

# 1 Description

The first we just calculated several Extended Barycentric Cubical Tori. The distribution of calculated toruses we can see in the Figure 1a.

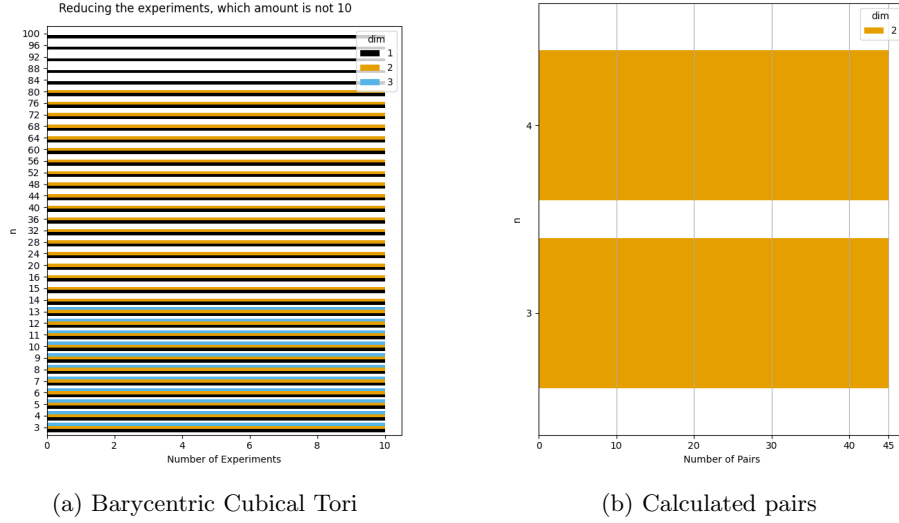


Figure 1: Size/dimension distribution

Then for each pair of filtrations with similar dimension and  $n$   $f_0, f_1 : \mathbb{T}_n^d \rightarrow \mathbb{R}$  we can define a linear homotopy  $h : [0, 1] \times \mathbb{T}_n^d \rightarrow \mathbb{R}$ :

$$h(t, \sigma) = (1 - t) \cdot f_0(\sigma) + t \cdot f_1(\sigma)$$

The moment of time  $t \in [0, 1]$  such there exist a pair of cells  $\sigma_0, \sigma_1 \in \mathbb{T}_n^d$  such that  $h(t, \sigma_0) = h(t, \sigma_1)$  is a moment of transposition of these cells during the homotopy (the probability, that this will be full segment is 0 for pair of independent barycentric cubical tori).

We have found all transposing pairs like this, classified them and calculated how these transpositions changes the Depth Poset. The distribution of calculated pairs of filtration we can see in the Figure 1b.

## 2 Switch Types Distributions

The distribution of transpositions types for the model  $\mathbb{T}_n^2$  we can see in the figure Fig. 2.

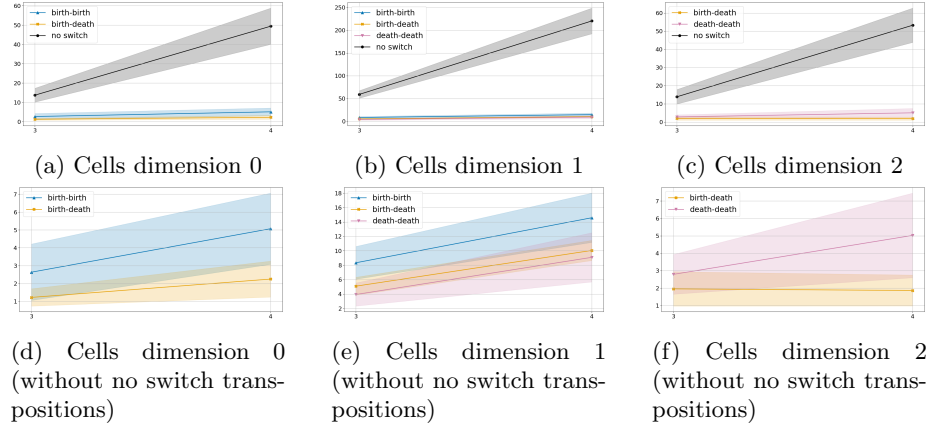


Figure 2: The switch type distribution for  $\mathbb{T}_n^2$

### 3 Similarity Scores

- **birth\_relation\_cell\_similarity** - The size of symmetric difference of arcs (edges) in the birth relation (given by row left to right reduction algorithm). Consider 2 birth-death pairs are similar if they correspond the similar cells.
- **death\_relation\_cell\_similarity** - The size of symmetric difference of arcs (edges) in the death relation (given by column bottom to top reduction algorithm). Consider 2 birth-death pairs are similar if they correspond the similar cells.
- **poset\_closure\_arcs\_cell\_similarity** - The size of symmetric difference of arcs (edges) in the transitive closure of the Depth Posets. Consider 2 birth-death pairs are similar if they correspond the similar cells.
- **poset\_reduction\_arcs\_cell\_similarity** - The size of symmetric difference of arcs (edges) in the transitive reduction of the Depth Posets. Consider 2 birth-death pairs are similar if they correspond the similar cells.

The distribution of the similarity scores for different models  $\mathbb{T}_n^{dim}$  we can see in the following figures:

dim	2
birth_relation_cell_similarity	Fig. 3
death_relation_cell_similarity	Fig. 4
poset_closure_arcs_cell_similarity	Fig. 5
poset_reduction_arcs_cell_similarity	Fig. 6

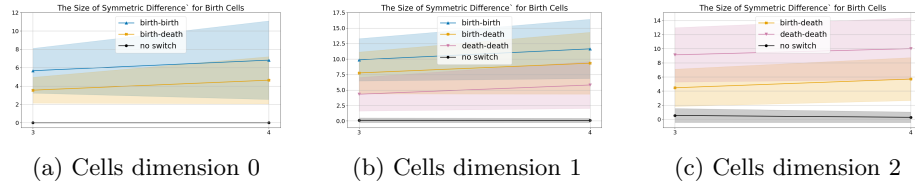


Figure 3: Similarity score birth\_relation\_cell\_similarity for  $\mathbb{T}_n^2$

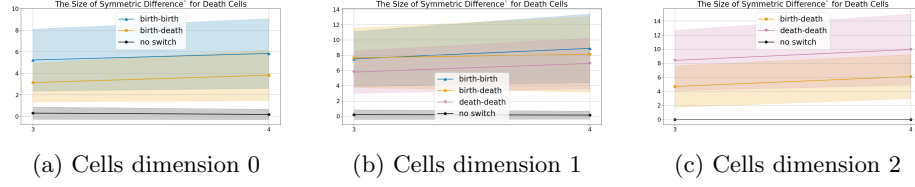


Figure 4: Similarity score death\_relation\_cell\_similarity for  $\mathbb{T}_n^2$

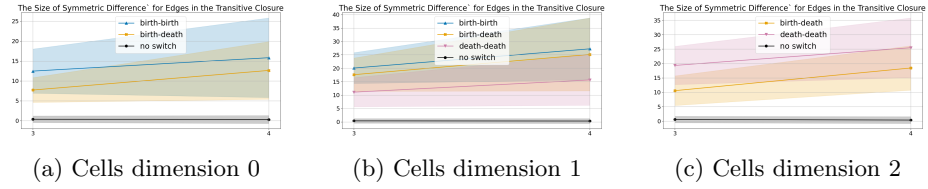


Figure 5: Similarity score poset\_closure\_arcs\_cell\_similarity for  $\mathbb{T}_n^2$

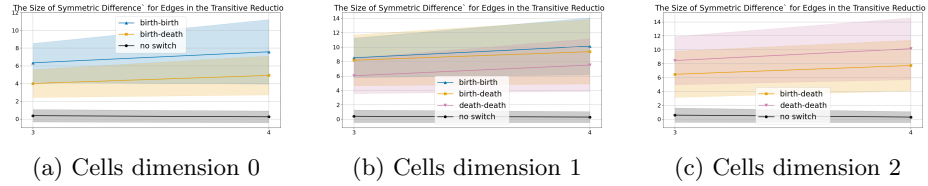


Figure 6: Similarity score poset\_reduction\_arcs\_cell\_similarity for  $\mathbb{T}_n^2$