Contents

1 Model

1.1 Complex, Filtration and Homotopy

In this model we define the simplicial complex by the Delauney triangulation of $n = 10$ points uniformly distributed in $[0, 1]^d$ for $d = 2$.

We defining the filtration on this complex, by assuming uniformly distributed in $[0, 1]$ height $h(f)$ for each vertex v. Then the filtration value of the simplex will be the maximum haight of its vertices.

$$f(\sigma) = \max_{v \in \sigma} h(v)$$

We define 2 filtrations like this and study the linear homotopy between them. In the Figure 1 we can see these 2 filtrations:

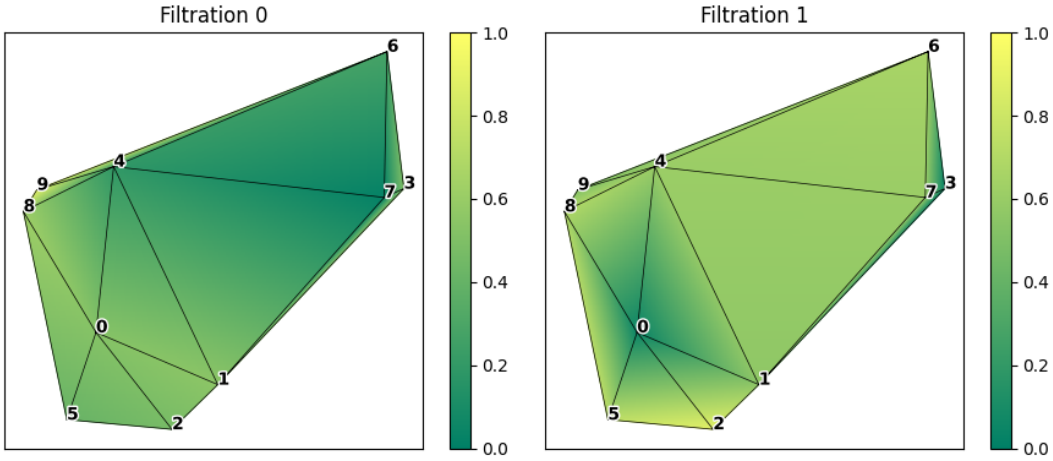

Figure 1: 2 filtrations on the defined complex.

Having these 2 filtrations we can define the homotopy between them by defining the linear homotopy between heights:

$$h_t(v) = h_0(v) \cdot (1 - t) + h_1(v) \cdot t$$

$$f_t(\sigma) = \max_{v \in \sigma} h_t(v)$$

1.2 Transpositions

In the Figure 2 we can see the vertices height $h_t(v)$ during this homotopy.

When there is a cross of lines $h_t(i)$ and $h_t(j)$ $(t : h_t(i) = h_t(j))$ there is transposition of heights of vertices i and j. This means that happens reordering in the filtration f_t. The order given by $f_{t-\varepsilon}$ changes to the order given by $f_{t+\varepsilon}$.

Let's $h_t(i) < h_t(j)$. We can define 3 groups of simplices moved in the order:

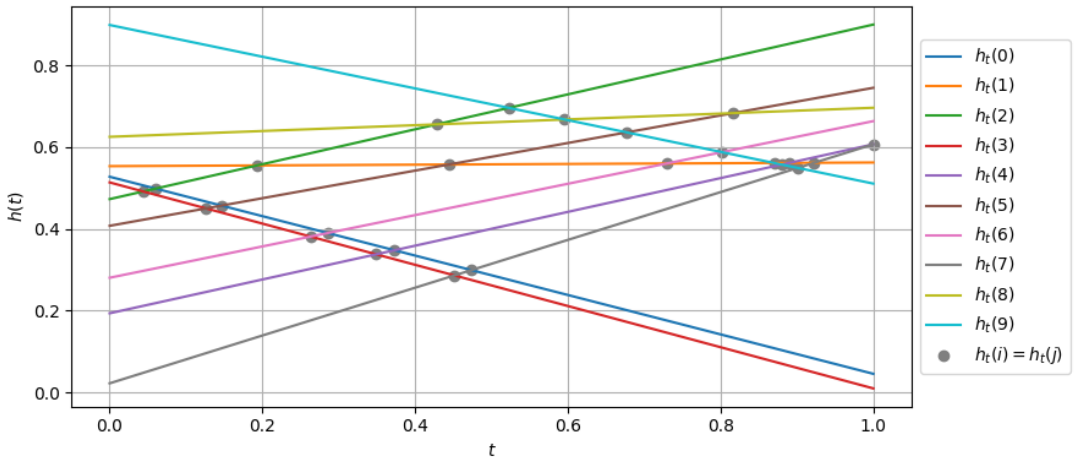

Figure 2: Heights of Vertices during the Homotopy.

1. $A = \{\sigma : i \in \sigma, j \notin \sigma, \nexists v \in \sigma : h(v) > h(j)\}$
2. $B = \{\sigma : i \notin \sigma, j \in \sigma, \nexists v \in \sigma : h(v) > h(j)\}$
3. $C = \{\sigma : i \in \sigma, j \in \sigma, \nexists v \in \sigma : h(v) > h(j)\}$

In the order given by $f_{t-\varepsilon}$ the group A stays on the first $\#A$ places, and in the order given by $f_{t+\varepsilon}$ the group B stays on the first $\#B$ places.

There are many paths of transpositions in the order, which brings us from the order $f_{t-\varepsilon}$ to the order $f_{t+\varepsilon}$ with the conctrain that σ_0 stays before σ_1 if $\sigma_0 \subset \sigma_1$. We difined 2 of them:

Up directed The first we move simplices of group B to the first places, and then we move simplices to group C to their places in $f_{t+\varepsilon}$.

Down directed The first we move simplices of group C to the last places, and then we move simplices of group A to their places in $f_{t+\varepsilon}$.

2 General Statistics

In 2 paths generated 722 unique transpositions: 341 in only upper path, 341 in only down path and 40 in both paths.

The distribution of the switch transposition types and dimensions is given in the table:

dim	0	1	2	undefined
birth-birth	1	1	0	0
birth-death	0	12	0	0
death-death	0	3	1	0
no switch	43	192	53	416

And we can see more detailed table about how many simplex transpositions of each type lies in each reordering defined by the transposition of heights on vertices:

				Type	birth-birth		birth-death	death-death		no switch
				Dim	0	1	1	1	2	
Time	Vertices	Value	Figure	Path						
0.044	$\langle 2,3 \rangle$	0.491	3	Up	0	0	0	0	0	8
				Down	0	0	0	0	0	8
0.060	$\langle 0,2 \rangle$	0.498	4	Up	0	0	2	1	0	6
				Down	0	0	2	1	0	6
0.126	$\langle 3,5 \rangle$	0.450	5	Up	0	0	0	0	0	4
				Down	0	0	0	0	0	4
0.146	$\langle 0,5 \rangle$	0.457	6	Up	0	0	0	0	0	2
				Down	0	0	0	0	0	2
0.193	$\langle 1,2 \rangle$	0.555	7	Up	0	0	0	0	0	48
				Down	0	0	0	0	0	48
0.263	$\langle 3,6 \rangle$	0.381	8	Up	0	0	2	0	1	10
				Down	0	1	2	0	0	10
0.286	$\langle 0,6 \rangle$	0.390	9	Up	0	0	0	0	0	12
				Down	0	0	0	0	0	12
0.349	$\langle 3,4 \rangle$	0.338	10	Up	0	0	0	0	0	4
				Down	0	0	0	0	0	4
0.373	$\langle 0,4 \rangle$	0.348	11	Up	0	0	0	1	0	2
				Down	1	0	0	0	0	2
0.428	$\langle 2,8 \rangle$	0.655	12	Up	0	0	0	0	0	36
				Down	0	0	0	0	0	36

				Type	birth-birth		birth-death	death-death		no switch
				Dim	0	1	1	1	2	
Time	Vertices	Value	Figure	Path						
0.444	$\langle 1, 5\rangle$	0.557	13	Up	0	0	0	0	0	16
				Down	0	0	0	0	0	16
0.451	$\langle 3, 7\rangle$	0.286	14	Up	0	0	0	0	0	1
				Down	0	0	0	0	0	1
0.474	$\langle 0, 7\rangle$	0.299	15	Up	0	0	0	0	0	2
				Down	0	0	0	0	0	2
0.523	$\langle 2, 9\rangle$	0.696	16	Up	0	0	0	0	0	36
				Down	0	0	0	0	0	36
0.596	$\langle 8, 9\rangle$	0.667	17	Up	0	0	0	0	0	24
				Down	0	0	0	0	0	24
0.677	$\langle 5, 9\rangle$	0.636	18	Up	0	0	0	0	0	8
				Down	0	0	0	0	0	8
0.730	$\langle 1, 6\rangle$	0.560	19	Up	0	0	0	0	0	48
				Down	0	0	0	0	0	48
0.802	$\langle 6, 9\rangle$	0.587	20	Up	0	0	0	0	0	12
				Down	0	0	0	0	0	12
0.817	$\langle 5, 8\rangle$	0.683	21	Up	0	0	0	0	0	16
				Down	0	0	0	0	0	16
0.870	$\langle 1, 9\rangle$	0.561	22	Up	0	0	0	0	0	16
				Down	0	0	0	0	0	16
0.880	$\langle 4, 9\rangle$	0.557	23	Up	0	0	0	0	0	3
				Down	0	0	0	0	0	3
0.890	$\langle 1, 4\rangle$	0.561	24	Up	0	0	1	0	0	28
				Down	0	0	1	0	0	28
0.901	$\langle 7, 9\rangle$	0.549	25	Up	0	0	0	0	0	2
				Down	0	0	0	0	0	2
0.923	$\langle 1, 7\rangle$	0.562	26	Up	0	0	1	0	0	6
				Down	0	0	1	0	0	6
1.000	$\langle 4, 7\rangle$	0.607	27	Up	0	0	0	0	0	22
				Down	0	0	0	0	0	22

3 Detalization of the process

Here we detalize the paths, how depth poset changed during the homotopy by different paths.

In the figures we can see graphs, which edges coresponds the transpositions of simplices, and nodes are orders, coresponding one of 31 Depth Posets we got.

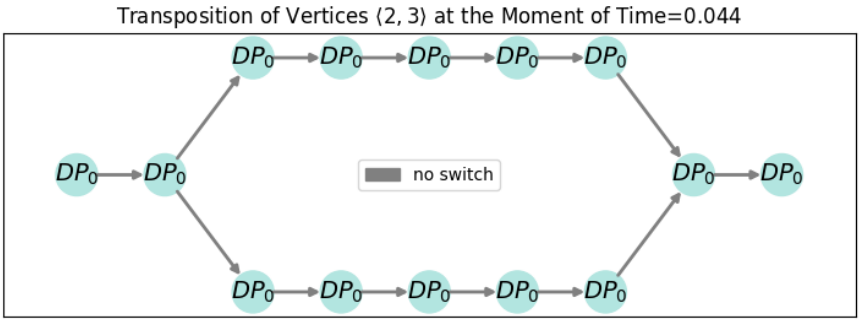

Figure 3: Reordering by transposition of vertices 2 and 3

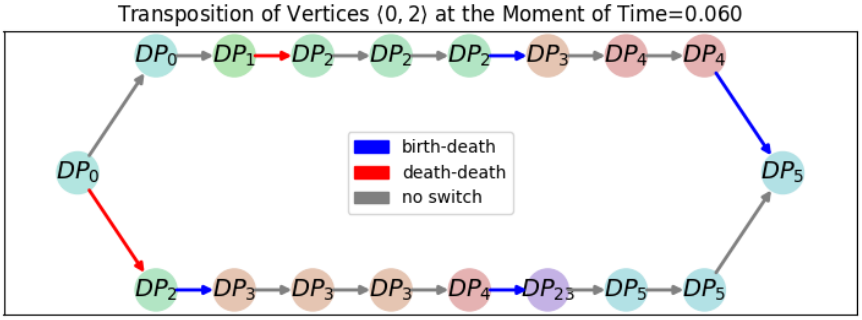

Figure 4: Reordering by transposition of vertices 0 and 2

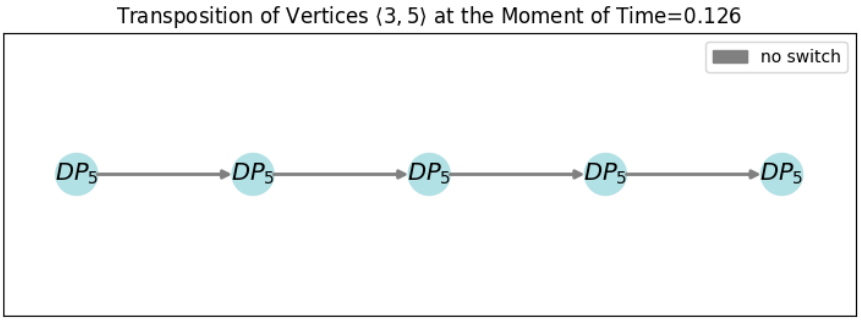

Figure 5: Reordering by transposition of vertices 3 and 5

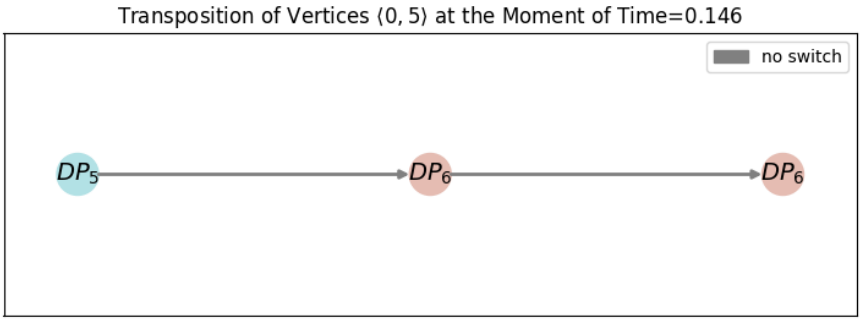

Figure 6: Reordering by transposition of vertices 0 and 5

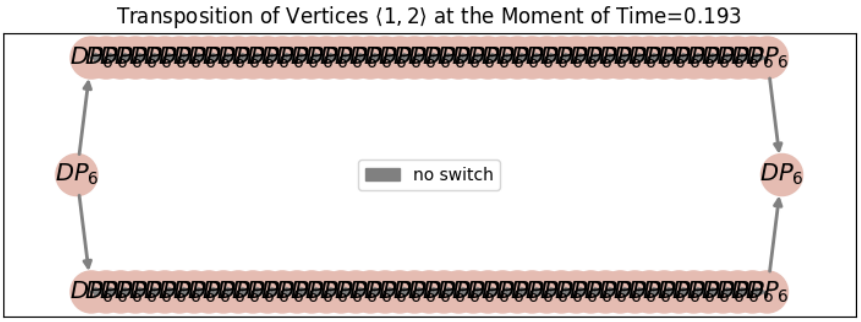

Figure 7: Reordering by transposition of vertices 1 and 2

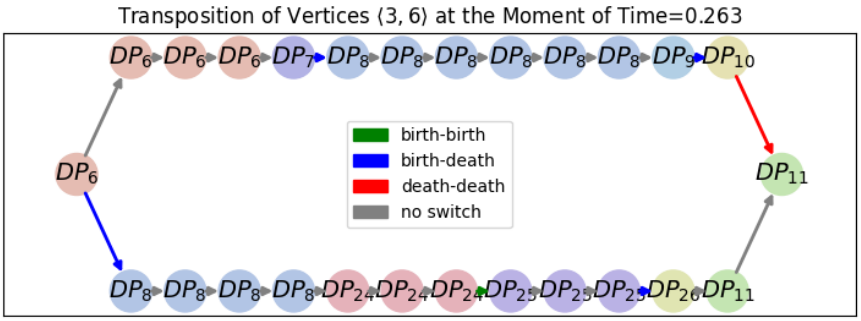

Figure 8: Reordering by transposition of vertices 3 and 6

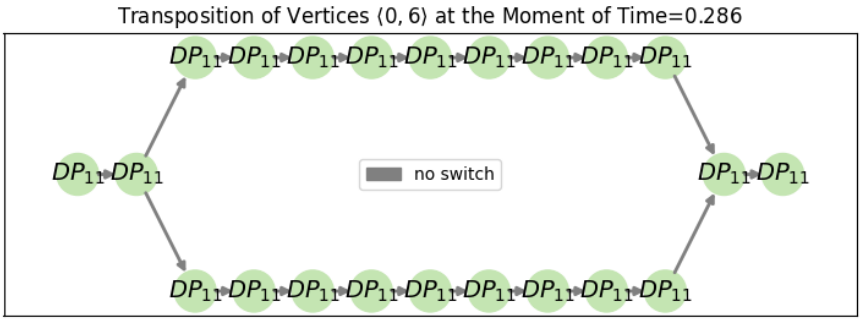

Figure 9: Reordering by transposition of vertices 0 and 6

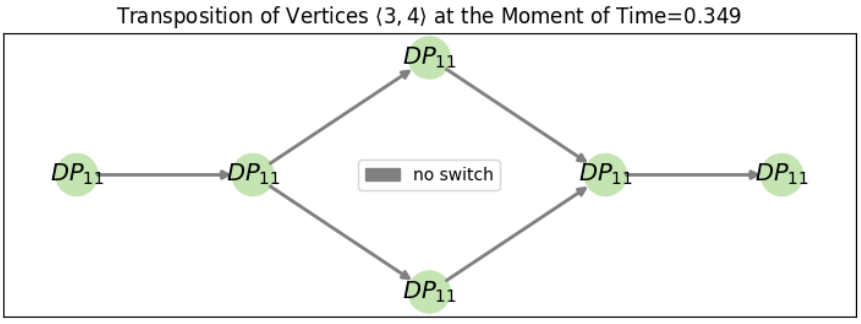

Figure 10: Reordering by transposition of vertices 3 and 4

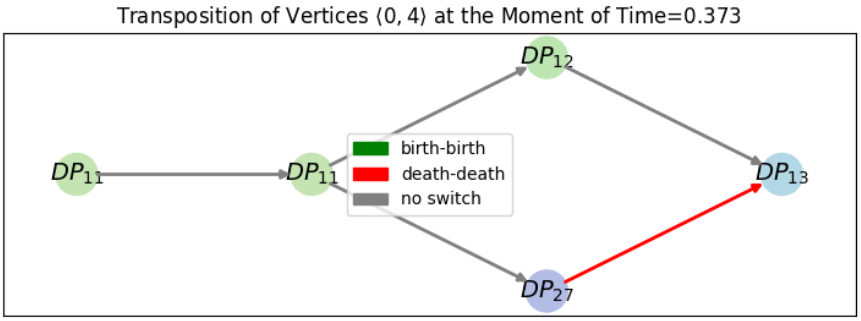

Figure 11: Reordering by transposition of vertices 0 and 4

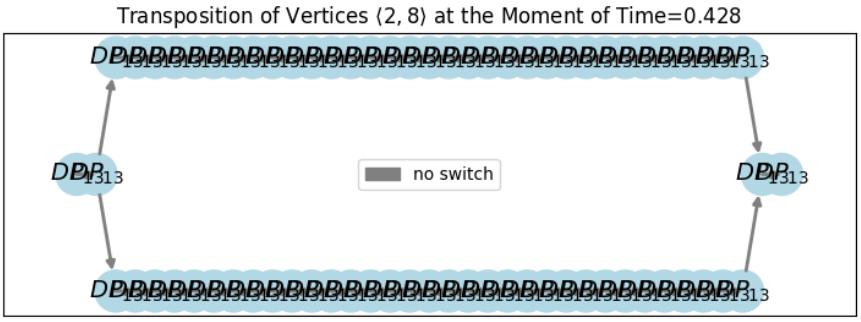

Figure 12: Reordering by transposition of vertices 2 and 8

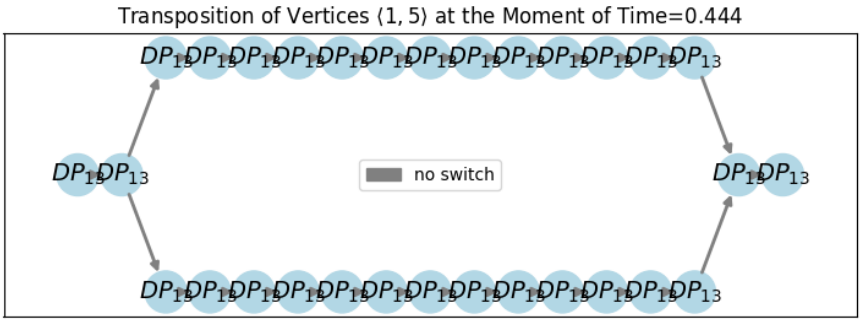

Figure 13: Reordering by transposition of vertices 1 and 5

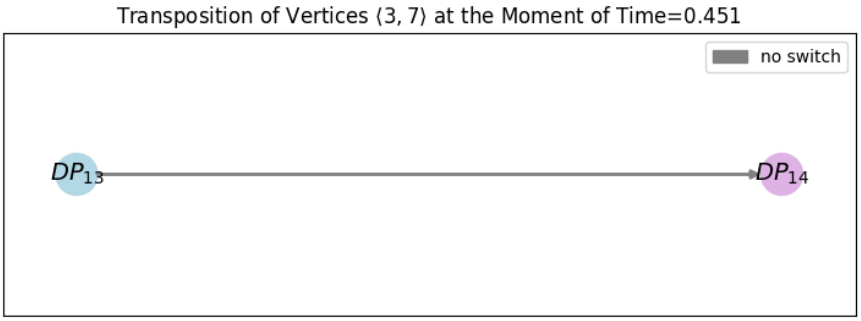

Figure 14: Reordering by transposition of vertices 3 and 7

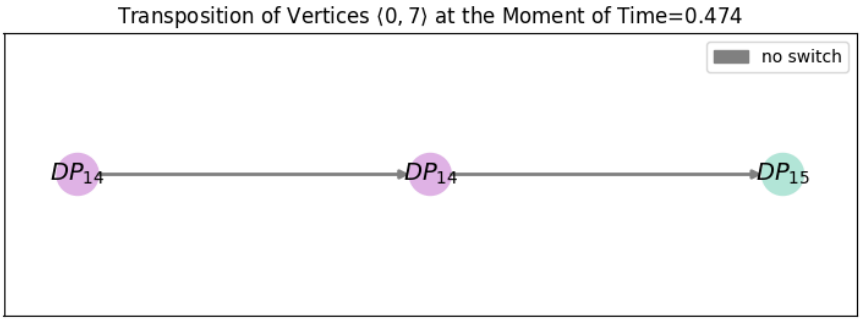

Figure 15: Reordering by transposition of vertices 0 and 7

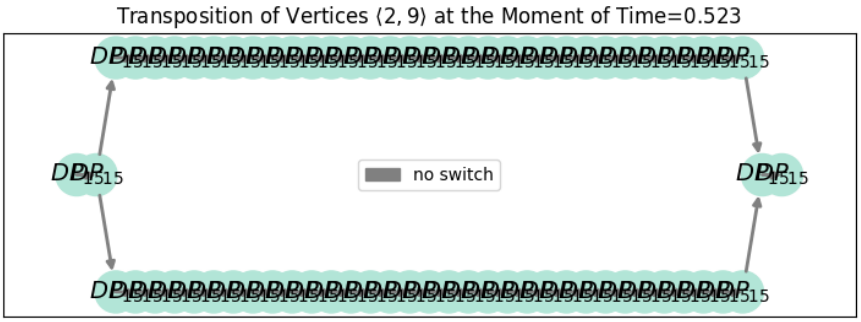

Figure 16: Reordering by transposition of vertices 2 and 9

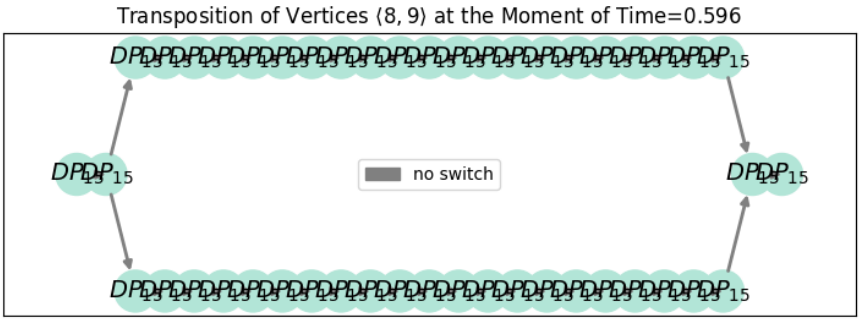

Figure 17: Reordering by transposition of vertices 8 and 9

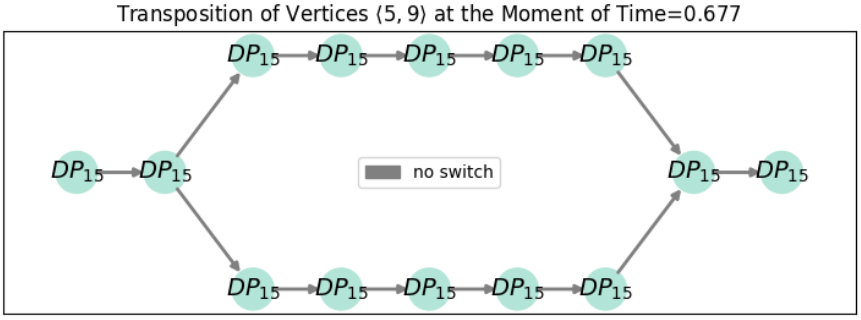

Figure 18: Reordering by transposition of vertices 5 and 9

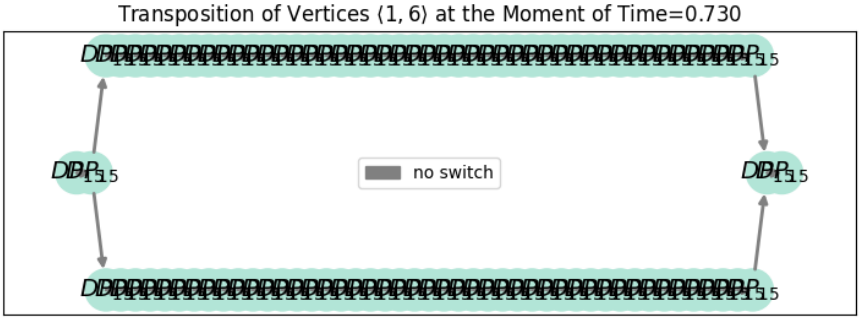

Figure 19: Reordering by transposition of vertices 1 and 6

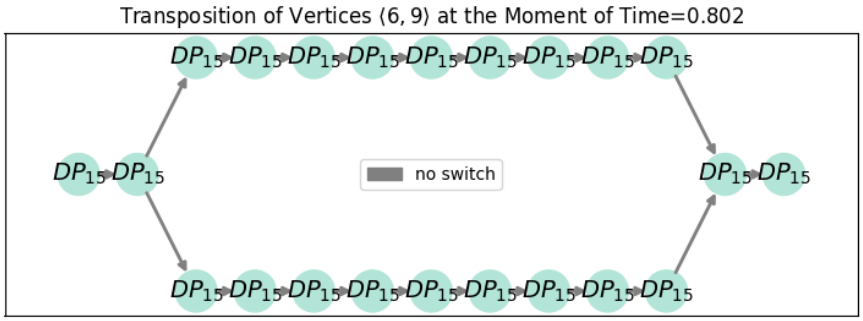

Figure 20: Reordering by transposition of vertices 6 and 9

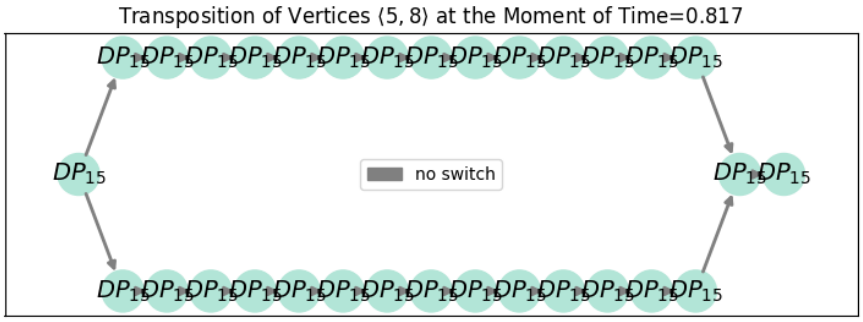

Figure 21: Reordering by transposition of vertices 5 and 8

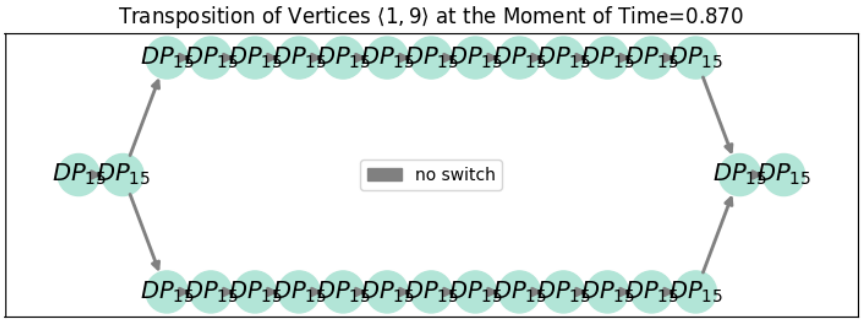

Figure 22: Reordering by transposition of vertices 1 and 9

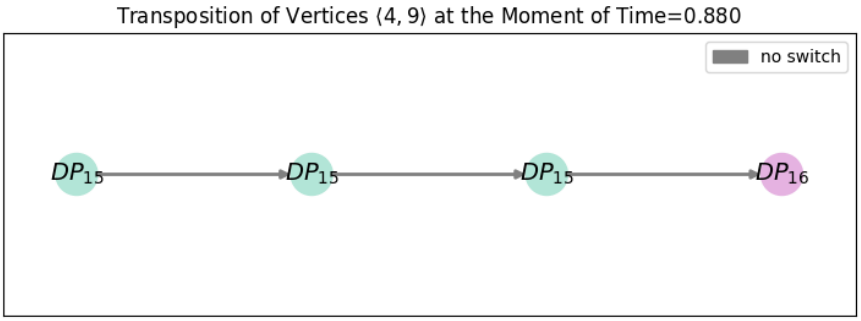

Figure 23: Reordering by transposition of vertices 4 and 9

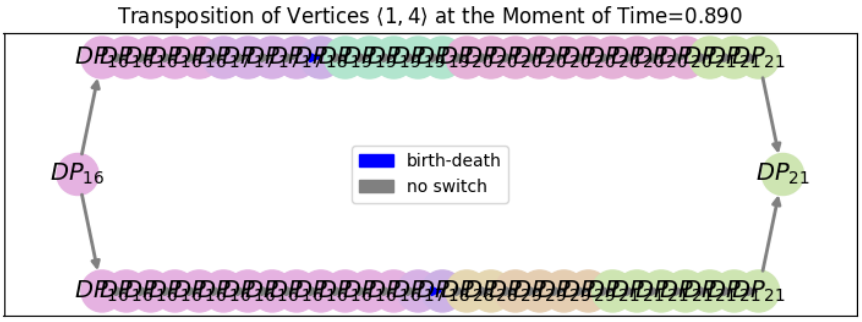

Figure 24: Reordering by transposition of vertices 1 and 4

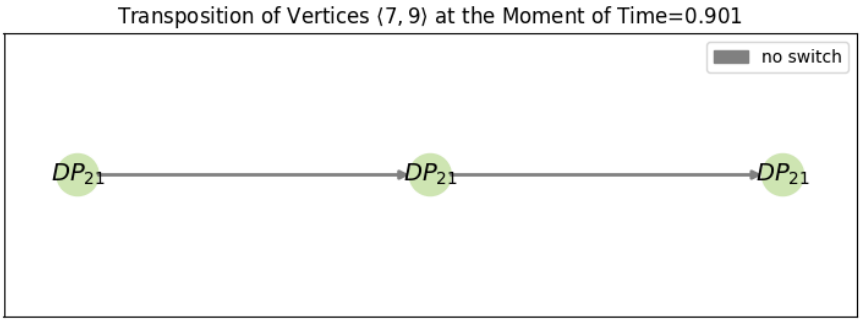

Figure 25: Reordering by transposition of vertices 7 and 9

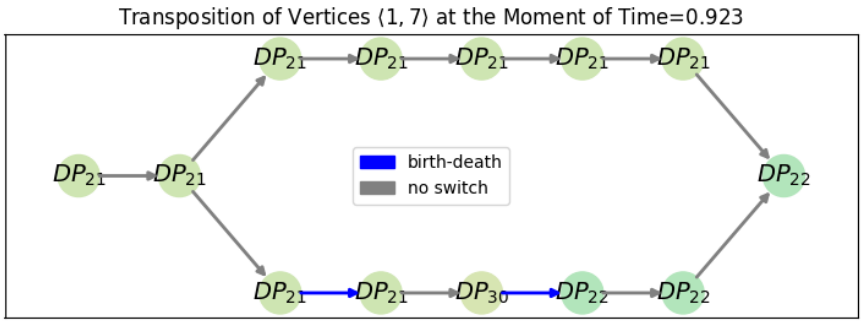

Figure 26: Reordering by transposition of vertices 1 and 7

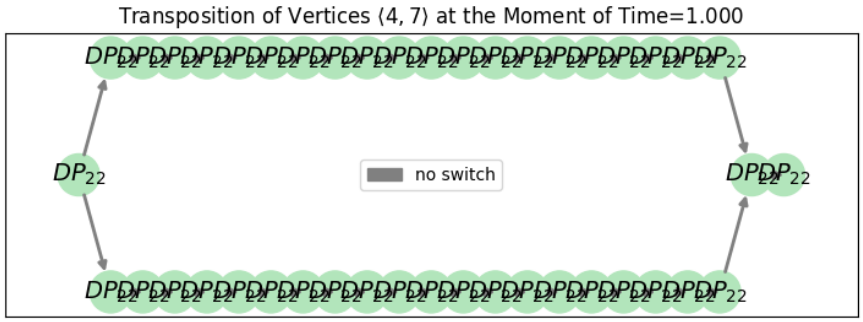

Figure 27: Reordering by transposition of vertices 4 and 7

4 Depth Posets

During the transposition we got 31 depth posets. In this section we list all of them.

The depth poset coresponding the first filtration we can see in Figure 28, and the depth poset coresponding the last filtration we can see in Figure 50.

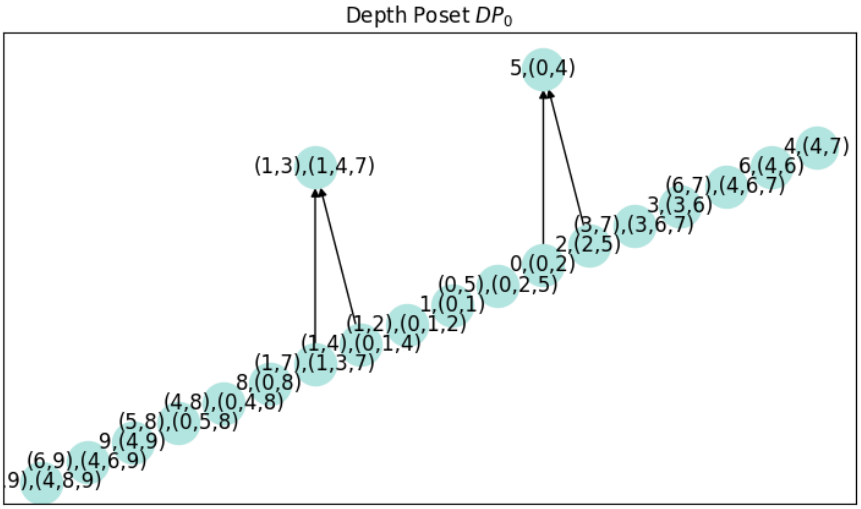

Figure 28: Depth Poset DP_0

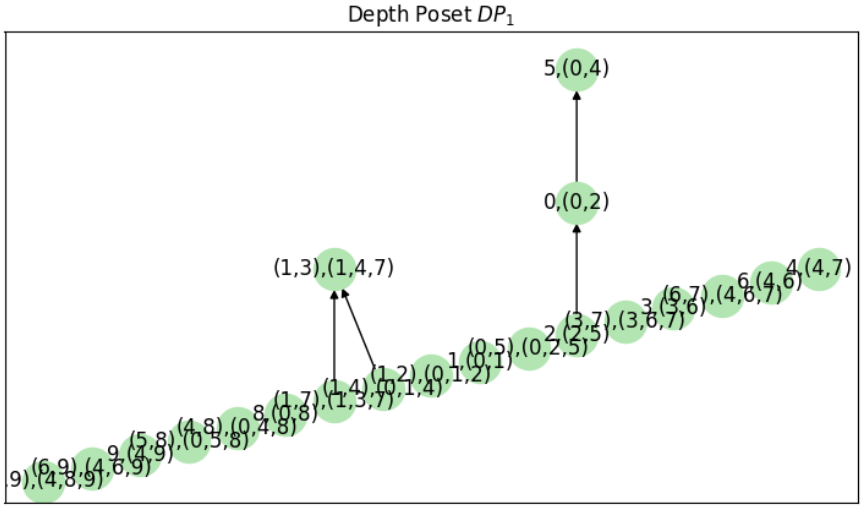

Figure 29: Depth Poset DP_1

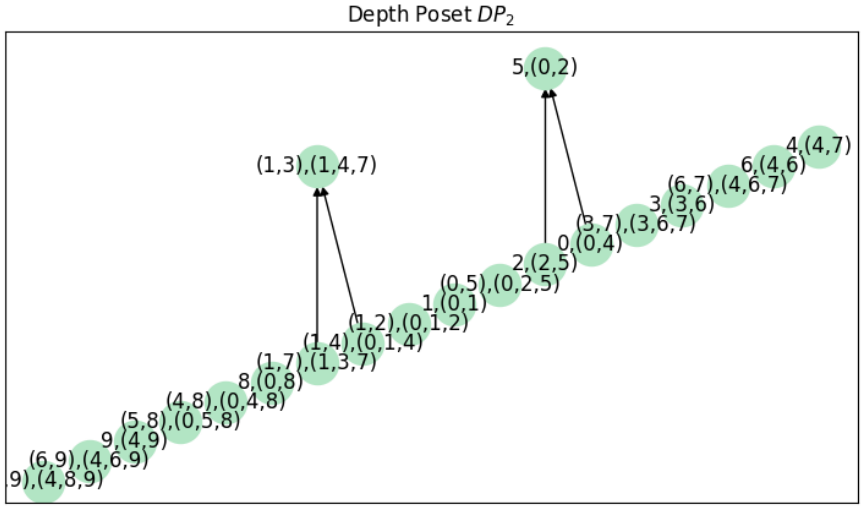

Figure 30: Depth Poset DP_2

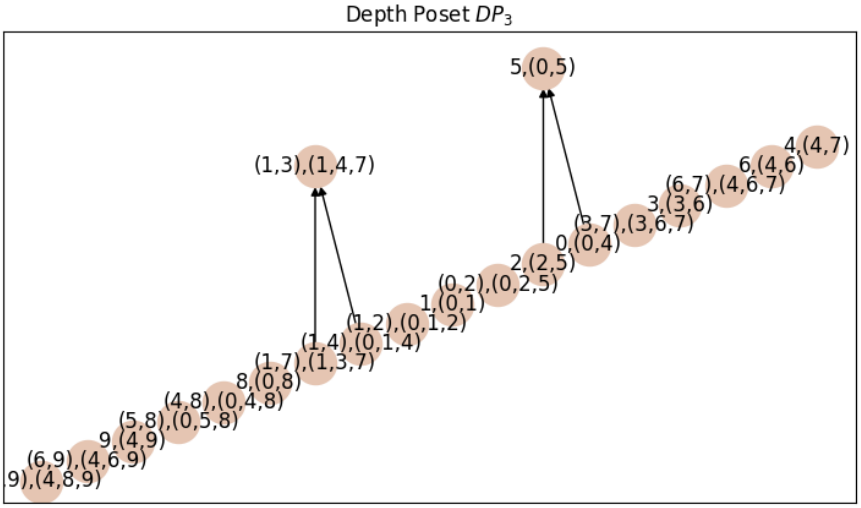

Figure 31: Depth Poset DP_3

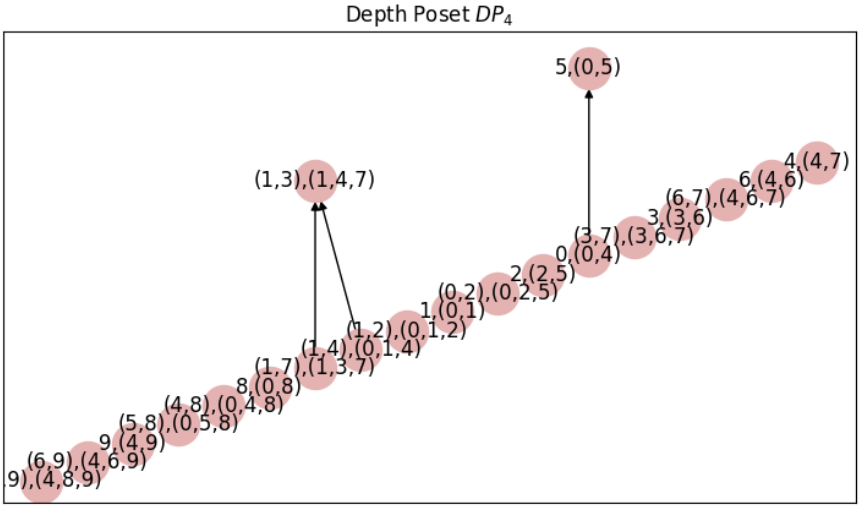

Figure 32: Depth Poset DP_4

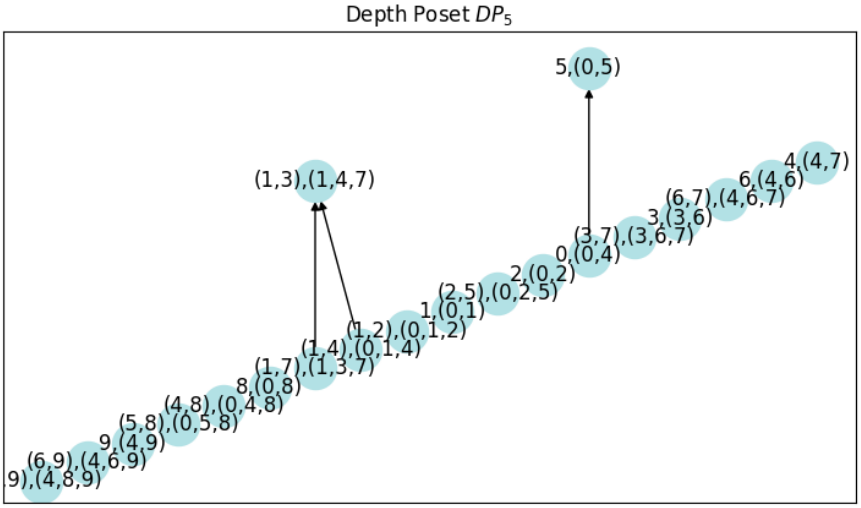

Figure 33: Depth Poset DP_5

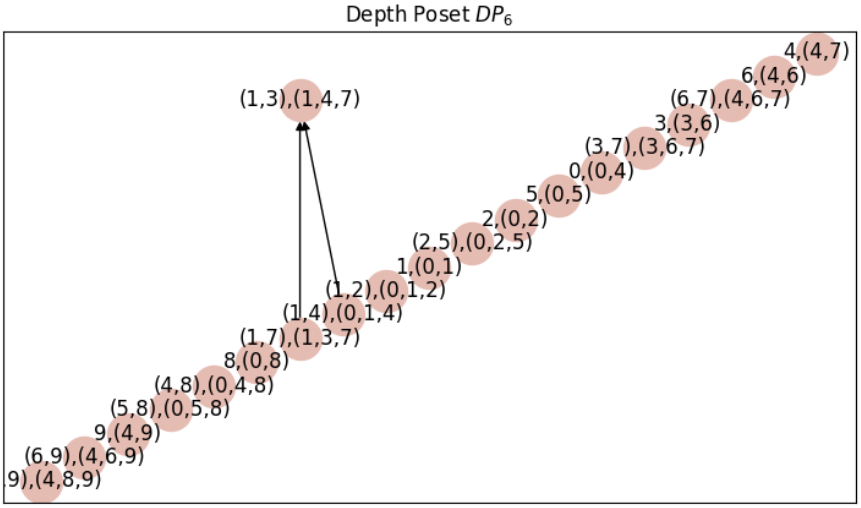

Figure 34: Depth Poset DP_6

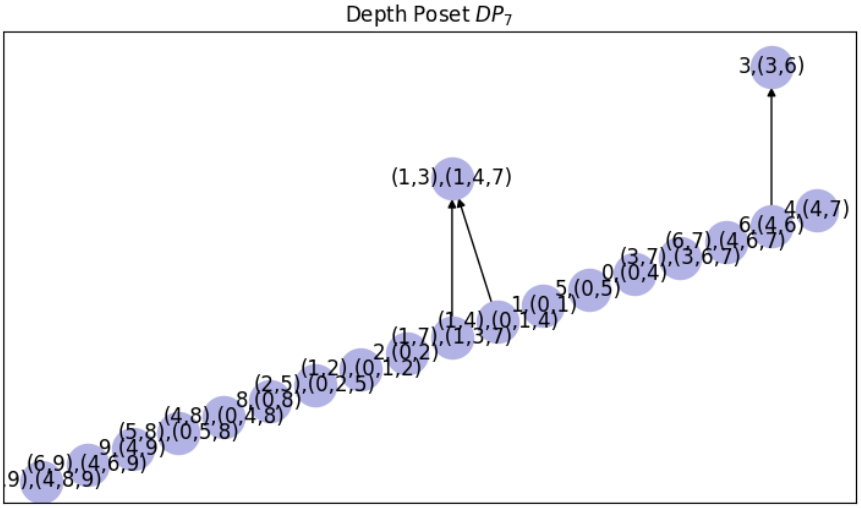

Figure 35: Depth Poset DP_7

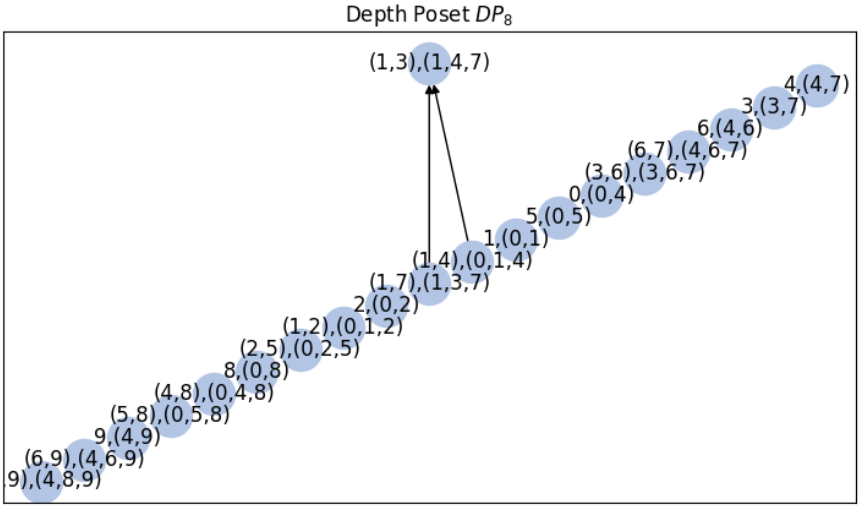

Figure 36: Depth Poset DP_8

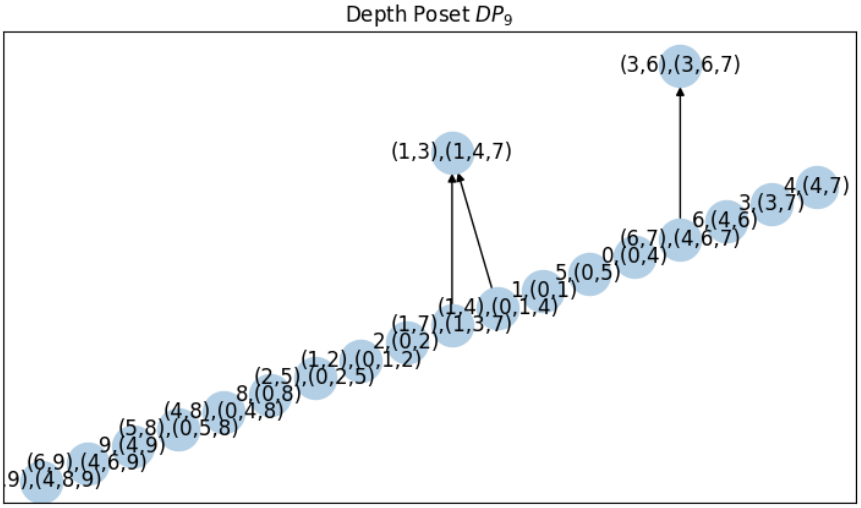

Figure 37: Depth Poset DP_9

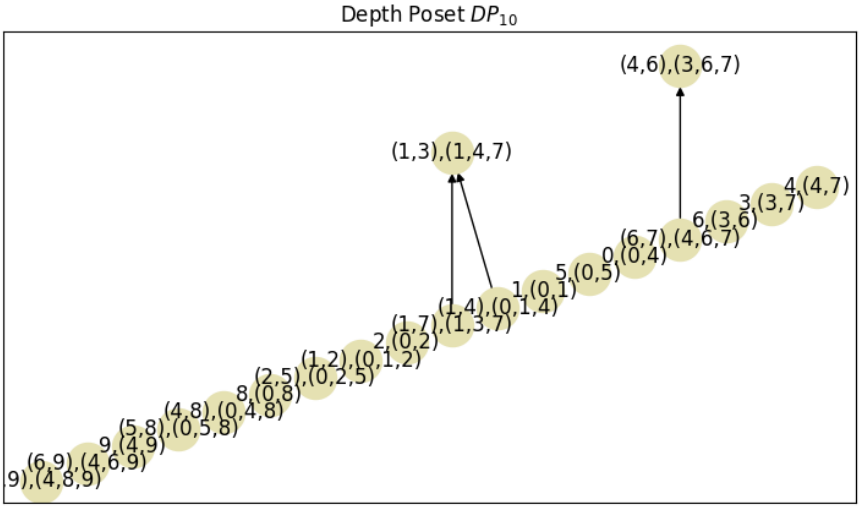

Figure 38: Depth Poset DP_{10}

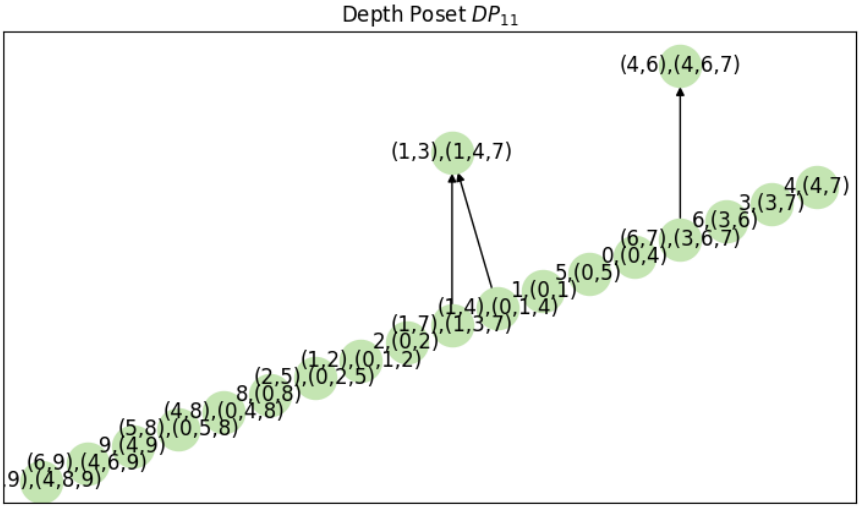

Figure 39: Depth Poset DP_{11}

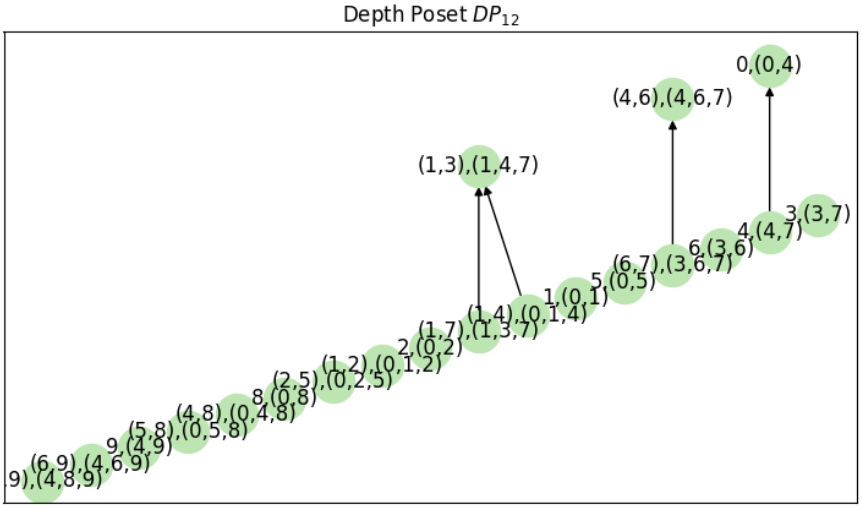

Figure 40: Depth Poset DP_{12}

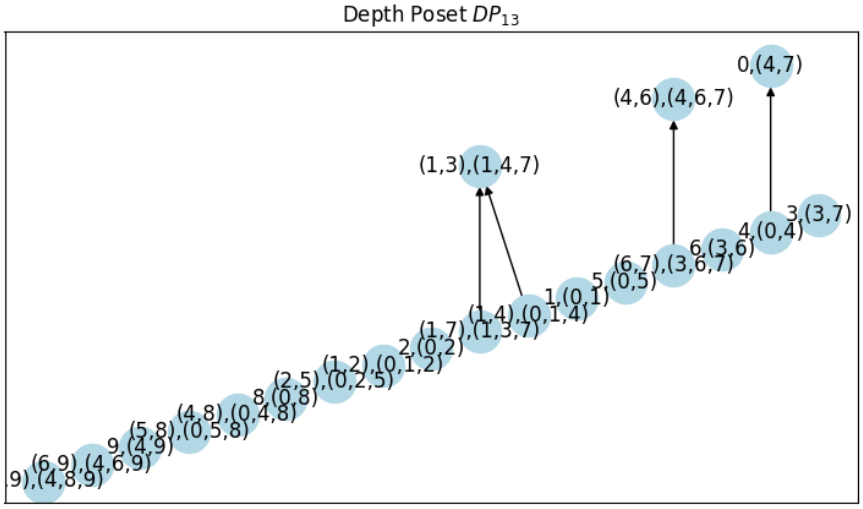

Figure 41: Depth Poset DP_{13}

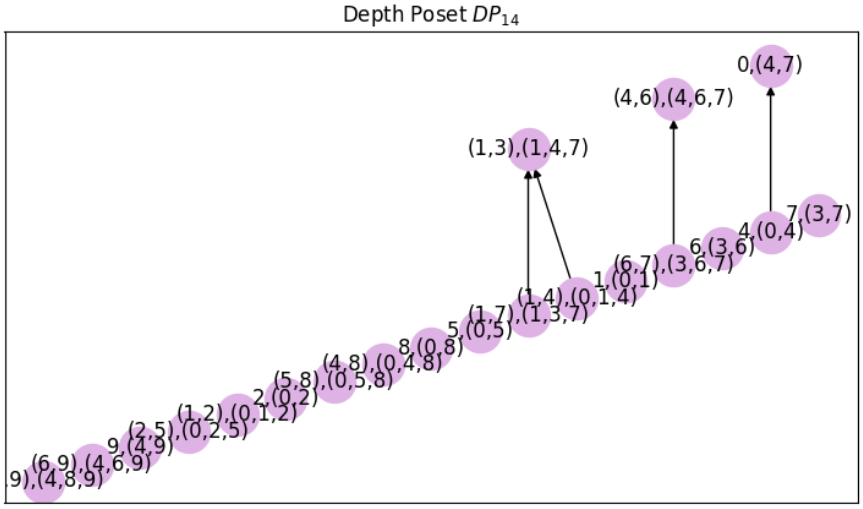

Figure 42: Depth Poset DP_{14}

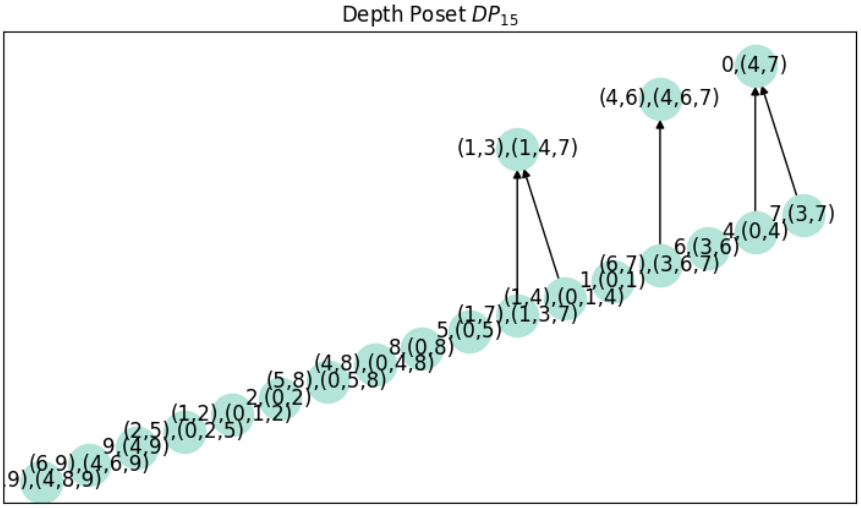

Figure 43: Depth Poset DP_{15}

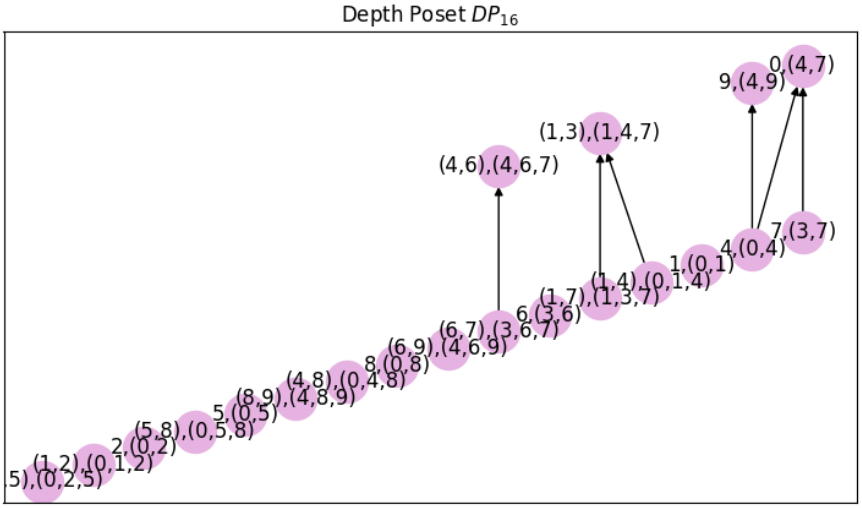

Figure 44: Depth Poset DP_{16}

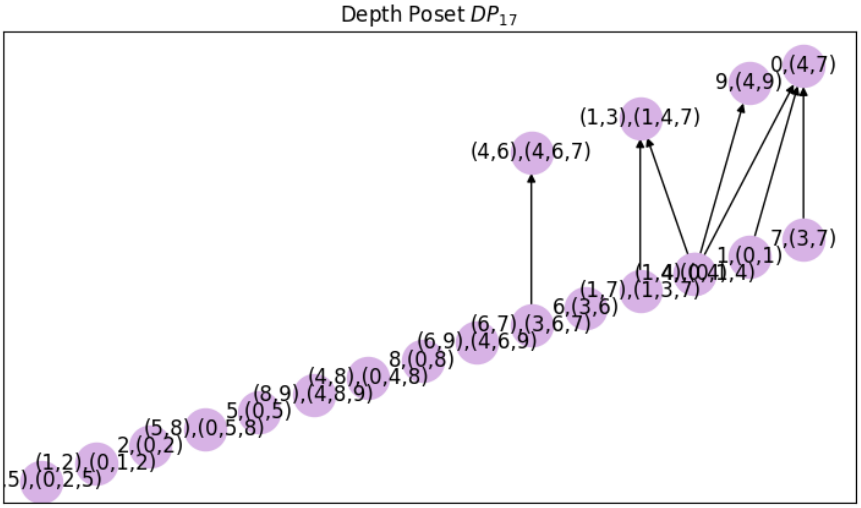

Figure 45: Depth Poset DP_{17}

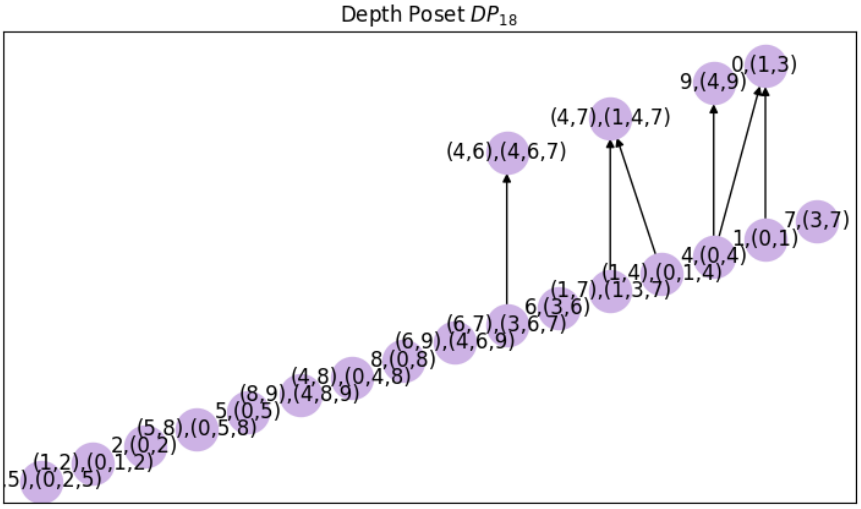

Figure 46: Depth Poset DP_{18}

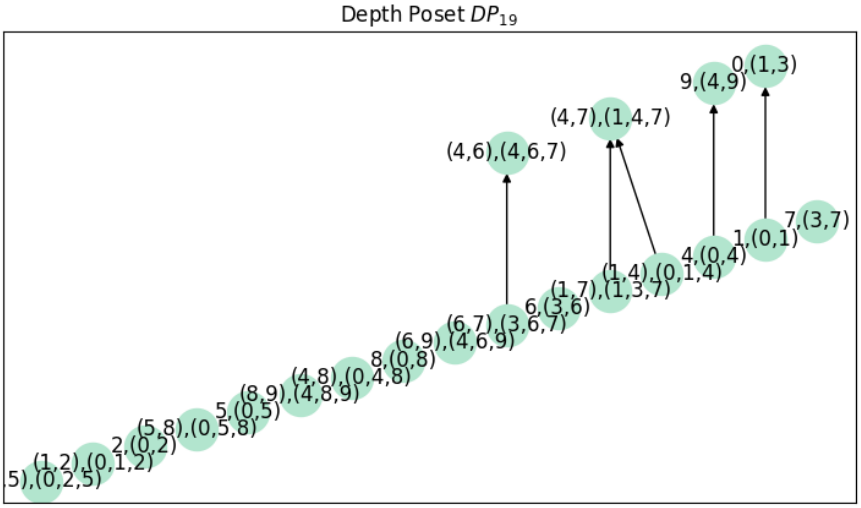

Figure 47: Depth Poset DP_{19}

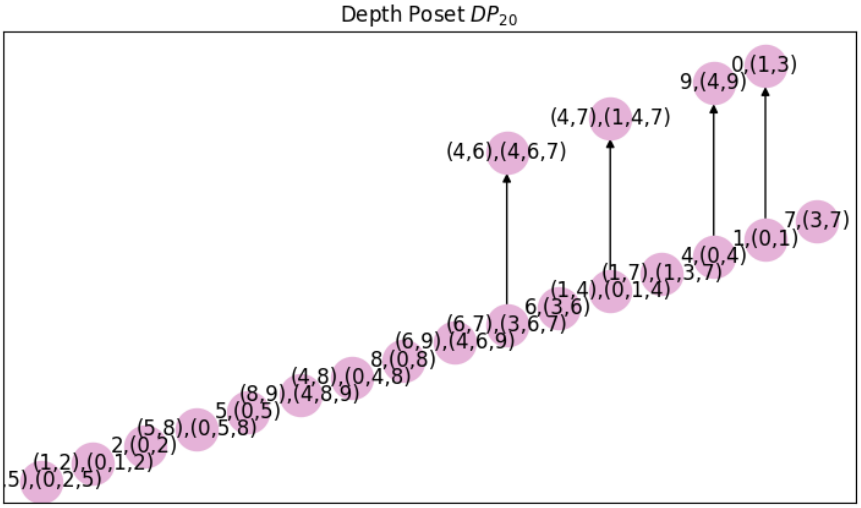

Figure 48: Depth Poset DP_{20}

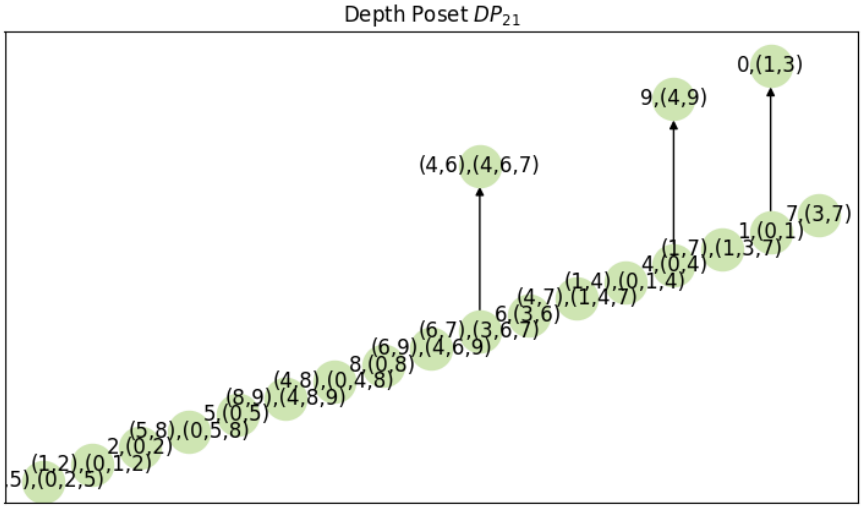

Figure 49: Depth Poset DP_{21}

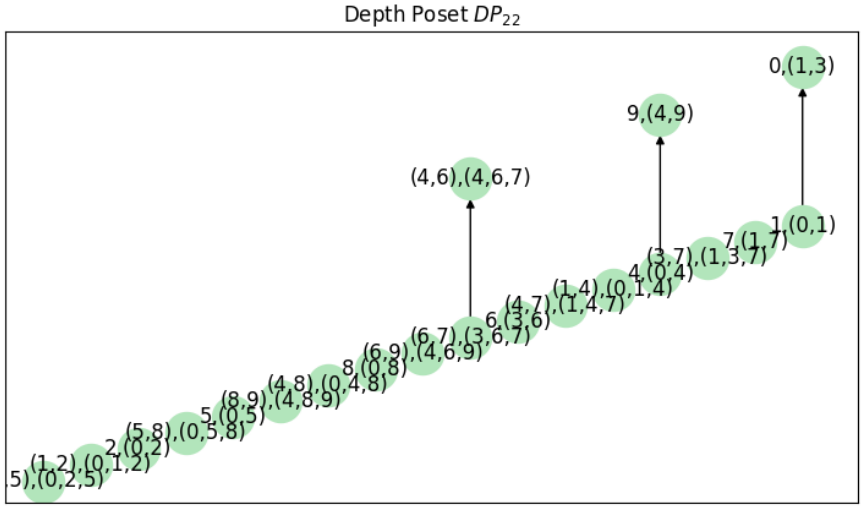

Figure 50: Depth Poset DP_{22}

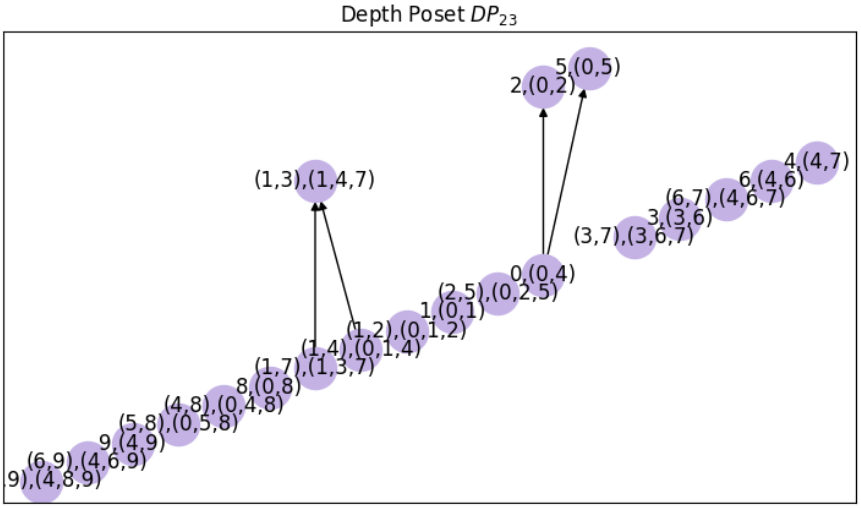

Figure 51: Depth Poset DP_{23}

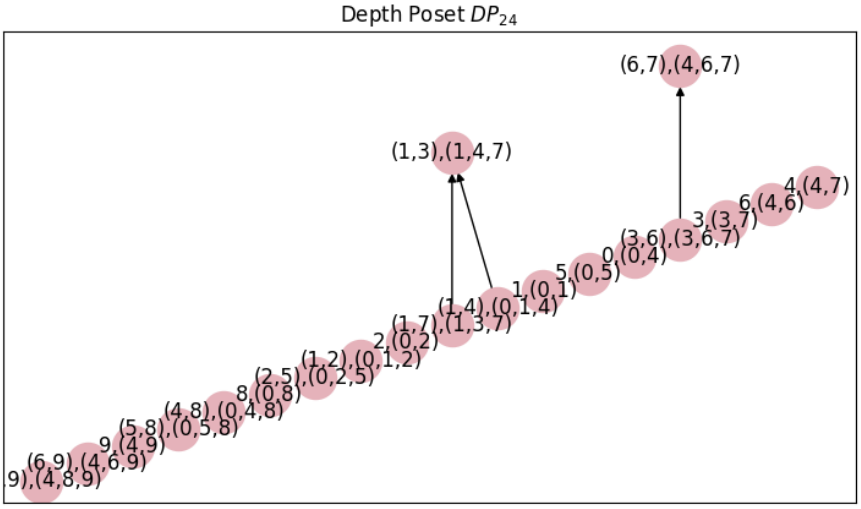

Figure 52: Depth Poset DP_{24}

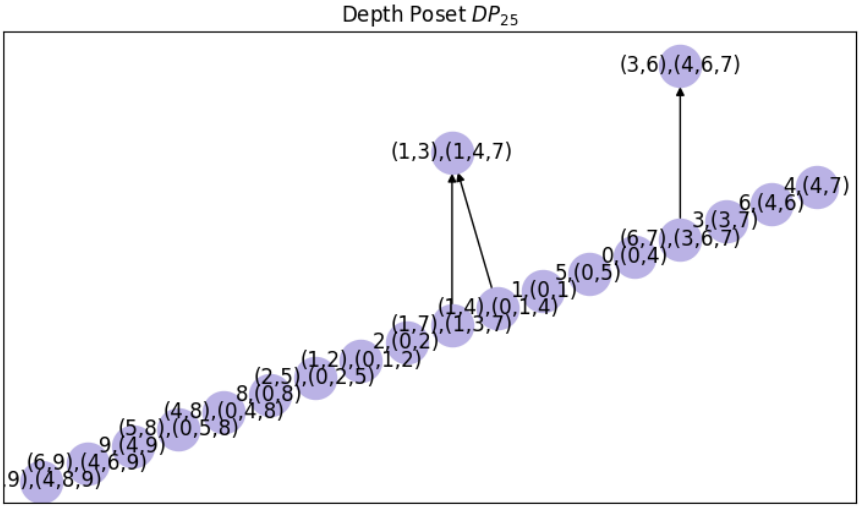

Figure 53: Depth Poset DP_{25}

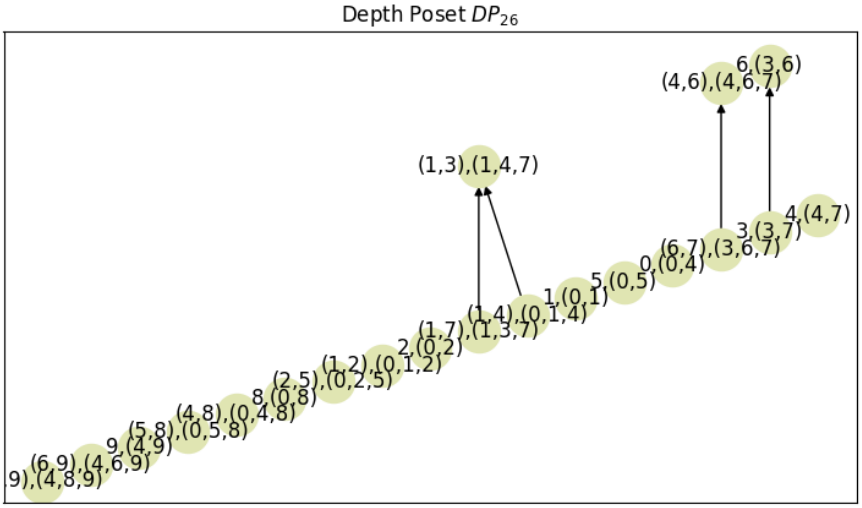

Figure 54: Depth Poset DP_{26}

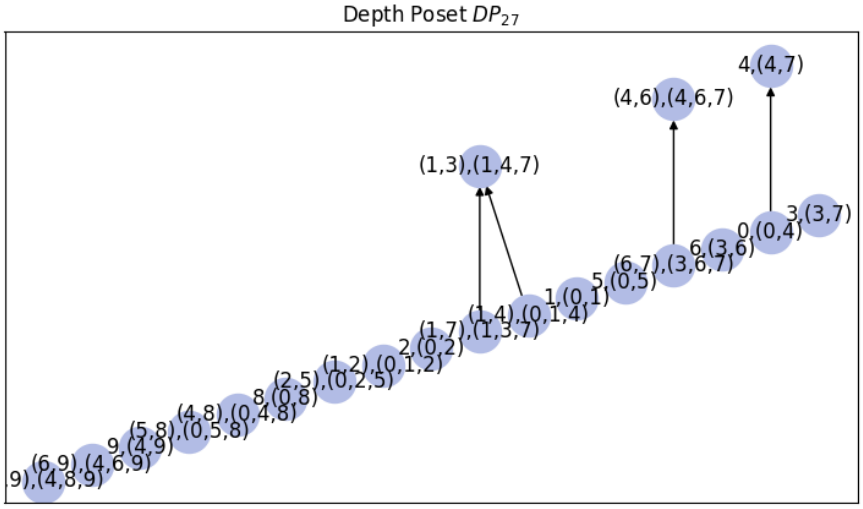

Figure 55: Depth Poset DP_{27}

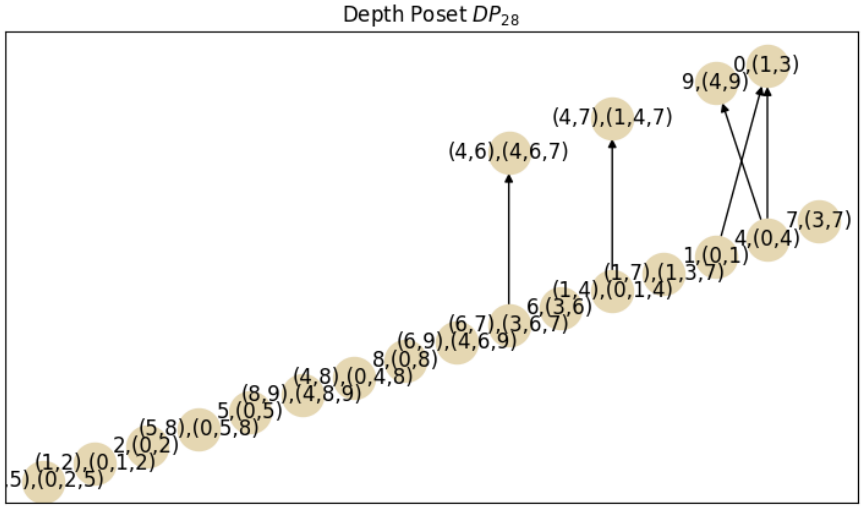

Figure 56: Depth Poset DP_{28}

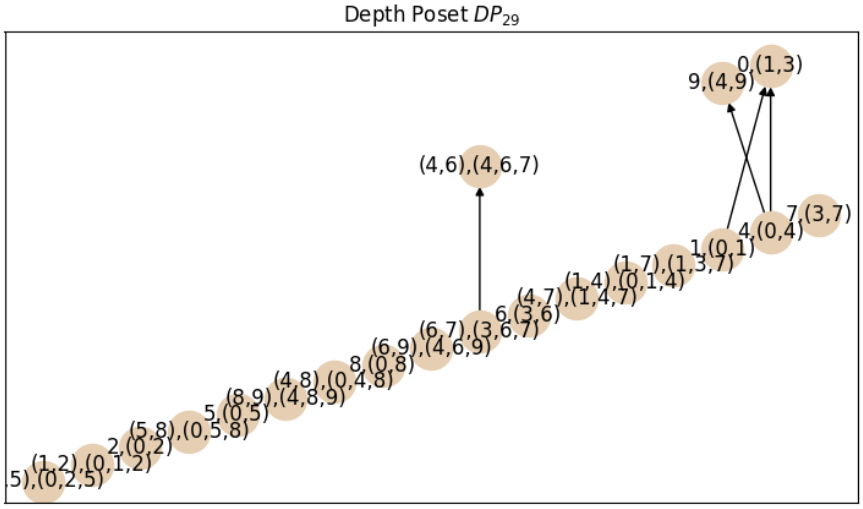

Figure 57: Depth Poset DP_{29}

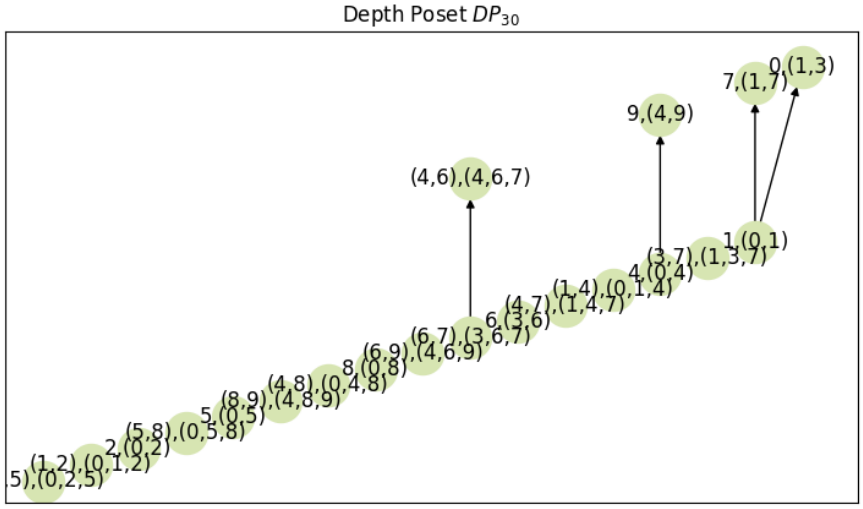

Figure 58: Depth Poset DP_{30}