IEORE4004: Optimization Models and Methods - Prof. Christian Kroer 09/11/2019

Assignment 1

Deadline: 09/24/2019, h: 11:40am

- This assignment sheet has 6 exercises. You must submit your solution via Canvas before the beginning of the class on the date it is due. The time constraint is strict. Late submissions will not be accepted.
- You are allowed to discuss the assignment with others but the write-up must be individual work. Please mention in your write-up all the people you have discussed the solution with.

Problem 1: This is problem 4 (page 114) from Chapter 3 in the textbook by Winston and Venkataraman.

Sunco processes oil into aviation fuel and heating oil. It costs $40 to purchase each 1,000 barrels of oil, which is then distilled and yields 500 barrels of aviation fuel and 500 barrels of heating oil. Output from the distillation may be sold directly or processed in the catalytic cracker. If sold after distillation without further processing, aviation fuel sells for $60 per 1,000 barrels, and heating oil sells for $40 per 1,000 barrels. It takes 1 hour to process 1,000 barrels of aviation fuel in the catalytic cracker, and these 1,000 barrels can be sold for $130. It takes 45 minutes to process 1,000 barrels of heating oil in the cracker, and these 1,000 barrels can be sold for $90. Each day, at most 20,000 barrels of oil can be purchased, and 8 hours of cracker time are available. Formulate an LP to maximize Sunco's daily profit, and use a software of your choice to compute the optimal solution.

Problem 2: This is problem 9 (page 114) from Chapter 3 in the textbook by Winston and Venkataraman.

Candy Kane Cosmetics (CKC) produces Leslie Perfume, which requires chemicals and labor. Two production processes are available: Process 1 transforms 1 unit of labor and 2 units of chemicals into 3 oz of perfume. Process 2 transforms 2 units of labor and 3 units of chemicals into 5 oz of perfume. It costs CKC $3 to purchase a unit of labor and $2 to purchase a unit of chemicals. Each year, up to 20,000 units of labor and 35,000 units of chemicals can be purchased. In the absence of advertising, CKC believes it can sell 1,000 oz of perfume. To stimulate demand for Leslie, CKC can hire a promoter, who is paid $100/hour. Each hour the promoter works for the company is estimated to increase the demand for Leslie Perfume by 200 oz. Each ounce of Leslie Perfume sells for $5. Formulate the problem as an LP and use a software of your choice to compute the optimal solution.

Problem 3: Find the set of ALL optimal solutions to the following LP:

$$
\begin{aligned}
\min z = \quad & 3x_1 - 2x_2 \\
\text{subject to} \quad & 3x_1 + x_2 \leq 12 \\
& 3x_1 - 2x_2 - x_3 = 12 \\
& x_1 \geq 2 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Problem 4: This is a modification from problem 44 (page 120) from Chapter 3 in the textbook by Winston and Venkataraman.

You have been put in charge of the Dawson Creek oil refinery. The refinery produces gas and heating oil from crude oil. Gas sells for $12 per barrel and must have an average grade level of at least 9. Heating

Method	Grade 6	8	10	Cost ($ per Barrel)
1	.3	.5	.8	3.40
2	.4	.2	.4	3.00
3	.1	.3	.2	2.60

Table 1: Data for Dawson Creek

Stock	Shares Owned	Price ($) Purchase	Current	In One Year
1	100	20	30	36
2	100	25	34	39
3	100	30	43	42
4	100	35	47	45
5	100	40	49	51
6	100	45	53	55
7	100	50	60	63
8	100	55	62	64
9	100	60	64	66
10	100	65	66	70

Table 2: Data for stock problem

oil sells for \$5 a barrel and must have an average grade level of at least 7. At most, 2,000 barrels of gas and 600 barrels of heating oil can be sold.

Incoming crude can be processed by one of three methods. The per barrel yield and per barrel cost of each processing method are shown in Table 1.

For example, if we refine one barrel of incoming crude by method 1, it costs us \$3.40 and yields .3 barrels of grade 6, .5 barrels of grade 8, and .8 barrels of grade 10. These costs include the costs of buying the crude oil.

Before being processed into gas and heating oil, grades 6 and 8 may be sent through the catalytic cracker to improve their quality. For \$1 per barrel, one barrel of grade 6 can be "cracked" into a barrel of grade 8. For \$1.50 per barrel, a barrel of grade 8 can be cracked into a barrel of grade 10. Formulate the problem as an LP and use a software of your choice to compute the optimal solution.

Problem 5: This is problem 45 (page 120) from Chapter 3 in the textbook by Winston and Venkataraman.

Currently we own 100 shares each of stocks 1 through 10. The original price we paid for these stocks, today's price, and the expected price in one year for each stock is shown in Table .

We need money today and are going to sell some of our stocks. The tax rate on capital gains is 30%. If we sell 50 shares of stock 1, then we must pay tax of $.3 \times 50(30 - 20) = \$150$. We must also pay transaction costs of 1% on each transaction. Thus, our sale of 50 shares of stock 1 would incur transaction costs of $.01 \times 50 \times 30 = \$15$. After taxes and transaction costs, we must be left with $\$30,000$ from our stock sales. Our goal is to maximize the expected (before-tax) value in one year of our remaining stock. What stocks should we sell? Assume it is all right to sell a fractional share of stock.

Problem 6: Which of the following statement is true, which is false? Briefly motivate your answers.

(i) There exists an LP with exactly two optimal solutions.

(ii) There exists an LP with exactly two optimal corner solutions.

(iii) If an LP is feasible, then it has an optimal solution that is a corner point.

(iv) Consider an LP with m constraints and n variables. We can transform it into an equivalent LP in standard form with at most $2m + 2n$ constraints and $2n + m$ variables.