统计学习：第十章

1 给定盒子和球组成的马尔可夫模型 $\lambda = (A, B, \pi)$，其中，

$$A = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}, B = \begin{bmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \\ 0.7 & 0.3 \end{bmatrix}, \pi = (0.2, 0.4, 0.4)^T$$

设 $T = 4, O = $（红，白，红，白），试用后向算法计算 $P(O|\lambda)$

答：$\beta_4 = (1\ 1\ 1)$，$\beta_3 = (0.46, 0.51, 0.43)$，$\beta_2 = (0.2461, 0.2312, 0.2577)$，$\beta_1 = (0.112462, 0.121737, 0.104881)$，$P(O|\lambda) = 0.04540746$

2 给定盒子和球组成的马尔可夫模型 $\lambda = (A, B, \pi)$，其中，

$$A = \begin{bmatrix} 0.5 & 0.1 & 0.4 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}, B = \begin{bmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \\ 0.7 & 0.3 \end{bmatrix}, \pi = (0.2, 0.3, 0.5)^T$$

设 $T = 8, O = $（红，白，红，红，白，红，白，白），用前向后向概率计算 $P(i_4 = q_3|O, \lambda)$

答：$\beta_8 = (1\ 1\ 1)$，$\beta_7 = (0.43, 0.51, 0.4)$，$\beta_6 = (0.1861, 0.2415, 0.1762)$，$\beta_5 = (0.105521, 0.100883, 0.111934)$，$\beta_4 = (0.04586531, 0.05280909, 0.04280618)$

$\alpha_1 = (0.1, 0.12, 0.35)$，$\alpha_2 = (0.078, 0.084, 0.0822)$，$\alpha_3 = (0.04032, 0.026496, 0.068124)$，$\alpha_4 = (0.0208668, 0.01236192, 0.04361112)$，

$\gamma_4(3) = 0.53695$

3 在习题 10.1 中，试用维特比算法求最优路径 $I^* = (i_1^*, i_2^*, i_3*, i_4*)$

答：$\delta_1 = (0.1\ 0.16\ 0.28), \Psi_1 = 0$，

$\delta_2 = (0.028\ 0.054\ 0.028), \Psi_2 = 3$，

$\delta_3 = (0.00756\ 0.01008\ 0.0147), \Psi_3 = (2, 2, 3)$，

$\delta_4 = (0.00189, 0.003024, 0.002205), \Psi_4 = (1, 2, 3)$

故最优路径为 $(3, 2, 2, 2)$

4 试用前向概率和后向概率推导答:

$$P(O|\lambda) = P(o_1, \ldots, o_T|\lambda)$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} P(o_1, \ldots, o_T, i_t = q_i, i_{t+1} = q_j|\lambda)$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} P(o_t+1, \ldots, o_T, i_{t+1} = q_j|o_1, \ldots, o_T, i_t = q_i, \lambda) P(o_1, \ldots, o_t, i_t = q_i|\lambda)$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} P(o_t+1, \ldots, o_T, i_{t+1} = q_j|i_t = q_i, \lambda)\alpha_t(i)$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} P(o_t+1, \ldots, o_T|, i_{t+1=q_j}, \lambda) P(i_{t+1} = q_j|i_t = q_i, \lambda)\alpha_t(i)$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} P(o_t+2, \ldots, o_T|, i_{t+1=q_j}, \lambda) P(o_{t+1}|i_{t+1} = q_j, \lambda) a_{ij}\alpha_t(i)$$

$$= \sum_{i=1}^{N}\sum_{j=1}^{N} \beta_{t+1}(j) b_j(o_{t+1}) a_{ij}\alpha_t(i)$$

4 比较维特比算法中变量 δ 的计算和前向算法中变量 α 的计算的主要区别。

答：维特比算法寻找最优路径，所以求得是确定路径的概率，在递推关系仅取特定路径的概率，计算的是最优路径的概率；前向算法计算的是概率，所以遍历了所有路径，求了概率之和，计算的是所有可能路径的概率和