IEOR4004

HW#1

Prof. Kroer

Due date: Sep 24 | 2019

1. var x_1 = aviation fuel sold without further processing (in 1000 barrels)

x_2 = heating fuel sold without further processing (in 1000 barrels)

x_3 = aviation fuel sold with further processing (in 1000 barrels)

x_4 = heating fuel sold with further processing (in 1000 barrels)

max obj: $60x_1 + 40x_2 + 130x_3 + 90x_4 - 40(x_1+x_2+x_3+x_4)$

subject to: $x_1 + x_3 = x_2 + x_4$

$x_1+x_2+x_3+x_4 \le 20000/1000$

$x_3 + 0.75x_4 \le 8$

$x_1, x_2, x_3, x_4 \ge 0$

Using Gurobi.

var $x_1 = 2$

$x_2 = 10$

$x_3 = 8$

$x_4 = 0$

obj: 760

Therefore, selling 2000 barrels aviation fuel without further processing, 10000 barrels heating fuel without further processing, 8000 barrels aviation fuel with further processing & 0 barrel of heating fuel with further processing, with \$760,000

2 Let x_1: unit of process one

x_2: unit of process two

x_3: # hours the promoter works.

obj: max $(3x_1+5x_2)\times 5 + 3\times(x_1+2x_2) - 2\times(2x_1+3x_2) - 100x_3$ * total earn

subject to $x_1 + 2x_2 \leq 20000$

$2x_1 + 3x_2 \leq 35000$

$3x_1 + 5x_2 \geq 1000 + 200x_3$

$x_1, x_2, x_3 \geq 0$

Using Gurobi: $x_1 = 10000$

$x_2 = 5000$

$x_3 = 270$

obj: 118000

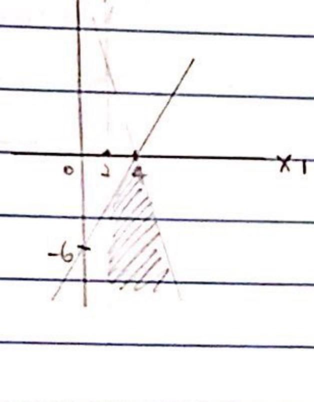
3. min $z = 3x_1 - 2x_2$ $\rightarrow x_2 = \frac{3}{2}x_1 - \frac{1}{2}z$

subject to $3x_1 + x_2 \leq 12$ ①

$3x_1 - 2x_2 - x_3 = 12$ ② $\rightarrow x_2 \leq \frac{3}{2}x_1 - 6$

$x_1 \geq 2$ ③

$x_1, x_2, x_3 \geq 0$



①②③ the constraints result in the shaded region. Since with ④. x_2 should be positive: only one point $(4,0)$ is left. Therefore, the optimal solution is.

$z = 3\times 4 - 2\times 0 = 12$.

9. m_i: unit using method i, $i=1,2,3$

g_i: the final amount of barrels of gas with the grade i, $i=6,8,10$

h_i: the final amount of barrels of heating oil with the grade i, $i=6,8,10$

x_6: barrels of grade 6 that are upgraded to grade 8

x_8: barrels of grade 8 that are upgrade to grade 10.

obj: $\max \; 12(g_6+g_8+g_{10})+5(h_6+h_8+h_{10})-(3.4m_1+3m_2+2.6m_3+x_6+1.5x_8)$

constraints:
$$g_6+h_6=0.3m_1+0.4m_2+0.1m_3-x_6$$
$$g_8+h_8=0.5m_1+0.2m_2+0.3m_3-x_8+x_6$$
$$g_{10}+h_{10}=0.8m_1+0.4m_2+0.2m_3+x_8$$
$$6g_6+8g_8+10g_{10}\geq 9(g_6+g_8+g_{10})$$
$$6h_6+8h_8+10h_{10}\geq 7(h_6+h_8+h_{10})$$
$$g_6+g_8+g_{10}\leq 2000$$
$$h_6+h_8+h_{10}\leq 600$$

Using Gurobi $(m_1,m_2,m_3)=(1625,0,0)$

$(g_6,g_8,g_{10})=(300,400,1300)$

$(h_6,h_8,h_{10})=(1187.5,412.5,0)$

$(x_6,x_8)=(0,0)$

obj: 21475

var

5 let x_i: # of stocks that we sell of stock i.

obj: max $(100-x_1)\times 36 + (100-x_2)\times 39 + (100-x_3)\times 42 + (100-x_4)\times 45$
$+ (100-x_5)\times 51 + (100-x_6)\times 55 + (100-x_7)\times 63$
$+ (100-x_8)\times 64 + (100-x_9)\times 66 + (100-x_{10})\times 70$

constraints: $x_1 \ldots x_{10} \geq 0$ & $x_1 \ldots x_{10} \leq 100$

$30x_1 + 34x_2 + 43x_3 + 47x_4 + 49x_5 + 53x_6 + 60x_7 + 62x_8 + 64x_9 + 66x_{10}$
$- [0.3\times x_1(30-20) + 0.3\times x_2(34-25) + 0.3\times x_3(43-30) +$
$0.3\times x_4(47-35) + 0.3\times x_5(49-40) + 0.3\times x_6(53-45) +$
$0.3\times x_7(60-50) + 0.3\times x_8(62-55) + 0.3\times x_9(64-60) + 0.3\times x_{10}(66-65)]$
$- 0.01\times[30x_1 + 34x_2 + 43x_3 + 47x_4 + 49x_5 + 53x_6 + 60x_7$
$+ 62x_8 + 64x_9 + 66x_{10}] \geq 30000$

Using Gurobi: $x_1 = 0$

$x_2 = 0$

$x_3 = 100$

$x_4 = 100$

$x_5 = 0$

$x_6 = 63.75075$

$x_7 = 0$

$x_8 = 100$

$x_9 = 100$

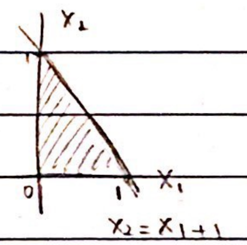
$x_{10} = 100$

obj: 20893.7088

6. 1) False. Since the LP model could either has one optimal solution ~~or~~ or more than 1 optimal solutions

Reason: the intersection of the moving isovalue curve that satisfies objective and the feasible region can only be one intersection point or ~~1 of~~ a line segments which result in more than 2 optimal solutions.

2) ~~False~~: True.



max $z = \cancel{x_1}\, x_2 - x_1$

constraints $x_1 \geq 0$

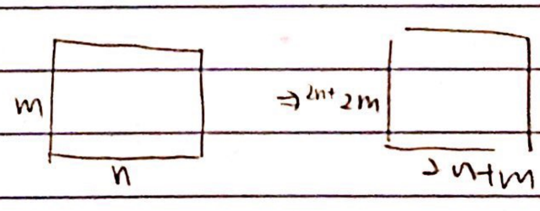
$x_2 \geq 0$

$x_2 \leq x_1 + 1 \Rightarrow x_2 - x_1 \leq 1$

$\exists$ only two optimal corner solutions $(1,0)$ & $(0,1)$

3) False. although LP is feasible, it might be unbounded. In this case, it does not ~~matter that it must be true~~ necessarily has an optimal solution $\Rightarrow$ the conclusion is false.

4) True



constrains

We can transform the original inequalities to equations by add m variables. and change the n variables to positive & negative $(x \to x^+, x^-)$

Thus, we'll have $2n+m$ variables with $2n+2m$ constraints