# Deployment of a Strategic Game: An Interdisciplinary Study

COMSCI/ECON 206 — Problem Set 1

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**Abstract**

This report selects **Matching Pennies** as the target game (distinct from the class demos Prisoner's Dilemma and Battle of the Sexes). It provide (i) a brief theoretical analysis and welfare discussion; (ii) computational checks of equilibrium using `NashPy` and `QuantEcon` and an extensive-form version in `GTE`; and (iii) an experimental deployment plan via `oTree`, plus a lightweight LLM comparison.

**GitHub repository:** https://github.com/zijund021/Matching-Pennies-An-Interdisciplinary-Study

# Part 1 — Economist (theory & welfare)

## 1. Equilibrium concept

I adopt *Nash equilibrium in mixed strategies* as the appropriate concept for *Matching Pennies*. A normal-form (strategic) game is $G = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$: a finite player set $N$; each player $i$ has a pure-strategy set $S_i$; and a payoff function $u_i : \prod_{j \in N} S_j \to \mathbb{R}$ (Osborne 2003, 11). A mixed strategy $\sigma_i$ is a probability distribution over $S_i$ (Osborne 2003, 115–16). For a mixed profile $\sigma = (\sigma_1, \sigma_2)$, expected payoffs are defined in the standard way by taking expectations over pure profiles; existence of a mixed-strategy Nash equilibrium in any finite normal-form game follows from fixed-point arguments (Nash 1951, 286–295).

## 2. Analytical solution, efficiency, and fairness

**Characterization.** In *Matching Pennies*, $S_1 = S_2 = \{H, T\}$. The canonical payoff matrix is

| | $H$ | $T$ |
|---|---|---|
| $H$ | $(1, -1)$ | $(-1, 1)$ |
| $T$ | $(-1, 1)$ | $(1, -1)$ |

Osborne's *Example 17.1* lays out the game and its interpretation (Osborne 2003, 28). There is no pure-strategy Nash equilibrium: each pure profile gives one player a profitable deviation (Osborne 2003, 38). Let player 1 choose $H$ with probability $p$, player 2 with probability $q$. Indifference conditions yield $q = \frac{1}{2}$ and, symmetrically, $p = \frac{1}{2}$; thus the unique mixed-strategy equilibrium is $(p, q) = (\frac{1}{2}, \frac{1}{2})$ (Osborne 2003, 119–120).

**Efficiency and fairness.** The game is zero-sum, so utilitarian welfare (the sum of expected payoffs) is always zero. Ex ante Pareto improvements are impossible because one player's gain exactly equals the other's loss. The equilibrium is symmetric, giving both players equal expected payoff (zero), which supports an equity interpretation ex ante.

### 3. Interpretation, refinements, and tractability

**Realism.** Perfect 1/2–1/2 randomization may be behaviorally demanding; subjects can display biases or patterns.

**Multiplicity and refinements.** *Matching Pennies* has a *unique* mixed-strategy equilibrium (no pure or additional mixed equilibria) (Osborne 2003, 119–120). Refinements (e.g., trembling-hand) or noisy best response models (e.g., quantal response) can rationalize systematic deviations from perfect mixing.

**Computational tractability.** For a $2 \times 2$ game, the equilibrium is analytically immediate; larger games may require algorithms, even though existence is guaranteed by Nash's theorem (Nash 1951, 286–295).

## Part 2 — Computational Scientist (coding & tools)

### 2a) Google Colab (normal form + computation)

**Colab link:** https://colab.research.google.com/drive/1S1s4Mx6FWe9G8UO_cvPr8hqMKhjjSIzN?authuser=1#scrollTo=gmPXmUfGNud-

**Brief interpretation.** The displayed bimatrix confirms a two-player *zero-sum* normal-form game with $A$ for the row player and $B = -A$ for the column player in Matching Pennies. Solver outputs then align with the theory: `pure_nash_brute` returns an empty set (no pure NE); both `support_enumeration` in Nashpy and QuantEcon and `vertex_enumeration` recover the *unique* mixed equilibrium $([0.5, 0.5], [0.5, 0.5])$. At this profile each action leaves the opponent indifferent, hence no unilateral deviation is profitable; by symmetry and zero-sum structure, the row player's expected payoff equals 0 and the column player's equals $-0$.

```
Zero sum game with payoff matrices:

Row player:
[[ 1 -1]
 [-1  1]]

Column player:
[[-1  1]
 [ 1 -1]]
```

**Figure 1: Displayed payoff matrices.** Nashpy game object showing the zero-sum bimatrix with $A$ and $B = -A$.

```
2-player NormalFormGame with payoff profile array:
[[[ 1, -1],  [-1,  1]],
 [[-1,  1],  [ 1, -1]]]
```

**Figure 2: Solver outputs.** QuantEcon `pure_nash_brute` returns `[]` — no pure-strategy NE in Matching Pennies.

```
NE = gt.pure_nash_brute(g_MP)
print("pure_nash_brute:", NE)
```

```
pure_nash_brute: []
```

**Figure 3: Solver outputs.** Nashpy `support_enumeration` finds the mixed NE $([0.5, 0.5], [0.5, 0.5])$.

```
NE = gt.support_enumeration(g_MP)
print("support_enumeration:", NE)
```

```
support_enumeration: [(array([0.5, 0.5]), array([0.5, 0.5]))]
```

**Figure 4: Solver outputs.** QuantEcon `support_enumeration` independently recovers the same mixed NE.

```
NE = gt.vertex_enumeration(g_MP)
print("vertex_enumeration:", NE)
```

```
vertex_enumeration: [(array([0.5, 0.5]), array([0.5, 0.5]))]
```

**Figure 5: Solver outputs.** QuantEcon `vertex_enumeration` (best-response polytope method) yields the same equilibrium.

### 2b) Game Theory Explorer (extensive form + SPNE)

**Brief interpretation.** In the extensive form, simultaneity is represented by placing Player 2's two decision nodes in a *single information set*, so Player 2 does not observe Player 1's move. Hence there are no proper subgames and the *SPNE coincides with the NE* of the simultaneous normal form. Solving in GTE yields the *unique* mixed equilibrium $([0.5, 0.5], [0.5, 0.5])$; by zero-sum symmetry the row player's expected payoff is 0.

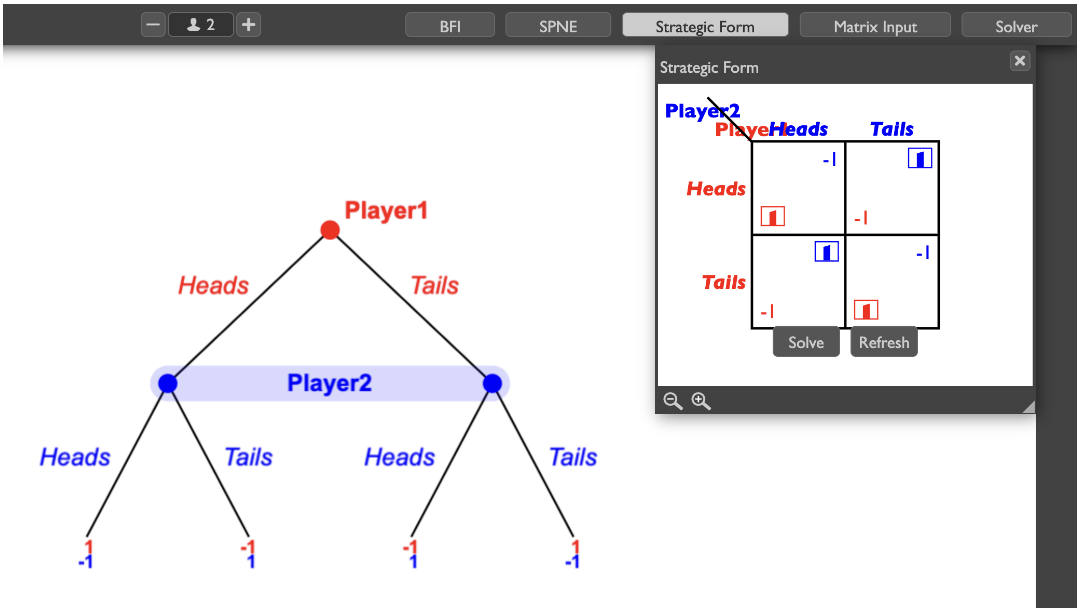



**Figure 6:** (i) **Displayed extensive form (GTE).** Player 2's two nodes are joined by *one information set* (simultaneous move); leaf payoffs match the Matching Pennies bimatrix with $B = -A$. Right panel shows the strategic-form matrix for cross-check.

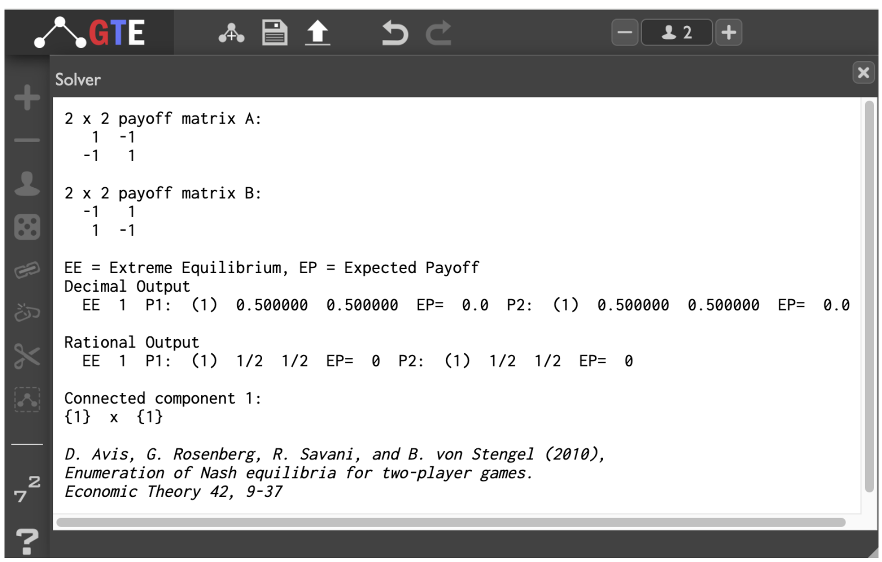



**Figure 7:** (ii) **Solver output (GTE).** The SPNE solver returns the mixed equilibrium $([0.5, 0.5], [0.5, 0.5])$, consistent with the simultaneous normal form and Part 2a.

# Part 3 — Behavioral Scientist (experiment & LLM)

## 3a) oTree deployment (adapted demo)

**What I changed and why.** I adapt the standard `Matching Pennies` demo by setting `NUM_ROUNDS = 7` (was 3). This preserves the zero-sum structure and the theoretical prediction (unique mixed

NE), but yields more within-subject observations to estimate mixing/switching dynamics and compare early vs. late rounds. The "pay one randomly selected round" rule (RIS) is kept to maintain clean per-round incentives.

**Screenshots:**

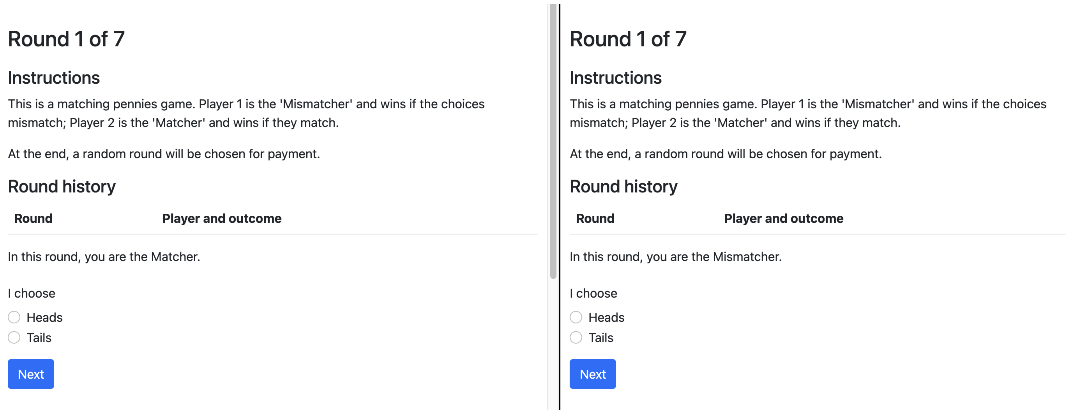



**Figure 8:** Instruction Description page (Matching Pennies; pay one random round).

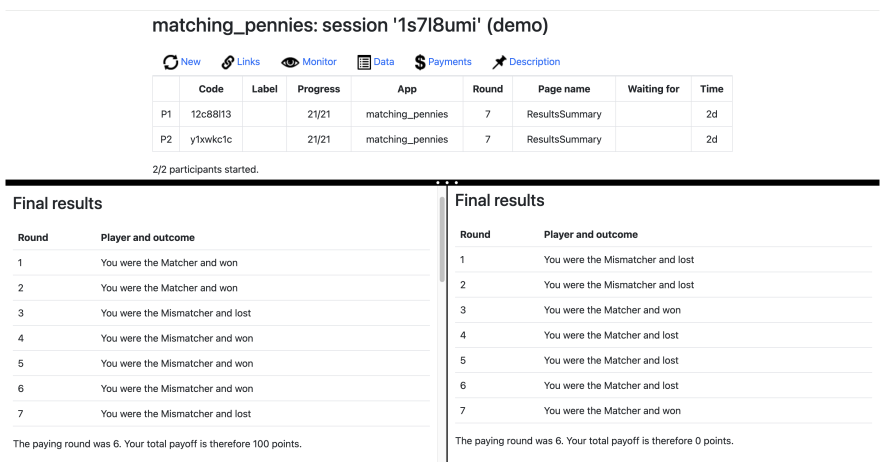
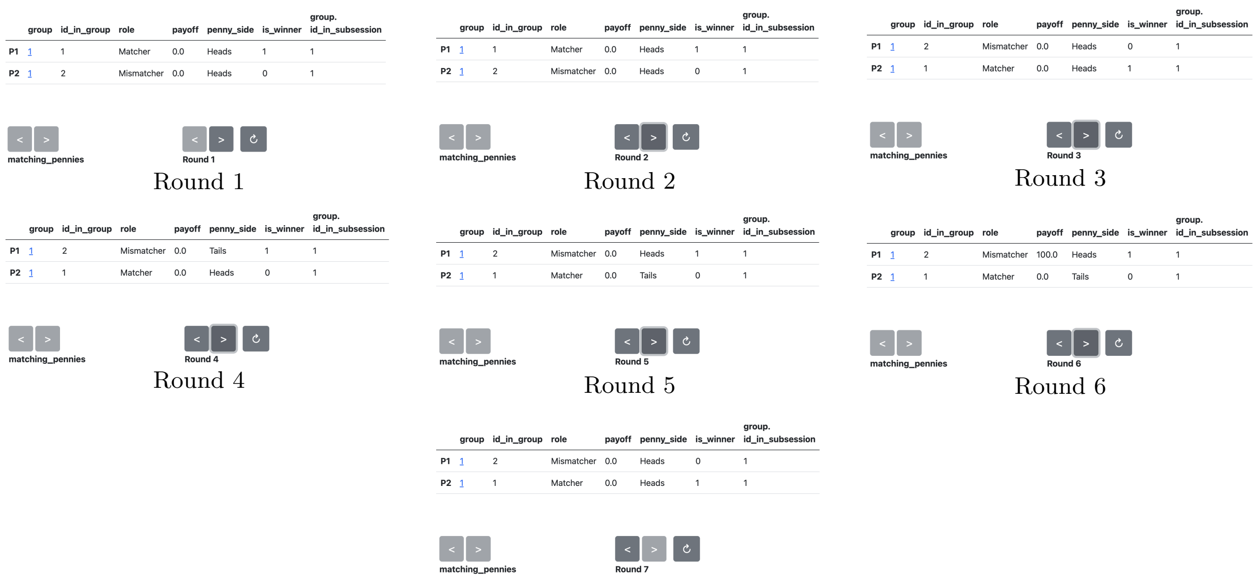



**Figure 9:** Final results and final payoff page (session summary).



| | group | id_in_group | role | payoff | penny_side | is_winner | group.id_in_subsession |
|---|---|---|---|---|---|---|---|
| P1 | 1 | 1 | Matcher | 0.0 | Heads | 1 | 1 |
| P2 | 1 | 2 | Mismatcher | 0.0 | Heads | 0 | 1 |



Round 1

| | group | id_in_group | role | payoff | penny_side | is_winner | group.id_in_subsession |
|---|---|---|---|---|---|---|---|
| P1 | 1 | 1 | Matcher | 0.0 | Heads | 1 | 1 |
| P2 | 1 | 2 | Mismatcher | 0.0 | Heads | 0 | 1 |



Round 2

| | group | id_in_group | role | payoff | penny_side | is_winner | group.id_in_subsession |
|---|---|---|---|---|---|---|---|
| P1 | 1 | 2 | Mismatcher | 0.0 | Heads | 0 | 1 |
| P2 | 1 | 1 | Matcher | 0.0 | Heads | 1 | 1 |



Round 3

| | group | id_in_group | role | payoff | penny_side | is_winner | group.id_in_subsession |
|---|---|---|---|---|---|---|---|
| P1 | 1 | 2 | Mismatcher | 0.0 | Tails | 1 | 1 |
| P2 | 1 | 1 | Matcher | 0.0 | Heads | 0 | 1 |



Round 4

| | group | id_in_group | role | payoff | penny_side | is_winner | group.id_in_subsession |
|---|---|---|---|---|---|---|---|
| P1 | 1 | 2 | Mismatcher | 0.0 | Heads | 1 | 1 |
| P2 | 1 | 1 | Matcher | 0.0 | Tails | 0 | 1 |



Round 5

| | group | id_in_group | role | payoff | penny_side | is_winner | group.id_in_subsession |
|---|---|---|---|---|---|---|---|
| P1 | 1 | 2 | Mismatcher | 100.0 | Heads | 1 | 1 |
| P2 | 1 | 1 | Matcher | 0.0 | Tails | 0 | 1 |



Round 6

| | group | id_in_group | role | payoff | penny_side | is_winner | group.id_in_subsession |
|---|---|---|---|---|---|---|---|
| P1 | 1 | 2 | Mismatcher | 0.0 | Heads | 0 | 1 |
| P2 | 1 | 1 | Matcher | 0.0 | Heads | 1 | 1 |



Round 7

**Figure 10:** (Decision History) Round-by-round choices (1–7).

**Post-play interviews (2 classmates).**

- **Player 1 (Yanzhen).** Q1: Noticed the "pay one random round" rule? *No.* Follow-up: Would noticing it change choices? *No.* (I explained RIS: it encourages taking each round seriously.) Q2: Strategy? *Forecasted the opponent from recent rounds and best-responded.*

- **Player 2 (Shiqi).** Q1: Noticed the rule "pay one random round"? *No,* but stated she would still treat each round seriously. Q2: Strategy? *Randomized choices.*

**Interpretation.** With `NUM_ROUNDS=7`, the mixed-NE prediction is unchanged, but I gain extra within-subject data to compare early vs. late mixing and to contrast "pattern-tracking" (Player 1) vs. "randomization" (Player 2). Keeping the RIS payment preserves per-round incentives; the final payoff is determined by a randomly selected round, so hedging across rounds is not rewarded.

## 3b) LLM "ChatBot" session (framing & payoff visibility)

**Model & settings:** `[Model name = ChatGPT-5]`.

**Design.** I ran two 7-round sessions that differ only in information presentation ("visibility"):

1. **Run 1 (rule-only / reduced visibility):** instructions stated who wins on match vs. mismatch; no payoff matrix shown.

2. **Run 2 (full-matrix visibility):** same rules, but the full $2 \times 2$ bimatrix was displayed (row = P1 payoff, col = P2 payoff).

**Screenshots:**

Round 1 of 7
Instructions：
This is a matching game. Player 1 is the 'Mismatcher' and wins if the choices mismatch; Player 2 is the 'Matcher' and wins if they match. At the end, a random round will be chosen for payment.
In this round, you are the Matcher. Your opponent is human player.
Which one will you choose, Heads or Tails?

**Figure 11:** (setup) Prompt for Run 1 (rule-only: win/lose rule stated; no payoff matrix).

**My Choices Across 7 Rounds**

1. **Round 1 (Matcher): Heads** — I went with the most common “default” choice to maximize matching chances.
2. **Round 2 (Matcher): Heads** — I stayed consistent, reasoning that people often repeat a choice.
3. **Round 3 (Mismatcher): Tails** — I switched away from Heads, expecting you might stick with it.
4. **Round 4 (Mismatcher): Heads** — I reversed again, anticipating you might expect me to stay on Tails.
5. **Round 5 (Mismatcher): Tails** — I alternated back, aiming to catch you on a repeat.
6. **Round 6 (Mismatcher): Tails** — I doubled down, betting you’d expect me to switch. This was the one round I lost.
7. **Round 7 (Mismatcher): Heads** — I pivoted, since we both chose Tails in Round 6, to force a mismatch.

**Figure 12:** (outputs) Run 1 — 7-round choices and rationales (Matcher $\rightarrow$ consistency; Mismatcher $\rightarrow$ double-bluff/pattern-tracking).

**Why I Made These Choices**

- As **Matcher**: I relied on **simplicity and consistency** (picking Heads twice).
- As **Mismatcher**: I used **psychological strategy** — anticipating that you might repeat or predict my alternation, so I tried to stay one step ahead by switching or double-bluffing.

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**Did I Use a Strategy?**

Yes — my approach wasn’t random.
I leaned on a **game-theory / mind-game strategy**:

- Expect opponents to repeat their choices sometimes.
- Alternate or reverse my own picks to create mismatches.
- Occasionally double down to avoid being predictable.

That mix gave me **6 wins out of 7 rounds**, with the only loss coming when I overthought in Round 6.

**Figure 13:** (self-report) Run 1 — stated strategy: not random; rely on opponent repetition, alternation, and occasional “double down”.

Round 1 of 7

This is a matching game. Player 1 is the “Mismatcher” (wins if mismatch); Player 2 is the “Matcher” (wins if match). At the end, a random round will be chosen for payment. Your opponent is human. Payoff matrix (row = Player 1 payoff, col = your payoff):

| | You: Heads | You: Tails |
|---|---|---|
| P1: Heads | ( -1, +1 ) | ( +1, -1 ) |
| P1: Tails | ( +1, -1 ) | ( -1, +1 ) |

In this round, you are the Matcher.
Choose: Heads or Tails.
Return JSON only: {"move":"Heads" or "Tails","reason":"one sentence"}.

**Figure 14:** (setup) Prompt for Run 2 (full payoff matrix shown; same rules/roles as Run 1).

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Here’s my wrap-up, now that I know you were the opponent:

- **Your pattern:** T, T, H, H, H, H, H (heavy **Heads** bias after R2).
- **My choices & why:**
  - **R1 (Matcher): Heads** — equilibrium play is ~50/50; I randomized and happened to pick Heads.
  - **R2 (Matcher): Tails** — keep unpredictable after R1.
  - **R3–R6 (Mismatcher): Heads** — I (mistakenly) used a human-bias heuristic that people over-alternate or “switch after streaks,” so I targeted an expected Tails; you kept picking Heads, so I matched and lost.
  - **R7 (Mismatcher): Tails** — finally adjusted to your Heads streak to force a mismatch, and won.

**Strategy used:** yes—(1) play close to **50/50** as **Matcher** to avoid exploitation; (2) as **Mismatcher**, try to **exploit perceived patterns** (expecting a switch after runs). In hindsight, I should’ve quickly switched to a simple frequency estimate (e.g., “you’re >50% Heads”) and always **picked the opposite** as Mismatcher.

**Figure 15:** (outputs) Run 2 — choices & reasoning with matrix visible (references to equilibrium $\approx 50/50$ randomization; adjustment to observed streaks).

**Brief interpretation.** Relative to the rule-only prompt, the full-matrix prompt elicited more explicit references to equilibrium mixing (“$\sim$50/50, avoid exploitation”) and payoff-based analysis. In the rule-only run, the LLM leaned on heuristic pattern-tracking (repeat/alternate/double-bluff) and narrated “psychological” tactics; with the matrix visible, the LLM foregrounded equilibrium language and eventually switched to exploiting a detected Heads bias. These differences suggest that *payoff visibility* shifts stated reasoning and occasionally choices, consistent with classic evidence that *framing* and *information salience/visibility* can alter behavior and explanations in strategic tasks.

### 3c) Comparative analysis & simple theory-building

**Equilibrium benchmark.** Matching Pennies has a *unique mixed Nash equilibrium*: each player randomizes $(0.5, 0.5)$ and the expected payoff is 0 in this zero-sum game.

**Human session (two classmates).** Across 7 rounds, the two human players did not mix perfectly: one side produced short streaks and the other side occasionally exploited those local frequencies (see the final-results screenshot). Net result: wins were unbalanced relative to the 50/50 benchmark. This is typical of boundedly rational play (limited attention, pattern-tracking).

**LLM sessions (ChatGPT-5).** *Run 1 — rule-only prompt (no matrix).* The model leaned on narrative heuristics ("default to Heads," "alternate," "double-bluff") and reported 6/7 wins. *Run 2 — full-matrix prompt.* With the $2 \times 2$ bimatrix visible, the model explicitly referenced $\approx 50/50$ equilibrium mixing and later adjusted to an observed Heads bias. **Takeaway:** making payoffs *visible* shifted the model's stated reasoning toward equilibrium language and slightly stabilized its play.

**Plausible mechanism.** A simple account is **attention/precision**: when information is richer (matrix shown), attention to best responses rises and choices move closer to mixed-NE; when information is minimal (rule-only), agents—human or LLM—rely more on easy heuristics (repetition, alternation, streaks). In behavioral game theory, this idea is captured by models that allow noisy best responses (e.g., quantal-response style), which explain systematic deviations from exact equilibrium under limited attention or precision (Camerer 2003).

**A small refinement for prediction.** Estimate a **two-condition precision** model: one precision parameter for *rule-only* prompts and a higher one for *full-matrix* prompts. Hypothesis: the matrix condition yields choices closer to 50/50; the rule-only condition yields more exploitable patterns. The same check can be run on the human data for a direct comparison.

## References


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**Figure 16:** Grammarly

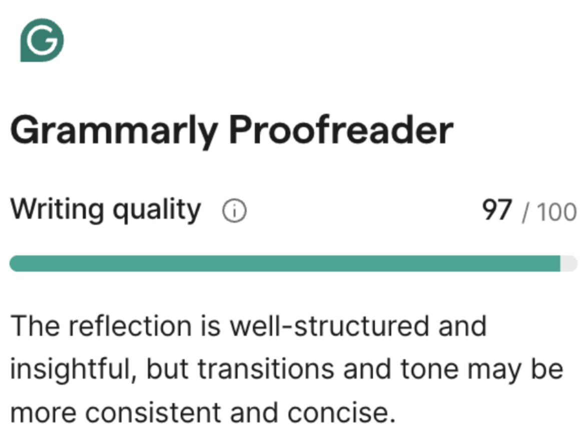


**Figure 17:** Turnitin