Extraction of the ^{12}C Longitudinal and Transverse Nuclear Electromagnetic Response Functions from Electron Scattering Cross Sections on Carbon

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(Dated: September 6, 2023)

We report on an extraction of the ^{12}C Longitudinal ($\mathcal{R}_T(Q^2,\nu)$) and Transverse ($\mathcal{R}_L(Q^2,\nu)$) nuclear electromagnetic response functions from an analysis of all available electron scattering cross sections on carbon. The response functions are extracted for a range of energy transfer ν (spanning the nuclear excitation, quasielastic, and Δ (1232 MeV) resonance regions) for a large range of the square of the four-momentum transfer Q^2 ($0.001 < Q^2 < 7.0GeV^2$). The data sample consists of $\approx$6600 ^{12}C differential cross section measurements including preliminary high precision cross section measurements from Jefferson Lab Hall C experiment E-04-00. Since extracted response functions cover a very large range Q^2 and ν, they can be used to validate nuclear models as well Monte Caro generators for electron and neutrino scattering experiments.

Note to Ziggy: Please do the analysis first without applying bin centering corrections, The cross sections are binned in bins of Q^2 and W^2. For each data set you need to specify for each value of ν the value of Q^2_{center} so Eric can generate those corrections,.

The cross sections should be binned in bins of Q^2. After all corrections are applied (bin centering corrections, Coulomb corrections, and normalization of each data set we then do the linear fits described in the tex in bins of W^2. If the bin size in W^2 is larger that the bin size of the data, there may be more than on cross section point from each experimental data set (for a fixed energy and scattering angle).

All the cross section data are in units of nanobarns per GeV. We use the constant hbar-c (0.1973269 GeV fm), convert nanobarns to GeV^2. This means that in the analysis each cross section is divided by (hbar-c)^2 = $(0.1973269)^2 10^7$ GeV^2 nb. Then the RL and RT will be in units of GeV^{-1}. After that we divide the extracted RL and RT by 1000 to get RL and RT units of MeV^{-1}.

Note fm^2 is 10^{-26} cm^2 and nanobarns is $10^{-9} \times 10^{-24} = 10^{-33}$ cm^2. Note M is the average nucleon mass which is the average of the proton (0.938272 GeV) and neutron (0.9406 GeV) masses.

I. INTRODUCTION

All electron scattering cross sections on nuclear targets can all be described in terms Transverse ($\mathcal{R}_T(Q^2,\nu)$) and Longitudinal ($\mathcal{R}_L(Q^2,\nu)$) nuclear electromagnetic response functions for each nuclear target. $\mathcal{R}_T(Q^2,\nu)$ and $\mathcal{R}_L(Q^2,\nu)$ are functions of the energy transfer ν and the the square of the 4-momentum transfer Q^2.

Therefore, theoretical models can tested by comparing the predictions of $\mathcal{R}_T(Q^2,\nu)$ and $\mathcal{R}_L(Q^2,\nu)$ to experimental data. Currently, extractions of $\mathcal{R}_T(Q^2\nu)$ and $\mathcal{R}_L(Q^2,\nu)$ from experimental data are available for only three values of momentum transfer of $\mathbf{q}$ of 30, 0.38, and 0.57 GeV and for energy transfer ν in the quasielastic region only. These correspond to values of the square of the 4-momentum transfer Q^2 of $\approx$ 0.005, 0.036 and 0.163 GeV^2. That 1996 analysis by Jourdan[1, 2] used experimental measurements from only two experiments.

In this communication we extract $\mathcal{R}_T(Q^2,\nu)$ and $\mathcal{R}_L(Q^2,\nu)$ from all available electron scattering data: $\approx$6600 ^{12}C differential cross section measurements including preliminary high precision cross section measurements from Jefferson Lab Hall C experiment E-04-00. We report on extractions of $\mathcal{R}_T(Q^2,\nu)$ and $\mathcal{R}_L(Q^2,\nu)$ over a larger range of energy transfer ν (including the quasielastic, nuclear excitations, and Δ-1232 MeV regions) for 16 Q^2 values of 0.010, 0.028, 0.050, 0.070, 0.100, 0.140, 0.190, 0.290, 0.370, 0.570, 0.750, 1.200, 1.970, 2.600, 3.700, 5.500 GeV^2. Since the extracted response functions cover a very large range Q^2 and ν, they can be compared to recent nuclear models, and also validate Monte Caro generators for electron and neutrino scattering experiments,

II. INCLUSIVE ELECTRON-NUCLEON SCATTERING

In terms of the incident electron energy, E_0, the scattered electron energy, $E^{'}$, and the scattering angle, θ, the absolute value of the exchanged 4-momentum squared in electron-nucleon scattering is given by

$$Q^2 = (-q)^2 = 4E_0E^{'}\sin^2\frac{\theta}{2}, \tag{1}$$

the mass of the undetected hadronic system is

$$W^2 = M^2 + 2M\nu - Q^2, \tag{2}$$

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and the square of the magnitude of 3-momentum transfer vector $\vec{\mathbf{q}}$ is

$$\mathbf{q}^2 = Q^2 + \nu^2 \tag{3}$$

Here M the average nucleon mass and $\nu = E_0 - E'$. In these expressions we have neglected the electron mass which is negligible for the kinematics studied.

Center	low	high
0.010	0.003	0.015
0.028	0.015	0.039
0.050	0.039	0.060
0.070	0.060	0.085
0.100	0.085	0.120
0.140	0.120	0.165
0.190	0.165	0.240
0.290	0.240	0.330
0.370	0.330	0.470
0.570	0.470	0.660
0.750	0.660	0.975
1.200	0.975	1.585
1.970	1.585	2.285
2.600	2.285	3.150
3.700	3.150	4.600
5.500	4.600	7.250

TABLE I: A summery of the bins in Q^2

A. Description in terms of structure functions

In the one-photon-exchange approximation, the spin-averaged cross section for inclusive electron-proton scattering can be expressed in terms of two structure functions as follows

$$\begin{aligned} \frac{d\sigma}{d\Omega dE'} &= \sigma_M[\mathcal{W}_2(W^2,Q^2) + 2\tan^2(\theta/2)\mathcal{W}_1(W^2,Q^2)] \\ \sigma_M &= \frac{\alpha^2\cos^2(\theta/2)}{[2E\sin^2(\theta/2)]^2} = \frac{4\alpha^2 E'^2}{Q^4}\cos^2(\theta/2) \end{aligned} \tag{4}$$

where σ_M is the Mott cross section, $\alpha = 1/137$ is the fine structure constant. The $\mathcal{F}_1$ and $\mathcal{F}_2$ structure functions are related to $\mathcal{W}_1$ and $\mathcal{W}_2$ by $\mathcal{F}_1 = M\mathcal{W}_1$ and $\mathcal{F}_2 = \nu\mathcal{W}_2$. The structure functions are typically expressed as functions of Q^2 and W^2 (or alternatively ν or $x = Q^2/(2M\nu)$.

B. Description in terms of response functions

The electron scattering differential cross section is written in terms of longitudinal ($\mathcal{R}_L(Q^2,\nu)$) and transverse ($\mathcal{R}_T(Q^2,\nu)$) nuclear response functions [3]

$$\frac{d\sigma}{d\nu d\Omega} = \sigma_M[A\mathcal{R}_L(Q^2,\nu) + B\mathcal{R}_T(Q^2,\nu)] \tag{5}$$

where σ_M is the Mott cross section, $A = (Q^2/\mathbf{q}^2)^2$ and $B = \tan^2(\theta/2) + Q^2/2\mathbf{q}^2$.

The relationship between the nuclear response functions, and structure functions is

$$\mathcal{R}_T(\mathbf{q},\nu) = \frac{2\mathcal{F}_1(\mathbf{q},\nu)}{M} \tag{6}$$

$$\mathcal{R}_L(\mathbf{q},\nu) = \frac{\mathbf{q}^2}{Q^2}\frac{\mathcal{F}_L(\mathbf{q},\nu)}{2Mx} \tag{7}$$

Where the units of $\mathcal{R}_L(\mathbf{q},\nu)$ and $\mathcal{R}_T(\mathbf{q},\nu)$ correspond to the units of M^{-1}.

The square of the electric and magnetic form factors for elastic scattering and nuclear excitations are obtained by the integration of the measured response functions over ν for each nuclear state. When form factors for nuclear elastic scattering and nuclear excitations are extracted from electron scattering data typically there is an additional factor of Z^2 in the definition of σ_M (where Z is the atomic number of the nucleus).

C. Experimental extraction of response functions

The method[1, 2] used to separate $\mathcal{R}_L(Q^2,\nu)$ and $\mathcal{R}_T(Q^2,\nu)$ is described below. The quantity

$$\begin{aligned} \Sigma(Q^2,\nu) &= H\frac{d\sigma}{d\nu d\Omega} \\ &= \epsilon\mathcal{R}_L(Q^2,\nu) + \frac{1}{2}(\frac{\mathbf{q}}{Q})^4\mathcal{R}_T(Q^2,\nu) \end{aligned} \tag{8}$$

$$\begin{aligned} H &= [\frac{1}{\sigma_M}\epsilon(\frac{\mathbf{q}}{Q})^4] \\ &= \frac{\mathbf{q}^4}{4\alpha^2 E'^2}\frac{1}{\cos^2(\theta/2) + 2(\frac{\mathbf{q}}{Q})^2\sin^2(\theta/2)} \end{aligned} \tag{9}$$

is plotted as a function of the virtual photon polarization ϵ. The virtual photon polarization ϵ is a function ν, Q^2 and θ as shown below.

$$\epsilon = \left[1 + 2(1 + \frac{\nu^2}{Q^2})\tan^2\frac{\theta}{2}\right]^{-1} \tag{10}$$

The virtual photon polarization ϵ varies from 0 to 1 as the scattering angle θ ranges from 180 to 0 degrees. We use equation 9 for H because it is valid for all scattering angles including 180 degrees.

As described below, we bin the data in Q^2 bins an apply an overall correction K_i to each cross section. The correction K_i corrects for for the small difference in the normalization of each data set, the effect of the nuclear Coulomb field, and a bin centering correction to the center of the bin in Q^2. We then perform a Rosenbluth linear for each value of ν. Here, $\mathcal{R}_L(Q^2,\nu)$ is the slope, and $\frac{1}{2}\frac{\mathbf{q}^2}{Q^2}\mathcal{R}_T(Q^2,\nu)$ is the intercept of the linear fit,

III. COULOMB CORRECTIONS

In modeling QE and inelastic (pion production) scattering from bound nucleons, Coulomb corrections to

QE and inelastic pion production processes are taken into account using the "Effective Momentum Approximation"(EMA) [4, 5]. The approximation is a simple energy gain/loss method, using a slightly higher incident and scattered electron energies at the vertex than measured in the lab. The effective incident energy is $E_{eff} = E_0 + V_{eff}$, and the effective scattered energy is $E'_{eff} = E' + V_{eff}$.

Assuming a spherical charge distribution in the nucleus (of radius R) the electrostatic potential inside the charged sphere can be defined as followed:

$$V(r) = \frac{3\alpha(Z-1)}{2R} + \frac{\alpha(Z-1)}{2R}\frac{r}{R} \tag{11}$$

where R (in units of GeV) is given by:

$$R = 1.1A^{(1/3)} + 0.86A^{(-1/3)}. \tag{12}$$

$$\begin{aligned} V_{eff} &= 0.775V(r=0) \\ &= 0.775\ \frac{3}{2}\ \alpha(Z-1)/R \end{aligned} \tag{13}$$

where Z and A are the atomic number and atomic weight of the nucleus, respectively. This value for V_{eff} is consistent with value of $V_{eff} = 3.1 \pm 0.25$ extracted from a comparison of positron and electron QE scattering cross sections on carbon[5]. The effective Q^2_{eff} is given by

$$Q^2_{eff} = 4(E_0 + V_{eff})(E' + V_{eff})\sin^2(\theta/2) \tag{14}$$

The structure functions are calculated with $Q^2 = Q^2_{eff}$ and $E' = E'_{eff}$. In addition, there is a focusing factor $F_{foc} = \frac{E_0+V_{eff}}{E_0}$ which modifies the Mott cross section The modified Mott cross section is

$$\begin{aligned} \sigma_{M-eff} &= F_{foc}\frac{\alpha^2\cos^2(\theta/2)}{[2E_{eff}\sin^2(\theta/2)]^2} \\ &= \sigma_M\frac{E_0}{E_0+V_{eff}}. \end{aligned} \tag{15}$$

In this analysis we use our overall fit to model all existing electron scattering data on ^{12}C. The fit includes the following components: (a) Nuclear-elastic scattering and excitation of nuclear states, (b) Quasielastic scattering and (c) Resonance production and the inelastic continuum. The parameterizations of the form factors for nuclear-elastic scattering and excitation of nuclear states are presented in [6]. A brief description of the quasielastic and resonance production fits and the measurement of the Coulomb Sum Rule are presented in [7]. .

The differential cross section model (σ_{model}) is used to correct the measured cross sections σ_{meas} and yield a coulomb corrected(CC).

IV. SUMMARY OF CORRECTIONS TO THE DATA

We use our cross section fit (σ_{model}) to evaluate the following corrections to the cross sections in each bin in Q^2.

1. From the overall fit we extract the relative normalization (N_i) of the various data sets.

2. We use the fit to apply Coulomb charge corrections (Q_i).

3. We use the fit to apply bin centering corrections (C_i) to all cross sections that are within the Q^2 range for each Q^2 bin. The bin centering corrections is done by only changing Q^2 and keeping the same values of W^2.

After the application of the above multiplicative correction all cross sections in each Q^2 bin we can assume that all cross section are for Q^2_{center}. When we apply these corrections, we keep the final state invariant mass W^2 fixed, and only correct for the change in Q^2.

All cross sections are multiplied by the product $K_1 = (N_i)(Q_i)(C_i)$ before we do the Rosenbluth linear fit for each value of W^2.

A. Q^2 versus q

The values of the response function then can be plotted versus W^2 or versus ν that corresponds to the the centered value Q^2_{center} using the following expression:

$$\nu = (W^2 - M^2 + Q^2_{center})/2M$$

In previous analyses the extraction of the response functions done for fixed value of $\mathbf{q}$. However, the correct extraction should be done in bins of Q^2. This is because for fixed Q^2 the peak position of the quasielastic distribution is at the same value of both W^2 and ν for all scattering angles. In contrast, for fixed value of $\mathbf{q}$ the he peak position of the quasielastic distribution in ν depends on the scattering angle.

$$\nu = (W^2 - M^2 + \mathbf{q}_{center} - \nu^2)/2M$$

Similarly, at fixed Q^2 each nuclear state with excitation energy E_x is at the same ν at all scattering angles.

$$\nu = Q^2/2M_{C12} + E_x$$

where M_{C12} is the mass of ^{12}C.

Appendix A SUMMARY OF CROSS SECTION DATA

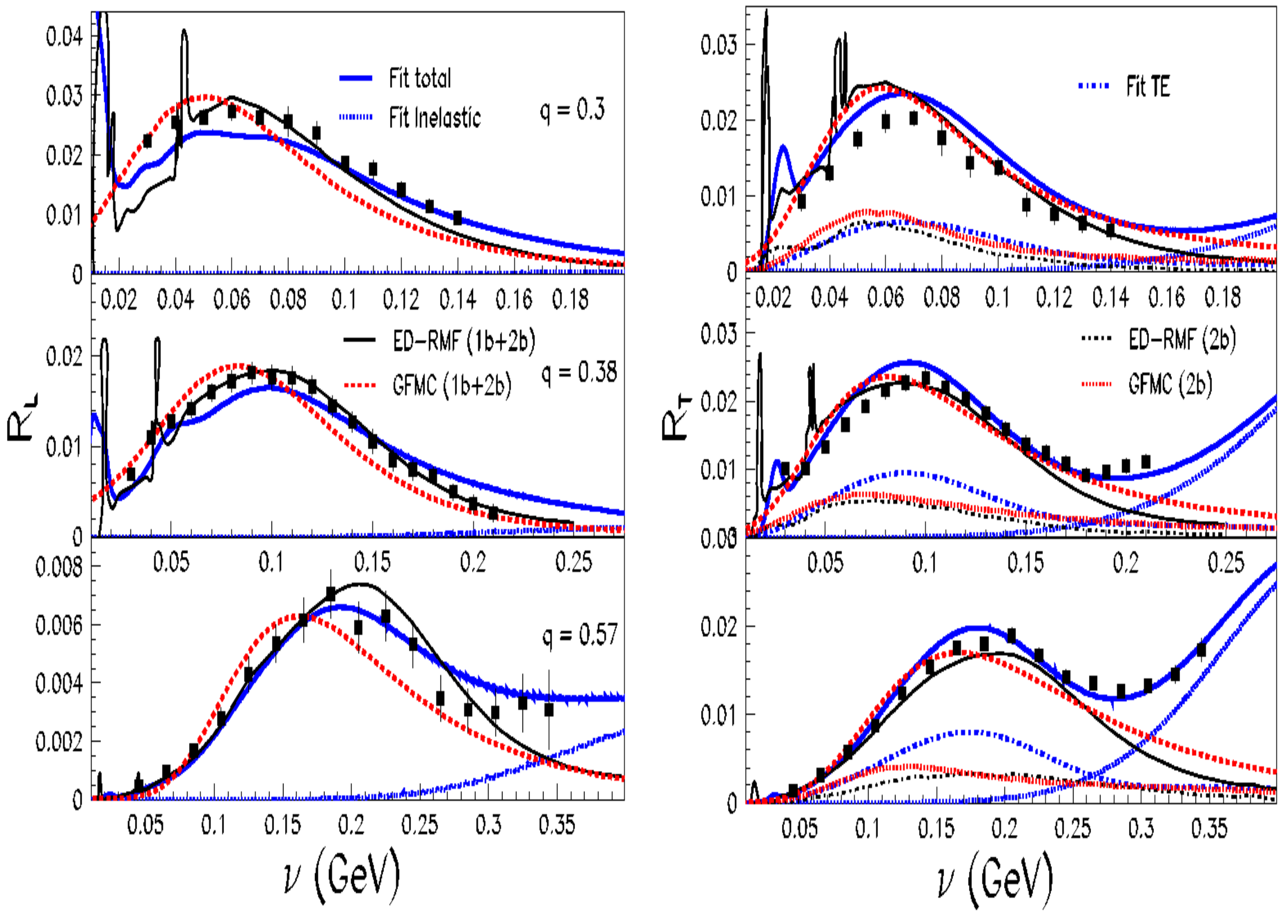

FIG. 1: Comparisons between our extraction of $\mathcal{R}_L(\mathbf{q},\nu)$ and $\mathcal{R}_T(\mathbf{q},\nu)$ (in units of MeV^{-1} and the extraction (for only three values of $\mathbf{q}$) by Jourdan[1, 2]. (The Jourdan analysis includes data from only two experiments). Also shown are comparisons to 1b+2b GFMC[8] and ED-RMF[9] theoretical predictions. In these two models the curves labeled 2b are the only contribution of 2-body currents to $TE(\mathbf{q},\nu)$. The transverse enhancement in both 1b and 2b currents is included in the total.

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Data Set	Q^2_{Min} (GeV^2)	Q^2_{Max} (GeV^2)	# Data Points	Normalization	χ^2_{pdf}
Yamaguchi71 [10]					
Gomez74 [11, 12]					
Whitney [13]					
Barreau83 [14]					
O'Connell87 [15]					
Baran88 [16]					
Bagdasaryan88 [17]					
Sealock89 [18]					
Day93 [19]					
Dai19 [20]					
Arrington96[21]					
Arrington99 [22]					
Fomin10 [23, 24]					
Gaskell [25, 26]					
Ryan [27]					
E04-001 (Preliminary) [28–30]					
Zeller73 [31] (not used)					

TABLE II: A summary table of the ^{12}C data sets used in the universal fit. Shown are the number of data points, the Q^2 range of each set, the normalization factor, and the universal fit χ^2 per degree of freedom for each set. The Zeller73 [31] data set is inconsistent with all other data sets and is not used.

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