Pricing Oil Futures Options under Heston's and Bates' Models

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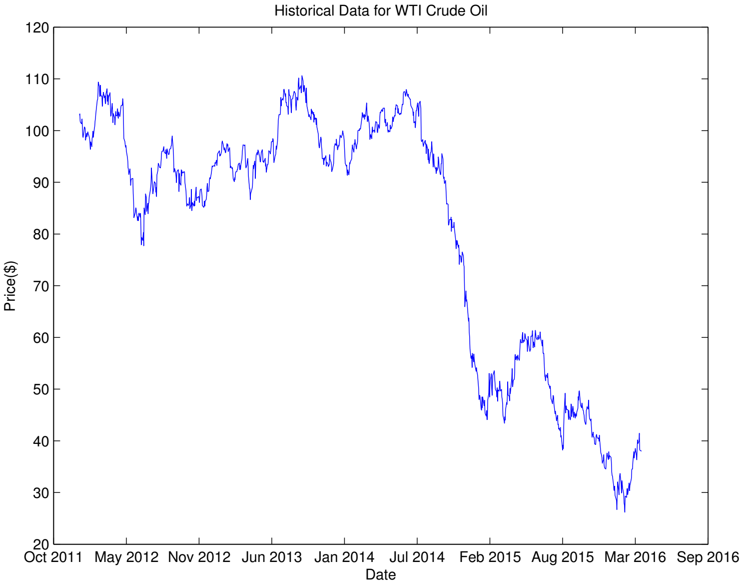
OUTLINE

Drop in Oil Prices



Reasons behind the Drop

The price of a barrel of oil fell more than 60% compared with June 2014 levels. The reasons behind the drop boils down to the simple economics of supply and demand.

- Oversupply
  - United States Domestic Production
  - Disagreement among OPEC
- Declining demand
  - Weak Economies
  - More Energy-Efficient Vehicles
- Strong US dollar

MOTIVATION

- We have noticed a very high volatility in oil prices over the last two years, which may indicate the existence of "jumps" in oil prices.
- We are interested in evaluating the performance of Heston's and Bates' models during the oil crisis.

HESTON'S MODEL

In Heston's model, the stock price S_t and its variance V_t are given by

$$\begin{cases} dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_{1t} \\ dV_t = \kappa[\theta - V_t]dt + \sigma\sqrt{V_t} dW_{2t} \\ \rho dt = \mathrm{cov}(dW_{1t}, dW_{2t}) \end{cases}$$

where

- W_{1t}, W_{2t} are Wiener processes with correlation ρ
- μ — growth rate or expected return for the asset
- κ — mean reversion speed
- θ — the long-run average of variance
- σ — the volatility of volatility.

Bates' Model

The general formula for Bates' model is given by

$$\begin{cases} dS_t = \mu S_t dt + \sqrt{V_t} S_t dW_{1t} - \lambda(e^{\mu_s + (1/2)\sigma_s^2} - 1) S_t dt + dQ_t \\ dV_t = \kappa(\theta - V_t)dt + \sigma\sqrt{V_t} dW_{2t} \\ \rho dt = \operatorname{cov}(dW_{1t}, dW_{2t}) \end{cases}$$

where the additional variable Q_t is a compound Poisson process with arrival intensity λ and *i.i.d* lognormal distributed jump size distribution e^{Z_j}, where $Z_j \sim \mathcal{N}(\mu_z, \sigma_z^2)$.

MCMC Simulation

Basic Idea We want to construct a Markov chain whose stationary distribution is the desired posterior distribution, and then we can run this chain to get draws that are approximately from the posterior distribution once the chain has converged.

Burn In Since convergence will occur regardless of the starting point, we can usually pick any feasible starting point. However, the time it takes for the chain to converge varies depending on the starting point.

MCMC SIMULATION

- The estimated parameters are $\mu, \kappa, \theta, \sigma, \mu_z, \sigma_z, \rho, \lambda$
- According to the Bayesian method, the posterior distribution is given by

$$\text{Posterior probability} \propto \text{Likelihood} \times \text{Prior probability}.$$

$$P(\mu|S, V, \kappa, \theta, \sigma, \mu_z, \sigma_z, \rho, \lambda) \propto P(S, V|\mu, \kappa, \theta, \sigma, \mu_z, \sigma_z, \rho, \lambda) \cdot P(\mu)$$

- All priors chosen are those frequently used in previous literature. Initial values for the MCMC algorithm were chosen based off on the observed data

MCMC SIMULATION

For the parameters $\{\mu, \kappa, \theta, \sigma, \mu_z, \sigma_z, \rho, \lambda\}$, a Gibbs sampler is used. The process involves mainly

1. Start from a set $\{\mu^{(0)}, \kappa^{(0)} ... V_0^{(0)}, V_1^{(0)} ... V_{T+1}^{(0)}\}$ of initial values.
2. Draw a value $\mu^{(1)}$ from the full conditional $P(\mu | S, \kappa^{(0)} ... V_0^{(0)}, V_1^{(0)} ... V_{T+1}^{(0)}\}$, then update the set to $\{\mu^{(1)}, \kappa^{(0)} ... V_0^{(0)}, V_1^{(0)} ... V_{T+1}^{(0)}\}$.
3. Apply step 2 to obtain draws for $\kappa^{(1)}, \theta^{(1)}$... and then update the current state of the Markov chain in step 1 with each draw.
4. The process in step 3 is repeated for $1, 2, ..., n$ iterations.

MCMC SIMULATION

For the state space $\{V_0, ..., V_T\}$ a Metropolis- Hastings approach is used. This involves

1. Starting with the initial values $\{V_0^{(0)}, ..., V_{T+1}^{(0)}\}$ for the 0th step;
2. For a fixed ith step, draw from the proposal density for $V_t^{(i)}$ for $t \in \{1, ..., T\}$ by $V_t^{*(i)} = V_t^{(i-1)} + \mathcal{N}_t$, where $\mathcal{N}_t \sim \mathcal{N}(0, \sigma_N^2)$.
3. Compute an acceptance ratio r, accept $V_t^{*(i)}$ as $V_t^{(i)}$ with probability $\min(r, 1)$. If $V_t^{*(i)}$ is not accepeted, then $V_t^{(i)} = V_t^{(i-1)}$.
4. Running the algorithm for n steps, to get $\{V_0^{(i)}, V_1^{(i)}, ..., V_{T+1}^{(i)}\}$, for $i \in \{1, ..., n\}$.

Data for Parameters Estimation

WTI crude oil futures CLM16 and CLJ16 were chosen as underlying assets.

- For Parameter Estimation: CLJ16 and CLM16 futures were chosen with maturities 21st March 2016 and 20th May 2016 respectively.
- For each of the futures, the historical data was collected from 28th March 2014 to 25th Feb 2016.
- The corresponding future call options CLJ16 and CLM16 are with maturity periods 9 days and 21 days respectively.

Parameter Estimates

Table: Parameters estimates for CLM16 and CLJ16 oil futures.

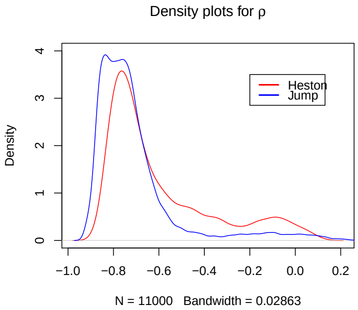
	CLM16		CLJ16	
	Bates	Heston	Bates	Heston
μ	-15.13%		-20.00%	
	(12.08%)		(11.43%)	
μ_z	0.50%		-0.01%	
	(4.17%)		(2.46%)	
σ_z	14.73%		11.87%	
	(9.00%)		(1.86%)	
λ	3.20%		5.70%	
	(1.14%)		(2.63%)	
κ	49.57%	45.29%	65.17%	56.23%
	(45.70%)	(41.42%)	(47.03%)	(43.82%)
θ	26.99%	40.02%	17.91%	29.26%
	(33.48%)	(39.92%)	(25.53%)	(33.12%)
σ_V	14.44%	31.71%	15.06%	24.26%
	(7.55%)	(17.24%)	(16.54%)	(3.08%)
ρ	-63.17%	-58.35%	-78.13%	-73.17%
	(12.17%)	(10.91%)	(8.33%)	(10.10%)
V	4.27%	6.89%	4.24%	7.36%
	(2.65%)	(4.99%)	(2.51%)	(5.10)

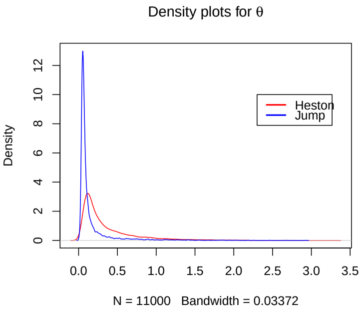
Parameter Estimates

From the table above we observe that:

1. Both of the estimates for the average return in the models (given by μ) are negative reflecting the persistent decline in prices.
2. The parameter ρ which measures the correlation between instantaneous volatility and returns indicates a strong negative correlation in the Bates' model.
3. The estimates of λ, the jump density indicates that jumps are very infrequent.

Comparing Distributions





Kolmogorov-Smirnov (K-S) Test

Despite the considerable differences observed in the plots we perform the K-S test.

- The hypothesis for each of the parameters is

$$\mathrm{H}_0 : F(X) = F(Y)$$
$$\mathrm{H}_\mathrm{A} : F(X) \neq F(Y)$$

where $F(X)$ and $F(Y)$ are cumulative distribution functions of each parameter under Heston's model and Bates' model respectively. We reject H_0 if the p-value is less than 0.05.

	ρ	κ	θ
D_{KS}	0.21909	0.24664	0.49491
p value	2.2e-16	2.2e-16	2.2e-16

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Using the estimated parameters, we got oil futures options prices under the strike prices between \$25 and \$55.

Table: Call Option Prices for CLJ16

Strikes	Actual	Heston	Bates
25	12.9	12.9579	12.907
26	11.91	11.9603	11.9076
27	10.91	10.9625	10.9083
28	9.92	9.9649	9.9090
29	8.92	8.9672	8.9097
30	7.76	7.9696	7.9106
31	6.60	6.9724	6.9119
...	...	...	...
42	0.13	0.0377	0.044
43	0.09	0.012	0.0189
44	0.08	0.0033	0.0098
45	0.04	0.0008	0.0067
46	0.03	0.0002	0.0055
47	0.03	0	0.0049
48	0.02	0	0.0045
...	...	...	...

Pricing oil futures options

Figure: CLJ16 Call Option Prices under Different Strikes

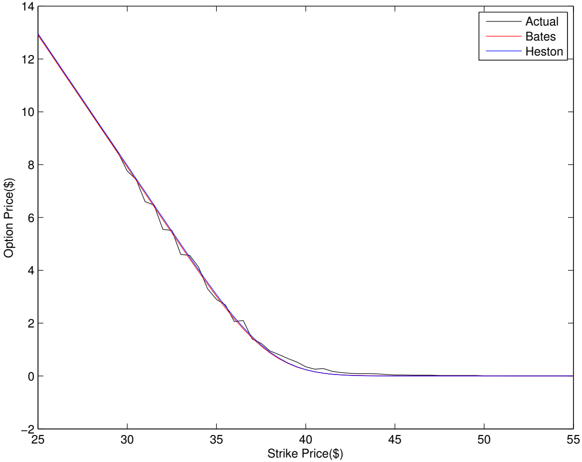

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Figure: CLM16 Call Option Prices under Different Strikes

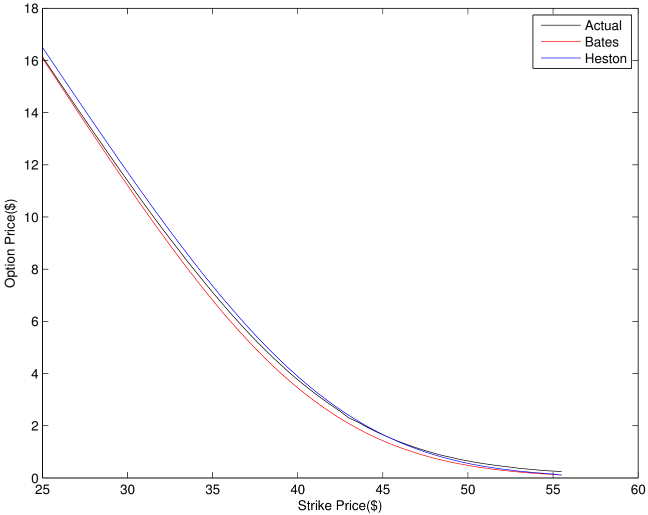

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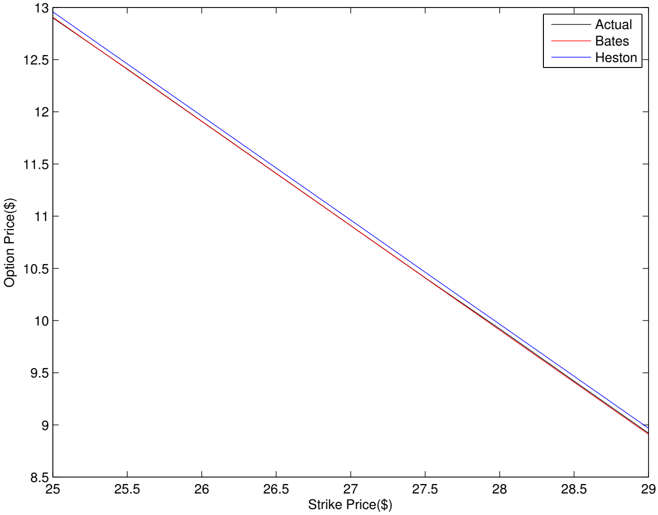
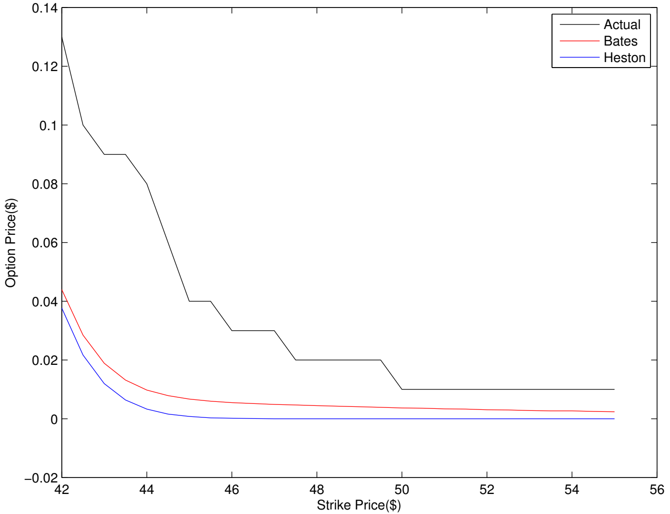
Figure: CLJ16 In the Money Call Option Prices



PRICING OIL FUTURES OPTIONS

Figure: CLJ16 Out of the Money Option Prices



Evaluating the goodness-of-fit

We employed the chi-square test.

- The hypothesis is

$$\begin{aligned} \mathrm{H}_0 &: \text{ Estimated prices have a good fit} \\ \mathrm{H}_A &: \text{ Estimated prices do not have a good fit} \end{aligned}$$

We reject H_0 if the p-value is less than 0.05 in each case.

Table: χ^2 Test for CLJ16 and CLM16

p-value	CLJ16	CLM16
Bates	0.246	0.238
Heston	0.0001	0.241

Conclusions

- The MCMC algorithm was successful in obtaining draws from the posterior distributions. The estimated values in Bates' model are smaller than those in Heston's model since part of the volatility in the model is accounted for using by the jump term.
- In our project, the Bates' model is better suited to short-term ITM and OTM options while the Heston's model performs better in medium to long term ATM cases.

Limitations

- Insufficient data
- The selection of prior parameters
- Difficulty in detecting jumps

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