1 Model

The d dimensional torus \mathbb{T}^d can be defined as $(\mathbb{R}/n\mathbb{Z})^d$ for some natural n. And we can represent this as a cell-complex with cubical d-dimensional cells $(\mathbb{Z}/n\mathbb{Z})^d + [0,1]^d$ and all their k-faces for k = 0, ..., d.

We randomly assume the filtration value for each k-face uniformly distributed in [k, k+1]. This filtration on segmented torus will corespond some real filtration $f\mathbb{T}^d \to \mathbb{R}$, s.t. the d-dimensional cells will corespond the local maximums, vertices will corespond the local minimums and other k-faces will be saddles.

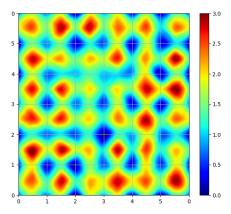


Figure 1: The example of the filtration $f: \mathbb{T}^2 \to \mathbb{R}$, s.t. there are local minimums in the vertices, saddles in the middle of edges, and the local maximums in the centers of square cells.

2 Scores

2.1 Poset Scores

- number_of_nodes : Returns the number of nodes in the poset.
- **number_of_relations** : Returns the number of relations in the transitive reduction.
- number_of_components : Returns the number of connected components in the poset
- cycles_dimension : Returns the dimension of space of cycles in reduction.

- number_of_minimal_nodes : Returns the number of minimal nodes.
- number_of_maximal_nodes : Returns the number of maximal nodes.
- height: Returns the poset height the length of the longest chain.
- width: Returns the poset width the length of the longest antichain (subset, s.t. all elements are pairwise incomparable). The algorithm is based on Dilworth's theorem and it's proof via Kőnig's theorem: link
- minimum_maximal_chain : Returns the minimum size of maximal chains in the poset.
- avarage_maximal_chain : Returns the avarage size of maximal chains in the poset.

2.2 Node Scores

- ancestors_number: Returns the number of nodes higher than given
- ancestors_height: Returns the size of maximum chain of subposet of nodes higher or equal than given
- ancestors_width: Returns the size of maximum chain of subposet of nodes higher or equal than given
- ancestors_cycles_dimension: Returns the dimension of space of cycles in reduction of subposet of nodes higher or equal than given
- successors_number: Returns the number of nodes higher than given
- successors_height: Returns the size of maximum chain of subposet of nodes lower or equal than given
- successors_width: Returns the size of maximum chain of subposet of nodes lower or equal than given
- successors_cycles_dimension: Returns the the dimension of space of cycles in reduction of subposet of nodes lower or equal than given

3 Experiments and Results

There are 339 experiments done. In the Figure 2 we can see how cases are distributed by size and dimension.

3.1 Depth Poset Features

In the Figure 3 we can see the avarage poset scores values for each number of points n in the depth poset.

In the Figure 4 we can see the avarage mean node scores values in poset for each number of points n in the depth poset.

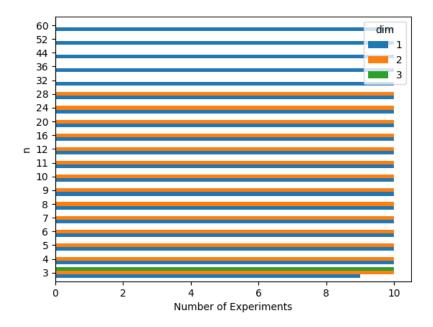


Figure 2: Size/dimension distribution of experiments

3.2 Column Reduction Poset Features

In the Figure 5 we can see the avarage poset scores values for each number of points n in the column reduction poset.

In the Figure 6 we can see the avarage mean node scores values in poset for each n in the column reduction poset.

3.3 Row Reduction Poset Features

In the Figure 7 we can see the avarage poset scores values for each number of points n in the row reduction poset.

In the Figure 8 we can see the avarage mean node scores values in poset for each number of points n in the row reduction poset.

Depth Poset: Mean Poset Scores

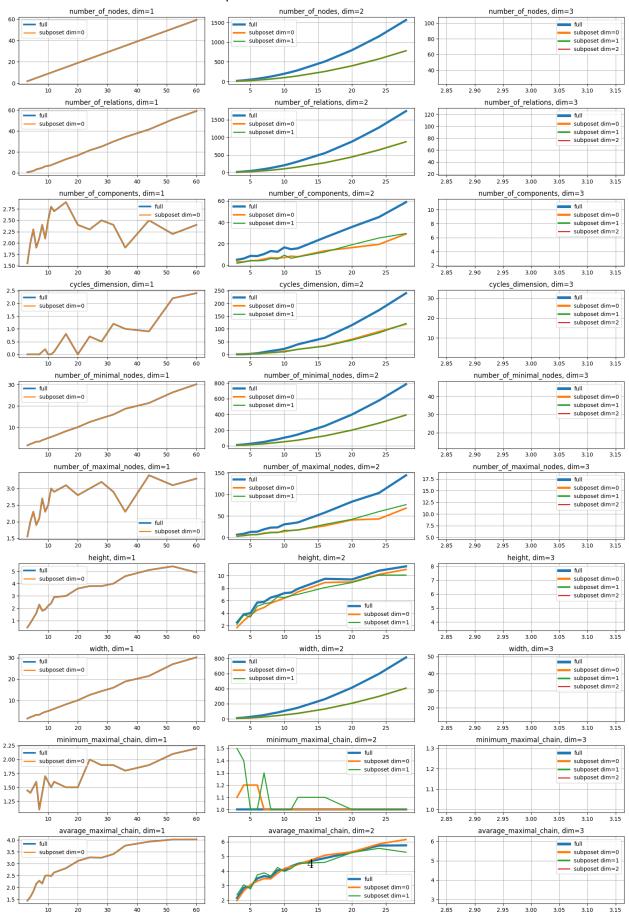


Figure 3: Depth Poset: Mean poset scores

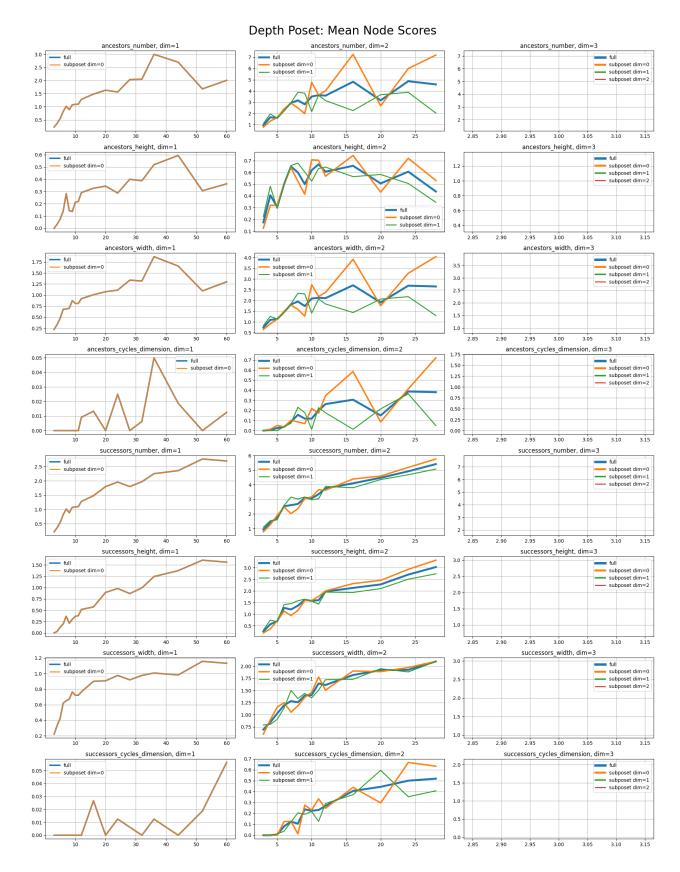


Figure 4: Depth Poset: Mean node scores

Column Reduction Poset: Mean Poset Scores

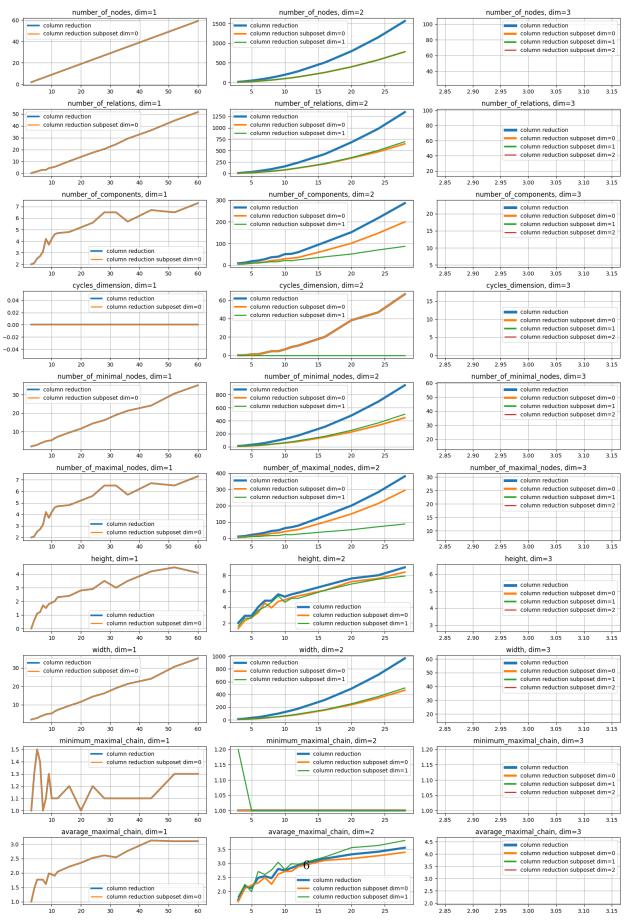


Figure 5: Column Reduction Poset: Mean poset scores

Column Reduction Poset: Mean Node Scores ancestors_number, dim=1 ancestors_number, dim=2 ancestors_number, dim=3 column reduction subposet dim=0 column reduction subposet dim=1 column reduction column reduction subposet dim=0 column reduction subposet dim=1 column reduction subposet dim=2 1.5 2.5 3.0 2.0 2.5 1.5 2.0 1.0 1.5 column reduction column reduction subposet dim=0 ancestors height, dim=1 ancestors height, dim=2 ancestors height, dim=3 column reduction column reduction column reduction column reduction subposet dim=0 column reduction subposet dim=0 column reduction subposet dim=0 0.7 0.3 column reduction subposet dim=1 0.6 column reduction subposet dim=2 0.3 0.5 0.2 0.4 0.1 0.3 0.1 0.2 3.15 ancestors_width, dim=1 ancestors_width, dim=2 ancestors_width, dim=3 2.00 column reduction 2.25 1.75 column reduction subposet dim=0 1.0 2.00 column reduction subposet dim=1 1.50 0.8 — column reduction subposet dim=2 1.75 0.6 1.25 1.00 0.4 1.25 column reduction column reduction subposet dim=0 column reduction subposet dim=1 0.75 0.2 1.00 40 15 2.90 2.95 3.00 3.10 3.15 ancestors_cycles_dimension, dim=1 ancestors_cycles_dimension, dim=2 ancestors_cycles_dimension, dim=3 column reduction column reduction subposet dim-column reduction subposet dimcolumn reduction column reduction subposet dim=0 column reduction column reduction subposet dim=0 column reduction subposet dim=1 column reduction subposet dim=2 0.04 0.15 0.02 0.10 0.00 -0.02 0.05 0.05 15 25 2.90 3.15 2.85 2.95 3.00 3.05 3.10 successors_number, dim=1 successors number, dim=2 successors_number, dim=3 column reduction column reduction subposet dim=0 column reduction subposet dim=1 column reduction subposet dim=2 column reduction column reduction subposet dim=0 2.5 3.0 2.0 1.0 2.5 1.5 2.0 column reduction 1.0 1.5 column reduction subposet dim=0 column reduction subposet dim=1 1.0 2.95 3.10 successors_height, dim=1 successors_height, dim=2 successors_height, dim=3 column reduction column reduction column reduction subposet dim=0 column reduction 0.8 column reduction subposet dim=0 column reduction subposet dim=0 1.50 olumn reduction subposet dim=1 column reduction subposet dim=1 column reduction subposet dim=2 1 25 1.00 1.0 0.4 0.75 0.5 0.50 0.0 0.25 25 2.90 2.95 3.10 3.15 successors_width, dim=1 successors_width, dim=2 successors_width, dim=3 column reduction column reduction subposet dim=0 1.2 1.6 column reduction subposet dim=1 0.6 1.0 1.4 column reduction subposet dim=2 0.8 1.0 0.2 0.6 0.8 0.6 2.85 2.95 3.00 3.05 3.15 $successors_cycles_dimension, \ dim{=}1$ successors_cycles_dimension, dim=2 successors_cycles_dimension, dim=3 column reduction column reduction subposet dim=0 column reduction subposet dim=1 column reduction column reduction subposet dim=0 column reduction subposet dim=1 column reduction subposet dim=2 0.04 0.05 0.02 0.04 0.3 0.00 0.03 0.2 0.02

Figure 6: Column Reduction Poset: Mean node scores

0.01

0.1

2.95

3.10

-0.02

-0.04

Row Reduction Poset: Mean Poset Scores

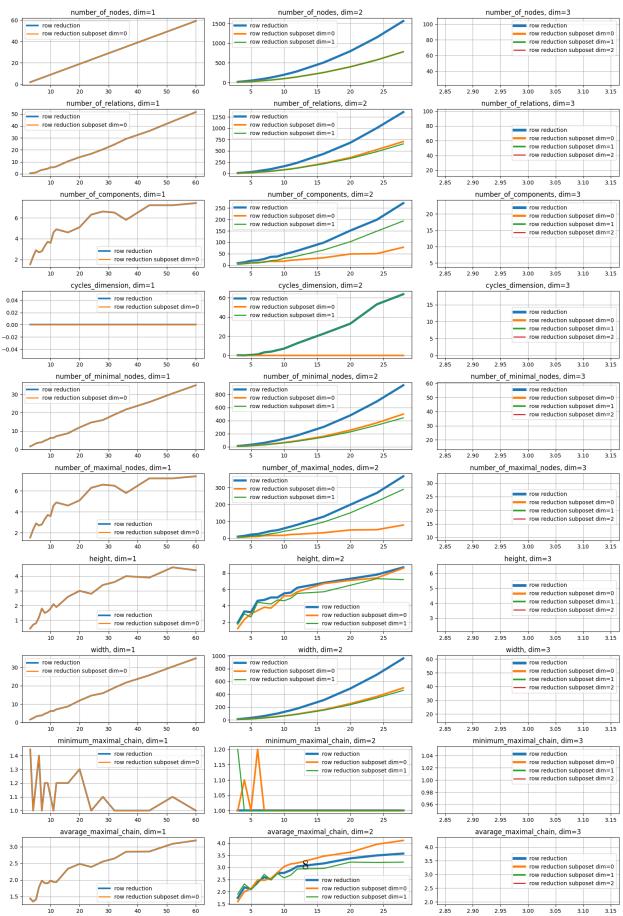


Figure 7: Row Reduction Poset: Mean poset scores

Row Reduction Poset: Mean Node Scores ancestors_number, dim=1 ancestors_number, dim=2 ancestors_number, dim=3 row reduction row reduction subposet dim=0 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction subposet dim=2 row reduction row reduction subposet dim=0 1.0 ancestors height, dim=2 ancestors height, dim=3 ancestors height, dim=1 row reduction 0.7 row reduction subposet dim=0 0.6 0.5 0.2 0.4 0.2 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction subposet dim=2 0.3 0.1 0.2 row reduction subposet dim=0 row reduction subposet dim=1 ancestors_width, dim=1 ancestors_width, dim=2 ancestors_width, dim=3 2.5 row reduction row reduction row reduction subposet dim=0 3.0 row reduction subposet dim=0 1.2 row reduction subposet din 2.0 2.5 row reduction 1.0 row reduction subposet dim=0 row reduction subposet dim=1 row reduction subposet dim=2 2.0 1.5 0.8 1.5 0.6 1.0 1.0 0.4 0.5 0.2 25 2.85 2.90 2.95 3.00 3.05 3.10 3.15 ancestors_cycles_dimension, dim=1 ancestors_cycles_dimension, dim=2 ancestors_cycles_dimension, dim=3 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction subposet dim=2 0.04 0.08 0.02 0.2 0.00 0.04 -0.02 0.1 0.02 25 2.90 3.15 2.85 2.95 3.00 3.05 3.10 successors_number, dim=1 successors_number, dim=2 successors_number, dim=3 2.00 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction subposet dim=2 row reduction row reduction subposet dim=0 1.75 3.5 1.50 2.0 3.0 1.25 1.00 2.5 1.5 0.75 2.0 1.0 0.50 row reduction subposet dim=0 1.5 row reduction subposet dim=1 0.5 0.25 1.0 2.95 3.10 successors_height, dim=1 successors_height, dim=2 successors_height, dim=3 - row reduction 2.0 row reduction 1.6 row reduction row reduction subposet dim=0 row reduction subposet dim=0 row reduction subposet dim=0 0.8 row reduction subposet dim=1 row reduction subposet dim=1 row reduction subposet dim=2 1.5 1.2 0.6 1.0 1.0 0.4 0.8 0.5 0.6 0.2 0.4 0.0 25 2.90 2.95 3.00 3.05 3.10 3.15 successors_width, dim=1 successors_width, dim=2 successors_width, dim=3 0.9 row reduction row reduction subposet dim=0 0.8 row reduction subposet dim=1 0.7 1.0 1.6 row reduction subposet dim=2 0.6 1.4 0.8 0.5 1.2 0.4 0.6 1.0 0.3 0.8 0.2 40 2.85 2.95 3.00 3.05 3.15 $successors_cycles_dimension, \, dim{=}1$ successors_cycles_dimension, dim=2 successors_cycles_dimension, dim=3 0.5 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction subposet dim=2 0.04 0.10 0.4 0.08 0.02 0.3 0.06 0.00 0.2 0.04 -0.02 0.1 0.02 -0.04

Figure 8: Row Reduction Poset: Mean node scores

2.85

2.95

3.10