1 Problems

Write $\text{Succ}(s,t) = \text{Succ}'(s,t) = \text{Succ}''(s,t)$ for the succesors and $\text{Pred}(s,t) = \text{Pred}'(s,t) = \text{Pred}''(s,t)$ for the predecessors (ancessors) of the pair in D, D', D'', respectively.

Since $D = D' \cup D''$, we also have $\text{Succ}(s,t) = \text{Succ}'(s,t) \cup \text{Succ}''(s,t)$ and $\text{Pred}(s,t) = \text{Pred}'(s,t) \cup \text{Pred}''(s,t)$.

1.1 Lemma 3.1.

Suppose (x, y) and (a, b) are birth-death pairs of $f : X \to R$, a, x are consecutive in the ordering of the cellls by f, and the transposition a, x is a switch. Then

$$\text{Succ}'(a,y) = \{(x,b)\} \cup \text{Succ}'(a,b) \cup \{(s,t) \in \text{Succ}'(x,y) | f(t) < f(b)\}$$

$$\text{Succ}'(x,b) = \{(s,t) \in \text{Succ}'(x,y) | f(t) > f(b)\}$$

1.2 Lemma 3.2.

Suppose (a, b) and (x, y) are birth-death pairs of $f : X \to R$, y, b are consecutive in the ordering by f, and transposition of y, b is a switch. Then

$$\text{Succ}''(x,b) = \{(a,y)\} \cup \text{Succ}''(a,b) \cup \{(s,t) \in \text{Succ}''(x,y) | f(a) < f(s) < f(x)\}$$

$$\text{Succ}''(a,y) = \{(s,t) \in \text{Succ}''(x,y) | f(s) < f(a)\}$$

1.3 Lemma 3.3.

Suppose (a, b) and (x, y) are birth-death pairs of f: $X \to R$, b, x are consecutive in the ordering by f, and the transposition of b, x is a switch. Then

$$\text{Succ}(a,x) = \text{Succ}(a,b) \ \text{ and } \ \text{Succ}(b,y) = \text{Succ}(x,y)$$

1.4 Hypothesis 1

:

Suppose a and b are 2-simplices consecutive in the ordering by f. And there is another Morse function f^*:

$$f^*(s) = \begin{cases} f(s), & \text{if } s \neq a, b \\ f(b), & \text{if } s = a \\ f(a), & \text{if } s = b \end{cases}$$

Let's denote DP_f the transitive reduction of the Depth Poset defined by the filtration f. And let's deonote $DP_f(s)$ the set of nodes in $DP_f(s)$ which are pairs containing cell s and the set of edges with these nodes.

Hypothesis: if the cell s has no faces and cofaces with a and b, then $DP_f(s) = DP_{f^*}(s)$.

2 Model

The probabilistic model is simple. The first we just generate the cloud of n points uniformly distributed in $[0,1]^d$. After this we calculate the Alpha complex with these points, and then find its depth poset. Then we itarate all neighbour pairs of simplices and check if their transposition will be possible filtration, calculating the scores for the switch-forward transpositions.

The number of points in the cloud n and the dimension $d = \dim$ for each generated cloud and coresponding complex are given in the table:

complex	n	dim	simplices
0	6	2	23
1	6	2	23
2	6	2	21
3	6	2	23
4	8	2	31
5	8	2	33
6	8	2	31
7	8	2	33
8	8	2	33
9	8	2	33
10	8	2	33
11	8	2	31
12	12	2	53
13	12	2	51
14	12	2	55
15	12	2	53
16	12	2	55
17	12	2	55
18	16	2	73
19	16	2	75
20	20	2	95
21	8	3	59
22	8	3	73
23	8	3	63
24	8	3	83
25	8	3	73
26	8	3	79
27	12	3	147
28	12	3	141
29	16	3	243
30	8	4	133
31	8	4	147
32	12	4	283

3 Scores

- **jacard_nodes_filtration**: The Jacard index of node sets from 2 depth posets. The birth-death pairs are equal in terms of filtration values.
- **jacard_nodes_simplex**: The Jacard index of nodes from 2 depth posets. The birth-death pairs are equal in terms of simplices.
- **jacard_edges_filtration**: The Jacard index of edge sets from transitive reductions of 2 depth posets. The birth-death pairs are equal in terms of filtration values.
- **jacard_edges_simplex**: The Jacard index of edge sets from transitive reductions of 2 depth posets. The birth-death pairs are equal in terms of simplices.
- **jacard_l31a**: Jacard Index of $\mathrm{Succ}'(x,b)$ and $\{(a,y)\} \cup \mathrm{Succ}'(a,b) \cup \{(s,t) \in \mathrm{Succ}'(x,y) | f(t) < f(b)\}$
- **jacard_l31b**: Jacard Index of $\mathrm{Succ}'(x,b)$ and $\{(s,t) \in \mathrm{Succ}'(x,y) | f(t) > f(b)\}$
- **jacard_l32a**: Jacard Index of $\mathrm{Succ}''(x,b)$ and $\{(a,y)\} \cup \mathrm{Succ}''(a,b) \cup \{(s,t) \in \mathrm{Succ}''(x,y) | f(a) < f(s) < f(x)\}$
- **jacard_l32b**: Jacard Index of $\mathrm{Succ}''(a,y)$ and $\{(s,t) \in \mathrm{Succ}''(x,y) | f(s) < f(a)\}$
- **jacard_l33a**: Jacard Index of $\mathrm{Succ}(a,x)$ and $\mathrm{Succ}(a,b)$
- **jacard_l33b**: Jacard Index of $\mathrm{Succ}(b,y)$ and $\mathrm{Succ}(x,y)$
- **jacard_nn_nodes**: The Jacard index of subsets of nodes (s,t) from 2 depth posets, s.t. $s,t \notin \nabla\partial\sigma_0 \cup \partial\nabla\sigma_0 \cup \nabla\partial\sigma_1 \cup \partial\nabla\sigma_1$, where σ_0 and σ_1 are transposing simplices.
- **jacard_nn_edges**: The Jacard index of subsets of edges $((s_0,t_0),(s_1,t_1))$ from 2 depth posets, s.t. $s_0,t_0,s_1,t_1 \notin \nabla\partial\sigma_0 \cup \partial\nabla\sigma_0 \cup \nabla\partial\sigma_1 \cup \partial\nabla\sigma_1$, where σ_0 and σ_1 are transposing simplices.

4 Results

Here are the tables, containing the scores for the switch transpositions in the experiment:

complex	dim	n	dim 0	dim 1	type	jacard_nodes_filtration	jacard_nodes_simplex
0	2	6	1	1	death-death	1.00	0.56
1	2	6	1	1	death-death	1.00	0.60
1	2	6	1	1	birth-death	1.00	0.60
1	2	6	1	1	birth-death	1.00	0.60
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	1	1	birth-death	1.00	0.50
3	2	6	1	1	birth-death	1.00	0.50
5	2	8	0	0	birth-birth	1.00	1.00
5	2	8	1	1	birth-death	1.00	0.64
6	2	8	1	1	birth-death	1.00	0.64
7	2	8	0	0	birth-birth	1.00	1.00
7	2	8	1	1	birth-death	1.00	0.69
8	2	8	1	1	birth-death	1.00	0.64
9	2	8	0	0	birth-birth	1.00	1.00
9	2	8	1	1	birth-death	1.00	0.69
9	2	8	1	1	birth-death	1.00	0.69
9	2	8	2	2	death-death	1.00	0.69
10	2	8	0	0	birth-birth	1.00	1.00
11	2	8	0	0	birth-birth	1.00	1.00
11	2	8	0	0	birth-birth	1.00	1.00
11	2	8	1	1	death-death	1.00	0.67
12	2	12	1	1	death-death	1.00	0.78
13	2	12	1	1	birth-death	1.00	0.78
13	2	12	1	1	birth-death	1.00	0.78
14	2	12	1	1	death-death	1.00	0.79
14	2	12	1	1	birth-death	1.00	0.79
14	2	12	1	1	birth-death	1.00	0.79
15	2	12	0	0	birth-birth	1.00	1.00
15	2	12	1	1	death-death	1.00	0.80
15	2	12	1	1	birth-death	1.00	0.80
15	2	12	1	1	birth-death	1.00	0.80
17	2	12	1	1	death-death	1.00	0.78
17	2	12	1	1	birth-death	1.00	0.78
18	2	16	0	0	birth-birth	1.00	1.00
18	2	16	1	1	birth-death	1.00	0.83
18	2	16	1	1	birth-death	1.00	0.83
18	2	16	1	1	death-death	1.00	0.83
19	2	16	1	1	death-death	1.00	0.86
19	2	16	2	2	death-death	1.00	0.86
20	2	20	0	0	birth-birth	1.00	1.00
20	2	20	1	1	birth-death	1.00	0.88
20	2	20	1	1	birth-death	1.00	0.88
20	2	20	1	1	death-death	1.00	0.88
21	3	8	0	0	birth-birth	1.00	1.00
21	3	8	1	1	birth-death	1.00	0.75
21	3	8	2	2	birth-death	1.00	0.75
22	3	8	0	0	birth-birth	1.00	1.00
22	3	8	1	1	birth-birth	1.00	0.81
22	3	8	2	2	birth-death	1.00	0.81
22	3	8	2	2	birth-death	1.00	0.81
23	3	8	1	1	birth-death	1.00	0.75
23	3	8	2	2	birth-death	1.00	0.75
24	3	8	0	0	birth-birth	1.00	1.00

complex	dim	n	dim 0	dim 1	type	jacard_edges_filtration	jacard_edges_simplex
0	2	6	1	1	death-death	1.00	0.20
1	2	6	1	1	death-death	1.00	0.14
1	2	6	1	1	birth-death	1.00	0.60
1	2	6	1	1	birth-death	1.00	0.60
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	1	1	birth-death	1.00	1.00
3	2	6	1	1	birth-death	0.67	0.67
5	2	8	0	0	birth-birth	1.00	1.00
5	2	8	1	1	birth-death	1.00	0.60
6	2	8	1	1	birth-death	0.57	0.38
7	2	8	0	0	birth-birth	1.00	1.00
7	2	8	1	1	birth-death	0.71	0.50
8	2	8	1	1	birth-death	1.00	1.00
9	2	8	0	0	birth-birth	1.00	1.00
9	2	8	1	1	birth-death	1.00	0.40
9	2	8	1	1	birth-death	1.00	0.40
9	2	8	2	2	death-death	1.00	0.75
10	2	8	0	0	birth-birth	1.00	1.00
11	2	8	0	0	birth-birth	1.00	1.00
11	2	8	0	0	birth-birth	1.00	1.00
11	2	8	1	1	death-death	1.00	0.25
12	2	12	1	1	death-death	0.75	0.40
13	2	12	1	1	birth-death	1.00	0.80
13	2	12	1	1	birth-death	0.80	0.50
14	2	12	1	1	death-death	1.00	0.67
14	2	12	1	1	birth-death	0.91	0.75
14	2	12	1	1	birth-death	1.00	0.54
15	2	12	0	0	birth-birth	1.00	1.00
15	2	12	1	1	death-death	1.00	0.67
15	2	12	1	1	birth-death	1.00	0.82
15	2	12	1	1	birth-death	1.00	0.67
17	2	12	1	1	death-death	1.00	0.50
17	2	12	1	1	birth-death	1.00	0.50
18	2	16	0	0	birth-birth	1.00	1.00
18	2	16	1	1	birth-death	0.94	0.72
18	2	16	1	1	birth-death	0.94	0.72
18	2	16	1	1	death-death	0.68	0.45
19	2	16	1	1	death-death	0.83	0.74
19	2	16	2	2	death-death	1.00	0.89
20	2	20	0	0	birth-birth	1.00	1.00
20	2	20	1	1	birth-death	1.00	0.89
20	2	20	1	1	birth-death	0.95	0.85
20	2	20	1	1	death-death	0.89	0.71
21	3	8	0	0	birth-birth	1.00	1.00
21	3	8	1	1	birth-death	1.00	0.43
21	3	8	2	2	birth-death	1.00	1.00
22	3	8	0	0	birth-birth	1.00	1.00
22	3	8	1	1	birth-birth	1.00	0.45
22	3	8	2	2	birth-death	1.00	1.00
22	3	8	2	2	birth-death	1.00	1.00
23	3	8	1	1	birth-death	1.00	0.56
23	3	8	2	2	birth-death	1.00	0.75
24	3	8	0	0	birth-birth	1.00	1.00

complex	dim	n	dim 0	dim 1	type	jacard_l31a	jacard_l31b
0	2	6	1	1	death-death	0.00	0.00
1	2	6	1	1	death-death	0.00	0.50
1	2	6	1	1	birth-death	NaN	NaN
1	2	6	1	1	birth-death	NaN	NaN
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	1	1	birth-death	NaN	NaN
3	2	6	1	1	birth-death	NaN	NaN
5	2	8	0	0	birth-birth	NaN	NaN
5	2	8	1	1	birth-death	NaN	NaN
6	2	8	1	1	birth-death	NaN	NaN
7	2	8	0	0	birth-birth	NaN	NaN
7	2	8	1	1	birth-death	NaN	NaN
8	2	8	1	1	birth-death	NaN	NaN
9	2	8	0	0	birth-birth	NaN	NaN
9	2	8	1	1	birth-death	NaN	NaN
9	2	8	1	1	birth-death	NaN	NaN
9	2	8	2	2	death-death	0.00	0.00
10	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	1	1	death-death	0.00	0.50
12	2	12	1	1	death-death	0.00	0.00
13	2	12	1	1	birth-death	NaN	NaN
13	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	death-death	0.50	0.50
14	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	birth-death	NaN	NaN
15	2	12	0	0	birth-birth	NaN	NaN
15	2	12	1	1	death-death	0.50	0.75
15	2	12	1	1	birth-death	NaN	NaN
15	2	12	1	1	birth-death	NaN	NaN
17	2	12	1	1	death-death	0.00	0.00
17	2	12	1	1	birth-death	NaN	NaN
18	2	16	0	0	birth-birth	NaN	NaN
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	1	1	death-death	0.00	0.00
19	2	16	1	1	death-death	0.00	0.67
19	2	16	2	2	death-death	0.00	0.00
20	2	20	0	0	birth-birth	NaN	NaN
20	2	20	1	1	birth-death	NaN	NaN
20	2	20	1	1	birth-death	NaN	NaN
20	2	20	1	1	death-death	0.00	0.00
21	3	8	0	0	birth-birth	NaN	NaN
21	3	8	1	1	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
22	3	8	0	0	birth-birth	NaN	NaN
22	3	8	1	1	birth-birth	1.00	1.00
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
23	3	8	1	1	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN
24	3	8	0	0	birth-birth	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard_l32a	jacard_l32b
0	2	6	1	1	death-death	1.00	1.00
1	2	6	1	1	death-death	1.00	1.00
1	2	6	1	1	birth-death	NaN	NaN
1	2	6	1	1	birth-death	NaN	NaN
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	1	1	birth-death	NaN	NaN
3	2	6	1	1	birth-death	NaN	NaN
5	2	8	0	0	birth-birth	NaN	NaN
5	2	8	1	1	birth-death	NaN	NaN
6	2	8	1	1	birth-death	NaN	NaN
7	2	8	0	0	birth-birth	NaN	NaN
7	2	8	1	1	birth-death	NaN	NaN
8	2	8	1	1	birth-death	NaN	NaN
9	2	8	0	0	birth-birth	NaN	NaN
9	2	8	1	1	birth-death	NaN	NaN
9	2	8	1	1	birth-death	NaN	NaN
9	2	8	2	2	death-death	1.00	1.00
10	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	1	1	death-death	1.00	1.00
12	2	12	1	1	death-death	1.00	1.00
13	2	12	1	1	birth-death	NaN	NaN
13	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	death-death	1.00	1.00
14	2	12	1	1	birth-death	NaN	NaN
14	2	12	1	1	birth-death	NaN	NaN
15	2	12	0	0	birth-birth	NaN	NaN
15	2	12	1	1	death-death	1.00	1.00
15	2	12	1	1	birth-death	NaN	NaN
15	2	12	1	1	birth-death	NaN	NaN
17	2	12	1	1	death-death	1.00	1.00
17	2	12	1	1	birth-death	NaN	NaN
18	2	16	0	0	birth-birth	NaN	NaN
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	1	1	birth-death	NaN	NaN
18	2	16	1	1	death-death	1.00	1.00
19	2	16	1	1	death-death	1.00	1.00
19	2	16	2	2	death-death	1.00	1.00
20	2	20	0	0	birth-birth	NaN	NaN
20	2	20	1	1	birth-death	NaN	NaN
20	2	20	1	1	birth-death	NaN	NaN
20	2	20	1	1	death-death	1.00	1.00
21	3	8	0	0	birth-birth	NaN	NaN
21	3	8	1	1	birth-death	NaN	NaN
21	3	8	2	2	birth-death	NaN	NaN
22	3	8	0	0	birth-birth	NaN	NaN
22	3	8	1	1	birth-birth	0.00	0.00
22	3	8	2	2	birth-death	NaN	NaN
22	3	8	2	2	birth-death	NaN	NaN
23	3	8	1	1	birth-death	NaN	NaN
23	3	8	2	2	birth-death	NaN	NaN
24	3	8	0	0	birth-birth	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard_l33a	jacard_l33b
0	2	6	1	1	death-death	NaN	NaN
1	2	6	1	1	death-death	NaN	NaN
1	2	6	1	1	birth-death	1.00	1.00
1	2	6	1	1	birth-death	1.00	1.00
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	0	0	birth-birth	NaN	NaN
2	2	6	1	1	birth-death	1.00	1.00
3	2	6	1	1	birth-death	1.00	1.00
5	2	8	0	0	birth-birth	NaN	NaN
5	2	8	1	1	birth-death	1.00	1.00
6	2	8	1	1	birth-death	1.00	1.00
7	2	8	0	0	birth-birth	NaN	NaN
7	2	8	1	1	birth-death	1.00	1.00
8	2	8	1	1	birth-death	1.00	1.00
9	2	8	0	0	birth-birth	NaN	NaN
9	2	8	1	1	birth-death	1.00	1.00
9	2	8	1	1	birth-death	1.00	1.00
9	2	8	2	2	death-death	NaN	NaN
10	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	0	0	birth-birth	NaN	NaN
11	2	8	1	1	death-death	NaN	NaN
12	2	12	1	1	death-death	NaN	NaN
13	2	12	1	1	birth-death	1.00	1.00
13	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	death-death	NaN	NaN
14	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	birth-death	1.00	1.00
15	2	12	0	0	birth-birth	NaN	NaN
15	2	12	1	1	death-death	NaN	NaN
15	2	12	1	1	birth-death	1.00	1.00
15	2	12	1	1	birth-death	1.00	1.00
17	2	12	1	1	death-death	NaN	NaN
17	2	12	1	1	birth-death	1.00	1.00
18	2	16	0	0	birth-birth	NaN	NaN
18	2	16	1	1	birth-death	1.00	1.00
18	2	16	1	1	birth-death	1.00	1.00
18	2	16	1	1	death-death	NaN	NaN
19	2	16	1	1	death-death	NaN	NaN
19	2	16	2	2	death-death	NaN	NaN
20	2	20	0	0	birth-birth	NaN	NaN
20	2	20	1	1	birth-death	1.00	1.00
20	2	20	1	1	birth-death	1.00	1.00
20	2	20	1	1	death-death	NaN	NaN
21	3	8	0	0	birth-birth	NaN	NaN
21	3	8	1	1	birth-death	1.00	1.00
21	3	8	2	2	birth-death	1.00	1.00
22	3	8	0	0	birth-birth	NaN	NaN
22	3	8	1	1	birth-birth	NaN	NaN
22	3	8	2	2	birth-death	1.00	1.00
22	3	8	2	2	birth-death	1.00	1.00
23	3	8	1	1	birth-death	1.00	1.00
23	3	8	2	2	birth-death	1.00	1.00
24	3	8	0	0	birth-birth	NaN	NaN

complex	dim	n	dim 0	dim 1	type	jacard_nn_nodes	jacard_nn_edges
0	2	6	1	1	death-death	1.00	1.00
1	2	6	1	1	death-death	1.00	1.00
1	2	6	1	1	birth-death	1.00	1.00
1	2	6	1	1	birth-death	1.00	1.00
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	0	0	birth-birth	1.00	1.00
2	2	6	1	1	birth-death	1.00	1.00
3	2	6	1	1	birth-death	1.00	1.00
5	2	8	0	0	birth-birth	1.00	1.00
5	2	8	1	1	birth-death	1.00	1.00
6	2	8	1	1	birth-death	1.00	1.00
7	2	8	0	0	birth-birth	1.00	1.00
7	2	8	1	1	birth-death	1.00	1.00
8	2	8	1	1	birth-death	1.00	1.00
9	2	8	0	0	birth-birth	1.00	1.00
9	2	8	1	1	birth-death	1.00	1.00
9	2	8	1	1	birth-death	1.00	1.00
9	2	8	2	2	death-death	1.00	1.00
10	2	8	0	0	birth-birth	1.00	1.00
11	2	8	0	0	birth-birth	1.00	1.00
11	2	8	0	0	birth-birth	1.00	1.00
11	2	8	1	1	death-death	1.00	1.00
12	2	12	1	1	death-death	1.00	1.00
13	2	12	1	1	birth-death	1.00	1.00
13	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	death-death	1.00	1.00
14	2	12	1	1	birth-death	1.00	1.00
14	2	12	1	1	birth-death	1.00	1.00
15	2	12	0	0	birth-birth	1.00	1.00
15	2	12	1	1	death-death	1.00	1.00
15	2	12	1	1	birth-death	1.00	1.00
15	2	12	1	1	birth-death	1.00	1.00
17	2	12	1	1	death-death	1.00	1.00
17	2	12	1	1	birth-death	1.00	1.00
18	2	16	0	0	birth-birth	1.00	1.00
18	2	16	1	1	birth-death	1.00	1.00
18	2	16	1	1	birth-death	1.00	1.00
18	2	16	1	1	death-death	1.00	1.00
19	2	16	1	1	death-death	1.00	1.00
19	2	16	2	2	death-death	1.00	1.00
20	2	20	0	0	birth-birth	1.00	1.00
20	2	20	1	1	birth-death	1.00	1.00
20	2	20	1	1	birth-death	1.00	1.00
20	2	20	1	1	death-death	1.00	1.00
21	3	8	0	0	birth-birth	1.00	1.00
21	3	8	1	1	birth-death	1.00	1.00
21	3	8	2	2	birth-death	1.00	1.00
22	3	8	0	0	birth-birth	1.00	1.00
22	3	8	1	1	birth-birth	1.00	1.00
22	3	8	2	2	birth-death	1.00	1.00
22	3	8	2	2	birth-death	1.00	1.00
23	3	8	1	1	birth-death	1.00	1.00
23	3	8	2	2	birth-death	1.00	1.00
24	3	8	0	0	birth-birth	1.00	1.00

4.1 Unexpected results

The Hypothesis problem can be measured by 2 scores with expected values: **jacard_nn_edges**, **jacard_nn_nodes** There are 25 cases, which does not corespond the expectations:

complex	dim 0	dim 1	type	jacard_nn_edges	jacard_nn_nodes	Figure
2	0	1	birth-death	1.00	0.67	Figure 3
3	0	1	birth-death	1.00	0.50	Figure 4
4	0	1	birth-death	0.33	0.50	
5	0	1	birth-death	0.00	0.43	
5	1	1	death-death	0.67	1.00	
6	0	1	birth-death	0.50	0.80	
7	0	1	birth-death	1.00	0.71	
9	0	1	birth-death	0.00	0.33	
10	0	1	birth-death	0.00	0.29	
13	0	1	birth-death	0.00	0.31	
15	0	1	birth-death	0.40	0.90	
16	0	1	birth-death	0.00	0.27	
16	2	1	birth-death	1.00	0.85	
17	0	1	birth-death	1.00	0.82	
17	2	1	birth-death	1.00	0.83	
18	0	1	birth-death	0.25	0.62	
19	0	1	birth-death	0.73	0.83	
20	0	1	birth-death	0.25	0.52	
21	0	1	birth-death	1.00	0.90	
24	0	1	birth-death	1.00	0.60	
25	0	1	birth-death	1.00	0.67	
27	0	1	birth-death	0.00	0.50	
28	0	1	birth-death	0.67	0.90	
29	0	1	birth-death	0.33	0.43	
30	0	1	birth-death	0.00	0.57	

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The Lemma 3.1 problem can be measured by 2 scores with expected values: **jacard_l31a**, **jacard_l31b** There are 20 cases, which does not corespond the expectations:

complex	dim 0	dim 1	type	jacard_l31a	jacard_l31b	Figure
2	0	0	birth-birth	NaN	NaN	Figure 1
2	0	0	birth-birth	NaN	NaN	Figure 2
5	0	0	birth-birth	NaN	NaN	
7	0	0	birth-birth	NaN	NaN	
9	0	0	birth-birth	NaN	NaN	
10	0	0	birth-birth	NaN	NaN	
11	0	0	birth-birth	NaN	NaN	
11	0	0	birth-birth	NaN	NaN	
15	0	0	birth-birth	NaN	NaN	
18	0	0	birth-birth	NaN	NaN	
20	0	0	birth-birth	NaN	NaN	
21	0	0	birth-birth	NaN	NaN	
22	0	0	birth-birth	NaN	NaN	
24	0	0	birth-birth	NaN	NaN	
25	0	0	birth-birth	NaN	NaN	
28	0	0	birth-birth	NaN	NaN	
29	0	0	birth-birth	NaN	NaN	
30	0	0	birth-birth	NaN	NaN	
31	0	0	birth-birth	NaN	NaN	
31	0	0	birth-birth	NaN	NaN	

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The Lemma 3.2 problem can be measured by 2 scores with expected values: **jacard_l32a**, **jacard_l32b** There are 0 cases, which does not corespond the expectations.

The Lemma 3.3 problem can be measured by 2 scores with expected values: **jacard_l33a**, **jacard_l33b** There are 0 cases, which does not corespond the expectations.

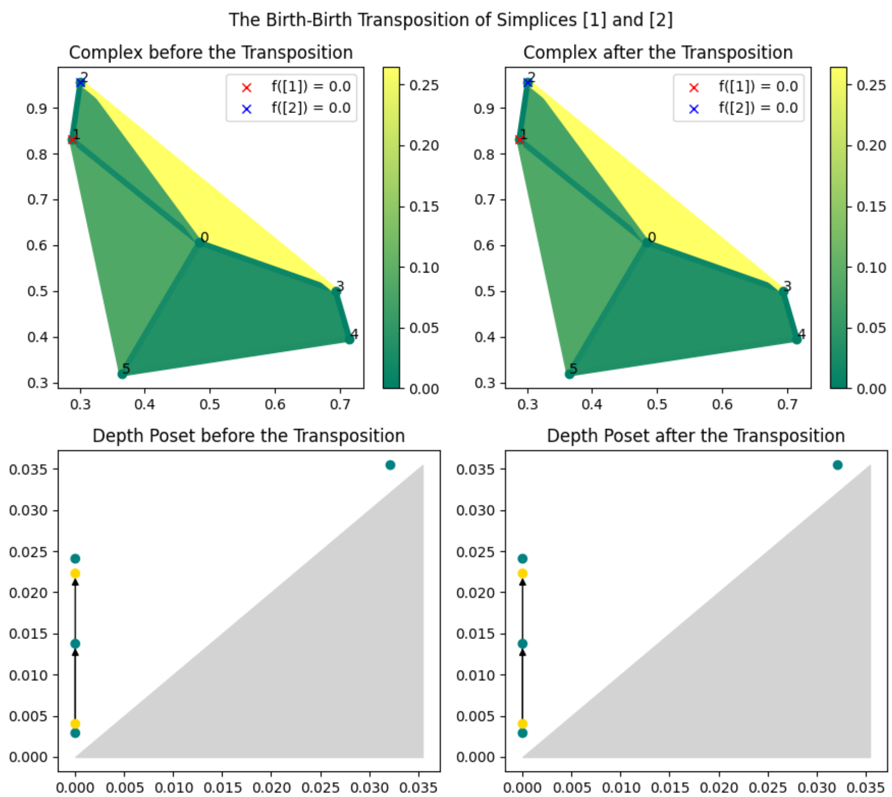

Figure 1: The Birth-Birth Transposition of simplices [1] and [2]

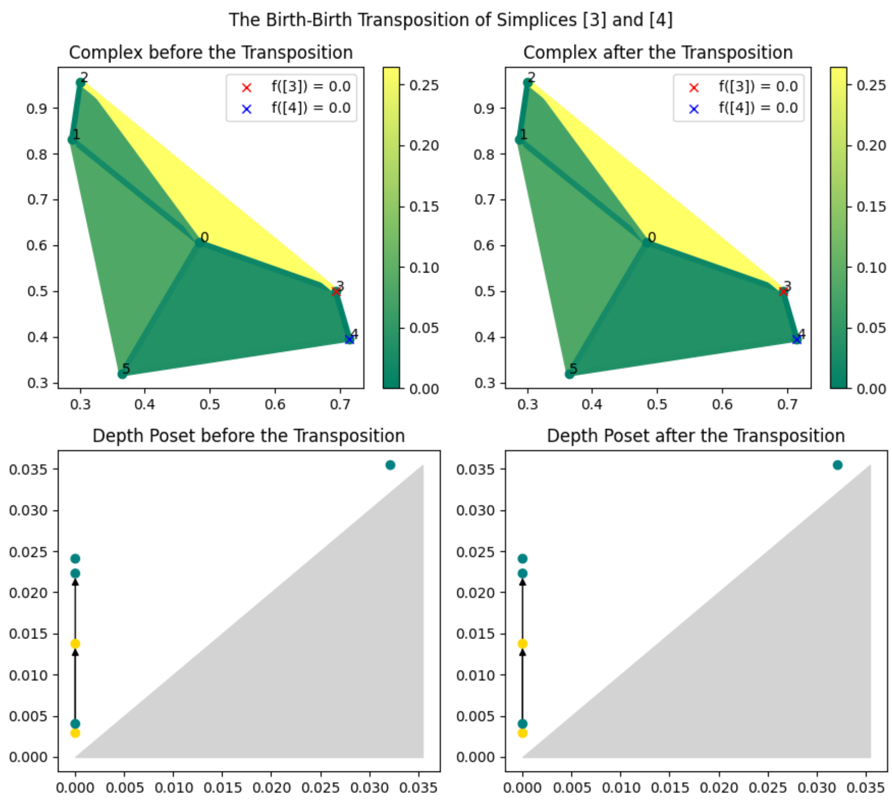

Figure 2: The Birth-Birth Transposition of simplices [3] and [4]

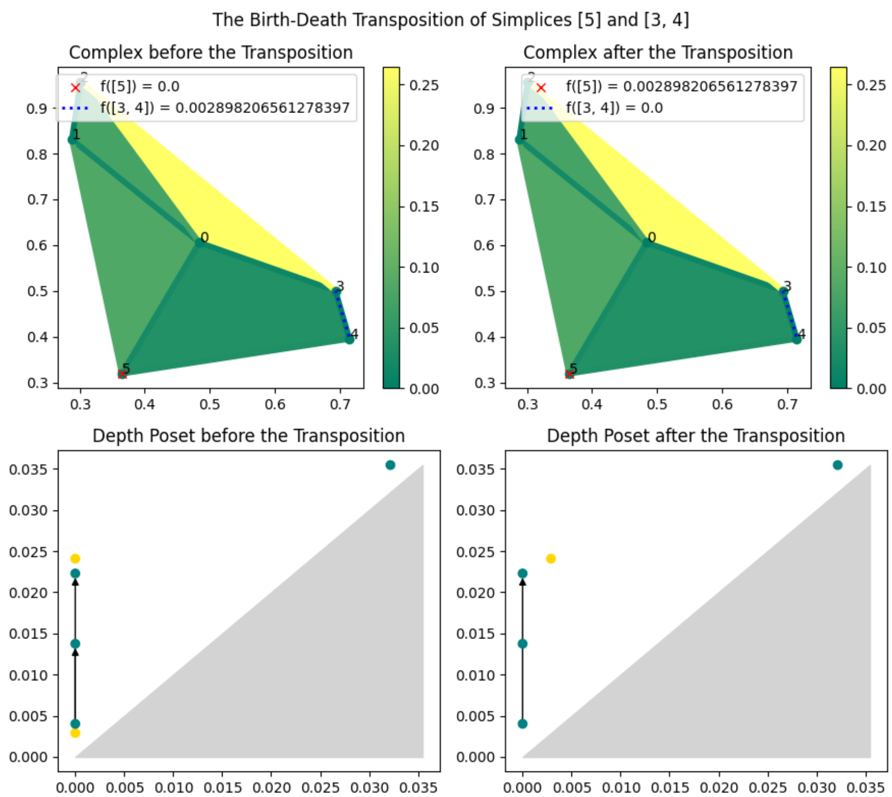

Figure 3: The Birth-Death Transposition of simplices [5] and [3, 4]

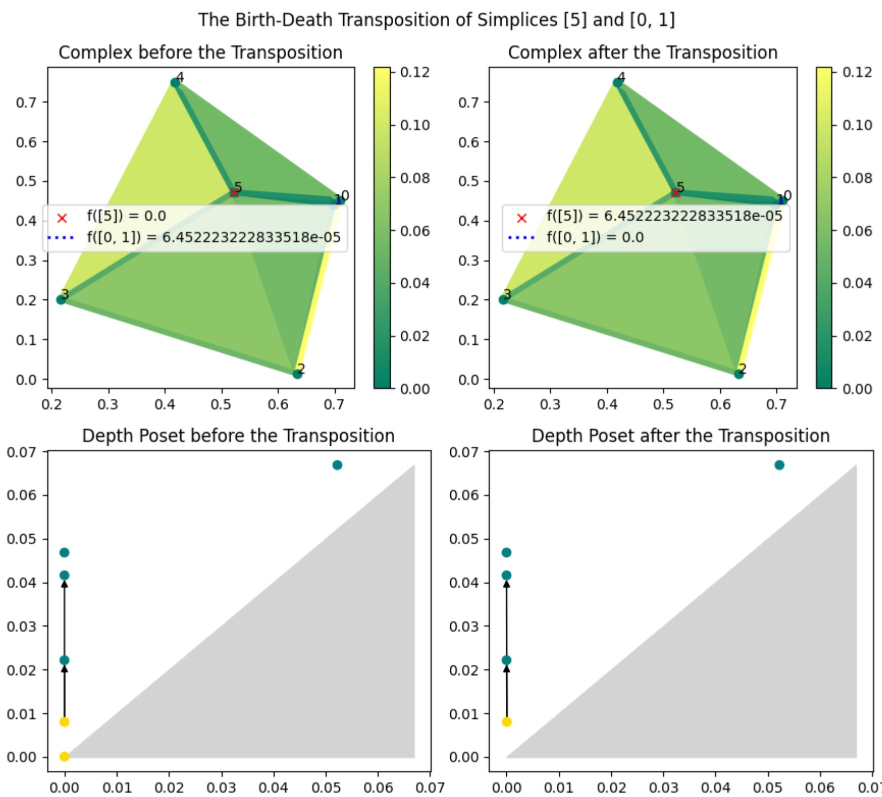

Figure 4: The Birth-Death Transposition of simplices [5] and [0, 1]