Faculty of Applied Science and Engineering

APS 502: Financial Engineering

Computational Assignment Winter 2021

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Question 1

Step1, Calculate Forward Rate from spot rate:

Spot Rate		Forward Rate	
1	1%	F(1,2)	0.02
2	1.50%	F(2,3)	0.03
3	2%	F(3,4)	0.04
4	2.50%	F(4,5)	0.05
5	3%	F(5,6)	0.06
6	3.50%		

Step 2, problem formulation:

Minimize 108*x1 + 94*x2 + 99*x3 + 92.7*x4 + 96.6*x5 +...
95.9*x6 + 92.9*x7 + 110*x8 + 104*x9 + 101*x10 + 107*x11 + 102*x12 + 95.2*x13

Subject to:
◎ 10*x1 + 7*x2 + 8*x3 + 6*x4 + 7*x5 + 6*x6 + 5*x7 + 10*x8 +...
8*x9 +6*x10 +10*x11 + 7*x12 + 100*x13 - z1 >= 500

◎ 10*x1 + 7*x2 + 8*x3 + 6*x4 + 7*x5 + 6*x6 + 5*x7 + 10*x8 +...
8*x9 +6*x10 +110*x11 + 107*x12 + 1.02*z1 - z2 >= 200

◎ 10*x1 + 7*x2 + 8*x3 + 6*x4 + 7*x5 + 6*x6 + 5*x7 + 110*x8 +...
108*x9 +106*x10 + 1.03*z2 - z3 >= 800;

◎ 10*x1 + 7*x2 + 8*x3 + 6*x4 + 7*x5 + 106*x6 + 105*x7 + 1.04*z3 - z4 >= 400

◎ 10*x1 + 7*x2 + 8*x3 + 106*x4 + 107*x5 + 1.05*z4 - z5 >= 700;
110*x1 + 107*x2 + 108*x3 + 1.06*z5 >= 900;

◎ Xi >= 0; Zj >=0 (i = 1..13; j = 1..5)

For Part 2:
(108*x1 + 94*x2 + 99*x3 + 92.7*x4 + 96.6*x5 + 95.9*x6) <= 0.5 *...
(108*x1 + 94*x2 + 99*x3 + 92.7*x4 + 96.6*x5 + 95.9*x6 + 92.9*x7 +...
110*x8 + 104*x9 + 101*x10 + 107*x11 + 102*x12 + 95.2*x13);

For Part 3:
(108*x1 + 94*x2 + 99*x3 + 92.7*x4 + 96.6*x5 + 95.9*x6) <= 0.25 *...
(108*x1 + 94*x2 + 99*x3 + 92.7*x4 + 96.6*x5 + 95.9*x6 + 92.9*x7 +...
110*x8 + 104*x9 + 101*x10 + 107*x11 + 102*x12 + 95.2*x13);

Part 1:

Matlab Code:

```
c = [108, 94, 99, 92.7,96.6,95.9,92.9,110,104,101,107,102,95.2,0,0,0,0,0]
A = -[10,7,8,6,7,6,5,10,8,6,10,7,100,-1,0,0,0,0;
10,7,8,6,7,6,5,10,8,6,110,107,0,1.02,-1,0,0,0;
10,7,8,6,7,6,5,110,108,106,0,0,0,0,1.03,-1,0,0;
10,7,8,6,7,106,105,0,0,0,0,0,0,0,0,1.04,-1,0;
10,7,8,106,107,0,0,0,0,0,0,0,0,0,0,1.05,-1;
110,107,108,0,0,0,0,0,0,0,0,0,0,0,0,0,1.06]

b = -[500,200,800,400,700,900]
Aeq = []
beq = []
lb = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
ub = [inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf]
[x,fval] = linprog(c,A,b,Aeq,beq,lb,ub)
```

Result:

x =[8.1818,0,0,0,5.7774,2.6202,0,0,6.1298,0,0.1180,0,3.1180]

fval = 2.6400e+03

Part 2:

```
c = [108, 94, 99, 92.7,96.6,95.9,92.9,110,104,101,107,102,95.2,0,0,0,0,0]
A = -[10,7,8,6,7,6,5,10,8,6,10,7,100,-1,0,0,0,0;
10,7,8,6,7,6,5,10,8,6,110,107,0,1.02,-1,0,0,0;
10,7,8,6,7,6,5,110,108,106,0,0,0,0,1.03,-1,0,0;
10,7,8,6,7,106,105,0,0,0,0,0,0,0,0,1.04,-1,0;
10,7,8,106,107,0,0,0,0,0,0,0,0,0,0,1.05,-1;
110,107,108,0,0,0,0,0,0,0,0,0,0,0,0,0,1.06;
-108*0.5,-94*0.5,-99*0.5,-92.7*0.5,-96.6*0.5,-95.9*0.5,92.9*0.5,110*0.5,104*0.5,101*
0.5,107*0.5,102*0.5,95.2*0.5,0,0,0,0,0]

b = -[500,200,800,400,700,900,0]
Aeq = []
beq = []
lb = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
ub = [inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf]
[x,fval] = linprog(c,A,b,Aeq,beq,lb,ub)
```

Result:

x =[8.1818,0,0,0,5.7774,0.0117,2.6333,0,6.1528,0,0.1389,0,3.1389]

fval = 2.6444e+03, which is **larger** than optimal value in Part 1

Part 3:

Matlab Code:

```
c = [108, 94, 99, 92.7,96.6,95.9,92.9,110,104,101,107,102,95.2,0,0,0,0,0]
```

```
A = -[10,7,8,6,7,6,5,10,8,6,10,7,100,-1,0,0,0,0;
10,7,8,6,7,6,5,10,8,6,110,107,0,1.02,-1,0,0,0;
10,7,8,6,7,6,5,110,108,106,0,0,0,0,1.03,-1,0,0;
10,7,8,6,7,106,105,0,0,0,0,0,0,0,0,1.04,-1,0;
10,7,8,106,107,0,0,0,0,0,0,0,0,0,0,1.05,-1;
110,107,108,0,0,0,0,0,0,0,0,0,0,0,0,0,1.06;
-108*0.75,-94*0.75,-99*0.75,-92.7*0.75,-96.6*0.75,-95.9*0.75,92.9*0.25,110*0.25,10
4*0.25,101*0.25,107*0.25,102*0.25,95.2*0.25,0,0,0,0,0]

b = -[500,200,800,400,700,900,0]
Aeq = []
beq = []
lb = [0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]
ub = [inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf,inf]
[x,fval] = linprog(c,A,b,Aeq,beq,lb,ub)
```

Result:

```
x = [0,7.1271,0,0,0,0,10.4068,0,6.4637,0,0.4215,0,3.4215]
fvalue  = 2.6798e+03
```

Conclusion:

Portfolio 3 (with at most 25% B rating bond) costs more than Portfolio 2 (with at most 50% B rating bond). And Portfolio 2 costs more than Portfolio 1 (With no restriction). Portfolio 3 costs the most and Portfolio 1 costs the least.

Question 2

Part 1 (a):

From Jan 2014 to Jan 2021, in a total of 84 months, the following result is obtained:

Stock	SPY	GOVT	EEMV
Arithmetic Average	0.0099	0.0032	0.0024
Geometric Average	0.0090	0.0032	0.0017
Standard Deviation	0.0428	0.0109	0.0380

Covariance	SPY	GOVT	EEMV
SPY	0.0018	-0.0002	0.0012
GOVT	-0.0002	0.0001	-0.000057
EEMV	0.0012	-0.000057	0.0014

Part 1 (b):

Formulation:

```
>>  Q = [0.0018, -0.0002, 0.0012;
     -0.0002, 0.0001, -0.000057;
     0.0012, -0.000057, 0.0014];
c = zeros(3,1);
A = -[0.0090, 0.0032, 0.0017];%Geometric Average
Aeq = [1 1 1];
beq = 1;
ub = [inf; inf; inf];
lb = [0; 0; 0];%Short Selling not allowed
% lb = [-inf,-inf,-inf] short allowed
var = [];
R = [];
W = [];

for i=0.0017:0.0006636:0.0090
b = -i;
R = [R,-b];
[x,fval] = quadprog(Q,c,A,b,Aeq,beq,lb,ub);
var = [var,fval];
W = [W,x];
end

sd = sqrt(2*var);
plot(sd,R);
hold all;
title('Efficient Frontier consisting of 3 assets');
xlabel('Standard Deviation');
ylabel('expected portfolio return');
hold all;
```

Case 1 (Short Selling Not allowed):

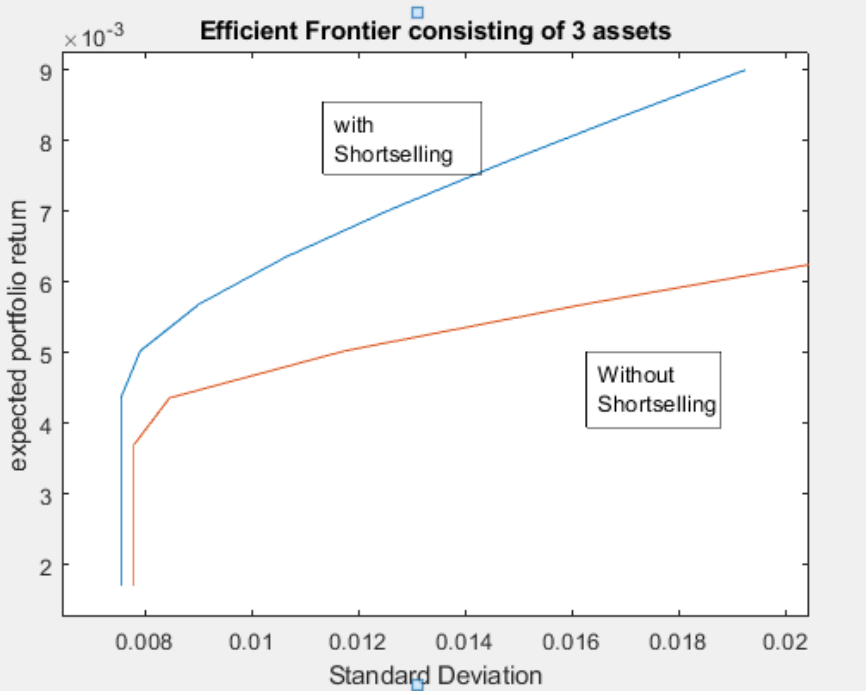
Expected Return of Portfolio	Weight of SPY	Weight of GOVT	Weight of EEMV	Variance of Portfolio
0.0017	0.130432917	0.869564322	0.00000276	0.0000304
0.0023636	0.130432472	0.869564108	0.00000342	0.0000304
0.0030272	0.130431041	0.86956342	0.00000554	0.0000304
0.0036908	0.130434353	0.869564993	0.00000065	0.0000304
0.0043544	0.199034524	0.800965387	0.00000009	0.0000358
0.005018	0.313448329	0.6865515	0.00000017	0.0000690
0.0056816	0.42786232	0.572136952	0.00000073	0.0001322
0.0063452	0.542275864	0.45772413	0.00000001	0.0002255
0.0070088	0.656689673	0.343310309	0.00000002	0.0003489
0.0076724	0.771103471	0.228896498	0.00000003	0.0005025
0.008336	0.885517243	0.114482754	0.00000000	0.0006861

0.0089996	0.999931042	6.90E-05	0.00000000	0.0008999

Case 2 (Short Selling Allowed):

Expected Return of Portfolio	Weight of SPY	Weight of GOVT	Weight of EEMV	Variance of Portfolio
0.0017	0.18616469	0.896154188	-0.082318877	0.0000285
0.0023636	0.186149596	0.8961522	-0.082301796	0.0000285
0.0036908	0.18615173	0.8961525	-0.082304229	0.0000285
0.0030272	0.186155793	0.896153055	-0.082308848	0.0000285
0.0043544	0.186454247	0.8961735	-0.082627746	0.0000285
0.005018	0.26923949	0.901720753	-0.170960243	0.0000314
0.0056816	0.358910111	0.907730395	-0.266640507	0.0000407
0.0063452	0.448574146	0.913739544	-0.36231369	0.0000566
0.0070088	0.538251401	0.919749683	-0.458001083	0.0000791
0.0076724	0.627913458	0.925758687	-0.553672145	0.0001081
0.008336	0.717582005	0.931768158	-0.649350163	0.0001436
0.0089996	0.807251186	0.937777683	-0.745028869	0.0001857

Efficient Frontier:



Part 1 (C)

From Yahoo Finance, we have obtained the following arithmetic return of asset for Feb, 2021:

SPY	GOVT	EEMV
0.02782	-0.01849	0.00989

Then we calculate the expected return of portfolio with the 4 different weights:

	Minimum Variance Portfolio Short Selling Not allowed (13.04% SPY, 86.96%GOVT)	Minimum Variance Portfolio Short Selling allowed (18.62% SPY, 89.62%GOVT, -8.23%EEV)	Equally Weighted Portfolio	60% SPY, 30% GOVT, 10% EEMV Portfolio
Return	-0.0124 Rank4	-0.0122 Rank3	0.0064 Rank2	0.0121 Rank1

Explanation:
In Feb 2021, the (60%, 30%,10%) Portfolio beat the equally weighted, and the Minimum Variance Portfolio performed the worst. The main reason is that for this single month, GOVT has a negative return which may be caused by the increase of the US Treasury Bond Yield. Therefore, a portfolio with a lower weight of GOVT tends to perform better. The minimum variance portfolio performs the worst since it has a large weight on GOVT which has the lowest volatility among those three ETFs. In the long run, SPY and EEMV tend to generate a higher return with higher volatility.

Part 2:

Similar to Part 1, we can obtain the following results:

Stock	ACN	ICE	CBOE	CME	BR
Arithmetic Average	0.0151	0.0134	0.0095	0.0136	0.0160
Geometric Average	0.0136	0.0120	0.0072	0.0121	0.0142
Standard Deviation	0.0547	0.0529	0.0670	0.0554	0.0605

Covariance Table:

	SPY	GOVT	EEMV	ACN	ICE	CBOE	CME	BR
SPY	0.0018	-0.0002	0.0012	0.0019	0.0013	0.0011	0.00096	0.0017
GOVT	-0.0002	0.00012	-0.000057	-0.00013	-0.00016	-0.00009	-0.00014	-0.00005
EEMV	0.0012	-0.000057	0.0014	0.00105	0.00052	0.00056	0.00032	0.00111
ACN	0.0019	-0.00013	0.00105	0.00295968 7	0.00166	0.00122	0.001067	0.002119
ICE	0.0013	-0.00016	0.00052	0.00166	0.002762	0.00164	0.001839	0.001436
CBOE	0.0011	-0.00009	0.00056	0.00122	0.00164	0.00444	0.002051	0.00119
CME	0.00096	-0.00014	0.00032	0.001067	0.001839	0.002051	0.00303	0.00132

BR	0.0017	-0.00005	0.00111	0.002119	0.001436	0.00119	0.00132	0.00362

By using the same quadratic model again:

```
>>  Q=[0.0018,-0.0002,0.0012,0.0019,0.0013,0.0011,0.00096,0.0017;
-0.0002,0.00012,-0.000057,-0.00013,-0.00016,-0.00009,-0.00014,-0.00005;
0.0012,-0.000057,0.0014,0.00105,0.00052,0.00056,0.00032,0.000111;
0.00019,-0.00013,0.000105,0.002959687,0.0016,0.00122,0.001067,0.002119;
0.0013,-0.00016,0.00052,0.00166,0.002762,0.00164,0.001839,0.001436;
0.0011,-0.00009,0.00056,0.00122,0.00164,0.00444,0.00205,0.00119;
0.00096,-0.00014,0.00032,0.001067,0.001839,0.002051,0.00303,0.00132;
0.0017,-0.00005,0.00111,0.002119,0.001436,0.00119,0.00132,0.00362]; %covariance
c = zeros(8,1);
A = -[0.0090, 0.0032, 0.0017, 0.0547, 0.0529, 0.067, 0.0554, 0.0605];%Geometric Average
Aeq = [1 1 1 1 1 1 1 1];
beq = 1;
ub = [inf; inf; inf;inf; inf; inf;inf;inf];
%lb = [-inf; -inf; -inf;-inf; -inf; -inf;-inf;-inf];%Short Selling allowed
lb = [0,0,0,0,0,0,0,0] %Short selling NOT allowed
var = [];
R = [];
W = [];

for i=0.0017:0.001136:0.0142
% From the lowest return to highest return asset: 10 steps
b = -i;
R = [R,-b];
[x,fval] = quadprog(Q,c,A,b,Aeq,beq,lb,ub);
var = [var,fval];
W = [W,x];
end

sd = sqrt(2*var);
plot(sd,R);
hold all;
title('Efficient Frontier consisting of 8 assets');
xlabel('Standard Deviation');
ylabel('expected portfolio return');

hold all;
```

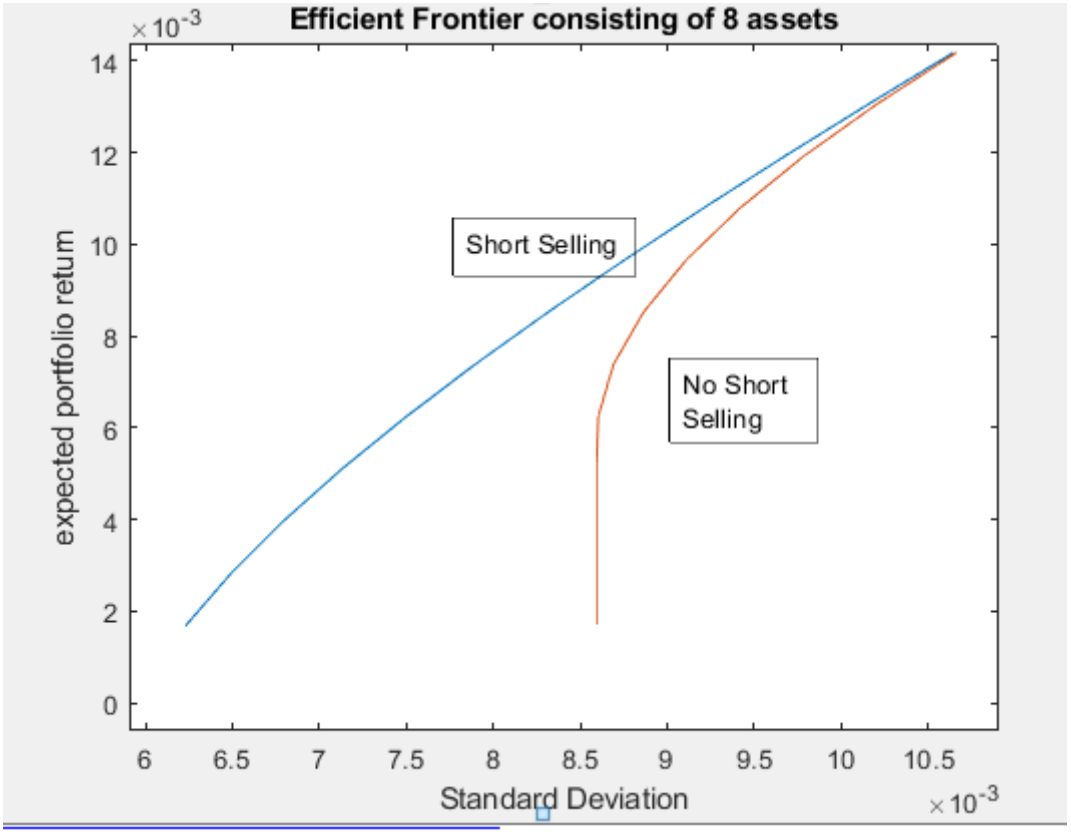
Portfolios weight and variance (Short Selling Allowed):

Return	W SPY	WGOVT	W EEMV	W ACN	W ICE	W CBOE	W CME	W BR	Variance
0.0017	0.386421	0.90562	-0.23027	0.118864	-0.0648	-0.01171	0.053786	-0.15791	1.95E-05
0.002836	0.354664	0.895211	-0.21215	0.114919	-0.05523	-0.00795	0.054226	-0.14368	2.11E-05
0.003972	0.323043	0.884846	-0.19411	0.110991	-0.0457	-0.00421	0.054664	-0.12951	2.31E-05
0.005108	0.291381	0.874467	-0.17605	0.107059	-0.03616	-0.00047	0.055103	-0.11533	2.54E-05
0.006244	0.259708	0.864085	-0.15797	0.103124	-0.02662	0.003271	0.055541	-0.10113	2.81E-05
0.00738	0.228032	0.853702	-0.1399	0.09919	-0.01708	0.007014	0.05598	-0.08694	3.12E-05
0.008516	0.196354	0.843318	-0.12183	0.095255	-0.00753	0.010758	0.056419	-0.07275	3.46E-05
0.009652	0.164677	0.832935	-0.10375	0.09132	0.002015	0.014502	0.056858	-0.05855	3.83E-05
0.010788	0.132945	0.822534	-0.08565	0.087378	0.011577	0.018252	0.057297	-0.04434	4.24E-05
0.011924	0.101292	0.812158	-0.06759	0.083447	0.021115	0.021993	0.057736	-0.03015	4.68E-05
0.01306	0.069628	0.801779	-0.04952	0.079514	0.030657	0.025735	0.058174	-0.01597	5.16E-05
0.014196	0.037957	0.791398	-0.03145	0.07558	0.0402	0.029479	0.058613	-0.00177	5.67E-05

Portfolios weight and variance (No Short Selling Allowed):

Return	W SPY	W GOVT	W EEMV	W CAN	W ICE	W CBOE	W CME	W BR	Variance
0.0017	0.115161	0.84873	2.63E-06	0.012898	0.001616	2.62E-05	0.021566	6.57E-07	3.69E-05
0.002836	0.115168	0.848728	2.66E-06	0.012906	0.001594	2.54E-05	0.021576	6.49E-07	3.69E-05
0.003972	0.115177	0.848723	2.72E-06	0.012924	0.001551	2.08E-05	0.0216	6.51E-07	3.69E-05
0.005108	0.115061	0.84863	3.70E-06	0.013034	0.001558	7.39E-06	0.021706	7.96E-07	3.69E-05
0.006244	0.109226	0.844232	1.53E-06	0.017777	0.002752	3.70E-05	0.025974	4.71E-07	3.70E-05
0.00738	0.095324	0.834531	3.66E-06	0.027463	0.008064	0.000449	0.034164	1.03E-06	3.78E-05
0.008516	0.081295	0.825302	3.39E-06	0.035853	0.014697	0.002664	0.040184	1.16E-06	3.93E-05
0.009652	0.067514	0.816377	4.07E-07	0.043496	0.020932	0.007096	0.044585	2.53E-07	4.15E-05
0.010788	0.053845	0.807566	7.48E-10	0.050857	0.026937	0.012439	0.048356	1.23E-07	4.44E-05
0.011924	0.040178	0.798765	8.13E-08	0.058194	0.032931	0.017829	0.052089	1.30E-05	4.79E-05
0.01306	0.02636	0.790146	3.13E-07	0.065118	0.038961	0.023171	0.055659	0.000584	5.21E-05
0.014196	0.01125	0.78308	1.18E-06	0.068534	0.045293	0.028097	0.057844	0.005902	5.69E-05

Efficient Frontier:



Observation:

Compared to the efficient frontier of Part 1, we notice that Part 2 yields a higher return than Part 1 when the volatility is similar. Since Part 2 includes 8 assets, and Part 1 includes 3, this shows within similar volatility a higher diversification could generate a better return.