

Deliverable4

Introduction

- Interest from a business perspective: helps bike rental businesses meet demands
- City planning perspective: helps cities to adapt to the change of number of bikers to enforce better traffic laws
- A way to sense mobility in the city

Backgrounds

The data is a two-year historical log corresponding to years 2011 and 2012 from Capital Bikeshare system, Washington D.C. containing the following datas: weathersit: 1: Clear, Few clouds, Partly cloudy, 2: Mist and Cloudy, Mist and Broken clouds, Mist and Few clouds, Mist 3: Light Snow, Light Rain and Thunderstorm and Scattered clouds, Light Rain and Scattered clouds 4: Heavy Rain and Ice Pellets and Thunderstorm and Mist, Snow and Fog instant: record index

dteday: date

season: season (1:spring, 2:summer, 3:fall, 4:winter)

yr: year (0: 2011, 1:2012)

mnth: month (1 to 12)

holiday: weather day is holiday or not (extracted from <http://dchr.dc.gov/page/holiday-schedule>)

weekday: day of the week

workingday: if day is neither weekend nor holiday is 1, otherwise is 0.

temp: Normalized temperature in Celsius. The values are divided to 41 (max)

atemp: Normalized feeling temperature in Celsius. The values are divided to 50 (max)

hum: Normalized humidity. The values are divided to 100 (max)

windspeed: Normalized wind speed. The values are divided to 67 (max)

casual: count of casual users

registered: count of registered users

cnt: count of total rental bikes including both casual and registered

Our goal is to use data in 2011 to predict bike rental behaviour in 2012.

Preprocessing

```
bikedata <- read.csv("day.csv",header=T)
names(bikedata)
```

```
## [1] "instant"      "dteday"        "season"        "yr"            "mnth"
## [6] "holiday"      "weekday"       "workingday"    "weathersit"     "temp"
## [11] "atemp"        "hum"           "windspeed"    "casual"        "registered"
## [16] "cnt"
```

```
#Transform temp, atemp, windspeed, and humidity to actual values
```

```
bikedata <-
  bikedata %>% mutate(actual.temp = temp*41) %>%
  mutate(actual.atemp = atemp*50) %>%
  mutate(actual.windspeed = windspeed*67) %>%
  mutate(actual.hum = hum*100)
```

```
#Combining summer, fall, and spring, winter
```

```
bikedata <- bikedata %>% mutate(season.2 = if_else(season == 2|season==3|season==4,0,if_else(season ==1
```

```
#process factor data
```

```
bikedata$season <- factor(format(bikedata$season, format="%A"),
  levels = c("1", "2","3","4") ,
  labels = c("Spring","Summer","Fall","Winter"))
```

```
bikedata$spring <- factor(format(bikedata$season.2, format="%A"),
  levels = c("0","1") ,
  labels = c("Not Spring","Spring"))
```

```
bikedata$holiday <-factor(format(bikedata$holiday, format="%A"),
  levels = c("0", "1") ,
  labels = c("Not Holiday","Holiday"))
```

```
bikedata$weathersit <- factor(format(bikedata$weathersit, format="%A"),
  levels = c("1", "2","3","4") ,
  labels = c("Good:Clear/Sunny","Moderate:Cloudy/Mist","Bad: Rain/Snow/Fog","Worse
```

```
bikedata$workingday <- factor(format(bikedata$workingday, format = "%A"),
  levels = c("0", "1"),
  labels = c("Not WorkingDay", "WorkingDay"))
```

```
bikedata$yr <- factor(format(bikedata$yr, format="%A"),
  levels = c("0", "1") , labels = c("2011","2012"))
```

```
bikedata <- bikedata %>% mutate(weekend = if_else(weekday == 0|weekday==6,0,if_else(weekday ==1|weekday
```

```
bikedata$weekend <- factor(format(bikedata$weekend, format = "%A"),
  levels = c(0,1),
  labels = c("Weekend", "Weekday"))
```

```
bikedata$mnth <- as.factor(bikedata$mnth)
```

```
#Generate days from start date values
```

```
start = "2011-01-01"
bikedata$date_diff <- as.Date(as.character(bikedata$dteday), format="%Y-%m-%d")-
                        as.Date(start, format="%Y-%m-%d")
```

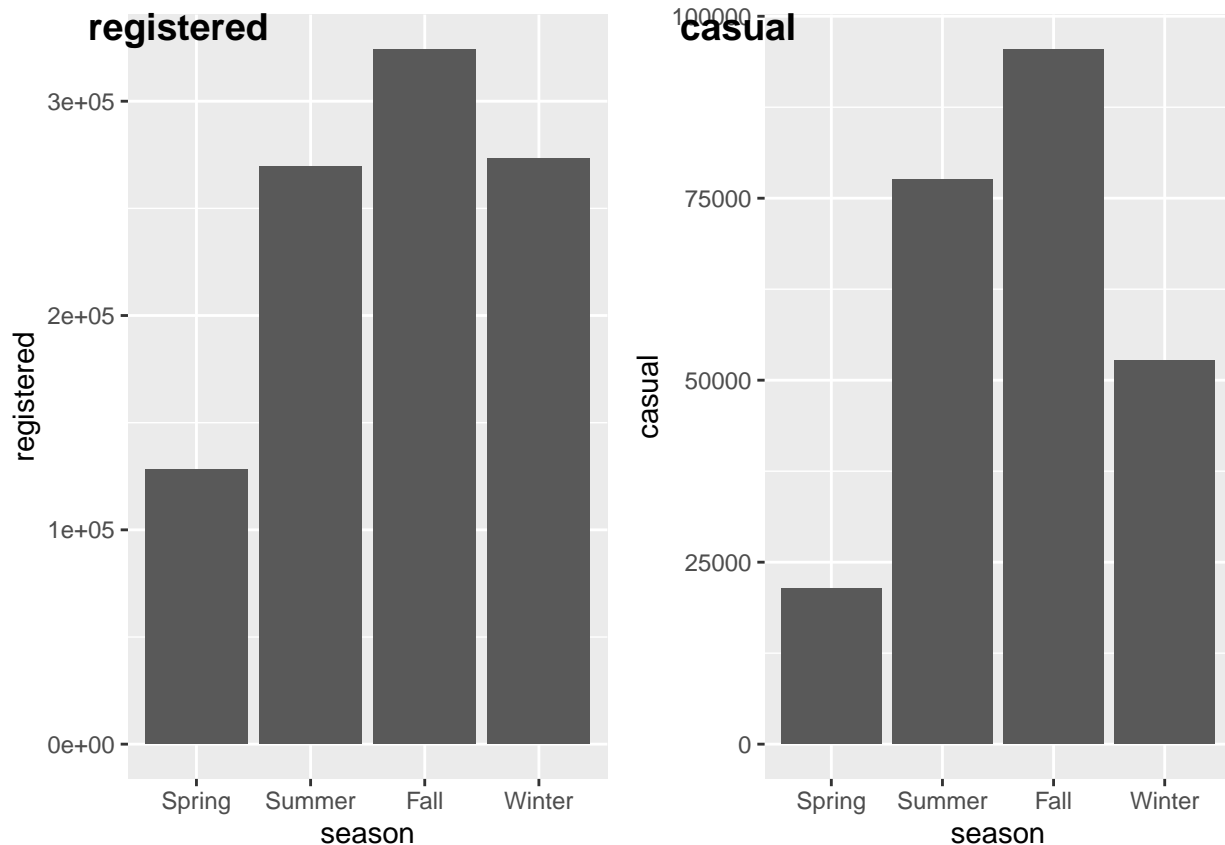
#Seperate training and validating datas base on year

```
training_d = bikedata %>% filter(yr == "2011")
set.seed(42)
#partitiontraining <- createDataPartition(y = train$cnt, p = 0.8, list = F)
#training_d <- train[partitiontraining, ]
#test_d <- train[-partitiontraining, ]
validate_d <- bikedata %>% filter(yr == "2012")
```

```
training.workingday = filter(training_d, workingday == "WorkingDay")
training.nworkingday = filter(training_d, workingday == "Not WorkingDay")
validate.workingday = filter(validate_d, workingday == "WorkingDay")
validate.nworkingday = filter(validate_d, workingday == "Not WorkingDay")
```

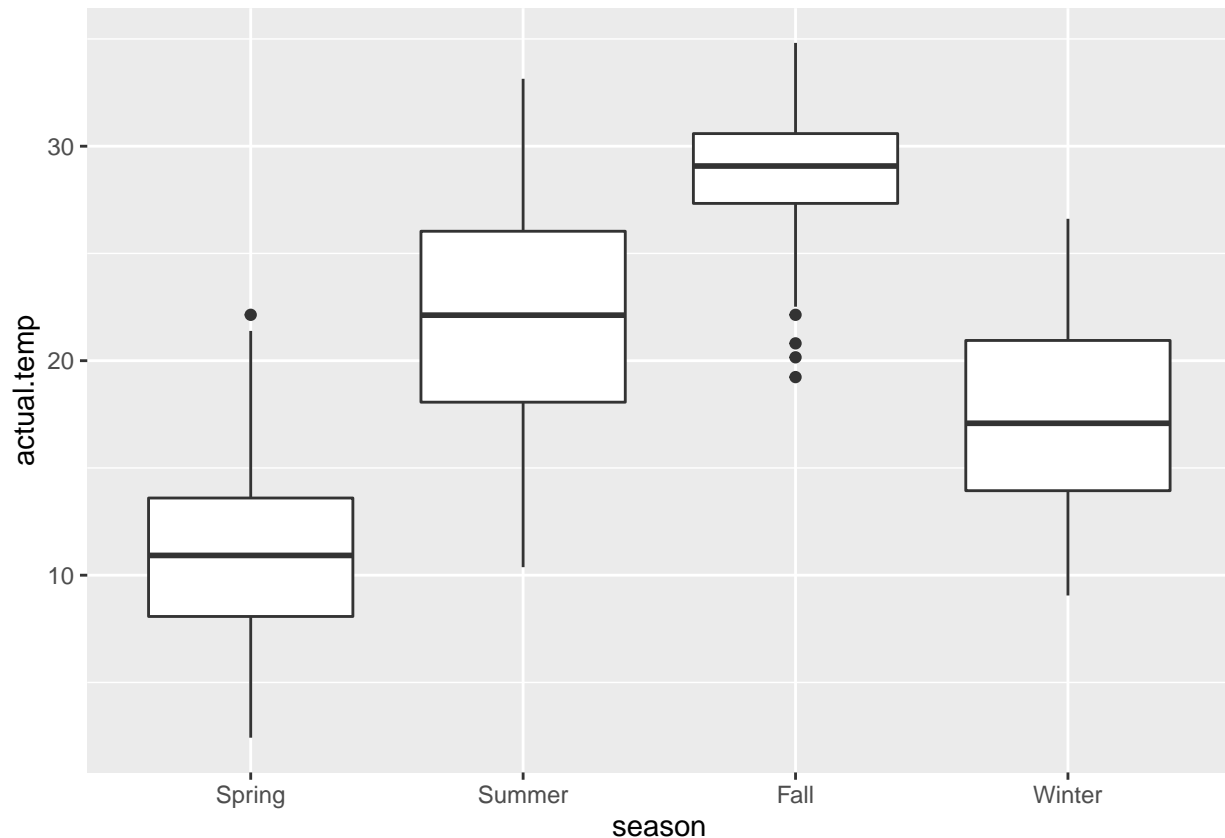
Season

```
plot1<- ggplot(training_d,aes(x=season,y=registered))+geom_col()
plot2<- ggplot(training_d,aes(x=season,y=casual ))+geom_col()
plot_grid(plot1, plot2, labels = c("registered", "casual"))
```



The graphs show that for both casual and registered bikers, there are the most rental counts during autumn season and the least during the spring season. However, for registered, there are about the same amount of count during summer and winter while for casual there are significantly less counts during winter than during summer. Therefore we think that we should fit different models for registered and casual.

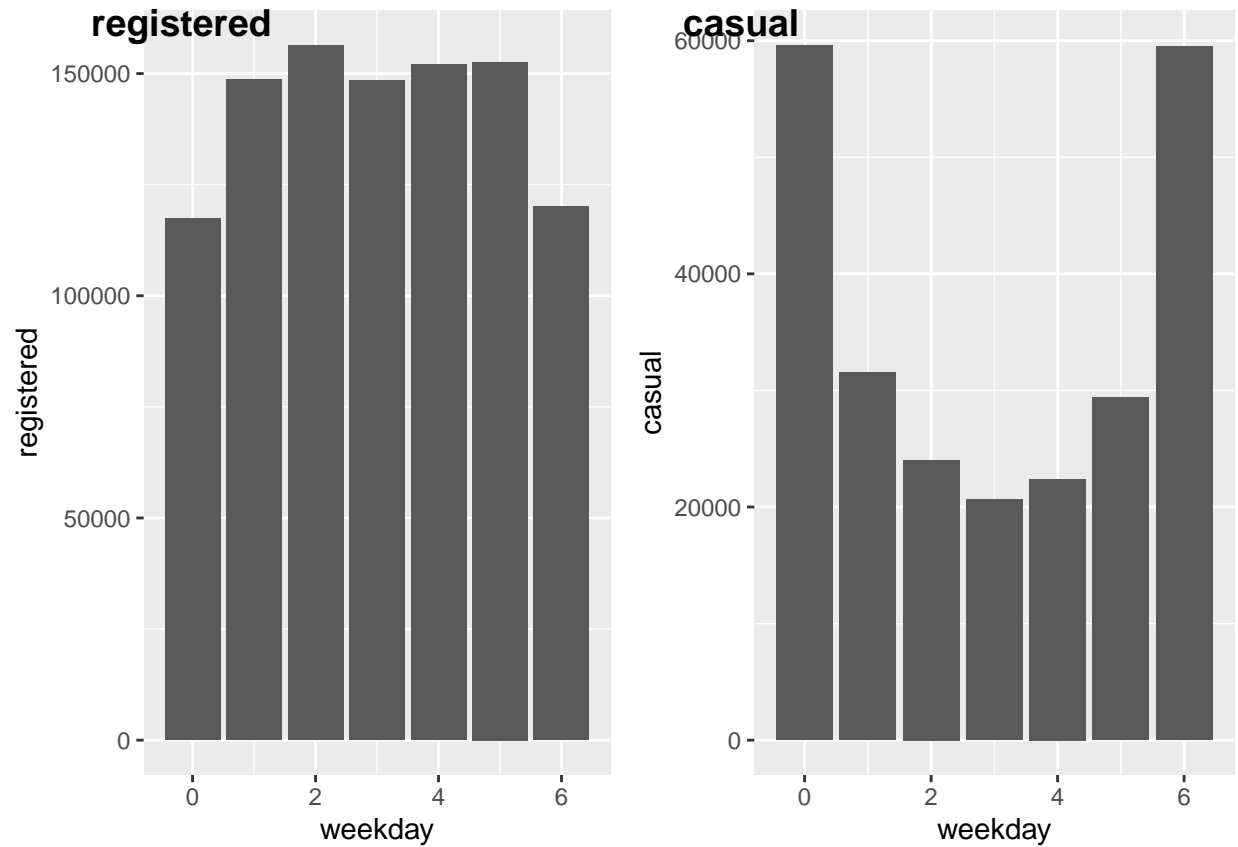
```
ggplot(training_d,aes(x=season,y=actual.temp))+geom_boxplot()
```



Temperature and seasons are strongly correlated. Spring has the lowest temperature while fall has the highest temperature.

Holiday, Weekday, Workingday

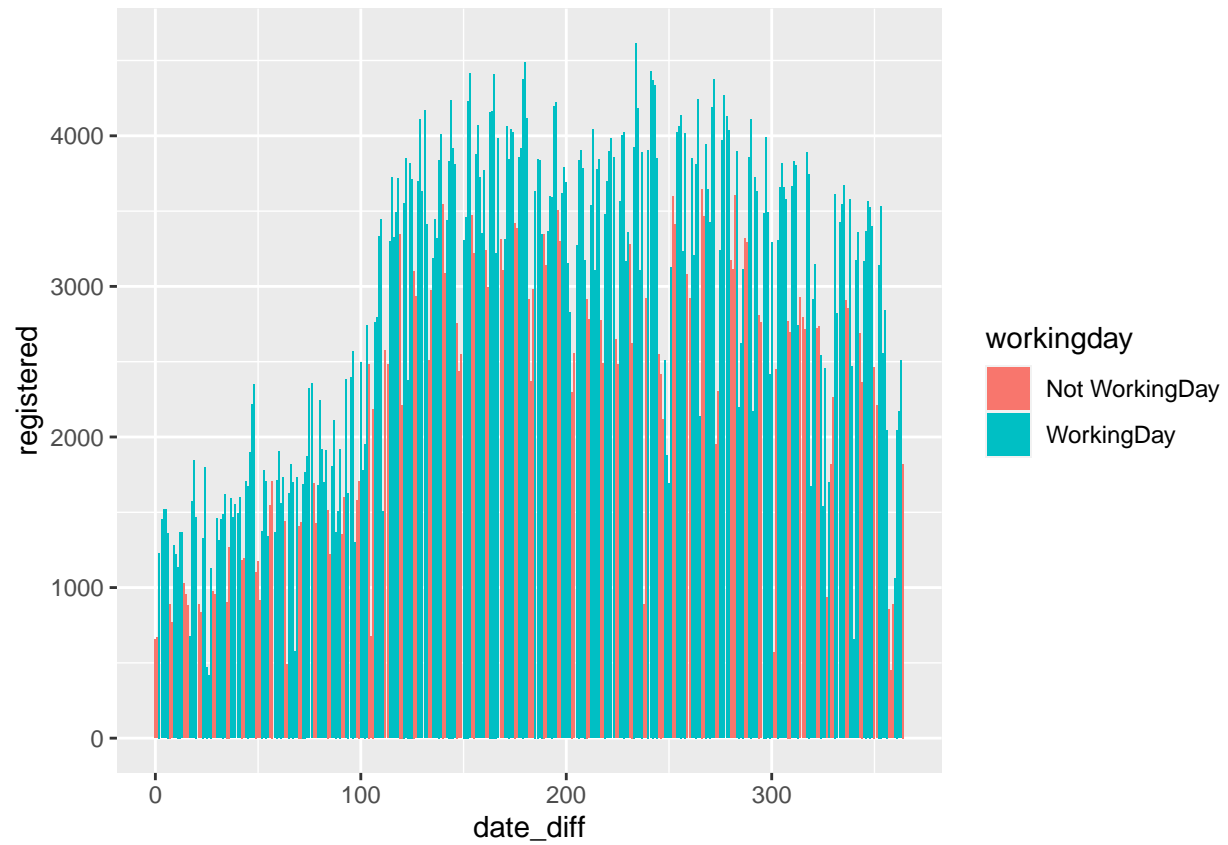
```
plot1<- ggplot(training_d,aes(x=weekday,y=registered))+geom_col()
plot2 <- ggplot(training_d,aes(x=weekday,y=casual))+geom_col()
plot_grid(plot1, plot2, labels = c("registered", "casual"))
```



Casual rental counts are higher on weekends compared to on weekdays while registered rental counts are lower on weekends than on weekdays.

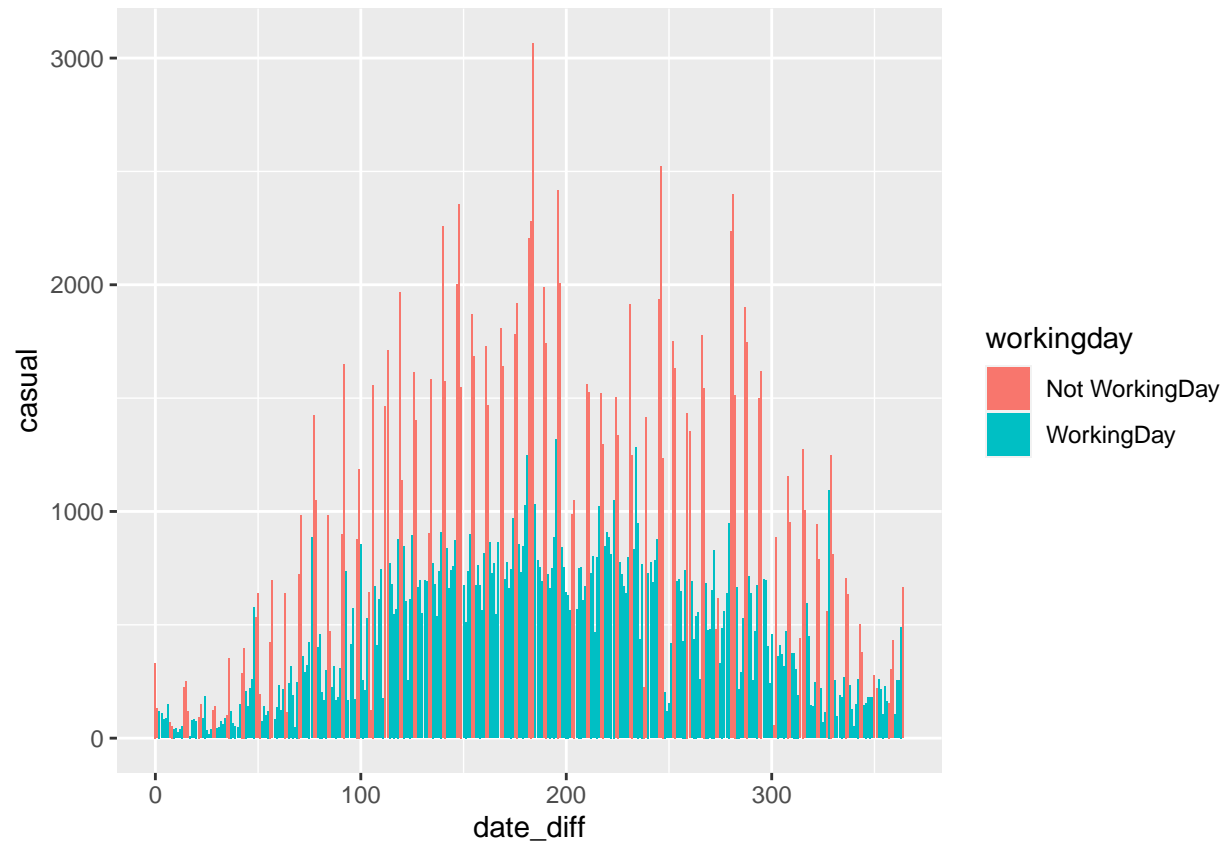
```
plot1 <- ggplot(data = training_d, aes(x=date_diff, y = registered)) + geom_col(aes(fill = workingday))
plot2 <- ggplot(data = training_d, aes(x=date_diff, y = casual)) + geom_col(aes(fill = workingday))
plot1
```

Don't know how to automatically pick scale for object of type difftime. Defaulting to continuous.



```
plot2
```

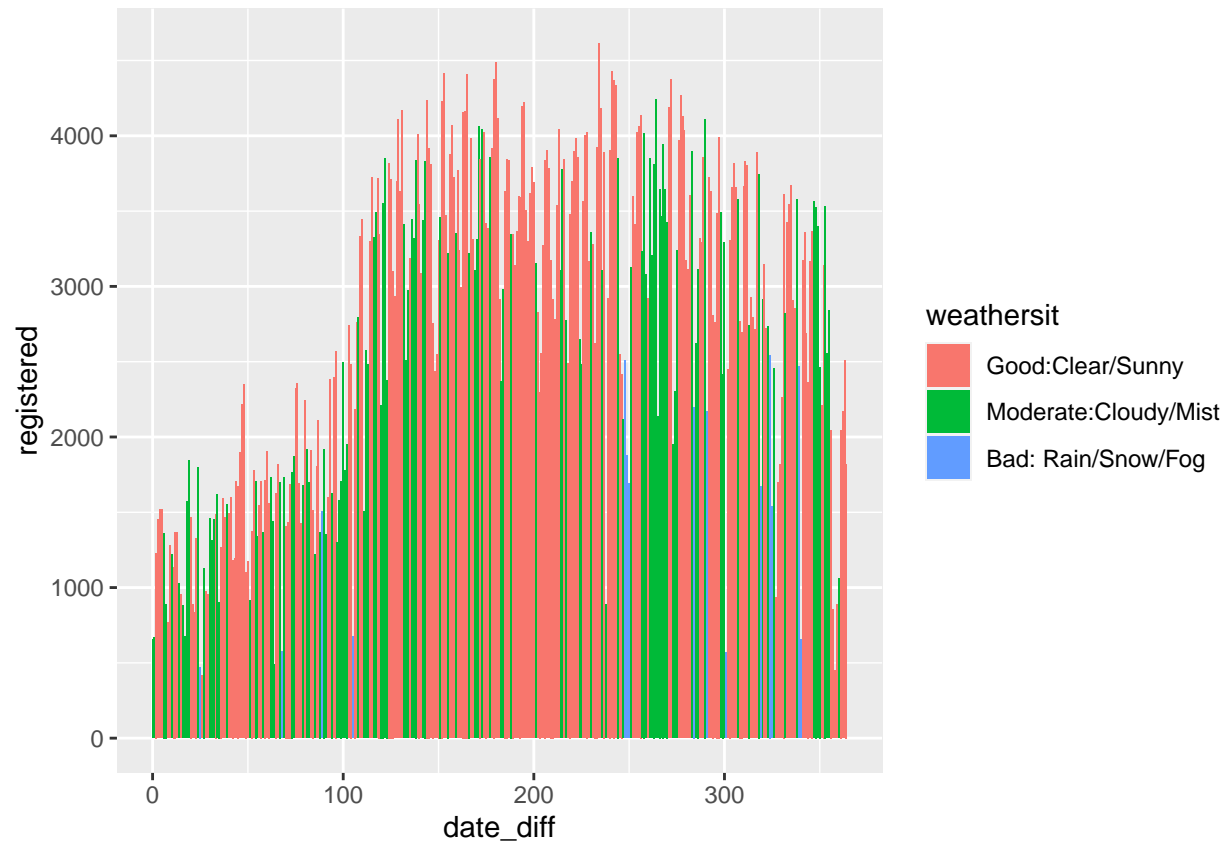
```
## Don't know how to automatically pick scale for object of type difftime. Defaulting to continuous.
```



There are more rental counts on not working days than on working days for casual bikers while there are more rental registered rental counts on working days than on not workingdays. There are also less rental counts for both registered and casual in the beginning of the year, then we see an increase of bikers during the summer and fall seasons, then a decrease during the end of the year. We suspect that this trend is due to temperature and other weather conditions.

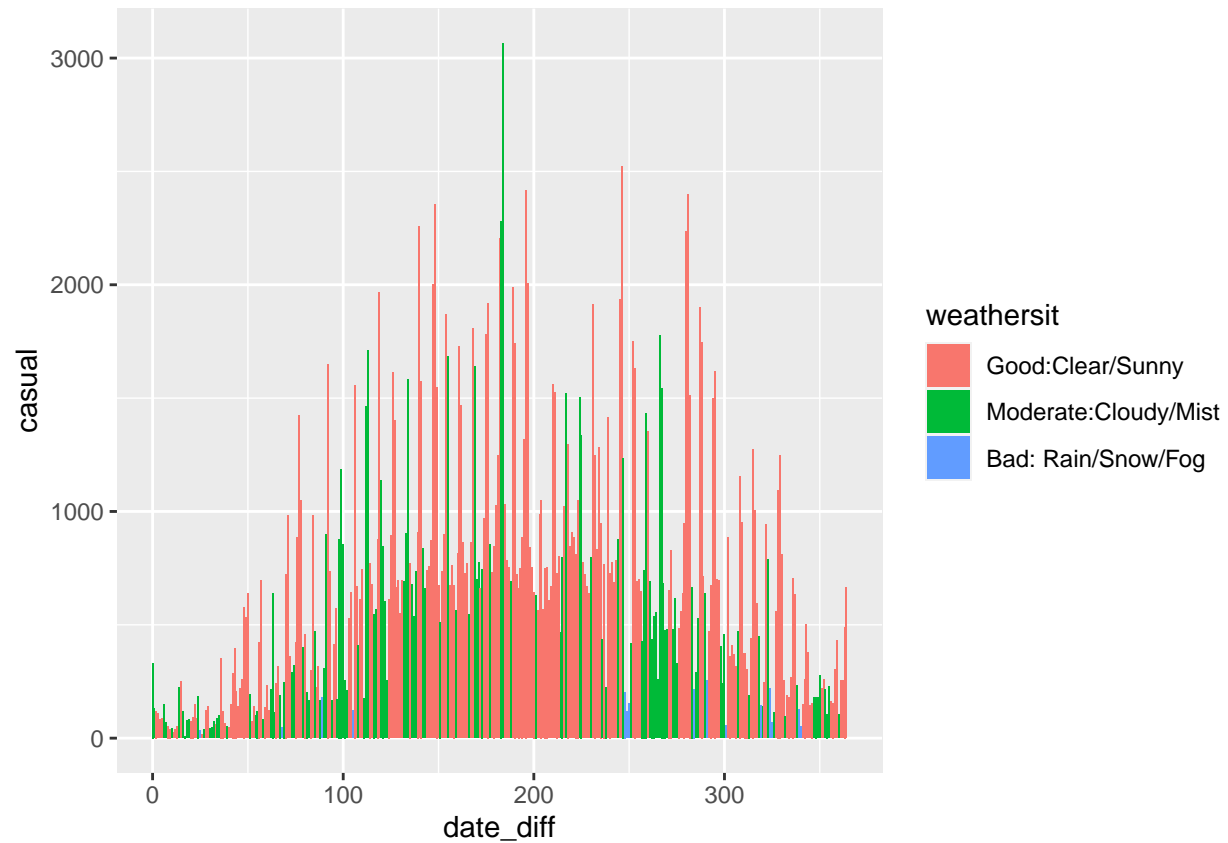
```
plot1 <- ggplot(data = training_d, aes(x=date_diff, y = registered)) + geom_col(aes(fill = workingday))
plot2 <- ggplot(data = training_d, aes(x=date_diff, y = casual)) + geom_col(aes(fill = workingday))
plot1
```

Don't know how to automatically pick scale for object of type difftime. Defaulting to continuous.



```
plot2
```

```
## Don't know how to automatically pick scale for object of type difftime. Defaulting to continuous.
```

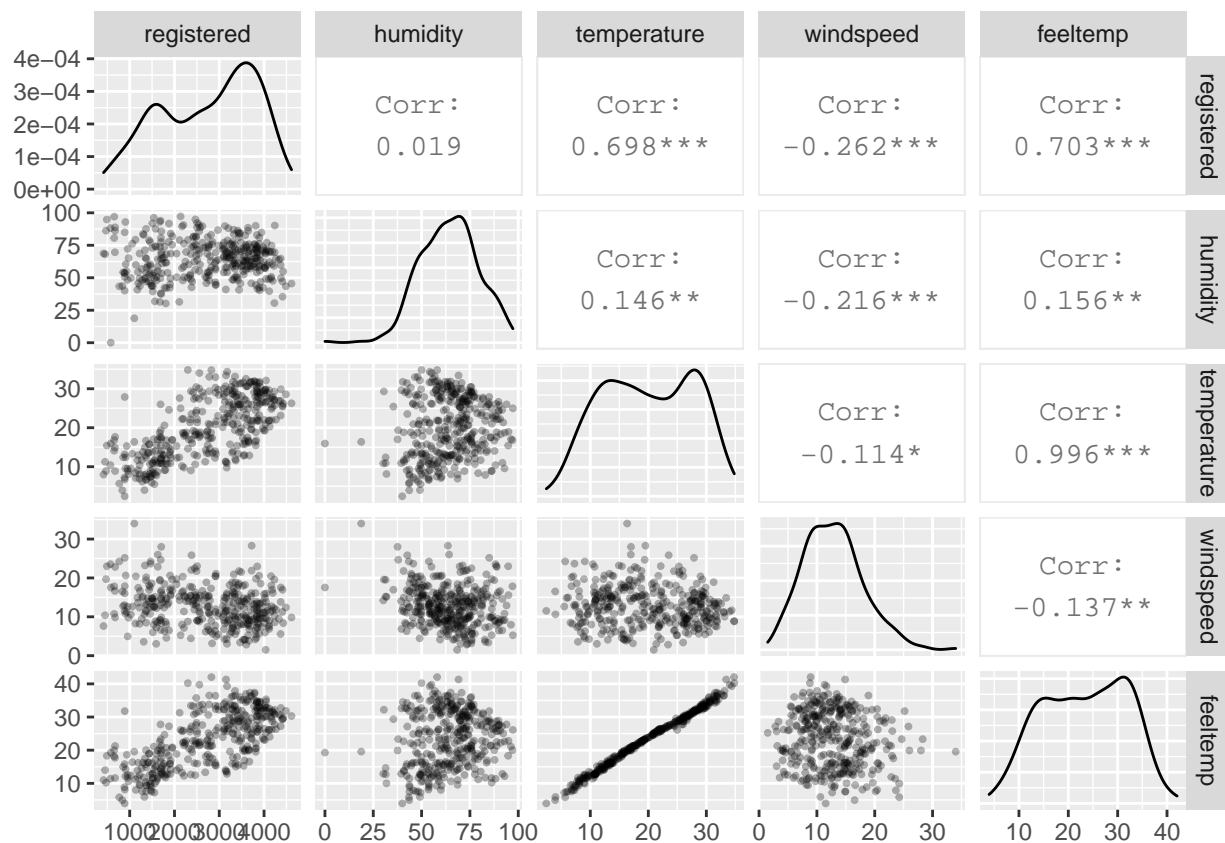



More bikers tend to bike on days with good and moderate weather conditions than on bad weather conditions.

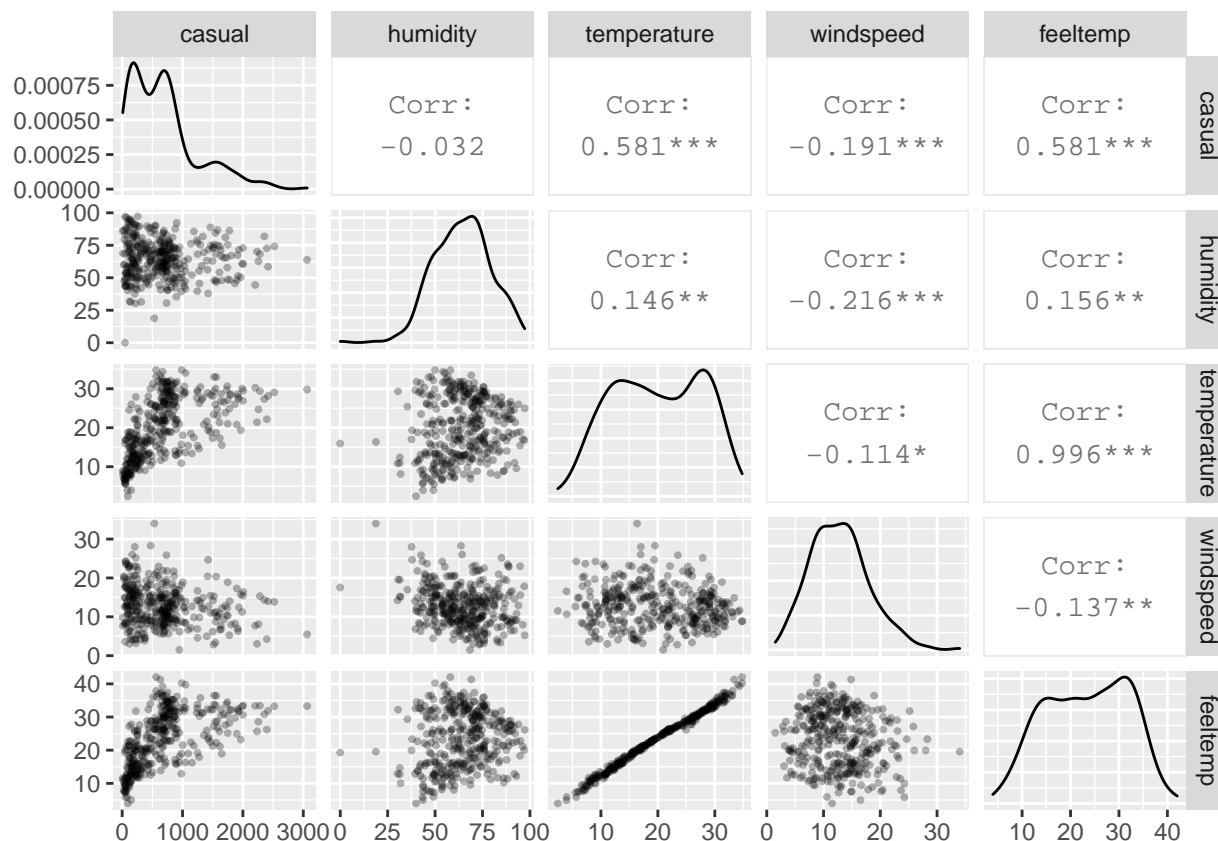
```
data <- data.frame(training_d$registered, training_d$actual.hum, training_d$actual.temp, training_d$actual.casual)
data = data%>% rename( registered = training_d.registered, humidity= training_d.actual.hum, temperature= training_d.actual.temp)
plot1 <- ggpairs(data, lower = list(continuous = wrap("points", alpha = 0.3, size= 0.7)))

data <- data.frame(training_d$casual, training_d$actual.hum, training_d$actual.temp, training_d$actual.registered)
data = data%>% rename( casual = training_d.casual, humidity= training_d.actual.hum, temperature= training_d.actual.temp, registered= training_d.actual.registered)
plot2 <- ggpairs(data, lower = list(continuous = wrap("points", alpha = 0.3, size= 0.7)))

plot1
```



plot2

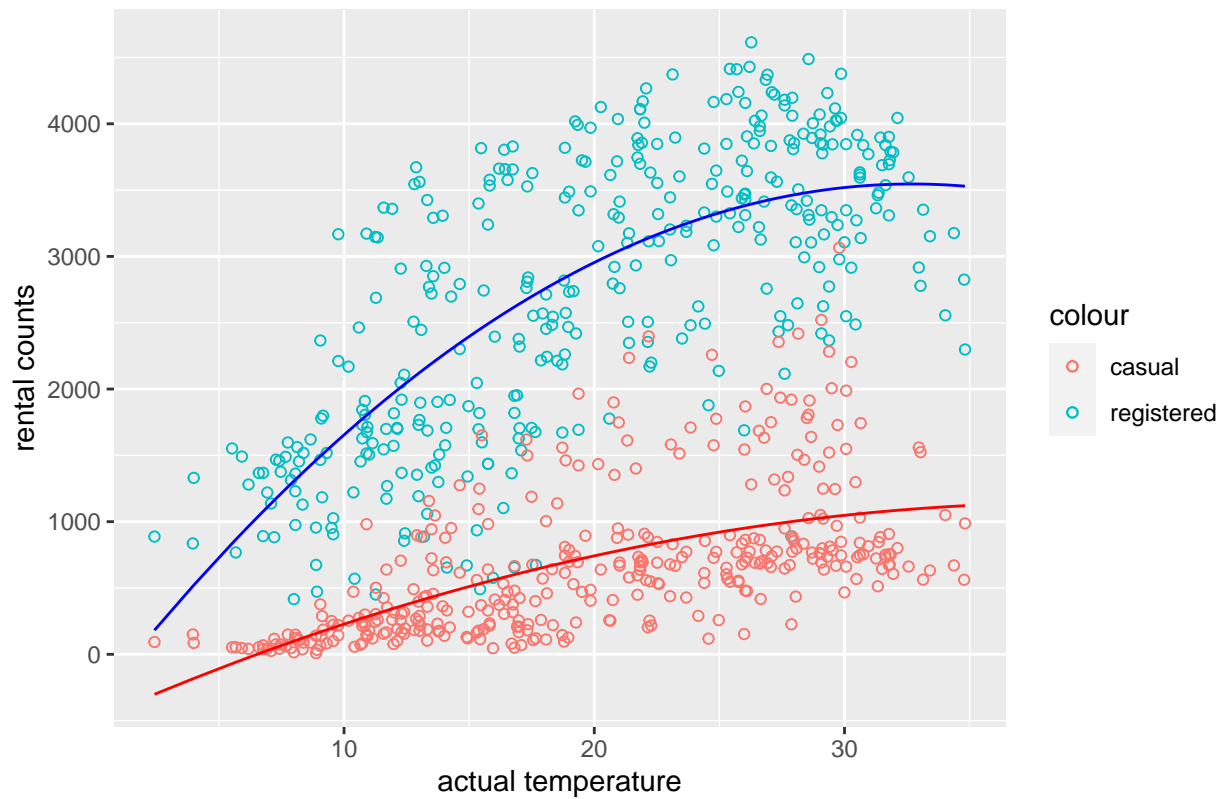


The graphs suggest that for both registered and casual bikers, there is a high correlation between temperature, windspeed and rental counts. There is strong correlation between temperature and feel temperature, so we decided to omit feel temperature to avoid collinearity.

```
m.quadls_casual <- lm(training_d$casual ~ training_d$actual.temp + I(training_d$actual.temp^2))
m.quadls_registered <- lm(training_d$registered ~ training_d$actual.temp + I(training_d$actual.temp^2))

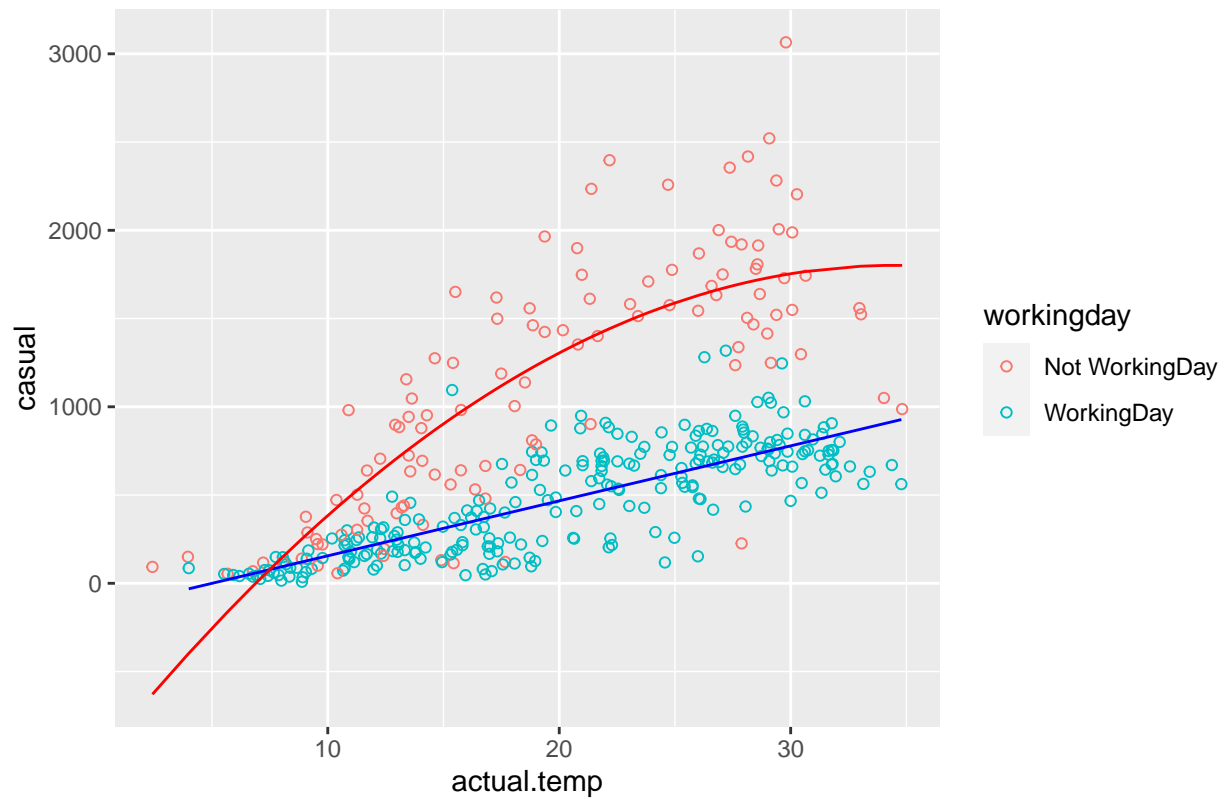
ggplot(training_d, aes(x = actual.temp)) + geom_point(aes(y = registered, color = "registered"), shape
```

Scatter plot with fitted models



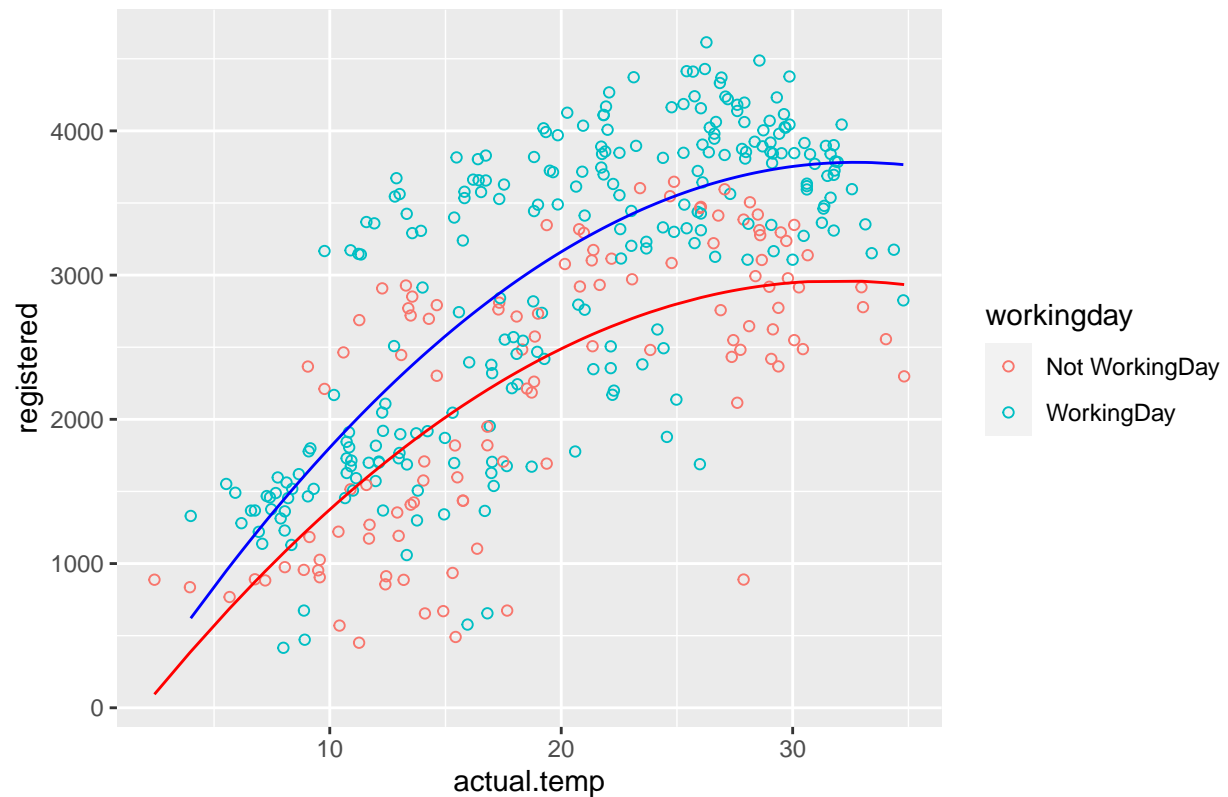
```
m.casual.workingday <- lm(training.workingday$casual ~ training.workingday$actual.temp)
m.quadls_casual.nworkingday <- lm(training.nworkingday$casual ~ training.nworkingday$actual.temp + I(tr
m.registered.nworkingday <- lm(training.nworkingday$registered ~ training.nworkingday$actual.temp + I(t
ggplot(training_d, aes(x = actual.temp)) + geom_point(aes(y = casual, color = workingday), shape = 1) +
```

Scatter plot of casual counts on weekdays and weekends with fitted mode



```
m.registered.workingday <- lm(training.workingday$registered ~ training.workingday$actual.temp + I(training.workingday$actual.temp^2))
m.registered.nworkingday <- lm(training.nworkingday$registered ~ training.nworkingday$actual.temp + I(training.nworkingday$actual.temp^2))
ggplot(training_d, aes(x = actual.temp)) + geom_point(aes(y = registered, color = workingday), shape = "circle") +
  geom_smooth(aes(color = workingday), method = "lm", se = FALSE)
```

Scatter plot of registered counts on weekdays and weekends with fitted models

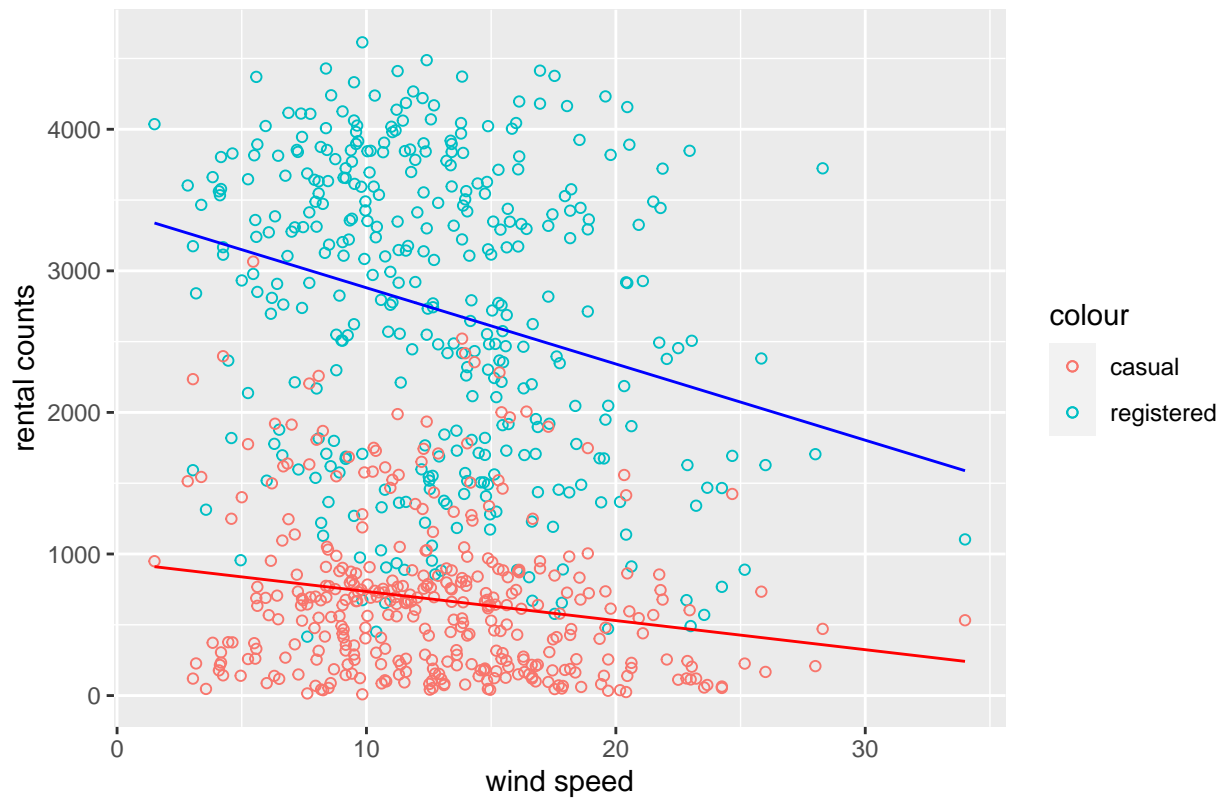


Wind speed and rental counts

```
m.lin_casual <- lm(training_d$casual ~ training_d$actual.windspeed)
m.lin_registered <- lm(training_d$registered ~ training_d$actual.windspeed)

ggplot(training_d, aes(x = actual.windspeed)) + geom_point(aes(y = registered, color = "registered"),
```

Scatter plot with fitted models

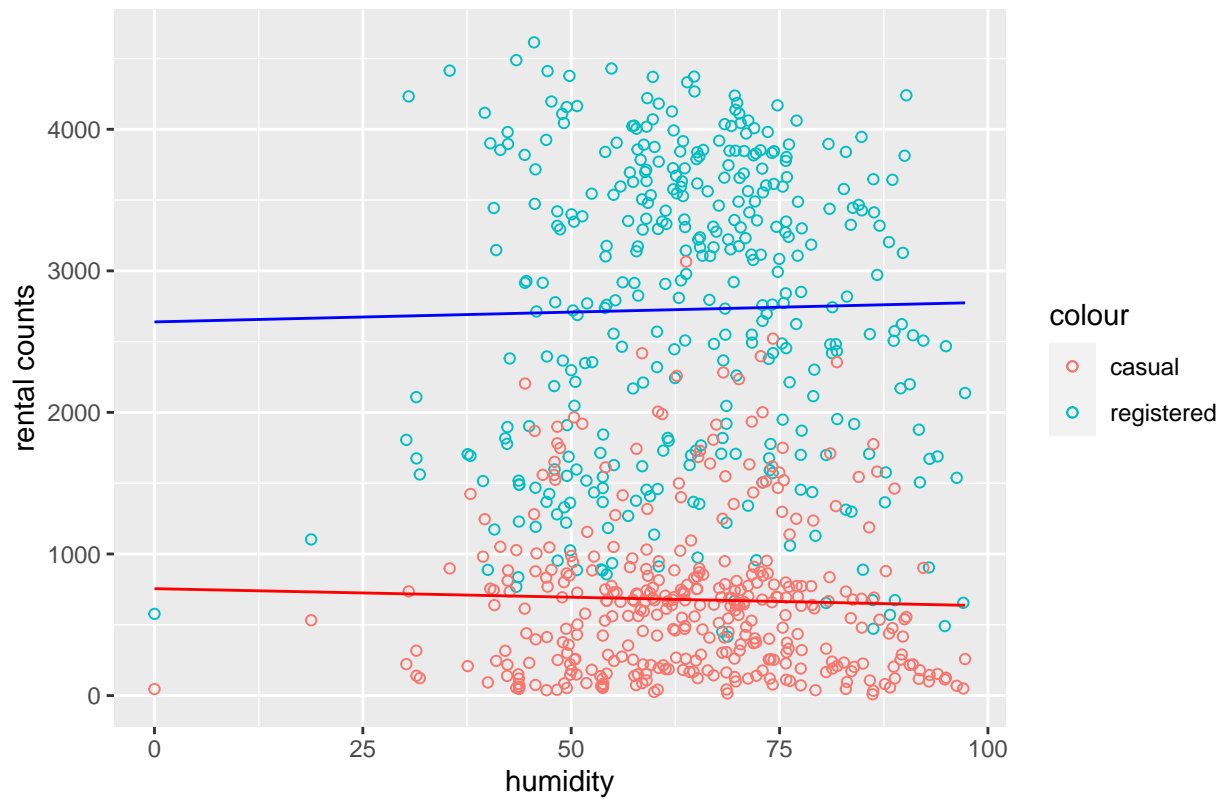


Humidity

```
m.lin_casual <- lm(training_d$casual ~ training_d$actual.hum)
m.lin_registered <- lm(training_d$registered ~ training_d$actual.hum)

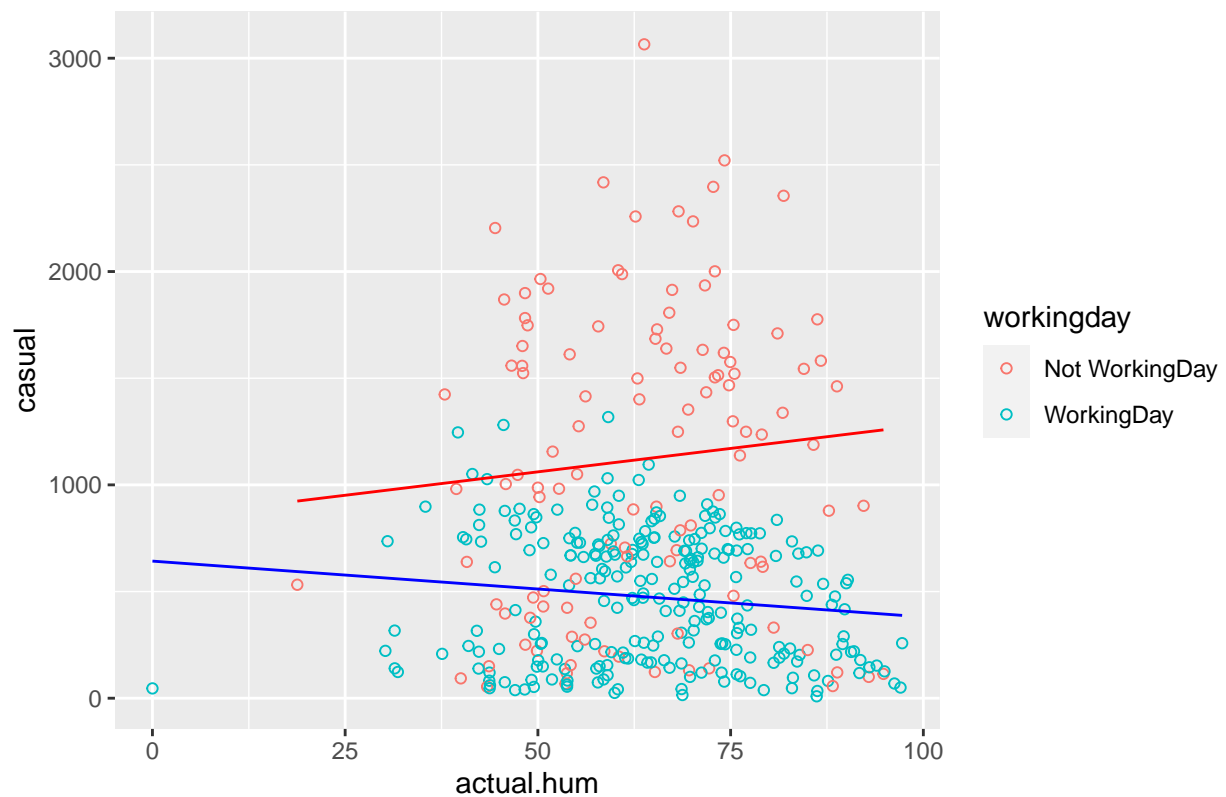
ggplot(training_d, aes(x = actual.hum)) + geom_point(aes(y = registered, color = "registered"), shape = "circle") +
```

Scatter plot with fitted models



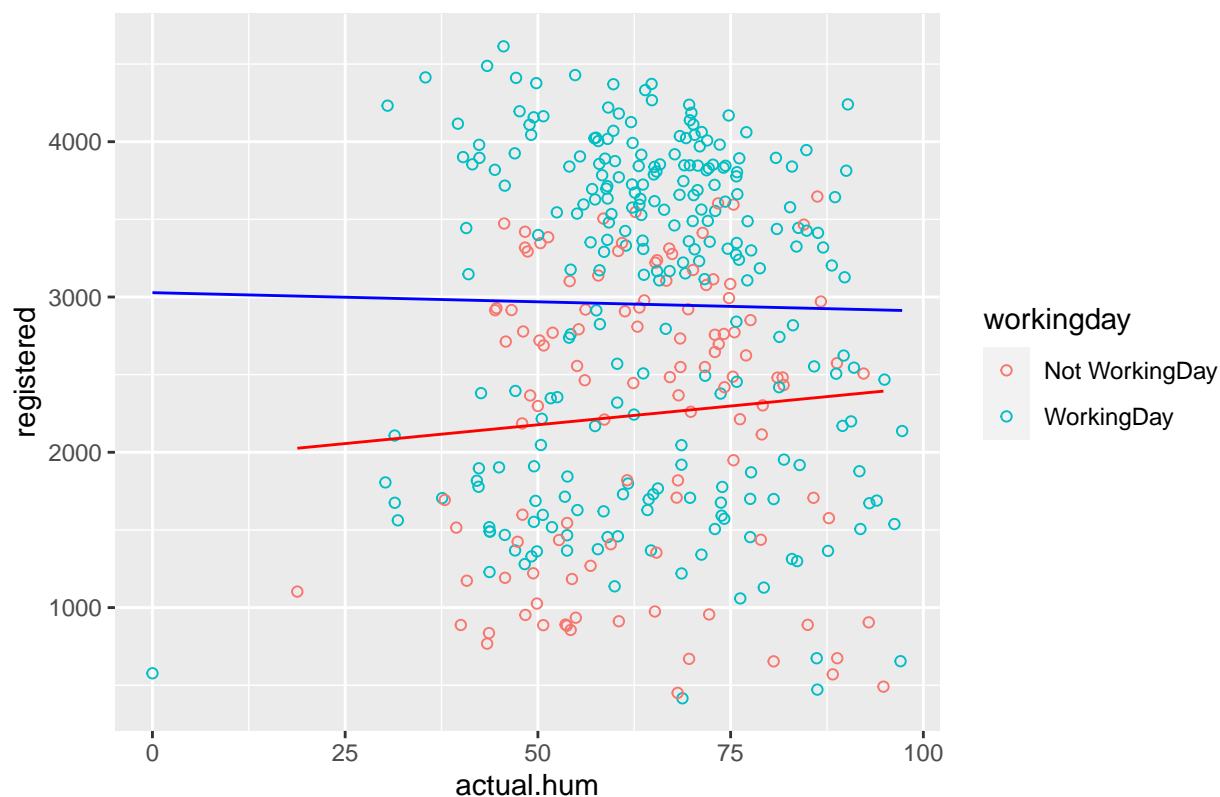
```
m.hum_casual.workingday <- lm(training.workingday$casual ~ training.workingday$actual.hum)
m.hum_casual.nworkingday <- lm(training.nworkingday$casual ~ training.nworkingday$actual.hum)
ggplot(training_d, aes(x = actual.hum)) + geom_point(aes(y = casual, color = workingday), shape = 1) +
```


Scatter plot of casual counts on weekdays and weekends with fitted mode



```
m.hum_registered.workingday <- lm(training.workingday$registered ~ training.workingday$actual.hum)
m.hum_registered.nworkingday <- lm(training.nworkingday$registered ~ training.nworkingday$actual.hum)
ggplot(training_d, aes(x = actual.hum)) + geom_point(aes(y = registered, color = workingday), shape = 1)
```

Scatter plot of registered counts on weekdays and weekends with fitted models



Model

```
model.casual.workingday <- lm(casual ~ actual.windspeed + actual.temp + I(actual.temp^2) + weathersit, data = training.workingday)
model.registered.workingday <- lm(registered ~ actual.temp + I(actual.temp^2) + actual.windspeed + weathersit, data = training.workingday)

summary(model.casual.workingday)
```

```
##
## Call:
## lm(formula = casual ~ actual.windspeed + actual.temp + I(actual.temp^2) + weathersit, data = training.workingday)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -369.62  -95.15  -20.24   70.12  636.87
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -316.7199    67.7510  -4.675 4.87e-06 ***
## actual.windspeed    -2.8500     1.9268  -1.479    0.14
## actual.temp       65.6497     7.2837   9.013 < 2e-16 ***
```

```
## I(actual.temp^2)          -0.9122      0.1828  -4.990 1.15e-06 ***
## weathersitModerate:Cloudy/Mist -144.9753    21.0026  -6.903 4.38e-11 ***
## weathersitBad: Rain/Snow/Fog   -390.4630    45.1814  -8.642 7.39e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 152.7 on 244 degrees of freedom
## Multiple R-squared:  0.7506, Adjusted R-squared:  0.7455
## F-statistic: 146.9 on 5 and 244 DF,  p-value: < 2.2e-16
```

```
summary(model.registered.workingday)
```

```
##
## Call:
## lm(formula = registered ~ actual.temp + I(actual.temp^2) + actual.windspeed +
##     weathersit + date_diff, data = training.workingday)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1760.06  -291.00    26.68   344.05  1084.55
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      50.2880   219.6676   0.229   0.8191
## actual.temp     221.4958    26.4657   8.369 4.60e-15 ***
## I(actual.temp^2)   -3.5440     0.6492  -5.459 1.18e-07 ***
## actual.windspeed  -16.6444     6.5696  -2.534  0.0119 *
## weathersitModerate:Cloudy/Mist -389.8799    67.7165  -5.758 2.56e-08 ***
## weathersitBad: Rain/Snow/Fog  -1651.5825   146.5403 -11.270 < 2e-16 ***
## date_diff         2.8943     0.3639   7.953 6.90e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 491.7 on 243 degrees of freedom
## Multiple R-squared:  0.7839, Adjusted R-squared:  0.7786
## F-statistic: 146.9 on 6 and 243 DF,  p-value: < 2.2e-16
```

```
model.casual.nworkingday <- lm(casual ~ actual.windspeed + actual.temp + I(actual.temp^2) + weathersit, data = training.nworkingday)
```

```
model.registered.nworkingday <- lm(registered ~ actual.temp + actual.windspeed + weathersit, data = training.nworkingday)
```

```
summary(model.casual.nworkingday)
```

```
##
## Call:
## lm(formula = casual ~ actual.windspeed + actual.temp + I(actual.temp^2) +
##     weathersit, data = training.nworkingday)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
##  -998.6  -272.4   -51.1   258.2  1329.4
##
```

```
## Coefficients:
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    -649.3944   253.0927  -2.566   0.0117 *
## actual.windspeed    -24.2122    7.2201  -3.353   0.0011 **
## actual.temp       170.4207   25.8870   6.583 1.67e-09 ***
## I(actual.temp^2)    -2.6486    0.6468  -4.095 8.13e-05 ***
## weathersitModerate:Cloudy/Mist -209.2066   80.9685  -2.584   0.0111 *
## weathersitBad: Rain/Snow/Fog  -536.2792  291.3608  -1.841   0.0684 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 391.3 on 109 degrees of freedom
## Multiple R-squared:  0.7062, Adjusted R-squared:  0.6927
## F-statistic: 52.4 on 5 and 109 DF, p-value: < 2.2e-16
```

```
summary(model.registered.nworkingday)
```

```
##
## Call:
## lm(formula = registered ~ actual.temp + actual.windspeed + weathersit,
##     data = training.nworkingday)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1402.68  -413.18   -12.15   435.30  1424.07
##
## Coefficients:
##
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    1509.210   225.096   6.705 9.02e-10 ***
## actual.temp       73.602    7.372   9.984 < 2e-16 ***
## actual.windspeed  -46.649   11.182  -4.172 6.06e-05 ***
## weathersitModerate:Cloudy/Mist -293.689   124.600  -2.357  0.0202 *
## weathersitBad: Rain/Snow/Fog  -839.067   450.831  -1.861  0.0654 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 606.1 on 110 degrees of freedom
## Multiple R-squared:  0.5861, Adjusted R-squared:  0.571
## F-statistic: 38.94 on 4 and 110 DF, p-value: < 2.2e-16
```

All of the p values on the coefficients of the regressors are less than 0.005. Therefore we are confident that all the regressors have an effect on the rental counts individually. Furthermore, the p value of the F-statistic is less than 0.005. Therefore we are very confident that all the regressors are jointly significant. The R^2 value is around 0.7, so the models explain around 70 percent of the variation in rental counts. (explain more in paper).

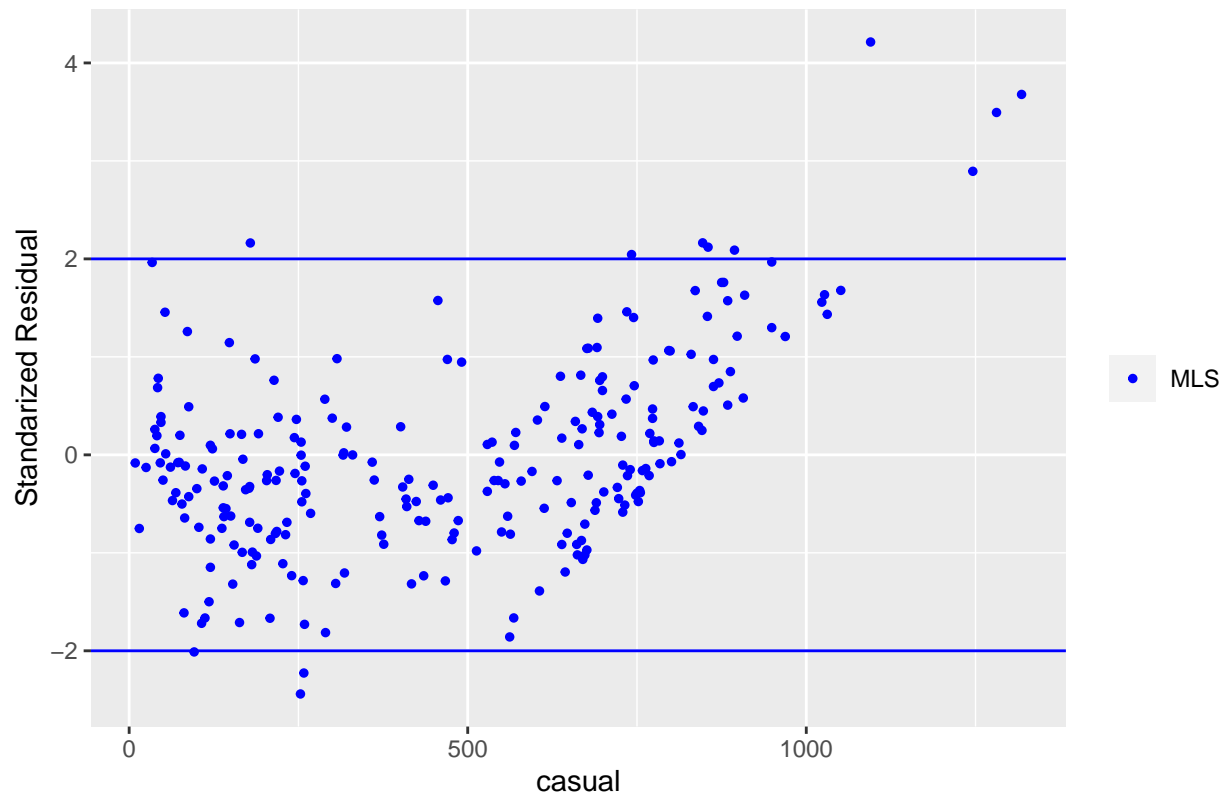
Model diagnosis

```
StanRes.casual.workingday <- rstandard(model.casual.workingday)
StanRes.registered.workingday <- rstandard(model.registered.workingday)
```

```
StanRes.casual.nworkingday <- rstandard(model.casual.nworkingday)
StanRes.registered.nworkingday <- rstandard(model.registered.nworkingday)
```

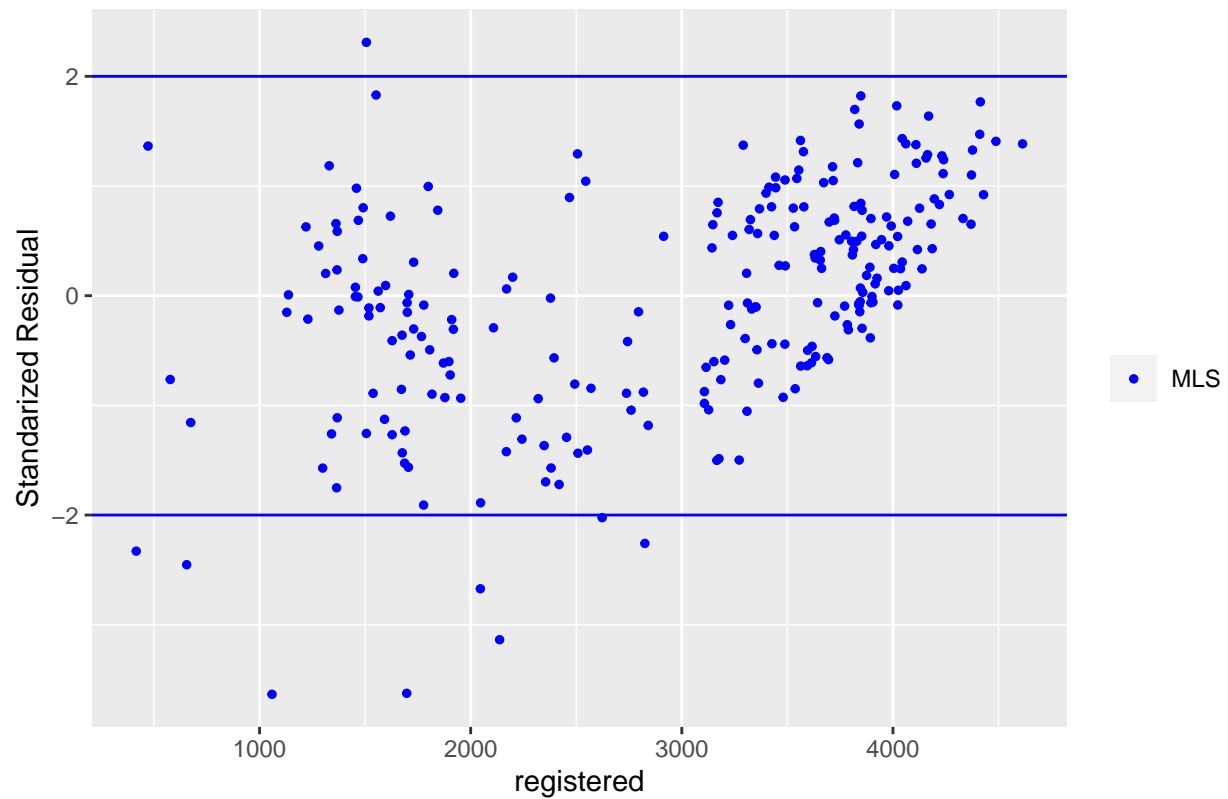
```
ggplot() +
  geom_point(data=training.workingday, aes(x=casual, y=StanRes.casual.workingday, color = "MLS"), size =
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') +
  scale_color_manual(name = element_blank(), labels = c("MLS"), values = c("blue")) +
  labs(y = "Standarized Residual") + ggtitle("Standarized Residuals MLS Plot for casual bikers on workingday")
```

Standarized Residuals MLS Plot for casual bikers on workingdays



```
ggplot() +
  geom_point(data=training.workingday, aes(x=registered, y=StanRes.registered.workingday, color = "MLS"),
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') +
  scale_color_manual(name = element_blank(), labels = c("MLS"), values = c("blue")) +
  labs(y = "Standarized Residual") + ggtitle("Standarized Residuals MLS Plot for registered bikers on workingday")
```

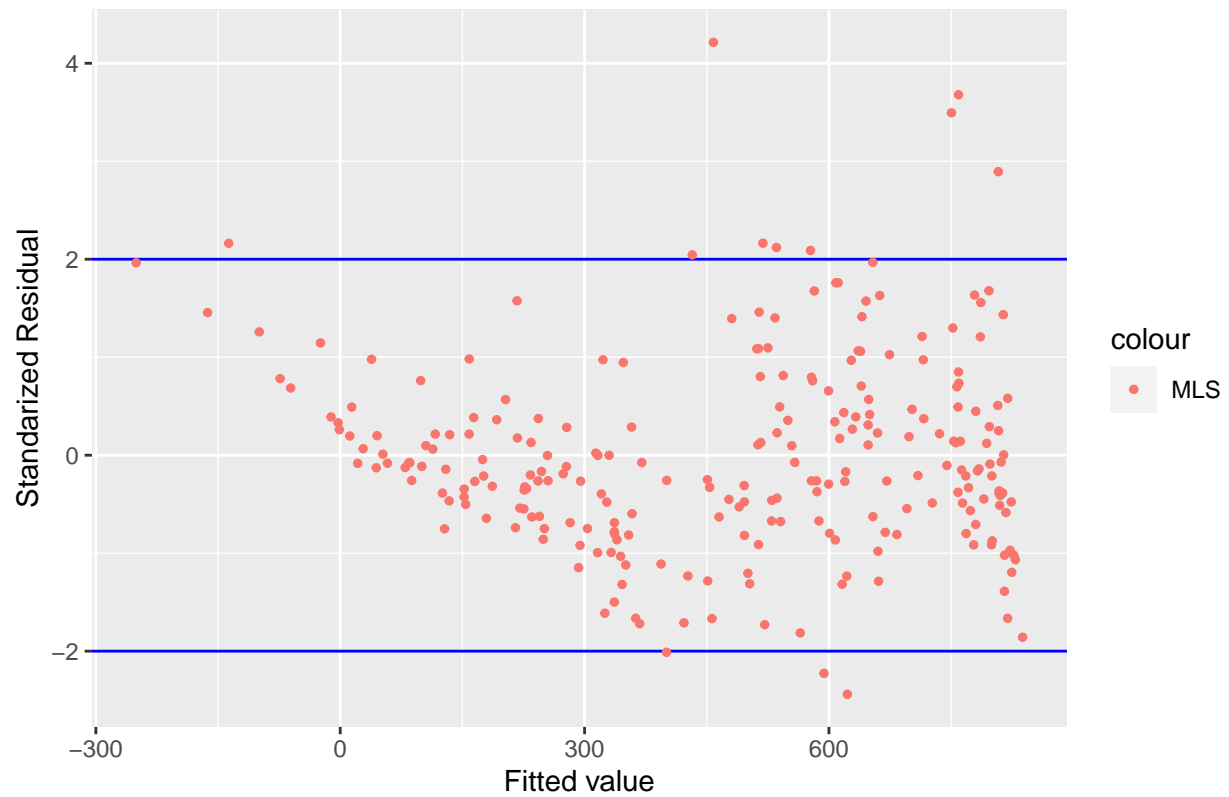
Standardized Residuals MLS Plot for registered bikers on workingdays



```
Fitted_casual.workingday = fitted(model.casual.workingday)

ggplot() +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') + geom_point(aes(x=Fitted_casual.workingday, y=Standardized_Residual)) +
  labs(y = "Standardized Residual") + labs(x = "Fitted value") +
  ggtitle("Standardized Residuals MLS Plot (Fitted) for casual bikers on workingdays")
```

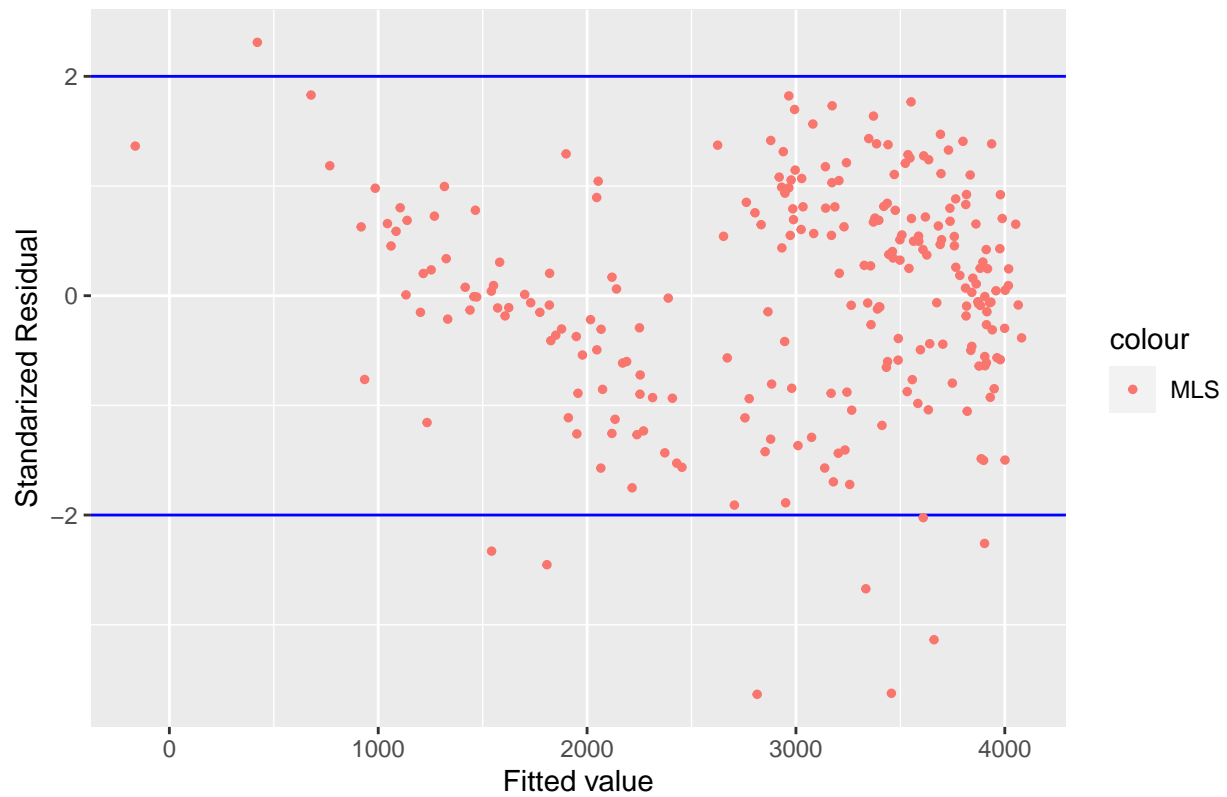
Standardized Residuals MLS Plot (Fitted) for casual bikers on workingdays



```
Fitted_registered.workingday = fitted(model.registered.workingday)

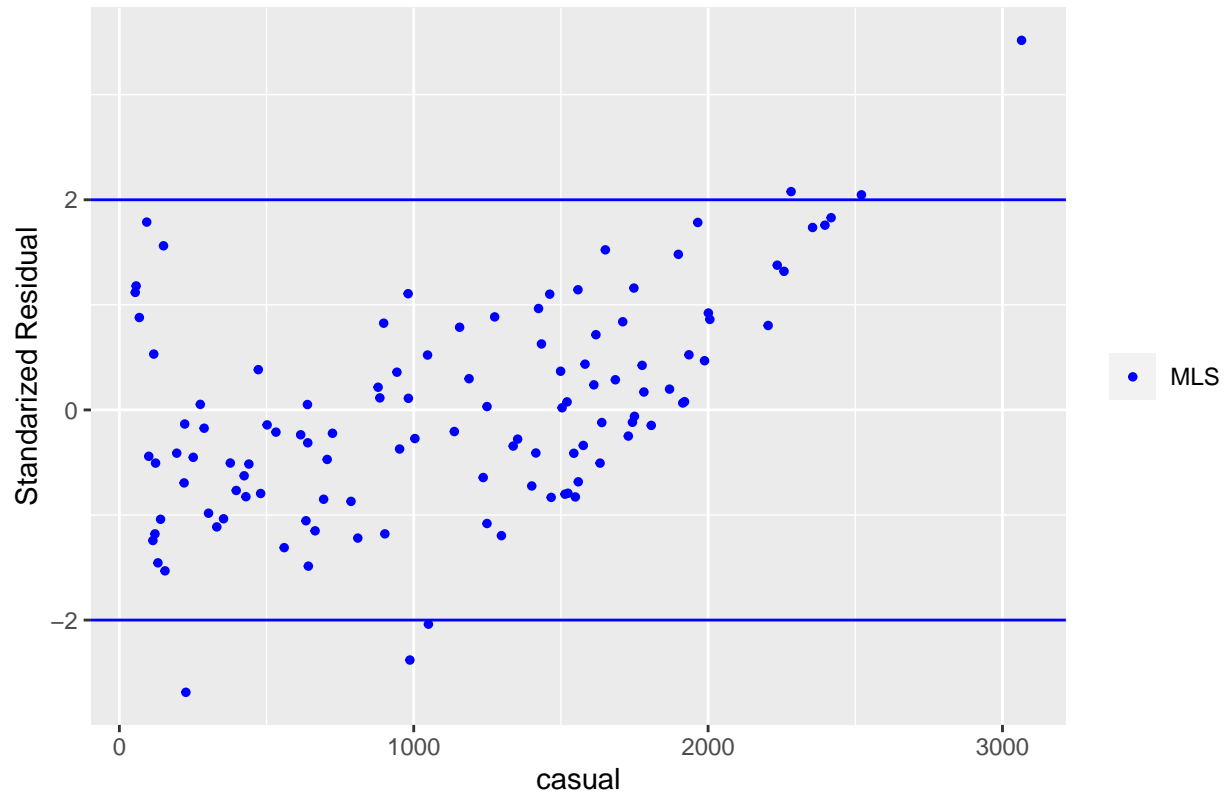
ggplot() +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') + geom_point(aes(x=Fitted value", y=Standardized Residual")) +
  labs(y = "Standardized Residual") + labs(x = "Fitted value") +
  ggtitle("Standardized Residuals MLS Plot (Fitted) for registered bikers on workingdays")
```

Standardized Residuals MLS Plot (Fitted) for registered bikers on workingday



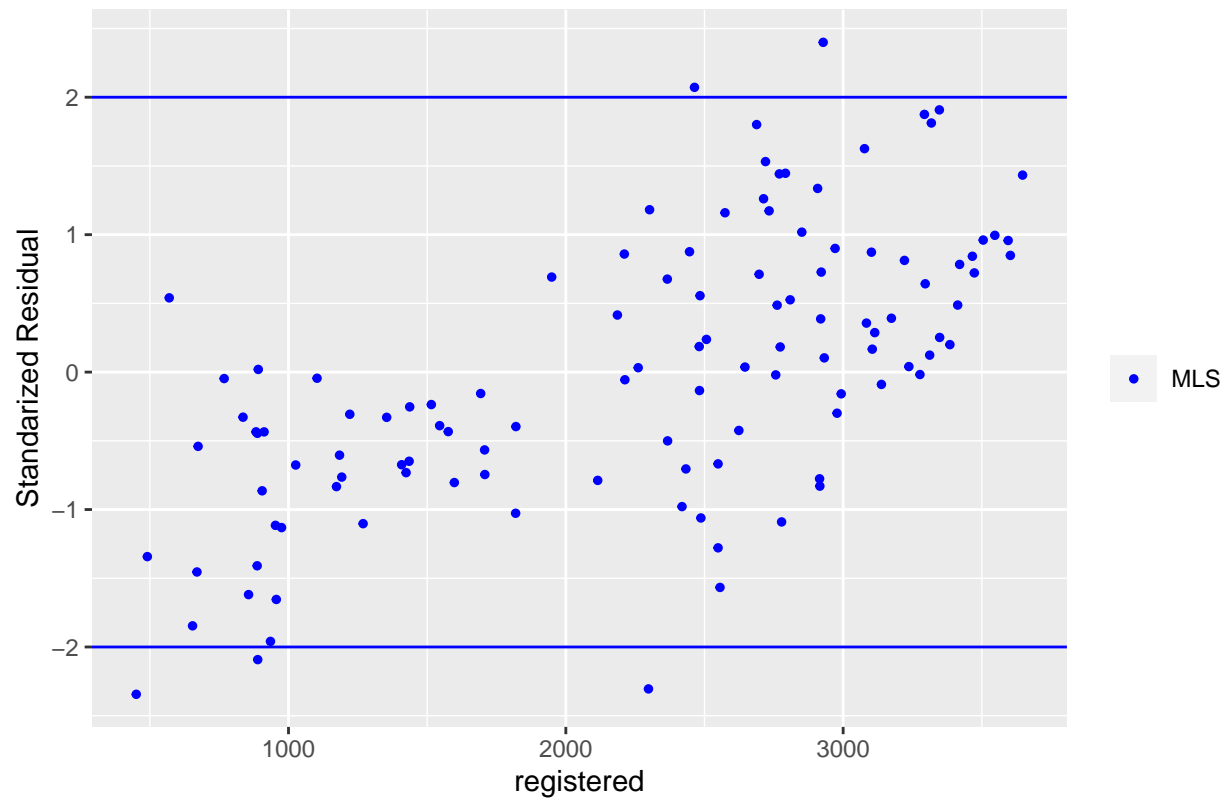
```
ggplot() +
  geom_point(data=training.nworkingday, aes(x=casual, y=StanRes.casual.nworkingday, color = "MLS"), size = 1) +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') +
  scale_color_manual(name = element_blank(), labels = c("MLS"), values = c("blue")) +
  labs(y = "Standardized Residual") + ggtitle("Standardized Residuals MLS Plot for casual bikers onnon-workday")
```


Standardized Residuals MLS Plot for casual bikers onnon-workingdays



```
ggplot() +
  geom_point(data=training.nworkingday, aes(x=registered, y=StanRes.registered.nworkingday, color = "MLS"),
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') +
  scale_color_manual(name = element_blank(), labels = c("MLS"), values = c("blue")) +
  labs(y = "Standarized Residual") + ggtitle("Standarized Residuals MLS Plot for registered bikers on non-working days")
```

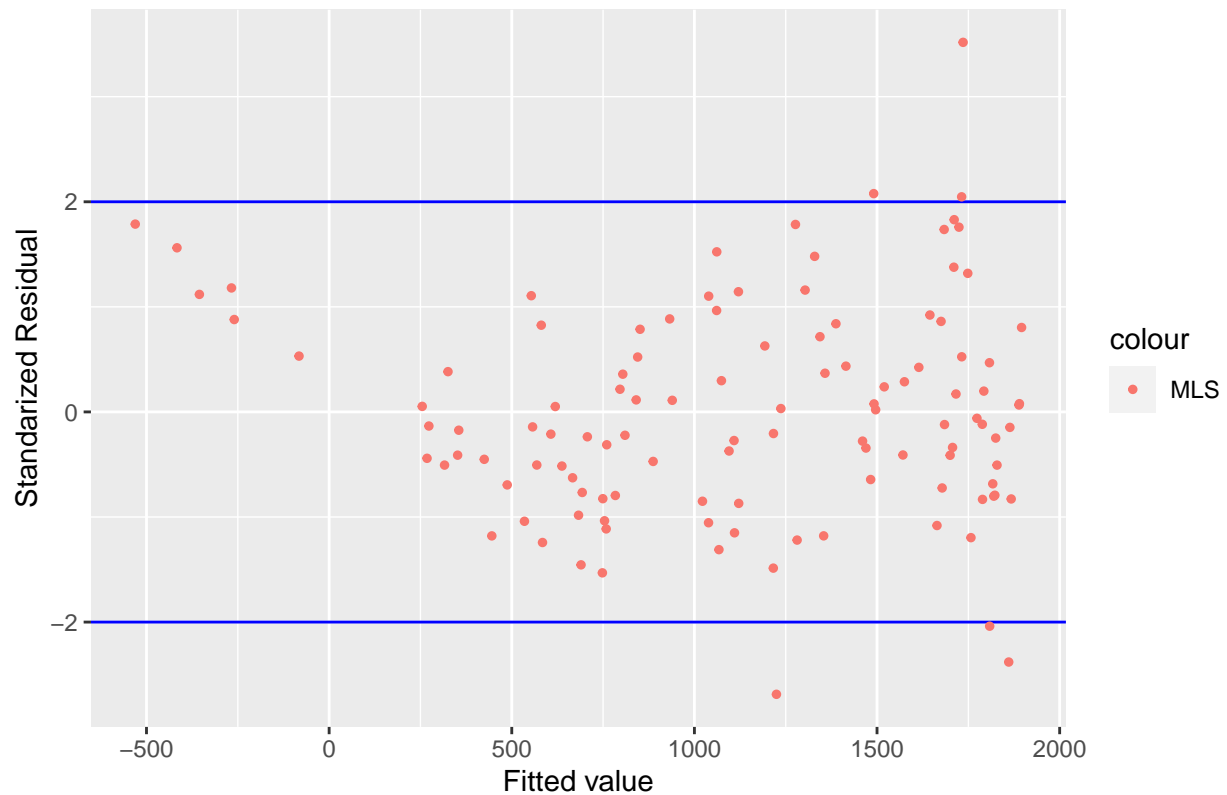
Standardized Residuals MLS Plot for registered bikers on non-workingdays



```
Fitted_casual.nworkingday = fitted(model.casual.nworkingday)

ggplot() +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') + geom_point(aes(x=Fitted_casual.nworkingday, y=Standardized_Residual)) +
  labs(y = "Standardized Residual") + labs(x = "Fitted value") +
  ggtitle("Standardized Residuals MLS Plot (Fitted) for casual bikers on non-workingdays")
```

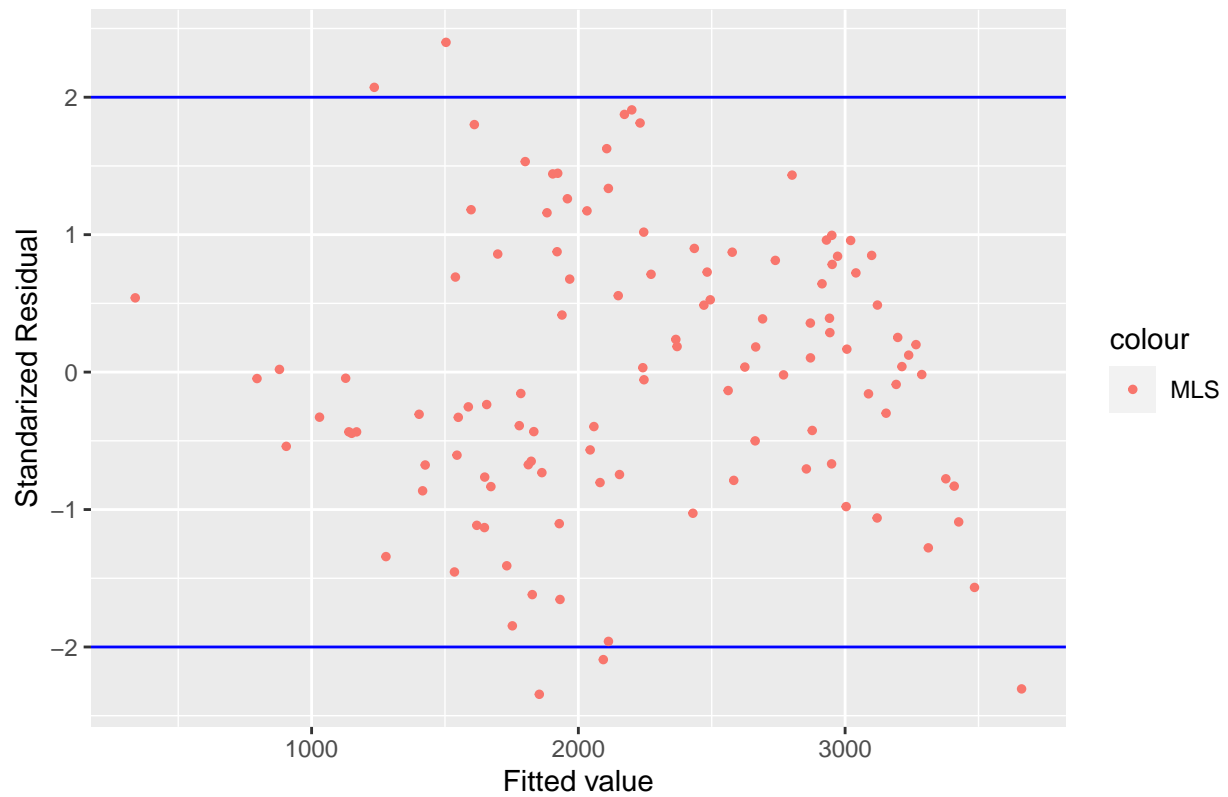
Standardized Residuals MLS Plot (Fitted) for casual bikers on non-workingdays



```
Fitted_registered.nworkingday = fitted(model.registered.nworkingday)

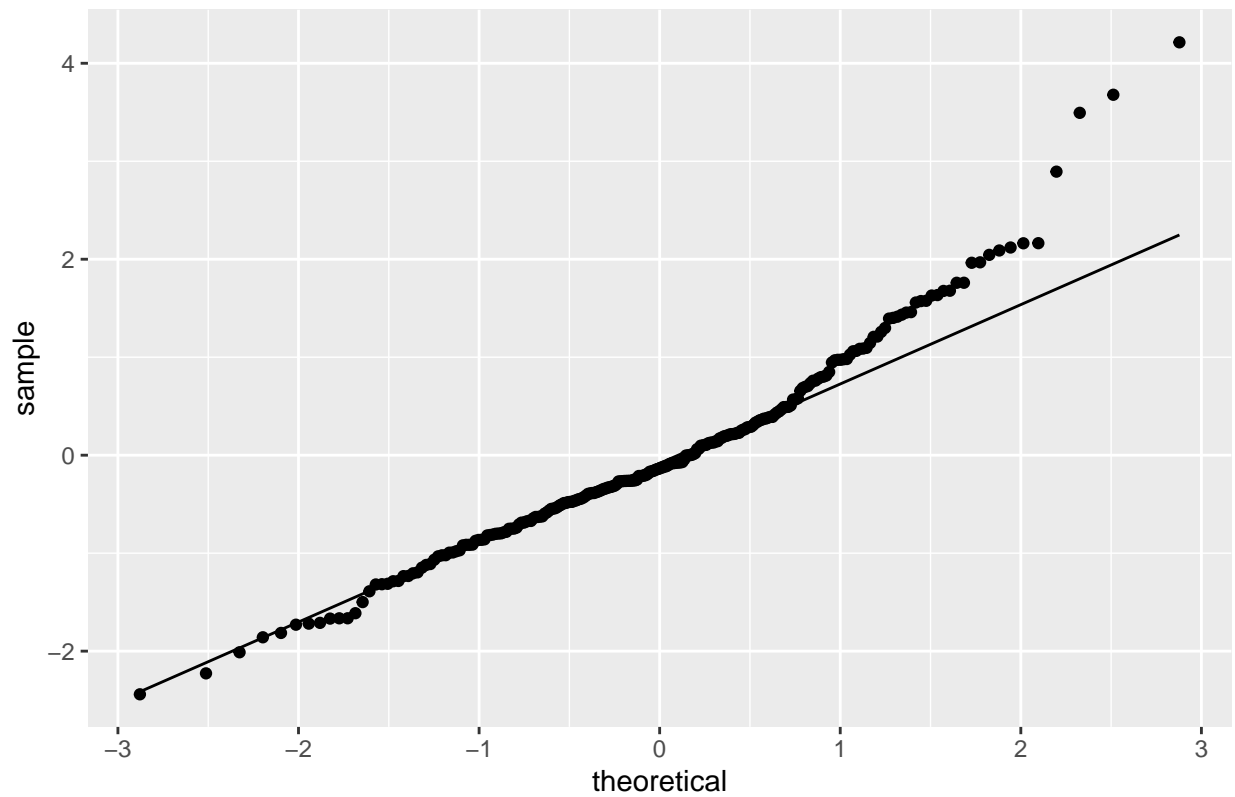
ggplot() +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') + geom_point(aes(x=Fitted value", y=Standardized Residual")) +
  labs(y = "Standardized Residual") + labs(x = "Fitted value") +
  ggtitle("Standardized Residuals WLS Plot (Fitted) for registered bikers on workingdays")
```

Standardized Residuals WLS Plot (Fitted) for registered bikers on workingday



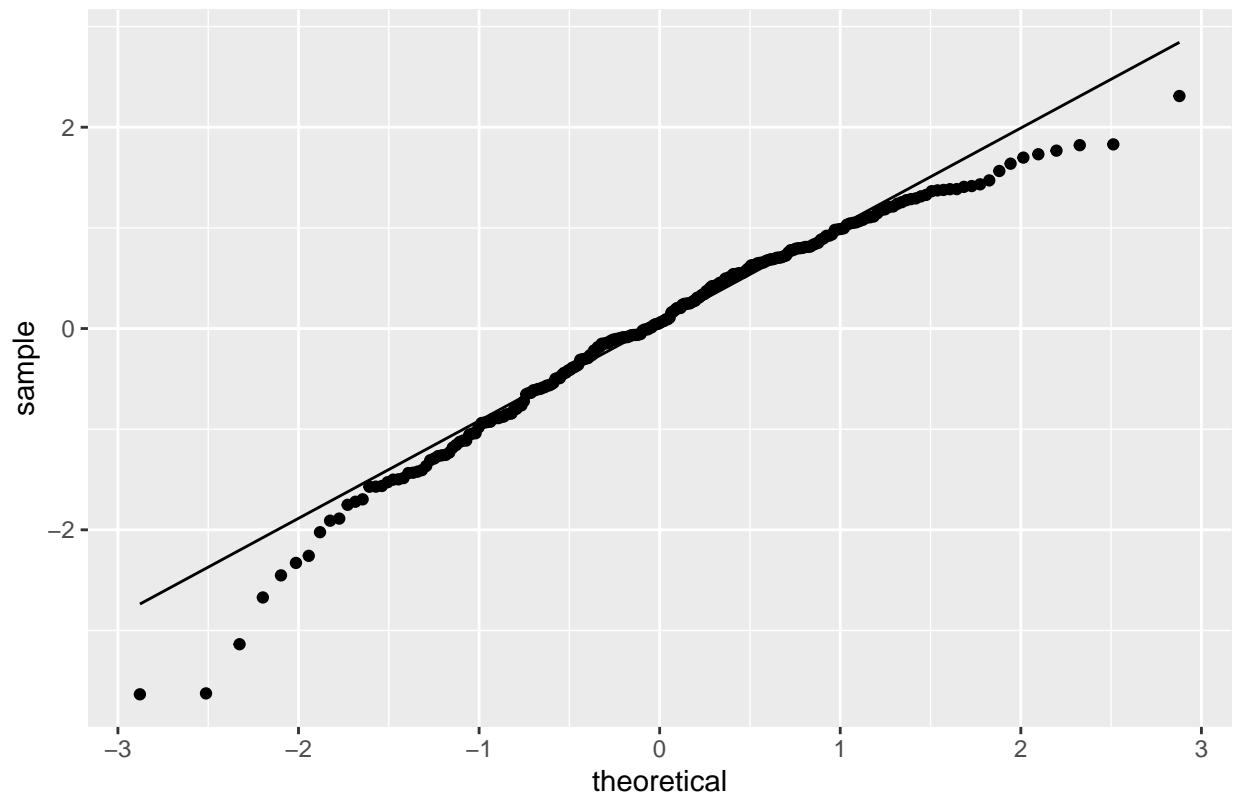
```
p <- ggplot(data.frame(StanRes.casual.workingday), aes(sample = StanRes.casual.workingday)) +  
  ggtitle("QQ MLS Plot for casual bikers on workingdays")  
p + stat_qq() + stat_qq_line()
```

QQ MLS Plot for casual bikers on workingdays



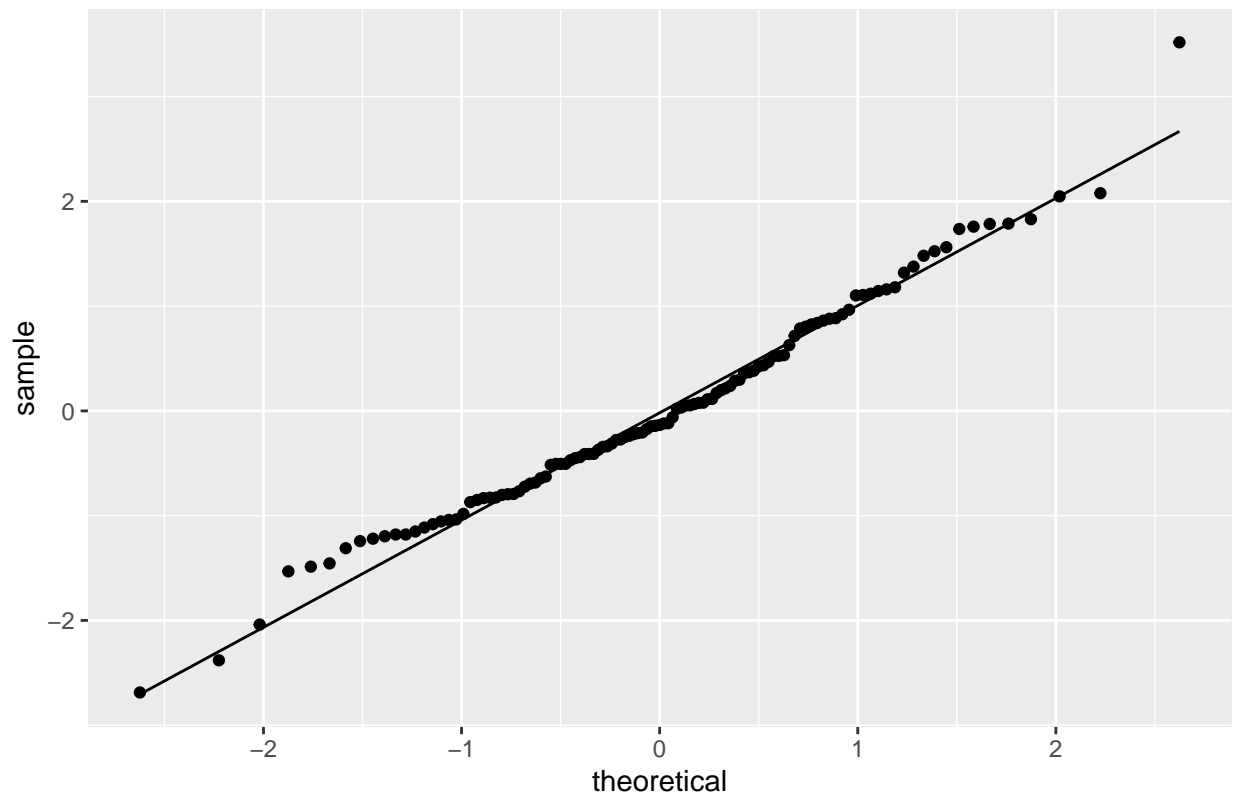
```
p <- ggplot(data.frame(StanRes.registered.workingday), aes(sample = StanRes.registered.workingday)) +  
  ggtitle("QQ MLS Plot for registered bikers on workingdays")  
p + stat_qq() + stat_qq_line()
```

QQ MLS Plot for registered bikers on workingdays

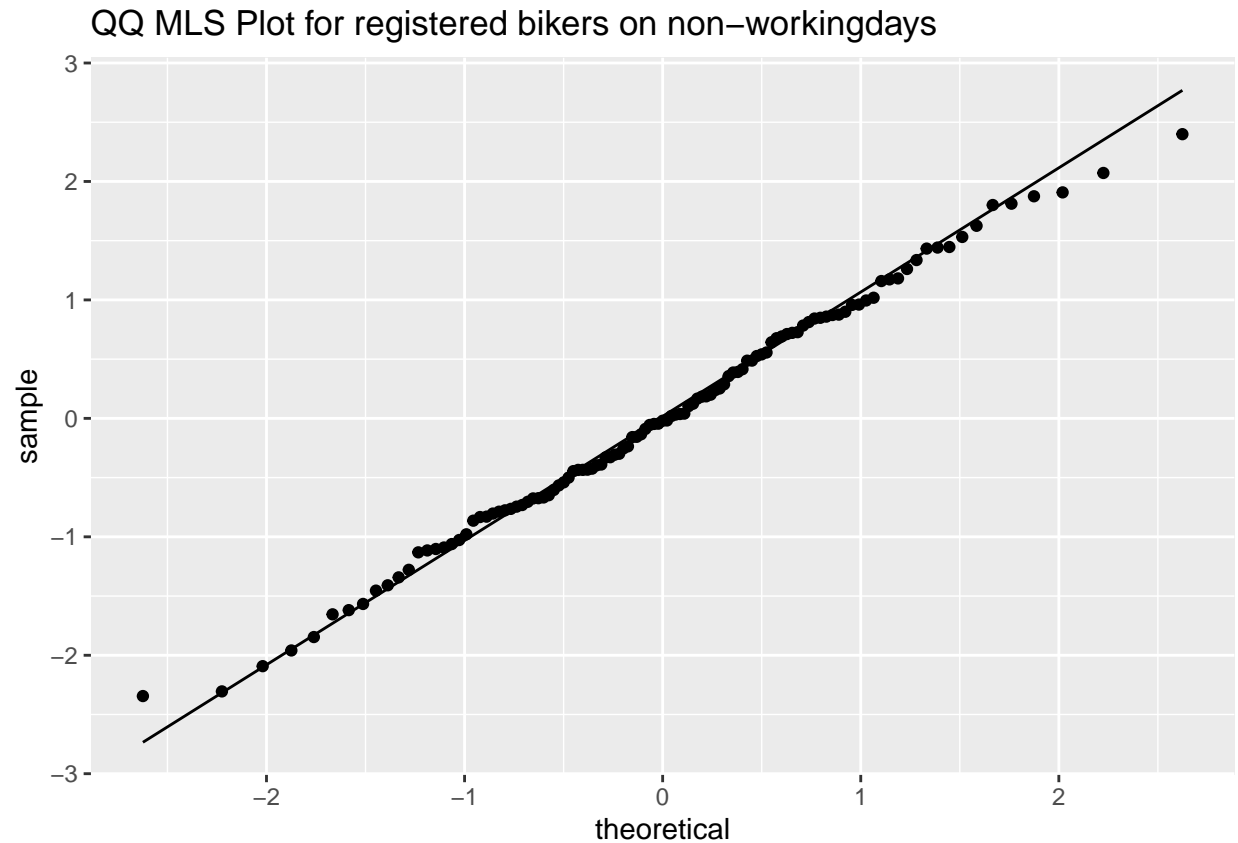


```
p <- ggplot(data.frame(StanRes.casual.nworkingday), aes(sample = StanRes.casual.nworkingday)) +  
  ggtitle("QQ MLS Plot for casual bikers on non-workingdays")  
p + stat_qq() + stat_qq_line()
```

QQ MLS Plot for casual bikers on non-workingdays



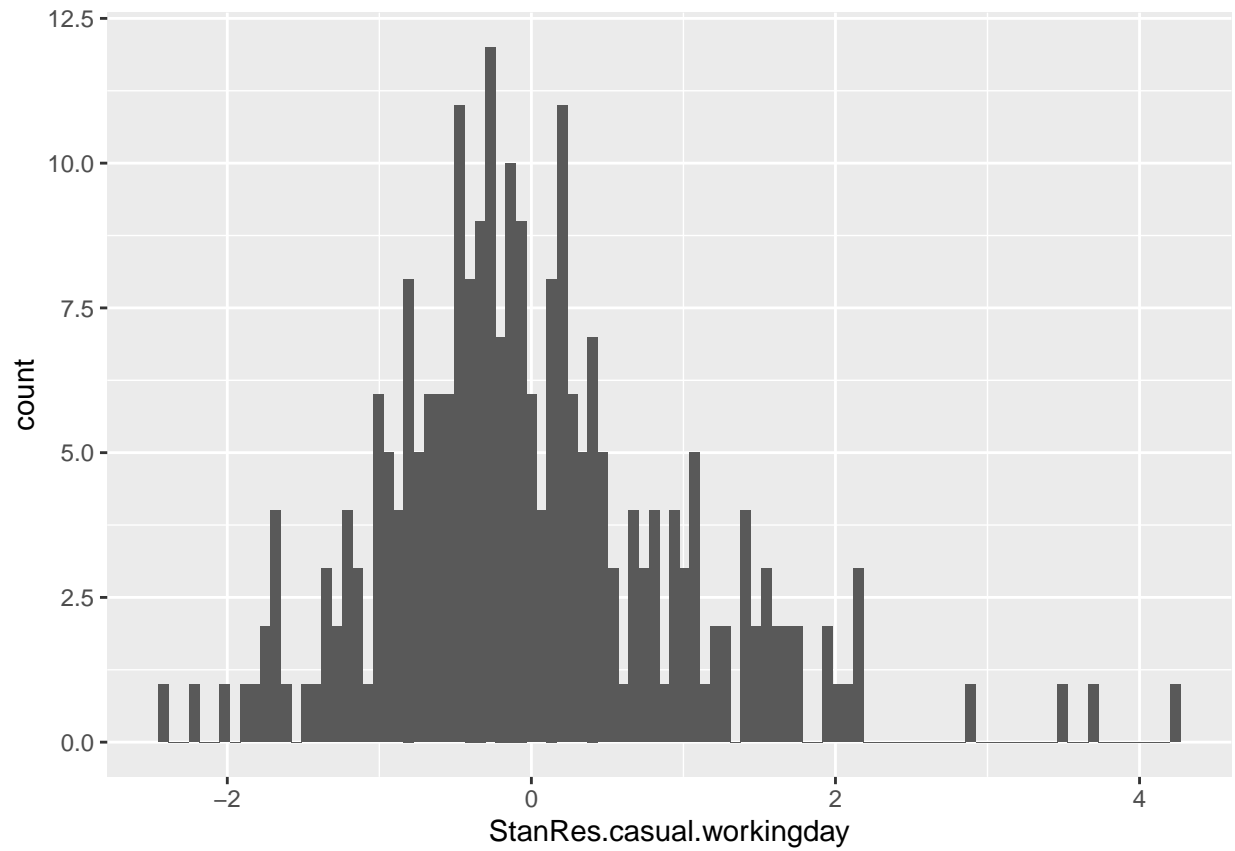
```
p <- ggplot(data.frame(StanRes.registered.nworkingday), aes(sample = StanRes.registered.nworkingday)) +  
  ggtitle("QQ MLS Plot for registered bikers on non-workingdays")  
p + stat_qq() + stat_qq_line()
```



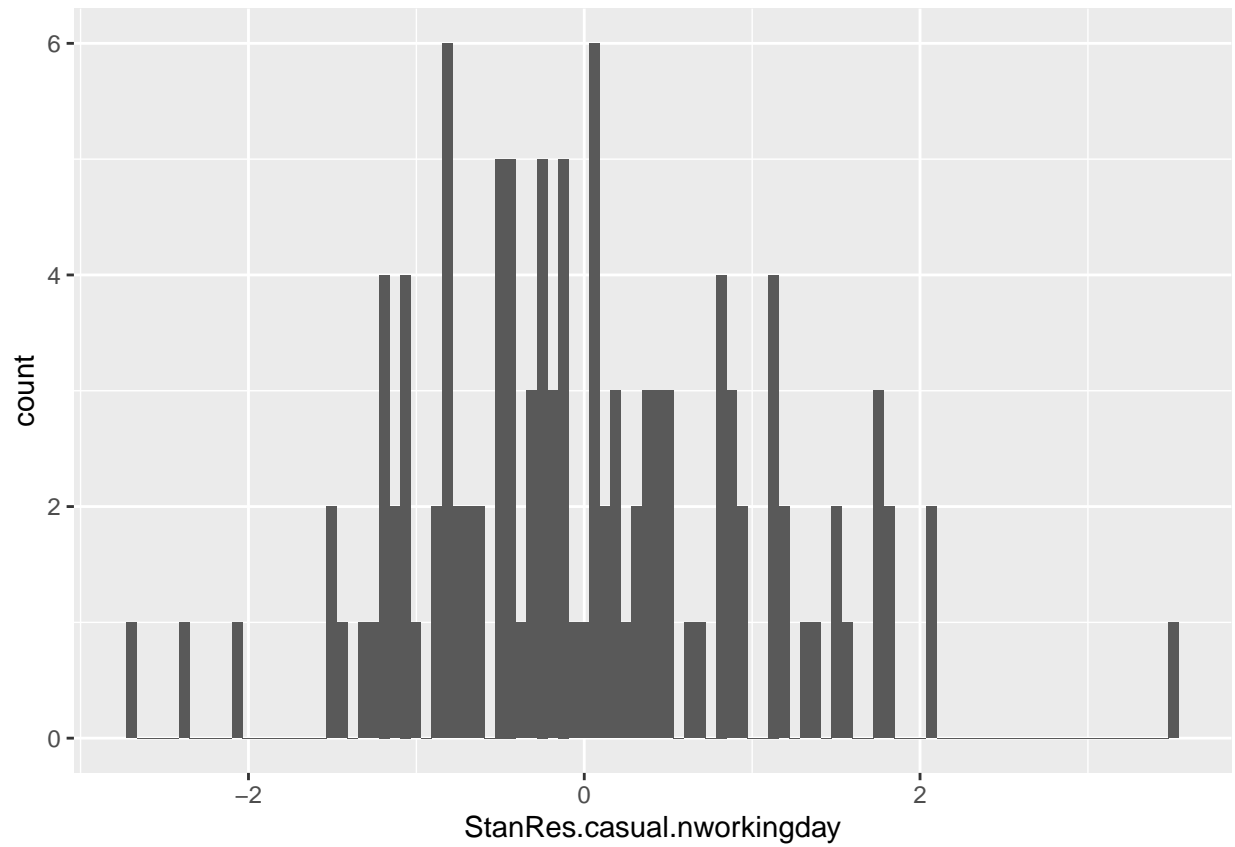
The fitted residual plot and the residual plot suggest that there are extreme outliers in the casual model and that the residual for both models are not evenly distributed around 0, therefore suggesting that there exists heterogeneity in the models.

The QQ plots show a line that is roughly straight, therefore we conclude that the data of registered bikers come from a normally distributed sample. We can also conclude the same for casual bikers, however, there exists some data points that do not come from a normal distribution as indicated by the few datapoints that deviate significantly from the straight line.

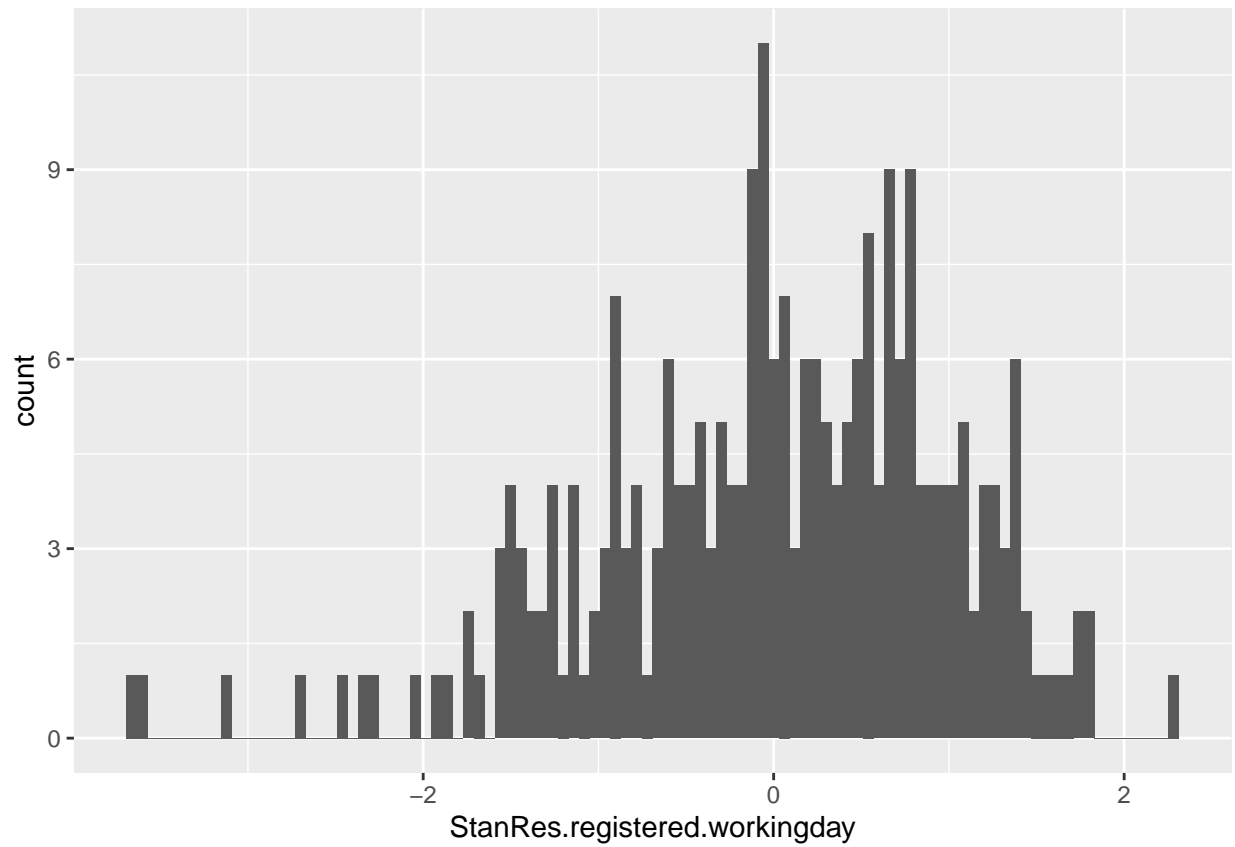
```
p1 <- ggplot(data = data.frame(StanRes.casual.workingday), aes(x = StanRes.casual.workingday)) + geom_h  
p1
```

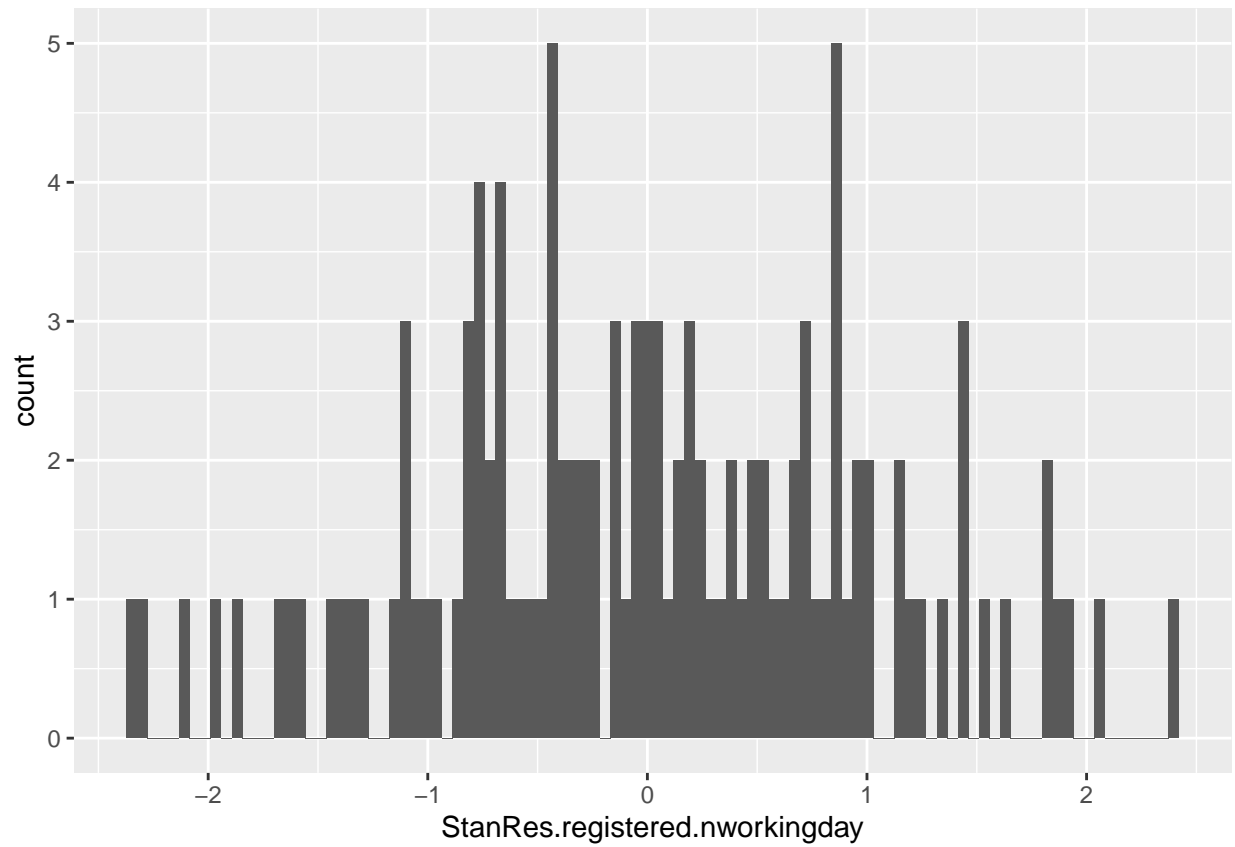
```
p2 <- ggplot(data = data.frame(StanRes.casual.nworkingday), aes(x = StanRes.casual.nworkingday)) + geom_histogram()
p2
```



```
p3 <- ggplot(data = data.frame(StanRes.registered.workingday), aes(x = StanRes.registered.workingday))  
p3
```

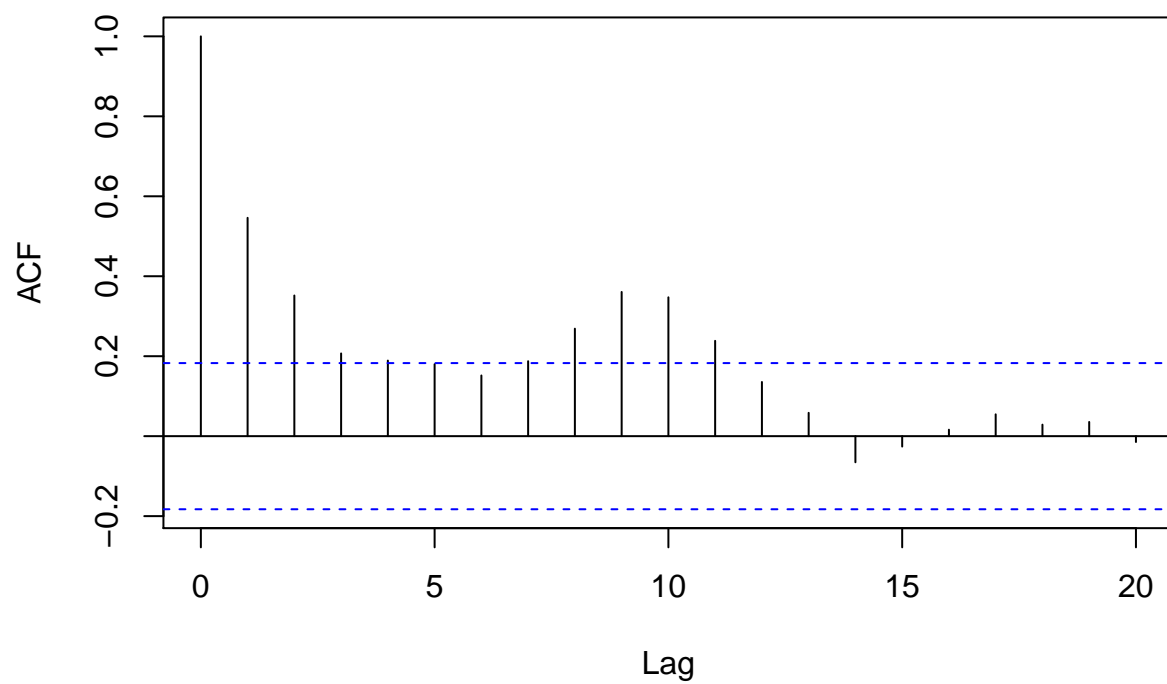


```
p4 <- ggplot(data = data.frame(StanRes.registered.nworkingday), aes(x = StanRes.registered.nworkingday))  
p4
```



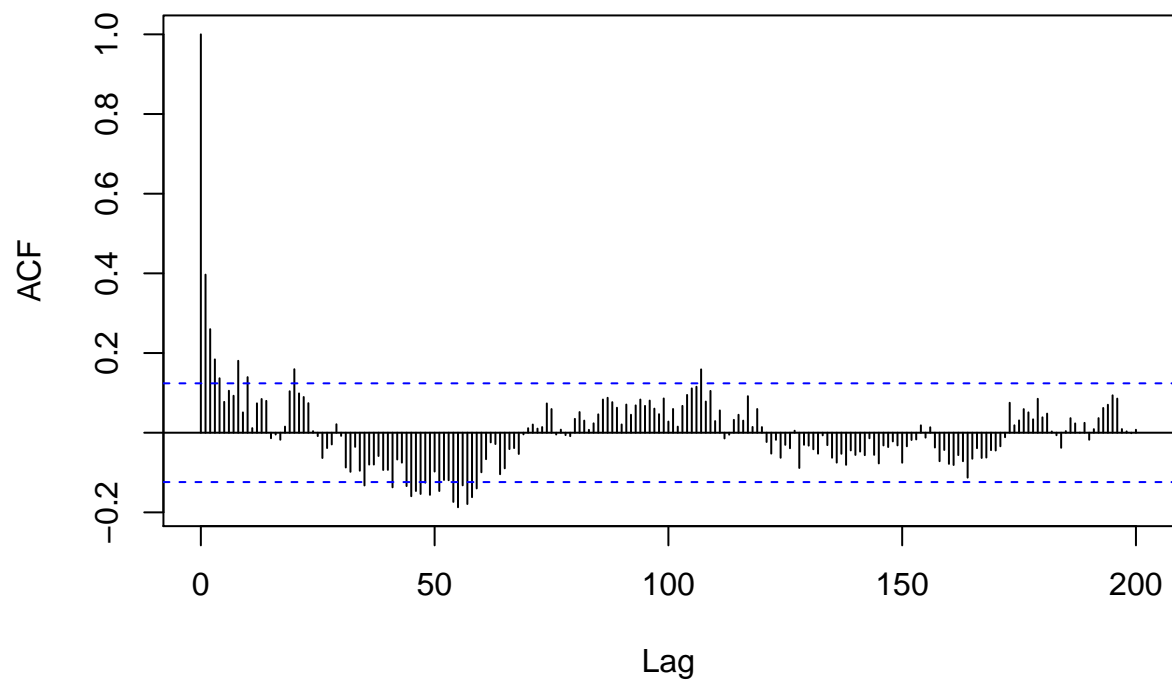
```
acf(StanRes.registered.nworkingday, main="ACF of standardised residuals")
```

ACF of standardised residuals



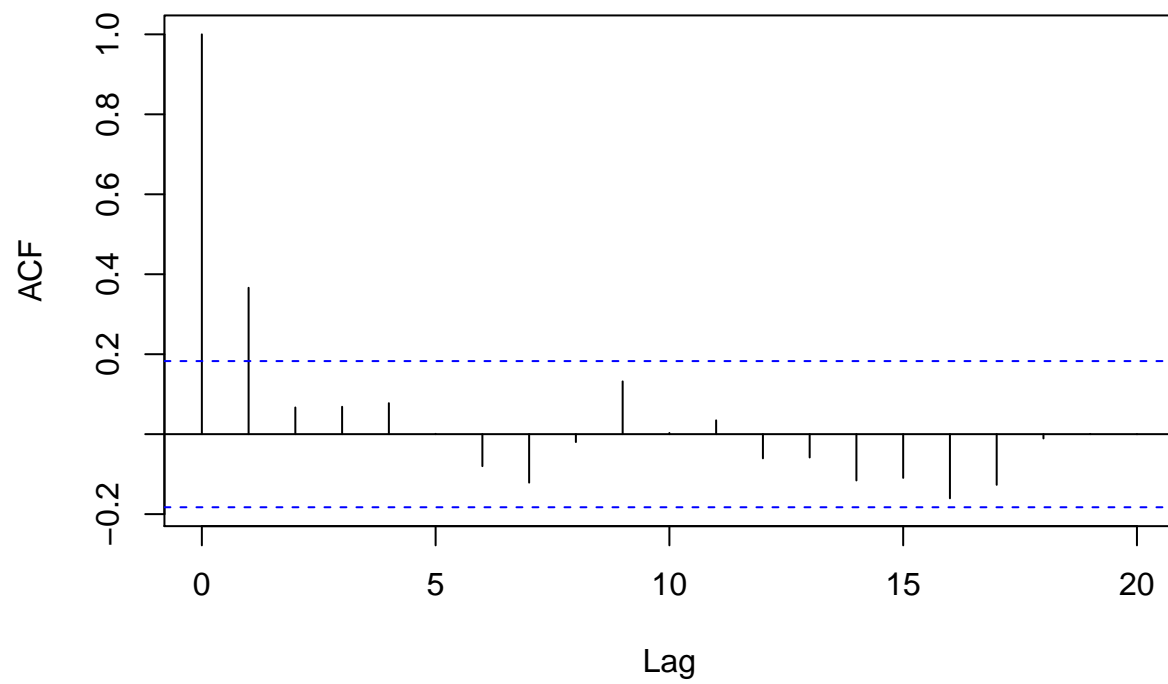
```
acf(StanRes.registered.workingday, main="ACF of standardised residuals", 200)
```

ACF of standardised residuals



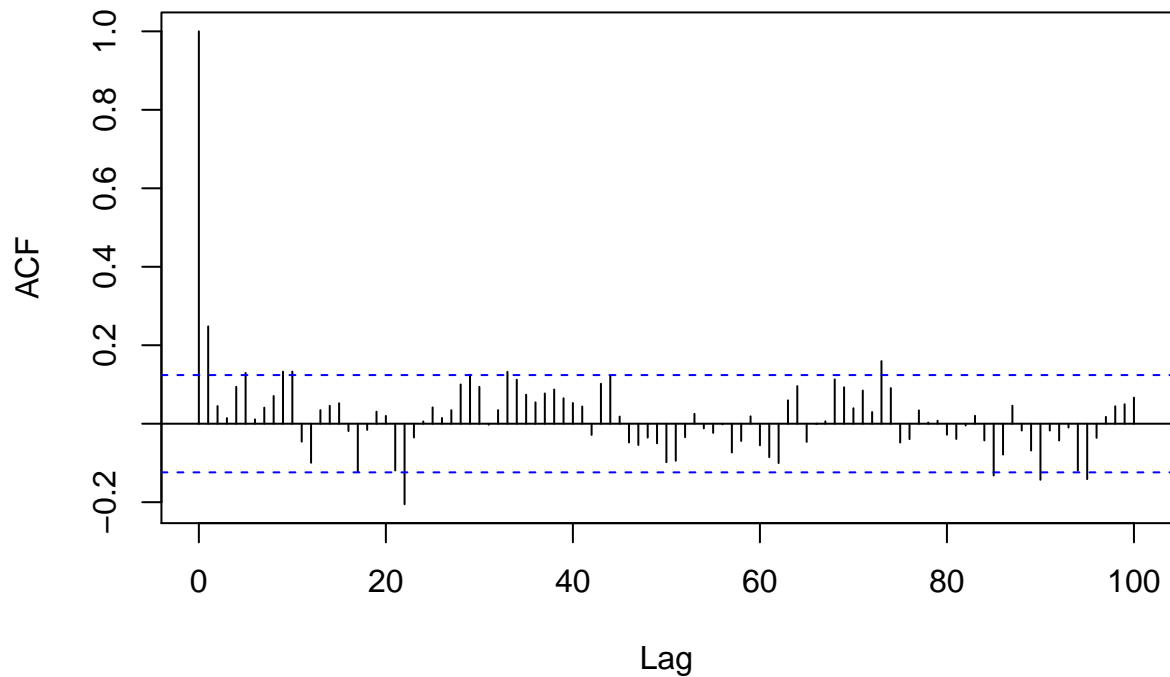
```
acf(StanRes.casual.nworkingday, main="ACF of standardised residuals")
```

ACF of standardised residuals



```
acf(StanRes.casual.workingday, main="ACF of standardised residuals", 100)
```

ACF of standardised residuals



Therefore using a gls With corrAR1 to correct correlations between y values in different periods.

model 2

```
m.gls.casual.workingday <- gls(casual ~ actual.windspeed + actual.temp + I(actual.temp^2) + weathersit,
correlation=corAR1(form=~instant), method="ML")

summary(m.gls.casual.workingday)
```

```
## Generalized least squares fit by maximum likelihood
## Model: casual ~ actual.windspeed + actual.temp + I(actual.temp^2) + weathersit
## Data: training.workingday
##      AIC      BIC    logLik
## 3208.902 3237.073 -1596.451
##
## Correlation Structure: ARMA(1,0)
## Formula: ~instant
## Parameter estimate(s):
##      Phi1
## 0.409339
##
## Coefficients:
##              Value Std.Error   t-value p-value
## (Intercept)  -292.1801  83.90200  -3.482397  0.0006
```



```
## actual.windspeed      -4.3055   1.83322 -2.348629  0.0196
## actual.temp           66.7114   9.19742  7.253271  0.0000
## I(actual.temp^2)      -0.9598   0.23163 -4.143686  0.0000
## weathersitModerate:Cloudy/Mist -137.8642  19.66188 -7.011750  0.0000
## weathersitBad: Rain/Snow/Fog  -341.1413  44.81133 -7.612836  0.0000
##
## Correlation:
##                      (Intr) act1.w act1.t I(.^2) wM:C/M
## actual.windspeed      -0.216
## actual.temp           -0.906 -0.075
## I(actual.temp^2)       0.841  0.083 -0.982
## weathersitModerate:Cloudy/Mist  0.049 -0.017 -0.171  0.188
## weathersitBad: Rain/Snow/Fog   0.109 -0.067 -0.161  0.167  0.299
##
## Standardized residuals:
##      Min      Q1      Med      Q3      Max
## -2.4735214 -0.6998380 -0.1837750  0.3880913  3.9953681
##
## Residual standard error: 154.4065
## Degrees of freedom: 250 total; 244 residual
```

```
m.gls.registered.workingday <- gls(registered ~ actual.temp + I(actual.temp^2)+actual.windspeed + weathersit + date_diff,
correlation=corAR1(form=~instant), method="ML")
```

```
summary(m.gls.registered.workingday)
```

```
## Generalized least squares fit by maximum likelihood
## Model: registered ~ actual.temp + I(actual.temp^2) + actual.windspeed + weathersit + date_diff
## Data: training.workingday
##      AIC      BIC    logLik
## 3782.694 3814.387 -1882.347
##
## Correlation Structure: ARMA(1,0)
## Formula: ~instant
## Parameter estimate(s):
##      Phi1
## 0.494695
##
## Coefficients:
##                      Value Std.Error   t-value p-value
## (Intercept)          232.7873  281.54230   0.826829  0.4091
## actual.temp           183.7581  33.35502   5.509157  0.0000
## I(actual.temp^2)      -2.7413   0.82643  -3.317067  0.0010
## actual.windspeed      -9.0304   5.78612  -1.560709  0.1199
## weathersitModerate:Cloudy/Mist -349.0268  60.52713  -5.766452  0.0000
## weathersitBad: Rain/Snow/Fog -1545.0791 138.93462 -11.120908  0.0000
## date_diff             3.2526   0.52415   6.205531  0.0000
##
## Correlation:
##                      (Intr) act1.t I(.^2) act1.w wM:C/M wB:R/S
## actual.temp          -0.862
## I(actual.temp^2)      0.810 -0.982
## actual.windspeed      -0.173 -0.148  0.145
## weathersitModerate:Cloudy/Mist  0.056 -0.167  0.184 -0.008
```

```
## weathersitBad: Rain/Snow/Fog      0.110 -0.133  0.141 -0.074  0.313
## date_diff                        0.051 -0.376  0.331  0.205  0.032 -0.039
##
## Standardized residuals:
##      Min      Q1      Med      Q3      Max
## -3.6582184 -0.4978623  0.1207504  0.7009100  1.9261065
##
## Residual standard error: 503.9427
## Degrees of freedom: 250 total; 243 residual
```

```
m.gls.casual.nworkingday <- gls(casual ~ actual.windspeed + actual.temp + I(actual.temp^2) + weathersit
correlation=corAR1(form=~instant), method="ML")
```

```
summary(m.gls.casual.nworkingday)
```

```
## Generalized least squares fit by maximum likelihood
## Model: casual ~ actual.windspeed + actual.temp + I(actual.temp^2) + weathersit
## Data: training.nworkingday
##      AIC      BIC    logLik
## 1693.409 1715.369 -838.7046
##
## Correlation Structure: ARMA(1,0)
## Formula: ~instant
## Parameter estimate(s):
##      Phi1
## 0.4745946
##
## Coefficients:
##              Value Std.Error   t-value p-value
## (Intercept)    -565.7639  284.55073  -1.988271  0.0493
## actual.windspeed    -23.3300   7.12136  -3.276056  0.0014
## actual.temp       162.5514  30.38286   5.350100  0.0000
## I(actual.temp^2)    -2.5204   0.76242  -3.305775  0.0013
## weathersitModerate:Cloudy/Mist -243.8728  76.67194  -3.180730  0.0019
## weathersitBad: Rain/Snow/Fog  -567.7572  239.91721  -2.366471  0.0197
##
## Correlation:
##              (Intr) act1.w act1.t I(.^2) wM:C/M
## actual.windspeed    -0.281
## actual.temp       -0.896 -0.060
## I(actual.temp^2)    0.830  0.080 -0.982
## weathersitModerate:Cloudy/Mist  0.002 -0.040 -0.088  0.095
## weathersitBad: Rain/Snow/Fog   0.004 -0.249  0.054 -0.044  0.044
##
## Standardized residuals:
##      Min      Q1      Med      Q3      Max
## -2.49539283 -0.70285493 -0.07174161  0.74973266  3.66719315
##
## Residual standard error: 380.7387
## Degrees of freedom: 115 total; 109 residual
```

```
m.gls.registered.nworkingday <- gls(registered ~ actual.temp + actual.windspeed + weathersit, data = t
```

```
summary(m.gls.registered.nworkingday)
```

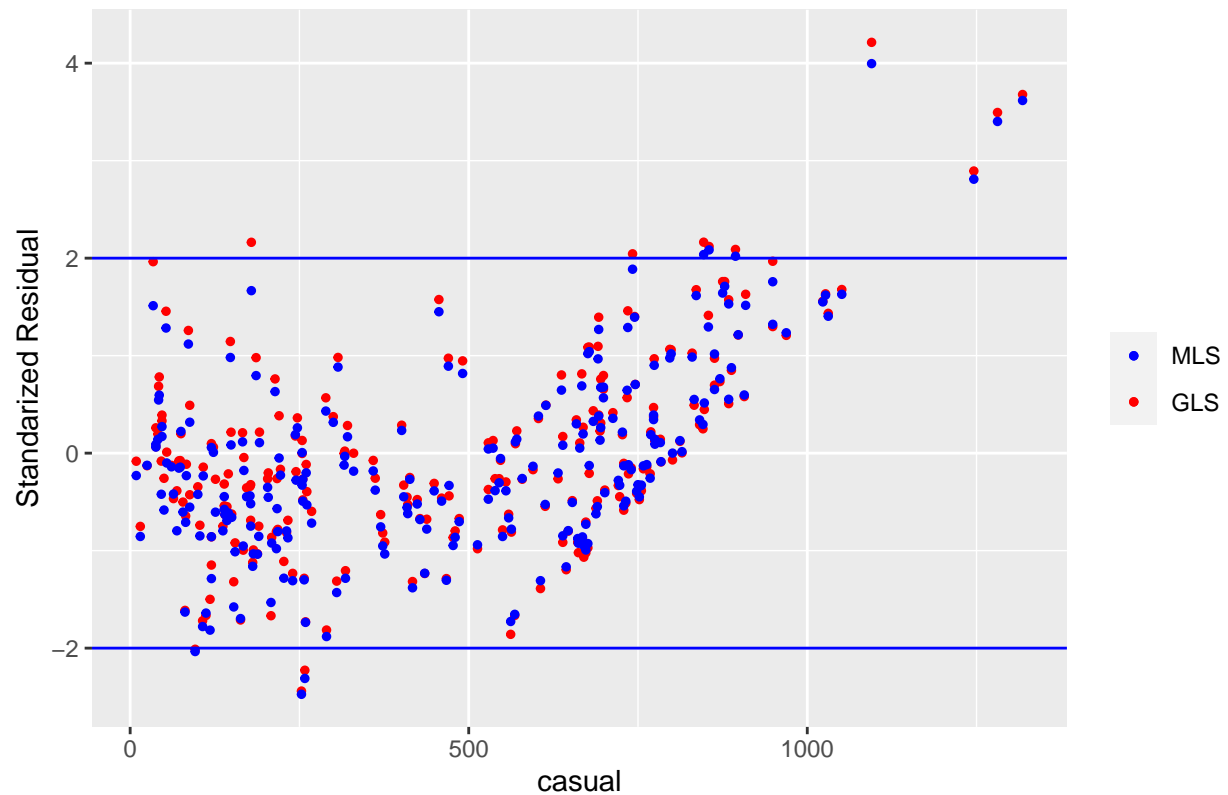
```
## Generalized least squares fit by maximum likelihood
## Model: registered ~ actual.temp + actual.windspeed + weathersit
## Data: training.nworkingday
##      AIC      BIC    logLik
## 1741.781 1760.996 -863.8907
##
## Correlation Structure: ARMA(1,0)
## Formula: ~instant
## Parameter estimate(s):
##      Phil
## 0.8274103
##
## Coefficients:
##                                Value Std.Error   t-value p-value
## (Intercept)                1303.4975  272.08118   4.790841  0.0000
## actual.temp                  67.1992   11.88276   5.655186  0.0000
## actual.windspeed             -20.1243    7.90017  -2.547327  0.0122
## weathersitModerate:Cloudy/Mist -265.4322   80.92342  -3.280042  0.0014
## weathersitBad: Rain/Snow/Fog  -1463.8807  226.05018  -6.475910  0.0000
##
## Correlation:
##                                (Intr) act1.t act1.w wM:C/M
## actual.temp                  -0.834
## actual.windspeed             -0.321 -0.048
## weathersitModerate:Cloudy/Mist -0.107  0.044 -0.060
## weathersitBad: Rain/Snow/Fog  -0.007  0.116 -0.280  0.026
##
## Standardized residuals:
##      Min      Q1      Med      Q3      Max
## -2.44652281 -0.72636708  0.03613507  0.81099354  1.86484084
##
## Residual standard error: 619.706
## Degrees of freedom: 115 total; 110 residual
```

Model2 diagnosis

```
StanResGLS.casual.nworkingday <- residuals(m.gls.casual.nworkingday,"pearson")
StanResGLS.casual.workingday <- residuals(m.gls.casual.workingday,"pearson")
StanResGLS.registered.nworkingday <- residuals(m.gls.registered.nworkingday,"pearson")
StanResGLS.registered.workingday <- residuals(m.gls.registered.workingday,"pearson")
```

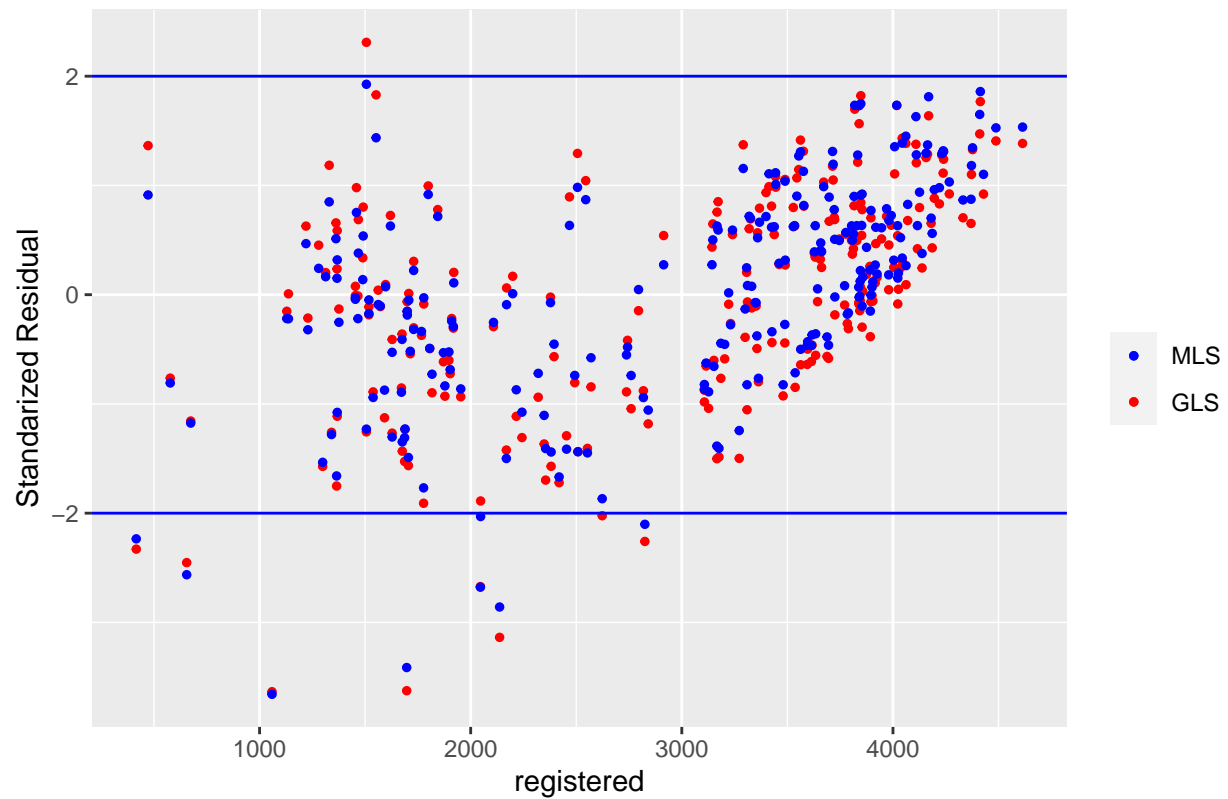
```
ggplot(data=training.workingday, aes(x=casual)) +
  geom_point(aes(y=StanRes.casual.workingday, color = "MLS"), size = 1) +
  geom_point(aes(y=StanResGLS.casual.workingday, color = "GLS"), size = 1) +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') +
  scale_color_manual(name = element_blank(), labels = c("MLS", "GLS"), values = c("blue", "red")) +
  labs(y = "Standardized Residual") + ggtitle("Standardized Residuals MLS Plot for casual bikers on workingday")
```

Standardized Residuals MLS Plot for casual bikers on workingdays



```
ggplot(data=training.workingday, aes(x=registered)) +
  geom_point(aes(y=StanRes.registered.workingday, color = "MLS"), size = 1) +
  geom_point(aes(y=StanResGLS.registered.workingday, color = "GLS"), size = 1) +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') +
  scale_color_manual(name = element_blank(), labels = c("MLS", "GLS"), values = c("blue", "red")) +
  labs(y = "Standardized Residual") + ggtitle("Standardized Residuals MLS Plot for registered bikers on workingdays")
```

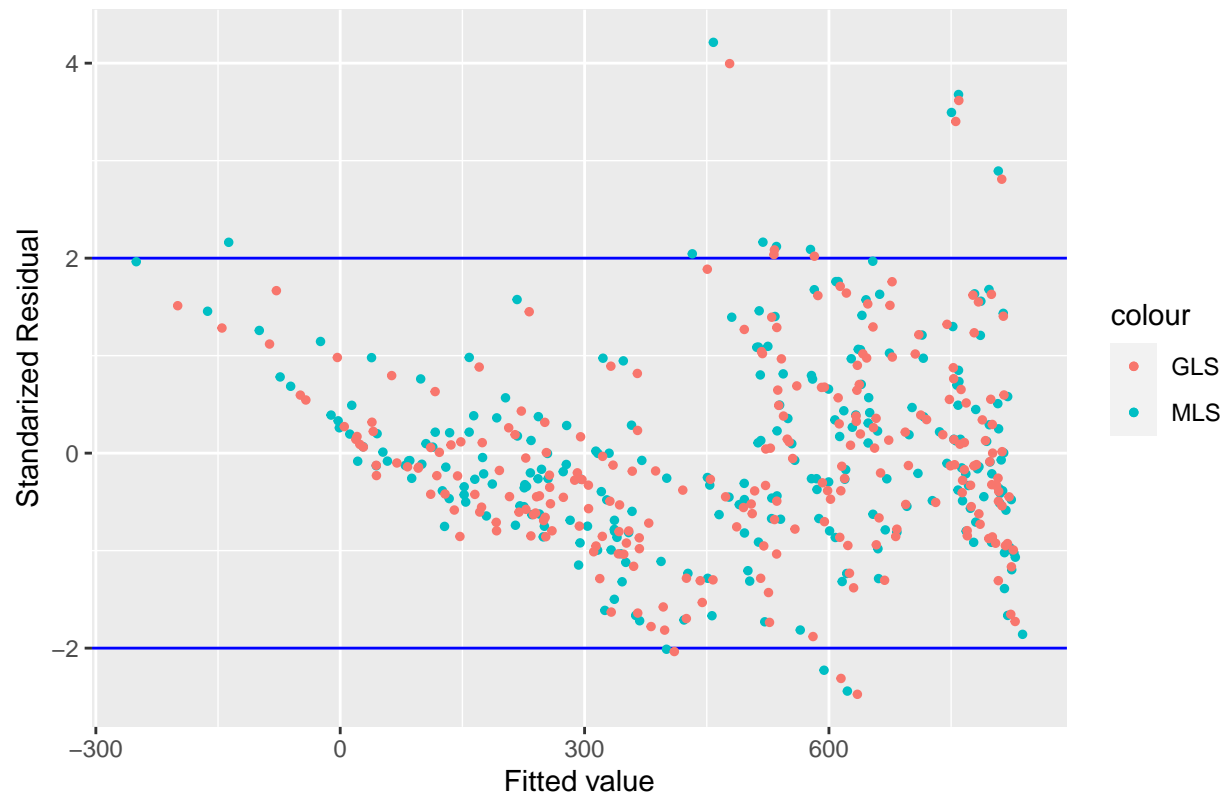
Standardized Residuals MLS Plot for registered bikers on workingdays



```
FittedGLS_casual.workingday = fitted(m.gls.casual.workingday)
```

```
ggplot() +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') + geom_point(aes(x=FittedGLS_casual.workingday, y=StanResGLS.casual.workingday, color = "GLS"), size = 1)
  labs(y = "Standardized Residual") + labs(x = "Fitted value") +
  ggtitle("Standardized Residuals Plot (Fitted) for casual bikers on workingdays")
```

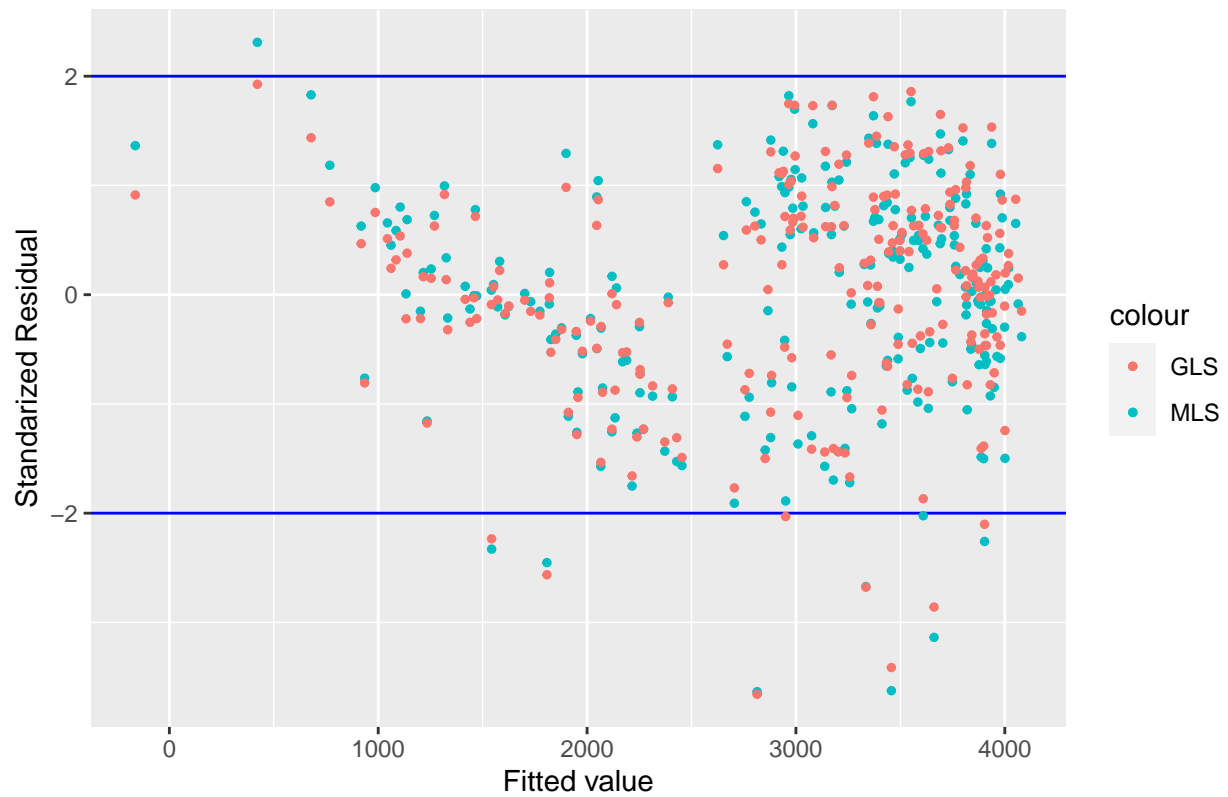
Standardized Residuals Plot (Fitted) for casual bikers on workingdays



```
FittedGLS_registered.workingday = fitted(model.registered.workingday)
```

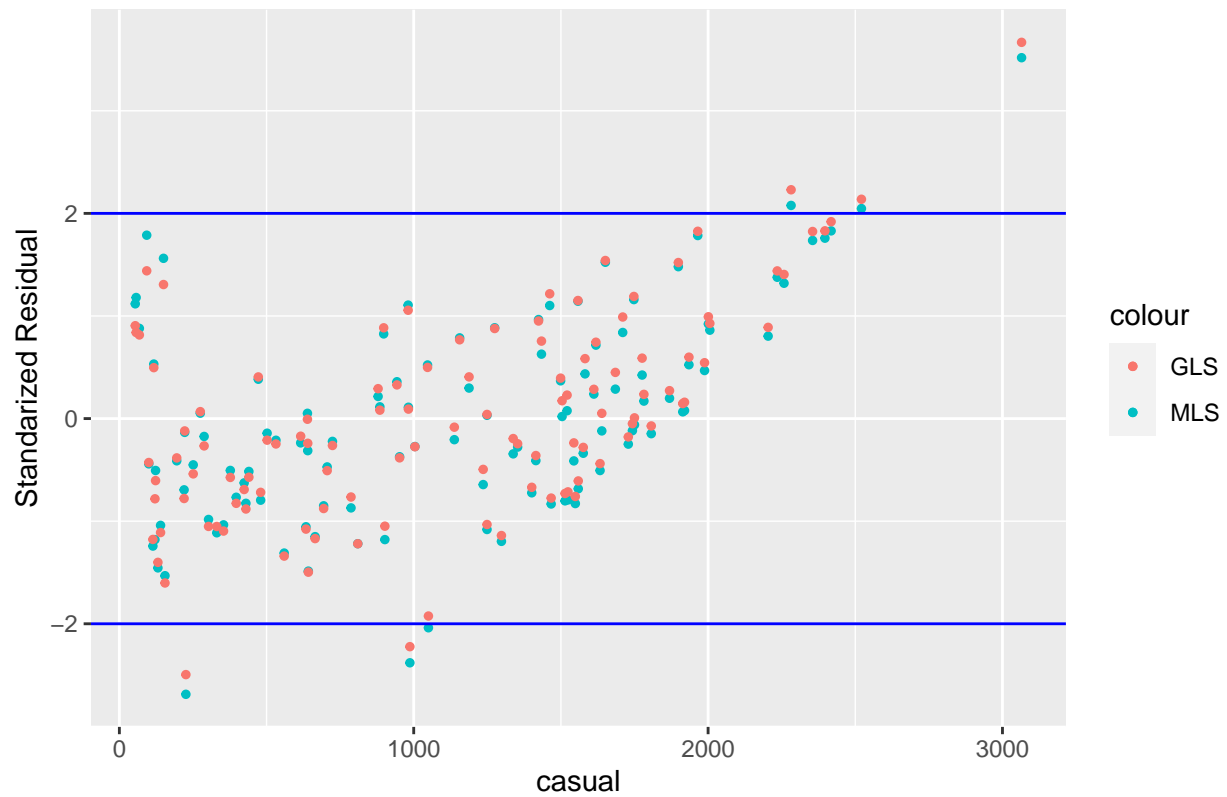
```
ggplot() +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') + geom_point(aes(x=FittedGLS_registered.workingday, y=StanResGLS_registered.workingday, color = "GLS"), size=1) +
  geom_point(aes(x=FittedMLS_registered.workingday, y=StanResMLS_registered.workingday, color = "MLS"), size=1) +
  labs(y = "Standardized Residual") + labs(x = "Fitted value") +
  ggtitle("Standardized Residuals Plot (Fitted) for registered bikers on workingdays")
```

Standardized Residuals Plot (Fitted) for registered bikers on workingdays



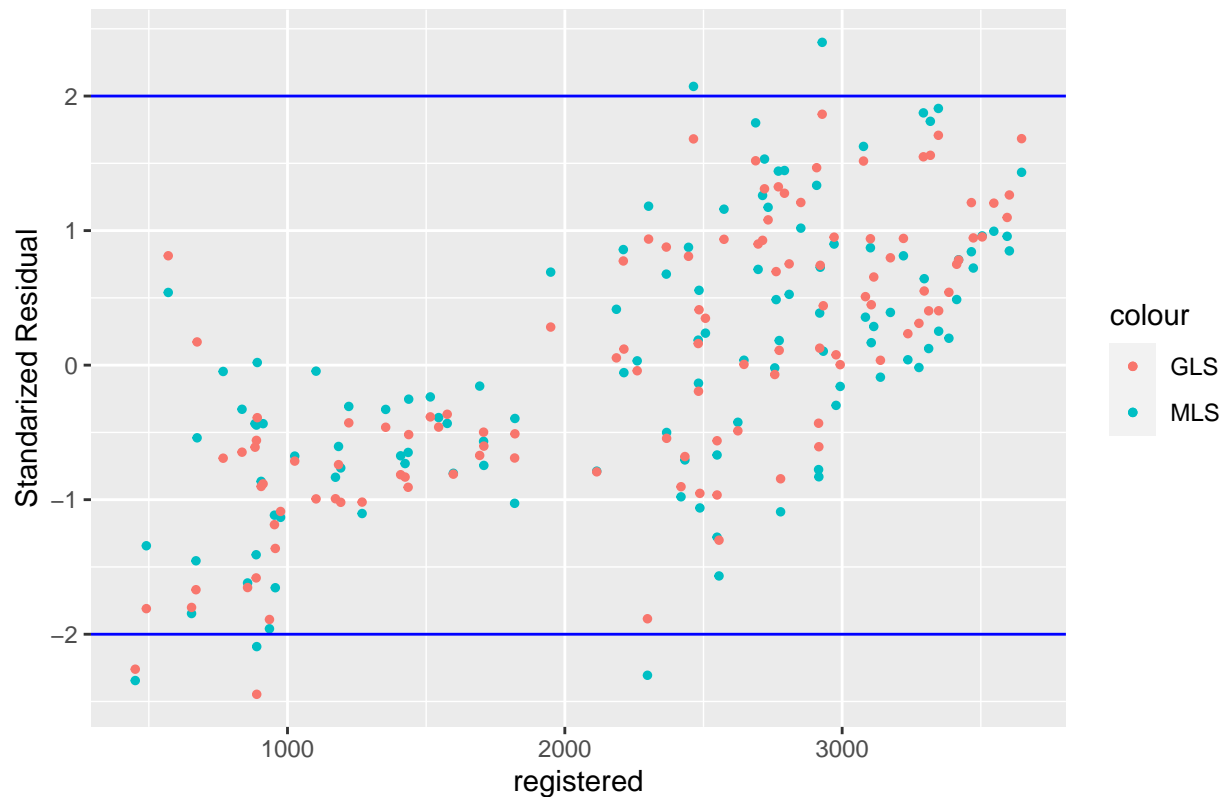
```
ggplot(data=training.nworkingday, aes(x=casual)) +
  geom_point(aes(y=StanRes.casual.nworkingday, color = "MLS"), size = 1) +
  geom_point(aes(y=StanResGLS.casual.nworkingday, color = "GLS"), size = 1) +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') +
  labs(y = "Standardized Residual") + ggtitle("Standardized Residuals MLS Plot for casual bikers onnon-work")
```

Standardized Residuals MLS Plot for casual bikers onnon-workingdays



```
ggplot(data=training.nworkingday, aes(x=registered)) +
  geom_point(aes(y=StanRes.registered.nworkingday, color = "MLS"), size = 1) +
  geom_point(aes(y=StanResGLS.registered.nworkingday, color = "GLS"), size = 1) +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') +
  labs(y = "Standardized Residual") + ggtitle("Standardized Residuals MLS Plot for registered bikers on non-
```

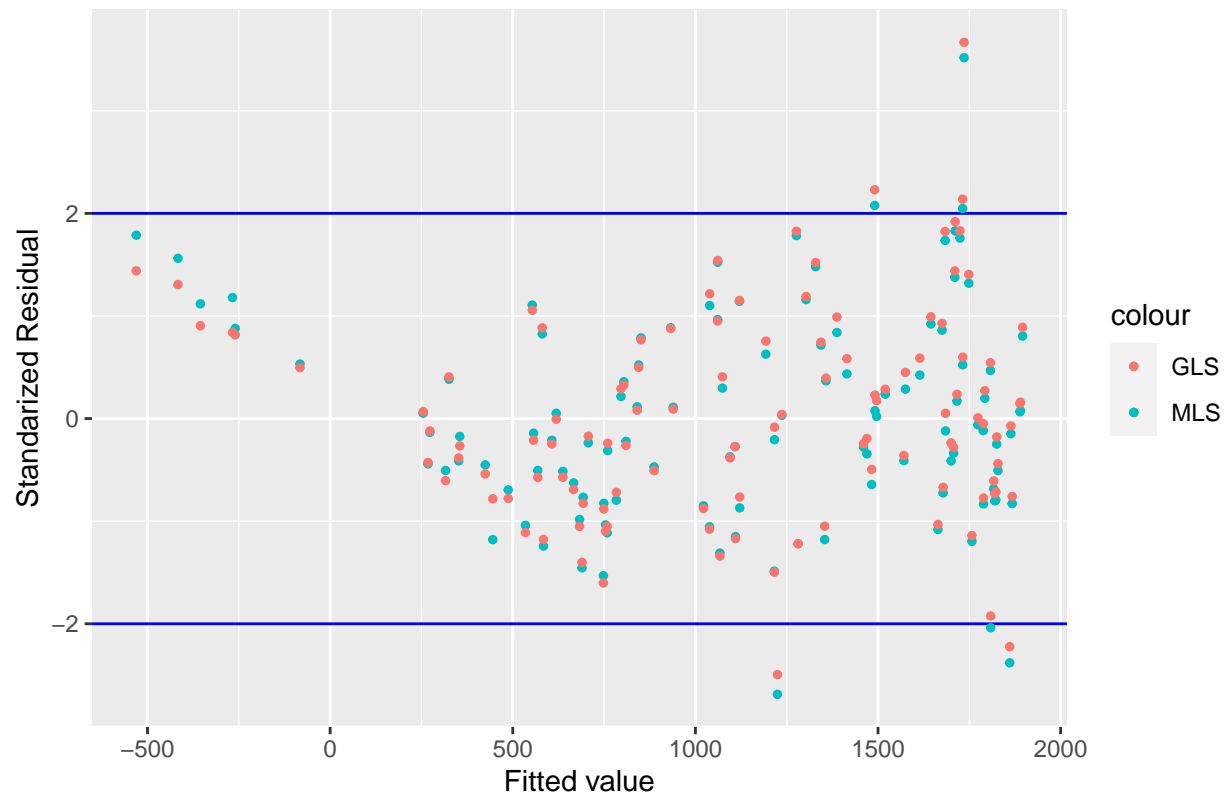

Standardized Residuals MLS Plot for registered bikers on non-workingdays



```
FittedGLS_casual.nworkingday = fitted(model.casual.nworkingday)
```

```
ggplot() +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') + geom_point(aes(x=FittedGLS_casual.nworkingday, y=StanResGLS.casual.nworkingday, color = "GLS"), size = 1) +
  geom_point(aes(x=FittedGLS_casual.nworkingday, y=StanResMLS.casual.nworkingday, color = "MLS"), size = 1) +
  labs(y = "Standardized Residual") + labs(x = "Fitted value") +
  ggtitle("Standardized Residuals WLS Plot (Fitted) for casual bikers on non-workingdays")
```

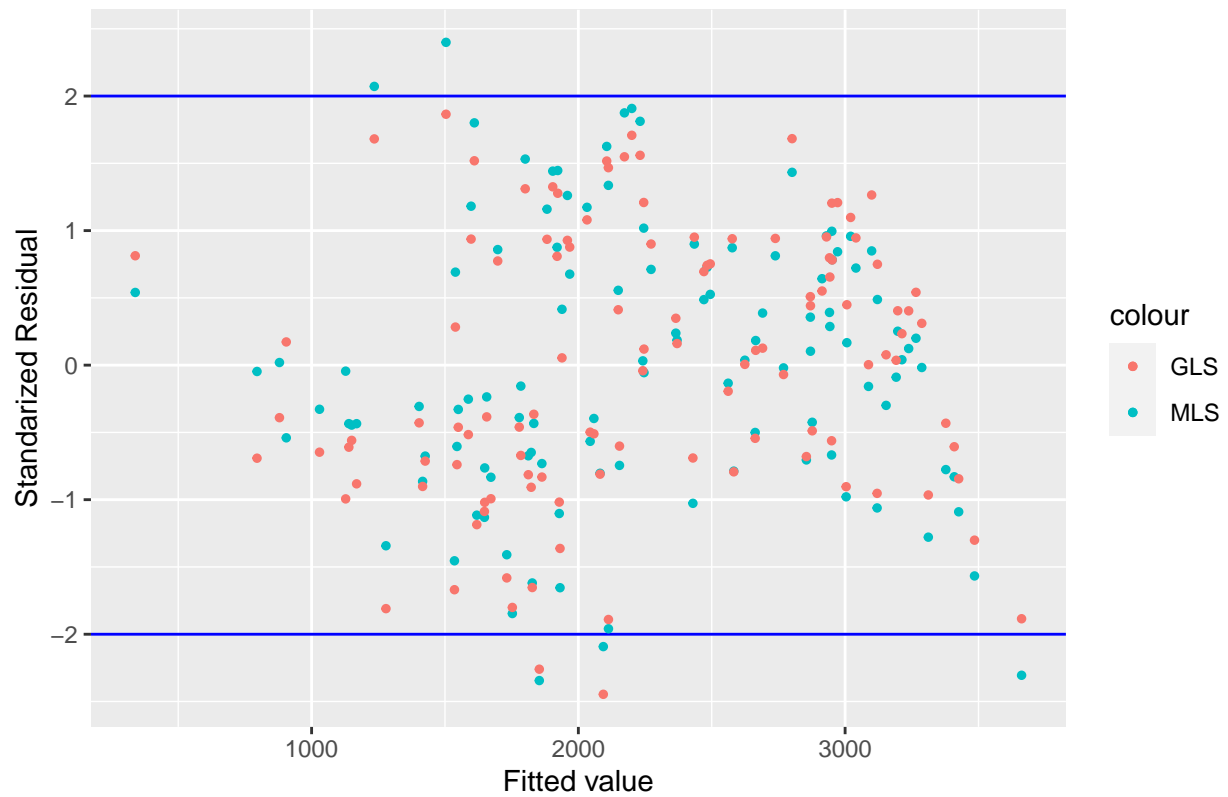
Standardized Residuals WLS Plot (Fitted) for casual bikers on non-workingd



```
FittedGLS_registered.nworkingday = fitted(model.registered.nworkingday)

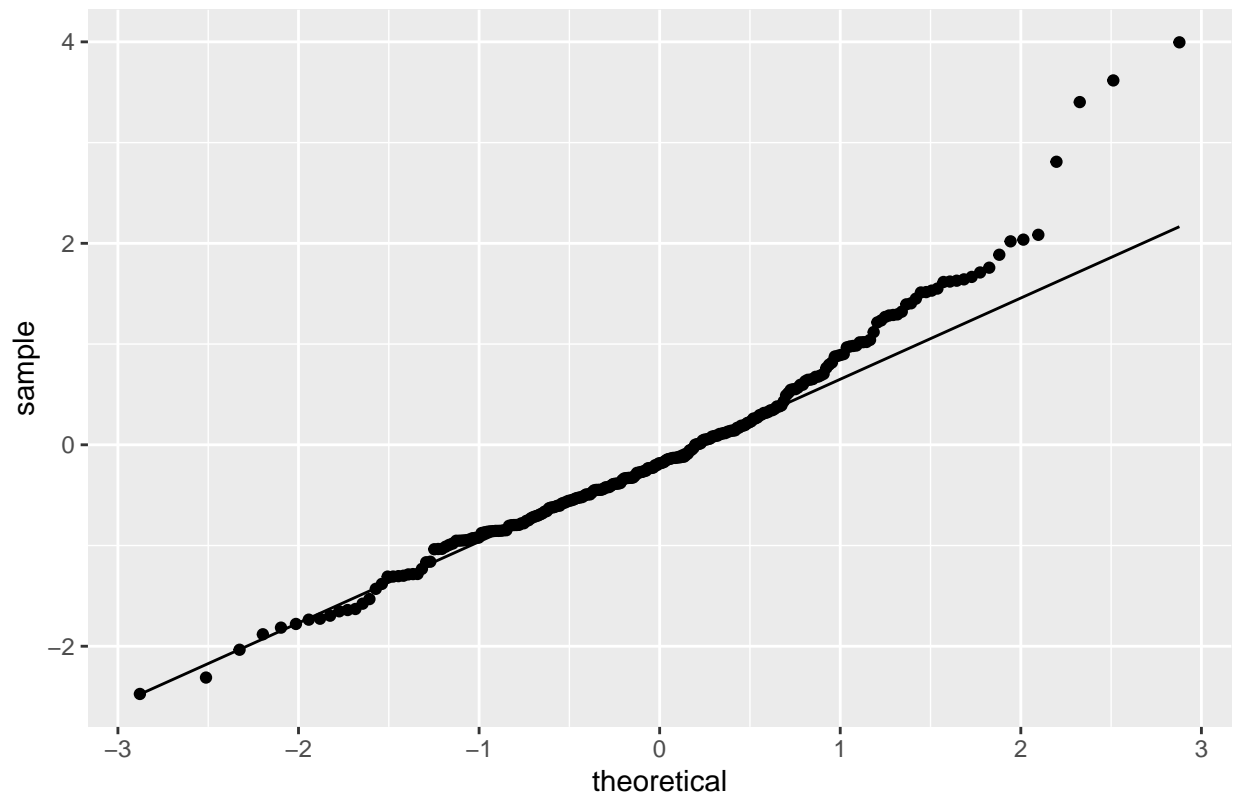
ggplot() +
  geom_hline(yintercept=2,color='blue') + geom_hline(yintercept=-2, color='blue') + geom_point(aes(x=FittedGLS_registered.nworkingday, y=StanResGLS_registered.nworkingday, color = "GLS"),
  geom_point(aes(x=FittedMLS_registered.nworkingday, y=StanResMLS_registered.nworkingday, color = "MLS"),
  labs(y = "Standardized Residual") + labs(x = "Fitted value") +
  ggtitle("Standardized Residuals WLS Plot (Fitted) for registered bikers on workingdays")
```

Standardized Residuals WLS Plot (Fitted) for registered bikers on workingday



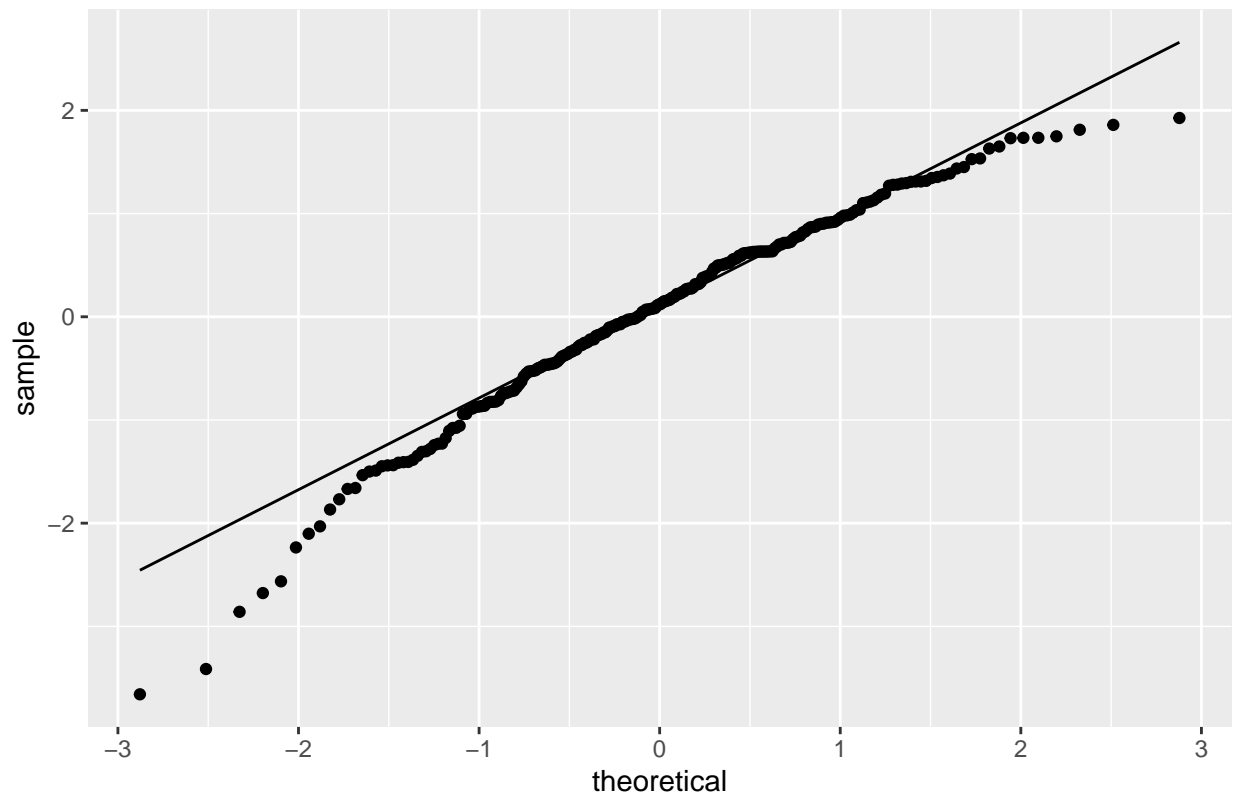
```
p <- ggplot(data.frame(StanResGLS.casual.workingday), aes(sample = StanResGLS.casual.workingday)) +  
  ggtitle("QQ MLS Plot for casual bikers on workingdays")  
p + stat_qq() + stat_qq_line()
```

QQ MLS Plot for casual bikers on workingdays



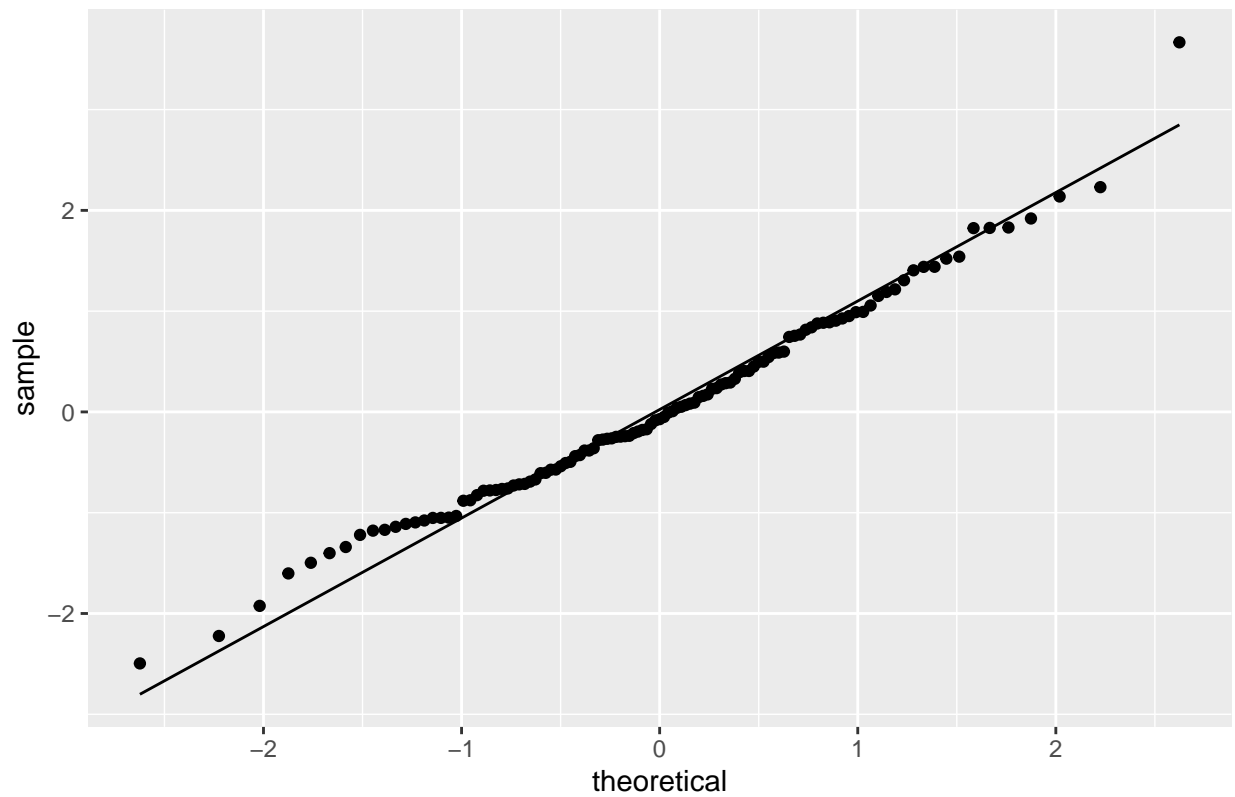
```
p <- ggplot(data.frame(StanResGLS.registered.workingday), aes(sample = StanResGLS.registered.workingday))  
ggtitle("QQ MLS Plot for registered bikers on workingdays")  
p + stat_qq() + stat_qq_line()
```

QQ MLS Plot for registered bikers on workingdays



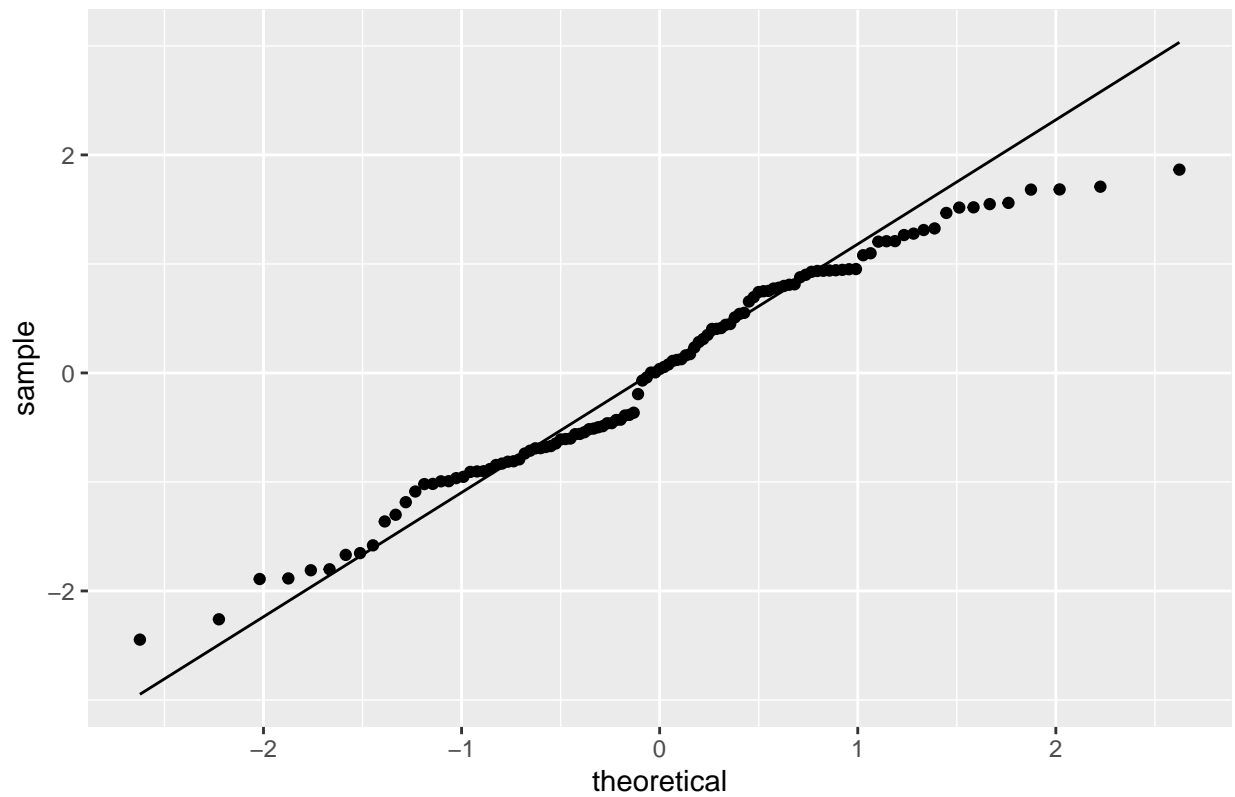
```
p <- ggplot(data.frame(StanResGLS.casual.nworkingday), aes(sample = StanResGLS.casual.nworkingday)) +  
  ggtitle("QQ MLS Plot for casual bikers on non-workingdays")  
p + stat_qq() + stat_qq_line()
```

QQ MLS Plot for casual bikers on non-workingdays

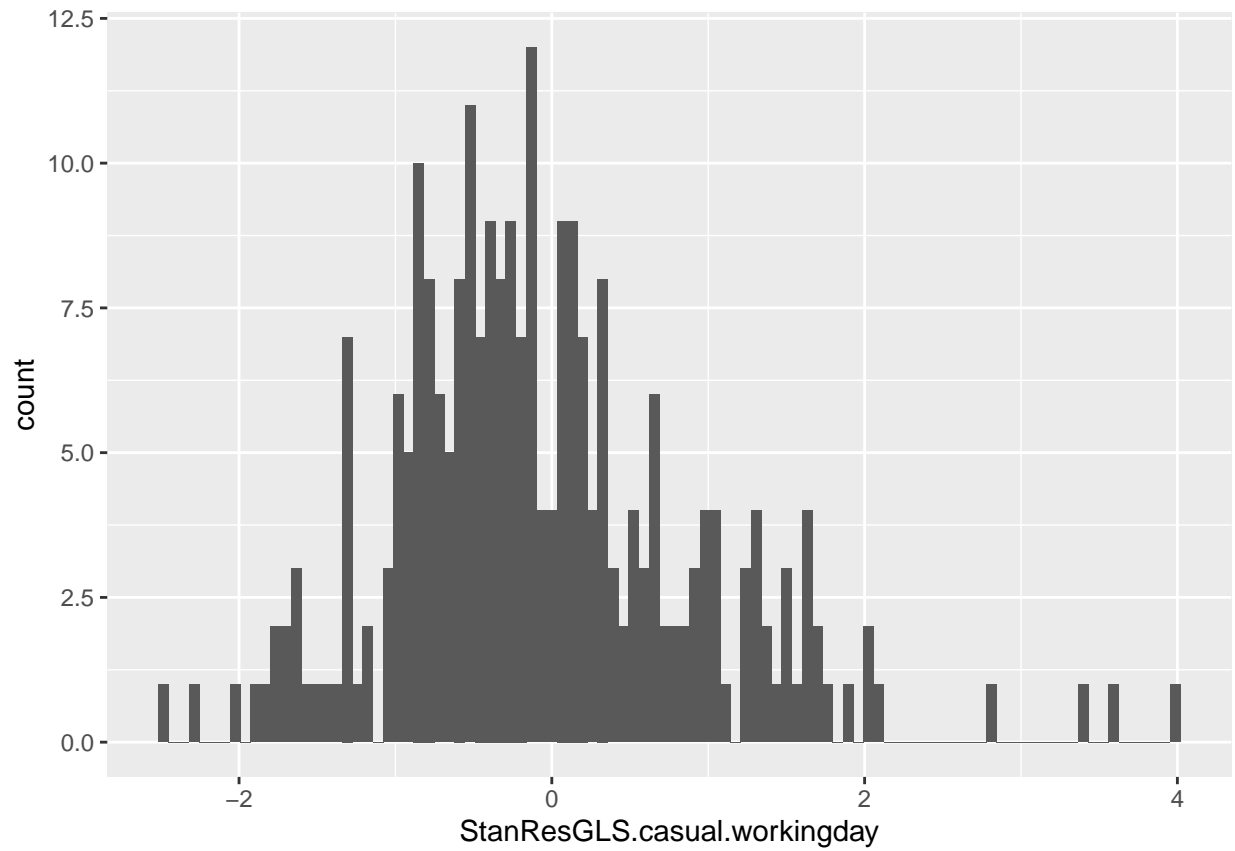


```
p <- ggplot(data.frame(StanResGLS.registered.nworkingday), aes(sample = StanResGLS.registered.nworkingday))
p + stat_qq() + stat_qq_line()
```

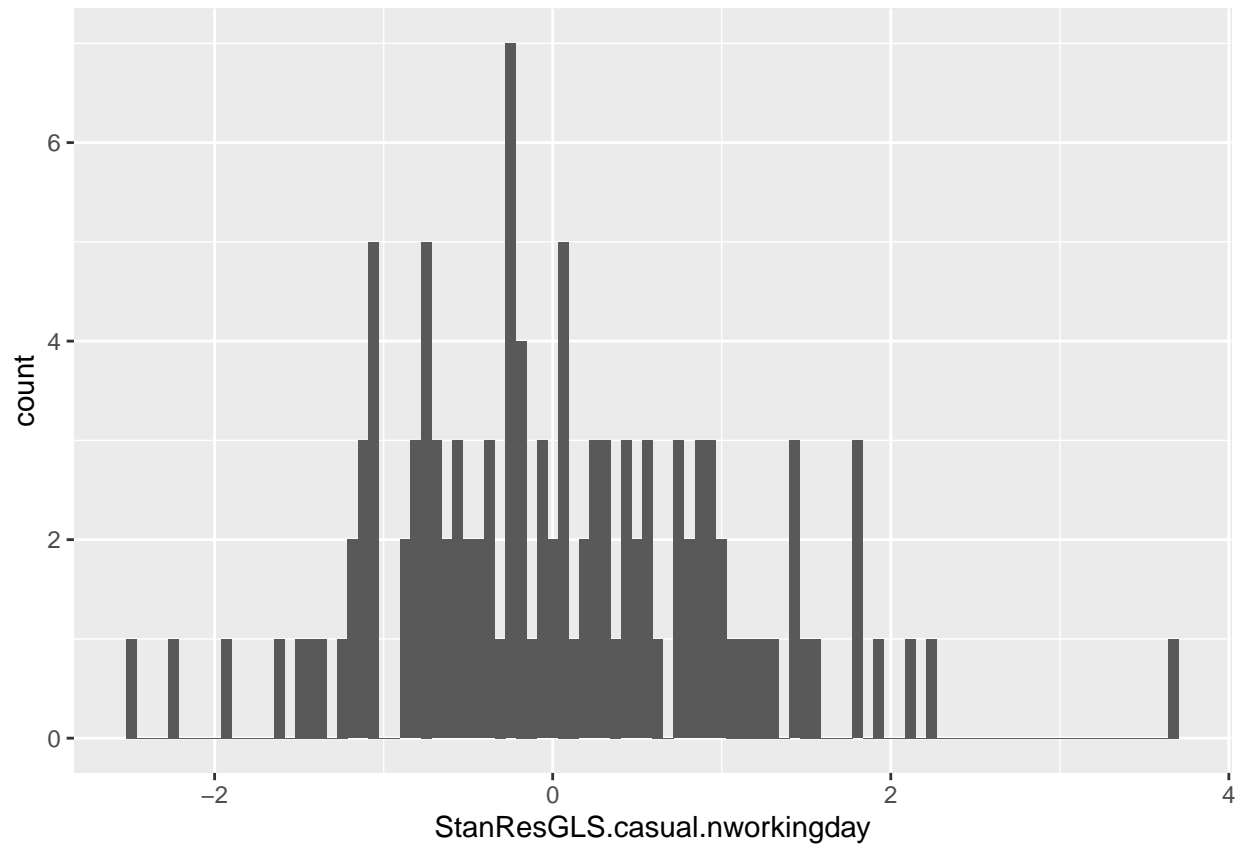
QQ MLS Plot for registered bikers on non-workingdays



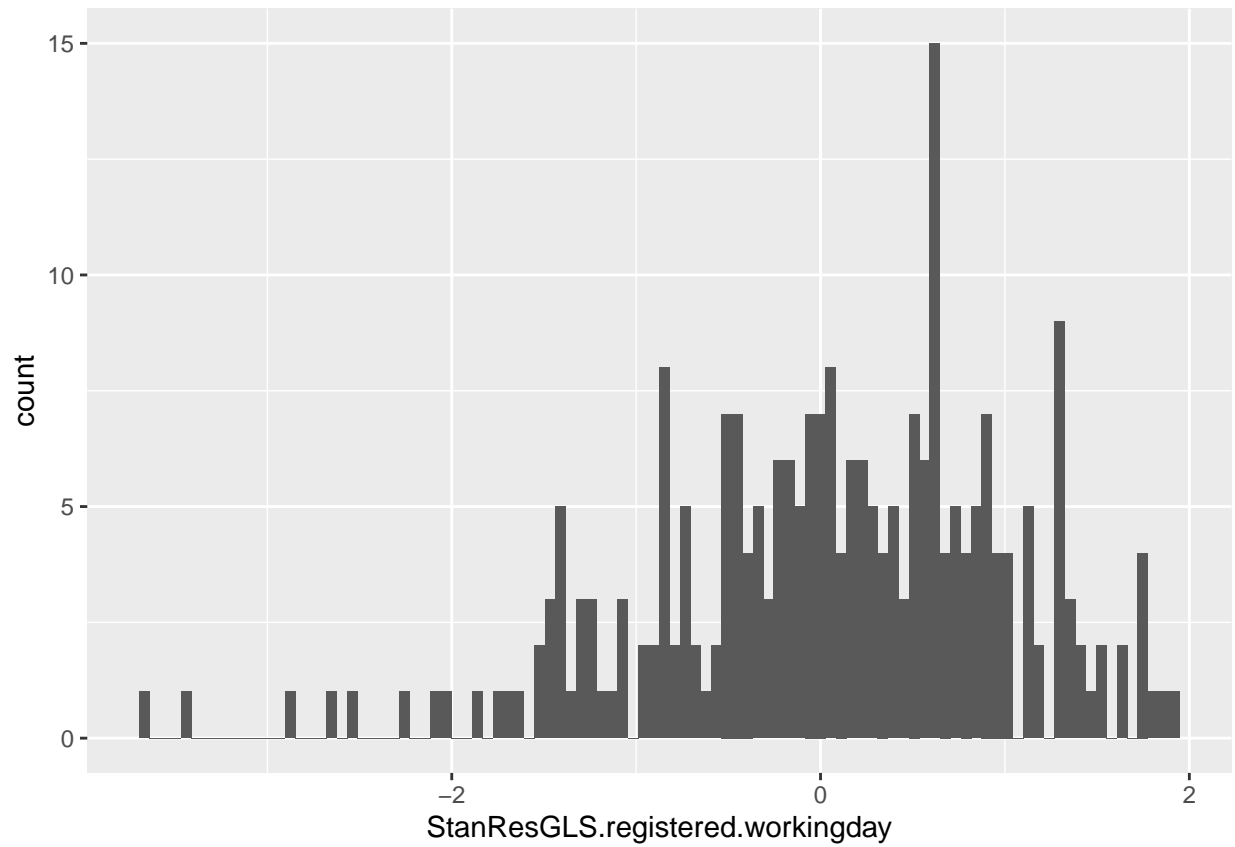
```
p1 <- ggplot(data = data.frame(StanResGLS.casual.workingday), aes(x = StanResGLS.casual.workingday)) +  
p1
```



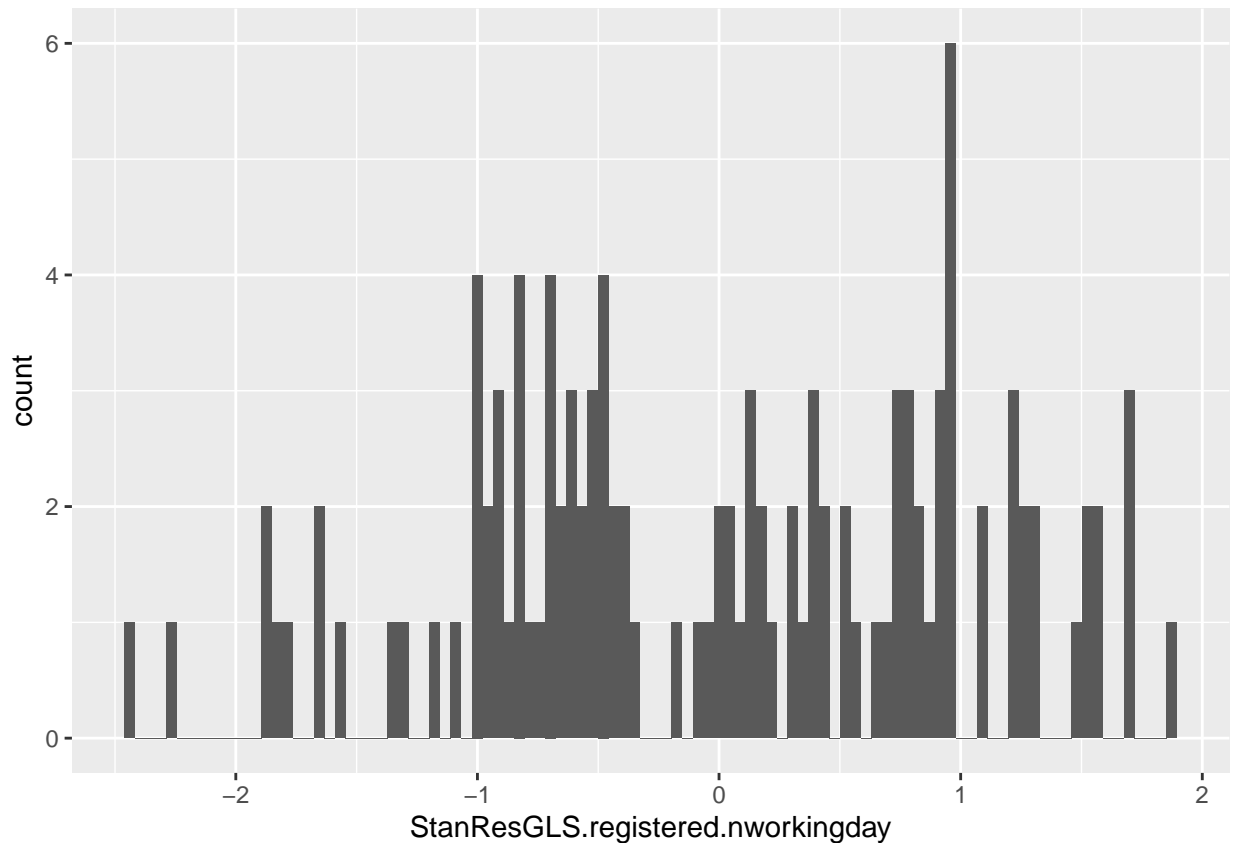
```
p2 <- ggplot(data = data.frame(StanResGLS.casual.nworkingday), aes(x = StanResGLS.casual.nworkingday))  
p2
```

```
p3 <- ggplot(data = data.frame(StanResGLS.registered.workingday), aes(x = StanResGLS.registered.workingday))  
p3
```



```
p4 <- ggplot(data = data.frame(StanResGLS.registered.nworkingday), aes(x = StanResGLS.registered.nworkingday))  
p4
```



Validation with model 2

```
p.casual.workingday <- predict(m.gls.casual.workingday, validate.workingday)
error.casual.workingday <- (p.casual.workingday - validate.workingday$casual)
RMSE_validation.causal.workingday <- sqrt(mean(error.casual.workingday^2))
RMSEGLS.casual.workingday <- sqrt(mean(resid(m.gls.casual.workingday)^2))

p.casual.nworkingday <- predict(m.gls.casual.nworkingday, validate.nworkingday)
error.casual.nworkingday <- (p.casual.nworkingday - validate.nworkingday$casual)
RMSE_validation.causal.nworkingday <- sqrt(mean(error.casual.nworkingday^2))
RMSEGLS.casual.nworkingday <- sqrt(mean(resid(m.gls.casual.nworkingday)^2))
```

Square root mean square error for validation data set

```
RMSE_validation.causal.workingday
```

```
## [1] 353.0044
```

```
RMSE_validation.causal.nworkingday
```

```
## [1] 761.3066
```

square root mean square error for training data set

```
RMSEGLS.casual.workingday
```

```
## [1] 151.8887
```

```
RMSEGLS.casual.nworkingday
```

```
## [1] 382.2737
```

```
p.registered.workingday <- predict(m.gls.registered.workingday, validate.workingday)
error.registered.workingday <- (p.registered.workingday - validate.workingday$registered)
RMSE_validation.registered.workingday <- sqrt(mean(error.registered.workingday^2))
RMSEGLS.registered.workingday <- sqrt(mean(resid(m.gls.registered.workingday)^2))

p.registered.nworkingday <- predict(m.gls.registered.nworkingday, validate.nworkingday)
error.registered.nworkingday <- (p.registered.nworkingday - validate.nworkingday$registered)
RMSE_validation.registered.nworkingday <- sqrt(mean(error.registered.nworkingday^2))
RMSEGLS.registered.nworkingday <- sqrt(mean(resid(m.gls.registered.nworkingday)^2))
```

Square root mean square error for validation data set

```
RMSE_validation.registered.workingday
```

```
## [1] 1109.121
```

```
RMSE_validation.registered.nworkingday
```

```
## [1] 1674.663
```

square root mean square error for training data set

```
RMSEGLS.registered.workingday
```

```
## [1] 490.0298
```

```
RMSEGLS.registered.nworkingday
```

```
## [1] 613.6237
```

Relative mean square error

```
mean((error.casual.workingday)^2) / mean((validate.workingday$casual)^2)
```

```
## [1] 0.1715408
```

```
mean((error.casual.nworkingday)^2) / mean((validate.nworkingday$casual)^2)
```

```
## [1] 0.1646588
```

```
mean((error.registered.workingday)^2) / mean((validate.workingday$registered)^2)
```

```
## [1] 0.04596131
```

```
mean((error.registered.nworkingday)^2) / mean((validate.nworkingday$registered)^2)
```

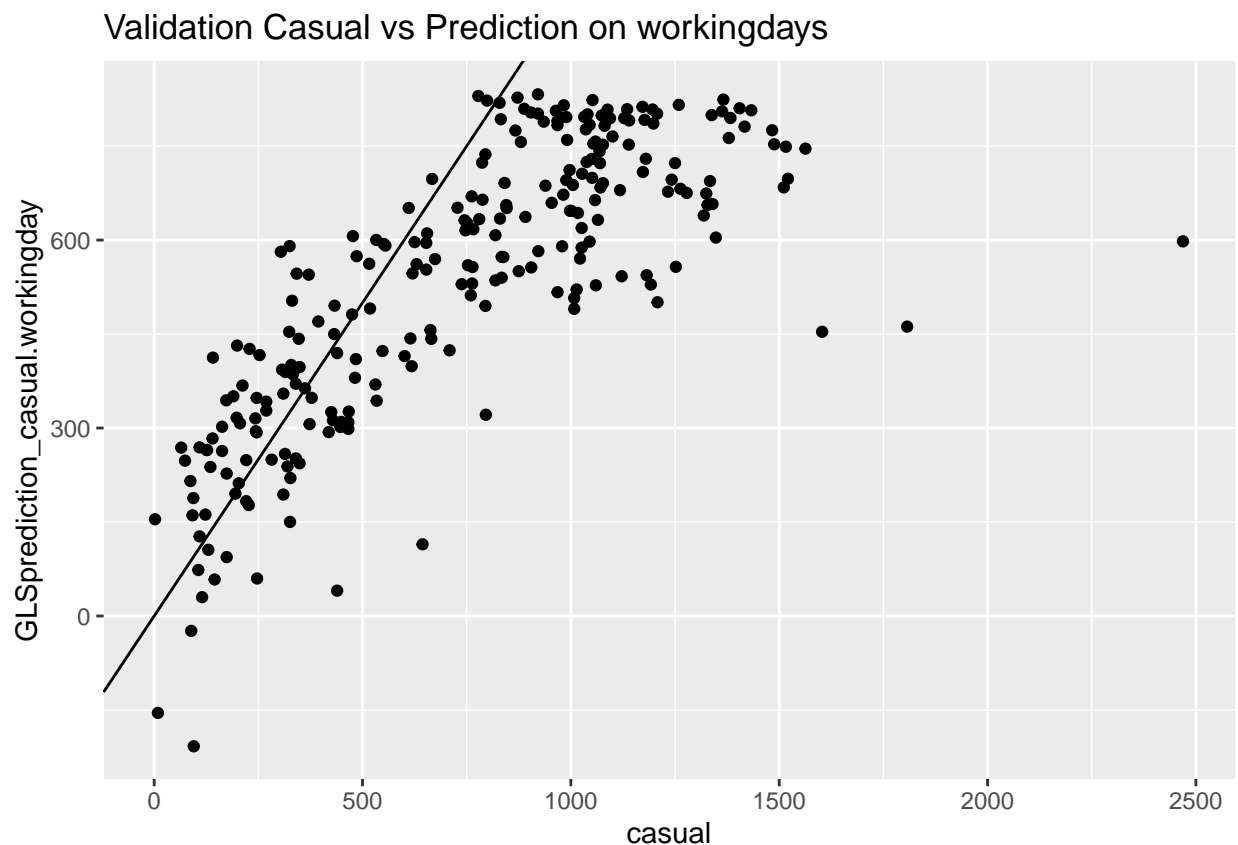
```
## [1] 0.1878861
```

Our model predicts the bike data in 2012 with mean error of 23 percent and 16 percent within the true value of casual and registered counts respectively. However, our model have twice as large of square root of mean square error with the validation data set than with the training data set.

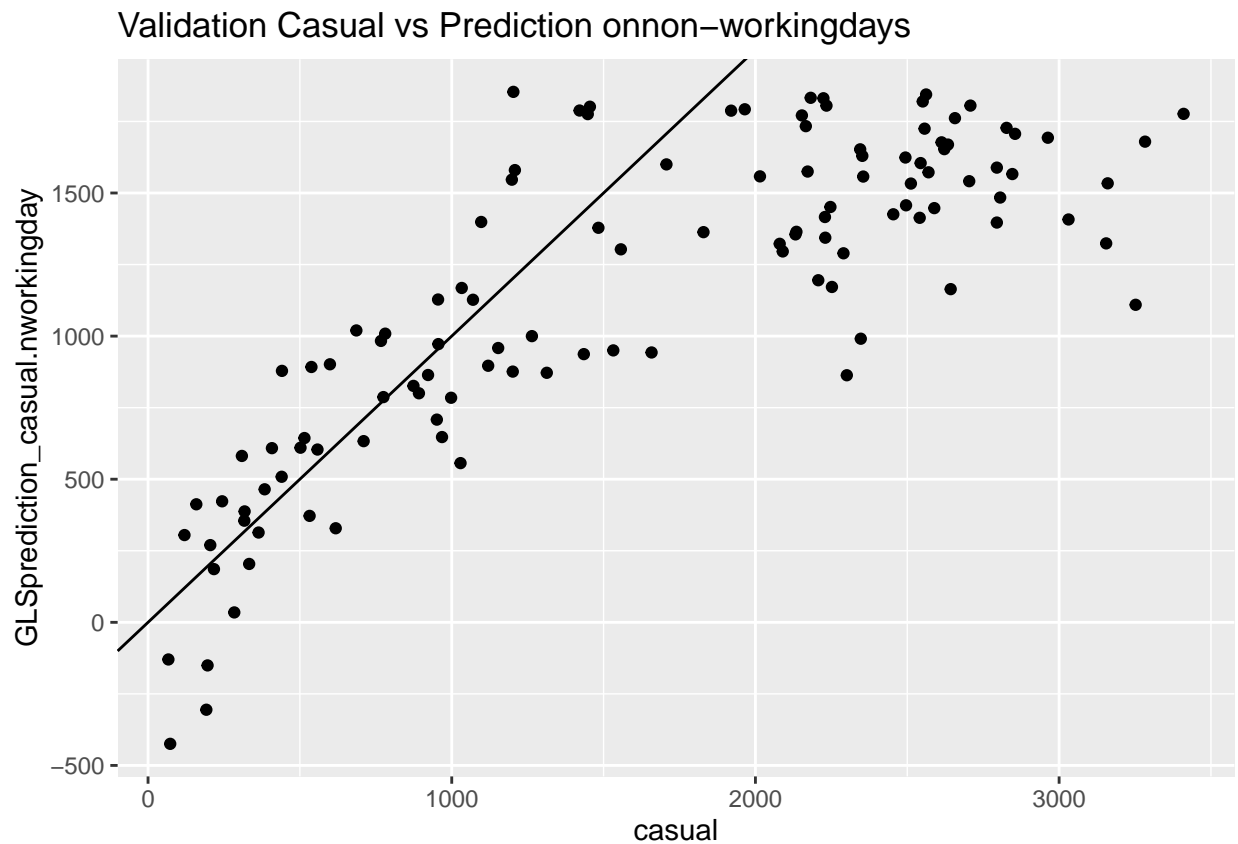
```
validate.workingday <- validate.workingday %>% mutate(GLSprediction_registered.workingday = predict(m.g
```

```
validate.nworkingday <- validate.nworkingday %>% mutate(GLSprediction_registered.nworkingday = predict(m
```

```
ggplot(validate.workingday, aes(x = casual, y = GLSprediction_casual.workingday)) + geom_point() +  
geom_abline(intercept = 0, slope = 1) +  
ggtitle("Validation Casual vs Prediction on workingdays")
```

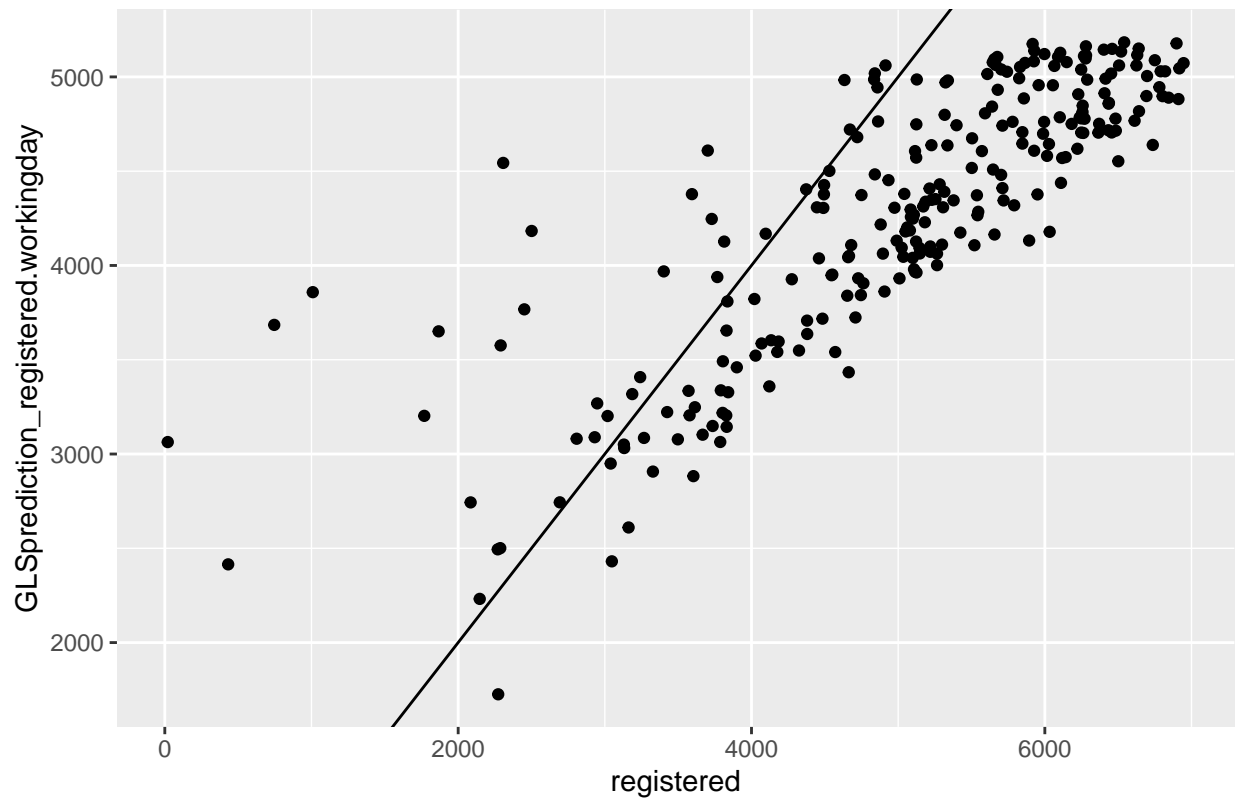


```
ggplot(validate.nworkingday, aes(x = casual, y = GLSprediction_casual.nworkingday)) + geom_point() +  
geom_abline(intercept = 0, slope = 1) +  
ggtitle("Validation Casual vs Prediction onnon-workingdays")
```



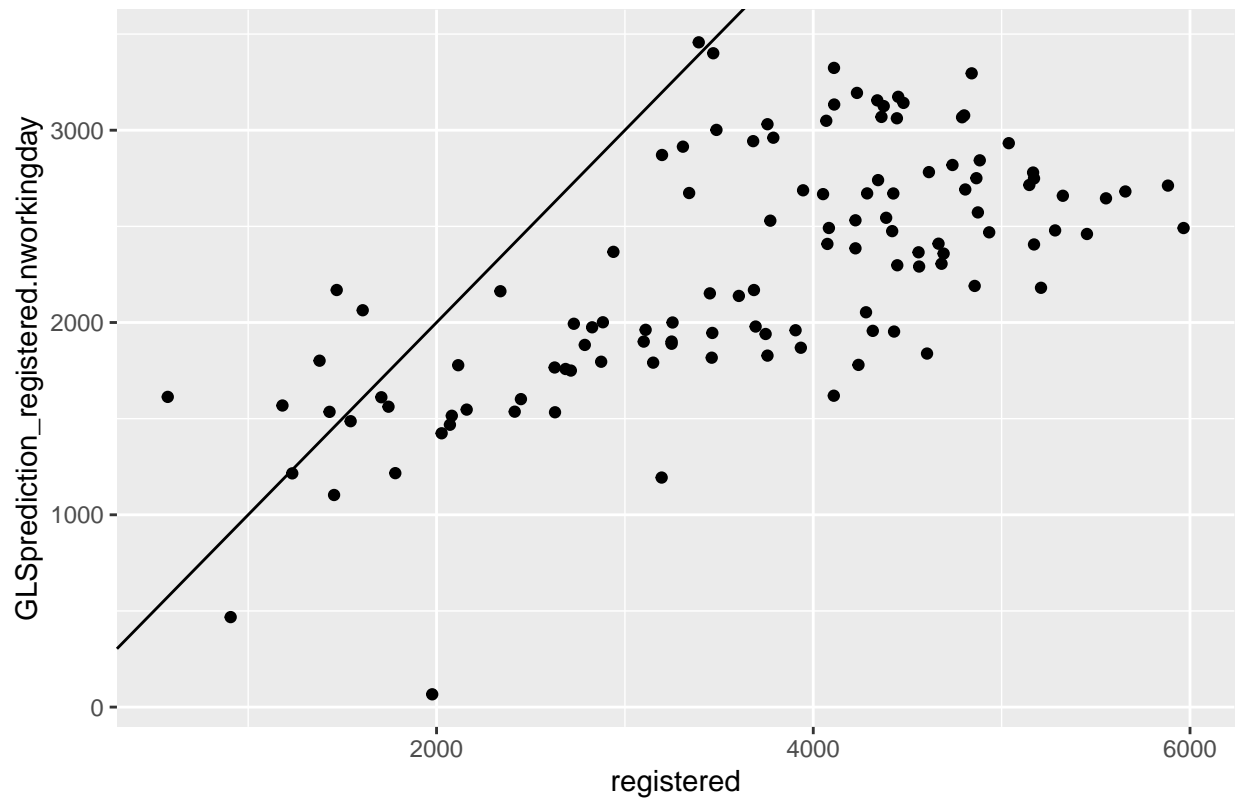
```
ggplot(validate.workingday, aes(x = registered, y = GLSprediction_registered.workingday)) + geom_point(  
geom_abline(intercept = 0, slope = 1) +  
ggtitle("Validation Registered vs Prediction on workingdays")
```

Validation Registered vs Prediction on workingdays



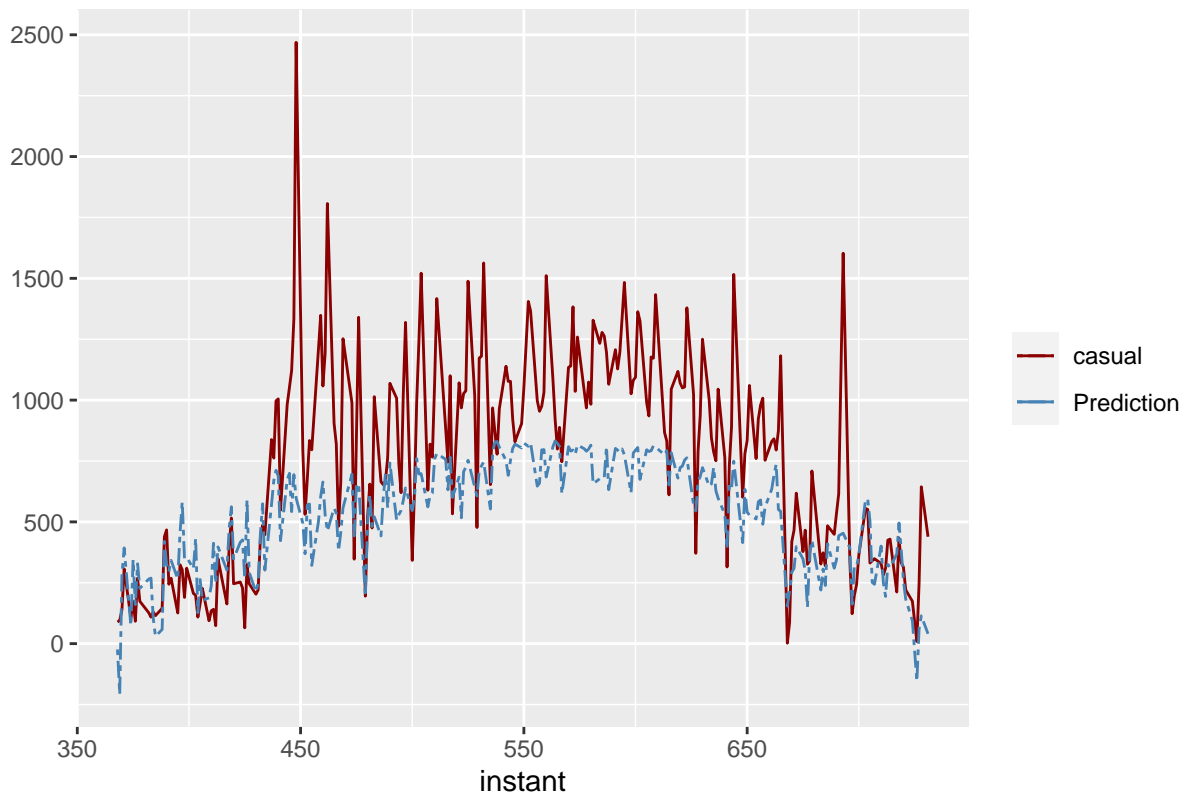
```
ggplot(validate.nworkingday, aes(x = registered, y = GLSprediction_registered.nworkingday)) + geom_point() +  
geom_abline(intercept = 0, slope = 1) +  
ggtitle("Validation Registered vs Prediction on non-workingdays")
```

Validation Registered vs Prediction on non-workingdays



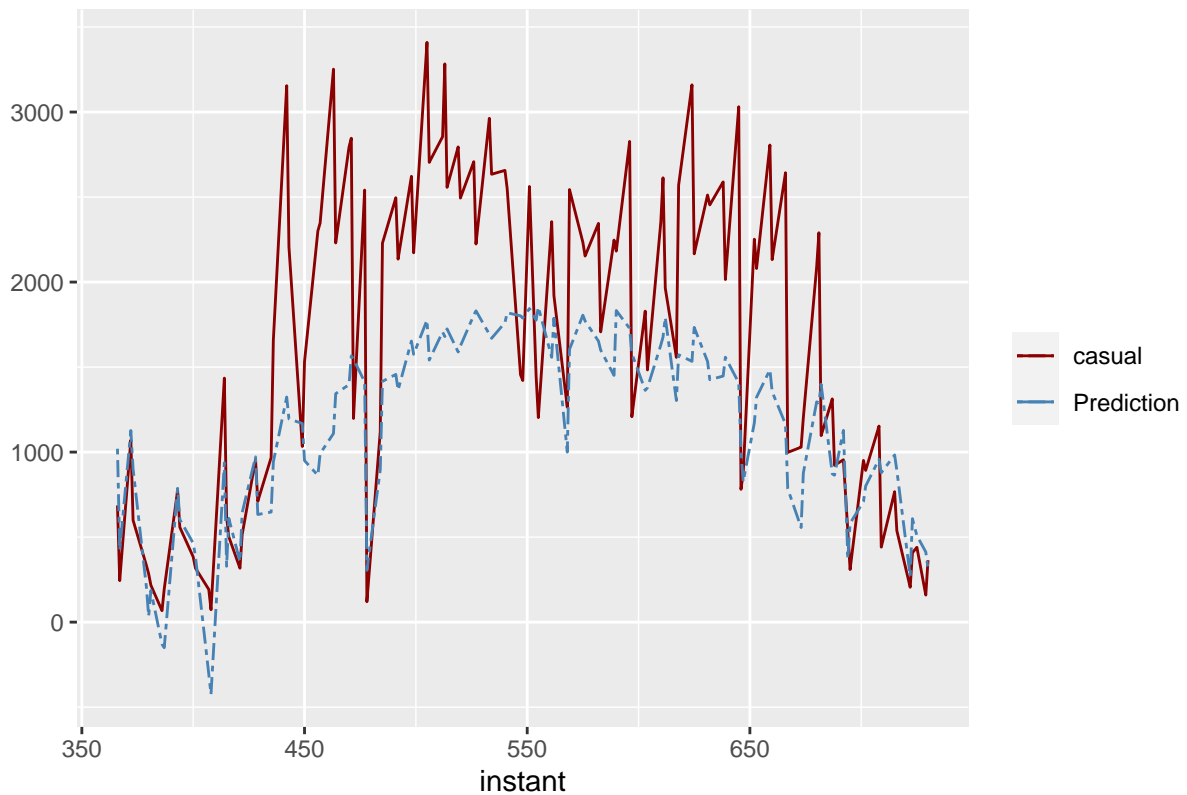
```
ggplot(data = validate.workingday, aes(x = instant)) +
  geom_line(aes(y = casual, color = "casual")) +
  geom_line(aes(y = GLSprediction_casual.workingday, color="Prediction"), linetype="twodash") +
  scale_color_manual(name = element_blank(), labels = c("casual","Prediction"),
    values = c("darkred", "steelblue")) + labs(y = "") +
  ggtitle("Validation of casual bikers on workingdays")
```


Validation of casual bikers on workingdays



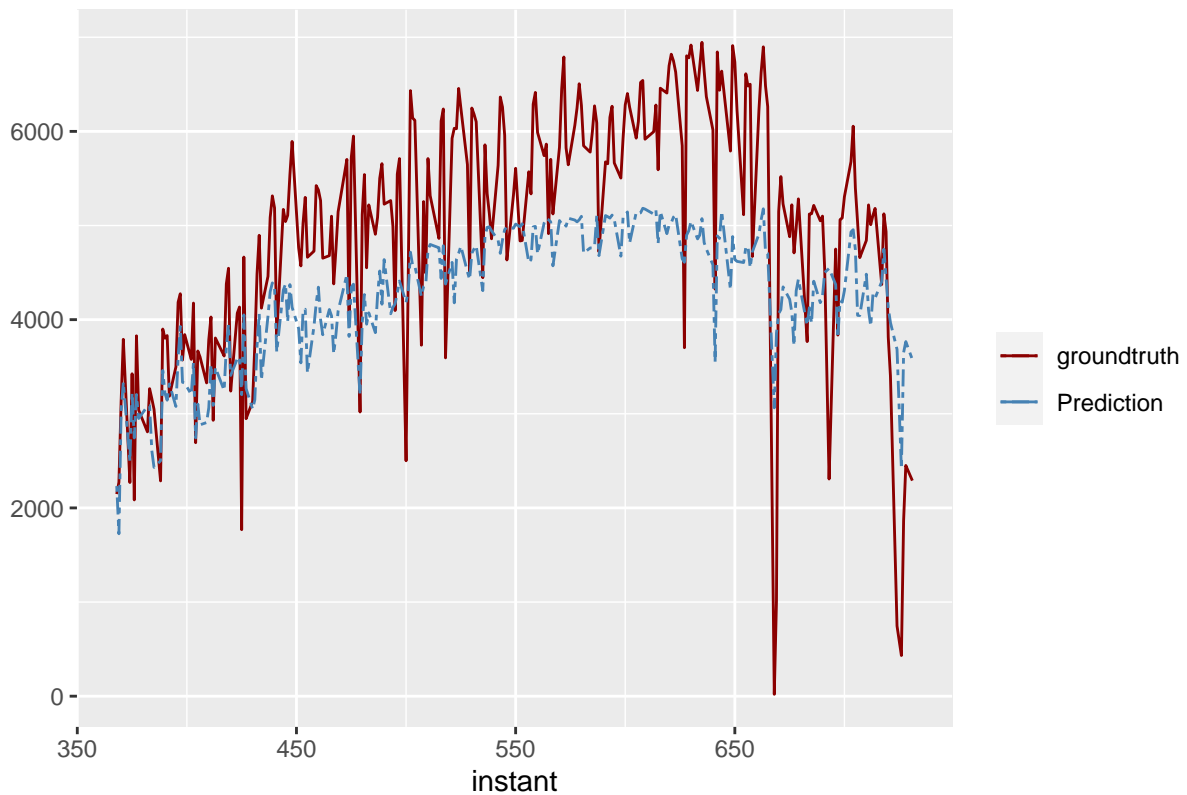
```
ggplot(data = validate.nworkingday, aes(x = instant)) +
  geom_line(aes(y = casual, color = "casual")) +
  geom_line(aes(y = GLSprediction_casual.nworkingday, color="Prediction"), linetype="twodash") +
  scale_color_manual(name = element_blank(), labels = c("casual","Prediction"),
    values = c("darkred", "steelblue")) + labs(y = "") +
  ggtitle("Validation of casual bikers on non-workingdays")
```

Validation of casual bikers on non-workingdays



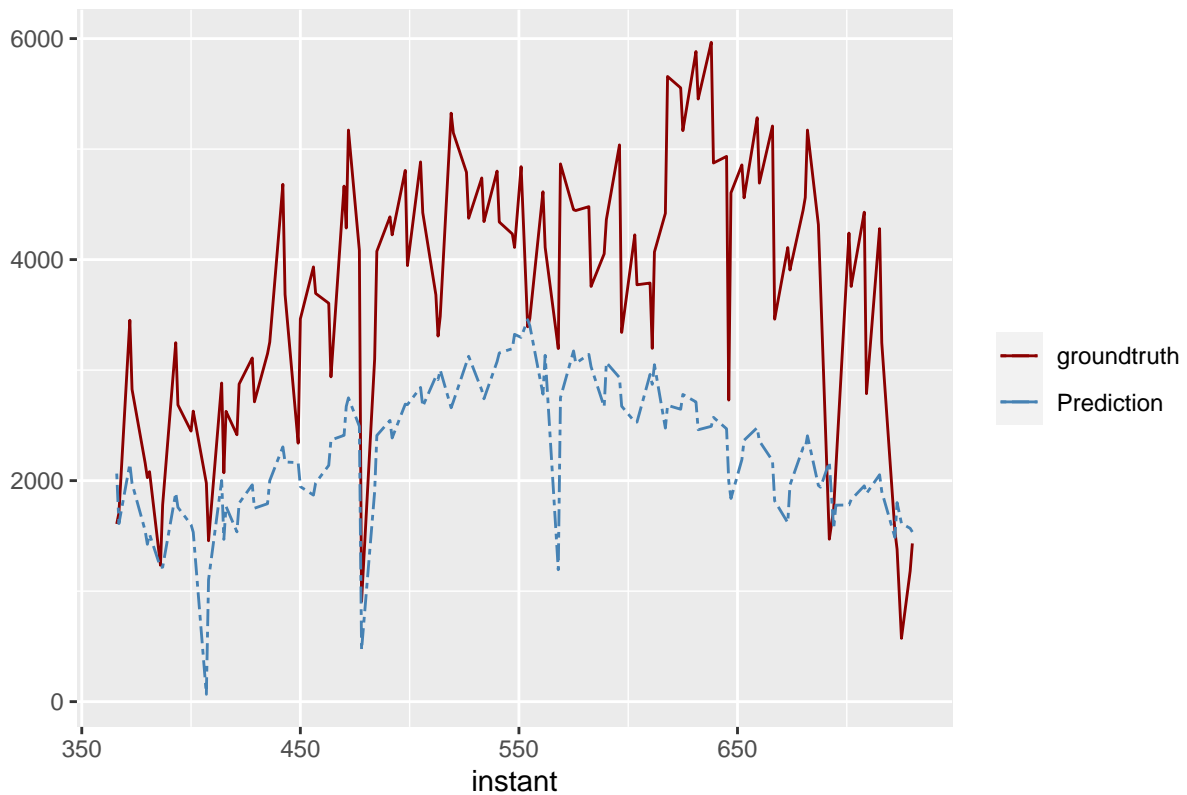
```
ggplot(data = validate.workingday, aes(x = instant)) +
  geom_line(aes(y = registered, color = "groundtruth")) +
  geom_line(aes(y = GLSprediction_registered.workingday, color="Prediction"), linetype="twodash") +
  scale_color_manual(name = element_blank(), labels = c("groundtruth","Prediction"),
    values = c("darkred", "steelblue")) + labs(y = "") +
  ggtitle("Validation of registered bikers on workingdays")
```

Validation of registered bikers on workingdays



```
ggplot(data = validate.nworkingday, aes(x = instant)) +
  geom_line(aes(y = registered, color = "groundtruth")) +
  geom_line(aes(y = GLSprediction_registered.nworkingday, color="Prediction"), linetype="twodash") +
  scale_color_manual(name = element_blank(), labels = c("groundtruth", "Prediction"),
    values = c("darkred", "steelblue")) + labs(y = "") +
  ggtitle("Validation of registered bikers on non-workingdays")
```

Validation of registered bikers on non-workingdays



```
validate.nworkingday<- validate.nworkingday %>% mutate(GLSpred.total = GLSprediction_registered.nworkingday)
validate.workingday<- validate.workingday %>% mutate(GLSpred.total = GLSprediction_registered.workingday)

temp1<- subset(validate.nworkingday, select = c(instant,GLSpred.total, cnt))
temp2<- subset(validate.workingday, select = c(instant,GLSpred.total, cnt))
GLStotal<- rbind(temp1, temp2)
```

```
ggplot(data = GLStotal, aes(x = instant)) +
  geom_line(aes(y = cnt, color = "GroundTruth")) +
  geom_line(aes(y = GLSpred.total, color="Prediction")) +
  scale_color_manual(name = element_blank(), labels = c("GroundTruth","Prediction"),
    values = c("darkred", "steelblue")) + labs(y = "") +
  ggtitle("Validation of total rental counts")
```

Validation of total rental counts

