1 Model

The d dimensional torus $\mathbb{T}^d$ can be defined as $(\mathbb{R}/n\mathbb{Z})^d$ for some natural n. And we can represent this as a cell-complex with cubical d-dimensional cells $(\mathbb{Z}/n\mathbb{Z})^d + [0,1]^d$ and all their k-faces for $k = 0, ..., d$.

We randomly assume the filtration value for each k-face uniformly distributed in $[k, k+1]$. This filtration on segmented torus will corespond some real filtration $f\mathbb{T}^d \to \mathbb{R}$, s.t. the d-dimensional cells will corespond the local maximums, vertices will corespond the local minimums and other k-faces will be saddles.

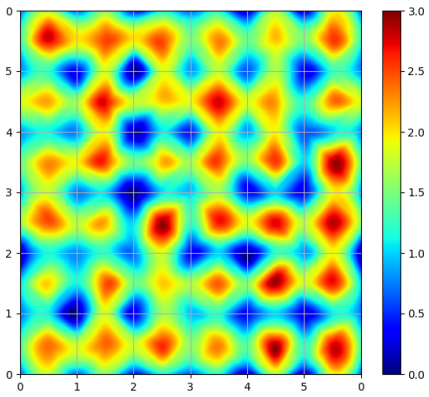

Figure 1: The example of the filtration $f : \mathbb{T}^2 \to \mathbb{R}$, s.t. there are local minimums in the vertices, saddles in the middle of edges, and the local maximums in the centers of square cells.

2 Scores

2.1 Poset Scores

- **number_of_nodes** : Returns the number of nodes in the poset.
- **number_of_relations** : Returns the number of relations in the transitive reduction.
- **number_of_components** : Returns the number of connetcted components in the poset
- **cycles_dimension** : Returns the dimension of space of cycles in reduction.

- **number_of_minimal_nodes** : Returns the number of minimal nodes.
- **number_of_maximal_nodes** : Returns the number of maximal nodes.
- **height** : Returns the poset height - the length of the longest chain.
- **width** : Returns the poset width - the length of the longest antichain (subset, s.t. all elements are pairwise incomparable). The algorithm is based on Dilworth's theorem and it's proof via Kőnig's theorem: link
- **minimum_maximal_chain** : Returns the minimum size of maximal chains in the poset.
- **avarage_maximal_chain** : Returns the avarage size of maximal chains in the poset.

2.2 Node Scores

- **ancestors_number**: Returns the number of nodes higher than given
- **ancestors_height**: Returns the size of maximum chain of subposet of nodes higher or equal than given
- **ancestors_width**: Returns the size of maximum chain of subposet of nodes higher or equal than given
- **ancestors_cycles_dimension**: Returns the the dimension of space of cycles in reduction of subposet of nodes higher or equal than given
- **successors_number**: Returns the number of nodes higher than given
- **successors_height**: Returns the size of maximum chain of subposet of nodes lower or equal than given
- **successors_width**: Returns the size of maximum chain of subposet of nodes lower or equal than given
- **successors_cycles_dimension**: Returns the the dimension of space of cycles in reduction of subposet of nodes lower or equal than given

3 Experiments and Results

There are 360 experiments done. In the Figure **??** we can see how cases are distributed by size and dimension.

3.1 Depth Poset Features

In the Figure **??** we can see the avarage poset scores values for each number of points n in the depth poset.

In the Figure **??** we can see the avarage mean node scores values in poset for each number of points n in the depth poset.

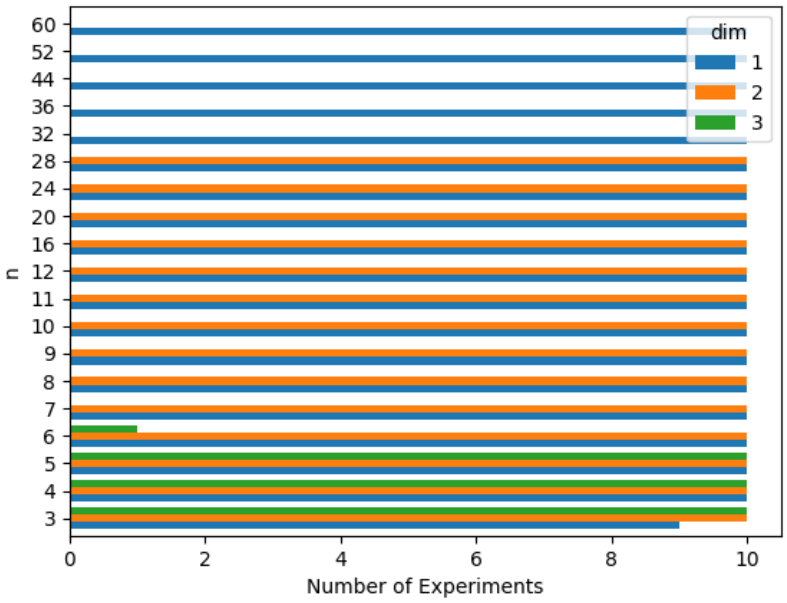

Figure 2: Size/dimension distribution of experiments

3.2 Column Reduction Poset Features

In the Figure **??** we can see the avarage poset scores values for each number of points n in the column reduction poset.

In the Figure **??** we can see the avarage mean node scores values in poset for each n in the column reduction poset.

3.3 Row Reduction Poset Features

In the Figure **??** we can see the avarage poset scores values for each number of points n in the row reduction poset.

In the Figure **??** we can see the avarage mean node scores values in poset for each number of points n in the row reduction poset.

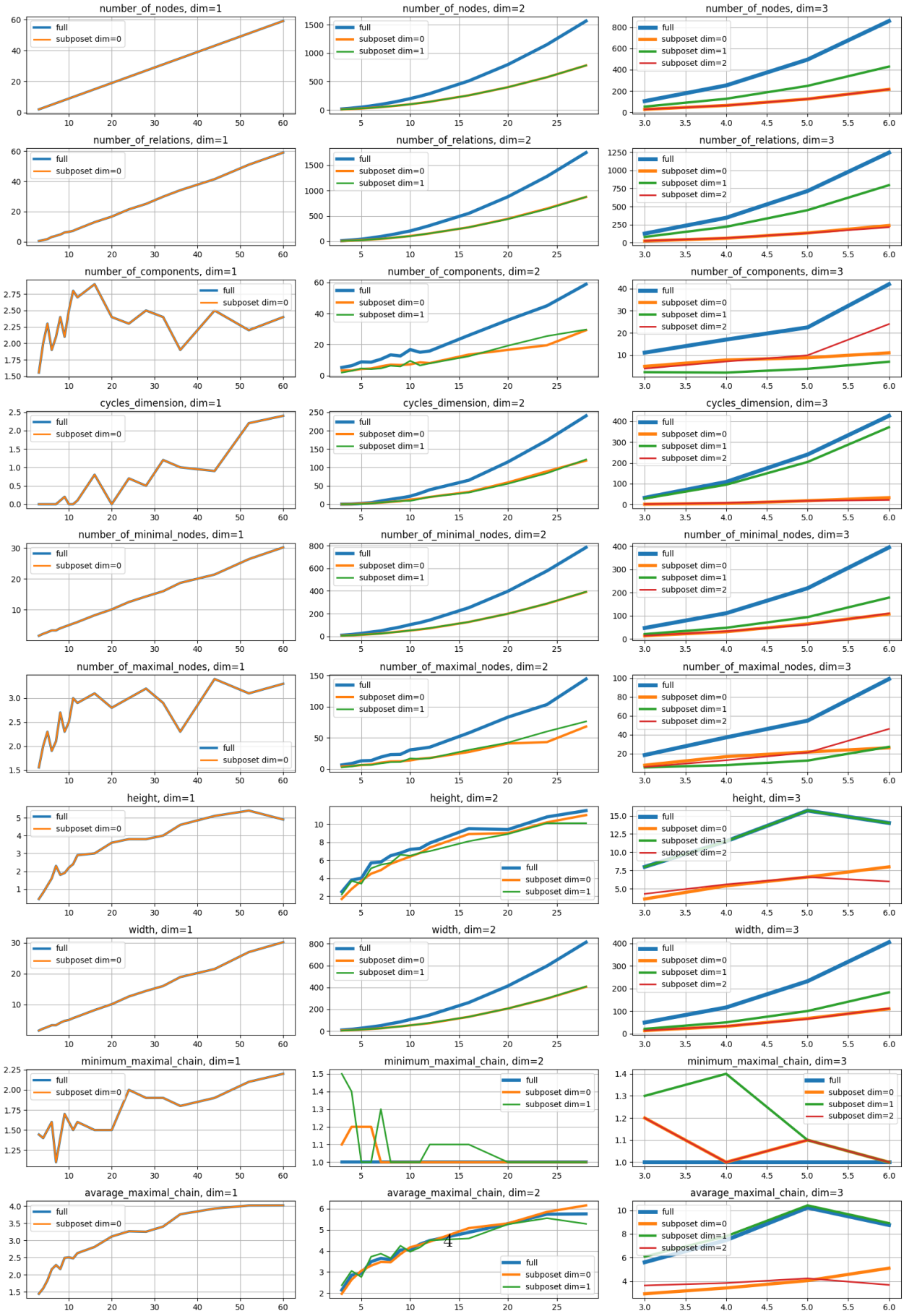

Figure 3: Depth Poset: Mean poset scores

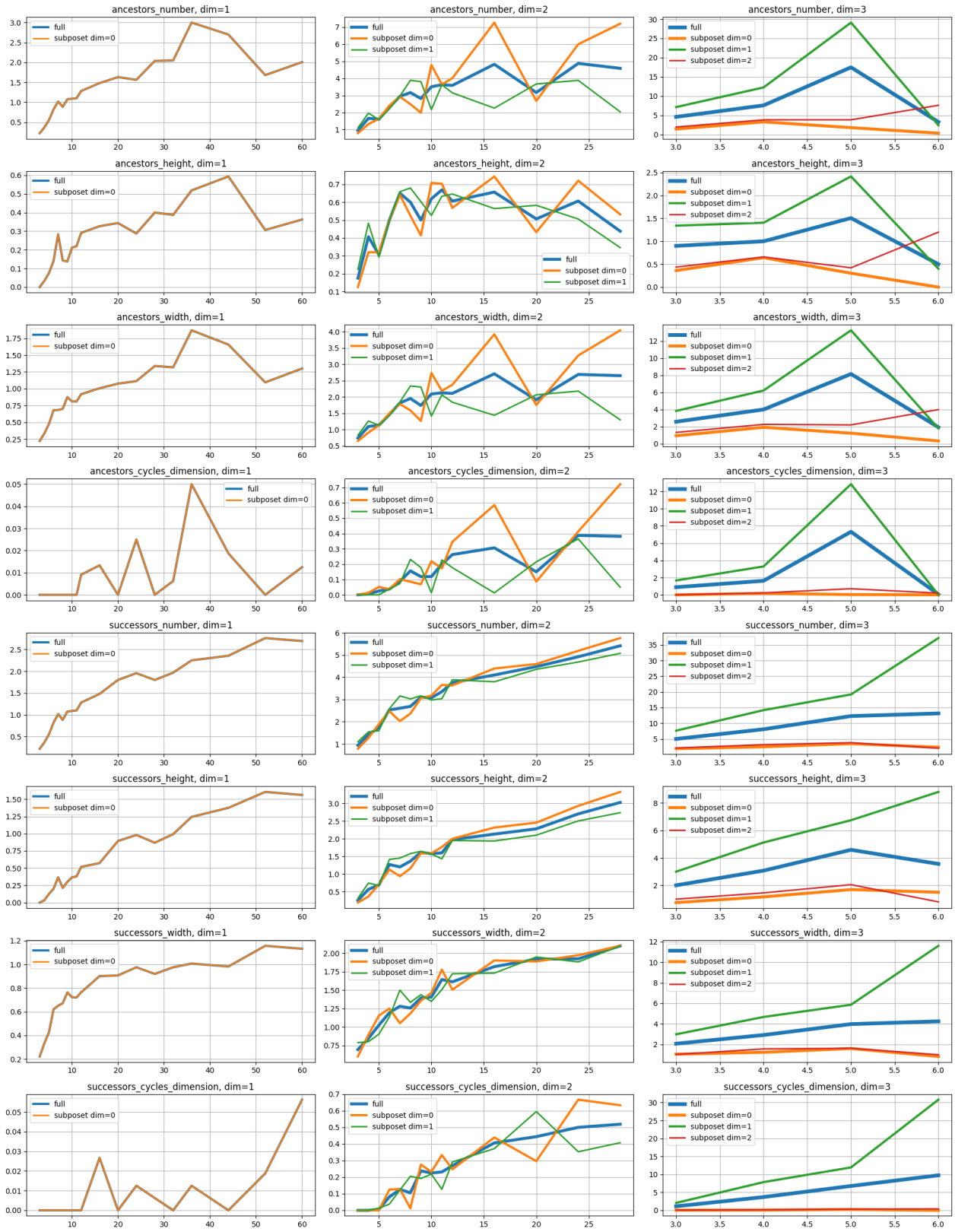

Figure 4: Depth Poset: Mean node scores

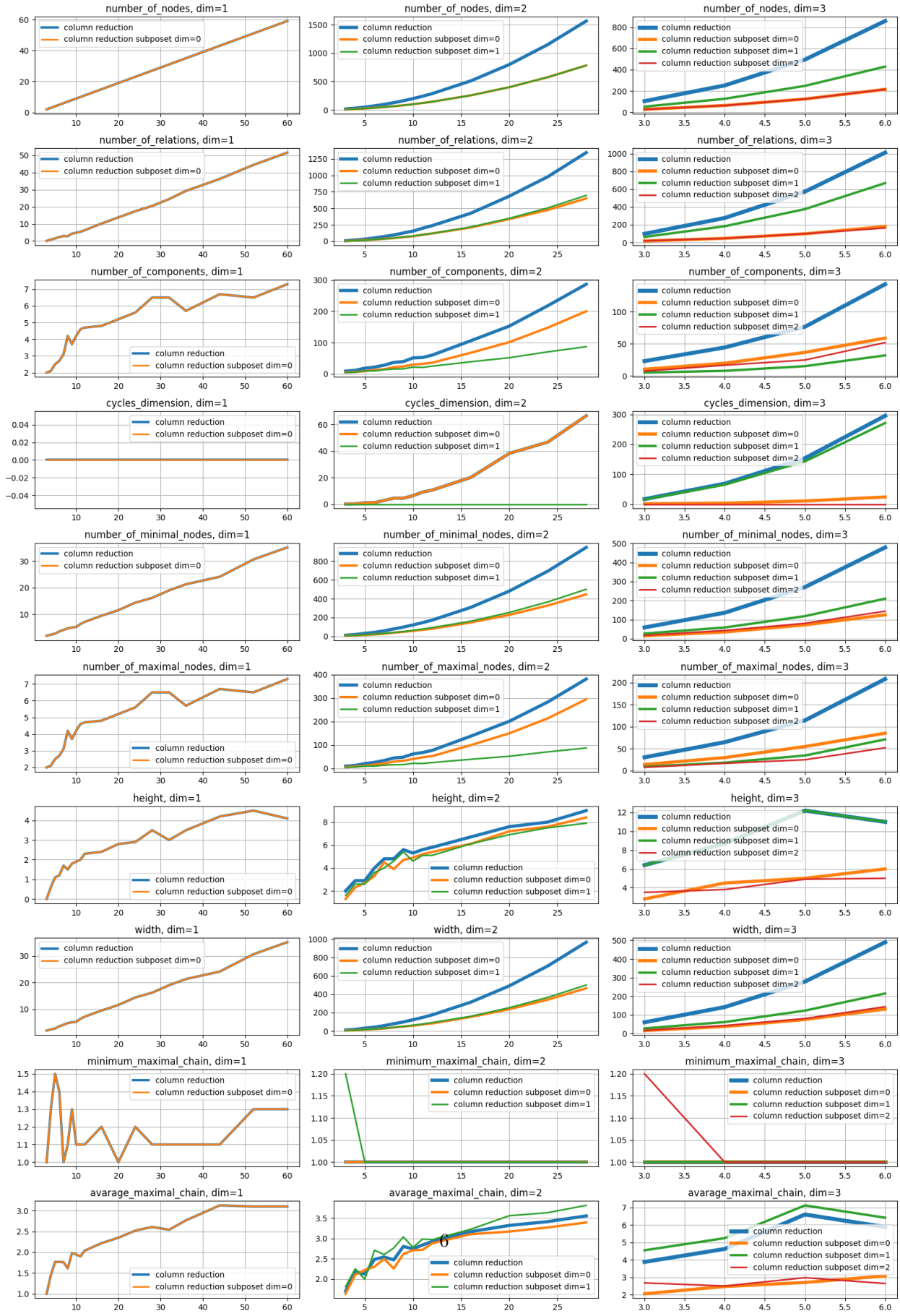

Figure 5: Column Reduction Poset: Mean poset scores

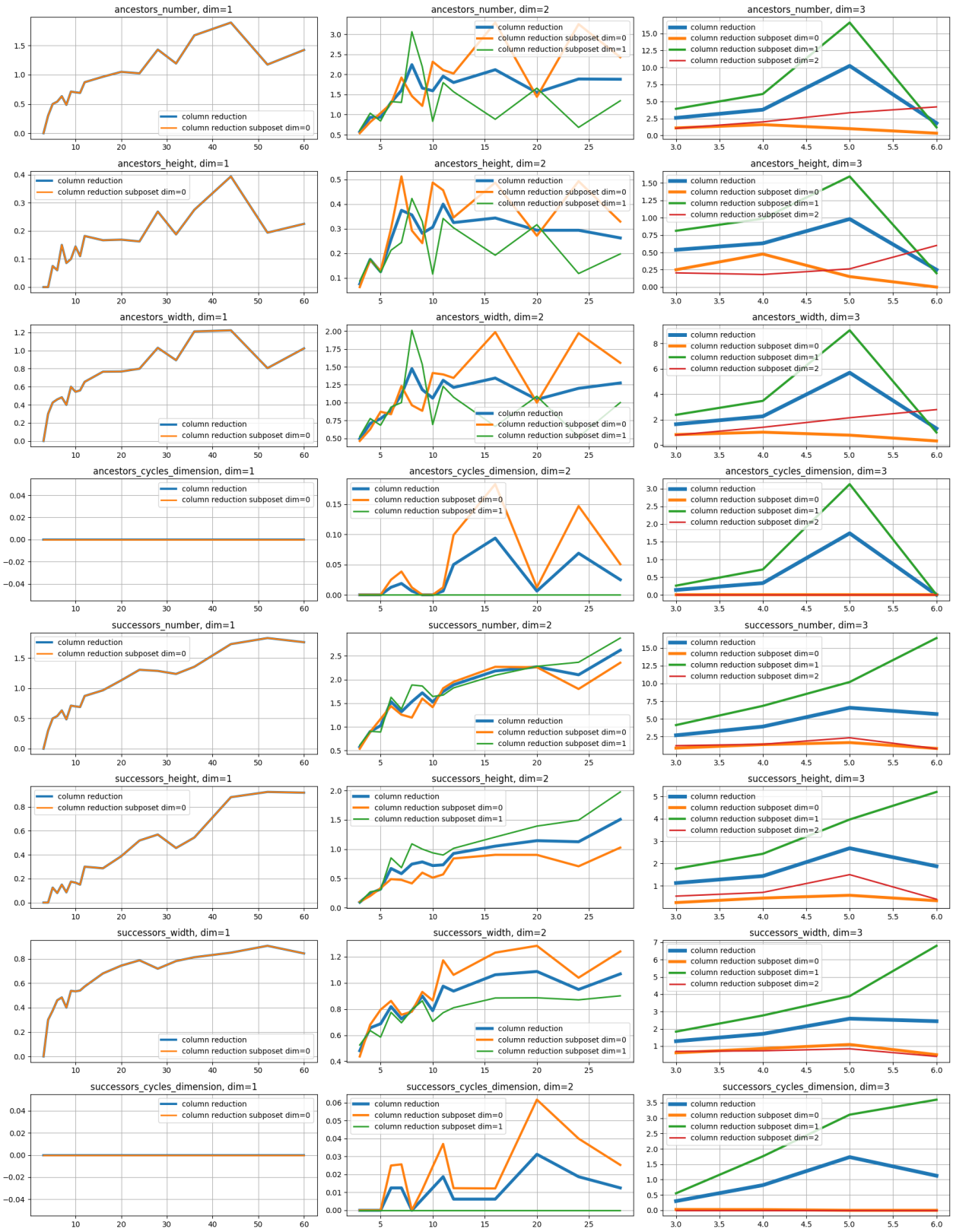

Figure 6: Column Reduction Poset: Mean node scores

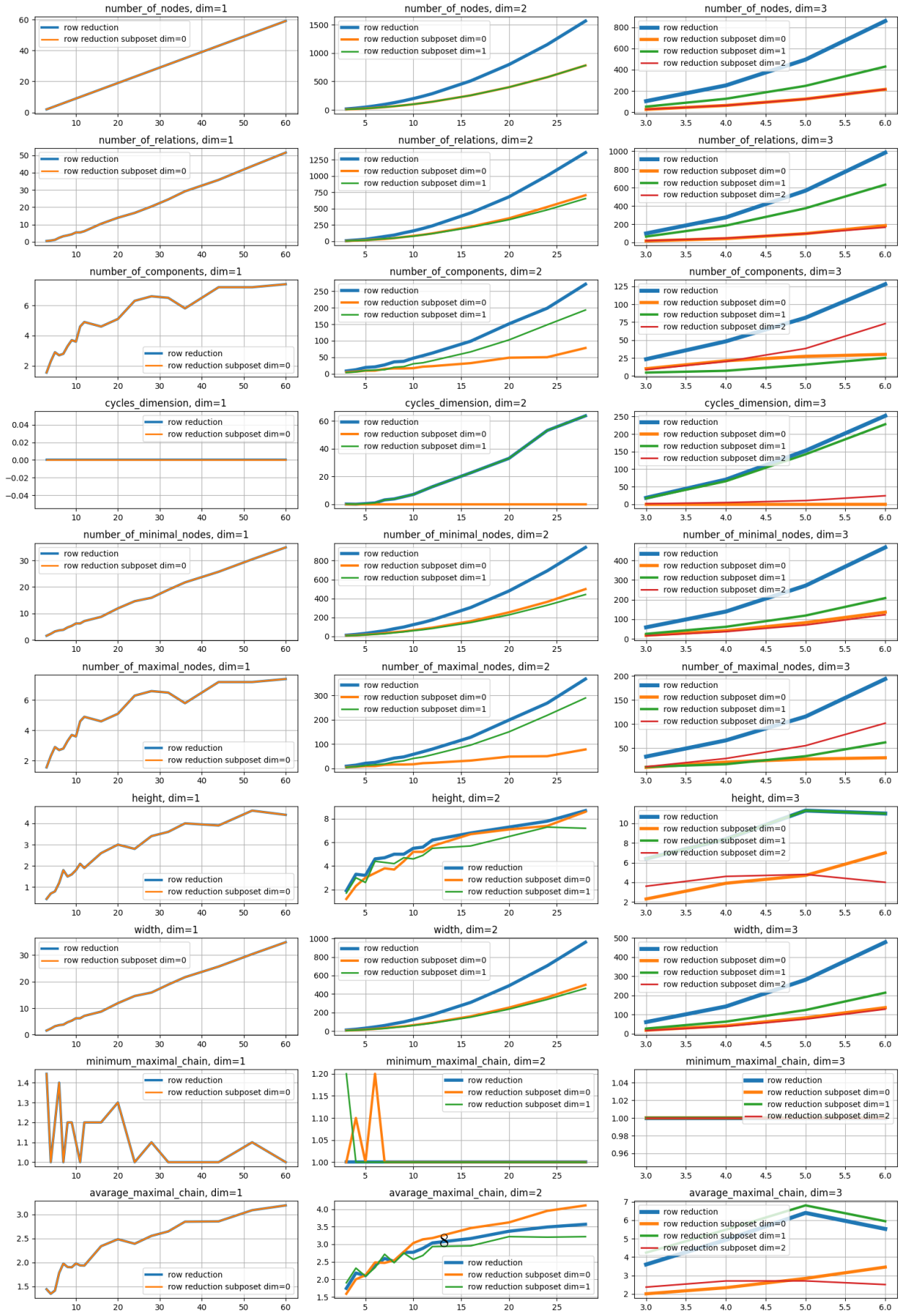

Figure 7: Row Reduction Poset: Mean poset scores

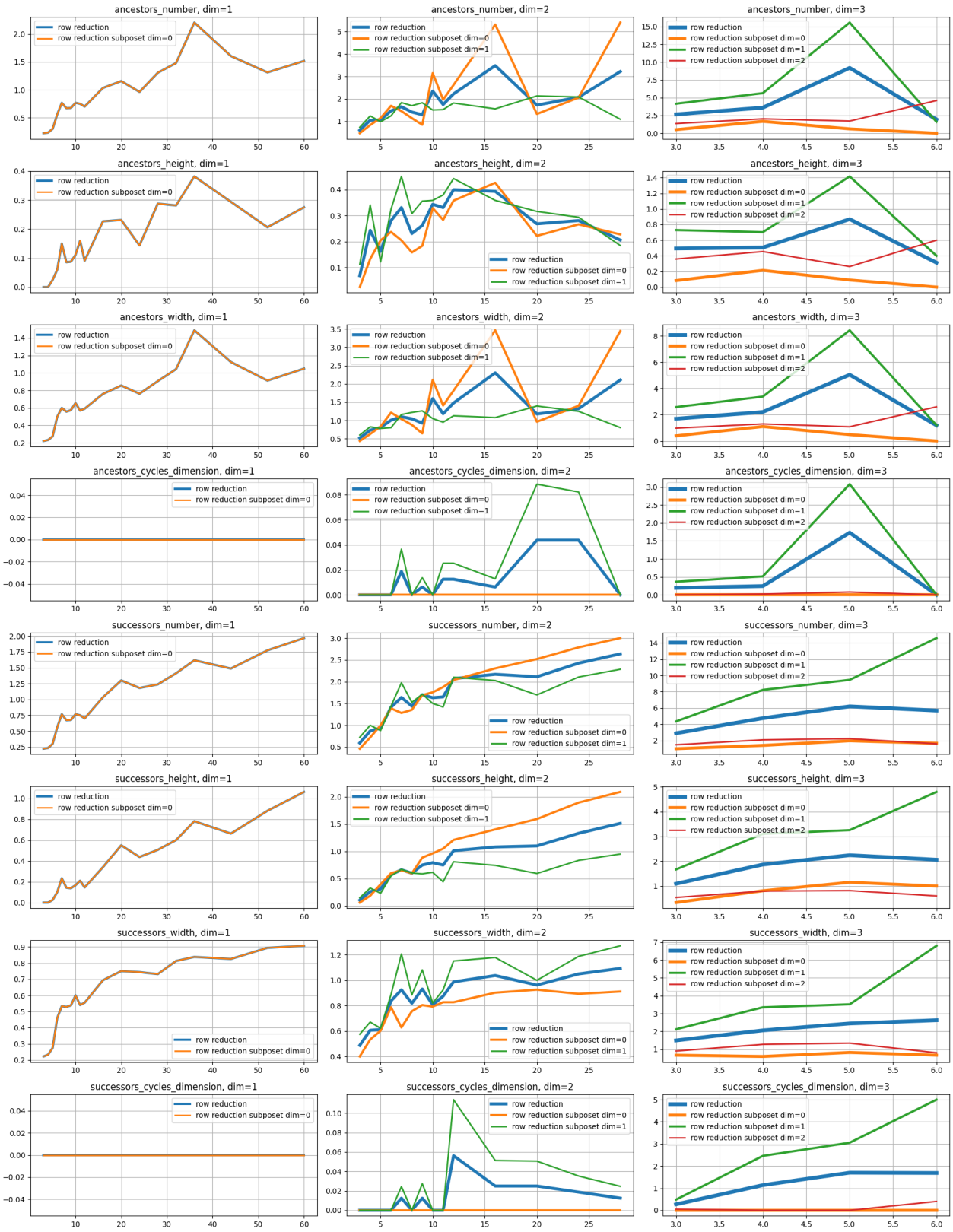

Figure 8: Row Reduction Poset: Mean node scores