1 Model

The d dimensional torus $\mathbb{T}^d$ can be defined as $(\mathbb{R}/n\mathbb{Z})^d$ for some natural n. And we can represent this as a cell-complex with cubical d-dimensional cells $(\mathbb{Z}/n\mathbb{Z})^d + [0,1]^d$ and all their k-faces for $k = 0, ..., d$.

We randomly assume the filtration value for each k-face uniformly distributed in $[k, k+1]$. This filtration on segmented torus will corespond some real filtration $f : mathbbT^d \to \mathbb{R}$, s.t. the d-dimensional cells will corespond the local maximums, vertices will corespond the local minimums and other k-faces will be saddles.

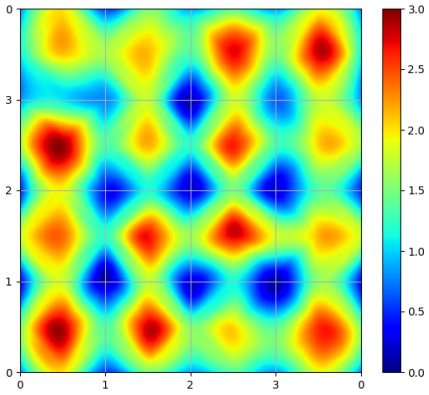

Figure 1: The example of the filtration $f : \mathbb{T}^2 \to \mathbb{R}$, s.t. there are local minimums in the vertices, saddles in the middle of edges, and the local maximums in the centers of square cells.

Let's call models like this as Barycentric Cubical Torus.

2 Scores

2.1 Poset Scores

- **number_of_nodes** : Returns the number of nodes in the poset.
- **number_of_relations** : Returns the number of relations in the transitive reduction.
- **number_of_components** : Returns the number of connetcted components in the poset
- **cycles_dimension** : Returns the dimension of space of cycles in reduction.

- **number_of_minimal_nodes** : Returns the number of minimal nodes.
- **number_of_maximal_nodes** : Returns the number of maximal nodes.
- **height** : Returns the poset height - the length of the longest chain.
- **width** : Returns the poset width - the length of the longest antichain (subset, s.t. all elements are pairwise incomparable). The algorithm is based on Dilworth's theorem and it's proof via Kőnig's theorem: link
- **minimum_maximal_chain** : Returns the minimum size of maximal chains in the poset.
- **avarage_maximal_chain** : Returns the avarage size of maximal chains in the poset.

2.2 Node Scores

- **ancestors_number**: Returns the number of nodes higher than given
- **ancestors_height**: Returns the size of maximum chain of subposet of nodes higher or equal than given
- **ancestors_width**: Returns the size of maximum chain of subposet of nodes higher or equal than given
- **ancestors_cycles_dimension**: Returns the the dimension of space of cycles in reduction of subposet of nodes higher or equal than given
- **successors_number**: Returns the number of nodes higher than given
- **successors_height**: Returns the size of maximum chain of subposet of nodes lower or equal than given
- **successors_width**: Returns the size of maximum chain of subposet of nodes lower or equal than given
- **successors_cycles_dimension**: Returns the the dimension of space of cycles in reduction of subposet of nodes lower or equal than given

3 Experiments and Results

There are 440 experiments done. In the Figure 2 we can see how cases are distributed by size and dimension.

3.1 Depth Poset Features

In the Figure 3 we can see the avarage poset scores values for each number of points n in the depth poset.

In the Figure 4 we can see the avarage mean node scores values in poset for each number of points n in the depth poset.

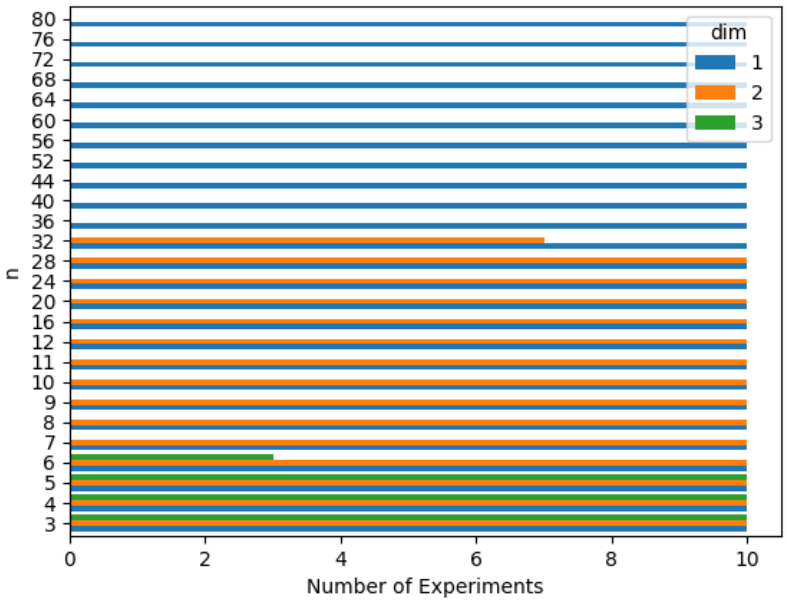

Figure 2: Size/dimension distribution of experiments

3.2 Column Reduction Poset Features

In the Figure 5 we can see the avarage poset scores values for each number of points n in the column reduction poset.

In the Figure 6 we can see the avarage mean node scores values in poset for each n in the column reduction poset.

3.3 Row Reduction Poset Features

In the Figure 7 we can see the avarage poset scores values for each number of points n in the row reduction poset.

In the Figure 8 we can see the avarage mean node scores values in poset for each number of points n in the row reduction poset.

Figure 3: Depth Poset: Mean poset scores

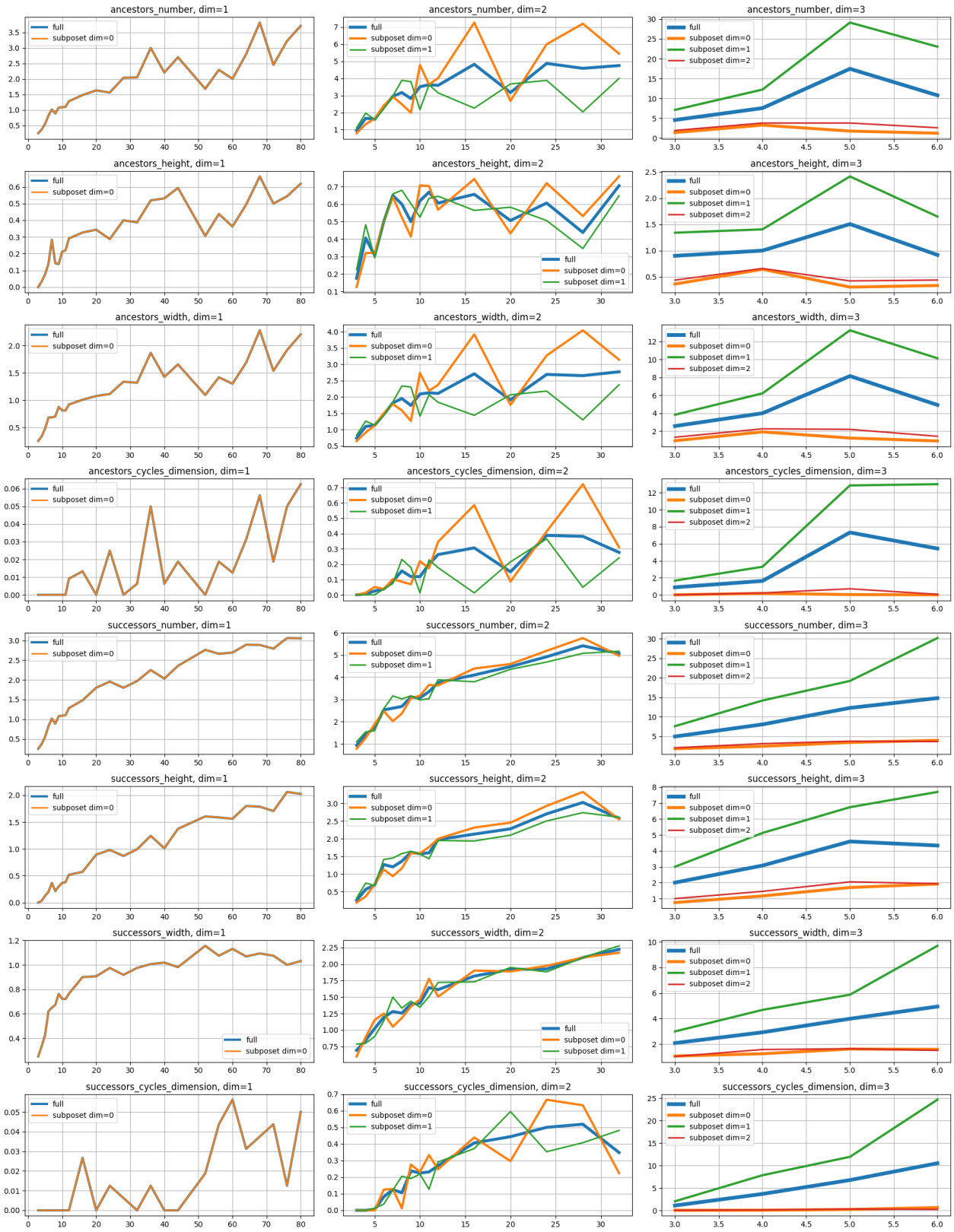

Figure 4: Depth Poset: Mean node scores

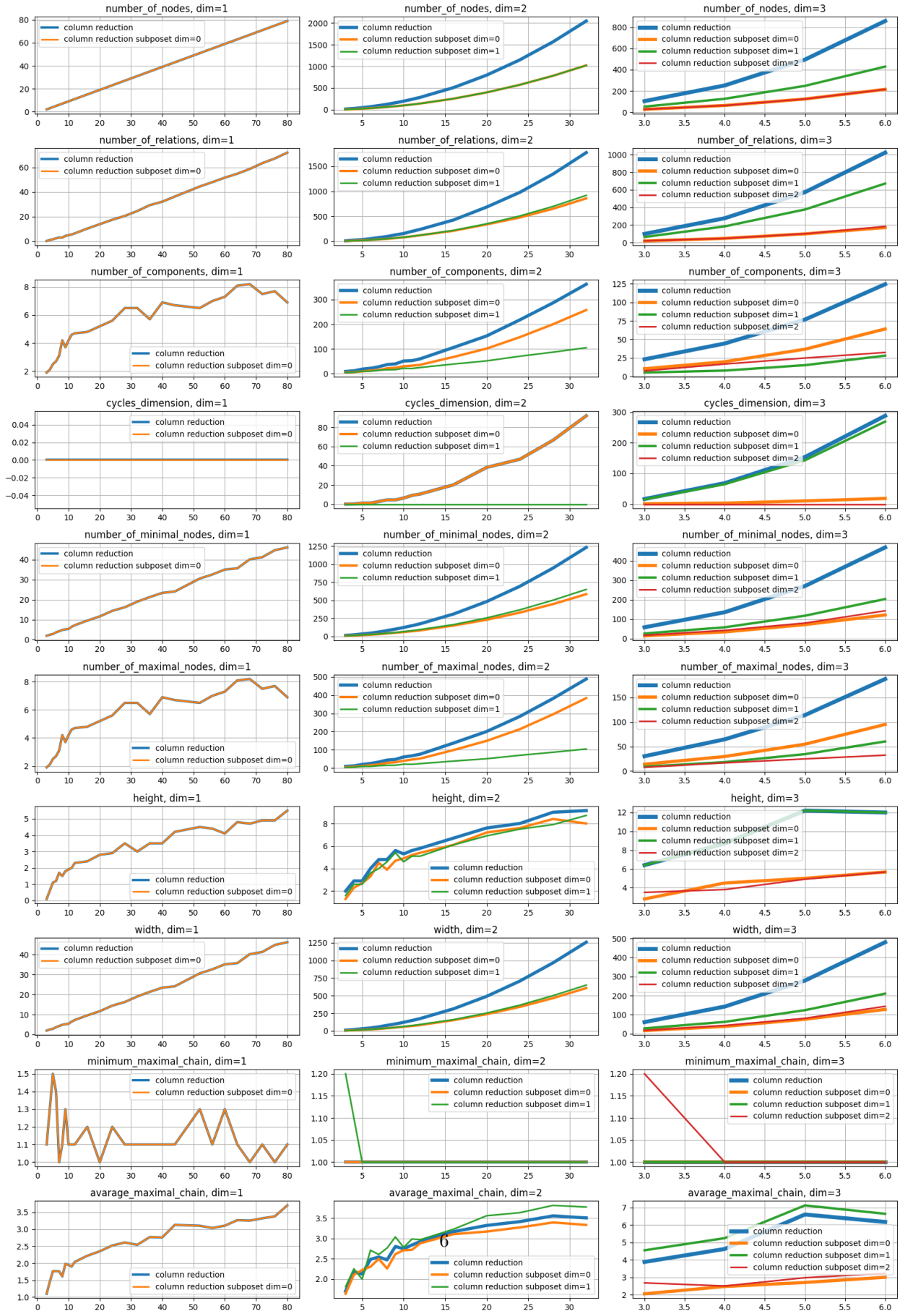

Figure 5: Column Reduction Poset: Mean poset scores

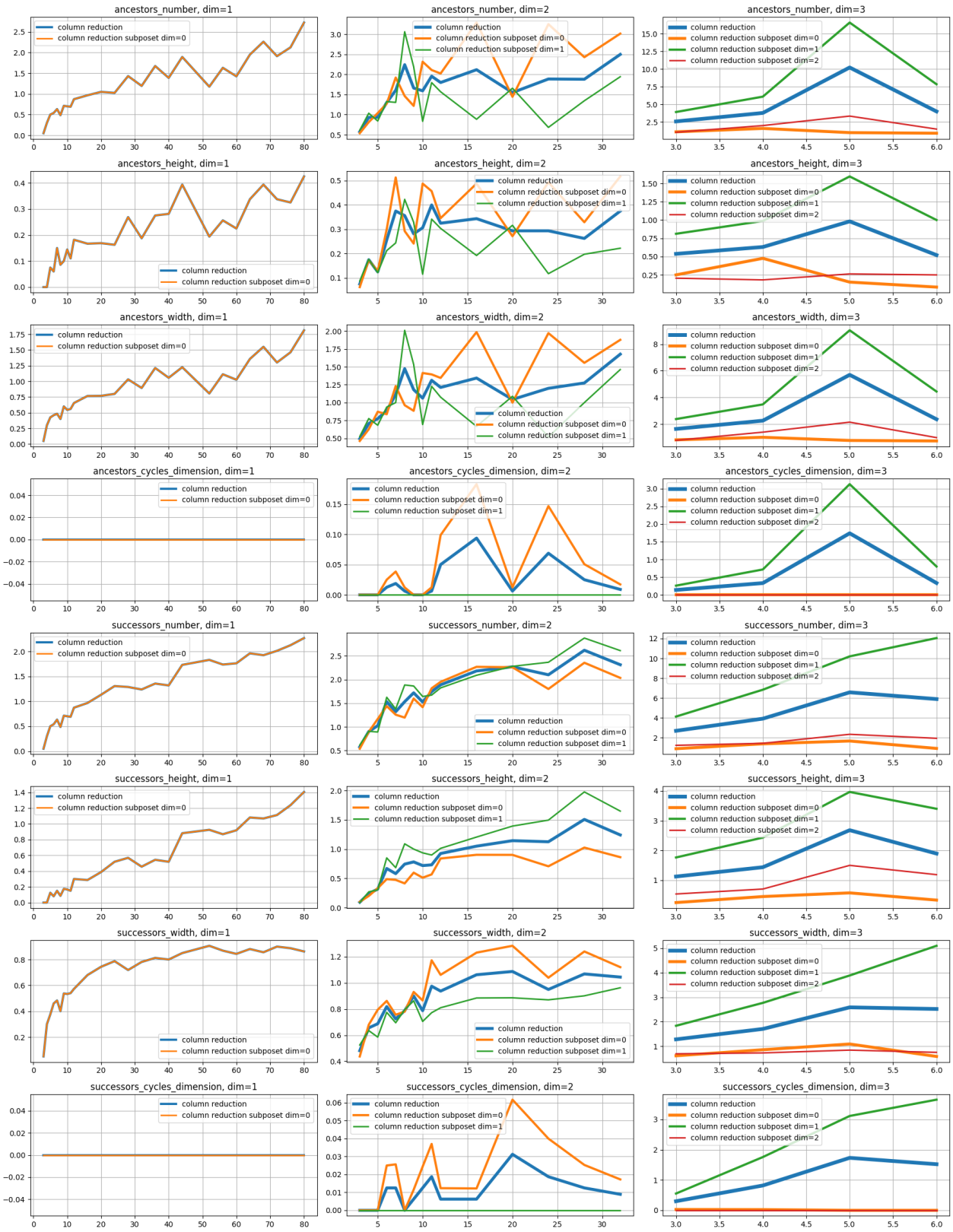

Figure 6: Column Reduction Poset: Mean node scores

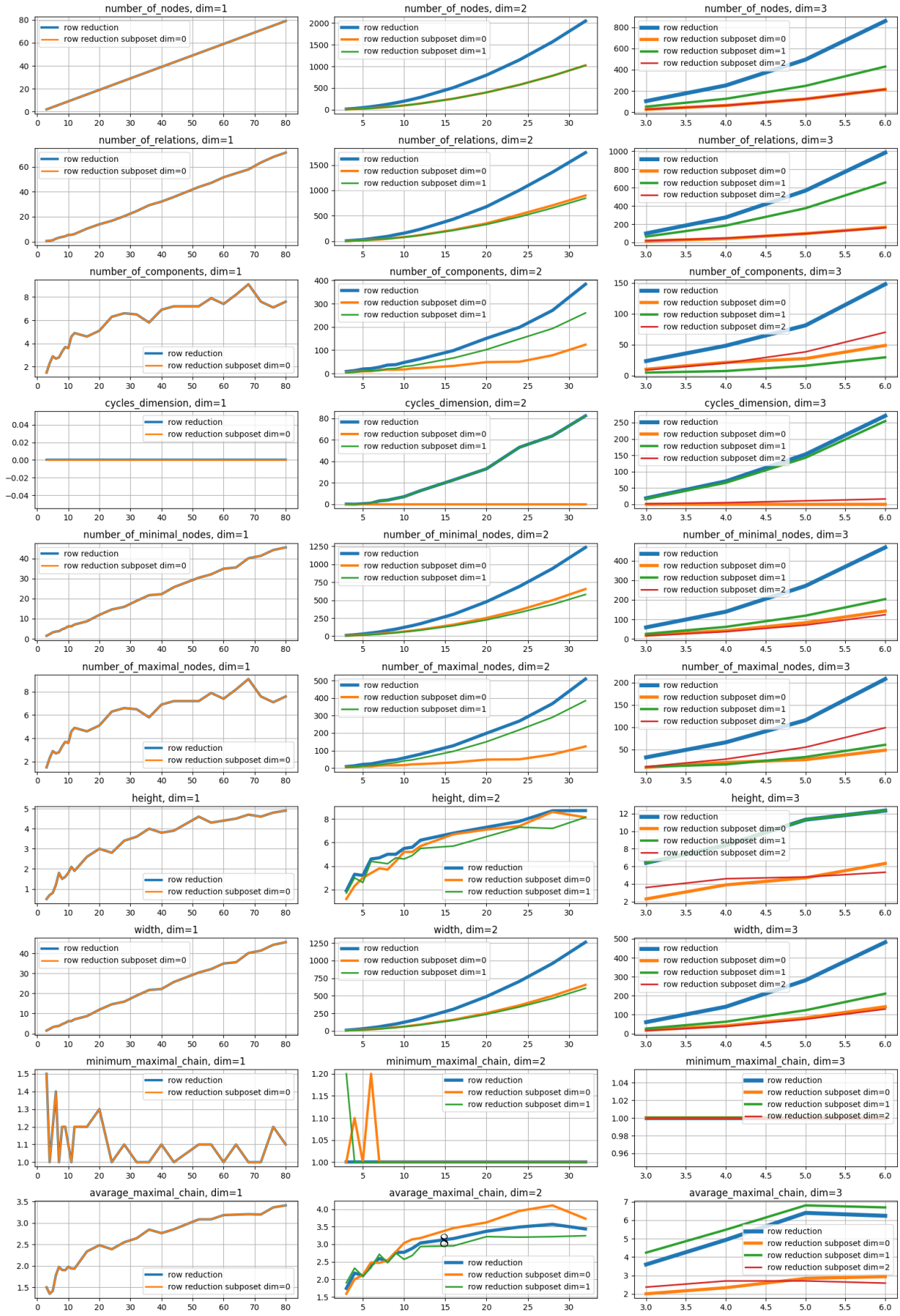

Figure 7: Row Reduction Poset: Mean poset scores

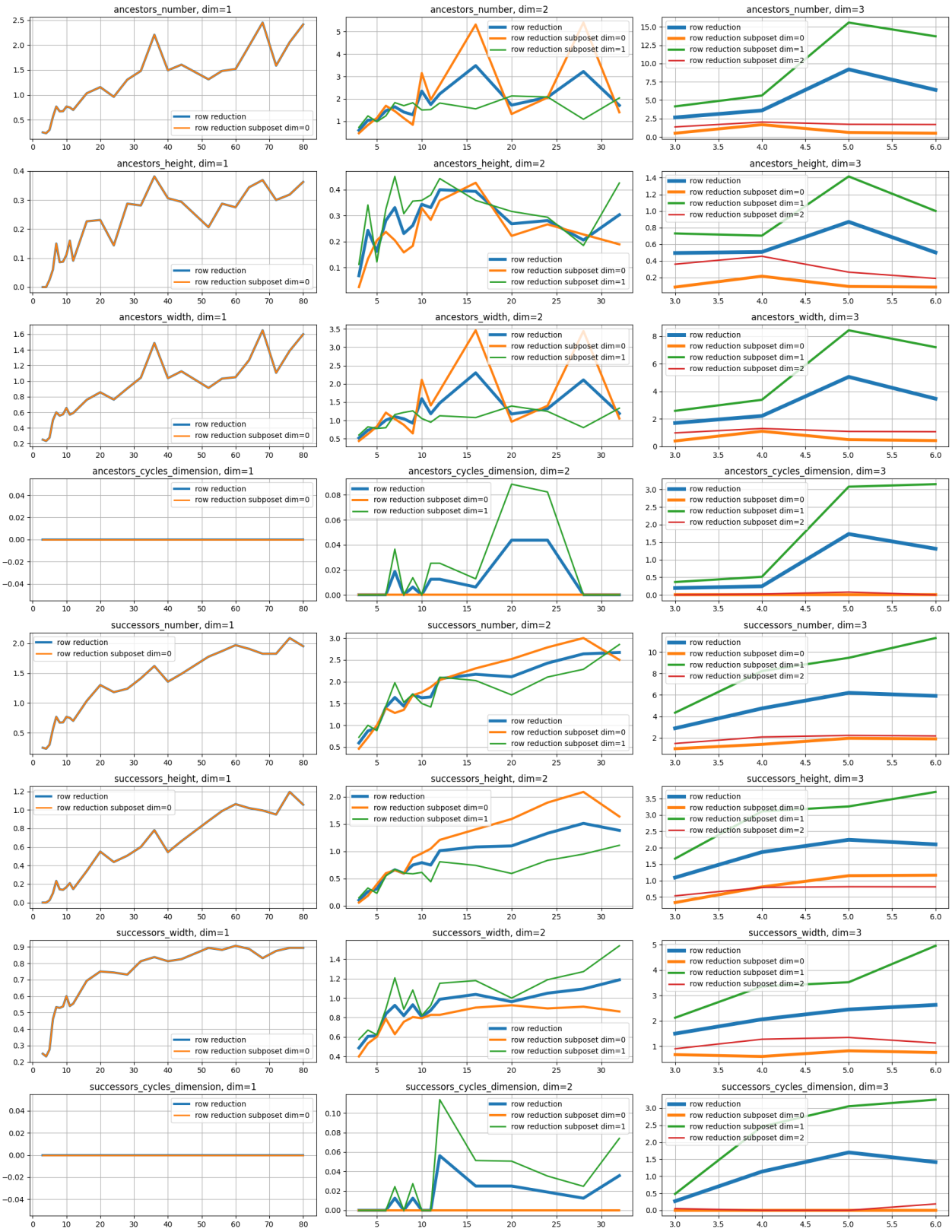

Figure 8: Row Reduction Poset: Mean node scores