## 1 Problems

Write  $\operatorname{Succ}(s,t) = \operatorname{Succ}'(s,t) = \operatorname{Succ}''(s,t)$  for the succesors and  $\operatorname{Pred}(s,t) = \operatorname{Pred}'(s,t) = \operatorname{Pred}''(s,t)$  for the predecessors (ancessors) of the pair in D, D', D'', respectively.

Since  $D = D' \cup D''$ , we also have  $\operatorname{Succ}(s,t) = \operatorname{Succ}'(s,t) \cup \operatorname{Succ}''(s,t)$  and  $\operatorname{Pred}(s,t) = \operatorname{Pred}''(s,t) \cup \operatorname{Pred}''(s,t)$ .

#### 1.1 Lemma 3.1.

Suppose (x, y) and (a, b) are birth-death pairs of  $f: X \to R$ , a, x are consecutive in the ordering of the cells by f, and the transposition a, x is a switch. Then

$$Succ'(a, y) = \{(x, b)\} \cup Succ'(a, b) \cup \{(s, t) \in Succ'(x, y) | f(t) < f(b)\}$$
$$Succ'(x, b) = \{(s, t) \in Succ'(x, y) | f(t) > f(b)\}$$

#### 1.2 Lemma 3.2.

Suppose (a, b) and (x, y) are birth-death pairs of  $f: X \to R$ , y, b are consecutive in the ordering by f, and transposition of y, b is a switch. Then

$$Succ''(x,b) = \{(a,y)\} \cup Succ''(a,b) \cup \{(s,t) \in Succ''(x,y) | f(a) < f(s) < f(x)\}$$
$$Succ''(a,y) = \{(s,t) \in Succ''(x,y) | f(s) < f(a)\}$$

#### 1.3 Lemma 3.3.

Suppose (a, b) and (x, y) are birth-death pairs of  $f: X \to R$ , b, x are consecutive in the ordering by f, and the transposition of b, x is a switch. Then

$$Succ(a, x) = Succ(a, b)$$
 and  $Succ(b, y) = Succ(x, y)$ 

### 1.4 Hypothesis 1

.

Suppose a and b are 2-simplices consecutive in the ordering by f. And there is another Morse function  $f^*$ :

$$f^*(s) = \begin{cases} f(s), & \text{if } s \neq a, b \\ f(b), & \text{if } s = a \\ f(a), & \text{if } s = b \end{cases}$$

Let's denote  $DP_f$  the transitive reduction of the Depth Poset defined by the filtration f. And let's denote  $DP_f(s)$  the set of nodes in  $DP_f(s)$  which are pairs containing cell s and the set of edges with these nodes.

**Hypothesis:** if the cell s has no faces and cofaces with a and b, then  $DP_f(s) = DP_{f^*}(s)$ .

# 2 Model

The probabilistic model is simple. The first we just generate the cloud of n points uniformly distributed in  $[0,1]^d$ . After this we calculate the Alpha complex with these points, and then find its depth poset. Then we itarate all neighbour pairs of simplices and check if their transposition will be possible filtration, calculating the scores for the switch-forward transpositions.

The number of points in the cloud n and the dimension  $d = \dim$  for each generated cloud and coresponding complex are given in the table:

complex	n	dim	simplices
0	6	2	23
1	6		23
2	6	2	21
3	6	2	23
4	8	2	31
5	8	2	33
6	8	2	31
7	8	2	33
8	8	2 2 2 2 2 2 2 2 2	33
9	8	2	33
10	8	2	33
11	8	2	31
12	12	2 2 2 2 2 2 2 2 2 2 2 2 2 2 3 2 3 2 3 3	53
13	12	2	51
14	12	2	55
15	12	2	53
16	12	2	55
17	12	2	55
18	16	2	73
19	16	2	75
20	20	2	95
21	8		59
22	8	3	73
23	8	3	63
24	8	3	83
25	8	3	73
26	8	3	79
27	12	3	147
28	12	3	141
29	16	3	243
30	8	4	133
31	8	4	147
32	12	4	283

#### 3 Scores

- jacard\_nodes\_filtration: The Jacard index of node sets from 2 depth posets. The birth-death pairs are equal in terms of filtration values.
- jacard\_nodes\_simplex: The Jacard index of nodes from 2 depth posets. The birth-death pairs are equal in terms of simplices.
- jacard\_edges\_filtration: The Jacard index of edge sets from transitive reductions of 2 depth posets. The birth-death pairs are equal in terms of filtration values.
- jacard\_edges\_simplex: The Jacard index of edge sets from transitive reductions of 2 depth posets. The birth-death pairs are equal in terms of simplices.
- jacard\_l31a: Jacard Index of Succ'(x,b) and  $\{(a,y)\}\cup$ Succ' $(a,b)\cup$  $\{(s,t)\in$ Succ' $(x,y)|f(t)< f(b)\}$
- jacard\_l31b: Jacard Index of Succ'(x,b) and  $\{(s,t) \in \text{Succ'}(x,y)|f(t) > f(b)\}$
- jacard\_l32a: Jacard Index of Succ"(x,b) and  $\{(a,y)\} \cup$  Succ" $(a,b) \cup$   $\{(s,t) \in$  Succ" $(x,y)|f(a) < f(s) < f(x)\}$
- jacard\_l32b: Jacard Index of Succ''(a, y) and  $\{(s,t) \in \text{Succ''}(x,y) | f(s) < f(a)\}$
- jacard\_l33a: Jacard Index of Succ(a, x) and Succ(a, b)
- jacard\_l33b: Jacard Index of Succ(b, y) and Succ(x, y)
- **jacard\_nn\_nodes**: The Jacard index of subsets of nodes (s,t) from 2 depth posets, s.t.  $s,t \notin \nabla \partial \sigma_0 \cup \partial \nabla \sigma_0 \cup \nabla \partial \sigma_1 \cup \partial \nabla \sigma_1$ , where  $\sigma_0$  and  $\sigma_1$  are transposing simplices.
- **jacard\_nn\_edges**: The Jacard index of subsets of edges  $((s_0, t_0), (s_1, t_1))$  from 2 depth posets, s.t.  $s_0, t_0, s_1, t_1 \notin \nabla \partial \sigma_0 \cup \partial \nabla \sigma_0 \cup \nabla \partial \sigma_1 \cup \partial \nabla \sigma_1$ , where  $\sigma_0$  and  $\sigma_1$  are transposing simplices.

## 4 Results

Here are the tables, containing the scores for the switch transpositions in the experiment:

0	complex	dim	n	dim 0	dim 1	type	jacard_nodes_filtration	jacard_nodes_simplex
1	0	2	6	1	1	death-death	1.00	0.56
1	1	2	6	1	1	death-death	1.00	0.60
2 2 6 0 0 birth-birth 1.00 1.00 2 2 2 6 1 1 birth-death 1.00 0.50 3 2 6 1 1 birth-death 1.00 0.50 5 2 8 0 0 birth-birth 1.00 0.50 5 2 8 1 1 birth-death 1.00 0.64 6 2 8 1 1 birth-death 1.00 0.64 6 2 8 1 1 birth-death 1.00 0.64 7 2 8 0 0 birth-birth 1.00 0.64 8 2 8 1 1 birth-death 1.00 0.69 9 2 8 0 birth-birth 1.00 1.00 9 2 8 1 birth-death 1.00 0.69 9 2 8 1 birth-death 1.00 0.69 9 2 8 1 birth-death 1.00 0.69 9 1 2 8 1 birth-death 1.00 0.69 9 1 2 8 0 birth-birth 1.00 1.00 11 2 8 0 birth-birth 1.00 0.69 11 2 8 0 birth-birth 1.00 0.69 11 2 1 1 birth-death 1.00 0.69 11 2 2 1 1 birth-death 1.00 0.79 11 2 1 1 birth-death 1.00 0.79 11 2 1 1 birth-death 1.00 0.79 11 2 1 1 birth-death 1.00 0.80 11 1 1 birth-death 1.00 0.80 11 1 1 birth-death 1.00 0.80 11 1 birth-death 1.00 0.80 11 1 birth-death 1.00 0.80	1	2	6	1	1	birth-death	1.00	0.60
2 2 6 0 0 0 birth-birth 1.00 0.50 3 2 6 1 1 birth-death 1.00 0.50 5 2 8 0 0 birth-birth 1.00 1.00 5 2 8 1 1 birth-death 1.00 0.64 6 2 8 1 1 birth-death 1.00 0.64 7 2 8 0 0 birth-birth 1.00 1.00 7 2 8 1 1 birth-death 1.00 0.68 8 2 8 1 1 birth-death 1.00 0.69 8 2 8 1 1 birth-death 1.00 0.69 8 2 8 1 1 birth-death 1.00 0.69 9 1 2 8 0 0 birth-birth 1.00 0.69 9 1 2 8 0 0 birth-birth 1.00 0.69 9 1 2 8 0 0 birth-birth 1.00 0.69 10 2 8 0 0 birth-birth 1.00 0.69 11 2 8 1 1 death-death 1.00 0.69 11 2 1 1 death-death 1.00 0.69 11 2 1 1 death-death 1.00 0.69 11 2 1 1 birth-death 1.00 0.69 12 2 1 1 birth-death 1.00 0.67 13 2 1 2 1 1 birth-death 1.00 0.67 14 2 12 1 1 birth-death 1.00 0.69 15 2 12 1 1 birth-death 1.00 0.69 16 0.69 17 2 12 1 1 birth-death 1.00 0.69 18 2 12 1 1 birth-death 1.00 0.69 19 2 10 0 birth-birth 1.00 0.69 10 0.69 11 0 0.69 11 0 0.69 12 0 0 0 birth-birth 1.00 0.69 12 0 0 0 birth-birth 1.00 0.69 13 0 0 0.69 14 0 0 0.69 15 0 0 0 0.69 15 0 0 0 0.69 16 0 0 0 0.69 17 0 0 0.69 18 0 0 0 0.69 18	1		6	1	1	birth-death	1.00	0.60
2 2 6 1 1 birth-death 1.00 0.50 3 2 6 1 1 birth-death 1.00 0.50 5 2 8 0 0 birth-birth 1.00 0.64 6 2 8 1 1 birth-death 1.00 0.64 6 2 8 1 1 birth-death 1.00 0.64 7 2 8 0 0 birth-birth 1.00 0.64 7 2 8 1 birth-death 1.00 0.66 8 2 8 1 1 birth-death 1.00 0.66 8 2 8 1 1 birth-death 1.00 0.66 9 9 2 8 1 1 birth-death 1.00 0.69 9 1 2 8 1 1 birth-death 1.00 0.69 9 2 8 1 1 birth-death 1.00 0.69 9 1 2 8 2 2 death-death 1.00 0.69 9 1 2 8 2 2 death-death 1.00 0.69 10 2 8 0 0 birth-birth 1.00 1.00 11 2 8 1 death-death 1.00 0.67 12 2 12 1 1 death-death 1.00 0.67 13 2 12 1 1 birth-death 1.00 0.67 14 2 12 1 1 birth-death 1.00 0.78 13 2 12 1 1 birth-death 1.00 0.78 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 15 2 12 1 1 birth-death 1.00 0.79 16 2 12 1 1 birth-death 1.00 0.79 17 2 12 1 1 birth-death 1.00 0.79 18 2 16 1 1 birth-death 1.00 0.80 18 2 16 1 1 birth-death 1.00 0.80 19 2 16 1 1 birth-death 1.00 0.80 10 0.80 11 0 0.80 12 12 1 1 birth-death 1.00 0.88 12 12 1 1 birth-death 1.00 0.88 13 1 1 birth-death 1.00 0.88 14 2 16 1 1 birth-death 1.00 0.88 18 2 16 1 1 birth-death 1.00 0.88 18 2 16 1 1 birth-death 1.00 0.88 20 2 20 1 1 birth-death 1.00 0.88 21 3 8 2 2 birth-death 1.00 0.88 22 3 8 1 1 birth-death 1.00 0.88 23 3 8 2 2 birth-death 1.00 0.88 24 2 2 birth-death 1.00 0.88 25 2 2 2 birth-death 1.00 0.88 26 3 3 8 2 2 birth-death 1.00 0.88 27 3 3 8 1 1 birth-death 1.00 0.88			6	0	0	birth-birth	1.00	1.00
3				0	0			
5         2         8         0         0         birth-birth         1.00         0.64           6         2         8         1         1         birth-death         1.00         0.64           7         2         8         0         0         birth-birth         1.00         0.69           8         2         8         1         1         birth-death         1.00         0.69           8         2         8         1         1         birth-birth         1.00         0.69           9         2         8         1         1         birth-birth         1.00         0.69           9         2         8         1         1         birth-birth         1.00         0.69           9         2         8         1         1         birth-death         1.00         0.69           9         2         8         2         2         death-death         1.00         0.69           10         2         8         0         0         birth-birth         1.00         0.60           11         2         8         0         0         birth-birth         1.00         0.0 </td <td></td> <td></td> <td></td> <td></td> <td>1</td> <td></td> <td></td> <td></td>					1			
5         2         8         1         1         birth-death         1.00         0.64           6         2         8         1         1         birth-death         1.00         0.64           7         2         8         1         1         birth-death         1.00         0.69           8         2         8         1         1         birth-death         1.00         0.64           9         2         8         0         0         birth-birth         1.00         0.69           9         2         8         1         1         birth-birth         1.00         0.69           9         2         8         1         1         birth-birth         1.00         0.69           9         2         8         2         2         death-death         1.00         0.69           10         2         8         0         0         birth-birth         1.00         1.00           11         2         8         0         0         birth-birth         1.00         1.00           11         2         8         1         1         death-death         1.00         0.67					1			
6 2 8 1 1 birth-death 1.00 0.64 7 2 8 0 0 birth-birth 1.00 1.00 7 2 8 1 1 birth-death 1.00 0.69 8 2 8 1 1 birth-death 1.00 0.64 9 2 8 0 0 birth-birth 1.00 1.00 9 2 8 1 1 birth-death 1.00 0.69 9 2 8 1 1 birth-death 1.00 0.69 9 2 8 1 1 birth-death 1.00 0.69 9 2 8 2 2 death-death 1.00 0.69 9 1 2 8 2 2 death-death 1.00 0.69 10 2 8 0 0 birth-birth 1.00 1.00 11 2 8 1 1 death-death 1.00 0.67 12 2 12 1 1 death-death 1.00 0.67 13 2 12 1 1 birth-death 1.00 0.67 13 2 12 1 1 birth-death 1.00 0.78 13 2 12 1 1 birth-death 1.00 0.78 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 15 2 12 1 1 death-death 1.00 0.79 16 2 12 1 1 birth-death 1.00 0.79 17 2 12 1 1 birth-death 1.00 0.79 18 2 12 1 1 birth-death 1.00 0.79 19 15 2 12 1 1 birth-death 1.00 0.79 19 15 2 12 1 1 death-death 1.00 0.79 19 15 2 12 1 1 death-death 1.00 0.79 19 15 2 12 1 1 death-death 1.00 0.79 19 15 2 12 1 1 death-death 1.00 0.79 19 15 2 12 1 1 death-death 1.00 0.79 19 15 2 12 1 1 death-death 1.00 0.79 19 15 2 12 1 1 death-death 1.00 0.80 10 0.80 11 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0				0	0			
7 2 8 0 0 0 birth-birth 1.00 1.00 7 2 8 1 1 birth-death 1.00 0.69 8 2 8 1 1 birth-death 1.00 0.64 9 2 8 0 0 0 birth-birth 1.00 1.00 9 2 8 1 1 birth-death 1.00 0.69 9 2 8 1 1 birth-death 1.00 0.69 9 2 8 2 1 birth-death 1.00 0.69 9 2 8 2 2 death-death 1.00 0.69 10 2 8 0 0 birth-birth 1.00 1.00 11 2 8 0 0 birth-birth 1.00 0.67 12 2 12 1 1 death-death 1.00 0.67 13 2 12 1 1 birth-death 1.00 0.78 13 2 12 1 1 birth-death 1.00 0.78 13 2 12 1 1 birth-death 1.00 0.78 14 2 12 1 1 birth-death 1.00 0.78 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 15 2 12 0 0 birth-birth 1.00 0.79 16 2 12 1 1 birth-death 1.00 0.79 17 2 12 1 1 death-death 1.00 0.79 18 2 12 1 1 birth-death 1.00 0.79 19 10 0.79 10 0.79 11 0.00 0.79 12 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.					1			
7         2         8         1         1         birth-death         1.00         0.69           8         2         8         1         1         birth-death         1.00         0.64           9         2         8         1         1         birth-death         1.00         0.69           9         2         8         1         1         birth-death         1.00         0.69           9         2         8         1         1         birth-death         1.00         0.69           10         2         8         0         0         birth-birth         1.00         1.00           11         2         8         0         0         birth-birth         1.00         1.00           11         2         8         0         0         birth-birth         1.00         1.00           11         2         8         0         0         birth-birth         1.00         0.67           12         2         12         1         1         death-death         1.00         0.78           13         2         12         1         1         birth-death         1.00 <td< td=""><td></td><td></td><td></td><td></td><td>1</td><td></td><td></td><td></td></td<>					1			
8         2         8         1         1         birth-birth         1.00         0.64           9         2         8         0         0         birth-death         1.00         0.69           9         2         8         1         1         birth-death         1.00         0.69           9         2         8         2         2         death-death         1.00         0.69           10         2         8         0         0         birth-birth         1.00         1.00           11         2         8         0         0         birth-birth         1.00         0.67           12         2         12         1         1         birth-death         1.00         0.67           13         2         12         1         1         birth-death         1.00 <t< td=""><td></td><td></td><td></td><td>0</td><td>0</td><td></td><td></td><td></td></t<>				0	0			
9 2 8 0 0 birth-birth 1.00 0.69 9 2 8 1 1 birth-death 1.00 0.69 9 2 8 2 0 0 birth-birth 1.00 0.69 10 2 8 0 0 birth-birth 1.00 1.00 11 2 8 0 0 birth-birth 1.00 1.00 11 2 8 0 0 birth-birth 1.00 1.00 11 2 8 1 1 death-death 1.00 0.67 11 2 8 1 1 death-death 1.00 0.67 12 2 12 1 1 birth-death 1.00 0.67 13 2 12 1 1 birth-death 1.00 0.78 13 2 12 1 1 birth-death 1.00 0.78 14 2 12 1 1 death-death 1.00 0.78 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 15 2 12 1 1 birth-death 1.00 0.79 16 2 12 1 1 birth-death 1.00 0.79 17 2 12 1 1 birth-death 1.00 0.79 18 2 12 1 1 birth-death 1.00 0.79 19 10 0.79 10 0.79 11 1 0 0.79 12 1 1 birth-death 1.00 0.79 13 2 1 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 15 2 12 1 1 birth-death 1.00 0.79 16 2 12 1 1 birth-death 1.00 0.79 17 2 12 1 1 birth-death 1.00 0.80 18 2 16 0 0 birth-birth 1.00 0.80 19 2 16 1 1 birth-death 1.00 0.88 18 2 16 1 1 birth-death 1.00 0.88 18 2 16 1 1 birth-death 1.00 0.88 19 2 16 1 1 birth-death 1.00 0.88 19 2 16 1 1 birth-death 1.00 0.88 20 2 20 1 1 birth-death 1.00 0.88 20 2 20 1 1 birth-death 1.00 0.88 21 3 8 2 2 birth-death 1.00 0.88 22 3 8 2 2 birth-death 1.00 0.88 23 3 8 2 2 birth-death 1.00 0.88 24 3 8 2 2 birth-death 1.00 0.88 25 3 8 2 2 birth-death 1.00 0.88 26 3 8 2 2 birth-death 1.00 0.88 27 5 2 3 8 2 2 birth-death 1.00 0.88 28 3 8 2 2 birth-death 1.00 0.88 29 3 8 2 2 birth-death 1.00 0.88					1			
9 2 8 1 1 birth-death 1.00 0.69 9 2 8 1 1 birth-death 1.00 0.69 9 2 8 2 2 death-death 1.00 0.69 10 2 8 0 0 birth-birth 1.00 1.00 11 2 8 0 0 birth-birth 1.00 1.00 11 2 8 0 0 birth-birth 1.00 1.00 11 2 8 1 1 death-death 1.00 0.67 11 2 8 1 1 death-death 1.00 0.67 12 2 12 1 1 death-death 1.00 0.78 13 2 12 1 1 birth-death 1.00 0.78 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 15 2 12 1 1 birth-death 1.00 0.79 16 2 12 1 1 birth-death 1.00 0.79 17 2 12 1 1 birth-death 1.00 0.79 18 2 10 0 birth-birth 1.00 0.80 18 2 10 1 1 birth-death 1.00 0.80 19 2 10 1 1 birth-death 1.00 0.80 19 2 10 1 1 birth-death 1.00 0.80 19 2 10 1 1 birth-death 1.00 0.88 19 2 16 1 1 birth-death 1.00 0.88 19 2 16 1 1 birth-death 1.00 0.88 20 2 20 1 1 birth-death 1.00 0.88 21 3 8 2 2 birth-death 1.00 0.88 22 3 8 2 2 birth-death 1.00 0.88 23 3 8 1 1 birth-death 1.00 0.88 24 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5								
9 2 8 1 1 birth-death 1.00 0.69 9 2 8 2 2 death-death 1.00 1.00 110 2 8 0 0 birth-birth 1.00 1.00 111 2 8 0 0 birth-birth 1.00 1.00 111 2 8 1 1 death-death 1.00 0.67 112 2 12 1 1 death-death 1.00 0.78 13 2 12 1 1 birth-death 1.00 0.78 13 2 12 1 1 birth-death 1.00 0.78 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 15 2 12 1 1 birth-death 1.00 0.79 16 2 12 1 1 birth-death 1.00 0.79 17 2 12 1 1 birth-death 1.00 0.79 18 2 12 1 1 birth-death 1.00 0.79 19 10 0.79 10 0.79 11 0.79								
9 2 8 2 0 0 birth-birth 1.00 1.00 11 2 8 0 0 birth-birth 1.00 1.00 111 2 8 0 0 birth-birth 1.00 1.00 111 2 8 0 0 birth-birth 1.00 1.00 111 2 8 1 1 death-death 1.00 0.67 11 2 8 1 1 death-death 1.00 0.78 12 2 12 1 1 death-death 1.00 0.78 13 2 12 1 1 birth-death 1.00 0.78 13 2 12 1 1 death-death 1.00 0.78 14 2 12 1 1 death-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 14 2 12 1 1 birth-death 1.00 0.79 15 2 12 1 1 birth-death 1.00 0.79 15 2 12 1 1 death-death 1.00 0.79 15 2 12 1 1 death-death 1.00 0.79 15 2 12 1 1 death-death 1.00 0.80 16 2 12 1 1 birth-death 1.00 0.80 17 2 12 1 1 death-death 1.00 0.80 18 2 16 1 death-death 1.00 0.88 19 2 16 1 death-death 1.00 0.83 18 2 16 1 birth-death 1.00 0.83 18 2 16 1 birth-death 1.00 0.83 18 2 16 1 death-death 1.00 0.83 19 2 16 1 death-death 1.00 0.83 19 2 16 1 death-death 1.00 0.83 19 2 16 1 death-death 1.00 0.83 20 2 20 1 death-death 1.00 0.88 20 2 20 1 death-death 1.00 0.88 20 2 20 1 death-death 1.00 0.88 21 3 8 0 0 birth-birth 1.00 0.88 22 3 8 2 2 birth-death 1.00 0.88 23 3 8 1 death-death 1.00 0.88 24 25 death-death 1.00 0.88 25 death-death 1.00 0.88 26 death-death 1.00 0.88 27 death-death 1.00 0.88 28 death-death 1.00 0.88 29 death-death 1.00 0.88 20 2 20 1 death-death 1.00 0.88 20 3 2 20 death-death 1.00 0.88 21 3 8 0 0 birth-birth 1.00 0.88 22 3 8 2 2 birth-death 1.00 0.88 22 3 8 2 2 birth-death 1.00 0.88 22 3 8 2 2 birth-death 1.00 0.88 23 3 8 1 1 birth-death 1.00 0.88								
10         2         8         0         0         birth-birth         1.00         1.00           11         2         8         0         0         birth-birth         1.00         1.00           11         2         8         0         0         birth-birth         1.00         1.00           11         2         8         1         1         death-death         1.00         0.78           12         2         12         1         1         birth-death         1.00         0.78           13         2         12         1         1         birth-death         1.00         0.78           14         2         12         1         1         birth-death         1.00         0.79           15         2         12         1         1         birth-death         1.00								
11         2         8         0         0         birth-birth         1.00         1.00           11         2         8         0         0         birth-birth         1.00         1.00           11         2         8         1         1         death-death         1.00         0.67           12         2         12         1         1         death-death         1.00         0.78           13         2         12         1         1         birth-death         1.00         0.78           14         2         12         1         1         birth-death         1.00         0.79           14         2         12         1         1         birth-death         1.00         0.79           14         2         12         1         1         birth-death         1.00         0.79           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.83           15         2         12         1         1         birth-death         1.00								
11         2         8         0         0         birth-birth         1.00         1.00           11         2         8         1         1         death-death         1.00         0.67           12         2         12         1         1         death-death         1.00         0.78           13         2         12         1         1         birth-death         1.00         0.78           14         2         12         1         1         birth-death         1.00         0.79           15         2         12         0         0         birth-birth         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00								
11         2         8         1         1         death-death         1.00         0.67           12         2         12         1         1         death-death         1.00         0.78           13         2         12         1         1         birth-death         1.00         0.78           14         2         12         1         1         birth-death         1.00         0.79           15         2         12         1         1         birth-death         1.00         0.79           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           17         2         12         1         1         birth-death         1.00 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
12         2         12         1         1         death-death         1.00         0.78           13         2         12         1         1         birth-death         1.00         0.78           14         2         12         1         1         birth-death         1.00         0.79           15         2         12         0         0         birth-birth         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           17         2         12         1         1         birth-death         1.00         0.78           18         2         16         0         0         birth-birth         1.00 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
13         2         12         1         1         birth-death         1.00         0.78           13         2         12         1         1         birth-death         1.00         0.79           14         2         12         1         1         death-death         1.00         0.79           14         2         12         1         1         birth-death         1.00         0.79           15         2         12         1         1         birth-death         1.00         0.79           15         2         12         0         0         birth-birth         1.00         0.80           15         2         12         1         1         death-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.78           17         2         12         1         1         birth-death         1.00 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
13         2         12         1         1         birth-death         1.00         0.78           14         2         12         1         1         death-death         1.00         0.79           14         2         12         1         1         birth-death         1.00         0.79           14         2         12         1         1         birth-death         1.00         0.79           15         2         12         0         0         birth-death         1.00         0.79           15         2         12         1         1         death-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           17         2         12         1         1         birth-death         1.00         0.78           17         2         12         1         1         birth-death         1.00         0.78           18         2         16         1         1         birth-death         1.00 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
14         2         12         1         1         death-death         1.00         0.79           14         2         12         1         1         birth-death         1.00         0.79           15         2         12         0         0         birth-birth         1.00         0.80           15         2         12         1         1         death-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           17         2         12         1         1         birth-death         1.00         0.78           17         2         12         1         1         birth-death         1.00         0.78           18         2         16         0         0         birth-death         1.00         0.83           18         2         16         1         1         birth-death         1.00 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
14         2         12         1         1         birth-death         1.00         0.79           14         2         12         1         1         birth-death         1.00         0.79           15         2         12         0         0         birth-birth         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           17         2         12         1         1         birth-death         1.00         0.78           17         2         12         1         1         birth-death         1.00         0.78           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         birth-death         1.00 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
14         2         12         1         1         birth-death         1.00         0.79           15         2         12         0         0         birth-birth         1.00         1.00           15         2         12         1         1         death-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           17         2         12         1         1         birth-death         1.00         0.78           17         2         12         1         1         birth-death         1.00         0.78           18         2         16         0         0         birth-death         1.00         0.83           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         death-death         1.00 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
15         2         12         0         0         birth-birth         1.00         1.00           15         2         12         1         1         death-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           17         2         12         1         1         birth-death         1.00         0.78           17         2         12         1         1         birth-death         1.00         0.78           18         2         16         0         0         birth-birth         1.00         0.78           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         death-death         1.00         0.86           19         2         16         2         2         death-death         1.00 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
15         2         12         1         1         death-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           17         2         12         1         1         death-death         1.00         0.78           18         2         16         0         0         birth-birth         1.00         0.78           18         2         16         0         0         birth-birth         1.00         0.83           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         death-death         1.00         0.83           18         2         16         1         1         death-death         1.00         0.86           19         2         16         2         2         death-death         1.00 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
15         2         12         1         1         birth-death         1.00         0.80           15         2         12         1         1         birth-death         1.00         0.80           17         2         12         1         1         death-death         1.00         0.78           17         2         12         1         1         birth-death         1.00         0.78           18         2         16         0         0         birth-birth         1.00         0.83           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         death-death         1.00         0.83           19         2         16         1         1         death-death         1.00         0.86           20         2         20         0         0         birth-birth         1.00 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
15         2         12         1         1         birth-death         1.00         0.80           17         2         12         1         1         death-death         1.00         0.78           17         2         12         1         1         birth-death         1.00         0.78           18         2         16         0         0         birth-birth         1.00         0.83           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         birth-death         1.00         0.83           18         2         16         1         1         death-death         1.00         0.83           18         2         16         1         1         death-death         1.00         0.83           18         2         16         1         1         death-death         1.00         0.86           19         2         16         1         1         death-death         1.00         0.86           20         2         20         0         0         birth-birth-birth         1								
17       2       12       1       1       death-death       1.00       0.78         17       2       12       1       1       birth-death       1.00       0.78         18       2       16       0       0       birth-birth       1.00       1.00         18       2       16       1       1       birth-death       1.00       0.83         18       2       16       1       1       birth-death       1.00       0.83         18       2       16       1       1       death-death       1.00       0.83         18       2       16       1       1       death-death       1.00       0.83         18       2       16       1       1       death-death       1.00       0.83         19       2       16       2       2       death-death       1.00       0.86         19       2       16       2       2       death-death       1.00       0.86         20       2       20       0       0       birth-death       1.00       0.88         20       2       20       1       1       birth-death <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>								
17       2       12       1       1       birth-death       1.00       0.78         18       2       16       0       0       birth-birth       1.00       1.00         18       2       16       1       1       birth-death       1.00       0.83         18       2       16       1       1       birth-death       1.00       0.83         18       2       16       1       1       death-death       1.00       0.83         19       2       16       1       1       death-death       1.00       0.86         19       2       16       2       2       death-death       1.00       0.86         20       2       20       0       0       birth-birth       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         21       3       8       0       0       birth-birth       1.00       0.75         21       3       8       1       1       birth-birth								
18       2       16       0       0       birth-birth       1.00       1.00         18       2       16       1       1       birth-death       1.00       0.83         18       2       16       1       1       birth-death       1.00       0.83         18       2       16       1       1       death-death       1.00       0.83         19       2       16       2       2       death-death       1.00       0.86         19       2       16       2       2       death-death       1.00       0.86         20       2       20       0       0       birth-birth       1.00       0.86         20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       death-death       1.00       0.88         21       3       8       0       0       birth-birth       1.00       0.75         21       3       8       1       1       4       birth-birt								
18       2       16       1       1       birth-death       1.00       0.83         18       2       16       1       1       birth-death       1.00       0.83         18       2       16       1       1       death-death       1.00       0.83         19       2       16       1       1       death-death       1.00       0.86         19       2       16       2       2       death-death       1.00       0.86         20       2       20       0       0       birth-birth       1.00       0.86         20       2       20       0       0       birth-death       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       death-death       1.00       0.88         21       3       8       0       0       birth-death       1.00       0.88         21       3       8       1       1       4       birth-death       1.00       0.75         21       3       8       1       1       birth-birth								
18       2       16       1       1       birth-death       1.00       0.83         18       2       16       1       1       death-death       1.00       0.83         19       2       16       1       1       death-death       1.00       0.86         19       2       16       2       2       death-death       1.00       0.86         20       2       20       0       0       birth-birth       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         21       3       8       0       0       birth-birth       1.00       0.75         21       3       8       1       1       birth-birth       1.00       0.75         22       3       8       0       0       birth-birth       1								
18       2       16       1       1       death-death       1.00       0.83         19       2       16       1       1       death-death       1.00       0.86         19       2       16       2       2       death-death       1.00       0.86         20       2       20       0       0       birth-birth       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       death-death       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         21       3       8       0       0       birth-death       1.00       0.75         21       3       8       1       1       4       birth-death       1.00       0.75         22       3       8       0       0       birth-birth       1.00       0.81         22       3       8       2       2       birth-death<								
19       2       16       1       1       death-death       1.00       0.86         19       2       16       2       2       death-death       1.00       0.86         20       2       20       0       0       birth-birth       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       death-death       1.00       0.88         20       2       20       1       1       death-death       1.00       0.88         20       2       20       1       1       death-death       1.00       0.88         21       3       8       0       0       birth-birth       1.00       0.75         21       3       8       1       1       4       birth-death       1.00       0.75         22       3       8       0       0       birth-birth       1.00       0.81         22       3       8       1       1       birth-death       1.00       0.81         22       3       8       2       2       birth-death </td <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
19       2       16       2       2 death-death       1.00       0.86         20       2       20       0       0 birth-birth       1.00       1.00         20       2       20       1       1 birth-death       1.00       0.88         20       2       20       1       1 death-death       1.00       0.88         20       2       20       1       1 death-death       1.00       0.88         21       3       8       0       0 birth-birth       1.00       0.75         21       3       8       1       1 4 birth-death       1.00       0.75         21       3       8       2       2 birth-death       1.00       0.75         22       3       8       0       0 birth-birth       1.00       0.81         22       3       8       1       1 birth-birth       1.00       0.81         22       3       8       2       2 birth-death       1.00       0.81         22       3       8       2       2 birth-death       1.00       0.81         22       3       8       2       2 birth-death       1.00       0.75 </td <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>								
20       2       20       0       0       birth-birth       1.00       1.00         20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         21       3       8       0       0       birth-birth       1.00       1.00         21       3       8       1       1       4       birth-death       1.00       0.75         21       3       8       2       2       birth-death       1.00       0.75         22       3       8       0       0       birth-birth       1.00       0.81         22       3       8       1       1       birth-birth       1.00       0.81         22       3       8       2       2       birth-death								
20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       death-death       1.00       0.88         21       3       8       0       0       birth-birth       1.00       1.00         21       3       8       1       1       4       birth-death       1.00       0.75         21       3       8       2       2       birth-death       1.00       0.75         22       3       8       0       0       birth-birth       1.00       0.81         22       3       8       1       1       birth-birth       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         23       3       8       1       1       birth-death       1.00       0.75								
20       2       20       1       1       birth-death       1.00       0.88         20       2       20       1       1       death-death       1.00       0.88         21       3       8       0       0       birth-birth       1.00       1.00         21       3       8       1       1       4       birth-death       1.00       0.75         21       3       8       2       2       birth-death       1.00       0.75         22       3       8       0       0       birth-birth       1.00       0.81         22       3       8       1       1       birth-death       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         23       3       8       1       1       birth-death       1.00       0.75								
20       2       20       1       1       death-death       1.00       0.88         21       3       8       0       0       birth-birth       1.00       1.00         21       3       8       1       1       4       birth-death       1.00       0.75         21       3       8       2       2       birth-death       1.00       0.75         22       3       8       0       0       birth-birth       1.00       1.00         22       3       8       1       1       birth-birth       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         23       3       8       1       1       birth-death       1.00       0.75								
21       3       8       0       0       birth-birth       1.00       1.00         21       3       8       1       1       4       birth-death       1.00       0.75         21       3       8       2       2       birth-death       1.00       0.75         22       3       8       0       0       birth-birth       1.00       1.00         22       3       8       1       1       birth-birth       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         23       3       8       1       1       birth-death       1.00       0.75								
21       3       8       1       1 4 birth-death       1.00       0.75         21       3       8       2       2 birth-death       1.00       0.75         22       3       8       0       0 birth-birth       1.00       1.00         22       3       8       1       1 birth-birth       1.00       0.81         22       3       8       2       2 birth-death       1.00       0.81         22       3       8       2       2 birth-death       1.00       0.81         23       3       8       1       1 birth-death       1.00       0.75								
21       3       8       2       2       birth-death       1.00       0.75         22       3       8       0       0       birth-birth       1.00       1.00         22       3       8       1       1       birth-birth       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         23       3       8       1       1       birth-death       1.00       0.75								
22       3       8       0       0       birth-birth       1.00       1.00         22       3       8       1       1       birth-birth       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         23       3       8       1       1       birth-death       1.00       0.75								
22       3       8       1       1       birth-birth       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         23       3       8       1       1       birth-death       1.00       0.75								
22       3       8       2       2       birth-death       1.00       0.81         22       3       8       2       2       birth-death       1.00       0.81         23       3       8       1       1       birth-death       1.00       0.75								
22 3 8 2 2 birth-death 1.00 0.81 23 3 8 1 1 birth-death 1.00 0.75								
23 3 8 1 1 birth-death 1.00 0.75								
	23	3	8	2	2	birth-death	1.00	0.75

complex	dim	n	dim 0	dim 1	type	jacard_edges_filtration	jacard_edges_simplex
0	2	6	1	1	death-death	1.00	0.20
1	2	6	1	1	death-death	1.00	0.14
1	2	6	1	1	birth-death	1.00	0.60
1	2	6	1	1	birth-death	1.00	0.60
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	1	1	birth-death	1.00	1.00
3	2	6	1	1	birth-death	0.67	0.67
5	2	8	0	0	birth-birth	1.00	1.00
5	2	8	1	1	birth-death	1.00	0.60
6	2	8	1	1	birth-death	0.57	0.38
7	2	8	0	0	birth-birth	1.00	1.00
7	2	8	1	1	birth-death	0.71	0.50
8	2	8	1	1	birth-death	1.00	1.00
9	2	8	0	0	birth-birth	1.00	1.00
9	2	8	1	1	birth-death	1.00	0.40
9	2	8	1	1	birth-death	1.00	0.40
9	2	8	2	2	death-death	1.00	0.75
10	$\overline{2}$	8	0	0	birth-birth	1.00	1.00
11	$\overline{2}$	8	0	0	birth-birth	1.00	1.00
11	2	8	0	0	birth-birth	1.00	1.00
11	2	8	1	1	death-death	1.00	0.25
12	2	12	1	1	death-death	0.75	0.40
13	2	12	1	1	birth-death	1.00	0.80
13	$\frac{2}{2}$	12	1	1	birth-death	0.80	0.50
14	$\frac{2}{2}$	12	1	1	death-death	1.00	0.67
14	$\frac{2}{2}$	12	1	1	birth-death	0.91	0.75
14	$\frac{2}{2}$	12	1	1	birth-death	1.00	$0.73 \\ 0.54$
15	$\frac{2}{2}$	12	0	0	birth-birth	1.00	1.00
15	$\frac{2}{2}$	$\frac{12}{12}$	1	1	death-death	1.00	0.67
15	$\frac{2}{2}$	12	1	1	birth-death	1.00	0.82
15	$\frac{2}{2}$	12	1	1	birth-death	1.00	0.62 $0.67$
17	$\frac{2}{2}$	12	1	1	death-death	1.00	0.50
17 17	$\frac{2}{2}$	12	1	1	birth-death	1.00	0.50
	$\frac{2}{2}$						
18		16	0	0	birth-birth	1.00	1.00
18	2	16	1	1	birth-death	0.94	0.72
18	2	16	1	1	birth-death	0.94	0.72
18	2	16	1	1	death-death	0.68	0.45
19	2	16	1	1	death-death	0.83	0.74
19	2	16	2	2	death-death	1.00	0.89
20	2	20	0	0	birth-birth	1.00	1.00
20	2	20	1	1	birth-death	1.00	0.89
20	2	20	1	1	birth-death	0.95	0.85
20	2	20	1	1	death-death	0.89	0.71
21	3	8	0	0	birth-birth	1.00	1.00
21	3	8	1	1	birth-death	1.00	0.43
21	3	8	2		6 birth-death	1.00	1.00
22	3	8	0	0	birth-birth	1.00	1.00
22	3	8	1	1	birth-birth	1.00	0.45
22	3	8	2	2	birth-death	1.00	1.00
22	3	8	2	2	birth-death	1.00	1.00
23	3	8	1	1	birth-death	1.00	0.56
23	3	8	2	2	birth-death	1.00	0.75
24	3	8	0	0	birth-birth	1.00	1.00



complex	$\dim$	n	$\dim 0$	dim 1	type	jacard_l31a	jacard_l31b
0	2	6	1	1	death-death	0.00	0.00
1	2	6	1	1	death-death	0.00	0.50
1	2	6	1	1	birth-death	NaN	NaN
1	2	6	1	1	birth-death	NaN	NaN
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	1	1	birth-death	NaN	NaN
3	2	6	1	1	birth-death	NaN	NaN
5	2	8	0	0	birth-birth	NaN	NaN
5	2	8	1	1	birth-death	NaN	NaN
6	2	8	1	1	birth-death	NaN	NaN
7	2	8	0	0	birth-birth	NaN	NaN
7	2	8	1	1	birth-death	NaN	NaN
8	2	8	1	1	birth-death	NaN	NaN
9	2	8	0	0	birth-birth	NaN	NaN
9	2	8	1	1	birth-death	NaN	NaN
9	2	8	1	1	birth-death	NaN	NaN
9	2	8	2	2	death-death	0.00	0.00
10	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	1	1	death-death	0.00	0.50
12	2	12	1	1	death-death	0.00	0.00
13	2	12	1	1	birth-death	NaN	NaN
13	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	death-death	0.50	0.50
14	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	birth-death	NaN	NaN
15	2	12	0	0	birth-birth	NaN	NaN
15	2	12	1	1	death-death	0.50	0.75
15	2	12	1	1	birth-death	NaN	NaN
15	2	12	1	1	birth-death	NaN	NaN
17	2	12	1	1	death-death	0.00	0.00
17	2	12	1	1	birth-death	NaN	NaN
18	2	16	0	0	birth-birth	NaN	NaN
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	1	1	death-death	0.00	0.00
19	2	16	1	1	death-death	0.00	0.67
19	2	16	2	2	death-death	0.00	0.00
20	2	20	0	0	birth-birth	NaN	NaN
20	2	20	1	1	birth-death	NaN	NaN
20	2	20	1	1	birth-death	NaN	NaN
20	2	20	1	1	death-death	0.00	0.00
21	3	8	0	0	birth-birth	NaN	NaN
21	3	8	1	1	birth-death	NaN	NaN
21	3	8	2		8 birth-death	NaN	NaN
$\frac{1}{22}$	3	8	0	0	birth-birth	NaN	NaN
22	3	8	1	1	birth-birth	1.00	1.00
22	3	8	2	$\frac{1}{2}$	birth-death	NaN	NaN
22	3	8	$\frac{2}{2}$	$\frac{2}{2}$	birth-death	NaN	NaN
23	3	8	1	1	birth-death	NaN	NaN
23	3	8	$\frac{1}{2}$	2	birth-death	NaN	NaN
$\frac{23}{24}$	3	8	0	0	birth-birth	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard_l32a	jacard_l32b
0	2	6	1	1	death-death	1.00	1.00
1	2	6	1	1	death-death	1.00	1.00
1	2	6	1	1	birth-death	NaN	NaN
1	2	6	1	1	birth-death	NaN	NaN
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	1	1	birth-death	NaN	NaN
3	2	6	1	1	birth-death	NaN	NaN
5	$\overline{2}$	8	0	0	birth-birth	NaN	NaN
5	2	8	1	1	birth-death	NaN	NaN
6	2	8	1	1	birth-death	NaN	NaN
7	2	8	0	0	birth-birth	NaN	NaN
7	$\frac{2}{2}$	8	1	1	birth-death	NaN	NaN
8	$\frac{2}{2}$	8	1	1	birth-death	NaN	NaN
9	$\frac{2}{2}$	8	0	0	birth-birth	NaN	NaN
9	$\frac{2}{2}$	8	1	1	birth-death	NaN	NaN
	$\frac{2}{2}$			1			
9		8	1		birth-death	NaN	NaN
9	2	8	2	2	death-death	1.00	1.00
10	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	1	1	death-death	1.00	1.00
12	2	12	1	1	death-death	1.00	1.00
13	2	12	1	1	birth-death	NaN	NaN
13	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	death-death	1.00	1.00
14	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	birth-death	NaN	NaN
15	2	12	0	0	birth-birth	NaN	NaN
15	2	12	1	1	death-death	1.00	1.00
15	2	12	1	1	birth-death	NaN	NaN
15	2	12	1	1	birth-death	NaN	NaN
17	2	12	1	1	death-death	1.00	1.00
17	2	12	1	1	birth-death	NaN	NaN
18	2	16	0	0	birth-birth	NaN	NaN
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	1	1	birth-death	NaN	NaN
18	$\frac{2}{2}$	16	1	1	death-death	1.00	1.00
19	$\frac{2}{2}$	16	1	1	death-death	1.00	1.00
19	$\frac{2}{2}$	16	$\frac{1}{2}$	$\frac{1}{2}$	death-death	1.00	1.00
20	$\frac{2}{2}$	20	0	0	birth-birth	NaN	NaN
20	$\frac{2}{2}$	20					
			1	1	birth-death	NaN	NaN
20	2	20	1	1	birth-death	NaN	NaN
20	2	20	1	1	death-death	1.00	1.00
21	3	8	0	0	birth-birth	NaN	NaN
21	3	8	1	1	birth-death	NaN	NaN
21	3	8	2		0 birth-death	NaN	NaN
22	3	8	0	0	birth-birth	NaN	NaN
22	3	8	1	1	birth-birth	0.00	0.00
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
23	3	8	1	1	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN
24	3	8	0	0	birth-birth	NaN	NaN

complex	$\dim$	n	dim 0	dim 1	type	jacard_l33a	jacard_l33b
0	2	6	1	1	death-death	NaN	NaN
1	2	6	1	1	death-death	NaN	NaN
1	2	6	1	1	birth-death	1.00	1.00
1	2	6	1	1	birth-death	1.00	1.00
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	1	1	birth-death	1.00	1.00
3	2	6	1	1	birth-death	1.00	1.00
5	2	8	0	0	birth-birth	NaN	NaN
5	2	8	1	1	birth-death	1.00	1.00
6	2	8	1	1	birth-death	1.00	1.00
7	2	8	0	0	birth-birth	NaN	NaN
7	2	8	1	1	birth-death	1.00	1.00
8	2	8	1	1	birth-death	1.00	1.00
9	2	8	0	0	birth-birth	NaN	NaN
9	2	8	1	1	birth-death	1.00	1.00
9	2	8	1	1	birth-death	1.00	1.00
9	2	8	2	2	death-death	NaN	NaN
10	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	1	1	death-death	NaN	NaN
12	2	12	1	1	death-death	NaN	NaN
13	2	12	1	1	birth-death	1.00	1.00
13	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	death-death	NaN	NaN
14	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	birth-death	1.00	1.00
15	2	12	0	0	birth-birth	NaN	NaN
15	2	12	1	1	death-death	NaN	NaN
15	2	12	1	1	birth-death	1.00	1.00
15	2	12	1	1	birth-death	1.00	1.00
17	2	12	1	1	death-death	NaN	NaN
17	2	12	1	1	birth-death	1.00	1.00
18	2	16	0	0	birth-birth	NaN	NaN
18	2	16	1	1	birth-death	1.00	1.00
18	2	16	1	1	birth-death	1.00	1.00
18	2	16	1	1	death-death	NaN	NaN
19	2	16	1	1	death-death	NaN	NaN
19	2	16	2	2	death-death	NaN	NaN
20	2	20	0	0	birth-birth	NaN	NaN
20	2	20	1	1	birth-death	1.00	1.00
20	2	20	1	1	birth-death	1.00	1.00
20	2	20	1	1	death-death	NaN	NaN
21	3	8	0	0	birth-birth	NaN	NaN
21	3	8	1	1	birth-death	1.00	1.00
21	3	8	2		2 birth-death	1.00	1.00
22	3	8	0	0	birth-birth	NaN	NaN
$\frac{-}{22}$	3	8	1	1	birth-birth	NaN	NaN
$\frac{-}{22}$	3	8	$\overline{2}$	$\stackrel{-}{2}$	birth-death	1.00	1.00
22	3	8	2	2	birth-death	1.00	1.00
23	3	8	1	1	birth-death	1.00	1.00
23	3	8	$\overset{1}{2}$	2	birth-death	1.00	1.00
24	3	8	0	0	birth-birth	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard_nn_nodes	jacard_nn_edges
0	2	6	1	1	death-death	1.00	1.00
1	2	6	1	1	death-death	1.00	1.00
1	2	6	1	1	birth-death	1.00	1.00
1	2	6	1	1	birth-death	1.00	1.00
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	1	1	birth-death	1.00	1.00
3	2	6	1	1	birth-death	1.00	1.00
5	2	8	0	0	birth-birth	1.00	1.00
5	2	8	1	1	birth-death	1.00	1.00
6	$\overline{2}$	8	1	1	birth-death	1.00	1.00
7	2	8	0	0	birth-birth	1.00	1.00
7	2	8	1	1	birth-death	1.00	1.00
8	$\frac{2}{2}$	8	1	1	birth-death	1.00	1.00
9	$\frac{2}{2}$	8	0	0	birth-birth	1.00	1.00
9	$\frac{2}{2}$	8	1	1	birth-death	1.00	1.00
9	$\frac{2}{2}$	8	1	1	birth-death	1.00	1.00
9	$\frac{2}{2}$	8	$\frac{1}{2}$	$\frac{1}{2}$	death-death	1.00	1.00
10	$\frac{2}{2}$	8	0	0	birth-birth	1.00	1.00
10	$\frac{2}{2}$	8	0		birth-birth		
				0		1.00	1.00
11 11	$\frac{2}{2}$	8	0	0	birth-birth	1.00	1.00
		8	1	1	death-death	1.00	1.00
12	2	12	1	1	death-death	1.00	1.00
13	2	12	1	1	birth-death	1.00	1.00
13	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	death-death	1.00	1.00
14	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	birth-death	1.00	1.00
15	2	12	0	0	birth-birth	1.00	1.00
15	2	12	1	1	death-death	1.00	1.00
15	2	12	1	1	birth-death	1.00	1.00
15	2	12	1	1	birth-death	1.00	1.00
17	2	12	1	1	death-death	1.00	1.00
17	2	12	1	1	birth-death	1.00	1.00
18	2	16	0	0	birth-birth	1.00	1.00
18	2	16	1	1	birth-death	1.00	1.00
18	2	16	1	1	birth-death	1.00	1.00
18	2	16	1	1	death-death	1.00	1.00
19	$\overline{2}$	16	1	1	death-death	1.00	1.00
19	2	16	2	2	death-death	1.00	1.00
20	2	20	0	0	birth-birth	1.00	1.00
20	2	20	1	1	birth-death	1.00	1.00
20	$\frac{2}{2}$	20	1	1	birth-death	1.00	1.00
20	$\frac{2}{2}$	20	1	1	death-death	1.00	1.00
21	3	8	0	0	birth-birth	1.00	1.00
21	3	8	1	1	birth-death	1.00	1.00
21	3 3	8	$\frac{1}{2}$		4 birth-death		1.00
						1.00	
22	3	8	0	0	birth-birth	1.00	1.00
22	3	8	1	1	birth-birth	1.00	1.00
22	3	8	2	2	birth-death	1.00	1.00
22	3	8	2	2	birth-death	1.00	1.00
23	3	8	1	1	birth-death	1.00	1.00
23	3	8	2	2	birth-death	1.00	1.00
24	3	8	0	0	birth-birth	1.00	1.00

## 4.1 Unexpected results

The Hypothesis problem can be measured by 2 scores with expected values: jacard\_nn\_edges, jacard\_nn\_nodes There are 25 cases, which does not corespond the expectations:

complex	dim 0	dim 1	type	jacard_nn_edges	jacard_nn_nodes	Figure
2	0	1	birth-death	1.00	0.67	Figure 3
3	0	1	birth-death	1.00	0.50	Figure 4
4	0	1	birth-death	0.33	0.50	
5	0	1	birth-death	0.00	0.43	
5	1	1	death-death	0.67	1.00	
6	0	1	birth-death	0.50	0.80	
7	0	1	birth-death	1.00	0.71	
9	0	1	birth-death	0.00	0.33	
10	0	1	birth-death	0.00	0.29	
13	0	1	birth-death	0.00	0.31	
15	0	1	birth-death	0.40	0.90	
16	0	1	birth-death	0.00	0.27	
16	2	1	birth-death	1.00	0.85	
17	0	1	birth-death	1.00	0.82	
17	2	1	birth-death	1.00	0.83	
18	0	1	birth-death	0.25	0.62	
19	0	1	birth-death	0.73	0.83	
20	0	1	birth-death	0.25	0.52	
21	0	1	birth-death	1.00	0.90	
24	0	1	birth-death	1.00	0.60	
25	0	1	birth-death	1.00	0.67	
27	0	1	birth-death	0.00	0.50	
28	0	1	birth-death	0.67	0.90	
29	0	1	birth-death	0.33	0.43	
30	0	1	birth-death	0.00	0.57	

The Lemma 3.1 problem can be measured by 2 scores with expected values: **jacard\_l31a**, **jacard\_l31b** There are 20 cases, which does not corespond the expectations:

complex	dim 0	dim 1	type	jacard_l31a	jacard_l31b	Figure
2	0	0	birth-birth	NaN	NaN	Figure 1
2	0	0	birth-birth	NaN	NaN	Figure 2
5	0	0	birth-birth	NaN	NaN	
7	0	0	birth-birth	NaN	NaN	
9	0	0	birth-birth	NaN	NaN	
10	0	0	birth-birth	NaN	NaN	
11	0	0	birth-birth	NaN	NaN	
11	0	0	birth-birth	NaN	NaN	
15	0	0	birth-birth	NaN	NaN	
18	0	0	birth-birth	NaN	NaN	
20	0	0	birth-birth	NaN	NaN	
21	0	0	birth-birth	NaN	NaN	
22	0	0	birth-birth	NaN	NaN	
24	0	0	birth-birth	NaN	NaN	
25	0	0	birth-birth	NaN	NaN	
28	0	0	birth-birth	NaN	NaN	
29	0	0	birth-birth	NaN	NaN	
30	0	0	birth-birth	NaN	NaN	
31	0	0	birth-birth	NaN	NaN	
31	0	0	birth-birth	NaN	NaN	

.

The Lemma 3.2 problem can be measured by 2 scores with expected values: **jacard\_l32a**, **jacard\_l32b** There are 0 cases, which does not corespond the expectations.

The Lemma 3.3 problem can be measured by 2 scores with expected values:  $jacard\_133a$ ,  $jacard\_133b$  There are 0 cases, which does not corespond the expectations.

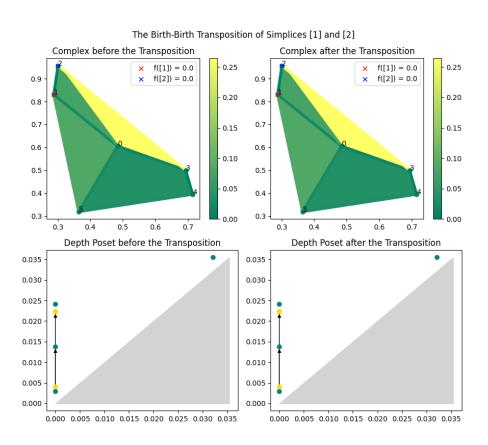


Figure 1: The Birth-Birth Transposition of simplices [1] and [2]

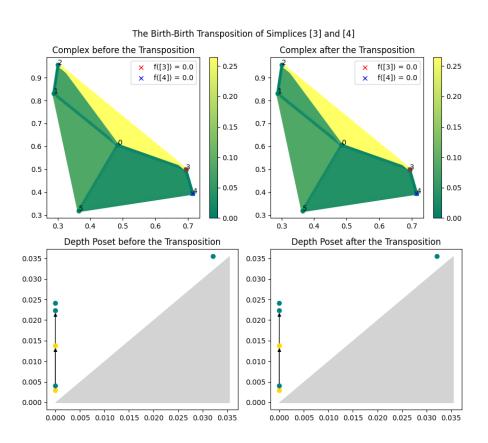


Figure 2: The Birth-Birth Transposition of simplices [3] and [4]

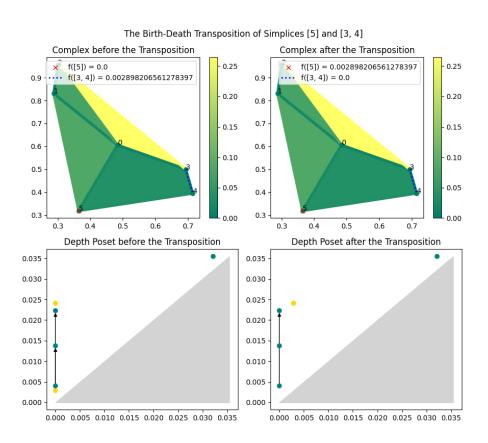


Figure 3: The Birth-Death Transposition of simplices [5] and [3, 4]

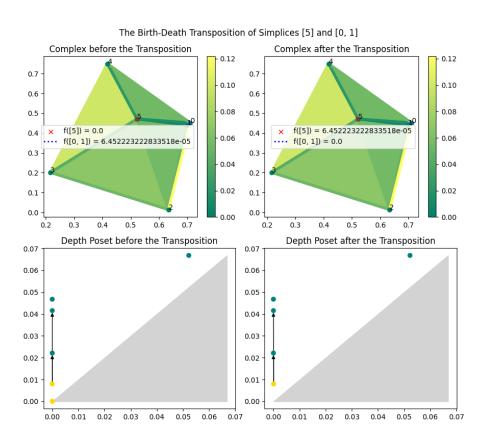


Figure 4: The Birth-Death Transposition of simplices [5] and [0, 1]