

(a)

When plotting a circle starting from $P(x_p, y_p)$ and moving y by 1,
we can pick from two possible pixels:

$$N = P(x_p, y_p + 1) \text{ or } NW = P(x_p - 1, y_p + 1).$$

Using the midpoint (M between N and NW) as the decision parameter,
we can decide which pixel to proceed.

$$\begin{aligned} M &= (x_p - 1/2, y_p + 1) \\ D &= F(M) = F(x_p - 1/2, y_p + 1) \\ &= (x_p - 1/2)^2 + (y_p + 1)^2 - r^2 \end{aligned}$$

If $D < 0$, M is inside the circle and we can proceed to N .

Otherwise, if $D \geq 0$, M is outside/on the circle and we can proceed to NW .

To find out D_{new} , we should consider two scenarios:

- 1) D_{new} after proceeding with N
- 2) D_{new} after proceeding with NW

1) D_{new} after proceeding with N ($x_p, y_p + 1$):

$$\begin{aligned} D_{new} &= (x_p - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 \\ D_{old} &= (x_p - 1/2)^2 + (y_p + 1)^2 - r^2 \\ D_{new} - D_{old} &= (x_p - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 - [(x_p - 1/2)^2 + (y_p + 1)^2 - r^2] \\ &= 2(y_p + 1) + 1 \\ \therefore D_{new} &= D_{old} + 2(y_p + 1) + 1 \end{aligned}$$

2) D_{new} after proceeding with NW ($x_p - 1, y_p + 1$):

$$\begin{aligned} D_{new} &= (x_{p-1} - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 \\ D_{old} &= (x_p - 1/2)^2 + (y_p + 1)^2 - r^2 \\ D_{new} - D_{old} &= (x_{p-1} - 1/2)^2 + (y_{p+1} + 1)^2 - r^2 - [(x_p - 1/2)^2 + (y_p + 1)^2 - r^2] \\ &= 2(y_p + 1) - 2(x_p + 1) + 1 \\ \therefore D_{new} &= D_{old} + 2(y_p + 1) - 2(x_p + 1) + 1 \end{aligned}$$

We can derive D_{start} by plugging in the initial coordinates ($r, 0$).

$$\begin{aligned} D_{start} &= F(r - 1/2, 1) \\ &= (r - 1/2)^2 + 1 - r^2 \\ &= 5/4 - r \\ &\cong 1 - r \quad (r \text{ is an integer}) \end{aligned}$$

Lastly, we can derive coordinates in 8 symmetric regions by considering eight symmetric points at:
(x,y), (-x,y), (x,-y), (-x,-y), (y,x), (-y,x), (y,-x), (-y,-x) on a circle.

(b)

OpenGL uses right-handed coordinate system, where positive x-axis is to viewer's right, positive y-axis is up. Origin (0, 0) is located at the bottom-left corner of the window. Therefore, if any of the x and y is less than 0, the coordinate won't be shown in the window.