### CS6533/CS4533 Lectures 11-12 Slides/Notes

## Shading and Illumination; Compositing (Notes, Ch 14, Notes)

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#### \* Continued on Shading and Illumination:

- First we reviewed the ``Overall Formula'' of the Phong Reflection Model (shown in the next 2 slides) as presented last time, which is implemented in the sample program ``Handout: rotate-cube-shading.cpp''.
- Discussed the sample program "Handout: rotate-cube-shading.cpp" (complete sample program has been posted at https://cse.engineering.nyu.edu/cs653/Rotate-Cube-Shading.tar.gz)
- Some screenshots of the sample program with annotations are then shown next.
- Showed a demo of the sample program ``Rotate-Cube-Shading".
- Then we finished up the last 2 pages (p.14 & p.15) of the last lecture notes/slides "Lecture-9-10.pdf" on Normal Matrix.
- \* New topic: Composition Techniques.

(3) If light is a spotlight then

(4) (Attenuation)  $\frac{1}{2} = \frac{1}{a+bd+cd^2}$ . (spotlight-attenuation)  $\frac{1}{2}$  (a) point source.

(5) point source.

(6) point source.

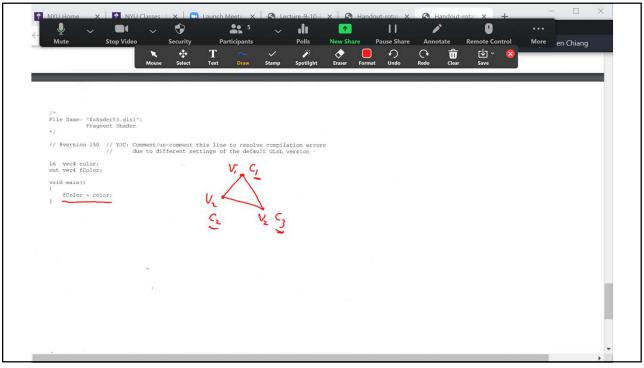
(7) In particular  $\frac{1}{2}$  [In particular  $\frac{1}{2}$  [In particular  $\frac{1}{2}$  [In particular  $\frac{1}{2}$  [In the range  $(0, 90^{\circ})$ ]

(a)  $\frac{1}{2}$   $\frac{1}{$ 

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Participants
                                                                                                                                                                                                                      Remote Control
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                                                                                                                                                                                                                                            Talking:
   #include "Angel-yjc.1
                                                                                                                                                                              void quad( int a, int b, int c, int d )
   typedef Angel::vec4 color4;
typedef Angel::vec4 point4;
                                                                                                                                                                                     // Initialize temporary vectors along the quad's edges to
// compute its face normal
vec4 u = vertices[b] - vertices[a];
vec4 v = vertices[d] - vertices[a];
   GLuint program; /* shader program object id */
GLuint cube_buffer; /* vertex buffer object id for cube */
   // Projection transformation parameters
GLfloat fovy = 45.0; // Field-of-view in Y direction angle (in degrees)
GLfloat aspect; // Viewport aspect ratio
GLfloat zNear = 0.5, zFar = 3.0;
                                                                                                                                                                                     vec3 normal = normalize( cross(u, v) );
                                                                                                                                                                                     normals[Index] = normal; points[Index] = vertices[a]; Ind
normals[Index] = normal; points[Index] = vertices[b]; Ind
normals[Index] = normal; points[Index] = vertices[c]; Ind
normals[Index] = normal; points[Index] = vertices[a]; Ind
normals[Index] = normal; points[Index] = vertices[c]; Ind
normals[Index] = normal; points[Index] = vertices[d]; Ind
   int animationFlag = 1; // 1: animation; 0: non-animation. Toggled by key 'a' or 'A'
   const int NumVertices = 36; //(6 faces)(2 triangles/face)(3 vertices/triangle)
   point4 points[NumVertices];
vec3 normals[NumVertices];
                                                                                                                                                                              // colorcube() generates 6 quad faces (12 triangles): 36 vert void colorcube()
   // Vertices of a unit cube centered at origin, sides aligned with axes
  // Vertices of a unit cube centered at oripoint4 vertices[8] = {
    point4(-0.5, -0.5, 0.5, 1.0), -0 }
    point4(-0.5, -0.5, 0.5, 1.0), -1 }
    point4(-0.5, 0.5, 0.5, 1.0), -1 }
    point4(-0.5, -0.5, 0.5, 1.0), -1 }
    point4(-0.5, -0.5, 0.5, 1.0), -1 }
    point4(-0.5, -0.5, -0.5, 1.0), -1 }
    point4(-0.5, 0.5, -0.5, 1.0), -1 }
    point4(-0.5, 0.5, -0.5, 1.0), -1 }
    point4(-0.5, 0.5, -0.5, 1.0), -1 }
}
                                                                                                                                                                                    quad( 1, 0, 3, 2 );
quad( 2, 3, 7, 6 );
quad( 3, 0, 4, 7 );
quad( 6, 5, 1, 2 );
quad( 4, 5, 6, 7 );
quad( 5, 4, 0, 1 );
   1.
                                                                                                                                                                              // OpenGL initialization void init()
 colorcube():
                                                                                                                                                                                    // Model-view and projection matrices uniform location 
GLuint ModelView, Projection;
           --- Shader Lighting Parameters
         color4 light_ambient( 0.2, 0.2, 0.2, 1.0 ); color4 light_diffuse( 1.0, 1.0, 1.0, 1.0 );
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and mouse() functions for
ting cube with shading.
rial properties & Normal Matrix are sent to the shader as
                                                                                            int Index = 0;
 computation is done in the Eye Frame (in shader).
                                                                                            /// quad() generates two triangles for each face and assigns normals
// to the vertices
void quad( int a, int b, int c, int d )
                                                                                                // Initialize temporary vectors along the quad's edges to // compute its face normal vec4 u = vertices[b] - vertices[a]; vec4 v = vertices[d] - vertices[a];
   /* shader program object id */
/* vertex buffer object id for cube */
                                                                                                 vec3 normal = normalize( cross(u, v) );
formation parameters
0; // Field-of-view in Y direction angle (in degrees)
// Viewport aspect ratio
5, zFar = 3.0;
                                                                                                 1; // 1: animation; 0: non-animation. Toggled by key 'a' or 'A'
 es = 36: //(6 faces)(2 triangles/face)(3 vertices/triangle)
                                                                                            ertices):
 it cube centered at origin, sides aligned with axes
                                                                                                                                                          normal vector is
[ (0.5, 0.5, 1.0), 0.5, 0.5, 1.0), 0.5, 0.5, 1.0), 0.5, 0.5, 1.0), 0.5, 0.5, 1.0), 0.5, -0.5, 1.0), 0.5, -0.5, 1.0), 0.5, -0.5, 1.0), 0.5, -0.5, 1.0), 0.5, -0.5, 1.0)
                                                                                                quad(1,0,3,2);
quad(2,3,7,6);
quad(3,0,4,7);
quad(6,5,1,2);
quad(4,5,6,7);
quad(5,4,0,1);
                                                                                                                                 as before pointry outward.
                                                                                            //---// OpenGL initialization void init()
n angles (in degrees) for each coordinate axis
axis = 1, Zaxis = 2, NumAxes = 3 };
                                                                                                colorcube();
is;
xes1 = { 0.0, 0.0, 0.0 };
                                                                                                  // Create and initialize a vertex buffer object
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ices = 36; //(6 faces)
Vertices];
mVertices];
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                                                                                                                                                                                                      // colorcube() generates 6 quad faces (12 triangles): 36 vertices & 36 normals void colorcube()
  unit cube centered at origin, sides aligned with axes
unit cube centered a 31 = {
    -0.5,    0.5,    1.0 },    0.5,    1.0 },    0.5,    1.0 },    0.5,    0.5,    1.0 },    0.5,    0.5,    1.0 },    0.5,    0.5,    1.0 },    0.5,    0.5,    1.0 },    0.5,    0.5,    1.0 },    0.5,    -0.5,    1.0 },    0.5,    -0.5,    1.0 },    0.5,    -0.5,    1.0 },    0.5,    -0.5,    1.0 },    -0.5,    -0.5,    1.0 }
                                                                                                                                                                                                               quad( 1, 0, 3, 2 );
quad( 2, 3, 7, 6 );
quad( 3, 0, 4, 7 );
quad( 6, 5, 1, 2 );
quad( 4, 5, 6, 7 );
quad( 5, 4, 0, 1 );
                                                                                                                                                                                                     //-
// OpenGL initialization
void init()
                                                                                                                                                                                                                                                                                                                                                                          TOP
ion angles (in degrees) for each coordinate axis
Yaxis = 1, Zaxis = 2, NumAxes = 3 );
coxis;
mmAxes] = ( 0.0, 0.0, 0.0 );
                                                                                                                                                                                                               colorcube();
                                                                                                                                                                                                    // Create and initialize a vertex buffer object
gldenBuffers(1, &cube_buffer);
glBindBuffer(GL_ARRAY_BUFFER, cube_buffer);
glBufferData(GL_ARRAY_BUFFER, sizeof(points) + sizeof(normals),
NULL, GL_STATIC_DRAW);
glBufferSubData(GL_ARRAY_BUFFER, 0, sizeof(points),
glBufferSubData(GL_ARRAY_BUFFER, 0, sizeof(points),
sizeof(normals), normals);
// Load shaders and create a shader program (to be used in display())
program = InitShader( 'vshader53.gls1', 'fshader53.gls1');
projection matrices uniform location , Projection;
ghting Parametere
ambient(0.2, 0.3, 0.2, 1.0,);
diffuse(1.0, 1.0, 1.0, 1.0);
specular(1.0, 1.0, 1.0, 1.0);
tt = 1.0;
att = 0.01;
tt = 0.01;
position(2.0, 2.0, 1.0, 1.0);
syrid frame.
                                                                                                                                                                                                               glEnable( GL_DEPTH_TEST );
glClearColor( 1.0, 1.0, 1.0, 1.0 );
Norld frame.
Is to transform it to Eye Frame ore sending it to the shader(s).
                                                                                                                                                                                                     ial_ambient( 1.0, 0.0, 1.0, 1.0 );
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Undo
                                                                                                                               glUniformlf(glGetUniformLocation(program, "LinearAtt"),
                                                                                                                                                                     linear_att);
                                                                                                                               glUniformlf(glGetUniformLocation(program, "OuadAtt"),
      /*
Pile Name: 'vshader53.glsl':
Vertex shader:
- Per vertex shading for a single point light source;
- distance attenuation is Vet To Be Completed.
- Entire shading computation is done in the Bye Prame.
                                                                                                                                                                    quad_att);
                                                                                                                              // #version 150 // YJC: Comment/un-comment this line to resol due to different settings of the defa
                                                                                                                        //-
// drawObj(buffer, num_vertices):
// drawObj(buffer, num_vertices):
// draw the object that is associated with the vertex buffer object "buffer"
// and has "num_vertices" vertices.
                                                                                                                        void drawObj(GLuint buffer, int num_vertices)
     uniform wec4 AmbientProduct, DiffuseProduct, SpecularProduct; uniform mat4 ModelVlew; uniform mat4 Projection; uniform mat3 Normal_Matrix; uniform mat3 Normal_Matrix; uniform wec4 LightPostition; // Must be in Bye Prame uniform wfo as Shininess;
                                                                                                                                         Activate the vertex buffer object to be drawn ---//
                                                                                                                               glBindBuffer(GL_ARRAY_BUFFER, buffer);
                                                                                                                             /*---- Set up vertex attribute arrays for each vertex attribute ----*/
GLuint vPosition = glGetAttribLocation( program, "vPosition" );
glEnableVertexAttribArray( vPosition );
glVertexAttribPointer( vPosition, 4, GL_FLOAT, GL_FALSE, 0,
BUFFER_OFFSET(0) );
      uniform float Constatt; // Constant Attenuation
uniform float LinearAtt; // Linear Attenuation
uniform float QuadAtt; // Quadratic Attenuation
                                                                                                                              GLuint vNormal = glGetAttribLocation( program, "vNormal");
           // Transform vertex position into eye coordinates
vec3 pos = (ModelView * vPosition).xyz;
                                                                                                                              GDuint vnormal = glosectetinbuccation; program, vigilianbleVetrexAttribPointer( vNormal ); glVertexAttribPointer( vNormal ), gl.FLOAT, GL.FALSE, 0, EMPFER, OPPSET(Gizeof(points)) ); // the offset is the (total) size of the previous vertex attribute array(s)
            vec3 L = normalize( LightPosition.xyz - pos );
vec3 E = normalize( -pos );
vec3 H = normalize( L + E );
           // Transform vertex normal into eye coordinates
// vec3 N = normalize( ModelView*vec4(vNormal, 0.0) ).xy
vec3 N = normalize(Normal_Matrix * vNormal);
                                                                                                                              /* Draw a sequence of geometric objs (triangles) from the (using the attributes specified in each enabled vertex glbrawArrays(GL_TRIANGLYS, 0, num_exi()); 120%
       // YJC Note: N must use the one pointing *toward* the viewe
                                                                                                                                /*--- Disable each vertex attribute array being enal
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                                                                                                                                                                                                                                                                                                RotateX( Theta[Xaxis]
RotateY( Theta[Yaxis]
RotateZ( Theta[Zaxis]
/*File Name: "vahader53.gls1'
Vertex shader:
- Per vertex shading for distance attenuation is - Entire shading computat
                                                                                                                                                                                                                  mat4 model_view = ( Translate( -viewer_pos )

Patatav [ Thata[Xaxis] ) *
                                                                                                                                                                                                                                                                                                RotateX( Theta[Xaxis] ) *
RotateY( Theta[Yaxis] ) *
RotateY( Theta[Yaxis] ) );
                                                                                        attribute ----*/
// #version 150 // YJC: Cc // di
                                                                                        :ion");
                                                                                                                                                                                                    #endif
                                                                                            0,
                                                                                                                                                                                                                  glUniformMatrix4fv(ModelView, 1, GL_TRUE, model_view);
in vec4 vPosition;
in vec3 vNormal;
out vec4 color;
                                                                                                                                                                                                                  // Set up the Normal Matrix from the model-view matrix mat3 normal_matrix = NormalMatrix(model_view, 1);

// Flag in NormalMatrix():

// 1: model_view involves non-uniform scaling,

// 0: otherwise.

// Using 1 is always correct.

// But if no non-uniform scaling,

// using 0 is faster (avoids matrix inverse computation).
                                                                                    : attribute array(s)
                                                                                      ne vertex buffer
ex attribute array) */
uniform float ConstAtt; /
uniform float LinearAtt; /
uniform float QuadAtt; /
                                                                                                                                                                                                                  glUniformMatrix3fv(glGetUniformLocation(program, "Normal_Matrix"),
                                                                                                                                                                                                                                                                                          1, GL_TRUE, normal_matrix );
void main()
           // Transform vertex po
vec3 pos = (ModelView
                                                                                                                                                                                                                  drawObj(cube_buffer, NumVertices); // draw the cube
                                                                                                                                                                                                                   glutSwapBuffers();
                                                                                                                                                                                                       void mouse( int button, int state, int x, int y )
         // Transform vertex no
// vec3 N = normalize
vec3 N = normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(Normalize(No
                                                                                                                                                                                                                 if ( state == GLUT_DOWN ) {
   switch( button ) (
  / YJC Note: N must use the
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# \* Normal Matrix

\* Typically we perform shading computation in the (eye frame) (it the right-handed eye frame where the eye/camera is at the origin looking at the -z direction. This is the frame obtained by applying Look At () to the world frame)

Let  $\vec{t}$  be the tangent at pt p being shaded.  $\underline{\vec{n}}$ ,  $\vec{t}$  are in the model frame  $\underline{\vec{n}}$ .  $\underline{\vec{n}}$  normal  $\underline{\vec{n}}$ .  $\underline{\vec{n}}$  the model-view matrix

(1) Suppose M involves (non-unisorm scaling) (ie scaling factors in x-, y-, z-dimensions are

eg. S(1,2) b  $\overrightarrow{t}$   $\overrightarrow{n}$  S(1,2) Mb  $\overrightarrow{t}'$   $\overrightarrow{n}'$   $\overrightarrow{t}' = Mb - Ma = M(b-a)$   $= M\overrightarrow{t}$  is the tangent

is. We can still apply M to I to obtain the new tangent I' correctly But applying M to \$\overline{n}\$ does NOT give the correct normal vector, since \$\overline{n}'\$ is NOT

after transformation

(2) Deriving the correct matrix for normal vector: the [normal matrix] perpendicular to t'

Let  $\vec{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$   $\vec{n} \cdot \vec{t} = 0$  The <u>dot product</u> can be expressed as <u>matrix multiplication</u>:  $\vec{n} \cdot \vec{t} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = (\vec{n})^t \vec{t} - (x) \begin{bmatrix} \vec{n} \end{bmatrix}^t : \text{ transpose } \vec{n}$ 

From (\*) we have  $0 = (\vec{n})^t \vec{t} = (\vec{n})^t M^{-1} M^{$ 

=) Then the 4th row of the remaining columns are 0 and can be ignored

i In (\*) we can use I to replace M:

 $(\vec{R})^{t} \ell^{-1})(\ell, \vec{t}) = 0$   $= \frac{1}{(\vec{X})^{t}} \text{ where } \vec{X} \text{ is the } \frac{1}{t} \text{ transformed pormal, in the form of } (*):$ 

 $(\vec{x})^{t} = (\vec{n})^{t} \ell^{-1} \Rightarrow \vec{x} = (\vec{n})^{t} \ell^{-1} = (\ell^{-1})^{t} (\vec{n})$ 

cf: In (\*): (n)+ 7 =0 Here: (x) + = = 0

> Pesired normal x is obtained by N [ nx my where the 3x3 matrix N (normal matrix)

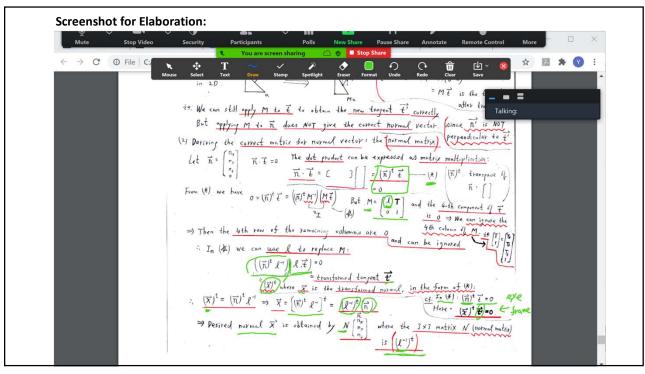
Simplification:

Simplification:

[3] If M only involves translations, rotations, uniform scaling and Look At()

then: translations have no effect on lLook At() has translation and rotation

But uniform scaling has no effect after we normalize the transformed normal  $l \equiv R$ . But  $R' = R^t$   $l \equiv R$ . But  $R' = R^t$   $l \equiv R$ . But  $R' = R^t$   $l \equiv R$ . We can use (l) to replace ( $l^{-1}$ )<sup>t</sup>  $l \equiv R$  we can use the model-view matrix M (4x4) to apply to normal  $l \equiv R^t$   $l \equiv R$ . But  $l \equiv R^t$   $l \equiv R^t$  l



#### **New Topic: Compositing Techniques**

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[Recall:
 * Fragments: generated by the rasterization of geometric primitives (polygons, etc.)
                 Each fragment corresponds to a single pixel
 * Compositing Techniques : compositing a-blending
    * How do we model transparent objects? - the alpha channel
        RGBA (RGBa) color: (r, g, b, a) pacity: 1 opaque
opacity: 1 transparent
                                             (transparency = 1-0)
    * & value controls how the RGB values are written to the frame buffer
                          B is opagne, blocking C invoverlapped portion.
                          A is transparent, the portron overlapped with B is blended with
                           the color of B (blending the colors of A & B)
          * Many fragments, each coming from a different object, may correspond
            to the same pixel = each such fragment contributes to the color of the pixel.
                                     the final color of the pixel is obtained by (blending)
                                                                          the fragment colors.
                                    The corresponding objects are (blended) or (composited) together
  * When a polygon is processed, pixel-size fragments are computed.
                     The fragments are assigned colors based on the shading model used
    Regard the fragment as the (source pixel)
           the frame - buffer pixel us the (destination pixel)
     Previously = 2-buffer, opaque: source pixel is closer to viewer => source pixel (replaces)
                                                                         the destination pixel
                                      destination pixel : .
                                                                     => source pixel is (blocked)
     Now: blend the source and destination pixels in various ways
                                                                        no action
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color of source pixel: S = [Sr Sg Sb Sa] source blending factor b = [br bg bb ba] destination; d = [dr dg db da], destination;  $c = [c_r c_g C_b c_a]$ Compositing: replace d with  $d' = [b_r s_r + c_r dr b_g s_g + c_g dg b_b s_b + c_b d_b b_a s_a + c_a da]$ the resulting r, g, b, a values are clamped to [0.0, 1.0]  $(21 \Rightarrow 1.0)$ Over operation: back-to-front. A  $C_{d'} = \alpha_s C_s + (1-\alpha_s) C_d$ transparency.  $\alpha_{d'} = \alpha_s + (1-\alpha_s) \alpha_d$ the fraction that
the 'behind color's urvives Depth Cueing and Fug \* Depth Cueing create illusion of depth by drawing objects farther from the viewer Aimmer

See (A over (B over C)): back to front

Handout for ((A over B) over C): front to back.

Full details between the object and the viewer, by blending in a (distance-dependent color) as each fragment is processed f: fog factor, given by the fog equation f(z), (f = f(z)) Z: distance between a (Sragment) rendered Cs: fragment color given in the (eye coordinates) Cf fog color

(\*\* Note: The Handout for the ``Over'' operation has been posted at NYU Brightspace: 
``Handouts -> Over-Op-Associativity.pdf'')

Resulting color: 
$$C_{s'} = \int C_s + (I - f) C_f$$
 (\*)

For mode

for equation 
$$f(z) = f$$

linear for

 $f = \frac{\text{end} - z}{\text{end} - \text{start}}$ 

linear, depth-cuesing effect

exponential for

 $f = e^{-(\text{density} \cdot z)}$ 

exponential square for

 $f = e^{-(\text{density} \cdot z)^2}$ 

Gaussian

for effect

\* I specified is clamped to [0,1] and then used in (\*) to compute Cs.

From fog equation:

Z / f / (f is clamped to [0,1])

Pluggin f into (\*)

Cs has more weight

i.e. when object is farther (Z /)

we see more of the fog color (Cs)