IEORE4004: Optimization Models and Methods	11/06/2019

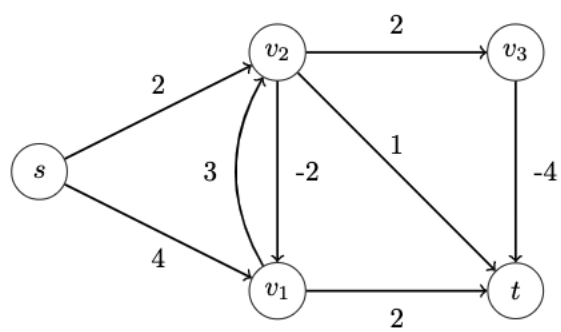
Assignment 4

Deadline: 11/19/2019, h: 11:40am — *Instructor: Christian Kroer*

- This assignment sheet has 7 exercises. You must submit your solution before the deadline. The time constraint is strict. Late submissions will not be accepted.
- You are allowed to discuss the assignment with others but the write-up must be individual work. Please mention in your write-up all the people you have discussed the solution with.
- Bonus questions are meant to compensate for missing points in this, past, or future assignments. The overall assignment score cannot exceed 100%.

Problem 1:

1. Compute the shortest path from s to t in the following network using the label-correcting algorithm.

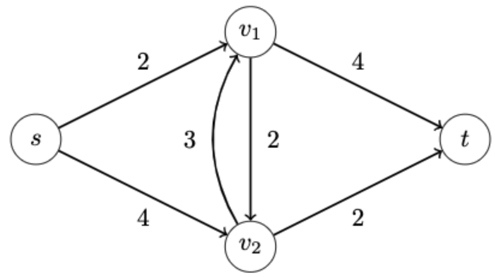


2. Now look at the final table of distances / predecessors only. Suppose you do NOT know the original network – indeed, suppose the original network may be different, but the final entries of d and p stay the same. Can you tell what the shortest path from v_1 to v_3 is? How about from v_2 to t?

Problem 2: Consider the (simplified) list of activities and predecessors that are involved in building a house in the table below. Draw a project network, determine the critical path.

Activity	Description	Immediate Predecessors	Duration (Days)
A	Build foundation	–	5
B	Build walls and ceilings	A	8
C	Build roof	B	10
D	Do electrical wiring	B	5
E	Put in windows	B	4
F	Put on siding	E	6
G	Paint house	C, F	3

Problem 3: Compute the maximum flow from s to t and the minimum $s-t$ cut in the following graph using the Ford and Fulkerson algorithm.



Problem 4: Suppose that a graph has more than one $s-t$ flow of maximum value. Can we deduce that it also has more than an $s-t$ cut of minimum capacity? Motivate.

Problem 5: Call the collection of the following steps of the Ford-Fulkerson algorithm an *iteration* of the algorithm:

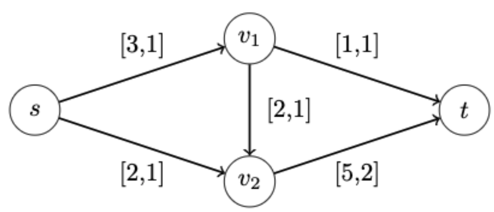
a) From the current flow, the residual network is constructed;

b) A path P from s to t in the residual network is found;

c) The original flow is augmented along P.

As in class, at a given iteration let ϵ be the minimum capacity of the arcs of a path from s to t in the residual network. Instead of augmenting by ϵ as seen in class, suppose that at each iteration you augment by $\epsilon/2$. How many iterations does the Ford and Fulkerson algorithm performs before finding the optimal solution?

Problem 6: Consider the maximum $s-t$ flow over time with source given by node s and sink by node t, and time horizon $T = 5$.



1. Build the time-expanded network for the problem;
2. Using the Ford-Fulkerson algorithm, compute the maximum flow over time.
3. In the flow you found, how many units of flows are traversing arc (v_2, t) at time $t = 3.7$?

Problem 7: After conducting business with each other, banks A,B,C,D,E,F,G owe to each other the following amounts of money (in thousands of USD).

	A	B	C	D	E	F	G
A	-	40	30	-120	12	60	40
B		-	30	-70	15	-40	-12
C			-	90	11	-20	60
D				-	40	-15	20
E					-	-20	-30
F						-	70

(In row i, column j, the amount of money bank i owes to bank j is reported. A negative number means that bank j owes to bank i). The banks now want to pay their debts fully. There is a fee of .003 USD for every dollar that will be exchanged among the banks.

1. Formulate the problem of paying off all debts by incurring in the smallest transaction fee as a min-cost flow problem.
2. (**bonus**) Write an LP for the min-cost flow problem from the previous step, feed it to Gurobi (or your favorite solver), and report the optimal solution you obtained.