1 Model

The d dimensional torus \mathbb{T}^d can be defined as $(\mathbb{R}/n\mathbb{Z})^d$ for some natural n. And we can represent this as a cell-complex with cubical d-dimensional cells $(\mathbb{Z}/n\mathbb{Z})^d + [0,1]^d$ and all their k-faces for k = 0, ..., d.

We randomly assume the filtration value for each k-face uniformly distributed in [k, k+1]. This filtration on segmented torus will corespond some real filtration $f\mathbb{T}^d \to \mathbb{R}$, s.t. the d-dimensional cells will corespond the local maximums, vertices will corespond the local minimums and other k-faces will be saddles.

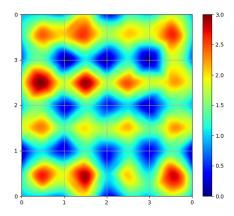


Figure 1: The example of the filtration $f: \mathbb{T}^2 \to \mathbb{R}$, s.t. there are local minimums in the vertices, saddles in the middle of edges, and the local maximums in the centers of square cells.

2 Scores

2.1 Poset Scores

- number_of_nodes : Returns the number of nodes in the poset.
- **number_of_relations** : Returns the number of relations in the transitive reduction.
- number_of_components : Returns the number of connected components in the poset
- cycles_dimension: Returns the dimension of space of cycles in reduction.

- number_of_minimal_nodes : Returns the number of minimal nodes.
- number_of_maximal_nodes : Returns the number of maximal nodes.
- height: Returns the poset height the length of the longest chain.
- width: Returns the poset width the length of the longest antichain (subset, s.t. all elements are pairwise incomparable). The algorithm is based on Dilworth's theorem and it's proof via Kőnig's theorem: link
- minimum_maximal_chain : Returns the minimum size of maximal chains in the poset.
- avarage_maximal_chain : Returns the avarage size of maximal chains in the poset.

2.2 Node Scores

- ancestors_number: Returns the number of nodes higher than given
- ancestors_height: Returns the size of maximum chain of subposet of nodes higher or equal than given
- ancestors_width: Returns the size of maximum chain of subposet of nodes higher or equal than given
- ancestors_cycles_dimension: Returns the dimension of space of cycles in reduction of subposet of nodes higher or equal than given
- successors_number: Returns the number of nodes higher than given
- successors_height: Returns the size of maximum chain of subposet of nodes lower or equal than given
- successors_width: Returns the size of maximum chain of subposet of nodes lower or equal than given
- successors_cycles_dimension: Returns the the dimension of space of cycles in reduction of subposet of nodes lower or equal than given

3 Experiments and Results

There are 312 experiments done. In the Figure 2 we can see how cases are distributed by size and dimension.

3.1 Depth Poset Features

In the Figure 3 we can see the avarage poset scores values for each number of points n in the depth poset.

In the Figure 4 we can see the avarage mean node scores values in poset for each number of points n in the depth poset.

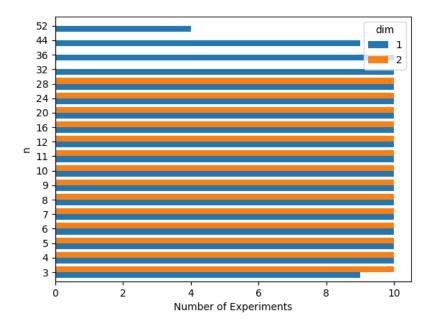


Figure 2: Size/dimension distribution of experiments

3.2 Column Reduction Poset Features

In the Figure 5 we can see the avarage poset scores values for each number of points n in the column reduction poset.

In the Figure 6 we can see the avarage mean node scores values in poset for each n in the column reduction poset.

3.3 Row Reduction Poset Features

In the Figure 7 we can see the avarage poset scores values for each number of points n in the row reduction poset.

In the Figure 8 we can see the avarage mean node scores values in poset for each number of points n in the row reduction poset.

Depth Poset: Mean Poset Scores

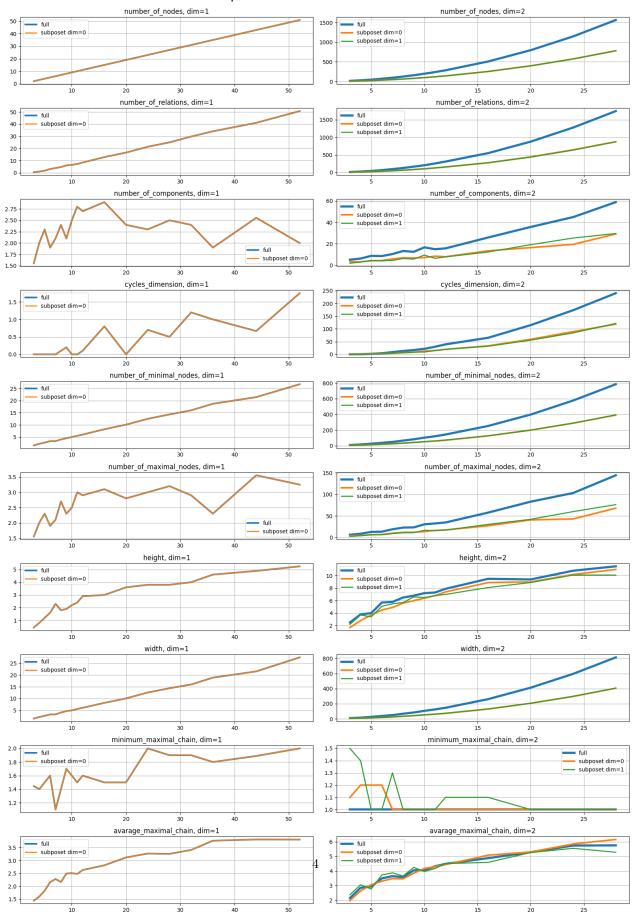


Figure 3: Depth Poset: Mean poset scores

Depth Poset: Mean Node Scores ancestors_number, dim=1 ancestors_number, dim=2 full subposet dim=0 2.5 2.0 1.5 1.0 0.5 ancestors_height, dim=1 ancestors height, dim=2 0.7 subposet dim=0 0.6 0.4 0.5 0.3 0.4 0.2 0.3 0.1 subposet dim=0 subposet dim=1 0.2 0.0 ancestors_width, dim=1 ancestors_width, dim=2 full subposet dim=0 subposet dim=1 3.5 1.50 3.0 1.25 2.5 1.00 2.0 0.75 1.5 0.50 1.0 0.25 25 ancestors_cycles_dimension, dim=1 ancestors_cycles_dimension, dim=2 0.05 0.6 0.04 0.5 0.03 0.4 0.3 0.02 0.2 0.01 0.1 successors_number, dim=1 successors_number, dim=2 full subposet dim=0 subposet dim=1 full subposet dim=0 2.0 1.0 successors_height, dim=2 successors_height, dim=1 full subposet dim=0 subposet dim=0 subposet dim=1 1.2 2.5 1.0 0.8 2.0 0.6 1.5 0.4 0.2 0.5 0.0 successors_width, dim=1 successors_width, dim=2 full subposet dim=0 subposet dim=1 1.75 0.8 1.50

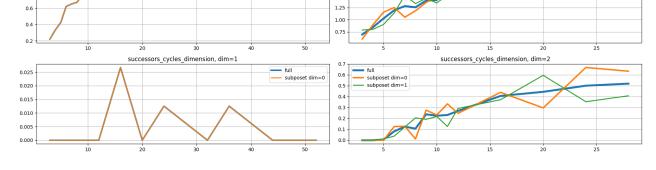


Figure 4: Depth Poset: Mean node scores

Column Reduction Poset: Mean Poset Scores

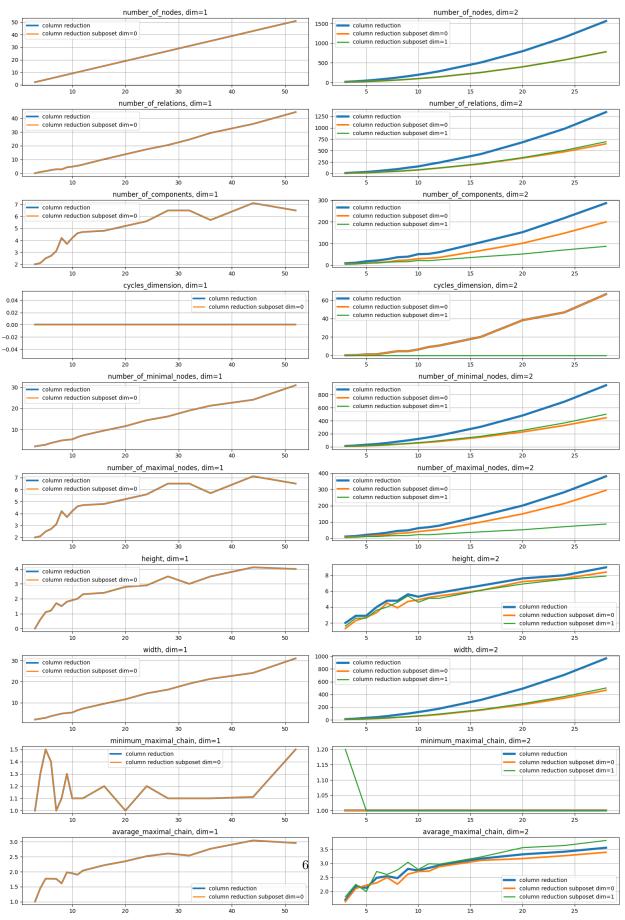


Figure 5: Column Reduction Poset: Mean poset scores

Column Reduction Poset: Mean Node Scores ancestors_number, dim=1 ancestors_number, dim=2 column reduction column reduction subposet dim=0 column reduction subposet dim=1 column reduction column reduction subposet dim=0 2.5 1.0 2.0 1.5 1.0 ancestors_height, dim=1 ancestors height, dim=2 column reduction 0.5 column reduction column reduction subposet dim=0 column reduction subposet dim=0 0.3 0.2 0.1 ancestors_width, dim=1 ancestors_width, dim=2 column reduction column reduction subposet dim=0 column reduction column reduction subposet dim=0 column reduction subposet dim=1 1.50 0.8 1.25 1.00 0.4 0.75 0.2 25 ancestors_cycles_dimension, dim=1 ancestors_cycles_dimension, dim=2 0.04 0.15 0.02 0.10 0.00 -0.02 0.05 10 25 successors_number, dim=1 successors_number, dim=2 1.75 column reduction column reduction subposet dim=0 column reduction subposet dim=1 column reduction column reduction subposet dim=0 1.50 2.5 1.25 2.0 1.00 0.75 1.5 0.50 1.0 0.25 0.00 successors_height, dim=2 successors_height, dim=1 column reduction column reduction subposet dim=0 column reduction subposet dim=1 column reduction column reduction subposet dim=0 1.0 0.5 25

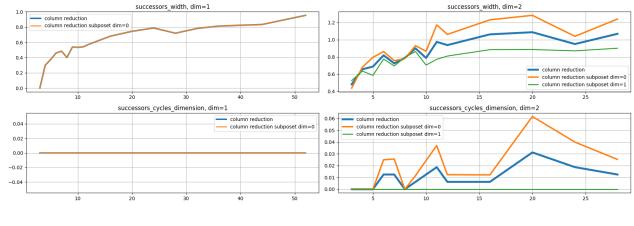


Figure 6: Column Reduction Poset: Mean node scores

Row Reduction Poset: Mean Poset Scores

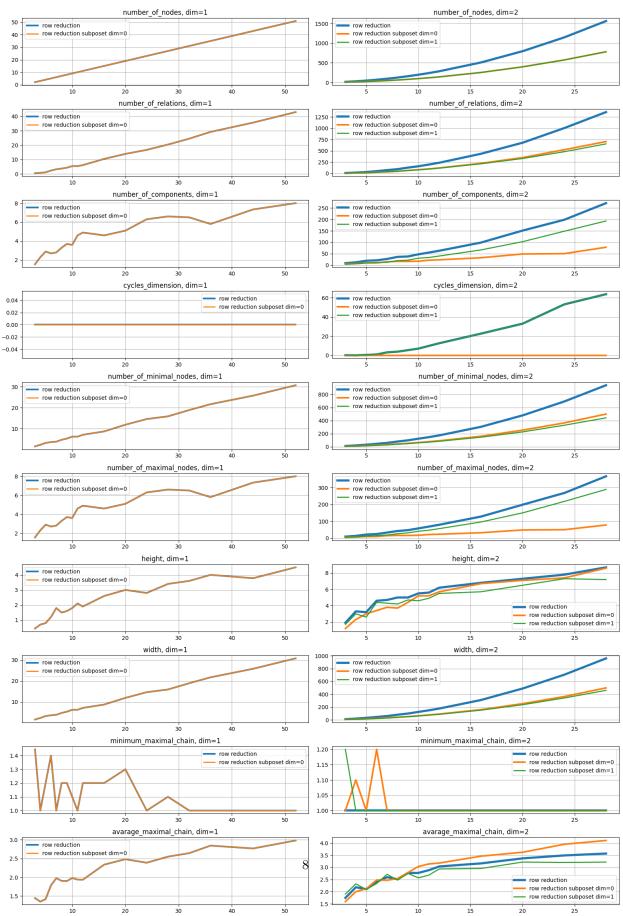


Figure 7: Row Reduction Poset: Mean poset scores

Row Reduction Poset: Mean Node Scores ancestors_number, dim=1 ancestors_number, dim=2 row reduction row reduction subposet dim=0 1.0 ancestors_height, dim=1 ancestors_height, dim=2 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction - row reduction subposet dim=0 0.2 0.2 0.1 ancestors_width, dim=1 ancestors_width, dim=2 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction row reduction subposet dim=0 3.0 1.2 2.5 1.0 2.0 0.8 1.5 1.0 0.4 0.5 0.2 25 ancestors_cycles_dimension, dim=1 ancestors_cycles_dimension, dim=2 0.08 0.04 0.02 0.00 0.04 -0.02 0.02 25 successors_number, dim=1 successors_number, dim=2 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction row reduction subposet dim=0 1.4 1.0 1.5 1.0 0.5 successors_height, dim=1 successors_height, dim=2 row reduction row reduction subposet dim=0 row reduction subposet dim=1 row reduction row reduction subposet dim=0 2.0 0.6 1.5 1.0 0.5 25 successors_width, dim=1 successors_width, dim=2 0.8 1.2 0.7 1.0 0.6 0.8 0.5 0.4 0.6 0.3 successors_cycles_dimension, dim=1 $successors_cycles_dimension, \, dim{=}2$ row reduction row reduction subposet dim=0 row reduction subposet dim=0 row reduction subposet dim=1 0.04 0.10 0.02 0.08 0.06 0.00 0.04 -0.02 0.02 -0.04

Figure 8: Row Reduction Poset: Mean node scores