HW 6

ECE 454

Group 12

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20) a

$$T(s) = \frac{16}{s^2+3s+16}$$

$$\mathcal{L}[\dot{c}(t)] = s\,C(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$\omega_n^2 = 16$ $2\zeta\omega_n = 3$

$\omega_n = 4$ $\zeta = 3/8$

$$T_s = \frac{4}{\omega_n \zeta} \qquad T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$T_s = 2.667\,s$ $T_p = 0.8472\,s$

$$\%OS = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100\%$$

$\%OS = 0.280596$

$\omega_n T_R = c(0.90) - c(0.10)$

$T_R = \left(c(0.90) - c(0.10)\right) / \omega_n$

$T_R = 0.300765\,s$

- See attached MATLAB code

23) b $\%OS = 10\%$ | $T_p = 5$ seconds

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2+\ln^2(\%OS/100)}}$$

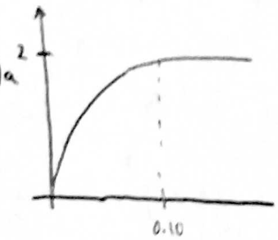
$$\zeta = \frac{-\ln(0.10)}{\sqrt{\pi^2+\ln^2(0.10)}} = 0.59115$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 5 \text{ seconds}$$

$$5 = \frac{\pi}{\omega_n(0.80655)}$$

$\omega_n = 0.7790$

32)a



- Time constant amplitude value

$\sim (0.63)(2) = 1.26$

time value $(1.26) \sim 0.03$ seconds

$$C(s) = \frac{K}{s(s+a)} = \frac{K/a}{s} - \frac{K/a}{(s+a)}$$

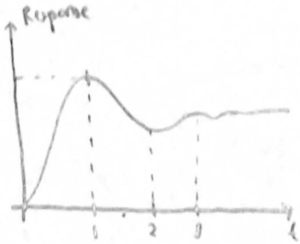
$$a = 1/\tau = 1/0.03 = 33.33$$

$$K/a = 2$$

$$K = 66.666$$

$$C(s) = \frac{6666}{(s+33.33)}$$

32)b



- Underdamped

$$\%OS = \frac{c(max) - c(ss)}{c(ss)} \times 100\%$$

$$\%OS \cong \left(\frac{13-11}{11}\right) \times 100\%$$

$$\%OS \cong 18.18\%$$

$$T_p = 1$$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

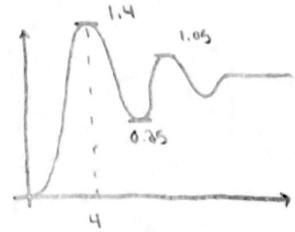
$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\frac{\pi}{T_p} = \omega_n\sqrt{1-\zeta^2}$$

$$\omega_n = \frac{\pi}{T_p\sqrt{1-\zeta^2}}$$

- See attached MATLAB script

32)c



- Underdamped

$$\%OS = \frac{c(max) - c(ss)}{c(ss)} \times 100\%$$

$$\%OS \cong \frac{1.4 - 1}{1} \times 100\%$$

$$\%OS \cong 40\%$$

$$T_p \cong 4 \text{ seconds}$$

- See attached MATLAB script

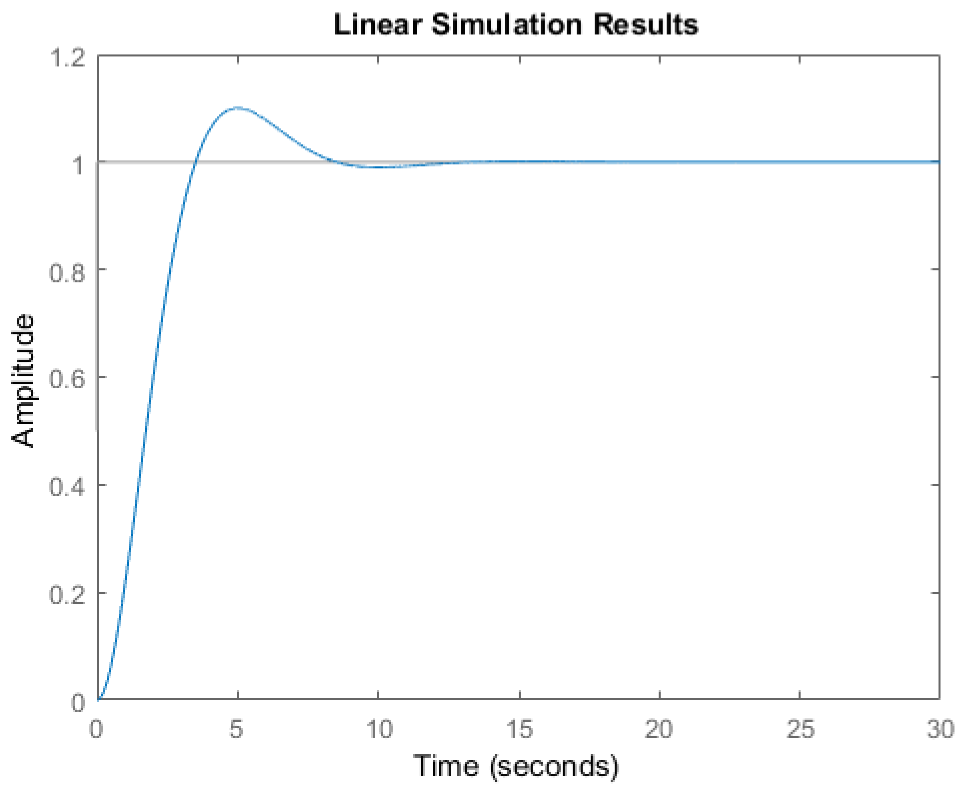
```
clc; clear; close all;
%Problem 20
%Part a
%Rise Time Calculation Approximation for Second Order System

fprintf('-----------------------------------------------------------------
\n');
fprintf('Problem 20\n');
fprintf('-----------------------------------------------------------------
\n');
fprintf('Part a\n');
wn = 4;
dR = 3/8;

const1 = sqrt(1-dR^2);
Tp = pi/(wn*const1);
fprintf('Peak Time | %g seconds\n',Tp);

OS = exp(-(dR*pi/const1))*100;
fprintf('Overshoot | %g Percent\n',OS);

Ts = 4/(dR*wn);
fprintf('Settling Time | %g seconds\n',Ts);

phi = atan(dR/const1);
timeSR = @(t)(1 - (exp(-(dR*wn*t))*(cos(wn*const1*t) + ...
                        (dR/const1)*sin(wn*const1*t))));

Tr = (timeSR(0.90) - timeSR(0.10))/wn;
fprintf('Rise Time | %g seconds\n',Tr);

%Problem 23
%Part b
fprintf('-----------------------------------------------------------------
\n');
fprintf('Problem 23\n');
fprintf('-----------------------------------------------------------------
\n');
fprintf('Part b\n');
t = linspace(0,30,10000);
OS_23B = 1/10;
Tp_23B = 5;
zeta23b = (-log(OS_23B) / sqrt((pi^2) + log(OS_23B)^2));
wn23B = (pi/(Tp_23B*sqrt(1-zeta23b^2)));
sys = tf([(wn23B^2)],[1,(2*zeta23b*wn23B), wn23B^2]);
sys %#ok<NOPTS>
inputResponse = heaviside(t);%u(t)
figure()
lsim(sys,inputResponse,t);

[z1,p1] = zpkdata(sys);
z1 = cell2mat(z1);
```

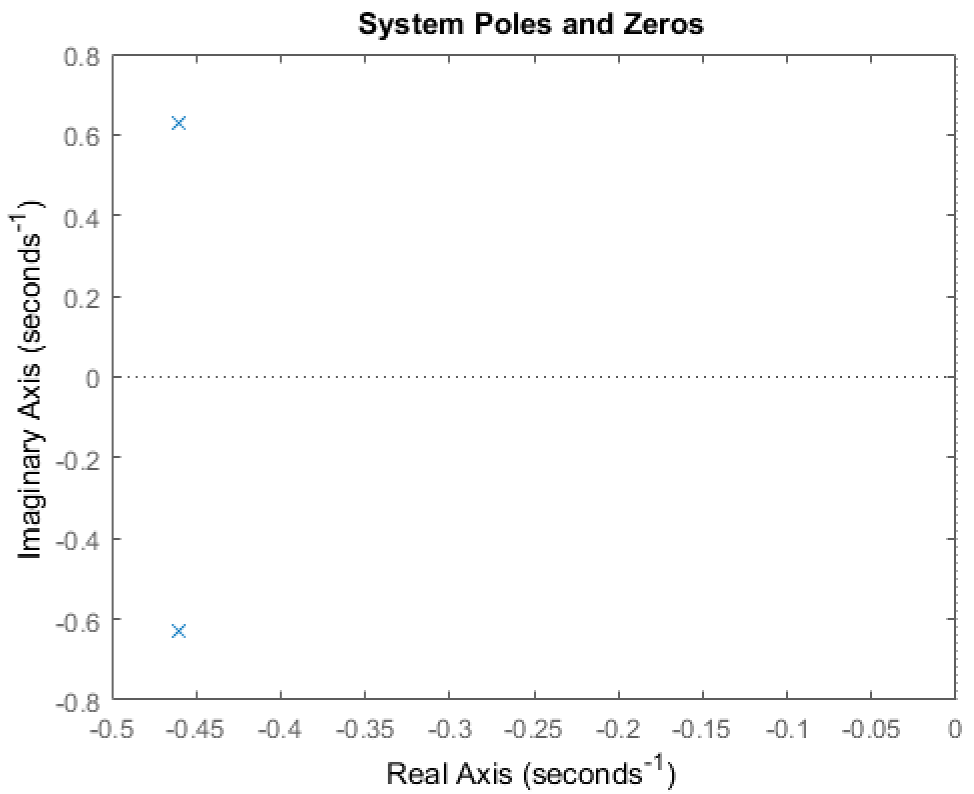
```
p1 = cell2mat(p1);
fprintf('Zeros | %g\n',z1);
fprintf('Poles | %g\n',p1);
figure()
pzmap(sys);
title('System Poles and Zeros')

%Problem 32
%Part a
fprintf('------------------------------------------------------------------
\n');
fprintf('Problem 32\n');
fprintf('------------------------------------------------------------------
\n');
fprintf('Part a\n');
tA = linspace(0,0.25,10000);
sysA = tf([(200/3)],[1,(100/3)]);
fprintf('Transfer Function Model\n');
sysA %#ok<NOPTS>
inputResponseA = heaviside(tA);%u(t)
figure()
lsim(sysA,inputResponseA,tA);

%Part b
fprintf('------------------------------------------------------------------
\n');
fprintf('Part b\n');
fprintf('------------------------------------------------------------------
\n');
tB = linspace(0,5,10000);
OS_B = 2/11;
Tp_B = 1;
zetab = (-log(OS_B) / sqrt((pi^2) + log(OS_B)^2));
wnB = (pi/(Tp_B*sqrt(1-zetab^2)));
sysB = tf([(wnB^2)],[1,(2*zetab*wnB), wnB^2]);
sysB %#ok<NOPTS>
inputResponseB = 11*heaviside(tB);%u(t)
figure()
lsim(sysB,inputResponseB,tB);

%Part c
fprintf('------------------------------------------------------------------
\n');
fprintf('Part c\n');
fprintf('------------------------------------------------------------------
\n');
tC = linspace(0,25,10000);
OS_C = 2/5;
Tp_C = 4;
zetac = (-log(OS_C) / sqrt((pi^2) + log(OS_C)^2));
wnC = (pi/(Tp_C*sqrt(1-zetac^2)));
sysC = tf([(wnC^2)],[1,(2*zetac*wnC), wnC^2]);
sysC %#ok<NOPTS>
inputResponseC = heaviside(tC);%u(t)
```

```
figure()
lsim(sysC,inputResponseC,tC);
```

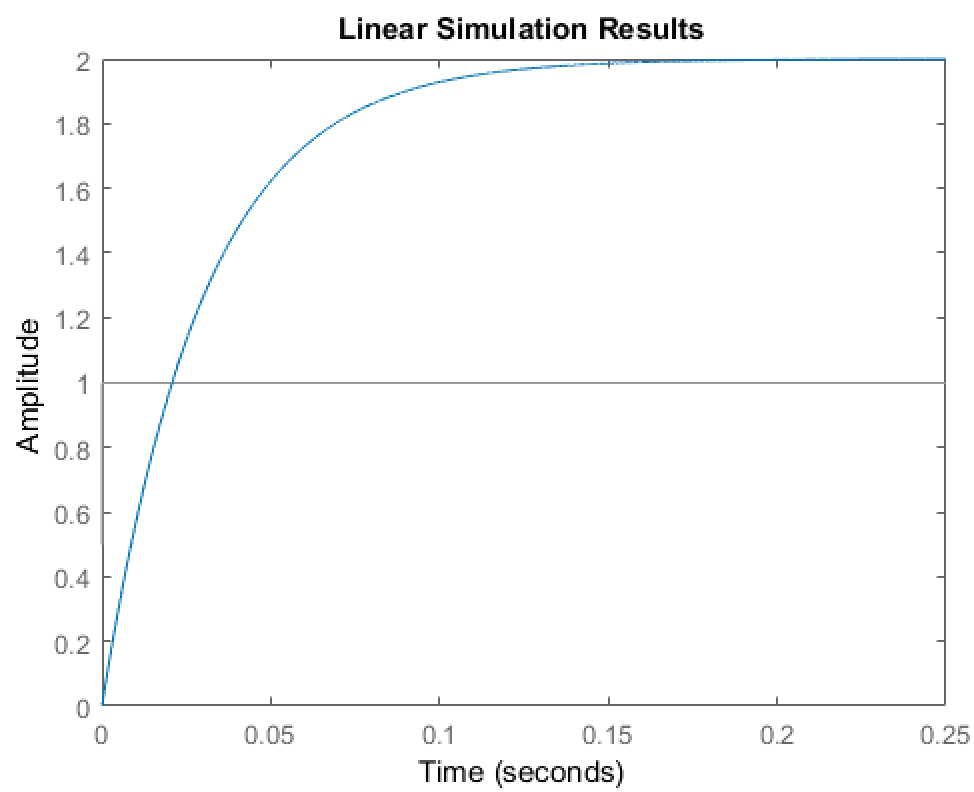
```
-----------------------------------------------------------------
Problem 20
-----------------------------------------------------------------
Part a
Peak Time | 0.847225 seconds
Overshoot | 28.0597 Percent
Settling Time | 2.66667 seconds
Rise Time | 0.300765 seconds
-----------------------------------------------------------------
Problem 23
-----------------------------------------------------------------
Part b

sys =

           0.6069
  ---------------------
  s^2 + 0.921 s + 0.6069

Continuous-time transfer function.

Zeros |
Poles | -0.460517
Poles | -0.460517
-----------------------------------------------------------------
Problem 32
-----------------------------------------------------------------
Part a
Transfer Function Model

sysA =

    66.67
  ---------
  s + 33.33

Continuous-time transfer function.

-----------------------------------------------------------------
Part b
-----------------------------------------------------------------

sysB =

          12.78
  --------------------
  s^2 + 3.409 s + 12.78

Continuous-time transfer function.

-----------------------------------------------------------------
```

```
Part c
--------------------------------------------------------------------

sysC =

            0.6693
  -----------------------
  s^2 + 0.4581 s + 0.6693

Continuous-time transfer function.
```

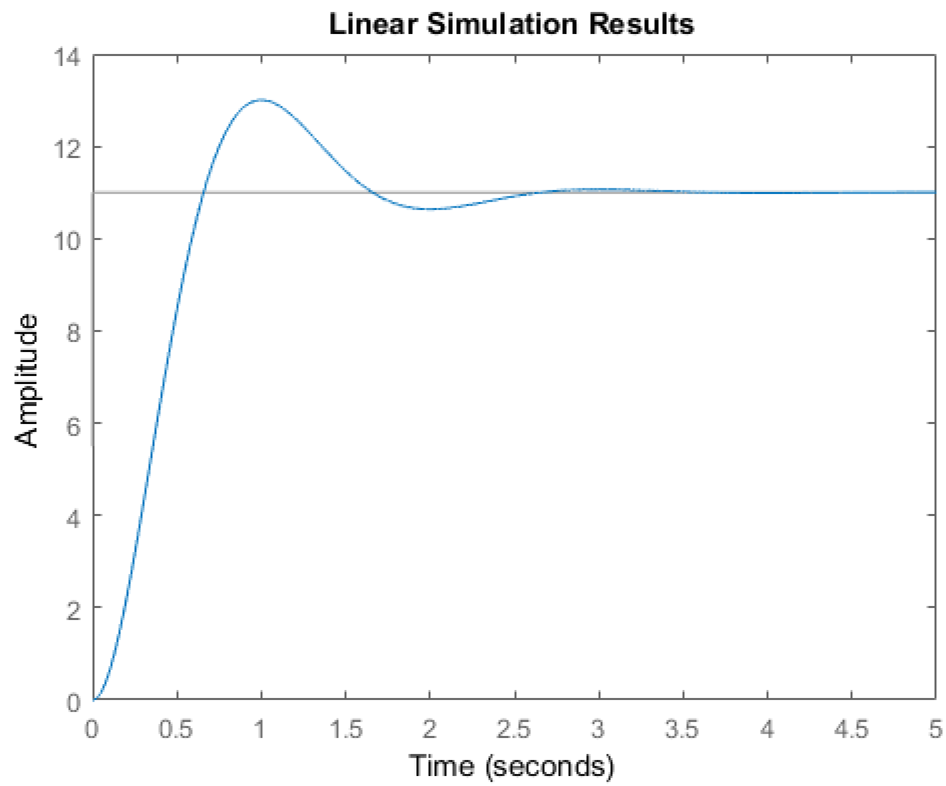


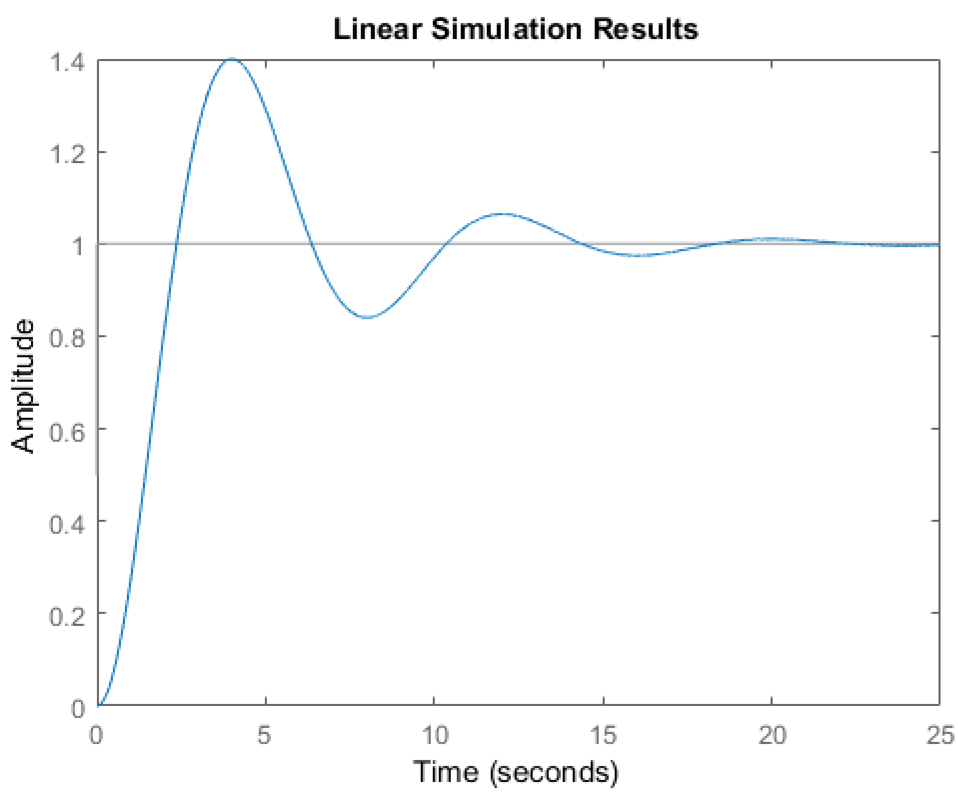


System Poles and Zeros
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
Imaginary Axis (seconds^{-1})
-0.5
-0.45
-0.4
-0.35
-0.3
-0.25
-0.2
-0.15
-0.1
-0.05
0
Real Axis (seconds^{-1})



Linear Simulation Results
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
0
0.05
0.1
0.15
0.2
0.25
Time (seconds)


Linear Simulation Results
Amplitude
Time (seconds)
0
2
4
6
8
10
12
14
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Linear Simulation Results
Amplitude
Time (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
5
10
15
20
25

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