

CS6533/CS4533 Lectures 11-12

Slides/Notes

Shading and Illumination; Compositing (Notes, Ch 14, Notes)

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*** Continued on Shading and Illumination:**

- First we reviewed the “Overall Formula” of the Phong Reflection Model (shown in the next 2 slides) as presented last time, which is implemented in the sample program “Handout: rotate-cube-shading.cpp”.
- Discussed the sample program “Handout: rotate-cube-shading.cpp” (complete sample program has been posted at <https://cse.engineering.nyu.edu/cs653/Rotate-Cube-Shading.tar.gz>)
- Some screenshots of the sample program with annotations are then shown next.
- Showed a demo of the sample program “Rotate-Cube-Shading”.
- Then we finished up the last 2 pages (p.14 & p.15) of the last lecture notes/slides “Lecture-9-10.pdf” on Normal Matrix.

*** New topic: Composition Techniques.**

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Overall formula:

$$I = K_a \cdot L_a \cdot \text{global} + \sum_{\text{light } i} (\text{Attenuation})_i \cdot [K_a \cdot L_a + K_d \cdot L_d \cdot \max\{\ell \cdot n, 0\} + [\text{if } \ell \cdot n \geq 0] \cdot K_s \cdot L_s \cdot (\max\{\ell \cdot n, 0\})^\alpha]_i$$

Note: component-wise multiplications:
 $K_a \cdot L_a, K_d \cdot L_d, K_s \cdot L_s$
 They are attenuated differently

(1) If light i is a distant (directional) light, then

① $(\text{Attenuation})_i = 1$

② vector ℓ (from pt p to light source i) = $-(\text{distant light direction } L)$ $\ell = -L$

(2) If light i is a point source, then

① $(\text{Attenuation})_i = \frac{1}{a + b \cdot d + c \cdot d^2}$ where d = distance from pt p to the light source
 a, b, c : constant, linear, quadratic attenuations

② $\ell = -L$ (Also, use this ℓ to compute $h = \text{normalize}(\ell + v)$)

replaced with $(n \cdot h)$

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(3) If light i is a spotlight then

① $(\text{Attenuation})_i = \frac{1}{a + b \cdot d + c \cdot d^2} \cdot (\text{spotlight-attenuation})_i$ a, b, c, d are as in (2) point source.

② $(\text{spotlight-attenuation})_i = ?$

θ : spotlight cut-off angle $\theta \in [0, 90^\circ]$

(a) If $\phi > \theta$ then contribution = 0

In the range $[0, 90^\circ]$
 $\phi > \theta \Leftrightarrow \cos \phi < \cos \theta$
 $\Leftrightarrow (L_s \cdot S) < \cos \theta$
 $\Leftrightarrow L_s \cdot (-\ell) < \cos \theta$

(b) Else
 $(\text{spotlight-attenuation})_i = (\cos \phi)^e = [L_s \cdot (-\ell)]^e$ e : spotlight exponent

Combining (a), (b): $(\text{spotlight-attenuation})_i = [\text{if } L_s \cdot (-\ell) \geq \cos \theta] \cdot [L_s \cdot (-\ell)]^e$

Diagram: A spotlight cone with vertex at point P_s and radius θ . A point P is shown. The vector from P_s to P is $\vec{P}P_s$. The vector from P to the light source is ℓ . The vector from P to the spotlight vertex is L_s . The angle between L_s and ℓ is ϕ . The angle between L_s and the cone's axis is θ . The angle between L_s and the horizontal is ψ . The angle between ℓ and the horizontal is γ . The angle between L_s and the horizontal is ψ . The angle between ℓ and the horizontal is γ . The angle between L_s and the horizontal is ψ . The angle between ℓ and the horizontal is γ .

In particular, $d = |\vec{P}P_s|$
 $\ell = \text{normalize}(\vec{P}P_s)$
 $= -S$
 $\therefore S = -\ell$
 If $L_s \cdot (-\ell) < \cos \theta$ then $(\text{spotlight-attenuation})_i = 0$

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```

#include "Angel-yjc.h"

typedef Angel::vec4 color4;
typedef Angel::vec4 point4;

GLuint program; /* shader program object id */
GLuint cube_buffer; /* vertex buffer object id for cube */

// Projection transformation parameters
GLfloat fovy = 45.0; // Field-of-view in Y direction angle (in degrees)
GLfloat aspect; // Viewport aspect ratio
GLfloat zNear = 0.5, zFar = 3.0;

int animationFlag = 1; // 1: animation; 0: non-animation. Toggled by key 'a' or 'A'

const int NumVertices = 36; // (6 faces) (2 triangles/face) (3 vertices/triangle)
point4 points[NumVertices];
vec3 normals[NumVertices];

// Vertices of a unit cube centered at origin, sides aligned with axes
point4 vertices[8] = {
    point4(-0.5, -0.5, 0.5, 1.0),
    point4(-0.5, 0.5, 0.5, 1.0),
    point4(0.5, 0.5, 0.5, 1.0),
    point4(0.5, -0.5, 0.5, 1.0),
    point4(-0.5, -0.5, -0.5, 1.0),
    point4(-0.5, 0.5, -0.5, 1.0),
    point4(0.5, 0.5, -0.5, 1.0),
    point4(0.5, -0.5, -0.5, 1.0)
};

// Array of rotation angles (in degrees) for each coordinate axis
enum { Xaxis = 0, Yaxis = 1, Zaxis = 2, NumAxes = 3 };
int Axis = Xaxis;
GLfloat Theta[NumAxes] = { 0.0, 0.0, 0.0 };

// Model-view and projection matrices uniform location
GLint ModelView, Projection;

/*----- Shader Lighting Parameters -----*/
color4 light_ambient( 0.2, 0.2, 0.2, 1.0 );
color4 light_diffuse( 1.0, 1.0, 1.0, 1.0 );
color4 light_specular( 1.0, 1.0, 1.0, 1.0 );

// to the vertices
void quad( int a, int b, int c, int d )
{
    // Initialize temporary vectors along the quad's edges to
    // compute its face normal
    vec4 u = vertices[b] - vertices[a];
    vec4 v = vertices[d] - vertices[a];

    vec3 normal = normalize( cross(u, v) );

    normals[Index] = normal; points[Index] = vertices[a]; Index++;
    normals[Index] = normal; points[Index] = vertices[b]; Index++;
    normals[Index] = normal; points[Index] = vertices[c]; Index++;
    normals[Index] = normal; points[Index] = vertices[d]; Index++;
    normals[Index] = normal; points[Index] = vertices[a]; Index++;
}

// colorcube() generates 6 quad faces (12 triangles): 36 vertices
void colorcube()
{
    quad( 1, 0, 3, 2 );
    quad( 2, 3, 7, 6 );
    quad( 3, 0, 4, 7 );
    quad( 6, 5, 1, 2 );
    quad( 4, 5, 6, 7 );
    quad( 5, 4, 0, 1 );
}

// OpenGL initialization
void init()
{
    colorcube();

    // Create and initialize a vertex buffer object
    glGenBuffers( 1, &cube_buffer );
    glBindBuffer( GL_ARRAY_BUFFER, cube_buffer );
    glBufferData( GL_ARRAY_BUFFER, sizeof(points) + sizeof(normals), NULL, GL_STATIC_DRAW );
    glBufferSubData( GL_ARRAY_BUFFER, 0, sizeof(points), points );
    glBufferSubData( GL_ARRAY_BUFFER, sizeof(points), sizeof(normals), normals );
}

```

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```

// user interface, etc.
// and mouse() functions for
// rotating cube with shading.

// Material properties & Normal Matrix are sent to the shader as
// uniforms. The normal computation is done in the Eye Frame (in shader).
// .h

color4;
point4;

/* shader program object id */
/* vertex buffer object id for cube */

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GLfloat aspect; // Viewport aspect ratio
GLfloat zNear = 0.5, zFar = 3.0;

int animationFlag = 1; // 1: animation; 0: non-animation. Toggled by key 'a' or 'A'

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    point4(0.5, -0.5, 0.5, 1.0),
    point4(-0.5, -0.5, -0.5, 1.0),
    point4(-0.5, 0.5, -0.5, 1.0),
    point4(0.5, 0.5, -0.5, 1.0),
    point4(0.5, -0.5, -0.5, 1.0)
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// to the vertices
void quad( int a, int b, int c, int d )
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    // Initialize temporary vectors along the quad's edges to
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    vec4 v = vertices[d] - vertices[a];

    vec3 normal = normalize( cross(u, v) );

    normals[Index] = normal; points[Index] = vertices[a]; Index++;
    normals[Index] = normal; points[Index] = vertices[b]; Index++;
    normals[Index] = normal; points[Index] = vertices[c]; Index++;
    normals[Index] = normal; points[Index] = vertices[d]; Index++;
    normals[Index] = normal; points[Index] = vertices[a]; Index++;
}

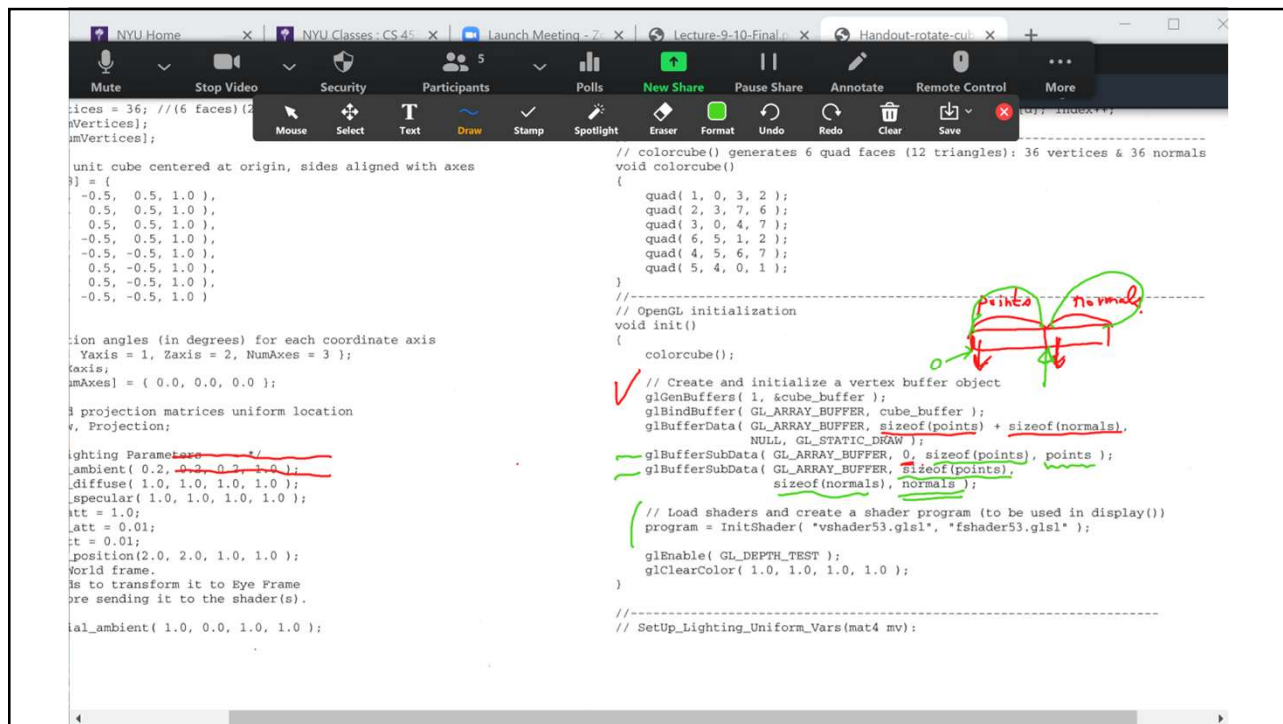
// colorcube() generates 6 quad faces (12 triangles): 36 vertices & 36 normals
void colorcube()
{
    quad( 1, 0, 3, 2 );
    quad( 2, 3, 7, 6 );
    quad( 3, 0, 4, 7 );
    quad( 6, 5, 1, 2 );
    quad( 4, 5, 6, 7 );
    quad( 5, 4, 0, 1 );
}

// OpenGL initialization
void init()
{
    colorcube();

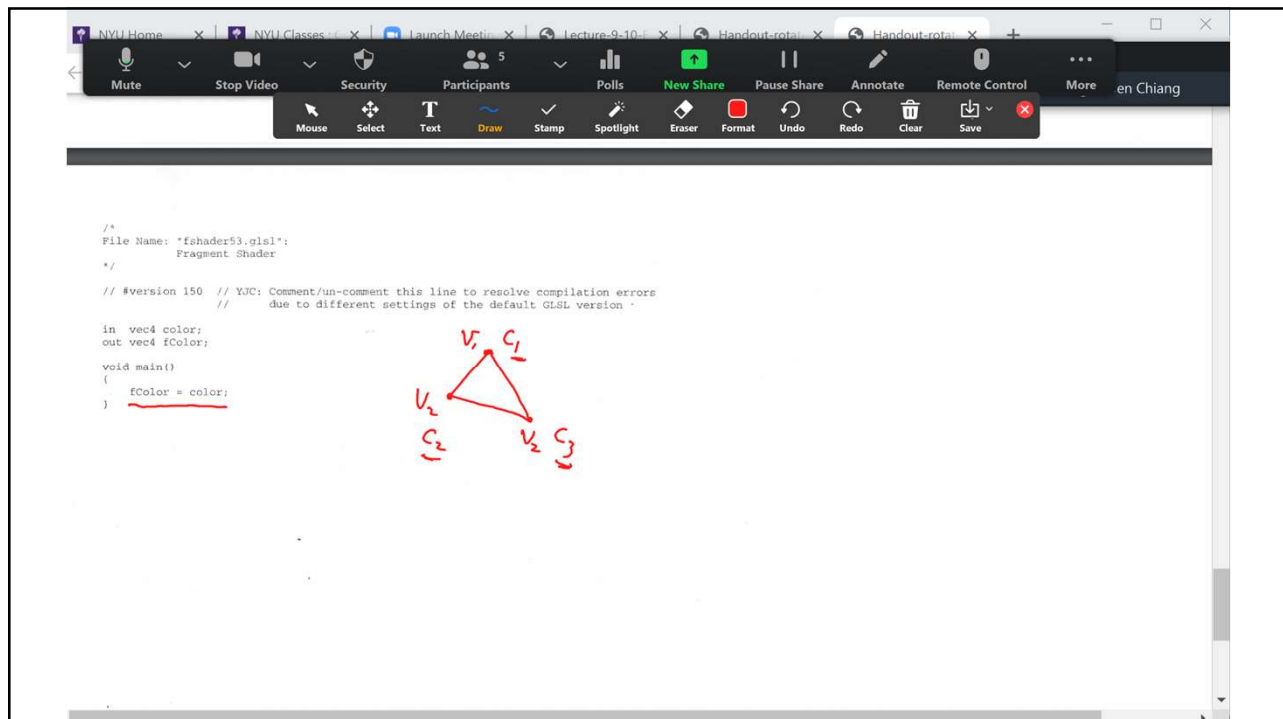
    // Create and initialize a vertex buffer object
}

```

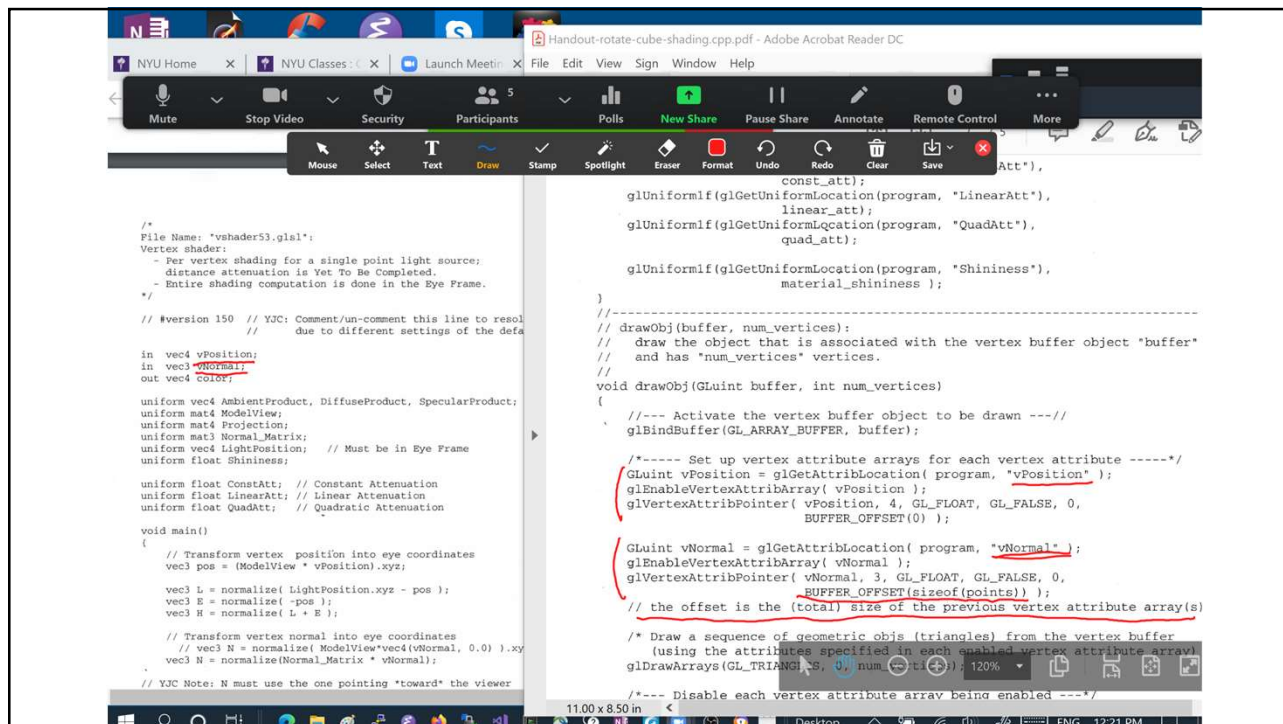
6



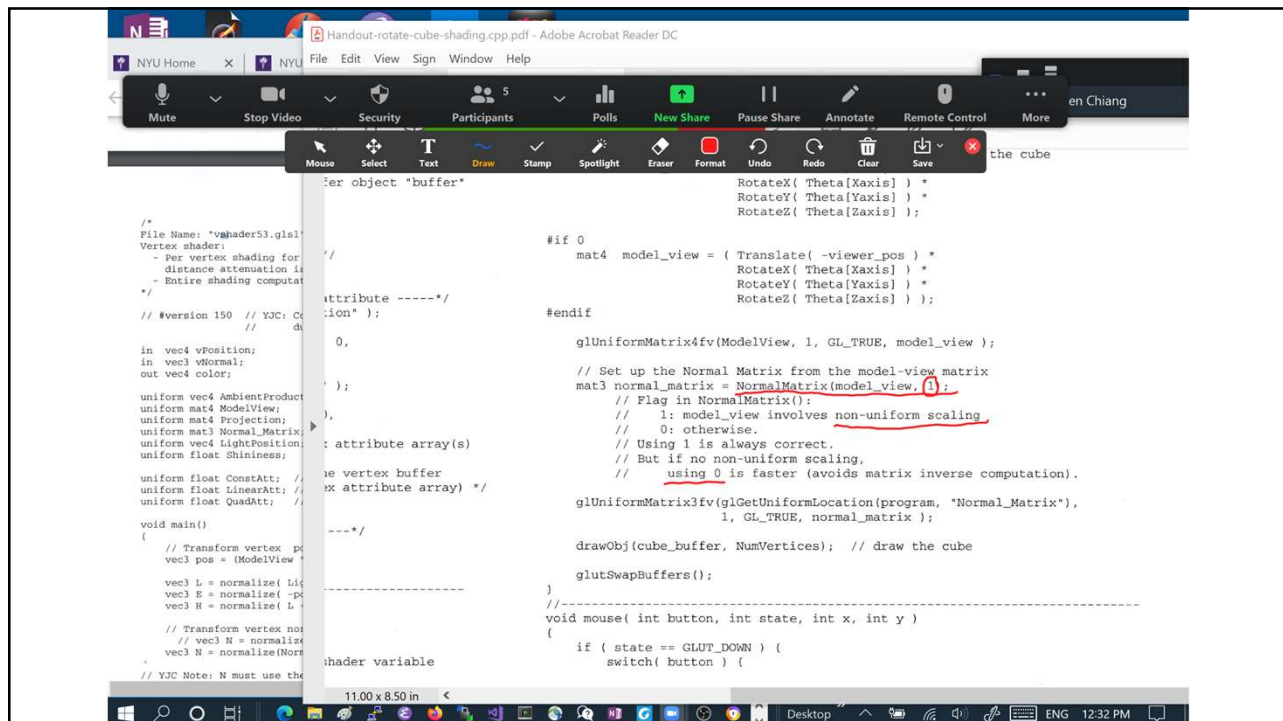
7



8



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* Normal Matrix

* Typically we perform shading computation in the (eye frame) (i.e. the right-handed eye frame where the eye/camera is at the origin looking at the -z direction. This is the frame obtained by applying LookAt() to the world frame).

Let \vec{T} be the tangent at pt p being shaded.

\vec{n} normal

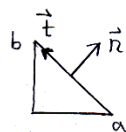
\vec{n}, \vec{T} are in the model frame

M the model-view matrix

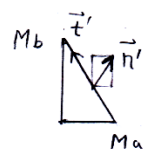
i.e. Mp puts p in the eye frame
in x, y, z -dimensions are different

(1) Suppose M involves (non-uniform scaling) (i.e. scaling factors

e.g. $S(1, 2)$
in 2D:



$S(1, 2)$



$$\begin{aligned} \vec{T}' &= M\vec{b} - M\vec{a} = M(\vec{b} - \vec{a}) \\ &= M\vec{T} \text{ is the tangent after transformation} \end{aligned}$$

i.e. We can still apply M to \vec{T} to obtain the new tangent \vec{T}' correctly.

But applying M to \vec{n} does NOT give the correct normal vector.

(since \vec{n}' is NOT perpendicular to \vec{T}')

(2) Deriving the correct matrix for normal vector: the (normal matrix)

$$\text{Let } \vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$$

$$\vec{n} \cdot \vec{T} = 0$$

The dot product can be expressed as matrix multiplication:

$$\vec{n} \cdot \vec{T} = \begin{bmatrix} \vec{n} \end{bmatrix}^T \begin{bmatrix} \vec{T} \end{bmatrix} = (\vec{n})^T \vec{T} \quad (*) \quad \begin{matrix} (\vec{n})^T: \text{transpose of } \vec{n} \\ \vec{n} = \begin{bmatrix} \end{bmatrix} \end{matrix}$$

From (*), we have

$$0 = (\vec{n})^T \vec{T} = (\vec{n})^T M^{-1} (M\vec{T})$$

$$\text{But } M = \begin{bmatrix} l & T \\ 0 & 1 \end{bmatrix}$$

and the 4th component of \vec{T} is 0 \Rightarrow We can ignore the 4th column of M , i.e. $\begin{bmatrix} T_x \\ T_y \\ T_z \\ 1 \end{bmatrix}$ and can be ignored

\Rightarrow Then the 4th row of the remaining columns are 0

\therefore In (*) we can use l to replace M :

$$\begin{aligned} ((\vec{n})^T l^{-1}) (l \vec{T}) &= 0 \\ &= \text{transformed tangent } \vec{T}' \end{aligned}$$

\vec{x} where \vec{x} is the transformed normal, in the form of (*):

$$\therefore (\vec{x})^T = (\vec{n})^T l^{-1} \Rightarrow \vec{x} = [(\vec{n})^T l^{-1}]^T = (l^{-1})^T (\vec{n})$$

cf. In (*): $(\vec{n})^T \vec{T} = 0$

Here: $(\vec{x})^T \vec{T}' = 0$

\Rightarrow Desired normal \vec{x} is obtained by $N \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$ where the 3x3 matrix N (normal matrix) is $(l^{-1})^T$

Simplification:

- (3) If M only involves translations, rotations, uniform scaling, and LookAt() then: translations have no effect on l LookAt() has translation and rotation $\Rightarrow l$ only involves rotations and uniform scaling
- But uniform scaling has no effect after we normalize the transformed normal vector

$\Rightarrow l \equiv R$. But $R^{-1} = R^t$

i $(l^{-1})^t \equiv (R^{-1})^t = (R^t)^t = R \equiv l$

ie ① We can use (l) to replace $(l^{-1})^t$

ie ② We can use the model-view matrix M (4×4) to apply to normal $\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$

① \equiv ②

(scaling factors in x , y , z -dim are all the same)

4 components

Screenshot for Elaboration:

in 2D.

is. We can still apply M to \vec{t} to obtain the new tangent \vec{t}' correctly. But applying M to \vec{n} does NOT give the correct normal vector. (since \vec{n}' is NOT perpendicular to \vec{t}')

(2) Deriving the correct matrix for normal vector: the (normal matrix)

Let $\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ 0 \end{bmatrix}$. $\vec{n} \cdot \vec{t} = 0$. The dot product can be expressed as matrix multiplication:

$$\vec{n} \cdot \vec{t} = \begin{bmatrix} n_x & n_y & n_z & 0 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ t_z \\ t_w \end{bmatrix} = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = 0 \quad (*)$$

From (*) we have $0 = (\vec{n})^t \vec{t} = (\vec{n})^t M^{-1} (M \vec{t})$. But $M = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}$ and the 4th component of \vec{t} is 0 \Rightarrow We can ignore the 4th column of M . is $\begin{bmatrix} R \\ T \end{bmatrix}$ and can be ignored \Rightarrow Then the 4th row of the remaining columns are 0

\therefore In (*) we can use L to replace M :

$$(\vec{n})^t L^{-1} L \vec{t} = 0$$

\vec{x}^t where \vec{x} is the transformed normal, in the form of (*):

$$\vec{x}^t = (\vec{n})^t L^{-1} \Rightarrow \vec{x} = (\vec{n})^t L^{-1} = \begin{bmatrix} L^{-1} \end{bmatrix}^t \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

\Rightarrow Desired normal \vec{x} is obtained by $N \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$ where the 3×3 matrix N (normal matrix) is $(L^{-1})^t$

cf. In (*): $(\vec{n})^t \vec{t} = 0$ Here: $(\vec{x})^t \vec{t} = 0$ \leftarrow same

New Topic: Compositing Techniques

Recall:

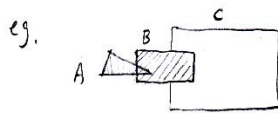
* Fragments : generated by the rasterization of geometric primitives (polygons, etc.)
Each fragment corresponds to a single pixel

* Compositing Techniques : compositing, α -blending

* How do we model transparent objects? — the alpha channel

RGBA (RGB α) color : (r, g, b, α)
 α → opacity : $\begin{matrix} 1 & \text{opaque} \\ \updownarrow & \\ 0 & \text{transparent} \end{matrix}$
 (transparency = $1 - \alpha$)

* α value controls how the RGB values are written to the frame buffer.



B is opaque, blocking C in ^{the} overlapped portion.

A is transparent, the portion overlapped with B is blended with the color of B (blending the colors of A & B)

* Many fragments, each coming from a different object, may correspond to the same pixel \Rightarrow each such fragment contributes ^{to} the color of the pixel.
the final color of the pixel is obtained by blending the fragment colors.

The corresponding objects are blended or composited together.

* When a polygon is processed, pixel-size fragments are computed.

The fragments are assigned colors based on the shading model used.

Regard the fragment as the (source pixel)

the frame-buffer pixel as the (destination pixel)

Previously : z-buffer, opaque : source pixel is closer to viewer \Rightarrow source pixel (replaces) the destination pixel.

destination pixel : \Rightarrow source pixel is (blocked) no action.

Now : blend the source and destination pixels in various ways

color of source pixel: $s = [s_r \ s_g \ s_b \ s_a]$ source blending factor $b = [b_r \ b_g \ b_b \ b_a]$

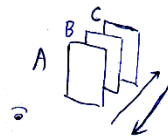
destination: $d = [d_r \ d_g \ d_b \ d_a]$ destination: $c = [c_r \ c_g \ c_b \ c_a]$

$$d' \leftarrow bs + cd$$

compositing: replace d with $d' = [b_r s_r + c_r d_r \ b_g s_g + c_g d_g \ b_b s_b + c_b d_b \ b_a s_a + c_a d_a]$

the resulting r, g, b, a values are clamped to $[0.0, 1.0]$ $\begin{cases} \geq 1 \Rightarrow 1.0 \\ \leq 0 \Rightarrow 0.0 \end{cases}$

Depth Cueing and Fog



"Over" operation:

back-to-front.

$$\begin{cases} C_d' = \alpha_s C_s + (1 - \alpha_s) C_d \\ \alpha_d' = \alpha_s + (1 - \alpha_s) \alpha_d \end{cases}$$

transparency: the fraction that the "behind color" survives

* Depth Cueing: create illusion of depth by drawing objects farther from the viewer dimmer

* Fog Effect: extend depth cueing.

See Handout for full details

$(A \text{ over } (B \text{ over } C))$: back to front
 $((A \text{ over } B) \text{ over } C)$: front to back.

create the illusion of partially translucent space (fog) between the object and the viewer, by blending in a (distance-dependent color) as each fragment is processed

f : fog factor, given by the fog equation $f(z)$, ($f = f(z)$)

C_s : fragment color

C_f : fog color

z : distance between a (fragment) being rendered and the (viewer) given in the (eye coordinates)

(** Note: The Handout for the "Over" operation has been posted at NYU Brightspace:

"Handouts -> Over-Op-Associativity.pdf")

Resulting color: $C_{s'} = f C_s + (1-f) C_f$ — (*)

fog-mode

fog equation $f(z) = f$

linear fog

$$f = \frac{\text{end} - z}{\text{end} - \text{start}}$$

linear, depth-cueing effect.

exponential fog

$$f = e^{-(\text{density} \cdot z)}$$

exponential

exponential square fog

$$f = e^{-(\text{density} \cdot z)^2}$$

Gaussian

} fog effect

* f specified is clamped to $[0,1]$ and then used in (*) to compute $C_{s'}$.

o From fog equation:
 $z \uparrow \Rightarrow f \downarrow$ (f is clamped to $[0,1]$)
Plugging f into (*)
 $\Rightarrow C_s$ has more weight
 \Rightarrow when object is farther ($z \uparrow$)
we see more of the fog color (C_f)