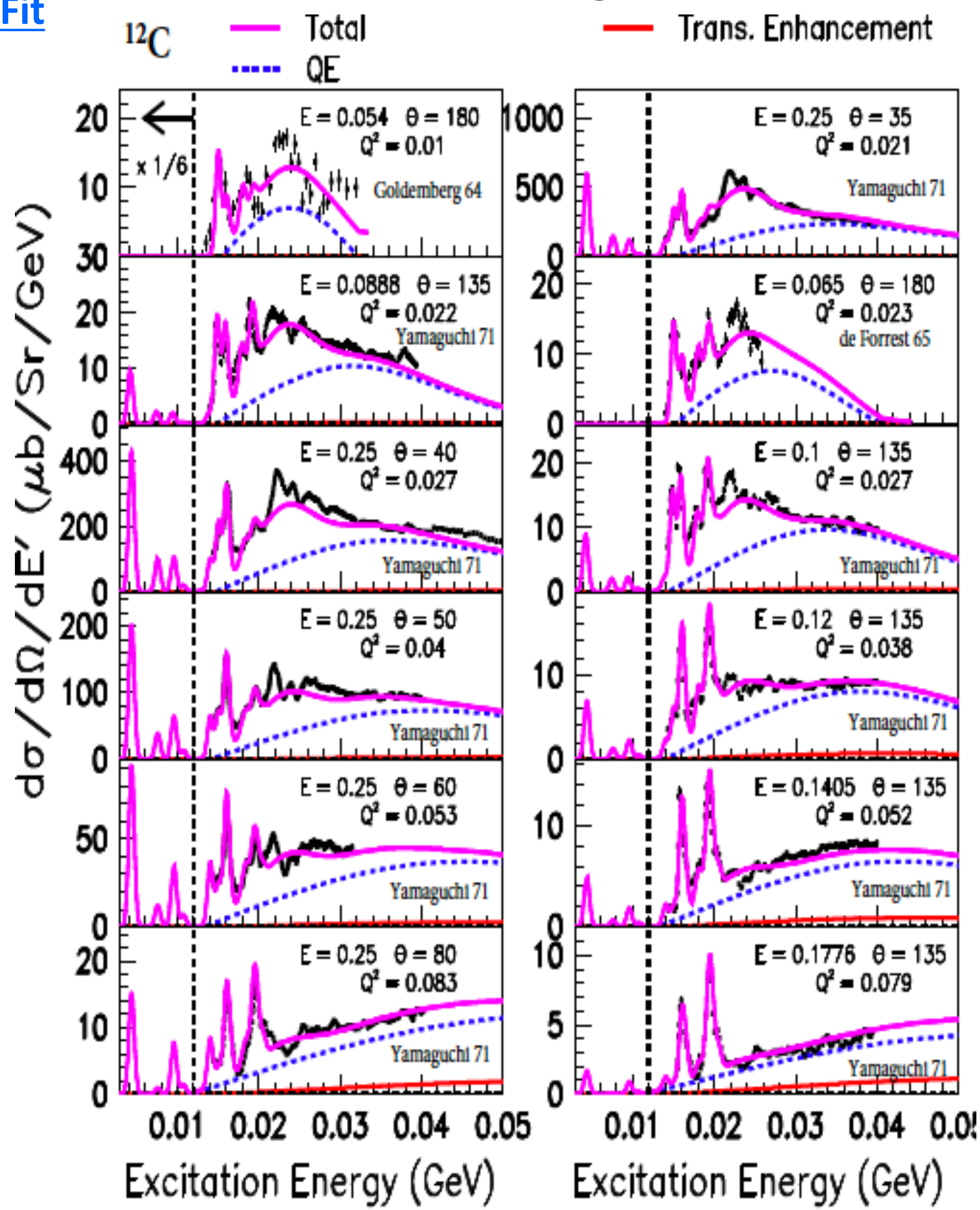
Comparison of our fit to representative e-C12 data for **$0.01<q^2< 0.08$ GeV^2** .

Shown: **Total including excitations : solid -------**

Quasielastic (QE) contribution: dashed------

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE response dashed------

Electron scattering cross sections



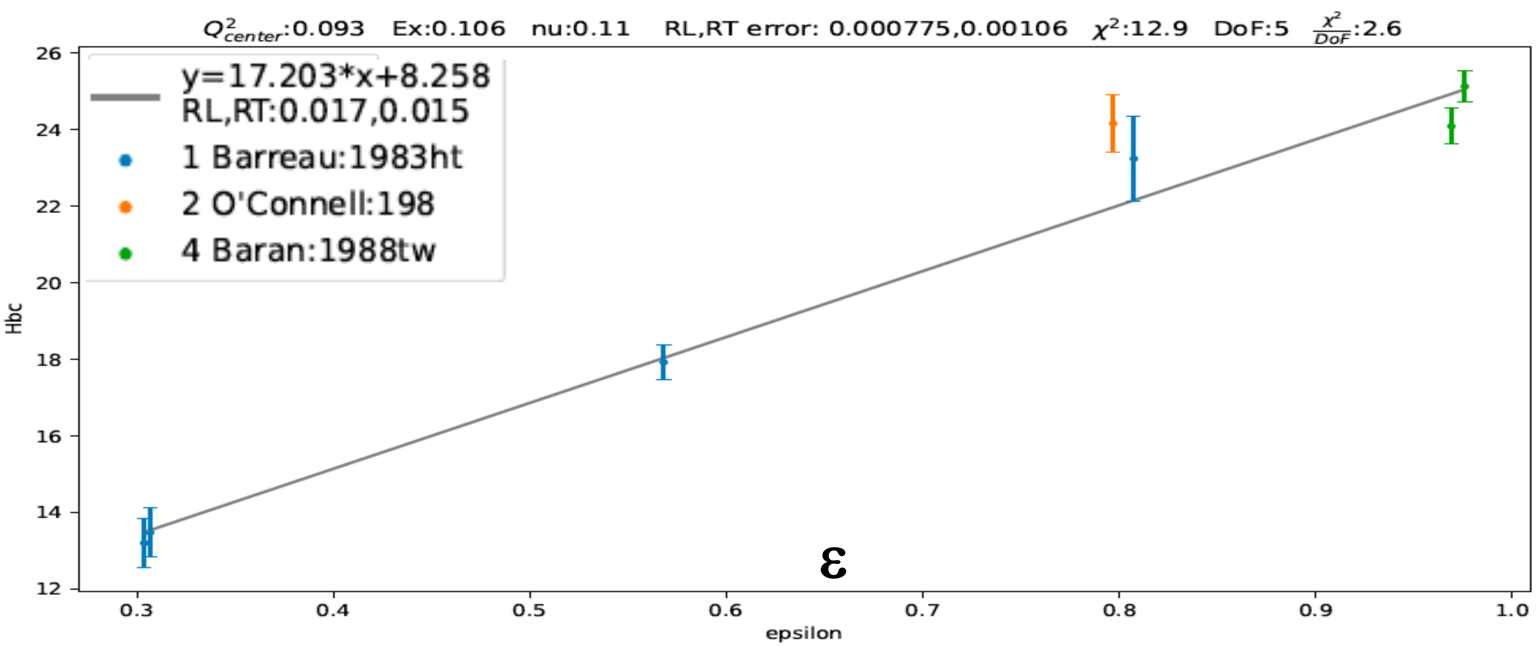
R_L R_T extractions. (Rosenbluth plots)

1. We apply Coulomb corrections in the analysis. We bin all cross sections in bins of **q** (we also do it for bins in Q^2), and apply bin centering corrections using the universal fit. Bin centers are at:

18 q values: 0.100, 0.148, 0.167, 0.205, 0.240, 0.300, 0.380, 0.475, 0.570, 0.649, 0.756, 0.991, 1.659, 1.921, 2.213, 2.500, 2.783, 3.500 GeV.

18 Q^{2} values: 0.00 (photoproduction), 0.010, 0.020, 0.026, 0.040, 0.056, 0.093, 0.120, 0.160, 0.265, 0.38, 0.50, 0.80, 1.25, 1.75, 2.25, 2.75, 3.25, 3.75 GeV^2.

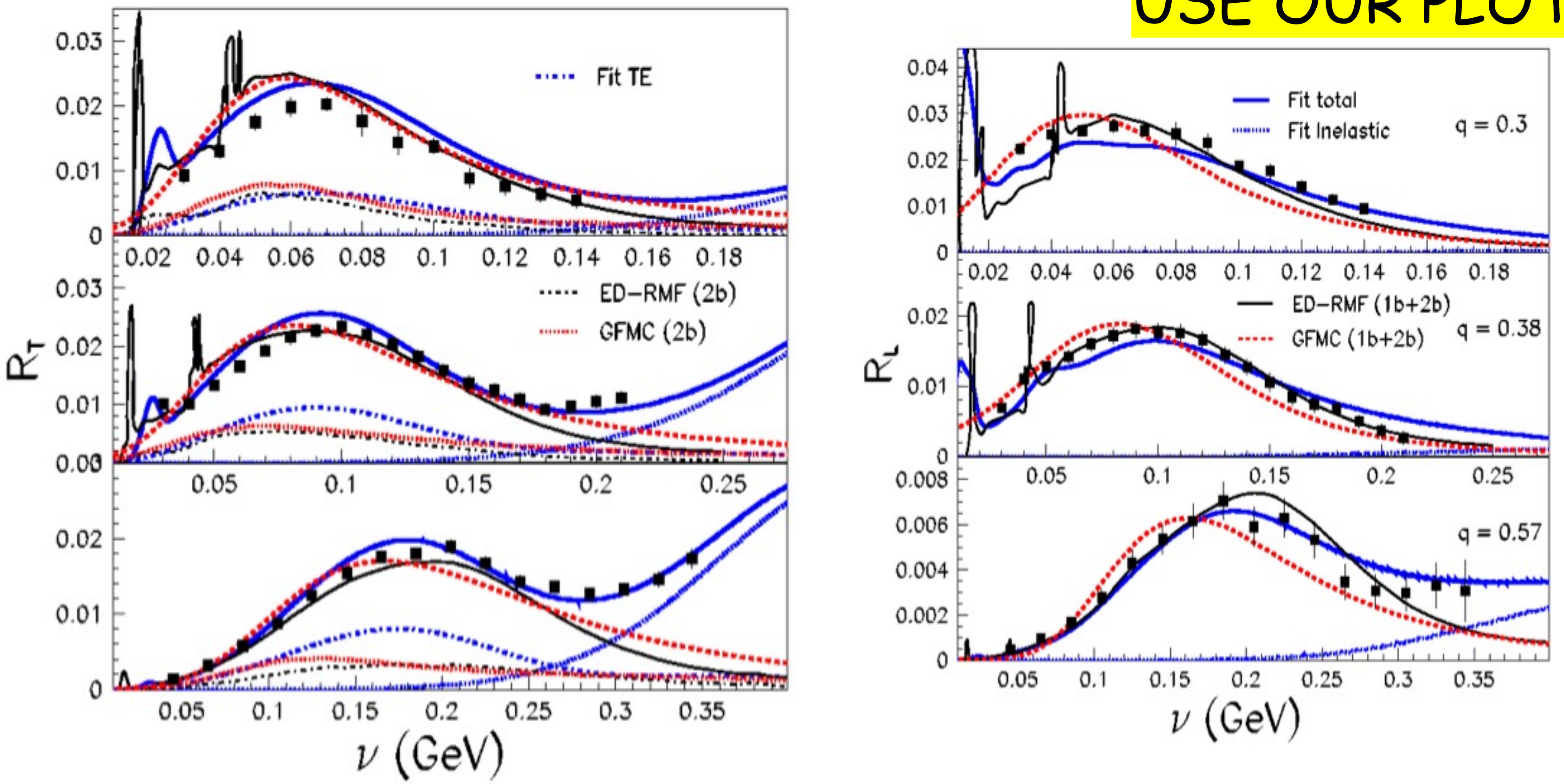
2. Bin centering correction in **q** (or Q^2). **For ν< 50 MeV** we bin center the data in bins of Excitation energy $\mathbf{E_x}$, and for ν >50 MeV we bin center the data in bins of $\mathbf{W^2}$. We then perform Rosenbluth fits versus virtual photon polarization ε extract $\mathbf{R_L}$ **and** $\mathbf{R_T}$. Laster convert Ex and $\mathbf{W^2}$ to ν.



Fixed q : Chirsty- Bodek Fit describes RL and RT

Comparison to nuclear theory for QE 1p1h process (available only for 3 q values)

USE OUR PLOTS



- **Shown are theory predictions for QE 1p1h process (including contribution from 1b and 2b currents). There fore, for comparison we also show our universal for QE 1p1ph.**
- **ED-RMF (Energy Depending Relativistic mean field .**
- **And GFMC (Green's Function Monte Carlo – first principle)**

We extract 18 fixed q values - However, at this time predictions are only available for 3 fixed q values.

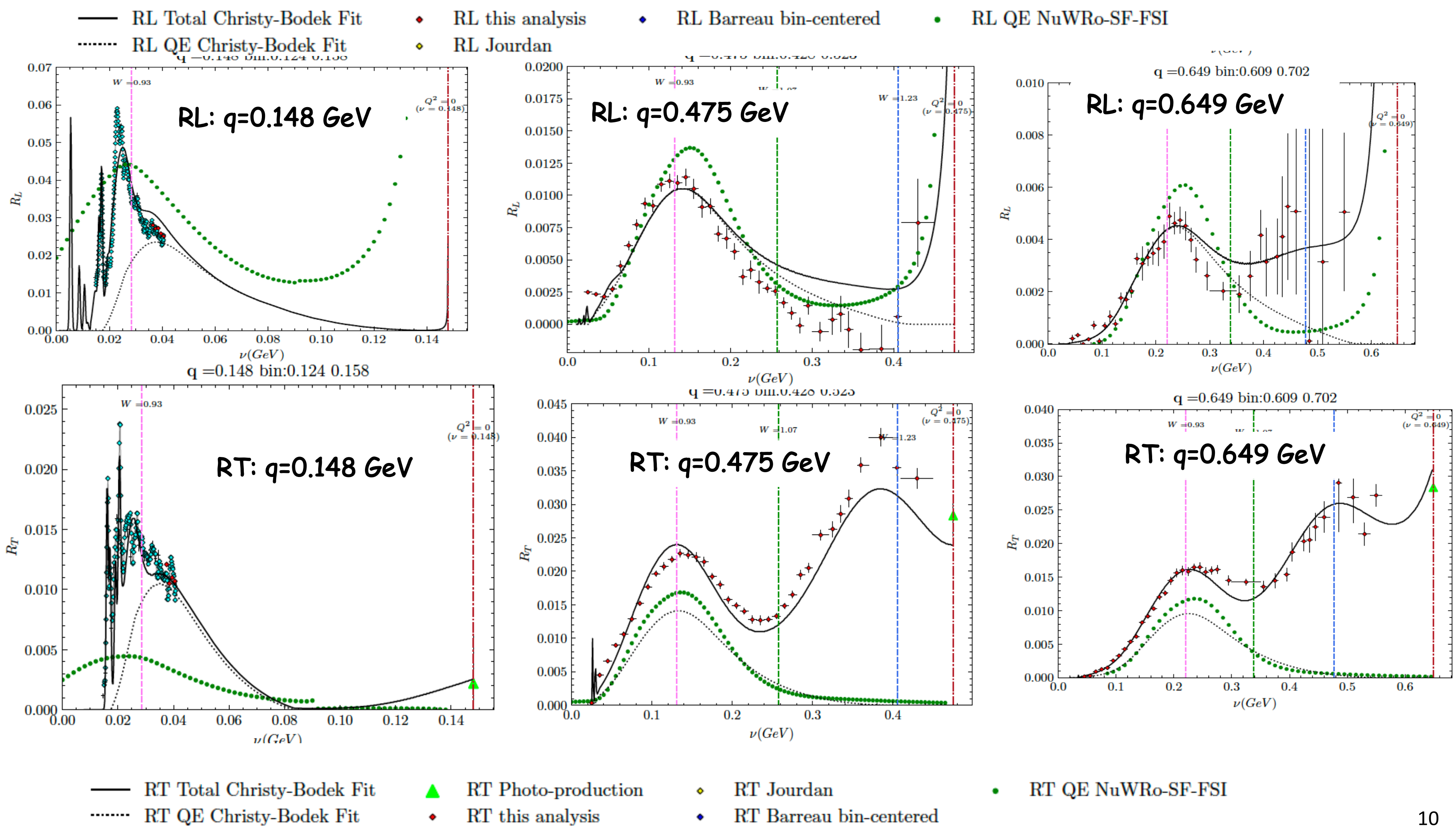
Fixed q : Chirsty- Bodek Fit describes RL and RT

Code, as well as tables of numbers for RL and RT for all q values will be provid

for easy validation of MC generators – **Example: Compare to NuWRo which only predicts QE response** • • •

18 q values: 0.100, 0.148, 0.167, 0.205, 0.240, 0.300, 0.380, 0.475, 0.570, 0.649, 0.756, 0.991, 1.659, 1.921, 2.213, 2.500, 2.783, 3.500 GeV. **are shown in Backup Slides**. Here we show a 3 examples: ***Start with fixed q (GeV).*** The maximum value of ν is ν=q (where it should match photoproduction for RT)

Low q – Large contribution from nuclear excitation. Not included in any MC generators. NuWRo breaks down at low q.



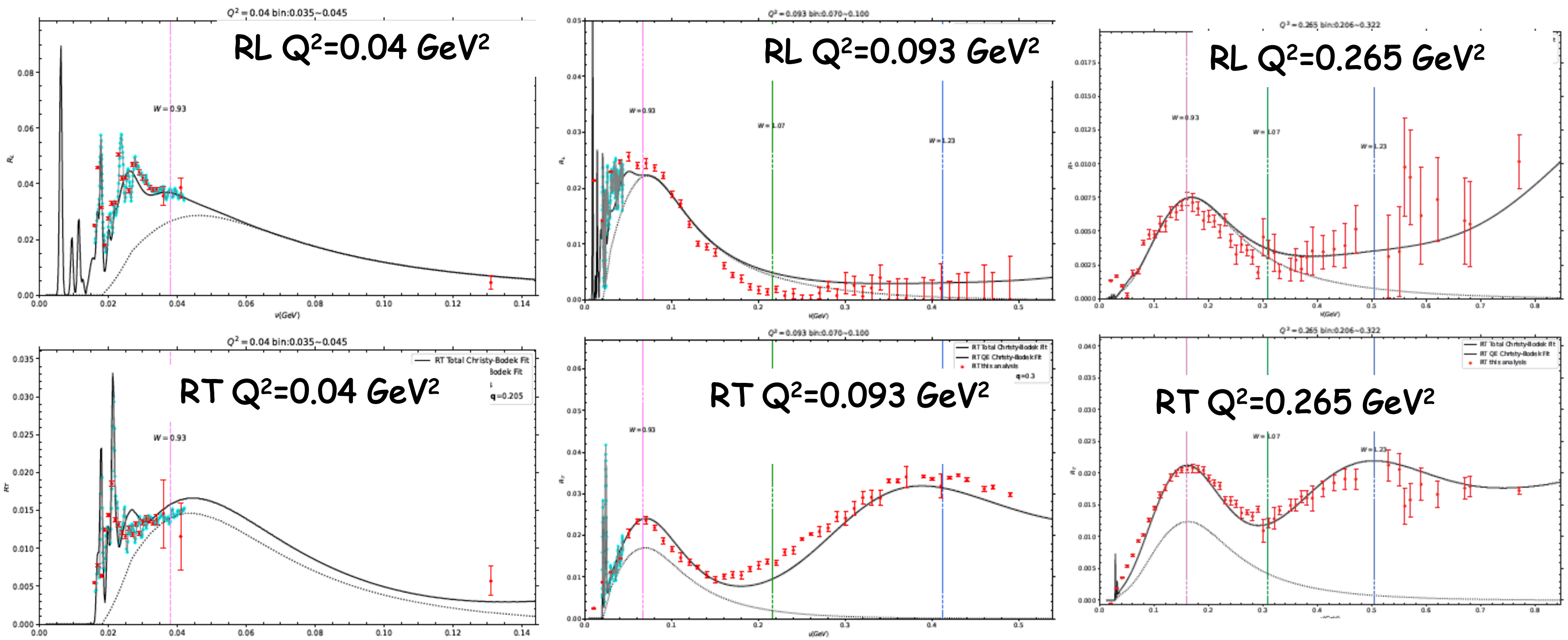
Fixed Q^2 : Chirsty- Bodek Fit describes RL and RT

Code, as well as tables of numbers for RL and RT for all q values will be provided

for easy validation of MC generators

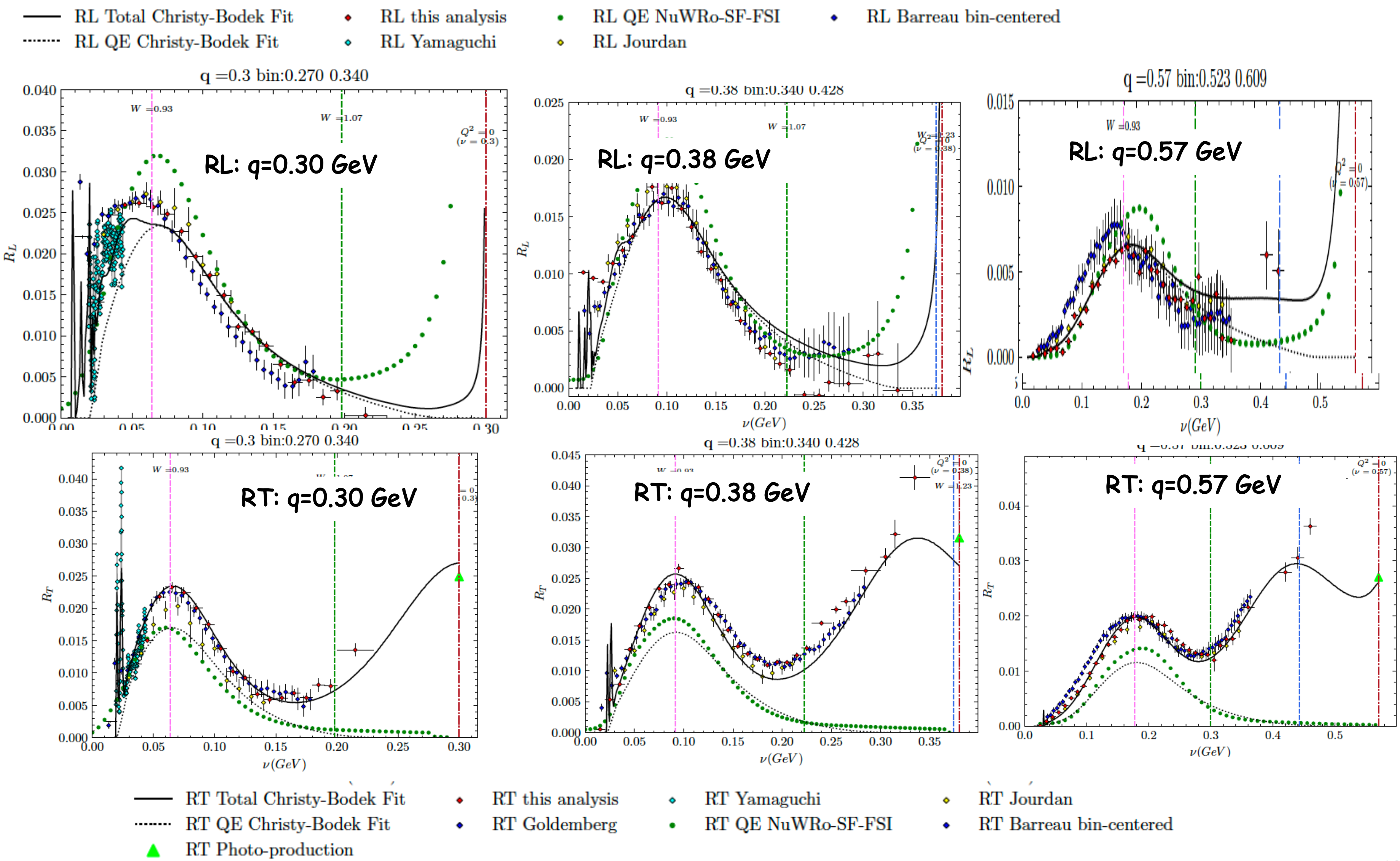
18 Q^2 values: 0.00 (photoproduction), 0.010, 0.020, 0.026, 0.040, 0.056, 0.093, 0.120, 0.160, 0.265, 0.38, 0.50, 0.80, 1.25, 1.75, 2.25, 2.75, 3.25, 3.75 GeV^2 **are shown in Backup Slides**. Here we show a 3 fixed Q^2 (GeV^2) examples. (there is no maximum value of ν)

Low q – Large contribution from nuclear excitation. Not included in any MC generators. NuWRo breaks down at low q.



Comparison to previous analyses
(which were only done at 3 values of q)

With all world's data we have 18 values of q (and Q2) with smaller error bars and larger kinematic range



Conclusions

- The 18 R_L and R_T extractions cover a very large kinematic range. The values are in excellent agreement with the Christy-Bodek Universal fit to all cross section values. Therefore, where there is no data the values form the universal fit can be used.

- Good agreement with nuclear theory for 3 values of q. Predictions for all other values of q not yet available.

- The R_L and R_T measurements as well as the universal fit provide a simple way to validate electron and neutrino MC generators over the entire kinematic range of interest.

- Tables of the fit values for R_L and R_T for the 18 q and Q^2 values will be provided. The contributions of nuclear excitations, QE, transverse enhancement and inelastic scattering will also be listed separately.

- Code for the universal fit will be provided.

Backup

Plots of RL and RT at all 18 values of q, and 18 values of Q^2

Comparison to options in NuWRo

(1) Relativistic Fermi Gas (RFG). **(2) Spectral Function (SF)**
(3) Spectral Function plus final state interaction (SF-FSI).

FSI is needs to be included. However, at large Q2 the effect of FSI is smaller

Bakcup: 2 Christy- Bodek Universal Fit

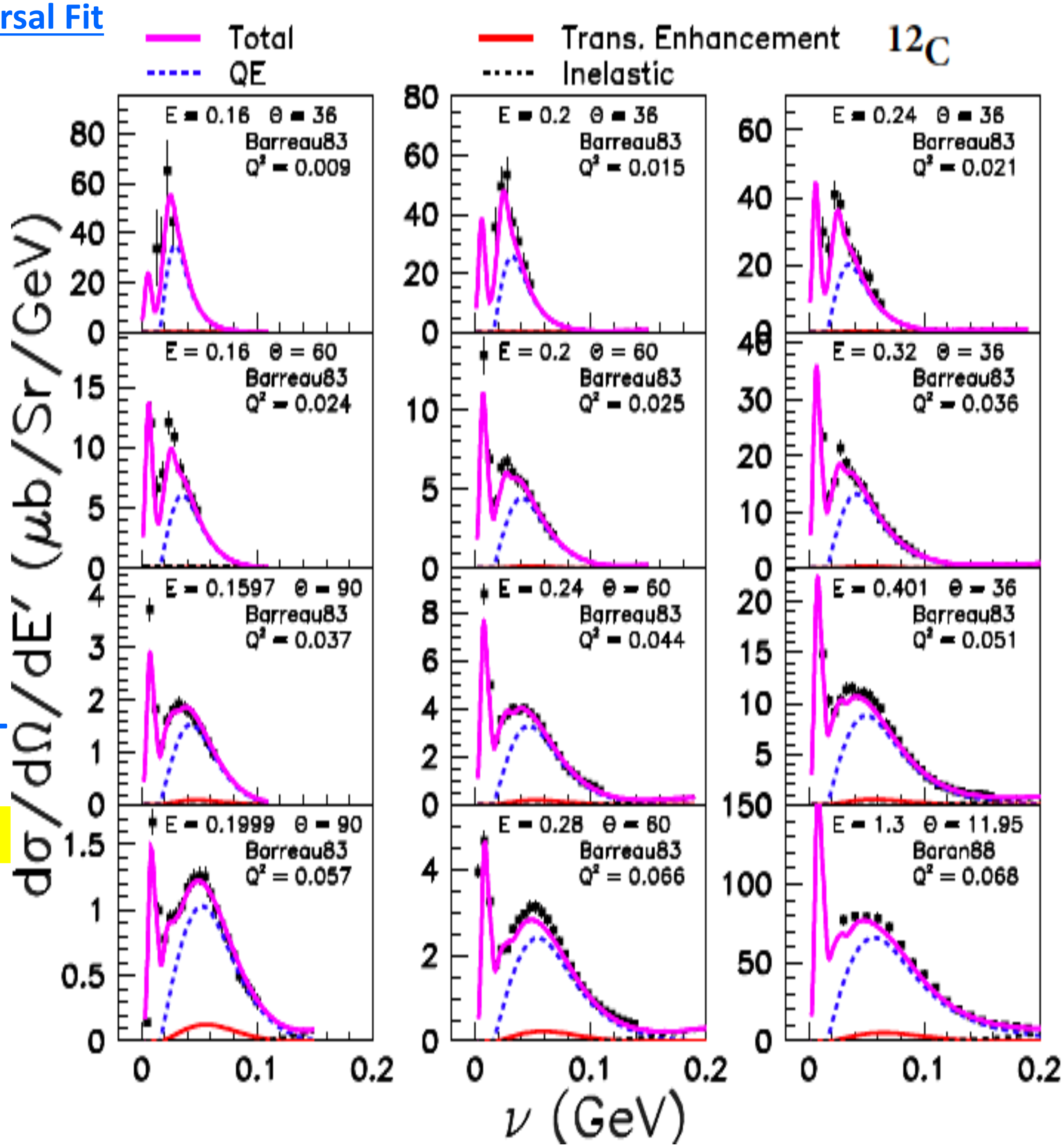
Quasielastic (QE) Region-I

Comparison of our fit to representative e-C12 data **For ν < 0.2 GeV and $0.01 < q^2 < 0.068$ GeV^2 .**

Shown: Total including excitations   solid -------

Quasielastic (QE) contribution   dashed ------

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE response dashed------



Bakcup: 3 Christy- Bodek Universal Fit

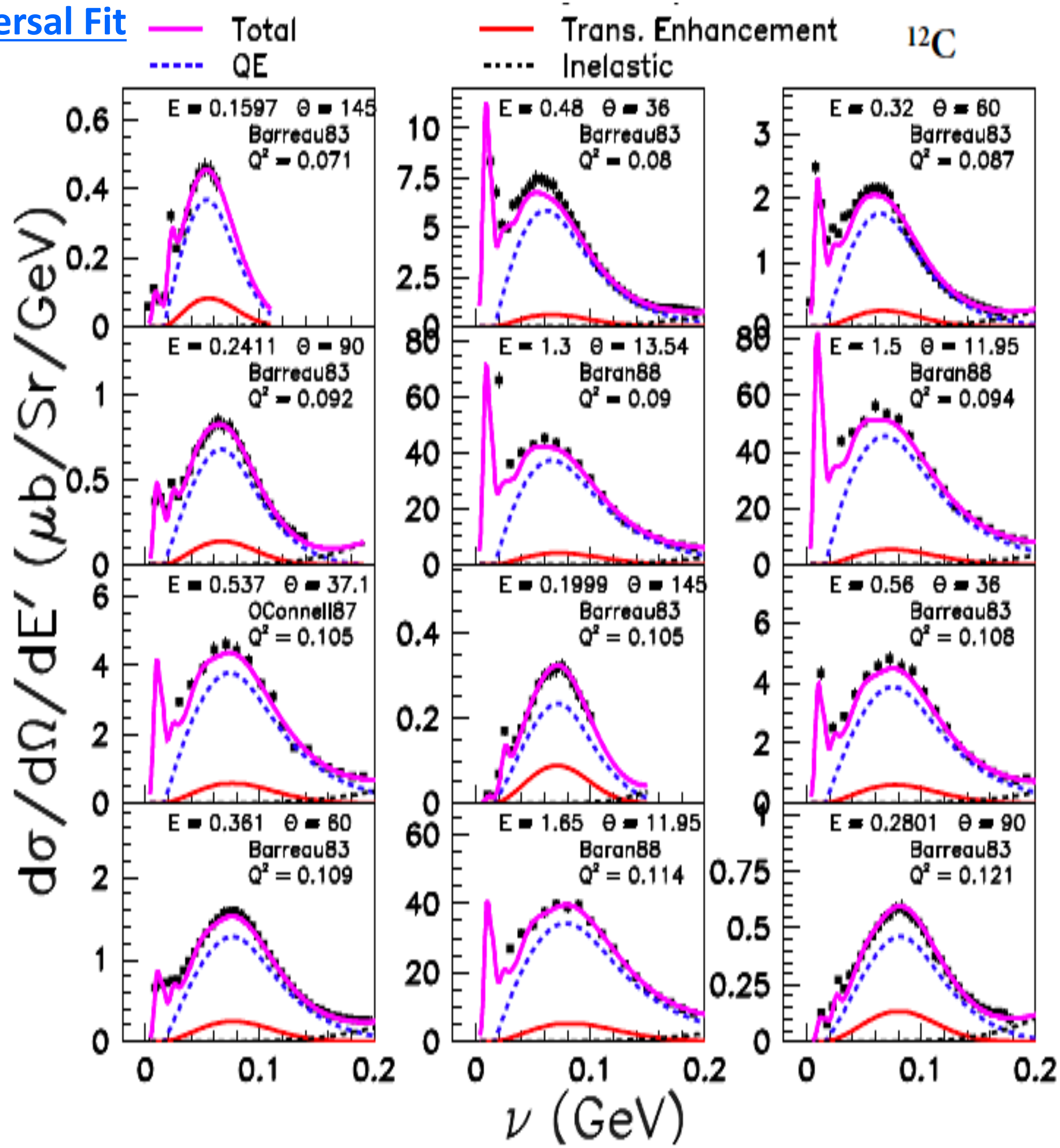
Quasielastic (QE) Region II

Comparison of our fit to representative e-C12 data for **< 0.2 GeV and 0.071 <q^2< 0.121 GeV^2 .**

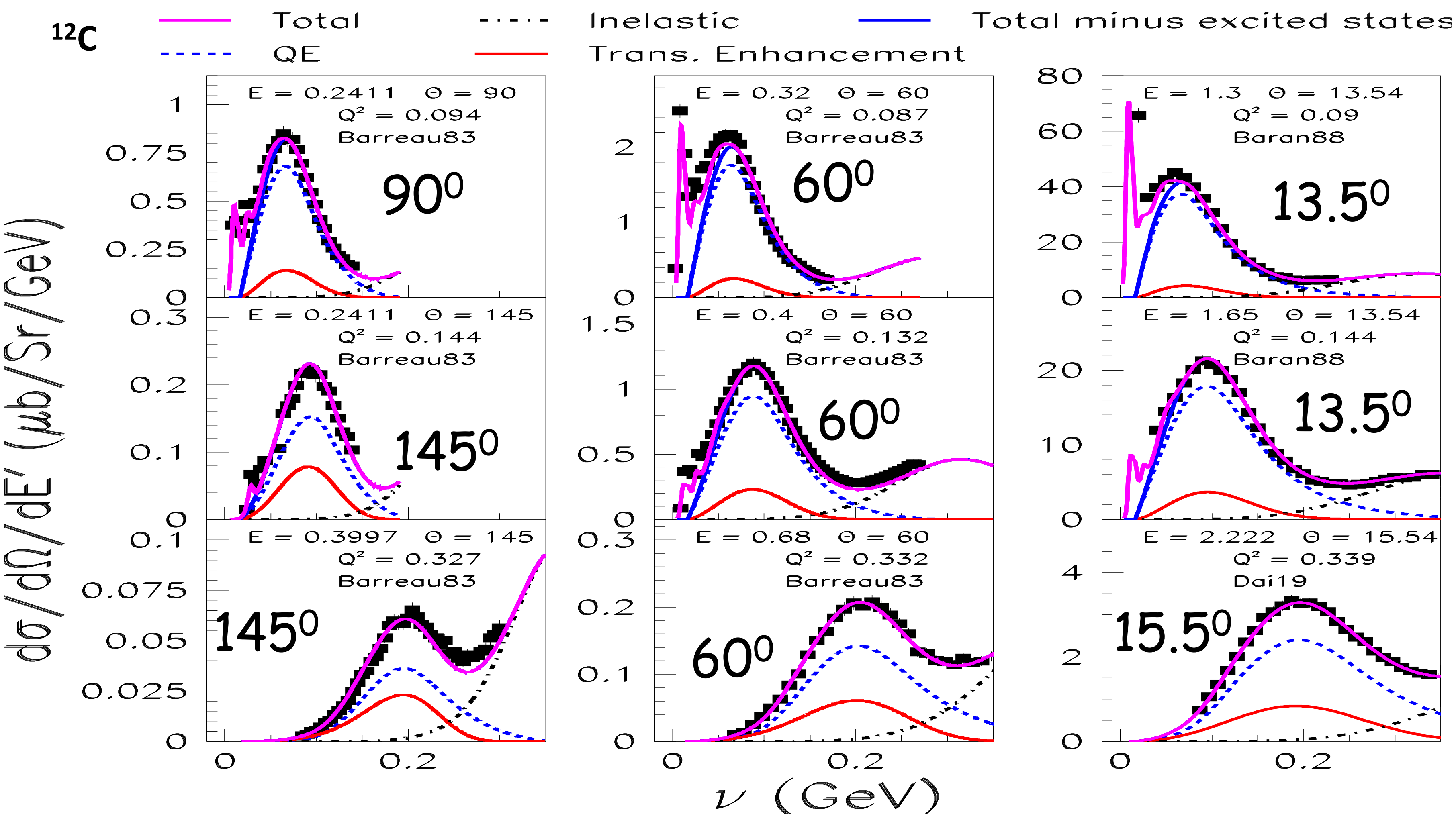
Shown: Total including excitations solid -------

Quasielastic (QE) contribution dashed ------

Transverse Enhancement at large angles accounts for Meson Exchange Currents and Enhancement of Transverse QE reponse dashed------



Backup 4: Christy- Bodek Universal Fit - Fit functional forms for RL RT

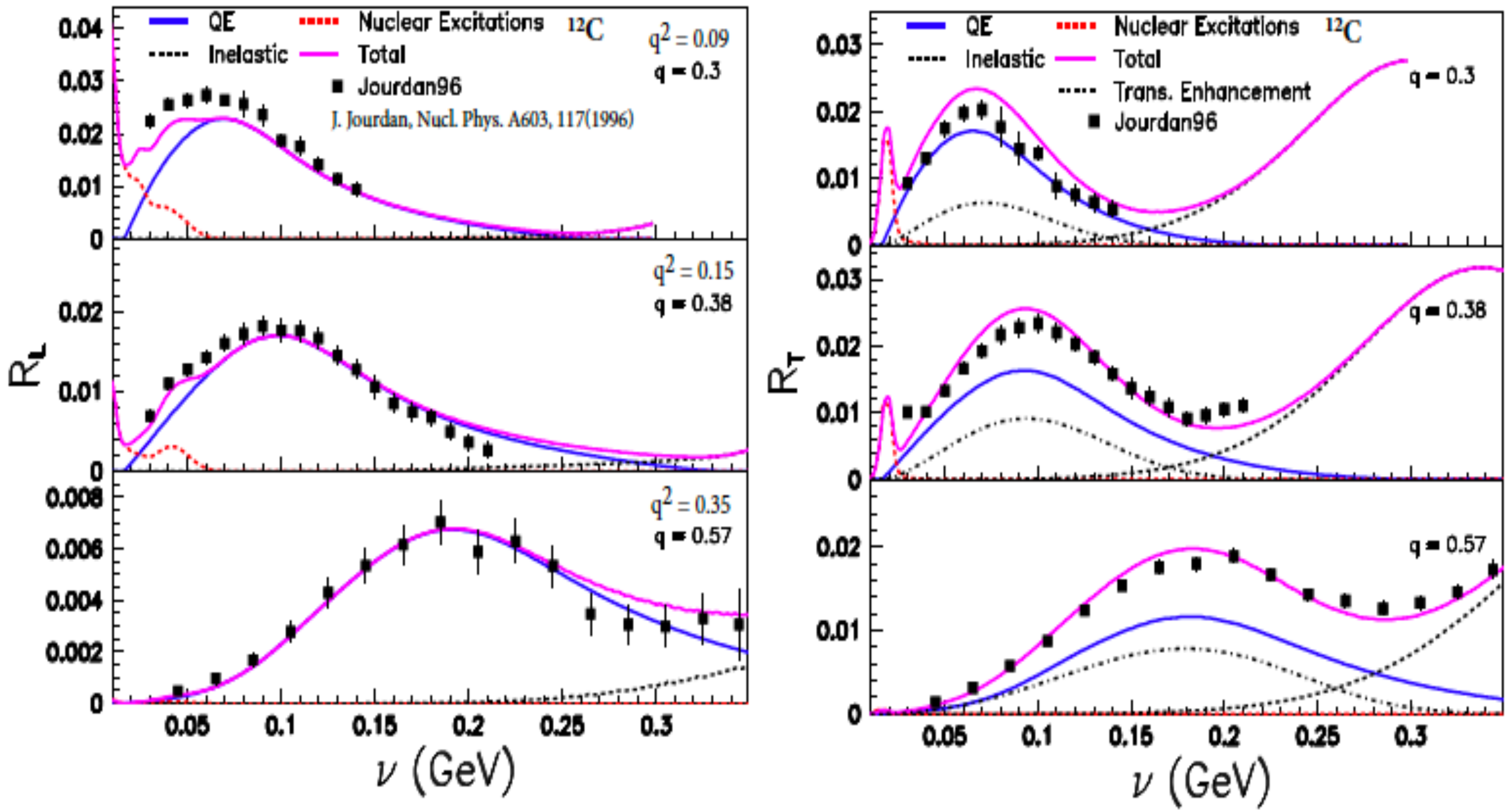


The overall fit provides R_L and R_T at all values of q

Shown are **large and small angle cross sections at the same q** that provide the major contribution to the extraction of R_L and R_T at

q^2=0.09, 0.15 and 0.35 GeV^2 (q=0.3, 0.38 and 0.57 GeV)

Bakcup: 5 Christy- Bodek Universal Fit



Comparison of our R_L and R_T from our universal fit to (~8000 cross sections) to previou extraction by Jourdan at **q^2=0.09, 0.15 and 0.35 GeV^2. (q=0.3, 0.38 and 0.57 GeV)**

Our extraction is more reliable since we include all of the world's data in the fit

- **At low q** the contribution of the **nuclear excitations** important.
- The **superscaling fit function** describes the QE distribution at higher ν.
- **Resonance region** is modeled with **Fermi Smeared H and D data**.