Q4.

(a) $b + \sum_{i=1}^{10} w_i x_i \leq 0$

$0.1 + 10 \cdot 0.1 \cdot 1 + 0.1 \cdot x_{11} \leq 0$

$x_{11} \leq -11$

$\max(x_{11}) = -11$

(b) 3/100 − 2.

input $\longrightarrow$ layer 1 $\longrightarrow$ layer 2.

3 $\longrightarrow$ 100 $\longrightarrow$ 2

W: 3×100   W: 100×2

b: 100   b: 2

$\left.\right\}$ $3\times100+100+100\times2+2 = 602.$

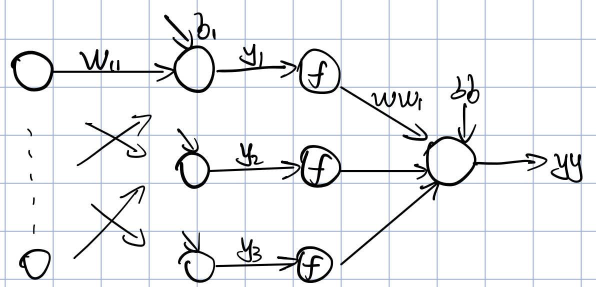
(c) input (7,1) $\longrightarrow$ 100 neurons $\longrightarrow$ 5 outputs

W: 7×100   W: 100×5

b: 100   b: 5

$\left.\right\}$ $7\times100+100+100\times5+5 = 1305$

(d)



$y_1 = b_1 + \sum_{i=1}^{101} W_{1i} \cdot x_i = 0.1 + 0.1 \cdot (51-50) = 0.2 \quad \Rightarrow f(y_1) = 0.2$

$y_2 = b_2 + \sum_{i=1}^{101} W_{2i} \cdot x_i = 0.1 + 0.1 \cdot (51-50) = 0.2 \quad \Rightarrow f(y_2) = 0.2$

$y_3 = b_3 + \sum_{i=1}^{101} W_{3i} \cdot x_i = 0.1 + 0.1 \cdot (51-50) = 0.2 \quad \Rightarrow f(y_3) = 0.2$

$yy = bb + \sum_{i=1}^{3} ww_i\, f(y_i) = 0.1 + 3 \times 0.1 \times 0.2 = 0.16$

$$\frac{\partial yy}{\partial w_{11}} = \frac{\partial yy}{\partial f(y_1)} \cdot \frac{\partial f(y_1)}{\partial y_1} \cdot \frac{\partial y_1}{\partial w_{11}} = ww_1 \cdot 1 \cdot x_1 = 0.1x_1$$

$$\Delta w_{11} = -\alpha e \frac{\partial yy}{\partial w_{11}}$$

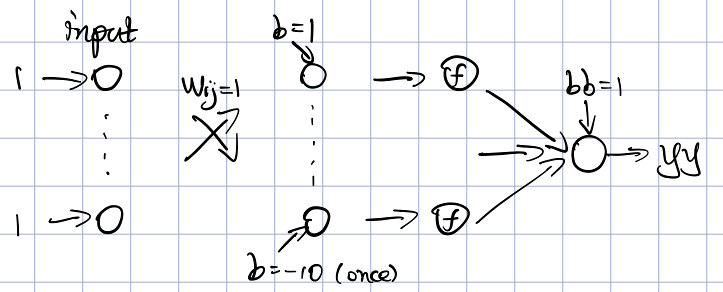
$$\Rightarrow 0.0014 = -0.01\,(yy - t) \cdot 0.1x_1$$

$$-1.4 = (0.16 - t)x_1$$

if $x_1 = 1 \quad \Rightarrow t = 1.56$

if $x_1 = -1 \quad \Rightarrow t = -1.24$

(e)



for the neuron with bias $= -10$ at layer 1:

$y = -10 + 9 \times 1 \times 1 = -1 \quad \Rightarrow f(y) = 0$

other neuron with bias $= 1$:

$y = 1 + 9 \times 1 \times 1 = 10 \quad \Rightarrow f(y) = 10$

$yy = bb + (100-1)\,ww \times 10 + 1 \times ww \times 0$

$100 = 1 + 990\,ww$

$ww = 0.1$

(f) $x = 0.5$

$y_1 = b_1 + w_1 x = -0.5 + 0.5 \times 0.5 = -0.25 \Rightarrow f(y_1) = 0$

$y_2 = b_2 + w_2 x = 0.5 + (-0.5) \times 0.5 = 0.25 \Rightarrow f(y_2) = 0.25$

$yy = ww_1 f(y_1) + ww_2 f(y_2) + bb = 0 + (-0.5) \times 0.25 + 0 = -0.125$

$t = g(0.5) = 1 - (0.5-1)^2 = 0.75$

$w_2 = w_2 - \alpha (yy - t) \frac{\partial yy}{\partial w_2} = -0.5 - 0.1(-0.125 - 0.75) \cdot \frac{\partial yy}{\partial f(y_2)} \cdot \frac{\partial f(y_2)}{\partial y_2} \cdot \frac{\partial y_2}{\partial w_2}$

$= -0.5 + 0.1 \times 0.875 \cdot ww_2 \cdot 1 \cdot x$

$= -0.5 + 0.1 \times 0.875 \cdot (-0.5) \cdot 1 \cdot 0.5$

$= -0.521875.$

Q5.

(a) $f(x,y) = (x-1)^4 + x + (y-1)^2 + 2$

$\frac{\partial f}{\partial x} = 4(x-1)^3 + 1$; $\frac{\partial f}{\partial y} = 2(y-1)$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} - \alpha \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} \Bigg|_{(1.8, 1.8)} = \begin{bmatrix} 1.8 \\ 1.8 \end{bmatrix} - 0.1 \begin{bmatrix} 4(1.8-1)^3+1 \\ 2(1.8-1) \end{bmatrix} = \begin{bmatrix} 1.4952 \\ 1.64 \end{bmatrix}$$

(b) by gradient descent : $\vec{p} = \vec{p} - \alpha \vec{\nabla} f$

for a particular parameter: $p_i = p_i - \alpha \frac{\partial f}{\partial p_i}$

$f = (yy - t)^2$

$\frac{\partial f}{\partial p_i} = \frac{\partial}{\partial p_i}[(yy-t)^2] = \frac{\partial}{\partial yy}[(yy-t)^2] \cdot \frac{\partial yy}{\partial p_i}$ // since t is constant.

$= 2(yy-t) \cdot \frac{\partial yy}{\partial p_i}$

$\Rightarrow p_i = p_i - 2\alpha(yy-t)\frac{\partial yy}{\partial p_i} = p_i - \alpha' e \frac{\partial yy}{\partial p_i}$ // let $\alpha' = 2\alpha$