1 Model

The d dimensional torus $\mathbb{T}^d$ can be defined as $(\mathbb{R}/n\mathbb{Z})^d$ for some natural n. And we can represent this as a cell-complex with cubical d-dimensional cells $(\mathbb{Z}/n\mathbb{Z})^d + [0,1]^d$ and all their k-faces for $k = 0, ..., d$.

We randomly assume the filtration value for each k-face uniformly distributed in $[k, k+1]$. This filtration on segmented torus will corespond some real filtration $f : mathbbT^d \to \mathbb{R}$, s.t. the d-dimensional cells will corespond the local maximums, vertices will corespond the local minimums and other k-faces will be saddles.

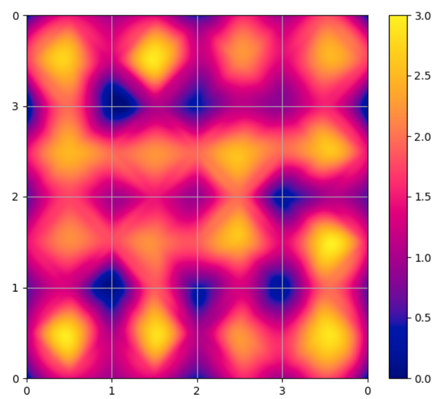

Figure 1: The example of the filtration $f : \mathbb{T}^2 \to \mathbb{R}$, s.t. there are local minimums in the vertices, saddles in the middle of edges, and the local maximums in the centers of square cells.

Let's call models like this as Barycentric Cubical Torus.

The d-dimensional torus model will contain 2^d essential cycles. So we will add some auxiliary cells to kill them: The first we will add -1-dimensional cell with the filtration value 0 which border will be all vertices. After this the cell which id birth of the first connected component will be paired with new auxiliary cell as the death of empty set. And then for each dimendion $k \in \{2, ..., d\}$ we will add C_d^k k-dimensional cells, which border will be all $(k-1)$ dimensial cells, satisfyed the $(d-k)$ constraints like $x_i = 0$. These auxiliary cells will be paired with previusly unpaired cells as deaths of essential cycles.

The distribution of computed cases by size and dimension we can see in the Fig. 2.

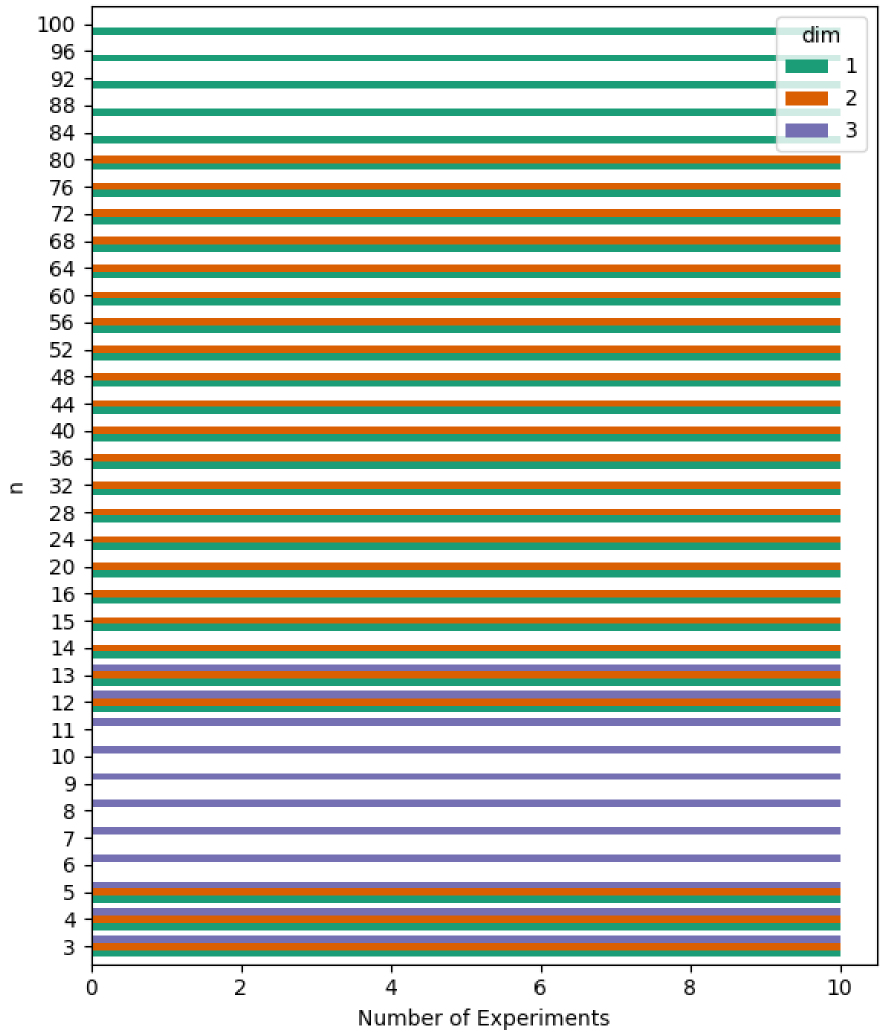

Figure 2: The distribution of computed cases by size and dimension

2 Poset Scores

2.1 Scores Description

We have computed the following scores for the objects in the depth poset of the extended barycentric cubical torus:

- **cycles_dimension** - Returns the dimension of space of cycles in reduction.
- **height** - Returns the poset height - the length of the longest chain.

- **number_of_components** - Returns the number of connetcted components in the poset
- **number_of_maximal_nodes** - Returns the number of maximal nodes.
- **number_of_minimal_nodes** - Returns the number of minimal nodes.
- **number_of_nodes** - Returns the number of nodes in the poset.
- **number_of_relations** - Returns the number of relations in the transitive reduction.

2.2 Scores

We can see the score values in the following figures:

dim	3	1			2
reduction	full	full	full	row reduction	column reduction
cycles_dimension	Fig. 3	Fig. 4	Fig. 5		
height	Fig. 6	Fig. 7	Fig. 8		
number_of_nodes	Fig. 19	Fig. 18	Fig. 20		
number_of_relations	Fig. 22	Fig. 21	Fig. 23		
number_of_components			Fig. 11	Fig. 10	Fig. 9
number_of_maximal_nodes			Fig. 14	Fig. 12	Fig. 13
number_of_minimal_nodes			Fig. 16	Fig. 15	Fig. 17

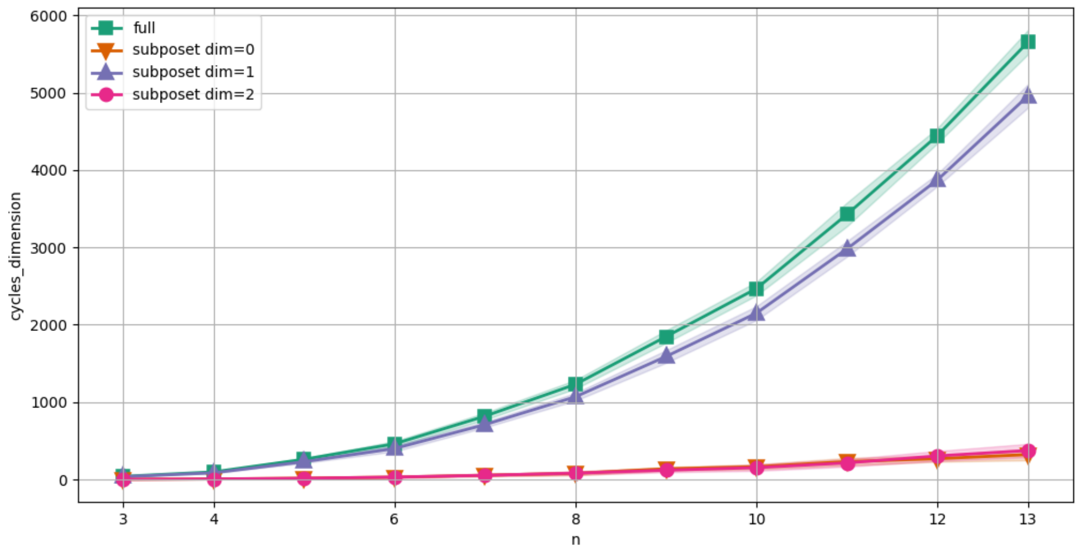

Figure 3: Score cycles_dimension values for the full poset of $\mathbb{T}^3_n$.

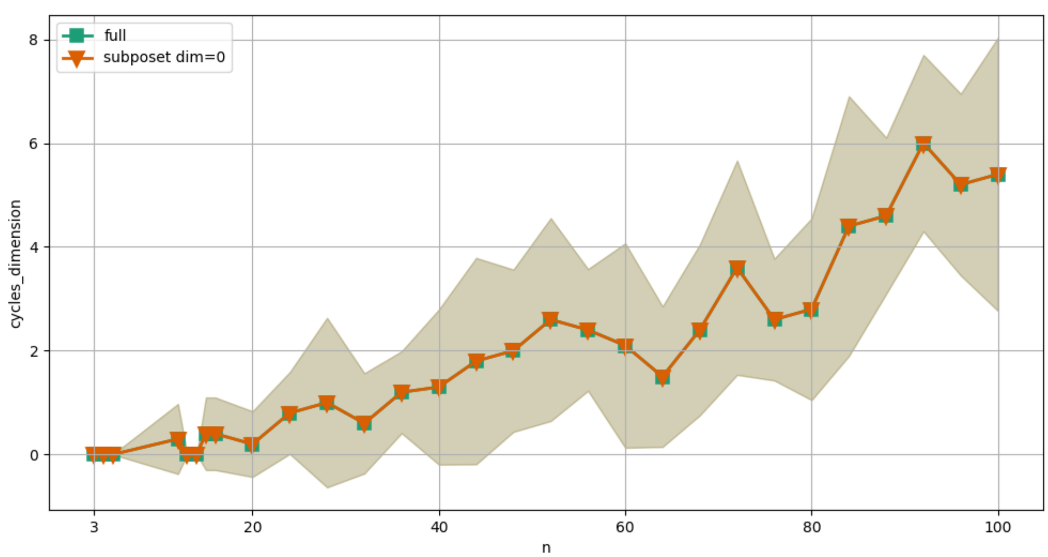

Figure 4: Score cycles_dimension values for the full poset of $\mathbb{T}^1_n$.

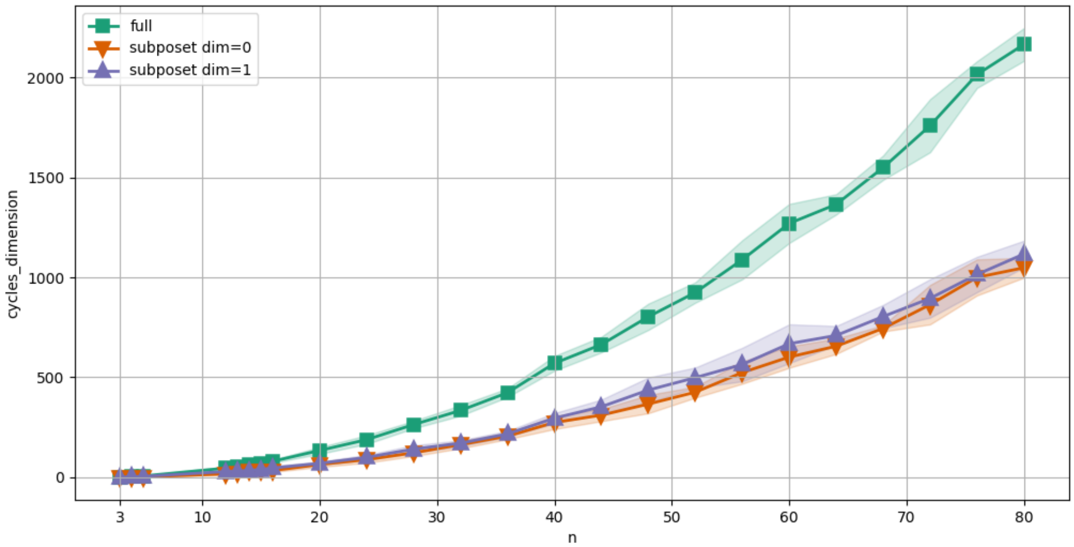

Figure 5: Score cycles_dimension values for the full poset of $\mathbb{T}^2_n$.

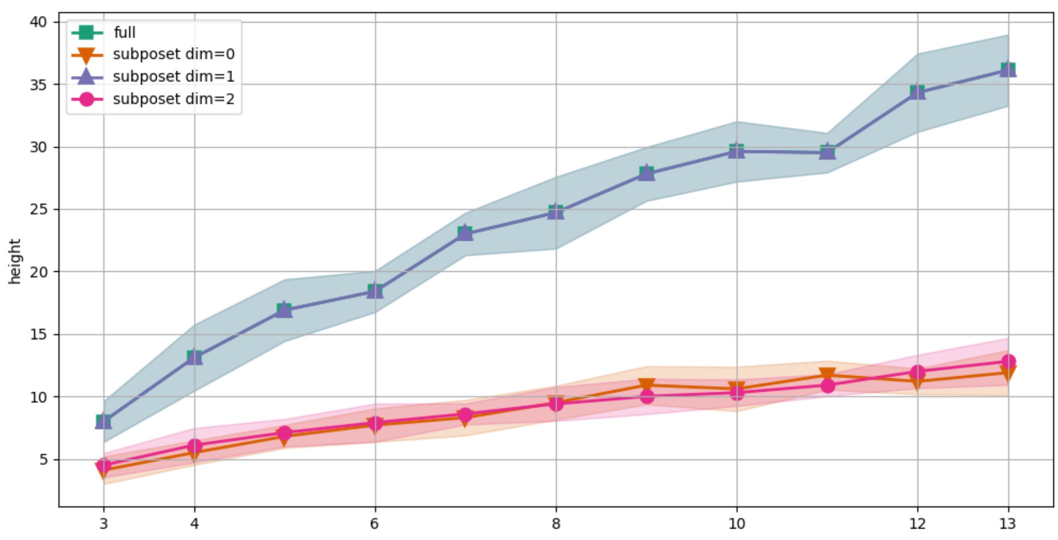

Figure 6: Score height values for the full poset of $\mathbb{T}^3_n$.

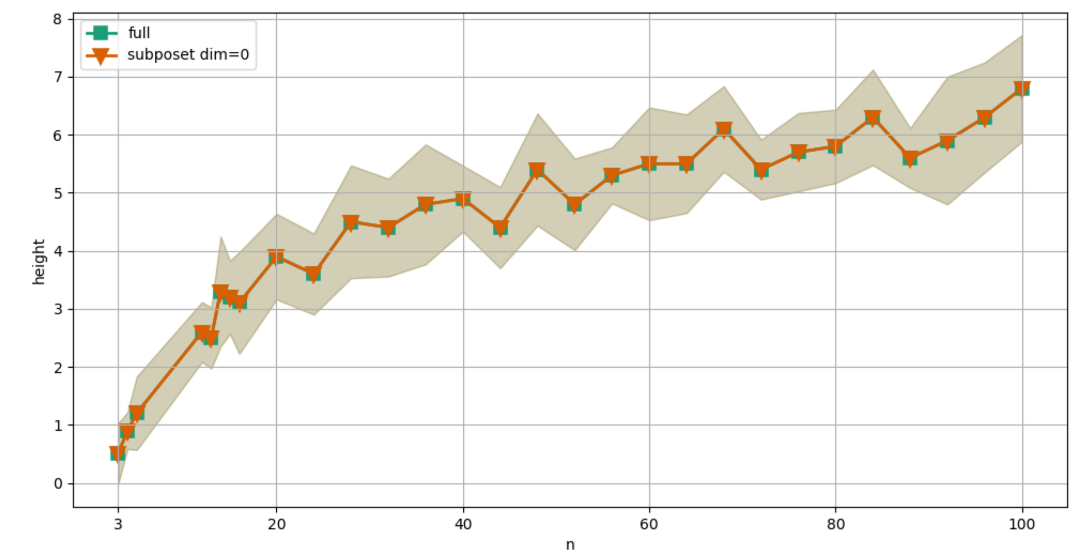

Figure 7: Score height values for the full poset of $\mathbb{T}_n^1$.

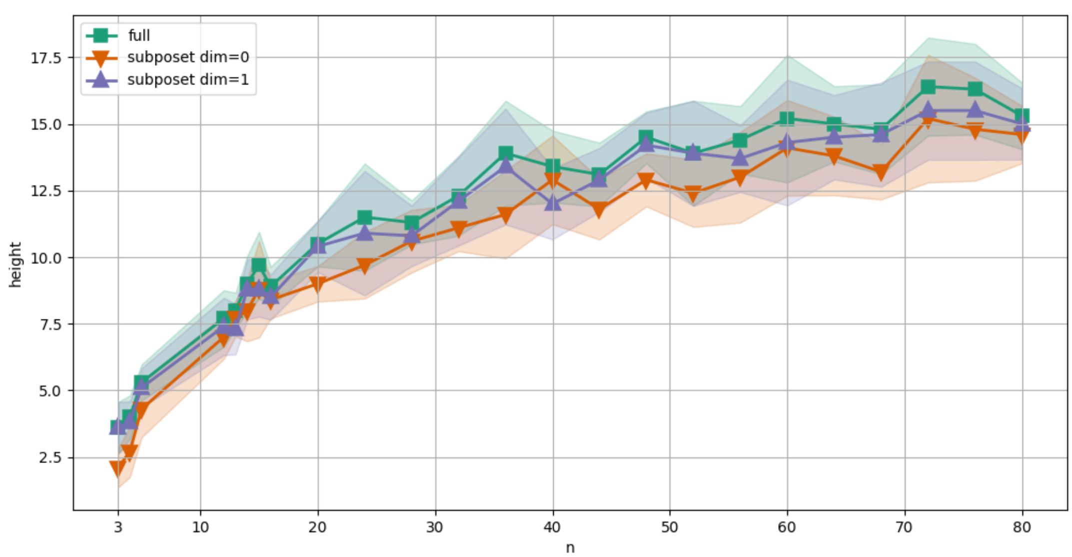

Figure 8: Score height values for the full poset of $\mathbb{T}_n^2$.

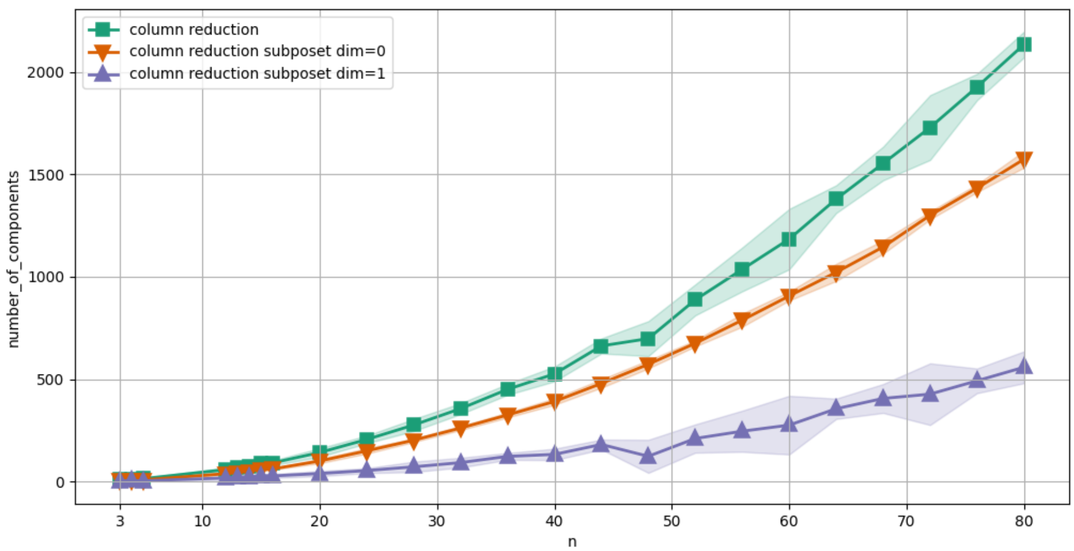

Figure 9: Score number_of_components values for the column reduction poset of $\mathbb{T}_n^2$.

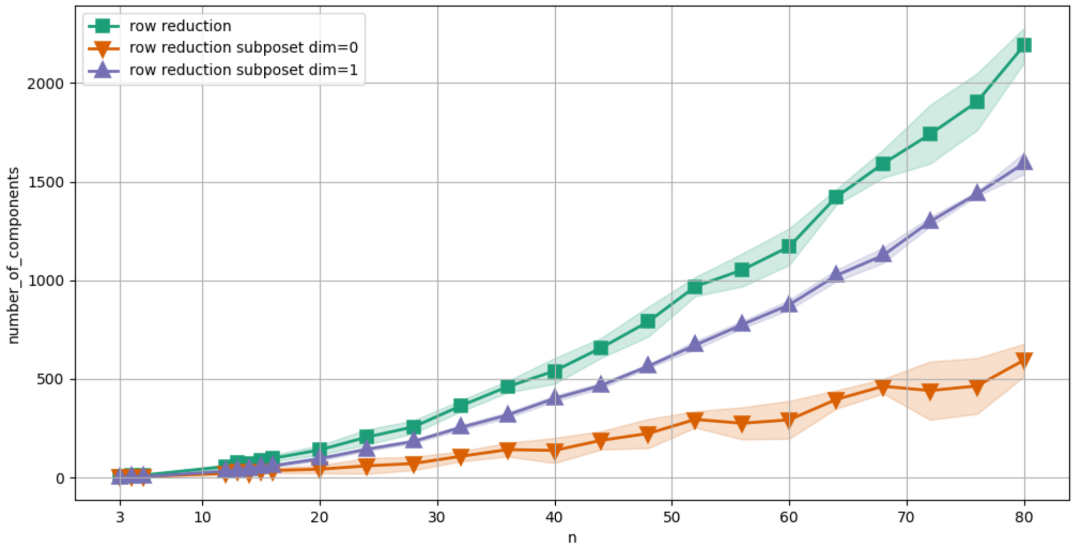

Figure 10: Score number_of_components values for the row reduction poset of $\mathbb{T}_n^2$.

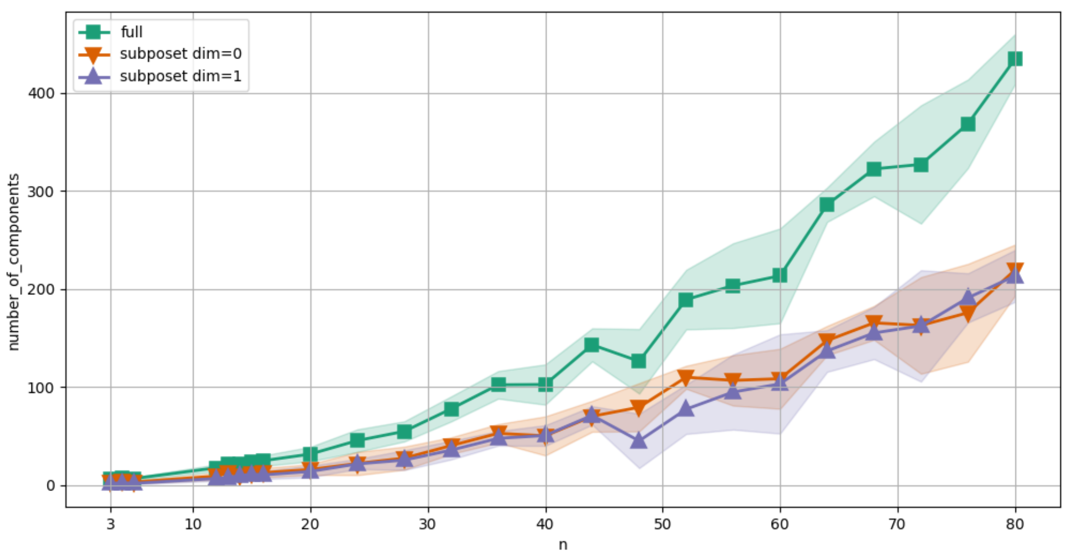

Figure 11: Score number_of_components values for the full poset of $\mathbb{T}_n^2$.

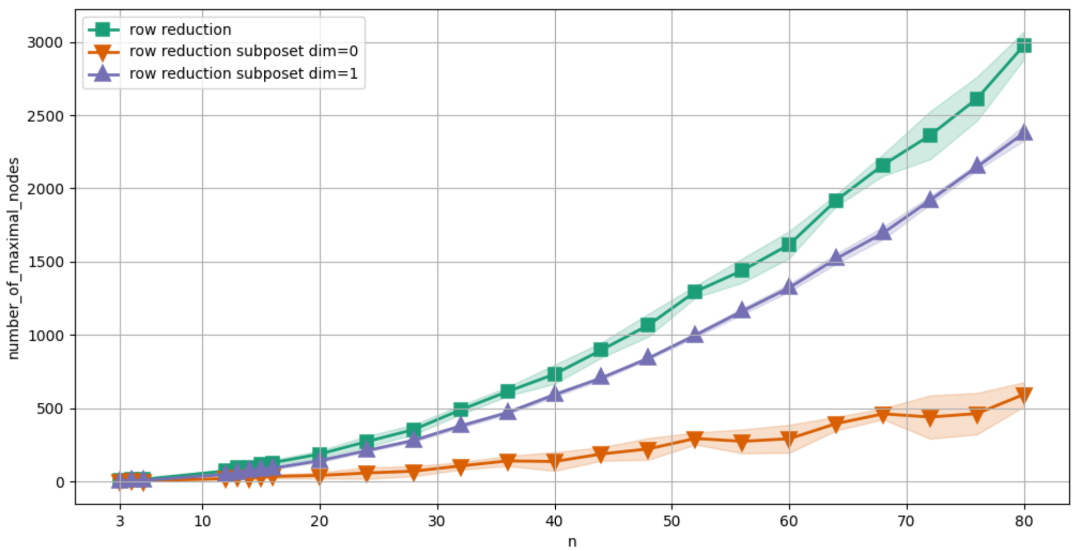

Figure 12: Score number_of_maximal_nodes values for the row reduction poset of $\mathbb{T}^2_n$.

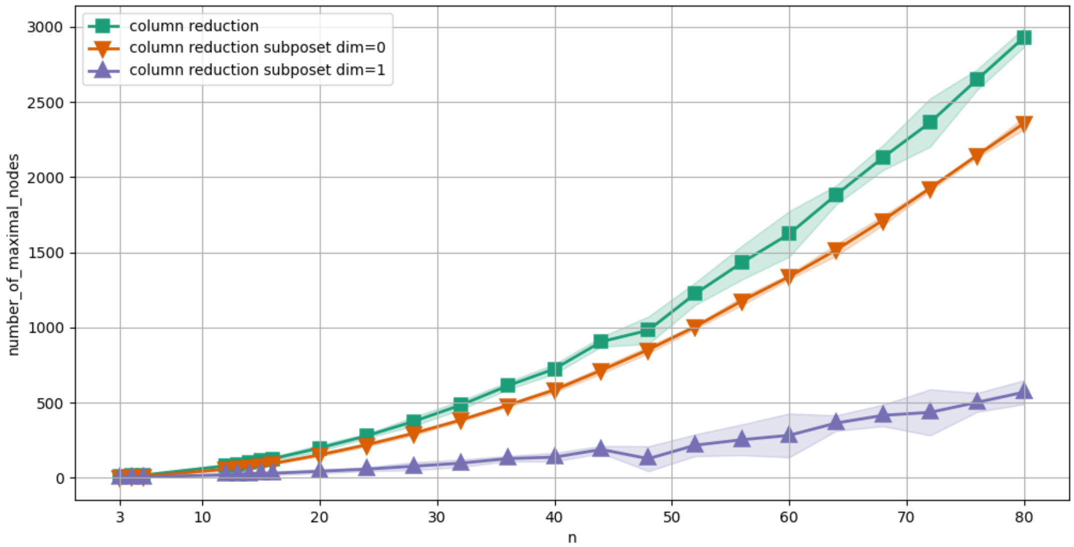

Figure 13: Score number_of_maximal_nodes values for the column reduction poset of $\mathbb{T}_n^2$.

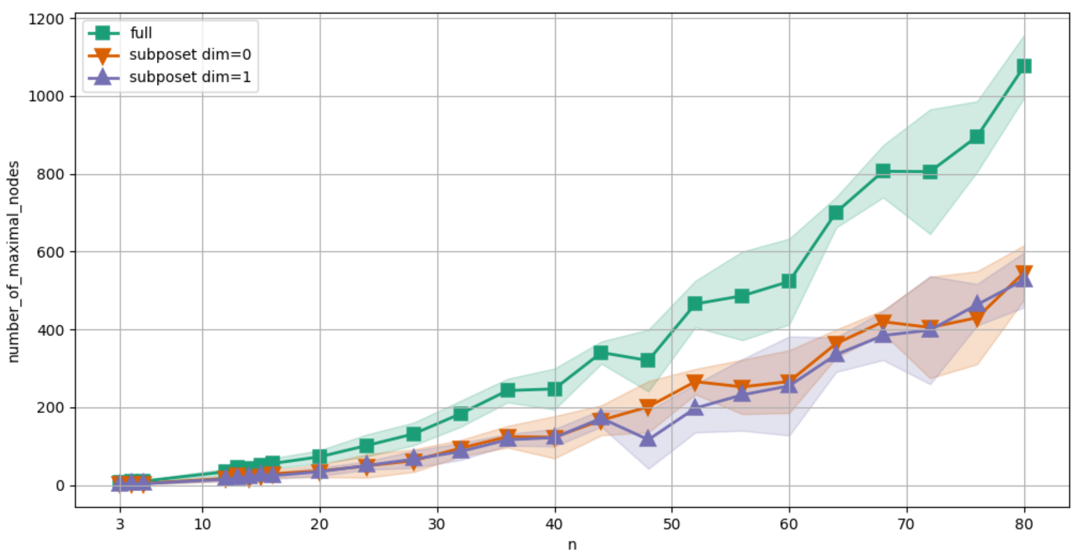

Figure 14: Score number_of_maximal_nodes values for the full poset of $\mathbb{T}_n^2$.

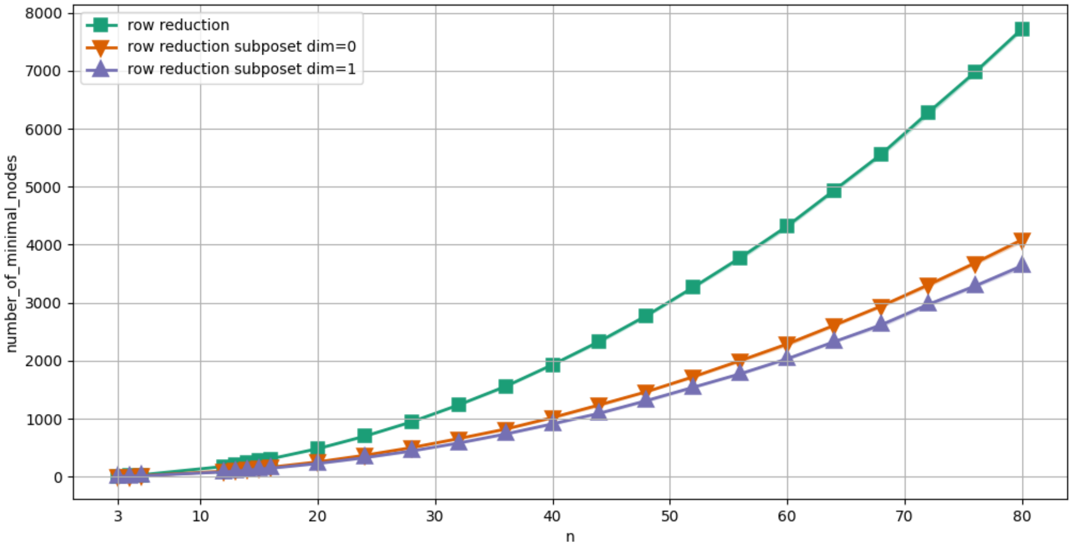

Figure 15: Score number_of_minimal_nodes values for the row reduction poset of $\mathbb{T}_n^2$.

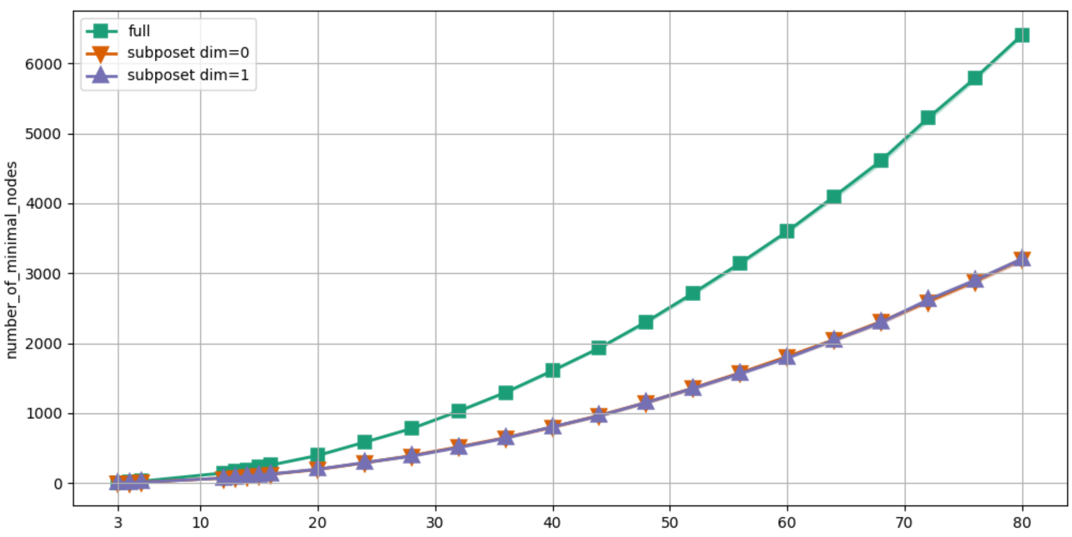

Figure 16: Score number_of_minimal_nodes values for the full poset of $\mathbb{T}_n^2$.

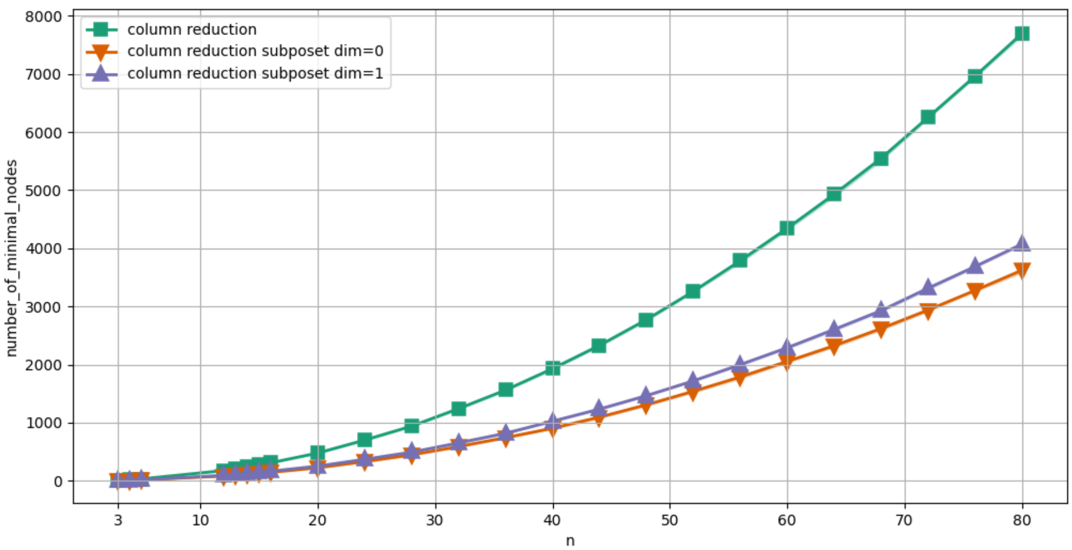

Figure 17: Score number_of_minimal_nodes values for the column reduction poset of $\mathbb{T}_n^2$.

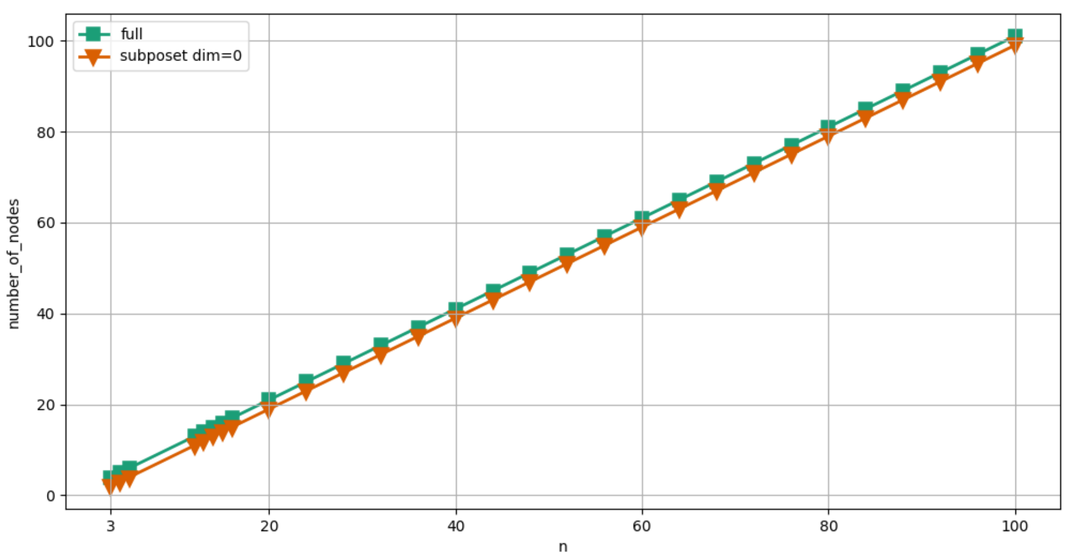

Figure 18: Score number_of_nodes values for the full poset of $\mathbb{T}_n^1$.

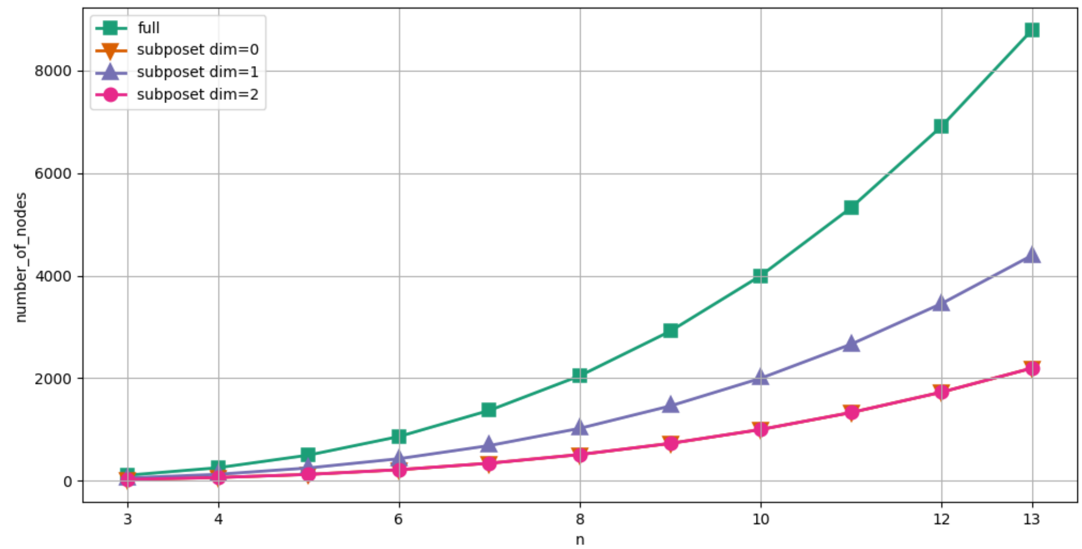

Figure 19: Score number_of_nodes values for the full poset of $\mathbb{T}^3_n$.

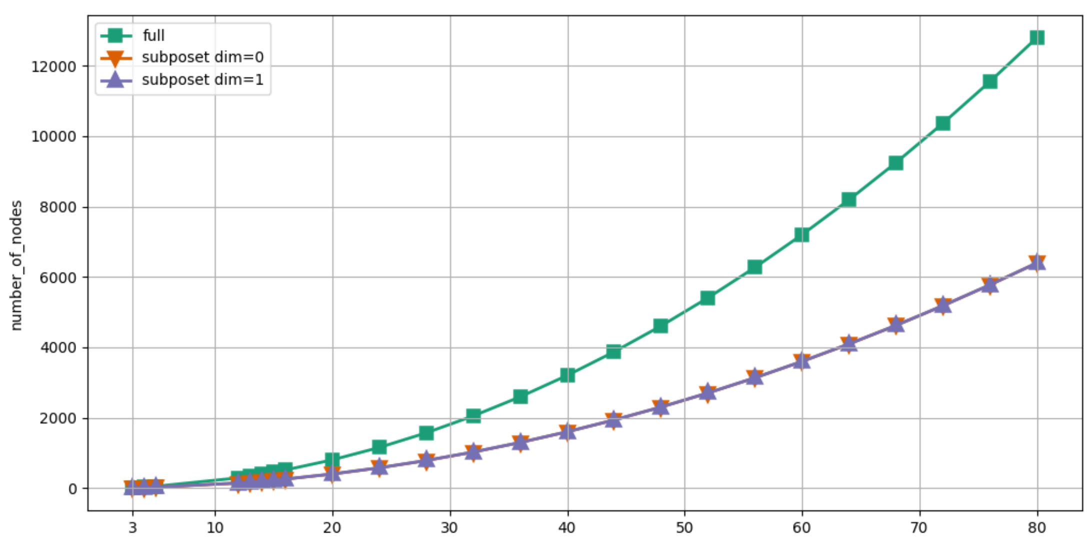

Figure 20: Score number_of_nodes values for the full poset of $\mathbb{T}^2_n$.

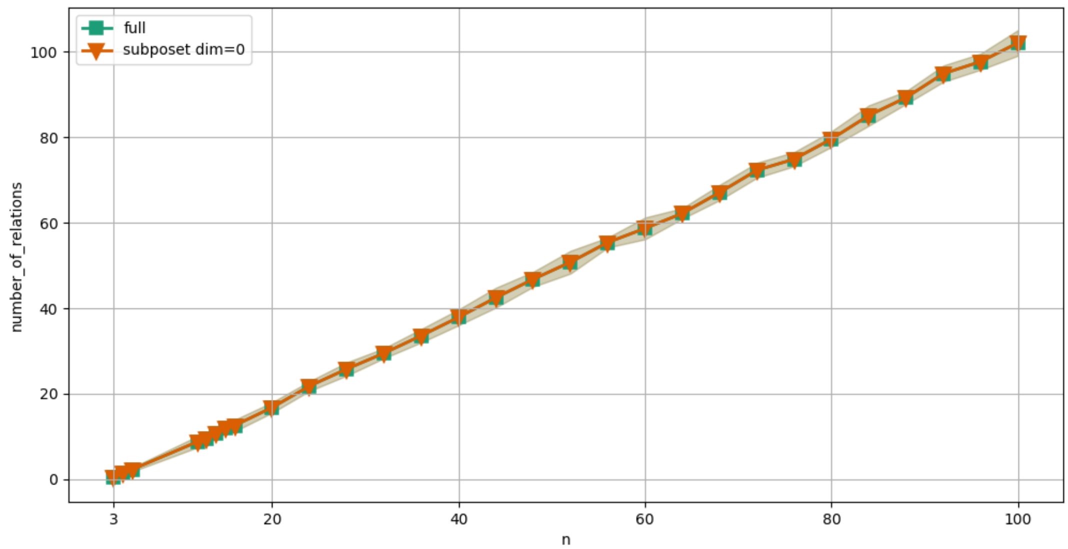

Figure 21: Score number_of_relations values for the full poset of $\mathbb{T}_n^1$.

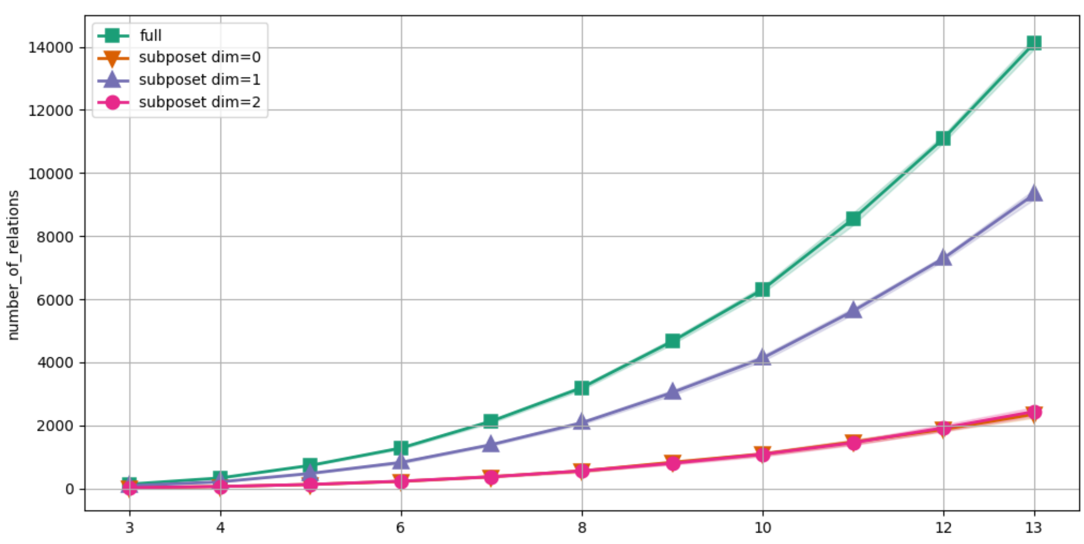

Figure 22: Score number_of_relations values for the full poset of $\mathbb{T}_n^3$.

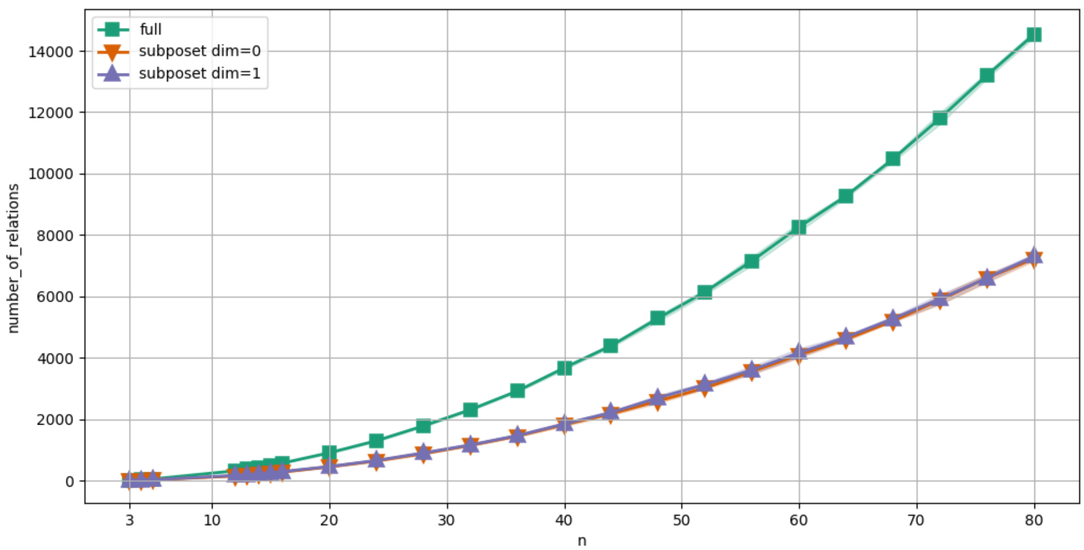

Figure 23: Score number_of_relations values for the full poset of $\mathbb{T}^2_n$.