Extraction of the ^{12}C Longitudinal (R_L) and (R_T) Nuclear Electromagnetic Response Functions from *all* Electron Scattering Measurements on Carbon

1. Testing first principle nuclear theory predictions
2. Provide a platform for verification of electron and neutrino MC generators over the entire kinematic range of interest

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10 min talk + 2 min questions

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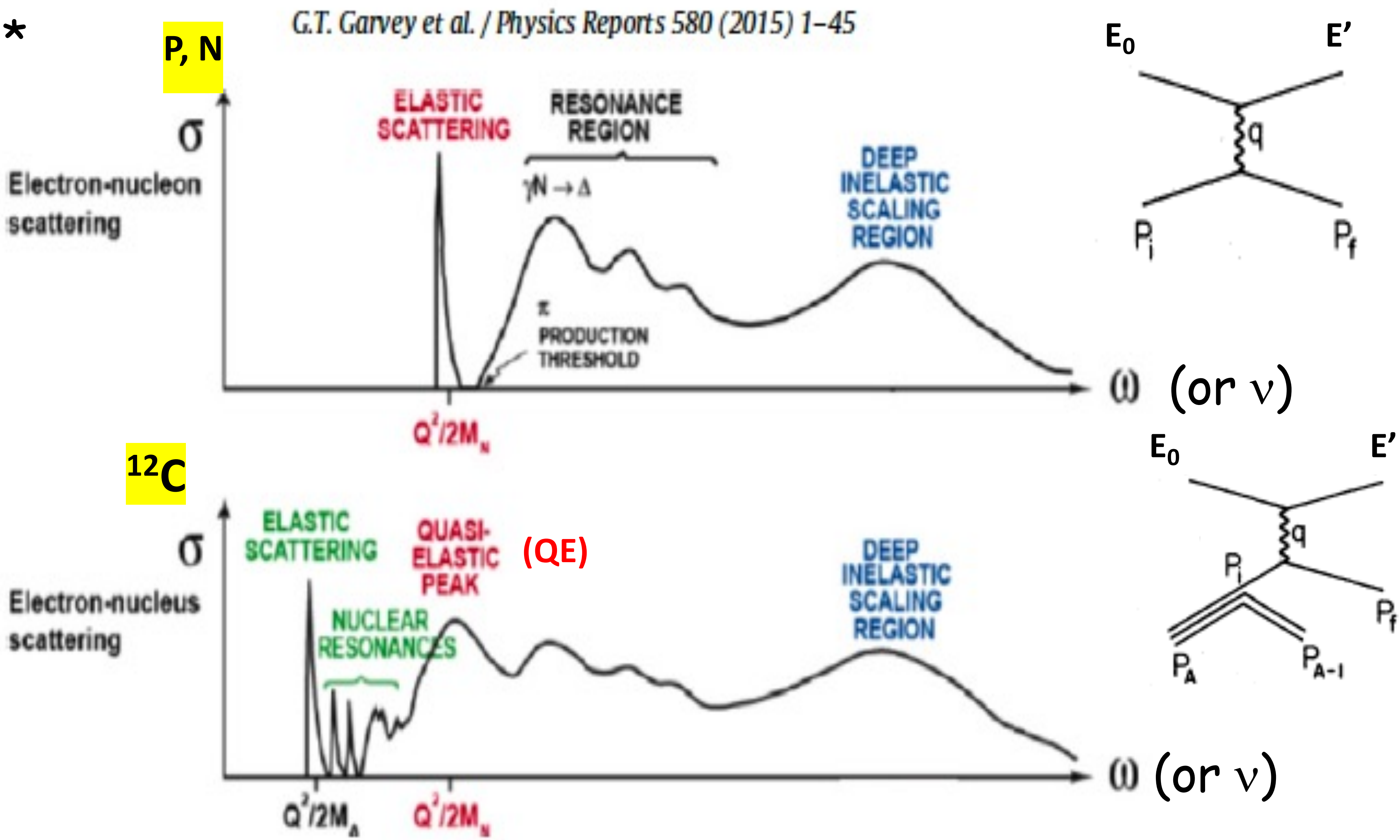

Nuclear corrections affect enhance the Transverse Response (R_T) and quench the Longitudinal Response (R_L) in QE scattering. There are three different formalisms to parametrize these two responses. (1) R_L and R_T (2) F_1 and F_L (3) σ_L and σ_T.

In this analysis we use R_L and R_T.

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Nuclear Physics

This description is primarily used in the nuclear excitation and QE regions. The electron scattering differential cross section is written in terms of longitudinal ($\mathcal{R}_L(Q^2,\nu)$) and transverse ($\mathcal{R}_T(Q^2,\nu)$) nuclear response functions [24]

energy transfer = ν (or ω)

$$\frac{d\sigma}{d\nu d\Omega} = \sigma_M[A\mathcal{R}_L(Q^2,\nu) + B\mathcal{R}_T(Q^2,\nu)] \quad (20)$$

where σ_M is the Mott cross section, $A = (Q^2/\mathbf{q}^2)^2$ and $B = \tan^2(\theta/2) + Q^2/2\mathbf{q}^2$.

Particle Physics

This description is primarily used in the inelastic continuum region. In the one-photon-exchange approximation, the spin-averaged cross section for inclusive electron-proton scattering can be expressed in terms of two structure functions as follows

$$\frac{d\sigma}{d\Omega dE'} = \sigma_M[\mathcal{W}_2(W^2,Q^2) + 2\tan^2(\theta/2)\mathcal{W}_1(W^2,Q^2)]$$

$$\sigma_M = \frac{\alpha^2\cos^2(\theta/2)}{[2E_0 sin^2(\theta/2)]^2} = \frac{4\alpha^2 E'^2}{Q^4}\cos^2(\theta/2) \quad (10)$$

Q^2 = 4-momentum transfer squared

$$Q^2 = (-q)^2 = 4E_0E'\sin^2\frac{\theta}{2},$$

$$\nu = E_0 - E'.$$

W^2 =final state invariant mass squared

$$W^2 = M^2 + 2M\nu - Q^2$$

$\mathbf{q}^2$ = 3-momentum transfer squared

$$\mathbf{q}^2 = Q^2 + \nu^2$$

E_x = Excitation energy

$$E_x = \nu - \nu_{elastic}$$

$$x = Q^2/(2M\nu).$$

$$\mathcal{F}_1 = M\mathcal{W}_1 \text{ and } \mathcal{F}_2 = \nu\mathcal{W}_2.$$

$$\mathcal{F}_L(x,Q^2) = \mathcal{F}_2\left(1+\frac{4M^2x^2}{Q^2}\right) - 2x\mathcal{F}_1,$$

$$\mathcal{R}_T(\mathbf{q},\nu) = \frac{2\mathcal{F}_1(\mathbf{q},\nu)}{M}$$

$$\mathcal{R}_L(\mathbf{q},\nu) = \frac{\mathbf{q}^2}{Q^2}\frac{\mathcal{F}_L(\mathbf{q},\nu)}{2Mx}$$

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- **We have initiated a program to extract RL and RT values on various nuclei using all available data**.
  Here we report on our extraction for Carbon.

- We extract extract RL and RT in all regions, nuclear elastic, nuclear excitations, quasielastic, resonance region and inelastic scattering.

- We extract at 18 values of fixed Q^2 values $0 < Q^2 < 3.45$ GeV^2 as a function of energy transfer ν .
- And at 18 values of fixed values of 3-momentum transfer $0.1 < \mathbf{q} < 2.78$ GeV as a function of ν

- We energy transfer ν, from ν=0 to the end of the resonance region W=2.0 GeV

For Carbon we use 16,000 electron scattering and photproduction cross sections

Goals:

- Test first-principle nuclear theories at fixed values of Q^2 and **q**.

- Use extracted RL and RT values to validate MC generators
  Since we covers all kinematic regions this is a much preferable way than comparison with a few cross section measurements in a limited sets of kinematic regions.

- **Where there is no data, we provide the values from our universal fit to all electron scattering data.**
  The fit will be also be made available for validation of electron and neutrino MC generators

* **First Step: Christy- Bodek Universal Fit (needed for this analysis))**

- We update the Bosted-Christy fit to **all of the world's electron scattering data** on H, D and nuclear targets to include the lo**west values of energy transfer ν and q^2** (for ^{12}**C we fit ~16,000 electron scattering and photoabsorption cross sections**.

- We fit for: **QE cross section (including Transverse Enhancement/MEC and longitudinal low q Quenching). Resonance and pion production**, **DIS, nuclear excitations,** elastic scattering data. Since the cross sections span a large range of energies and scattering angles, we **extract both the longitudinal RL and transverse RT** contributions, and also get the Coulomb Sum Rule.

- We parameterize both the **Enhancement of the Transverse QE cross section** and the **Quenching of the Longitudinal QE cross** section. **We extract the most precise Coulomb Sum rule as a function of q and compare to theoretical calculations.**

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- The fit can be used **in lieu of data to benchmark Monte Carlo predictions** (e.g. for e-H, e-D and e-^{12}C and e-^{1}O cross sections, and to is being used **compute radiative corrections for electron scattering** experiments..

* NEW: Individual R_L R_T extractions (Rosenbluth plots)

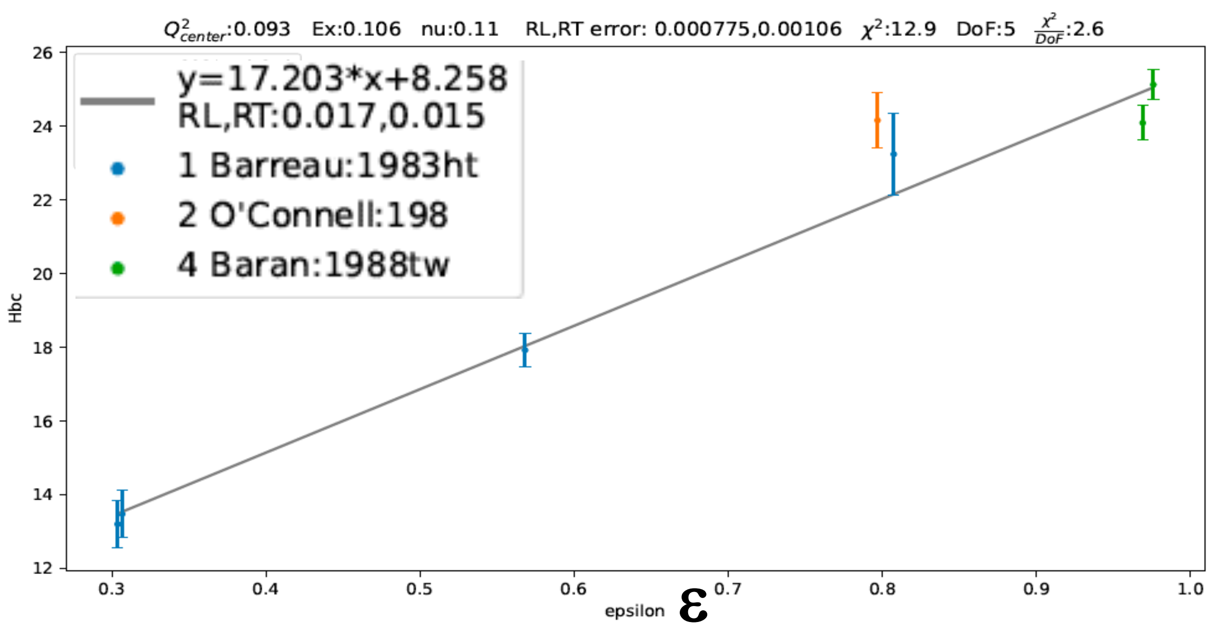
We apply Coulomb corrections in the analysis. We bin all cross sections in bins of q (we also do it for bins in Q^2) and apply bin centering corrections using the universal fit. Bin centers are at:

18 **q** values: 0.100, 0.148, 0.167, 0.205, 0.240, 0.300, 0.380, 0.475, 0.570, 0.649, 0.756, 0.991, 1.659, 1.921, 2.213, 2.500, 2.783, 3.500 GeV

18 Q^2. values: 0.00 (photoproduction), 0.010, 0.020, 0.026, 0.040, 0.056, 0.093, 0.120, 0.160, 0.265, 0.38, 0.50, 0.80, 1.25, 1.75, 2.25, 2.75, 3.25, 3.75 GeV^2

2. Bin centering correction in **q** (or Q^2). **For ν< 50 MeV** we bin-center the data in bins of Excitation energy $\mathbf{E_x}$, and for ν >50 MeV we bin-center the data in bins of $\mathbf{W^2}$ **.** We then perform Rosenbluth fits versus virtual photon polarization ε extract $\mathbf{R_L}$ **and** $\mathbf{R_T}$ **.** Later convert Ex and $\mathbf{W^2}$ to ν.

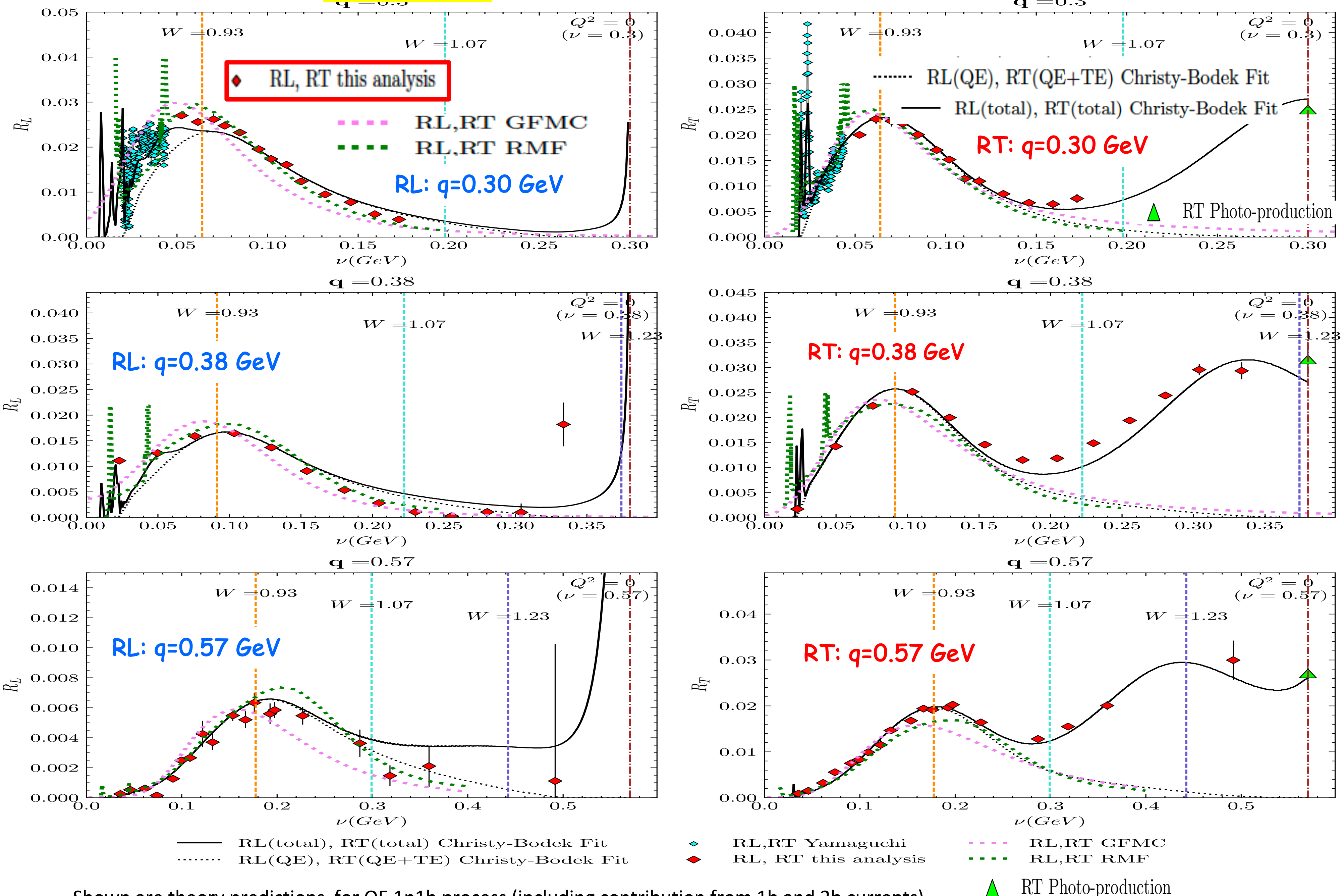
2. The universal fit includes all **16,000 cross sections**, The individual R_L R_T only use a **subset of the data** in which for the same values ν and Q^2 there is data spanning a a significant range of angles. Therefore the Universal Fit provide RL and RT for a more extended kinematic range



ε = virtual photon polarization

$$\epsilon = \left[1 + 2(1 + \frac{\nu^2}{Q^2})\tan^2\frac{\theta}{2}\right]^{-1}$$

*Comparison to nuclear theory for QE 1p1h process (available for q= 0.3, 0.38, 0.57 GeV) only



Shown are theory predictions for QE 1p1h process (including contribution from 1b and 2b currents).

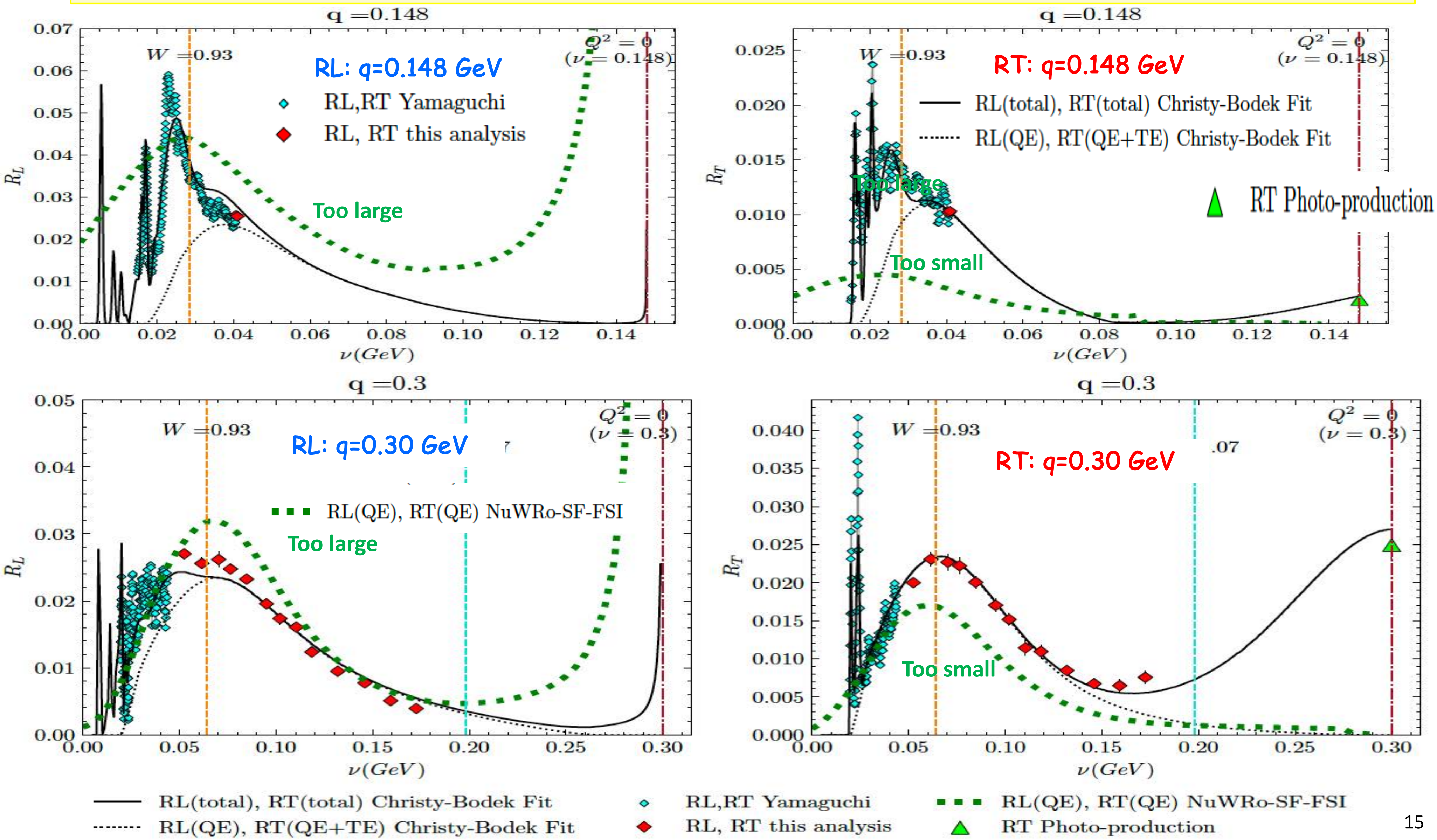
GFMC (Green's Function Monte Carlo – first principle)………. No nuclear states

ED-RMF (Energy Depending Relativistic mean field ………… Nuclear states grouped at 2 values of fixed

QE predictions reasonable but not perfect

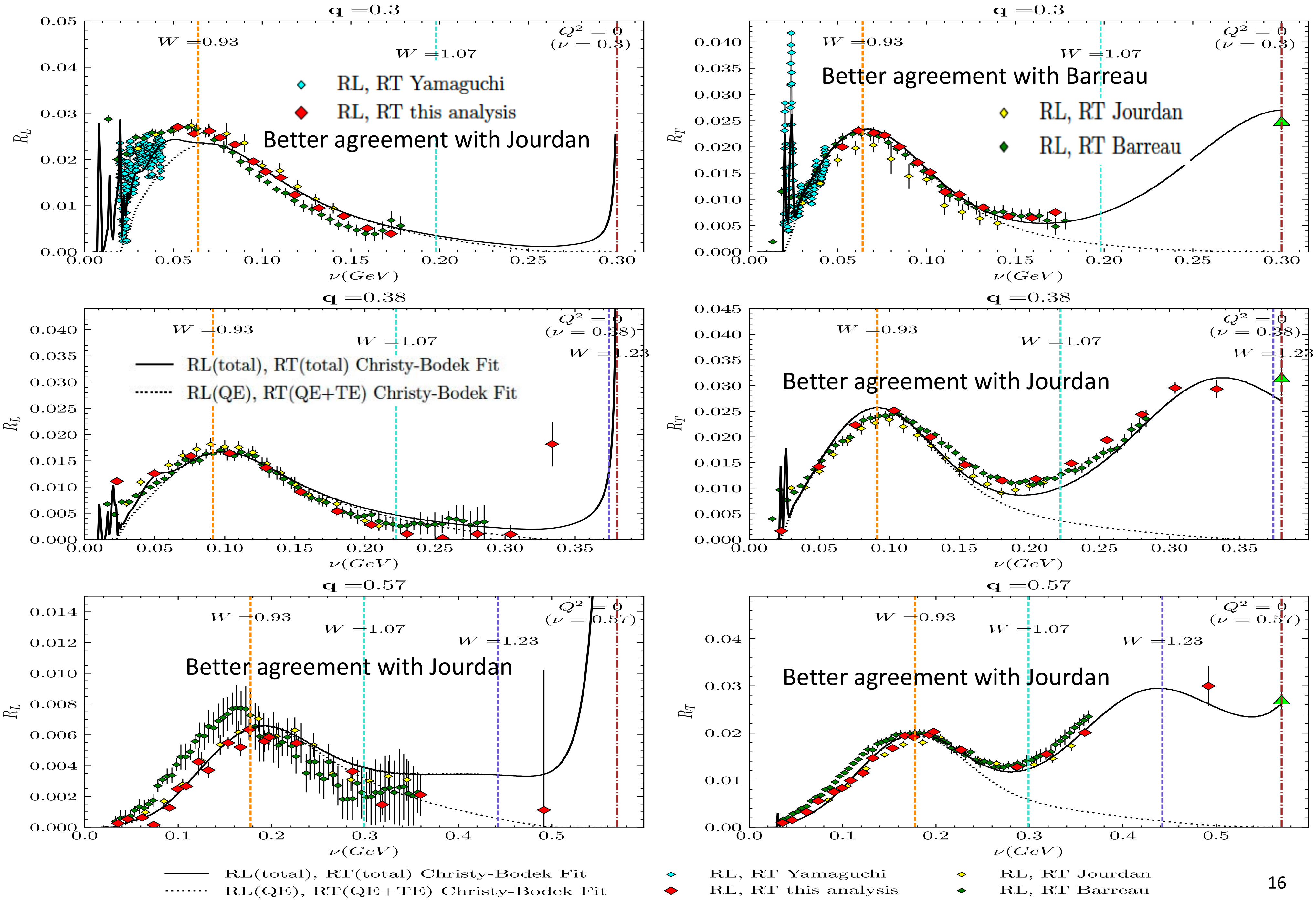
* Comparison to NuWRo Neutrino MC generator for QE (in electron scattering mode)

NuWRo; **R_L is too high (needs longitudinal quenching),** **R_T is too low (needs Transverse Enhancement/ME**C). No nuclear excitations are modeled in any NuWRo. Note: **NuWRo has Transverse Enhancement/MEC in neutrino but not in electron mode**. **---If MEC is implemented in electron mode, these data can be used to validate the model**.

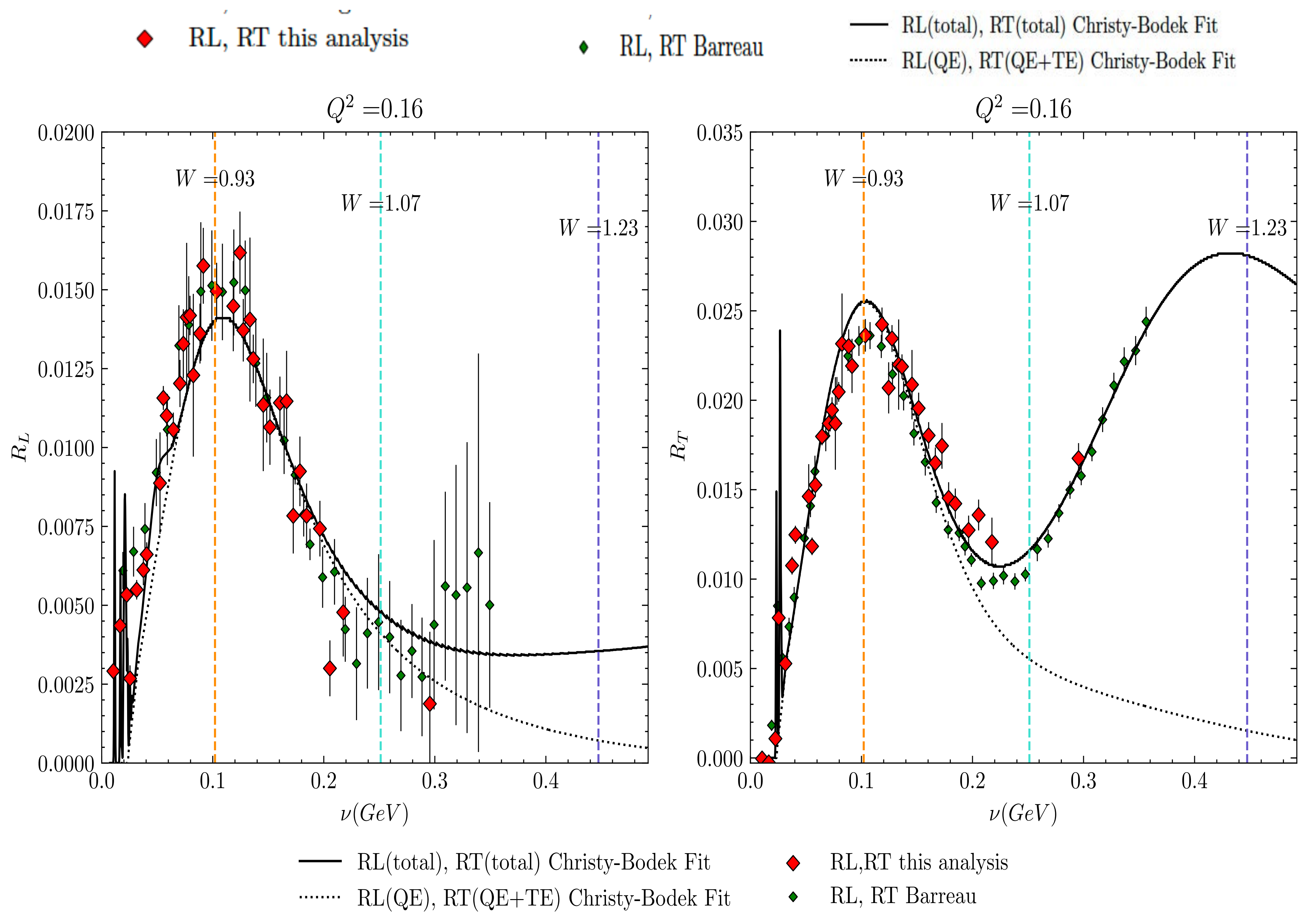


Comparisons with GENIE (in electron mode) will be available shortly

Comparison to previous R_L R_T extractions q = 0.3, 0.38, 0.57 GeV



Comparison to previous R_L R_T extractions Q^2= 0. 16 GeV^2



Comments

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- The extracted RL and RT are in good agreement with the Christy-Bodek Universal Fit. The fit can be used for validation of MC generators over a more extended range of **q** and ν.
- The Christy-Bodek fit includes **Longitudinal Quenching** at low **q** and **Transverse Enhancement** at intermediate q. It also includes nuclear excitations
- The Δ (1.23 GeV) peak is only seen in R_T (since the Δ is mostly Transverse)**.**
- For fixed **q** the maximum value of ν is $\boldsymbol{\nu}$**=q** (where R_T can also be extracted from photoproduction data).
- Validation of Neutrino Generator NuWRo in electron model: NuWRo only models QE with a spectral function, and adds Final State Interaction (FSI)
- NuWRo (spectral function) requires FSI for better agreement with the data.
- However, even with FSI for q<0.3 GeV NuWRo overestimates R_L **(requires Longitudinal Quenching)**
- NuWRo underestimates R_T (in neutrino mode it has Transverse Enhancement/MEC, but not in electron mode).
- Nuclear excitations are not included
- Comparisons with GENIE (in electron mode) will be available shortly.

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Conclusions

- The 18 R_L and R_T extractions cover a large kinematic range. The values are in excellent agreement with the Christy-Bodek Universal fit to all cross section values. The universal fit covers an even larger kinematic range.

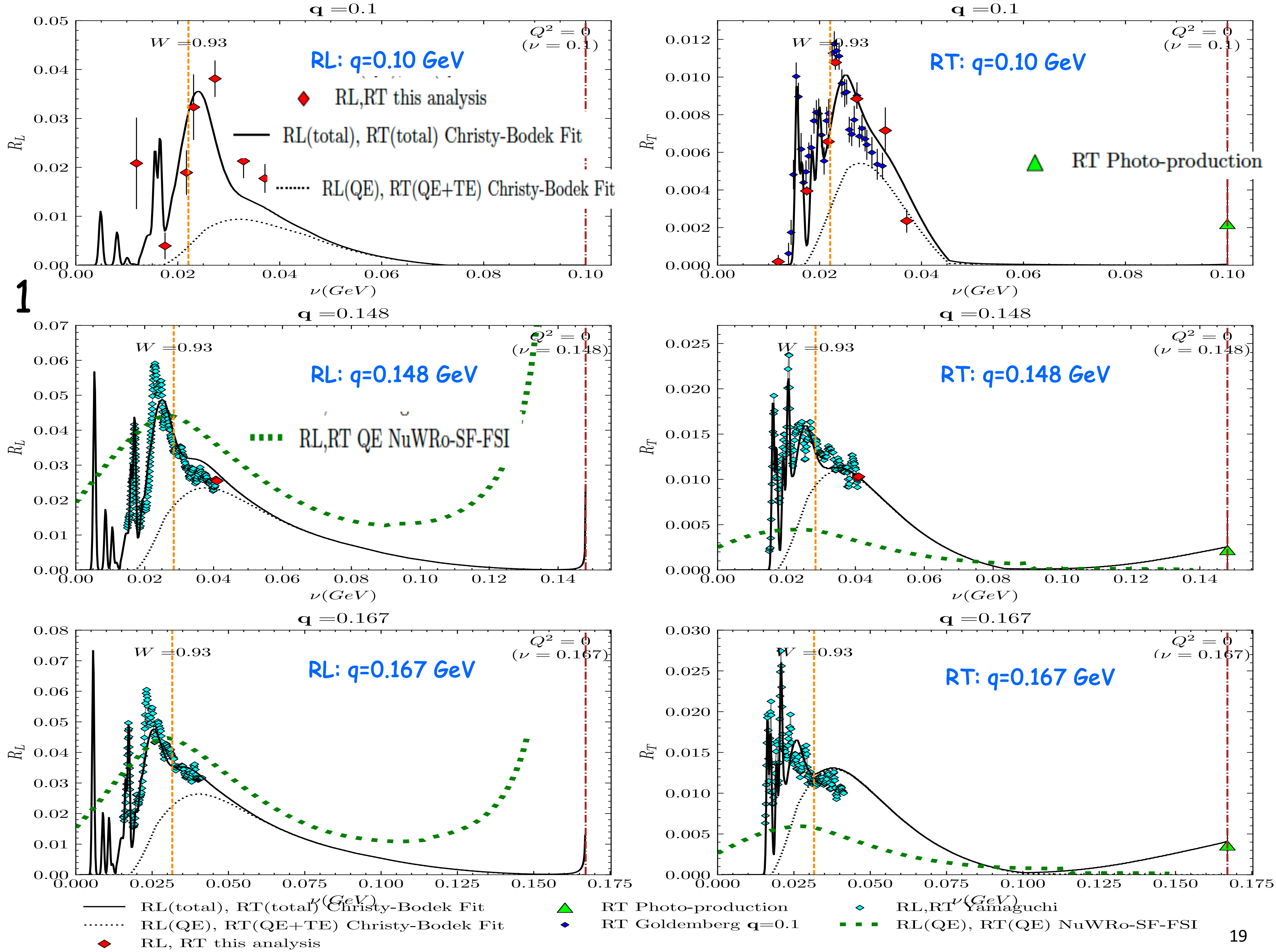
- The R_L and R_T measurements as well as the universal fit provide a simple way to validate electron and neutrino MC generators over the entire kinematic range of interest.
- Good agreement in the QE region with nuclear theory for 3 values of q. Predictions for all other values of q not yet available.

<u>In Supplemental Materials we will provide</u>

Tables of the extracted values of R_L and R_T

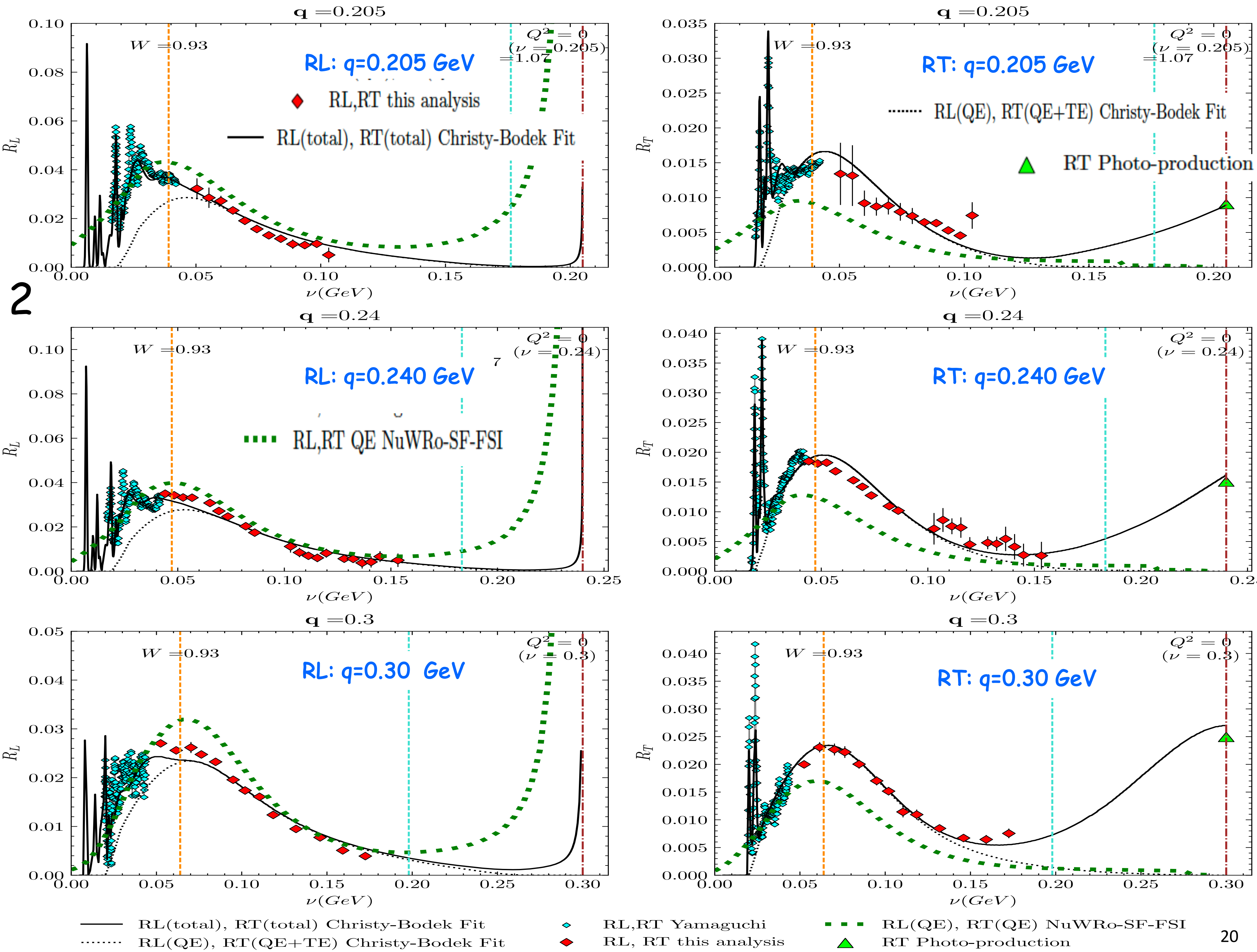
- Tables of the Christy-Bodek Universal fit values for R_L and R_T for the 18 q and Q^2 values. The contributions of nuclear excitations, QE, transverse enhancement and inelastic scattering will be listed separately.
- Code for the Christy-Bodek Universal fit

Backup

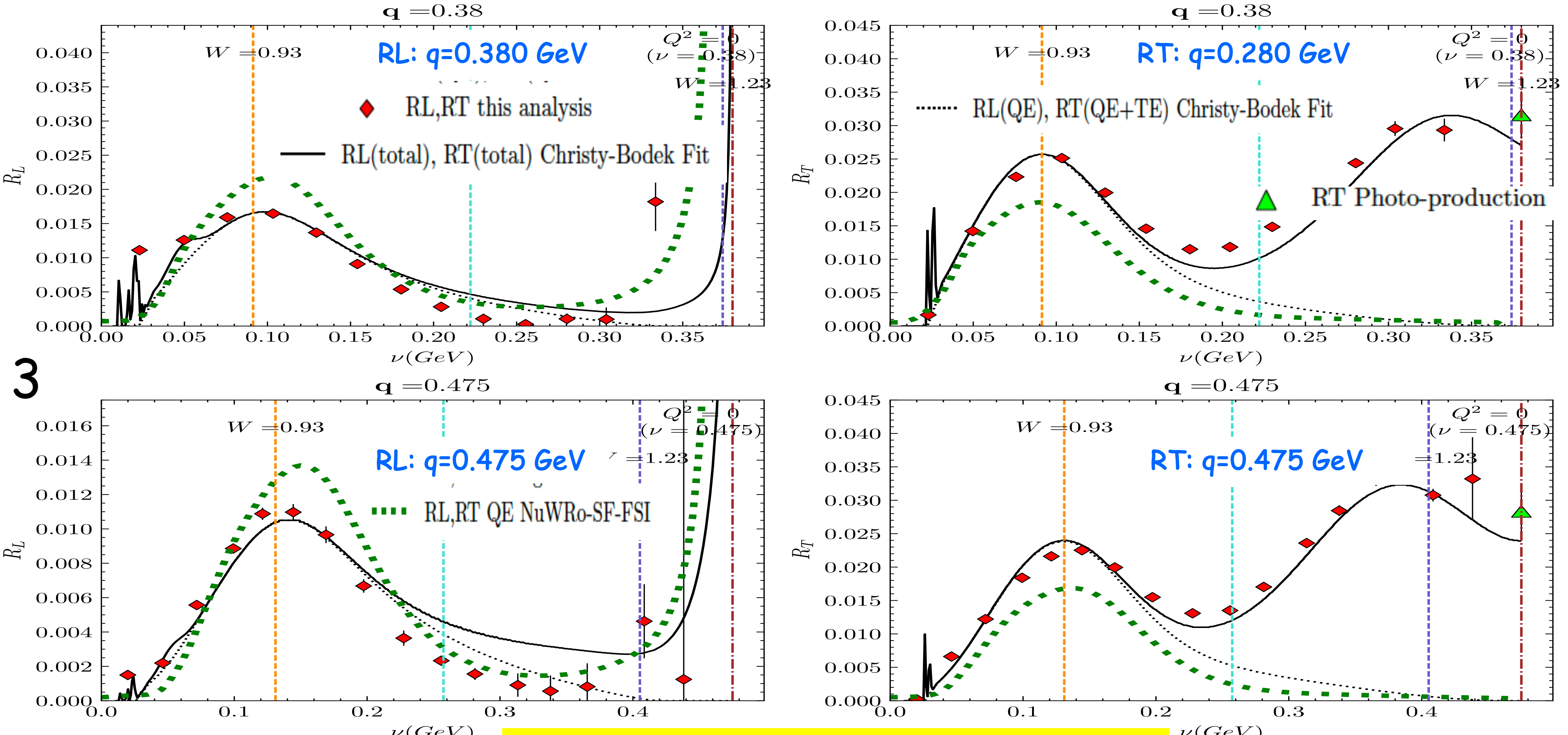


RL: q=0.10 GeV
RT: q=0.10 GeV
RL: q=0.148 GeV
RT: q=0.148 GeV
RL: q=0.167 GeV
RT: q=0.167 GeV
RL,RT this analysis
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RT Photo-production
RL,RT QE NuWRo-SF-FSI
RL,RT Yamaguchi
RT Goldemberg q=0.1
RL(QE), RT(QE) NuWRo-SF-FSI
RL, RT this analysis
1

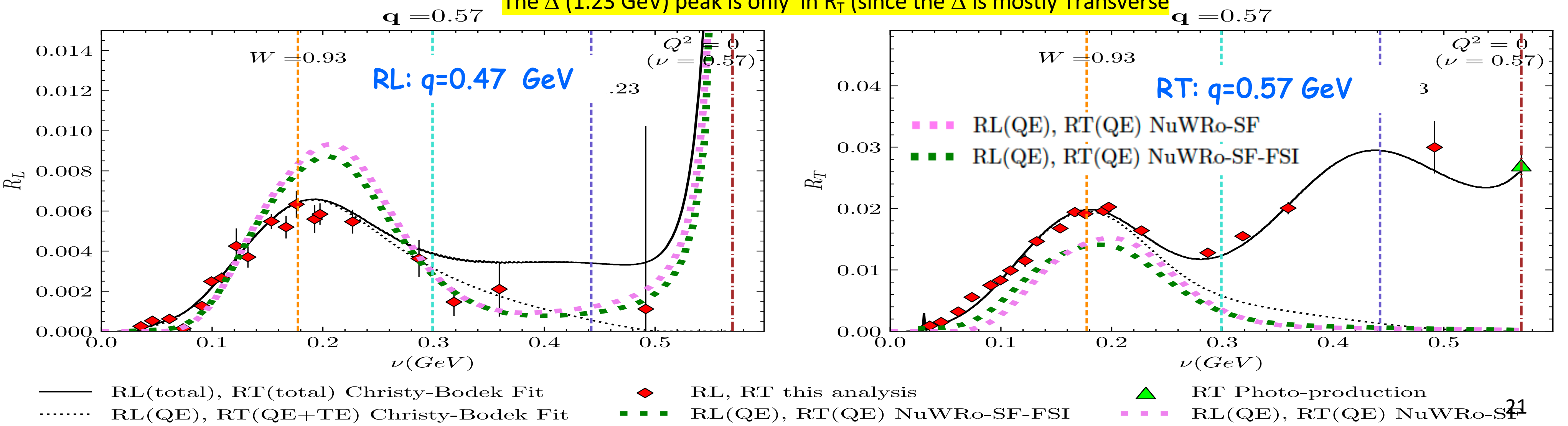
2


q =0.205
RL: q=0.205 GeV
RL,RT this analysis
RL(total), RT(total) Christy-Bodek Fit
RT: q=0.205 GeV
RL(QE), RT(QE+TE) Christy-Bodek Fit
RT Photo-production
q =0.24
RL: q=0.240 GeV
RL,RT QE NuWRo-SF-FSI
RT: q=0.240 GeV
q =0.3
RL: q=0.30 GeV
RT: q=0.30 GeV
W =0.93
$Q^2 = 0$
$\nu(GeV)$
R_L
R_T
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL,RT Yamaguchi
RL, RT this analysis
RL(QE), RT(QE) NuWRo-SF-FSI
RT Photo-production

3



The Δ (1.23 GeV) peak is only in R_T (since the Δ is mostly Transverse



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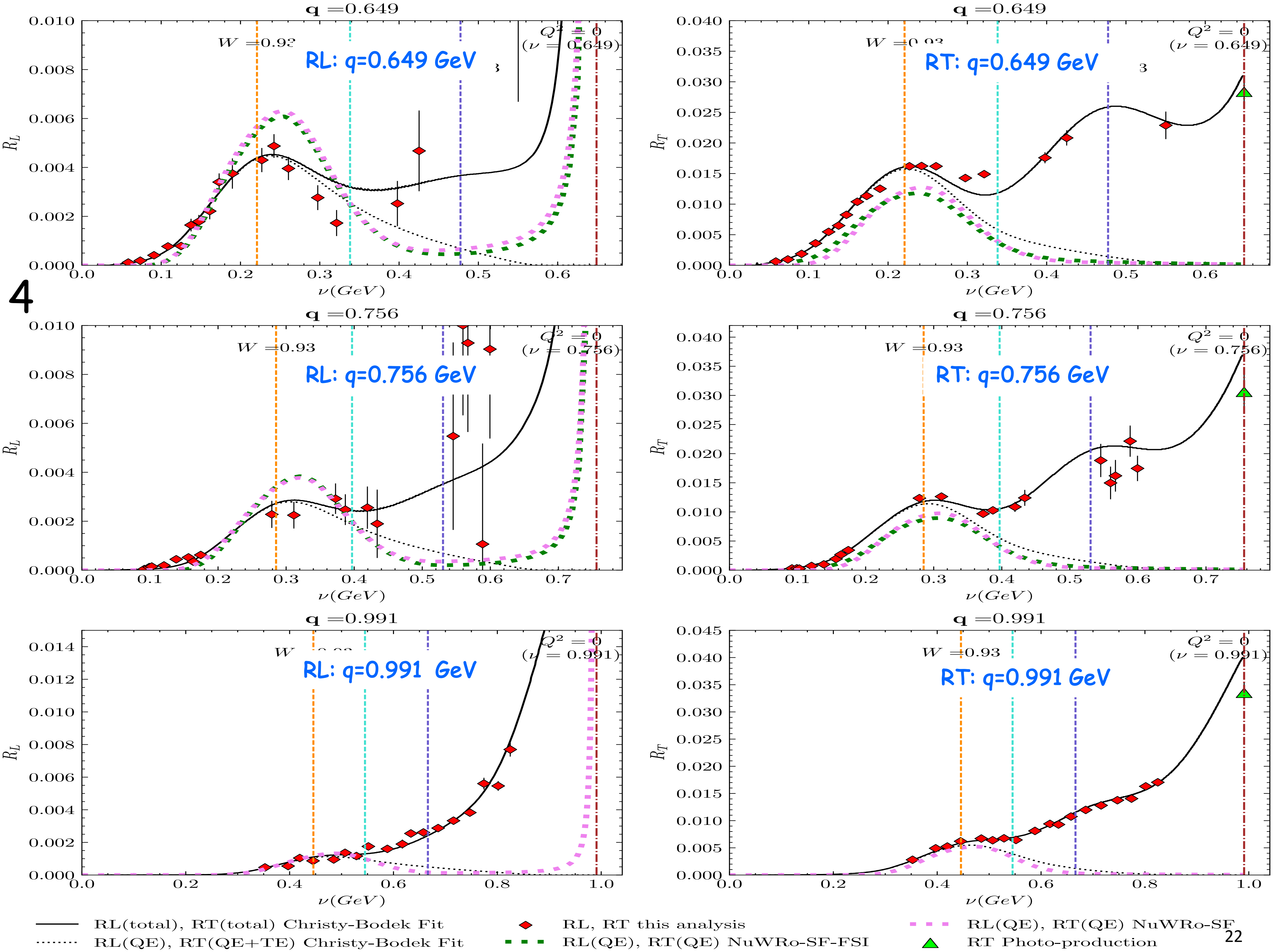
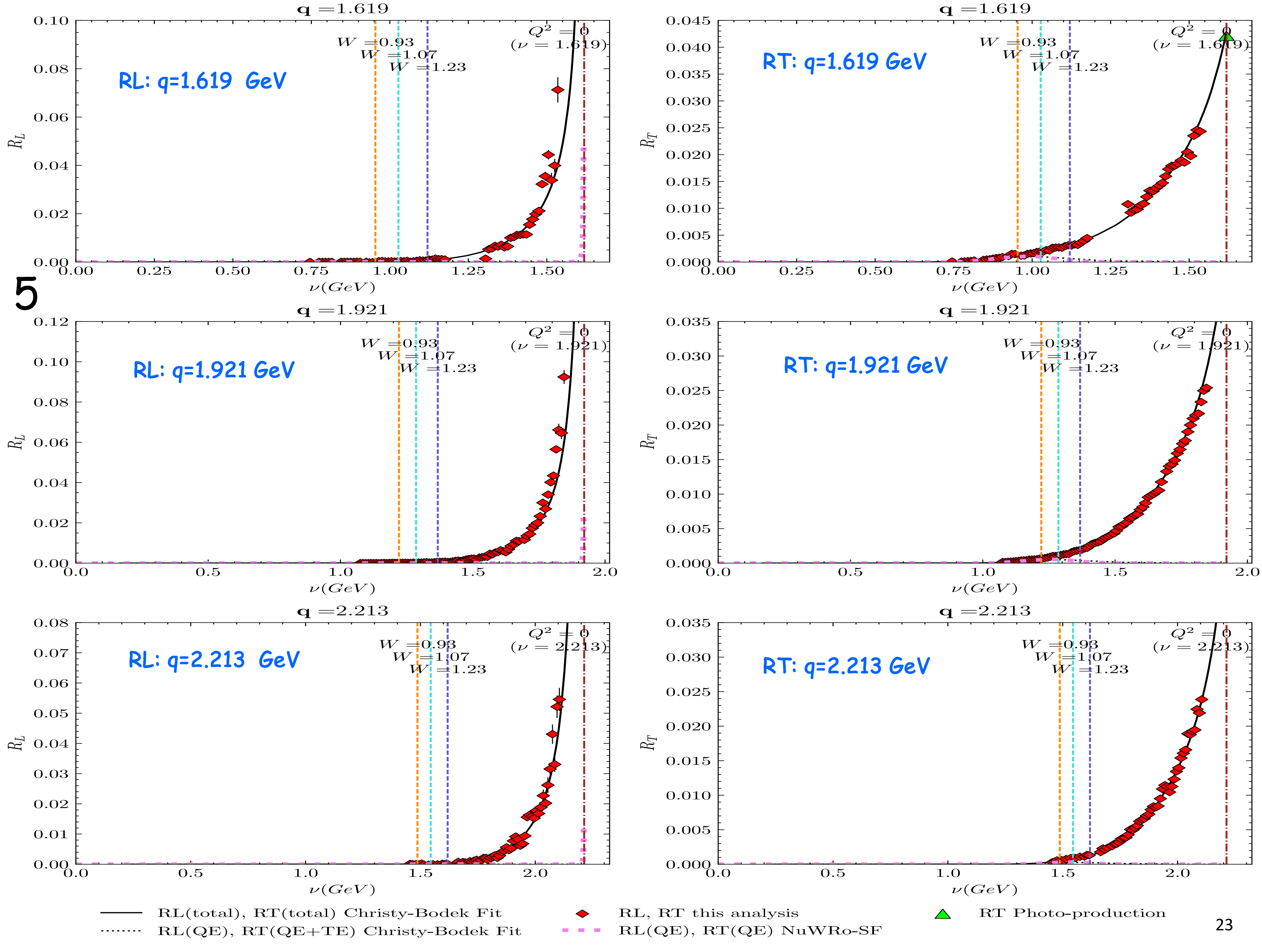
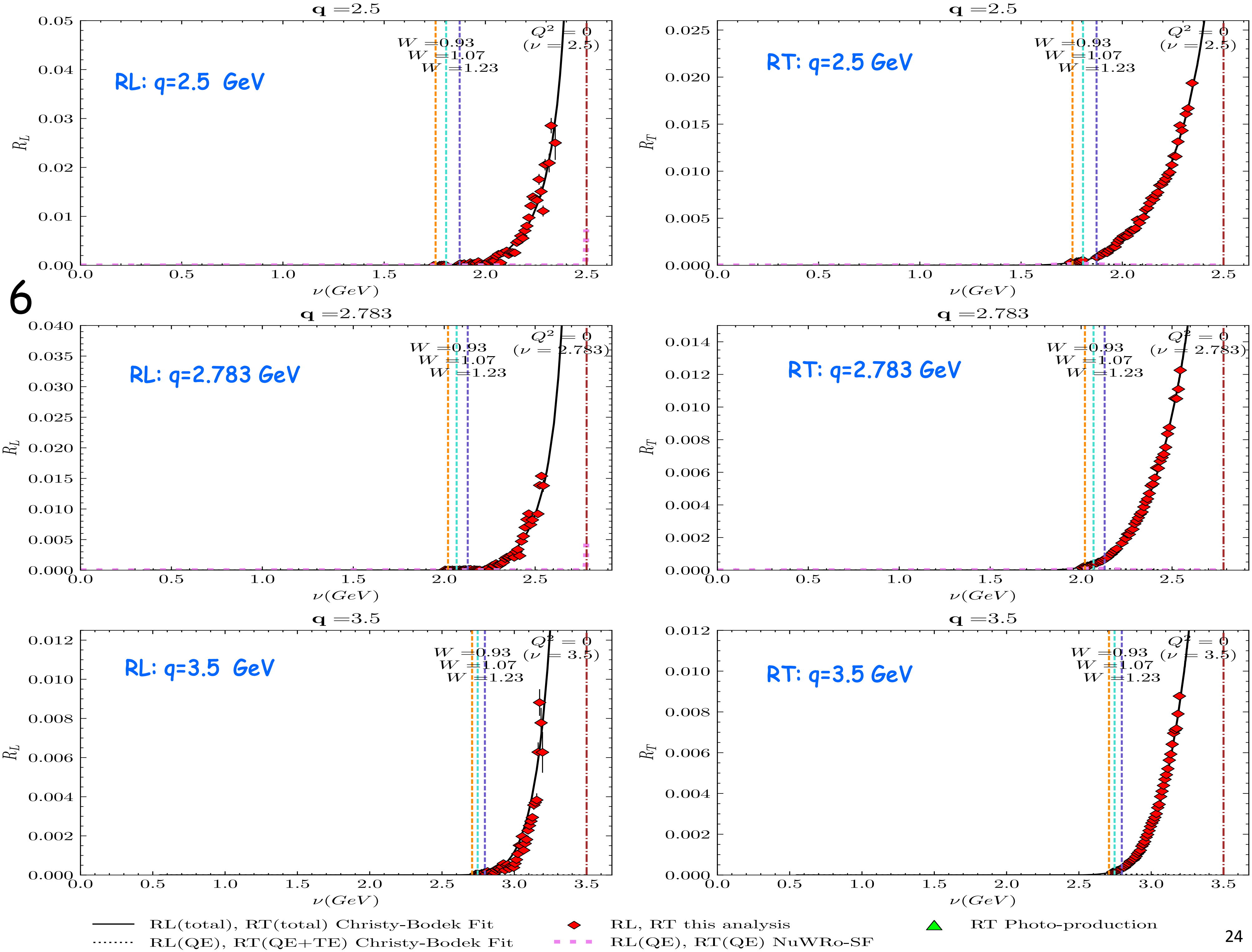
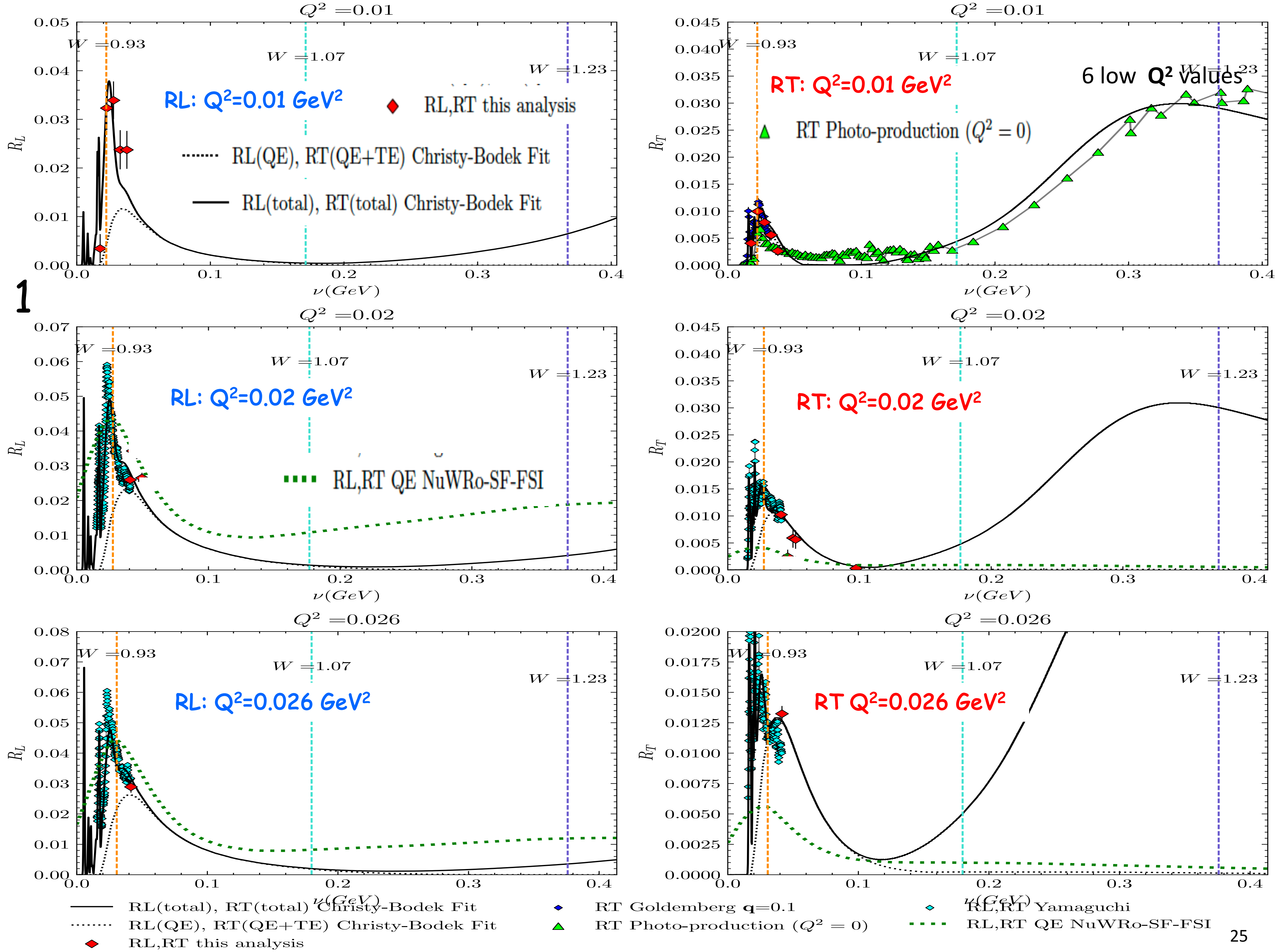

RL: q=0.649 GeV
RT: q=0.649 GeV
RL: q=0.756 GeV
RT: q=0.756 GeV
RL: q=0.991 GeV
RT: q=0.991 GeV
$\mathbf{q}$ =0.649
$\mathbf{q}$ =0.756
$\mathbf{q}$ =0.991
W =0.93
$Q^2=0$
$(\nu = 0.649)$
$(\nu = 0.756)$
$(\nu = 0.991)$
R_L
R_T
$\nu(GeV)$
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL, RT this analysis
RL(QE), RT(QE) NuWRo-SF-FSI
RL(QE), RT(QE) NuWRo-SF
RT Photo-production



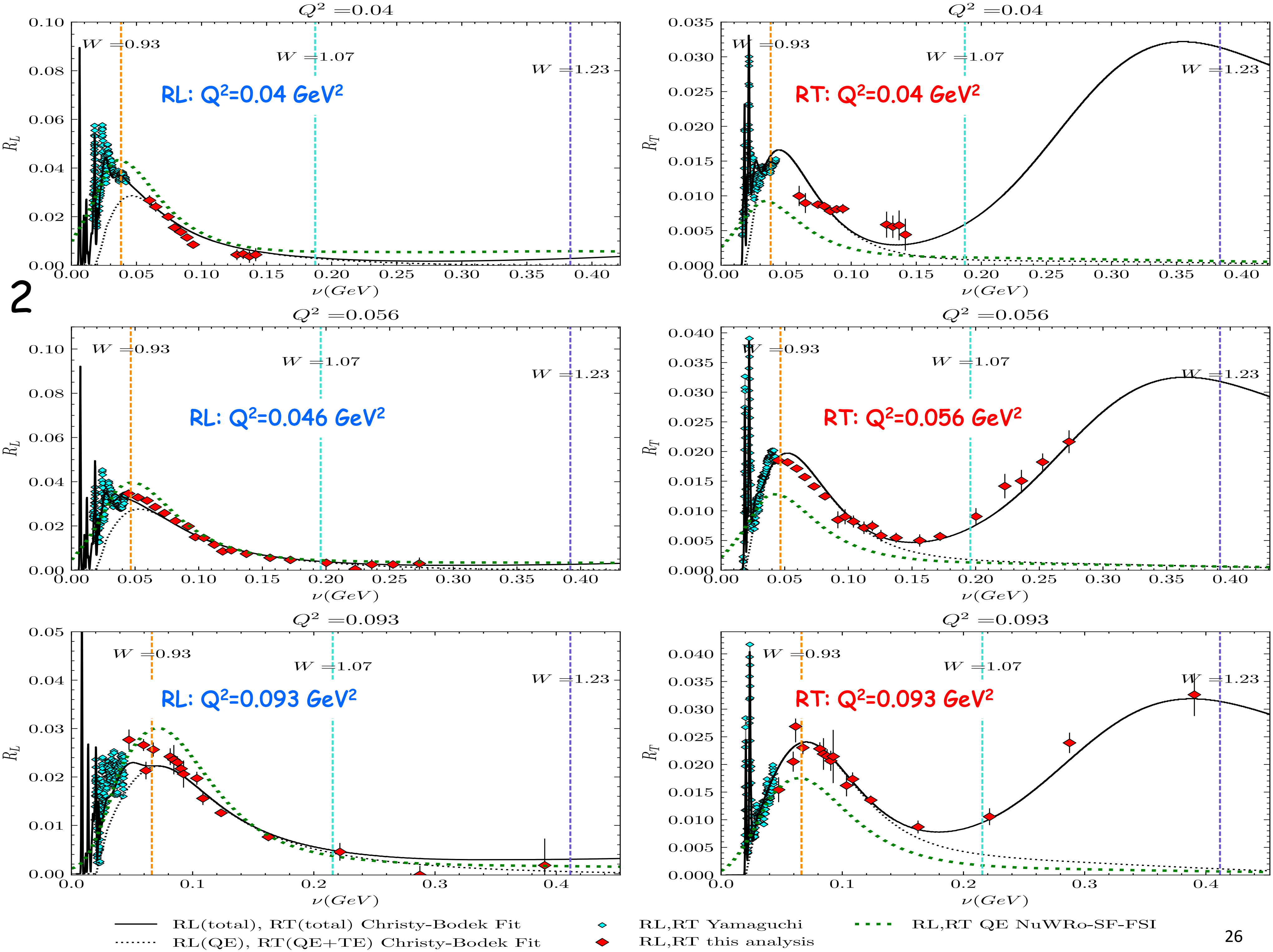
5
q =1.619
RL: q=1.619 GeV
RT: q=1.619 GeV
q =1.921
RL: q=1.921 GeV
RT: q=1.921 GeV
q =2.213
RL: q=2.213 GeV
RT: q=2.213 GeV
W =0.93
W =1.07
W =1.23
$Q^2 = 0$
R_L
R_T
$\nu(GeV)$
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL, RT this analysis
RL(QE), RT(QE) NuWRo-SF
RT Photo-production

6

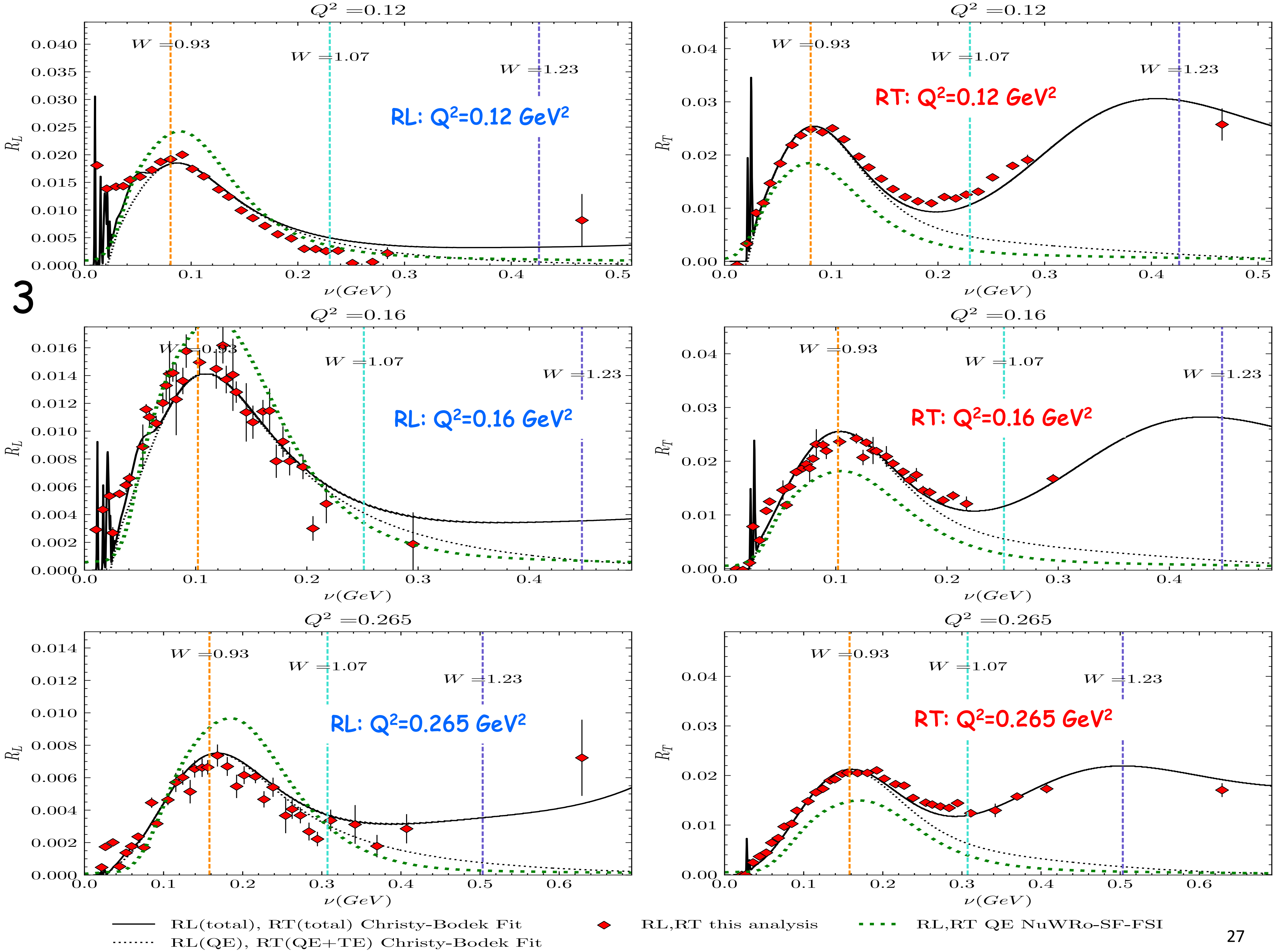

q =2.5
RL: q=2.5 GeV
RT: q=2.5 GeV
q =2.783
RL: q=2.783 GeV
RT: q=2.783 GeV
q =3.5
RL: q=3.5 GeV
RT: q=3.5 GeV
W =0.93
W =1.07
W =1.23
$Q^2 = 0$
R_L
R_T
$\nu(GeV)$
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL, RT this analysis
RL(QE), RT(QE) NuWRo-SF
RT Photo-production

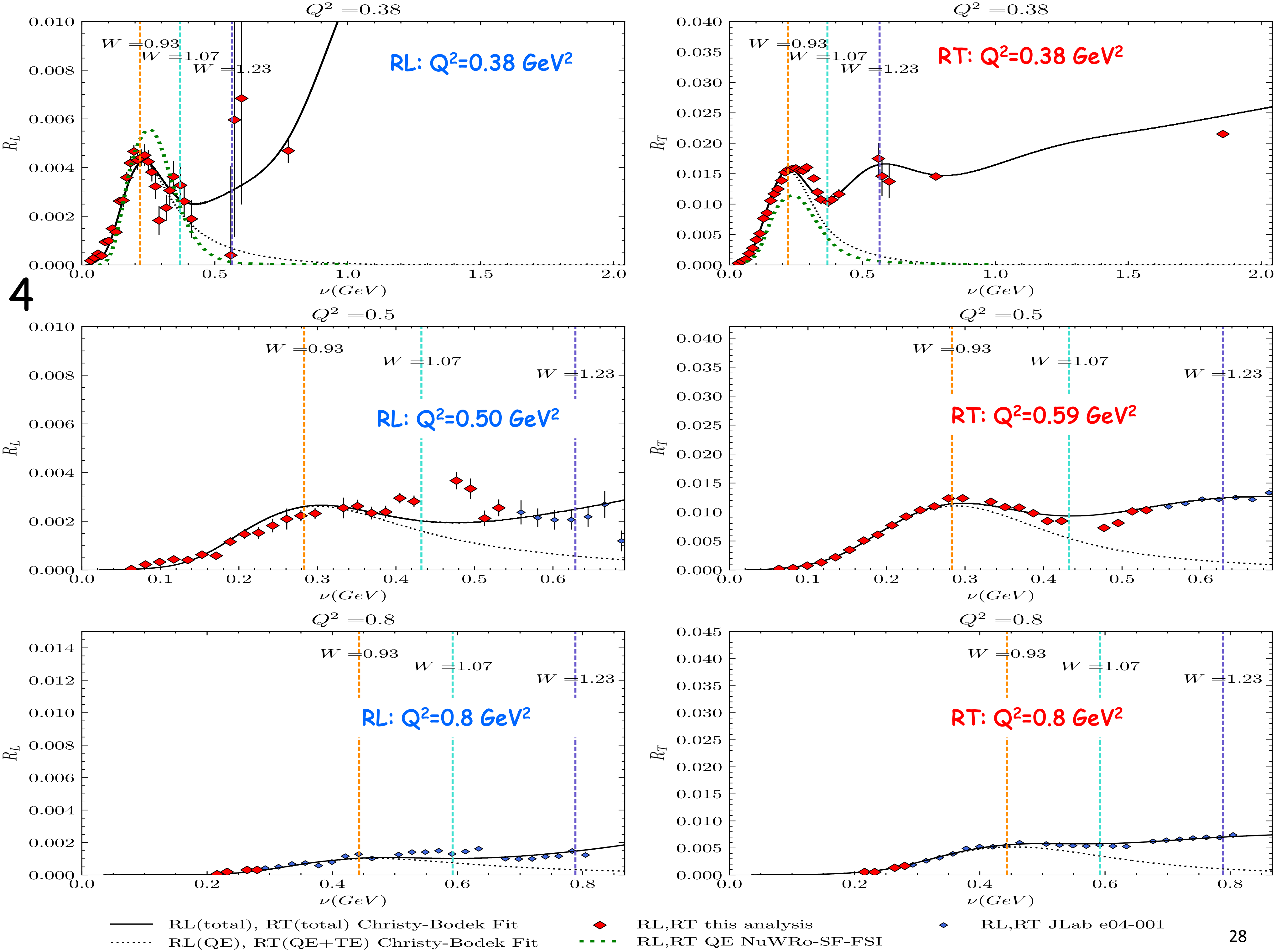


1
6 low Q^2 values
$Q^2 = 0.01$
RL: Q^2=0.01 GeV^2
RT: Q^2=0.01 GeV^2
RL,RT this analysis
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL(total), RT(total) Christy-Bodek Fit
RT Photo-production ($Q^2 = 0$)
$Q^2 = 0.02$
RL: Q^2=0.02 GeV^2
RT: Q^2=0.02 GeV^2
RL,RT QE NuWRo-SF-FSI
$Q^2 = 0.026$
RL: Q^2=0.026 GeV^2
RT Q^2=0.026 GeV^2
$W = 0.93$
$W = 1.07$
$W = 1.23$
R_L
R_T
$\nu(GeV)$
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL,RT this analysis
RT Goldemberg q=0.1
RT Photo-production ($Q^2 = 0$)
RL,RT Yamaguchi
RL,RT QE NuWRo-SF-FSI



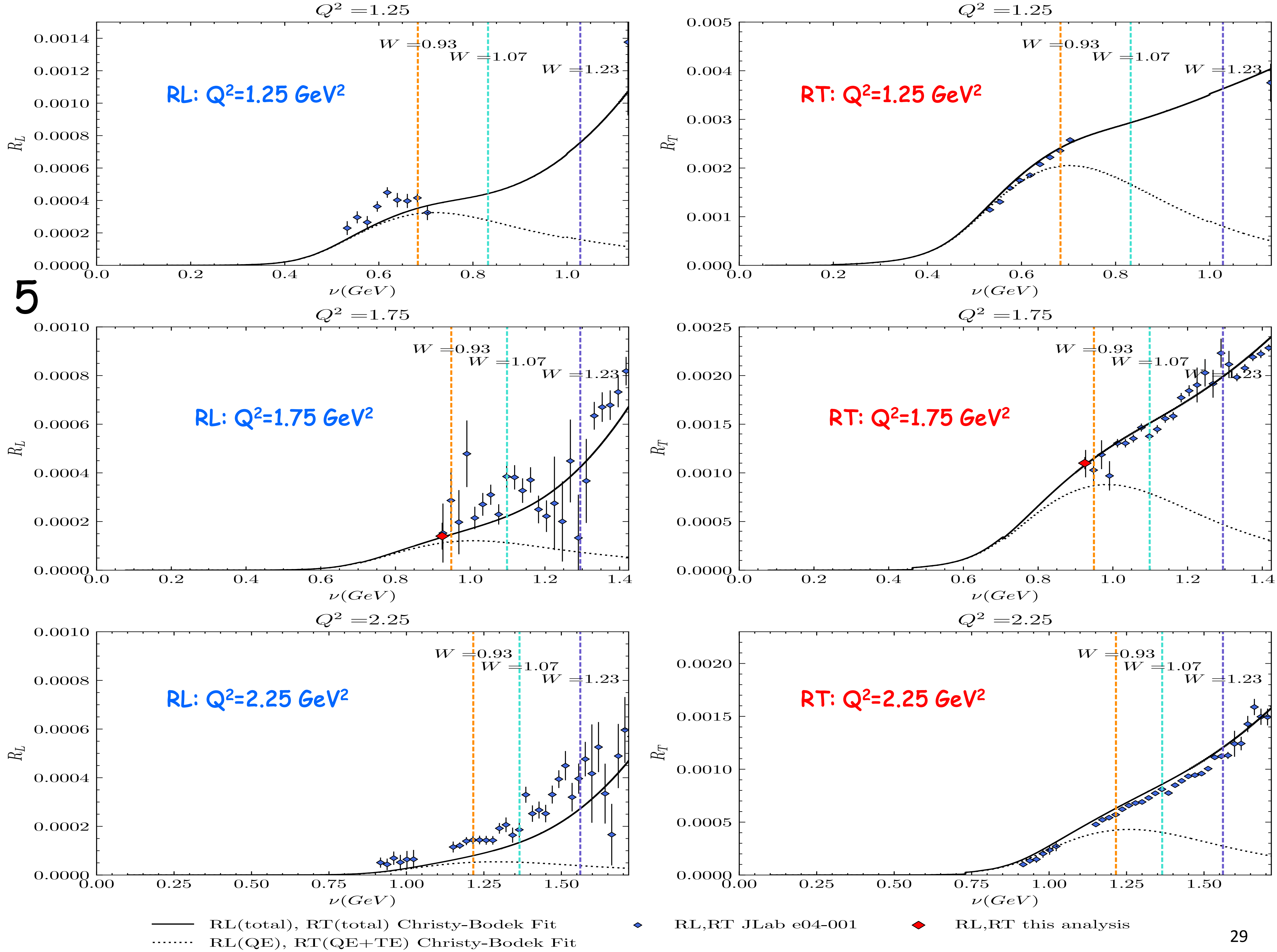
$Q^2 = 0.04$
RL: Q^2=0.04 GeV^2
RT: Q^2=0.04 GeV^2
2
$Q^2 = 0.056$
RL: Q^2=0.046 GeV^2
RT: Q^2=0.056 GeV^2
$Q^2 = 0.093$
RL: Q^2=0.093 GeV^2
RT: Q^2=0.093 GeV^2
$W = 0.93$
$W = 1.07$
$W = 1.23$
R_L
R_T
$\nu(GeV)$
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL,RT Yamaguchi
RL,RT this analysis
RL,RT QE NuWRo-SF-FSI


3
Q² =0.12
RL: Q²=0.12 GeV²
RT: Q²=0.12 GeV²
Q² =0.16
RL: Q²=0.16 GeV²
RT: Q²=0.16 GeV²
Q² =0.265
RL: Q²=0.265 GeV²
RT: Q²=0.265 GeV²
W =0.93
W =1.07
W =1.23
R_L
R_T
ν(GeV)
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL,RT this analysis
RL,RT QE NuWRo-SF-FSI

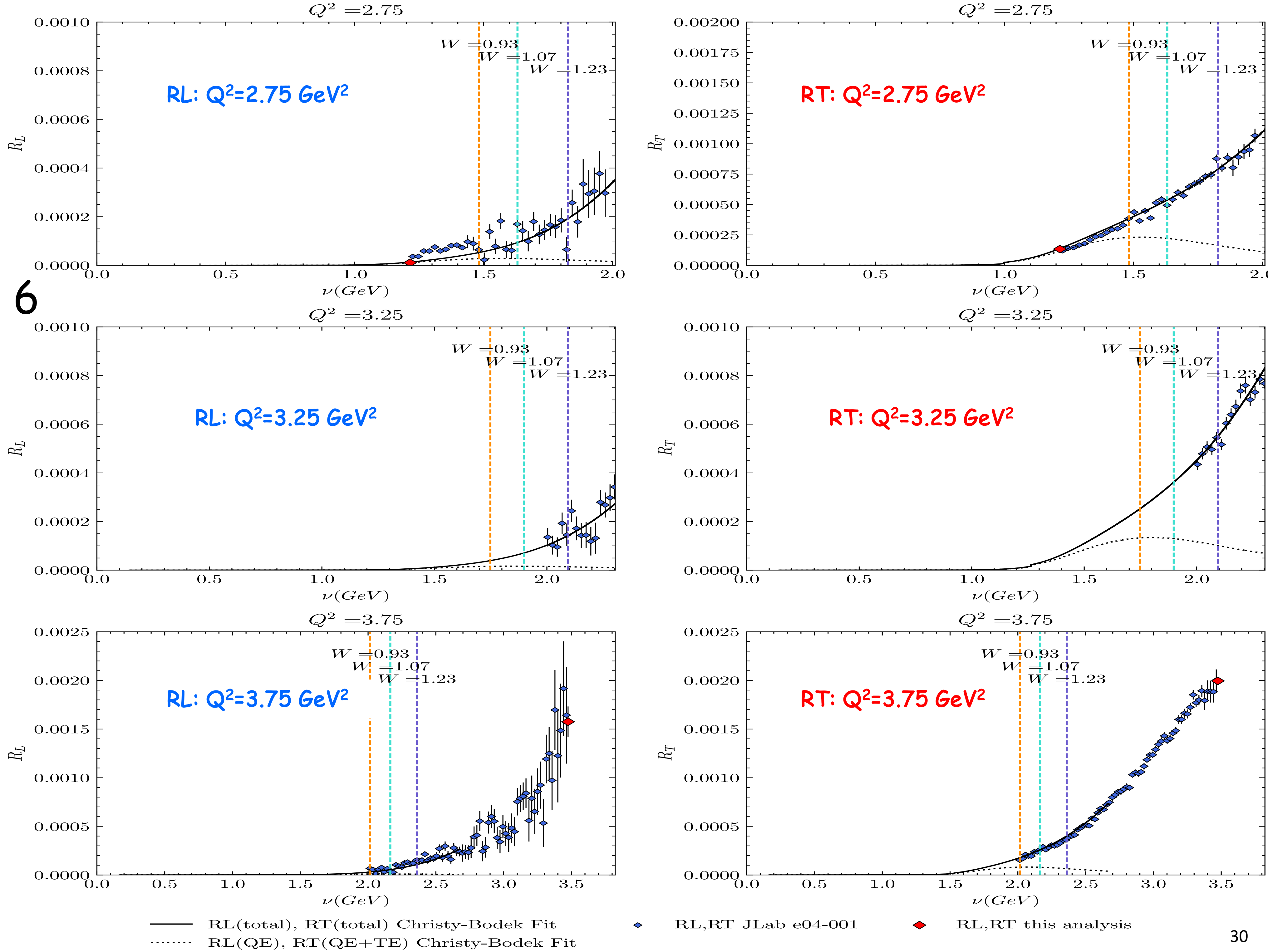


4
RL: Q²=0.38 GeV²
RT: Q²=0.38 GeV²
RL: Q²=0.50 GeV²
RT: Q²=0.59 GeV²
RL: Q²=0.8 GeV²
RT: Q²=0.8 GeV²
Q² =0.38
Q² =0.5
Q² =0.8
W =0.93
W =1.07
W =1.23
R_L
R_T
ν(GeV)
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL,RT this analysis
RL,RT QE NuWRo-SF-FSI
RL,RT JLab e04-001

5



Q^2 =1.25
RL: Q^2=1.25 GeV^2
RT: Q^2=1.25 GeV^2
Q^2 =1.75
RL: Q^2=1.75 GeV^2
RT: Q^2=1.75 GeV^2
Q^2 =2.25
RL: Q^2=2.25 GeV^2
RT: Q^2=2.25 GeV^2
W =0.93
W =1.07
W =1.23
R_L
R_T
$\nu(GeV)$
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL,RT JLab e04-001
RL,RT this analysis



$Q^2 = 2.75$
RL: Q²=2.75 GeV²
RT: Q²=2.75 GeV²
6
$Q^2 = 3.25$
RL: Q²=3.25 GeV²
RT: Q²=3.25 GeV²
$Q^2 = 3.75$
RL: Q²=3.75 GeV²
RT: Q²=3.75 GeV²
$W = 0.93$
$W = 1.07$
$W = 1.23$
R_L
R_T
$\nu(GeV)$
RL(total), RT(total) Christy-Bodek Fit
RL(QE), RT(QE+TE) Christy-Bodek Fit
RL,RT JLab e04-001
RL,RT this analysis