

# 1 Model

The  $d$  dimensional torus  $\mathbb{T}^d$  can be defined as  $(\mathbb{R}/n\mathbb{Z})^d$  for some natural  $n$ . And we can represent this as a cell-complex with cubical  $d$ -dimensional cells  $(\mathbb{Z}/n\mathbb{Z})^d + [0, 1]^d$  and all their  $k$ -faces for  $k = 0, \dots, d$ .

We randomly assume the filtration value for each  $k$ -face uniformly distributed in  $[k, k + 1]$ . This filtration on segmented torus will correspond some real filtration  $f : \mathbb{T}^d \rightarrow \mathbb{R}$ , s.t. the  $d$ -dimensional cells will correspond the local maximums, vertices will correspond the local minimums and other  $k$ -faces will be saddles.

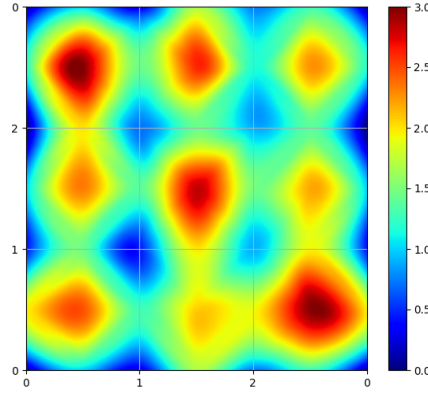


Figure 1: The example of the filtration  $f : \mathbb{T}^2 \rightarrow \mathbb{R}$ , s.t. there are local minimums in the vertices, saddles in the middle of edges, and the local maximums in the centers of square cells.

## 2 Scores

### 2.1 Poset Scores

- **number\_of\_nodes** : Returns the number of nodes in the poset.
- **number\_of\_relations** : Returns the number of relations in the transitive reduction.
- **number\_of\_components** : Returns the number of connected components in the poset
- **cycles\_dimension** : Returns the dimension of space of cycles in reduction.

- **number\_of\_minimal\_nodes** : Returns the number of minimal nodes.
- **number\_of\_maximal\_nodes** : Returns the number of maximal nodes.
- **height** : Returns the poset height - the length of the longest chain.
- **width** : Returns the poset width - the length of the longest antichain (subset, s.t. all elements are pairwise incomparable). The algorithm is based on Dilworth's theorem and it's proof via König's theorem: [link](#)
- **minimum\_maximal\_chain** : Returns the minimum size of maximal chains in the poset.
- **avarage\_maximal\_chain** : Returns the avarage size of maximal chains in the poset.

## 2.2 Node Scores

- **ancestors\_number**: Returns the number of nodes higher than given
- **ancestors\_height**: Returns the size of maximum chain of subposet of nodes higher or equal than given
- **ancestors\_width**: Returns the size of maximum chain of subposet of nodes higher or equal than given
- **ancestors\_cycles\_dimension**: Returns the the dimension of space of cycles in reduction of subposet of nodes higher or equal than given
- **successors\_number**: Returns the number of nodes higher than given
- **successors\_height**: Returns the size of maximum chain of subposet of nodes lower or equal than given
- **successors\_width**: Returns the size of maximum chain of subposet of nodes lower or equal than given
- **successors\_cycles\_dimension**: Returns the the dimension of space of cycles in reduction of subposet of nodes lower or equal than given

## 3 Experiments and Results

There are 361 experiments done. In the Figure 2 we can see how cases are distributed by size and dimension.

### 3.1 Depth Poset Features

In the Figure 3 we can see the avarage poset scores values for each number of points  $n$  in the depth poset.

In the Figure 4 we can see the avarage mean node scores values in poset for each number of points  $n$  in the depth poset.

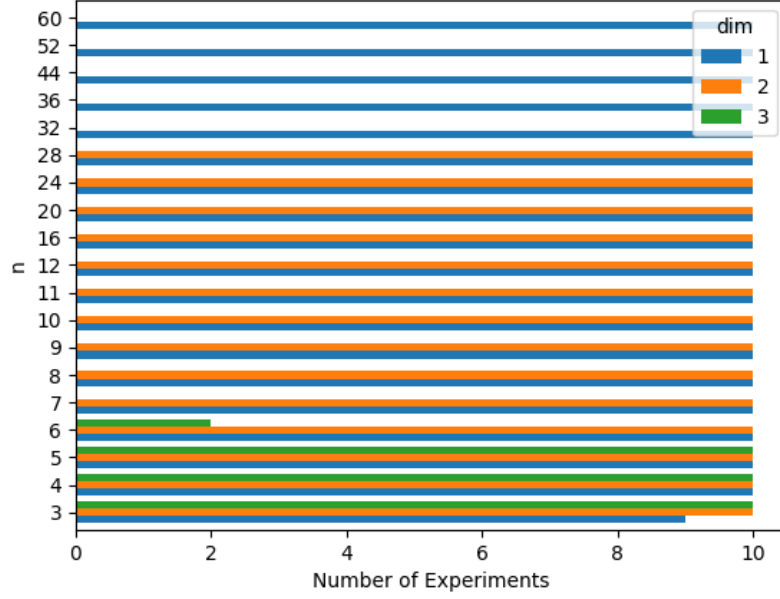


Figure 2: Size/dimension distribution of experiments

### 3.2 Column Reduction Poset Features

In the Figure 5 we can see the average poset scores values for each number of points  $n$  in the column reduction poset.

In the Figure 6 we can see the average mean node scores values in poset for each  $n$  in the column reduction poset.

### 3.3 Row Reduction Poset Features

In the Figure 7 we can see the average poset scores values for each number of points  $n$  in the row reduction poset.

In the Figure 8 we can see the average mean node scores values in poset for each number of points  $n$  in the row reduction poset.

## Depth Poset: Mean Poset Scores

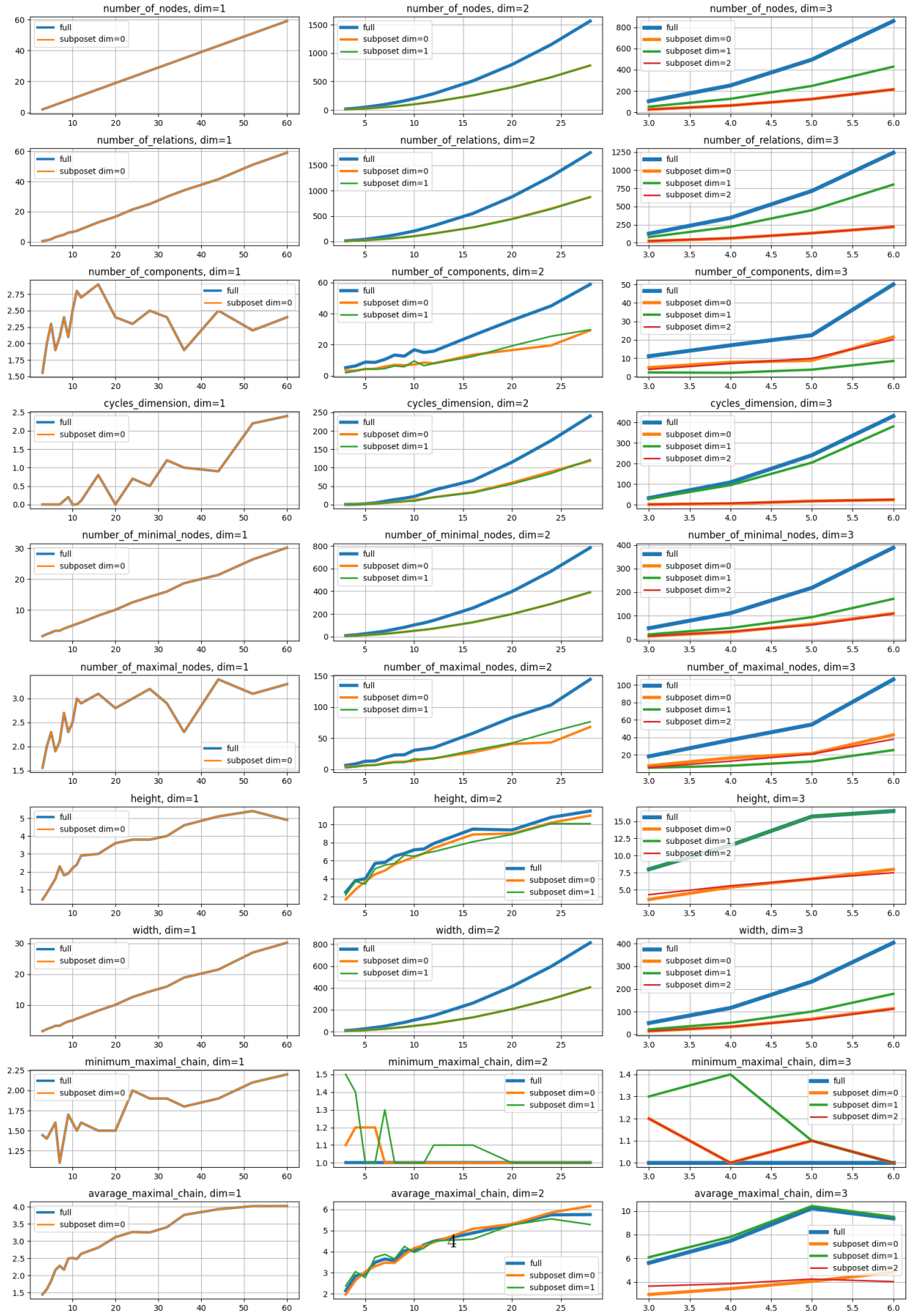


Figure 3: Depth Poset: Mean poset scores

Depth Poset: Mean Node Scores

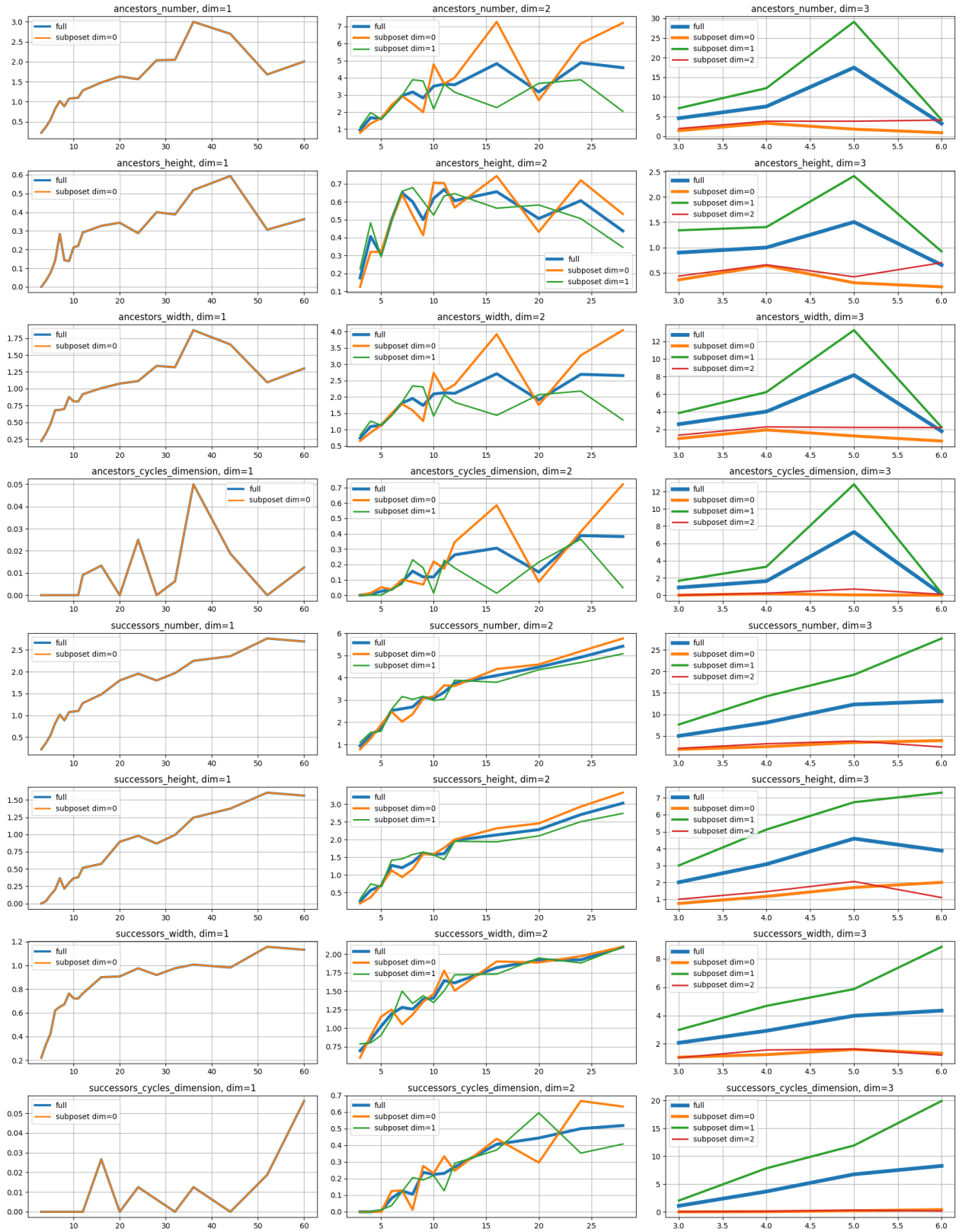


Figure 4: Depth Poset: Mean node scores

## Column Reduction Poset: Mean Poset Scores

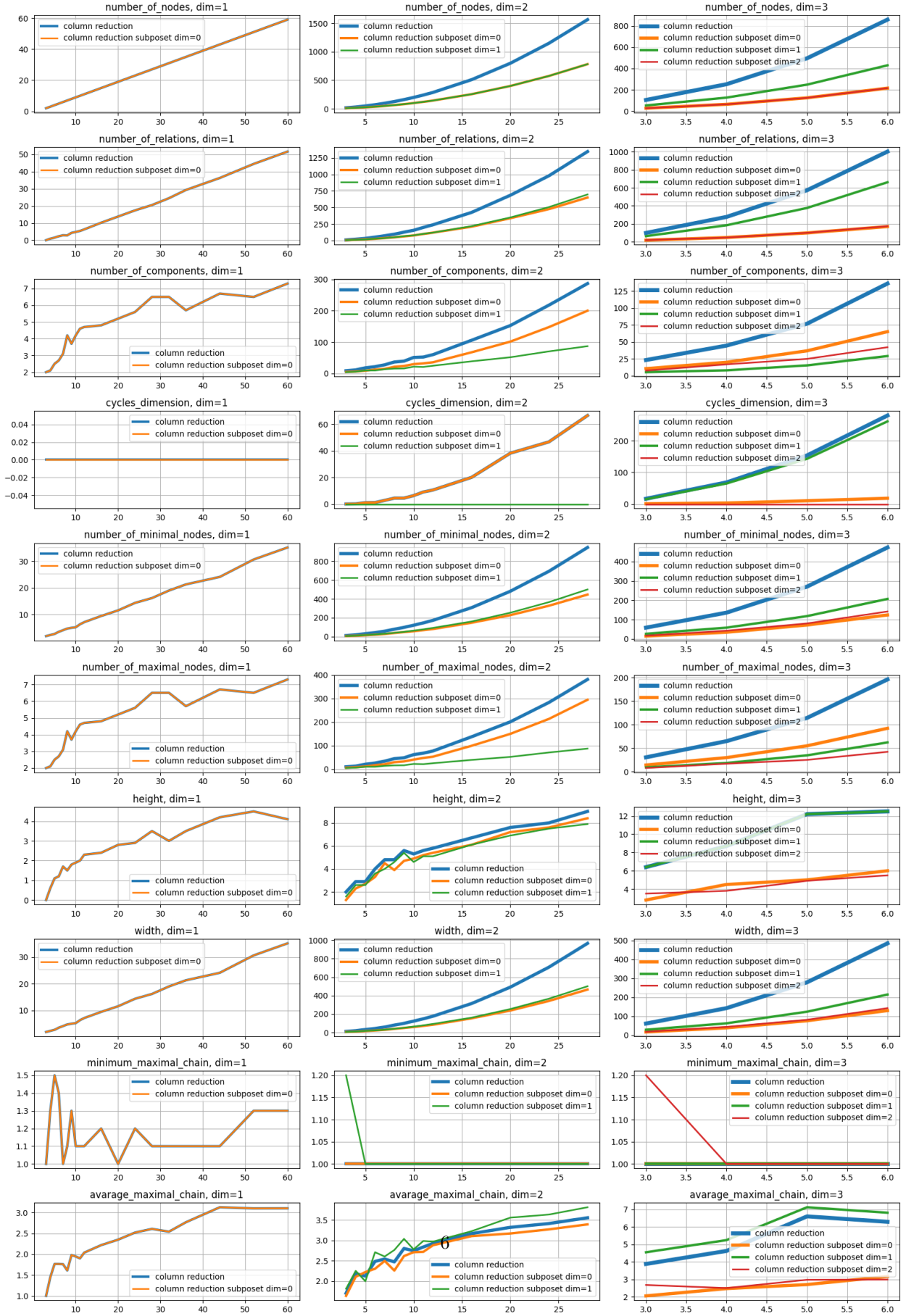


Figure 5: Column Reduction Poset: Mean poset scores

Column Reduction Poset: Mean Node Scores

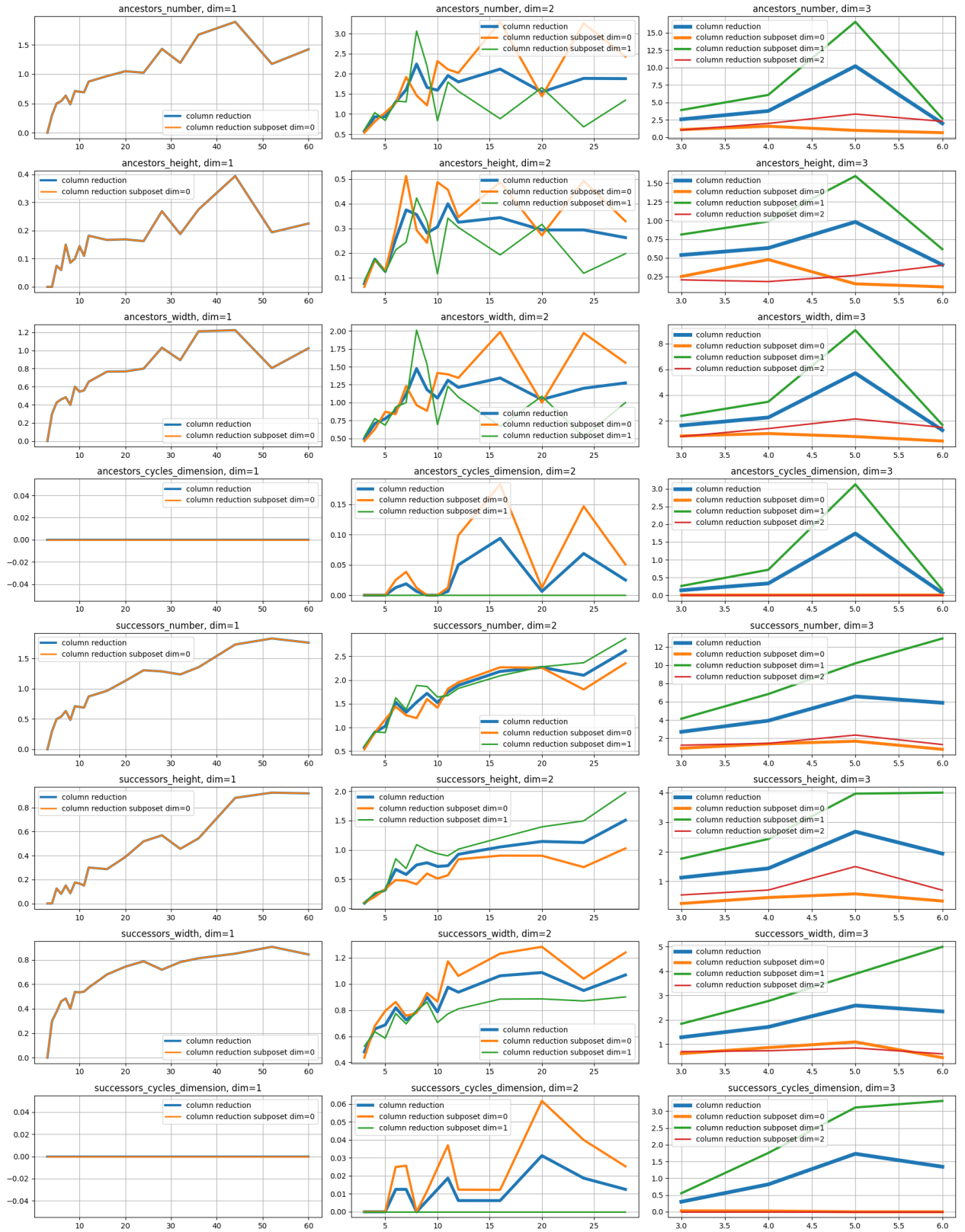


Figure 6: Column Reduction Poset: Mean node scores



Row Reduction Poset: Mean Poset Scores

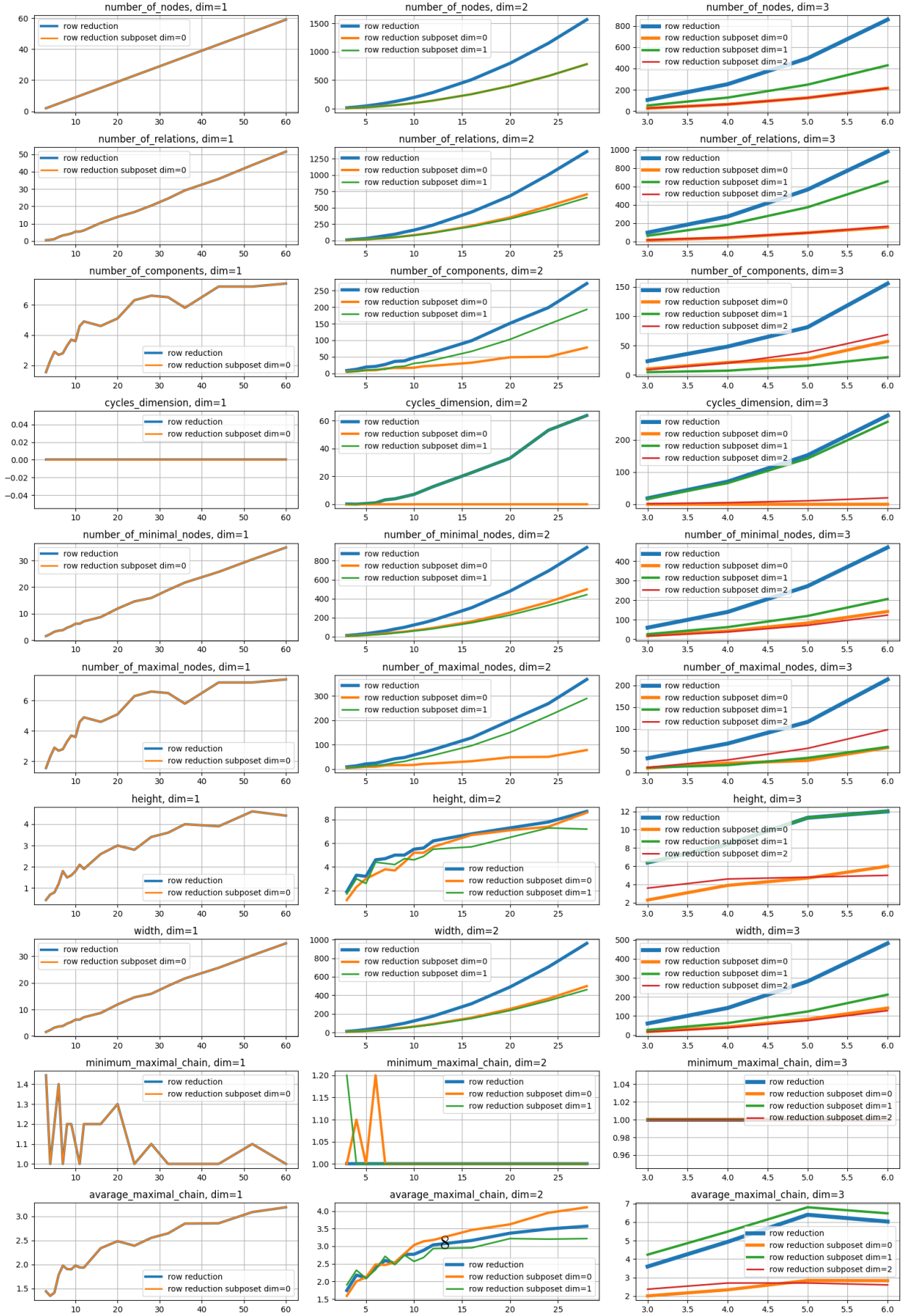


Figure 7: Row Reduction Poset: Mean poset scores



Row Reduction Poset: Mean Node Scores

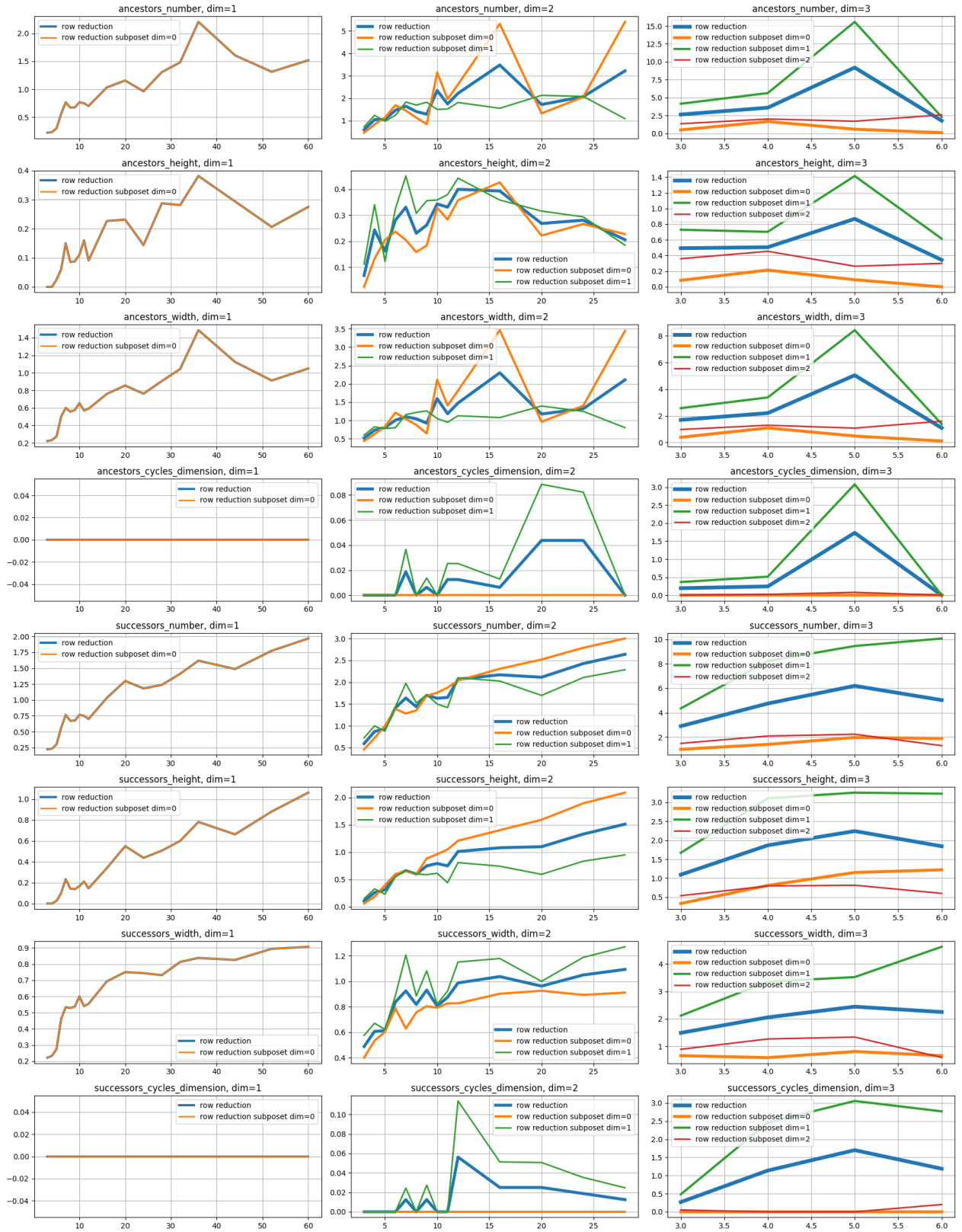


Figure 8: Row Reduction Poset: Mean node scores