1 Theory

1.1 Persistent Homology and Giant Cycles

The first we should understand, which objects topological data analysis research.

People call cloud a collection of points $\{x_\alpha\} \subset X$, where X is a metric space. That's interesting to convert that data to some structure, so points can be representated as vertices of some combinatorial graph. And that graph can become scaffold of a simplicial complex. That's a good way to research data, ignoring high dimension of the space [3].

A simplicial complex (I mean abstract simplicial complex) is a set of vertices $\{v_\alpha\}$ and a collection of its subsets, called simplices, S such that, $\forall a \in S \ \forall b \subset a \ b \in S$ [2]. The dimension of a simplicial complex is the maximal dimension of it's simplices ($\max_{a \in S} |a| - 1$).

... Bla-bla-bla about simplicial homology...

One of the popular methods to represent a cloud as a simplicial complex is the Cech complex.

For a given cloud $\{x_\alpha\} \subset \mathbb{E}^n$ the Cech complex C_ϵ is the simplicial complex whose k-simplices (the simplices dimension k: $a : |a| - 1 = k$) are determined by unordered $(k+1)$-tuples of points $\{x_\alpha\}_0^k$ whose closed $\epsilon/2$-ball neighbourhoods have a point of common intersection [3].

... (the Cech (Nerve) theorem)

...

1.2 Lattices Voronoi Cells and their Interpretations on Thorus

In this chapter we will show default definitions about lattices in $\mathbb{R}^n$, talk about Voronoi cells and then extropolate their definitions to the thorus case.

A lattice in $\mathbb{R}^n$ is a subset $\Gamma \subset \mathbb{R}^n$ with the property that there exxists a basis $(e_1, ..., e_n)$ of $\mathbb{R}^n$ s.t. $\Gamma = \mathbb{Z}e_1 \oplus \cdots \oplus \mathbb{Z}e_n$. [1]

Let's throw few examples of lattices, which will be interesting in this work:

The Lattices Z_n: ...

The Lattices A_n: ...

The Lattices D_n: ...

Let's define a Γ^* dual to Γ as $\{x \in \mathbb{R}^\ltimes : \ x \cdot y \in \mathbb{Z} \ \forall y \in \Gamma\}$.

...

Let's define d-dimensional thorus as $\mathbb{R}^n/\mathbb{Z}^n$ or $(\mathbb{R}/\mathbb{Z})^n$. Not hard to see, that $\mathbb{R}/a_1\mathbb{Z} \times \cdots \times \mathbb{R}/a_n\mathbb{Z}$ $(a_1, ..., a_n \in \mathbb{R}_{>0})$ will be the homeomorphically-same object.

Let's redefine lattice thinking, that' lattices lie not just on $\mathbb{R}^n$, but on some thorus with defined equivalence relation. So...

1.3 Random Filtration on Cells

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References

[1] Ebeling, Wolfgang. (2002). Lattices and Codes. 10.1007/978-3-322-90014-2.

[2] Prasolov, V. V. (2006), Elements of combinatorial and differential topology, American Mathematical Society, ISBN 0-8218-3809-1, MR 2233951

[3] Ghrist, Robert. (2008). Barcodes: The persistent topology of data. BULLETIN (New Series) OF THE AMERICAN MATHEMATICAL SOCIETY. 45. 10.1090/S0273-0979-07-01191-3.