Homework 6

Assigned: October November 15, 2022

Due: 2:00pm on November 22, 2022

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1. (10 points) The following code is written in C, where elements within the same row are stored contiguously. Assume each word is a 32-bit integer.

```
for (I=0; I<8; I++)
    for (J=0; J<8000; J++)
        A[I][J]=B[I][0]+A[J][I];
```

(1) Which variable references exhibit temporal locality? (5 points)

Answer: I, J, B[I][0]

(2) Which variable references exhibit spatial locality? (5 points)

Answer: A[I][J]

2. (40 points) Below is a list of 32-bit memory address references, given as word addresses:

0x03, 0xB4, 0x2B, 0x02, 0xBF, 0x58, 0xBE, 0x0E, 0xB5, 0x2C, 0xBA, 0xFD

(1) For each of these references, identify the tag and the cache index given a direct-mapped cache with 8 one-word blocks. Also list if each reference is a hit or a miss, assuming the cache is initially empty. (10 points)

Word Address	Index	Tag (27 bits)	Hit or Miss
0x0000_0011	011	00000	M
0x1011_0100	100	10110	M
0x0010_1011	011	00101	M
0x0000_0010	010	00000	M
0x1011_1111	111	10111	M
0x0101_1000	000	01011	M
0x1011_1110	110	10111	M
0x0000_1110	110	00001	M
0x1011_0101	101	10110	M
0x0010_1100	100	00101	M
0x1011_1010	010	10111	M
0x1111_1101	101	11111	M

Index	Valid	Tag	Data
000	1	01011	
001	0		
010	1	10111	
011	1	00101	
100	1	00101	
101	1	11111	
110	1	00001	
111	1	10111	

(2) For each of these references, identify the tag and the cache index given a direct-mapped cache with two-word blocks and a total size of 4 blocks. Also list if each reference is a hit or a miss, assuming the cache is initially empty. (10 pints)

Word Address	Index	Tag (27 bits)	Hit or Miss
0x0000_0011	01	00000	M
0x1011_0100	10	10110	M
0x0010_1011	01	00101	M
0x0000_0010	01	00000	M
0x1011_1111	11	10111	M
0x0101_1000	00	01011	M
0x1011_1110	11	10111	H
0x0000_1110	11	00001	M
0x1011_0101	10	10110	H
0x0010_1100	10	00101	M
0x1011_1010	01	10111	M
0x1111_1101	10	11111	M

Index	Valid	Tag	Word 0	Word 1
00	1	01011	0x58	0x59
01	1	10111	0x02	0x03
10	1	11111	0xFC	0xFD
11	1	00001	0x0E	0x0F

(3) You are asked to optimize a cache design for the given references. There are three direct-mapped cache designs possible, all with a total of 8 words of data: C1 has 1-word blocks, C2 has 2-word blocks, and C3 has 4-word blocks. In terms of miss rate, which cache design is the best? If the miss stall time is 35 cycles, and C1 has an access time of 2 cycles, C2 takes 3 cycles, and C3 takes 5 cycles, which is the best cache design? (20 points)

Word Address	Index	Tag (27 bits)	Hit or Miss
0x0000_0011	0	00000	M
0x1011_0100	1	10110	M
0x0010_1011	0	00101	M
0x0000_0010	0	00000	M
0x1011_1111	1	10111	M
0x0101_1000	0	01011	M
0x1011_1110	1	10111	H
0x0000_1110	1	00001	M
0x1011_0101	1	10110	M
0x0010_1100	1	00101	M
0x1011_1010	0	10111	M
0x1111_1101	1	11111	M

Index	Valid	Tag	Word 0	Word 1	Word 2	Word 3
0	1	01011	0xB8	0xB9	0xBA	0xBB
1	1	11111	0xFC	0xFD	0xFE	0xFF

In terms of miss rate, C1's miss rate is 100%, C2's miss rate is 83.3%, C3's miss rate is 91.7%. C2 is the best.

In terms of total time:

For C1: Total time = 12 * 2 cycles + 12 * 35 cycles = 444 cycles

For C2: Total time = 12 * 3 cycles + 10 * 35 cycles = 386 cycles

For C3: Total time = 12 * 5 cycles + 11 * 35 cycles = 445 cycles

C2 is the best design.

3. (50 points) For a direct-mapped cache design with a 32-bit byte address, the following bits of the address are used to access the cache.

Tag	Index	Offset
31 – 10	9 – 5	4 – 0

(1) What is the cache block size (in words)? (5 points)

Answer: 8 words

(2) How many blocks does the cache have? (5 points)

Answer: 32 blocks

(3) What is the ratio between total bits required for such a cache implementation over the data storage bits? (5 points)

Answer: Assume only valid bit is implemented.

Each block has 1 valid and 21 bits of tag, total number of bits = 32 * (21 + 1) = 704 bits

Each block has 8 words of data, total bits = 32 * 8 * 32 = 8192 bits

Ratio = 704 / 8192 = 8.59%

Beginning from power on, the following byte addresses for cache references are recorded.

Address											
0x00	0x04	0x10	0x84	0xE8	0xA0	0x400	0x1E	0x8C	0xC1C	0xB4	0x884

(4) (20 points) For each reference, list

    a) its tag, index, and offset

    b) whether it is a hit or a miss, and

    c) How many blocks were replaced (if any)?

Byte Address	Tag (21 bits)	Index	Offset	Hit or Miss	Blocks replaced
0x0000_0000	0	00000	00000	M	0
0x0000_0100	0	00000	00100	H	0
0x0001_0000	0	00000	10000	H	0
0x1000_0100	0	00100	00100	M	0
0x1110_1000	0	00111	01000	M	0
0x0100_0000_0000	0…01	00000	00000	M	1
0x0001_1110	0	00000	11110	M	1
0x1000_1100	0	00100	01100	H	0
0x1100_0001_1100	0…011	00000	11100	M	1
0x1011_0100	0	00101	10100	M	0
0x1000_1000_0100	0…010	00100	00100	M	1

(5) What is the hit ratio? (5 points)

Answer: hit ratio = 3/11 = 27.3%

(6) Show the final state of the cache, with each valid line represented as <index, tag, data>.
(10 points)

Index	Valid	Tag	W0	W1	W2	W3	W4	W5	W6	W7
00000	1	0…011	0xC00	0xC04	0xC08	0xC0C	0xC10	0xC14	0xC18	0xC1C
00001	0									
00010	0									
00011	0									
00100	1	0…010	0x880	0x884	0x888	0x88C	0x890	0x894	0x898	0x89C
00101	1	0	0xA0	0xA4	0xA8	0xAC	0xB0	0xB4	0xB8	0xBC
00110	0									
00111	1	0	0xE0	0xE4	0xE8	0xEC	0xF0	0xF4	0xF8	0xFC
01000	0									
01001	0									
01010	0									
01011	0									
01100	0									
01101	0									
01110	0									
01111	0									
10000	0									
10001	0									
10010	0									
10011	0									
10100	0									
10101	0									
10110	0									
10111	0									
11000	0									
11001	0									
11010	0									
11011	0									
11100	0									
11101	0									
11110	0									
11111	0									