

1 Problems

Write $\text{Succ}(s, t) = \text{Succ}'(s, t) = \text{Succ}''(s, t)$ for the successors and $\text{Pred}(s, t) = \text{Pred}'(s, t) = \text{Pred}''(s, t)$ for the predecessors (ancestors) of the pair in D , D' , D'' , respectively.

Since $D = D' \cup D''$, we also have $\text{Succ}(s, t) = \text{Succ}'(s, t) \cup \text{Succ}''(s, t)$ and $\text{Pred}(s, t) = \text{Pred}'(s, t) \cup \text{Pred}''(s, t)$.

1.1 Lemma 3.1.

Suppose (x, y) and (a, b) are birth-death pairs of $f : X \rightarrow R$, a, x are consecutive in the ordering of the cells by f , and the transposition a, x is a switch. Then

$$\begin{aligned}\text{Succ}'(a, y) &= \{(x, b)\} \cup \text{Succ}'(a, b) \cup \{(s, t) \in \text{Succ}'(x, y) | f(t) < f(b)\} \\ \text{Succ}'(x, b) &= \{(s, t) \in \text{Succ}'(x, y) | f(t) > f(b)\}\end{aligned}$$

1.2 Lemma 3.2.

Suppose (a, b) and (x, y) are birth-death pairs of $f : X \rightarrow R$, y, b are consecutive in the ordering by f , and transposition of y, b is a switch. Then

$$\begin{aligned}\text{Succ}''(x, b) &= \{(a, y)\} \cup \text{Succ}''(a, b) \cup \{(s, t) \in \text{Succ}''(x, y) | f(a) < f(s) < f(x)\} \\ \text{Succ}''(a, y) &= \{(s, t) \in \text{Succ}''(x, y) | f(s) < f(a)\}\end{aligned}$$

1.3 Lemma 3.3.

Suppose (a, b) and (x, y) are birth-death pairs of $f : X \rightarrow R$, b, x are consecutive in the ordering by f , and the transposition of b, x is a switch. Then

$$\text{Succ}(a, x) = \text{Succ}(a, b) \quad \text{and} \quad \text{Succ}(b, y) = \text{Succ}(x, y)$$

1.4 Hypothesis 1

:

Suppose a and b are 2-simplices consecutive in the ordering by f . And there is another Morse function f^* :

$$f^*(s) = \begin{cases} f(s), & \text{if } s \neq a, b \\ f(b), & \text{if } s = a \\ f(a), & \text{if } s = b \end{cases}$$

Let's denote DP_f the transitive reduction of the Depth Poset defined by the filtration f . And let's denote $DP_f(s)$ the set of nodes in $DP_f(s)$ which are pairs containing cell s and the set of edges with these nodes.

Hypothesis: if the cell s has no faces and cofaces with a and b , then $DP_f(s) = DP_{f^*}(s)$.

2 Model

The probabilistic model is simple. The first we just generate the cloud of n points uniformly distributed in $[0, 1]^d$. After this we calculate the Alpha complex with these points, and then find its depth poset. Then we iterate all neighbour pairs of simplices and check if their transposition will be possible filtration, calculating the scores for the switch-forward transpositions.

The number of points in the cloud n and the dimension $d = \dim$ for each generated cloud and corresponding complex are given in the table:

complex	n	dim	simplices	complex	n	dim	simplices
0	6	2	23	13	12	2	51
1	6	2	21	14	12	2	55
2	6	2	23	15	12	2	53
3	6	2	23	16	12	2	55
4	8	2	33	17	12	2	55
5	8	2	33	18	16	2	77
6	8	2	31	19	16	2	77
7	8	2	29	20	8	3	65
8	8	2	31	21	8	3	65
9	8	2	37	22	8	3	73
10	8	2	33	23	8	3	69
11	8	2	35	24	8	3	73
12	12	2	53	25	8	3	65
13	12	2	51				

3 Scores

- **jacard_nodes_filtration:** The Jacard index of node sets from 2 depth posets. The birth-death pairs are equal in terms of filtration values.
- **jacard_nodes_simplex:** The Jacard index of nodes from 2 depth posets. The birth-death pairs are equal in terms of simplices.
- **jacard_edges_filtration:** The Jacard index of edge sets from transitive reductions of 2 depth posets. The birth-death pairs are equal in terms of filtration values.
- **jacard_edges_simplex:** The Jacard index of edge sets from transitive reductions of 2 depth posets. The birth-death pairs are equal in terms of simplices.
- **jacard_l31a:** Jacard Index of $\text{Succ}'(x, b)$ and $\{(a, y)\} \cup \text{Succ}'(a, b) \cup \{(s, t) \in \text{Succ}'(x, y) | f(t) < f(b)\}$
- **jacard_l31b:** Jacard Index of $\text{Succ}'(x, b)$ and $\{(s, t) \in \text{Succ}'(x, y) | f(t) > f(b)\}$

- **jacard_l32a**: Jacard Index of $\text{Succ}''(x, b)$ and $\{(a, y)\} \cup \text{Succ}''(a, b) \cup \{(s, t) \in \text{Succ}''(x, y) | f(a) < f(s) < f(x)\}$
- **jacard_l32b**: Jacard Index of $\text{Succ}''(a, y)$ and $\{(s, t) \in \text{Succ}''(x, y) | f(s) < f(a)\}$
- **jacard_l33a**: Jacard Index of $\text{Succ}(a, x)$ and $\text{Succ}(a, b)$
- **jacard_l33b**: Jacard Index of $\text{Succ}(b, y)$ and $\text{Succ}(x, y)$
- **jacard_nn_nodes**: The Jacard index of subsets of nodes (s, t) from 2 depth posets, s.t. $s, t \notin \nabla\partial\sigma_0 \cup \partial\nabla\sigma_0 \cup \nabla\partial\sigma_1 \cup \partial\nabla\sigma_1$, where σ_0 and σ_1 are transposing simplices.
- **jacard_nn_edges**: The Jacard index of subsets of edges $((s_0, t_0), (s_1, t_1))$ from 2 depth posets, s.t. $s_0, t_0, s_1, t_1 \notin \nabla\partial\sigma_0 \cup \partial\nabla\sigma_0 \cup \nabla\partial\sigma_1 \cup \partial\nabla\sigma_1$, where σ_0 and σ_1 are transposing simplices.

4 Results

Here are the tables, containing the scores for the switch transpositions in the experiment:

complex	dim	n	dim 0	dim 1	type	jacard_nodes_filtration	jacard_nodes_simplex
0	2	6	0	0	birth-birth	1.00	0.69
0	2	6	1	1	birth-death	1.00	0.69
0	2	6	1	1	birth-death	1.00	0.69
1	2	6	0	0	birth-birth	1.00	0.67
1	2	6	1	1	birth-death	1.00	0.67
2	2	6	0	0	birth-birth	1.00	0.69
2	2	6	1	1	birth-death	1.00	0.69
2	2	6	1	1	birth-death	1.00	0.69
3	2	6	1	1	birth-death	1.00	0.69
3	2	6	1	1	birth-death	1.00	0.69
4	2	8	0	0	birth-birth	1.00	0.78
4	2	8	1	1	birth-death	1.00	0.78
4	2	8	1	1	birth-birth	1.00	0.78
5	2	8	1	1	birth-death	1.00	0.78
5	2	8	2	2	death-death	1.00	0.78
5	2	8	1	1	birth-death	1.00	0.78
6	2	8	1	1	death-death	1.00	0.76
6	2	8	1	1	birth-death	1.00	0.76
6	2	8	1	1	death-death	1.00	0.76
7	2	8	1	1	birth-birth	1.00	0.75
7	2	8	2	2	death-death	1.00	0.75
8	2	8	1	1	birth-death	1.00	0.76
8	2	8	1	1	death-death	1.00	0.76
8	2	8	1	1	birth-death	1.00	0.76
8	2	8	2	2	death-death	1.00	0.76
9	2	8	1	1	birth-death	1.00	0.80
9	2	8	1	1	birth-death	1.00	0.80
9	2	8	1	1	birth-death	1.00	0.80
10	2	8	1	1	birth-death	1.00	0.78
11	2	8	1	1	birth-death	1.00	0.79
12	2	12	0	0	birth-birth	1.00	0.86
12	2	12	1	1	birth-death	1.00	0.86
12	2	12	2	2	death-death	1.00	0.86
12	2	12	1	1	birth-birth	1.00	0.86
13	2	12	0	0	birth-birth	1.00	0.85
13	2	12	1	1	birth-death	1.00	0.85
13	2	12	1	1	birth-death	1.00	0.85
14	2	12	1	1	birth-death	1.00	0.86
14	2	12	1	1	death-death	1.00	0.86
14	2	12	2	2	death-death	1.00	0.86
14	2	12	1	1	birth-birth	1.00	0.86
15	2	12	0	0	birth-birth	1.00	0.86
15	2	12	1	1	birth-death	1.00	0.86
16	2	12	0	0	birth-birth	1.00	0.86
16	2	12	1	1	death-death	1.00	0.86
16	2	12	1	1 4	birth-death	1.00	0.86
17	2	12	1	1	birth-death	1.00	0.86
17	2	12	1	1	birth-death	1.00	0.86
18	2	16	0	0	birth-birth	1.00	0.90
18	2	16	1	1	birth-death	1.00	0.90

complex	dim	n	dim 0	dim 1	type	jacard_nodes_filtration	jacard_nodes_simplex
18	2	16	1	1	birth-death	1.00	0.90
18	2	16	1	1	birth-death	1.00	0.90
18	2	16	2	2	death-death	1.00	0.90
19	2	16	0	0	birth-birth	1.00	0.90
19	2	16	1	1	death-death	1.00	0.90
20	3	8	0	0	birth-birth	1.00	0.88
20	3	8	1	1	death-death	1.00	0.88
20	3	8	1	1	birth-death	1.00	0.88
20	3	8	2	2	birth-death	1.00	0.88
20	3	8	2	2	birth-death	1.00	0.88
20	3	8	2	2	birth-death	1.00	0.88
20	3	8	2	2	birth-death	1.00	0.88
20	3	8	2	2	birth-death	1.00	0.88
20	3	8	2	2	birth-death	1.00	0.88
20	3	8	2	2	birth-death	1.00	0.88
21	3	8	0	0	birth-birth	1.00	0.88
21	3	8	0	0	birth-birth	1.00	0.88
21	3	8	2	2	birth-death	1.00	0.88
21	3	8	2	2	birth-death	1.00	0.88
21	3	8	1	1	birth-death	1.00	0.88
21	3	8	2	2	birth-death	1.00	0.88
21	3	8	2	2	birth-death	1.00	0.88
21	3	8	2	2	birth-death	1.00	0.88
21	3	8	2	2	birth-death	1.00	0.88
21	3	8	2	2	birth-death	1.00	0.88
21	3	8	2	2	birth-death	1.00	0.88
21	3	8	2	2	birth-death	1.00	0.88
22	3	8	0	0	birth-birth	1.00	0.89
22	3	8	1	1	birth-death	1.00	0.89
22	3	8	1	1	birth-death	1.00	0.89
22	3	8	1	1	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	death-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
22	3	8	2	2	birth-death	1.00	0.89
23	3	8	0	0	birth-birth	1.00	0.89
23	3	8	1	1	death-death	1.00	0.89
23	3	8	1	1	birth-death	1.00	0.89
23	3	8	2	2	death-death	1.00	0.89
23	3	8	2	2	birth-death	1.00	0.89
23	3	8	2	2	birth-death	1.00	0.89
23	3	8	2	2 ⁵	birth-death	1.00	0.89
23	3	8	2	2	birth-death	1.00	0.89
23	3	8	2	2	birth-death	1.00	0.89
23	3	8	2	2	birth-death	1.00	0.89

complex	dim	n	dim 0	dim 1	type	jacard_nodes_filtration	jacard_nodes_simplex
23	3	8	2	2	birth-death	1.00	0.89
24	3	8	1	1	birth-death	1.00	0.89
24	3	8	1	1	birth-death	1.00	0.89
24	3	8	2	2	birth-death	1.00	0.89
24	3	8	2	2	birth-death	1.00	0.89
24	3	8	2	2	birth-death	1.00	0.89
24	3	8	2	2	birth-death	1.00	0.89
24	3	8	2	2	birth-death	1.00	0.89
24	3	8	2	2	birth-death	1.00	0.89
25	3	8	0	0	birth-birth	1.00	0.88
25	3	8	0	0	birth-birth	1.00	0.88
25	3	8	1	1	birth-death	1.00	0.88
25	3	8	2	2	death-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88
25	3	8	2	2	birth-death	1.00	0.88

complex	dim	n	dim 0	dim 1	type	jacard_edges_filtration	jacard_edges_simplex
0	2	6	0	0	birth-birth	1.00	0.20
0	2	6	1	1	birth-death	1.00	0.20
0	2	6	1	1	birth-death	1.00	1.00
1	2	6	0	0	birth-birth	0.50	0.20
1	2	6	1	1	birth-death	1.00	0.20
2	2	6	0	0	birth-birth	1.00	0.33
2	2	6	1	1	birth-death	0.60	0.14
2	2	6	1	1	birth-death	1.00	0.33
3	2	6	1	1	birth-death	1.00	0.20
3	2	6	1	1	birth-death	1.00	0.20
4	2	8	0	0	birth-birth	1.00	0.67
4	2	8	1	1	birth-death	0.80	0.50
4	2	8	1	1	birth-birth	1.00	0.67
5	2	8	1	1	birth-death	0.60	0.00
5	2	8	2	2	death-death	1.00	0.60
5	2	8	1	1	birth-death	0.80	0.29
6	2	8	1	1	death-death	1.00	0.50
6	2	8	1	1	birth-death	0.71	0.50
6	2	8	1	1	death-death	1.00	0.50
7	2	8	1	1	birth-birth	1.00	0.45
7	2	8	2	2	death-death	1.00	0.45
8	2	8	1	1	birth-death	0.67	0.43
8	2	8	1	1	death-death	1.00	0.43
8	2	8	1	1	birth-death	1.00	0.67
8	2	8	2	2	death-death	1.00	0.67
9	2	8	1	1	birth-death	1.00	0.25
9	2	8	1	1	birth-death	1.00	0.43
9	2	8	1	1	birth-death	0.67	0.67
10	2	8	1	1	birth-death	0.75	0.56
11	2	8	1	1	birth-death	0.75	0.75
12	2	12	0	0	birth-birth	1.00	0.45
12	2	12	1	1	birth-death	1.00	0.78
12	2	12	2	2	death-death	1.00	0.78
12	2	12	1	1	birth-birth	1.00	0.78
13	2	12	0	0	birth-birth	1.00	0.67
13	2	12	1	1	birth-death	0.82	0.67
13	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	birth-death	0.92	0.79
14	2	12	1	1	death-death	1.00	0.60
14	2	12	2	2	death-death	1.00	0.85
14	2	12	1	1	birth-birth	1.00	0.71
15	2	12	0	0	birth-birth	0.83	0.47
15	2	12	1	1	birth-death	1.00	0.57
16	2	12	0	0	birth-birth	0.80	0.38
16	2	12	1	1	death-death	0.80	0.29
16	2	12	1	1	birth-death	0.70	0.70
17	2	12	1	1	birth-death	1.00	0.60
17	2	12	1	1	birth-death	0.70	0.31
18	2	16	0	0	birth-birth	0.88	0.52
18	2	16	1	1	birth-death	0.88	0.67

[illegible]

complex	dim	n	dim 0	dim 1	type	jacard_edges_filtration	jacard_edges_simplex
23	3	8	2	2	birth-death	1.00	1.00
24	3	8	1	1	birth-death	1.00	0.43
24	3	8	1	1	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
25	3	8	0	0	birth-birth	1.00	0.33
25	3	8	0	0	birth-birth	1.00	0.50
25	3	8	1	1	birth-death	1.00	0.50
25	3	8	2	2	death-death	1.00	0.71
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00

complex	dim	n	dim 0	dim 1	type	jacard_l31a	jacard_l31b
0	2	6	0	0	birth-birth	1.00	1.00
0	2	6	1	1	birth-death	NaN	NaN
0	2	6	1	1	birth-death	NaN	NaN
1	2	6	0	0	birth-birth	1.00	1.00
1	2	6	1	1	birth-death	NaN	NaN
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	1	1	birth-death	NaN	NaN
2	2	6	1	1	birth-death	NaN	NaN
3	2	6	1	1	birth-death	NaN	NaN
3	2	6	1	1	birth-death	NaN	NaN
4	2	8	0	0	birth-birth	1.00	1.00
4	2	8	1	1	birth-death	NaN	NaN
4	2	8	1	1	birth-birth	1.00	1.00
5	2	8	1	1	birth-death	NaN	NaN
5	2	8	2	2	death-death	0.00	0.00
5	2	8	1	1	birth-death	NaN	NaN
6	2	8	1	1	death-death	0.50	0.50
6	2	8	1	1	birth-death	NaN	NaN
6	2	8	1	1	death-death	0.00	0.00
7	2	8	1	1	birth-birth	1.00	1.00
7	2	8	2	2	death-death	0.00	0.00
8	2	8	1	1	birth-death	NaN	NaN
8	2	8	1	1	death-death	0.00	0.00
8	2	8	1	1	birth-death	NaN	NaN
8	2	8	2	2	death-death	0.00	0.00
9	2	8	1	1	birth-death	NaN	NaN
9	2	8	1	1	birth-death	NaN	NaN
9	2	8	1	1	birth-death	NaN	NaN
10	2	8	1	1	birth-death	NaN	NaN
11	2	8	1	1	birth-death	NaN	NaN
12	2	12	0	0	birth-birth	1.00	1.00
12	2	12	1	1	birth-death	NaN	NaN
12	2	12	2	2	death-death	0.00	0.00
12	2	12	1	1	birth-birth	1.00	1.00
13	2	12	0	0	birth-birth	1.00	1.00
13	2	12	1	1	birth-death	NaN	NaN
13	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	death-death	0.00	0.00
14	2	12	2	2	death-death	0.00	0.00
14	2	12	1	1	birth-birth	1.00	1.00
15	2	12	0	0	birth-birth	1.00	1.00
15	2	12	1	1	birth-death	NaN	NaN
16	2	12	0	0	birth-birth	1.00	1.00
16	2	12	1	1	death-death	0.00	0.50
16	2	12	1	1	birth-death	NaN	NaN
17	2	12	1	10	birth-death	NaN	NaN
17	2	12	1	1	birth-death	NaN	NaN
18	2	16	0	0	birth-birth	1.00	1.00
18	2	16	1	1	birth-death	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard_l31a	jacard_l31b
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	2	2	death-death	0.00	0.00
19	2	16	0	0	birth-birth	1.00	1.00
19	2	16	1	1	death-death	0.67	0.67
20	3	8	0	0	birth-birth	1.00	1.00
20	3	8	1	1	death-death	0.33	0.67
20	3	8	1	1	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
21	3	8	0	0	birth-birth	1.00	1.00
21	3	8	0	0	birth-birth	1.00	1.00
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	1	1	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
22	3	8	0	0	birth-birth	1.00	1.00
22	3	8	1	1	birth-death	NaN	NaN
22	3	8	1	1	birth-death	NaN	NaN
22	3	8	1	1	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	death-death	0.00	0.00
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
23	3	8	0	0	birth-birth	1.00	0.00
23	3	8	1	1	death-death	0.00	0.50
23	3	8	1	1	birth-death	NaN	NaN
23	3	8	2	2	death-death	0.00	0.00
23	3	8	2	2	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN
23	3	8	2	2 ¹¹	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard_l31a	jacard_l31b
23	3	8	2	2	birth-death	NaN	NaN
24	3	8	1	1	birth-death	NaN	NaN
24	3	8	1	1	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
25	3	8	0	0	birth-birth	1.00	1.00
25	3	8	0	0	birth-birth	1.00	0.50
25	3	8	1	1	birth-death	NaN	NaN
25	3	8	2	2	death-death	0.00	0.00
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard_l32a	jacard_l32b
0	2	6	0	0	birth-birth	0.00	0.00
0	2	6	1	1	birth-death	NaN	NaN
0	2	6	1	1	birth-death	NaN	NaN
1	2	6	0	0	birth-birth	0.00	0.00
1	2	6	1	1	birth-death	NaN	NaN
2	2	6	0	0	birth-birth	0.50	0.50
2	2	6	1	1	birth-death	NaN	NaN
2	2	6	1	1	birth-death	NaN	NaN
3	2	6	1	1	birth-death	NaN	NaN
3	2	6	1	1	birth-death	NaN	NaN
4	2	8	0	0	birth-birth	0.00	0.00
4	2	8	1	1	birth-death	NaN	NaN
4	2	8	1	1	birth-birth	0.00	0.00
5	2	8	1	1	birth-death	NaN	NaN
5	2	8	2	2	death-death	1.00	1.00
5	2	8	1	1	birth-death	NaN	NaN
6	2	8	1	1	death-death	1.00	1.00
6	2	8	1	1	birth-death	NaN	NaN
6	2	8	1	1	death-death	1.00	1.00
7	2	8	1	1	birth-birth	0.00	0.00
7	2	8	2	2	death-death	1.00	1.00
8	2	8	1	1	birth-death	NaN	NaN
8	2	8	1	1	death-death	1.00	1.00
8	2	8	1	1	birth-death	NaN	NaN
8	2	8	2	2	death-death	1.00	1.00
9	2	8	1	1	birth-death	NaN	NaN
9	2	8	1	1	birth-death	NaN	NaN
9	2	8	1	1	birth-death	NaN	NaN
10	2	8	1	1	birth-death	NaN	NaN
11	2	8	1	1	birth-death	NaN	NaN
12	2	12	0	0	birth-birth	0.00	0.00
12	2	12	1	1	birth-death	NaN	NaN
12	2	12	2	2	death-death	1.00	1.00
12	2	12	1	1	birth-birth	0.00	0.00
13	2	12	0	0	birth-birth	0.50	0.50
13	2	12	1	1	birth-death	NaN	NaN
13	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	death-death	1.00	1.00
14	2	12	2	2	death-death	1.00	1.00
14	2	12	1	1	birth-birth	0.00	0.00
15	2	12	0	0	birth-birth	0.50	0.50
15	2	12	1	1	birth-death	NaN	NaN
16	2	12	0	0	birth-birth	0.00	0.00
16	2	12	1	1	death-death	1.00	1.00
16	2	12	1	1	birth-death	NaN	NaN
17	2	12	1	1 ¹³	birth-death	NaN	NaN
17	2	12	1	1	birth-death	NaN	NaN
18	2	16	0	0	birth-birth	0.00	0.00
18	2	16	1	1	birth-death	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard_l32a	jacard_l32b
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	2	2	death-death	1.00	1.00
19	2	16	0	0	birth-birth	0.50	0.50
19	2	16	1	1	death-death	1.00	1.00
20	3	8	0	0	birth-birth	0.00	0.00
20	3	8	1	1	death-death	1.00	1.00
20	3	8	1	1	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
20	3	8	2	2	birth-death	NaN	NaN
21	3	8	0	0	birth-birth	0.00	0.00
21	3	8	0	0	birth-birth	0.50	0.50
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	1	1	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
22	3	8	0	0	birth-birth	0.00	0.00
22	3	8	1	1	birth-death	NaN	NaN
22	3	8	1	1	birth-death	NaN	NaN
22	3	8	1	1	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	death-death	1.00	1.00
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
23	3	8	0	0	birth-birth	0.50	0.50
23	3	8	1	1	death-death	1.00	1.00
23	3	8	1	1	birth-death	NaN	NaN
23	3	8	2	2	death-death	1.00	1.00
23	3	8	2	2	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN
23	3	8	2	2 ¹⁴	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard_l32a	jacard_l32b
23	3	8	2	2	birth-death	NaN	NaN
24	3	8	1	1	birth-death	NaN	NaN
24	3	8	1	1	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
24	3	8	2	2	birth-death	NaN	NaN
25	3	8	0	0	birth-birth	0.00	0.00
25	3	8	0	0	birth-birth	0.50	0.50
25	3	8	1	1	birth-death	NaN	NaN
25	3	8	2	2	death-death	1.00	1.00
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN
25	3	8	2	2	birth-death	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard.l33a	jacard.l33b
0	2	6	0	0	birth-birth	NaN	NaN
0	2	6	1	1	birth-death	1.00	1.00
0	2	6	1	1	birth-death	1.00	1.00
1	2	6	0	0	birth-birth	NaN	NaN
1	2	6	1	1	birth-death	1.00	1.00
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	1	1	birth-death	1.00	1.00
2	2	6	1	1	birth-death	1.00	1.00
3	2	6	1	1	birth-death	1.00	1.00
3	2	6	1	1	birth-death	1.00	1.00
4	2	8	0	0	birth-birth	NaN	NaN
4	2	8	1	1	birth-death	1.00	1.00
4	2	8	1	1	birth-birth	NaN	NaN
5	2	8	1	1	birth-death	1.00	1.00
5	2	8	2	2	death-death	NaN	NaN
5	2	8	1	1	birth-death	1.00	1.00
6	2	8	1	1	death-death	NaN	NaN
6	2	8	1	1	birth-death	1.00	1.00
6	2	8	1	1	death-death	NaN	NaN
7	2	8	1	1	birth-birth	NaN	NaN
7	2	8	2	2	death-death	NaN	NaN
8	2	8	1	1	birth-death	1.00	1.00
8	2	8	1	1	death-death	NaN	NaN
8	2	8	1	1	birth-death	1.00	1.00
8	2	8	2	2	death-death	NaN	NaN
9	2	8	1	1	birth-death	1.00	1.00
9	2	8	1	1	birth-death	1.00	1.00
9	2	8	1	1	birth-death	1.00	1.00
10	2	8	1	1	birth-death	1.00	1.00
11	2	8	1	1	birth-death	1.00	1.00
12	2	12	0	0	birth-birth	NaN	NaN
12	2	12	1	1	birth-death	1.00	1.00
12	2	12	2	2	death-death	NaN	NaN
12	2	12	1	1	birth-birth	NaN	NaN
13	2	12	0	0	birth-birth	NaN	NaN
13	2	12	1	1	birth-death	1.00	1.00
13	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	death-death	NaN	NaN
14	2	12	2	2	death-death	NaN	NaN
14	2	12	1	1	birth-birth	NaN	NaN
15	2	12	0	0	birth-birth	NaN	NaN
15	2	12	1	1	birth-death	1.00	1.00
16	2	12	0	0	birth-birth	NaN	NaN
16	2	12	1	1	death-death	NaN	NaN
16	2	12	1	1	birth-death	1.00	1.00
17	2	12	1	16	birth-death	1.00	1.00
17	2	12	1	1	birth-death	1.00	1.00
18	2	16	0	0	birth-birth	NaN	NaN
18	2	16	1	1	birth-death	1.00	1.00

[illegible]

complex	dim	n	dim 0	dim 1	type	jacard_l33a	jacard_l33b
23	3	8	2	2	birth-death	1.00	1.00
24	3	8	1	1	birth-death	1.00	1.00
24	3	8	1	1	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
25	3	8	0	0	birth-birth	NaN	NaN
25	3	8	0	0	birth-birth	NaN	NaN
25	3	8	1	1	birth-death	1.00	1.00
25	3	8	2	2	death-death	NaN	NaN
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00

complex	dim	n	dim 0	dim 1	type	jacard_nn_nodes	jacard_nn_edges
0	2	6	0	0	birth-birth	1.00	1.00
0	2	6	1	1	birth-death	1.00	1.00
0	2	6	1	1	birth-death	1.00	1.00
1	2	6	0	0	birth-birth	1.00	1.00
1	2	6	1	1	birth-death	1.00	1.00
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	1	1	birth-death	1.00	1.00
2	2	6	1	1	birth-death	1.00	1.00
3	2	6	1	1	birth-death	1.00	1.00
3	2	6	1	1	birth-death	1.00	1.00
4	2	8	0	0	birth-birth	1.00	1.00
4	2	8	1	1	birth-death	1.00	1.00
4	2	8	1	1	birth-birth	1.00	1.00
5	2	8	1	1	birth-death	1.00	1.00
5	2	8	2	2	death-death	1.00	1.00
5	2	8	1	1	birth-death	1.00	1.00
6	2	8	1	1	death-death	1.00	1.00
6	2	8	1	1	birth-death	1.00	1.00
6	2	8	1	1	death-death	1.00	1.00
7	2	8	1	1	birth-birth	1.00	1.00
7	2	8	2	2	death-death	1.00	1.00
8	2	8	1	1	birth-death	1.00	1.00
8	2	8	1	1	death-death	1.00	1.00
8	2	8	1	1	birth-death	1.00	1.00
8	2	8	2	2	death-death	1.00	1.00
9	2	8	1	1	birth-death	1.00	1.00
9	2	8	1	1	birth-death	1.00	1.00
9	2	8	1	1	birth-death	1.00	1.00
10	2	8	1	1	birth-death	1.00	1.00
11	2	8	1	1	birth-death	1.00	1.00
12	2	12	0	0	birth-birth	1.00	1.00
12	2	12	1	1	birth-death	1.00	1.00
12	2	12	2	2	death-death	1.00	1.00
12	2	12	1	1	birth-birth	1.00	1.00
13	2	12	0	0	birth-birth	1.00	1.00
13	2	12	1	1	birth-death	1.00	1.00
13	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	death-death	1.00	1.00
14	2	12	2	2	death-death	1.00	1.00
14	2	12	1	1	birth-birth	1.00	1.00
15	2	12	0	0	birth-birth	1.00	1.00
15	2	12	1	1	birth-death	1.00	1.00
16	2	12	0	0	birth-birth	1.00	1.00
16	2	12	1	1	death-death	1.00	1.00
16	2	12	1	1	birth-death	1.00	1.00
17	2	12	1	19	birth-death	1.00	1.00
17	2	12	1	1	birth-death	1.00	1.00
18	2	16	0	0	birth-birth	1.00	1.00
18	2	16	1	1	birth-death	1.00	1.00

[illegible]

complex	dim	n	dim 0	dim 1	type	jacard_nn_nodes	jacard_nn_edges
23	3	8	2	2	birth-death	1.00	1.00
24	3	8	1	1	birth-death	1.00	1.00
24	3	8	1	1	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
24	3	8	2	2	birth-death	1.00	1.00
25	3	8	0	0	birth-birth	1.00	1.00
25	3	8	0	0	birth-birth	1.00	1.00
25	3	8	1	1	birth-death	1.00	1.00
25	3	8	2	2	death-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00
25	3	8	2	2	birth-death	1.00	1.00

4.1 Unexpected results

The Hypothesis problem can be measured by 2 scores with expected values: **jacard_nn_edges**, **jacard_nn_nodes** There are 2 cases, which does not corespond the expectations:

complex	dim 0	dim 1	type	jacard_nn_edges	jacard_nn_nodes	Figure
11	1	1	death-death	0.00	1.00	Figure 1
25	1	1	birth-birth	0.00	1.00	

The Lemma 3.1 problem can be measured by 2 scores with expected values: **jacard_l31a**, **jacard_l31b** There are 2 cases, which does not corespond the expectations:

complex	dim 0	dim 1	type	jacard_l31a	jacard_l31b	Figure
23	0	0	birth-birth	1.00	0.00	
25	0	0	birth-birth	1.00	0.50	

The Lemma 3.2 problem can be measured by 2 scores with expected values: **jacard_l32a**, **jacard_l32b** There are 0 cases, which does not corespond the expectations.

The Lemma 3.3 problem can be measured by 2 scores with expected values: **jacard_l33a**, **jacard_l33b** There are 0 cases, which does not correspond the expectations.

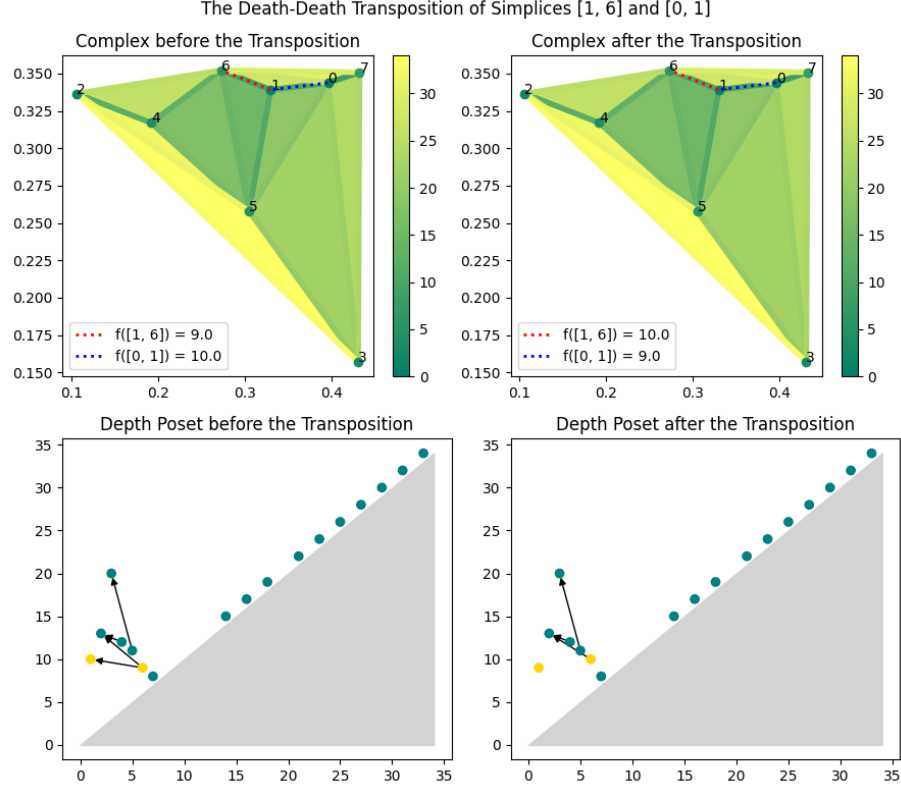


Figure 1: The Death-Death Transposition of simplices $[1, 6]$ and $[0, 1]$

4.1.1 Complex 11

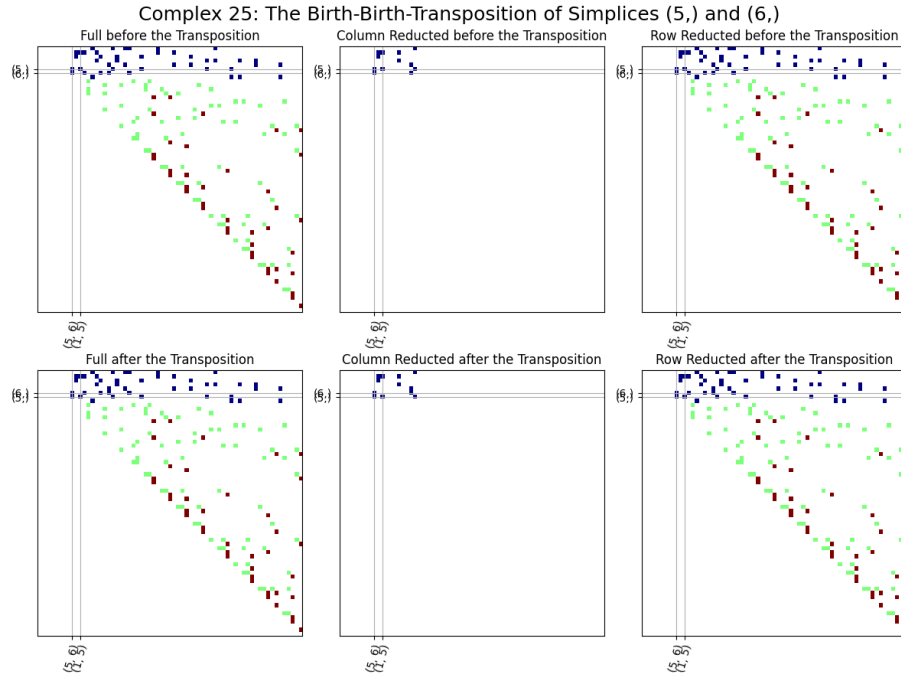
The switch transposition and unexpected scores are given in the table:

simplex 0	simplex 1	type
$(3, 5)$	$(0, 3)$	birth-death

4.1.2 Complex 25

The switch transposition and unexpected scores are given in the table:

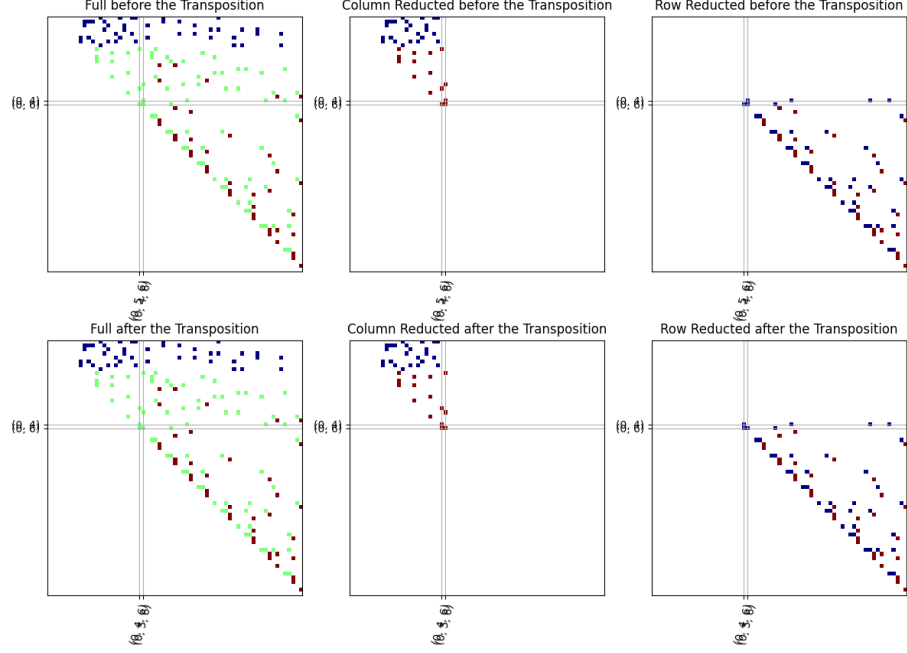
simplex 0	simplex 1	type	jacard_l31b
(1)	(2)	birth-birth	1.00
(5)	(6)	birth-birth	0.50
(1, 5)	(1, 6)	birth-death	NaN
(0, 5, 6)	(0, 4, 6)	death-death	0.00
(1, 2, 5)	(2, 5, 6)	birth-death	NaN
(0, 1, 5)	(0, 1, 6)	birth-death	NaN
(1, 4, 6)	(0, 1, 4)	birth-death	NaN
(2, 4, 6)	(1, 2, 4)	birth-death	NaN
(0, 1, 3)	(0, 3, 5)	birth-death	NaN
(0, 3, 7)	(3, 5, 7)	birth-death	NaN
(1, 3, 4)	(0, 3, 4)	birth-death	NaN
(0, 4, 7)	(3, 4, 7)	birth-death	NaN



4.1.3 Complex 23

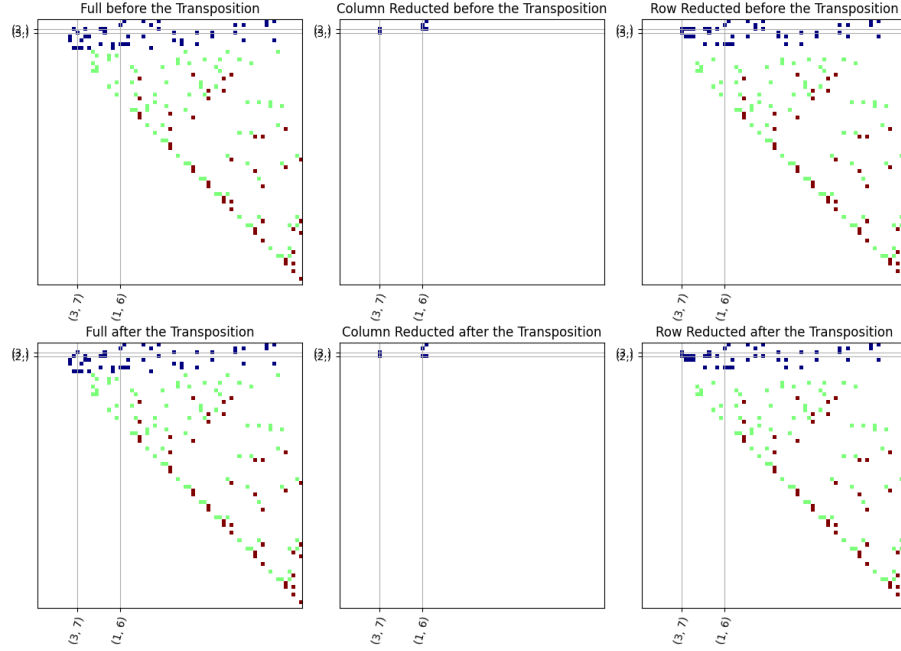
The switch transposition and unexpected scores are given in the table:

Complex 25: The Death-Death-Transposition of Simplices (0, 5, 6) and (0, 4, 6)

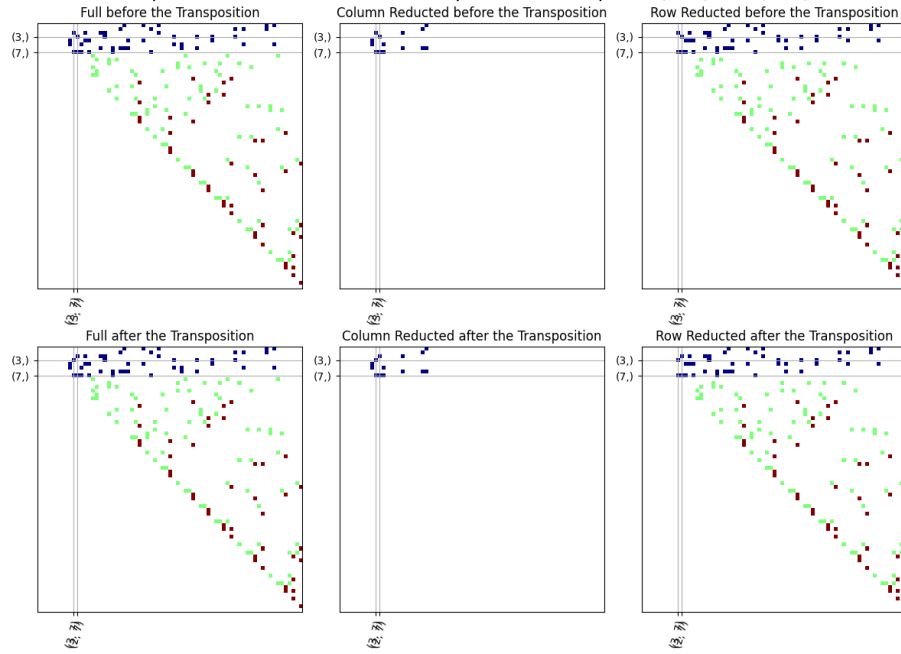


simplex 0	simplex 1	type	jacard_l31b
(2)	(3)	birth-birth	0.00
(2, 7)	(3, 7)	death-death	0.50
(5, 7)	(2, 4)	birth-death	NaN
(4, 5, 7)	(2, 4, 7)	death-death	0.00
(2, 4, 6)	(4, 6, 7)	birth-death	NaN
(0, 2, 4)	(0, 2, 6)	birth-death	NaN
(4, 5, 6)	(5, 6, 7)	birth-death	NaN
(2, 3, 6)	(3, 6, 7)	birth-death	NaN
(2, 3, 4)	(3, 4, 7)	birth-death	NaN
(1, 5, 6)	(1, 5, 7)	birth-death	NaN
(0, 4, 5)	(0, 5, 6)	birth-death	NaN

Complex 23: The Birth-Birth-Transposition of Simplices (2,) and (3,)



Complex 23: The Death-Death-Transposition of Simplices (2, 7) and (3, 7)



Complex 23: The Death-Death-Transposition of Simplices (4, 5, 7) and (2, 4, 7)

