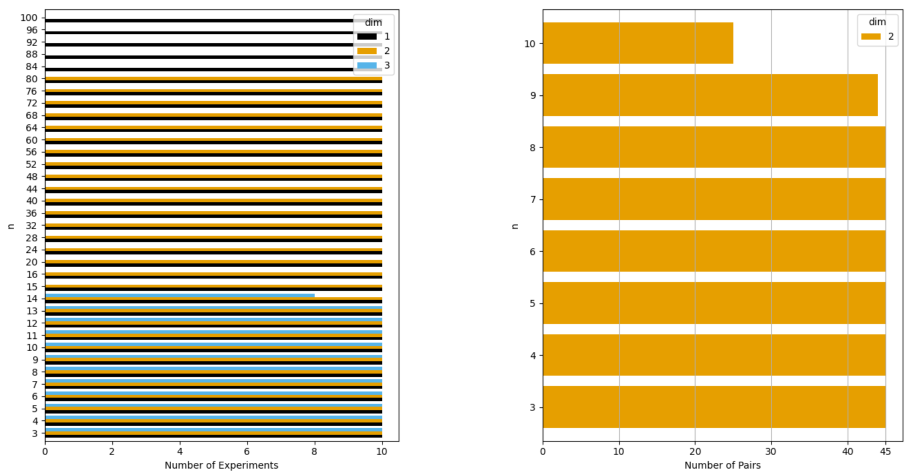
# 1 Description

The first we just calculated several Extended Barycentric Cubical Tori. The distribution of calculated toruses we can see in the Figure 1a.




(a) Barycentric Cubical Tori

(b) Calculated pairs

Figure 1: Size/dimension distribution

Then for each pair of filtrations with similar dimension and $n$ $f_0, f_1 : \mathbb{T}^d_n \to \mathbb{R}$ we can define a linear homotopy $h : [0,1] \times \mathbb{T}^d_n \to \mathbb{R}$:

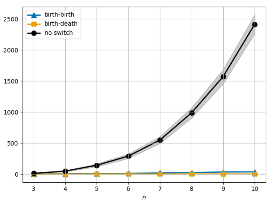
$$h(t,\sigma) = (1-t) \cdot f_0(\sigma) + t \cdot f_1(\sigma)$$

The moment of time $t \in [0,1]$ such there exist a pair of cells $\sigma_0, \sigma_1 \in \mathbb{T}^d_n$ such that $h(t,\sigma_0) = h(t,\sigma_1)$ is a moment of transposition of these cells during the homotopy (the probability, that this will be full segment is 0 for pair of independent barycentric cubical tori).

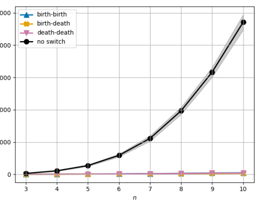
We have found all transposing pairs like this, classified them and calculated how these transpositions cahnges the Depth Poset. The distribution of calculated pairs of filtration we can see in the Figure 1b.

## 2 Switch Types Distributions

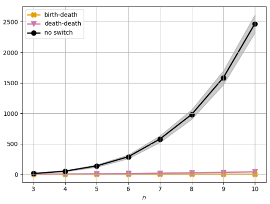
The distribution of transpositions types for the model $\mathbb{T}_n^2$ we can see in the figure Fig. 3.



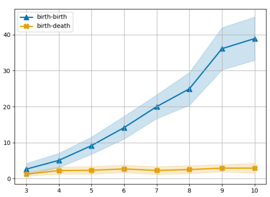

(a) Cells dimension 0



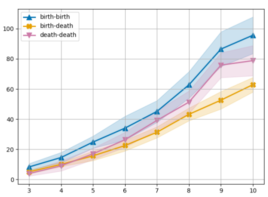

(b) Cells dimension 1



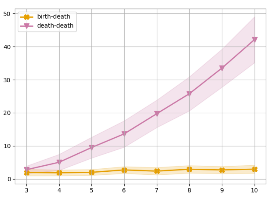

(c) Cells dimension 2




(d) Cells dimension 0 (without no switch transpositions)




(e) Cells dimension 1 (without no switch transpositions)




(f) Cells dimension 2 (without no switch transpositions)

Figure 2: The switch type distribution for $\mathbb{T}_n^2$

# 3 Similarity Scores

- **birth_relation_cell_similarity** - The Jacard index of arcs (edges) in the birth relation (given by row left to right reduction algorithm). Consider 2 birth-death pairs are similar if they corespond the similar cells.

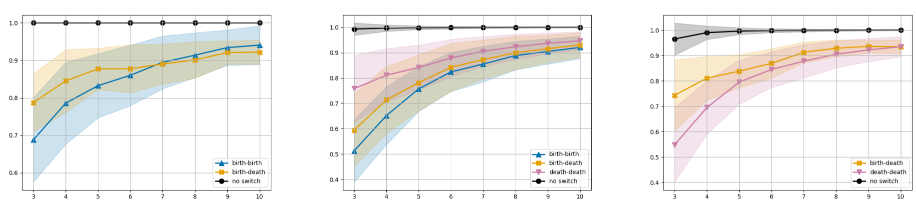
- **death_relation_cell_similarity** - The Jacard index of arcs (edges) in the death relation (given by column bottom to top reduction algorithm). Consider 2 birth-death pairs are similar if they corespond the similar cells.

- **poset_closure_arcs_cell_similarity** - The Jacard index of arcs (edges) in the transitive closure of the Depth Posets. Consider 2 birth-death pairs are similar if they corespond the similar cells.

- **poset_reduction_arcs_cell_similarity** - The Jacard index of arcs (edges) in the transitive reduction of the Depth Posets. Consider 2 birth-death pairs are similar if they corespond the similar cells.

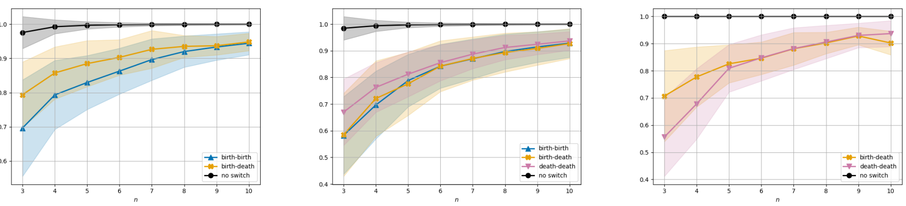
The distribution of the similarity scores for different models $\mathbb{T}_n^{dim}$ we can see in the following figures:

| dim | 2 |
|---|---|
| birth_relation_cell_similarity | Fig. 5 |
| death_relation_cell_similarity | Fig. 7 |
| poset_closure_arcs_cell_similarity | Fig. 9 |
| poset_reduction_arcs_cell_similarity | Fig. 11 |



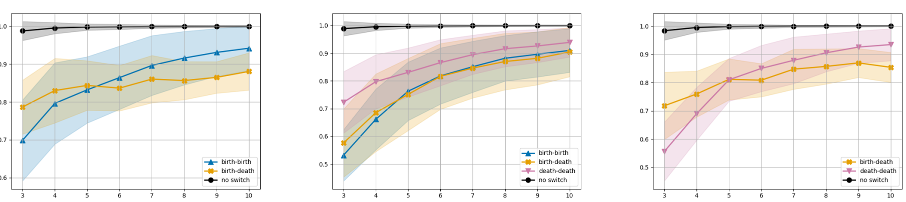

(a) Cells dimension 0 (b) Cells dimension 1 (c) Cells dimension 2

Figure 3: Similarity score birth_relation_cell_similarity for $\mathbb{T}_n^2$



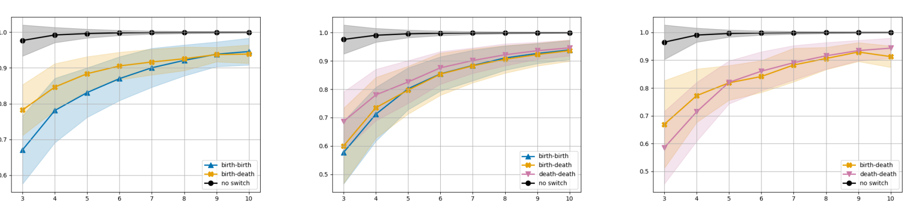

(a) Cells dimension 0 (b) Cells dimension 1 (c) Cells dimension 2

Figure 4: Similarity score death_relation_cell_similarity for $\mathbb{T}_n^2$




(a) Cells dimension 0 (b) Cells dimension 1 (c) Cells dimension 2

Figure 5: Similarity score poset_closure_arcs_cell_similarity for $\mathbb{T}_n^2$




(a) Cells dimension 0 (b) Cells dimension 1 (c) Cells dimension 2

Figure 6: Similarity score poset_reduction_arcs_cell_similarity for $\mathbb{T}_n^2$