

# 1 Model

The probabilistic model is simple. The first we just generate the cloud of  $n$  points uniformly distributed in  $[0, 1]^d$ . After this we calculate the Alpha complex with these points, and then find its depth poset.

# 2 Scores

## 2.1 Poset Scores

- **number\_of\_nodes** : Returns the number of nodes in the poset.
- **number\_of\_relations** : Returns the number of relations in the transitive reduction.
- **number\_of\_components** : Returns the number of connected components in the poset
- **cycles\_dimension** : Returns the dimension of space of cycles in reduction.
- **number\_of\_minimal\_nodes** : Returns the number of minimal nodes.
- **number\_of\_maximal\_nodes** : Returns the number of maximal nodes.
- **height** : Returns the poset height - the length of the longest chain.
- **width** : Returns the poset width - the length of the longest antichain (subset, s.t. all elements are pairwise incomparable). The algorithm is based on Dilworth's theorem and it's proof via König's theorem: link
- **minimum\_maximal\_chain** : Returns the minimum size of maximal chains in the poset.
- **avarage\_maximal\_chain** : Returns the average size of maximal chains in the poset.

## 2.2 Node Scores

- **ancestors\_number**: Returns the number of nodes higher than given
- **ancestors\_height**: Returns the size of maximum chain of subposet of nodes higher or equal than given
- **ancestors\_width**: Returns the size of maximum chain of subposet of nodes higher or equal than given
- **ancestors\_cycles\_dimension**: Returns the the dimension of space of cycles in reduction of subposet of nodes higher or equal than given

- **successors\_number**: Returns the number of nodes higher than given
- **successors\_height**: Returns the size of maximum chain of subposet of nodes lower or equal than given
- **successors\_width**: Returns the size of maximum chain of subposet of nodes lower or equal than given
- **successors\_cycles\_dimension**: Returns the dimension of space of cycles in reduction of subposet of nodes lower or equal than given

### 3 Experiments and Results

There are 1119 experiments done. In the Figure 1 we can see how cases are distributed by size and dimension.

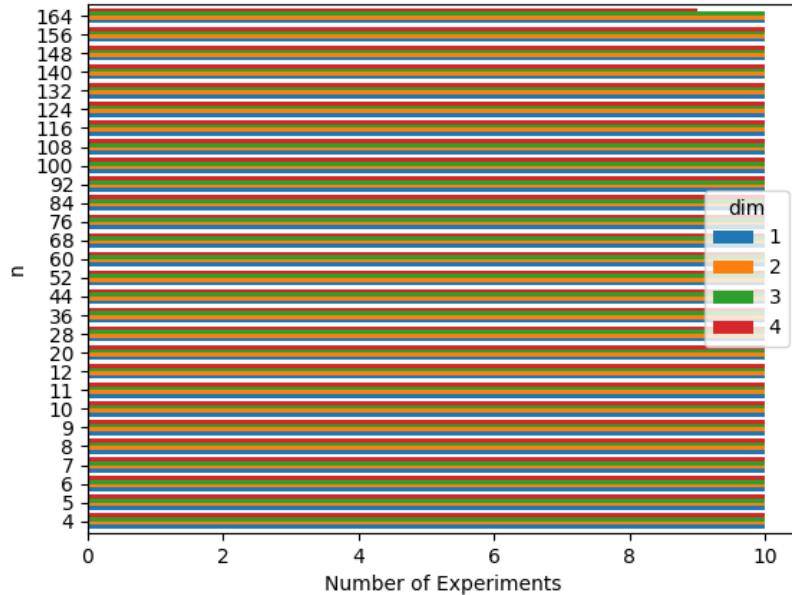


Figure 1: Size/dimension distribution of experiments

#### 3.1 Depth Poset Features

In the Figure 2 we can see the average poset scores values for each number of points  $n$  in the depth poset.

In the Figure 3 we can see the avarage mean node scores values in poset for each number of points  $n$  in the depth poset.

In the Figure 4 we can see the avarage maximum node scores values in poset for each number of points  $n$  in the depth poset.

### 3.2 Column Reduction Poset Features

In the Figure 5 we can see the avarage poset scores values for each number of points  $n$  in the column reduction poset.

In the Figure 6 we can see the avarage mean node scores values in poset for each number of points  $n$  in the column reduction poset.

In the Figure 7 we can see the avarage maximum node scores values in poset for each number of points  $n$  in the column reduction poset.

### 3.3 Row Reduction Poset Features

In the Figure 8 we can see the avarage poset scores values for each number of points  $n$  in the row reduction poset.

In the Figure 9 we can see the avarage mean node scores values in poset for each number of points  $n$  in the row reduction poset.

In the Figure 10 we can see the avarage maximum node scores values in poset for each number of points  $n$  in the row reduction poset.

## 4 Conclusion

Row and Column reduction posets for d-dimensional case can have cycles, but their subposets with pairs dimension  $d-1$  (where the dimension of a pair corresponds to the lower dimension of a simplex) are always trees.

## Depth Poset: Mean Poset Scores

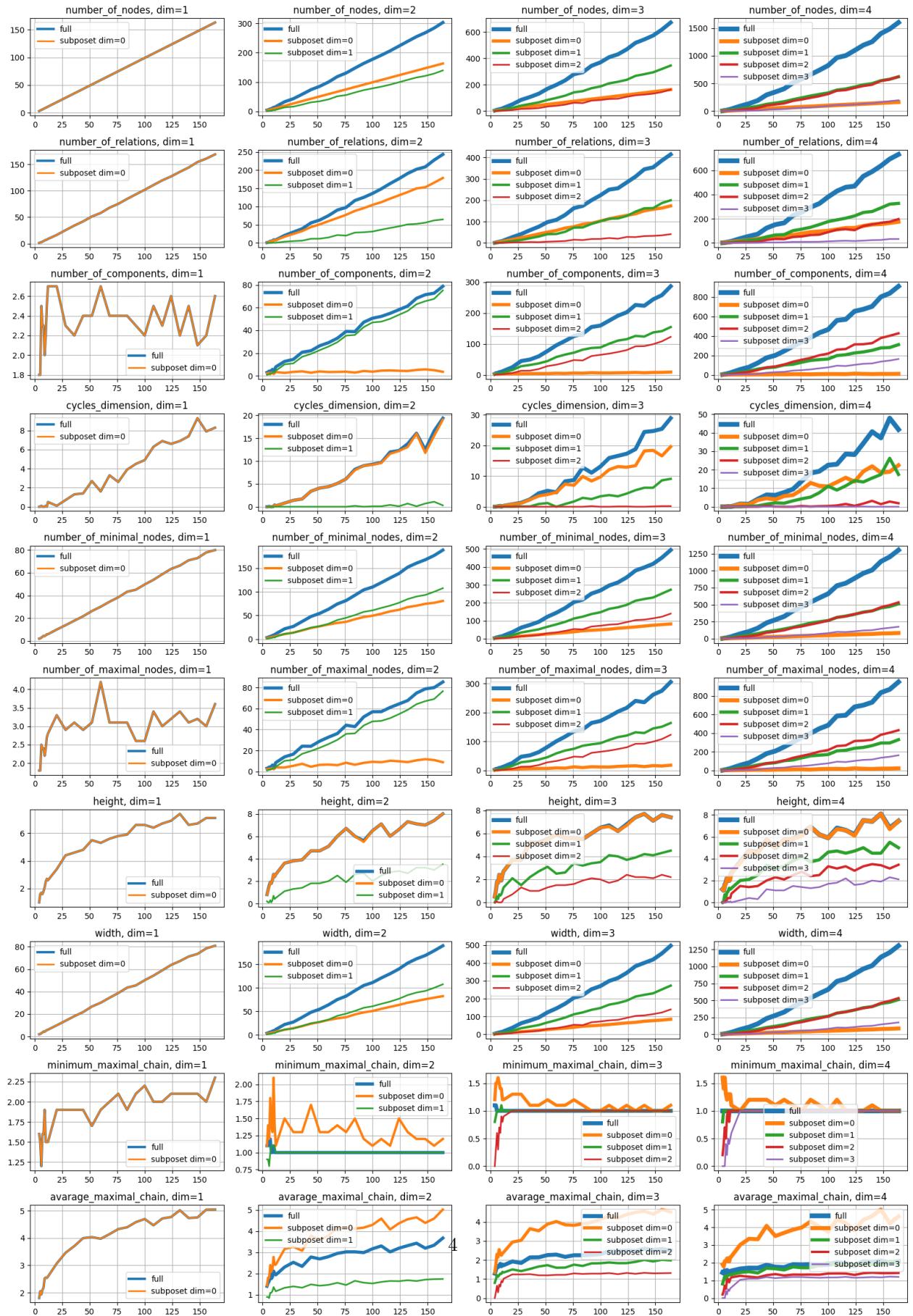


Figure 2: Depth Poset: Mean poset scores

### Depth Poset: Mean Node Scores

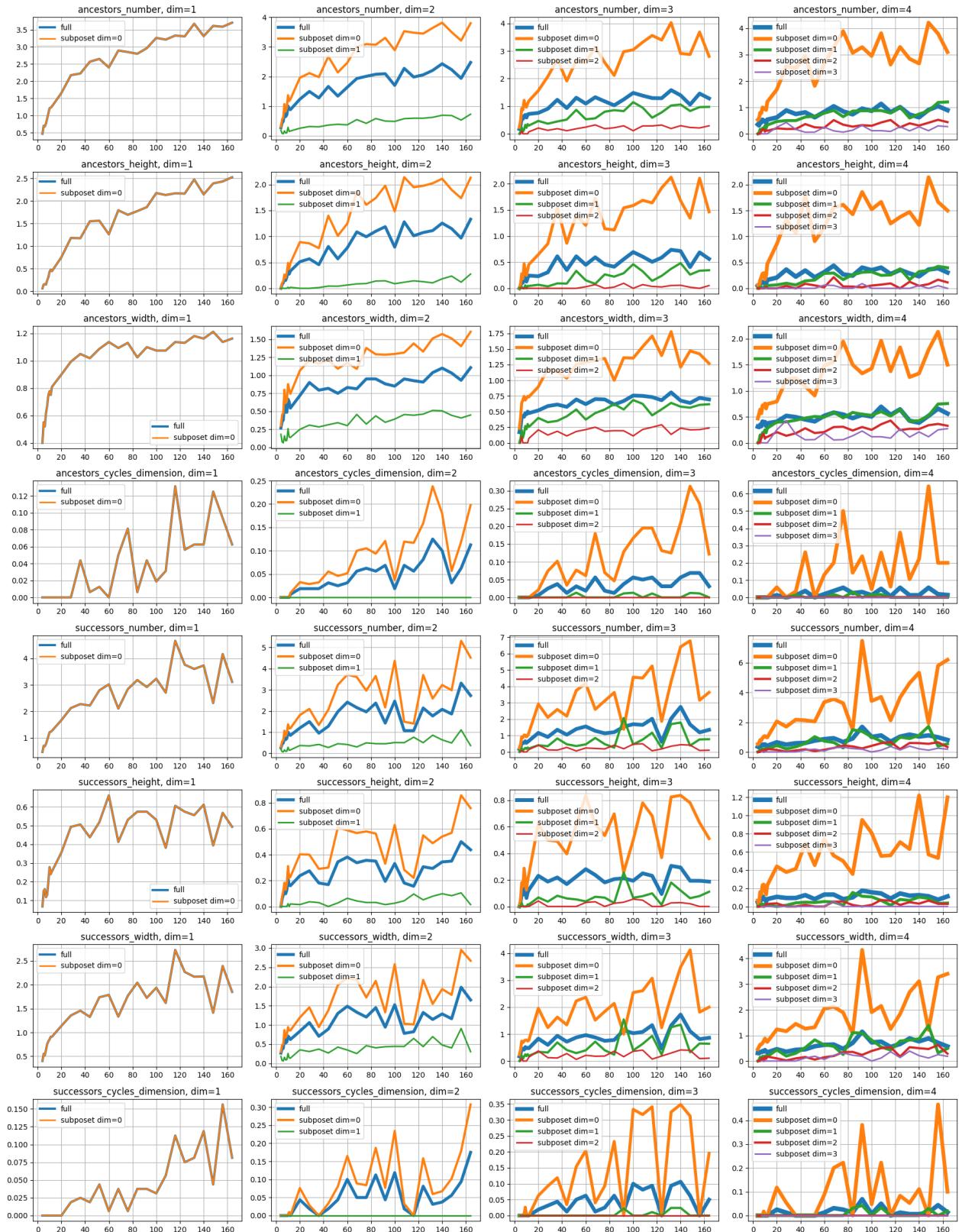


Figure 3: Depth Poset: Mean node scores

## Depth Poset: Max Node Scores

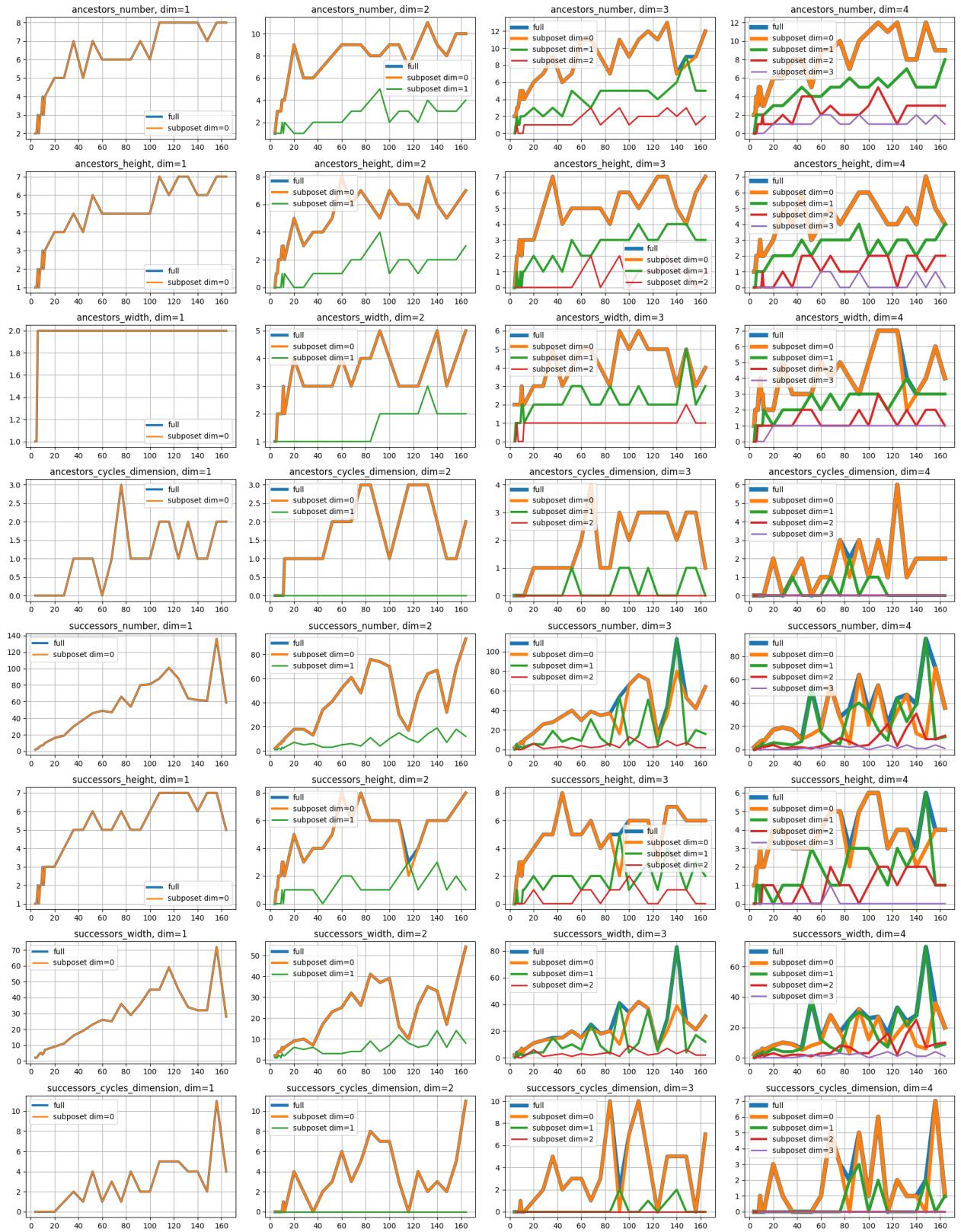


Figure 4: Depth Poset: Max node scores

## Column Reduction Poset: Mean Poset Scores

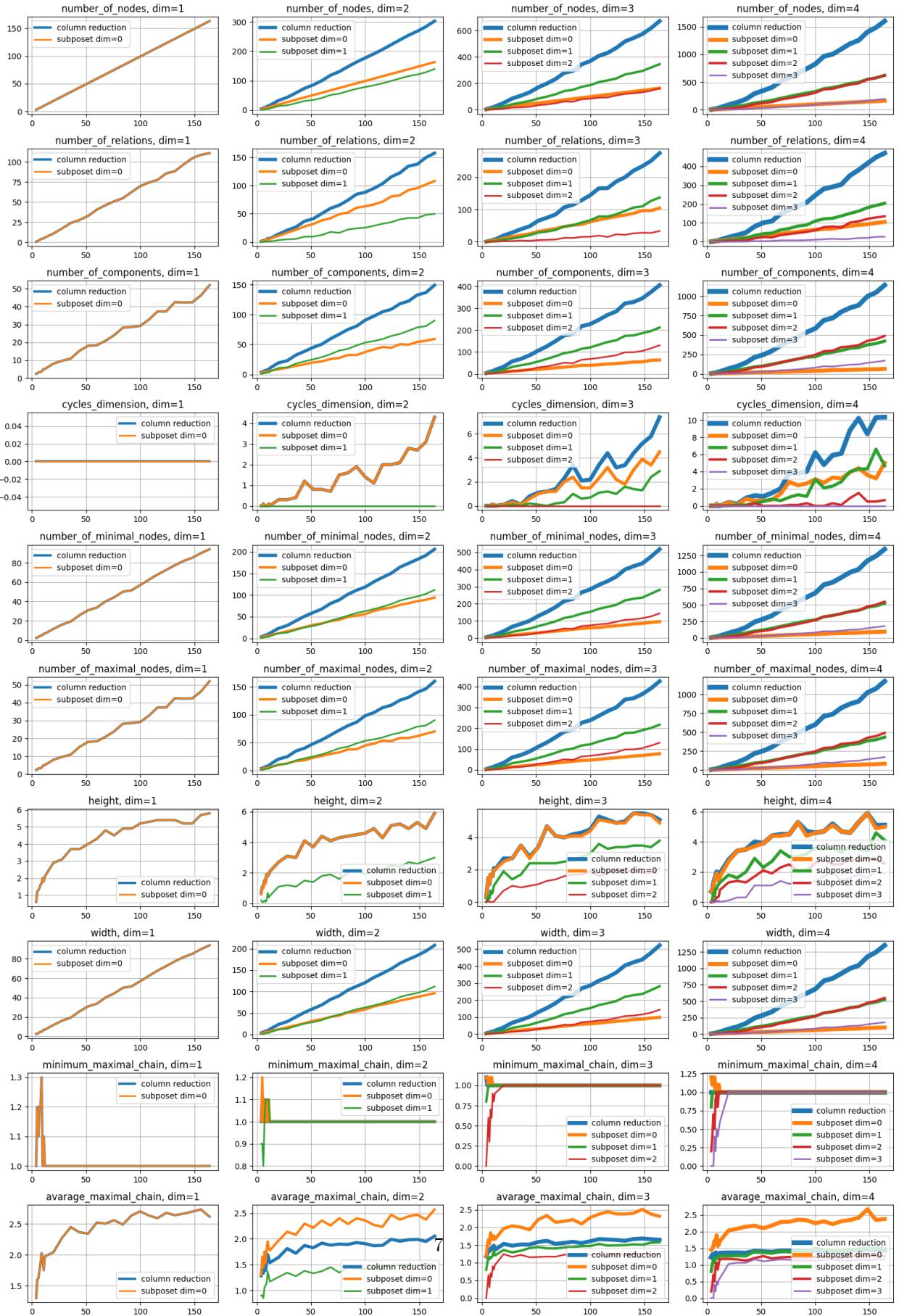


Figure 5: Column Reduction Poset: Mean poset scores

## Column Reduction Poset: Mean Node Scores

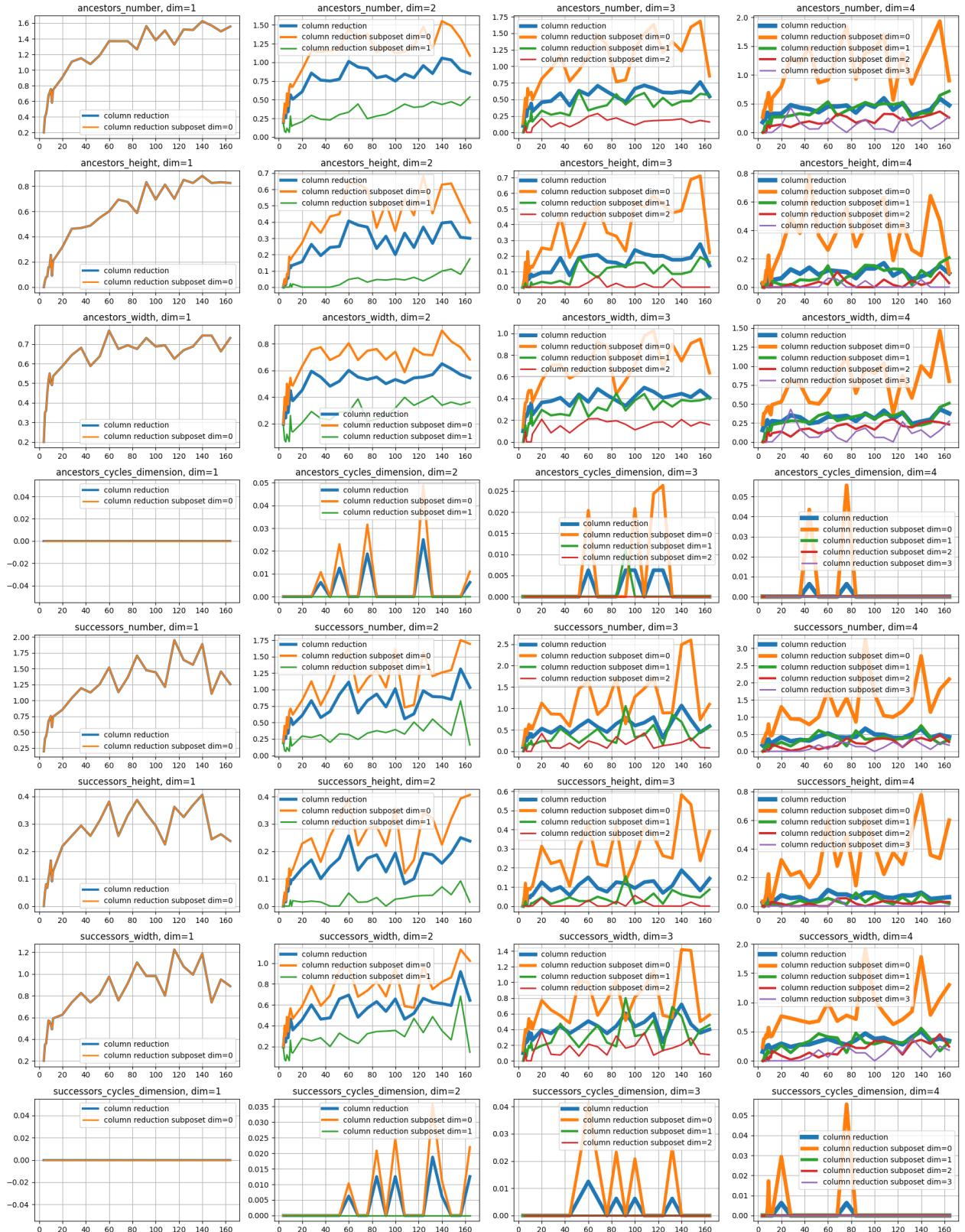


Figure 6: Column Reduction Poset: Mean node scores

## Column Reduction Poset: Max Node Scores

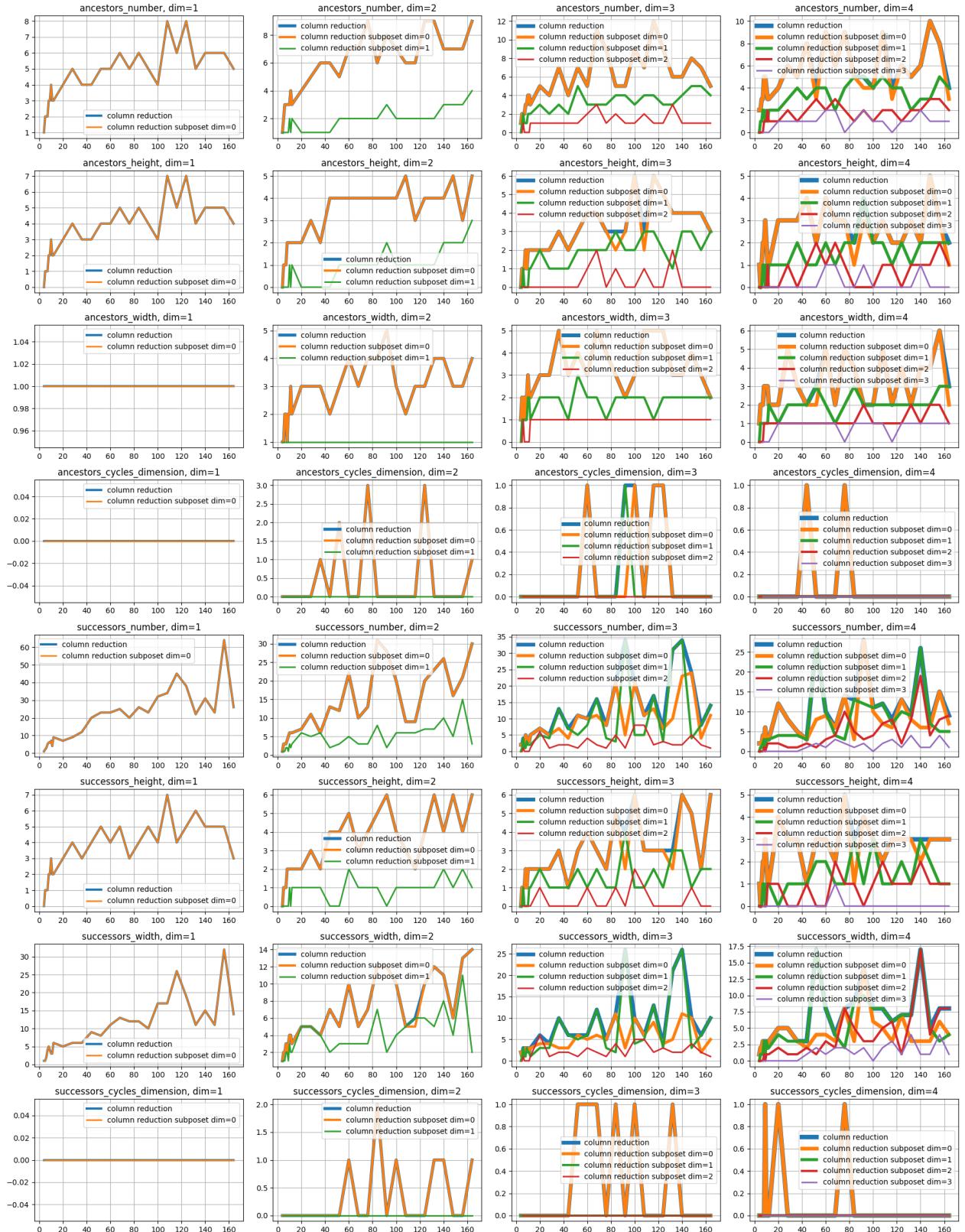


Figure 7: Column Reduction Poset: Max node scores

## Row Reduction Poset: Mean Poset Scores

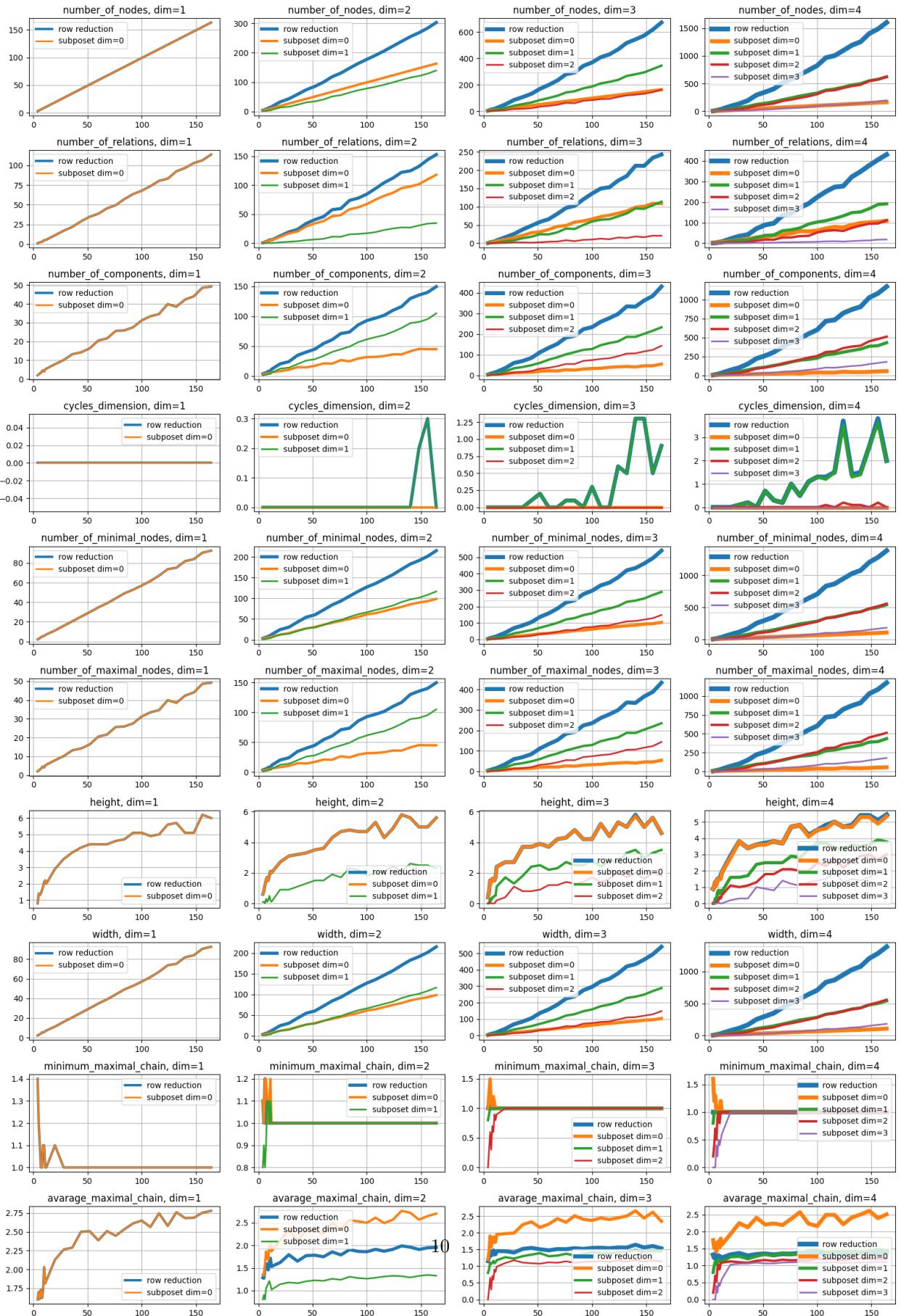


Figure 8: Row Reduction Poset: Mean poset scores

## Row Reduction Poset: Mean Node Scores

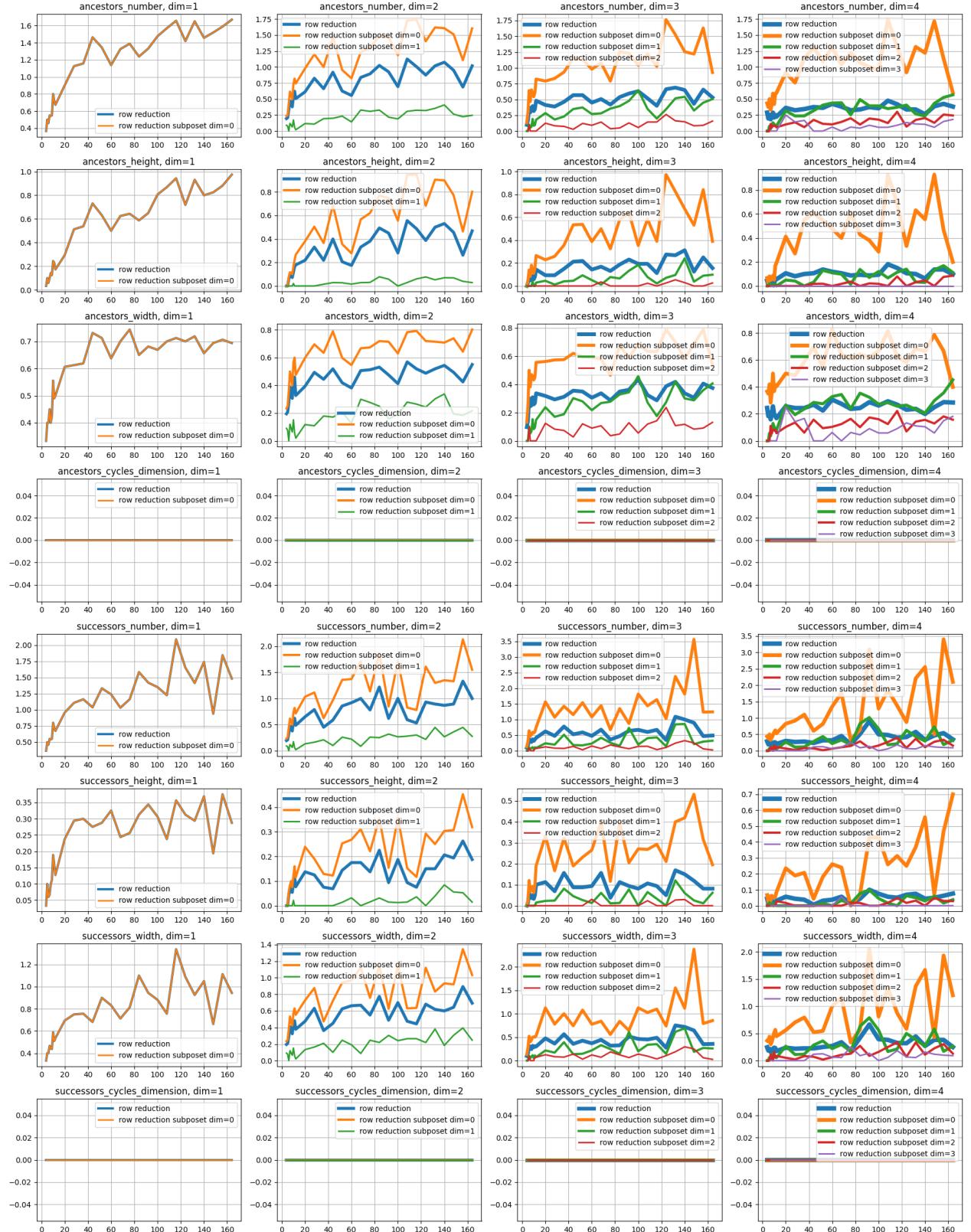


Figure 9: Row Reduction Poset: Mean node scores

## Row Reduction Poset: Max Node Scores

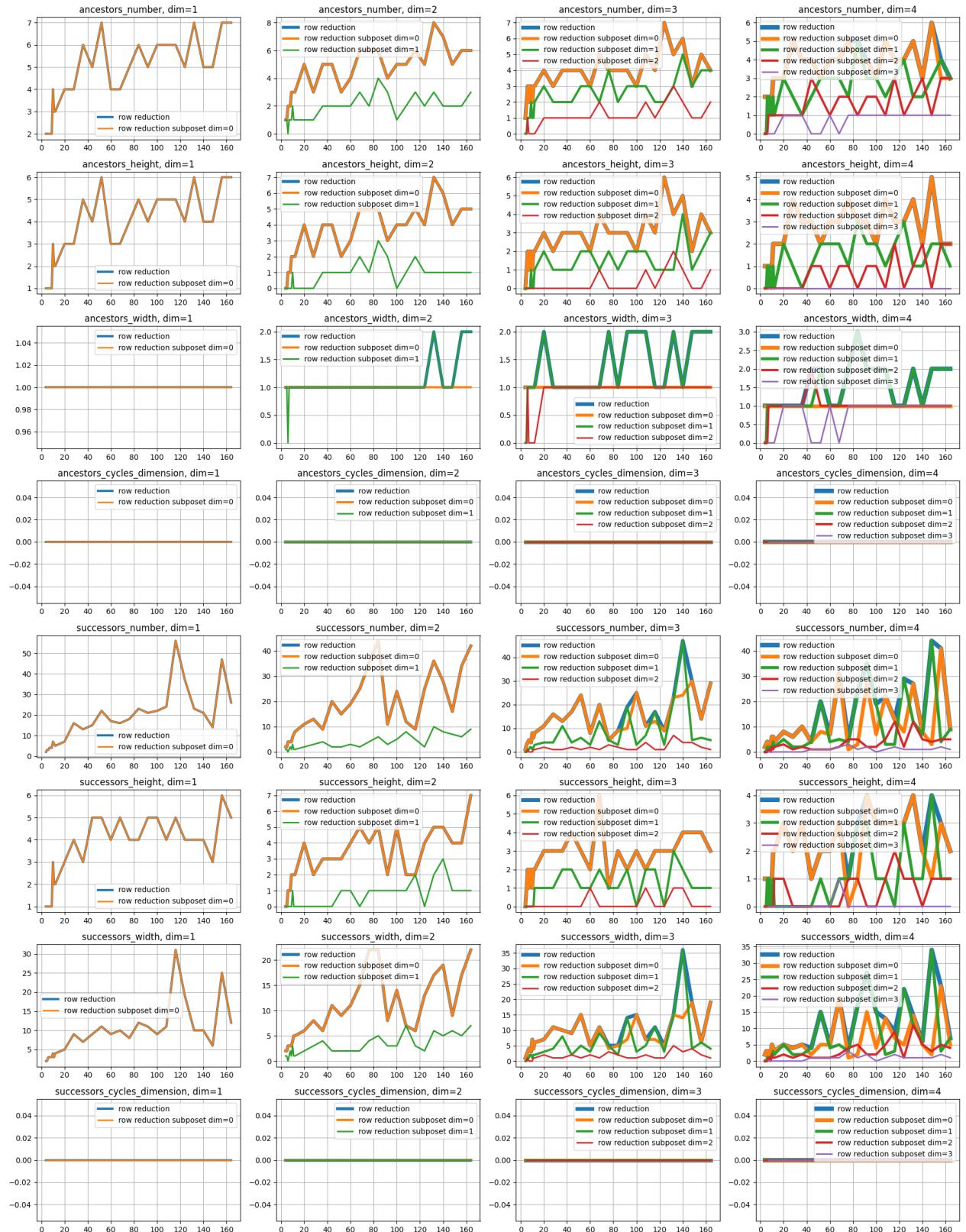


Figure 10: Row Reduction Poset: Max node scores