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### 1 Model

### 1.1 Complex, Filtration and Homotopy

In this model we define the simplicial complex by the Delauney triangulation of n = 24 points uniformly distributed in  $[0, 1]^d$  for d = 2.

We defining the filtration on this complex, by assuming uniformly distributed in [0,1] height h(f) for each vertex v. Then the filtration value of the simplex will be the maximum haight of its vertices.

$$f(\sigma) = \max_{v \in \sigma} h(v)$$

We define 2 filtrations like this and study the linear homotopy between them. In the Figure 1 we can see these 2 filtrations:

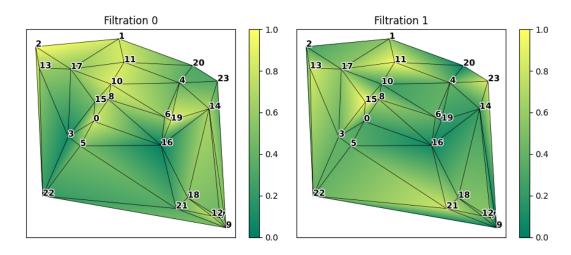


Figure 1: 2 filtrations on the defined complex.

Having these 2 filtrations we can define the homotopy between them by defining the linear homotopy between heights:

$$h_t(v) = h_0(v) \cdot (1 - t) + h_1(v) \cdot t$$
$$f_t(\sigma) = \max_{v \in \sigma} h_t(v)$$

#### 1.2 Transpositions

In the Figure 2 we can see the vertices height  $h_t(v)$  during this homotopy.

When there is a cross of lines  $h_t(i)$  and  $h_t(j)$   $(t:h_t(i)=h_t(j))$  there is transposition of heights of vertices i and j. This means that happens reordering in the filtration  $f_t$ . The order given by  $f_{t-\varepsilon}$  changes to the order given by  $f_{t+\varepsilon}$ .

Let's  $h_t(i) < h_t(j)$ . We can define 3 groups of simplices moved in the order:

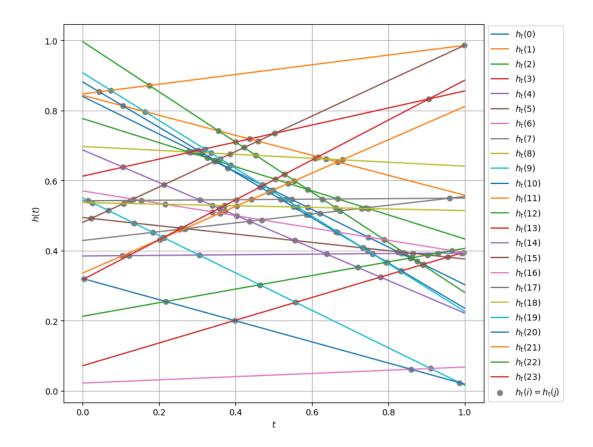


Figure 2: Heights of Vertices during the Homotopy.

- 1.  $A = \{ \sigma : i \in \sigma, j \notin \sigma, \exists v \in \sigma : h(v) > h(j) \}$
- 2.  $B = \{ \sigma : i \notin \sigma, j \in \sigma, \not\exists v \in \sigma : h(v) > h(j) \}$
- 3.  $C = \{ \sigma : i \in \sigma, j \in \sigma, \not\exists v \in \sigma : h(v) > h(j) \}$

In the order given by  $f_{t-\varepsilon}$  the group A stays on the first #A places, and in the order given by  $f_{t+\varepsilon}$  the group B stays on the first #B places.

There are many paths of transpositions in the order, which brings us from the order  $f_{t-\varepsilon}$  to the order  $f_{t+\varepsilon}$  with the conctrain that  $\sigma_0$  stays before  $\sigma_1$  if  $\sigma_0 \subset \sigma_1$ . We difined 2 of them:

Up directed The first we move simplices of group B to the first places, and then we move simplices to group C to their places in  $f_{t+\varepsilon}$ .

Down directed The first we move simplices of group C to the last places, and then we move simplices of group A to their places in  $f_{t+\varepsilon}$ .

### 2 General Statistics

In 2 paths generated 9245 unique transpositions: 4504 in only upper path, 4504 in only down path and 237 in both paths.

The distribution of the switch transposition types and dimensions is given in the table:

dim	0	1	2	undefined
birth-birth	7	21	0	0
birth-death	0	48	0	0
death-death	0	14	23	0
no switch	267	2483	1031	5351

And we can see more detailed table about how many simplex transpositions of each type lies in each reordering defined by the transposition of heights on vertices:

				Type	birth-birth	birth-death	dea	th-death	no switch
				Dim	0 1		1	2	
Time	Vertices	Value	Figure	Path		_	_	_	
0.003	$\langle 20, 23 \rangle$	0.319	3	Up	1 (	0	0	0	0
				Down	1 (	0	0	0	0
0.014	$\langle 9, 17 \rangle$	0.542	??	Up	0 (	0	0	0	24
				Down	0 (	0	0	0	24
0.022	$\langle 5, 15 \rangle$	0.492	??	Up	0 (	0	0	0	16
				Down	0 (	0	0	0	16
0.025	$\langle 9, 18 \rangle$	0.536	??	Up	0 (	1	0	0	29
				Down	0 (	1	0	0	29
0.043	$\langle 10, 11 \rangle$	0.853	??	Up	0 (	0	0	2	100
				Down	0 (	0	0	2	100
0.067	$\langle 9, 15 \rangle$	0.514	??	Up	0 (	0	0	0	12
	, , ,			Down	0 (	0	0	0	12
0.073	$\langle 11, 19 \rangle$	0.857	??	Up	0 (	0	0	0	99
				Down	0 (	0	0	0	99
0.104	$\langle 4, 21 \rangle$	0.385	??	Up	0 (	0	0	0	12
				Down	0 (	0	0	0	12
0.105	$\langle 1, 10 \rangle$	0.813	??	Up	0 (	0	0	0	30
				Down	0 (	0	0	0	30
	$\langle 13, 14 \rangle$	0.638	??	Up	0 (	0	0	0	72
	. ,			Down	0 (	0	0	0	72

				Type	birth-birth		birth-death		ath-death	no switch
Time	Vertices	Value	Figure	Dim Path	0	1	1	1	2	
0.107	$\langle 15, 18 \rangle$	0.534	??	Up	0 (	0	0	0	0	10
	, , ,			Down	0	0	0	0	0	10
0.120	$\langle 4, 23 \rangle$	0.385	4	Up	0 (	0	1	0	0	4
	, ,			Down	0	0	1	0	0	4
0.124	$\langle 15, 17 \rangle$	0.543	??	Up	0 (	0	0	0	0	4
				Down	0	0	0	0	0	4
0.131	$\langle 6, 15 \rangle$	0.547	??	Up	0 (	0	0	0	0	12
				Down	0	0	0	0	0	12
0.133	$\langle 5, 9 \rangle$	0.478	??	Up	0 (	0	0	0	0	48
				Down	0 (	0	0	0	0	48
0.153	$\langle 6, 17 \rangle$	0.543	??	Up	0 (	0	0	0	0	6
				Down	0 (	0	0	0	0	6
0.162	$\langle 1, 19 \rangle$	0.796	??	Up	0 (	0	0	0	0	27
				Down	0 (	0	0	0	0	27
0.174	$\langle 2, 11 \rangle$	0.871	??	Up	0 (	0	0	0	0	88
				Down	0 (	0	0	0	0	88
0.183	$\langle 7, 9 \rangle$	0.452	5	Up	1 (	0	0	0	0	8
				Down	0 (	0	0	1	0	8
0.201	$\langle 21, 23 \rangle$	0.432	??	Up	0 (	0	0	0	0	12
				Down	0 (	0	0	0	0	12
0.211	$\langle 9, 23 \rangle$	0.437	??	Up	0 (	0	0	0	0	18
				Down	0 (	0	0	0	0	18
0.212	$\langle 9, 21 \rangle$	0.437	6	Up	0 (	0	2	1	0	6
				Down	0 (	0	2	1	0	6
0.213	$\langle 14, 15 \rangle$	0.588	??	Up	0 (	0	0	0	0	48
				Down		0	0	0	0	48
0.216	$\langle 6, 18 \rangle$	0.532	??	Up	0 (	0	0	0	0	15
				Down	0 (	0	0	0	0	15
0.217	$\langle 20, 22 \rangle$	0.255	??	Up	0 (	0	0	0	0	2
				Down	0 (	0	0	0	0	2
0.252	$\langle 7, 23 \rangle$	0.460	??	Up	0 (	0	0	0	0	10
				Down		0	0	0	0	10
0.258	$\langle 5, 23 \rangle$	0.464	??	Up	0 (	0	0	0	0	40
	•			Down	0	0	0	0	0	40
0.265	$\langle 7, 21 \rangle$	0.462	??	Up	0 (	0	0	0	0	10
				Down	0	0	0	0	0	10

				Type		h-birth	birth-death		ath-death	no switch
Time	Vertices	Value	Figure	Dim Path	0	1	1	1	2	
0.267	$\langle 5, 21 \rangle$	0.463	7	Up	0	1	2	0	1	32
	, ,			Down	0	2	2	0	0	32
0.269	$\langle 5, 7 \rangle$	0.462	??	Up	0	0	0	0	0	10
				Down	0	0	0	0	0	10
0.277	$\langle 8, 12 \rangle$	0.681	??	Up	0	0	0	0	0	45
				Down	0	0	0	0	0	45
0.280	$\langle 12, 13 \rangle$	0.680	??	Up	0	0	0	0	0	54
				Down	0	0	0	0	0	54
0.283	$\langle 8, 13 \rangle$	0.681	??	Up	0	0	0	0	0	30
				Down	0	0	0	0	0	30
0.291	$\langle 0, 13 \rangle$	0.683	??	Up	0	0	0	0	0	66
				Down	0	0	0	0	0	66
0.296	$\langle 0, 8 \rangle$	0.680	??	Up	0	1	1	0	1	50
				Down	0	1	1	0	1	50
0.302	$\langle 10, 13 \rangle$	0.686	??	Up	0	0	0	0	0	60
				Down	0	0	0	0	0	60
0.305	$\langle 4, 9 \rangle$	0.387	??	Up	0	0	0	0	0	4
				Down	0	0	0	0	0	4
0.306	$\langle 14, 17 \rangle$	0.544	??	Up	0	0	0	0	0	24
				Down	0	0	0	0	0	24
0.312	$\langle 8, 10 \rangle$	0.679	??	Up	0	0	0	0	1	76
				Down	0	1	0	0	0	76
0.319	$\langle 13, 19 \rangle$	0.690	??	Up	0	0	0	0	0	54
				Down	0	0	0	0	0	54
0.326	$\langle 0, 12 \rangle$	0.664	??	Up	0	0	0	0	0	72
				Down	0	0	0	0	0	72
0.336	$\langle 8, 19 \rangle$	0.678	??	Up	0	0	0	0	0	99
				Down	0	0	0	0	0	99
0.339	$\langle 14, 18 \rangle$	0.529	??	Up	0	1	0	0	0	61
				Down	0	1	0	0	0	61
0.340	$\langle 6, 23 \rangle$	0.510	??	Up	0	0	1	0	0	14
				Down	0	0	1	0	0	14
0.345	$\langle 0, 15 \rangle$	0.654	??	Up	0	0	1	0	0	32
				Down	0	0	1	0	0	32
	$\langle 10, 12 \rangle$	0.658	??	Up	0	0	0	0	0	63
				Down	0	0	0	0	0	63

-				Type	birth-bir	th	birth-death	de	ath-death	no switch
				$\operatorname{Dim}$	0	1	1	1	2	
Time	Vertices	Value	Figure	Path						
0.348	$\langle 10, 15 \rangle$	0.656	??	Up	0	1	0	0	0	32
				Down	0	1	0	0	0	32
0.349	$\langle 12, 15 \rangle$	0.657	??	Up	0	0	0	0	0	72
				Down	0	0	0	0	0	72
0.355	$\langle 1, 2 \rangle$	0.741	8	Up	0	1	2	0	0	26
				Down	0	0	2	0	1	26
0.358	$\langle 14, 23 \rangle$	0.520	??	Up	0	1	0	0	0	35
				Down	0	1	0	0	0	35
0.360	$\langle 15, 19 \rangle$	0.662	??	Up	0	0	0	0	0	72
				Down	0	0	0	0	0	72
•	$\langle 6, 21 \rangle$	0.507	??	Up	0	0	0	0	0	24
				Down	0	0	0	0	0	24
0.372	$\langle 18, 23 \rangle$	0.528	??	Up	0	0	0	0	0	64
				Down	0	0	0	0	0	64
0.373	$\langle 14, 21 \rangle$	0.513	??	Up	0	0	0	0	0	48
				Down	0	0	0	0	0	48
0.380	$\langle 0, 10 \rangle$	0.636	??	Up	0	0	0	0	0	30
				Down	0	0	0	0	0	30
0.386	$\langle 8, 15 \rangle$	0.675	??	Up	0	0	0	0	2	82
				Down	0	0	0	0	2	82
0.387	$\langle 12, 19 \rangle$	0.643	??	Up	0	0	0	0	1	80
				Down	0	0	0	0	1	80
0.397	$\langle 3, 20 \rangle$	0.200	??	Up	0	0	0	0	0	1
				Down	0	0	0	0	0	1
0.400	$\langle 2, 13 \rangle$	0.709	9	Up	0	0	2	0	2	26
				Down	0	2	2	0	0	26
0.401	$\langle 17, 23 \rangle$	0.545	??	Up	0	0	0	0	0	16
				Down	0	0	0	0	0	16
0.404	$\langle 18, 21 \rangle$	0.528	??	Up	0	0	0	0	0	54
				Down	0	0	0	0	0	54
	$\langle 6, 14 \rangle$	0.499	10	Up	0	1	1	0	0	16
				Down	0	1	1	0	0	16
0.422	$\langle 2, 15 \rangle$	0.694	??	Up	0	0	0	0	0	33
				Down	0	0	0	0	0	33
0.436	$\langle 1, 13 \rangle$	0.718	??	Up	0	0	0	0	0	45
				Down	0	0	0	0	0	45

				Type	birth-b		birth-death		th-death	no switch
Time	Vertices	Value	Figure	Dim Path	0	1	1	1	2	
0.437	$\langle 7, 14 \rangle$	0.484	??	Up	0	0	1	0	0	10
0.10.	(*, = =/	0.101		Down	0	0	1	0	0	10
0.441	$\langle 17, 21 \rangle$	0.545	??	Up	0	0	0	0	0	20
	( - ) /			Down	0	0	0	0	0	20
0.453	$\langle 2, 8 \rangle$	0.671	??	Up	0	0	0	0	0	24
	( / /			Down	0	0	0	0	0	24
0.459	$\langle 1, 15 \rangle$	0.712	??	Up	0	0	0	0	0	55
	, ,			Down	0	0	0	0	0	55
0.463	$\langle 9, 22 \rangle$	0.302	??	Up	0	0	0	0	0	2
	, , ,			Down	0	0	0	0	0	2
0.464	$\langle 10, 23 \rangle$	0.581	??	Up	0	0	0	0	0	40
				Down	0	0	0	0	0	40
0.469	$\langle 0, 19 \rangle$	0.587	??	Up	0	0	0	0	0	54
				Down	0	0	0	0	0	54
	$\langle 6,7 \rangle$	0.488	??	Up	0	0	0	0	0	12
				Down	0	0	0	0	0	12
0.472	$\langle 19, 23 \rangle$	0.586	??	Up	0	0	0	0	0	72
				Down	0	0	0	0	0	72
0.473	$\langle 0, 23 \rangle$	0.586	??	Up	0	0	0	0	0	48
				Down	0	0	0	0	0	48
0.486	$\langle 10, 21 \rangle$	0.567	??	Up	0	0	0	0	0	50
				Down	0	0	0	0	0	50
0.494	$\langle 19, 21 \rangle$	0.571	??	Up	0	0	0	0	0	90
				Down	0	0	0	0	0	90
0.498	$\langle 0, 21 \rangle$	0.572	??	Up	0	0	0	0	0	60
				Down	0	0	0	0	0	60
0.503	$\langle 13, 15 \rangle$	0.734	??	Up	0	0	0	0	0	99
				Down	0	0	0	0	0	99
0.504	$\langle 12, 23 \rangle$	0.603	??	Up	0	0	0	0	0	72
				Down	0	0	0	0	0	72
0.519	$\langle 10, 17 \rangle$	0.546	??	Up	0	0	0	0	0	8
				Down	0	0	0	0	0	8
0.529	$\langle 2, 23 \rangle$	0.618	??	Up	0	0	0	0	0	24
				Down	0	0	0	0	0	24
0.530	$\langle 17, 19 \rangle$	0.546	??	Up	0	0	0	0	0	27
				Down	0	0	0	0	0	27

				Type		-birth	birth-death		th-death	no switch
m.	<b>T</b> 7	37.1	ъ.	Dim	0	1	1	1	2	
Time	Vertices	Value	Figure	Path						
0.538	$\langle 12, 21 \rangle$	0.592	??	Up	0	0	0	0	2	76
				Down	0	0	0	0	2	76
0.546	$\langle 0, 17 \rangle$	0.546	??	Up	0	0	0	0	0	18
				Down	0	0	0	0	0	18
0.552	$\langle 10, 18 \rangle$	0.524	??	Up	0	0	0	0	0	24
				Down	0	0	0	0	0	24
0.554	$\langle 2, 21 \rangle$	0.599	??	Up	0	0	0	0	0	39
				Down	0	0	0	0	0	39
0.555	$\langle 5, 14 \rangle$	0.429	??	Up	0	0	1	1	0	18
				Down	0	0	1	1	0	18
0.557	$\langle 3, 9 \rangle$	0.252	11	Up	1	0	0	0	0	0
				Down	1	0	0	0	0	0
0.563	$\langle 18, 19 \rangle$	0.524	??	Up	0	0	0	0	0	54
				Down	0	0	0	0	0	54
0.586	$\langle 7, 10 \rangle$	0.502	??	Up	0	0	0	0	0	16
				Down	0	0	0	0	0	16
0.589	$\langle 0, 18 \rangle$	0.523	??	Up	0	0	0	0	0	36
				Down	0	0	0	0	0	36
0.590	$\langle 2, 12 \rangle$	0.574	??	Up	0	0	0	0	0	18
				Down	0	0	0	0	0	18
0.593	$\langle 7, 19 \rangle$	0.503	??	Up	0	0	0	0	0	36
				Down	0	0	0	0	0	36
0.608	$\langle 8, 23 \rangle$	0.663	??	Up	0	0	0	0	0	64
				Down	0	0	0	0	0	64
0.616	$\langle 1, 23 \rangle$	0.667	??	Up	0	0	0	0	0	40
				Down	0	0	0	0	0	40
0.620	$\langle 0,7 \rangle$	0.507	??	Up	0	0	0	0	0	24
				Down	0	0	0	0	0	24
0.628	$\langle 2, 17 \rangle$	0.547	12	Up	0	0	2	0	0	7
				Down	0	0	2	0	0	7
0.637	$\langle 1, 8 \rangle$	0.661	??	Up	0	0	0	0	0	40
				Down	0	0	0	0	0	40
0.638	$\langle 4, 14 \rangle$	0.390	13	Up	0	0	0	3	0	6
				Down	0	0	0	3	0	6
0.662	$\langle 6, 10 \rangle$	0.453	??	Up	0	0	0	0	0	8
	•			Down	0	0	0	0	0	8

				Type	birth-bir		birth-death		ath-death	no switch
m.	3.7	37.1	ъ.	Dim	0	1	1	1	2	
Time	Vertices	Value	Figure	Path						
0.663	$\langle 2, 18 \rangle$	0.522	??	Up	0	0	0	0	0	12
				Down	0	0	0	0	0	12
0.667	$\langle 1, 21 \rangle$	0.653	??	Up	0	0	0	0	0	65
				Down	0	0	0	0	0	65
0.668	$\langle 12, 17 \rangle$	0.547	??	Up	0	0	0	0	0	24
				Down	0	0	0	0	0	24
•	$\langle 6, 19 \rangle$	0.452	??	Up	0	1	0	0	0	38
				Down	0	0	0	0	1	38
0.674	$\langle 2,7 \rangle$	0.513	??	Up	0	0	0	0	0	8
				Down	0	0	0	0	0	8
0.680	$\langle 8, 21 \rangle$	0.659	??	Up	0	0	0	0	0	104
				Down	0	0	0	0	0	104
0.720	$\langle 14, 22 \rangle$	0.352	??	Up	0	0	0	0	0	9
				Down	0	0	0	0	0	9
0.729	$\langle 7, 18 \rangle$	0.520	??	Up	0	0	1	0	0	20
				Down	0	0	1	0	0	20
0.733	$\langle 5, 10 \rangle$	0.407	??	Up	0	0	0	0	0	10
				Down	0	0	0	0	0	10
0.734	$\langle 5, 19 \rangle$	0.407	??	Up	0	0	0	0	0	30
				Down	0	0	0	0	0	30
0.742	$\langle 7, 12 \rangle$	0.522	??	Up	0	0	1	0	1	28
				Down	0	1	1	0	0	28
0.747	$\langle 0, 6 \rangle$	0.439	??	Up	0	0	0	0	0	48
				Down	0	0	0	0	0	48
	$\langle 10, 19 \rangle$	0.399	??	Up	0	0	0	0	0	12
				Down	0	0	0	0	0	12
	$\langle 12, 18 \rangle$	0.520	??	Up	0	0	1	0	0	10
				Down	0	0	1	0	0	10
0.758	$\langle 4, 19 \rangle$	0.391	??	Up	0	0	1	0	0	18
				Down	0	0	1	0	0	18
0.759	$\langle 4, 10 \rangle$	0.391	14	Up	0	0	0	1	0	8
				Down	1	0	0	0	0	8
0.779	$\langle 3, 14 \rangle$	0.324	??	Up	0	0	0	0	0	3
				Down	0	0	0	0	0	3
0.789	$\langle 2, 6 \rangle$	0.431	??	Up	0	0	0	0	0	16
				Down	0	0	0	0	0	16

				Type	birth-		birth-death		th-death	no switch
				Dim	0	1	1	1	2	
Time	Vertices	Value	Figure	Path						
0.794	$\langle 19, 22 \rangle$	0.366	??	Up	0	0	0	0	0	12
	, , ,			Down	0	0	0	0	0	12
0.796	$\langle 10, 22 \rangle$	0.367	??	Up	0	0	0	0	0	3
				Down	0	0	0	0	0	3
0.825	$\langle 0, 5 \rangle$	0.397	??	Up	0	1	1	0	0	24
				Down	0	1	1	0	0	24
0.832	$\langle 3, 19 \rangle$	0.341	??	Up	0	0	0	0	0	4
				Down	0	0	0	0	0	4
0.834	$\langle 0, 4 \rangle$	0.392	??	Up	0	0	0	0	0	18
				Down	0	0	0	0	0	18
0.835	$\langle 3, 10 \rangle$	0.342	??	Up	0	0	0	0	0	1
				Down	0	0	0	0	0	1
0.840	$\langle 2, 5 \rangle$	0.395	??	Up	0	0	0	0	0	16
				Down	0	0	0	0	0	16
0.844	$\langle 2, 4 \rangle$	0.392	??	Up	0	0	0	0	0	12
				Down	0	0	0	0	0	12
0.858	$\langle 0, 22 \rangle$	0.379	15	Up	0	0	1	0	0	8
				Down	0	0	1	0	0	8
0.860	$\langle 16, 20 \rangle$	0.061	??	Up	0	0	0	0	0	1
				Down	0	0	0	0	0	1
0.862	$\langle 2, 22 \rangle$	0.379	16	Up	0	0	0	1	0	4
				Down	1	0	0	0	0	4
0.864	$\langle 4, 5 \rangle$	0.392	??	Up	0	0	0	0	0	48
				Down	0	0	0	0	0	48
0.876	$\langle 0, 2 \rangle$	0.369	??	Up	0	0	0	0	0	3
				Down	0	0	0	0	0	3
0.889	$\langle 2, 3 \rangle$	0.360	??	Up	0	0	0	0	0	1
				Down	0	0	0	0	0	1
0.892	$\langle 0, 3 \rangle$	0.361	17	Up	1	0	0	0	0	2
				Down	0	0	0	1	0	2
0.903	$\langle 5, 22 \rangle$	0.387	??	Up	0	0	0	0	0	26
				Down	0	0	0	0	0	26
0.906	$\langle 13, 23 \rangle$	0.832	??	Up	0	0	0	0	0	72
				Down	0	0	0	0	0	72
0.911	$\langle 9, 16 \rangle$	0.063	18	Up	1	0	0	0	0	0
				Down	1	0	0	0	0	0

				Type	birth-birth	birth-death	death	-death	no switch
				Dim	0 1	1	1	2	
Time	Vertices	Value	Figure	Path					
0.930	$\langle 4, 22 \rangle$	0.393	??	Up	0 0	0	0	0	36
				Down	0 0	0	0	0	36
0.955	$\langle 3, 5 \rangle$	0.381	??	Up	0 0	0	0	0	12
				Down	0 0	0	0	0	12
0.961	$\langle 7, 17 \rangle$	0.549	??	Up	0 0	0	0	0	36
				Down	0 0	0	0	0	36
0.967	$\langle 6, 22 \rangle$	0.400	??	Up	0 0	0	0	0	48
				Down	0 0	0	0	0	48
0.987	$\langle 9, 20 \rangle$	0.023	??	Up	0 0	0	0	0	1
				Down	0 0	0	0	0	1
0.992	$\langle 3, 4 \rangle$	0.393	??	Up	0 0	0	0	0	24
				Down	0 0	0	0	0	24
0.997	$\langle 3, 6 \rangle$	0.395	??	Up	0 0	0	0	0	32
				Down	0 0	0	0	0	32
0.999	$\langle 11, 15 \rangle$	0.985	??	Up	0 0	0	0	0	121
				Down	0 0	0	0	0	121

## 3 present the details

Here we detalize the paths, how depth poset changed during the homotopy by different paths.

In the figures we can see graphs, which edges coresponds the transpositions of simplices, and nodes are orders, coresponding one of 240 Depth Posets we got.

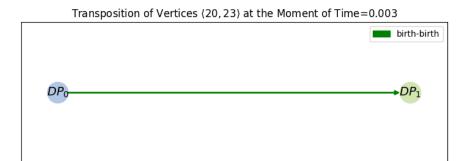


Figure 3: Reordering by transposition of vertices 20 and 23

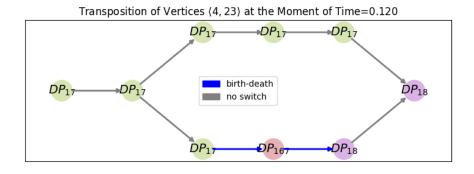


Figure 4: Reordering by transposition of vertices 4 and 23

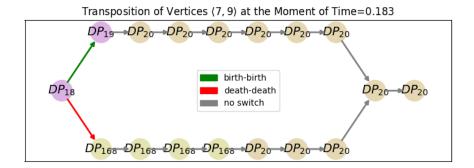


Figure 5: Reordering by transposition of vertices 7 and 9

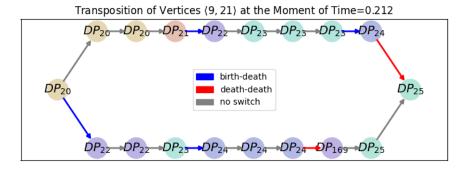


Figure 6: Reordering by transposition of vertices 9 and 21

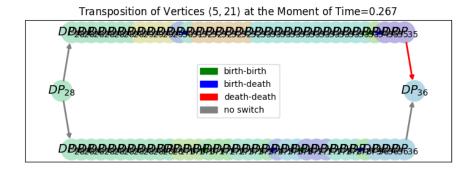


Figure 7: Reordering by transposition of vertices 5 and 21

#### Transposition of Vertices (1, 2) at the Moment of Time=0.355

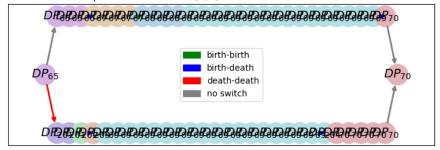
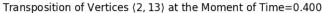


Figure 8: Reordering by transposition of vertices 1 and 2



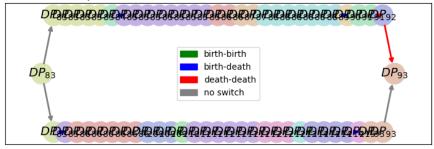


Figure 9: Reordering by transposition of vertices 2 and 13

Transposition of Vertices (6, 14) at the Moment of Time=0.404

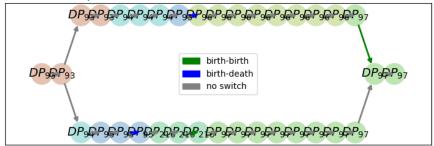
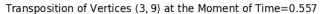


Figure 10: Reordering by transposition of vertices 6 and 14



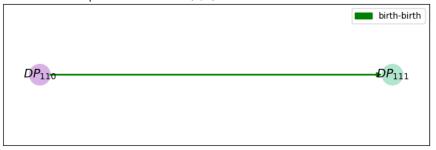


Figure 11: Reordering by transposition of vertices 3 and 9

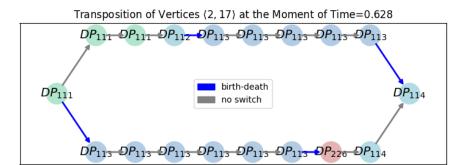


Figure 12: Reordering by transposition of vertices 2 and 17

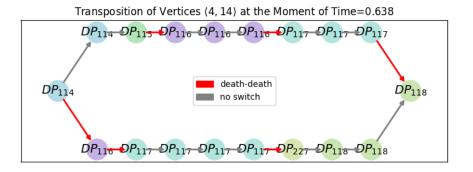


Figure 13: Reordering by transposition of vertices 4 and 14

#### Transposition of Vertices (4, 10) at the Moment of Time=0.759

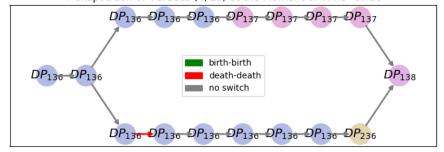
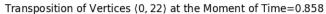


Figure 14: Reordering by transposition of vertices 4 and 10



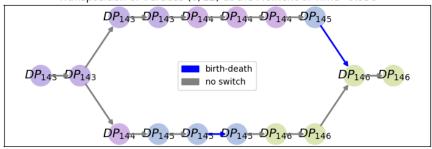


Figure 15: Reordering by transposition of vertices 0 and 22

Transposition of Vertices (2, 22) at the Moment of Time=0.862

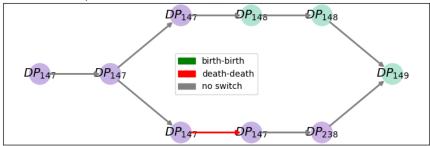


Figure 16: Reordering by transposition of vertices 2 and 22

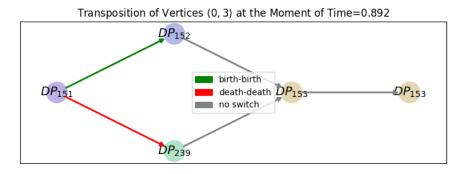


Figure 17: Reordering by transposition of vertices 0 and 3

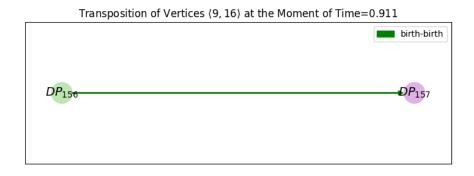


Figure 18: Reordering by transposition of vertices 9 and 16  $\,$ 

## 4 Depth Posets

During the transposition we got 240 depth posets. In this section we listed few of them.

The depth poset coresponding the first filtration we can see in Figure 19, and the depth poset coresponding the last filtration we can see in Figure 20.

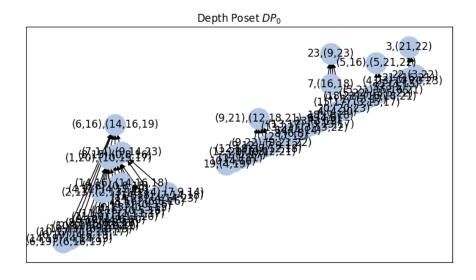


Figure 19: Depth Poset  $DP_0$ 

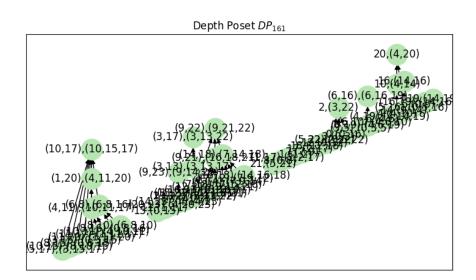


Figure 20: Depth Poset  $DP_{161}$ 

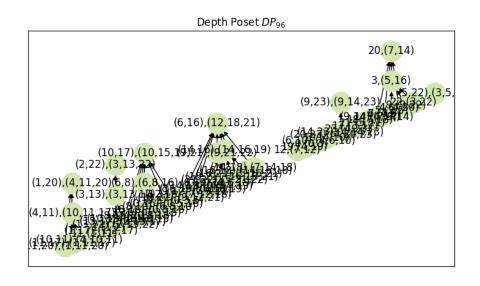


Figure 21: Depth Poset  $DP_{96}$ 

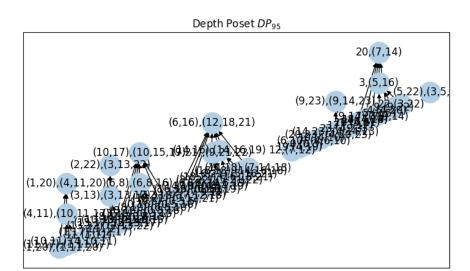


Figure 22: Depth Poset  $DP_{95}$ 

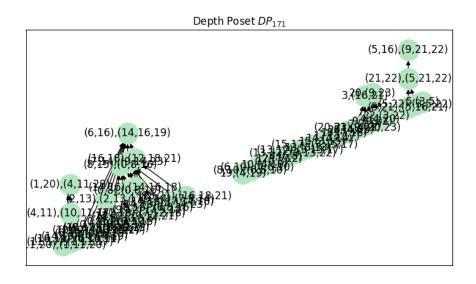


Figure 23: Depth Poset  $DP_{171}$ 

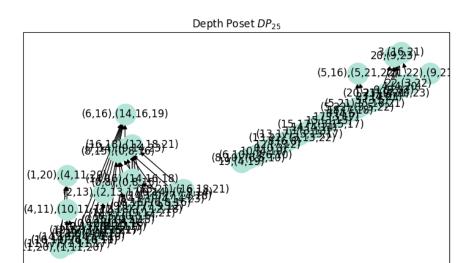


Figure 24: Depth Poset  $DP_{25}$ 

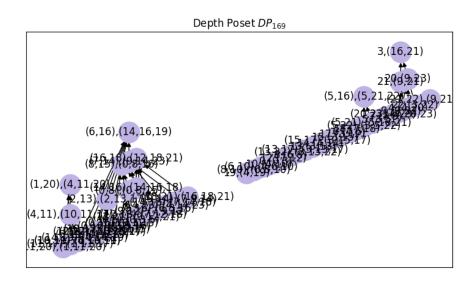


Figure 25: Depth Poset  $DP_{169}$ 

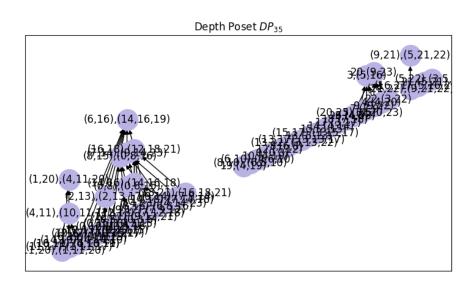


Figure 26: Depth Poset  $DP_{35}$ 

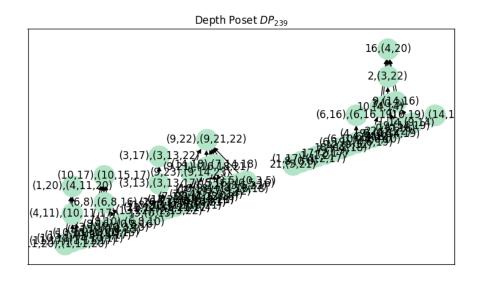


Figure 27: Depth Poset  $DP_{239}$ 

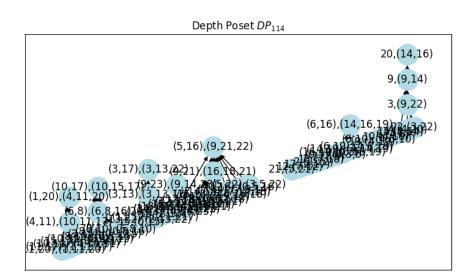


Figure 28: Depth Poset  $DP_{114}$ 

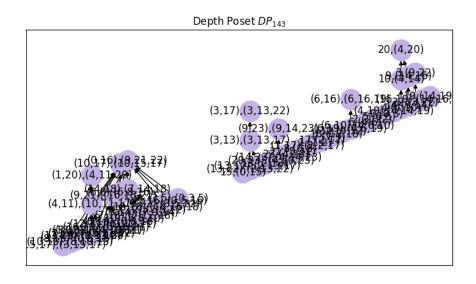


Figure 29: Depth Poset  $DP_{143}$ 

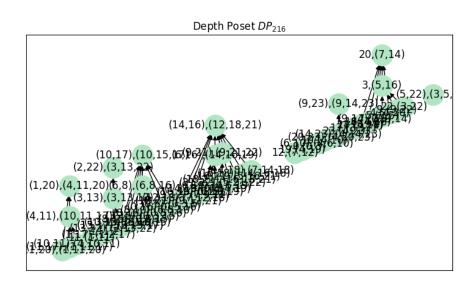


Figure 30: Depth Poset  $DP_{216}$