RANDOMIZATION IN ZERO-SUM PLAY

A CASE STUDY OF MATCHING PENNIES

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SESSION#1 THE FUTURE OF INTERDISCIPLINARY GAME THEORY FOUNDATIONS

INTRODUCTION

- **Bridging Theory & Practice:** Connects game theory with real human and AI behavior in a canonical zero-sum game.
- **The Visibility Nudge:** Introduces a payoff-matrix toward equilibrium play.
- **Broad Relevance:** Offers insights for AI safety, behavioral science, and experiential education.

MOST INSPIRING NOBEL PRIZE WINNER

- **John F. Nash Jr.**, Nobel Prize in Economic Sciences, 1994
- For formalizing the Nash equilibrium, the backbone of non-cooperative game theory, and the theoretical anchor for mixed strategies in Matching Pennies.

EQUILIBRIUM FOUNDATIONS: THEORY, WELFARE, AND INTERPRETATION

- Nash Equilibrium Derivation
  - No pure-strategy Nash Equilibrium exists.
  - Unique mixed-strategy NE: both players randomize with p(H) = q(H) = 0.5.
  - Expected payoffs sum to zero, defining a zero-sum game.
- Welfare & Equity Analysis
  - Utilitarian welfare is always zero; ex ante Pareto improvements are impossible.
  - The symmetric equilibrium ensures ex ante equity, granting both players equal expected payoffs (zero).

Player1\Player2	Heads	Tails
Heads	(+1, -1)	(-1, +1)
Tails	(-1, +1)	(+1, -1)

Figure 1: Matching Pennis Payoff Metric

COMPUTATIONAL VERIFICATION: TOOLCHAIN CONSENSUS AND SPNE

- Normal-Form Solution (Nashpy & QuantEcon)
  - Brute-force search confirms no pure-strategy equilibrium.
  - Solver output verifies the unique mixed-strategy NE: [0.5, 0.5] for both players.
- Extensive-Form Solution (Game Theory Explorer)
  - Simultaneity is modeled via information sets, eliminating proper subgames.
  - Confirms Subgame Perfect Nash Equilibrium (SPNE) coincides with NE, imposing no additional restrictions.

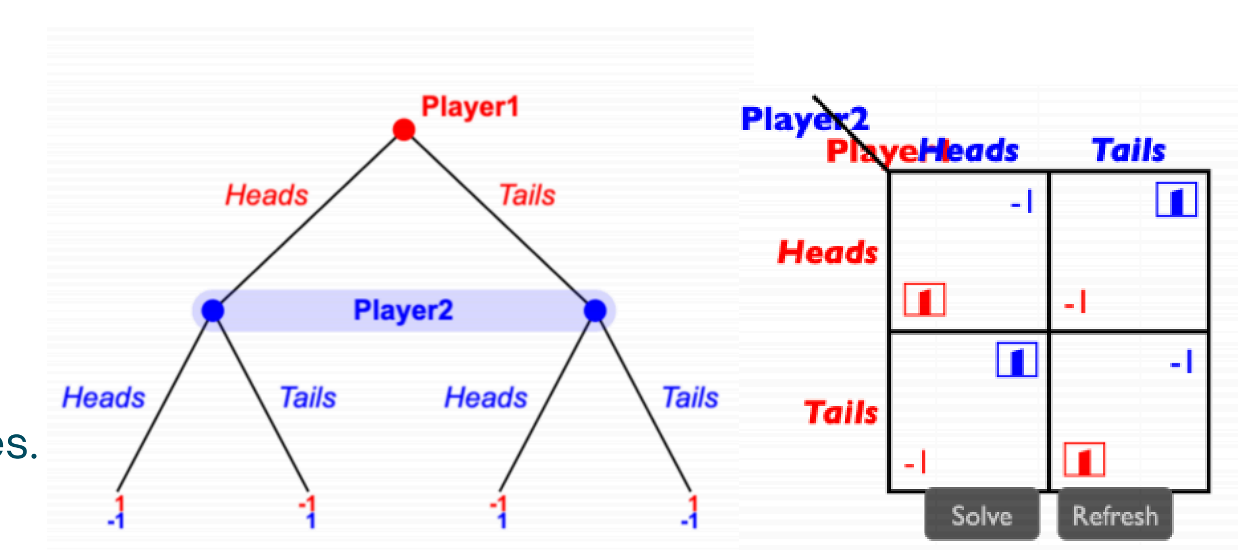

Figure 2: Matching Pennis Extensive Form in GTE

BEHAVIORAL AND AI EXPERIMENTS: HUMAN AND LLM NUDGES

- Human Subject Sessions
  - **Observation:** Play deviated from perfect 50/50 mixing; short streaks were observed and exploited.
  - **Insight:** Highlights the behavioral difficulty of true randomization and the exploitability of small biases.
- LLM Agent Sessions with Visibility Manipulation
  - **Run 1 (Rule-Only):** LLM played with only rule description. Result: Mean p(H) = 0.43.
  - **Run 2 (Full-Matrix):** LLM played with full payoff matrix visible. Result: Mean p(H) = 0.51 (Δ = +0.08).
  - **Key Finding:** Payoff visibility nudged LLM's play towards the theoretical equilibrium and shifted its stated reasoning to use more equilibrium-like language.
  - **Implication:** Information presentation is a critical lever in strategic environments for both humans and AI.

SDG CONTRIBUTION

- **SDG 4: Quality Education:** Transforms abstract game theory into reproducible, open-access learning artifacts.
- **SDG 9: Industry, Innovation & Infrastructure:** Demonstrates lightweight, testable mechanism design prototypes.

References

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