1 Model

1.1 Complex, Filtration and Homotopy

In this model we define the simplicial complex by the Delauney triangulation of $n = 10$ points uniformly distributed in $[0,1]^d$ for $d = 2$.

We defining the filtration on this complex, by assuming uniformly distributed in $[0,1]$ height $h(f)$ for each vertex v. Then the filtration value of the simplex will be the maximum haight of its vertices.

$$f(\sigma) = \max_{v \in \sigma} h(v)$$

We define 2 filtrations like this and study the linear homotopy between them. In the Figure 1 we can see these 2 filtrations:

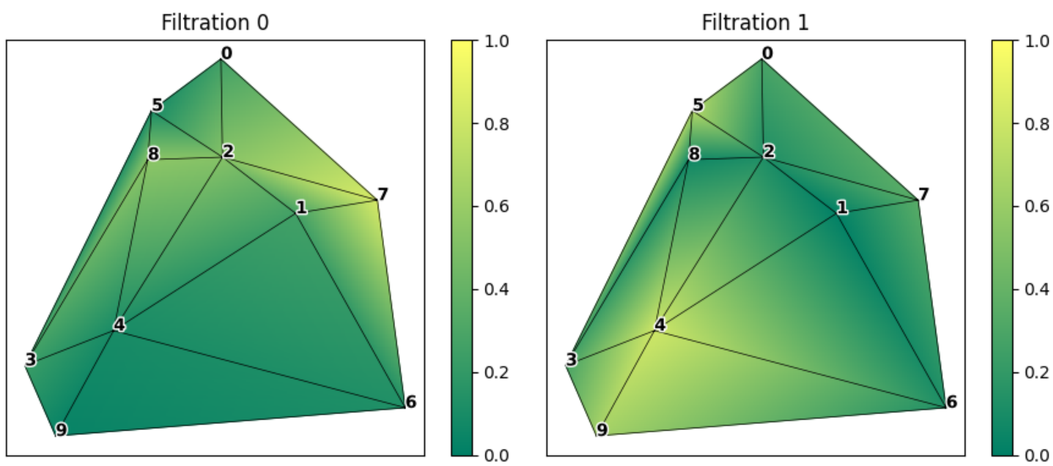

Figure 1: 2 filtrations on the defined complex.

Having these 2 filtrations we can define the homotopy between them by defining the linear homotopy between heights:

$$h_t(v) = h_0(v) \cdot (1-t) + h_1(v) \cdot t$$

$$f_t(\sigma) = \max_{v \in \sigma} h_t(v)$$

1.2 Transpositions

In the Figure 2 we can see the vertices height $h_t(v)$ during this homotopy.

When there is a cross of lines $h_t(i)$ and $h_t(j)$ $(t : h_t(i) = h_t(j))$ there is transposition of heights of vertices i and j. This means that happens reordering in the filtration f_t. The order given by $f_{t-\varepsilon}$ changes to the order given by $f_{t+\varepsilon}$.

Let's $h_t(i) < h_t(j)$. We can define 3 groups of simplices moved in the order:

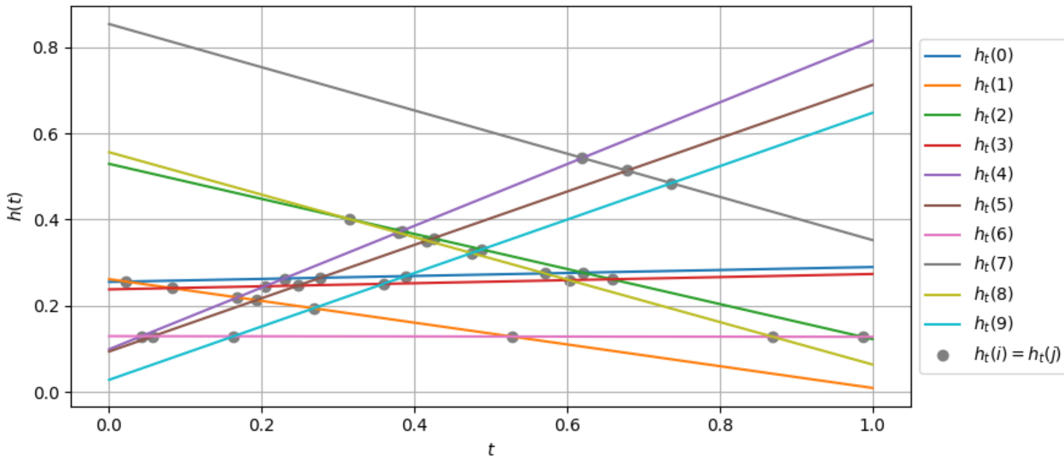

Figure 2: Heights of Vertices during the Homotopy.

1. $A = \{\sigma : i \in \sigma, j \notin \sigma, \nexists v \in \sigma : h(v) > h(j)\}$
2. $B = \{\sigma : i \notin \sigma, j \in \sigma, \nexists v \in \sigma : h(v) > h(j)\}$
3. $C = \{\sigma : i \in \sigma, j \in \sigma, \nexists v \in \sigma : h(v) > h(j)\}$

In the order given by $f_{t-\varepsilon}$ the group A stays on the first $\#A$ places, and in the order given by $f_{t+\varepsilon}$ the group B stays on the first $\#B$ places.

There are many paths of transpositions in the order, which brings us from the order $f_{t-\varepsilon}$ to the order $f_{t+\varepsilon}$ with the conctrain that σ_0 stays before σ_1 if $\sigma_0 \subset \sigma_1$. We difined 2 of them:

Up directed The first we move simplices of group B to the first places, and then we move simplices to group C to their places in $f_{t+\varepsilon}$.

Down directed The first we move simplices of group C to the last places, and then we move simplices of group A to their places in $f_{t+\varepsilon}$.