

1 Model

The d dimensional torus \mathbb{T}^d can be defined as $(\mathbb{R}/n\mathbb{Z})^d$ for some natural n . And we can represent this as a cell-complex with cubical d -dimensional cells $(\mathbb{Z}/n\mathbb{Z})^d + [0, 1]^d$ and all their k -faces for $k = 0, \dots, d$.

We randomly assume the filtration value for each k -face uniformly distributed in $[k, k + 1]$. This filtration on segmented torus will correspond some real filtration $f: \mathbb{T}^d \rightarrow \mathbb{R}$, s.t. the d -dimensional cells will correspond the local maximums, vertices will correspond the local minimums and other k -faces will be saddles.

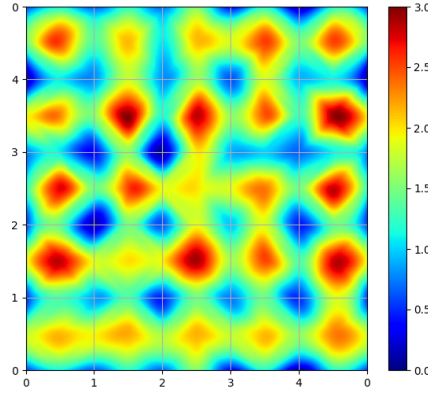


Figure 1: The example of the filtration $f : \mathbb{T}^2 \rightarrow \mathbb{R}$, s.t. there are local minimums in the vertices, saddles in the middle of edges, and the local maximums in the centers of square cells.

2 Scores

2.1 Poset Scores

- **number_of_nodes** : Returns the number of nodes in the poset.
- **number_of_relations** : Returns the number of relations in the transitive reduction.
- **number_of_components** : Returns the number of connected components in the poset
- **cycles_dimension** : Returns the dimension of space of cycles in reduction.

- **number_of_minimal_nodes** : Returns the number of minimal nodes.
- **number_of_maximal_nodes** : Returns the number of maximal nodes.
- **height** : Returns the poset height - the length of the longest chain.
- **width** : Returns the poset width - the length of the longest antichain (subset, s.t. all elements are pairwise incomparable). The algorithm is based on Dilworth's theorem and it's proof via König's theorem: [link](#)
- **minimum_maximal_chain** : Returns the minimum size of maximal chains in the poset.
- **avarage_maximal_chain** : Returns the avarage size of maximal chains in the poset.

2.2 Node Scores

- **ancestors_number**: Returns the number of nodes higher than given
- **ancestors_height**: Returns the size of maximum chain of subposet of nodes higher or equal than given
- **ancestors_width**: Returns the size of maximum chain of subposet of nodes higher or equal than given
- **ancestors_cycles_dimension**: Returns the the dimension of space of cycles in reduction of subposet of nodes higher or equal than given
- **successors_number**: Returns the number of nodes higher than given
- **successors_height**: Returns the size of maximum chain of subposet of nodes lower or equal than given
- **successors_width**: Returns the size of maximum chain of subposet of nodes lower or equal than given
- **successors_cycles_dimension**: Returns the the dimension of space of cycles in reduction of subposet of nodes lower or equal than given

3 Experiments and Results

There are 362 experiments done. In the Figure 2 we can see how cases are distributed by size and dimension.

3.1 Depth Poset Features

In the Figure 3 we can see the avarage poset scores values for each number of points n in the depth poset.

In the Figure 4 we can see the avarage mean node scores values in poset for each number of points n in the depth poset.

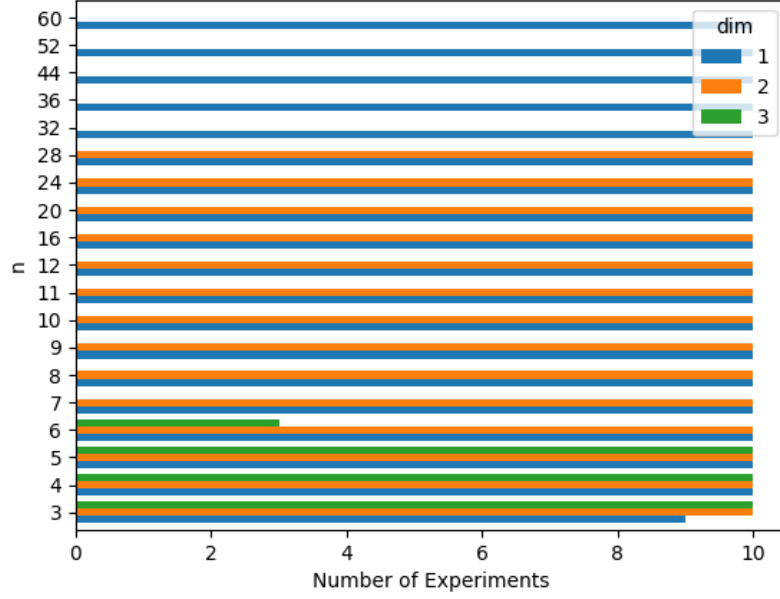


Figure 2: Size/dimension distribution of experiments

3.2 Column Reduction Poset Features

In the Figure 5 we can see the average poset scores values for each number of points n in the column reduction poset.

In the Figure 6 we can see the average mean node scores values in poset for each n in the column reduction poset.

3.3 Row Reduction Poset Features

In the Figure 7 we can see the average poset scores values for each number of points n in the row reduction poset.

In the Figure 8 we can see the average mean node scores values in poset for each number of points n in the row reduction poset.

Depth Poset: Mean Poset Scores

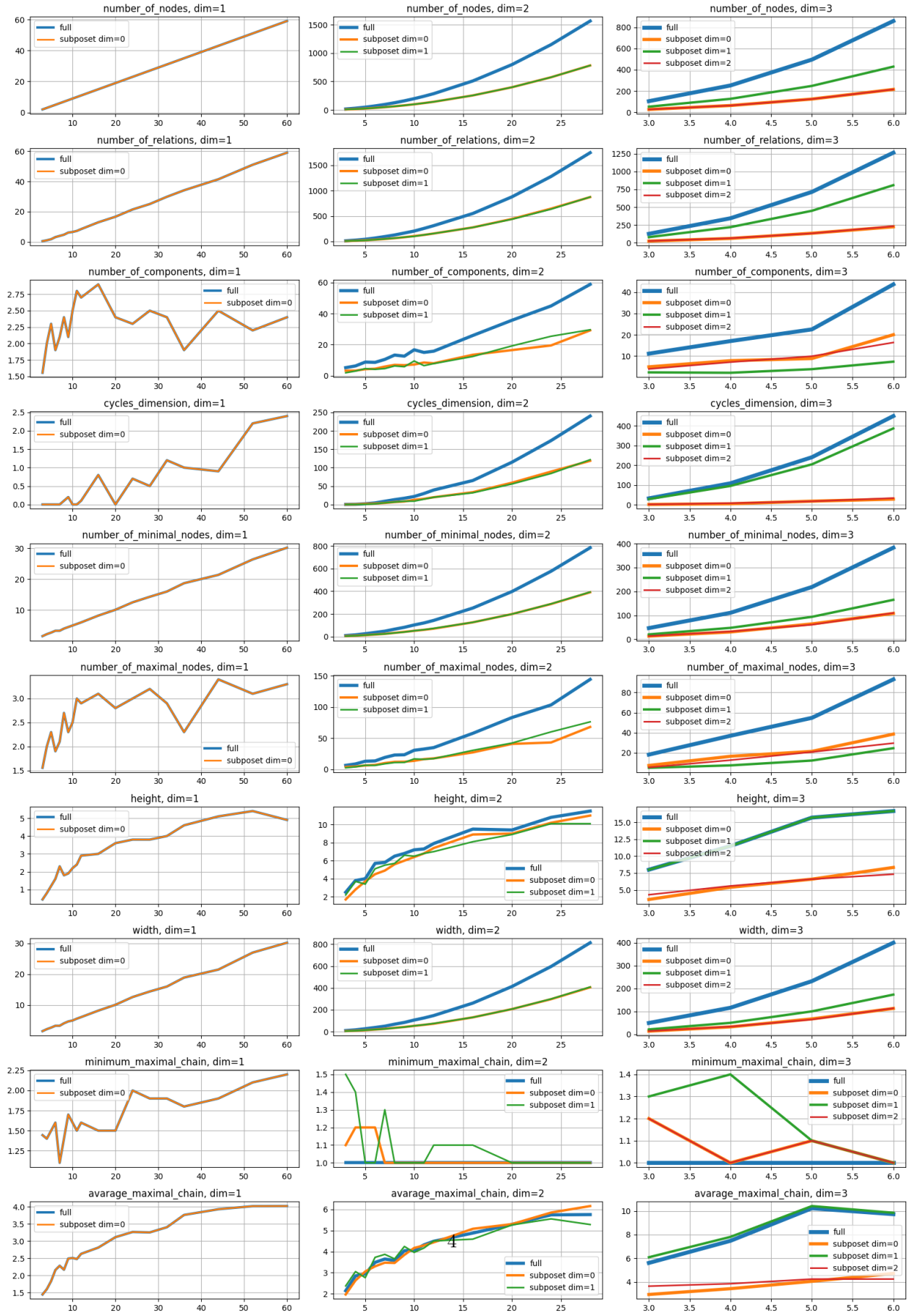


Figure 3: Depth Poset: Mean poset scores

Depth Poset: Mean Node Scores

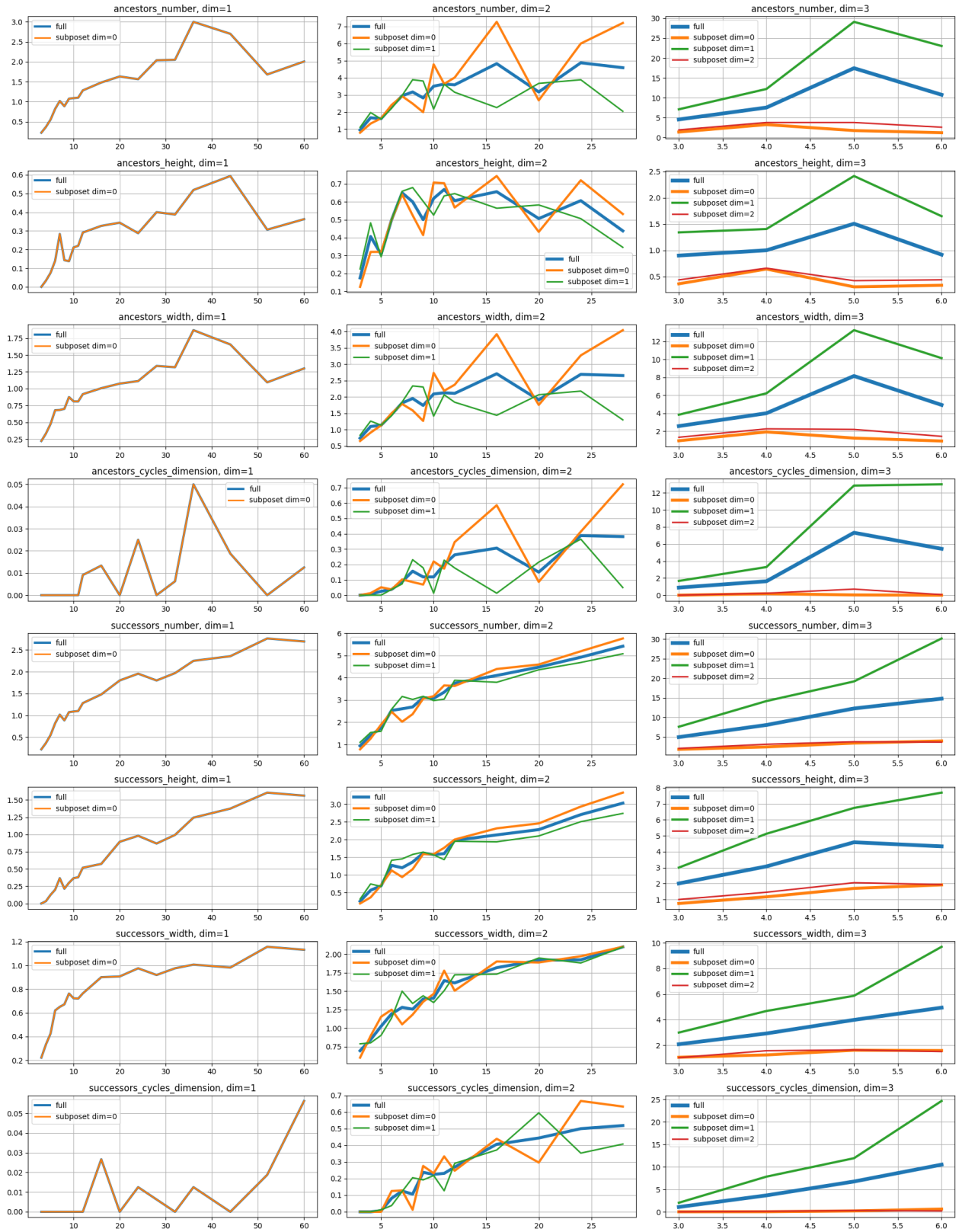


Figure 4: Depth Poset: Mean node scores

Column Reduction Poset: Mean Poset Scores

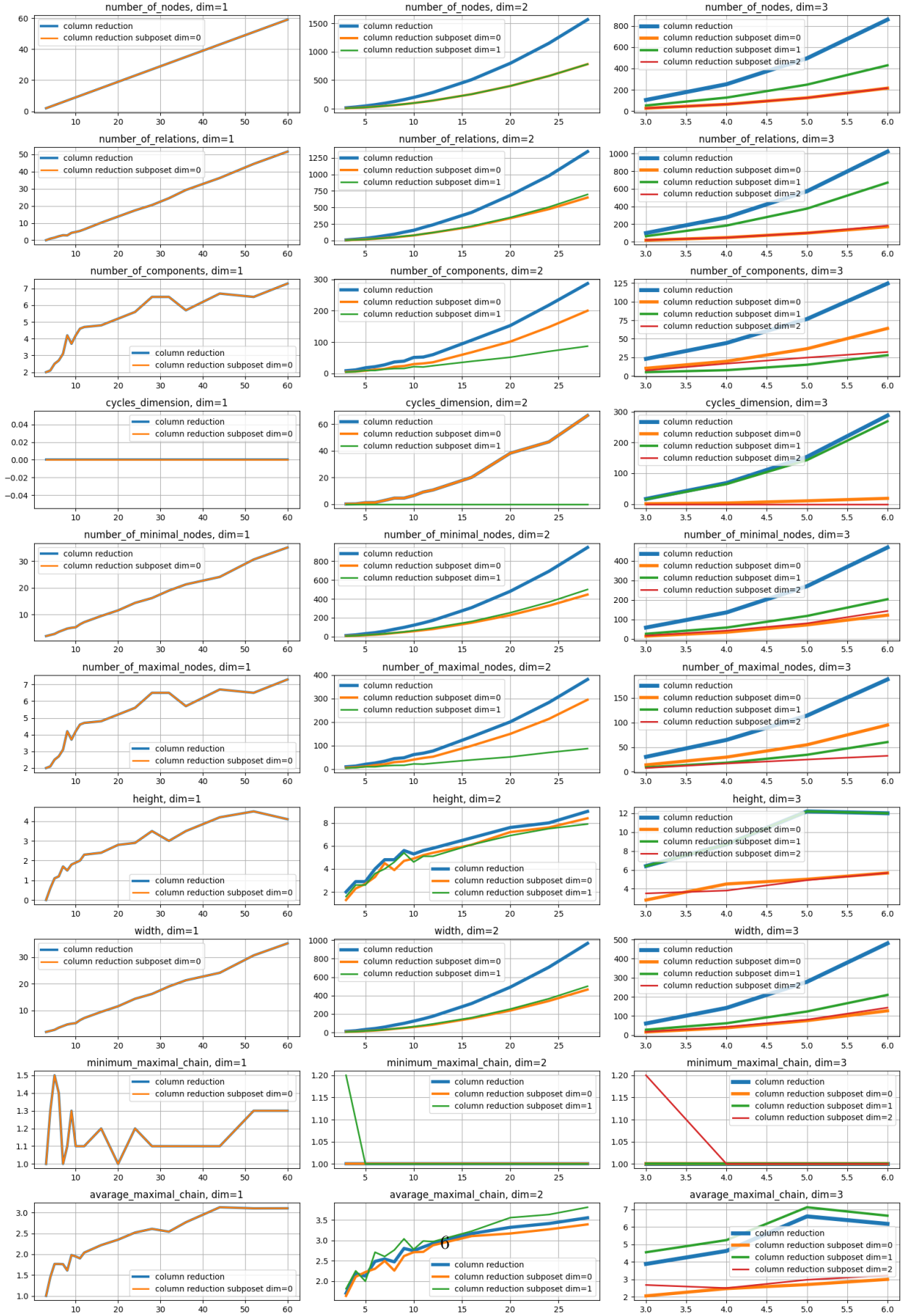


Figure 5: Column Reduction Poset: Mean poset scores

Column Reduction Poset: Mean Node Scores

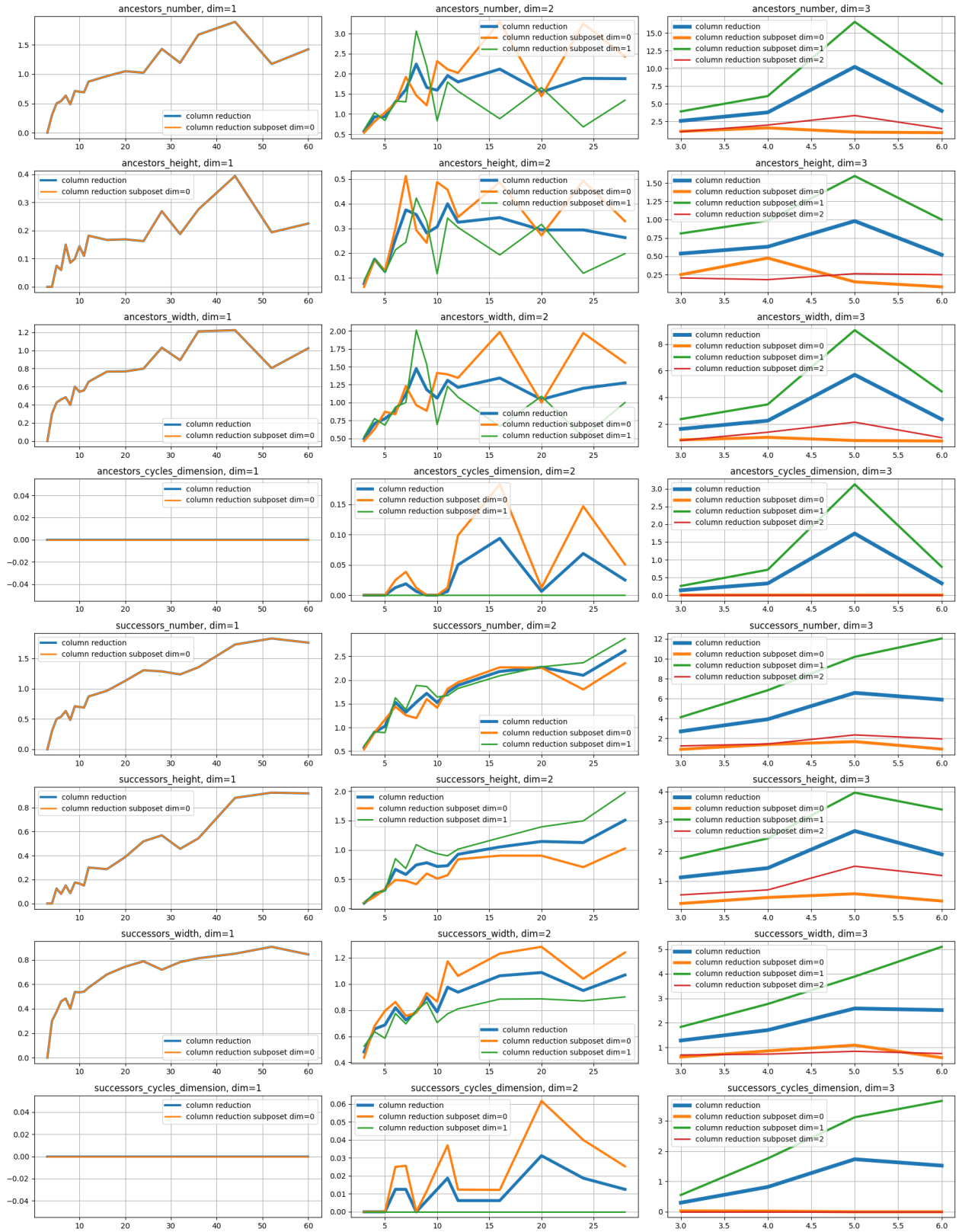


Figure 6: Column Reduction Poset: Mean node scores

Row Reduction Poset: Mean Poset Scores

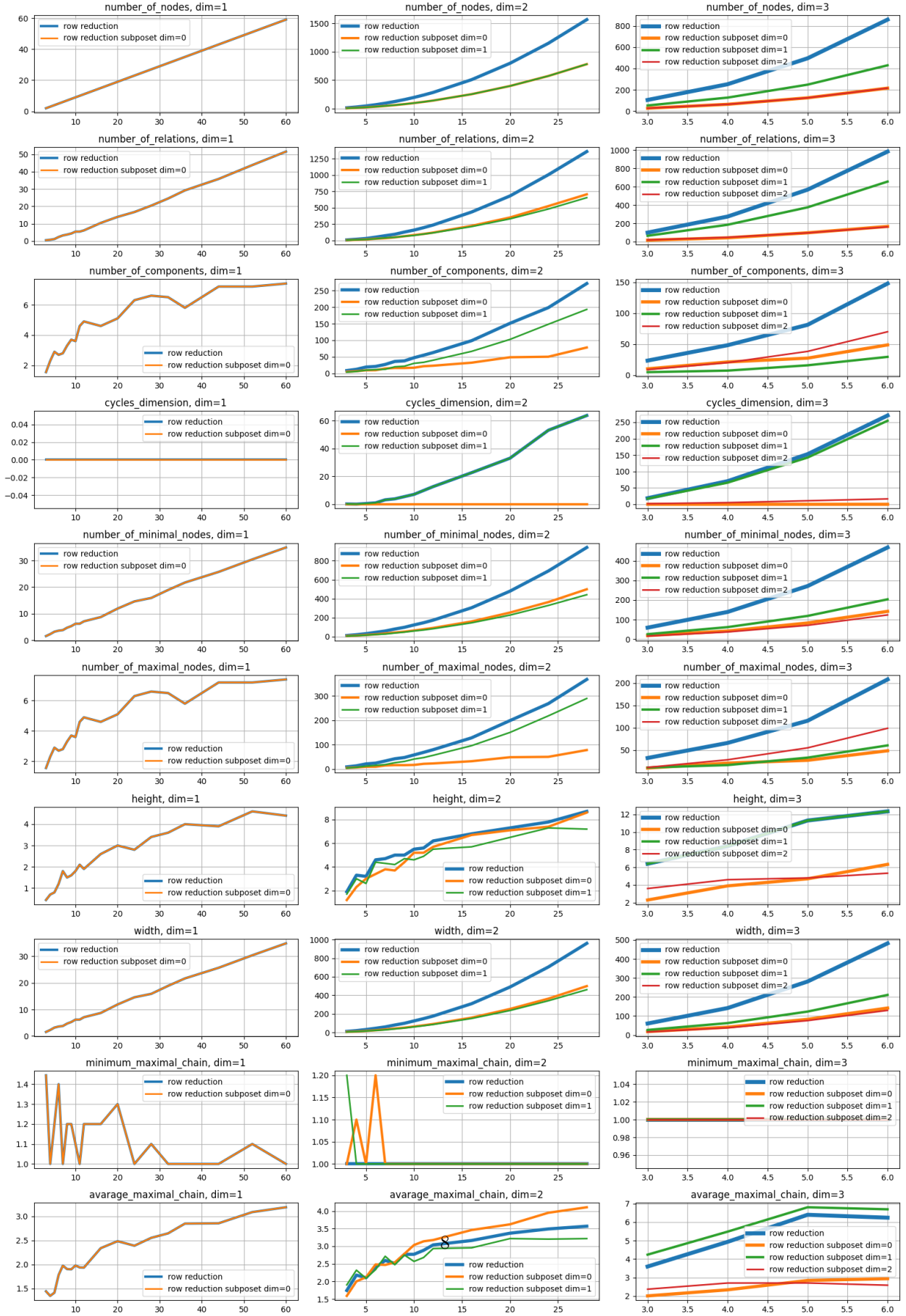


Figure 7: Row Reduction Poset: Mean poset scores

Row Reduction Poset: Mean Node Scores

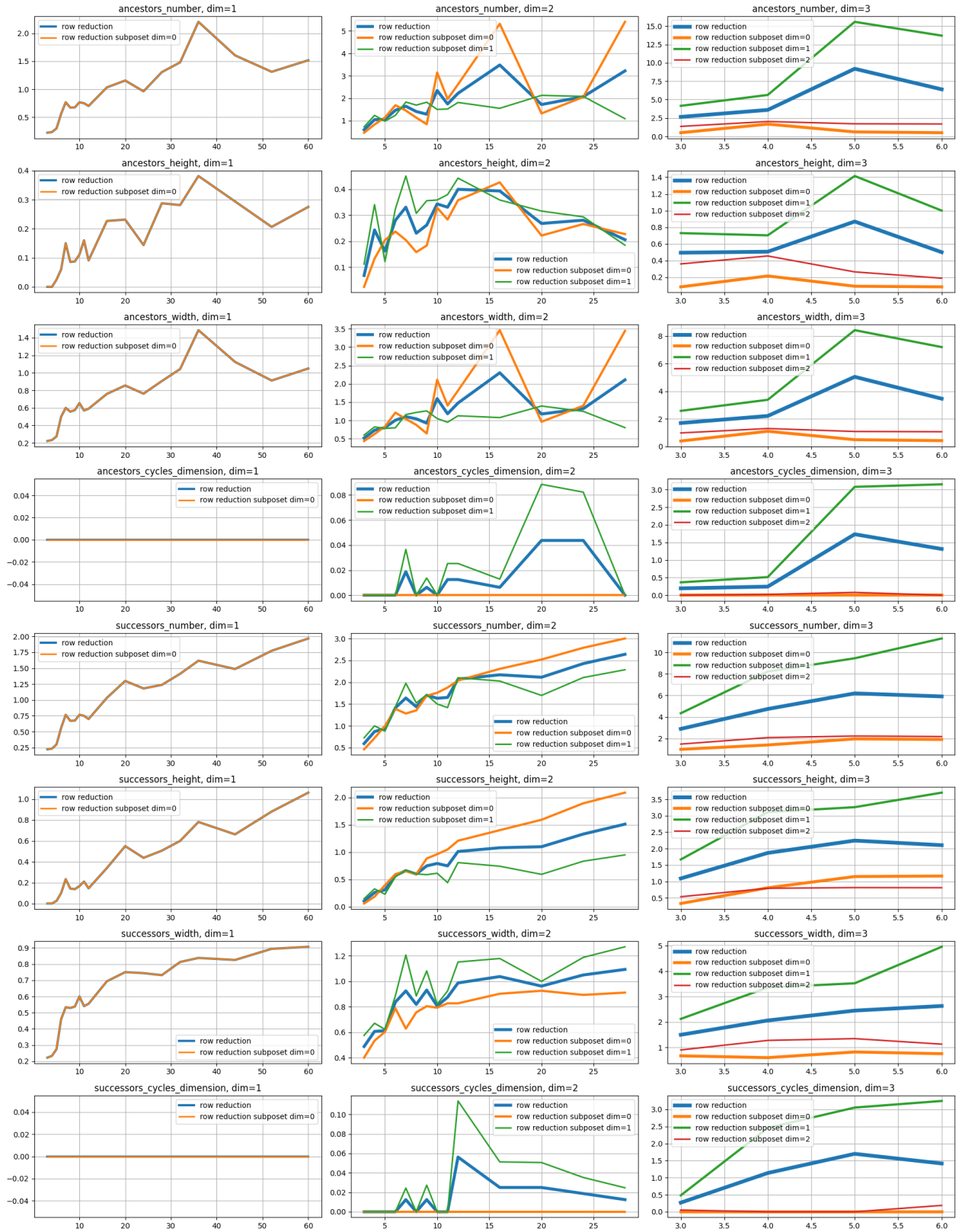


Figure 8: Row Reduction Poset: Mean node scores