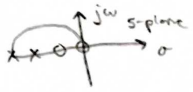
HW 9 (Chapter 8, 1, 2, 19, 21)

Jannathan Schaff

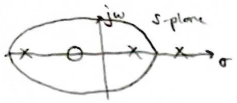
1) a



Not Root Locus

- System is asymmetric about real axis
- Real axis branching not shown

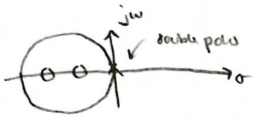
1) b



Not Root Locus

- Real axis branch is to the left of 4 poles / zeros (even)

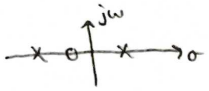
1) c



Not Root Locus

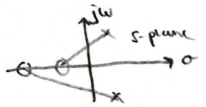
- real axis branch is to left of 4 poles / zeros (even)

1) d



Root Locus

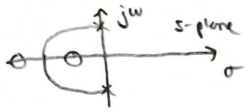
1) e



Not Root Locus

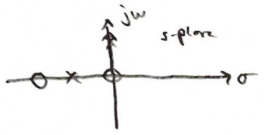
- System is asymmetric about real axis
- No real axis branch between zeros

1) f



Root Locus

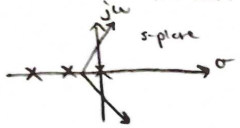
1) g



Not Root Locus

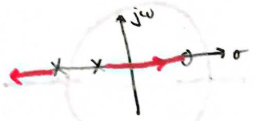
- asymmetric about real axis (no conjugate pair)
- real axis branch is to the left of two poles / zeros (even)

1) h

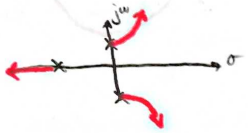


Root Locus

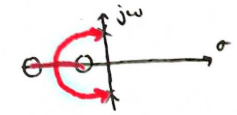
2) a



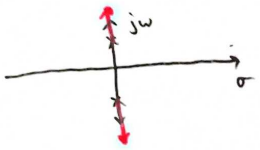
2) b



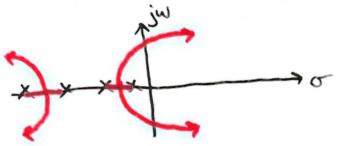
2) c



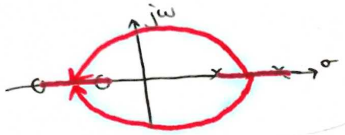
2) d



2) e



2) f



19) $G(s) = \frac{K(s+1)}{s(s+1)(s+3)(s+5)} = \frac{K(s+2)}{(s^4+9s^3+23s^2+15s)}$

O.L poles | $s = 0, -1, -3, -5$, $n = 4$

O.L zeros | $s = -2$, $m = 1$

Breakaway - Breakin Pts

$$\sum_{i=1}^{m} \frac{1}{\sigma+z_i} = \sum_{i=1}^{n} \frac{1}{\sigma+p_i} = \frac{1}{\sigma+2} = \frac{1}{\sigma} + \frac{1}{\sigma+1} + \frac{1}{\sigma+3} + \frac{1}{\sigma+5}$$

$$\sigma(\sigma+1)(\sigma+3)(\sigma+5) = \sigma+2\left[(\sigma+1)(\sigma+3)(\sigma+5) + \sigma(\sigma+3)(\sigma+5) + \sigma(\sigma+1)(\sigma+3)\right]$$

$$3\sigma^4 + 26\sigma^3 + 77\sigma^2 + 92\sigma + 30 = 0$$

$\sigma_1 = -4.04$, $[\sigma_2 = -0.505]$

jω crossings

$$1 + L(s) = 0 \rightarrow 1 + k\left(\frac{s+2}{s(s+1)(s+3)(s+5)}\right) = 0$$

$$s(s+1)(s+3)(s+5) + k(s+2) = 0$$

$$\omega^4 - 9j\omega^3 - 23\omega^2 + (15+k)j\omega + 2k = 0$$

$$\left.\begin{aligned} \mathrm{Re}[\omega^4 - 23\omega^2 + 2k = 0] \\ \mathrm{Im}[-9j\omega^3 + (15+k)j\omega = 0] \end{aligned}\right\} \begin{aligned} \omega &= \pm 2.91 \\ K &= 61.69 \end{aligned}$$

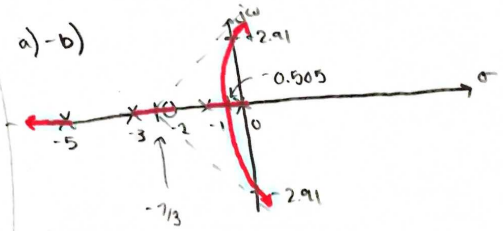
Asymptotes

$$\sigma = \frac{\Sigma(0,-1,-3,-5) - \Sigma(-2)}{4-1} = -7/3$$

$$\theta_k = \left(\frac{2k+1}{3}\right)180^\circ$$

$\theta_1 = 180^\circ$, $\theta_2 = 300^\circ$, $\theta_3 = 420^\circ = 60^\circ$

a) - b)



c) System is marginally stable between $0 \leq K \leq 61.69$

d) $$\left.1 + \frac{K(s+2)}{s(s+1)(s+3)(s+5)}\right|_{s=-0.50} = 0$$

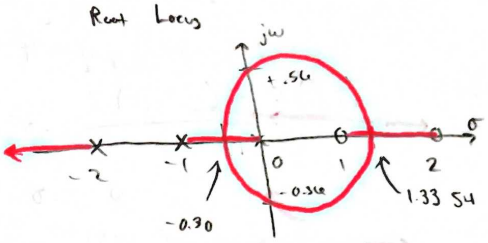
$$1 + \frac{K(1.5)}{(-0.50)(0.50)(2.5)(4.5)} = 0$$

$$\frac{-8K}{15} = -1, \quad K = \frac{15}{8} = 1.875$$

21) $G(s) = \dfrac{K(s-1)(s-2)}{s(s+1)(s+2)}$

O.L Poles | $s = 0, -1, -2$ $\quad n = 3$

O.L zeros | $s = 1, 2$ $\quad m = 2$



a)

Breakaway - Breakin Points

$$\frac{1}{\sigma-1} + \frac{1}{\sigma-2} = \frac{1}{\sigma} + \frac{1}{\sigma+1} + \frac{1}{\sigma+2}$$

$$\frac{2\sigma-3}{\sigma^2-3\sigma+2} = \frac{\sigma^2+3\sigma+2+\sigma^2+2\sigma+\sigma^2+\sigma}{\sigma^3+2\sigma^2+\sigma^2+2\sigma}$$

$$\frac{2\sigma-3}{\sigma^2-3\sigma+2} = \frac{3\sigma^2+6\sigma+2}{\sigma^3+3\sigma^2+2\sigma}$$

$$\sigma^4 + 6\sigma^3 - 5\sigma^2 + 4 = 0$$

$[\sigma_1 = -0.30]$ Break-away

$\sigma_2 = 6.47$

$\sigma_3 = -1.49$

$[\sigma_4 = 1.3354]$ Break-in

b)

Jω Crossings

$$1 + L(s) = 0$$

$$s(s+1)(s+2) + K(s-1)(s-2) = 0$$

$$s^3 + (3+K)s^2 + (2-3K)s + 2K = 0$$

$$(j\omega)^3 + (3+K)(j\omega)^2 + (2-3K)(j\omega) + 2K = 0$$

$$-j\omega^3 - (3+K)\omega^2 + j(2-3K)\omega + 2K = 0$$

$$\text{Re}\left[-(3+K)\omega^2 + 2K = 0\right] \qquad \text{Im}\left[-\omega^3 + (2-3K)\omega = 0\right]$$

$K = 3.56$, $\omega = \pm 3.56$

$[K = 0.56$, $\omega = \pm 0.56]$

c) Range of K to maintain stability

$0 \leq K \leq 0.56$

d) Determined using MATLAB

$\zeta = 0.50$

pole | $0.92 - 0.77i$

$[K = 2.09]$